

A NOVEL FUZZY C-REGRESSION MODEL ALGORITHM USING A NEW ERROR MEASURE AND PARTICLE SWARM OPTIMIZATION

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This paper presents a new algorithm for fuzzy c-regression model clustering. The proposed methodology is based on adding a second regularization term in the objective function of a Fuzzy C-Regression Model (FCRM) clustering algorithm in order to take into account noisy data. In addition, a new error measure is used in the objective function of the FCRM algorithm, replacing the one used in this type of algorithm. Then, particle swarm optimization is employed to finally tune parameters of the obtained fuzzy model. The orthogonal least squares method is used to identify the unknown parameters of the local linear model. Finally, validation results of two examples are given to demonstrate the effectiveness and practicality of the proposed algorithm.

Keywords: Takagi–Sugeno fuzzy model, noise clustering algorithm, fuzzy c-regression model, orthogonal least squares, particle swarm optimization.

1. Introduction

In the past few years, fuzzy modeling algorithms have been widely used in many research areas because of their excellent ability of describing nonlinear systems. Fuzzy modeling is an effective tool for the approximation of uncertain systems on the basis of measured data (Hellendoorn and Driankov, 1997). The Takagi–Sugeno (T–S) model (Takagi and Sugeno, 1985) has been widely applied in many fields, such as modeling (Boukhris *et al.*, 1999; Alci, 2008; Soltani *et al.*, 2010a), control (Ying, 2000; Brdyś and Littler, 2002; Kościelny and Syfert, 2006; Qi and Brdys, 2009; Kluska, 2009) and fault tolerant control (Marx *et al.*, 2007; Ichalal *et al.*, 2010). In many studies, T–S based approaches such as the Gustafson–Kessel (GK) clustering algorithm (Gustafson and Kessel, 1979), the Gath–Geva (GG) algorithm (Gath and Geva, 1989), the fuzzy c-regression model clustering algorithm (Hathaway and Bezdek, 1993), enhanced fuzzy system models (Celikyilmaz and Burhan Turksen, 2008), the new FCRM clustering algorithm (NFCRMA) (Chaoshun *et al.*, 2009; 2010) and the Fuzzy C-Means (FCM) clustering algorithm (Bezdek, 1981) are often used for the description of complex systems in a human intuitive way (especially the last one). A modified version of FCM, called the fuzzy c-regression model clustering

algorithm, has been proposed by Hathaway and Bezdek (1993).

The FCRM algorithm develops hyper-plane-shaped clusters, while the FCM algorithm develops hyper-spherical-shaped clusters. The FCRM algorithm suffers from two major problems:

- It uses an alternating optimization technique, whose iterative nature makes it sensitive to noise (Nasraoui and Krishnapuram, 1996; Sumit and Dave, 1998; Ichihashi and Honda, 2004; Leski, 2004). In addition, if the noisy data have significant high values, they can severely affect the model parameter estimation (Bezdek *et al.*, 1999).
- Derived from Bezdek’s fuzzy c-means algorithm, it is sensitive to initialization and may converge to a local minimum of the objective function (Frigui and Krishnapuram, 1999; Zhang *et al.*, 2006; Xu and Zhang, 2009; Ying *et al.*, 2011). Hence, different initializations may lead, easily, to different results.

The algorithms reported by Kim *et al.* (1998), Hathaway and Bezdek (1993), Chaoshun *et al.* (2009), Chaoshun *et al.* (2010), Chen *et al.* (1998), Rezaee and Zarandi (2010) as well as Andri and Ennu (2011) did not treat the two points cited above. Consequently, in order

to deal with these two points, firstly we design a new objective function for a robust version of the FCRM algorithm based on the Noise Clustering (NC) algorithm (Dave, 1991). This latter has a robust capability against noise and is quite successful in improving the robustness of a variety of fuzzy clustering algorithms (Dave and Krishnapuram, 1997; Tran and Wagner, 1999; Ichihashi et al., 2005; Soltani et al., 2010b; Honda et al., 2010). Moreover, a new error measure is introduced into the objective function to get a good result. Secondly, many studies have proposed the evolutionary computation technique based on Particle Swarm Optimization (PSO). They have been successfully applied to solve various optimization problems (Zhang et al., 2006; 2011; Panchal et al., 2009; Niknam and Amiri, 2010). Thus, we introduce PSO into the FCRM clustering to achieve global optimization.

In this paper, new methods of the FCRM clustering algorithm are proposed using a modified objective function, a new error measure, and parameter estimation based particle swarm optimization. Based on these methods, we develop a fuzzy modeling approach for unknown nonlinear systems, in order to provide a good model approximation. The optimal consequent parameters of the local linear T-S model are estimated using the Orthogonal Least Squares (OLS) method.

The rest of this paper is organized as follows. In Section 2, a brief review of the FCRM clustering algorithm formulation is given, and in Section 3 the PSO algorithm is introduced. Three novel FCRM clustering algorithms are detailed in Section 4 considering the noise of the data set. Simulation results compared with other learning algorithms are shown in Section 5, and Section 6 summarizes the important features of our approach.

2. Fuzzy c-regression model clustering algorithm

A Takagi–Sugeno model consists of a set of fuzzy rules, each describing a local input–output relation as follows:

$$\begin{aligned}
 R_i : & \text{ IF } x_{k1} \text{ is } A_{i1} \text{ and } \dots \text{ and } x_{kM} \text{ is } A_{iM}, \\
 \text{ THEN } & y_i = a_{i1}x_{k1} + a_{i2}x_{k2} + \dots + a_{iM}x_{kM} + b_{i0}, \\
 & i = 1, \dots, c,
 \end{aligned} \tag{1}$$

where R_i denotes the i -th IF-THEN rule, A_{ij} is the fuzzy subset, and c is the number of rules.

The affine T-S fuzzy model based on the FCRM belongs to the range of clustering algorithms with a linear prototype.

