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FACTORIZATION THEOREM FOR 1-SUMMING OPERATORS

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Abstract. We study some classes of summing operators between spaces of integrable functions with respect to a vector measure in order to prove a factorization theorem for 1-summing operators between Banach spaces.

Keywords: vector measures, integrable functions, sequences on Banach spaces, summing operators

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1. Introduction

Let (Ω, Σ, μ) be a *positive finite measure space*, where Ω is a set, Σ is a σ-algebra of subsets of Ω and μ is a finite positive measure. A set function $m\colon \Sigma \to X$ defined on a Banach space X is called a *vector measure* whenever it is σ-additive. Throughout this work λ will stand for a Rybakov's control measure for m. Further references about vector measure theory can be found in [5].

Integrability of scalar functions with respect to a vector measure was first studied by Dunford, Bartle and Schwartz in [1]. Several years later, Lewis gave an equivalent definition in [8]. He showed that a real λ-measurable function f is integrable with respect to m if the following two conditions hold. The function f is $\langle m, x^*\rangle$-integrable for every x^* in the dual of X, X^*, where $\langle m, x^*\rangle$ is the scalar measure defined by $\langle m, x^*\rangle(A) := \langle m(A), x^*\rangle$ for $A \in \Sigma$. Moreover, for each set A in Σ there is a unique element $m_f(A) \in X$ such that $\langle m_f(A), x^*\rangle = \int_A f \, \mathrm{d}\langle m, x^*\rangle$ for every $x^* \in X^*$. In this case $m_f(A)$ corresponds to the integral of f over the set A with respect to the measure m, and it is denoted by $\int_A f \, \mathrm{d}m$. We denote by $L^1(m)$ the space of equivalence

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classes of (λ-a.e. equal) functions that are integrable with respect to m. The space $L^1(m)$ endowed with the norm given by $\|f\|_{L^1(m)} := \sup_{x^*\in B(X^*)} \int_\Omega |f| \,\mathrm{d}|\langle m, x^*\rangle|$ for $f \in L^1(m)$ is a Banach function space over the measure space $(\Omega, \Sigma, \lambda)$ and the norm is order continuous. The constant function χ_Ω is a weak order unit. These spaces are particularly interesting because they represent a large class of Banach lattices; G. Curbera proved in [4, Theorem 8] that every order continuous Banach lattice X with weak order unit is lattice isomorphic and isometric to $L^1(m)$ for m an X-valued vector measure. Further properties of these spaces can be found in [9]

For $p = \infty$, the space $L^\infty(m)$ consists of the real valued functions that are Σ-measurable and m-essentially bounded. When equipped with the essential supremum norm $\|\cdot\|_{L^\infty(m)}$, $L^\infty(m)$ is a Banach function space over $(\Omega, \Sigma, \lambda)$. Bounded Σ-measurable functions are integrable with respect to m.

The aim of this article is to characterize the 1-summing operators as those that factorize through a space of integrable functions with respect to a vector measure. The operators that appear in the decomposition will have particular properties of summability. We will begin by introducing a space of summable sequences in the space $L^1(m)$, the space of m-r-summable sequences. This space is an intermediate space between the classical spaces of strong and weakly summable sequences in Banach spaces and has been already studied in a more general setting in [2], [3]. The reference for the study of classical spaces of summable sequences on Banach spaces is [6]. As usual we will denote by $\ell_r(X)$ and $\ell_r^w(X)$ the spaces of strongly and weakly summable X-valued sequences, respectively. The ideal of r-summing operators is denoted by Π_r, and we write $\pi_r(T)$ for the norm of an operator T in the space of r-summing operators. We use standard Banach space notation; if Y is a Banach space, $B(Y)$ denotes its unit ball and Y^* its topological dual.

2. Definitions

Let $\Psi\colon X \times Y \to Z$ be a bounded bilinear map and X, Y and Z Banach spaces. In [3, Section 2.2] the author defines the space $\ell_r^\Psi(X)$ of X-valued sequences $(x_n)_n$ so that $(\Psi(x_n, y))_n$ is strongly r-summable in Z for every $y \in B(Y)$. This space is endowed with a norm naturally defined by

$$\|(x_n)_n\|_{\ell_r^\Psi(X)} := \sup\{\|(\Psi(x_n, y))_n\|_{\ell_r(Z)} \colon y \in B(Y)\}. \tag{2.1}$$

The space X is Ψ-*normed* whenever there is $K \geqslant 0$ such that

$$\|x\|_X \leqslant K \sup\{\|\Psi(x, y)\|_Z \colon y \in B(Y)\}.$$

The following proposition is proved in [2] for $r = 1$ and in [3] for $r > 1$.

