Section C

INVESTIGATIONS OF HIGHER ATMOSPHERE

Секция Ц

ИССЛЕДОВАНИЕ ВЕРХНЕЙ АТМОСФЕРЫ

Density Scale Height Determined from the Motion of Dash 2 Satellite

by

Y.E. Helali and M.Y. Tawadraus
Helwan Observatory of Astronomy and Geophysics
Helwan- Cairo- Egypt

Abstract:

The density scale height H is determined at low as well as high altitudes using the equation of change rate of the orbital period of Dash 2 satellite. The computed values of H are compared with those statistical values obtained by Jacchia. Jacchia's values of H are given up to altitude 2 500 km. We extend Jacchia's values of H up to altitude 3 110.7 km since our values of H are computed from altitude 316.7 km to altitude 3 110.7 km. Finally a comparison between our values and Jacchia's values of H at different altitudes and different MJD are given. For higher altitudes, our values of H are much greater than Jacchia's values while at smaller altitudes, our values are approximately the same as Jacchia's values.

1. Introduction

The problem of the determination of the density scale height H has been studied by many authors. King-Hele /1963/ stated two methods for determining H; the first: knowing the perigee distance and eccentricity e of the satellite orbit for two different dates, we can compute the mean value of H during the time interval between the two dates; the second for finding H from the changes in the orbital period for small eccentricity, and the equation for this purpose applied for $C < 3$ with $e < 0.02$, where $C = ae/H$ (a is the semi-major axis of the orbit) and $H = - \rho / (d\rho/dz)$, where ρ is the density of the upper atmosphere and z is the altitude. However, the 44 average day/night values of H obtained by King-Hele were up to about 450 km altitude. His conclusion is that the increase of H with height becomes much less rapid at heights above 350 km.

2 - The data used

The present paper deals with the determination of H for low as well as high altitudes above the Earth's surface for some selected MJD[x] between 38397 & 41045, which correspond to altitudes from 3 464 to 300 km; using the equation of the rate of change of the orbital period derived by Stern (1960), under the effect of air drag only. The rate of change of the period $\dot{P}$ is related to a , e of the satellite's orbit as well as to ρ_p density at perigee, the cross-sectional area to mass ratio of the satellite (A/m), and the air drag coefficient C_D . For this purpose we used the data of the balloon satellite 1963 30D (Dash 2), which was derived by Slowey (1974). Slowey computed densities using J 71 model to represent the variation in density around the orbit, improving the J 71 model for use in the beginning interval (MJD 38397 to 38814) which corresponds to altitudes from 3 464 to 3 112 km. Also the end interval beginning with MJD 40237 to 41045, which corresponds to altitudes from 1 811.4 to 300 km during five time periods, each seperated from the next by a period when the orbit was partly in shadow.

We selected 103 different MJD distributed, as much as possible, over the mentioned two intervals for computing H using the equation of $\dot{P}$. Of the 103 MJD, 20 MJD

---

x/ MJD = JD-2400000.5

lie in the beginning interval where the perigee height is very large and the remaining MJD lie in the end interval.

3 - Determination of H from the rate of change of the orbital period

The rate of change of the period $\dot{P}$ for an artificial satellite, given by Sterne (1960), as

$$\dot{P} = -3\, C_D\, (A/m)\, a \pi \rho_p \{ e^{-c} \left(1 + \frac{3}{4}e^2 + ce(1+ \frac{3}{8} e^2) + \frac{1}{4} c^2 (1 + \frac{9}{8} e^2) + c^3 (e/8)(1+ \frac{5}{12} e^2) + (1/64)\, c^4 (1 + \frac{5}{4} e^2 + \ldots) \right) \} \quad (1)$$

We used the above equation when c was small ($c<2$), and e very much smaller ($e < 0.07$).