Let $S = \{(\mathbf{x}_1, y_1), \dots, (\mathbf{x}_N, y_N)\} = \{(\mathbf{x}_k, y_k), k = 1, \dots, N\}$ be a set of input–output sample data pairs. Assume that the data pairs in S are drawn from c different fuzzy regression models. The hyper-plane of the i -th clus-

ter representative is expressed as follows:

$$\begin{aligned}
 y_k &= f_i(\mathbf{x}_k, \boldsymbol{\theta}_i) + E_{ik}(\boldsymbol{\theta}_i), \\
 &= a_{i1}x_{k1} + a_{i2}x_{k2} + \dots + a_{iM}x_{kM} \\
 &\quad + b_{i0} + E_{ik}(\boldsymbol{\theta}_i), \\
 &= [\mathbf{x}_k \ 1] \cdot \boldsymbol{\theta}_i^T + E_{ik}(\boldsymbol{\theta}_i), \quad i = 1, 2, \dots, c,
 \end{aligned} \tag{2}$$

where $\mathbf{x}_k = [x_{k1}, \dots, x_{kM}] \in \mathbb{R}^M$ is the input vector, $y_k \in \mathbb{R}$ is the output and $\boldsymbol{\theta}_i = [a_{i1}, \dots, a_{iM}, b_{i0}] \in \mathbb{R}^{M+1}$ is the parameter vector of the corresponding local linear model.

The distance (error measure) between the value predicted by the model $f_i(\mathbf{x}_k, \boldsymbol{\theta}_i)$ and the output y_k is defined by

$$E_{ik}(\boldsymbol{\theta}_i) = |y_k - [\mathbf{x}_k \ 1] \cdot \boldsymbol{\theta}_i^T|. \tag{3}$$

The distances ($E_{ik}(\boldsymbol{\theta}_i)$) are weighted with the membership values μ_{ik} in the objective function that is minimized by the clustering algorithm and is given as

$$J(S; U, \boldsymbol{\theta}) = \sum_{k=1}^N \sum_{i=1}^c (\mu_{ik}^m) E_{ik}^2(\boldsymbol{\theta}_i), \tag{4}$$

where m is the weighting exponent and μ_{ik} is the membership degree of \mathbf{x}_k to the i -th cluster. The membership values μ_{ik} have to satisfy the following conditions:

$$\mu_{ik} \in [0 \ 1], \quad i = 1, 2, \dots, c, \quad k = 1, 2, \dots, N, \tag{5}$$

$$0 < \sum_{k=1}^N \mu_{ik} < N, \quad i = 1, 2, \dots, c, \tag{6}$$

$$\sum_{i=1}^c \mu_{ik} = 1, \quad k = 1, 2, \dots, N. \tag{7}$$

The identification procedure of the FCRM algorithm is summarized as follows (Hathaway and Bezdek, 1993). Given data S , set $m > 1$ and specify regression models (Eqn. (2)), choose an error measure (Eqn. (3)). Select a termination threshold $\epsilon > 0$ and initialize $U^{(0)}$ (e.g., at random).

Repeat for $l = 1, 2, \dots$

Step 1. Calculate values for c model parameters $\boldsymbol{\theta}_i^{(l)}$ in Eqn. (2) that globally minimize the restricted function Eqn. (4).

Step 2. Update $U^{(l)}$ with $E_{ik}(\boldsymbol{\theta}_i^{(l)})$, to satisfy

$$\begin{aligned}
 U_{ik}^{(l)} &= \begin{cases} \left[\sum_{j=1}^c \left(\frac{E_{ik}}{E_{jk}} \right)^{\frac{2}{m-1}} \right]^{-1} & \text{if } E_{ik} > 0 \text{ for } 1 \leq i \leq c, \\ 0 & \text{otherwise.} \end{cases}
 \end{aligned} \tag{8}$$

Until $\|U^{(l)} - U^{(l-1)}\| \leq \epsilon$, then stop. Otherwise, set $l = l + 1$ and return to Step 1.

3. New FCRM clustering algorithm

Several authors have shown that the clustering results can be severely distorted when they are based on noisy data (Ohashi, 1984; Chen and Wang, 1999; Kim *et al.*, 2004; Yang *et al.*, 2005). To overcome this problem, many studies on the robust fuzzy modeling technique have been reported, one of them being the NC algorithm. In this approach, noise is considered a separate class. It is represented by a fictitious prototype that has a constant distance δ from all the data points. The membership μ_{*k} of point x_k in the noise cluster is given by

$$\mu_{*k} = 1 - \sum_{i=1}^c \mu_{ik}. \quad (9)$$

Thus, the membership constraint for the good clusters is effectively relaxed to

$$\sum_{i=1}^c \mu_{ik} < 1. \quad (10)$$

Dave's objective function is given by

$$J_{NC}(U, V) = \sum_{i=1}^c \sum_{k=1}^N \mu_{ik}^m D_{ik}^2 + \sum_{k=1}^N \delta^2 (1 - \sum_{i=1}^c \mu_{ik})^m, \quad (11)$$

for any input \mathbf{x}_k in subspace i denoted by center \mathbf{v}_i , $D_{ik} = \|\mathbf{x}_k - \mathbf{v}_i\|$.

The combination of the noise clustering algorithm with the FCRM algorithm can lead to a new FCRM objective function as follows:

$$J_{\text{new}}(S; U, \theta) = \sum_{k=1}^N \sum_{i=1}^c \mu_{ik}^m E_{ik}^2(\theta_i) + \sum_{k=1}^N \delta^2 \mu_{*k}^m. \quad (12)$$

In Eqn. (12), the first term on the left hand side is the same as the original FCRM objective function, while the second term is due to the noise clustering extension. Here δ is a scale parameter and may be used based on the idea presented by Dave (1991) as

$$\delta^2 = \gamma \frac{1}{cN} \sum_{k=1}^N \sum_{i=1}^c E_{ik}^2(\theta_i), \quad (13)$$

where γ is a user-defined parameter depending on the example type.

