MODERN PUZZLES

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AND HOW TO SOLVE THEM

BY

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"THE CANTERBURY PUZZLES," "AMUSEMENTS IN MATHEMATICS,"
"THE WORLD'S BEST WORD PUZZLES," ETC.

"Amusement is one of the fields of applied mathematics."

W. F. WHITE.

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PREFACE

FREQUENT requests received from readers of the *Strand Magazine* and others, in all parts of the world, have encouraged me to put together a new selection from the best pastime puzzles that I have given during the last decade, together with some new problems. I have tried, as is my habit, to present the puzzles in a manner that may attract the reader who has perhaps only an elementary acquaintance with Mathematics and at the same time to make them acceptable to those learned in the science.

The puzzles are roughly classified in the manner that I have previously introduced, so that the reader can easily satisfy his preference for any particular type. Space does not allow me to give the full method of solution to every puzzle, but a more lengthy solution here and there will frequently be found to afford a clue to the way of attacking other problems in the book. I have had to exclude Chess puzzles from this volume, but I contemplate producing, at some future time, a new collection of these.

AUTHORS' CLUB,
29 July, 1926.

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MODERN PUZZLES

ARITHMETICAL AND ALGEBRAICAL PROBLEMS

MONEY PUZZLES

1.—Concerning a Cheque

A man went into a bank to cash a cheque. In handing over the money the cashier, by mistake, gave him pounds for shillings and shillings for pounds. He pocketed the money without examining it, and spent half a crown on his way home, when he found that he possessed exactly twice the amount of the cheque. He had no money in his pocket before going to the bank, and it is an interesting puzzle to find out what was the exact amount of that cheque.

2.—Pocket-money

I went down the street with a certain amount of money in my pocket, and when I returned home I discovered that I had spent just half of it, and that I now had just as many shillings as I previously had pounds, and half as many pounds as I then had shillings. How much money had I spent?

3.—Dollars and Cents

An American correspondent tells me that a man went into a store and spent one-half of the money that was in his pocket. When he came out he found that he had just as many cents as he had dollars when he went in and half as many dollars as he had cents when he went in. How much money did he have on him when he entered?

4.—Loose Cash

What is the largest sum of money—all in current silver coins and no four-shilling-piece—that I could have in my pocket without being able to give change for half a sovereign?

5.—Doubling the Value

It is a curious fact that if you double £6 13s. you get £13 6s., which is merely changing the shillings and the pounds. Can you find another sum of money that has the same peculiarity that, when multiplied by any number you may choose to select, will merely exchange the shillings and the pounds? There is only one other multiplier and sum of money, beside the case shown, that will work. What is it?

6.—Generous Gifts

A generous man set aside a certain sum of money for equal distribution weekly to the needy of his acquaintance. One day he remarked, "If there are five fewer applicants next week, you will each receive two shillings more." Unfortunately, instead of there being fewer there were actually four more persons applying for the gift. "This means," he pointed out, "that you will each receive one shilling less." Now, how much did each person receive at that last distribution?

7.—Selling Eggs

A woman took a certain number of eggs to market and sold some of them. The next day, through the industry of her hens, the number left over had been doubled, and she sold the same number as the previous day. On the third day the new remainder was trebled, and she sold the same number as before. On the fourth day the remainder was quadrupled, and her sales the same as before. On the fifth day what had been left over were quintupled, yet she sold exactly the same as on all the previous occasions, and so disposed of her entire stock. What is the smallest number of eggs she could have taken to market the first day, and how many did she sell daily?

8.—Buying Buns

Buns were being sold at three prices: one a penny, two a penny, and three a penny. Some children (there were as many boys as girls) were given sevenpence to spend on these buns, each receiving exactly alike. How many buns did each receive? Of course no buns were divided.

9.—Fractional Value

What part of threepence is one-third of twopence.

10.—Unrewarded Labour

A man persuaded Weary Willie, with some difficulty, to try to work on a job for thirty days at eight shillings a day, on the condition that he would forfeit ten shillings a day for every day that he idled. At the end of the month neither owed the other anything, which entirely convinced Willie of the folly of labour. Now, can you tell just how many days' work he put in, and on how many days he idled?

11.—The Perplexed Banker

A man went into a bank with a thousand sovereigns and ten bags. He said, "Place this money, please, in the bags in such a way that if I call and ask for a certain number of sovereigns you can hand me over one or more bags, giving me the exact amount called for without opening any of the bags." How was it to be done? We are, of course, only concerned with a single application, but he may ask for any exact number of pounds from £1 to £1000.

12.—A Weird Game

Seven men engaged in play. Whenever a player won a game he doubled the money of each of the other players. That is, he gave each player just as much money as each had in his pocket. They played seven games and, strange to say, each won a game in turn in the order of their names, which began with the letters A, B, C, D, E, F, and G. When they had finished it was found that each man had exactly two shillings and eightpence in his pocket. How much had each man in his pocket before play?

13.—Find the Coins

Three men, Abel, Best, and Crewe, possessed money, all in silver coins. Abel had one coin fewer than Best and one more than Crewe. Abel gave Best and Crewe as much money as they already had, then Best gave Abel and Crewe the same amount of money as they then held, and finally Crewe gave Abel and Best as much money as they then had. Each man then held exactly ten shillings. To find what amount each man started with is not difficult. But the sting of the puzzle is in the tail. Each man held exactly the *same coins* (the fewest possible) amounting to ten shillings. What were the coins and how were they originally distributed?

14.—An Easy Settlement

Three men, Andrews, Baker, and Carey, sat down to play at some game. When they put their money on the table it was found that they each possessed two coins only, making altogether £1 4s. 6d. At the end of play Andrews had lost five shillings and Carey had lost sixpence, and they all squared up by simply exchanging the coins. What were the exact coins that each held on rising from the table?

15.—Sawing Logs

"Your charge," said Mr. Grigsby, "was thirty shillings for sawing up three cords of wood made up of logs three feet long, each log to be cut into pieces one foot in length."

"That is so," the man replied.

"Well, here are four cords of logs, all of the same thickness as before, only they are in six-feet lengths, instead of three feet. What will your charge be for cutting them all up into similar one-foot lengths?"

It is curious that they could not at once agree as to the fair price for the job. What does the reader think the charge ought to be?

16.—Digging a Ditch

Here is a curious question that is more perplexing than it looks at first sight. Abraham, an infirm old man, undertook to dig a ditch for two pounds. He engaged Benjamin, an able-bodied fellow, to assist him and share the money fairly according to their capacities. Abraham could dig as fast as Benjamin could shovel out the dirt, and Benjamin could dig four times as fast as Abraham could do the shovelling. How should they divide the money? Of course, we must assume their relative abilities for work to be the same in digging or shovelling.

17.—Name their Wives

A man left a legacy of £1000 to three relatives and their wives. The wives received together £396. Jane received £10 more than Catherine, and Mary received £10 more than Jane. John Smith was given just as much as his wife, Henry Snooks got half as much again as his wife, and Tom Crowe received twice as much as his wife. What was the Christian name of each man's wife?

18.—A Curious Paradox

A man went into a shop to pay a little bill that he owed. On placing the money on the counter he found that he had not quite sufficient, owing to a small purchase that he had thoughtlessly made on the way.

"I am sorry," he said, "but you see I am a little short."

"Oh, that is all right," replied the tradesman, after looking at the money, "it won't make any difference to me."

"My good man!" exclaimed the customer. "How can that be? If I am the gainer, as I certainly shall be, by paying you an insufficient amount, you must be the loser."

"Not at all, I assure you."

"Do you mean that you will allow me a discount?"

"No, for then I should certainly be the loser. But really it will not affect my pocket in the slightest."

Can you explain the mystery? It may come to you in a flash. The tradesman was certainly correct.

19.—Market Transactions

A farmer goes to market and buys a hundred animals at a total cost of £100. The price of cows being £5 each, sheep £1 each, and rabbits 1s. each, how many of each kind does he buy? Most people will solve this, if they succeed at all, by more or less laborious trial, but there are several direct ways of getting the solution.

20.—The Seven Applewomen

Here is an old puzzle that people are frequently writing to me about. Seven applewomen, possessing respectively 20, 40, 60, 80, 100, 120, and 140 apples, went to market and sold all their apples at the same price, and each received the same sum of money. What was the price?

AGE AND KINSHIP PUZZLES

21.—Their Ages

If you add the square of Tom's age to the age of Mary, the sum is 62; but if you add the square of Mary's age to the age of Tom, the result is 176. Can you say what are the ages of Tom and Mary?

22.—Mrs. Wilson's Family

Mrs. Wilson had three children, Edgar, James, and John. Their combined ages were half of hers. Five years later, during which time Ethel was born, Mrs. Wilson's age equalled the total of all her children's ages. Ten years more have now passed, Daisy appearing during that interval. At the latter event Edgar was as old as John and Ethel together. The combined ages of all the children are now double Mrs. Wilson's age, which is, in fact, only equal to that of Edgar and James together. Edgar's age also equals that of the two daughters. Can you find all their ages?

23.—De Morgan and Another

Augustus de Morgan, the mathematician, who died in 1871, used to boast that he was x years old in the year x^2. My living friend, Jasper Jenkins, wishing to improve on this, tells me he was a^2+b^2 in a^4+b^4; that he was $2m$ in the year $2m^2$; and that he was $3n$ years old in the year $3n^4$. Can you give the years in which De Morgan and Jenkins were respectively born?

24.—"Simple" Arithmetic

When visiting a lunatic asylum, I asked two inmates to give me their ages. They did so, and then, to test their arithmetical powers, I asked them to add the two ages together. One gave me 44 as the answer, and the other gave 1280. I immediately saw that the first had subtracted one age from the other, while the second person had multiplied them together. What were their ages?

CLOCK PUZZLES

25.—A Dreamland Clock

In a dream, I was travelling in a country where they had strange ways of doing things. One little incident was fresh in my memory when I awakened. I saw a clock and announced the time as it appeared to be indicated, but my guide corrected me. He said, "You are apparently not aware that the minute-hand always moves in the opposite direction to the hour-hand. Except for this improvement, our clocks are precisely the same as those you have been accustomed to." Now, as the hands were exactly together between the hours of four and five o'clock, and they started together at noon, what was the real time?

26.—What is the Time?

At what time are the two hands of a clock so situated that, reckoning as minute points past XII, one is exactly the square of the distance of the other?

27.—The First-born's Legacy

Mrs. Goodheart gave birth to twins. The clock showed clearly that Tommy was born about an hour later than Freddy. Mr. Goodheart, who died a few months earlier, had made a will leaving £8400, and had taken the precaution to provide for the possibility of there being twins. In such a case the money was to be divided in the following proportions: two-thirds to the widow, one-fifth to the first-born, one-tenth to the other twin, and one-twelfth to his brother. Now, what is the exact amount that should be settled on Freddy?

LOCOMOTION AND SPEED PUZZLES

28.—Hill Climbing

Weary Willie went up a certain hill at the rate of one and a half miles per hour and came down at the rate of four and a half miles per hour, so that it took him just six hours to make the double journey. Now, how far was it to the top of the hill?

29.—Timing the Motor-car

"I was walking along the road at three and a half miles an hour," said Mr. Pipkins, "when the motor-car dashed past me and only missed me by a few inches."

"Do you know at what speed it was going?" asked his friend.

"Well, from the moment it passed me to its disappearance round a corner I took twenty-seven steps, and walking on reached that corner with one hundred and thirty-five steps more."

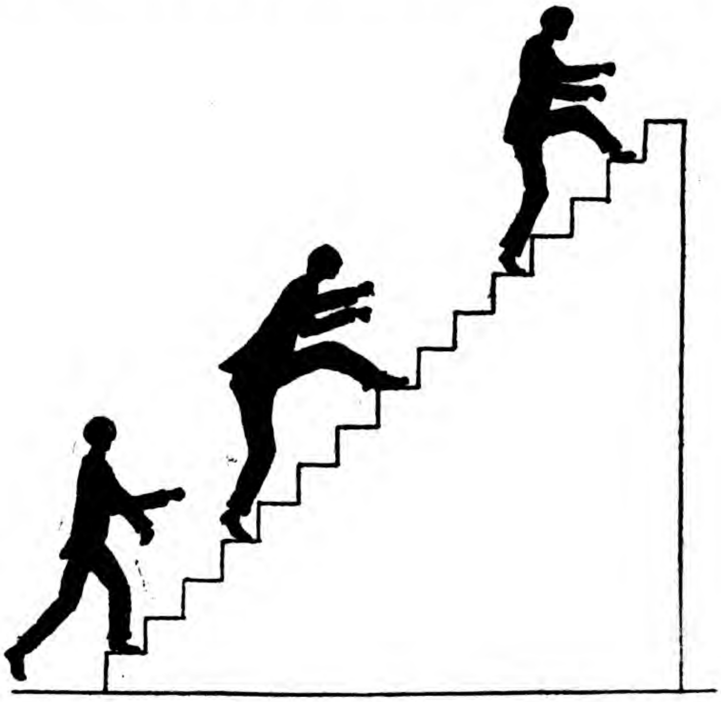
"Then, assuming that you walked, and the car ran, each at a uniform rate, we can easily work out the speed."

30.—The Staircase Race

This is a rough sketch of the finish of a race up a staircase in which three men took part. Ackworth, who is leading, went up three risers at a time, as arranged; Barnden, the second man, went four risers at a time, and Croft, who is last, went five at a time.

Undoubtedly Ackworth wins. But the point is, How many risers are there in the stairs, counting the top landing as a riser?

I have only shown the top of the stairs. There may be scores,



or hundreds, of risers below the line. It was not necessary to draw them, as I only wanted to show the finish. But it is possible to tell from the evidence the fewest possible risers in that staircase. Can you do it?

31.—A Walking Puzzle

A man set out at noon to walk from Appleminster to Boneyham, and a friend of his started at two p.m. on the same day to walk from Boneyham to Appleminster. They met on the road at five minutes past four o'clock and each man reached his destination at exactly the same time. Can you say at what time they both arrived?

32.—Riding in the Wind

A man on a bicycle rode a mile in 3 minutes with the wind at his back, but it took him 4 minutes to return against the wind. How long would it take him to ride a mile if there was no wind?

Some will say that the average of 3 and 4 is $3\frac{1}{2}$, and it would take him $3\frac{1}{2}$ minutes. That answer is entirely wrong.

33.—A Rowing Puzzle

A crew can row a certain course upstream in $8\frac{4}{7}$ minutes, and, if there were no stream, they could row it in 7 minutes less than it takes them to drift down the stream. How long would it take to row down with the stream?

34.—The Moving Stairway

On one of the moving stairways on the London Tube Railway I find that if I walk down twenty-six steps I require thirty seconds to get to the bottom, but if I make thirty-four steps I require only eighteen seconds to reach the bottom. What is the height of the stairway in steps? The time is measured from the moment the top step begins to descend to the time I step off the last step at the bottom on to the level platform.

35.—Sharing a Bicycle

Two brothers had to go a journey and arrive at the same time. They had only a single bicycle, which they rode in turns, each rider leaving it in the hedge when he dismounted for the one walking behind to pick up, and walking ahead himself, to be again overtaken. What was their best way of arranging their distances? As their walking and riding speeds were the same, it is extremely easy. Simply divide the route into any *even* number of equal stages and drop the bicycle at every stage, using the cyclometer. Each man would then walk half-way and ride half-way.

But here is a case that will require a little more thought. Anderson and Brown have to go twenty miles and arrive at exactly the same time. They have only one bicycle. Anderson can only walk four miles an hour, while Brown can walk five miles an hour, but Anderson can ride ten miles an hour to Brown's eight miles an hour. How are they to arrange the journey? Each man always either walks or rides at the speeds mentioned, without any rests.

36.—More Bicycling

Referring to the last puzzle, let us now consider the case where a third rider has to share the same bicycle. As a matter of fact, I understand that Anderson and Brown have taken a man named Carter into partnership, and the position to-day is this: Anderson, Brown, and Carter walk respectively four, five, and three miles per hour, and ride respectively ten, eight, and twelve miles per hour. How are they to use that single bicycle so that all shall complete the twenty miles' journey at the same time?

37.—A Side-car Problem

Atkins, Baldwin, and Clarke had to go a journey of fifty-two miles across country. Atkins had a motor-bicycle with side-car for one passenger. How was he to take one of his companions a certain distance, drop him on the road to walk the remainder of the way, and return to pick up the second friend, who, starting at the same time, was already walking on the road, so that they should all arrive at their destination at exactly the same time? The motor-bicycle could do twenty miles an hour, Baldwin could walk five miles an hour, and Clarke could walk four miles an hour. Of course, each went at his proper speed throughout and there was no waiting. I might have complicated the problem by giving more passengers, but I have purposely made it easy, and all the distances are an exact number of miles—without fractions.

38.—The Despatch-Rider

If an army forty miles long advances forty miles while a despatch-rider gallops from the rear to the front, delivers a despatch to the commanding general, and returns to the rear, how far has he to travel?

39.—The Two Trains

Two railway trains, one four hundred feet long and the other two hundred feet long, ran on parallel rails. It was found that when they went in opposite directions they passed each other in five seconds, but when they ran in the same direction the faster train would pass the other in fifteen seconds. Now, a curious passenger worked out from these facts the rate per hour at which each train ran. Can the reader discover the correct answer? Of course, each train ran with a uniform velocity.

40.—Pickleminster to Quickville

Two trains, A and B, leave Pickleminster for Quickville at the same time as two trains, C and D, leave Quickville for Pickleminster. A passes C 120 miles from Pickleminster and D 140 miles from Pickleminster. B passes C 126 miles from

Quickville and D half-way between Pickleminster and Quickville. Now, what is the distance from Pickleminster to Quickville? Every train runs uniformly at an ordinary rate.

41.—The Damaged Engine

We were going by train from Anglechester to Clinkerton, and an hour after starting some accident happened to the engine.

We had to continue the journey at three-fifths of the former speed, and it made us two hours late at Clinkerton, and the driver said that if only the accident had happened fifty miles farther on the train would have arrived forty minutes sooner. Can you tell from that statement just how far it is from Anglechester to Clinkerton?

42.—The Puzzle of the Runners

Two men ran a race round a circular course, going in opposite directions. Brown was the best runner and gave Tompkins a start of one-eighth of the distance. But Brown, with a contempt for his opponent, took things too easily at the beginning, and when he had run one-sixth of his distance he met Tompkins, and saw that his chance of winning the race was very small. How much faster than he went before must Brown now run in order to tie with his competitor? The puzzle is quite easy when once you have grasped its simple conditions.

43.—The Two Ships

A correspondent asks the following question. Two ships sail from one port to another—two hundred nautical miles—and return. The *Mary Jane* travels outwards at twelve miles an hour and returns at eight miles an hour, thus taking forty-one and two-third hours for the double journey. The *Elizabeth Ann* travels both ways at ten miles an hour, taking forty hours on the double journey. Now, seeing that both ships travel at the average speed of ten miles per hour, why does the *Mary Jane* take longer than the *Elizabeth Ann*? Perhaps the reader could explain this little paradox.

44.—Find the Distance

A man named Jones set out to walk from A—— to B——, and on the road he met his friend Kenward, ten miles from A——, who had left B—— at exactly the same time. Jones executed his commission at B—— and, without delay, set out on his return journey, while Kenward as promptly returned from A—— to B——. They met twelve miles from B——. Of course, each walked at a uniform rate throughout. Now, how far is A—— from B—— ?

I will show the reader a simple rule by which the distance may be found by anyone in a few seconds without the use of a pencil. In fact, it is quite absurdly easy—when you know how to do it.

45.—The Man and the Dog

" Yes ; when I take my dog for a walk," said a mathematical friend, " he frequently supplies me with some interesting puzzle to solve. One day, for example, he waited, as I left the door, to see which way I should go, and when I started he raced along to the end of the road, immediately returning to me ; again racing to the end of the road and again returning. He did this four times in all, at a uniform speed, and then ran at my side the remaining distance, which according to my paces measured 27 yards. I afterwards measured the distance from my door to the end of the road and found it to be 625 feet. Now, if I walk 4 miles per hour, what is the speed of my dog when racing to and fro ? "

46.—Baxter's Dog

This is an interesting companion to the " Man and Dog " puzzle. Anderson set off from an hotel at San Remo at nine o'clock and had been walking an hour when Baxter went after him along the same road. Baxter's dog started at the same time as his master and ran uniformly forwards and backwards between him and Anderson until the two men were together. Anderson's speed is two, Baxter's four, and the dog's ten miles an hour. How far had the dog run when Baxter overtook Anderson ? My correspondent in Italy who sends me this is an exact man, and he says, " Neglect length of dog and time spent in turning." I will merely add, neglect also the dog's name and the day of the month.

47.—The Runner's Refreshment

A man runs n times round a circular track whose radius is t miles. He drinks s quarts of beer for every mile that he runs. Prove that he will only need one quart !

48.—Railway Shunting

How are the two trains in our illustration to pass one another, and proceed with their engines in front? The small side-track is only large enough to hold one engine or one carriage at a time,

and no tricks, such as ropes and flying-switches, are allowed. Every reversal—that is, change of direction—of an engine is counted as a move in the solution. What is the smallest number of moves necessary?

49.—Exploring the Desert

Nine travellers, each possessing a motor-car, meet on the eastern edge of a desert. They wish to explore the interior, always going due west. Each car can travel forty miles on the contents of the engine tank, which holds a gallon of petrol, and each can carry nine extra gallon tins of petrol and no more. Unopened tins can alone be transferred from car to car. What is the greatest distance at which they can enter the desert without making any depots of petrol for the return journey?

50.—Exploring Mount Neverest

Professor Walkingholme, one of the exploring party, was allotted the special task of making a complete circuit of the base of the mountain at a certain level. The circuit was exactly a hundred miles in length and he had to do it all alone on foot. He could walk twenty miles a day, but he could only carry rations for two days at a time, the rations for each day being packed in sealed boxes for convenience in dumping. He walked his full twenty miles every day and consumed one day's ration as he walked. What is the shortest time in which he could complete the circuit? This simple question will be found to form one of the most fascinating puzzles that we have considered for some time. It made a considerable demand on Professor Walkingholme's well-known ingenuity. The idea was suggested to me by Mr. H. F. Heath.

DIGITAL PUZZLES

51.—An Exceptional Number

A number is formed of five successive digits (not necessarily in regular order) so that the number formed by the first two multiplied by the central digit will produce the number expressed by the last two. Thus, if it were 1 2 8 9 6, then 12 multiplied by 8 produces 96. But, unfortunately, 1, 2, 6, 8, 9 are not successive numbers, so it will not do.

52.—The Five Cards

I have five cards bearing the figures 1, 3, 5, 7, and 9. How can I arrange them in a row so that the number formed by the first pair multiplied by the number formed by the last pair, with the central number subtracted, will produce a number composed

of repetitions of one figure? Thus, in the example I have shown, 31 multiplied by 79 and 5 subtracted will produce 2 4 4 4, which would have been all right if that 2 had happened to be another 4. Of course, there must be two solutions, for the pairs are clearly interchangeable.

53.—Squares and Digits

What is the smallest square number that terminates with the greatest possible number of similar digits? Thus the greatest possible number might be five and the smallest square number with five similar digits at the end might be 24677777. But this is certainly not a square number. Of course, 0 is not to be regarded as a digit.

54.—The Two Additions

Can you arrange the following figures in two groups of four figures each so that each group shall add to the same sum?

1 2 3 4 5 7 8 9

If you were allowed to reverse the 9 so as to change it into the missing 6 it would be very easy. For example, 1, 2, 7, 8 and 3, 4, 5, 6 add up to 18 in both cases. But you are not allowed to make any such reversal.

55.—The Repeated Quartette

If we multiply 64253 by 365 we get the product 23452345, where the first four figures are repeated. What is the largest number that we can multiply by 365 in order to produce a similar product of eight figures with the first four figures repeated in the same order? There is no objection to a repetition of figures—that is, the four that are repeated need not be all different, as in the case shown.

56.—Easy Division

To divide the number 8,1 0 1,2 6 5,8 2 2,7 8 4 by 8, all we need do is to transfer the 8 from the beginning to the end! Can you find a number beginning with 7 that can be divided by 7 in the same simple manner?

57.—A Misunderstanding

An American correspondent asks me to find a number composed of any number of digits that may be correctly divided by 2 by simply transferring the last figure to the beginning. He has apparently come across our last puzzle with the conditions wrongly stated. If you are to transfer the *first* figure to the *end* it is solved by 3 1 5 7 8 9 4 7 3 6 8 4 2 1 0 5 2 6, and a solution may easily be found from this with any given figure at the beginning. But if the figure is to be moved from the *end* to the *beginning,* there is no possible solution for the divisor 2. But there is a solution for the divisor 3. Can you find it?

58.—The Two Fours

I am perpetually receiving inquiries about the old "Four Fours" puzzle. I published it in 1899, but have since found that it first appeared in the first volume of *Knowledge* (1881). It has since been dealt with at some length by various writers. The point is to express all possible whole numbers with four fours (no more and no fewer), using the various arithmetical signs. Thus $4\times4+\frac{4}{4}$ equals 17, and $44+4+\sqrt{4}$ equals 50. All numbers up to 112 inclusive may be solved, using only the signs for addition, subtraction, multiplication, division, square root, decimal points, and the factorial sign $\underline{4|}$ which means $1\times2\times3\times4$, or 24, but 113 is impossible.

It is necessary to discover which numbers can be formed with one four, with two fours, and with three fours, and to record these for combination as required. It is the failure to find some of these that leads to so much difficulty. For example, I think very few discover that 64 can be expressed with only two fours. Can the reader do it ?

59.—The Two Digits

Write down any two-figure number (different figures and no 0) and then express that number by writing the same figures in reverse order, with or without arithmetical signs. For example, $45=5\times 9$ would be correct if only the 9 had happened to be a 4. Or $81=(1+8)^2$ would do, except for the fact that it introduces a third figure—the 2.

60.—Digital Coincidences

If I multiply, and also add, 9 and 9 I get 81 and 18, which contain the same figures. If I multiply and add 2 and 47 I get 94 and 49—the same figures. If I multiply and add 3 and 24 I get the same figures—72 and 27. Can you find two numbers that when multiplied and added will, in this simple manner, produce the same *three* figures ? There are two cases.

61.—Palindromic Square Numbers

This is a curious subject for investigation—the search for square numbers the figures of which read backwards and forwards alike. Some of them are very easily found. For example, the squares of 1, 11, 111, and 1111 are respectively 1, 121, 12321, and 1234321, all palindromes, and the rule applies for any number of 1's provided the number does not contain more than nine. But there are other cases that we may call irregular, such as the square of 264=69696 and the square of 2285=5221225. Now, all the examples I have given contain an *odd* number of digits. Can the reader find a case where the square palindrome contains an *even* number of figures ?

62.—Factorizing

What are the factors (the numbers that will divide it without any remainder) of this number—1 0 0 0 0 0 0 0 0 0 0 0 1 ? This is easily done if you happen to know something about numbers of this peculiar form. In fact, it is just as easy for me to give two factors if you insert, say, one hundred and one noughts, instead of eleven, between the two ones. There is a curious, easy, and beautiful rule for these cases. Can you find it ?

63.—Find the Factors

Find two whole numbers with the smallest possible difference between them which, when multiplied together, will produce 1234567890.

64.—Dividing by Eleven

If the nine digits are written at haphazard in any order, for example, 4 1 2 5 3 9 7 6 8, what are the chances that the number that happens to be produced will be divisible by 11 without remainder? The number I have written at random is not, I see, so divisible, but if I had happened to make the 1 and the 8 change places it would be.

65.—Dividing by 37

I want to know whether the number 49,129,308,213 is exactly divisible by 37, or, if not, what is the remainder when so divided. How may I do this quite easily without any process of actual division whatever? It can be done by inspection in a few seconds—if you know how.

66.—Another 37 Division

Here is an interesting extension of the last puzzle. If the nine digits are written at haphazard in any order, for example, 412539768, what are the chances that the number that happens to be produced will be divisible by 37 without remainder?

67.—A Digital Difficulty

Arrange the ten digits, 1 2 3 4 5 6 7 8 9 0, in such order that they shall form a number that may be divided by every number from 2 to 18 without in any case a remainder. As an example, if I arrange them thus, 1 2 7 4 9 5 3 6 8 0, this number can be divided by 2, 3, 4, 5, and so on up to 16, without any remainder, but it breaks down at 17.

68.—Threes and Sevens

What is the smallest number composed only of the digits 3 and 7 that may be divided by 3 and by 7, and also the sum of its digits by 3 and by 7, without any remainder. Thus, for example, 7 7 3 3 7 3 3 is divisible by 3 and by 7 without remainder, but the sum of its digits (33), while divisible by 3, is not divisible by 7 without remainder. Therefore it is not a solution.

69.—Root Extraction

In a conversation I had with Professor Simon Greathead, the eminent mathematician, now living in retirement at Colney Hatch, I had occasion to refer to the extraction of the cube root. "Ah," said the professor, "it is astounding what ignorance prevails on that elementary matter! The world seems to have made little advance in the process of the extraction of roots since the primitive method of employing spades, forks, and trowels for the purpose. For example, nobody but myself has ever discovered the simple fact that, to extract the cube root of a number, all you have to do is to add together the digits. Thus, ignoring the obvious case of the number 1, if we want the cube root of 512, add the digits—8, and there you are!" I suggested that that was a special case. "Not at all," he replied. "Take another number at random—4913—and the digits add to 17, the cube of which is 4913." I did not presume to argue the point with the learned man, but I will just ask the reader to discover all the other numbers whose cube root is the same as the sum of their digits. They are so few that they can be counted on the fingers of one hand.

70.—The Solitary Seven

```
* * * ) * * * * * * * * * ( * 7 * * *
        * * * *
        -------
          * * *
          * * *
          -----
          * * * *
            * * *
          -------
              * * * *
              * * * *
              -------
```

Here is a little puzzle, sent me by the Rev. E. F. O. It is the first example I have seen of one of these missing-figure puzzles in which only one figure is given, and there appears to be only one possible solution. And, curiously enough, it is not difficult to reconstruct the simple division sum. For example, as the divisor when multiplied by 7 produces only three figures we know the first figure in the divisor must be 1. We can then prove that the first figure in the dividend must be 1, that, in consequence of bringing down together the last two figures of the dividend, the last but one figure in the quotient must be 0, that the first and last figures in the quotient must be greater than 7, because they each produce four figures in the sum, and so on.

71.—A Complete Skeleton

Here is an arrangement without any figure at all, constructed by Mr. A. Corrigan. Note the decimal dot in the quotient. The extension to four places of decimals makes it curiously easy to solve.

```
* * *)* * * * * * *(* * * * · * * * *
      * * *
      -----
        * * *
        * * *
        -----
          * * *
          * * *
          -----
            * * *
            * * *
            -----
              * * * *
              * * * *
              -------
```

72.—Alphabetical Sums

There is a family resemblance between puzzles where an arithmetical working has to be reconstructed from a few figures and a number of asterisks, and those in which every digit is represented by a letter, but they are really quite different. The resemblance lies in the similarity of the process of solving. Here is a little example of the latter class. It can hardly be called difficult.

```
P R ) M T V V R ( R S R
      M V R
      -----
        K K V
        K M D
        -----
          M V R
          M V R
          -----
```

Can you reconstruct this simple division sum? Every digit is represented by a different letter.

73.—Alphabetical Arithmetic

Here is a puzzle that will keep the reader agreeably employed for several minutes.

	F	G
Less A B multiplied by C =	D	E
Leaving	H	I

Each letter stands for a different figure (1, 2, 3, 4, 5, 6, 7, 8, or 9) and 0 is not allowed.

74.—Queer Division

The following is a rather curious puzzle in which we are restricted to the use of only two digits. Find the smallest number which when divided successively by 45, 454, 4545, and 45454 leaves the remainders 4, 45, 454, and 4545 respectively. This is perhaps not very easy but it affords a good arithmetical exercise.

75.—A Teasing Legacy

Professor Rackbrain left his typist what he called a trifle of a legacy if she was able to claim it. The legacy was the largest amount that she could find in an addition sum, where pounds, shillings, and pence were all represented and no digit used more than once. Every digit must be used once, a single nought may or may not appear, as in the examples below, and the dash may be employed in the manner shown.

£	s.	d.
–	3	7
–	4	8
–	5	9
1	6	–
£2	–	–

£	s.	d.
4	2	5
6	7	3
£10	9	8

The young lady was cleverer than he thought. What was the largest amount that she could claim?

76.—THE NINE VOLUMES

In a small bookcase were arranged nine volumes of some big work, numbered from 1 to 9 inclusive, on the three shelves exactly as shown in the illustration.

The nine digits express money value. You will see that they are so arranged that £26 5s. 9d. multiplied by 7 will produce £184 0s. 3d. Every digit represented once, and yet they form a correct sum in the multiplication of money.

But the blank shillings space in the bottom row is a slight defect, and I want to correct it. The puzzle is to use the multiplier 3, instead of 7, and get a correct result with the nine volumes, without any blank space; with pounds, shillings and pence all represented in both the top and bottom line.

77.—THE TEN VOLUMES

As an extension of the last puzzle, let us introduce a tenth volume marked 0. If we arrange the ten volumes as follows, we get a sum of money correctly multiplied by 2.

54	3	9
		2
108	7	6

Can you do the same thing with the multiplier 4 so that the nine digits and 0 are all represented, once and once only?

VARIOUS ARITHMETICAL AND ALGEBRAICAL PROBLEMS

78.—THE MILLER'S TOLL

Here is a very simple puzzle, yet I have seen people perplexed by it for a few minutes. A miller was accustomed to take as toll one-tenth of the flour that he ground for his customers. How much did he grind for a man who had just one bushel after the toll had been taken?

79.—Egg Laying

The following is a new variation of an old friend. Though it looks rather complicated and difficult, it is absurdly easy if properly attacked. If a hen and a half lays an egg and a half in a day and a half, how many and a half who lay better by half will lay half a score and a half in a week and a half?

80.—The Flocks of Sheep

Four brothers were comparing the number of sheep that they owned. It was found that Claude had ten more sheep than Dan. If Claude gave a quarter of his sheep to Ben, then Claude and Adam would together have the same number as Ben and Dan together. If, then, Adam gave one-third to Ben, and Ben gave a quarter of what he then held to Claude, who then passed on a fifth of his holding to Dan, and Ben then divided one-quarter of the number he then possessed equally amongst Adam, Claude, and Dan, they would all have an equal number of sheep. How many sheep did each son possess?

81.—Pussy and the Mouse

"There's a mouse in one of these barrels," said the dog.

"Which barrel?" asked the cat.

"Why, the five-hundredth barrel."

"What do you mean by the five-hundredth? There are only five barrels in all."

"It's the five-hundredth if you count backwards and forwards in this way."

And the dog explained that you count like this:

1	2	3	4	5
9	8	7	6	
	10	11	12	13

So that the seventh barrel would be the one marked 3 and the twelfth barrel the one numbered 4.

"That will take some time," said the cat, and she began a laborious count. Several times she made a slip, and had to begin again.

"Rats!" exclaimed the dog. "Hurry up or you will be too late!"

"Confound you! You've put me out again, and I must make a fresh start."

Meanwhile the mouse, overhearing the conversation, was working madly at enlarging a hole, and just succeeded in escaping as the cat leapt into the correct barrel.

"I knew you would lose it," said the dog. "Your education has been sadly neglected. A certain amount of arithmetic is

necessary to every cat, as it is to every dog. Bless me! Even some snakes are adders!"

Now, which was the five-hundredth barrel? Can you find a quick way of arriving at the answer without making the actual count?

82.—Army Figures

A certain division in an army was composed of a little over twenty thousand men, made up of five brigades. It was known that one-third of the first brigade, two-sevenths of the second brigade, seven-twelfths of the third, nine-thirteenths of the fourth, and fifteen-twenty-seconds of the fifth brigade happened in every case to be the same number of men. Can you discover how many men there were in every brigade?

83.—A Critical Vote

A meeting of the Amalgamated Society of Itinerant Askers (better known as the "Tramps' Union") was held to decide whether the members should strike for reduced hours and larger donations. It was arranged that during the count those in favour of the motion should remain standing, and those who voted against should sit down.

"Gentlemen," said the chairman in due course, "I have the pleasure to announce that the motion is carried by a majority equal to exactly a quarter of the opposition." (Loud cheers.)

"Excuse me, guv'nor," shouted a man at the back, "but some of us over here couldn't sit down."

"Why not?"

"'Cause there ain't enough chairs."

"Then perhaps those who wanted to sit down but couldn't will hold up their hands. . . . I find there are a dozen of you, so the motion is lost by a majority of one." (Hisses and disorder.)

Now, how many members voted at that meeting?

84.—The Three Brothers

The discussion arose before one of the tribunals as to which of a tradesman's three sons could best be spared for service in the Army. "All I know as to their capacities," said the father, "is this: Arthur and Benjamin can do a certain quantity of work in eight days, which Arthur and Charles will do in nine days, and which Benjamin and Charles will take ten days over." Of course, it was at once seen that as longer time was taken over the job whenever Charles was one of the pair, he must be the slowest worker. This was all they wanted to know, but it is an interesting puzzle to ascertain just how long each son would require to do that job alone. Can you discover?

85.—The House Number

A man said the house of his friend was in a long street, numbered on his side one, two, three, and so on, and that all the numbers on one side of him added up exactly the same as all the numbers on the other side of him. He said he knew there were more than fifty houses on that side of the street, but not so many as five hundred.

Can you discover the number of that house?

86.—A New Street Puzzle

Brown lived in a street which contained more than twenty houses, but fewer than five hundred, all numbered one, two, three, four, etc., throughout. Brown discovered that all the numbers from one upwards to his own number inclusive summed to exactly half the sum of all the numbers from one up to, and including, the last house. Now what was the number of his house?

87.—Another Street Puzzle

A long street in Brussels has all the odd numbers of the houses on one side and all the even numbers on the other—a method of street numbering quite common in our own country. (1) If a man lives in an odd-numbered house and all the numbers on one side of him, added together, equal the numbers on the other side, how many houses are there, and what is the number of his house? (2) If a man lives on the even side and all the numbers on one side of him equal those on the other side, how many houses are there, and what is his number? We will assume that there are more than fifty houses on each side of the street and fewer than five hundred.

88.—Correcting an Error

Hilda Wilson was given a certain number to multiply by 409, but she made a blunder that is very common with children when learning the elements of simple arithmetic: she placed the first figure of her product by 4 below the second figure from the right instead of below the third. We have all done that as youngsters when there has happened to be a 0 in the multiplier. The result of Hilda's mistake was that her answer was wrong by 328,320, entirely in consequence of that little slip. Now, what was the multiplicand—the number she was given to multiply by 409?

89.—The Seventeen Horses

"I suppose you all know this old puzzle," said Jeffries. "A farmer left seventeen horses to be divided amongst his three sons in the following proportions: To the eldest, one-half; to the second, one-third; and to the youngest, one-ninth. How should they be divided?"

"Yes; I think we all know that," said Robinson, "but it can't be done. The answer always given is a fallacy."

"I suppose you mean," Prodgers suggested, "the answer where they borrow another chap's horse to make eighteen for the purpose of the division. Then the three sons take 9, 6, and 2 respectively, and return the borrowed horse to the lender."

"Exactly," replied Robinson, and each son receives more than his share."

"Stop!" cried Benson. "That can't be right. If each man received more than his share the total must exceed seventeen horses, but 9, 6, and 2 add up to 17 correctly."

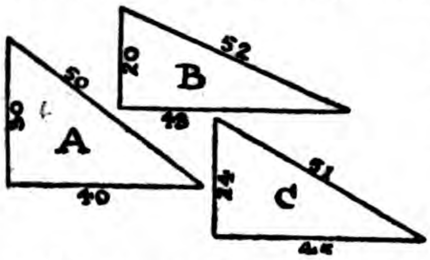
"At first sight that certainly looks queer," Robinson admitted, " but the explanation is that if each man received his true fractional share, these fractions would add up to less than seventeen. In fact, there would be a fraction left undistributed. The thing can't really be done."

" That's just where you are all wrong," said Jeffries. " The terms of the will can be exactly carried out, without any mutilation of a horse."

To their astonishment, he showed how it was possible. How should the horses be divided in strict accordance with the directions ?

90.—Equal Perimeters

Rational right-angled triangles have been a fascinating subject for study since the time of Pythagoras, before the Christian era. Every schoolboy knows that the sides of these,



generally expressed in whole numbers, are such that the square of the hypotenuse is exactly equal to the sum of the squares of the other two sides. Thus, in the case of Diagram A, the square of 30 (900), added to the square of 40 (1600), is the square of 50 (2500), and similarly with B and C. It will be found that the three triangles shown have each the same perimeter. That is the three sides in every case add up to 120. Now, can you find six rational right-angled triangles each with a common perimeter, and the smallest possible ? It is not a difficult puzzle like my " Four Princes " (in the *Canterbury Puzzles*), in which you had to find four such triangles of equal *area*.

91.—Counting the Wounded

When visiting with a friend one of our hospitals for wounded soldiers, I was informed that exactly two-thirds of the men had lost an eye, three-fourths had lost an arm, and four-fifths had lost a leg. " Then," I remarked to my friend," it follows that at least twenty-six of the men must have lost all three—an eye, an arm, and a leg." That being so, can you say exactly how many men were in the hospital ? It is a very simple calculation, but I have no doubt it will perplex a good many readers.

92.—A Cow's Progeny

"Supposing," said my friend Farmer Hodge, "that cow of mine to have a she-calf at the age of two years, and supposing she goes on having the like every year, and supposing every one of her young to have a she-calf at the age of two years, and afterwards every year likewise, and so on. Now, how many do you suppose would spring from that cow and all her descendants in the space of twenty-five years?" I understood from Hodge that we are to count from the birth of the original cow, and it is obvious that the family can produce no feminine beef or veal during the period stated.

93.—Sum Equals Product

"This is a curious thing," a man said to me. "There are two numbers whose sum equals their product. That is, they give the same result whether you add them together or multiply them together. They are 2 and 2, for if you add them or multiply them, the result is 4." Then he tripped badly, for he added, "These are, I find, the only two numbers that have this peculiarity."

I asked him to write down any number, as large as he liked, and I would immediately give him another that would give a like result by addition or multiplication. He selected the number 987654321, and I promptly wrote down the second number. What was it? The fact is, no matter what number you may select there is always another to which that peculiarity applies in combination with it. If this is new to the reader it cannot fail to be interesting to him. He should try to find the rule.

94.—Adding their Cubes

The numbers 407 and 370 have this peculiarity, that they exactly equal the sum of the cubes of their digits. Thus the cube of 4 is 64, the cube of 0 is 0, and the cube of 7 is 343. Add together 64, 0, and 343, and you get 407. Again, the cube of 3 (27), added to the cube of 7 (343), is 370. Can you find a number not containing a nought that will work in the same way? Of course, we bar the absurd case of 1.

95.—Squares and Cubes

Can you find two whole numbers, such that the difference of their squares is a cube and the difference of their cubes is a square? What is the answer in the smallest possible numbers?

96.—Concerning a Cube

What is the length in feet of the side of a cube when (1) the superficial area equals the cubical contents; (2) when the superficial area equals the square of the cubical contents; (3) when the square of the superficial area equals the cubical contents?

97.—A Common Divisor

Here is a puzzle that has been the subject of frequent inquiries by correspondents, only, of course, the actual figures are varied considerably. A country newspaper stated that many schoolmasters have suffered in health in their attempts to master it! Perhaps this is merely a little journalistic exaggeration, for it is really a very simple question if only you have the cunning to hit on the method of attacking it. This is the question: Find a common divisor for the three numbers, 480,608, 508,811, and 723,217, so that the remainder shall be the same in every case.

98.—Curious Multiplication

I have frequently been asked to explain the following, which will doubtless interest many readers who have not seen it. If a person can add correctly but is incapable of multiplying or dividing by a number higher than 2, it is possible to obtain the product of any two numbers in this curious way. Multiply 97 by 23.

