



Bodleian Libraries

UNIVERSITY OF OXFORD

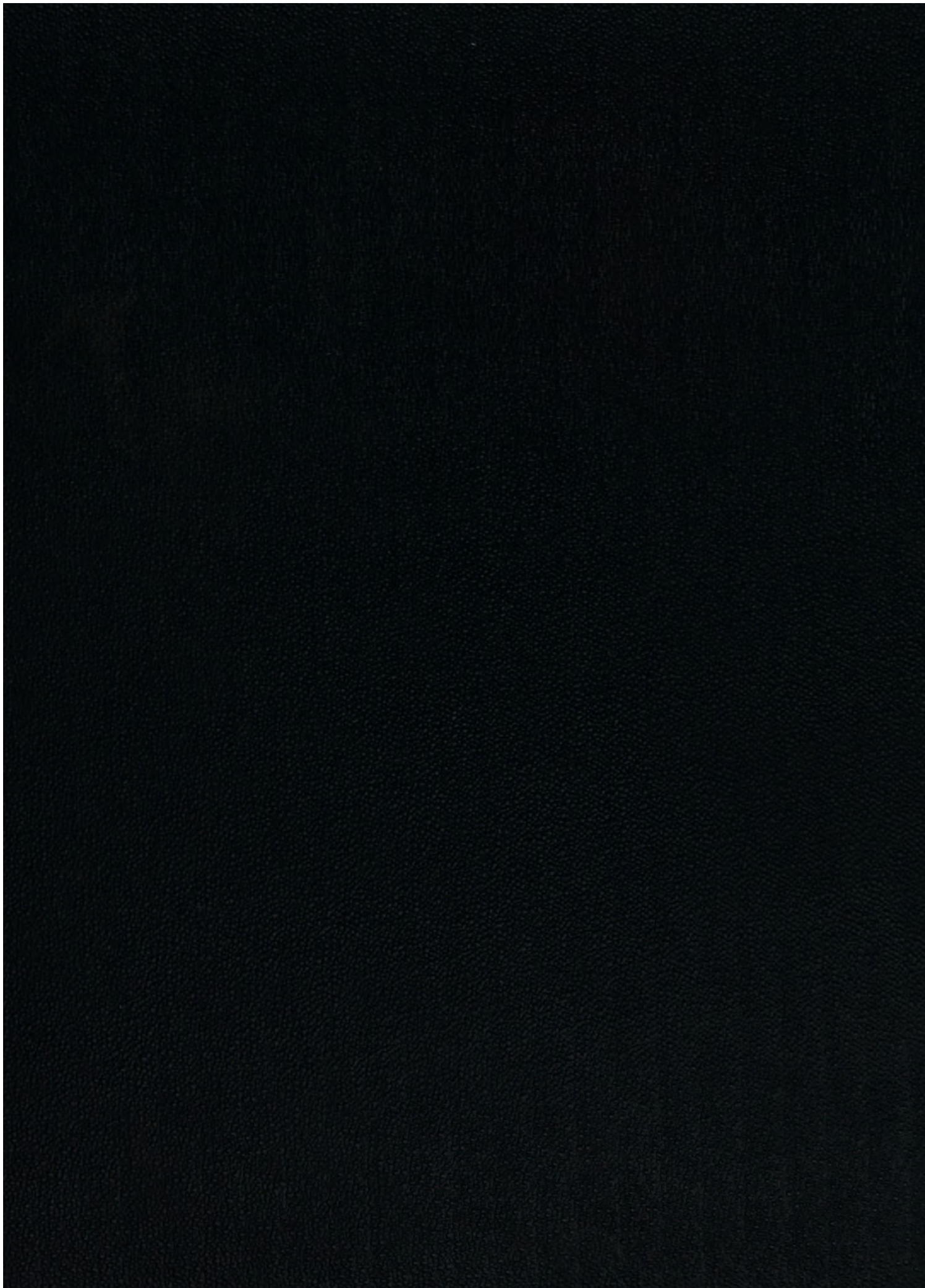
This book is part of the collection held by the Bodleian Libraries and scanned by Google, Inc. for the Google Books Library Project.

For more information see:

<http://www.bodleian.ox.ac.uk/dbooks>



This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 2.0 UK: England & Wales (CC BY-NC-SA 2.0) licence.



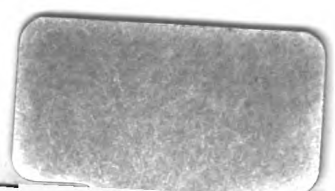


600025014I

12/21

C
L

22





THE
ELEMENTARY PARTS

OF

Revised
Dr. SMITH'S COMPLEAT SYSTEM OF OPTICKS,

SELECTED AND ARRANGED

FOR THE USE OF STUDENTS AT THE UNIVERSITIES:

TO WHICH ARE ADDED

IN THE FORM OF NOTES

SOME

EXPLANATORY PROPOSITIONS FROM OTHER AUTHORS.

C A M B R I D G E,

Printed by J. ARCHDEACON, Printer to the UNIVERSITY;

And sold by T. & J. MERRILL, J. WOODYER, and J. NICHOLSON, at Cambridge;
J. & J. FLETCHER, and D. PRINCE, at Oxford; J. NOURSE, in the Strand,
and B. WHITE, in Fleetstreet, London.

M. DCC. LXXVIII.

1854 . d . 22



December 1777, St. John's College, Cambridge.

T H E
P R E F A C E.

WHEN the Editor of the following Treatise drew the outlines of his plan, he had no Intention either of printing so large a volume or of publishing any. His work was then calculated to facilitate the progress of science in this society alone, by accommodating the students with those parts of Dr. SMITH'S OPTICKS, which are explained to them in the lectures on that Branch of philosophy. The scarcity of that Author's SYSTEM had long been a subject of complaint; and it was at length become impossible to procure a number of copies sufficient for the wants of so large and industrious a society. But the Editor had not proceeded far, before he was diverted from his first design, by being repeatedly informed that the same difficulty prevailed in almost every other College, and that no person could be found, who was willing to hazard a Republication of the whole of Dr. SMITH'S Treatise. He therefore determined, though already oppressed by accumulated business, to extend his plan; rather than suffer the scarcity complained of to be an Impediment to Industry, or become an Excuse for Idleness. To remove the former of these ill consequences, and obviate the latter in the most effectual manner possible, he communicated his design to the Tutors of other Colleges, and added to the stock, he had formerly collected, all the parts of Dr. SMITH'S Treatise, in which he found their lectures differed from his own.

But though the Editor was possessed of the requisite materials, the chief difficulty still remained. For the primitive form of these deranged Elements resembled more the chaos of the poets, than a well-connected series of philosophical Reasonings. His next attempt was therefore to reduce the whole into System, by classing its parts and

ranging the classes in such order as should best correspond with the plan of Lectures given by the Tutors in this university, whose exemplary diligence in the cultivation of science entitles them to this and every other assistance. — After extricating these materials from disorder and confusion, in which his success has far exceeded his expectations; he tried also to efface every other mark of deformity, and to give them the appearance of a new creation. But here he met with an insuperable obstacle. In some chapters of the Original the subject is popularly explained, and in others geometrically demonstrated. It was therefore impossible to diffuse over the Whole an uniformity of Style without endless Interpolations and a proportionable Delay, neither of which was consistent with the Reasons, which first induced him to undertake the Work.

Although the following Treatise was intended in its origin, and calculated in its progress, for the sole use of the Academick, the Editor now ventures, on a review of its contents, to recommend it to the use of others, who desire to be instructed in the first principles of Opticks. These principles are demonstrated in it with as much easiness and perspicuity, if not with so much elegance and accuracy, as in other works of the same nature; with so much easiness, indeed, as to be intelligible to every Reader, who is previously furnished with the mere Rudiments of Geometry. And the order in which they are ranged, though peculiarly adapted to the original design for which they were selected, is in the Editor's opinion the most natural, in which the subject can be treated. The following is a slight sketch of this arrangement. In the three first chapters are digested the articles which contain Dr. SMITH'S Explanation of the general properties of *Light*, and Sir ISAAC NEWTON'S Experiments to prove that Light consists of different *Colours*. In the next chapter is traced the motion of a single *Ray* of Light in its passage through refracting surfaces of a spherical figure, and in the four subsequent chapters the motion of a *Pencil* of Rays, till they unite again, after Reflection or Refraction, to form an *Image* of the object from whence they proceeded. And as we are made to see external objects by means of their Images formed upon the *Retina*, in the two next chapters are explained the whole structure of the human *Eye*, the whole process of *Vision* with the naked Eye, what assistances the sight receives from
Telescopes,

Telescopes, Microscopes, and Spectacles, and the method of *constructing* these and other optical Instruments. The eleventh Chapter treats of the *Imperfection* in Telescopes and Microscopes, which is caused by the *aberration* of Rays from their geometrical focus in consequence of the spherical figure of a Lens and the different Refrangibility of different kinds of Light: and the twelfth chapter contains the known *Rules*, which, in constructing Telescopes, it is necessary to observe on account of these aberrations. The Elements of the science being demonstrated, they are applied in the two last chapters to solve the phenomena of *the Rainbow*, and *the annual aberration of the fixed stars*.