To solve the constrained problem J_{new} with respect to μ_{ik} , we introduce N Lagrange multipliers λ_k , $k = 1, \dots, N$. The minimization of J_{new} starts by forming the Lagrangian

$$F = J_{\text{new}} - \sum_{k=1}^N \lambda_k \left(\sum_{i=1}^c \mu_{ik} + \mu_{*k} - 1 \right). \quad (14)$$

By differentiating the Lagrangian with respect to μ_{ik} , μ_{*k} and λ_k and setting the derivatives to zero, we obtain

$$\frac{\partial F}{\partial \mu_{ik}} = m \mu_{ik}^{m-1} E_{ik}^2 - \lambda_k = 0, \quad (15)$$

$$\frac{\partial F}{\partial \mu_{*k}} = m \delta^2 \mu_{*k}^{m-1} - \lambda_k = 0, \quad (16)$$

$$\frac{\partial F}{\partial \lambda_k} = \sum_{i=1}^c \mu_{ik} + \mu_{*k} - 1 = 0. \quad (17)$$

From Eqns. (15) and (16), we get

$$\mu_{ik} = \left[\frac{\lambda_k}{m} \right]^{\frac{1}{m-1}} \left[\frac{1}{E_{ik}^2} \right]^{\frac{1}{m-1}}, \quad (18)$$

and

$$\mu_{*k} = \left[\frac{\lambda_k}{m} \right]^{\frac{1}{m-1}} \left[\frac{1}{\delta^2} \right]^{\frac{1}{m-1}}. \quad (19)$$

Using Eqns. (17)–(19), we get

$$\left[\frac{\lambda_k}{m} \right]^{\frac{1}{m-1}} = \frac{1}{\sum_{j=1}^c \left(\frac{1}{E_{jk}} \right)^{\frac{2}{m-1}} + \left(\frac{1}{\delta} \right)^{\frac{2}{m-1}}}, \quad (20)$$

and then, by substituting this into Eqn. (18), the following equation can be obtained:

$$\mu_{ik} = \frac{1}{\sum_{j=1}^c \left(\frac{E_{ik}}{E_{jk}} \right)^{\frac{2}{m-1}} + \left(\frac{E_{ik}}{\delta} \right)^{\frac{2}{m-1}}}. \quad (21)$$

From Eqns. (2) and (12), the objective function of the New FCRM (NFCRM) clustering algorithm is defined as

$$J_{\text{new}}(S; U, \theta) = \sum_{k=1}^N \sum_{i=1}^c \mu_{ik}^m (y_k - [\mathbf{x}_k \ 1] \theta_i^T)^2 + \sum_{k=1}^N \delta^2 (1 - \sum_{i=1}^c \mu_{ik})^m, \\ = \sum_{k=1}^N \sum_{i=1}^c \mu_{ik}^m \left(y_k - \sum_{j=1}^{M+1} \theta_{ij} \hat{x}_{kj} \right)^2 + \sum_{k=1}^N \delta^2 (1 - \sum_{i=1}^c \mu_{ik})^m, \quad (22)$$

where $\hat{x}_k = [\mathbf{x}_k \ 1]$ and \hat{x}_{kj} is j -th coordinate of the vector $[\mathbf{x}_k \ 1]$.

The partial derivative of the objective function in Eqn. (22) is

$$\frac{\partial J_{\text{new}}}{\partial \theta_{ij}} = -2 \sum_{k=1}^N \mu_{ik}^m \left(y_k - \sum_{t=1}^{M+1} \theta_{it} \hat{x}_{kt} \right) \hat{x}_{kj}, \quad (23)$$

and then

$$\theta_{ij} = \frac{\sum_{k=1}^N \mu_{ik}^m (y_k - \sum_{t \neq j} \theta_{it} \hat{x}_{kt}) \hat{x}_{kj}}{\sum_{k=1}^N \mu_{ik}^m \hat{x}_{kj}^2},$$

$$i = 1, 2, \dots, c, \quad j = 1, 2, \dots, M + 1. \quad (24)$$

Based on the optimization conditions (Eqns. (21) and (24)), the identification algorithm for Type 1 NFCRM (NFCRM1) via iterative optimization is given as follows (Soltani et al., 2011).

Algorithm: NFCRM1

Given a data S , set $m > 1$. Fix $\gamma > 0$ and the parameter vectors θ_i at random. Pick a termination threshold $\epsilon > 0$ and an initial partition $U^{(0)}$.
Repeat for $l = 1, 2, \dots$.
Step 1. Compute error measure $E_{ik}(\theta_i)$ via Eqn. (3).
Step 2. Calculate δ^2 via Eqn. (13).
Step 3. Compute $\mu_{ik}^{(l)}$ and $\theta_{ij}^{(l)}$ via Eqns. (21) and (24), respectively.
Step 4. Compute $\text{err} = \|U^{(l)} - U^{(l-1)}\|$.
Until $\text{err} \leq \epsilon$, then stop. Otherwise, set $l = l + 1$ and return to Step 1.

As mentioned by Wu and Yang (2002), the non-Euclidean distance is more robust than the Euclidean one. Then we give an extension of the non-Euclidean distance in the case of the NFCRM1 algorithm. By transforming (Eqn. (3)), the new error measure of is defined as

$$E_{ik}(\theta_i) = \sqrt{1 - \exp(-\rho |y_k - [\mathbf{x}_k \ 1] \cdot \theta_i^T|)}, \quad (25)$$

where ρ is a positive constant. Then the NFCRM1 objective function (Eqn. (12)) is rewritten as follows:

$$J_{\text{new}}(S; U, \theta) = \sum_{k=1}^N \sum_{i=1}^c \mu_{ik}^m E_{ik}^2(\theta_i) + \sum_{k=1}^N \delta^2 (1 - \sum_{i=1}^c \mu_{ik})^m. \quad (26)$$

Equations (21) and (13) can be respectively rewritten as

$$\mu_{ik} = \frac{1}{\sum_{j=1}^c \left(\frac{E_{ik}}{E_{jk}} \right)^{\frac{2}{m-1}} + \left(\frac{E_{ik}}{\delta} \right)^{\frac{2}{m-1}}} \quad (27)$$

and

$$\delta^2 = \gamma \frac{1}{cN} \sum_{k=1}^N \sum_{i=1}^c E_{ik}^2(\theta_i). \quad (28)$$

Algorithm: NFCRM2

Fix $\rho > 0$, $\gamma > 0$ and choose parameter vector θ_i at random. Set a termination threshold $\epsilon > 0$ and an initial partition $U^{(0)}$.
Repeat for $l = 1, 2, \dots$.
Step 1. Compute error measure $E_{ik}(\theta_i)$ via Eqn. (25).
Step 2. Calculate δ^2 via Eqn. (28).
Step 3. Compute $\mu_{ik}^{(l)}$ and $\theta_{ij}^{(l)}$ via Eqn. (27) and Weighted Recursive Least-Squares (WRLS), respectively.
Step 4. Compute $\text{err} = \|U^{(l)} - U^{(l-1)}\|$.
Until $\text{err} \leq \epsilon$, then stop. Otherwise set $l = l + 1$ and return to Step 1.