For $c>2$ and e larger than 0.07, we used the equation of Sterne (1960)

$$\dot{P} = -3 C_D\, (A/m)\, a\, \rho_p\, (1+e)^{3/2} / (1-e)^{1/2}\, (\pi/2c)^{1/2} \left(1+ \frac{1}{8c} f_1 + \frac{9}{128c^2} f_2 + \ldots \right) \quad (2)$$

where

$$f_1(e) = (1-8e +3e^2) \,/\, (1-e^2) \quad \text{and}$$

$$f_2(e) = (3- 16\, e^2 + 16\, e^3 - 5\, e^4)\, /3(1-e^2)^2$$

Equations (1) & (2) can be solved separately, knowing the values of a, e, (A/m) , C_D , ρ_p , and $\dot{P}$, to find c, and hence H from $H = ae/c$. However, because e and c are small quantities, we retained in equation (1) powers of c up to c^4 only. It should be noted that equation (1) is suitable for large perigee distances in the beginning interval, while the second equation is applied to smaller perigee distances in the end interval.

Now the part $\{\cdots\}$ between braces in equation (1) depends on c; let it be K(c), then equation (1) is writen in the form $\dot{P} = -3C_D\ (A/m)\ a\ \pi\ \rho_p\ K(c)$, from which we can compute K(c) which, in turn, is solvable for c, since all other parameteres are regarded to be known. Therefore H = ae/c is the value of the density scale height computed for given MJD and the corresponding altitude.

Equations (1) & (2) are derived without taking into account the atmospheric rotation and the Earth's flattening. The rotation effect, as pointed out by Sterne (1960), can alter the density deduced from $\dot{P}$ by as much as one part in ten. Therefore the negligence of the effect of rotation of the atmosphere is to affect the computed values of H by as much as one part in 500 only.

Now, we amend equation (2) to permit computation of H in the following way:

$$\dot{P} = -3\ C_D\ (A/m)\ (1+e)^{3/2}\ /(1-e)^{1/2}\ a\ \rho_p\ (\pi/2)^{\frac{1}{2}}\ K(L), \qquad (3)$$

where $L = (1\ /c^{1/2})$ and ,

$$K(L) = L + \tfrac{1}{8}L^3\ f_1 + \tfrac{9}{128}\ L^5\ \ f_2 \qquad (4)$$

We can obtain K(L) from equation (3), as a, e, $\dot{P}$, C_D, A/m, and ρ_p are all known, and then L from equation (4), and hence H from H = ae /c . It should be noted that $\dot{P}$ used in equations (1) & (2) is that part of $\dot{P}$, which is related to drag force only. Also C_D is considered to be 3.6 for the beginning interval, 3 for the early portion of the end interval, and 2.2 at the last part of the end interval; A/m is taken to be 37.9 cm^2/m; Slowey (1974).

The semi-major axis a and density scale height H are measured in km, the density ρ_p is measured in gm/cm^3. We shall denote our density scale height computed from either equation (1) or (2) by H_ρ and those statistical values of pressure scale height given by Jacchia by H_p', while the density scale height H_ρ' can be computed from values of H_p'.

4 - Computation of H_ρ' from H_p'

Indeed, the values of scale height given by Jacchia (1970) are pressure scale height H_p', and it is important to convert them from pressure to density scale height to be able to compare our results of H_ρ with the corresponding values H_ρ' at different altitudes. Thus if we assume that density ρ is an empirically determined function of altitude z in the form

$$\rho = \rho_p \cdot e^{-(z-z_o)/H_\rho} \tag{5}$$

where z_o is the altitude at perigee, H_ρ the density scale height, and ρ_p the density at perigee. We can compute the pressure p (dyn $/cm^2$) at different altitudes, using the ideal gas law, $p = \rho RT/\mu$; where R is the universal gas constant, μ is the mean molecular weight and T is the absolute temperature, or using $p = \rho RT_n/\mu_o$, where μ_o is the sea-level molecular weight 28.960 gm, R equals 8.31439 Joules $/^oK$; T_n is a fictitous "molecular temperature" given physically by $T_n = T\ (\mu_o/\mu)$. It is thus possible to compute the pressure p for corresponding known values of ρ at different values of temperatures.

As the pressure p decreases with height, similar to that of the density, one can assume an exponential form for the pressure similar to equation (5). Therefore, we can write,

$$\rho/\rho_p = e^{-(z-z_o)\ /\ H_\rho}, \quad \text{and } P/P_p = e^{-(z-z_o)\ /\ H_p}, \text{ where}$$

H_ρ and H_p are the density and pressure scale height respectively, and P_p is the pressure at perigee. Taking ln of both sides of the two above equations, we can write

$\ln(P/P_p)/\ln(\rho/\rho_p) = H_\rho/ H_p$, so H_ρ may be computed from values of H_p given by Jacchia /1970) if we know the pressure, density, temperature, and altitude.