The only modern authors, who have treated geometrically the principles of this science, and whose works are still in print, are Mr. EMERSON and Mr. HARRIS. But the philosophical writings of a superior genius are seldom adapted to the capacity of an unassisted Learner; and the demonstrations of the former of these authors are not yet adopted by the Tutors in our universities. The elementary Treatise of the latter is entirely silent upon some of the most important subjects belonging to this science, such as the construction of Telescopes and the phenomena of the Rainbow. Had Mr. Harris lived to finish his Work, it would have precluded the necessity of this and every other publication of the same extent.

It is not intended, in this character of the writings of others, to intimate in the slightest degree that the following Treatise is faultless. It contains many Inaccuracies and even some Errors, of which the Editor was fully sensible before he sent it to the press, but was restrained from correcting them by the dread of Reprehension. The only method of correction was a compleat commentary on the Text, or frequent alterations of it. But, besides that such a commentary would have been as tedious and troublesome as a new Treatise on the subject, there were other objections against it too obvious to be mentioned: and to have erased and corrected the Text of an eminent Writer however judiciously, might have been deemed by some an impertinent presumption and an unjust Treatment of the author.

These Inaccuracies might indeed have been prevented, and an uniformity introduced by composing a new system of the same materials. But as the want of Dr. SMITH'S OPTICAL ELEMENTS was become

too pressing an Inconvenience to allow sufficient time for executing a regular and well-digested plan, the Editor was reduced to the alternative either of garbling the works of that Author or of publishing some crudities of his own. Besides, he had neither health nor leisure to engage in any publication more laborious than the present: He prepared it for the press without the Trouble of copying any part of its contents, except a few propositions, which he has borrowed from other Books of eminence not easily to be procured, and which the Tutors in this university have introduced into their lectures for a more ample explanation of what Dr. SMITH has but slightly touched upon. To have circumscribed his collection of notes within a narrower limit would hardly have been possible; and he judged it more adviseable that the Learner should be left to consult the entire works of *modern* authors, than that this volume should be swelled with Extracts from their writings; it being of the greatest assistance to the student, during his noviciate in philosophy, to have the same Truth represented to him in a variety of lights. For the principal notes, which are subjoined to the following pages, the Reader is indebted to Dr. BARROW and DES CARTES.

Such is the Nature and Intention of the following treatise, and such the Editor's apologies for presuming to publish so irregular and incorrect a composition. But if these Reasons be insufficient to defend him against private cavils, he hopes the following considerations will secure him from public Censure: — that it could not be a desire of Fame, which induced him to undertake the mechanical office of an *Editor*; or the Hope of Profit, to be the Instrument of a publication, the Expence of which must be great, and the purchasers few; that he could have no View to his own Improvement in forming a system of principles, which it has been his business for several years successively to explain to others; and lastly, had amusement been his object, that he certainly would have directed his attention to some other province in the intellectual world less frequented by him than the present, less barren and more beautiful. His only motives were public utility and a deep sense of the duty incumbent on every member of these Societies to promote the designs of those venerable Benefactors, whose Endowments they have the honour and happiness to participate.

S U B S C R I B E R S.

Where no University is mentioned in the following List, that of Cambridge is understood.

A.

THE Right Honourable
Lord Viscount Althorp of
Trinity Coll.

Rev. John Arden, B. A. of St.
John's Coll. and Rector of
Oxnead, Norfolk

Pepper Arden, Esq; Fellow of
Trinity Coll.

Rev. Mr. Atcherley, M. A. Head
Master of Shrewsbury School

Rev. Mr. Atlay, M. A. Fellow of
St. John's Coll.

Mr. Atwood, M. A. Fellow of
Trinity Coll.

B.

Honourable Thomas Broderick,
Member of Parliament for
Midleton in Ireland

Peter Burrell, Esq; M. A. Member
of Parliament for Haslemere,
3 Copies

Rev. James Browne, D. D. Master
of Pembroke Hall

Mr. Babington of St. John's Coll.

Rev. Mr. Baines, M. A. Fellow
of Sidney Coll.

Henry Banks, Esq; Fellow-Com-
moner of Trinity Hall

Rev. Mr. Baldwin, M. A. Fellow
of St. Peter's Coll.

Mr. Bartholomew, B. A. of St.
John's Coll.

Mr. Barnard, B. A. of Trinity Coll.

Mr. Barnard of St. John's Coll.

Mr. Barlow of Trinity Coll.

Rev. Edward Beadon, Chaplain
to his Majesty

Rev. Richard Beadon, Public
Orator in the University of
Cambridge

Mr. Bell of St. John's Coll.

Mr. Beloe of Bene't Coll.

Mr. Bevil, B. A. of St. Peter's Coll.

Mr. Beeston of St. John's Coll.

Mr. Beauvoir of St. John's Coll.

Rev. Edward Blakeway, M. A.
late Fellow of Magdalen Coll.

Mr. George Blount, B. A. of St.
John's Coll.

Rev. Mr. Brand, M. A. Fellow
of Christ's Coll.

Mr. Bramston of Trinity Coll.

Rev. Mr. Brundish, M. A. Fellow
of Caius Coll.

Mr. Brereton of St. John's Coll.

Mr. Burne of St. John's Coll.

Mr. Burroughes of Queen's Coll.

Mr. Burroughes, B. A. of Caius
Coll.

Mr. Butcher of St. John's Coll.

Rev. William Burslem, M. A.
Fellow of St. John's Coll.

Right

S U B S C R I B E R S.

C.

Right Honourable Earl of Clarendon
 Frederick Cornewall, Esq; Diddlebury, 2 Copies
 Frederick Cornewall, Esq; Member of Parliament for Leominster, 2 Copies
 Rev. John Chevallier, D. D. Master of St. John's Coll. Caius College Library, 2 Copies
 Rev. John Carr, M. A. Fellow of St. John's Coll.
 Mr. Carlyle of Christ's Coll.
 Mr. Carpindale of St. John's Coll.
 Mr. Capper of St. John's Coll.
 Mr. Christian of St. John's Coll.
 Mr. Clapham, B. A. of Clare Hall
 Mr. John Clapham of Christ's Coll.
 Rev. Mr. Clarke, M. A. Fellow of Clare Hall
 Mr. Coffin of Sidney Coll.
 Mr. Coffin of Clare Hall
 Mr. William Colquitt of Christ's Coll.
 Mr. Cookson, Fellow of St. John's Coll.
 Mr. Cookson of St. John's Coll.
 Mr. Constable, B. A. of St. Peter's Coll.
 Rev. Mr. Coulthurst, B. A. of St. John's Coll.
 Rev. F. H. Cornewall, Fellow of St. John's Coll. 2 Copies
 Rev. Mr. Cook of St. John's Coll.
 Rev. William Craven, B. D. President of St. John's Coll. and

Professour of Arabic in the University of Cambridge
 Rev. John Cranke, M. A. Fellow of Trinity Coll.
 Rev. Thomas Crafter, M. A. of St. John's Coll.
 Mr. Curtis of St. John's Coll.

D.

Honourable Richard Dawson of St. John's Coll. 2 Copies
 Somersfet Davies, Esq; M. A. of St. John's Coll. 2 Copies
 Mr. Charles Davy of Caius Coll.
 Mr. Dealtry, B. A. of Trinity Coll.
 Rev. Mr. Deane, B. D. late Fellow of St. John's Coll. 3 Copies
 Rev. T. Drake, M. A. Fellow of St. John's Coll.

E.

Right Rev. Dr. Edmund Keene, Bishop of Ely
 Rev. William Ellifston, D. D. Master of Sidney Coll.
 Rev. William Easton, M. A. Fellow of St. John's Coll. 3 Copies
 Mr. Eaton of St. John's Coll.
 Rev. Mr. Ellis, M. A. Fellow of Trinity Coll.
 Thomas Eyton, Esq; of Wellington, 3 Copies

F.

Right Honourable Lord De Ferrars
 Mr. E. Fagg of Clare Hall
 Rev. Mr. Fallowfield, B. A. Fellow of Clare Hall

Rev.

S U B S C R I B E R S.