Proposition 2.1. *Let X be a Banach space, $\Psi\colon X \times Y \to Z$ a bounded bilinear map and $1 \leqslant r < \infty$.*

i) *X is Ψ-normed if and only if $\ell_r(X) \subseteq \ell_r^{\Psi}(X) \subseteq \ell_r^w(X)$.*

ii) *If X is Ψ-normed then $\ell_r^B(X)$ is a Banach space when endowed with the norm $\|\cdot\|_{\ell_r^{\Phi}(X)}$.*

In order to study the sequences in $L^1(m)$, let Φ be the bilinear map defined by:

$$\Phi\colon\ L^1(m) \times L^\infty(m) \to X,$$
$$(f, g) \mapsto \int_\Omega fg\,\mathrm{d}m.$$

A sequence $(f_n)_n \subset L^1(m)$ is *m-r-summable* whenever for each $g \in L^\infty(m)$, the X-valued sequence $(\Phi(f_n, g))_n$ is strongly r-summable in X. In what follows we denote by $\ell_r^m(L^1(m))$ the space of m-r-summable sequences in $L^1(m)$. Following the previous definitions we have that a suitable norm is given by

$$\|(f_n)\|_{\ell_r^m(L^1(m))} := \sup\biggl\{\biggl(\sum_n \biggl\|\int_\Omega f_n g\,\mathrm{d}m\biggr\|_X^r\biggr)^{1/r} :\ g \in B(L^\infty(m))\biggr\}. \tag{2.2}$$

By [9, Proposition 3.31 (i)] we have that the norm of $f \in L^1(m)$ can be computed as

$$\|f\|_{L^1(m)} := \sup_{g \in B(L^\infty(m))} \biggl\|\int_\Omega fg\,\mathrm{d}m\biggr\|_X,$$

therefore the space $L^1(m)$ is Φ-normed. By Proposition 2.1 we conclude that $(\ell_r^m(L^1(m)),\ \|\cdot\|_{\ell_r^m(L^1(m))})$ is a Banach space. Moreover, the following chain of inclusions holds:

$$\ell_r(L^1(m)) \subseteq \ell_r^m(L^1(m)) \subseteq \ell_r^w(L^1(m)).$$

In the following we will study those operators that transform m-r-summable sequences of (a closed subspace of) $L^1(m)$ into strongly r-summable sequences in a Banach space Y and also those that transform weakly r-summable sequences in Y into m-r-summable ones into $L^1(m)$.

We say that $T\colon L^1(m) \to Y$ is *m-r-summing* if there is a constant $C \geqslant 0$ such that for every natural number n and regardless of the choice of functions $f_1, \ldots, f_n$ in $L^1(m)$ we have

$$\biggl(\sum_{i=1}^n \|T(f_i)\|_Y^r\biggr)^{1/r} \leqslant C \sup_{g \in B(L^\infty(m))} \biggl(\sum_{i=1}^n \biggl\|\int_\Omega f_i g\,\mathrm{d}m\biggr\|_X^r\biggr)^{1/r}. \tag{2.3}$$

The least C for which the inequality (2.3) always holds is denoted by $\pi_r^m(T)$. We shall write $\Pi_r^m(L^1(m),Y)$ for the set of m-r-summing operators in $\mathscr{L}(L^1(m),Y)$. We clearly have that $\Pi_r^m(L^1(m),Y)$ is a linear subspace of $\mathscr{L}(L^1(m),Y)$ and that π_r^m defines a norm in $\Pi_r^m(L^1(m),Y)$ with $\|T\| \leqslant \pi_r^m(T)$ for each $T \in \Pi_r^m(L^1(m),Y)$.

Remark 2.2. Let L be a closed subspace of $L^1(m)$. We say that $T\colon L \to Y$ is m-r-summing if for every finite choice of functions $f_1,\ldots,f_n$ in L, inequality (2.3) holds for some positive constant C.

Notice that for m a scalar measure the notion of m-r-summability coincides with classical r-summability; for a general vector measure m the inclusion $\Pi_r(L^1(m),Y) \subset \Pi_r^m(L^1(m),Y)$ always holds.

