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A

VINDICATION

OF

Sir *ISAAC NEWTON*'s Principles of FLUXIONS,

AGAINST THE

OBJECTIONS

Contained in the

ANALYST.

By *J. WALTON.*

——— *Si quid novisti rectius istis,*
Candidus imperti: Si non, his utere mecum. Hor.

In the Fulness of his Sufficiency he shall be in Straits: Every Hand of the WICKED *shall come upon him.* JOB.

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A VINDICATION, &c.

UNDER Pretence of ſome Abuſes committed by Mathematicians, in Virtue of the Authority they derive from their Profeſſion, the Author of the *Minute Philoſopher*, in a *Libel* called the *Analyſt*, has declared 'em Infidels, Makers of Infidels, and Seducers of Mankind in Matters of the higheſt Concernment: This he profeſſes to have done, not from any real Knowlege of his own, but from the credible Information of others; but he has neither produc'd his Informers, nor proved the Accuſation in any one Inſtance; and therefore it is Defamatory.

But

But they assume an Authority, it seems, in Things foreign to their Profession, and undertake to decide in Matters whereof their Knowledge can by no Means qualify them to be competent Judges: And as this Practice, if not prevented, may be of dangerous Consequence; he has undertaken to enquire into the Object, Principles and Method of Demonstration, admitted by the Mathematicians of the present Age, with the same Freedom, he says, they presume to treat the Principles and Mysteries of Religion, to the end, that all Men may see what Right they have to lead or what Encouragement others have to follow them.

And whereas Sir *Isaac Newton* has presum'd to interpose in *Prophecies* and *Revelations*, and to decide in religious Affairs, it has been thought proper to begin with his Method of *Fluxions*, and to try what cou'd be done with that Method, with the Inventor himself, and with his Followers: And what has been done with 'em every intelligent Reader is able to judge.

If

If this Writer may be credited, the Objects about which the Method of *Fluxions* is conversant, are difficult to conceive or imagine distinctly; the Notions are most abstracted incomprehensible Metaphysics, not to be admitted for the Foundations of clear and accurate Science; the Principles are obscure, repugnant, precarious; the Arguments admitted in Proofs, are fallacious, indirect, illogical; and the Inferences and Conclusions not more just, than the Conceptions of the Principles are clear.

How far the Credulous and Injudicious may become infected by this uncommon Way of treating Mathematics and Mathematicians, is not easy to foresee, and therefore it will be necessary to give a short Account of the Nature of *Fluxions*, and of the Objects about which the Method is conversant; and when it shall be made apparent, that this Author has not understood the Metaphysics he wou'd refute; it will not be difficult to defend the Principles and their Demonstrations, from any Imputations of Fallacy or Repugnancy, which

which have yet been pointed at by him or any other Writer.

" In the Method of *Fluxions*, Sir
" *Isaac Newton* considers mathematical
" Quantities, not as composed of the
" smallest Parts, but as described or ge-
" nerated by continual Motion. Lines
" are described, and by being described
" are generated, not by an Apposition
" of Parts, but by the Motion of Points;
" Surfaces by the Motion of Lines, So-
" lids by the Motion of Surfaces, Angles
" by the Rotation of their Sides, Times
" by a continual Flux, and so of the rest.
" And by considering that Quantities, in-
" creasing in equal Times, and generated
" by increasing, become greater or less,
" according as the Velocity with which
" they increase, and are generated, is greater
" or less, he found a Method of deter-
" mining the Quantities themselves from
" the Velocities of the Motions, or of
" the Increments, with which they are
" generated; calling the Velocities of
" the Motions or of the Increments
" *Fluxions*, and the Quantities generated
" *Fluents*.

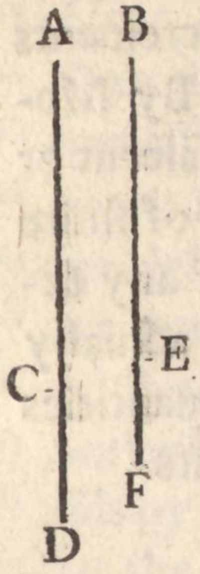
The

The momentaneous Increments or Decrements of flowing Quantities, he elsewhere calls by the Name of *Moments*, and considers the Increments as added or affirmative Moments, and the Decrements as subducted or negative ones: By Moments we may understand the nascent or evanescent Elements or Principles of finite Magnitudes, but not Particles of any determinate Size, or Increments actually generated; for all such are Quantities themselves, generated of Moments.