97	23
48	(46)
24	(92)
12	(184)
6	(368)
3	736
1	1472
	2231

In the first column we divide by 2, rejecting the remainders, until 1 is reached. In the second column we multiply 23 by 2 the same number of times. If we now strike out those products that are opposite to the even numbers in the first column (we have enclosed these in brackets for convenience in printing) and add up the remaining numbers we get 2231, which is the correct answer. Why is this?

99.—The Rejected Gun

Here is a little military puzzle that may not give you a moment's difficulty. It is such a simple question that a child can understand it and no knowledge of artillery is required. Yet some of my readers may find themselves perplexed for quite five minutes. An inventor offered a new large gun to the committee appointed by our Government for the consideration of such things. He declared that when once loaded it would fire sixty shots at the rate of a shot a minute. The War Office put it to the test and found that it fired sixty shots an hour, but declined it, "as it did not fulfil the promised condition." "Absurd," said the inventor, "for you have shown that it clearly does all that we undertook it should do." "Nothing of the sort," said the experts. "It has failed." Now, can you explain this extraordinary mystery? Was the inventor, or were the experts, right?

100.—Odds and Evens

Here is a little parlour trick, the explanation of which is quite easy. Ask a friend to take an even number of coins in one hand and an odd number in the other. You then undertake to tell him which hand holds the odd and which the even. Tell him to multiply the number in the right hand by 7 and the number in the left by 6, add the two products together, and tell you the result. You can then immediately give him the required answer. How are you to do it? In practice, after he had given me the result, I should say, "Let us add your age, now deduct the day of the month," and so on—a lot of absurd apparent calculations merely devised for his mystification, for you already know the answer.

101.—Twenty Questions

I am reminded of an interesting old game I used to play as a bachelor. Somebody thinks of an object—say Big Ben, or the knocker on the front door, or the gong of the clock in the next room, or the top button of his friend's coat, or Mr. Baldwin's tobacco-pipe. You have then to discover the object, by putting not more than twenty questions, each of which must be answered by " yes " or " no." You have to word your questions discreetly, because if you ask, for example, " Is it animal, vegetable, or mineral ? " you might get the unsatisfactory answer " Yes," and so waste a question. We found that the expert rarely failed to get an exact solution, and I have known some most remarkably difficult cases solved by the twenty questions.

A novel limitation of the game is suggested to me, which will call for some ingenuity, and the puzzle will doubtless be attacked in various ways by different persons. It is simply this. I think of a number containing six figures. Can you discover what it is by putting to me twenty questions, each of which can only be answered by " yes " or " no " ? After the twentieth question you must give the number.

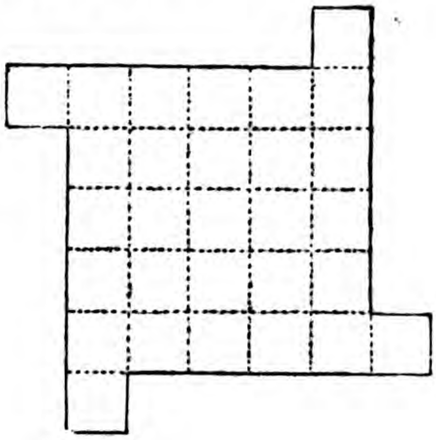
102.—The Nine Barrels

In how many different ways may these nine barrels be arranged in three tiers of three so that no barrel shall have a smaller number than its own below it or to the right of it ? The first correct arrangement that will occur to you is 1 2 3 at the top, then 4 5 6 in the second row, and 7 8 9 at the bottom, and my sketch gives a second arrangement. How many are there altogether ?

GEOMETRICAL PROBLEMS

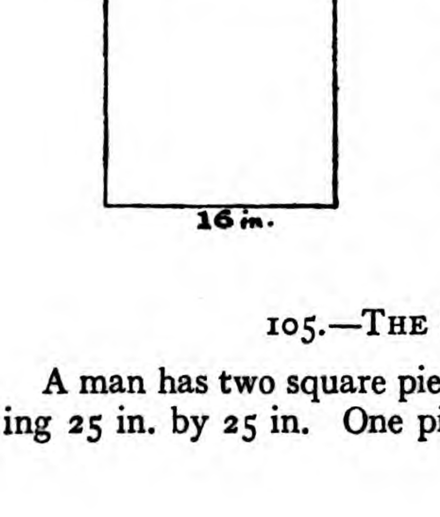
DISSECTION PUZZLES

103.—A New Cutting-out Puzzle



Cut the figure into four pieces that will fit together and form a square.

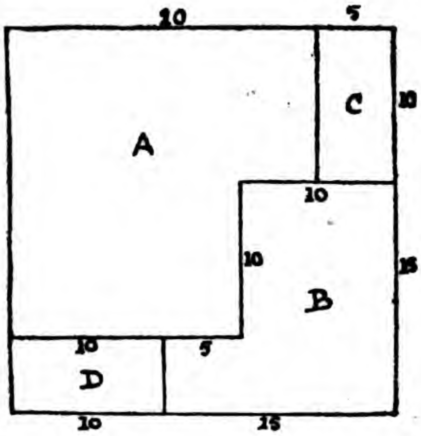
104.—The Square Table-top



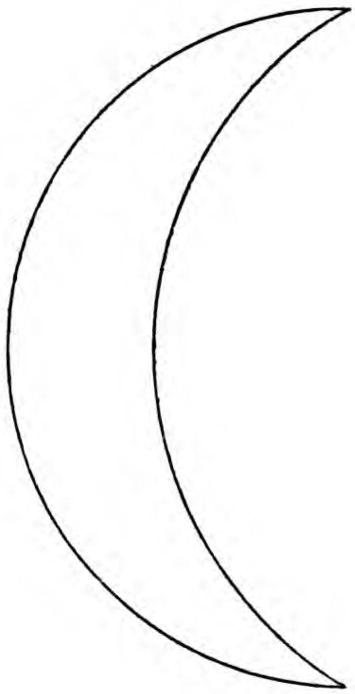
A man had three pieces of beautiful wood, measuring 12 in., 15 in., and 16 in. square respectively. He wanted to cut these into the fewest pieces possible that would fit together and form a small square table-top 25 in. by 25 in. How was he to do it? I have found several easy solutions in six pieces, very pretty, but have failed to do it in five pieces. Perhaps the latter is not possible. I know it will interest my readers to examine the question.

105.—The Squares of Veneer

A man has two square pieces of valuable veneer, each measuring 25 in. by 25 in. One piece he cut, in the manner shown in

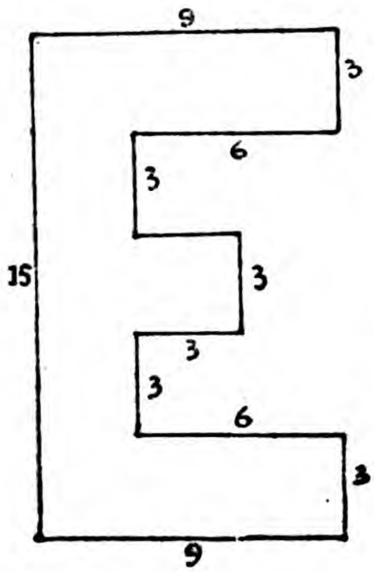


our illustration, in four parts that will form two squares, one 20 in. by 20 in. and the other 15 in. by 15 in. Simply join C to A and D to B. How is he to cut the other square into four pieces that will form again two other squares with sides in exact inches, but not 20 and 15 as before?



106.—Dissecting the Moon

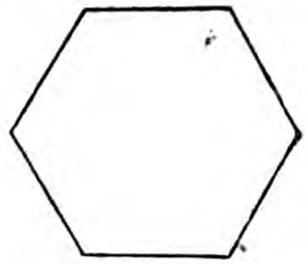
In how large a number of pieces can this crescent moon be cut with five straight cuts of the knife? The pieces may not be piled or shifted after a cut.



107.—Dissecting the Letter E

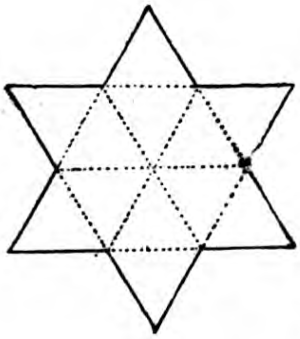
Can you cut this letter E into only five pieces so that they will fit together to form a perfect square? I have given all the measurements in inches so that there should be no doubt as to the correct proportions of the letter. In this case you are not allowed to turn over any piece.

108.—Hexagon to Square



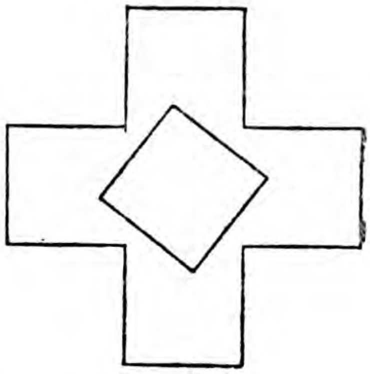
Can you cut this perfect hexagon into five pieces that will fit together and form a square?

109.—Squaring a Star



This six-pointed star can be cut into as few as five pieces that will fit together and form a perfect square. To perform the feat in seven pieces is quite easy, but to do it in five is more difficult. I introduce the dotted lines merely to show the true proportions of the star, which is thus built up of twelve equilateral triangles.

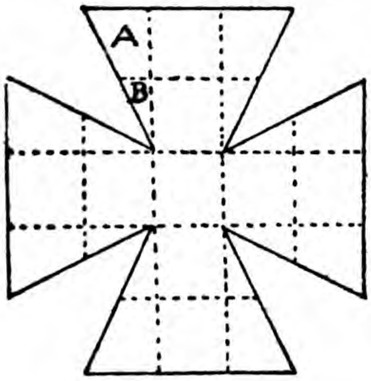
110.—The Mutilated Cross



Here is a symmetrical Greek Cross from which has been cut a square piece exactly equal to one of the arms of the cross. The puzzle is to cut what remains into four pieces that will fit together and form a square. This is a pleasing but particularly easy cutting-out puzzle.

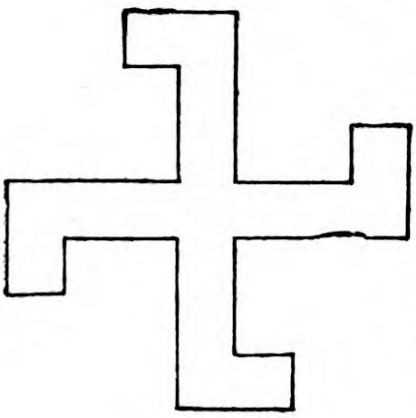
111.—The Victoria Cross

We have shown elsewhere how innumerable puzzles may be devised on the Greek, St. George, or Red Cross, so familiar to us all, composed as it is of five equal squares assembled together. Let us now do homage to the Maltese or Victoria Cross. Cut the cross shown into seven pieces that will fit together and form a perfect square. Of course, there must be no trickery or waste of material.



In order that the reader may have no doubt as to the exact proportions of the cross as given, I have inserted the dotted lines. As the pieces A and B will fit together to form one of those little squares, it is clear that the area of the cross is equal to seventeen such squares.

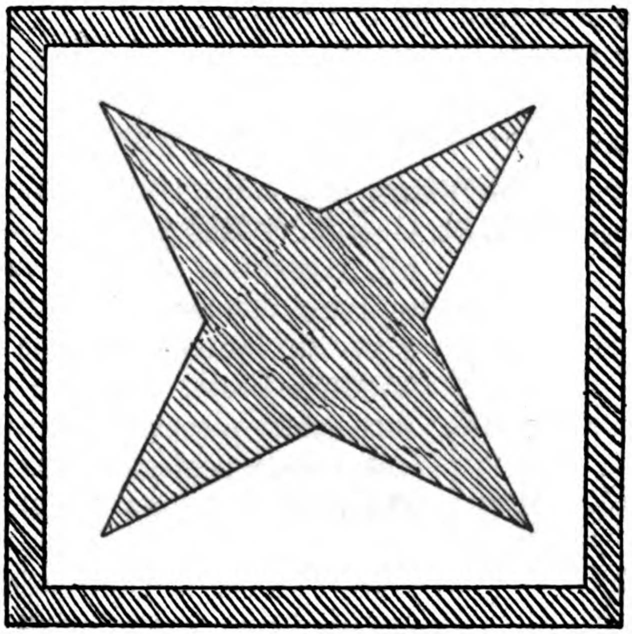
112.—Squaring the Swastika



Cut out the swastika and then cut it up into four pieces that will fit together and form a square. There can be no question as to the proportions of the figure if we regard it as built up of seventeen equal squares. You can divide it up into these seventeen squares with your pencil without difficulty. Now try to cut it into four pieces to form a single square.

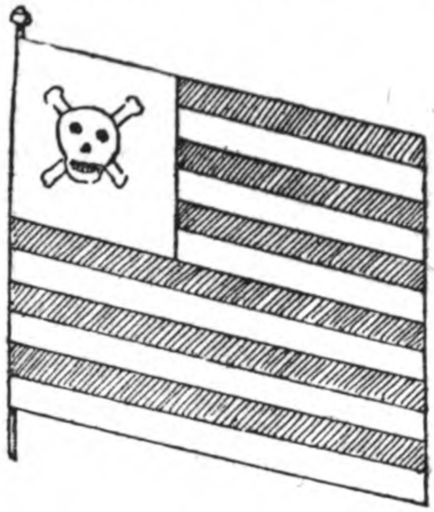
113.—The Maltese Cross

Can you cut the star into four pieces and place them inside the frame so as to show a perfect Maltese Cross?



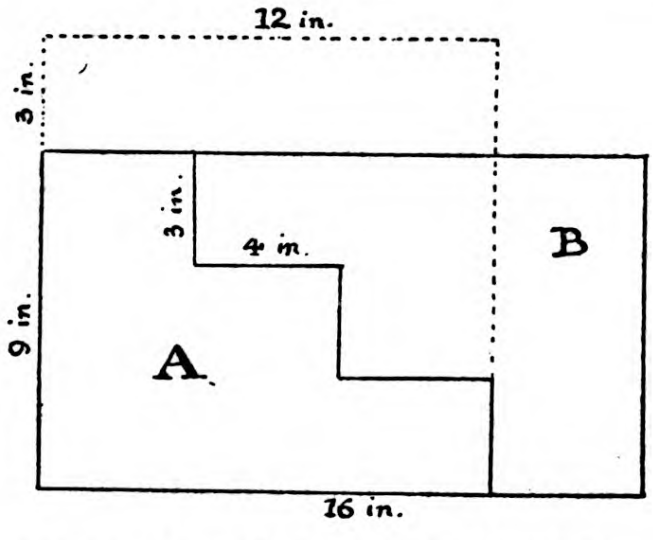
114.—The Pirates' Flag

Here is a flag taken from a band of pirates on the high seas. The twelve stripes represented the number of men in the band, and when a new man was admitted or dropped out a new stripe was added or one removed, as the case might be. Can you discover how the flag should be cut into as few pieces as possible so that they may be put together again and show only ten stripes? No part of the material may be wasted, and the flag must retain its oblong shape.



115.—The Carpenter's Puzzle

Here is a well-known puzzle, given in all the old books. A ship's carpenter had to stop a hole twelve inches square, and the only piece of wood that was available measured 9 in. in breadth

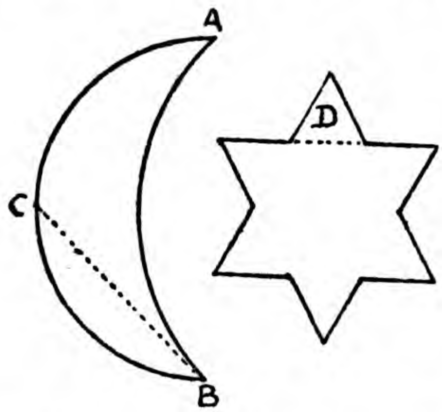


by 16 in. in length. How did he cut it into only two pieces that would exactly fit the hole? The answer is based on what I call the "step principle," as shown in the diagram. If you move the piece marked B up one step to the left, it will exactly fit on A and form a perfect square measuring twelve inches on every side.

This is very simple and obvious. But nobody has ever attempted to explain the general law of the thing. As a consequence, the notion seems to have got abroad that the method will apply to any rectangle where the proportion of length to breadth is within reasonable limits. This is not so, and I have had to expose some bad blunders in the case of published puzzles that were supposed to be solved by an application of this step principle, but were really impossible of solution. Let the reader take different measurements, instead of 9 in. by 16 in., and see if he can find other cases in which this trick will work in two pieces and form a perfect square.

116.—The Crescent and the Star

Here is a little puzzle on the Crescent and the Star. Look at the illustration, and see if you can determine which is the larger, the Crescent or the Star. If both were cut out of a sheet of solid gold, which would be the more valuable? As it is very difficult to guess by the eye, I will state that the outer arc, A C B, is a semicircle; the radius of the inner arc is equal to the straight line B C; the distance in a straight line from A to B is twelve inches; and the point of the star, D, contains three square inches. Now it is quite easy to settle the matter at a glance—when you know how.

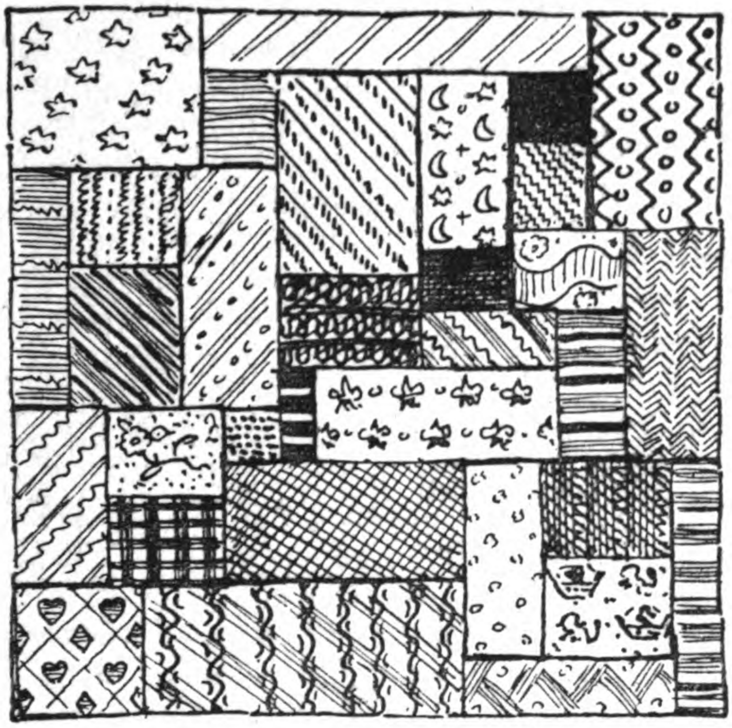


PATCHWORK PUZZLES

117.—The Patchwork Quilt

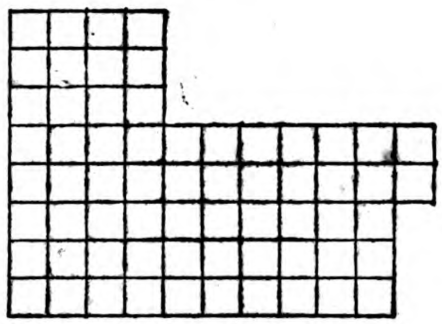
Here is a patchwork quilt that was produced by two young ladies for some charitable purpose. When they came to join their work it was found that each lady had contributed a portion of exactly the same size and shape. It is an amusing puzzle to discover just where these two portions are joined together. Can you divide the quilt into two parts, simply by cutting the stitches, so that the portions shall be of the same size and shape?

You may think you have solved it in a few minutes, but—wait and see!



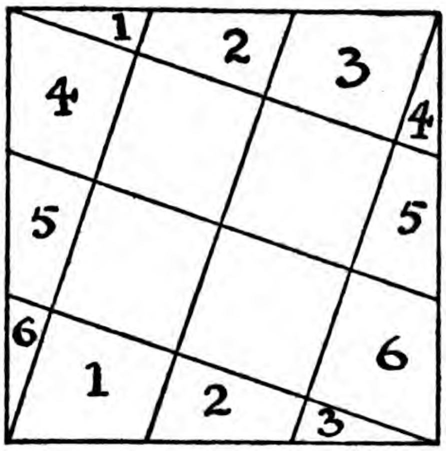
118.—THE IMPROVISED DRAUGHTS-BOARD

Some Englishmen at the front during the Great War wished to pass a restful hour at a game of draughts. They had coins and small stones for the men, but no board. However, one of them found a piece of linoleum as shown in the illustration, and, as it contained the right number of squares, it was decided to cut it and fit the pieces together to form a board, blacking some of the squares afterwards for convenience in playing. An ingenious Scotsman showed how this could be done by cutting the stuff in two pieces only, and it is a really good puzzle to discover how he did it. Cut the linoleum along the lines into two pieces that will fit together and form the board, eight by eight.



119.—Tessellated Pavements

The reader must often have noticed, in looking at tessellated pavements and elsewhere, that a square space had sometimes to be covered with square tiles under such conditions that a certain number of the tiles have to be cut in two parts. A familiar example is shown in our illustration, where a square has been formed with ten square tiles. As ten is not a square number a certain number of tiles must be cut. In this case it is six. It will be seen that the pieces 1 and 1 are cut from one tile, 2 and 2 from another, and so on.



Now, if you had to cover a square space with exactly twenty-nine square tiles of equal size, how would you do it? What is the smallest number of tiles that you need cut in two parts?

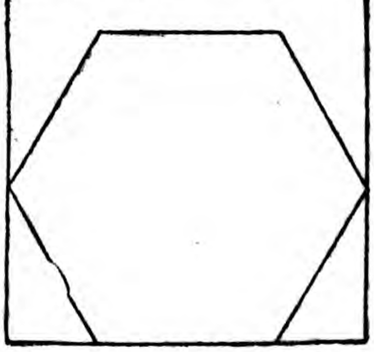
PAPER-FOLDING PUZZLES

120.—The Ribbon Pentagon

The solution to the following will be found very interesting if the reader has not seen it before. I want to form a regular pentagon, but the only thing at hand happens to be a rectangular strip of paper. How am I to do it without pencil, compasses, scissors, or anything else whatever but my fingers?

121.—Paper Folding

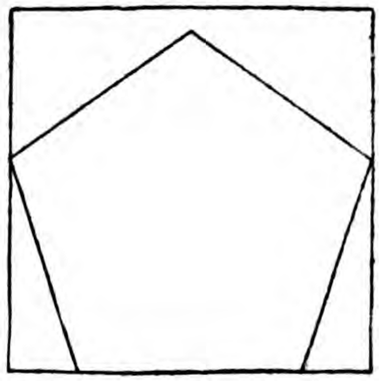
This is a branch of puzzledom both instructive and interesting. I do not refer to folding paper into the forms of boxes, boats, frogs, and such things, for these are toys rather than puzzles, but to the solving of certain geometrical problems with paper and fingers alone. I will give a comparatively easy example. Suppose you are given a perfectly square piece of paper, how are you going to fold it so as to



indicate by creases a regular hexagon, as shown in the illustration, all ready to be cut out? Of course, you must use no pencil, measure, or instrument of any kind whatever. The hexagon may be in any position in the square.

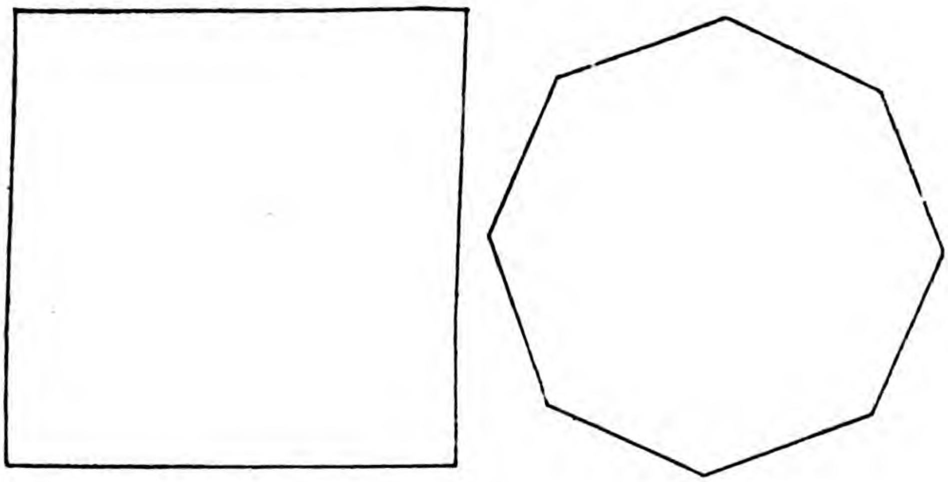
122.—Folding a Pentagon

Here is another puzzle in paper folding of a rather more difficult character than the hexagon example that we have considered. If you are given a perfectly square piece of paper, how are you to fold it so as to indicate by creases a regular pentagon, as in our illustration, all ready to be cut out? Remember that you must use your fingers alone, without any instrument or measure whatever.



123.—Making an Octagon

Can you cut the regular octagon from a square piece of paper without using compasses or ruler, or anything but scissors? You can fold the paper so as to make creases.



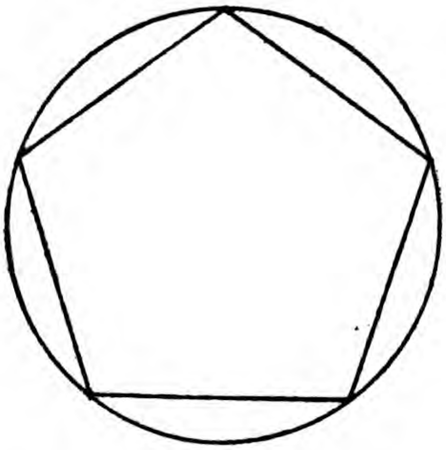
VARIOUS GEOMETRICAL PUZZLES

124.—Drawing a Straight Line

If we want to describe a circle we use an instrument that we call a pair of compasses, but if we need a straight line we use no such instrument—we employ a ruler or other straight edge. In other words, we first seek a straight line to produce our required straight line, which is equivalent to using a coin, saucer, or other circular object to draw a circle. Now, imagine yourself to be in such a position that you cannot obtain a straight edge—not even a piece of thread. Could you devise a simple instrument that would draw your straight line, just as the compasses describe a circle? It is an interesting abstract question, but, of course, of no practical value. We shall continue to use the straight edge.

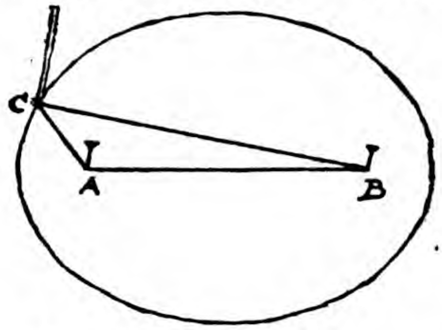
125.—Making a Pentagon

"I am about to start on making a silk patchwork quilt," said a lady, "all composed of pieces in the form of a pentagon. How am I to cut out a true pentagon in cardboard, the sides of which must measure exactly an inch? Of course, I can draw a circle, and then by trial with the compass find five points equidistant on the circumference" (see the illustration), "but unless I know the correct size of my circle the pentagon is just as it happens, and the sides are always a little more, or a little less, than an exact inch." Could you show her a simple and direct way of doing it without any trial?



126.—Drawing an Oval

I suppose a large proportion of my readers are familiar with this trick for drawing an oval. It is very useful if you have to cut a mount for a portrait or to make an oval flower-bed. You drive in two pins or nails (or, in the case of the flower-bed, two stakes) and enclose them with an endless band of thread or string, as shown in our diagram, where the pins are at A and B, and the pencil point, at C, stretches the loop of thread. If you keep the thread taut and pass the



pencil all round until you come back to the starting-point you will describe the perfect oval shown.

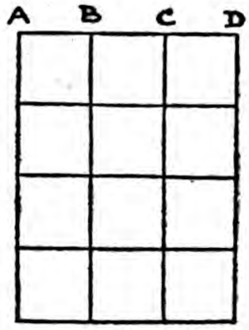
But I have sometimes heard the complaint that the method is too haphazard: that it was only by a lot of trials that you can draw an oval of the exact dimensions required. This is a delusion, and it will make an interesting little puzzle to show at what distance apart the pins should be placed, and what length the string should be, to draw an oval, say, twelve inches in length by eight inches in breadth. Can you discover the very simple rule for doing this?

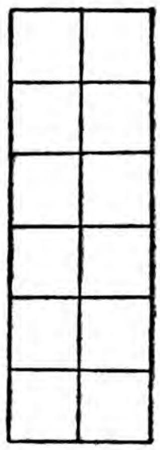
127.—With Compasses only

Can you show how to mark off the four corners of a square, using the compasses only? You simply use a sheet of paper and the compasses, and there is no trick, such as folding the paper.

128.—Lines and Squares

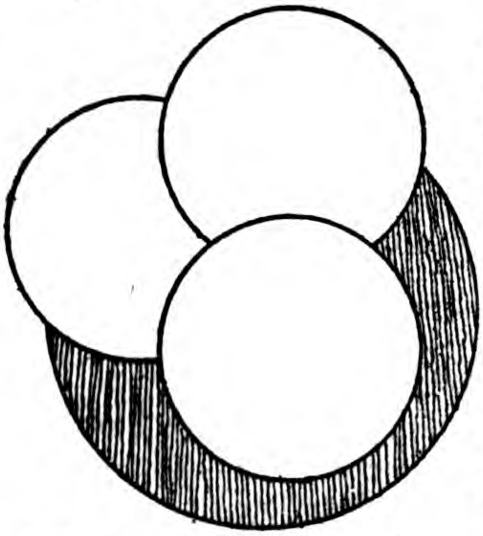
Here is a simple question. With how few straight lines can you make exactly one hundred squares? Thus, in the first diagram it will be found that with nine straight lines I have made twenty squares (twelve with sides of the length A B, six with sides A C, and two with sides of the length A D). In the second diagram, although I use one more line, I only get seventeen squares. So, you see, everything depends on how the lines are drawn. Remember there must be exactly one hundred squares—neither more nor fewer.





129.—The Circle and Discs

During a recent visit to a fair we saw a man with a table, on the oil-cloth covering of which was painted a large red circle, and he invited the public to cover this circle entirely with five tin discs which he provided, and offered a substantial prize to anybody who was successful. The circular discs were all of the same size, and each, of course, smaller than the red circle. The diagram, where three discs are shown placed, will make everything clear.



He showed that it was "quite easy when you know how" by covering up the circle himself without any apparent difficulty, but many tried over and over again and failed every time. I should explain that it was a condition that when once you had placed any disc you were not allowed to shift it, otherwise, by sliding them about after they had been placed, it might be tolerably easy to do.

Let us assume that the red circle is six inches in diameter. Now, what is the smallest possible diameter (say, to the nearest half-inch) for the five discs in order to make a solution possible?

130.—Mr. Grindle's Garden

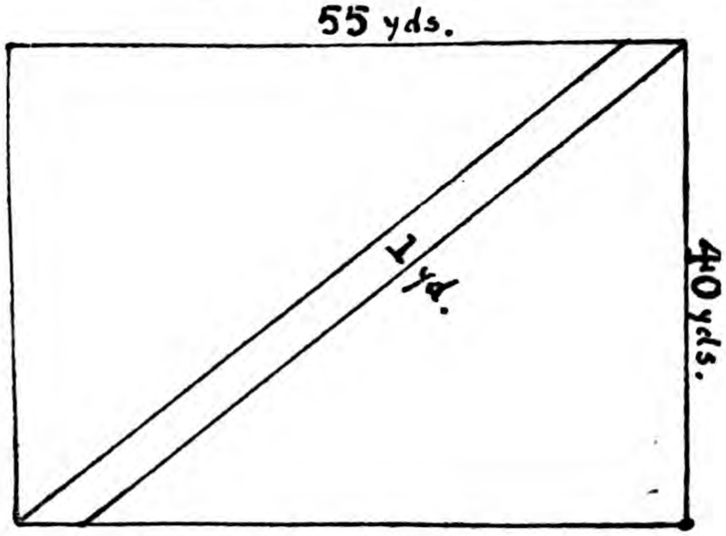
"My neighbour," said Mr. Grindle, "generously offered me, for a garden, as much land as I could enclose with four straight walls measuring 7, 8, 9, and 10 rods in length respectively."

"And what was the largest area you were able to enclose?" asked his friend.

Perhaps the reader can discover Mr. Grindle's correct answer. You see, in the case of three sides the triangle can only enclose one area, but with four sides it is quite different. For example, it is obvious that the area of Diagram A is greater than that of B, though the sides are the same.

131.—The Garden Path

This is an old puzzle that I find frequently cropping up. Many find it perplexing, but it is easier than it looks. A man has a rectangular garden, 55 yds. by 40 yds., and he makes a diagonal path, one yard wide, exactly in the manner indicated in the diagram. What is the area of the path? Dimensions for the garden are generally given that only admit of an approximate answer, but I select figures that will give an answer that is quite



exact. The width of the path is exaggerated in the diagram for the sake of clearness.

132.—The Garden Bed

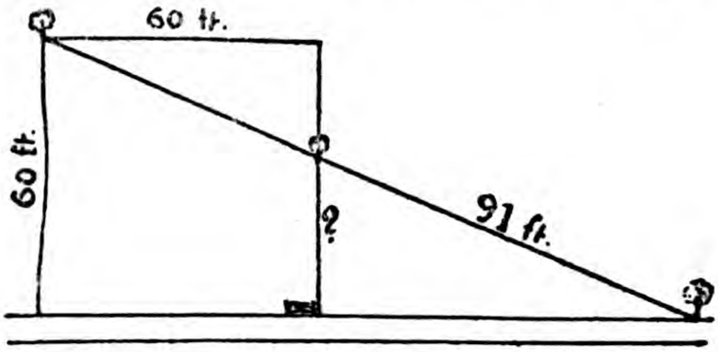
Here is quite a simple little puzzle. A man has a triangular lawn of the proportions shown, and he wants to make the largest possible rectangular flower-bed without enclosing the tree. How is he to do it? This will serve to teach the uninitiated a simple rule that may prove useful on occasion. For example, it would equally apply to the case of a carpenter who had a triangular board and wished to cut out the largest possible rectangular table-top without including a bad knot in the wood.

133.—A Problem for Surveyors

There are tricks in every trade, and the science of numbers contains an infinite number of them. In nearly every vocation of life there are little wrinkles and short cuts that are most useful when known. For example, a man bought a little field, and here is the scale map (one inch to the rod) that was given to me. I asked my surveyor to tell me the area of the field, but he said it was impossible without some further measurements; the mere length of one side, seven rods, was insufficient. What was his surprise when I showed him in about two minutes what was the area! Can you tell how it is to be done?

134.—A Fence Problem

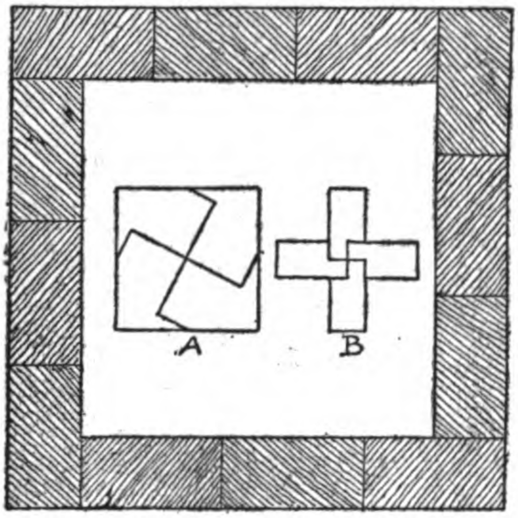
This is a problem that is very frequently brought to my notice in various forms. It is generally difficult, but in the form in which I present it it should be easy to the cunning solver. A man has a square field, 60 ft. by 60 ft., with other property, ad-



joining the highway. For some reason he put up a straight fence in the line of the three trees, as shown, and the length of fence from the middle tree to the tree on the road was just 91 feet. What is the distance in exact feet from the middle tree to the gate on the road?

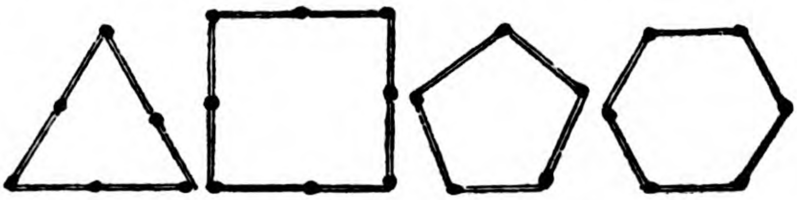
135.—The Domino Swastika

Here is a little puzzle by Mr. Wilfred Bailey. Form a square frame with twelve dominoes, as shown in the illustration. Now, with only four extra dominoes, form within the frame a swastika. The reader may hit on the idea at once, or it may give him considerable trouble. In any case he cannot fail to be pleased with the solution. For the benefit of those who do not happen to be familiar with the swastika—the most ancient of all symbols, meaning "Good luck to you"—I give two examples. The first, A, is from one of our old cutting-out puzzles, where, if you make the swastika cuts as shown, the four pieces of the square fit together and form a perfect Greek Cross. In the second case, B, four playing-cards are so placed together that the swastika is indicated in the centre. How are you to indicate a swastika with the four extra dominoes?



136.—A New Match Puzzle

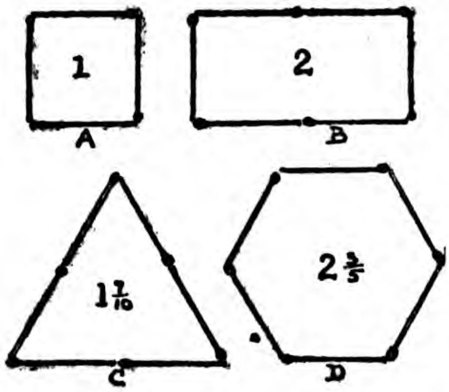
I have a box of matches. I find that I can form with them any given pair of these four regular figures, using all the matches every time. Thus, if there were eleven matches, I could form with them, as shown, the triangle and pentagon or the pentagon



and hexagon, or the square and triangle (by using only three matches in the triangle); but could not with eleven matches form the triangle and hexagon, or the square and pentagon, or the square and hexagon. Of course there must be the same number of matches in every side of a figure. Now, what is the smallest number of matches I can have in the box?

137.—Hurdles and Sheep

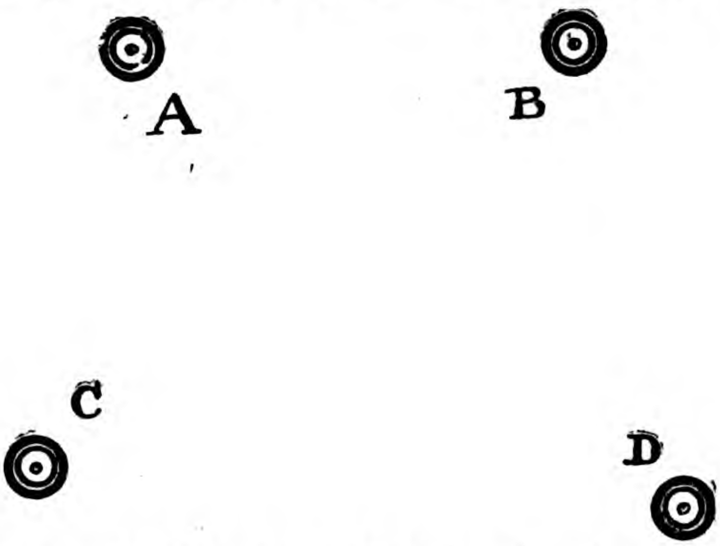
This is a little puzzle that you can try with matches. A farmer says that four of his hurdles will form a square enclosure just sufficient for one sheep. That being so, what is the smallest number of hurdles that he will require for enclosing ten sheep?



Everything depends on the shape of your enclosure. The only other way of placing the four matches (or hurdles) in A is to form a diamond-shaped figure, and the more attenuated this diamond becomes the smaller will be its area, until the sides meet, when there will be no area enclosed at all. If you place six matches, as in B, you will have room for two sheep. But if you place them as in C, you will only have room for one sheep, for seven-tenths of a sheep will only exist as mutton. And if you place them as in D, you can still only accommodate two sheep, which is the maximum for six hurdles. Now, how many hurdles do you require for ten sheep?

138.—The Four Draughtsmen

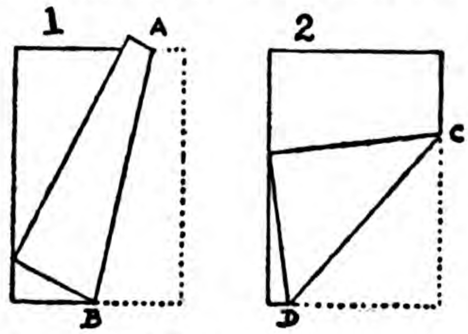
Here is a queer new puzzle that I know will interest my readers considerably. The four draughtsmen are shown exactly as they stood on a square chequered board—not necessarily eight squares



by eight—but the ink with which the board was drawn was evanescent, so that all the diagram except the men has disappeared. How many squares were there in the board and how am I to reconstruct it? I know that each man stood in the middle of a square, one on the edge of each side of the board and no man in a corner. It is a real puzzle, until you hit on the method of solution, and then to get the correct answer is absurdly easy.

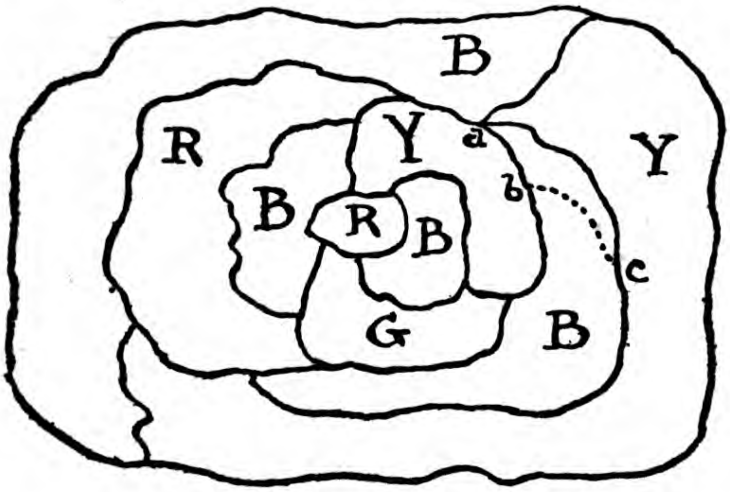
139.—A Crease Problem

Fold a page, so that the bottom outside corner touches the inside edge and the crease is the shortest possible. That is about as simple a question as we could put, but it will puzzle a good many readers to discover just where to make that fold. I give two examples of folding. It will be seen that the crease A B is considerably longer than C D, but the latter is not the shortest possible.



140.—The Four-colour Map Theorem

For just about fifty years various mathematicians, including De Morgan, Cayley, Kempe, Heawood, Heffter, Wernicke, Birkhoff, Franklin, and many others have attempted to prove the truth of this theorem, and in a long and learned article in the

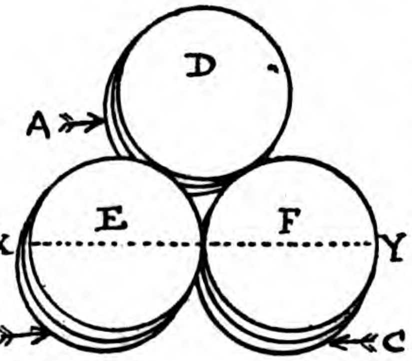


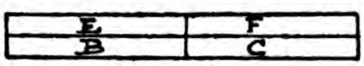
American Mathematical Monthly for July-August, 1923, Professor Brahana, of the University of Illinois, states that "the problem is still unsolved." It is simply this, that in colouring any map under the condition that no contiguous countries shall be coloured alike, not more than four colours can ever be necessary. Countries only touching at a point, like the two Blues and two Yellows at *a* in the diagram, are not contiguous. If the boundary line *c a* had been, instead, at *c b*, then the two Yellows would be contiguous, but that would simply be a different map, and I should have only to substitute Green for that outside Yellow to make it all right. In fact, that Yellow might have been Green as the map at present stands.