The algorithm based new error measure is called Type 2 NFCRM (NFCRM2).

Two NFCRM algorithms are presented and developed taking into account the noisy data. The random initialization leads to the convergence to a local minimum of the objective function. To overcome this problem, we introduce PSO into the NFCRM2 algorithm to achieve global optimization.

4. NFCRM based on PSO (PSO-NFCRM)

4.1. PSO algorithm. Particle swarm optimization was first introduced by Kennedy and Eberhat (1995). The PSO algorithm has been successfully applied to solve various optimization problems. Panchal et al. (2009) proposed PSO based clustering algorithms for remote image classification. A Multi-swarm Cooperative PSO (MCPSO) was used to adjust the parameters of the T-S fuzzy model and for the control of nonlinear dynamical systems (Ben et al., 2008). Qiang and Xinjian (2011) proposed a PSO-based FCM clustering algorithm encoded by membership in order to handle data sets with dimensions smaller than the number of samples. Liang et al. (2009) used a PSO algorithm to optimize the initial clustering centers of the possibilistic c-means algorithm for image segmentation.

The PSO algorithm is initialized with a population of random solutions, called particles, to find an optimization result. Each particle has a position and a velocity, representing a possible solution to the optimization problem and a search direction in the search space. In each iterative process, the particle adjusts the velocity and position according to the best experience called the **pbest**, found by itself, and **gbest**, found by all its neighbors (Liang et al., 2009). For every generation, the velocity and po-

sition can be updated by the following equations:

$$\mathbf{V}_{pd}^{k+1} = \omega \mathbf{V}_{pd}^k + c_1 r_1 (\mathbf{pbest}^k - \mathbf{X}_{pd}^k) + c_2 r_2 (\mathbf{gbest}^k - \mathbf{X}_{pd}^k), \quad (29)$$

$$\mathbf{X}_{pd}^{k+1} = \mathbf{X}_{pd}^k + \mathbf{V}_{pd}^{k+1}, \quad (30)$$

where ω is the inertia weight, which often changes from 0.2 to 0.9; k is the iteration number; \mathbf{V}_{pd}^k is the velocity in the d -th dimension of the p -th particle; \mathbf{x}_{id}^k is the position in the d -th dimension of the p -th particle; \mathbf{pbest} and \mathbf{gbest} are the memory of the particle; c_1 and c_2 are the cognition and the social factor, respectively; r_1 and r_2 are random functions uniformly distributed in $[0, 1]$.

4.2. PSO-NFCRM algorithm. The PSO-NFCRM algorithm combines the advantages of the new fuzzy c-regression model clustering algorithm of Type 2 and the PSO algorithm. To evaluate each particle, the fitness function is defined as follows:

$$\text{Fitness} = \frac{G}{J_{\text{new}}(S; U, \theta)}, \quad (31)$$

where G is a user-defined parameter.

The PSO-NFCRM clustering algorithm is summarized in 7 steps.

Algorithm: PSO-NFCRM

Fix $\rho > 0$, $\gamma > 0$ and choose parameter vectors θ_i at random. Select a termination threshold $\epsilon > 0$ and an initial partition $U^{(0)}$. Choose the number of particles N_P ; Initialize the position and velocity of each particle, fix learning factors c_1 and c_2 and the inertia weight ω .

Repeat for $l = 1, 2, \dots$

Step 1. Compute error measure $E_{ik}(\theta_i)$ via Eqn. (25).

Step 2. Calculate δ^2 via Eqn. (28).

Step 3. Compute $\mu_{ik}^{(l)}$ and $\theta_{ij}^{(l)}$ via Eqn. (27) and WRLS, respectively.

Step 4. Calculate the fitness value of each particle according to Eqn. (31).

Step 5. Find the individual best \mathbf{pbest} for each particle and the global best \mathbf{gbest} .

Step 6. Update the velocity and the position of each particle using Eqns. (29) and (30), respectively.

Step 7. Compute $\text{err} = \|\mathbf{V}^{(l)} - \mathbf{V}^{(l-1)}\|$.

Until $\text{err} \leq \epsilon$, then stop. Otherwise, set $l = l + 1$ and return to Step 1.

4.3. Estimation of antecedent and consequent parameters. We use the novel fuzzy c-regression models for

decomposition of the input-output space into multiple linear structures. Gaussian membership functions are usually chosen to represent the fuzzy sets in the premise part of each fuzzy rule. As mentioned by Hathaway and Bezdek (1993) as well as Chaoshun *et al.* (2009), the antecedent parameters can be easily obtained using μ_{ik} . The fuzzy sets centers ν_{ik} and the standard deviations σ_{ik} are calculated as follows:

$$\nu_{ij} = \frac{\sum_{k=1}^N \mu_{ik} x_{kj}}{\sum_{k=1}^N \mu_{ik}}, \quad i = 1, 2, \dots, c, \quad j = 1, 2, \dots, M, \quad (32)$$

$$\sigma_{ij} = \sqrt{\frac{2 \sum_{k=1}^N \mu_{ik} (x_{kj} - \nu_{ij})^2}{\sum_{k=1}^N \mu_{ik}}}. \quad (33)$$

Once the antecedent parameters have been fixed, the OLS method (Chen *et al.*, 1989; Wu *et al.*, 2005) can be applied to estimate the consequent parameters for each rule. Using OLS, the consequent parameters are estimated by transforming the model (1) into an equivalent auxiliary one

$$\mathbf{Y} = P \Theta + \mathbf{e}, \quad (34)$$

where $\mathbf{Y} = [y_1, \dots, y_N]^T$, $P = [\mathbf{p}_1, \dots, \mathbf{p}_M]$ with $\mathbf{p}_i = [p_i(x_1), \dots, p_i(x_N)]$, $\Theta = [\Theta_1, \dots, \Theta_M]^T$ and $\mathbf{e} = [e_1, \dots, e_N]^T$.