5 - Figures

We have plotted four figures to represent the altitudes on the x-axis, with the density scale heights H_ρ and $H_\rho^`$ on the y-axis. It should be noted that we extend Jacchia´s values of $H_\rho^`$ for altitudes greater than 2 500 km; values of H_ρ and $H_\rho^`$ are computed at the same altitudes.

Fig (1) shows that H_ρ values are much greater than $H_\rho^`$ values, maybe due to the different values of the density used by Slowey and Jacchia. Altitudes are ranging from 3 064 to 3 110.7 km and corresponding to MJD from 38760 to 38810. We noted from Fig(1) that $H_\rho$ values are increasing with decreasing altitude, while $H_\rho^`$ values are approximately constant. H_ρ values in Fig (2) are greater than $H_\rho^`$ values for altitudes between 1 800 and 1 200 km and for MJD from 40240 to 40608.

Fig (3) indicates that H_ρ values are slightly greater than $H_\rho^`$ values for altitudes from 983.0 to 895.9 km, except for altitude 983 km, where $H_\rho = H_\rho^` = 230$ km, and approximately the same from altitudes 696.4 to 600.5 km. These altitudes correspond to MJD from 40720 to 40894.

Fig (4), gives an clear picture for changing H_ρ and $H_\rho^`$ at altitudes from 316 to 600 km and MJD from 40850 to 41000. It also seems that $H_\rho$ values oscillate around $H_\rho^`$ values with differences between them ranging from 0 to 10 km.

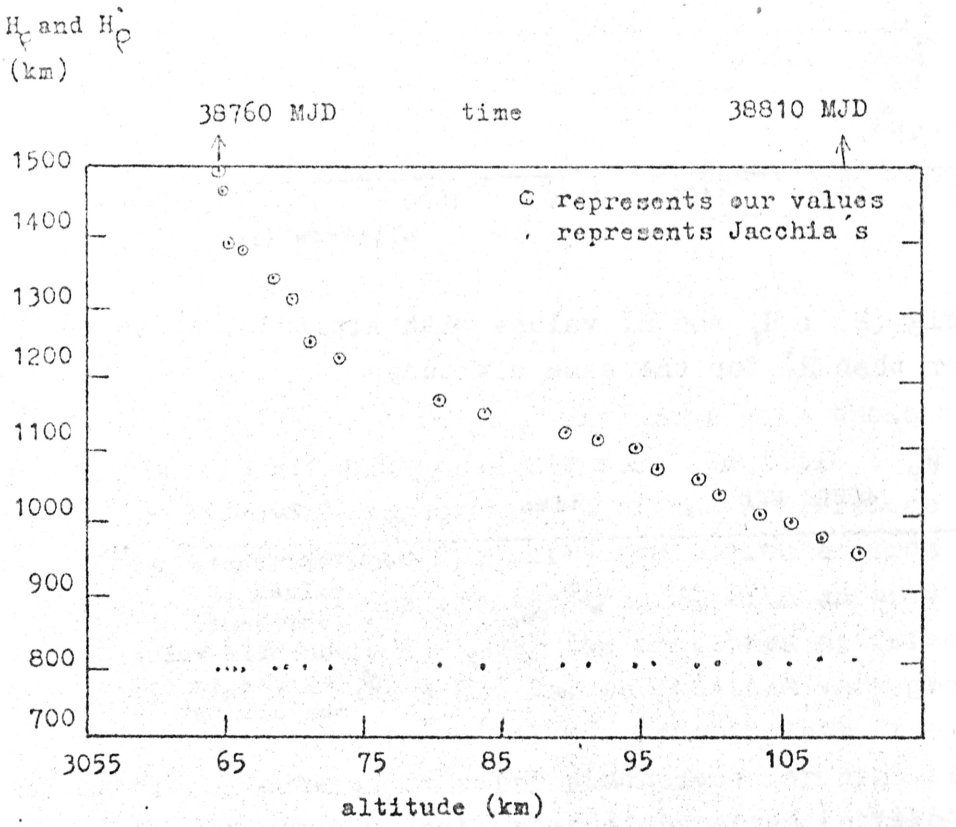

Fig (1) : H_ρ and $H_\rho^`$ with altitude. $H_\rho$ values are increasing with decreasing altitude, while $H_\rho^`$ values indicate approximately constant values for different altitudes.