I will give, in condensed form, a suggested proof of my own which several good mathematicians to whom I have shown it accept as quite valid. Two others, for whose opinion I have great respect, think it fails for a reason that the former maintain will not " hold water." The proof is in a form that anybody can understand. It should be remembered that it is one thing to be convinced, as everybody is, that the thing is true, but quite another to give a rigid proof of it.

141.—The Six Submarines

Readers may remember a puzzle, to place five pennies so that every penny shall touch every other penny, that is given in my book, *Amusements in Mathematics*, and a correspondent has suggested that as many as six coins can be placed under the conditions if we arrange them as shown in the upper diagram—



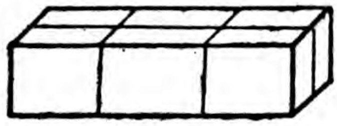


that is, with A, B, and C in the form of a triangle, and D, E, and F respectively on the top of A, B, and C. If we take a section of the coins at X Y (see the lower diagram), he held that E and C and also B and F meet at a " mathematical point," and are therefore in contact. But he was wrong, for if E touches C a barrier is set up between B and F. If B touches F, then E cannot touch C. It is a subtle fallacy that I know will interest my readers. When we say that a number of things meet at a point (like the spokes of a wheel) only three can be in contact (each with each) on the same plane.

This has led me to propound a new " touching " problem. If five submarines, sunk on the same day, all went down at the same spot where another had previously been sunk, how might they all lie at rest so that every one of the six U-boats should touch every other one? To simplify we will say, place six ordinary wooden matches so that every match shall touch every other match. No bending or breaking allowed.

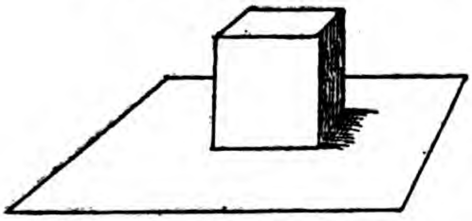
142.—Economy in String

Owing to the scarcity of string a lady found herself in this dilemma. In making up a parcel for her son, a prisoner in Germany, she was limited to using twelve feet of string, exclusive of knots, which passed round the parcel once lengthways and twice round its girth, as shown in the illustration. What was the largest rectangular parcel that she could make up, subject to these conditions?



143.—The Stone Pedestal

In laying the base and cubic pedestal for a certain public memorial, the stonemason used cubic blocks of stone all measuring one foot on every side. There was exactly the same number of these blocks (all uncut) in the pedestal as in the square base on the centre of which it stood. Look at the sketch and try to determine the total number of blocks actually used.



The base is only a single block in depth.

144.—The Bricklayer's Task

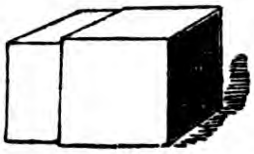
When a man walled in his estate, one of the walls was partly level and partly over a small rise or hill, precisely as shown in the drawing herewith, wherein it will be observed that the distance from A to B is the same as from B to C. Now, the

master-builder desired and claimed that he should be paid more for the part that was on the hill than for the part that was level, since (at least, so he held) it demanded the use of more material. But the employer insisted that he should pay less for that part.

It was a nice point, over which they nearly had recourse to the law. Which of them was in the right?

145.—A Cube Paradox

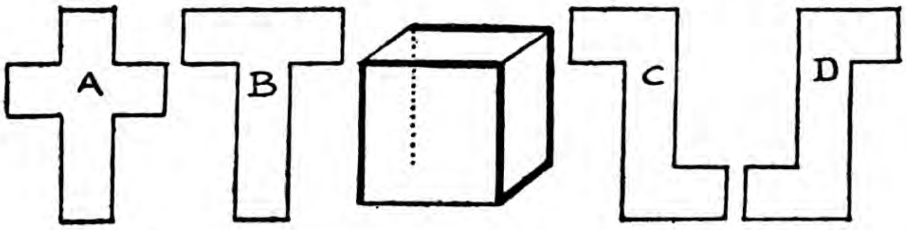
I had two solid cubes of lead, one very slightly larger than the other, just as shown in the illustration.



Through one of them I cut a hole (without destroying the continuity of its four sides) so that the other cube could be passed right through it. On weighing them afterwards it was found that the larger cube was still the heavier of the two! How was this possible?

146.—The Cardboard Box

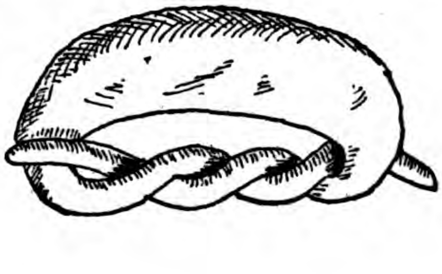
Readers must have often remarked on the large number of little things that one would have expected to have been settled generations ago, and yet never appear to have been considered. Here is a case that has just occurred to me. If I have a closed cubical cardboard box, by running the penknife along seven of



the twelve edges (it must always be seven) I can lay it out in one flat piece in various shapes. Thus, in the diagram, if I pass the knife along the darkened edges and down the invisible edge indicated by the dotted line, I get the shape A. Another way of cutting produces B or C. It will be seen that D is simply C turned over, so we will not call that a different shape. Now, how many different shapes can be produced?

147.—The Austrian Pretzel

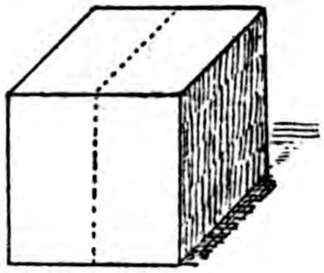
Here is a twisted Vienna bread roll, known as a Pretzel. The twist, like the curl in a pig's tail, is entirely for ornament. The Wiener Pretzel, like some other things, is doomed to be cut up or broken, and the interest lies in the number of resultant pieces.



Suppose you had the Pretzel depicted in the illustration lying on the table before you, what is the greatest number of pieces into which you could cut it with a single straight cut of a knife? In what direction would you make the cut?

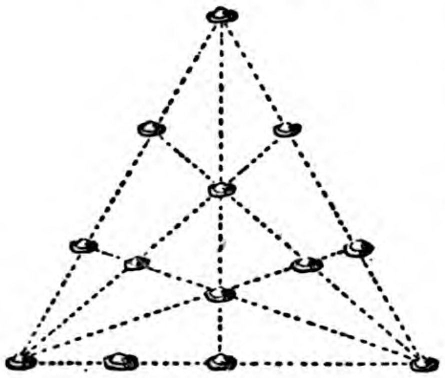
148.—Cutting the Cheese

Here is a simple question that will require just a few moments' thought to get an exact answer. I have a piece of cheese in the shape of a cube. How am I to cut it in two pieces with one straight cut of the knife so that the two new surfaces produced by the cut shall each be a perfect hexagon? Of course, if cut in the direction of the dotted line the surfaces would be squares. Now produce hexagons.



149.—A Tree-planting Puzzle

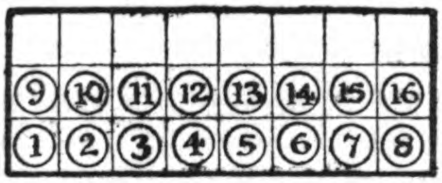
A man planted thirteen trees in the manner shown, and so formed eight straight rows with four trees in every row. But he was not satisfied with that second tree in the horizontal row. As he quaintly put it, "it was not doing enough work—seemed to be a sort of loafer." It certainly does appear to be somewhat out of the game, as the only purpose it serves is to complete one row. So he set to work on a better arrangement, and in the end discovered that he could plant thirteen trees so as to get nine rows of four. Can the reader show how it might be done?



MOVING COUNTER PROBLEMS

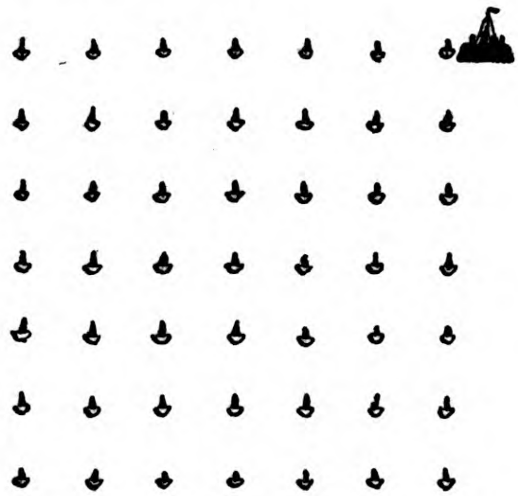
150.—COUNTER SOLITAIRE

This simplification of the board of the old game of Solitaire lends itself to many entertaining little pastimes of patience. Copy the simple diagram on a sheet of paper or cardboard and use sixteen counters, numbered and placed as shown. The puzzle is to remove all but one counter by a succession of leaps. A counter can leap over another adjoining it to the next square beyond, if vacant, and in making the leap you remove the one jumped over. But no leap can be made in a diagonal direction. The following is a solution in eight moves: 5—13, (6—14, 6—5), 16—15, (3—11, 3—6), 2—10, (8—7, 8—16, 8—3), (1—9, 1—2, 1—8), (4—12, 4—1). This means that 5 leaps over 13 and 13 is removed, 6 then leaps over 14 and 14 is removed, and so on. The leaps within a bracket count as one move, because the leaps are made with the same counter in succession. It will be seen that No. 4 makes the last leap. Now try to find a solution, in seven moves, in which No. 1 makes the last leap.



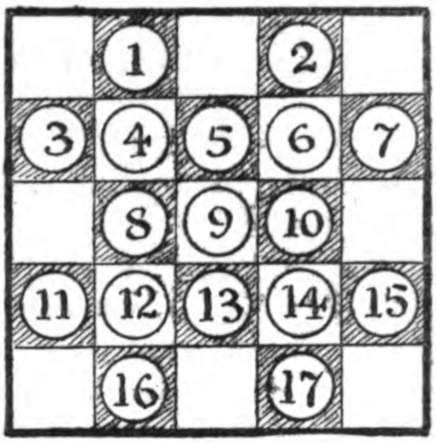
151.—SINKING THE FISHING-BOATS

There are forty-nine fishing-boats in the North Sea. How could an enemy ram and sink the lot in twelve straight courses, starting from the place shown and finishing up at the same place?



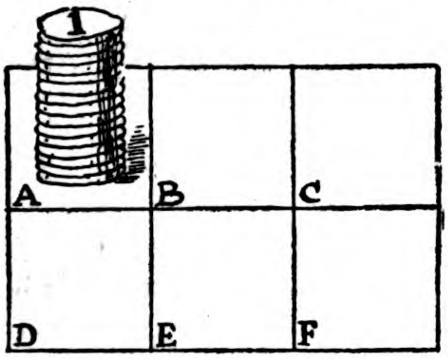
152.—A New Leap-Frog Puzzle

Make a rough board, as shown, and place seventeen counters on the squares indicated. The puzzle is to remove all but one by a series of leaping moves, as in draughts or solitaire. A counter can be made to leap over another to the next square beyond, if vacant, and you then remove the one jumped over. It will be seen that the first leap must be made by the central counter, No. 9, and one has the choice of eight directions. A continuous series of leaps with the same counter will count as a single move. It is required to take off sixteen counters in four moves, leaving the No. 9 on its original central square. Every play must be a leap.



153.—Transferring the Counters

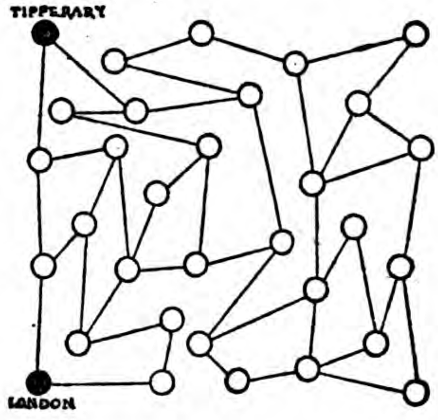
Divide a sheet of paper into six compartments, as shown in the illustration, and place a pile of fifteen counters, numbered consecutively 1 2, 3 . . . 15 downwards, in compartment A. The puzzle is to transfer the complete pile, in the fewest possible moves, to compartment F. You can move the counters one at a time to any compartment, but may never place a counter on one that bears a smaller number than itself. Thus, if you place 1 on B and 2 on C, you can then place 1 on 2, but not 2 on 1.



UNICURSAL AND ROUTE PROBLEMS

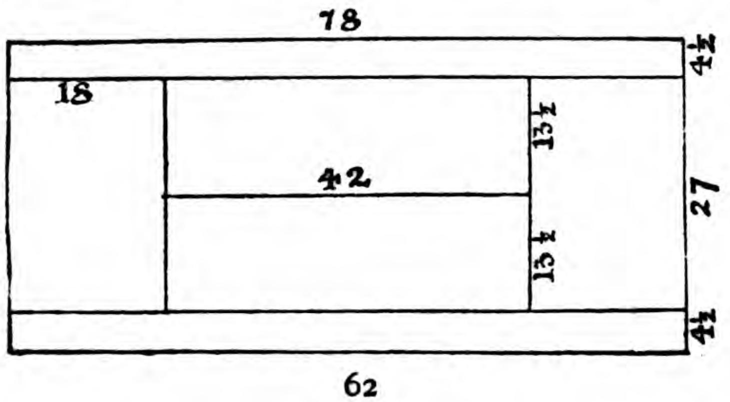
154.—The Way to Tipperary

The popular bard assures us that "it is a long, long way to Tipperary." Look at the accompanying chart and see if you can discover the best way from London to "the sweetest girl I know." The lines represent stages from town to town, and it is necessary to get from London to Tipperary in an even number of stages. You will find no difficulty in getting there in 3, 5, 7, 9, or 11 stages, but these are odd numbers and will not do. The reason they are odd is that they all omit the sea passage, a very necessary stage. If you get to your destination in an even number of stages, it will be because you have crossed the Irish Sea. Which stage is the Irish Sea?



155.—Marking a Tennis Court

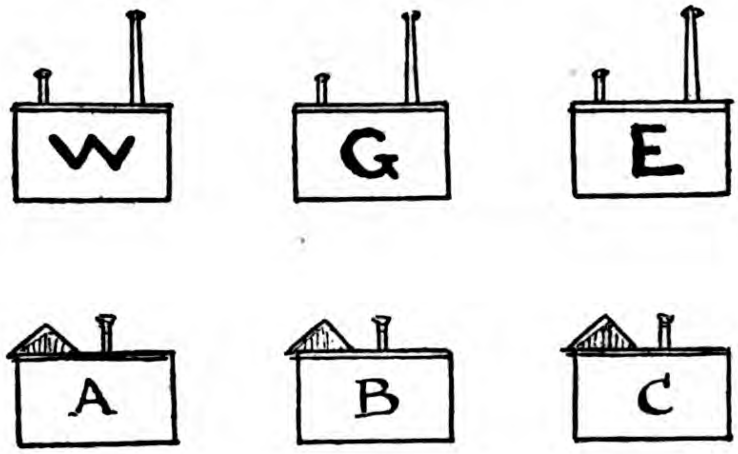
The lines of our tennis court are faint and want re-marking. My marker is of such a kind that, though I can start anywhere and finish anywhere, it cannot be lifted off the lines when working without making a mess. I have therefore to go over some



of the lines twice. Where should I start and what route should I take, without lifting the marker, to mark the court completely and yet go over the minimum distance twice? I give the correct proportions of a tennis court in feet. What is the best route?

156.—Water, Gas, and Electricity

I think I receive, on an average, about ten letters a month from unknown correspondents respecting this puzzle which I published some years ago under the above title. They invariably say that someone has shown it to them who did not know the answer, and they beg me to relieve their minds by telling them



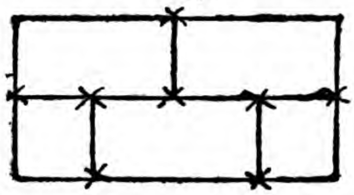
whether there is, or is not, any possible solution. As many of my readers may have come across the puzzle and be equally perplexed, I will try to clear up the mystery for them in a more complete way than I have done in *Amusements in Mathematics.* First of all here is the thing, as I originally gave it, for their consideration.

It is required to lay on water, gas, and electricity from W, G, and E to each of the three houses, A, B, and C, without any pipe crossing another. Take your pencil and draw lines showing how this should be done. You will soon find yourself landed in difficulties.

157.—Crossing the Lines

There is a little puzzle about which, for many years, I have perpetually received enquiries as to its possibility of solution. You are asked to draw the diagram in Fig. 1 (exclusive of the little crosses) with three continuous strokes of the pencil, without removing the pencil from the paper during a stroke, or going over a line twice. As generally understood, it is quite impossible. Wherever I have placed a cross there is an "odd node," and the law for all such cases is that half as many lines will be necessary as there are odd nodes—that is, points from which you can depart in an odd number of ways. Here we have, as indicated, eight nodes, from each of which you can proceed in three directions (an odd number), and, therefore, *four* lines will be required. But, as I have shown in my book of *Amusements*, it may be solved by a trick, overriding the conditions as understood. You first fold the paper, and with a thick lead-pencil

1



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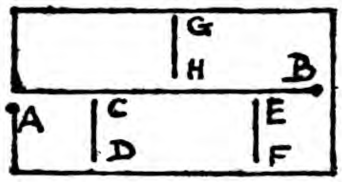

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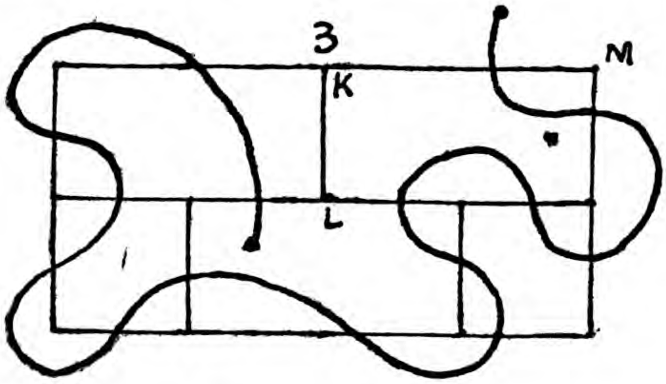
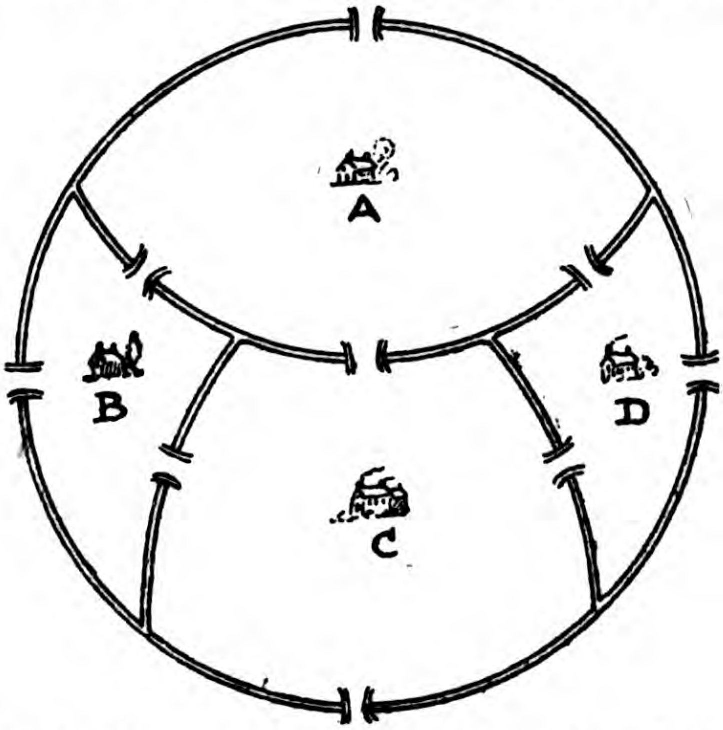


draw C D and E F, in Fig. 2, with a single stroke. Then draw the line from A to B as the second stroke, and G H as the third !

During the last few years this puzzle has taken a new form. You are given the same diagram and asked to start where you like and try to pass through every short line comprising the figure, once and once only, without crossing your own path. Fig. 3 will make quite clear what is meant. It is an attempted solution, but it fails because the line from K to L has not been crossed. We might have crossed it instead of K M, but that would be no better. Is it possible ? Many who write to me about the puzzle say that though they have satisfied themselves as a "pious opinion" that it cannot be done, yet they see no way whatever of *proving* the impossibility, which is quite another matter. I will show my way of settling the question.

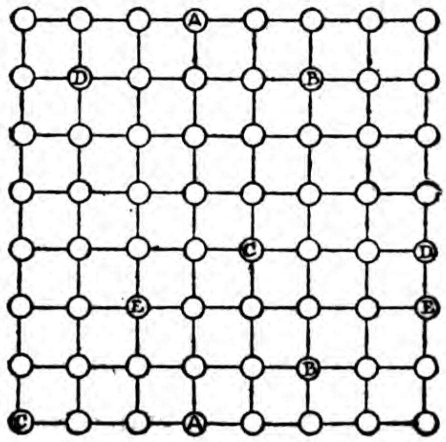
158.—The Nine Bridges

The illustration represents the map of a district with a peculiar system of irrigation. The lines are waterways enclosing the four islands, A, B, C, and D, each with its house, and it will be seen that there are nine bridges available. Whenever Tompkins leaves his house to visit his friend Johnson, who lives in one of



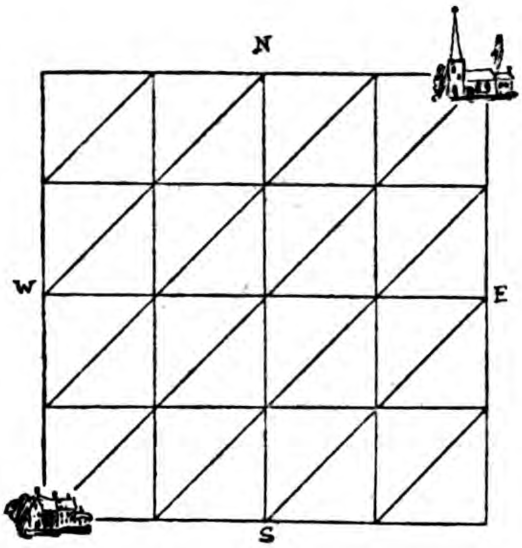
the others, he always carries out the eccentric rule of crossing every one of the bridges once, and once only, before arriving at his destination. How many different routes has he to select from? You may choose any house you like as the residence of Tompkins.

159.—The Five Regiments



The illustration represents a map (considerably simplified for our purpose) of a certain district on the Continent. The circles are towns and the lines roads. During the war five regiments marched to new positions on the same night. The body stationed at the upper A marched to the lower A, that at the upper B to the lower B, that at the upper C to the lower C, that at the upper D to the lower D, and the regiment at the left-hand E marched to the right-hand E. Yet no regiment ever saw anything of any other regiment. Can you mark out the route taken by each so that no two regiments ever go along the same road anywhere?

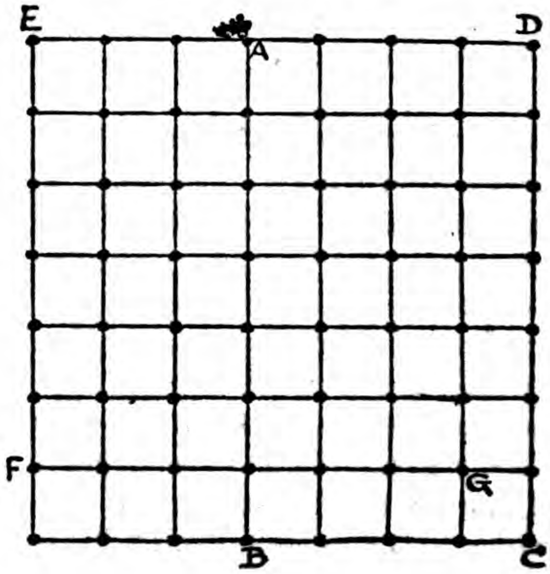
160.—Going to Church



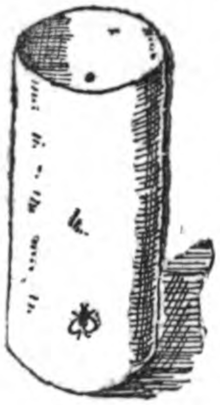
A man living in the house shown in the diagram wants to know what is the greatest number of different routes by which he can go to the church. The possible roads are indicated by the lines, and he always walks either due N, due E. or N.E.; that is, he goes so that every step brings him nearer to the church. Can you count the total number of different routes from which he may select?

161.—A Motor-car Puzzle

A traveller starts in his car from the point A and wishes to go as far as possible while making only fifteen turnings, and never going along the same road twice. The dots represent towns and are one mile apart. Supposing, for example, that he went straight to B, then straight to C, then to D, E, F, and G, then you will find that he has gone thirty-seven miles in five turnings. How far can he go in fifteen turnings?



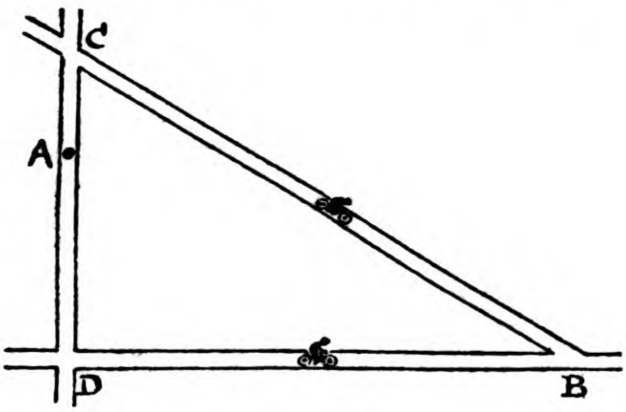
162.—The Fly and the Honey



I have a cylindrical cup four inches high and six inches in circumference. On the inside of the vessel, one inch from the top, is a drop of honey, and on the opposite side of the vessel, one inch from the bottom on the outside, is a fly. Can you tell exactly how far the fly must walk to reach the honey?

163.—The Russian Motor-cyclists

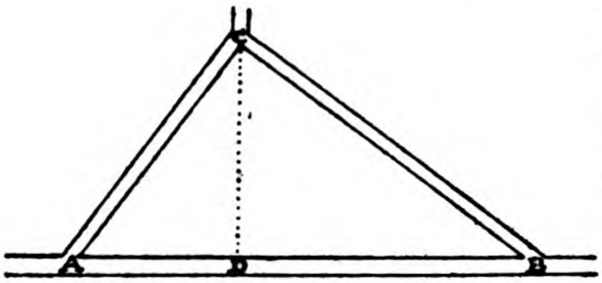
Two Army motor-cyclists, on the road at Adjbkmlprzll, wish to go to Brczrtwxy, which, for the sake of brevity, are marked in the accompanying map as A and B. Now, Pipipoff said: "I shall go to D, which is six miles, and then take the straight road to B, another fifteen miles." But Sliponsky thought he would try the upper road by way of C. Curiously enough, they found



on reference to their cyclometers that the distance either way was exactly the same. This being so, they ought to have been able easily to answer the General's simple question, "How far is it from A to C?" It can be done in the head in a few moments, if you only know how. Can the reader state correctly the distance?

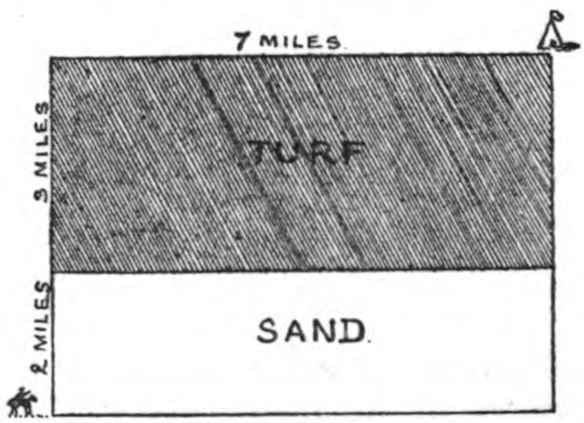
164.—Those Russian Cyclists again

Here is another little experience of the two Russian Army motor-cyclists that I described in our last puzzle. In the section from a map given in our illustration we are shown three long straight roads, forming a right-angled triangle. The General asked the two men how far it was from A to B. Pipipoff replied that all he knew was that in riding right round the triangle, from A to B, from there to C and home to A, his cyclometer registered exactly sixty miles, while Sliponsky could only say that he happened to know that C was exactly twelve miles from the road A to B —that is, to the point D, as shown by the dotted line. Whereupon the General made a very simple calculation in his head and declared that the distance from A to B must be—— Can the reader discover so easily how far it was?



165.—The Despatch-rider in Flanders

A despatch-rider on horseback, somewhere in Flanders, had to ride with all possible speed from the position in which he is shown to the spot indicated by the tent. The distances are marked on the plan. Now, he can ride just twice as fast over the soft turf (the shaded ground) as he can ride over the loose sand. Can you show what is the quickest possible route for him to take? This is just one of those practical problems with which the soldier is faced from day to day when on active service. Important results may hang on the rider taking the right or the wrong route. Which way would you have gone? Of course, the turf and the sand extend for miles to the right and the left with the same respective depths of three miles and two miles, so there is no trick in the puzzle.



COMBINATION AND GROUP PROBLEMS

166.—Picture Presentation

A wealthy collector had ten valuable pictures. He proposed to make a presentation to a public gallery, but could not make up his mind as to how many he would give. So it amused him to work out the exact number of different ways. You see, he could give any one picture, any two, any three, and so on, or give the whole ten. The reader may think it a long and troublesome calculation, but I will give a little rule that will enable him to get the answer in all such cases without any difficulty and only trivial labour.

167.—A General Election

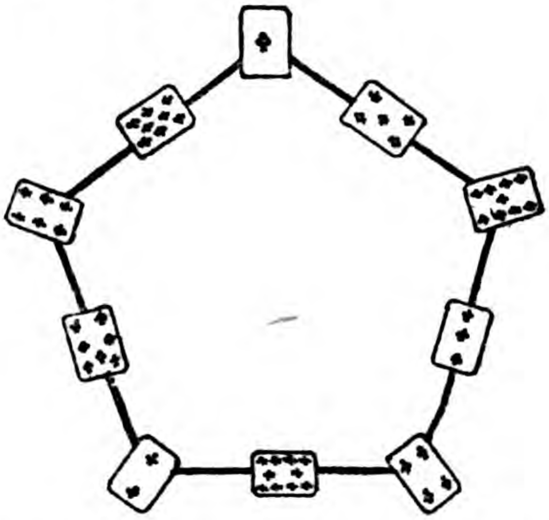
In how many different ways may a Parliament of 615 members be elected if there are only four parties : Conservatives, Liberals, Socialists, and Independents ? You see you might have C. 310, L. 152, S. 150, I. 3 ; or C. 0, L. 0, S. 0, I. 615 ; or C. 205, L. 205, S. 205, I. 0 ; and so on. The candidates are indistinguishable, as we are only concerned with the party numbers.

168.—The Magisterial Bench

A friend at Singapore asked me some time ago to give him my solution to this problem. A bench of magistrates (he does not say where) consists of two Englishmen, two Scotsmen, two Welshmen, one Frenchman, one Italian, one Spaniard, and one American. The Englishmen will not sit beside one another, the Scotsmen will not sit beside one another, and the Welshmen also object to sitting together. Now, in how many different ways may the ten men sit in a straight line so that no two men of the same nationality shall ever be next to one another ?

169.—The Card Pentagon

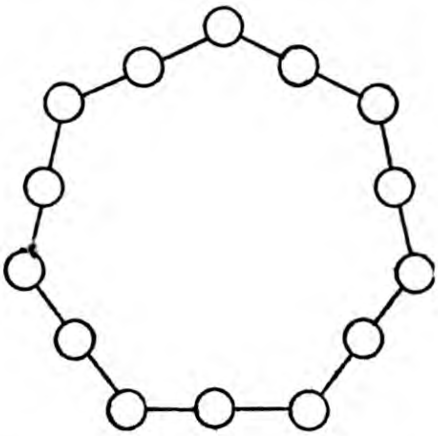
Make a rough pentagon on a large sheet of paper. Then throw down the ten non-court cards of a suit at the places indicated in the illustration, so that the pips on every row of three cards on the sides of the pentagon shall add up alike. The example will



be found faulty. After you have found the rule you will be able to deal the cards into their places without any thought. And there are very few ways of placing them.

170.—A Heptagon Puzzle

Using the fourteen numbers, 1, 2, 3, up to 14, place a different



number in every circle so that the three numbers in every one of the seven sides add up 19.

MAGIC SQUARE PROBLEMS

171.—An Irregular Magic Square

Here we have a perfect magic square composed of the numbers 1 to 16 inclusive. The rows, columns, and two long diagonals all add up 34. Now, supposing you were forbidden to use the two numbers 2 and 15, but allowed, in their place, to repeat any two numbers already used, how would you construct your square so that rows, columns, and diagonals should still add up 34? Your success will depend on which two numbers you select as substitutes for the 2 and 15.

1	14	7	12
15	4	9	6
10	5	16	3
8	11	2	13

172.—A Magic Square Delusion

Here is a magic square of the fifth order. I have found that a great many people who have not gone very profoundly into these things believe that the central number in all squares of this order must be 13. One correspondent who had devoted years to amusing himself with this particular square was astounded when I told him that any number from 1 to 25 might be in the centre. I will show that this is so. Try to form such a magic square with 1 in the central cell.

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

173.—Difference Squares

Can you rearrange the nine digits in the square so that in all the eight directions the difference between one of the digits and the sum of the remaining two shall always be the same? In the example shown it will be found that all the rows and columns give the difference 3; (thus $4+2-3$, and $1+9-7$, and $6+5-8$, etc.), but the two diagonals are wrong, because $8-(4+1)$ and $6-(1+2)$ is not allowed: the sum of two must not be taken from the single digit, but the single digit from the sum. How many solutions are there?

4	3	2
7	1	9
6	5	8

174.—Swastika Magic Square

A correspondent sent me this little curiosity. It is a magic square, the rows, columns, and two diagonals all adding up 65, and all the prime numbers that occur between 1 and 25 (viz. 1, 2, 3, 5, 7, 11, 13, 17, 19, 23) are to be found within the swastika except 11. "This number," he says, "in occult lore is ominous and is associated with the eleven Curses of Ebal, so it is just as well it does not come into this potent charm of good fortune." He is clearly under the impression that 11 cannot be got into the swastika with the other primes. But in this he is wrong, and the reader may like to try to reconstruct the square so that the swastika contains all the ten prime numbers and yet forms a correct magic square, for it is quite possible.

24	3	17	11	10
12	6	25	4	18
5	19	13	7	21
8	22	1	20	14
16	15	9	23	2

175.—Is it very easy?

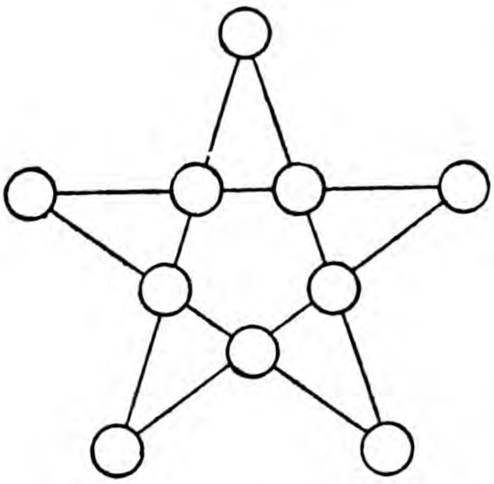
Here is a simple magic square, the three columns, three rows, and two diagonals adding up 72. The puzzle is to convert it into a multiplying magic square, in which the numbers in all the eight lines if *multiplied* together give the same product in every case. You are not allowed to change, or add to, any of the figures in a cell or use any arithmetical sign whatever!

27	20	25
22	24	26
23	28	21

But you may shift the two figures within a cell. Thus, you may write 27 as 72, if you like. These simple conditions make the puzzle absurdly easy, if you once hit on the idea ; if you miss it, it will appear to be an utter impossibility.

MAGIC STAR PROBLEMS

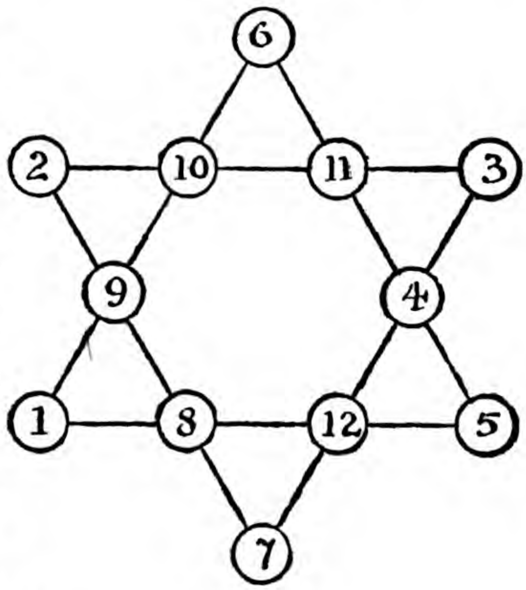
176.—The Five-pointed Star



There is something very fascinating about star puzzles. I give an example, taking the case of the simple five-pointed star. It is required to place a different number in every circle so that the four circles in a line shall add up 24 in all the five directions. No solution is possible with ten consecutive numbers, but you can use any whole numbers you like.

177.—The Six-pointed Star

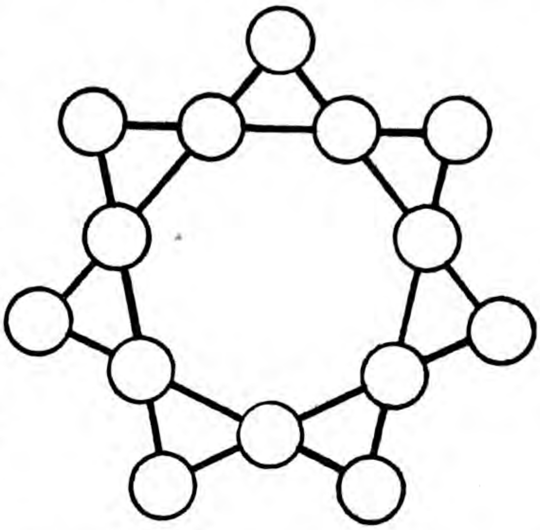
We have considered the question of the five-pointed star. We shall now find the six-pointed star even more interesting. In this case we can always use the twelve consecutive numbers 1 to 12 and the sum of the four numbers in every line will always be 26. The numbers at the six points of the star may add up to any even number from 24 to 54 inclusive, except 28 and 50, which are impossible. It will be seen that in the example I have given the six points add up to 24. If for every number in its



present position you substitute its difference from 13 you will get another solution, its complementary, with the points adding up 54, which is 78 less 24. The two complementary totals will always sum to 78. I will give the total number of different solutions and point out some of the pretty laws which govern the problem, but I will leave the reader this puzzle to solve. There are six arrangements, and six only, in which all the lines of four and the six points also add up to 26. Can you find one or all of them?

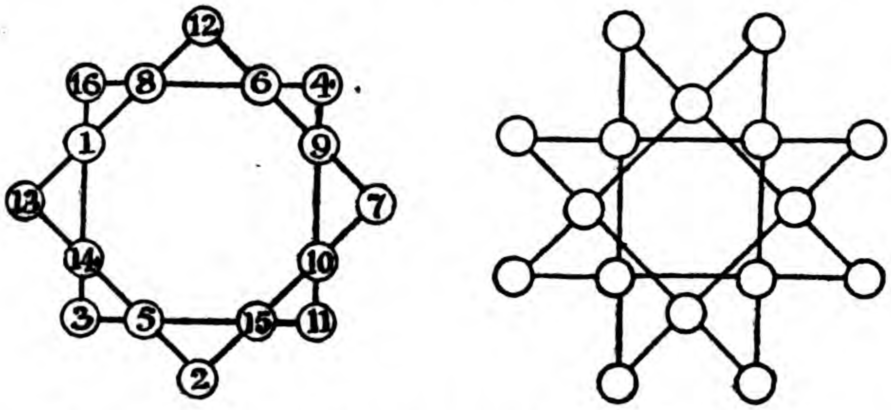
178.—The Seven-pointed Star

We have already dealt briefly with stars of five and six points. The case of the seven-pointed star is particularly interesting. All you have to do is to place the numbers 1, 2, 3, up to 14 in the fourteen discs so that every line of four discs shall add up to 30. If you make a rough diagram and use numbered counters, you will soon find it difficult to break away from the fascination of the thing. Possibly, however, not a single reader will hit upon a simple method of solution; his answer, when found, will be obtained by mere patience and luck. Yet, like those of the large majority of the puzzles given in these pages, the solution is subject to law, if you can unravel it.



179.—Two Eight-pointed Stars

The puzzles of stars with five, six, and seven points that I have given lead us to the eight-pointed star. The star may be formed in two different ways, as shown in our illustration, and the first example is a solution. The numbers 1 to 16 are so placed that every straight line of four adds up 34. If you substitute for every number its difference from 17 you will get the complementary solution. Let the reader try to discover some of the other solutions, and he will find it a very hard nut, even with this one to help him. But I will present the puzzle in an easy and entertaining form. When you know how, every arrangement in the first star can be transferred to the second one auto-

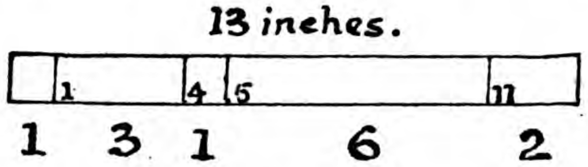


matically. Every line of four numbers in the one case will appear in the other, only the *order* of the numbers will have to be changed. Now, with this information given, it is not a difficult puzzle to find a solution for the second star.

MEASURING, WEIGHING, AND PACKING PUZZLES

180.—The Damaged Measure

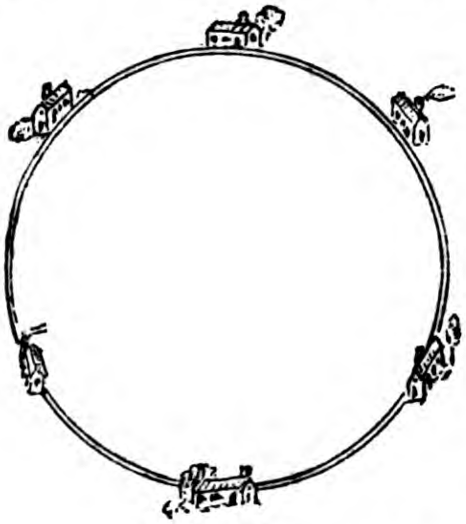
Here is a new puzzle that is interesting, and it reminds one, though it is really very different, of the classical problem by Bachet concerning the weight that was broken in pieces which would then allow of any weight in pounds being determined from one pound up to the total weight of all the pieces. In the present case a man has a yard-stick from which 3 inches have been broken off, so that it is only 33 inches in length. Some of the graduation marks are also obliterated, so that only eight of these marks are legible; yet he is able to measure any given number of inches from 1 inch up to 33 inches. Where are these marks placed?



As an example, I give in the illustration the case of a 13-inch rod with four markings. If I want to measure 4 inches, I take 1 and 3; for 8 inches, 6 and 2; for 10 inches, 3, 1, and 6; and so on. Of course, the exact measure must be taken at once on the rod; otherwise the single mark of 1 inch repeated a sufficient number of times would measure any length, which would make the puzzle absurd!

181.—The Six Cottages

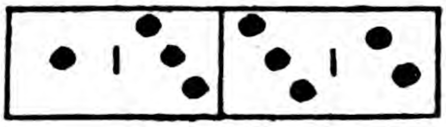
A circular road, twenty-seven miles long, surrounds a tract of wild and desolate country, and on this road are six cottages so placed that one cottage or another is at a distance of one, two, three up to twenty-six miles inclusive from some other cottage.



Thus, Brown may be a mile from Stiggins, Jones two miles from Rogers, Wilson three miles from Jones, and so on. Of course, they can walk in either direction as required. Can you place the cottages at distances that will fulfil these conditions? The illustration is intended to give no clue as to the relative distances.