The Magnitudes of the momentaneous Increments or Decrements of Quantities are not regarded in the Method of Fluxions, but their first or last Proportions only; that is, the Proportions with which they begin or cease to exist: These are not their Proportions immediately before or after they begin or cease to exist, but the Proportions with which they begin to exist, or with which they vanish. If the Lines A C and B E are supposed to be generated in the same Time, by the Motions of the Points A and B, to C and E; and if by continuing the Motions of

those

those Points to D and F, they generate D C and E F, synchronal Increments of A C and B E; it is evident that the Points



D and F may flow back in the same Time to C and E, and by flowing back perpetually lessen the Magnitudes of those Increments till at last they vanish together, when the Points D and F come to coincide with C and E: Now the ultimate Ratio of those Increments is that Ratio with which they vanish and become nothing; or the Ratio with which they cease to be: And the first Ratio of them is the Ratio with which they begin to exist, at the very first setting out of the Points from C and E towards D and F.

Hence, if the describing Points move back to C and E, in the same Time wherein by moving forward they generated the Increments D C and E F; and in returning have every where the same Velocities, at certain Distances from C and E, which they had at those Distances in in going forward; the laſt and firſt Ratios of the Increments will be equal, or they will vaniſh, and become nothing, with the very ſame Ratio with which they began to exiſt.

Hence likewiſe it appears, that to obtain the laſt Ratio of ſynchronal Increments, the Magnitudes of thoſe Increcrements muſt be infinitely diminiſh'd. For their laſt Ratio is the Ratio with which they vaniſh or become nothing: But they cannot vaniſh or become nothing, by a conſtant Diminution, till they are infinitely diminiſh'd; for without an infinite Diminution they muſt have finite or aſſignable Magnitudes, and while they have finite or aſſignable Magnitudes they cannot vaniſh.

The ultimate Ratios with which ſynchronal Increments of Quantities vaniſh, are not the Ratios of finite Increments, but Limits which the Ratios of thoſe Increments attain, by having their Magnitudes infinitely diminiſh'd: The Proportions of Quantities which grow leſs and leſs by Motion, and at laſt ceaſe to be, will continually change, and become different in every ſucceſſive Diminution of the Quantities themſelves: And there are certain determinate Limits to which all ſuch Proportions perpetually tend, and approach nearer than by any aſſignable Difference, but never attain before the Quantities themſelves are infinitely diminiſh'd; or till the Inſtant they evaneſce and become nothing. Theſe Limits are the laſt Ratios with which ſuch Quantities or their Increments vaniſh or ceaſe to exiſt; and they are the firſt Ratios with which Quantities or the Increments of Quantities, begin to ariſe or come into being.

Quantities, and the Ratios of Quantities, which conſtantly tend to an Equality, by a Diminution of their Difference, and before the End of ſome finite Time approach nearer to an Equality than by any aſſignable Difference, at laſt become equal. For they become equal when the Difference between them vaniſhes or becomes nothing; and it will

vaniſh

vanish or become nothing by being infinitely diminished: If the Quantities A C and A D perpetually tend to an Equality, either by the Motion of the Point D to C, or by that of C to D; they will become equal, and their Ratio a Ratio of Equality, when their Difference C D, by a constant Diminution, vanishes and becomes nothing, which it will do under a Coincidence of the two Points in C or D; and then either A D becomes A C, and so $\frac{AD}{AC}$ or $\frac{AC}{AC}$ is a Ratio of Equality, or else A C becomes A D and $\frac{AD}{AC}$ becomes $\frac{AD}{AD}$; which is also a Ratio of Equality.

The Fluxions of Quantities are very nearly as the Increments of their Fluents generated in the least equal Particles of Time: If C D and E F be Increments of the Fluents A C and B E, described in the least equal Particles of Time; the Fluxions in the Points C and E will be nearly as the Increments D C and E F. For from the exceeding Smallness of the

Times it is evident that the Points D and F, must be extreamly near to C and E; and by Consequence however the Velocities are accelerated or retarded thro' the Spaces CD and EF, they will be very nearly the same in D and F as they were in C and E: But Velocities which are very nearly uniform, will be very nearly proportional to the Spaces described by them in equal Times; and therefore the Velocities in the Points C and E, which are the Fluxions of AC and BE in those Points, will be very nearly as the Increments DC and EF, described in the least equal Particles of Time.

The Fluxions of Quantities are accurately in the first or last Proportions of their nascent or evanescent Increments: Thus the Fluxions of AC and BE, in the Points C and E, are in the first or last Ratio of the Increments CD and EF. For the first or last Ratio of the Increments CD and EF, is the Ratio with which they begin or cease to exist: But the Ratio with which they begin or cease to exist, is the same with the Ratio of the

Velocities in C and E, which are the Fluxions in thoſe Points; and conſequently the Fluxions in C and E are in the firſt or laſt Ratio of the Increments CD and EF.