Rev. Mr. Farnham, M. A. of
St. John's Coll.
Rev. Charles Favell, M. A. Fel-
low of Clare Hall
Rev. James Fawcett, M. A. Fel-
low of St. John's Coll.
Rev. Thomas Ferris, B.D. Fellow
of St. John's Coll.
Mr. Ferraby of Clare Hall
Rev. John Fisher, M. A. Fellow
of St. John's Coll.
Rev. Mr. Fisher, M. A. Fellow
of Caius Coll.
Rev. Robert Fiske, M. A. Fellow
of St. John's Coll.
Mr. Forby of Caius Coll.
Mr. Friend of Christ's Coll.
Mr. Frost of Caius Coll.
A. Frere, Esq; M. A. of St. John's
Coll.
Rev. Edward Frewen, M. A. Fel-
low of St. John's Coll.

G.

Thomas Gisborne, Esq; Fellow-
Commoner of St. John's Coll.
Rev. Dr. Gordon, Precentor of
Lincoln.
James Gordon, Esq; Fellow-
Commoner of St. John's Coll.
2 Copies
Rev. Mr. Gould, M. A. Fellow
of Clare Hall
Mr. Greenwood, B. A. of St.
John's Coll.
Rev. Mr. Gretton, B. A. of Tri-
nity Coll.
Mr. Griffith of St. John's Coll.
Mr. Grigson of Caius Coll.
Rev. Harry Grove, M. A. Fellow
of St. John's Coll. and Junior

Proctor in the University of
Cambridge
Robert Lovell Gwatkin, Esq;
B. A. Fellow-Commoner of
St. John's Coll. 2 Copies

H.

Right Honourable Lord Hyde
Noel Hill, Esq; Member of Par-
liament for the County of
Salop, 4 Copies
Rev. Anthony Hamilton, D. D.
Archdeacon of Colchester
Mr. Hale of Clare Hall
Rev. Mr. Hall, B. A. of St. John's
Coll.
Mr. Hammond, B. A. of Queen's
Coll.
Mr. Hammond of St. John's Coll.
Mr. William Hamond of St.
John's Coll.
Mr. Hanley of Christ's Coll.
Mr. Harley of Trinity Coll.
Mr. George Harrison of St.
John's Coll.
Mr. Hart, B.A. of St. John's Coll.
Mr. Heath of St. John's Coll.
Mr. Heberden, Fellow of St.
John's Coll. 3 Copies
Mr. Henson of St. John's Coll.
Mr. Heslop, B. A. of Sidney Coll.
Rev. Mr. Hetley, M. A. of St.
John's Coll.
Rev. John Hey, B. D. Fellow of
Sidney Coll.
Rev. S. Hey, M. A. Fellow of
Magdalen Coll.
Mr. Holcombe of St. John's Coll.
Rev. B. Holmes, M. A. Fellow
of St. John's Coll.
Mr. How of St. Peter's Coll.

S U B S C R I B E R S.

Rev. T. Hughes, B. A. of St. John's Coll.
 Rev. Lewis Hughes, M. A. Fellow of St. John's Coll.
 Rev. Mr. Humphries, M. A. Second Master of Shrewsbury School
 Rowland Hunt, Esq; of Boreaton
 Rev. Mr. Hunter, B. D. Fellow of Sidney Coll.

J.

Mr. James, M. A. Fellow of King's Coll.
 St. John's College Library
 Rev. Samuel Johnson, Third Master of Shrewsbury School
 Mr. Jones of Trinity Coll.
 Rev. Mr. Jordan, M. A. Fellow of Queen's Coll.
 Mr. Jordan of Pembroke Hall
 Rev. Mr. Jowett, LL. B. Fellow of Trinity Hall

K.

Mr. Kerrick, M. A. Fellow of Magdalen Coll.
 Mr. Kilvert, B. A. of Emanuel Coll.
 T. Kinnerfley, Esq; of Leighton, 2 Copies
 Mr. R. Kynaston of St. John's Coll.
 Mr. Ed. Kynaston of St. John's Coll.

L.

Mr. S. Lawrence, M. A. Fellow of St. John's Coll. 2 Copies
 Mr. Lawrence of Christ's Coll.
 Mr. Latter of St. John's Coll.
 Mr. S. Leacroft, London, 3 Copies

Charleton Leighton, Esq; of Loton, 3 Copies
 Thomas Lempriere, Esq; Jersey
 Mr. Lens of St. John's Coll.
 Mr. Littlehales of St. John's Coll.
 Mr. Lodington, B. A. of Sidney Coll.
 Mr. Lovering of St. John's Coll.
 Mr. Loxham of St. John's Coll.
 Mr. Lund of St. John's Coll. 2 Copies

M.

Right Honourable Lord Viscount Midleton
 Right Honourable Lord Maxwell
 Rev. Owen Manning, B. D. of Godalmin
 Mr. Mantell, B. A. of Bene't Coll.
 Rev. Mr. Marth, M. A. Fellow of Trinity Coll.
 Mr. Marsh of St. John's Coll.
 Rev. Mr. Metcalfe, M. A. Fellow of Christ's Coll.
 Rev. Isaac Milner, M. A. Fellow of Queen's Coll.
 George Milner, Esq; Fellow-Commoner of St. John's Coll.
 Mr. Moises of Trinity Coll.
 Rev. Mr. Mountain, M. A. of Caius Coll.
 Rev. Mr. Mounsey, M. A. Fellow of Jesus Coll. and Moderator in the Univerfity of Cambridge
 Mr Munnings of Caius Coll.

N.

Rev. Charles Newling, Prebendary of Litchfield and Cvo.
 Mr. William Nelson, B. A. of Christ's Coll.

Mr. Nesfield

S U B S C R I B E R S.

Mr. Nesfield of Caius Coll.
Mr. North, M.A. Fellow of Caius Coll.

O.

Rev. Mr. Owen, B. A. of Trinity Coll.

P.

Right Honourable Earl of Powis,
4 Copies

Mr. Paley of Christ's Coll.
Mr. Palmer, B. A. of St. John's Coll.

Rev. Mr. Parkinson, M. A. Fellow of Christ's Coll.

Rev. William Pearce, M. A. Fellow of St. John's Coll.

J. Pennington, M. D. Fellow of St. John's Coll.

Mr. Pyndar, Fellow of Queen's Coll.

Mr. Popple, B. A. of Trinity Coll.

Rev. Mr. Porter, M. A. Fellow of Trinity Coll.

Mr. William Pott of St. John's Coll.

Charles Poole, Esq; Fellow-Commoner of Pembroke Hall

Rev. Mr. Postlethwaite, M. A. Fellow of Trinity Coll.

Thomas Jelf Powys, Esq; of Berwick, 2 Copies

Rev. Edward Powys, Fellow of St. John's Coll. 2 Copies

Mr. Praed of St. John's Coll.

Rev. Mr. Prettyman, M. A. Fellow of Pembroke Hall

Rev. R. Pritchett, B. D. Fellow of St. John's Coll.

Mr. Prichard of St. John's Coll.

Mr. Pym of Christ's Coll.

Q.

Queen's College Library

R.

Rev. Mr. Raines, M. A. Fellow of Jesus Coll.

Mr. Raper of St. John's Coll.

Mr. Reaston of St. John's Coll.

Mr. Rippon of Trinity Coll.

Mr. J. Rocke of St. John's Coll.
2 Copies

M. Robinson of St. John's Coll.

Rev. John Robinson, M. A. late Fellow of Clare Hall

S.

Right Honourable Lord St. John of St. John's Coll.

Honourable St. Andrew St. John of St. John's Coll.

Rev. Mr. Sandiford, M. A. Fellow of Trinity Hall

Rev. Mr. Seale, M. A. Fellow of Christ's Coll.

Mr. Sheild of St. John's Coll.

Mr. Shipley, B. A. of Clare Hall

Mr. Skeels, B. A. of Pembroke Hall

Rev. William Smith, M. A. Fellow of St. John's Coll.

Mr. Joseph Smith of Caius Coll.

Mr. Sneyd, B. A. of St. John's Coll.

Mr. Sparhauke, B. A. of St. John's Coll.

Rev. Mr. Squire, M. A. Fellow of Caius Coll.

Mr. Standen of Queen's Coll.

Mr. Sturges of Clare Hall

Rev. Mr. Stephenson, M. A. Fellow of Clare Hall, 2 Copies

Rev. Thomas Starkie, M. A. Fellow of St. John's Coll.

Mr. Sutton of St. John's Coll.

Honourable

S U B S C R I B E R S.

T.