In [6, p. 36] the authors proved that the weak norm of a sequence can be computed by taking the supremum in a norming subset. Moreover, by [7] we know that the set $\Gamma := \{\langle \Phi(\cdot,g),\, x^*\rangle\colon\ g \in B(L^\infty(m)),\ x^* \in B(X^*)\} \subset L^1(m)^*$ is norming. Then it is easy to conclude that each bounded linear operator $T\colon\ L^1(m) \to Y$ induces a bounded linear map $\hat{T}\colon\ (f_n)_n \mapsto (T(f_n))_n$ between the spaces $\ell_r^m(L^1(m))$ and $\ell_r^w(Y)$.

The following result shows that m-r-summing operators are exactly those that transform m-r-summable sequences in $L^1(m)$ into strongly r-summable ones in the range Y. We present the proof for completeness.

Theorem 2.3. *An operator $T \in \mathscr{L}(L^1(m),Y)$ is m-r-summing if and only if $\hat{T}(\ell_r^m(L^1(m))) \subset \ell_r(Y)$. Moreover, $\|\hat{T}\| = \pi_r^m(T)$.*

Proof. Suppose first that T is m-r-summing. Then for each finite collection $f_1,\ldots,f_k \in L^1(m)$ we have

$$\Big(\sum_{i=1}^k \|T(f_i)\|_Y^r\Big)^{1/r} \leqslant \pi_r^m(T) \sup_{g\in B(L^\infty(m))} \Big(\sum_{i=1}^k \Big\|\int_\Omega f_i g\,\mathrm{d}m\Big\|_X^r\Big)^{1/r}.$$

Take a sequence $(f_n)_n \in \ell_r^m(L^1(m))$. We claim that $\hat{T}((f_n)_n) \in \ell_r(Y)$, hence

$$\begin{aligned}
\|\hat{T}((f_n)_n)\|_{\ell_r(Y)} = \|(T(f_n))_n\|_{\ell_r(Y)} &= \sup_{k\in\mathbb{N}} \Big(\sum_{n\leqslant k} \|T(f_n)\|_Y^r\Big)^{1/r} \\
&\leqslant \pi_r^m(T) \sup_{k\in\mathbb{N}} \sup_{g\in B(L^\infty(m))} \Big(\sum_{n\leqslant k} \Big\|\int_\Omega f_n g\,\mathrm{d}m\Big\|_X^r\Big)^{1/r} \\
&= \pi_r^m(T) \sup_{g\in B(L^\infty(m))} \Big(\sum_{n=1}^\infty \Big\|\int_\Omega f_n g\,\mathrm{d}m\Big\|_X^r\Big)^{1/r} \\
&= \pi_r^m(T) \|(f_n)_n\|_{\ell_r^m(L^1(m))},
\end{aligned}$$

therefore $\|\hat{T}\| \leqslant \pi_r^m(T)$.

We prove the converse implication by a closed graph argument. Suppose that $\hat{T}(\ell_r^m(L^1(m))) \subset \ell_r(Y)$. Since $\hat{T}\colon \ell_r^w(L^1(m)) \to \ell_r(Y)$ is continuous and the $\ell_r(Y)$ norm dominates the $\ell_r^w(Y)$ norm we have that the corresponding operator $\hat{T}\colon \ell_r^m(L^1(m)) \to \ell_r(Y)$ has closed graph and is bounded. Thus for a finite sequence $(f_i)_{i=1}^k \subset L^1(m)$ we get

$$\|(T(f_i))_{i=1}^k\|_{\ell_r(Y)} \leqslant \|\hat{T}\| \|(f_i)_{i=1}^n\|_{\ell_r^m(L^1(m))}.$$

Therefore T is m-r-summing and $\pi_r^m(T) \leqslant \|\hat{T}\|$. □

It is direct, as a consequence of the previous characterization, that the space of m-r-summing operators endowed with their respective norms are Banach spaces.

Theorem 2.4. *Let Y be a Banach space, and $1 \leqslant r < \infty$. The space of m-r-summing operators, $\Pi_r^m(L^1(m), Y)$ endowed with the norm π_r^m is a Banach space.*

We say that an operator $T\colon Y \to L^1(m)$ is *weakly m-r-summing* if there is a constant $C > 0$ such that for every finite set of elements $y_1, \dots, y_n \in Y$,

$$\sup_{g \in B(L^\infty(m))} \left(\sum_{i=1}^n \left\| \int_\Omega T(y_i) g \, \mathrm{d}m \right\|^r \right)^{1/r} \leqslant C \sup_{y^* \in B(Y^*)} \left(\sum_{i=1}^n |\langle y_i, y^* \rangle|^r \right)^{1/r}. \tag{2.4}$$

We write $\pi_r^{w-m}(T)$ for the least constant such that the inequality above holds and denote by $\Pi_r^{w-m}(Y, L^1(m))$ the space of weakly m-r-summing operators. Applying arguments similar to those used in the proof of Theorem 2.3 we get that weakly m-r-summing operators are exactly those that transform weakly summable Y-valued sequences into m-r-summable sequences in $L^1(m)$. As a consequence, the space $\Pi_r^{w-m}(Y, L^1(m))$ is a Banach space when endowed with the norm $\pi_r^{w-m}(\cdot)$.