The Fluxions of Quantities are only the Velocities with which thoſe Quantities begin to be generated or increaſed; or the Velocities with which the generating Quantities begin to ſet out; not the Velocities they have after moving thro' Spaces of any finite or aſſignable Magnitudes: And therefore if two mathematical Quantities ſet out together, and begin to move with Velocities which are as *a* and *b*, they muſt begin to deſcribe Spaces in the ſame Proportion with *a* and *b*; or the Proportion with which the Spaces begin to exiſt or to be deſcribed, muſt be the ſame with that which the Velocities have at the very Beginning of the Motion. For in the very Beginning of the Motion there is neither any Change of Velocity from Acceleration or Retardation, nor Difference of Time.

Hence

Hence it appears that to obtain the Ratios of Fluxions, the correſponding ſynchronal or iſochronal Increments muſt be leſſened *in infinitum.* For the Magnitudes of ſynchronal or iſochronal Increments muſt be infinitely diminiſhed and become evaneſcent, in order to obtain their firſt or laſt Ratios, to which Ratios the Ratios of their correſponding Fluxions are equal.

Hence likewiſe it appears that the Moments of like Quantities, compared with each other, are in Ratios compounded of the Ratios of the generating Quantities, taken when firſt they begin to move, and of the Velocities with which they ſet out: Or in Ratios compounded of the Ratios of the generating Quantities when firſt they begin to move, and of the firſt Ratios of their ſynchronal naſcent Increments. The Moments of Lines therefore are as the generating Points and as the Velocities with which they begin to move taken together: The Moments of Surfaces, which become greater or leſs by carrying of moveable Lines along immoveable

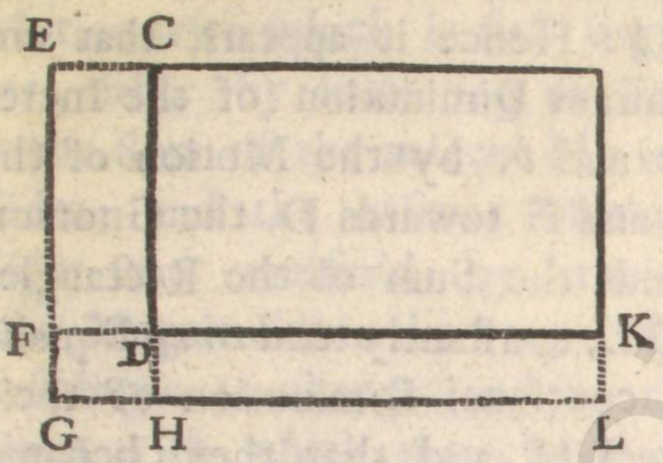
moveable ones, are in Ratios compounded of the Ratios of the moving Lines, and of their firſt Velocities, or firſt Ratios of the Increments which begin to riſe with thoſe Velocities: And the whole Motion by which Squares or Rectangles begin to alter, either from an Augmentation or Diminution of their Sides, is the Sum of the naſcent Motions of thoſe Sides, or the Sum of the naſcent Increments ariſing with the firſt Motions of the Sides: For the Proportion of naſcent Increments is the ſame with that of the Motions with which they begin to be generated.

From this ſhort Account of the Nature of Fluxions, compared with the *Analyſt*, it appears that the Author of that Paper is greatly miſtaken in the Object of 'em; and he is alſo miſtaken in the Principles: For he thinks the Moment or Fluxion of a Rectangle, contain'd under two indeterminate Quantities A and B, from whence are deduc'd Rules for obtaining the Moments or Fluxions of all other Products or Powers whatever, is no where truly determin'd by Sir *Isaac Newton:* But he ought to have read Sir *Isaac* with more Care and Attention than he seems to have done, before he set up to decide and dictate in Matters of this Nature; and he wou'd do well yet to read him with Attention.

If any Rectangle CK be increased from an Augmentation of its Sides by Motion, so as that DK becomes LG in the same Time that DC becomes EG; the Moment of that Rectangle is the Sum of the Rectangles of DK into the Moment of DC, and of DC into the Moment of DK: That is, putting A and B for the Sides DK and DC, and a and b for their respective Moments, the Moment of the Rectangle AB will be $Ab+Ba$.

