

The maximum likelihood method is used for study of galactic rotation which utilizes the whole 21 cm line profile. The rotation curve is obtained in the region from  $R = 0.4 R_0$  up to  $2.3 R_0$  and we do not find evidence for a fall-off below a flat rotation curve.

Neutral hydrogen 21 cm line observations may be used for the study of the global kinematic characteristics of the Galaxy. Earlier in the works of the soviet authors a method has been suggested for investigation of the galactic rotation which utilizes the whole 21 cm line profile in contrast to the use of only the cut-off identified with the tangential point when  $R < R_0$  (Agekyan et al., 1964; Petrovskaya, 1970; Petrovskaya, 1979; Bektasova and Petrovskaya, 1983). This method may be utilized both for inner ( $R < R_0$ ) and for outer ( $R > R_0$ ) regions of the Galaxy, and also for z-gradient studies of the rotation velocity (Agekyan et al., 1965).

The expression for the optical depth at the frequency corresponding to the radial velocity  $v$  in the galactic longitude  $l$  is as follows

$$(1) \quad \tau = \frac{kN}{|dv/dr|}.$$

Here  $N$  is the hydrogen density,  $r$  is the heliocentric distance of the radiating point,  $k$  is a constant of proportionality.

We assume the region of the galactic plane ( $z = 0$ ), and as the simplest model that the angular velocity is the monotonically decreasing function of  $R$ ,  $\omega = \omega(R)$ , and the density is a function of  $R$  only,  $N = N(R)$ . Then using the formula  $v = R \sin l$ ,  $\Omega = R_0(\omega - \omega_0)$ ,  $x = R/R_0$  where  $\omega_0$  is the angular velocity of the local standard of rest, one may write Eq. (1) in the form

$$(2) \quad \tau(\Omega, l) = \frac{kN(R)}{|d\Omega/dR| |\sin l| \sqrt{1-x^2 \sin^2 l}}.$$

In the earlier paper we studied the rotation law,  $\Omega = \Omega(x)$ , using the least mean square method both for  $R < R_0$  and  $R \geq R_0$  (Petrovskaya, 1979). Teerikorpi (1985) suggested a graphical method for the determination of the rotation curve for  $R > R_0$ . This later method was utilized by Petrovskaya and Teerikorpi (1986) for the 1-st and 4-th galactic quadrants.

In the present study we use the method of maximum likelihood method in contrast to the least mean square method because the equation (2) is non-linear relative to  $x$ . Let us assume the gaussian distribution for the observed depth  $\tau_i = \tau(\Omega, l_i)$  near the theoretical value (2)

$$(3) \quad f_i(\tau_i, \Omega, x) = \frac{\sqrt{P_i}}{\sqrt{2\pi}} e^{-\frac{P_i(\tau_i - \tau)^2}{2\sigma^2}}.$$

In (3)  $\Omega = kN/|d\Omega/dR|$ ,

$$(4) \quad \tau = \Omega / (|\sin l| \sqrt{1-x^2 \sin^2 l}),$$

$P_i$  - the weight of  $i$ -th value  $\tau_i$ .

The values of the parameters  $\Omega$  and  $x$  have to be determined from the condition that the likelihood function  $L$  obtains its maximum value:

$$(5) \quad L = (2\pi)^{-\frac{n}{2}} 6^{-n} (p_1 \dots p_n)^{\frac{1}{2}} \exp \left\{ -\frac{1}{2\sigma^2} \sum_{i=1}^n P_i (\tau_i - \tau)^2 \right\};$$

$$(6) \quad \partial \ln L / \partial \Omega = 0; \quad \partial \ln L / \partial x = 0.$$

The solution of the equations (6) gives

$$(7) \quad \Omega = \frac{\sum_{i=1}^n P_i \tau_i}{\sum_{i=1}^n \frac{P_i}{\sin^2 l_i (1-x^2 \sin^2 l_i)}};$$

$$(8) \quad \sum_{i=1}^n P_i (\tau_i - \Omega) \frac{1}{\sin^2 l_i (1-x^2 \sin^2 l_i)} = 0$$

The expressions (7) and (8) were used for the determination of the rotation curve beyond the solar circle ( $R > R_0, x > 1$ ). In that case one can put  $p_i \equiv 1$  (because  $|\sin l| \neq x$  when  $x > 1$ ). We have used the observations of Kerr and Hindman (1970) and Weaver and Williams (1974).

For the fixed value  $\Omega = \Omega_*$  we found  $v_i = \Omega_* \sin l_i$  and  $\tau_i$  for each  $i$ -th of  $n$  profiles. Then for several values of  $x$  we calculated  $\Omega$  and  $F$  from (7) and (8). The distance  $x = x_*$  corresponding to  $\Omega_*$  we find from the condition  $F(x_*) = 0$ .

The results of our calculation are shown in Fig. 1 for the 1-st (crosses), 2-nd (open crosses), 3-rd (triangles) and 4-th (diagonal circles) galactic quadrants separately. The results of the graphical method (Petrovskaya and Teerikorpi, 1986) are also presented (dots). The solid line is the least mean square solution. The dashed line represent a flat rotation curve ( $\omega R = \text{const}$ ). In agreement with other authors (Blitz, 1979; Chini, 1985) we do not find any evidence for a fall-off below a flat rotation curve within  $R \leq 2.3 R_0$ .