Remark 2.5. Notice that every r-summing operator $T\colon Y \to L^1(m)$ is weakly m-r-summing, and $\pi_r^{w-m}(T) \leqslant \pi_r(T)$. For a linear and continuous operator T between spaces of integrable functions with respect to a vector measure, $T\colon L^1(m_1) \to L^1(m_2)$, we have that T is weakly m-r-summing whenever it is m-r-summing.

Examples of weakly m-r-summing operators are easy to find. More interesting are spaces $L^1(m)$ such that the identity map has this property. The canonical case happens when m is a scalar positive finite measure μ. Clearly, the identity $\mathrm{Id}\colon L^1(\mu) \to L^1(\mu)$ has this property since in this case the integrals in the left hand side term of inequality (2.4) give exactly the usual duality, the one that appears in the right hand side term.

In the following we present a characterization of weakly m-r-summing operators in terms of a Pietsch type domination theorem. As we will show, it is required that the composition of T with the integration map for every $g \in L^\infty(m)$ be r-summing, together with some sort of uniform behavior of the associated r-summing norms.

Proposition 2.6. *Let $T\colon Y \to L^1(m)$ with Y be a Banach space. The following statements are equivalent.*

(i) *T is weakly m-r-summing.*

(ii) *There is a constant $C > 0$ such that for every $g \in B(L^\infty(m))$, the operator $I_g \circ T\colon Y \to X$ is r-summing, and*

$$\pi_r(I_g \circ T) \leqslant C.$$

(iii) *There is a constant $C > 0$ such that for every $g \in B(L^\infty(m))$, there is a probability measure η_g defined on the σ-algebra of Borel subsets of $B(Y^*)$ (endowed with the weak*-topology) such that, for every $y \in Y$,*

$$\Big\| \int_\Omega T(y) g \,\mathrm{d}m \Big\|_X \leqslant C\Big(\int_{B(Y^*)} |\langle y, y^*\rangle|^r \,\mathrm{d}\eta_g(y^*)\Big)^{1/r}. \tag{2.5}$$

Moreover, the least C appearing in (i), (ii) *and* (iii) *coincides with*

$$\sup_{g\in B(L^\infty(m))} \pi_r(I_g \circ T) = \pi_r^{w-m}(T).$$

Proof. For the implication (i) $\Rightarrow$ (ii) it is enough to use the definition of an r-summing operator. The converse is also obvious. The equivalence between (iii) and (ii) is obtained just by applying the Pietsch Domination Theorem to each one of the maps $I_g \circ T$. The formula for the norm is also a direct consequence of the definitions. $\square$

Remark 2.7. The lattice properties of the sets of Pietsch measures appearing in (iii) of Proposition 2.6 provide a criterion for an operator to be weakly m-r-summing. In fact, a weakly m-r-summing operator $T\colon Y \to L^1(m)$ is r-summing if and only if the set of Pietsch measures is order bounded. Indeed, let $T \in \Pi_r^{w-m}(Y, L^1(m))$. If $\mathscr{M}(B(Y^*))$ is the usual space of Radon measures over the σ-algebra of Borel subsets of $B(Y^*)$, where Y^* is endowed with the weak*-topology, then $\mathscr{M}(B(Y^*)) = \mathscr{C}(B(Y^*))^*$. As a consequence of Proposition 2.6 there is a set of Pietsch measures $\{\eta_g\colon g \in B(L^\infty(m))\}$ associated with the operator T such that for each $g \in L^\infty(m)$ inequality (2.5) holds. Assuming that the set $\{\eta_g\colon g \in B(L^\infty(m))\}$ is order bounded in $\mathscr{M}(B(Y^*))$ by an element η, we obtain that for every $y \in Y$,

$$\|T(y)\|_{L^1(m)} \leqslant K\Big(\int_{B(Y^*)} |\langle y, y^*\rangle|^r \,\mathrm{d}\eta\Big)^{1/r}.$$

Consequently, T is r-summing. The converse is also obvious, since every r-summing operator $T\colon Y \to L^1(m)$ is weakly m-r-summing.