For the Gnomon CGK in the Instant it begins or ceases to exist is the Moment of the Rectangle CK: But the first or last Ratio of that Gnomon to the Sum of the Rectangles LD and FC is a Ratio of Equality: For the Difference between

between the Gnomon and the Sum of those Rectangles perpetually lessens, by a constant Diminution of the Increments



F D and D H, or by an Approach of the Points F and H towards D; as will be manifest on taking the Ratio between the said Gnomon and the Sum of the Rectangles, at several Distances of the Points F and H from D: For whatever be the Magnitudes of *a* and *b*, when F and H first begin to move back towards D, the Gnomon C G K and Sum of the Rectangles L D and F C, will be as $Ab + Ba + ba$ and $Ab + Ba$; when those Points, by moving towards D, have lessen'd the Increments of D K and D C to $\frac{1}{2}a$ and $\frac{1}{2}b$, the Gnomon and Sum of the Rectangles will be as $Ab + Ba + \frac{1}{2}ba$ and $Ab + Ba$; when they have lessen'd the Increments to $\frac{1}{4}a$ and $\frac{1}{4}b$, the Gnomon and Sum of the Rectangles will be as $Ab+Ba+\frac{1}{4}ab$ and $Ab+Ba$; and as $Ab+Ba+\frac{1}{8}ab$ and $Ab+Ba$, when they have leſſen'd thoſe Increments to $\frac{1}{8}a$ and $\frac{1}{8}b$: Hence it appears, that under a conſtant Diminution of the Increments a and b, by the Motion of the Points F and H towards D, the Gnomon C G K and the Sum of the Rectangles C F and D L, conſtantly tend to an Equality by a continual Diminution of their Difference F H, and that they become equal, and their Ratio becomes a Ratio of Equality, in the Inſtant that Difference vaniſhes and the Points F and H coincide with D; or in other Words the Gnomon and Sum of the Rectangles L D and F C begin or ceaſe to *be* under a Ratio of Equality: And therefore the Sum of thoſe Rectangles, or $Ab+Ba$, is the Moment of A B.

Hence, the Gnomon C G K, or $Ab+Ba+ab$, found by taking the Difference between the Rectangles E L and C K, or by deducting the Rectangle A B from a Rectangle contain'd under the Sides A and B increaſed by their whole Increments, is not the Moment or Fluxion of of the Rectangle A B, except in the very Instant when it begins or ceases to exist: And this will also appear by considering it in another Light. For the Moment of the Rectangle C K, or the Motion with which it first begins to alter, either by increasing or decreasing, is the Sum of the nascent Motions of its Sides; and the nascent Motions of its Sides, are measur'd by their respective Magnitudes in the very Instant they first begin to change, and by the Velocities with which they begin to move taken together; and the Velocities with which the Sides begin to move being in the first Ratio of the momentaneous Spaces which arise with 'em; it follows that the Sum of the nascent Motions of the Sides, is the Sum of D K multiply'd into D H in its nascent State, and of C D multiply'd into D F in its nascent State: But D H and D F in their nascent States, are the Moments of D C and D K: And therefore the whole Moment of the Rectangle A B, is $A b + B a$.

In determining the Moments of Quantities, Sir *Isaac Newton* expresly tells us, that we are only to consider the Ratios with which they begin or cease to exist; and to obtain those Ratios, it is not necessary that the insochronal Increments shou'd have finite Magnitudes. "*Cave tamen intellexeris particulas* "*finitas*, says he, *Particulæ finitæ non* "*sunt Momenta, sed Quantitates ipsæ* "*ex Momentis genitæ. Intelligenda* "*sunt Principia jamjam nascentia* "*finitarum Magnitudinum. Neque* "*enim spectatur in hoc Lemmate mag-* "*nitudo Momentorum, sed prima na-* "*scentium proportio.* And in another Place, "*Fluxiones sunt quam proximè* "*ut Fluentium Augmenta æqualibus* "*Temporis particulis quam minimis* "*genita, et, ut accurate loquar, sunt* "*in primâ ratione Augmentorum na-* "*scentium; exponi autem possunt per* "*lineas quascunque, quæ sunt ipsis* "*proportionales.* And again, *Siquan-* "*do facili rerum conceptui consulens* "*dixero Quantitates quam minimas,* "*vel evanescentes, vel ultimas; cave* "*intelligas*

" *intelligas quantitates magnitudine*
" *determinatas, ſed cogita ſemper di-*
" *minuendas ſine limite.*

From theſe Paſſages it appears, that the Gnomon CGK in its naſcent or evaneſcent State only, or in the Inſtant it begins or ceaſes to exiſt, is the Moment or Fluxion of the Rectangle CK; and in a naſcent or evaneſcent State, when only the Increments of Quantities become their Moments, its Ratio to $Ab+Ba$, which is the Sum of the Rectangles LD and FC, is a Ratio of Equality. By diminiſhing the Magnitudes of a and b, which are Increments of DK and DC, it is obvious that the Gnomon CGK diminiſhes faſter in Proportion, than the Sum of the Rectangles FC and DL does; and by diminiſhing faſter, it continually approaches to an Equality with that Sum, and attains the Equality only, when their Difference FH becomes evaneſcent, that is, when the Points F and H come to coincide with D; ſo that here is no Artifice or falſe Reaſoning uſed, to get rid of HF, or ab, that Term having no Exiſtence

Exiſtence at the very Beginning of the Motion, or in the naſcent State of the Augments.