182.—A New Domino Puzzle

It will be seen that I have selected and placed together two dominoes so that by taking the pips in unbroken conjunction I can get all the numbers from 1 to 9 inclusive. Thus, 1, 2, and 3 can be taken alone; then 1 and 3 make 4; 3 and 2 make 5; 3 and 3 make 6; 1, 3, and 3 make 7; 3, 3, and 2 make 8; and 1, 3, 3, and 2 make 9. It would not have been allowed to take the 1 and the 2 to make 3, nor to take the first 3 and the 2 to make 5. The numbers would not have been in conjunction. Now try to arrange four dominoes so that you can make the pips in this way sum to any number from 1 to 23 inclusive. The dominoes need not be placed 1 against 1, 2 against 2, and so on, as in play.



183.—At the Brook

In introducing liquid measuring puzzles in my book, *Amusements in Mathematics*, I have said, "It is the general opinion that puzzles of this class can only be solved by trial, but I think formulæ can be constructed for the solution generally of certain related cases. It is a practically unexplored field for investigation." So far as I know, the hint has not been taken and the field is still unexplored, so I recently took advantage of a little unexpected leisure to look into the matter. The result, as I thought probable, was that I struck some new and very interesting things. For example, let us take the simplest possible case of a man who goes to a brook with only two vessels with which to measure a given quantity of water. When we are dealing, say, with a barrel of wine we may have complications arising from the barrel being full or empty, from its capacity and contents being known or unknown, from waste of wine being permitted or not permitted, and from pouring back into the barrel being allowed. All these points are eliminated. Is it then possible that any puzzle remains? Let us see. A man goes to the brook with two measures of 15 pints and 16 pints. How is he to measure exactly 8 pints of water, in the fewest possible transactions? Filling or emptying a vessel or pouring any quantity from one vessel to another counts as a transaction. The puzzle is not difficult, but I think the reader will find it very entertaining and instructive. I need hardly add that no tricks, such as marking or tilting the vessels, are allowed.

184.—A Prohibition Poser

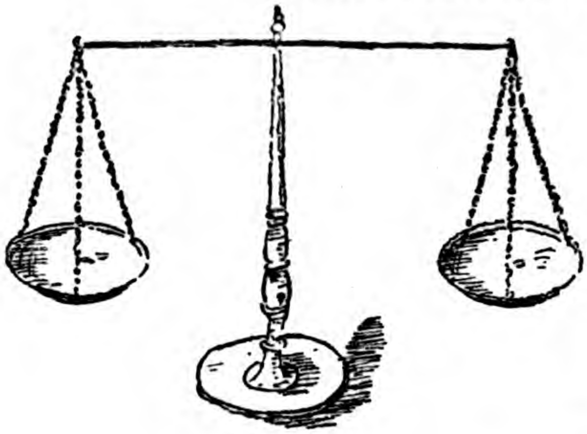
Let us now take another step and look at those cases where we are still allowed any amount of waste, though the liquid is now limited to a stated quantity.

The American Prohibition authorities discovered a full barrel of beer, and were about to destroy the liquor by letting it run down a drain when the owner pointed to two vessels standing by and begged to be allowed to retain in them a small quantity for the immediate consumption of his household. One vessel was a 7-quart and the other a 5-quart measure. The officer was a wag, and, believing it to be impossible, said that if the man could measure an exact quart into each vessel (without any pouring back into the barrel) he might do so. How was it to be done in the fewest possible transactions without any marking or other tricks? Perhaps I should state that an American barrel of beer contains exactly 120 quarts.

185.—Prohibition again

Let us now try to discover the fewest possible manipulations under the same conditions as in the last puzzle, except that we may now pour back into the barrel.

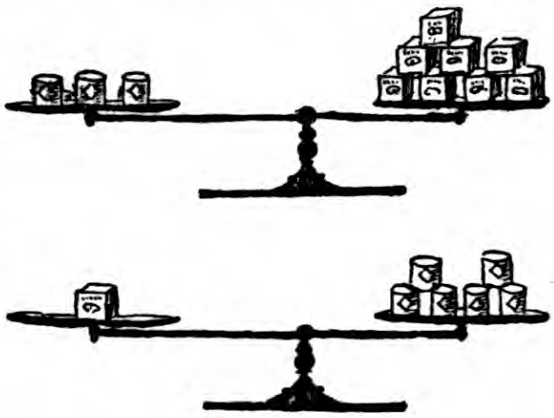
186.—The False Scales



A pudding, when put into one of the pans of these scales, appeared to weigh four ounces more than nine-elevenths of its true weight, but when put into the other pan it appeared to weigh three pounds more than in the first pan. What was its true weight?

187.—Weighing the Goods

A tradesman whose morals had become corrupted during the war by a course of profiteering went to the length of introducing a pair of false scales. It will be seen from the illustration that one arm is longer than the other, though they are purposely so drawn as to give no clue to the answer. As a consequence, it happened that in one of the cases exhibited eight of the little packets (it does not matter what they contain) exactly balanced three of the canisters, while in the other case one packet appeared to be of the same weight as six canisters. Now, as the true weight of one canister was known to be exactly one ounce, what was the true weight of the eight packets?



188.—Monkey and Pulley

Here is a funny tangle. It is a mixture of Lewis Carroll's "Monkey and Pulley," Loyd's "How old was Mary?" and some other trifles. But it is quite easy if you have a pretty clear head.

A rope is passed over a pulley. It has a weight at one end and a monkey at the other. There is the same length of rope on either side and equilibrium is maintained. The rope weighs four ounces per foot. The age of the monkey and the age of the monkey's mother together total four years. The weight of the monkey is as many pounds as the monkey's mother is years old. The monkey's mother is twice as old as the monkey was when the monkey's mother was half as old as the monkey will be when the monkey is three times as old as the monkey's mother was when the monkey's mother was three times as old as the monkey. The weight of the rope and the weight at the end was half as much again as the difference in weight between the weight of the weight and the weight and the weight of the monkey. Now, what was the length of the rope?

189.—Weighing the Baby

"I saw a funny incident at the railway station last summer," said a friend. "There was a little family group in front of the automatic weighing machine, that registered up to 200 lb., and they were engaged in the apparently difficult task of weighing the baby. Whenever they attempted to put the baby alone on the machine she always yelled and rolled off, while the father was holding off the dog, who always insisted on being included in the operations. At last the man, with the baby and Fido, were on the machine together, and I took this snapshot of them with my camera."

He produced a photograph, from which I have simply copied the dial, as that is all we need.

"Then the man turned to his wife and said, 'It seems to me, my dear, that baby and I together weigh 162 lb. more than the dog, while the dog weighs 70 per cent less than the baby. We must try to work it out at home.' I also amused myself by working it out from those figures. What do you suppose was the actual weight of that dear infant?"

190.—Packing Cigarettes

A manufacturer sends out his cigarettes in boxes of 160; they are packed in eight rows of twenty each, and exactly fill the box. Could he, by packing differently, get more cigarettes than 160 into the box? If so, what is the greatest number that he could add? At first sight it sounds absurd to expect to get more cigarettes into a box that is already exactly filled, but a moment's consideration should give you the key to the paradox.

CROSSING RIVER PROBLEMS

191.—Crossing the Ferry

Six persons, all related, have to cross a river in a small boat that will only hold two. Mr. Webster, who had to plan the little affair, had quarrelled with his father-in-law and his son, and, I am sorry to say, Mrs. Webster was not on speaking terms with her own mother or her daughter-in-law. In fact, the relations were so strained that it was not safe to permit any of the belligerents to pass over together or to remain together on the same side of the river. And to prevent further discord, no man was to be left with two women or two men with three women. How are they to perform the feat in the fewest possible crossings? No tricks, such as making use of a rope or current, or swimming across, are allowed.

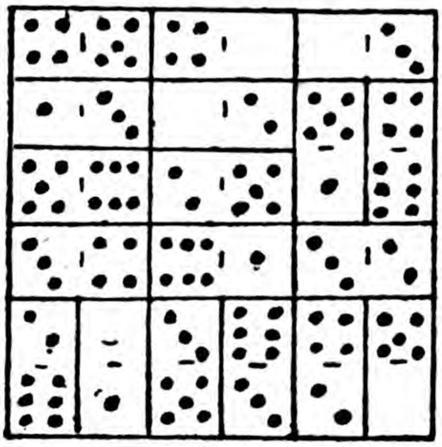
192.—Missionaries and Cannibals

There is a strange story of three missionaries and three cannibals, who had to cross a river in a small boat that would only carry two men at a time. Being acquainted with the peculiar appetities of the cannibals, the missionaries could never allow their companions to be in a majority on either side of the river. Only one of the missionaries and one of the cannibals could row the boat. How did they manage to get across?

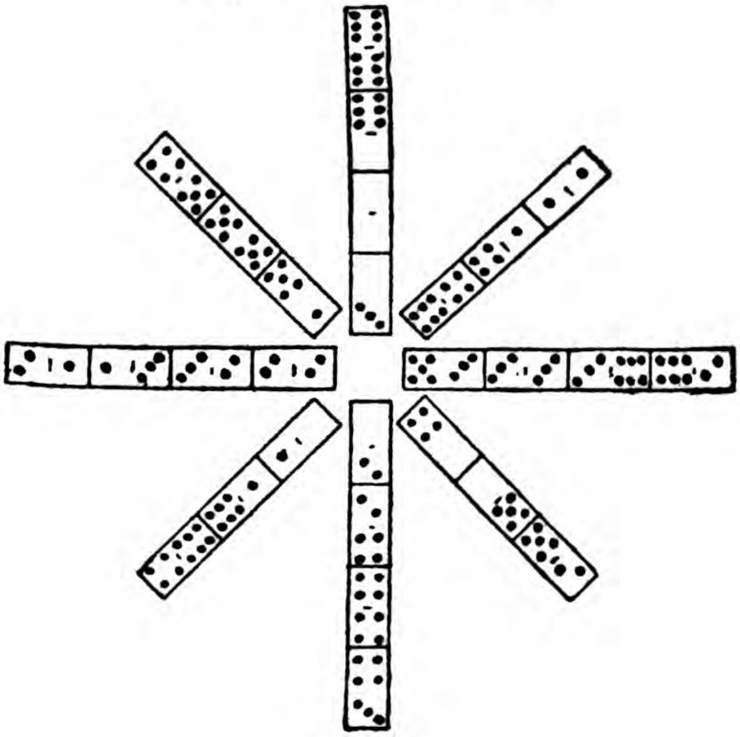
PROBLEMS CONCERNING GAMES

193.—A Domino Square

Select any eighteen dominoes you please from an ordinary box, and arrange them any way you like in a square so that no number shall be repeated in any row or any column. The example given is imperfect, for it will be seen that though no number is repeated in any one of the columns yet three of the rows break the condition. There are two 4's and two blanks in the first row, two 5's and two 6's in the third row, and two 3's in the fourth row. Can you form an arrangement without such errors? Blank counts as a number.



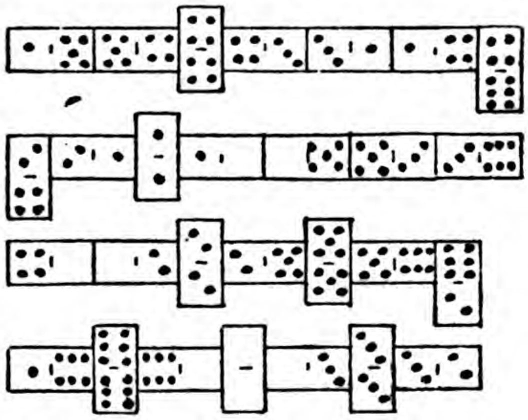
194.—A Domino Star



Place the twenty-eight dominoes, as shown in the illustration, so as to form a star with alternate rays of four and three dominoes. Every ray must contain twenty-one pips (in the example only one ray contains this number) and the central numbers must be 1, 2, 3, 4, 5, 6, and two blanks, as at present, and these may be in any order. In every ray the dominoes must be placed according to the ordinary rule, six against six, blank against blank, and so on.

195.—Domino Groups

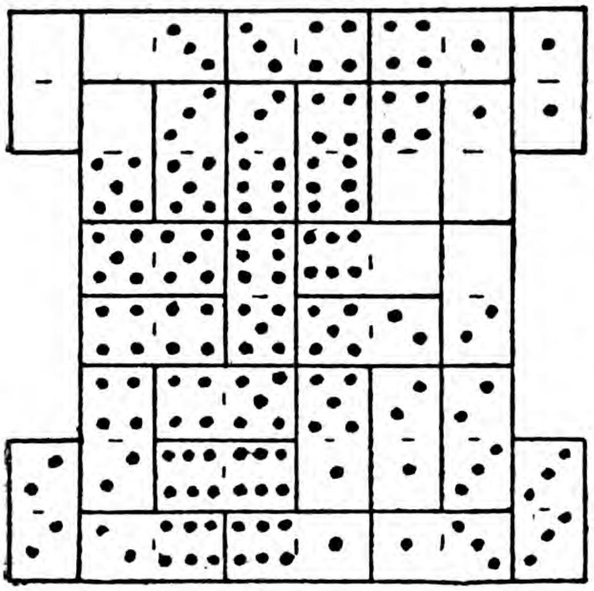
I wonder how many of my readers know that if you lay out the twenty-eight dominoes in line according to the ordinary rule—six against six, two against two, blank against blank, and so on—the last number must always be the same as the first, so that they will really always form a circle. It is a very ancient trick to conceal one domino (but do not take a double) and then ask him to arrange all the others in line without your



seeing. It will astonish him when you tell him, after he has succeeded, what the two end numbers are. They must be those on the domino that you have withdrawn, for that domino completes the circle. If the dominoes are laid out in the manner shown in our illustration and I then break the line into four lengths of seven dominoes each, it will be found that the sum of the pips in the first group is 49, in the second 34, in the third 46, and in the fourth 39. Now I want to play them out so that all the four groups of seven when the line is broken shall contain the same number of pips. Can you find a way of doing it?

196.—Les Quadrilles

This old French puzzle will, I think, be found very interesting. It is required to arrange a complete set of twenty-eight dominoes so as to form the figure shown in our illustration, with all the numbers forming a series of squares. Thus, in the upper two rows we have a square of blanks, and a square of four 3's, and a square of 4's, and a square of 1's; in the third and fourth rows



we have squares of 5, 6, and blank, and so on. This is, in fact, a perfect solution under the conditions usually imposed, but what I now ask for is an arrangement with no blanks anywhere on the outer edge. At present every number from blank to 6 inclusive will be found somewhere on the margin. Can you construct an arrangement with all the blanks inside?

197.—A Puzzle with Cards

Take from the pack the thirteen cards forming the suit of diamonds and arrange them in this order face downwards with the 3 at the top and 5 at the bottom: 3, 8, 7, ace, queen, 6, 4, 2, jack, king, 10, 9, 5. Now play them out in a row on the table in this way. As you spell "ace" transfer for each letter a card from the top to the bottom of the pack—A-C-E—and play the fourth card on to the table. Then spell T-W-O, while transferring three more cards to the bottom, and place the next card on the table. Then spell T-H-R-E-E, while transferring five to the bottom, and so on until all are laid out in a row, and you will find they will be all in regular order. Of course, you will spell out the knave as J-A-C-K. Can you arrange the whole pack so that they will play out correctly in order, first all the diamonds, then the hearts, then the spades, and lastly the clubs?

198.—A Card Trick

Take an ordinary pack of playing-cards and regard all the court cards as tens. Now, look at the top card—say it is a seven—place it on the table face downwards and play more cards on top of it, counting up to twelve. Thus, the bottom card being seven, the next will be eight, the next nine, and so on, making six cards in that pile. Then look again at the top card of pack—say it is a queen—then count 10, 11, 12 (three cards in all), and complete the second pile. Continue this, always counting up to twelve, and if at last you have not sufficient cards to complete a pile, put these apart. Now, if I am told how many piles have been made and how many unused cards remain over, I can at once tell you the sum of all the bottom cards in the piles. I simply multiply by 13 the number of piles less 4, and add the number of cards left over. Thus, if there were 6 piles and 5 cards over, then 13 times 2 (i.e. 6 less 4) added to 5 equals 31, the sum of the bottom cards. Why is this? That is the question.

199.—A Golf Competition Puzzle

I was asked to construct some schedules for players in American golf competitions. The conditions are: (1) Every player plays every other player once, and once only. (2) There are half as many links as players, and every player plays twice on every links except one, on which he plays but once. (3) All the players play simultaneously in every round, and the last round is the one in which every player is playing on a links for the first time. I have written out schedules for a long series of even numbers of players up to twenty-six, but the problem is too difficult for this page except in its most simple form—for six players. Can the reader, calling the players A, B, C, D, E, and F, and pairing these in all possible ways, such as A B, C D, E F, A F, B D, C E, etc., complete the above simple little table for six players? For such a small number it is easy but interesting.

	ROUNDS				
	1	2	3	4	5
1st LINKS					
2nd LINKS					
3rd LINKS					

200.—Cricket Scores

In a country match Great Muddleton, who went in first, made a score of which they were proud. Then Little Wurzleford had their innings and scored a quarter less. The Muddletonians in their next attempt made a quarter less than their opponents, who, curiously enough, were only rewarded on their second effort by a quarter less than the last score. Thus, every innings was a quarter less fruitful in runs than the one that preceded it. Yet the Muddletonians won the match by fifty runs. Can you give the exact score for every one of the four innings?

201.—Football Results

Near the close of a football season a correspondent informed me that when he was returning from Glasgow after the international match between Scotland and England the following table caught his eye in a newspaper :—

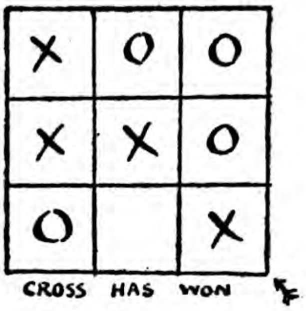
	Played	Won	Lost	Drawn	Goals. For	Goals. Against	Points
Scotland . .	3	3	0	0	7	1	6
England . .	3	1	1	1	2	3	3
Wales . .	3	1	1	1	3	3	3
Ireland . .	3	0	3	0	1	6	0

As he knew, of course, that Scotland had beaten England by 3—0, it struck him that it might be possible to find the scores in the other five matches from the table. In this he succeeded. Can you discover from it how many goals were won, drawn, or lost by each side in every match ?

PUZZLE GAMES

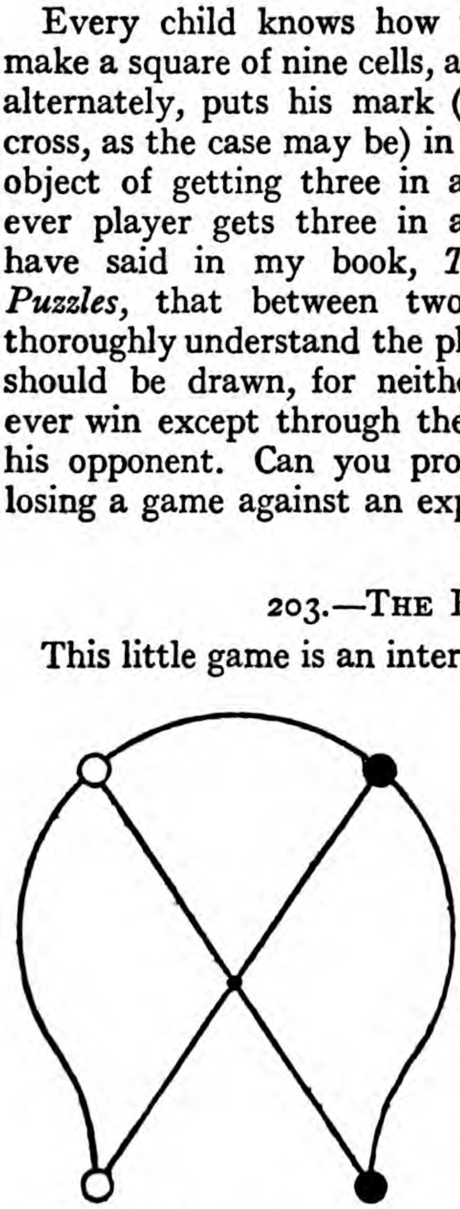
202.—Noughts and Crosses

Every child knows how to play this ancient game. You make a square of nine cells, and each of the two players, playing alternately, puts his mark (a nought or a cross, as the case may be) in a cell with the object of getting three in a line. Whichever player gets three in a line wins. I have said in my book, *The Canterbury Puzzles*, that between two players who thoroughly understand the play every game should be drawn, for neither party could ever win except through the blundering of his opponent. Can you prove this? Can you be sure of not losing a game against an expert opponent?



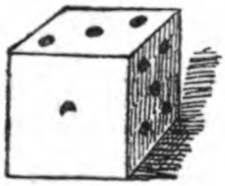
203.—The Horse-shoe Game

This little game is an interesting companion to our "Noughts and Crosses." There are two players. One has two white counters, the other two black. Playing alternately, each places a counter on a vacant point, where he leaves it. When all are played, you slide only, and the player is beaten who is so blocked that he cannot move. In the example, Black has just placed his lower counter. White now slides his lower one to the centre, and wins. Black should have played to the centre himself, and won. Now, which player ought to win at this game?



204.—Turning the Die

This is played with a single die. The first player calls any number he chooses, from 1 to 6, and the second player throws the die at hazard. Then they take it in turns to roll over the die in any direction they choose, but never giving it more than a quarter turn. The score increases as they proceed, and the player wins who manages to score 25 or force his opponent to score beyond 25. I will give an example game.

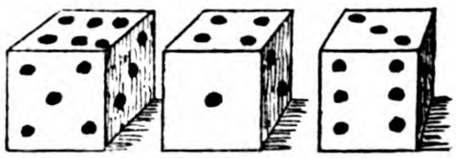


A calls 6, and B happens to throw a 3 (as shown in our illustration), making the score 9. Now A decides to turn up 1, scoring 10; B turns up 3, scoring 13; A turns up 6, scoring 19; B turns up 3, scoring 22; A turns up 1, scoring 23; and B turns up 2, scoring 25 and winning.

What call should A make in order to have the best chance of winning? Remember that the numbers on opposite sides of a correct die always sum to 7, that is, 1—6, 2—5, 3—4.

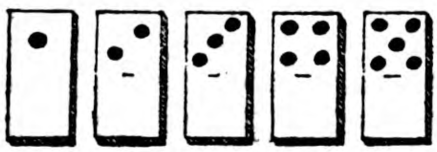
205.—The Three Dice

Mason and Jackson were playing with three dice. The player won whenever the numbers thrown added up to one of two numbers he selected at the beginning of the game. As a matter of fact, Mason selected seven and thirteen, and one of his winning throws is shown in the illustration. What were his chances of winning a throw? And what two other numbers should Jackson have selected for his own throws to make his chances of winning exactly equal?



206.—The 37 Puzzle Game

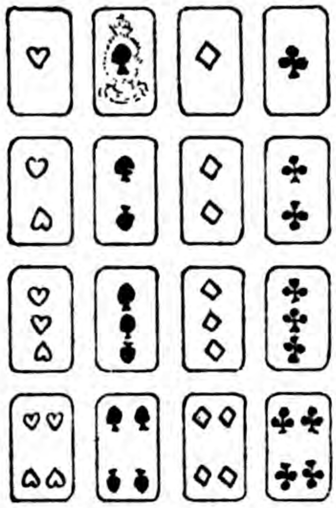
Here is a beautiful new puzzle game, absurdly simple to play but quite fascinating. To most people it will seem to be practically a game of chance—equal for both players—but there are pretty subtleties in it, and I will show how to win with certainty.



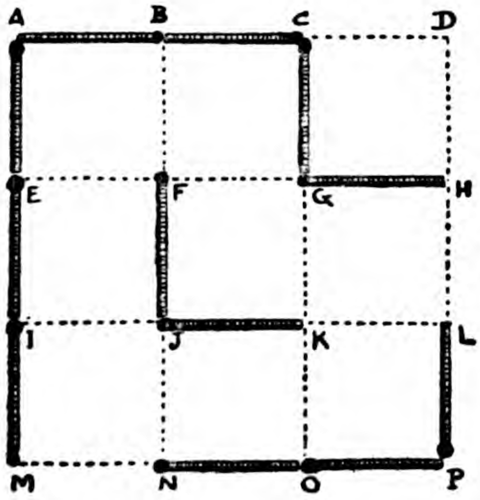
Place the five dominoes, 1, 2, 3, 4, 5, on the table. There are two players, who play alternately. The first player places a coin on any domino, say the 5, which scores 5; then the second player removes the coin to another domino, say to the 3, and adds that domino, scoring 8; then the first player removes the coin again, say to the 1, scoring 9; and so on. The player who scores 37, or forces his opponent to score more than 37, wins. Remember, the coin must be removed to a different domino at each play.

207.—The Twenty-two Game

Here is a variation of our little "Thirty-one Game" (Canterbury Puzzles: No. 79). Lay out the sixteen cards as shown. Two players alternately turn down a card and add it to the common score, and the player who makes the score of twenty-two, or forces his opponent to go beyond that number, wins. For example, A turns down a 4, B turns down a 3 (counting 7), A turns down a 4 (counting 11), B plays a 2 (counting 13), A plays 1 (14), B plays 3 (17), and whatever A does, B scores the winning 22 next play. Again, supposing the play was 3—1, 1—2, 3—3, 1—2, 1—4, scoring 21, the second player would win again, because there is no 1 left and his opponent must go beyond 22. Now, which player should always win, and how?



208.—The Nine Squares Game

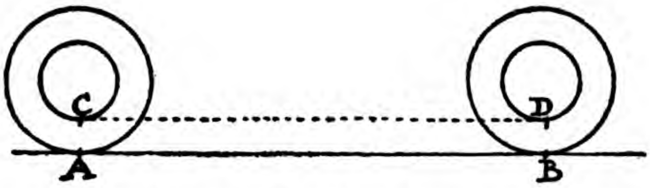


Make the simple square diagram shown and provide a box of matches. The side of the large square is three matches in length. The game is, playing one match at a time alternately, to enclose more of those small squares than your opponent. For every small square that you enclose you not only score one point, but you play again. The illustration shows an illustrative game in progress. Twelve matches are placed, my opponent and myself having made six plays each, and, as I had first play, it is now my turn to place a match. What is my best line of play in order to win most squares? If I play F G my opponent will play B F and score one point. Then, as he has the right to play again, he will score another with E F and again with I J, and still again with G K. If he now plays C D, I have nothing better than D H (scoring one), but, as I have to play again, I am compelled, whatever I do, to give him all the rest. So he will win by 8 to 1—a bad defeat for me. Now, what should I have played instead of that disastrous F G? There is room for a lot of skilful play in the game, and it can never end in a draw.

WHEEL PARADOX PROBLEMS

209.—A WHEEL FALLACY

Here is a curious fallacy that I have found to be very perplexing to many people. The wheel shown in the illustration makes one complete revolution in passing from A to B. It is therefore obvious that the line (A B) is exactly equal in length to the circumference of the wheel. What that length is cannot be stated with accuracy for any diameter, but we can get it near enough for all practical purposes. Thus, if it is a bicycle wheel with a diameter of 28 inches, we can multiply by 22 and divide by 7, and get the length—88 inches. This is a trifle too much, but if we multiply by 355 and divide by 113 we get 87.9646,

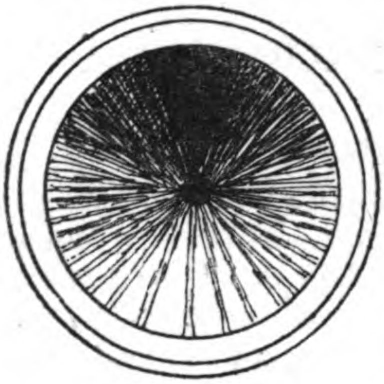


which is nearer; or by multiplying by 3.1416 we get 87.9648, which is still more nearly exact. This is just by the way.

Now the inner circle (the large hub in the illustration) also makes one complete revolution along the imaginary dotted line (C D) and, since the line (C D) is equal to the line (A B), the circumference of the larger and smaller circles are the same! This is certainly not true, as the merest child can see at a glance. Yet, wherein lies the fallacy? Try to think it out. There can be no question that the hub makes one complete revolution in passing from C to D. Then why does not C D equal in length its circumference?

210.—A FAMOUS PARADOX

There is a question that one is perpetually hearing asked, but to which I have never heard or read an answer that was satisfactory or really convincing to the ordinary man. It is this, "When a bicycle is in motion, does the upper part of each wheel move faster than the bottom part near the ground?" People who are not accustomed to the habit of exact thought will invariably dismiss the subject with a laugh, and the reply, "Of course not!" They regard it as too absurd for serious consideration. A wheel, they say, is a rigid whole, revolving round a central axis, and if one part went faster than another it would simply break in pieces.



Then you draw attention of your sceptic to a passing cart and ask him to observe that, while you can clearly distinguish the spokes as they pass the bottom, and count them as they go by, those at the top are moving so fast that they are quite indistinguishable. In fact, a wheel in motion looks something like our rough sketch, and artists will draw it in this way. Our friend has to admit that it is so, but as he cannot explain it he holds to his original opinion, and probably says, "Well, I suppose it is an optical illusion." Now, I invite the reader to consider the matter: Does the upper part of a wheel move faster than the lower part?

211.—Another Wheel Paradox

Two cyclists were resting on a railway bridge somewhere in Sussex, when a railway train went by.

"That's a London train, going to Brighton," said Henderson.

"Most of it is," replied Banks, "but parts of it are going direct towards London."

"What on earth are you talking about?"

"I say that if a train is going from London to Brighton, then parts of that train are all the time going in the opposite direction—from Brighton to London."

"You seriously tell me that while I am cycling from Croydon to Eastbourne, parts of my machine are flying back to Croydon?"

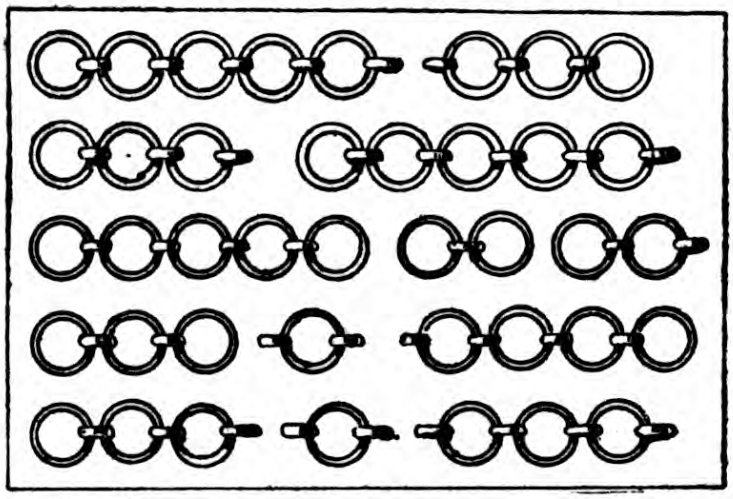
"Steady on, old man," said Banks calmly. "I said nothing about bicycles. My statement was confined to railway trains."

Henderson thought it was a mere catch and suggested the smoke or steam of the engine, but his friend pointed out that there might be a strong wind in the direction the train was going. Then he tried "the thoughts of the passengers," but here there was no evidence, and these would hardly be parts of the train! At last he gave it up. Can the reader explain this curious paradox?

UNCLASSIFIED PROBLEMS

212.—A Chain Puzzle

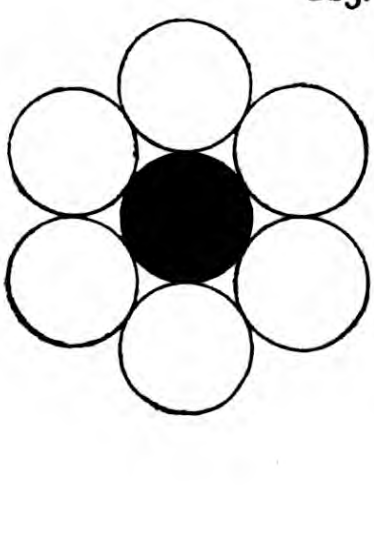
A man has eighty links of old chain in thirteen fragments, as shown here. It will cost him 1d. to open a link and 2d. to weld



one together again. What is the lowest price it must cost him to join all the pieces together so as to form an endless chain?

A new chain will cost him 3s. What is the cheapest method of procedure? Remember that the large and small links must run alternately.

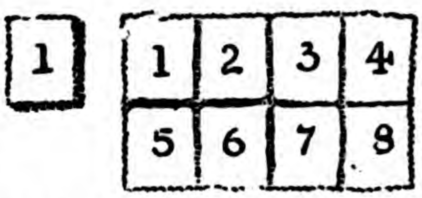
213.—The Six Pennies



Lay six pennies on the table, and then arrange them as shown by the six white circles in the illustration, so that if a seventh penny (the black circle) were produced it could be dropped in the centre and exactly touch each of the six. It is required to get it exact, without any dependence on the eye. In this case you are not allowed to lift any penny off the table—otherwise there would be no puzzle at all—nor can any measuring or marking be employed. You require only the six pennies.

214.—Folding Postage Stamps

If you have eight postage stamps, 4 by 2, as in the diagram, it is very interesting to discover the various ways in which they can be folded so that they will lie under one stamp, as shown. I will say at once that they can actually be folded in forty different ways so that No. 1 is face upwards and all the others invisible beneath it. Nos. 5, 2, 7, and 4 will always be face downwards, but you may arrange for any stamp except No. 6 to lie next to No. 1, though there are only two ways each in which Nos. 7 and 8 can be brought into that position. From a little law



that I discovered, I was convinced that they could be folded in the order 1, 5, 6, 4, 8, 7, 3, 2, and also 1, 3, 7, 5, 6, 8, 4, 2, with No. 1 at the top, face upwards, but it puzzled me for some time to discover how. Can the reader so fold them without, of course, tearing any of the perforation? Try it with a piece of paper well creased like the diagram, and number the stamps on both sides for convenience. It is a fascinating puzzle. Do not give it up as impossible!

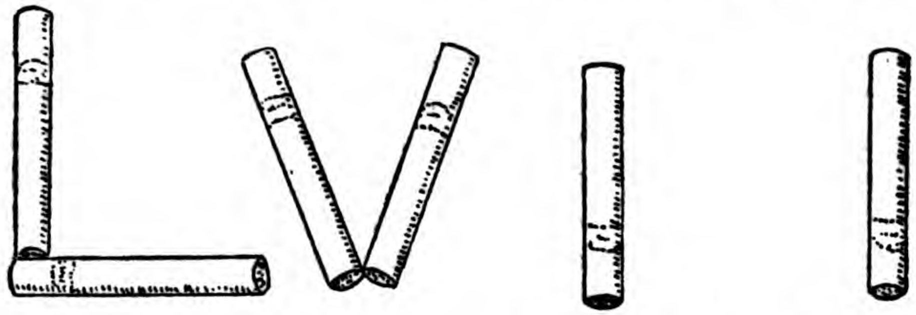
215.—An Ingenious Match Puzzle

Place six matches as shown, and then shift one match without touching the others so that the new arrangement shall represent an arithmetical fraction equal to 1. The match forming the horizontal fraction bar must not be the one moved.

216.—Fifty-seven to Nothing

After the last puzzle, this one should be easy.

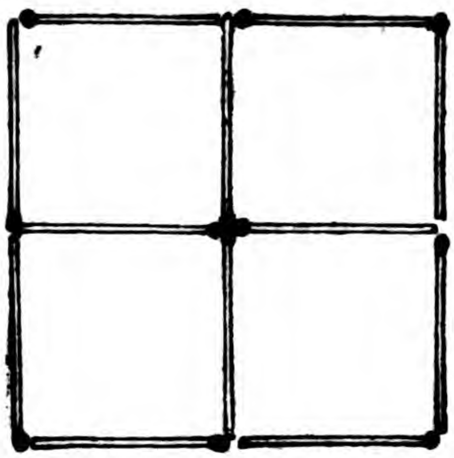
It will be seen that we have arranged six cigarettes so as to represent the number 57. The puzzle is to remove any two of them



you like (without disturbing any of the others) and so replace them as to represent 0, or nothing. Remember that you can only shift two cigarettes. There are two entirely different solutions. Can you find one or both ?

217.—The Five Squares

Here is a new little match puzzle that will perplex a good many readers, though they will smile when they see the answer. It will be seen that the twelve matches are so arranged that they form four squares. Can you rearrange the same number of matches (all lying flat on the table) so that they enclose five squares ? Every square must be entirely "empty" or the illustration itself will show five squares if we were allowed to count the large square forming the boundary. No duplicated match or loose ends are allowed.



218.—A Square with Four Pennies

Can you place four pennies together so as to show a square ? They must all lie flat on the table.

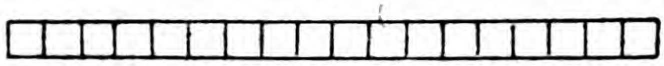
219.—A Calendar Puzzle

I have stated in my book, *Amusements in Mathematics*, that, under our present calendar rules, the first day of a century can never fall on a Sunday or a Wednesday or a Friday. As I have not given the proof, I am frequently asked the reason why. I will try to explain the mystery in as simple a way as possible.

Note that 1901 was the first day of a century: not 1900.

220.—The Fly's Tour

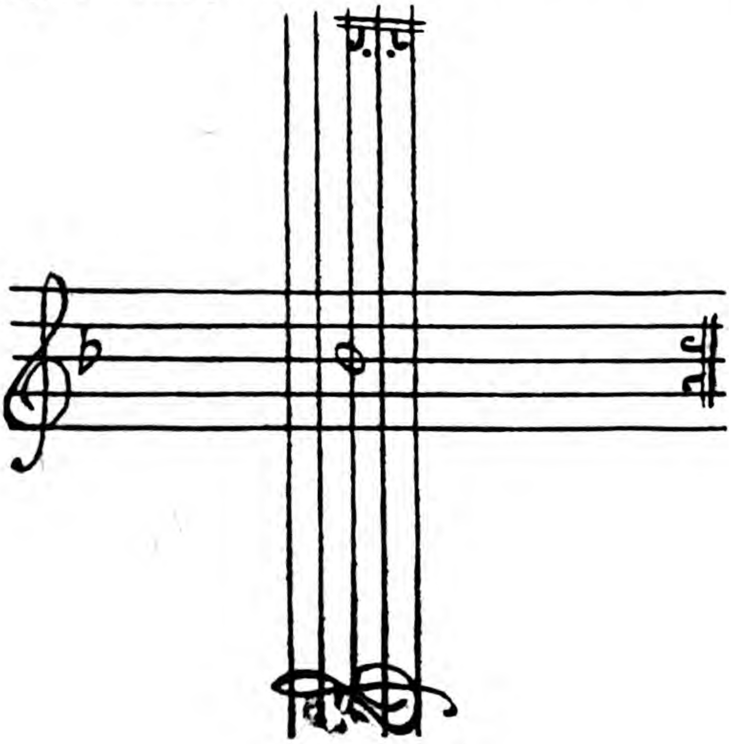
I had a ribbon of paper, divided into squares on each side, as shown in the illustration. I joined the two ends together to make a ring, which I threw on the table. Later I noticed that



a fly pitched on the ring and walked in a line over every one of the squares on both sides, returning to the point from which it started, *without ever passing over the edge of the paper!* Its course passed through the centres of the squares all the time. How was this possible?

221.—A Musical Enigma

Here is an old musical enigma that has been pretty well known in Germany for some years.



222.—A Mechanical Paradox

A remarkable mechanical paradox, invented by James Ferguson about the year 1751, ought to be known by everyone, but, unfortunately, it is not. It was contrived by him as a challenge to a sceptical watchmaker during a metaphysical controversy. "Suppose," Ferguson said, "I make one wheel as thick as three others and cut teeth in them all, and then put the three wheels all loose upon one axis and set the thick wheel to turn them, so that its teeth may take into those of the three thin ones. Now, if I turn the thick wheel round, how must it turn the others?" The watchmaker replied that it was obvious that all three must be turned the contrary way. Then Ferguson produced his simple machine, which anybody can make in a few hours, showing that, turning the thick wheel which way you would, one of the thin wheels revolved the *same way*, the second the *contrary way*, and the third *remained stationary*. Although the watchmaker took the machine away for careful examination, he failed to detect the cause of the strange paradox.

SOLUTIONS

1.—Concerning a Cheque

If you set to work under the notion that there were only pounds and shillings—no pence—in the amount, a solution is impossible. The amount must have been £5 11s. 6d. He received £11 5s. 6d., and after he had spent half a crown there would remain the sum of £11 3s., which is twice the amount of the cheque.

2.—Pocket-money

The amount originally in pocket was £19 18s., so that after spending one-half I had remaining £9 19s., or as many shillings as previously pounds and half as many pounds as shillings.

3.—Dollars and Cents

The man must have entered the store with 99.98 dollars in his pocket.

4.—Loose Cash

The largest sum is 15s. 9d., composed of a crown and a half-crown (or three half-crowns), four florins, and a threepenny-piece.

5.—Doubling the Value

The only answer is £2 17s. multiplied by six, which will produce £17 2s., where the pounds change places with the shillings as required.

6.—Generous Gifts

At first there were twenty persons, and each received 6s. Then fifteen persons (five fewer) would have received 8s. each. But twenty-four (four more) appeared and only received 5s. each. The amount distributed weekly was thus 120s.

7.—Selling Eggs

The smallest possible number of eggs is 103, and the woman sold 60 every day. Any multiple of these two numbers will work. Thus, she might have started with 206 eggs and sold 120 daily; or with 309 and sold 180 daily. But we required the smallest possible number.

8.—Buying Buns

There must have been three boys and three girls, each of whom received two buns at three a penny and one bun at two a penny, the cost of which would be exactly sevenpence.

9.—Fractional Value

One-third of twopence is the same as two-thirds of a penny, and therefore equal to two-ninths of threepence.

10.—Unrewarded Labour

Weary Willie must have worked $16\frac{2}{3}$ days and idled $13\frac{1}{3}$ days. Thus the former time, at 8s. a day, amounts to exactly the same as the latter at 10s. a day, that is £6 13s. 4d.

11.—The Perplexed Banker

The contents of the ten bags should be as follows: 1, 2, 4, 8, 16, 32, 64, 128, 256, 489. The first nine numbers are in geometrical progression, and their sum, deducted from 1000, gives the contents of the tenth bag.

12.—A Weird Game

The seven men, A, B, C, D, E, F, and G, had respectively in their pockets before play the following sums, reduced to farthings for the sake of simplicity: 449, 225, 113, 57, 29, 15, and 8. The answer may be found by laboriously working backwards, but a simpler method is as follows: $7+1=8$; $2\times7+1=15$; $4\times7+1=29$; and so on, where the multiplier increases in powers of 2, that is, 2, 4, 8, 16, 32, and 64.

13.—Find the Coins

Abel had at first 16s. 3d., Best had 8s. 9d., and Crewe had 5s. Abel and Best must each have had a threepenny-piece, but as there must have been an even number of these pieces, divisible by three, the fewest possible threepenny-pieces is six. Then the 8s. 9d. first paid to B. in the fewest possible coins would be 5s., 2s. 6d., 1s., and 3d.—four coins—the 5s. paid to C. would be a crown, and the 2s. 6d. left with A. would be a half-crown. If A. thus had the minimum of six coins, B. had seven coins and C. five coins, and each had six coins at the finish. In short, A. started with 2 crowns, 2 half-crowns, a shilling, and a threepenny-piece, B. with a crown, 3 shillings, and 3 threepenny-pieces, and C. with a half-crown, 2 shillings, and 2 threepenny-pieces. The reader will now have no difficulty in making the various payments so that each man is left with 1 crown, 1 half-crown, 2 shillings, and 2 threepenny-pieces.

14.—An Easy Settlement

At the start of play Andrews held a half-sovereign and a shilling, Baker held a crown and a florin, and Carey held a double florin and a half-crown. After settlement, Andrews held double florin and florin, Baker the half-sovereign and half-crown, and Carey held crown and shilling. Thus, Andrews lost 5s., Carey lost 6d., and Baker won 5s. 6d. The selection of the coins is obvious, but their allotment requires a little judgment and trial.