The OLS algorithm is described as follows (Wang and Mendel, 1992):

Step 1. For $1 \leq i \leq M$, compute

$$\mathbf{w}_1^{(i)} = \mathbf{p}_i, \quad g_1^{(i)} = \frac{(\mathbf{w}_1^{(i)})^T \mathbf{Y}}{(\mathbf{w}_1^{(i)})^T \mathbf{w}_1^{(i)}}, \quad (35)$$

$$[\text{err}]_1^{(i)} = \frac{(g_1^{(i)})^2 (\mathbf{w}_1^{(i)})^T \mathbf{w}_1^{(i)}}{\mathbf{Y}^T \mathbf{Y}}. \quad (36)$$

Find

$$[\text{err}]_1^{(i_1)} = \max([\text{err}]_1^{(i)}), \quad (37)$$

and select

$$\mathbf{w}_1 = \mathbf{w}_1^{(i_1)} = \mathbf{p}_{(i_1)}, \quad g_1 = g_1^{(i_1)}. \quad (38)$$

Step 2. For $2 \leq i \leq M$, for $1 \leq k \leq N$, $i \neq i_1, \dots, i \neq i_k$, compute

$$\alpha_{jk}^{(i)} = \frac{\mathbf{w}_j^T \mathbf{p}_i}{\mathbf{w}_j^T \mathbf{w}_j}, \quad 1 \leq j < i, \quad (39)$$

$$\mathbf{w}_k^{(i)} = \mathbf{p}_i - \sum_{j=1}^{k-1} \alpha_{jk}^{(i)} \mathbf{w}_j, \quad (40)$$

$$g_k^{(i)} = \frac{(\mathbf{w}_k^{(i)})^T \mathbf{Y}}{(\mathbf{w}_k^{(i)})^T \mathbf{w}_k^{(i)}}, \quad (41)$$

$$[\text{err}]_k^{(i)} = \frac{(g_k^{(i)})^2 (\mathbf{w}_k^{(i)})^T \mathbf{w}_k^{(i)}}{\mathbf{Y}^T \mathbf{Y}}. \quad (42)$$

Find

$$[\text{err}]_k^{(i_k)} = \max([\text{err}]_k^{(i)}), \quad (43)$$

and select

$$\mathbf{w}_k = \mathbf{w}_k^{(i_k)}, \quad g_k = g_k^{(i_k)}. \quad (44)$$

Step 3. Solve the triangular system $A \Theta = \mathbf{g}$, where

$$A = \begin{pmatrix} 1 & \alpha_{12}^{i_2} & \alpha_{13}^{i_3} & \cdots & \alpha_{1M}^{i_M} \\ 0 & 1 & \alpha_{23}^{i_3} & \cdots & \alpha_{2M}^{i_M} \\ \vdots & \ddots & \ddots & \cdots & \vdots \\ \vdots & \cdots & \ddots & 1 & \alpha_{M-1,M}^{i_M} \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix}, \quad (45)$$

$$\mathbf{g} = [g_1, \dots, g_M]^T.$$

5. Simulation results

In this section, we are going to examine the performance of the proposed clustering algorithms developed above.

In this paper, the Mean Square Error (MSE) is used as the Performance Index (PI), which is defined as

$$\text{MSE} = \frac{1}{N} \sum_{k=1}^N (y_k - \hat{y}_k)^2. \quad (46)$$

5.1. Benchmark problem. We consider the nonlinear system given as (Bidyadhar and Debashisha, 2011)

$$y_k = \frac{y_{k-1} (y_{k-2} + 2) (y_{k-1} + 2.5)}{8.5 + y_{k-1}^2 + y_{k-2}^2} + u_k + v_k, \quad (47)$$

which is used as a test for identification techniques introduced in this paper, to demonstrate the effectiveness of the proposed algorithms in a noisy environment. Here y_k is the output, u_k is the input which is uniformly bounded in the region $[-1, 1]$ and v_k is a white noise with zero mean

and variance σ^2 , which is added to the output system at different SNR levels (SNR = 1, 5, 10, 15 and 20 dB).

We simulated two experimental cases: Case 1 and Case 2. The training data set contains 500 input–output pairs while for the testing 1000 data pairs are generated by the following input signal:

$$u_k = \begin{cases} \sin(\frac{2k\pi}{250}) & \text{if } k \leq 500, \\ 0.8 \sin(\frac{2k\pi}{250}) + 0.2 \sin(\frac{2k\pi}{25}) & \text{otherwise.} \end{cases} \quad (48)$$

Tables 1–6 compare our results with those obtained with different algorithms such as Gustafson–Kessel (GK) (Gustafson and Kessel, 1979), the New FCRM Algorithm (NFCRMA) (Chaoshun et al., 2009), FCM (Hoppner et al., 1999) and the Fuzzy Model Identification (FMI) clustering algorithm (Chen et al., 1998). We choose $\{y(k-1), y(k-2), u(k), u(k-1)\}$ as input variables, and the number of fuzzy rules is four. The parameter settings are $\gamma = 0.1$ and $\{\gamma = 0.01, \rho = 0.1\}$ for the NFCRM1 and NFCRM2 algorithms, respectively. In addition, the PSO-NFCRM algorithm performs best under the following settings: $\omega = 0.94, G = 1, c_1 = c_2 = 2, N_P = 50$. In Case 1, we compare our results with those cited above

Table 1. Comparison results (Case 1).