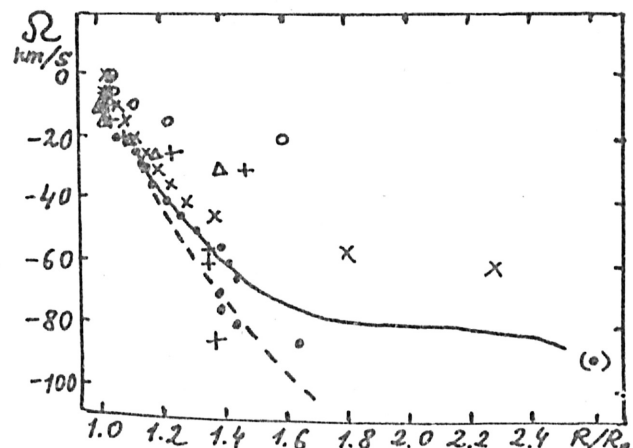


Fig. 1.

The maximum likelihood method was also applied to the inner region of the galactic plane,  $R < R_0$  ( $x < 1$ ). In this case one may expect that the weights  $p_i$  are important in Eqs. (3)-(8) because of the infinite increase of  $\tau$  when  $|\sin l| \rightarrow x$  in (2).

As for  $x \geq 1$ , the calculations of  $\Omega$  and  $\tau$  were performed for each  $\Omega = \text{constant}$  and for different values of  $x$ . Following variants were used: 1)  $p_i \equiv 1$ ; 2)  $p_i = 1/\sqrt{x}$ . The variant 2 was taken to compensate the increase of  $\tau$  when  $|\sin l| \rightarrow x$ , however, the results did not differ from the results using the variant 1. On the other hand, the number  $n$  of the profiles varies with  $x$ , and there are jumps in the dependence  $\tau = \tau(x)$ . Therefore, for each  $\Omega = \Omega_*$ , we considered only such a profiles for which  $|\sin l| \leq \alpha$ ,  $\alpha$  being chosen to be somewhat smaller than the expected value  $x_*$ . This procedure also eliminates the influence of the large  $\tau$  discussed above.

Figure 2 shows our results for the region  $R < R_0$  in the form  $\Omega$  vs  $x = R/R_0$  for 1-st (crosses) and 4-th (diagonal crosses) quadrants separately. One can see the well-known difference between the results for two quadrants. Our curve is somewhat lower than the rotation curve determined from the tangential points as one can see in Fig. 3 where the results are presented in the form  $\Omega = \omega R$  vs  $x = R/R_0$  (solid line for our curve).

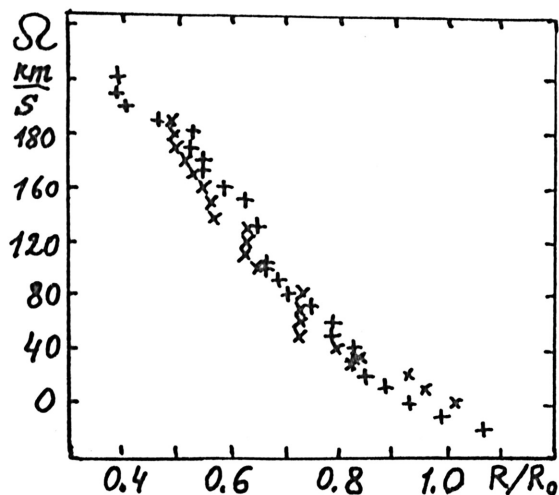


Fig. 2.

Figure 3 also shows the mean square curve of Fig. 1 (present results and the results of Petrovskaya and Teerikorpi, 1986) noted by solid line for  $x > 1$ . We have represented in Fig. 3 the rotation curves based on HII regions (Georgelin and Georgelin, 1976 - dot-dashed line; Chini, 1985 - crosses), on CO clouds (Blitz, 1979 - dashed line,  $x > 1$ ), HI+CO clouds (Burton and Gordon, 1978 - dashed line,  $x < 1$ ).

The least mean square rotation curve for  $R > R_0$ , based on all the data represented in Fig. 3 (dotted line) agrees with our HI results (solid line) and does not show any plateau up to a distance  $R \approx 2.3 R_0$  from the galactic centre.

The author would like to thank I.G. Murashkina and Yu.N. Malahova for help with the calculations.

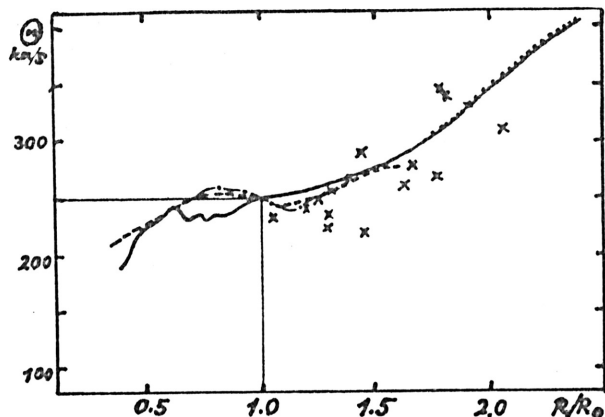


Fig. 3.

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## DISCUSSION

G. LYN<sup>0</sup>GA: In this paper, Dr Petrovskaya, uses a model with the full width of the 21 cm line profile.

P.O.LINDBLAD: Apparently Dr Petrovskaya derives a rather steeply rising rotation curve going far out from the centre. Could you say anything of what total mass would imply for the galaxy?

S. KUTUZOV: To estimate mass, one has to adopt some mass distribution. It may be infinite or finite with certain radius which

is of great importance. One can speak of the mass inside of the sphere or cylinder with given radius. No mass estimates were made in this work, so I can't name any number.

S. NINKOVIČ: Until what distance from the axis of the galactic rotation can one say that the form of the galactic rotation curve is established according to Dr Petrovskaya's work?

S. KUTUZOV: From this work we know the rotation curve until 2.3 solar distances.