15.—Sawing Logs

In the first case the charge was 10s. a cord for short logs. Four cords of long logs would equal in number of logs two cords of short logs, but every long log would need five cuts to two cuts in the case of short logs. Therefore, the charge should be in the proportion of 5 to 4, or 12s. 6d. a cord, as compared with 10s. a cord, and four cords at 12s. 6d. would be 50s., the correct charge.

16.—Digging a Ditch

A. should receive one-third of two pounds (13s. 4d.), and B. two-thirds (£1 6s. 8d.). Say B. can dig all in 2 hours and shovel all in 4 hours; then A. can dig all in 4 hours and shovel all in 8 hours. That is, their ratio of digging is as 2 to 4 and their ratio of shovelling as 4 to 8 (the same ratio), and A. can dig in the same time that B. can shovel (4 hours), while B. can dig in a quarter of the time that A. can shovel. Any other figures will do that fill these conditions and give two similar ratios for their working ability. Therefore, A. takes one-third and B. twice as much—two-thirds.

17.—Name their Wives

As it is evident that Catherine, Jane, and Mary received respectively £122, £132, and £142, making together the £396 left to the three wives, if John Smith receives as much as his wife Catherine, £122; Henry Snooks half as much again as his wife Jane, £198; and Tom Crowe twice as much as his wife Mary, £284, we have correctly paired these married couples and exactly accounted for the £1000.

18.—A Curious Paradox

The amount of the bill was £2. The man bought a newspaper on the way to the shop, and so had only £1 19s. 11d. If the full amount had been paid the shopkeeper would have had to spend one penny on a receipt stamp, so by paying the smaller amount the customer saved a penny and the Inland Revenue lost that amount. With the receipt stamp now costing twopence, the short payment of £1 19s. 11d. would be a gain of one penny each to both customer and shopkeeper, but £1 19s. 10d. would save the former twopence and leave the latter unaffected.

19.—Market Transactions

The man bought 19 cows for £95, 1 sheep for £1, and 80 rabbits for £4, making together 100 animals at a cost of £100.

A purely arithmetical solution is not difficult by a method of averages, the average cost per animal being the same as the cost of a sheep.

By algebra we proceed as follows, working in shillings:

$$\begin{aligned} 100x+20y+z&=2000 \\ x+y+z&=100 \\ \hline 99x+19y\quad&=1900 \end{aligned}$$

by subtraction. We have therefore to solve this indeterminate equation, when we find that the only answer is $x=19$, $y=1$. Then, to make up the 100 animals, z must$=80$.

20.—The Seven Applewomen

Each woman sold her apples at seven for 1d., and 3d. each for the odd ones over. Thus, each received the same amount, 1s. 8d. Without questioning the ingenuity of the thing, I have always thought the solution unsatisfactory, because really indeterminate, even if we admit that such an eccentric way of selling may be fairly termed a "price." It would seem just as fair if they sold them at different rates and afterwards divided the money; or sold at a single rate with different discounts allowed; or sold different kinds of apples at different values; or sold the same rate per basketful; or sold by weight, the apples being of different sizes; or sold by rates diminishing with the age of the apples; and so on. That is why I have never held a high opinion of this old puzzle.

In a general way, we can say that n women, possessing $an+(n-1)$, $(a+b)n+(n-2)$, $(a+2b)n+(n-3)\ldots\{a+(n-1)b\}n$ apples respectively, can sell at n for a penny and b pence for each odd one over, and each receive $a+b(n-1)$ pence. In the case of our puzzle $a=2$, $b=3$, and $n=7$.

21.—Their Ages

Tom's age was seven years and Mary's thirteen years.

22.—Mrs. Wilson's Family

The ages must have been as follows: Mrs. Wilson, 39; Edgar, 21; James, 18; John, 18; Ethel, 12; Daisy, 9. It is clear that James and John were twins.

23.—De Morgan and Another

De Morgan was born in 1806. When he was 43, the year was the square of his age—1849. Jenkins was born in 1860. He was 5^2+6^2 (61) in the year 5^4+6^4 (1921). Also he was 2×31 (62) in the year 2×31^2 (1922). Again, he was 3×5 (15) in the year 3×5^4 (1875).

24.—"Simple" Arithmetic

Their ages were respectively 64 and 20.

25.—A Dreamland Clock

The hour indicated would be exactly $23\frac{1}{13}$ minutes after 4 o'clock. But as the minute-hand moved in the opposite direction, the real time would be $36\frac{12}{13}$ minutes after 4. You must deduct the number of minutes indicated from 60 to get the real time.

26.—What is the Time?

The time is $6\frac{3}{4}$ minutes past IX, when the hour-hand is $45\frac{9}{16}$ minutes past XII. Then $45\frac{9}{16}$ is the square of $6\frac{3}{4}$. If we allow fractions *less* than a minute point then there is also the solution, five seconds (one-twelfth of a minute) past XII o'clock.

27.—The First-born's Legacy

The fractions $\frac{2}{3}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{12}$, equal $\frac{40}{60}$, $\frac{12}{60}$, $\frac{6}{60}$, $\frac{5}{60}$, which together make $\frac{63}{60}$, which is greater than unity, but the legacies were to be in the "*proportions*" of those fractions. Therefore, the widow receives $\frac{40}{63}$=£5333 6s. 8d., the first-born $\frac{12}{63}$=£1600, the second-born $\frac{6}{63}$=£800, and the brother $\frac{5}{63}$=£666 13s. 4d. But the sting is in the tail, and the question is indeterminate until we learn just *when* the twins were born. Tommy was actually born just before 2 a.m. on 21st September, 1924, immediately after which time the clocks were set back an hour, summer time being ended, and Freddy was born a little after 1 a.m. "by the clock." So we have the curious paradox that the first-born was born later than the second-born! Freddy therefore only receives £800.

28.—Hill Climbing

It must have been $6\frac{3}{4}$ miles to the top of the hill. He would go up in $4\frac{1}{2}$ hours and descend in $1\frac{1}{2}$ hours.

29.—Timing the Motor-car

As the man can walk 27 steps while the car goes 162, the car is clearly going six times as fast as the man. The man walks $3\frac{1}{2}$ miles an hour: therefore the car was going at 21 miles an hour.

30.—The Staircase Race

If the staircase were such that each man would reach the top in a certain number of full leaps, without taking a reduced number at his last leap, the smallest possible number of risers would, of course, be 60 (that is, $3\times4\times5$). But the sketch showed us that A. taking three risers at a leap, has one odd step at the end; B. taking four at a leap, will have three only at the end; and C. taking five at a leap, will have four only at the finish. Therefore, we have to find the smallest number that, when divided by 3, leaves a remainder 1, when divided by 4 leaves 3, and when divided by 5 leaves a remainder 4. This number is 19. So there were 19 risers in all, only four being left out in the sketch.

31.—A Walking Puzzle

It will be found (and it is the key to the solution) that the man from B. can walk 7 miles while the man from A. can walk 5 miles. Say the distance between the towns is 24 miles, then the point of meeting would be 14 miles from A. and the man from A. walked $3\frac{3}{7}$ miles per hour, while the man from B. walked $4\frac{4}{5}$ miles per hour. They both arrived at 7 p.m. exactly.

32.—Riding in the Wind

He could ride one mile in $3\frac{3}{7}$ minutes, or $\frac{7}{24}$ mile per minute. The wind would help or retard him to the extent of $\frac{1}{24}$ mile per minute. Therefore, with the wind he could ride $\frac{8}{24}$ mile per minute and against the wind $\frac{6}{24}$ mile per minute; so that is 1 mile in 3 minutes, or 4 minutes respectively, as stated.

33.—A Rowing Puzzle

The correct answer is $3\frac{9}{17}$ minutes. The crew can row $\frac{1}{5}$ of the distance per minute on still water, and the stream does $\frac{1}{12}$ of the distance per minute. The difference and sum of these two fractions are $\frac{7}{60}$ and $\frac{17}{60}$. Therefore, against the stream would take $\frac{60}{7}$ minutes (or $8\frac{4}{7}$ minutes), and with the stream $\frac{60}{17}$ (or $3\frac{9}{17}$ minutes).

34.—The Moving Stairway

If I walk 26 steps I require 30 seconds, and if I walk 34 steps I require only 18 seconds. Multiply 30 by 34 and 26 by 18 and we get 1020 and 468, the difference between which is 552. Divide this by the difference between 30 and 18 (that is, by 12) and the answer is 46, the number of steps in the stairway, which descends at the rate of 1 step in $1\frac{1}{2}$ seconds. The speed at which I walk on the stairs does not affect the question, as the step from which I alight will reach the bottom at a given moment, whatever I do in the meantime.

35.—Sharing a Bicycle

Let Anderson ride $11\frac{1}{9}$ miles, drop the bicycle, and walk the rest of the way. Brown will walk until he picks up the bicycle, and then rides to their destination, getting there at exactly the same time as Anderson. The journey takes them 3 hours 20 minutes. Or you can divide the 20 miles into nine stages of $2\frac{2}{9}$ miles each, and drop the machine at every stage, only you must make Anderson ride at the start. Anderson will then ride each of his five stages in $\frac{2}{9}$ hour and walk each of his four stages in $\frac{5}{9}$ hour, making his total time $3\frac{1}{3}$ hours. Brown will ride each of his four stages in $\frac{5}{18}$ hour and walk each of his five stages in $\frac{4}{9}$ hour, making his total time also $3\frac{1}{3}$ hours. The distances that Anderson and Brown ride respectively must be in the proportion of 5 to 4; the distances they walk in the proportion of 4 to 5.

36.—More Bicycling

A. rides $7\frac{11}{27}$ miles, B. rides $1\frac{13}{27}$ miles, and C. rides $11\frac{3}{27}$ miles, making the twenty miles in all. They may ride in any order, only each man should complete his ride in one mount and the second rider must always walk both before and after riding. They will each take $3\frac{8}{9}$ hours on the journey, and therefore will all arrive together.

37.—A Side-car Problem

Atkins takes Clarke 40 miles in his car and leaves him to walk the remaining 12 miles. He then rides back and picks up Baldwin at a point 16 miles from the start and takes him to their destination. All three arrive in exactly 5 hours. Or Atkins might take Baldwin 36 miles and return for Clarke, who will have walked his 12 miles. The side-car goes 100 miles in all, with no passenger for 24 miles.

38.—The Despatch-Rider

The answer is the square root of twice the square of 40, added to 40. This is 96·568 miles, or, roughly, $96\frac{1}{2}$ miles.

39.—The Two Trains

In 5 seconds both trains (together) go 600 feet, or $81\frac{9}{11}$ miles per hour. In 15 seconds the faster train gains 600 feet, or

$27\frac{3}{11}$ miles per hour. From this we get $54\frac{6}{11}$ miles per hour as the rate of the faster train; and it is clear that $27\frac{3}{11}$ miles per hour is the rate of the other.

40.—Pickleminster to Quickville

There are two possible distances that will fit the conditions—210 miles and 144 miles, only I barred out the latter by the words, "at an ordinary rate." With 144 miles A would run 140 miles, while B and D ran 4 miles; so if the latter went 2 miles per hour, the former would have to go 70 miles per hour—rates which are certainly not "ordinary"! With 210 miles B and D go half the speed of A, and C goes three-quarters the speed of A, so you can give them reasonable rates.

41.—The Damaged Engine

The distance from Anglechester to Clinkerton must be 200 miles. The train went 50 miles at 50 m.p.h. and 150 miles at 30 m.p.h. If the accident had occured 50 miles farther on, it would have gone 100 miles at 50 m.p.h. and 100 miles at 30 m.p.h.

42.—The Puzzle of the Runners

While Brown has only run $\frac{1}{6}$ or $\frac{4}{24}$ of the course, Tompkins has run the remainder $\frac{5}{6}$, less $\frac{1}{8}$, or $\frac{17}{24}$. Therefore Tompkins's pace is $\frac{17}{4}$ times that of Brown. Brown has now $\frac{5}{6}$ of the course to run, whereas Tompkins has only $\frac{1}{8}$. Therefore Brown must go five times as fast as Tompkins, or increase his own speed to five times $\frac{17}{4}$, that is $\frac{85}{4}$ times as fast as he went at first. But the question was not how many times as fast, but "how much faster," and $\frac{85}{4}$ times as fast is equal to $\frac{81}{4}$ times faster than Brown's original speed. The correct answer is therefore $20\frac{1}{4}$ times faster, though in practice probably impossible.

43.—The Two Ships

The error lies in assuming that the average speeds are equal. They are not. The first ship does a mile in a twelfth of an hour outwards and in an eighth of an hour homewards. Half of the sum of these fractions is five forty-eighths. Therefore the ship's average speed for the four hundred miles is a mile in five forty-eighths of an hour. The average speed of the second ship is a mile in one-tenth of an hour.

44.—Find the Distance

The distance between the two places must have been 18 miles. The meeting-points were 10 miles from A—— and 12 miles from B——. Simply multiply 10 (the first distance) by 3 and deduct the second distance, 12. Could anything be simpler? Try other distances for the meeting-points (taking care that the first meeting distance is more than two-thirds of the second) and you will find the little rule will always work.

45.—The Man and the Dog

The dog's speed was 16 miles per hour. The following facts will give the reader clues to the general solution. The distance remaining to be walked side by side with the dog was 81 feet, the fourth power of 3 (for the dog returned four times), and the distance to the end of the road was 625 feet, the fourth power of 5. Then the difference between the speeds (in miles per hour) of man and dog (that is, 12) and the sum of the speeds (20) must be in the same ratio, 3 to 5, as is the case.

46.—Baxter's Dog

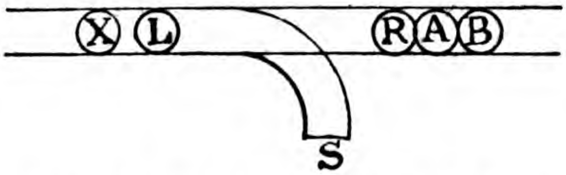
It is obvious that Baxter will overtake Anderson in one hour, for each will be four miles from the hotel in the same direction. Then, as the dog has been running uniformly at ten miles an hour during that hour, he must have run ten miles! When a friend put this problem before a French professor of mathematics, he exclaimed: "*Mon Dieu, quelle série!*" quite overlooking the simple manner of solution.

47.—The Runner's Refreshment

As the radius is *t*, the diameter is 2*t*. The diameter multiplied by Π (the Greek letter *pi*) gives us the circumference, 2*t*Π miles. As he goes round *n* times, 2*t*Π*n* equals the number of miles run, and, as he drinks *s* quarts per mile, he consumes 2*t*Π*ns* quarts. We can put the factors in any order: therefore the answer is 2Π*nts* (two pints) or one quart!

48.—Railway Shunting

Make a rough sketch like our diagram and use five counters marked X, L, R, A, and B. The engines are L and R, and the two cars on the right A and B. The three cars on the left are never separated, so we call them X. The side-track is marked S. Now, play as follows: R to left, R to S, X L to right, R to left, X L A to left, L takes A to S, L to left, X L to right, R to A, R A to left,



X L B to left, L takes B to S, L to left, L X right away, R A to B, R A B right away. Fourteen moves, because the first and third moves (R to left and X L to right) do not involve a change of direction. It cannot be done in fewer moves.

49.—Exploring the Desert

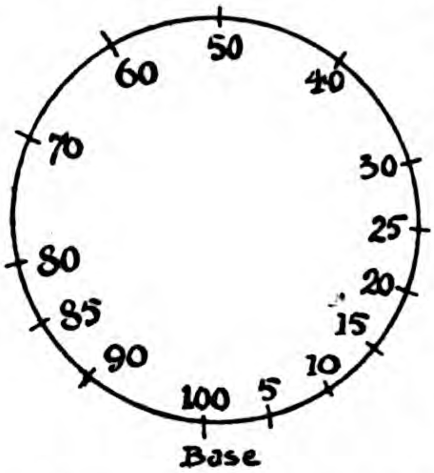
The nine men, A, B, C, D, E, F, G, H, J, all go 40 miles together on the 1 gall. in their engine tanks, when A transfers 1 gall. to each of the other eight and has 1 gall. left to return home. The eight go another 40 miles, when B transfers 1 gall. to each of the other seven and has 2 galls. to take him home. The seven go another 40 miles, when C transfers 1 gall. to each of the six others and returns home on the remaining 3 galls. The six go another 40 miles, when D gives each of five 1 gall. and returns home. The five go 40 miles, when E gives each of four 1 gall. and returns home. The four go another 40 miles, when F gives each of three 1 gall. and returns home. The three go 40 miles, when G gives each of two 1 gall. and returns home. The two go 40 miles, when H gives 1 gall. to J and returns home. Finally, the last man, J, goes another 40 miles and then has 9 galls. to take him home. Thus J has gone 360 miles out and home, the greatest distance in a straight line that could be reached under the conditions.

50.—Exploring Mount Neverest

Dump 5 rations at 90-mile point and return to base (5 days). Dump 1 at 85 and return to 90 (1 day). Dump 1 at 80 and return to 90 (1 day). Dump 1 at 80, return to 85, pick up 1 and dump at 80 (1 day). Dump 1 at 70 and return to 80 (1 day). Return to base (1 day). We have thus left 1 ration at 70 and 1 at 90. Dump 1 at 5 and return to base (1 day). If he must walk 20 miles he can do so by going to 10 and returning to base.

Dump 4 at 10 and return to base (4 days). Dump 1 at 10 and return to 5; pick up 1 and dump at 10 (1 day). Dump 2 at 20 and return to 10 (2 days). Dump 1 at 25 and return to 20 (1 day). Dump 1 at 30, return to 25, pick up 1 and dump at 30 (1 day). March to 70 (2 days). March to base (1½ days). Total, 23½ days.

A few claims have been made for fewer than 23½ days, but they were all based on tricks that were actually, or in spirit, forbidden, such as dumping only portions of the sealed boxes, making the man take a long march fasting, or making him eat his day's ration before starting so as to carry and dump two rations. In the last case he would really be carrying three rations, one inside and two on his back!



If the route had been in a straight march across a desert, the shortest time necessary would be 86 days, as follows:—

Dump 42 at 10, return to base (42 days). Dump 1 at 15, return to 10 (1 day). Dump 20 at 20 and return to 10 (20 days). Dump 1 at 20 and return to 15, pick up 1 and dump at 20 (1 day). Dump 10 at 30 and return to 20 (10 days). Dump 1 at 35, return to 30 (1 day). Dump 4 at 40 and return to 30 (4 days). Dump 1 at 40 and return to 35. Pick up 1 and dump at 40 (1 day). Dump 2 at 50 and return to 40 (2 days). Dump 1 at 55 and return to 50 (1 day). Dump 1 at 60 and return to 55. Pick up 1 and dump at 60 (1 day). March to base (2 days). Total, 86 days.

51.—An Exceptional Number

1 3 4 5 2. The successive numbers are 1, 2, 3, 4, 5, and 13 multiplied by 4 equals 52.

52.—The Five Cards

The number is either 3 9 1 5 7 or 5 7 1 3 9. In either case the product of the two pairs 39 and 57, minus 1, results in 2 2 2 2.

53.—Squares and Digits

If a square number terminates in similar digits those digits, must be 4, as in the case of 144, the square of 12. But there cannot be more than three equal digits, and therefore the smallest answer is 1444, the square of 38.

54.—The Two Additions

Arrange the figures in the following way:

173	85
4	92
177	177

and both sums add up alike.

55.—The Repeated Quartette

Multiply 273863 by 365 and the product is 99959995. Working the problem backwards, any number whatever consisting of eight figures with the first four repeated is divisible by 73 (and by 137) without remainder, and if it ends in 5 or 0 is divisible by 365 (or by 50005). Knowing this, the highest possible product can be written down at once.

56.—Easy Division

All we need do is to proceed as follows:

7—7101449275362318840579

1014492753623188405797

Divide 7 by 7, carry the 1 to the top line, divide again by 7 and carry the 0 to the top line, and so on until we come to a 7 in the bottom line without any remainder. Then stop, for the correct number is found. A solution may be found for any divisor and with any given figure at the beginning. A general examination of the question is very interesting.

If we take the divisor 2 we get the number 2–1 0–5 2–6–3 1 5 7 8–9 4–7 3 6–8–4–.

This is a complete circuit. The hyphens are at the places where there is no remainder when divided by 2. Note that the successive figures immediately following a hyphen are 1, 5, 6, 3, 9, 7, 8, 4, 2, so if I want my number to begin with an 8 I start

8 4 2 1 0 5, etc., taking the 8 that follows a hyphen. Where there is a complete circuit, as in this case, and in the case of the divisors, 3, 6, and 11, the number of figures in the number will always be 10 times the divisor less 2. If you try the divisor 4, there are five broken circuits. Thus, 4–1 0 2 5 6– will give you numbers beginning with 4 or 1; or 2 0–5 1 2–8– with 2, 5 or 8; or 7 1 7 9 4 8– beginning with 7; or 3 0 7 6–9 2– with 9 or 3; or 6 1 5 3 8 4– for a number beginning with 6. Some divisors, like 5 and 9, though producing broken circuits, require the same total of figures as if they were complete circuits. Our divisor 7 gives three broken circuits: the one shown above for the initial figures, 7, 1 or 4, another for 5, 8 or 2, and a third for 6, 9 or 3.

57.—A Misunderstanding

We may divide 8 5 7 1 4 2 by 3 by simply transferring the 2 from the end to the beginning. Or 4 2 8 5 7 1, by transferring the 1.

58.—The Two Fours

This is how 64 may be expressed by the use of only two fours with arithmetical signs:

The process of simplification shown should make everything quite clear.

$$\sqrt{\left(\sqrt{\sqrt{4}}\right)^{\underline{|4}}} = \sqrt{\left(\sqrt{2}\right)^{24}} = \sqrt{2^{12}} = \sqrt{4096} = 64$$

59.—The Two Digits

Of course it is entirely a matter of individual taste what arithmetical forms and signs are admissible, but I should personally draw the line at expressions introducing "log" and "anti-log," while perhaps admitting the combination sign as in the last example, where 8, taken 2 at a time, equals 28.

Solutions are as follows:

$$25=5^2 \text{ and } 36=6\times\underline{|3}.$$

$$64=(\sqrt{4})^6 \text{ or } \sqrt{(4^6)};\ 24=\underline{|\sqrt{4+2}};\ 4^2=2^4;$$

$$71=\sqrt{1+\underline{|7}};\ 15=5\div\sqrt{\cdot\dot{1}};\ 25=5\div\cdot 2;\ 28={}^8C^2.$$

60.—Digital Coincidences

If we multiply 497 by 2 we get the product 994. If we add together 497 and 2 we get 499. The figures are the same in both cases. Also 263 multiplied by 2 and added to 2 will give 526 and 265 respectively.

61.—Palindromic Square Numbers

The square of 836 is 698896, which contains an even number of digits and reads backwards and forwards alike. I do not know of any smaller square number containing an even number of figures that is a palindrome.

62.—Factorizing

If the number of noughts enclosed by the two ones is 2 added to any multiple of 3, two factors can always be written down at once in this curious way. $1001=11\times 91$; $1000001=101\times 9901$; $1000000001=1001\times 999001$; $1000000000001=10001\times 99990001$. The last is our required answer, and $10001=73\times 137$. The multiple of 3 in 11 is 3: therefore we insert 3 noughts in each factor and one more 9. If our number contained 101 noughts, as I suggested, then the multiple of 3 is 33 and the factors will contain 33 noughts and 34 nines in the form shown. If the number of noughts in the number be even you can get two factors in this way: $1001=11\times 91$; $100001=11\times 9091$; $10000001=11\times 909091$, and so on.

63.—Find the Factors

The factors of 1234567890 are $2\times 3\times 3\times 5\times 3607\times 3803$. If we multiply 3607 by 10 and 3803 by 9 we get the two composite factors 36070 and 34227, which multiplied together produce 1234567890 and have the least possible difference between them.

64.—Dividing by Eleven

To be divisible by 11, four of the alternate digits must sum to 17 and the remaining five to 28, or four to 28 and five to 17, Thus, in the example I gave (4 8 2 5 3 9 7 6 1), 4, 2, 3, 7, 1 sum to 17, and 8, 5, 9, 6 to 28. Now four digits will sum to 17 in 9 different ways and five to 17 in 2 ways, making 11 together. In each of the 11 cases 4 may be permuted in 24 ways and 5 in

120 ways, or together in 2880 ways. So that 2880×11=31,680 ways. As the nine digits can be permuted in 362,880 ways, the chances are just 115 to 11 against a haphazard arrangement being divisible by 11.

65.—Dividing by 37

Write beneath the number successively, from right to left, the numbers 1, 10, 11, as follows:

4	9	1	2	9	3	0	8	2	1	3
10	1	11	10	1	11	10	1	11	10	1

Now, regarding the lower figures as multipliers, add together all the products of 1 and 10 and deduct all the products by 11. This is the same as adding 13, 08, 29, and 49 together (99) and deducting eleven times 2 plus 3 plus 1 (66). The difference, 33, will be the remainder when the large number is divided by 37. Here is the key. If we divide 1, 10, 100, 1000, etc., by 37 we get successively the remainders 1, 10, 26, but for convenience we deduct the 26 from 37 and call it *minus* 11. If you try 49,629,708,213 you will find the minus or negative total 165, or in excess of the positive 99. The difference is 66. Deduct 37 and you get 29. But as the result is minus, deduct it from 37 and you have 8 as correct answer. You can now find the method for other prime divisors. The cases of 7 and 13 are easy. In the former case you write 1, 3, 2 (1, 3, 2), 1, 3, 2, etc., from right to left, the bracketed numbers being minus. In the latter case, 1 (3, 4, 1), 3, 4, 1 (3, 4, 1), etc.

66.—Another 37 Division

Call the required numbers ABCABCABC. If the sum of the A digits, the B digits, and the C digits respectively are:

A	B	C
18	19	8
15	15	15
12	11	22
19	8	18
22	12	11
8	18	19
11	22	12

then in the first three groups 11A—10 B=C. In the next two groups 11A—10B—C=111 (3×37); and in the last two groups 10 B+C—11 A=111. It does not matter what the figures are, but if they comply with these conditions we can always divide by 37. Here is an example of the first case—ABCABCABC, 984763251, where the 3 A's sum to 18, the 3 B's to 19, and the 3 C's to 8. You will find 22 cases with the first equation, 10 with the second, and 10 with the third, making 42 fundamental cases in all. But in every case the A figures may be permuted in 6 ways, and the B figures in 6 ways, and the C figures in 6 ways, making 6×6×6=216, which multiplied by 42 gives the answer 9072 ways divisible by 37. As the 9 digits may be permuted in 362880 ways the chances are $\frac{9072}{362880}$ or $\frac{1}{40}$ or 39 to 1 against divisibility.

67.—A Digital Difficulty

There are four solutions, as follows: 2,438,195,760; 3,785,942,160; 4,753,869,120; 4,876,391,520. The last figure must be 0. Any arrangement with an even figure next to the 0 will be divisible by 2, 3, 4, 5, 6, 9, 10, 12, 15, and 18. We have therefore only to consider 7, 11, 13, 16, and 17. To be divisible by 11 the odd digits must sum to 28 and the even to 17, or vice versa. To be divisible by 7×11×13=1001, if we ignore the 0, the numbers formed by the first three and the last three digits must sum to the middle three. (Note that the third case above is really 474—1386—912, with the 1 carried forward and added to the 4.) But, as the large majority have found, we cannot do better than take the lowest multiple (82) of the L.C.M. of the divisors (12,252,240), which gives ten figures (this is 1,004,683,680), and keep on adding that L.C.M. until all digits are different. The 199th multiple will give us the first answer, 309 the second, 388 the third, and 398 the fourth. The work can be considerably shortened by leaping over groups where figures will obviously be repeated, and all the answers may be obtained in about twenty minutes by the use of a calculating machine.

68.—Threes and Sevens

The smallest number possible is 3,333,377,733, which is divisible by 3 and by 7, and the sum of its digits (42) also divisible by 3 and by 7. There must be at fewest three 7's and seven 3's, and the 7's must be placed as far to the right as possible.

69.—Root Extraction

The only other numbers are 5832, 17,576, and 19,683, the cube roots of which may be correctly obtained by merely adding the digits, which come to 18, 26, and 27 respectively.

70.—The Solitary Seven

The restored simple division sum is as follows :

```
124)12128316(97809
    1116
    ----
     968
     868
     ---
     1003
      992
     ----
       1116
       1116
       ----
```

71.—A Complete Skeleton

```
625)631938(1011·1008
    625
    ---
      693
      625
      ---
       688
       625
       ---
        630
        625
        ---
          5000
          5000
          ----
```

The three o's that must occur at the bottom show that the divisor is a multiple of 1000. The factors therefore can only be 5, 5, 5, 2, 2, 2, x, where x is less than 10. To form the three-figure divisor, one factor at least must be 5, and therefore the last figure must be 5 or 0. The subtraction from the single o near the bottom shows that it is a 5 and at once gives us the 5000. The factor 2 being excluded from the divisor (or it could not end in a 5), the final figure in the quotient must be 8 ($2 \times 2 \times 2$), and the divisor 625, making x a fourth 5. The rest is quite easy.

72.—Alphabetical Sums

The answer is as follows:

```
35)19775(565
   175
   ---
    227
    210
    ---
     175
     175
     ---
```

It is clear that R cannot be 1: it must therefore be either 5 or 6 to produce the R in the second line. Then D must be 0 to give the V in the fifth line. Also M must be 1, 2, 3, or 4, if R is 5, but may be 5 if R is 6. Again, S must be an even number if R is 5, to make D a 0, and if R is 6, then S must be 5. When we have discovered and noted these facts, only a little trial is necessary.

73.—Alphabetical Arithmetic

```
         93
4 × 17 = 68
         --
         25
         --
```

74.—Queer Division

The smallest number that fulfils the conditions is 35641667749. Other numbers that will serve may be obtained by adding 4689557361o or any multiple of it.

75.—A Teasing Legacy

The best answer, so far as I have been able to discover, is the following:

£	s.	d.
296	-	5
8	-	7
£304	1	-

76.—The Nine Volumes

Arrange the books in the following order on the three shelves

21	8	7
		3
64	5	9

and the little multiplication sum in money is perfect.

There is a second solution:

8	5	7
		3
24	16	9

77.—The Ten Volumes

The answer is as follows:

81	6	9
		4
325	7	0

78.—The Miller's Toll

There must have been one bushel and one-ninth, which after taking the one-tenth as toll, would leave exactly one bushel.

79.—Egg Laying

The answer is half a hen and a half hen; that is, one hen. If one and a half hens lay one and a half eggs in one and a half days, one hen will lay one egg in one and a half days. And a hen who lays better by half will lay one and a half eggs in one and a half days, or one egg per day. So she will lay ten and a half (half a score and a half) in ten and a half days (a week and a half).

80.—The Flocks of Sheep

Adam must have possessed 60 sheep, Ben 50, Claude 40, and Dan 30. If the distributions described had taken place, each brother would have then had 45 sheep.

81.—Pussy and the Mouse

You have simply to divide the given number by 8. If there be no remainder, then it is the second barrel. If the remainder be 1, 2, 3, 4, or 5, then that remainder indicates the number of the barrel. If you get a remainder greater than 5, just deduct it from 10 and you have the required barrel. Now 500 divided by 8 leaves the remainder 4, so that the barrel marked 4 was the one that contained the mouse.

82.—Army Figures

The five brigades contained respectively 5670, 6615, 3240, 2730, and 2772 men. Represent all the fractions with the common denominator 12,012, and the numerators will be 4004, 3432, 7007, 8316, and 8190. Combining all the *different* factors contained in these numbers, we get 7,567,560, which, divided by each number in turn, gives us 1890, 2205, 1080, 910, and 924. To fulfil the condition that the division contained a "little over 20,000 men," we multiply these by 3 and have the correct total—21,027.

83.—A Critical Vote

There must have been 207 voters in all. At first 115 voted for the motion and 92 against, the majority of 23 being just a quarter of 92. But when the 12 who could not sit down were transferred to the other side, 103 voted for the motion and 104 against. So it was defeated by 1 vote.

84.—The Three Brothers

Arthur could do the work in $14\frac{34}{49}$ days, Benjamin in $17\frac{23}{41}$ days, and Charles in $23\frac{7}{31}$ days.

85.—The House Number

The numbers of the houses on each side will add up alike if the number of the house be 1 and there are no other houses; if the number be 6, with 8 houses in all; if 35, with 49 houses; if 204, with 288 houses; if 1189, with 1681 houses; and so on. But it was known that there were more than 50 and fewer than 500 houses, so we are limited to a single case, and the number of the house must have been 204.

Find the integral solutions of $\frac{x^2+x}{2}=y^2$. Then we get the answers:—

x=Number of houses.	y=Number of particular house.
1	1
8	6
49	35
288	204
1681	1189

and so on.

86.—A New Street Puzzle

Brown's number must have been 84 and there were 119 houses. The numbers from 1 to 84 sum to 3570 and those from 1 to 119 to 7140, which is just double, as stated.

Write out the successive solutions to the Pellian equation (explained on page 164 in my book, *Amusements in Mathematics*) $2x^2-1=y^2$, thus:—

x	y
1	1
5	7
29	41
169	239
985	1393

and so on. Then the integral half of any value of x will give you the house number and the integral half of y the total number of houses. Thus (ignoring the values 0—0) we get 2–3, 14–20, 84–119, 492–696, etc.

87.—Another Street Puzzle

On the odd side of the street the house must have been No. 239, and there were 169 houses on that side. On the even side of the street the house must have been No. 408, and there were 288 houses.

In the first case, find integral solution of $2x^2-1=y^2$. Then we get the answers :—

x=Number of houses.	y=Number of particular house.
1	1
5	7
29	41
169	239
985	1393

and so on.

In the second case, find integral solution of $2(x^2+x)=y^2$. Then we get the answers :—

x=Number of houses.	y=Number of particular house.
1	2
8	12
49	70
288	408
1681	2378

and so on.

These two cases, and the two previous puzzles, all involve the well-known Pellian equation and are related.

88.—Correcting an Error

Hilda's blunder amounted to multiplying by 49, instead of by 409. Divide the error by the difference (328,320 by 360) and you will get the required number—912.

89.—The Seventeen Horses

The farmer's seventeen horses were to be divided in the *proportions* one-half, one-third, and one-ninth. It was not stated that the sons were to receive those *fractions* of seventeen. The proportions are thus nine-eighteenths, six-eighteenths, and two-eighteenths, so if the sons receive respectively nine, six, and two horses each, the terms of the legacy will be exactly carried out. Therefore, the ridiculous old method described does happen to give a correct solution.

A correspondent suggested to me the ingenious solution :—

$$\begin{aligned} \tfrac{1}{2}, \text{ i.e. } 2 \text{ and } 1 \text{ over} &= 3 \\ \tfrac{1}{3}, \text{ i.e. } 3 \text{ and } 1 \text{ over} &= 4 \\ \tfrac{1}{9}, \text{ i.e. } 9 \text{ and } 1 \text{ over} &= 10 \\ & \overline{17} \end{aligned}$$

90.—Equal Perimeters

The six right-angled triangles having each the same, and the smallest possible, perimeter (720), are the following : 180, 240, 300 ; 120, 288, 312 ; 144, 270, 306 ; 72, 320, 328 ; 45, 336, 339 ; 80, 315, 325.

91.—Counting the Wounded

The three fractions are respectively $\frac{40}{60}$, $\frac{45}{60}$, and $\frac{48}{60}$. Add together 40, 45, and 48, and deduct twice 60. The result is 13, as the minimum number for every 60 patients. Therefore as the minimum (who could have each lost an eye, an arm, and a leg) was 26, the number of patients must have been 120.

92.—A Cow's Progeny

Note the following series of numbers, first considered by Leonardo Fibonacci (born at Pisa in 1175), who practically introduced into Christian Europe our Arabic numerals :—

0, 1, 1, 2, 3, 5, 8, 13, 21, 34 . . . 46,368.

Each successive number is the sum of the two preceding it. The sum of all numbers from the beginning will always equal 1 less than the next but one term. Twice any term, added to the preceding term, equals the next but one term. Now, there would be 0 calf in the first year, 1 in the second, 1 in the third, 2 in the fourth, and so on, as in the series. The twenty-fifth term is 46,368, and if we add all the twenty-five terms or years together we get the result, 121,392, as the correct answer. But we need not do that addition. When we have the twenty-fourth and twenty-fifth terms we simply say (46,368 multiplied by 2) plus 28,657 equals 121,393, from which we deduct 1.

93.—Sum Equals Product

If you take any number in combination with 1 and a fraction whose numerator is 1 and its denominator 1 less than the given number, then the sum and product will always be the same. Thus, 3 and $1\frac{1}{2}$, 4 and $1\frac{1}{3}$, 5 and $1\frac{1}{4}$, and so on. Therefore, when I was given 987654321, I immediately wrote down $1\frac{1}{987654320}$ and their sum and product is $987654322\frac{1}{987654320}$.

Now, the reason why 2 and 2 are often regarded as an exceptional case is that the denominator is 1, thus 2 and $1\frac{1}{1}$, which happens to make the second a whole number, 2. But it will be seen that it is really subject to the universal rule. A number may be fractional as well as whole, and I did not make it a condition that we must find a whole number, for that would be impossible except in the case of 2 and 2. Of course, decimal fractions may be used, such as 6 and 1·2, or 11 and 1·1, or 26 and 1·04.

In short, the general solution is $n \genfrac{}{}{0pt}{}{+}{\times} \frac{n}{n-1} = (n+1) + \frac{1}{n-1}$

94.—Adding their Cubes

The required number is 153. The cubes of 1, 5, and 3 are respectively 1, 125, and 27, and these added together make 153.

95.—Squares and Cubes

The solution in the smallest possible numbers appears to be this :—

$$10^2-6^2=100-36=64=4^3.$$
$$10^3-6^3=1000-216=784=28^2.$$

96.—Concerning a Cube

(1). 6 feet. (2). 1·57 feet nearly. (3). $\frac{1}{36}$ foot.

97.—A Common Divisor

Since the numbers have a common factor plus the same remainder, if the numbers are subtracted from one another in the manner shown below the results must contain the common factor without the remainder

508,811	723,217
480,608	508,811
28,203	214,406

Here the prime factors of 28,203 are 3, 7, 17, 79, and those of 214,406 are 2, 23, 59, 79. And the only factor common to both is 79. Therefore the required divisor is 79, and the common remainder will be found to be 51. Simple, is it not ?

98.—Curious Multiplication

In the first column write in the successive remainders, which are 1 0 0 0 0 1 1, or reversed, 1 1 0 0 0 0 1. This is 97 in the binary scale of notation, or 1 plus 2^5 plus 2^6. In the second column (after rejecting the numbers opposite to the remainder 0) we add together 23×1, 23×2^5, 23×2^6, equals 2231. The whole effect of the process is now obvious. It is merely an operation in the binary scale.

99.—The Rejected Gun

The experts were right. The gun ought to have fired sixty shots in fifty-nine minutes if it really fired a shot a minute. The time counts from the first shot, so that the second would be fired at the close of the first minute, the third at the close of the second minute, and so on. In the same way, if you put up sixty posts in a straight line, a yard apart, they will extend a length of fifty-nine yards, not sixty.

100.—Odds and Evens

If the result given is odd, the odd number is in the right hand ; if even, the even number is in the right hand. An even number multiplied by either an odd or even number will produce an even number. An odd number multiplied by an odd number will alone produce odd. And if the result given is even, both products added must be even ; if the result is odd, one product is even and the other odd. The former result can only happen when the even number is multiplied by the 7 ; the latter when the odd number is multiplied by 7.

101.—Twenty Questions

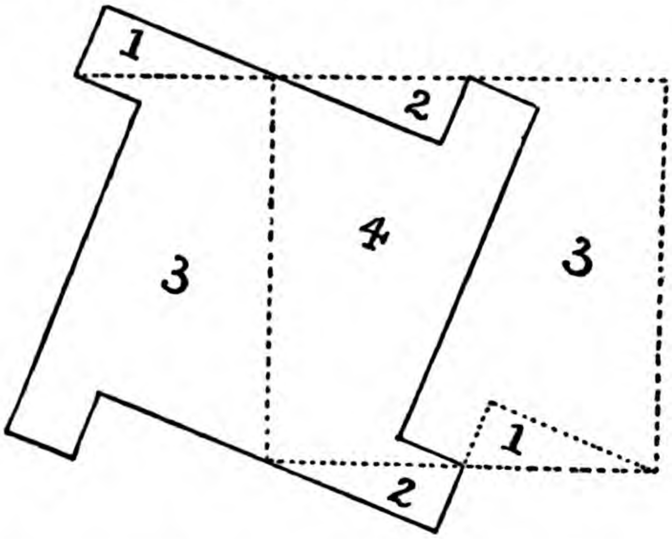
There are various ways of solving this puzzle, but the simplest is, I think, the following. Suppose the six-figure number to be 843712. (1) If you divide it by 2, is there any remainder? No. (2) If you divide the quotient by 2, is there any remainder? No. (3) If you divide again by 2, is there any remainder? No. Your twenty questions will be all the same, and writing from right to left, you put down a nought for the answer "No," and 1 for the answer "Yes." The result will be that after the twentieth question you will get 11001101111111000000. This is 843712 written in the binary scale. Dropping the final 0 in the units place, the first 1 is the sixth figure backwards. Add together the 6th, 7th, 8th, 9th, 10th, 11th, 12th, 14th, 15th, 18th, and 19th powers of 2 and you will get 843712 in our denary scale. If the number is a low one like 100000, seventeen questions would be sufficient if only you knew that the 0 had been reached in the quotient, but the three final questions will merely add three noughts to the left of your binary number. But to prevent quibbles as to infinities, etc., it is best to state before beginning your questions that nought divided by 2 is understood to mean nought with no remainder.

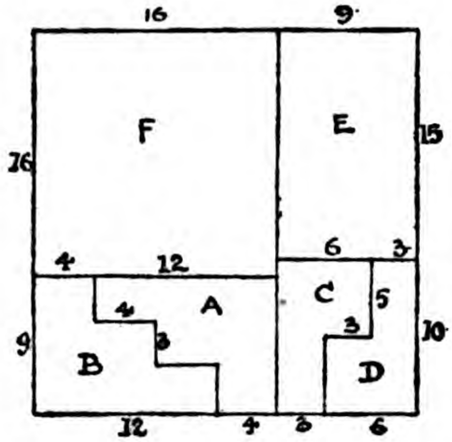
102.—The Nine Barrels

There are forty-two different arrangements. The positions of the 1 and 9 are fixed. Always place the 2 beneath the 1. Then, if the 3 be beneath the 2 there are five arrangements. If the 3 be to the right of the 1 there are five arrangements with 4 under the 2, five with 5 under the 2, four with 6 under 2, two with 7 under 2. We have thus twenty-one arrangements in all. But the 2 might have been always to the right of 1, instead of beneath, and then we get twenty-one reversed and reflected arrangements (practically similar), making forty-two in all. Either the 4, 5, or 6 must always be in the centre.

103.—A New Cutting-out Puzzle

Make the cuts as shown in the illustration and fit the pieces into the places enclosed by the dotted lines.



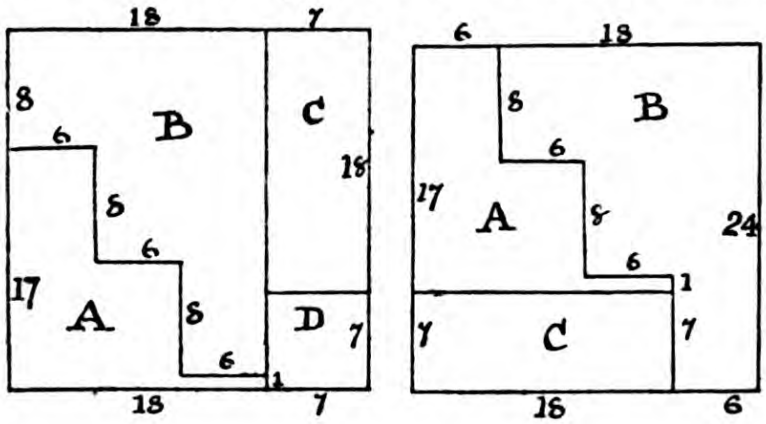


104.—The Square Table-top

The illustration shows the simplest, and I think the prettiest, solution in six pieces. Move piece A up a step on B and you have the original piece 12×12. Move C up a step on D and the two pieces will join E and form the square 15×15. The piece 16×16 is not cut.