After Sir *Iſaac* had ſo expreſly told us what he meant by Moments and Fluxions, and by naſcent or evaneſcent Quantities, one wou'd imagine it impoſſible to have miſtaken and miſrepreſented him in the Manner this Author has done. He ſeems indeed to have been lead, or rather to have been deceived, by an Opinion that there can be no firſt or laſt Ratios of mathematical Quantities or of their iſochronal Increments generated or deſtroy'd by Motion; imagining that no ſuch Quantities, by any Diviſion or Diminution whatever, can be exhauſted or reduc'd to nothing: But if Lines, Surfaces and Solids can be generated or augmented by the Motion of Points, Lines, and Surfaces, they may likewiſe be deſtroy'd or diminiſh'd by the Motion of the ſame Points, Lines and Surfaces, in returning to the Places from whence they firſt ſet out. While a generating Quantity moves back thro' the ſame

Space it before described in moving forward, the Quantity generated, or its Augment, continually lessens; and by persevering in a State of decreasing, it must in some finite Time vanish and become nothing; and therefore mathematical Quantities, by a constant Diminution, may be reduc'd to nothing: And such as are thus generated or destroy'd in equal Times by Motion, or which arise and vanish together, will arise or vanish under certain Ratios, which are their first or last Ratios; or the Ratios with which they begin or cease to be: But it may be necessary to persue this Case a little farther, and see whether Sir *Isaac Newton*'s Demonstration of it cannot be defended, and proved to be geometrical.

" Suppose any Rectangle A B augmented by continual Motion; and the momentaneous Increments of its Sides A and B to be denoted by a and b; the Moment of the generated Rectangle will be measured by $Ab+Ba$.

" For

" For when the Sides A and B wanted half of their Moments, the Rectangle was $\overline{A-\frac{1}{2}a}\times\overline{B-\frac{1}{2}b}$ or $AB-\frac{1}{2}Ab-\frac{1}{2}Ba+\frac{1}{4}ab$: And as soon as the Sides A and B are augmented by the other halves of their Moments, it becomes $\overline{A+\frac{1}{2}a}\times\overline{B+\frac{1}{2}b}$, or $AB+\frac{1}{2}Ab+\frac{1}{2}Ba+\frac{1}{4}ab$: From this Rectangle deduct the former, and there will remain $Ab+Ba$: Therefore the Increment of the Rectangle AB, generated with a and b the whole Increments of the Sides, is $Ab+Ba$.

Now, in determining the Moment of a Rectangle, there is nothing to be considered, when it first begins to be augmented by the Motions of its Sides, but the Sides themselves and the Velocities with which they begin to move; or the Sides and the first Ratio of the Spaces described by them. And therefore the true Moment of the Rectangle AB, or the Law according to which it begins to be augmented, on the Principles of Sir *Isaac Newton*, will only be the Sum of the

the Rectangles Ab and Ba; for the Sides A and B begin to move with Velocities which are as b and a: But this Moment Ab+Ba, is manifestly equal to the Difference between the Rectangles $\overline{A+\frac{1}{2}a} \times \overline{B+\frac{1}{2}b}$ and $\overline{A-\frac{1}{2}a} \times \overline{B-\frac{1}{2}b}$; and therefore Sir *Isaac*'s determination of it is geometrical.

From the foregoing Principle so demonstrated, the general Rule for finding the Moment or Fluxion of any Power of a flowing Quantity, is easily deduc'd: It is easy, from hence, to infer that the Moment or Fluxion of A^n is as nA^{n-1}, or that the Fluxion of x^n is as nx^{n-1}: But because this is also determined in a manner seemingly different, by Sir *Isaac*, in his Introduction to the *Quadrature of Curves*, the Author of the *Analyst* observes, " That there seems to have been " some inward Scruple or Consciousness " of Defect in the foregoing Demon- " stration." And he repeats the same Reflection in another Place, adding withal, " That Sir *Isaac* was not enough " pleased with any one Notion steadily

" to adhere to it: But Reflections of this Nature deserve no Regard unless it be allowable, by way of Return, to observe that the Person who makes 'em has very often been guilty of like Practices himself.*