Honourable John Townshend,
M. A. of St. John's Coll.
William Tayleur, Esq; of Shrewsbury
Rev. Mr. Therond, M. A. Fellow
of Trinity Coll.
Mr. Tremenheere of Pembroke
Hall
Mr. Tucker of Trinity Coll.
Robert Tunstall, Esq; Fellow-
Commoner of Trinity Coll.
Rev. Joseph Turner, M. A. Fel-
low of Pembroke Hall
George Turner, Esq; Fellow-
Commoner of Trinity Coll.
Mr. Twigge, B. A. of St. John's
Coll.
Richard Tylden, Esq; B. A. Fel-
low-Commoner of Jesus Coll.

V.

Honourable John Charles Vil-
liers, M. A. of St. John's Coll.
Mr. Vince, B. A. of Sidney Coll.

W.

Rev. William Wade, Fellow of
St. John's Coll.
Mr. Wakefield, Fellow of Jesus
Coll.

Rev. Mr. Walsby, M. A. Fellow
of Bene't Coll.
Rev. Peploe Ward, M. A. Fellow
of Queen's Coll. and Preben-
dary of Ely
Rev. Richard Watson, D.D.F.R.S.
Regius Professor of Divinity in
the University of Cambridge,
and Prebendary of Ely
Thomas Lucas Wheeler, Esq;
Fellow-Commoner of Trinity
Coll.
Mr. Whinfield, B. A. of St. Pe-
ter's Coll.
Rev. George Whitmore, M. A.
Fellow of St. John's Coll.
Mr. George Wilson, B. A. of
Christ's Coll.
Rev. John Wingfield, B.A. of St.
John's Coll.
Rev. Mr. Woodrooffe, M. A.
Fellow of Clare Hall
Mr. Woodman of Trinity Coll.
Rev. James Wood, M.A. Fellow
of St. John's Coll.
Mr. Wright, Fellow of St. John's
Coll.

Z.

Rev. Thomas Zouch, M. A.
Rector of Wycliffe

T H E
C O N T E N T S.

CHAP. I.	CONCERNING Light.	—	—	Page 1
	II. Concerning the Origin and Cause of Colours.	—	—	8
	III. Concerning the Causes of Refraction, Reflection, Inflection and Emission of Light, and concerning Transparency, Opacity, and Colours in Bodies.	—	—	16
	IV. Concerning the Refractions of a single Ray of Light in its Passage through a Prism, Globe, or Lens.	—	—	27
	V. To find the Focus of Rays reflected from any given Surface	—	—	32.
	VI. To determine the Place, Magnitude, and Situation of Images formed by reflected Rays.	—	—	36
	VII. To find the Focus of Rays falling almost perpendicularly upon any refracting Surface, Sphere or Lens.	—	—	38
	VIII. To determine the Place, Magnitude, and Situation of Images formed by refracted Rays.	—	—	49
	IX. Concerning the Eye and Manner of Vision.	—	—	51
	X. Concerning Vision with Glasses.	—	—	62.
	XI. To determine the Aberration of Rays from the geometrical Focus caused by their unequal Refrangibility, and also by the Sphericalness of the Figure of reflecting and refracting Surfaces.	—	—	80
	XII. A refracting or reflecting Telescope being given, whose Aperture and Eye-glass are adjusted by Experience, to determine the Length, Aperture and Eye-glass of another Telescope, through which an Object shall appear as bright and distinct as in the given one, and magnified as much as shall be required.	—	—	88
	XIII. Concerning the Rain-bow.	—	—	98
	XIV. Telescopical Discoveries in the fixt Stars.	—	—	110

ERRATA.

E R R A T A.

Article

40. line 6. *for* reflected, *read* refracted.
47. l. 8. *for* the refracting, *r.* their refracting.
64. in the margin *for* Fig. 56 to 59. *r.* Fig. 56 & 59.
70. at the beginning of the 2d part, the word DEFINITIONS is wanting.
79. line 7. *for* the Focus Q. *r.* the Focus q.
129. l. 1. *for* E q, *r.* E Q.
140. l. 34. *for* perhaps a hundred miles or more, *r.* some hundreds of miles.
172. l. 8. *for* p l, *r.* q l.
181. l. 21. *for* T c s, *r.* T c S.
228. l. 1. *dele* and double microscopes.
244. in the 4th column of the Table, *for* 199 and 398, *r.* 200 and 400.
258. in the margin, *for* Art. 31. *r.* Newton's Opt. p. 114. 8vo.

Plate

5. Fig. 67. C is wanting at the vertex of the parabola.
9. Fig. 127. r is wanting at the upper extremity of p q r.
11. Fig. 157. C is wanting where E O cuts the circle.

T H E

O P T I C K S.

CHAP. I. CONCERNING LIGHT.

1. **W**HOWER has considered what a number of properties Light consists of parts. and effects of light are exactly similar to the properties and effects of bodies of a sensible bulk, will find it difficult to conceive that light is any thing else but very small and distinct particles of matter * : which being incessantly thrown out from shining substances, and every way dispersed by reflection from all others, do impress upon our organs of seeing that peculiar motion, which is requisite to excite in our minds the sensation of light. But for the present purpose it is sufficient to observe that light consists of parts, both successive in the same lines and contemporary in several lines: because in the same place, you may stop that which comes one moment, and let pass that which comes presently after; and at the same time, you may stop it in one place, and let it pass in another. For that part of the light which is stopt cannot be the same with that which is let pass.

2. The least light or part of light, which may be stopt alone without the rest of the light, or propagated alone, or do or suffer any thing alone, which the rest of the light doth not or suffereth not, is called a Ray of light †. That rays of light are straight, is evident enough from the shadows of bodies; or from the appearance of light passing through little holes into a dark room full of dust or smoke; or because bodies cannot be seen through the bore of a bended

* Newt. Opt. Qu. 29. p. 345. 8°. Edit.

† Newton's definition. Opt. p. 1.

pipe, or because they cease to be seen by the interposition of other bodies, as the fixt stars by the interposition of the moon and planets; and the parts of the sun by the interposition of the Moon, Mercury or Venus. Rays of light may therefore be represented by straight lines, not Mathematical but Physical, which are described by the motion of the parts or particles of light: and the point which a ray possesses in falling upon any surface may be considered as a Physical Point.

The manner of reflection and refraction of a ray described. Fig. 1.

Fig. 2.

Angles and sines of incidence and refraction what. Fig. 1, 2.

A medium what.

Laws of reflection and refraction what.

3. When a ray of light falls obliquely upon a smooth polished surface, it is turned out of its way either by reflection or refraction in the following manner. Imagine the paper upon which this figure is drawn to be perpendicular to the surface of stagnating water, and to cut it in the line RS , and that a ray of light, coming in the air along the line AC , falls upon RS at the point C . Then supposing the line PCQ to be perpendicular to the surface of the water, if the ray be reflected, or turned back at C into the air again, it will describe a straight line CB , inclined to the perpendicular CP at an angle PCB exactly equal to the angle PCA .

But if the ray that came along AC goes into the water at C , it will not proceed straight forward, but being refracted or bent at C , it will describe another straight line CE inclined to the perpendicular CQ at a lesser angle ECQ than the angle ACP ; and the line CE will always be so situated, that when any circle, described about the center C , cuts the line CA in A and CE in E , the perpendiculars AD and EF , drawn from A and E to the line PQ , shall always bear the same proportion to each other; whatever be the magnitude of the angle ACP . In water the line EF is always three quarters of AD .

4. In both these cases the line AC is called the Incident Ray, CB the Reflected Ray, CE the Refracted Ray, C the point of incidence, PCQ the perpendicular (at the point) of incidence, the angle ACP the Angle of Incidence, BCP the Angle of Reflection, ECQ the Angle of Refraction; the line AD the Sine of Incidence, that is, of the angle of incidence; and EF the Sine of Refraction, that is, of the angle of refraction.

5. Empty space, or any transparent body, is called a Medium; and mediums are denser in proportion as they are heavier bulk for bulk; and their power to reflect and refract light is found to be greater in proportion as they are denser, very nearly*.

6. The foregoing properties of Reflection and Refraction being discovered and established by repeated experiments upon light and bodies of all sorts both fluid and solid, without any exception yet

* Newt. Opt. p. 245. 8°.

known;

known; and being the principal foundation of the whole science of Opticks, are called the Laws of Reflection and Refraction; and are expressed by Sir *Isaac Newton* in the following words.

7. *The angles of reflection and refraction lye in one and the same plane with the angle of incidence; that is, in the plane drawn through the incident ray and the perpendicular at the point of incidence, as represented in the figures, 1, 2.* First law.

8. *The angle of reflection is equal to the angle of incidence.* Second law.