Remark 2.8. When the previous argument is applied to the identity map $I_d\colon L^1(m) \to L^1(m)$, we obtain that it is weakly m-r-summing with a set of Pietsch measures that is uniformly order bounded if and only if $L^1(m)$ is finite dimensional. This is a consequence of the Dvoretsky-Rogers Theorem and the following calculations. If η is the required order bound, for every $f \in L^1(m)$ we have

$$\begin{aligned}\|f\|_{L^1(m)} &= \sup_{g\in B(L^\infty(m))} \left\| \int_\Omega fg \,\mathrm{d}m \right\|_X \\ &\leqslant K \sup_{g\in B(L^\infty(m))} \left(\int_{B((L^1(m))^*)} |\langle f,h\rangle|^r \,\mathrm{d}\eta_g \right)^{1/r} \\ &\leqslant K \left(\int_{B((L^1(m))^*)} |\langle f,h\rangle|^r \,\mathrm{d}\eta \right)^{1/r}.\end{aligned}$$

The previous remark shows that uniform boundedness of the integrals $\| \int_\Omega (\cdot) g \,\mathrm{d}m\|_X$ by an integral $\left(\int_{B((L^1(m))^*)} |\langle f,h\rangle|^r \,\mathrm{d}\eta\right)^{1/r}$ only holds for finite dimensional $L^1(m)$ spaces. In the same direction, the following result shows that $L^1(m)$ spaces where m-r-summable sequences and weakly r-summable sequences coincide (i.e. the identity map is weakly m-r-summing) for some $1 \leqslant r < \infty$, have strong restrictions on the properties of the integration maps $\int_\Omega (\cdot) g \,\mathrm{d}m$, $g \in L^\infty(m)$.

Recall that an operator T between Banach spaces X and Y is said to be *strictly singular* if, for every infinite dimensional (closed) subspace M of X, the restriction $T|_M$ is not an isomorphism into Y.

Proposition 2.9. *If* $\mathrm{Id}\colon L^1(m) \to L^1(m)$ *is weakly m-r-summing for some* $1 \leqslant r < \infty$*, then for every* $g \in L^\infty(m)$ *the integration operator* I_g *is strictly singular.*

Proof. Let $g \in L^\infty(m)$. Suppose that there is a subspace S such that the restriction $I_g|_S\colon S \to X$ is an isomorphism into the range. Let us write i for the inclusion map $i\colon S \to L^1(m)$ and $R\colon I_g|_S(S) \to S$ for the inverse map $(I_g|_S)^{-1}\colon I_g|_S(S) \to S$. Since Id is weakly m-r-summing, each I_g is r-summing as a consequence of (ii) in Proposition 2.6. Therefore, $I_g|_S = I_g \circ i\colon S \to L^1(m) \to X$ is an r-summing isomorphism into the range, and since the identity in S can be factorized as

$$R \circ I_g|_S\colon S \to I_g(i(S)) \to S,$$

the ideal property of the r-summing operators and the Dvoretsky-Rogers Theorem yield that S is finite dimensional. $\square$

3. Main result

Recall that, by Remark 2.2, the definition of an m-r-summing operator can be extended to the operators defined on closed subspaces of $L^1(m)$ in a natural way. Therefore the composition $T = R \circ U$ of a weakly m-r-summing operator $U\colon Y \to L^1(m)$ and an m-r-summing one $R\colon S \to Z$, where S is a subspace of $L^1(m)$ such that $U(Y) \subseteq S$, is r-summing. The main result characterizes the 1-summing operators as those that factorize through a space of integrable functions with respect to a vector measure m. It shows that in a sense, regarding the structure properties of $L^1(m)$ spaces and factorizations through them, 1-summability can be decomposed in m-1-summability and weakly m-1-summability.