The Proof given in the Introduction to the *Quadratures*, is said to be a most inconsistent way of arguing; as proceeding to a certain Point of the Demonstration upon Supposition of an Increment, and then in a fallacious Manner, shifting the Supposition to that of no Increment; and to shew the Inconsisteny with greater Force, a Lemma is premised by Way of Axiom; as if some very obvious and natural Application of an apparent Truth, wou'd at once overturn the Whole of Sir *Isaac*'s Demonstration: But that Lemma, however true in it self, is no Way pertinent to the Case for which it was intended; and therefore

* See his new *Theory of Vision*; his *Treatise on the Principles of Human Knowlege*; and some later Undertakings of equal Importance.

therefore ſuch Inferences as are made in Virtue of it, with relation to the Point in diſpute, are illegitimate, and inconſiſtent with the Rules of true reaſoning.

Nothing is more plain and obvious, than that Quantities which begin to exiſt together under certain Proportions, and with certain Velocities; may become evaneſcent and ceaſe to exiſt, under the ſame Proportions and with the ſame Velocities; and this is all Sir *Iſaac* ſuppoſes in that Determination of the Fluxion of x^n; and it is not very obvious, that the Lemma which this Author has hit upon, is applicable to Caſes of ſuch a Nature.

That the Reader may ſee how ſtrictly Sir *Iſaac Newton* has kept to the ſame Principle in this Determination, how ſteadily he adheres to the ſame Method, and how ill the Author of the *Analyſt* has proved his Imputations; it will be neceſſary to perſue this Point, and conſider the Proof it ſelf.

Let it be required to find the Fluxion of x^n, supposing x to increase uniformly.

Suppose x in any finite Particle of Time, to become greater than before, by a finite Increment, whose Magnitude is expreſs'd by o. Then, in the same Time that x, by flowing becomes $x+o$, the n Power of x will become $x^n + nox^{n-1} + \frac{n^2-n}{2} o^2 x^{n-2} +$ &c. Consequently the Magnitudes of the synchronal Increments of x and of x^n, are to each other as 1 and $nx^{n-1} + \frac{n^2-n}{2} ox^{n-2} +$ &c. Now, let the Increments decrease by flowing back, in like Manner as they increas'd before by flowing forward, and continually grow less and less till they vanish; and their ultimate Ratio, that is, the Ratio with which they become evanescent, will be expreſs'd by 1 and nx^{n-1}: But the Fluxions of Quantities are in the last Ratio of their evanescent Arguments; and by Consequence the Fluxion of x is to that of x^n, as 1 to nx^{n-1}.

In

In this Computation, Sir *Isaac* endeavours to collect the Proportion with which the isochronal Increments of x and of x^n, begin or cease to exist: Their Proportion obtain'd on Supposition that o is something, is allowed to be the same with that of 1 and $nx^{n-1} + \frac{n^2-n}{2} ox^{n-2} +$ &c. And it must be acknowleg'd that this Ratio has a Limite dependent on the Magnitude of o, which Limite it cannot attain before the Increments are infinitely diminish'd and become evanescent; and when, by an infinite Diminution, they become evanescent, no other Terms of their Ratio will be affected, so as to vanish with 'em, but such as are govern'd or regulated by them: In the Instant therefore that o vanishes, $\frac{n^2-n}{2} ox^{n-2}$ and all ensuing Terms of the Series absolutely vanish together; but the Terms 1 and nx^{n-1} remain invariable under all possible Changes of the Increments, from any finite Degrees of Magnitude whatever, even till they become

come evanescent: They therefore express the last Ratio, under which the isochronal Increments of x and x^n vanish, or the Proportion of the Velocities with which those Increments cease to exist: Sir *Isaac Newton* then rightly retain'd 'em for the Measures of the Ratio of the Fluxions of x and x^n, tho' got in Virtue of his first Supposition; and the Fallacy, the Inconsistency, lies on the Side of this Author; who wou'd have them rejected on the Authority of a Lemma not to the purpose.

To make this Point still more plain and obvious, I shall propose the reasoning in a stronger Light: It amounts therefore to this, or may in other Words, be thus expressed: If x be suppos'd to flow uniformly, the Fluxions of x and x^n, will be as 1 and nx^{n-1}. For in the same Time that x by flowing, becomes $x+o$, x^n will become $\overline{x+o}|^n$, which by the Method of infinite Series, is equal to $x^n+nox^{n-1}+\frac{n^2-n}{2}o^2x^{n-2}+$ &c. Consequently,