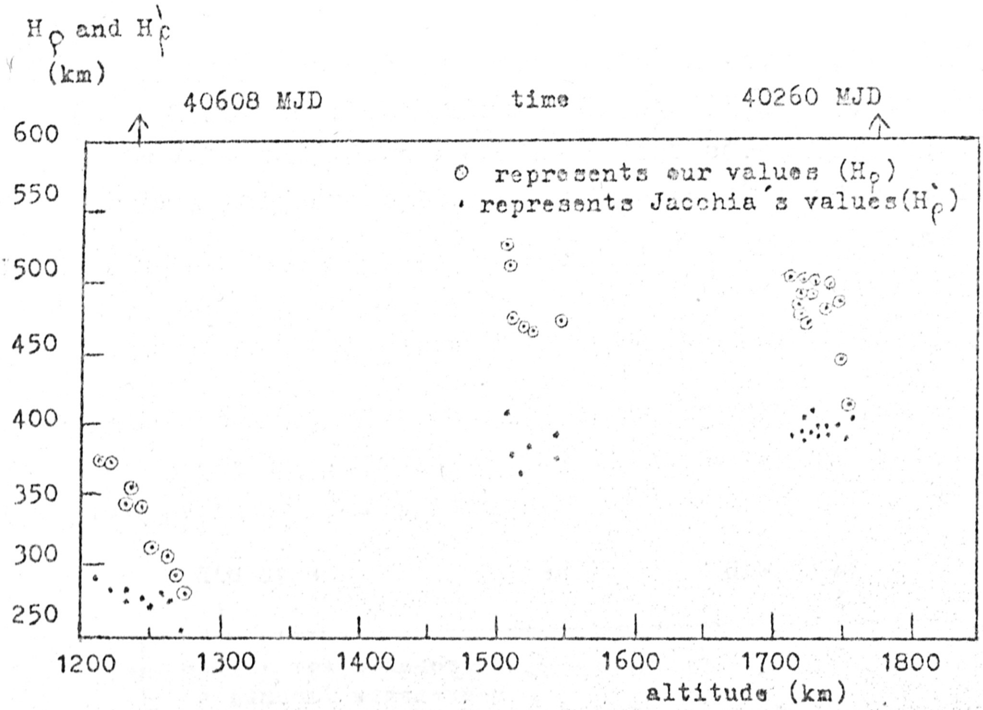

Fig (2) : H_ρ and H'_ρ values with altitude. H_ρ values are greater than H'_ρ for the same altitudes.

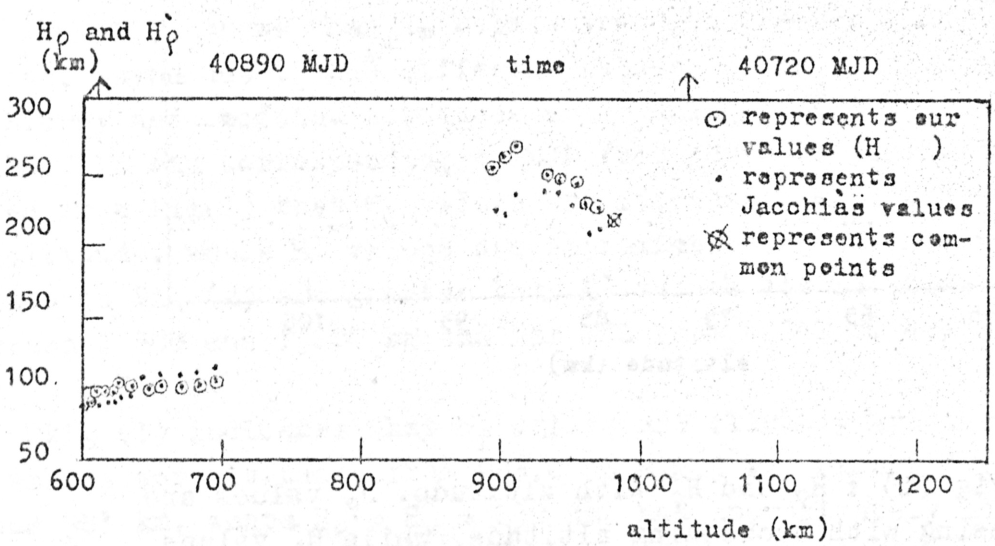

Fig (3) : H_ρ and H'_ρ values with altitude . H_ρ and H'_ρ values are approximately the same for altitudes from 605 to 750 km, and H_ρ values are greater than H'_ρ values for other altitudes, except for altitude 983 km where they are equal.