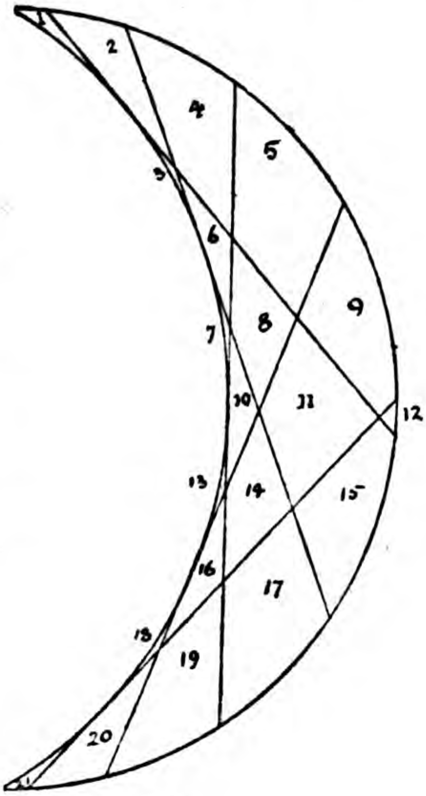
105.—The Squares of Veneer

The sides of the two squares must be 24 inches and 7 inches



respectively. Make the cuts as in the first diagram and the pieces A, B, and C will form a perfect square as in the second diagram. The square D is cut out intact.

106.—Dissecting the Moon

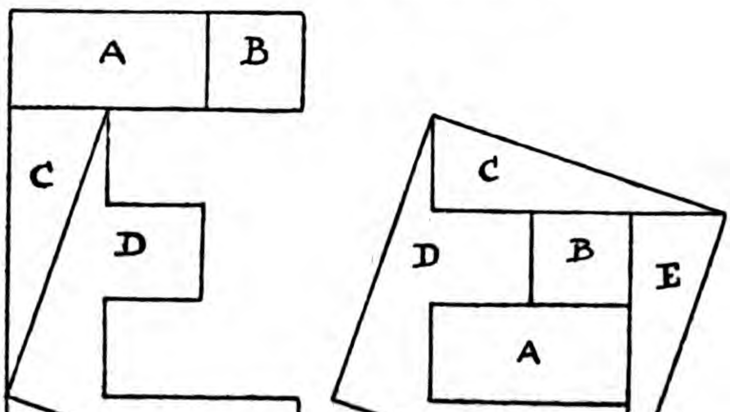


The illustration shows that the five cuts can be so cunningly made as to produce as many as twenty-one pieces.

Calling the number of cuts n, then in the case of a circle the maximum number of pieces will be $\frac{n^2+n}{2}+1$, but in the case of the crescent it will be $\frac{n^2+3n}{2}+1$.

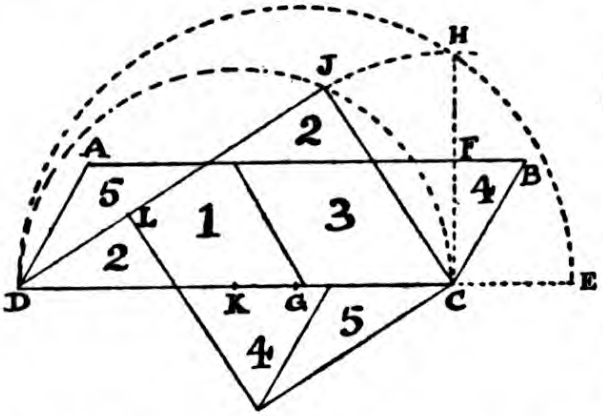
107.—Dissecting the Letter E

The illustration shows how to cut the letter into five pieces that will fit together to form a perfect square. It can be done in four pieces if you are allowed to turn pieces over. Readers may like to find for themselves the method.



108.—HEXAGON TO SQUARE

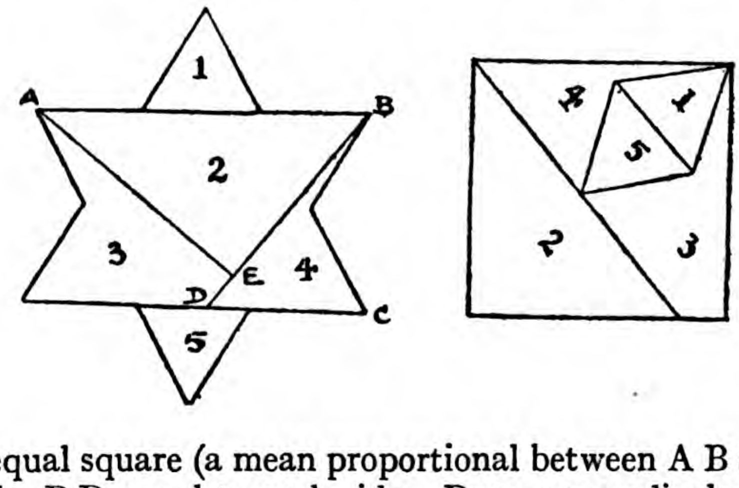
Cut your hexagon in half and place the two parts together to form the figure A B C D. Continue the line D C to E, making C E equal to the height C F. Then, with the point of your compasses at G, describe the semicircle D H E, and draw the line C H perpendicular to D E. Now, C H is the mean proportional between D C and C E, and therefore the side of the required square. From C describe the arc H J, and with the point of your compasses at K describe the semicircle D J C. Draw C J and D J. Make J L equal to J C, and complete the square. The rest requires no explanation.



This solution was first published by me in the *Weekly Dispatch*, in August, 1901.

109.—SQUARING A STAR

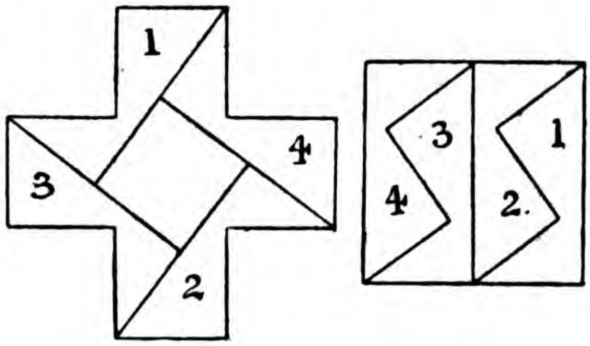
I give the very neat solution by Mr. E. B. Escott, of Chicago, Illinois. The five pieces of the star form a perfect square. Find



side of equal square (a mean proportional between A B and B C) and make B D equal to such side. Drop perpendicular from A on B D at E and A E will equal B D. The rest is obvious.

110.—The Mutilated Cross

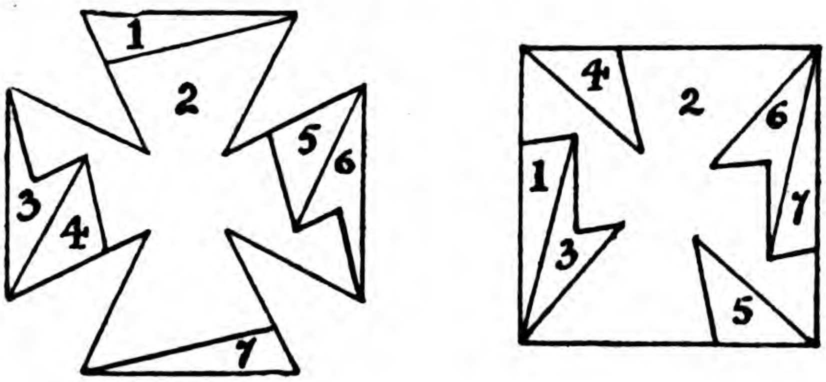
The illustration shows clearly how to cut the mutilated cross into four pieces to form a square. Just continue each side of the square until you strike a corner, and there you are!



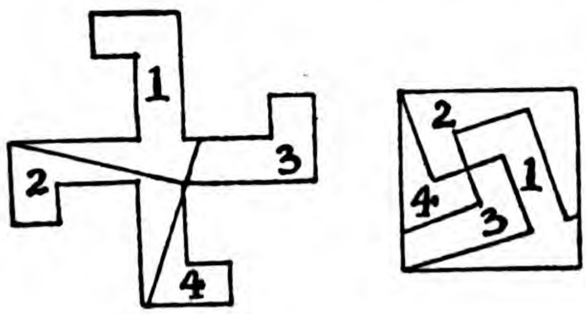
111.—The Victoria Cross

The illustration will show how to cut the cross into seven pieces to form a square.

This solution was sent to me by Mr. A. E. Hill.



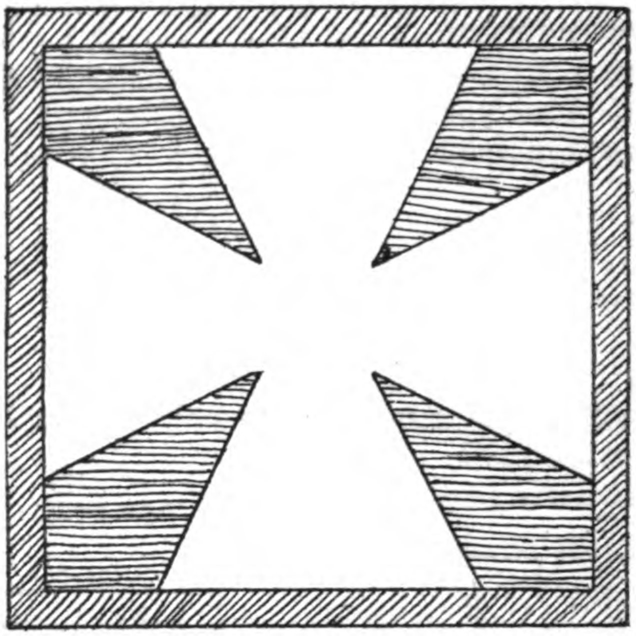
112.—Squaring the Swastika

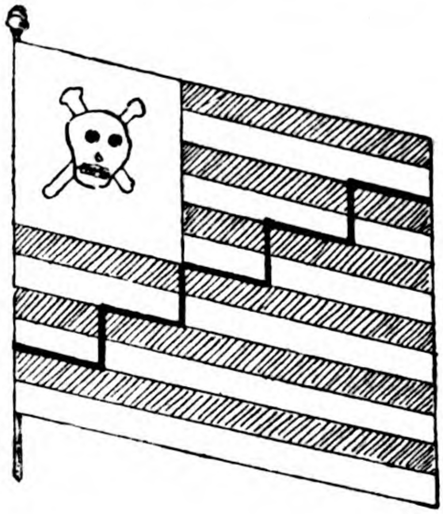


The illustration shows how the swastika should be cut into four parts and placed together to form a square. The direction of the nearly horizontal cut is obvious, the other is at right angles to it.

113.—The Maltese Cross

Cut the star in four pieces across the centre, and place them in the four corners of the frame. Then you have a perfect Maltese Cross in white, as indicated.





114.—The Pirates' Flag

The illustration will show that the flag need only be cut in two pieces—along the zigzag line. If the lower piece is then moved up one step we shall get a flag with the required ten stripes.

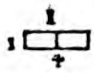
115.—The Carpenter's Puzzle

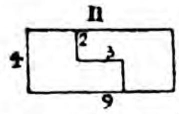
In order that it may be cut in two pieces, on the step principle, to form a square, take any rectangular board whose sides are the squares of two consecutive whole numbers. Thus, in the following table, the square of 1 (1) and the square of 2 (4); or

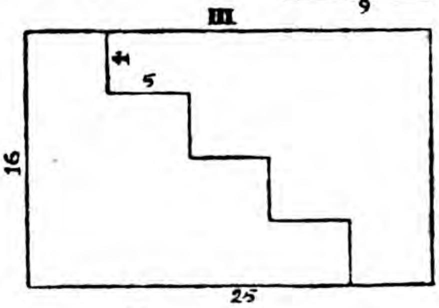
2 (4) and 3 (9); or 3 (9) and 4 (16);—and so on. The table may be extended to any length desired.

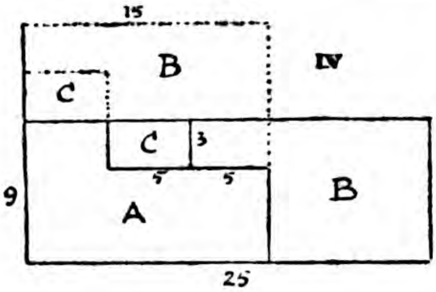
Sides.	No. of Steps.	Side of Square.
1×4	1	2
4×9	2	6
9×16	3	12
16×25	4	20
25×36	5	30

Now, in Figure I is shown the simple case of a board 1×4, in Figure II the case of 4×9, and Figure III shows the case of 16×25. It will be seen that the number of steps increases regularly as we advance, but with the table they are easily found. Thus, for the case 16×25, as the side of the square will be 20, the steps will be 20—16=4 in height and 25—20=5 in breadth.







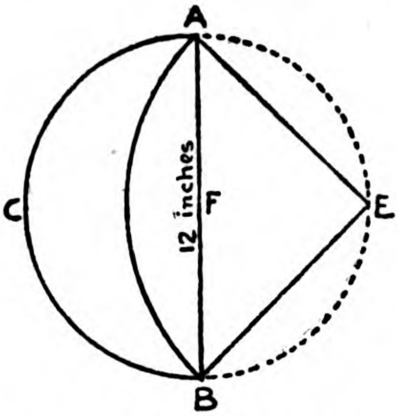


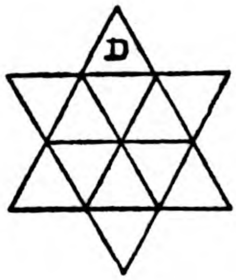
As the sides are square numbers, and two square numbers multiplied together always make another square, the area will always be a perfect square. But we must not conclude from this that a board, say, 9×25 would work just because its area is a square with side 15. Figure IV shows the best that can be done in that case, but there are three pieces instead of two as required. This is because 9 is not a multiple of the added height, 6, nor 25 a multiple of the reduced length, 10. Consequently, the steps cannot be formed.

Of course, any multiple of the sides will work. Thus, a board 8×18 is solved exactly like 4×9, in two steps, by just doubling all the measurements. Similarly, a board 4×6¼ will work, for it is the same ratio as 16×25, the steps being 1 in height and 1¼ in breadth. In the former case we should reduce it like a fraction, and in the second case multiply it by 4 to get rid of the fraction. Then we should see that 4×9 and 16×25 were, in each case, squares of consecutive numbers and know that a solution is possible.

116.—The Crescent and the Star

Though we cannot square a circle, certain portions of a circle may be squared, as Hippocrates of old first discovered. If we draw the circle in the diagram and then, with the point of the compasses at E, draw the arc B A, the area of the lune or crescent

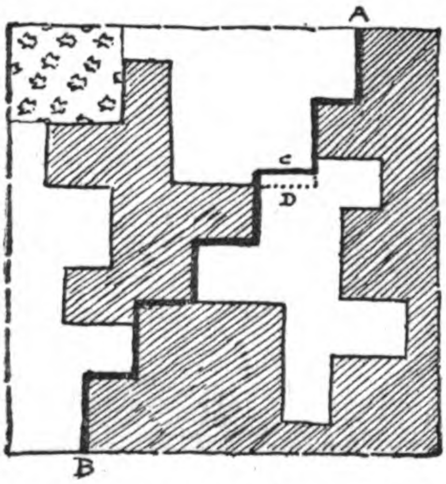




is exactly the same as the area of the triangle A B E. As we know the line A B to be 12 inches, the area of the triangle (and therefore of the crescent) is obviously 36 square inches. Also, as the triangle D is known to contain 3 square inches, the star, which is built up of twelve such triangles, contains 36 square inches. Therefore the areas of the crescent and the star were exactly the same.

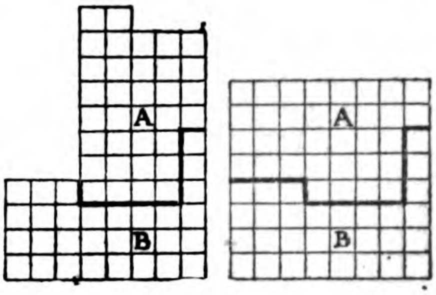
117.—The Patchwork Quilt.

Except for my warning the reader might have supposed that the dark zigzag line from A to B would solve the puzzle. But it will not, because the pieces are not of the same size and shape. It would be all right if we could go along the dotted line D instead of C, but that would mean cutting a piece. We must cut out all the shaded portion in one piece, which will exactly match the other. One portion of the patchwork is drawn in just to guide the eye when comparing with the original.



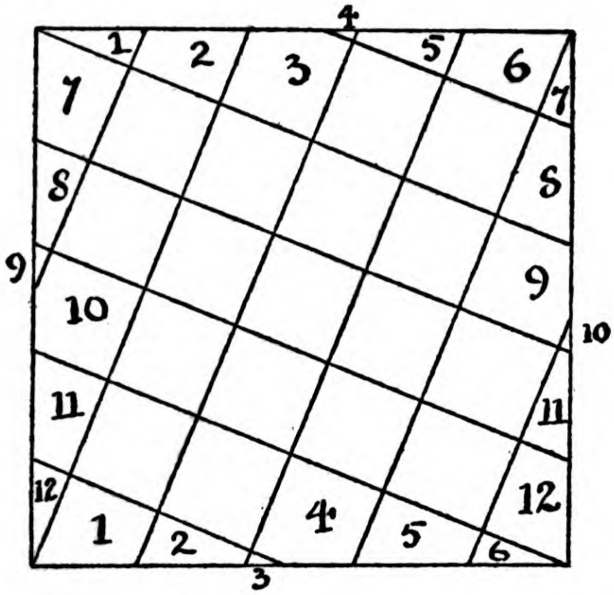
118.—The Improvised Draughts-board

The illustration shows how to cut into two pieces, A and B, that will fit together and form the square board.



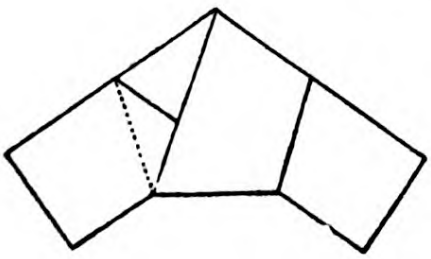
119.—Tessellated Pavements

The illustration shows how the square space may be covered with twenty-nine square tiles by laying down seventeen whole



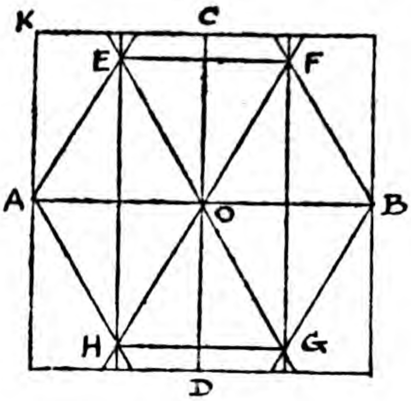
and cutting each of the remaining twelve tiles in two parts. Two parts having a similar number form a whole tile.

120.—The Ribbon Pentagon



Tie a ribbon of paper into a simple, ordinary knot and press flat, as shown in the illustration, and fold back at the dotted line. Then you have a regular pentagon, obtained with very little trouble.

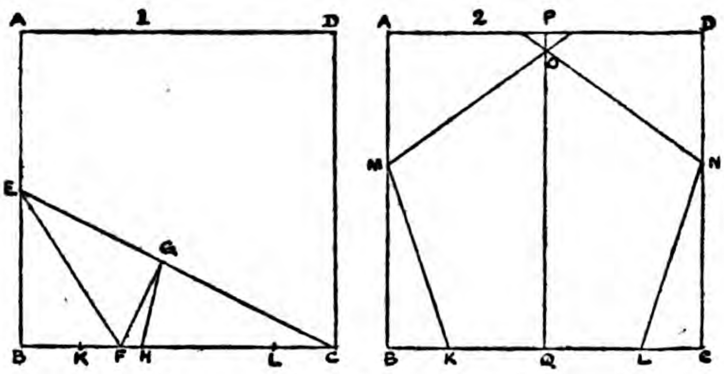
121.—Paper Folding



Fold through the mid-points of the opposite sides and get the lines A O B and C O D. Also fold E H and F G, bisecting A O and O B. Turn over A K so that K lies on the line E H, at the point E, and then fold A E and E O G. Similarly find H and fold A H and H O F. Now fold B F, B G, E F and H G, and E F B G H A E is the regular hexagon required.

122.—Folding a Pentagon

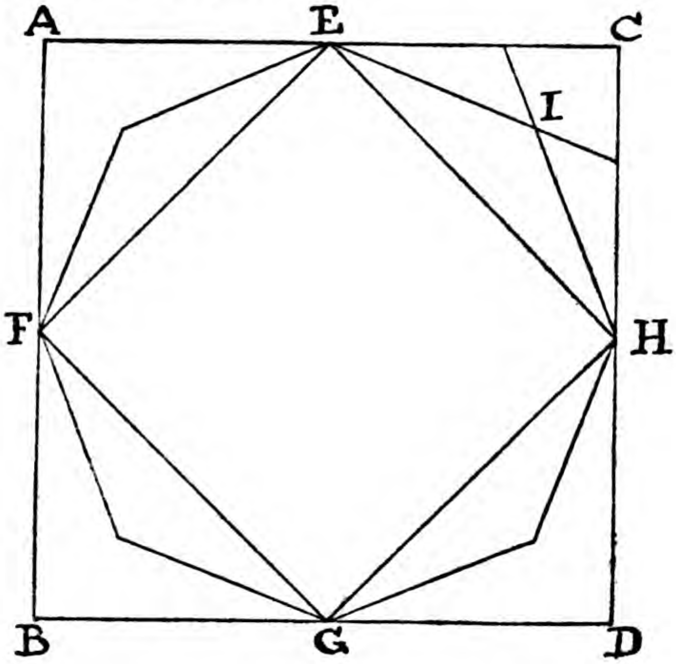
Fold A B on itself and find the mid-point E. Fold through E C. Lay E B on E C and fold so as to get E F and F G. Make C H equal to C G. Find K, the mid-point on B H, and make C L equal to B K. B C is said to be divided in medial section,



and we have found K L, the side of the pentagon. Now (see second diagram) lay K M and L N equal to K L, so that M and N may lie on B A and C D respectively. Fold P Q and lay M O and N O equal to K M and L N. Then K M O N L is the pentagon required. For this solution I am indebted to a little book, *Geometrical Exercises in Paper Folding*, by T. Sundara Row (Madras, 1893).

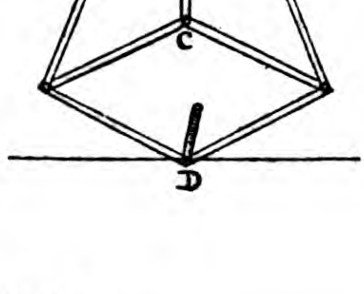
123.—Making an Octagon

By folding the edge C D over A B we can crease the middle points E and G. In a similar way we can find the points F and H, and then crease the square E H G F. Now fold C H on E H and E C on E H, and the point where the creases cross will be I. Proceed in the same way at the other three corners, and the regular octagon, as shown, will be marked out by the creases, and may be at once cut out.



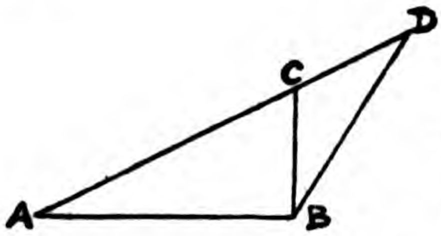
124.—Drawing a Straight Line

Take pieces of stout cardboard (they need not have straight edges !) and join them with shoemakers' eyelets, as in the illustration. The two long pieces are of equal length from centre of eyelet to eyelet, and the four pieces at the bottom forming a diamond are all of equal length. Nails or pins at A and B fasten the instrument to the table, B being so fixed that the distance from A to B is the same as from B to C. Then the pencil at D (if all is accurately made and adjusted) will draw the straight line shown.



125.—Making a Pentagon

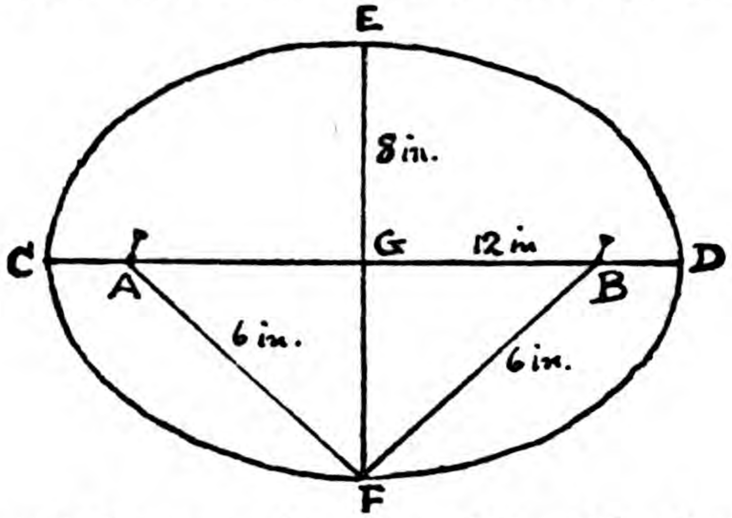
Let A B be the required 1 inch in length. Make B C perpendicular to A B and equal to half A B. Draw A C, which produce



until C D equals C B. Then join B D, and B D is the radius of the circumscribing circle. If you draw the circle the sides of the pentagon can be marked off—1 inch in length.

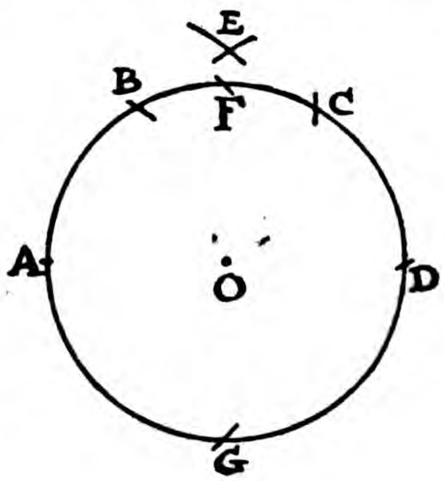
126.—DRAWING AN OVAL

Draw the two lines C D and E F at right angles (C D being equal to the required length, 12 inches, and E F to the required breadth, 8 inches), intersecting midway. Find the points A and B, so that A F and F B each equals half the length C D, that is



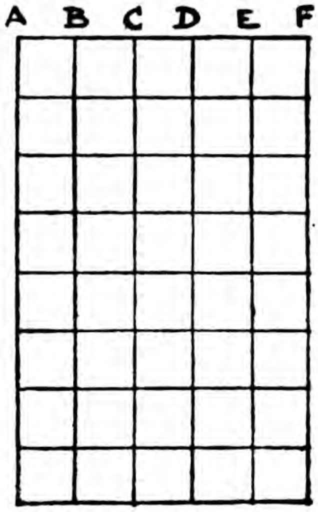
6 inches, and place your pins at A and B, making the length of your loop of string equal to A B F A. Say the distance C A=x. Then, when the pencil is at F the length of string is $12+(12-2x)=24-2x$, and when the pencil is at C the length of string is $2(12-x)=24-2x$ also, proving the correctness of the solution.

127.—WITH COMPASSES ONLY



In order to mark off the four corners of a square, using the compasses only, first describe a circle, as in the diagram. Then, with the compasses open at the same distance, and starting from any point, A, in the circumference, mark off the points B, C, D. Now, with the centres A and D and the distance A C, describe arcs at E, and the distance E O is the side of the square sought. If, therefore, we mark off F and G from A with this distance, the points A, F, D, G will be the four corners of a perfect square.

128.—Lines and Squares



If you draw fifteen lines in the manner shown in the diagram, you will have formed exactly one hundred squares. There are forty with sides of the length A B, twenty-eight of the length A C, eighteen of the length A D, ten of the length A E, and four squares with sides of the length A F, making one hundred in all. It is possible with fifteen straight lines to form 112 squares, but we were restricted to one hundred. With fourteen straight lines you cannot form more than ninety-one squares.

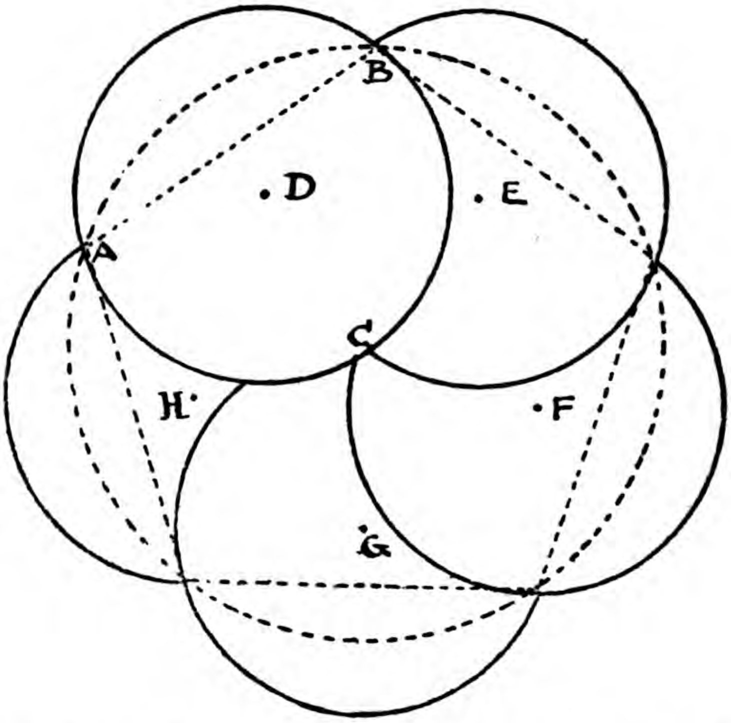
The general formula is that with n straight lines we can form as many as $\frac{(n-3)\ (n-1)\ (n+1)}{24}$ squares if n be odd, and $\frac{(n-2)\,n\,(n-1)}{24}$ if n be even.

If there are m straight lines at right angles to n straight lines, m being less than n, then $\frac{m(m-1)\ (3n-m-1)}{6}$=number of squares.

129.—The Circle and Discs

In our diagram the dotted lines represent the circumference of the red circle and an inscribed pentagon. The centre of both is C. Find D, a point equidistant from A, B, and C, and with radius A D draw the circle A B C. Five discs of this size will cover the circle if placed with their centres at D, E, F, G, and H. If the diameter of the large circle is 6 inches, the diameter of the discs is a little less than 4 inches, or 4 inches "to the nearest half-inch." It requires a little care and practice correctly to place the five discs without shifting, unless you make some secret markings that would not be noticed by others.

If readers require a closer approximation or further information as to the manner of solving this puzzle, I cannot do better than refer them to a paper, on "Solutions of Numerical Func-



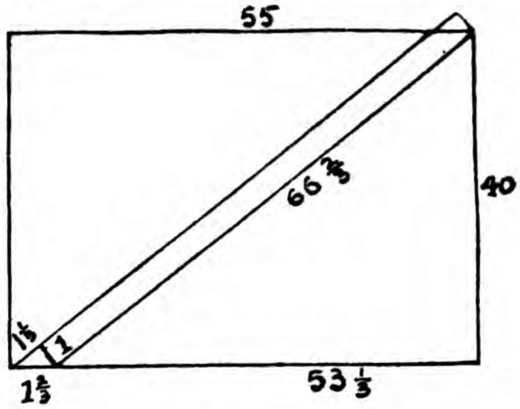
tional Equations, illustrated by an account of a Popular Puzzle and its Solution," by Mr. Eric H. Neville, in the *Proceedings of London Mathematical Society*, Series II, Vol. 14, Part 4. I will just add that covering is possible if the ratio of the two diameters exceeds ·6094185, and impossible if the ratio is less than ·6094180. In my case above, where all five discs touch the centre, the ratio is ·6180340.

130.—Mr. Grindle's Garden

The rule is this. When the four sides are in arithmetical progression the greatest area is equal to the square root of their continual product. The square root of $7 \times 8 \times 9 \times 10$ is 70·99, or very nearly 71 square rods. This is the correct answer.

131.—The Garden Path

The area of the path is exactly $66\frac{2}{3}$ square yards, which is clearly seen if you imagine the little triangular piece cut off at the bottom and removed to the top right-hand corner. Here is the proof. The area of the garden is $55 \times 40 = 2200$. And $(53\frac{1}{3} \times 40) + 66\frac{2}{3}$ also equals 2200. Finally, the sum of the squares of $53\frac{1}{3}$ and 40 must equal the square of $66\frac{2}{3}$, as it does.

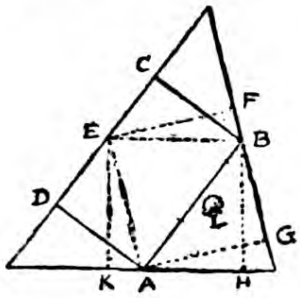


The general solution is as follows: Call breadth of rectangle B, length of rectangle L, width of path C, and length of path x. Then $x = \frac{\pm B\sqrt{(B^2 - C^2)(B^2 + L^2) + C^2L^2} - BCL}{B^2 - C^2}$

In the case given above $x = 66\frac{2}{3}$, from which we find the length $53\frac{1}{3}$.

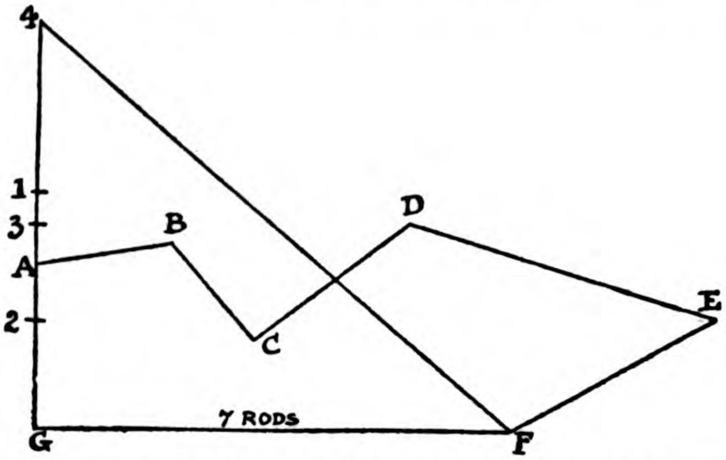
132.—The Garden Bed

Bisect the three sides in A, B, and E. If you join A B and drop the perpendiculars A D and B C, then A B C D will be the largest possible rectangle and exactly half the area of the triangle. The two other solutions, F E A G and K E B H, would also serve (all these rectangles being of the same area) except for the fact that they would enclose the tree. This applies to any triangle with acute angles, but in the case of a right-angled triangle there are only two equal ways of proceeding.



133.—A Problem for Surveyors

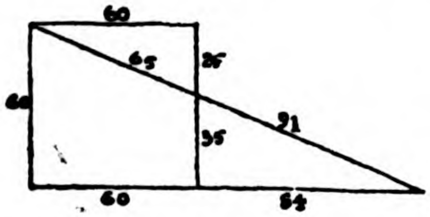
A rectilinear figure of any number of sides can be reduced to a triangle of equal area, and as A G F happens to be a right-angle the thing is quite easy in this way. Continue the line G A. Now lay a parallel ruler from A to C, run it up to B and mark the point 1. Then lay the ruler from 1 to D and run it down to C, marking point 2. Then lay it from 2 to E, run it up to D and mark point 3. Then lay it from 3 to F, run it up to E and mark point 4. If you now draw the line 4 to F the triangle G 4 F is equal in area to the irregular field. As our scale map shows G F to be 7 inches (rods), and we find the length G 4 in this case to



be exactly 6 inches (rods), we know that the area of the field is half of 7 times 6, or 21 square rods. The simple and valuable rule I have shown should be known by everybody—but is not.

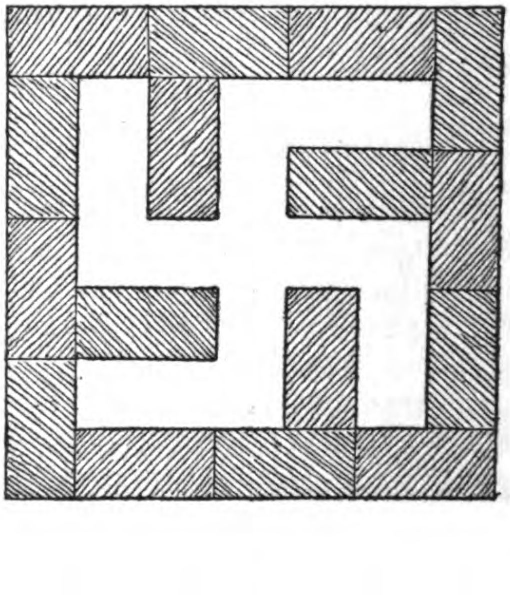
134.—A Fence Problem

The diagram gives all the measurements. Generally a solution involves a biquadratic equation, but as I said the answer was in "*exact* feet," the square of 91 is found to be the sum of two squares in only one way—the squares of 84 and 35. Insert these numbers as shown and the rest is easy and proves itself. The required distance is 35 feet.



135.—The Domino Swastika

It will be seen that by placing the four extra dominoes in the positions shown a perfect swastika is formed within the frame.

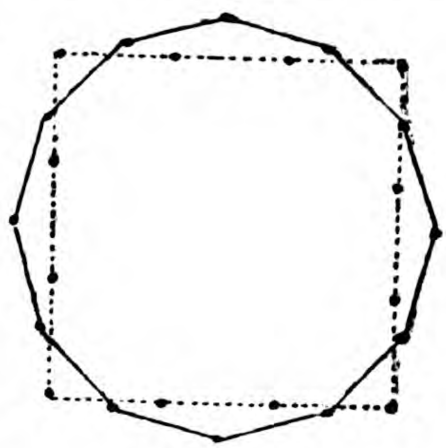


136.—A New Match Puzzle

The smallest possible number is 36 matches. We can form triangle and square with 12 and 24 respectively, triangle and pentagon with 6 and 30, triangle and hexagon with 6 and 30, square and pentagon with 16 and 20, square and hexagon with 12 and 24, and pentagon and hexagon with 30 and 6. The pairs of numbers may be varied in all cases except the fourth and last. There cannot be fewer than 36. The triangle and hexagon require a number divisible by 3: the square and hexagon require an even number. Therefore the number must be divisible by 6, such as 12, 18, 24, 30, 36, but this condition cannot be fulfilled for a pentagon and hexagon with fewer than 36 matches.

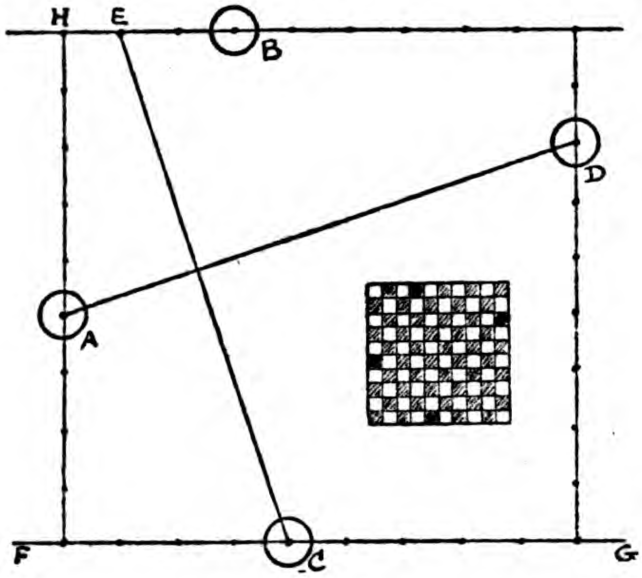
137.—Hurdles and Sheep

If the enclosure is to be rectangular, the nearer the rectangle approaches to the form of a square the greater will be the area. But the greatest area of all will always be when the hurdles are arranged in the form of a regular polygon, inscribed in a circle, and if this can be done in more than one way the greatest area will be when there are as many sides as hurdles. Thus, the hexagon given on page 54 had a greater area than the triangle. The twelve-sided figure or regular dodecagon therefore encloses the largest possible area for twelve hurdles—room for about eleven sheep and one-fifth. Eleven hurdles would only accommodate a maximum of about nine and nine-twenty-fifths, so that twelve hurdles are necessary for ten sheep. If you arrange the hurdles in the form of a square, as shown by the dotted lines, you only get room for nine sheep.

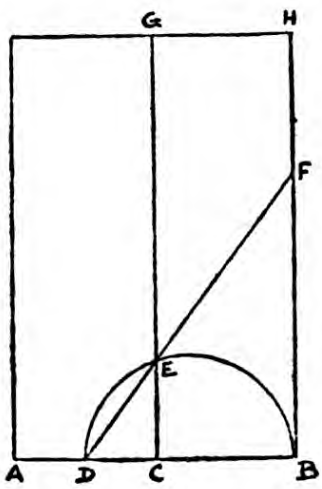


138.—The Four Draughtsmen

Draw a line from A to D. Then draw C E perpendicular to A D, and equal in length to A D. Then E will be the centre of another square. Draw a line from E to B and extend it on both sides. Also draw a line F G through C and parallel to E B and the lines through A and D perpendicular to E B and F G. Now, as H is the centre of a corner square, we can mark off the length H E all round the square and we find the board is 10 × 10.



If the size of the men were not given we might subdivide into more squares, but the men would be too large for the squares. As the distance between the centres of squares is the same as the width of the squares, we can now complete the board with ease, as shown in the diagram inset.

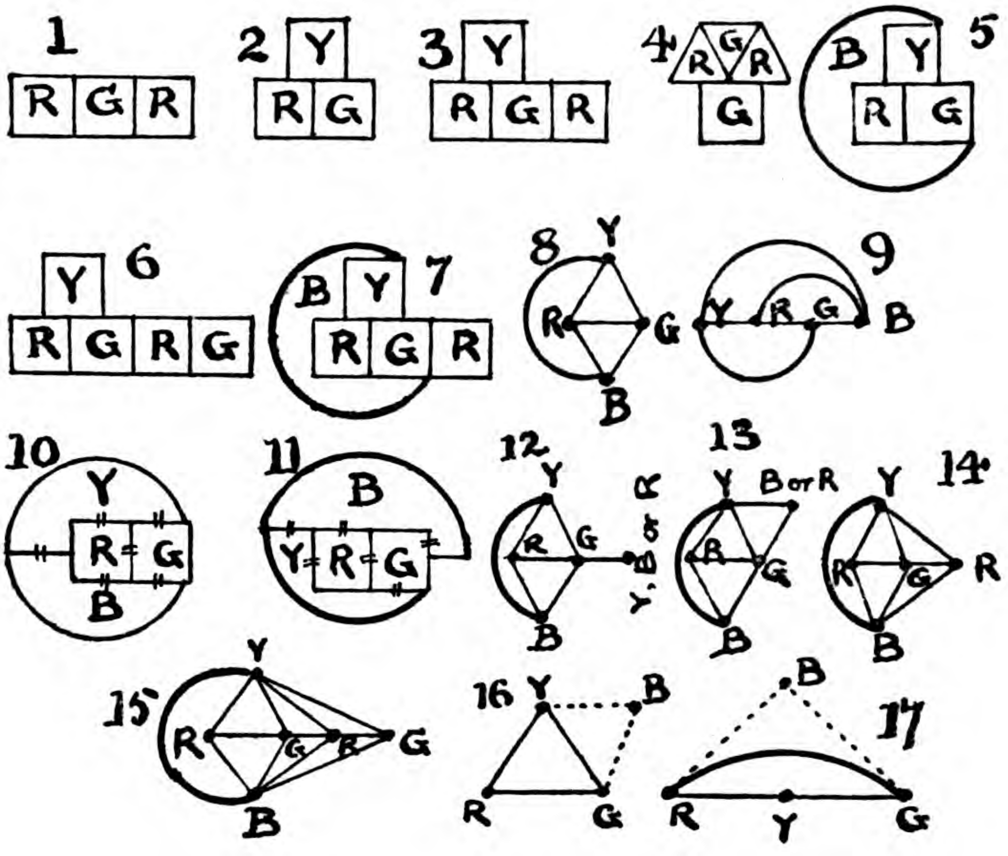


139.—A Crease Problem

Bisect A B in C and draw the line C G, parallel to B H. Then bisect A C in D and draw the semicircle D B, cutting the line C G in E. Now the line D E F gives the direction of the shortest possible crease under the conditions.