quently the Increments of x and x^n, generated in the same Time, are o and $nox^{n-1}+\frac{n^2-n}{2}o^2x^{n-2}+$ &c. But the nascent or evanescent Increment of x^n, is as its Fluxion; and in either of these States the Ratio of $nox^{n-1}+\frac{n^2-n}{2}o^2x^{n-2}+$ &c. to nox^{n-1} is a Ratio of Equality: For as the Magnitude of o becomes less and less, the Quantities $nox^{n-1}+\frac{n^2-n}{2}o^2x^{n-2}+$ &c. and nox^{n-1} constantly tend to an Equality, by a continual Diminution of their Difference; and they become equal, and their Ratio becomes a Ratio of Equality, when their Difference vanishes; that is, in the Instant o becomes evanescent, or in the Instant that the Increment of x^n first begins to exist: For as they vanish together under a Ratio of Equality, so they begin to exist together under the same Ratio; and therefore in the nascent or evanescent State of o, the Fluxions of x and x^n, are as o and nox^{n-1}, which are manifestly to each other as 1 and nx^{n-1}.

Hence

Hence it appears, that this Method of finding the Fluxion of x^n, upon a Supposition that x flows uniformly, is the very ſame with that of finding the Fluxion of a Rectangle, as it is deſcribed in the ſecond Book of the mathematical Principles: For, as ab the Difference between $Ab + Ba + ab$ and $Ab + Ba$ grows leſs and leſs perpetually, by diminiſhing the ſynchronal Increments of the Sides of the Rectangle, and at laſt evaneſces, and in the Inſtant of its Evaneſcence, the Gnomon $Ab + Ba + ab$ becomes equal to the Sum of the Rectangles Ab and Ba; ſo $\frac{n^2-n}{2}o^2x^{n-2} +$ &c. the Difference between $nox^{n-1} + \frac{n^2-n}{2}o^2x^{n-2} +$ &c. and nox^{n-1} grows leſs and leſs perpetually, by diminiſhing the Increment o, and at laſt evaneſces, and in the Inſtant of its Evaneſcence $nox^{n-1} + \frac{n^2-n}{2}o^2x^{n-2} +$ &c. becomes equal to nox^{n-1}: And as the Gnomon $Ab + Ba + ab$ is not the Moment or Fluxion of the Rectangle AB, but in the Inſtant of its be-

becoming equal to $Ab + Ba$, so $nox^{n-1} + \frac{n^2-n}{2}o^2x^{n-2} +$ &c. is not the Moment or Fluxion of x^n, but in the Instant of its becoming equal to nox^{n-1}.

The Author of the *Analyst* therefore, is greatly mistaken, in thinking Sir *Isaac* found the Fluxion of x^n, by a Method different from that he used in finding the Fluxion of a Rectangle, contain'd under two flowing Quantities: He steadily adheres to one and the same Method; namely, that of taking the first or last Ratios of Quantities, or of their isochronal Increments, for the Measures of the Ratios of their Fluxions; and uses no illegitimate Artifice to obtain these first or last Ratios; unless it be accounted illegitimate to suppose that mathematical Quantities can be generated and destroyed by Motion.

It is pretended, " That the Method " for finding the Fluxion of a Rectangle " of two flowing Quantities, as it is " set forth in the *Treatise of Quadra-*

" *tures*, differs from that found in the " *ſecond Book of the Principles*, and is " in Effect the ſame with that uſed in " the *Calculus differentialis*: For the " ſuppoſing a Quantity infinitely di- " miniſh'd and therefore rejecting it, " is in Effect the rejecting an Infiniteſi- " mal." But if this Author deduces the Rule from the firſt Propoſition in the *Treatiſe of Quadratures*, and conſiders it ever ſo little, he will find it the very ſame with that ſet down in the *ſecond Book of the Principles*: And it is doubtleſs in Effect too the ſame with that uſed in the *differential Calculus*, ſo far as different Methods *can* effect the ſame Thing, but no farther: For Quantities are not rejected in the Method of Fluxions, as in the *differential Calculus*, on Account of their exceeding Smallneſs.

" But according to the received Prin- " ciples it is evident, ſays he, that no " geometrical Quantity, by being infi- " nitely diminiſh'd can ever be exhauſt- " ed or become nothing." Now, on the received Principles of Fluxions, this is a direct Absurdity For these Principles suppose that mathematical Quantities can be generated by Motion, which he has not yet thought proper to contradict; and consequently they may also by Motion be destroy'd: For Quantities, and the Augments of Quantities, which in some finite Time are produc'd by Motion, may perpetually grow less and less by reverting that Motion; and by constantly growing less and less, they may come to be infinitely diminished, or to be less than any assignable Quantities; and from being less than any assignable Quantities, the Motion still persevering, they must at last vanish and become nothing; otherwise it might be contended that a Body setting out from any Place, and, in any finite Time, describing a certain length, cou'd never by moving back and returning in the same Line, arrive at the Place from whence it first set out.