Algorithms	MSE _{Tr}	MSE _{Ts}
FCM	0.0090	0.2220
GK	0.0046	0.1347
FMI	0.0013	0.0181
NFCRMA	5.20e-4	0.0096
NFCRM1	4.76e-4	0.0052
NFCRM2	3.94e-4	0.0045
PSO-NFCRM	1.80e-4	0.0020

with regard to the noisy data. Table 1 shows the various modeling performance results obtained by different algorithms. MSE_{Tr} and MSE_{Ts} are the MSE for training and testing data, respectively. The comparison results demonstrate that the best MSE is obtained by the proposed methods. In the absence of noise, the positive scalar parameter δ can be regarded as a regulatory factor to reduce the sensitivity of the model to the identification data.

In Case 2, the noise influence is analyzed with different SNR levels (SNR=1, 5, 10, 15 and 20 dB). The parameter settings are: $\gamma = 0.1$ and $\{\gamma = 0.1, \rho = 1\}$ for the NFCRM1 and NFCRM2 algorithms, respectively. In addition, the PSO-NFCRM algorithm performs best under the following settings: $\omega = 0.9, G = 1, c_1 = c_2 = 2, N_P = 50$. As shown in Tables 2–6 both algorithms (FCRM1 and FCRM2) present almost similar performance for the lower level of noise regarding Figs. 2 and 3. However, only the PSO-FCRM algorithm retained good performance with a higher level of noise. On the whole, we

Table 2. Comparison results with $SNR = 20$ dB (Case 2).

Algorithms	MSE_{T_r}	MSE_{T_s}
FCM	0.0285	0.2417
GK	0.0258	0.1867
FMI	0.0134	0.0802
NFCRMA	0.0133	0.0621
NFCRM1	0.0126	0.0473
NFCRM2	0.0116	0.0442
PSO-NFCRM	0.0107	0.0321

Table 5. Comparison results with $SNR = 5$ dB (Case 2).

Algorithms	MSE_{T_r}	MSE_{T_s}
FCM	0.4577	0.8167
GK	0.3364	0.8094
FMI	0.3356	0.4859
NFCRMA	0.3309	0.4465
NFCRM1	0.3158	0.4276
NFCRM2	0.3094	0.4261
PSO-NFCRM	0.2794	0.4042

Table 3. Comparison results with $SNR = 15$ dB (Case 2).

Algorithms	MSE_{T_r}	MSE_{T_s}
FCM	0.0533	0.3164
GK	0.0471	0.2212
FMI	0.0373	0.1110
NFCRMA	0.0363	0.0806
NFCRM1	0.0355	0.0786
NFCRM2	0.0342	0.0770
PSO-NFCRM	0.0322	0.0732

Table 6. Comparison results with $SNR = 1$ dB (Case 2).

Algorithms	MSE_{T_r}	MSE_{T_s}
FCM	2.1292	2.1640
GK	1.0079	1.3765
FMI	0.9171	1.1649
NFCRMA	0.9046	1.1395
NFCRM1	0.8505	0.9491
NFCRM2	0.8092	0.9194
PSO-NFCRM	0.7141	0.8914

Table 4. Comparison results with $SNR = 10$ dB (Case 2).

Algorithms	MSE_{T_r}	MSE_{T_s}
FCM	0.1155	0.5505
GK	0.1100	0.3042
FMI	0.1068	0.2136
NFCRMA	0.1039	0.1926
NFCRM1	0.0963	0.1836
NFCRM2	0.0947	0.1762
PSO-NFCRM	0.0913	0.1683

note that, whatever the noise level is, our proposed algorithms always keep the best performance (Figs. 2 and 3). As shown in Fig. 1, our algorithms give the best accuracy models compared with other existing algorithms in the literature. Thus, they are more robust to noise, especially the PSO-NFCRM algorithm.

Figure 1(a) shows original and the identified data obtained using FCM for the testing data set, and Fig. 1(b) presents the respective errors. Figures 1(c) and (e) show the original and the identified data obtained using the NFCRMA and PSO-NCFRM algorithms for the testing data set, and Figs. 1(d) and (f) present the respective errors.

5.2. Box–Jenkins system. We consider the Box–Jenkins gas furnace data set (Box and Jenkins, 1970), which is used as a standard test for identification techniques. The data set consists of 296 pairs of input–output measurements. The input u is the gas flow rate into a fur-

nace; the output y is the CO_2 concentration in the outlet gases. In order to take all the above-mentioned issues into account, we simulated two experimental cases: Case 1 and Case 2. In Case 1, all the 296 data pairs are used as training data and $\{y(k-1), u(k-4)\}$ are selected as input variables to NFCRM1, NFCRM2 and PSO-NFCRM algorithms. The parameter settings are $\gamma = 0.01$ and $\{\gamma = 1, \rho = 1\}$ for the NFCRM1 and NFCRM2 algorithms, respectively. In addition, the PSO-NFCRM algorithm performs best under the following settings: $\omega = 0.9$, $G = 10$, $c_1 = c_2 = 2$, $N_P = 50$.

Figure 4 shows the modeling performance of the proposed clustering algorithms. Table 7 compares the results

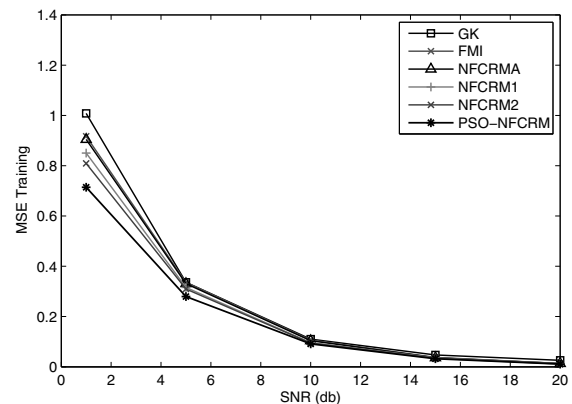


Fig. 2. MSE training performance of GK, NFCRMA, FMI, NFCRM1, NFCRM2 and PSO-NFCRM for different cases of SNR.