9. Hence it follows that the incident and reflected rays are equally inclined to the reflecting plane; that is, the angles *ACR* and *BCS* are equal; as appears by taking the equal angles *PCA* and *PCB* from the equal angles *PCR* and *PCS*. First consequence. Fig. 1.

10. It follows also that when the incident ray is perpendicular to the reflecting surface, it shall be reflected directly back along the same perpendicular; as appears by diminishing the equal angles of incidence and reflection till the rays *AC*, *CB* coincide with the perpendicular *CP*. Second consequence.

11. *If the reflected or refracted ray be returned directly back to the point of incidence, it shall be reflected or refracted into the same line before described by the incident ray.* Third law.

12. *Refraction out of a rarer medium into a denser^a is made towards the perpendicular; that is, so that the angle of refraction be less than the angle of incidence.* Fourth law.
^a Art. 5.

13. *The sine of incidence, AD, is to the sine of refraction, EF, either accurately or very nearly in a given ratio; that is, supposing any other incident ray *aC* to be refracted into the line *Ce*, and the sines *ad* and *ef* to be drawn perpendicular to *PQ*, the ratio of *ad* to *ef* is the same as the ratio of *AD* to *EF*. It is found by experience, that if the refraction be made out of air into water, the sine of incidence of red light is to the sine of its refraction as 4 to 3: if out of air into glass as 17 to 11, or nearly as 3 to 2. In light of other colours the sines have other proportions, but the difference is so little that it seldom need be considered.* Fifth law.
Fig. 4.

14. Hence it appears by inspection of the figures (2, 3, 4.) that when the angle of incidence *ACP* is increased, the corresponding angle of refraction *ECQ* will also be increased; because the ratio of their sines, *AD*, *EF*, cannot continue the same unless they be both increased. Consequently if two angles of incidence be equal to each other, the angles of refraction will also be equal to each other. On the contrary, when the angle of incidence is diminished, the angle of refraction will also be diminished; insomuch that when one of these angles becomes infinitely small the other also becomes infinitely small. First consequence.

Second consequence.

15. And so it comes to pass that when the incident ray coincides with the perpendicular to the refracting surface, it will proceed straight forward into the other medium without any bending at all.

Third consequence.

16. From which it is reasonable enough to conclude back again, that while the angle of incidence is continually increasing, the refracted ray will be continually more and more bent and diverted from the course of the incident ray produced: I mean if AC be continued to G , the arc EG and the angle ECG will continually increase*: especially considering that when the angle of incidence in air becomes very nearly a right one, and consequently the incident ray goes almost parallel to the surface of the water, this ray is as much bent at C into the line CE as the 3^d figure represents. In which EF , the sine of refraction, being always three quarters^a of AD , is now three quarters of the radius of the circle. Hence we find † that this angle of refraction, ECQ , is about $48\frac{1}{2}$ degrees: and so the angle ECS (being its complement to 90 degrees) is about $41\frac{1}{2}$ degrees; which in this case measures the deviation of the ray from its first course along the surface of the water. The deviation at the surface of glass is greater than at the surface of water; the ratio of the sines.

Fig. 2.

Fig. 3.

^a Art. 13.

* PROPOSITION. *The greater the angle of incidence is, the greater will be the angle of deviation.*

Fig. 1.
^a Art. 8.

1. In the case of reflection, the angle of deviation is that angle which is contained by the incident and reflected rays. But if the angle of incidence ACP increases, the double of that angle, or ACB ^a, must increase also.

Fig. 5.

2. In the case of refraction, the angle of deviation is that angle which is contained by the refracted ray and the course of the incident ray produced. Draw QBP perpendicular to the refracting plane EBF ; let ABG , DBH , be two incident rays, of which AB falls more obliquely than DB ; and let Ba , Bd , be the directions in which they move respectively after refraction: I affirm that the angle GBa is greater than HBd .

^b Euc.III.31.

In the perpendicular QB produced take any point P , and upon the diameter BP describe the semicircle BGP , cutting AB , DB produced in the points G , H , and Ba , Bd in the points a , d : Join PG , Pa , PH , Pd . And PGB , PaB , PHB , PdB being right angles^b, PG , PH are the right sines of the angles of incidence PBG , PBH , to the radius BP ; also Pa , Pd are the right sines of the angles of refraction PBa , PBd , to the same radius. Next, make the angle GBK equal to HBd , or the arc GK equal to the arc Hd ; and join Gd , cutting PK in X : lastly, draw the chords Ga , Hd . Now the angles PGd , PHd being equal^c, and the angles GPK , HPd being also equal, by construction, the triangles GPX , HPd are similar. Therefore PG is to PX as PH to Pd : but,

^c Euc.III.27.

^d Art. 13.

by the law of refraction^d, PH is to Pd as PG to Pa ; consequently PG is to PX as PG to Pa , and therefore PX equals Pa . But PX is less than PK , because the chord Gd lies wholly within the circle; and therefore Pa is less than PK : consequently PK cuts the angle GPa , and the arc Ga is greater than the arc GK , or Hd . Wherefore the angle GBa is greater than HBd . Q. E. D.

In the preceding demonstration, the angle of incidence is supposed to be greater than the angle of refraction: But the truth of the proposition may be easily deduced from it, when the ray passes out of a denser medium into a rarer. For if aB , dB are made the incident rays, BA , BD become the refracted rays^d; and therefore aBG , dBH are still the angles of deviation.

† By a Table of sines.

being

being greater, that is, as 3 to 2, or nearer as 31 to 20. Hence we find that the angle ECQ is about 40 and ECS about 50 degrees.

17. The bending and deviation is the same when the ray goes back again along the same lines EC, CA ; and if an angle of incidence eCQ be any thing greater than about $48\frac{1}{2}$ degrees in water, or any thing greater than about 40 in glass, this ray eC will not be refracted into air, but will be reflected into the line Cf , making the angle of reflection $Q Cf$ equal to the angle of incidence $Q Ce$.

Refraction changed into reflection.

18. The truth of these laws and of all the consequences drawn from them may be easily examined in the manner following. Upon a smooth board $KLMN$, about a center C with any opening of the compasses (the larger the better) describe a circle $PRQS$; and having drawn two diameters PQ and RS perpendicular to each other, from the point P , with any opening of the compasses, cut off equal arches PA, PB , and draw the lines CA, CB ; then sticking three pins perpendicular to the board at the points A, B, C , dip the board into water as far as the line RS ; and holding it perpendicular to the surface of the water, look along the pins A, C ; and an image of the pin B will appear in the water in the line AC produced. Which shews that the ray which came from the pin B is reflected from the water, at the point C , along the line CA to the eye of the spectator. If the pin at C touches the water, it will disturb the smoothness of its surface; and therefore it is better not to place it in the center, but a little higher in the line CA . The event will be the same if the reflection be made by any other fluid or solid body, as may be tried by cutting off the lower semicircle, and by placing the diameter, RS , of the upper semicircle upon the surface of the solid.

Experimental proof of these Laws of reflection and refraction.
Fig. 4.

Upon the same board draw the line AB cutting CP in D , and from the lines DB and CS cut off DH and CI , each equal to three quarters of DA , and through the points H, I , draw the line HIE , cutting the circumference in E ; and the perpendicular EF drawn from E upon PQ will be equal to DH , or three quarters of DA . Then stick another pin at E , and the board being dipped into water, as before, the pin at E will appear to the eye to be in the same line with the pins at A and C . Which shews that the ray which comes from the pin E is so refracted at C , as to advance to the eye along the line CA ; and therefore when the refraction is made out of water into air, EF the sine of incidence, is to AD the sine of refraction, as 3 to 4. If other pins be fixed any where in the line CE , they will all appear in the line AC produced: and the whole line CE will appear in the water as if it were a continuation of AC straight forward. Which shews that the ray which comes from the pin E , describes a straight line in the water; and that it is bent at the surface only. On the

con-

contrary, if an opportunity be taken when the Sun is just so high, that the shadow of the pin *A* shall coincide with the line *AC*, the refracted shadow will coincide with the line *CE*. Or whatever be the Sun's height, move the pin *A* higher or lower till the shadow falls upon the center *C*, and there fix it, suppose at *a*; then sticking the compasses into any point of the refracted shadow, take up the board, and through this point and the center *C* draw a line *Ce*, cutting the circle in a new point *e*; and the ratio of the new perpendiculars, *ad* and *ef*, will be the same as before; that is, as 4 to 3, as near as can be measured.

This proof applied to spherical surfaces. Fig. 6, 7.