Upon the whole then it appears, that the Method of Fluxions, as describ'd by Sir *Isaac Newton* in his Introduction to the *Quadrature of Curves*, and in the *ſecond Book of his mathematical Principles*, is not that wretched un-ſcientifical Knack ſet forth in the *Analyſt*; but a Method founded upon obvious, rational, accurate and demonſtrative Principles: It likewiſe appears, that the Concluſions do not ariſe from illegitimate tentative Ways or Inductions, but follow from ſuch Premiſes, and by ſuch Arguments, as are moſt conformable to the Rules of Logic and right Reaſon: All the Skill and Dexterity therefore by this Author ſhewn, in the Inveſtigation of contrary Errors correcting each other, are vain and impertinent. He has miſtaken the Doctrine of Fluxions, and by not rightly diſtinguiſhing its Principles from thoſe of the *differential Calculus*, has impoſed a falſe Meaſure of Moments upon his Readers, and arguing from that falſe Meaſure, has unjuſtly charg'd Sir *Iſaac* with Errors ariſing from it; and, to mend the Matter, has inſtituted Computations to ſhew how thoſe Errors redreſs one another, and how Mathematicians by Means of Errors bring forth Truth and Science.

The

The Dispute between the Followers of Sir *Isaac Newton*, and the Author of the *Analyst*, is not about the Principles of the *differential Calculus*, but about those of Fluxions; and it is whether these Principles in themselves are clear or obscure, and whether the Inferences from them are just or unjust, true or false, scientific or otherwise: We are not concerned about Infinitesimals or minute Differences, but about the Ratios with which mathematical Quantities begin or cease to exist by Motion; and to consider the first or last Proportions of Quantities does not imply that such Quantities have any finite Magnitudes: They are not the Proportions of first or last Quantities, but Limits of Ratios; which Limits, the Ratios of Quantities attain *only* by an infinite Diminution of their Magnitudes, by which infinite Diminution of their Magnitudes they become evanescent and cease to exist. If therefore Quantities may cease to exist by Motion, and if the Ratios with which they become evanescent be truly determin'd, it will follow that there are no Errors, however

ever ſmall, admitted in the Principles of Fluxions; and if no Errors be admitted in the Principles; there can be none in the Concluſions, nor any to be accounted for in the Arguments by which thoſe Concluſions are deduc'd from their Premiſes: The Hints therefore, which this Author has condeſcended to give the Mathematicians for aſcertaining the Truth of their Concluſions, by means of contrary Errors deſtroying each other, will probably be left to be further extended and apply'd by himſelf, to all the *good Purpoſes* he pleaſes to extend and apply them; as having more Leiſure, and a Science more tranſcendental *, and perhaps a much greater Curioſity for ſuch Matters, than they have.

It has been obſerv'd before, that Fluxions may be expounded by any Lines which are proportional to them; and ſo the Analyſis may be inſtituted, by conſidering

* A *Philoſophia prima*, a certain tranſcendental Science ſuperior to and more extenſive than Mathematics, which, he ſays, it might behove our modern *Analyſts* rather to learn than deſpiſe.

ſidering the mutual Relations or Proportions of finite Quantities, as the Proportions of Fluxions themſelves. To this it is objected, "That if, in order "to arrive at theſe finite Lines proportional to Fluxions, there be certain Steps "made uſe of which are obſcure and inconceivable, it muſt be acknowleged, "that the Proceeding is not clear, nor "the Method ſcientific." But there may be many Steps obſcure and inconceivable to Perſons, who are unacquainted with Sir *Iſaac Newton*'s Method of firſt and laſt Ratios, with his Doctrine of Fluxions, and with his Principles of Motion; and yet thoſe Steps may appear very different to others who have duly conſider'd 'em: And therefore, till it be made apparent from geometrical Principles that the fluxional Triangle, which evaneſces upon the returning of the Ordinate of any Curve to the Place from whence it firſt ſet out, cannot in its laſt Form, that is, in the Form it has at the Inſtant it becomes evaneſcent, be ſimilar to a Triangle contain'd between the Tangent, the Abſciſs extended and the Ordinate of the

ſame

ſame Curve; or till it be proved that no Triangle, which is capable of becoming evaneſcent by a Diminution of its Sides from Motion, can be ſimilar in its laſt Form to any plain Triangle whatſoever; we ſhall continue to expound Fluxions by ſuch Right Lines as are proportional to them; and do aſſert, that the Proceeding is clear, and the Method ſcientific.

FINIS.

ERRATA.

PAGE 10. L. 2. r. will in moſt Caſes continually change. Page 14. L. 20. r. of the Spaces deſcribed by them in equal Times.