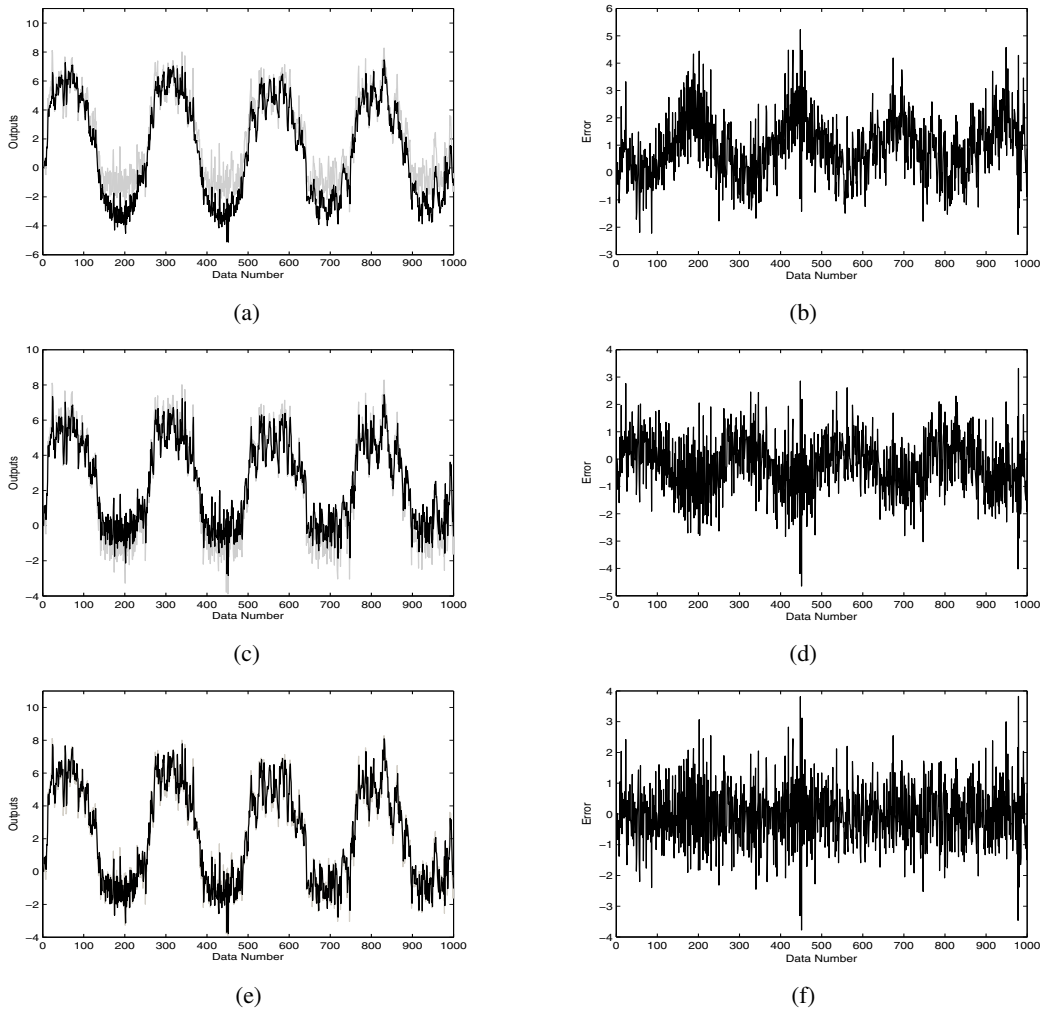


Fig. 1. FCM, NFCRMA and PSO-NFCRM performance for the testing data set with $SNR = 1$ dB.

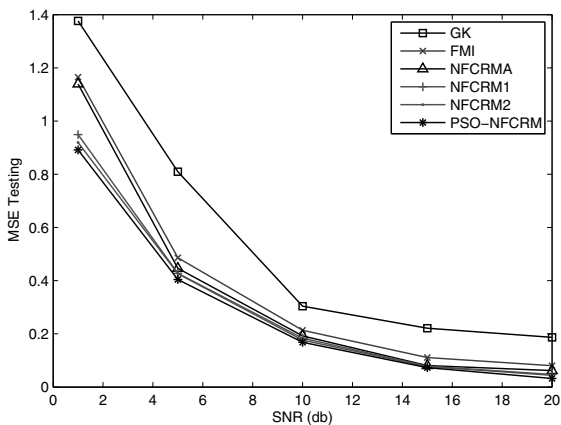


Fig. 3. MSE testing performance of GK, NFCRMA, FMI, NFCRM1, NFCRM2 and PSO-NFCRM for different cases of SNR.

of the proposed models with those of other models reported by Zhang *et al.* (2006) as well as Andri and Ennu (2011).

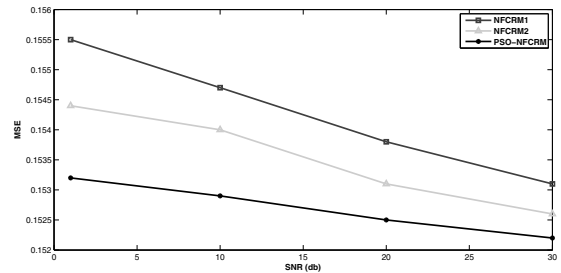


Fig. 4. MSE performance analysis of NFCRM1, NFCRM2 and PSO-NFCRM for different cases of SNR.

In Case 2, the first 148 input-output data were taken as training data and the last 148 as test data. We choose $\{y(k-1), y(k-2), y(k-3), u(k), u(k-1), u(k-2)\}$ as the variables of the fuzzy model, while the number of

Table 7. Comparison results for gas furnace process identification (Case 1).