19. Lastly it is to be observed, that a ray of light is reflected or refracted at a spherical surface according to the same laws as if it were reflected or refracted at a plane, touching the spherical surface at the point of incidence. Let *AC* be a ray of light falling upon any point *C* of a spherical surface *MCN*, represented by the arc *MCN*, whose center is *O*; through the points *O* and *C* draw the line *PQ*, and the line *RCS* perpendicular to it, representing a plane surface touching the spherical surface at *C*. Now because a ray of light is considered as a physical line, and is refracted or reflected at a physical point^a, which is common to both surfaces, *MCN* and *RCS*, it follows that the refracted or reflected ray will take the same course in both cases. And this argument is also confirmed by universal experience.

^a Art. 18.

An object what and how it radiates.

20. As rays of light are incessantly thrown out and dispersed in all possible directions from every point of a luminous body; so when they illuminate other bodies, on which they fall, they are also incessantly thrown back from every point of these bodies. For the points of opaque bodies so enlightened are visible to the eye at any point of space and in any point of time, as well as the points of the luminous body that enlightened them. The numberless rays which flow from all visible bodies, called objects, may be methodically distributed in this manner. The surface of the object is considered as consisting of physical lines, and these lines as consisting of physical points, and these points are conceived to radiate all manner of ways. It is usual to make use of nothing else for an object but a physical line. For by how much that line is increased or diminished in apparent magnitude or brightness or distinctness, so much the diameter or length of any object, in its place, would be increased or diminished.

A focus, pencil, parallel rays what. Fig. 10.

21. The point *Q* from which rays diverge, or towards which they converge (being made to go back towards the same point though they may never meet at it) is called their focus. And in both cases any parcel of these rays, as *QBC*, or *QBA*, considered apart from the rest, is called a pencil of rays; and these rays are said to belong to that

that focus; whether it be near at hand or at an immense distance; and in the latter case the rays are called, and considered as, parallel or equidistant from each other; because the difference of their distances at any two given places is insensible.

22. The 8th and 9th figures represent a pencil of rays, QC , which falling in parallel lines upon a plane polished surface, represented by the line ACB , are reflected from it into as many other parallel lines, Cq ; because they are inclined to that plane just as much as the incident rays were inclined to it^a.

Reflection of a pencil of parallel rays at a plane surface.

^a Art. 9.

23. In the 11th figure QC represents a pencil of parallel rays falling obliquely upon a straight line ACB , or upon a plane surface represented by it, which after refraction are also parallel among themselves; every one being equally bent. Because when the angles of incidence are all equal among themselves the angles of refraction are also equal among themselves^b. For the same reason if these rays be refracted again at another plane, either parallel or oblique to the former, they will still be parallel among themselves after every refraction. In strictness this is only to be understood of rays of the same colour: as will be explained in the next chapter.

Refraction of a pencil of parallel rays at a plane surface.

^b Art. 14; Fig. 11, 12.

24. Let the light which flows from a point A and passes through a square hole $bcde$ be received upon a plane, $BCDE$, parallel to the plane of the hole; or if you please let the figure BD be the shadow of the plane bd ; and when the distance AB is double of Ab , the length and breadth of the shadow BD will each be double the length and breadth of the plane bd ; and treble, when AB is treble of Ab ; and so on: which may be easily examined by the light of a candle placed at A .

Experiment: To shew that the breadths of a pencil of diverging rays are as their distances from the focus.

Fig. 13.

25. Therefore the surface of the shadow BD , at the distance AB double of Ab , is divisible into four squares, and at a treble distance, into nine squares, severally equal to the square bd , as represented in the figure. The light then which falls upon the plane bd , being suffered to pass to a double distance, will be uniformly spread over four times the space, and consequently will be four times thinner in every part of that space, and at a treble distance it will be nine times thinner, and at a quadruple distance sixteen times thinner, than it was at first; and so on according to the increase of the square surfaces $bcde$, $BCDE$, &c, or of the square surfaces $Abfg$, $ABFG$, &c, built upon the distances Ab , AB , &c. Consequently the quantities of this rarified light received upon a surface of any given size and shape whatever, removed successively to those several distances, will be but one quarter, one ninth, one sixteenth, of the whole quantity received by it at the first distance Ab . Or in general words the densities and quantities

Hence the density and quantity of light received upon a given plane are reciprocally as the squares of its distances from the luminous body.

quantities of light, received upon any given plane, are diminished in the same proportion as the squares of the distances of that plane, from the luminous body, are increased: and on the contrary, are increased in the same proportion as those squares are diminished. For the lights of the several points of the body, which severally follow this rule, will compose a light which will still follow the same rule.

C H A P. II.

CONCERNING THE ORIGIN AND CAUSE OF COLOURS.

Design.

THE preceding chapter contains such properties as belong to all kinds of light; in this are related some of the experiments by which Sir *Isaac Newton* discovered that light consists of different colours.

A prism
what.

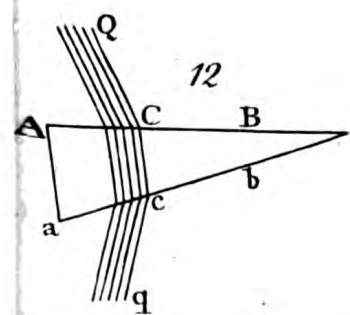
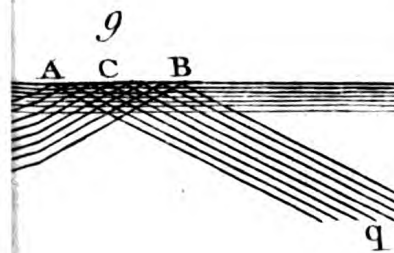
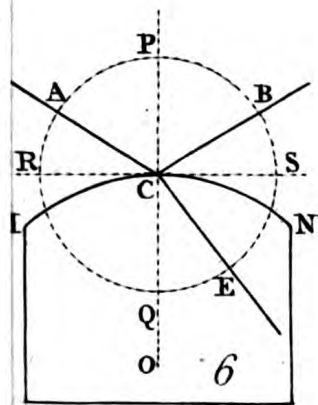
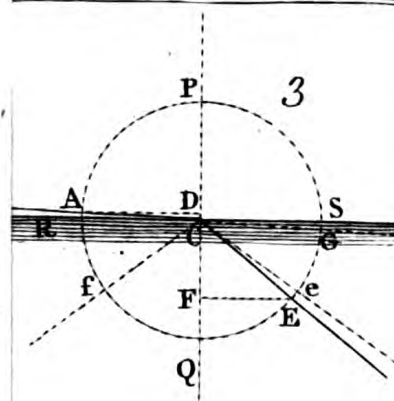
Fig. 14, 15.

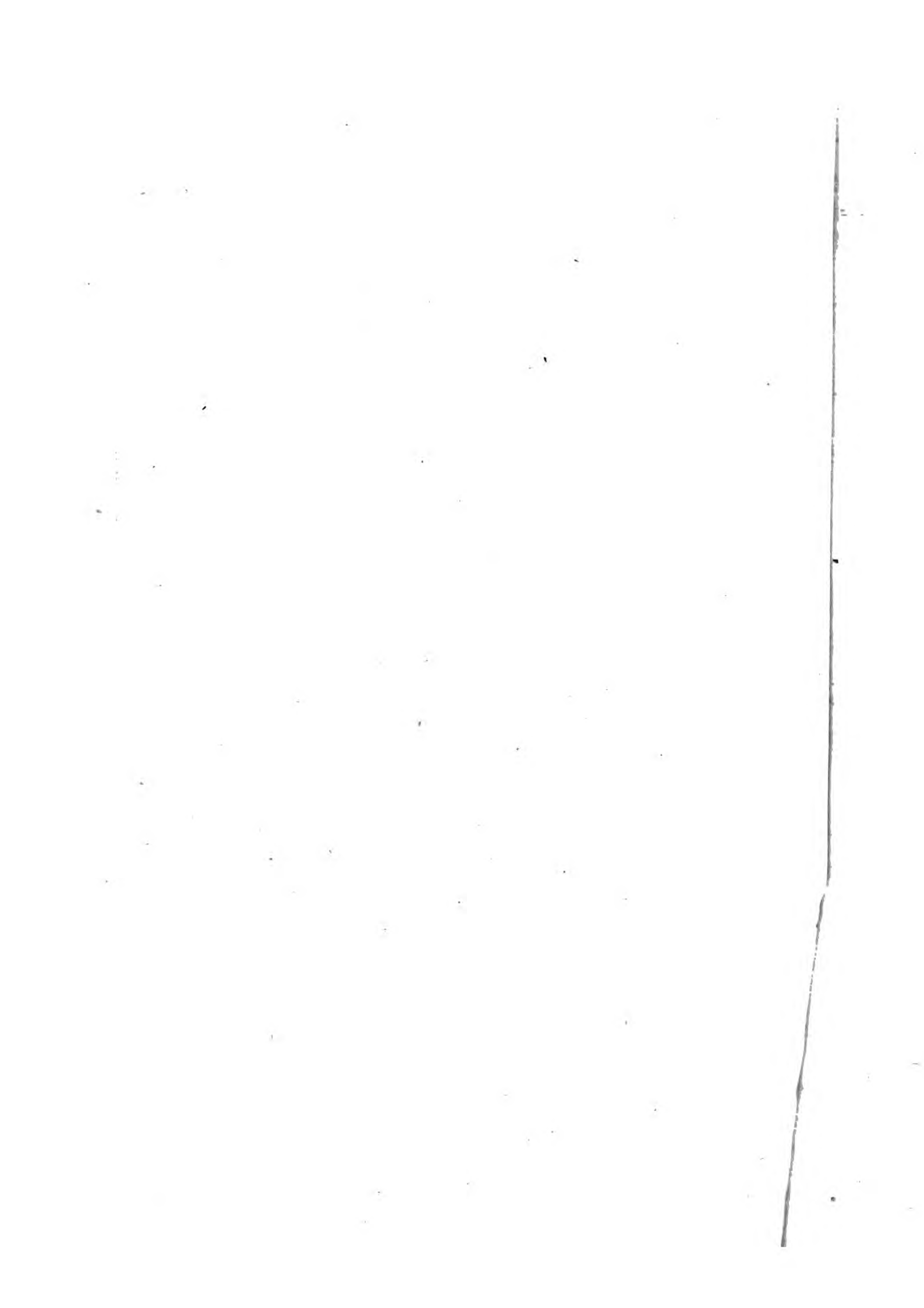
26. A glass prism is a body, shaped like a wedge, that has three edges, being bounded with two equal and parallel triangular ends ABC and abc , and three plane and well polished sides, which meet in three parallel lines Aa , Bb , Cc , running from the three angles of one end to the three angles of the other: and when it is viewed endways it is represented only by a triangle ABC , as in the 15th figure.