140.—The Four-colour Map Theorem

With two or more contiguous countries, two colours at least are obviously necessary (Fig. 1). If three countries are contiguous each with each, three colours are necessary (Fig. 2). With four countries, three will be required if the fourth (Y) is contiguous with each of two that are already contiguous (Fig. 3). (For, as in Fig. 4, G may be contiguous with two countries not contiguous with one another, when only two colours are needed.) And four colours will be necessary if the fourth country is con-



tiguous with each of three already contiguous each with each (Fig. 5).

With five contiguous countries, three will be required if one country is contiguous with two already contiguous (Fig. 6). And four will be necessary if the fifth country is contiguous with each of three countries that are contiguous each with each (Fig. 7). Yet five colours would be needed if a fifth country were contiguous with each of four that are contiguous each with each. If such a map is possible, the theorem breaks down.

First, let us consider four countries contiguous each with each. We will use a simple transformation and suppose every two contiguous countries to be connected by a bridge across the boundary line. The bridge may be as long as we like, and the countries may be reduced to mere points, without affecting the conditions. In Figs. 8 and 9 I show four countries (points) connected by bridges (lines), each with each. The relative positions of these points is quite immaterial, and it will be found in every possible case that one country (point) must be unapproachable from the outside.

The proof of this is easy. If three points are connected each with each by straight lines these points must either form a triangle or lie in a straight line. First suppose they form a triangle, Y R G, as in Fig. 16. Then a fourth contiguous country, B, must lie either within or without the triangle. If within, it is obviously enclosed. Place it outside and make it contiguous with Y and G, as shown: then B cannot be made contiguous with R without enclosing either Y or G. Make B contiguous with Y and R: then B cannot be made contiguous with G without enclosing either Y or R. Make B contiguous with R and G: then B cannot be made contiguous with Y without enclosing either R or G.

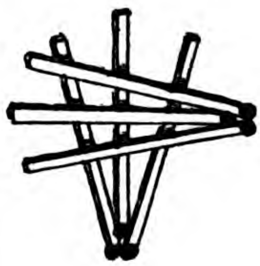
Take the second case, where R Y G lie in a straight line, as in Fig. 17. If B lies within the figure it is enclosed. Place B outside and make it contiguous with R and G, as shown: then B cannot be made contiguous with Y without enclosing either R or G. Make B contiguous with R and Y: then B cannot be made contiguous with G without enclosing either R or Y. Make B contiguous with Y and G: then B cannot be made contiguous with R without enclosing either Y or G.

We have thus taken every possible case and found that if three countries are contiguous each with each a fourth country cannot be made contiguous with all three without enclosing one country.

Fig. 10 is Fig. 8 before the transformation, and Fig. 11 is the same as Fig. 9, and it will be seen at once that you cannot possibly reach R from the outside. Therefore four countries cannot possibly be drawn so that a fifth may be contiguous with every one of them, and consequently the fifth country may certainly repeat the colour R. And if you cannot draw five countries, it is quite obvious that you cannot draw any greater number contiguous each with each.

It is now clear that, at each successive addition of a new country, all the countries previously drawn must be contiguous each with each to prevent the employment of a *repeated* colour. We can draw four countries under this condition, only one country must always be enclosed. Now, we can make the fifth country contiguous with only one country (as in Fig. 12), with two countries (as in Fig. 13), or with three countries (as in Fig. 14). In the first case the new country can be Y or B or R, in the second case B or R, and in the third case R only. We take the last case (Fig. 14) and "bring out," or repeat, R. But in doing so we have been compelled to enclose G. In drawing a sixth country the best we can do (in trying to upset the theorem) is to "bring out" G (as in Fig. 15), and the result is that we must enclose R. And so on into infinity. We can never avoid enclosing a colour at every successive step, and thus making the colour available for the next step. If you cannot artificially *construct* a map that will require a fifth colour, such a map cannot possibly *occur*. Therefore a fifth colour can never be necessary, and the truth of the theorem is proved.

141.—The Six Submarines



It will be seen from the illustration that this puzzle is absurdly easy—when you know how to do it! And yet I have not the slightest doubt that many readers found it a hard nut to crack. It will be seen that every match undoubtedly touches every other match.

142.—Economy in String

The total length of string that passes along the length, breadth, or depth must in every case be the same to allow of the maximum dimensions—that is, 4 feet. When the reader is told this, or has found it for himself (and I think the point will be found interesting), the rest is exceedingly easy. For the string passes 2 times along length, 4 times along breadth, and 6 times along depth. Therefore 4 feet divided by 2, 4, and 6 will give us 2 feet, 1 foot, and $\frac{2}{3}$ foot respectively for the length, breadth, and depth of the largest possible parcel.

The following general solution is by Mr. Alexander Fraser. Let the string pass a times along length x, b times along breadth y, and c times along depth z, and let length of string be m.

Then $ax+by+cz=m$. Find maximum value of xyz.

First find maximum area of xy.

Put $ax+by=n, x=\frac{n-by}{a}, xy=\frac{n}{a}y-\frac{b}{a}y^2, \frac{dxy}{dy}=\frac{n}{a}-\frac{2b}{a}y=0,$

$\therefore y=\frac{n}{2b}$, or $by=\frac{n}{2}.$

$\therefore ax$ also $=\frac{n}{2}$, and $ax=by$.

Similarly, $ax=by=cz=\frac{m}{3}.$

$\therefore x=\frac{m}{3a}, y=\frac{m}{3b}, z=\frac{m}{3c}$, and $xyz=\frac{m^3}{27abc}.$

In the case of the puzzle, $a=2, b=4, c=6, m=12.$

$\therefore x=2, y=1, z=\frac{2}{3}.$

$xyz=1\frac{1}{3}.$

143.—The Stone Pedestal

The cube of a square number is always a square. Thus :—

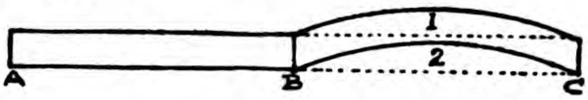
The cube of 1 is 1, the square of 1.
The cube of 4 is 64, the square of 8.
The cube of 9 is 729, the square of 27.
The cube of 16 is 4096, the square of 64.

And so on.

We were told to look at the illustration. If there were one block in pedestal and one in base, the base would be entirely covered, which it was not. If 64 in pedestal and base, the side of the former would measure 4 feet, and the side of square 8 feet. A glance will show that this is wrong. But 729 blocks in each case is quite in agreement with the illustration, for the width of the pedestal (9 feet) would be one-third of the width of the square (27 feet). In all the successive higher cases the square will be increasingly too large for the pedestal to be in agreement with the illustration.

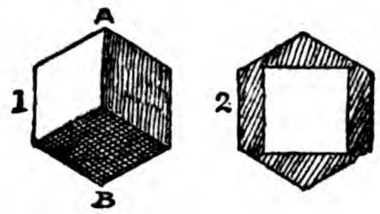
144.—The Bricklayer's Task

A glance at the illustration will show that if you could cut off the portion of wall marked 1 and place it in the position indicated by 2, you would have a piece of straight wall, B C, enclosed by the dotted lines, exactly similar to the wall A B. Therefore, both men were wrong, and the price should be the same for the portion of wall that went over the hill as for the part on the level. Of course, the reader will see at a glance that this will only apply within a certain limitation. But an actual drawing of the wall was given.



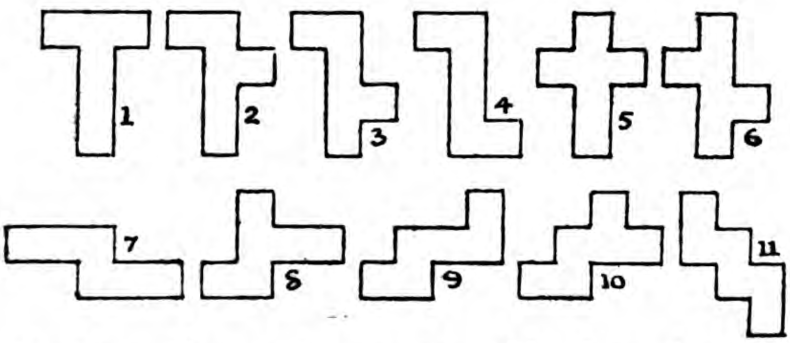
145.—A Cube Paradox

It is a curious fact that a cube can be passed through another cube of smaller dimensions. Suppose a cube to be raised so that its diagonal A B is perpendicular to the plane on which it rests, as in Figure 1. Then the resulting projection will be a regular hexagon, as shown. In Figure 2 the square hole is cut for the passage of a cube of the same dimensions. But it will be seen that there is room for cutting a hole that would pass a cube of even larger dimensions. Therefore, the one through which I cut a hole was not, as the reader may have hastily supposed, the larger one, but the smaller! Consequently, the larger cube would obviously remain the heavier. This could not happen if the smaller were passed through the larger.

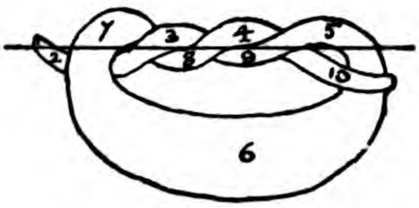


146.—The Cardboard Box

There are eleven different shapes in all, if turning over is allowed, and they are as shown. If the outside of the box is blue



and the inside white, and every possible shape has to be laid out with white uppermost, then there are twenty different ways, for all except Nos. 1 and 5 can be reversed to be different.

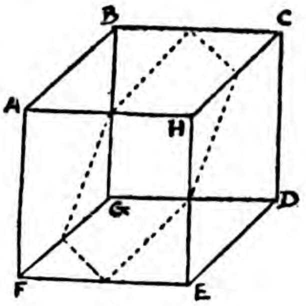


147.—The Austrian Pretzel

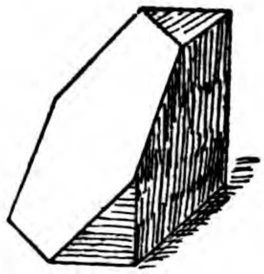
The Pretzel may be divided into as many as ten pieces by one straight cut of the knife in the direction indicated in the illustration.

148.—Cutting the Cheese

Mark the mid-points in B C, C H, H E, E F, F G, and G B. Then insert the knife at the top and follow the direction indi-



1

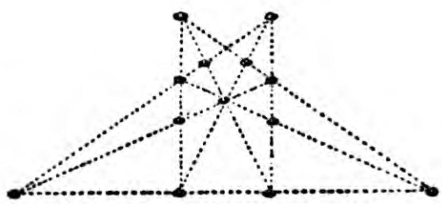


2

cated by the dotted plane. Then the two surfaces will each be a perfect hexagon, and the piece on the right will, in perspective, resemble Figure 2.

149.—Tree-planting Puzzle

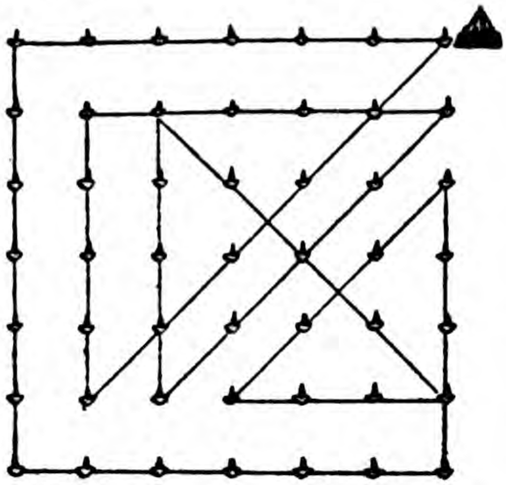
The illustration shows the graceful manner of planting the trees so as to get nine rows with four trees in every row.



150.—Counter Solitaire

Play in the following manner and all the counters except one will be removed in seven moves, and the final leap will be made by No. 1, as required: 2–10, 4–12, 6–5, 3–6, 7–15 (8–16, 8–7, 8–14, 8–3), (1–9, 1–2, 1–11, 1–8, 1–13, 1–4).

151.—Sinking the Fishing-boats



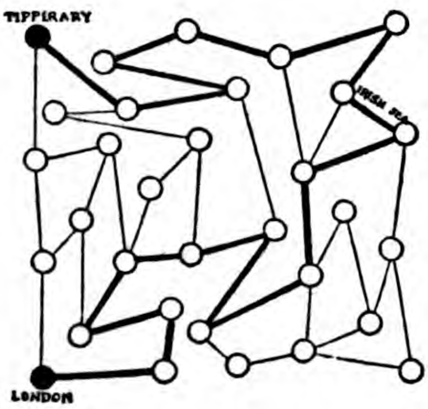
The diagram shows how the warship sinks all the forty-nine boats in twelve straight courses, ending at the point from which she sets out. Follow every line to its end before changing your direction.

152.—A New Leap-Frog Puzzle

Play 9 over 13, 14, 6, 4, 3, 1, 2, 7, 15, 17, 16, 11. Play 12 over 8. Play 10 over 5 and 12. Play 9 over 10.

153.—Transferring the Counters

Make a pile of five counters (1 to 5) on B in 9 moves. Make a pile of four (6 to 9) on C in 7 moves. Make a pile of three (10 to 12) on D in 5 moves. Make a pile of two (13 and 14) on E in 3 moves. Place one (15) on F in 1 move. Replace 13 and 14 on F in 3, 10 to 12 on F in 5, 6 to 9 in 7, and 1 to 5 in 9 moves. Forty-nine moves in all.



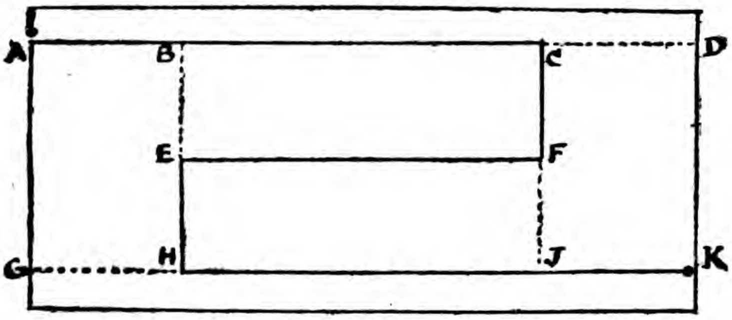
154.—The Way to Tipperary

The thick line in the illustra tion shows a route from London to Tipperary in eighteen moves. It is absolutely necessary to include the stage marked "Irish Sea" in order to perform the journey in an even number of stages.

155.—Marking a Tennis Court

The ten points lettered in the illustration are all "odd nodes," that is points from which you can go in an odd number of directions

—three. Therefore we know that five lines (one-half of 10) will be required to draw the figure. The dotted lines will be the four shortest possible between nodes. Note that you cannot here use a node twice or it would be an improvement to make

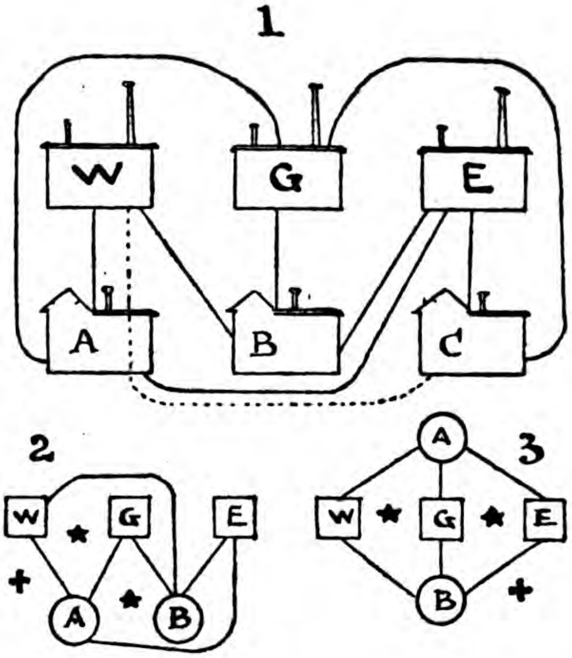


E H and C F dotted lines instead of C D and G H. Having fixed our four shortest lines, the remainder may all be drawn in one continuous line from A to K, as shown. When you get to D you must run up to C and back to D, from G go to H and back, and so on. Or you can wait until you get to C and go to D and back, etc. The dotted lines will thus be gone over twice and the method shown gives us the minimum distance that must be thus repeated.

156.—Water, Gas, and Electricity

This puzzle can only be solved by a trick. If one householder will allow one pipe for a neighbour to pass through his house there is no difficulty, and the conditions did not prohibit this not very unreasonable arrangement. Look at Diagram 1, and you will see that the water-pipe for supplying house C passes through house A, but no pipe anywhere crosses another pipe. I am, however, often asked to *prove* that there is no solution without any trick, and I will now give such a proof for the first time in a book.

Assume that only two houses, A and B, are to be supplied. The relative positions of the various buildings clearly make no difference whatever. I give two positions for the two houses in Diagrams 2 and 3. Wherever you build those houses the effect will be the same—one of the supply stations will be cut off. In the examples it will be seen that if you build a third house on the outside (say in the position indicated by one of the black crosses) the gas can never reach you without crossing a pipe. Whereas if you put the house inside one of the enclosures (as indicated by the stars), then you must be cut off either from the water or the electricity—one or the other. But the house must be either inside or outside. Therefore a position is impossible in which

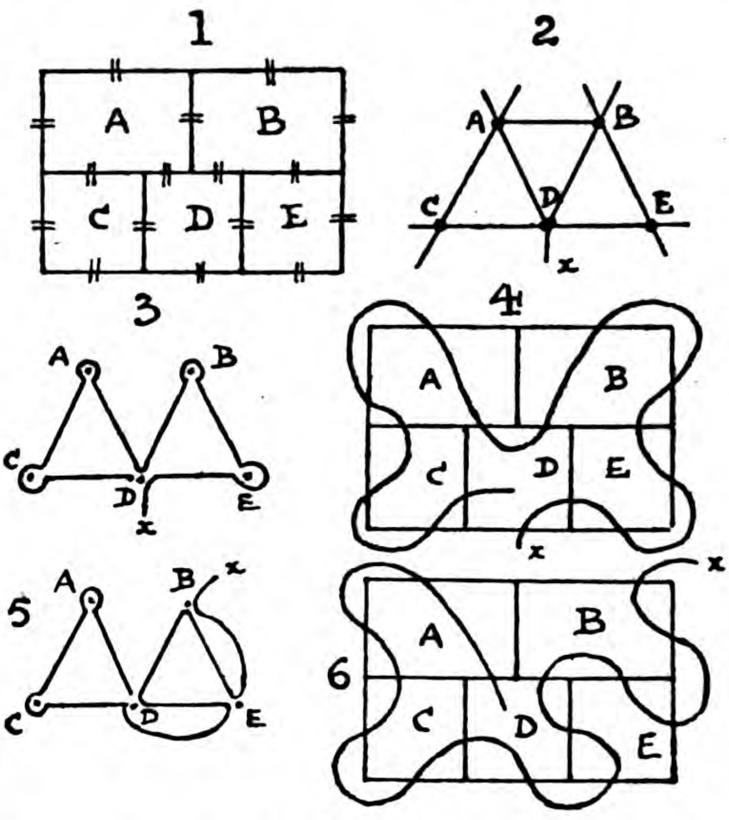


it can be supplied from all three stations without one pipe crossing another. I hope that is thoroughly convincing. Build your two houses wherever you like and you will find that those conditions that I have described will always obtain.

157.—Crossing the Lines

Let us suppose that we cross the lines by bridges, represented in Fig. 1 by the little parallels. Now, in Fig. 2, I transform the diagram, reducing the spaces A, B, C, D, E to mere points, and representing the bridges that connect these spaces by lines or roads. This transformation does not affect the conditions, for there are 16 bridges or roads in one case, and 16 roads or lines in the other, and they connect with A, B, C, D, E in precisely the same way. It will be seen that 9 bridges or roads connect with the outside. Obviously we are free to join these up in pairs in any way we choose, provided the roads do not cross one another. The simplest way is shown in Fig. 3, where on coming out from A, B, C, or E, we immediately return to the same point by the adjacent bridge, leaving one point, X, necessarily in the open. In Fig. 2 there are 4 odd nodes, A, B, D, and X (if we decide on the exits and entrances, as in Fig. 3), so, as I have already explained, we require 2 strokes (half of 4) to go over all the roads, proving a perfect solution to be impossible.

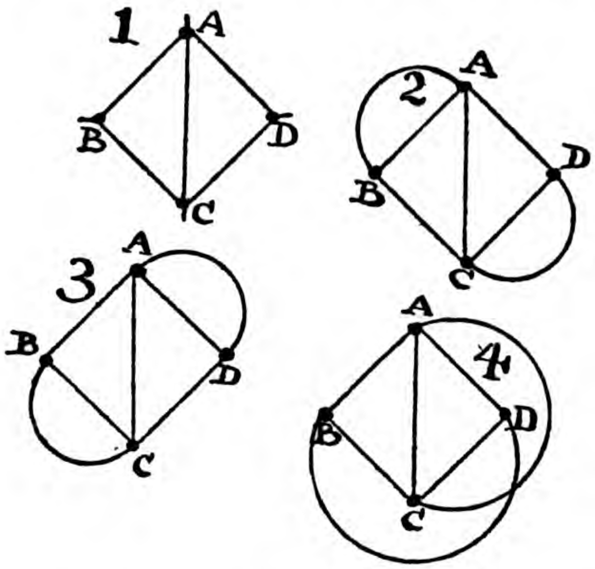
Now, let us cancel the line A B. Then A and B become even nodes, but we must begin and end at the odd nodes, D



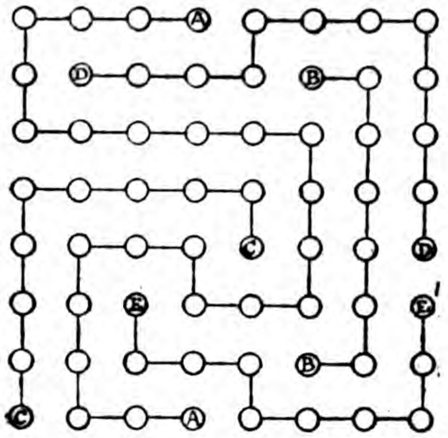
and X. Follow the line in Fig. 3 and you will see that this can be done, omitting the line from A to B. This route the reader will easily transform into Fig. 4 if he says to himself, "Go from X to D, from D to E, from E to the outside and return into E," and so on. The route can be varied by linking up those outside bridges differently, by making X an outside bridge to A or B, instead of D, and by taking the cancelled line either at A B, A D, B D, X A, X B, or X D. In Fig. 5 I make X lead to B. We still omit A B, but we must start and end at D and X. Transformed in Fig. 6, this will be seen to be the precise example that I gave on page 64. The reader can now write out as many routes as he likes for himself, but he will always find it necessary to omit one line or crossing. It is thus seen how easily sometimes a little cunning, like that of the transformation shown, will settle a perplexing question of this kind.

158.—The Nine Bridges

Transform the map as follows. Reduce the four islands, A, B, C, and D, to mere points and extend the bridges into lines, as in Fig. 1, and the conditions are unchanged. If you link A and B for outside communication, and also C and D, the conditions are as in Fig. 2; if you link A and D, and also B and



C, you get Fig. 3; if you link A and C, and also B and D, you get Fig. 4. In each case B and D are "odd nodes" (points from which you can proceed in an odd number of ways, three), so in every route you must start and finish at B or D, to go over every line once, and once only. Therefore, Tompkins must live at B or D: we will say B, and place Johnson at D. There are 44 routes by scheme 2, 44 by scheme 3, and 44 by scheme 4, making 132 in all, not counting reverse routes as different. Taking Fig. 2, and calling the outside curved lines O, if you start B O A B, B O A C, B A O B, or B A C, there are 6 ways of continuing in each case. If you start B O A D, B A D, B C O D, B C A, or B C D, there are always 4 ways of continuing. In the case of Fig. 3, B O C A, B O C B, B C A, or B C O B give 6 ways. B O C D, B A O D, B A C, B A D, or B C D give 4 ways each. Similarly in the case of Fig. 4.

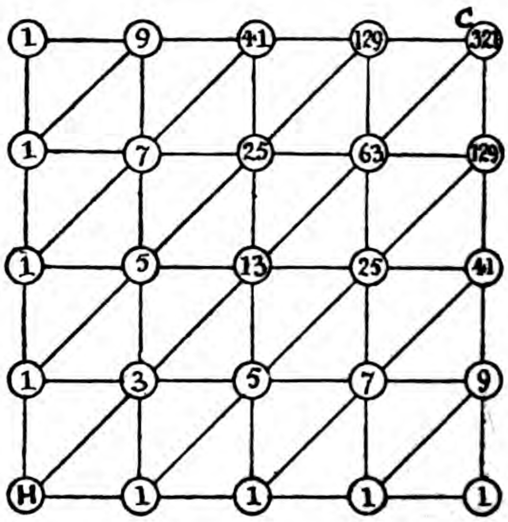


159.—The Five Regiments

In the illustration, in which the roads not used are omitted for the sake of clearness, the routes of the five regiments are shown. No two regiments ever go along the same road.

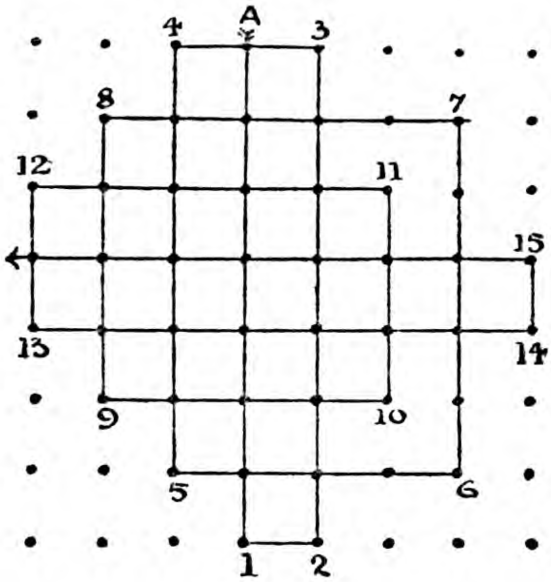
160.—Going to Church

Starting from the house, H, there is only one way of getting to each of the points in a northerly direction, and also going direct east, so I write in the figure 1, as shown. Now take the second column, and you will find that there are three ways of going to the second point from the bottom, five ways to the next above, seven to the next, and so on, continually adding two. The same applies to the second row from the bottom. Write in these numbers. Then the central point of all can be reached in thirteen ways, because we can enter it either from the point below that can be reached in five ways, from the point to the left that can also be reached in five ways, or from the diagonal point below that can be reached in three ways, making together thirteen. So all we have to do is to write in turn at every point the sum of those three numbers from which it can be immediately reached. We thus find that the total number of different routes from H to C is 321.



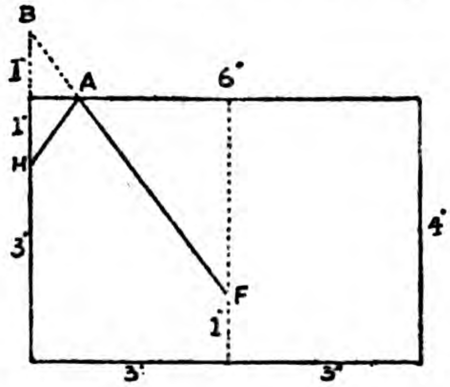
161.—A Motor-car Puzzle

It will be seen from the illustration (where the roads not used are omitted) that the traveller can go as far as seventy miles in fifteen turnings. The turnings are all numbered in the order in which they are taken. He never visits nineteen of the towns. He might visit them all in fifteen turnings, never entering any town twice, and end at A, from which he starts, but such a tour would only take him sixty-four miles.



162.—The Fly and the Honey

The drop of honey is represented by H, the fly by F. The fly clearly has to go over the edge to the other side. Now, imagine we are dealing with a cylinder of cardboard. If we cut it we can lay it out flat. If we then extend the line of the side 1 inch to B, the line F B will cut the edge at A, which will be the point at which the fly must go over. The shortest distance is thus the hypotenuse of a right-angled triangle, whose height is 4 and base 3. This we know is 5, so that the fly has to go exactly 5 inches.

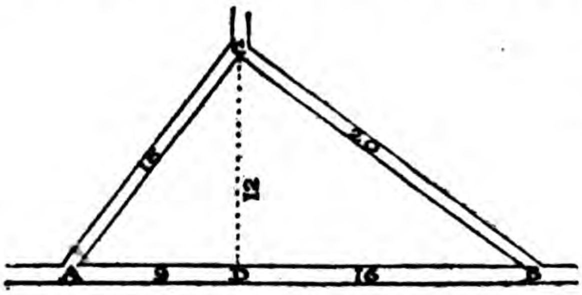


163.—The Russian Motor-cyclists

The two distances given were 15 miles and 6 miles. Now, all you need do is to divide 15 by 6 and add 2, which gives us $4\frac{1}{2}$. Now divide 15 by $4\frac{1}{2}$, and the result ($3\frac{1}{3}$ miles) is the required distance between the two points. This pretty little rule applies to all such cases where the road forms a right-angled triangle. A simple solution by algebra will show why that constant 2 is added. And we can prove the answer in this way. The three sides of the triangle are 15 miles, $9\frac{1}{3}$ miles (6 plus $3\frac{1}{3}$ miles) and $17\frac{2}{3}$ miles (to make it 21 miles each way). Multiply by 3 to get rid of the fractions, and we have 45, 28, and 53. Now, if the square of 45 (2025) added to the square of 28 (784) equal the square of 53 (2809) then it is correct—and it will be found that they do so.

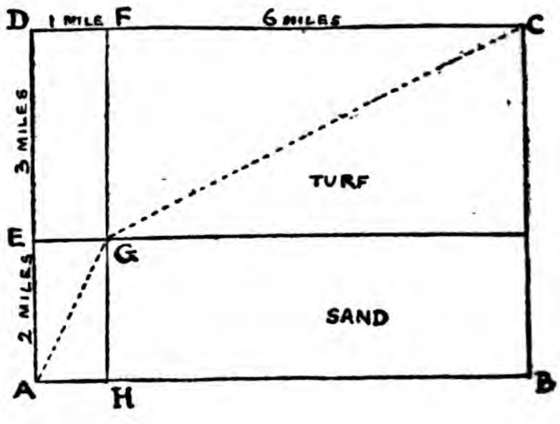
164.—Those Russian Cyclists again

The diagram gives all the correct distances. All the General had to do was to square Pipipoff's 60 miles (3600) and divide by twice the sum of that 60 and Sliponsky's 12 miles —that is, by 144. Doing it in his head, he, of course, saw that this is the same as dividing 300 by 12, which at once gave him the correct answer, 25 miles, as the distance from A to B. I need not show how all the other distances are now easily obtained, if we want them.



165.—The Despatch-rider in Flanders

Of course, a straight line from A to C would not be the quickest route. It would be quicker to ride from A to E and then direct to C. The quickest possible route of all is that shown in the diagram by the dotted line from A to G (exactly 1 mile from E) and then direct to C.



It is necessary that the sine of the angle F G C shall be double the sine of A G H. In the first case the sine is 6 divided by the square root of 6^2+3^2, which is 6 divided by the square root of 45, or the same as 2 divided by the square root of 5. In the second case the sine is 1 divided by the square root of 1^2+2^2, which is 1 divided by the square root of 5. Thus the first is exactly double the second.

166.—A Picture Presentation

Multiply together as many 2's as there are pictures and deduct 1. Thus 2 raised to the tenth power is 1024, and deducting 1 we get 1023 as the correct answer. Suppose there had been only three pictures. Then one can be selected in 3 ways, two in 3 ways, and three in 1 way, making together 7 ways, which is the same as the cube of 2 less 1.

167.—A General Election

The answer is 39,147,416 different ways. Add 3 to the number of members (making 618) and deduct 1 from the number of parties (making 3). Then the answer will be the number of ways in which 3 things may be selected from 618. That is

$$\frac{618\times617\times616}{1\times2\times3}=39{,}147{,}416 \text{ ways.}$$

The general solution is as follows. Let p=parties and m=members. Then C^{p-1}_{m+p-1} =number of ways.

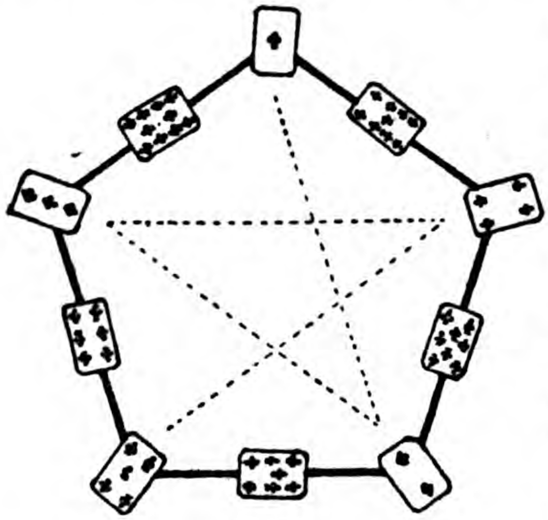
168.—The Magisterial Bench

Apart from any conditions, ten men can be arranged in line in $\underline{|10}$ ways=3,628,800. Now how many of these cases are barred? Regard two of a nationality in brackets as one item. (1) Then (E E) (S S) (W W) F I S A can be permuted in $\underline{|7}\times2^3$ ways=40,320. Remember the two E's can change places within their bracket wherever placed, and so with the S's and the W's. Hence the 2^3. (2) But we may get (E E) (S S) W W F I S A, where the W's are not bracketed, but free. This gives $\underline{|8}\times2^2$ cases, but we must deduct result (1) or these will be included a second time. Result, 120,960. (3) Deal similarly with the two S's unbracketed. Result, 120,960. (4) Deal again, with the E's unbracketed. Result, 120,960. (5) But we may have (E E) S S W W F I S A, where both S and W are unbracketed. This gives $\underline{|9}\times2$ cases, but we must deduct results (1), (2), and (3) for reasons that will now be obvious. Result, 443,520.

(6) When only S is bracketed, deducting (1), (2), and (4). Result, 443,520. (7) When only W is bracketed, deducting (1), (3), and (4). Result, 443,520. Add these seven results together and you get 1,733,760, which deducted from the number first given above leaves 1,895,040 as the number of ways in which the ten men may sit.

169.—The Card Pentagon

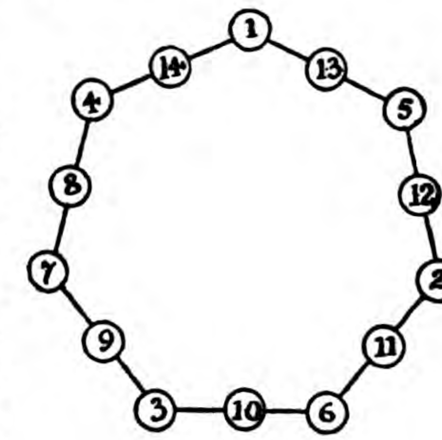
Deal the cards, 1, 2, 3, 4, 5, in the manner indicated by the dotted lines (that is, drop one at every alternate angle in a clockwise direction round the pentagon), and then deal the 6, 7, 8, 9, 10 in the opposite direction, as shown, taking care to start with the 6 on the correct side of the 5. The pips on every side add to 14. If you deal the 6, 7, 8, 9, 10 in the first manner, and the 1, 2, 3, 4, 5 in the second manner, you will get another solution, adding up 19. Now work with the two sets of numbers, 1, 3, 5, 7, 9, and 2, 4, 6, 8, 10, in the same way and you will get two more solutions, adding respectively to 16 and 17.



There are six different solutions in all. The last two are peculiar. Write in, in the same order, 1, 4, 7, 10, 13 and 6, 9, 12, 15, 18; also write in 8, 11, 14, 17, 20 and 3, 6, 9, 12, 15. Then deduct 10 from every number greater than 10.

170.—A Heptagon Puzzle

The diagram shows the solution. Starting at the highest point, write in the numbers 1 to 7 in a clockwise direction at alternate points. Then, starting just above the 7, write 8 to 14 successively in the opposite direction, taking every vacant circle in turn. If instead you write in



1, 3, 5, 7, 9, 11, 13, and then 2, 4, 6, 8, 10, 12, 14, you will get a solution with the sides adding to 22 instead of 19. If you substitute for every number in these solutions its difference from 15 you will get the complementary solutions, adding respectively to 26 and 23 (the difference of 19 and 22 from 45).

171.—An Irregular Magic Square

1	10	9	14
13	10	5	6
8	3	16	7
12	11	4	7

If for the 2 and 15 you substitute 7 and 10, repeated, the square can be formed as shown. Any sixteen numbers can be arranged to form a magic square if they can be written in this way, so that all the horizontal differences are alike and all the vertical differences also alike. The differences here are 3 and 2:

1	4	7	10
3	6	9	12
5	8	11	14
7	10	13	16

172.—A Magic Square Delusion

If you make nine squares precisely similar to this one and then place them together to form a larger square, then you can pick out a square of 25 cells in any position and it will always be a magic square, so it is obvious you can arrange for any number you like to be in the central cell. It is, in fact, what is called a Nasik square (so named by the late Mr. Frost after the place in India where he resided), and it is only perfect squares of this character that can be treated in the manner described.

9	11	18	5	22
3	25	7	14	16
12	19	1	23	10
21	8	15	17	4
20	2	24	6	13

173.—DIFFERENCE SQUARES

The three examples I give are, I believe, the only cases possible. The difference throughout is 5.

2	1	4
3	5	7
6	9	8

8	1	4
3	5	7
6	9	2

2	1	6
3	5	7
4	9	8

174.—SWASTIKA MAGIC SQUARE

It will be seen that the square is perfectly "magic" and that all the prime numbers are placed within the swastika.

17	5	13	21	9
4	12	25	8	16
11	24	7	20	3
10	18	1	14	22
23	6	19	2	15

175.—IS IT VERY EASY?

All that is necessary is to push up the second figure in every cell and so form powers of 2, as in the first square. Then the

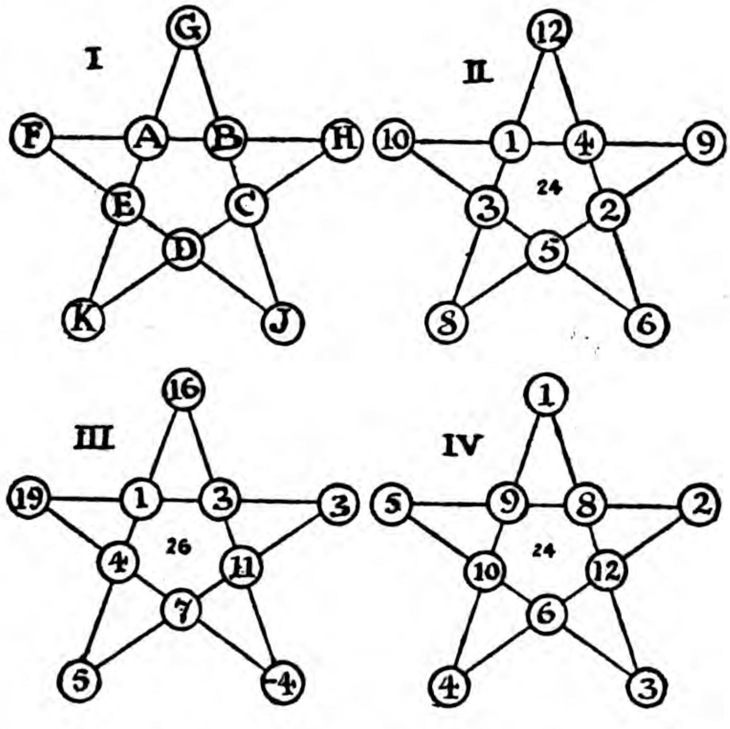
2^7	2^0	2^5
2^2	2^4	2^6
2^3	2^8	2^1

128	1	32
4	16	64
8	256	2

numbers become those in the second square, where all the eight rows give the same product—4096. Of course, every arithmetician knows that 2^0 equals 1.

176.—The Five-pointed Star

Referring to Diagram I, we will call A, B, C, D, E the "pentagon," and F, G, H, J, K the "points." Write in the numbers 1, 2, 3, 4, 5 in the pentagon in the order shown in Diagram II, where you go round in a clockwise direction, starting with 1 and jumping over a disc to the place for 2, jumping over another for 3, and so on. Now to complete the star for the constant summation of 24, as required, use this simple rule. To find H subtract the sum of B and C from half the constant plus E. That is,



subtract 6 from 15. We thus get 9 as the required number for H. Now you are able to write in successively 10 at F (to make 24), 6 at J, 12 at G, and 8 at K. There is your solution.

You can write any five numbers you like in the pentagon, in any order, and with any constant summation that you wish, and you will always get, by the rule shown, the only possible solution for that pentagon and constant. But that solution may require the use of repeated numbers and even negative numbers. Suppose, for example, I make the pentagon 1, 3, 11, 7, 4, and the constant 26, as in Diagram III, then I shall find the 3 is repeated, and the repeated 4 is negative and must be deducted instead of added. You will also find that if we had written our pentagon numbers in Diagram II in any other order we should always get repeated numbers.

Let us confine our attention to solutions with ten different positive whole numbers. Then 24 is the smallest possible constant. A solution for any higher constant can be derived from it. Thus, if we want 26, add 1 at each of the points; if we want 28 add 2 at every point or 1 at every place in both points and pentagon. Odd constants are impossible unless we use fractions. Every solution can be "turned inside out." Thus, Diagram IV is simply a different arrangement of Diagram II. Also the four numbers in G, K, D, J may always be changed, if repetitions do not occur. For example, in Diagram II substitute 13, 7, 6, 5 for 12, 8, 5, 6 respectively. Finally, in any solution the constant will be two-fifths of the sum of all the ten numbers. So, if we are given a particular set of numbers we at once know the constant, and for any constant we can determine the sum of the numbers to be used.

177.—The Six-pointed Star

I have insufficient space to explain fully the solution to this interesting problem, but I will give the reader the main points.

1. In every solution the sum of the numbers in the triangle A B C (Fig. I) must equal the sum of the triangle D E F. This sum may be anything from 12 to 27 inclusive, except 14 and 25, which are impossible. We need only obtain solutions for 12, 13, 15, 16, 17, 18, and 19, because from these all the complementaries, 27, 26, 24, 23, 22, 21, and 20, may be derived by substituting for every number in the star its difference from 13.

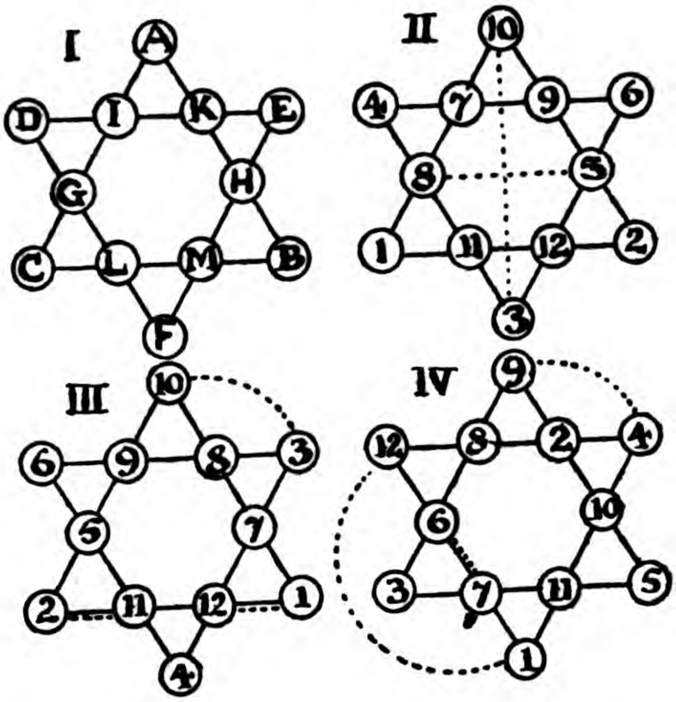
2. Every arrangement is composed of three independent diamonds, A G H F, D K B L, and E M C I, each of which must always sum to 26.