Algorithms	No. of inputs	No. of rules	MSE
Tong (1980)	2	19	0.469
Pedrycz (1984)	2	81	0.320
Xu (1987)	2	25	0.328
Sugeno and Tanaka (1991)	2	2	0.359
Yoshinari (1993)	2	6	0.299
Joo (1997)	2	6	0.166
Chen (1998)	2	3	0.2678
Delgado (1999)	2	2	0.396
Liu (2002)	2	2	0.1653
Zhang (2006)	2	2	0.1600
Glowaty (2008)	2	2	0.391
Andri (2011)	2	10	0.167
NFCRM1	2	2	0.1528
NFCRM1-SNR = 30 dB	2	2	0.1531
NFCRM1-SNR = 20 dB	2	2	0.1538
NFCRM1-SNR = 10 dB	2	2	0.1547
NFCRM1-SNR = 1 dB	2	2	0.1555
NFCRM2	2	2	0.1524
NFCRM2-SNR = 30 dB	2	2	0.1526
NFCRM2-SNR = 20 dB	2	2	0.1531
NFCRM2-SNR = 10 dB	2	2	0.1540
NFCRM2-SNR = 1 dB	2	2	0.1544
PSO-NFCRM	2	2	0.1509
PSO-NFCRM-SNR = 30 dB	2	2	0.1522
PSO-NFCRM-SNR = 20 dB	2	2	0.1525
PSO-NFCRM-SNR = 10 dB	2	2	0.1529
PSO-NFCRM-SNR = 1 dB	2	2	0.1532

rules in our model is two. The parameter settings are $\gamma = 0.1$ and $\{\gamma = 0.1, \rho = 1\}$ for NFCRM1 and NFCRM2 algorithms, respectively. In addition, the PSO-NFCRM algorithm performs best under the following settings: $\omega = 0.9, G = 10, c_1 = c_2 = 2, N_P = 50$. Table 8 provides the performance of the proposed approaches compared with that of other models in the same case. The hyper-planes obtained by the PSO-NFCRM clustering algorithm are

$$\begin{aligned}
 y_k &= 0.8510y_{k-1} - 0.0202y_{k-2} - 0.1024y_{k-3} \\
 &\quad - 0.0398u_k - 0.0072u_{k-1} - 0.8256u_{k-2} \\
 &\quad + 14.4647, \\
 y_k &= 1.2172y_{k-1} - 0.4905y_{k-2} + 0.0881y_{k-3} \\
 &\quad + 0.3219u_k - 0.7976u_{k-1} - 0.1038u_{k-2} \\
 &\quad + 9.8267. \tag{49}
 \end{aligned}$$

Figure 5 shows a comparative MSE analysis for Case 2. It can be clearly shown that the PSO-NFCRM algorithm is more robust to noise than the NFCRM1 and NFCRM2 algorithms (Figs. 4 and 5). However, when the noise variance increases, the PSO-NFCRM clustering satisfies

Table 8. Comparison results for gas furnace process identification (Case 2).

Algorithms	No. of rules	MSE _{Tr}	MSE _{TS}
Kim <i>et al.</i> (1998)	2	0.034	0.244
Tsekouras (2005)	2	0.0164	0.145
Rezaee <i>et al.</i> (2010)	2	0.0162	0.1318
NFCRM1	2	0.0153	0.1047
NFCRM1-SNR = 30 dB	2	0.0159	0.1218
NFCRM1-SNR = 20 dB	2	0.0171	0.2317
NFCRM1-SNR = 10 dB	2	0.0180	0.3816
NFCRM1-SNR = 1 dB	2	0.0193	0.4414
NFCRM2	2	0.0151	0.0955
NFCRM2-SNR = 30 dB	2	0.0153	0.0961
NFCRM2-SNR = 20 dB	2	0.0165	0.1639
NFCRM2-SNR = 10 dB	2	0.0172	0.2030
NFCRM2-SNR = 1 dB	2	0.0189	0.2208
PSO-NFCRM	2	0.0148	0.0937
PSO-NFCRM-SNR = 30 dB	2	0.0151	0.0952
PSO-NFCRM-SNR = 20 dB	2	0.0155	0.1546
PSO-NFCRM-SNR = 10 dB	2	0.0161	0.1887
PSO-NFCRM-SNR = 1 dB	2	0.0183	0.2084

the convergence conditions. Consequently, the PSO-NFCRM algorithm becomes more robust to noise with MSE = 0.0183. The other two algorithms, NFCRM1 and NFCRM2, exhibit poor performance with the MSE equal to 0.0193 and 0.0189, respectively. A similar analysis can be seen also in Table 7. In the absence of noise, it is clear that PSO-NFCRM performs better than the other algorithms reported in the literature (Tables 7 and 8). A good approximation accuracy of PSO-NFCRM is shown in Fig. 5.

6. Conclusions

In this paper, a new fuzzy c-regression clustering algorithm is proposed using a modified objective function, a new error measure, and a parameter estimation based particle swarm optimization. The application of a modified objective function improves the robustness of the FCRM method, based on the noise clustering algorithm and the new error measure. Yet, the PSO procedure allows achieving the global minimum of the new objective function. The proposed modifications of the FCRM method provide better results of fuzzy modeling of unknown nonlinear systems. The robustness and the quality of these modifications in the FCRM method are demonstrated by simulation results of two benchmark problems. Thus, the proposed methods show favorable results compared with the techniques reported in the literature. Consequently, they may be extended to identification of complex applications regarding various types of modeling problems in

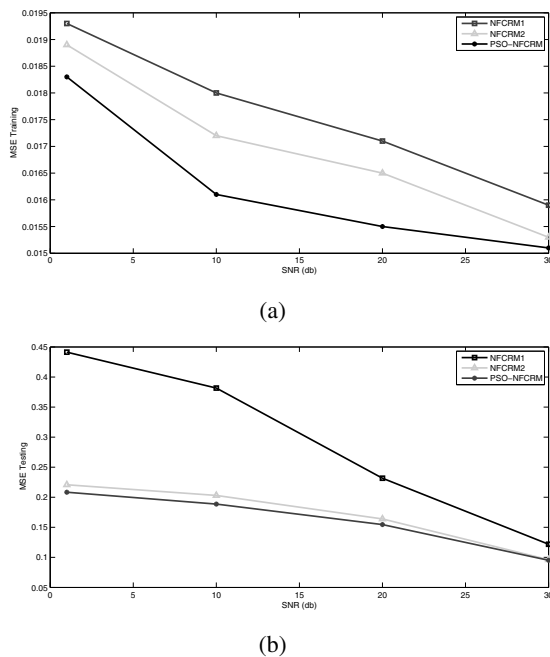


Fig. 5. MSE training and testing performance analysis of NFCRM1, NFCRM2 and PSO-NFCRM for different cases of SNR.

noisy environments.

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