I.
Experiment.
A description
of the sun's
image made
by a prism.
Newt. Opt.
p. 21.
Fig. 15.

27. In a very dark chamber at a round hole F , about one third of an inch broad, made in the shut of a window, I placed a glass prism ABC whereby the beam of the sun's light SF , which came in at that hole, might be refracted upwards, toward the opposite wall of the chamber, and there form a coloured image of the sun, represented at PT . The axis of the prism, (that is the line passing through the middle of the prism, from one end of it to the other end, parallel to the edge of the refracting angle) was in this and the following experiments perpendicular to the incident rays. About this axis I turned the prism slowly, and saw the refracted light on the wall, or coloured image of the sun, first to descend, and then to ascend. Between the descent and ascent when the image seemed stationary, I stopped the prism and fixt it in that posture.

Then I let the refracted light fall perpendicularly upon a sheet of white paper MN , placed at the opposite wall of the chamber, and observed the figure and dimensions of the solar image, PT , formed on the paper by that light. This image was oblong and not oval, but terminated by two rectilinear and parallel sides and two semi-circular ends. On its sides it was bounded pretty distinctly, but on its





its ends very confusedly and indistinctly, the light there decaying and vanishing by degrees. At the distance of $18\frac{1}{2}$ feet from the prism the breadth of the image was about $2\frac{1}{8}$ inches, but its length was about $10\frac{1}{4}$ inches, and the length of its rectilinear sides about 8 inches; and ACB the refracting angle of the prism, whereby so great a length was made, was 64 degrees. With a less angle the length of the image was less, the breadth remaining the same. It is farther to be observed that the rays went on in straight lines from the prism to the image, and therefore at their going out of the prism had all that inclination to one another from which the length of the image proceeded. This image PT was coloured, and the more eminent colours lay in this order from the bottom at T to the top at P ; red, orange, yellow, green, blue, indigo, violet; together with all their intermediate degrees in a continual succession perpetually varying.

28. Our author concludes from this experiment, and some others to be mentioned hereafter, that the light of the sun consists of a mixture of several sorts of coloured rays, some of which at equal incidences are more refracted than others, and therefore are called more refrangible. The red at T , being nearest to the place \mathcal{Y} , where the rays of the sun would go directly if the prism was taken away, is the least refracted of all the rays; and the orange, yellow, green, blue, indigo and violet are continually more and more refracted, as they are more and more diverted from the course of the direct light. For by mathematical reasoning he has proved, that when the prism is fixt in the posture above mentioned, so that the place of the image shall be the lowest possible, or at the limit between its descent and ascent, the figure of the image ought then to be round like the spot at \mathcal{Y} , if all the rays that tended to it were equally refracted. Therefore seeing by experience it is found that this image is not round, but about 5 times longer than broad, it follows that all the rays are not equally refracted.

Hence the sun's rays are differently refrangible.

For the discovery of this fundamental property of light, which has opened the whole mystery of colours, we see our author was not only beholden to the experiments themselves, which many others had made before him, but also to his skill in geometry; which was absolutely necessary to determine what the figure of the refracted image ought to be upon the old principle of an equal refraction of all the rays: but having thus made the discovery he contrived the following experiment to prove it at sight.

In the middle of two thin boards, DE, de , I made a round hole in each, at G and g , a third part of an inch in diameter; and in the window-shut a much larger hole being made, at F , to let into my

II.
Experiment.
Newt. Opt.
p. 37.
Fig. 16.

B

darkened

darkened chamber a large beam of the sun's light, I placed a prism, ABC , behind the shut in that beam, to refract it towards the opposite wall; and close behind this prism I fixed one of the boards DE , in such manner that the middle of the refracted light might pass through the hole made in it at G , and the rest be intercepted by the board. Then at the distance of about 12 feet from the first board I fixed the other board de , in such manner that the middle of the refracted light, which came through the hole in the first board, and fell upon the opposite wall, might pass through the hole g in this other board de , and the rest being intercepted by the board might paint upon it the coloured spectrum of the sun. And close behind this board I fixed another prism abc to refract the light which came through the hole g . Then I returned speedily to the first prism ABC and by turning it slowly to and fro about its axis, I caused the image which fell upon the second board de to move up and down upon that board, that all its parts might pass successively through the hole in that board, and fall upon the prism behind it. And in the mean time I noted the places, M, N , on the opposite wall, to which that light after its refraction in the second prism did pass; and by the difference of the places at M and N , I found that the light, which being most refracted in the first prism ABC , did go to the blue end of the image, was again more refracted by the second prism abc , than the light which went to the red end of that image. For when the lower part of the light which fell upon the second board de , was cast through the hole g , it went to a lower place M on the wall; and when the higher part of that light was cast through the same hole g , it went to a higher place N on the wall; and when any intermediate part of the light was cast through that hole, it went to some place in the wall between M and N . The unchanged position of the holes in the boards made the incidence of the rays upon the second prism to be the same in all cases. And yet in that common incidence some of the rays were more refracted and others less: and those were more refracted in this prism, which by a greater refraction in the first prism were more turned out of their way; and therefore for their constancy of being more refracted are deservedly called more refrangible.

Our author shews also, by experiments made with a convex glass, that lights (reflected from natural bodies) which differ in colour, differ also in degrees of refrangibility^a: and that they differ in the same manner as the rays of the sun do.

^a Newt. Opt. p. 16.

Definitions
Newt. Opt.
p. 4.

The light whose rays are all alike refrangible I call simple homogeneous and similar, and that whose rays are some more refrangible than

than

than others I call compound, heterogeneal and dissimilar. The former light I call homogeneal not because I would affirm it so in all respects; but because the rays which agree in refrangibility agree at least in all their other properties which are considered in this chapter.

The colours of homogeneal lights I call primary, homogeneal and simple, and those of heterogeneal lights, heterogeneal and compound. For these are always compounded of homogeneal lights, as will appear in the following articles.

The homogeneal light and rays which appear red, or rather make objects appear so, I call rubrifick or red-making; those which make objects appear yellow, green, blue and violet, I call yellow-making, green-making, blue-making, violet-making; and so the rest. And if at any time I speak of light and rays as coloured or endued with colours, I would be understood to speak not philosophically and properly but grossly, and according to such conceptions as vulgar people in seeing all these experiments would be apt to frame. For the rays to speak properly are not coloured. In them there is nothing else than a certain power and disposition to stir up a sensation of this or that colour. For as sound in a bell or musical string or other sounding body, is nothing but a trembling motion, and in the air nothing but that motion propagated from the object, and in the sensorium it is a sense of that motion under the form of sound; so colours in the object are nothing but a disposition to reflect this or that sort of rays more copiously than the rest; in the rays they are nothing but their dispositions to propagate this or that motion into the sensorium; and in the sensorium they are sensations of those motions under the forms of colours.