Theorem 3.1. *Let $T\colon Y \to Z$ be an operator between Banach spaces. The following statements are equivalent.*

(i) *T is 1-summing.*

(ii) *There is a vector measure m such that T factorizes through a subspace of $L^1(m)$ as $T = R \circ U$, where U is weakly m-1-summing and R is m-1-summing.*

Proof. For the proof of (i) $\Rightarrow$ (ii), consider the factorization of T as a 1-summing operator through the map $i\colon \mathscr{C}(B(Y^*)) \to L^1(B(Y^*), \eta)$ given by the classical Pietsch domination theorem. Recall that we consider $B(Y^*)$ endowed with the weak*-topology. Here η is a Radon probability measure and $i(f) = f$ is the identification map of continuous functions as integrable functions. Take the vector measure defined on $\mathscr{B}$, the σ-algebra of the Borel subsets of $B(Y^*)$, with range in $L^1(B(Y^*), \eta)$ given by $m(A) = \chi_A$, $A \in \mathscr{B}$. Then $L^1(m) = L^1(B(Y^*), \eta)$ isometrically. Consider the map $U\colon Y \to F \subset L^1(m)$ given by $U(y) = \langle y, \cdot \rangle$, where F is the closure of the functions $\langle y, \cdot \rangle$ in $L^1(\eta)$. Recall that $L^\infty(m) = L^\infty(\eta)$. The following calculations show that U is weakly m-1-summing. For a finite set $y_1, \ldots, y_n \in Y$,

$$\begin{aligned}
\sup_{g \in B(L^\infty(m))} \sum_{i=1}^n \Big\| \int U(y_i) g \,\mathrm{d}m \Big\|_{L^1(\eta)} &= \sup_{g \in B(L^\infty(m))} \sum_{i=1}^n \Big\| \int \langle y_i, \cdot \rangle g \,\mathrm{d}m \Big\|_{L^1(\eta)} \\
&= \sup_{g \in B(L^\infty(\eta))} \sum_{i=1}^n \int_{B(Y^*)} |\langle y_i, \cdot \rangle g| \,\mathrm{d}\eta \\
&= \sum_{i=1}^n \int_{B(Y^*)} |\langle y_i, \cdot \rangle| \,\mathrm{d}\eta \\
&\leqslant \sup_{y^* \in B(Y^*)} \sum_{i=1}^n |\langle y_i, y^* \rangle|.
\end{aligned}$$

Now take the map $R\colon F \to Z$ given by $R(\langle x, \cdot\rangle) = T(x)$ and extended by density to the elements of the closure of the range of U. Let us show that it is m-1-summing. It is enough to prove it for the elements of the range of U. Take $\langle y_1, \cdot\rangle, \dots, \langle y_n, \cdot\rangle$. Then, having in mind that there is a constant K such that for every $y \in Y$, $\|T(y)\|_Z \leqslant K\|\langle y, \cdot\rangle\|_{L^1(\eta)}$, we obtain

$$\begin{aligned}\sum_{i=1}^n \|R(\langle y_i, \cdot\rangle)\|_Z = \sum_{i=1}^n \|T(y_i)\| &\leqslant K \sum_{i=1}^n \|\langle y_i, \cdot\rangle\|_{L^1(\eta)} \\ &\leqslant K \sup_{g \in B(L^\infty(m))} \sum_{i=1}^n \|\langle y_i, \cdot\rangle g\|_{L^1(\eta)}.\end{aligned}$$

Consequently, the map is m-1-summing.

Implication (ii) $\Rightarrow$ (i) follows directly by the definitions of m-r-summing and weakly m-r-summing operators. $\square$

References

[1] *R. G. Bartle, N. Dunford, J. Schwartz*: Weak Compactness and Vector Measures. Canad. J. Math. *7* (1955), 289–305. zbl

[2] *O. Blasco, J. M. Calabuig, T. Signes*: A bilinear version of Orlicz-Pettis Theorem. J. Math. Anal. Appl. *348* (2008), 150–164. zbl

[3] *J. M. Calabuig*: Integración bilineal. Tesis doctoral, 2004.

[4] *G. P. Curbera*: Operators into L^1 of a vector measure and applications to Banach lattices. Math. Ann. *293* (1992), 317–330. zbl

[5] *J. Diestel, J. J. Uhl Jr.*: Vector Measures. Amer. Math. Soc. Surveys 15, Providence, R.I., 1977. zbl

[6] *J. Diestel, H. Jarchow, A. Tonge*: Absolutely Summing Operators. Cambridge Studies in Advanced Mathematics, Cambridge, 1995. zbl

[7] *I. Ferrando, J. Rodríguez*: The weak topology on L^p of a vector measure. vol. 55, Topology and its Applications, 2008, pp. 1439–1444. zbl

[8] *D. R. Lewis*: Integration with respect to vector measures. Pacific J. Math. *33* (1970), 157–165. zbl

[9] *S. Okada, W. Ricker, E. A. Sánchez-Pérez*: Optimal Domain and Integral Extension of Operators Acting in Function Spaces. Operator Theory: Advances and Applications, Vol. 180, Birkhäuser, 2008. zbl

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