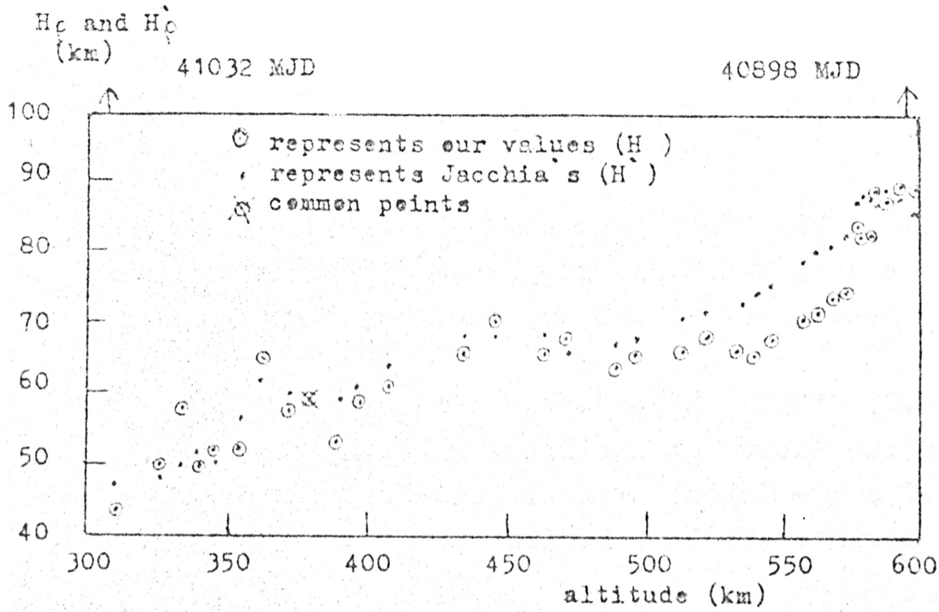

Fig (4) : H_ρ & $H_\rho^`$ values with altitude. The differences between $H_\rho$ & $H_\rho^`$ values are small and ranging from 0 to 10 km.

6- Conclusion

Our H_ρ values are in good agreement with those values of Jacchia $H_\rho^`$ for altitudes ranging from about 316 to 700 km with an arithmetic mean percentage 5 % of the differences between $H_\rho$ & $H_\rho^`$ with respect to H_ρ, while for altitudes from 700 to 1,760 km, H_ρ values are greater than $H_\rho^`$ with an arithmetic mean percentage about 14 % for the mentioned differences. For altitudes greater than 3 000 km, we obtained very great values for $H_\rho$ compared with those of $H_\rho^`$. Nevertheless, the inaccurate values of the density of upper atmosphere for high altitudes may be the reason for such great differences between H_ρ and $H_\rho^`$ values.

7- Acknowledgment

It is a pleasure to thank Prof. Dr. L. Sehnal, for his valuable discussions when we were at Ondřejov Observatory near Prague last year, September, 1983 :

- References

Jacchia,L.G. (1970), "New Static Models of the Thermosphere and Exosphere with Empirical Temperature Profiles". Smithsonian Astrophys. Obs. Spec. Rep. No. 313.

King-Hele.D.G. (1963), "The Contraction of Satellite Orbit under the Action of Air Drag, Allowing for the Variation of Scale Height with Altitude". Dynamics of Satellite, Springer-Verlag, Berlin.

Sterne,E.P. (1960), "Atmospheric Drag on Non-Spherical A.E.S". Space science Review, Rep. No. 65 .

Slowey,W.J. "Radiation Pressure and Air Drag effects on the Orbit of the Balloon Satellite 1963 30 D". Research in Space Science, Spec. Rep. No. 356. Smithsonian Institution, Astrophys. Obs. Cambridge, Massachusetts, (1974).