3. The sum of the numbers in opposite external triangles will always be equal. Thus A I K equals L M F.

4. If the difference between 26 and the triangle sum A B C be added to any number at a point, say A, it will give the sum of the two numbers in the relative positions of L and M. Thus (in Fig. II) $10+13=11+12$, and $6+13=8+11$.

5. There are six pairs summing to 13; they are $12+1$, $11+2$, $10+3$, $9+4$, $8+5$, $7+6$, and one pair, or two pairs, may occur among the numbers at the points, but never three. The relative positions of these pairs determine the type of solution. In the

regular type, as in Fig. II, A and F and also G and H, as indicated by the dotted lines, always sum to 13, though I subdivide this class. Fig. III and IV are examples of the two irregular types. There are 37 solutions in all (or 74, if we count the com-



plementaries described in my first paragraph), of which 32 are regular and 5 irregular.

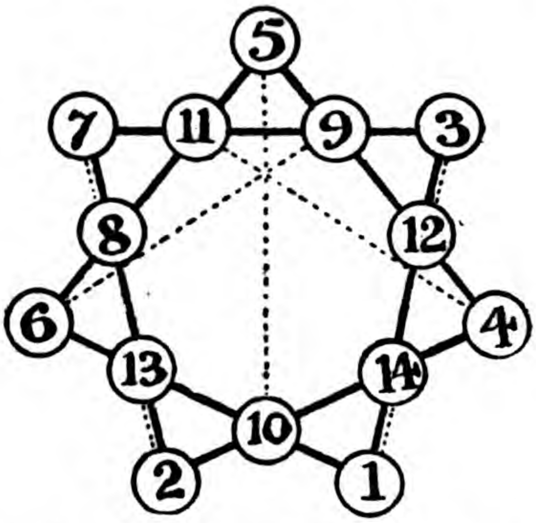
Of the 37 solutions, 6 have their points summing to 26. They are as follows :—

10	6	2	3	1	4	7	9	5	12	11	8
9	7	1	4	3	2	6	11	5	10	12	8
5	4	6	8	2	1	9	12	3	11	7	10
5	2	7	8	1	3	11	10	4	12	6	9
10	3	1	4	2	6	9	8	7	12	11	5
8	5	3	1	2	7	10	4	11	9	12	6

The first is our Fig. II, and the last but one our Fig. III, so a reference to those diagrams will show how to write the numbers in the star. The reader should write them all out in star form and remember that the 6 are increased to 12 if you also write out their complementaries. The first four are of the regular type and the last two of the irregular. If the reader should be tempted to find all the 37 (or 74) solutions to the puzzle it will help him to know that, where the six points sum to 24, 26, 30, 32, 34, 36, 38, the respective number of solutions is 3, 6, 2, 4, 7, 6, 9, making 37 in all.

178.—The Seven-pointed Star

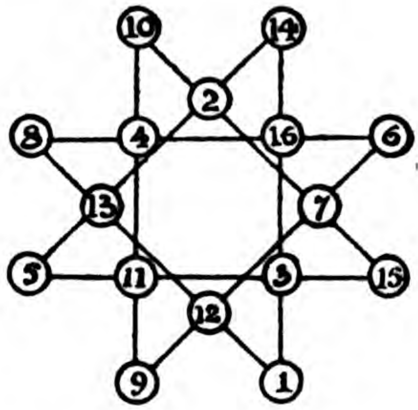
Place 5 at the top point, as indicated in diagram. Then let the four numbers in the horizontal line (7, 11, 9, 3) be such that the two outside numbers shall sum to 10 and the inner numbers to 20, and that the difference between the two outer numbers shall be twice the difference between the two inner numbers. Then their complementaries to 15 are placed in the relative



positions shown by the dotted lines. The remaining four numbers (13, 2, 14, 1) are easily adjusted. From this fundamental arrangement we can get three others. (1) Change the 13 with the 1 and the 14 with the 2. (2 and 3) Substitute for every number in the two arrangements already found its difference from 15. Thus, 10 for 5, 8 for 7, 4 for 11, and so on. Now, the reader should be able to construct a second group of four solutions for himself, by following the rules.

The general solution is too lengthy to be given here in full, but there are, in all, 56 different arrangements, counting complementaries. I divide them into three classes. Class I includes all cases like the above example, where the pairs in the positions of 7–8, 13–2, 3–12, 14–1 all sum to 15, and there are 20 such cases. Class II includes cases where the pairs in the *positions* of 7–2, 8–13, 3–1, 12–14 all sum to 15. There are, again, 20 such cases. Class III includes cases where the pairs in the *positions* of 7–8, 13–2, 3–1, 12–14 all sum to 15. There are 16 such cases. Thus we get 56 in all.

179.—Two Eight-pointed Stars



The illustration is the required solution. Every line of four numbers adds up 34. If you now find any solution to one of the stars, you can immediately transfer it to the other by noting the relative positions in the case given.

I have not succeeded in enumerating the stars of this order. The task is, I think, a particularly difficult one. Perhaps readers may like to attempt the solution.

180.—The Damaged Measure

Let the eight graduation marks divide the 33-inch measure into the following nine sections: 1, 3, 1, 9, 2, 7, 2, 6, 2, and any length can be measured from 1 inch up to 33 inches. Of course, the marks themselves will be at 1, 4, 5, 14, 16, 23, 25, and 31 inches from one end. Another solution is 1, 1, 1, 1, 6, 6, 6, 6, 5.

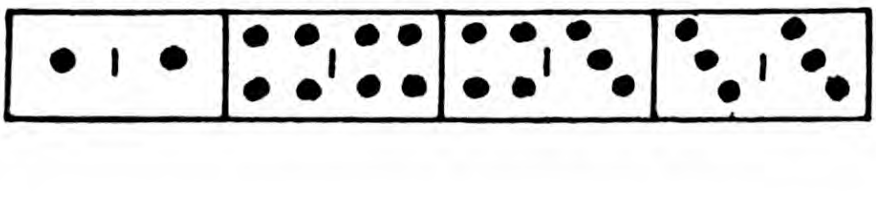
This puzzle may be solved in, at fewest, sixteen different ways. I have sought a rule for determining the fewest possible marks for any number of inches, and for at once writing out a solution, but a general law governing all the multiplicity of answers has still to be found.

181.—The Six Cottages

If the distances between the cottages are as follows, in the order given, any distance from one mile up to twenty-six inclusive may be found as from one cottage to another: 1, 1, 4, 4, 3, 14 miles round the circular road.

182.—A New Domino Puzzle

These four dominoes fulfil the conditions. It will be found that, taking contiguous pips, we can make them sum to any number from 1 to 23 inclusive.



183.—At the Brook

A

15	16		
0	16*	15	5*
15	1*	0	5
0	1	5	0
1	0	5	16
1	16	15	6*
15	2*	0	6
0	2	6	0
2	0	6	16
2	16	15	7*
15	3*	0	7
0	3	7	0
3	0	7	16
3	16	15	8*
15	4*	0	8
0	4	8	0
4	0	8	16
4	16		

B

15	16		
*15	0	0	11
0	15	15	11
15	15	*10	16
*14	16	10	0
14	0	0	10
0	14	15	10
15	14	*9	16
*13	16	9	0
13	0	0	9
0	13	15	9
15	13	*8	16
*12	16		
12	0		
0	12		
15	12		
*11	16		
11	0		

Every line shows a transaction. Thus, in column A, we first fill the 16 measure; then fill the 15 from the 16, leaving 1, if we want it; then empty the 15; then transfer the 1 from 16 to 15; and so on. The asterisks show how to measure successively 1, 2, 3, 4, etc. Or we can start, as in B column, by first filling the 15 and so measure in turn, 14, 13, 12, 11, etc. If we continue A we get B read upwards, or vice versa. It will thus be seen that to measure from 1 up to 7 inclusive in the fewest transactions we must use the method A, but to get from 8 to 14 we must use method B. To measure 8 in the A direction will take 30 transactions, but in the B manner only 28, which is the correct answer. It is a surprising fact that with any two measures that are prime to each other (that have no common divisor, like 15 and 16) we can measure any whole number from 1 up to the largest measure. With measures 4 and 6 (each divisible by 2) we can only measure 2, 4, and 6. With 3 and 9 we could only measure 3, 6, and 9. In our tables the quantities measured come in regular numerical order, because the difference between 15 and 16 is 1. If I had given the measures 9 and 16, under A we should get the order 7, 14, 5, 12, 3, etc., a cyclical difference of 7 (since 16—9=7). After adding 7 to 14 we must deduct 16 to get 5, and after adding 7 to 12 we must deduct 16 to get 3, and so on.

184.—A Prohibition Poser

First fill and waste the 7-quart measure 14 times and you will have thrown away 98 and leave 22 quarts in the barrel in 28 transactions. (Filling and emptying are 2 transactions.) Then, fill 7-qt.; fill 5-qt. from 7-qt., leaving 2 in 7-qt.; empty 5-qt.; transfer 2 from 7-qt. to 5-qt.; fill 7-qt.; fill up 5-qt. from 7-qt., leaving 4 in 7-qt.; empty 5-qt.; transfer 4 to 5-qt.; fill 7-qt.; fill up 5-qt. from 7-qt., leaving 6 in 7-qt.; empty 5-qt.; fill 5-qt. from 7-qt., leaving 1 in 7-qt.; empty 5-qt., leaving 1 in 7-qt.; draw off remaining 1-qt. from barrel into 5-qt., and the thing is done in 14 more transactions, making, with the 28 above, 42 transactions. Or you can start by wasting 104 and leaving 16 in barrel. These 16 can be dealt with in 10 transactions, and the 104 require 32 in the wasting (12 times 7 and 4 times 5 is the quickest way).

185.—Prohibition again

Fill 7-qt.; fill 5-qt.; empty 108 quarts from barrel; empty 5-qt. into barrel; fill 5-qt. from 7-qt.; empty 5-qt. into barrel; pour 2 quarts from 7-qt. into 5-qt.; fill 7-qt. from barrel; fill up 5-qt. from 7-qt.; empty 5-qt. into barrel; pour 4 quarts from 7-qt. into 5-qt.; fill 7-qt. from barrel; fill up 5-qt. from 7-qt.; throw away contents of 5-qt.; fill 5-qt. from barrel; throw away 5 quarts from 5-qt.; empty 1 quart from barrel into 5-qt. The feat is thus performed in 17 transactions—the fewest possible.

186.—The False Scales

If the scales had been false on account of the pans being unequally weighted, then the true weight of the pudding would be 154 oz., and it would have weighed 130 oz. in one pan and 178 oz. the other. Half the sum of the apparent weights (the arithmetic mean) equals 154. But the illustration showed that the pans weighed evenly, and that the error was in the unequal lengths of the arms of the balance. Therefore, the apparent weights were 121 oz. and 169 oz., and the real weight 143 oz. Multiply the apparent weights together and we get the square of 143—the geometric mean. The lengths of the arms were in the ratio 11 to 13.

If we call the true weight x in each case, then we get the equations :—

$$\frac{\left(\frac{9}{11}x+4\right)+\left(\frac{9}{11}x+52\right)}{2}=x, \text{ and } x=154.$$

$$\sqrt{\left(\frac{9}{11}x+4\right)\times\left(\frac{9}{11}x+52\right)}=x, \text{ and } x=143.$$

187.—Weighing the Goods

Since one canister weighs an ounce, the first illustration shows that in one pan eight packets equal three ounces, and, therefore, one packet will weigh three-eighths of an ounce. The second illustration shows that in the other pan one packet equals six ounces. Multiply $\frac{3}{8}$ by 6 and we get $\frac{9}{4}$, the square root of which is $\frac{3}{2}$, or $1\frac{1}{2}$ oz. as the real weight of one packet. Therefore, eight packets weigh twelve ounces, which is the correct answer.

188.—Monkey and Pulley

We find the age of the monkey works out at $1\frac{1}{2}$ years, and the age of the mother $2\frac{1}{2}$ years, the monkey therefore weighing $2\frac{1}{2}$ lb., and the weight the same. Then we soon discover that the rope weighed $1\frac{1}{4}$ lb., or 20 oz.; and, as a foot weighed 4 oz., the length of the rope was 5 feet.

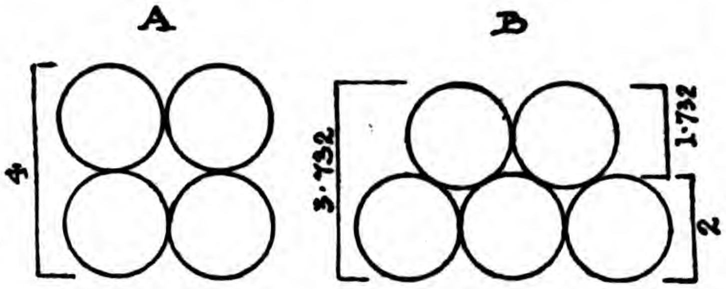
189.—Weighing the Baby

It is important to notice that the man, baby, and dog weigh together 180 lb., as recorded on the dial in the illustration. Now, the difference between 180 and 162 is 18, which equals twice the weight of the dog, whose weight is 9 lb. Therefore the baby weighs 30 lb., since 30 less 70 per cent is 9.

190.—Packing Cigarettes

Say the diameter of a cigarette is 2 units and that 8 rows of 20 each, as in Fig. A (that is, 160 cigarettes) exactly fit the box. The inside length of the box is therefore 40 and the depth 16.

Now, if we place 20 in the bottom row, and, instead of placing 20 in the next row, we drop 19 into the position shown in Fig. B, we save ·268 (i.e. $2-\sqrt{3}$) in height. This second row, and every additional row of 20 and 19 alternately, will increase the height



by 1·732. Therefore, we shall have 9 rows reaching to a height of $2+8\times1{\cdot}732$ or 15·856, which is less than our depth of 16. We shall thus increase the number of cigarettes by 20 (through the additional row), and reduce it by 4 (1 in each row of 19), making a net increase of 16 cigarettes.

191.—Crossing the Ferry

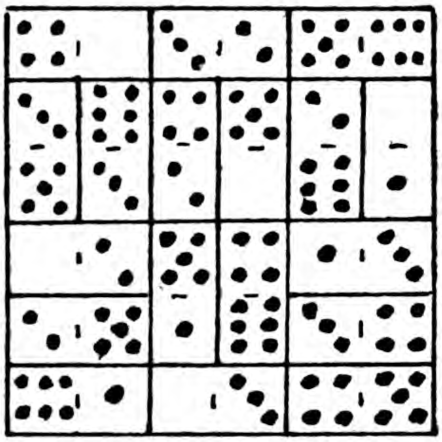
The puzzle can be solved in as few as nine crossings, as follows: (1) Mr. and Mrs. Webster cross. (2) Mrs. Webster returns. (3) Mother and daughter-in-law cross. (4) Mr. Webster returns. (5) Father-in-law and son cross. (6) Daughter-in-law returns. (7) Mr. Webster and daughter-in-law cross. (8) Mr. Webster returns. (9) Mr. and Mrs. Webster cross.

192.—Missionaries and Cannibals

Call the three missionaries M m m, and the three cannibals C c c, the capitals denoting the missionary and the cannibal who can row the boat. Then C c row across; C returns with the boat; C c row across; C returns; M m row across; M c return; M C row across; M c return; M m row across; C returns; C c row across; C returns; C c row across; and all have crossed the river within the conditions stated.

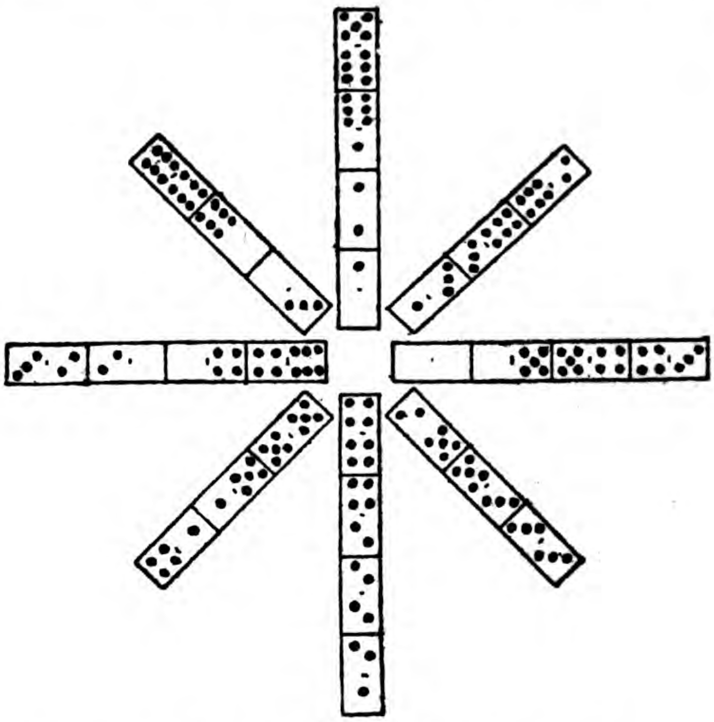
193.—A Domino Square

The illustration explains itself. The eighteen dominoes are arranged so as to form the required square, and it will be found that in no column or row is a number repeated. There are, of course, many other ways of doing it.



194.—A DOMINO STAR

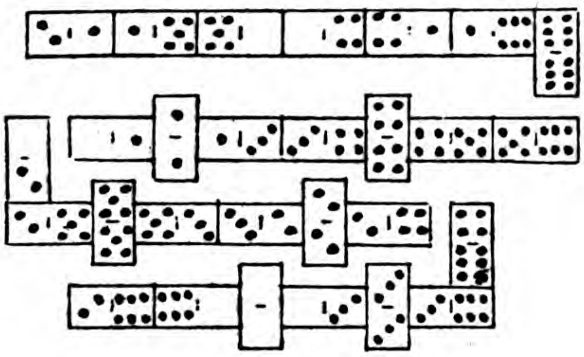
The illustration shows a correct solution. The dominoes are placed together according to the ordinary rule, the pips in every



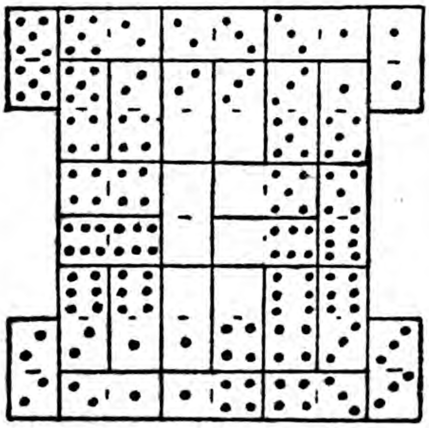
ray sum to 21, and the central numbers are 1, 2, 3, 4, 5, 6, and two blanks.

195.—Domino Groups

The illustration shows one way in which the dominoes may be laid out so that, when the line is broken in four lengths of seven dominoes each, every length shall contain forty-two pips.



196.—Les Quadrilles



The illustration shows a correct solution, the two blank squares being on the inside. If in the example shown on page 86 all the numbers had not happened to be found somewhere on the edge, it would have been an easy matter, for we should have had merely to exchange that missing number with a blank wherever found. There would thus have been no puzzle. But in the circumstances it is impossible to avail oneself of such a simple manœuvre.

197.—A Puzzle with Cards

Arrange the pack in the following order face downwards with 9 of Clubs at the top and 5 of Spades at bottom : 9 C., Jack D., 5 C., ace D., King H., King S., 7 H., 2 D., 6 S., Queen D., 10 S., ace S., 3 C., 3 D., 8 C., King D., 8 H., 7 C., 4 D., 2 S., ace H., ace C., 7 S., 5 D., 9 H., 2 H., Jack S., 6 D., Queen C., 6 C., 10 H., 3 S., 3 H., 7 D., 4 C., 2 C., 8 S., Jack H., 4 H., 8 D., Jack C., 4 S., Queen S., King C., 9 D., 5 H., 10 C., Queen H., 10 D., 9 S., 6 H., 5 S.

198.—A Card Trick

Every pile must contain thirteen cards, less the value of the bottom card. Therefore, thirteen times the number of piles less the sum of the bottom cards, and plus the number of cards left over, must equal fifty-two, the number in the pack. Thus thirteen times the number of piles plus number of cards left over, less fifty-two, must equal sum of bottom cards. Or, which is the same thing, the number of piles less four, multiplied by thirteen, and plus the cards left over gives the answer as stated. The algebraically inclined reader can easily express this in terms of his familiar symbols.

199.—A Golf Competition Puzzle

The players may be paired and arranged as follows :—

	ROUNDS				
	1	2	3	4	5
1st LINKS	BC	BF	EF	CE	AD
2nd LINKS	FA	CD	CA	DF	BE
3rd LINKS	DE	EA	DB	AB	CF

200.—Cricket Scores

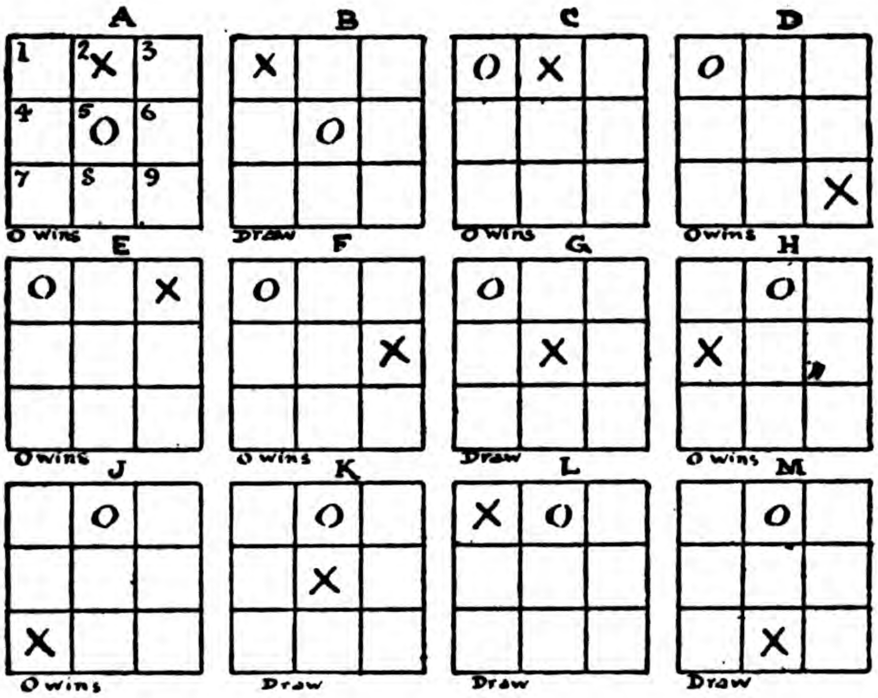
The four innings must have secured 128, 96, 72, and 54 runs respectively. Therefore, the Muddletonians scored 200 against their opponents' 150 and beat them by 50 runs.

201.—Football Results

We see at once from the table that England beat Ireland and drew with Wales. As E. scored 2 goals to 0 in these games, they must have won 2—0 and drawn 0—0. This disposes of E. and leaves three games, W. *v.* I., S. *v.* I., and S. *v.* W., to be determined. Now, S. had only 1 goal scored against them—by W. or I. I. scored only 1 goal, and that must have been against W. or S. Assume it was against S. In that case W. did not score against S. But W. scored 3 goals altogether; therefore these must have been scored against I. We find I. had 6 goals against them: 2 scored by E., as shown, 3 by W. (if we assume that I. scored *v.* S.), and the remaining goal was scored by S. But, as we have just assumed I. scored 1 goal against S., the match would have been drawn. It was won by S., and therefore I. could not have scored against S. Thus the goal against S. must have been scored by W. And as W. scored 3 goals, the other two must have been *v.* I., who must have scored their only goal against W. Thus S. beat W. by 2—1 and I. by 2—0, while W. won by 2—1 *v.* I.

202.—Noughts and Crosses

Number the board, as in Fig. A. Mr. Nought (the first player) can open in one of three ways: he can play to the centre, 5, or to a corner, 1, 3, 7, or 9, or to a side, 2, 4, 6, or 8. Let us take these openings in turn. If he leads with a centre, then Mr. Cross



has the option of a corner or a side. If he takes a side, such as 2 in Fig. A, then Nought plays 1 and 4 successively (or 1 and 7), and wins. Cross must take a corner, as in Fig. B, and then Nought cannot do better than draw. If Nought leads with a corner, say 1, Cross has five different replies, as in Figs. C, D, E, F, and G (for 4 here is the same as 2, 7 the same as 3, and 8 the same as 6). If he plays as in Fig. C, Nought wins with 5 and

4; if he plays as in D, Nought wins with 7 and 3; if as in E, Nought wins with 9 and 7; if as in F, Nought wins with 5 and 3. Cross is compelled to take the centre, as in Fig. G, to save the game, for this will result in a draw. If Nought opens with a side, say 2, as in Figs. H, J, K, L, and M, and Cross plays as in H, Nought wins with 5 and 1; and if he plays as in J, Nought wins with 1 and 5. Cross must play as in K, L, or M to secure a draw.

I have thus shown the play for Nought to win in seven cases where Cross makes a bad first move, but I have not space to prove the draws in the remaining five positions B, G, K. L, and M. But the reader can easily try each of these cases for himself and be convinced that neither player can win without the bad play of his opponent. Of course, either player can throw away the game. For example, if in Fig. L Nought stupidly plays 3 on his second move, Cross can play 7 and 9 and win. Or if Nought plays 8, Cross can play 5 and 7 and win.

Now, if I were playing with an equally expert player I should know that the best I could possibly do (barring my opponent's blunders) would be to secure a draw. As first player, Nought, I should know that I could safely lead with any square on the board. As second player, Cross, I should take a corner if Nought led with a centre and take the centre if he led with anything else. This would avoid many complexities and should always draw. The fact remains that it is a capital little game for children, and even for adults who have never analysed it, but two experts would be merely wasting their time in playing it. To them it is not a game, but a mere puzzle that they have completely solved.

203.—The Horse-shoe Game

Just as in "Noughts and Crosses," every game should be a draw. Neither player can win except by the bad play of his opponent.

204.—Turning the Die

The best call for the first player is either "two" or "three," as in either case only one particular throw should defeat him. If he called "one," the throw of either 3 or 6 should defeat him. If he called "two," the throw of 5 only should defeat him. If he called "three," the throw of 4 only should defeat him. If he called "four," the throw of either 3 or 4 should defeat him. If he called "five," a throw of either 2 or 3 should defeat him. And if he called "six," the throw of either 1 or 5 should defeat him. It is impossible to give here a complete analysis of the play, but I will just state that if at any time you score either 5, 6, 9, 10, 14, 15, 18, 19, or 23, with the die any side up, you ought to lose. If you score 7 or 16 with any side up you should win. The chance of winning with the other scores depends on the lie of the die.

205.—The Three Dice

Mason's chance of winning was one in six. If Jackson had selected the numbers 8 and 14 his chances would have been exactly the same.

206.—The 37 Puzzle Game

The first player (A) can always win, but he must lead with 4. The winning scores to secure during the play are 4, 11, 17, 24, 30, 37. In the first game below the second player (B) puts off defeat as long as possible. In the second game he prevents A scoring 17 or 30, but has to give him 24 and 37. In the third game he prevents A scoring 11 or 24, but has to give him 17, 30, and 37. Notice the important play of the 3 and the 5.

A	B	A	B	A	B
4	1 (*a*)	4	1	4	1
3	1 (*b*)	3	1	3	4
(11) 2	1	(11) 2	3 (*d*)	(17) 5	1
(17) 5	1 (*c*)	5	1	3	4
3	2	(24) 4	3 (*e*)	(30) 5 (*f*)	1
(24) 1	2	5	1	3	1
(30) 4	1	(37) 4		(37) 2	
3	2				
(37) 1					

(*a*) Or A will score 11 next move. (*b*) B could not prevent A scoring 11 or 17 next move. (*c*) Again, to prevent A immediately scoring 24. (*d*) Preventing A scoring 17, but giving him 24. (*e*) Preventing A scoring 30, but giving him the 37. (*f*) Thus A can always score 24 (as in the last game) or 30 (as in this), either of which commands the winning 37.

207.—The Twenty-two Game

Apart from the exhaustion of cards, the winning series is 7, 12, 17, 22. If you can score 17 and leave at least one 5-pair of both kinds (4—1, 3—2), you must win. If you can score 12 and leave two 5-pairs of both kinds, you must win. If you can score 7 and leave three 5-pairs of both kinds, you must win. Thus, if the first player plays a 3 or 4, you play a 4 or 3, as the case may be, and score 7. Nothing can now prevent the second player from scoring 12, 17, and 22. The lead of 2 can also always be defeated if you reply with a 3 or a 2. Thus, 2—3, 2—3, 2—3, 2—3 (20), and, as there is no remaining 2, second player wins. Again, 2—3, 1—3, 3—2, 3—2 (19), and second player wins. Again, 2—3, 3—4 (12), or 2—3, 4—3 (12), also win for second player. The intricacies of the defence 2—2 I leave to the reader. The best second play of first player is a 1.

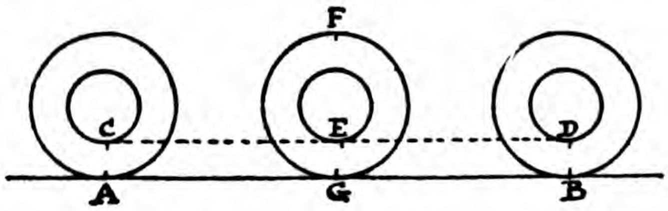
The first player can always win if he plays 1, and in no other way. Here are specimen games: 1—1, 4—1, 4—1, 4 (16) wins. 1—3, 1—2, 4—1, 4—1, 4 (21) wins. 1—4, 2 (7) wins. 1—2, 4 (7) wins.

208.—The Nine Squares Game

I should play M N. My opponent may play H L, and I play C D. (If he had played C D, I should have replied H L, leaving the same position.) The best he can now do is D H (scoring one), but, as he has to play again, I win the remaining eight squares.

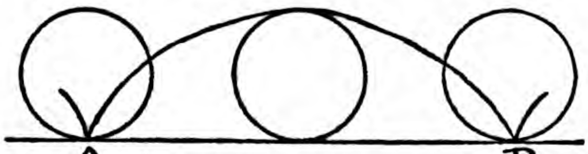
209.—A Wheel Fallacy

The inner circle has half the diameter of the whole wheel, and therefore has half the circumference. If it merely ran along the imaginary line C D it would require two revolutions: after the



first, the point D would be at E. But the point B would be at F, instead of at G, which is absurd. The fact is the inner circle makes only one revolution, but in passing from one position to the other it progresses partly by its own revolution and partly by carriage on the wheel. The point A gets to B entirely by its own revolution, but if you imagine a point at the very centre of the wheel (a point has no dimensions and therefore no circumference), it goes the same distance entirely by what I have called carriage. The curve described by the passage of the point A to B is a common cycloid, but the point C in going to D describes a curtate trochoid.

We have seen that if a bicycle wheel makes one complete revolution, so that the point A touches the ground again at B, the distance A B is the exact length of the circumference, though we cannot, if we are given the length of the diameter, state it in exact figures. Now that point A travels in the direction of the curved line shown in our illustration. This curve is called, as I have said, a " common cycloid." Now, if the diameter of the

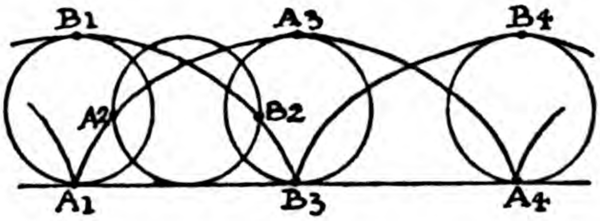


wheel is 28 inches, we can give the exact length of that curve. This is remarkable—that we cannot give exactly the length from A to B in a straight line, but can state exactly the length of the curve. What is that length? I will give the answer at once. The length of the cycloid is exactly four times that of the diameter. Therefore, four times 28 gives us 112 inches as its length. And the area of the space enclosed by the curve and the straight line A B is exactly three times the area of the circle. Therefore, the enclosed space on either side of the circle is equal in area to the circle.

210.—A Famous Paradox

Of course, every part of the wheel revolves round the axle at a uniform speed, and, therefore, in the case of a *fixed* wheel, such as a grindstone, the answer is in the negative. But in the case of a bicycle wheel in *motion* along a road it is an undoubted fact that what is the upper part for the time being always moves faster *through space* than the lower part. If it did not do so, no progress would be made and the cyclist would have to remain as stationary as the grindstone.

Look at our diagram and you will see the wheel in four different positions that occur during one complete revolution from A_1 to A_4. I have elsewhere explained the peculiar curve, called a common cycloid, that is described by a point on the edge of the tyre. The curve is shown here for two points at A_1 and B_1. Note that in a half-revolution A_1 goes to A_3 and B_1 to



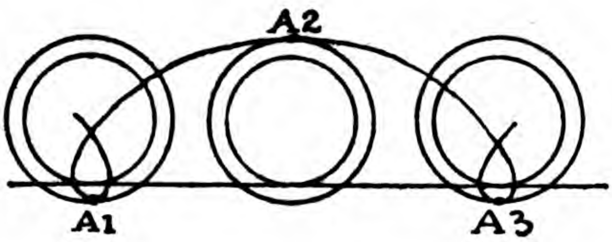
B_3, equal distances. But neither point moves *throughout* at a uniform speed. This is at once seen if we examine the quarter-revolution, where A_1 has only moved as far as A_2, while B_1 has gone all the way to B_2. We thus see that a point on the rim moves slowest through space when at the bottom, and fastest when near the top.

And here is a simple practical way of demonstrating it to your unbelieving friends without the aid of my diagrams. Draw a straight line on a sheet of paper and lay down a penny with Britannia's toe on the line and the upper part of the helmet at the top. Now make the penny run along the line a very short distance to the right and then to the left. That the toe hardly leaves the original point on the line, while the head of Britannia travels a considerable distance, ought to be at once obvious to everybody, and should be quite convincing that the part of the wheel that is for the time being at the top moves faster through space than the part at the bottom.

211.—Another Wheel Paradox

I have already shown that, if you mark a spot on the circumference of a bicycle wheel, that spot, when the wheel is progressing, will describe in space a curve known as a common cycloid. If, however, you mark the edge of the flange of a locomotive or railway-carriage wheel, the spot will describe a curtate cycloid curve, terminating in nodes or loops, as shown in the diagram. I have shown a wheel, with flanges below the railway line, in three positions—the start, a half-revolution, and a complete revolution. The spot marked at A1, has gone to

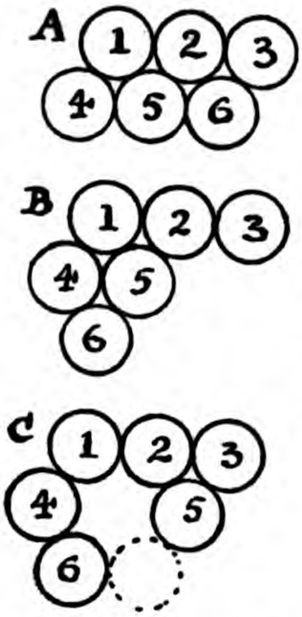
A2 and A3. As the wheel is supposed to move from left to right, trace with your pencil the curve in that direction. You will then find that at the lower part of the loop you are actually going from right to left. The fact is that "at any given moment"



certain points at the bottom of the loop must be moving in the opposite direction to the train. As there is an infinite number of such points on the flange's circumference, there must be an infinite number of these loops being described while the train is in motion. In fact, certain points on the flanges are always moving in a direction opposite to that in which the train is going.

212.—A Chain Puzzle

To open a link and join it again will cost 3d. By opening one link at the end of each of the thirteen pieces the cost will be 3s. 3d., so it would be cheaper than that to buy a new chain. If there happened to be a piece of twelve links, all these twelve could be opened to join the remaining twelve pieces at a cost also of 3s. If there had happened to be two pieces together, containing eleven links, all these could be opened to join the remaining eleven pieces at a cost of 2s. 9d. The best that can be done is to open three pieces containing together ten links to join the remaining ten pieces at a cost of 2s. 6d. This is possible if we break up the piece of four links and two pieces of three links. Thus, if we include the piece of three links that was shown in the middle row as one of the three link pieces, we shall get altogether five large links and five small ones. If we had been able to find four pieces containing together nine links we should save another 3d., but this is not possible, nor can we find five pieces containing together eight links, and so on, therefore the correct answer is as stated, 2s. 6d.



213.—The Six Pennies

First arrange the pennies as in Diagram A. Then carefully shift 6 and get position B. Next place 5 against 2 and 3 to get the position C. No. 3 can now be placed in the position indicated by the dotted circle.

214.—Folding Postage Stamps

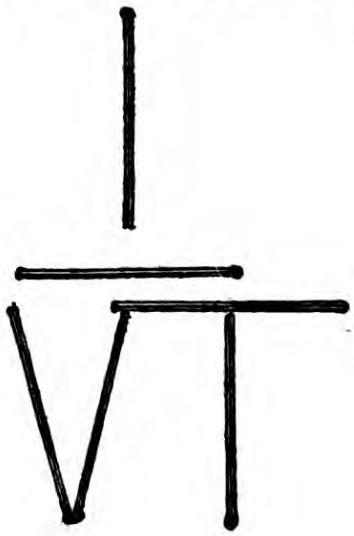
Numbering the stamps as in the diagram on p. 97—that is, 1 2 3 4 in the first row and 5 6 7 8 in the second row, to get the order 1 5 6 4 8 7 3 2 (with No. 1 face upwards, only visible), hold this way, with all faces downwards:

5 6 7 8
1 2 3 4

Fold 7 over 6. Lay 4 flat on 8 and tuck them both in between 7 and 6 so that these four are in the order 7 8 4 6. Now bring 5 and 1 under 6, and it is done. The order 1 3 7 5 6 8 4 2 is more difficult and might well have been overlooked, if one had not been convinced, that according to law, it must be possible. First fold so that 5 6 7 8 only are visible with their faces uppermost. Then fold 5 on 6. Now, between 1 and 5 you have to tuck in 7 and 8, so that 7 lies on the top of 5, and 8 bends round under 6. Then the order will be as required.

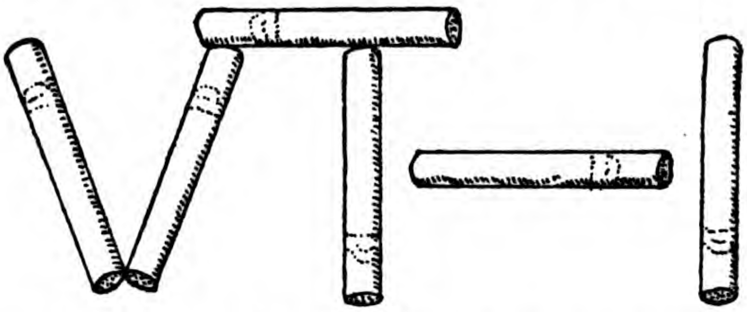
215.—An Ingenious Match Puzzle

It will be seen that the second I in VII has been moved, so as to form the sign of square root. The square root of 1 is, of course, 1, so that the fractional expression itself represents 1.



216.—Fifty-seven to Nothing

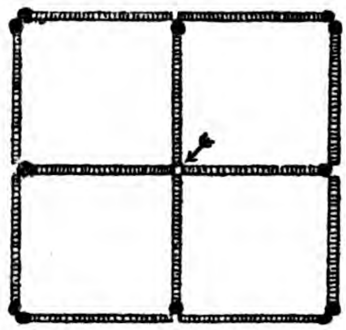
Remove the two cigarettes forming the letter L in the original arrangement, and replace them in the way shown in our illustration. We have the square root of 1 minus 1 (that is 1 less 1),



which clearly is 0. In the second case we can remove the same two cigarettes and, by placing one against the V and the other against the second I, form the word NIL, or nothing.

217.—The Five Squares

Place the twelve matches as in the diagram and five squares are enclosed. It is true that the one in the centre (indicated by the arrow) is very small, but no conditions were imposed as to dimensions.



218.—A Square with Four Pennies

The illustration indicates how we may show a rectilinear square with four pennies. The sides of the square are the lines beneath Britannia.

219.—A Calendar Puzzle

Every year divisible by 4 without remainder is bissextile (leap year), ***except*** that every year divisible by 100 without remainder is ***not*** leap year, unless it be also divisible by 400 without remainder, when it *is* leap year. This is not generally understood. Thus 1800 was not leap year, nor was 1900; but 2000, 2400, 2800, etc., will all be leap years. The first day of the present century, 1st January, 1901, was *Tuesday.*

Now, the present century will contain 25 leap years, because 2000 is leap year, and therefore 36525 ($365 \times 100 + 25$) days, or 5217 weeks and 6 days; so that 1st January, 2001, will be 6 days later than Tuesday—that is Monday. The century beginning 1st January, 2001, will contain only 24 leap years, because 2100 is not leap year, and 1st January, 2101, will be 5 days later than Monday, last mentioned, that is Saturday, because there are 5217 weeks and only 5 days. It will now be convenient to put the results into tabular form, thus:—

1st	January,	1901—Tuesday.	
1st	„	2001—Monday.	6 days later (2000, leap year)
1st	„	2101—Saturday.	5 „ „
1st	„	2201—Thursday.	5 „ „
1st	„	2301—Tuesday.	5 „ „
1st	„	2401—Monday.	6 „ „ (2400, leap year)

It will thus be seen that the first days of successive centuries will be Tuesday, Monday, Saturday, and Thursday—perpetually recurring—so that the first day of a century can never occur on a Sunday, Wednesday, or a Friday, as I have stated.

220.—The Fly's Tour

Before you join the ends give one end of the ribbon a half-turn, so that there is a twist in the ring. Then the fly can walk over all the squares without going over the edge, for we have the curious paradox of a piece of paper with only one side and one edge!

221.—A Musical Enigma

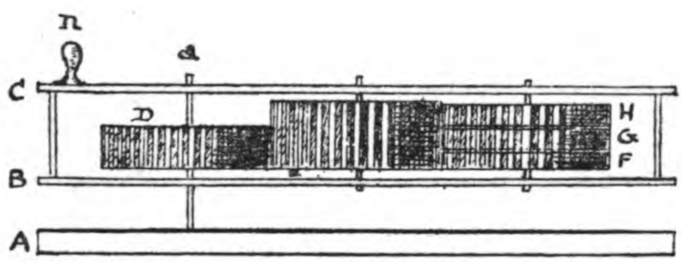
The undoubtedly correct solution to this enigma is B A C H. If you turn the cross round, you get successively B flat (treble clef), A (tenor clef), C (alto clef), and B natural (treble clef). In

German B flat is called " B " and B natural " H," making it read B A C H.

It reminds me of an organ fugue by C. P. Emmanuel Bach, based on the family name and beginning as in the illustration.

222.—A Mechanical Paradox

The machine shown in our illustration consists of two pieces of thin wood, B, C, made into a frame by being joined at the corners. This frame, by means of the handle, *n*, may be turned round an axle, *a*, which pierces the frame and is fixed in a stationary board or table, A, and carries within the frame an immovable wheel. This first wheel, D, when the frame revolves,



turns a second and thick wheel, E, which, like the remaining three wheels, F, G, and H, moves freely on its axis. The thin wheels, F, G, and H, are driven by the wheel E in such a manner that when the frame revolves H turns the same way as E does, G turns the contrary way, and F remains stationary. The secret lies in the fact that though the wheels may be all of the same diameter, and D, E, and F may (D and F *must*) have an equal number of teeth, yet G must have at least one tooth fewer, and H at least one tooth more, than D. Readers will find a full account of this paradox and its inventor in a little book, *Remarkable Men*, published by the S.P.C.K.

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