A COUPLED DISCRETE-HOMOGENIZED APPROACH TO STUDY THE BEHAVIOR OF BALLAST UNDER RAILWAYS

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1. Abstract

Modeling of granular materials is an important research area especially in civil engineering. Granular materials are strongly present in nature, and are involved in many industrial processes, such as the ballast used under the railways of a high-speed trains. Using only a discrete approach to simulate a large medium that employs an enormous number of grains of ballast seems very difficult in terms of calculations and implementation. Moreover, a homogeneous approach considered like a continuum approach, does not give the exact response in a zone where particular and localized phenomena can occur. The purpose of this paper is to present a formulation for coupling between a discrete approach at the microscopic scale and a homogenized approach deduced from the discrete approach at the macroscopic scale. Numerical methods that are enable to treat such situations where the domain can be decomposed in sub domains described by approaches on various scales, are proposed. It is at the same time a question of clarifying the bases of these approaches and of proposing numerical tools adapted to this kind of situations.

Keywords: Discrete approach, Homogenized approach, grains of ballast, static equilibrium, deflection.

2. Introduction

A primary objective of modern materials modeling, is to predict the materials response and failure governed by deformation mechanisms. Modeling the ballast all along a line at high speed (TGV) by using a Discrete Elements Method (DEM) seems very difficult in 3D. This difficulty is due to the long time of simulation and which carries out at a high cost. However, by using continuum mechanics for zones where singular phenomena occur, we can not obtain exact behavior of the studied material. It is clear that some coupled methodology must be established to combine the strengths of both discrete and continuum modeling. Although this field has acquired a substantial history, it remains an active area of research [1], [2], [3].

The majority of methods of coupling between the discrete and continuum modeling consider firstly a microscopic approach on fine scale, and deduce the coarse approach on macroscopic scale from the microscopic approach.

In the objective of basing a clear idea on the behavior of the ballast under the rails of a TGV in 2D or 3D, we propose a 1D model composed of a beam resting on springs, and on which we apply a load F.

In this work, the deflection of the beam (as well as all node parameters) that minimizes the energy of the system is calculated using two approaches; discrete approach and homogenized approach deduced from the discrete approach. A comparison between the response of the sytem obtained using these approaches will be always made in order to illustrate the cases where the homogenized approach can not replace the discrete approach. This difference will bring us to apply the coupled approach.

3. Discrete and Homogenized Approach

We note that the beam respresents a rail, under which the track tie and the grains of ballast are modelled by springs with elastic behavior. The applied load is supposed fix, so we are interested to the static problem. The static equilibrium equation of the discrete approach is written:

$$EIu^{(4)}(x) + \sum_{i=1}^{N} h\, k_i u(x_i)\delta\left(x - x_i\right) = F\delta(x - D) \tag{1}$$

D, h and k_i means respectively the distance between the end of the beam and the point of application of the applied load, the spacing between consecutive track ties and the stiffness of springs. N is the number of track ties and x_i is the node position.

For the homogenized approach we proceed by the homogenization of the beam compared to the stiffnesses of the springs. The philosophy of this approach returns to the fact, that in the microscopic scale we started from an enormous degree of freedom (dof), whereas the homogenisation is used to replace the zones that have homogeneous dof by only one dof, which will have like consequence to reduce the nedeed computing time.

After a numerical implementation of these approaches, several test were elaborated. We tested many cases where we have heterogeneous and homogeneous stiffness. In the case of heterogeneities under the rail way, it was clear that the two approaches lead to different results, especially when the ratio between the number of the elements of two approaches increases. This difference is illustrated more particularly in the zones presenting heterogeneities. Because of this difference, a coupled approach between the homogenized and discrete approahes that is enable to produce a similar behavior of ballast like that produced by the discrete approach, is proposed.

4. Coupled Approach

The first stage in the numerical solution of the coupled approach is a homogenized approach where the ratio between the size of a homogenized element and another discrete element is very high. Firstly the mechanical parameters on the first node are calculated and the criteria of coupling is applied. This criterion can be summarized as follows: If the deflection and rotation errors calculated by two approaches is lower than 10%, the scale of computation is not changed, else the discretization is refined that it means a decreasing in the size of the homogenized element. This procedure of refinement is used as long as it is necessary in order to be placed on the scale of the discrete elements.

5. Conclusion

After applying the coupled approach in the cases where the homogenized and discrete approaches do not give an identical behavior of the ballast, we could show the efficiency of this approach and it can be summarized in two points. Firstly, the good agreement between the discrete and the coupled behavior and secondly, the reduction of the number of discrete nodes that implies a reduction in the computation time compared to the discrete approach.

6. References

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