29. Homogeneal light is refracted regularly without any dilatation splitting or shattering of the rays, and the confused vision of objects seen through refracting bodies by heterogeneal light, arises from the different refrangibility of several sorts of rays. This will appear by the experiments which follow. In the middle of a black paper I made a round hole about a fifth or a sixth part of an inch in diameter. Upon this paper I caused the spectrum of homogeneal light described in the former article, so to fall that some part of the light might pass through the hole in the paper. This transmitted part of the light I refracted with a prism placed behind the paper, and letting this refracted light fall perpendicularly upon a white paper two or three feet distant from the prism, I found that the spectrum formed on the paper by this light was not oblong, as when it is made, in the first experiment, by refracting the sun's compound

Newt. Opt.
p. 108.

III.
Experiment.
Homogeneal
light is re-
fracted regu-
larly, &c.
Newt. Opt.
p. 62.

light, but was (so far as I could judge by my eye) perfectly circular, the length being no where greater than the breadth; which shews that this light is refracted regularly without any dilatation of the rays; and is an ocular demonstration of the mathematical proposition mentioned in the 28th article.

IV.
Experiment.
Newt. Opt.
p. 63.

In the homogeneal light I placed a paper circle of a quarter of an inch in diameter; and in the sun's unrefracted, heterogeneal, white light I placed another paper circle of the same bigness; and going from these papers to the distance of some feet I viewed both circles through a prism. The circle illuminated by the sun's heterogeneal light appeared very oblong, the length being many times greater than the breadth. But the other circle illuminated with homogeneal light appeared circular and distinctly defined, as when it is viewed by the naked eye; which proves the whole proposition mentioned at the beginning of this article.

V.
Experiment.
Ibid.

In the homogeneal light I placed flies and such like minute objects, and viewing them through a prism I saw their parts as distinctly defined as if I had viewed them with the naked eye. The same objects placed in the sun's unrefracted heterogeneal light which was white, I viewed also through a prism, and saw them most confusedly defined, so that I could not distinguish their smaller parts from one another. I placed also the letters of a small print one while in the homogeneal light and then in the heterogeneal, and viewing them through a prism they appeared in the latter case so confused and indistinct that I could not read them; but in the former they appeared so distinct that I could read readily, and thought I saw them as distinct as when I viewed them with my naked eye; in both cases I viewed the same objects through the same prism at the same distance from me and in the same situation. There was no difference but in the lights by which the objects were illuminated and which in one case was simple in the other compound; and therefore the distinct vision in the former case and confused in the latter could arise from nothing else than from that difference in the lights. Which proves the whole proposition.

The colour of
homogeneal
light cannot
be changed
by refractions
nor by re-
flexions.
Newt. Opt.
p. 107.

30. In these three experiments it is farther very remarkable that the colour of homogeneal light was never changed by the refraction: and as these colours were not changed by refractions, so neither were they by reflexions. For all white, grey, red, yellow, green, blue, violet bodies, as paper, ashes, red lead, orpiment, indigo, bise, gold, silver, copper, grafs, blue flowers, violets, bubbles of water tinged with various colours, peacocks' feathers, the tincture of *lignum-nephriticum* and such like, in red homogeneal light appeared totally red,

red, in blue light totally blue, in green light totally green, and so of other colours. In the homogeneal light of any colour they all appeared totally of that same colour, with this only difference, that some of them reflected that light more strongly, others more faintly. I never yet found any body which by reflecting homogeneal light could sensibly change its colour.

From all which it is manifest, that if the sun's light consisted of but one sort of rays, there would be but one colour in the world. Nor would it be possible to produce any new colour by reflexions and refractions: and by consequence that the variety of colours depends upon the composition of light.

31. Every homogeneal ray considered apart is refracted according to one and the same rule^a, so that its sine of incidence is to its sine of refraction in a given ratio: that is, every different coloured ray has a different ratio belonging to it. This our author has proved by experiment, and by other experiments has determined by what numbers those given ratios are expressed. For instance, if an heterogeneal white ray of the sun emerges out of glass into air, or which is the same thing, if rays of all colours be supposed to succeed one another in the same line *AC*, and *AD* their common sine of incidence in glass be divided into 50 equal parts, then *EF* and *GH* the sines of refraction into air, of the least and most refrangible rays, will be 77 and 78 such parts respectively. And since every colour, has several degrees, the sines of refraction of all the degrees of red will have all intermediate degrees of magnitude from 77 to $77\frac{1}{5}$, of all the degrees of orange from $77\frac{1}{5}$ to $77\frac{1}{3}$, of yellow from $77\frac{1}{3}$ to $77\frac{1}{2}$, of green from $77\frac{1}{2}$ to $77\frac{2}{3}$, of blue from $77\frac{2}{3}$ to $77\frac{3}{4}$, of indigo from $77\frac{3}{4}$ to $77\frac{7}{8}$, and of violet from $77\frac{7}{8}$ to 78^b.

32. Colours may be produced by composition which shall be like to the colours of homogeneal light, as to the appearance of colour, but not as to the immutability of colour and constitution of light. And those colours, by how much they are more compounded, by so much are they less full and intense; and by too much composition they may be diluted and weakened till they cease, and the mixture becomes white or grey. There may be also colours produced by composition, which are not fully like any of the colours of homogeneal light. For a mixture of homogeneal red and yellow compounds an orange, like in appearance of colour to that orange which in the series of unmixed prismatic colours lyes between them. But the light of one orange is homogeneal as to refrangibility, that of the other is heterogeneal; and the colour of the one, if viewed through a prism remains unchanged, that of the other is changed and resolved.

The sine of incidence of every homogeneal ray is to its sine of refraction in a given ratio. Newt. Opt. p. 64. Fig. 17. Art. 13.

^b Newt. Opt. p. 109. The different properties of simple and compound colours. Newt. Opt. p. 115.

solved into its component colours red and yellow. And after the same manner other neighbouring homogeneous colours may compound new colours, like the intermediate homogeneous ones: as yellow and green the colour between them both; and afterwards if blue be added there will be made a green, the middle colour of the three which enter the composition. For the yellow and blue on either hand, if they are equal in quantity, draw the intermediate green equally toward themselves, and so keep it as it were in æquilibrium, that it verge not more to the yellow on one hand, than to the blue on the other, but by their mixed actions remain still a middle colour. To this mixed green there may be farther added some red and violet, and yet the green will not presently cease but only grow less full and vivid; and by increasing the red and violet, it will grow more and more dilute, until by the prevalence of the added colours it be overcome and turned into whiteness or some other colour. So if to the colour of any homogeneous light, the sun's white light composed of all sorts of rays be added, that colour will not vanish or change its species, but be diluted, and by adding more and more white it will be diluted more and more perpetually. Lastly if red and violet be mingled there will be generated according to their various proportions various purples: such as are not like in appearance to the colour of any homogeneous light; and of these purples mixed with yellow and blue may be made other new colours.

VI.
Experiment.
Whiteness
may be com-
pounded of
colours.
Newt. Opt.
p. 117.
Fig. 18.

33. Whiteness and all grey colours between white and black, may be compounded of colours; and the whiteness of the sun's light is compounded of all the primary colours mixed in a due proportion.

For let the solar image PT fall upon a lens MN above four inches broad and about six feet distant from the prism ABC , and so figured that it may cause the coloured light which divergeth from the prism to converge and meet again at its focus G about 6 or 8 feet distant from the lens, and there to fall perpendicular upon a white paper DE . And if you move this paper to and fro, you will perceive that near the lens, as at de , the whole solar image, suppose at pt , will appear upon it intensely coloured after the manner above explained: and that by receding from the lens those colours will perpetually come towards one another, and by mixing more and more dilute one another continually, until at length the paper comes to the focus G , where by a perfect mixture they will wholly vanish and be converted into whiteness, the whole light appearing now upon the paper like a little white circle.

VII.
Experiment.
Newt. Opt.
p. 129.

In the foregoing experiment I have produced whiteness by mixing the prismatic colours. If now the colours of natural bodies are to be

be mingled, let a little water thickened with soap be agitated to raise a froth, and after that froth has stood a little, there will appear to one that shall view it intently various colours every where in the surface of the several bubbles, but to one that shall go so far off that he cannot distinguish the colours from one another, the whole froth will grow white with a perfect whiteness.

34. The colours of natural bodies arise from hence, that some of them reflect some sort of rays, others other sorts more copiously than the rest. Minium reflects the least refrangible or red-making rays most copiously and thence appears red. Violets reflect the most refrangible most copiously, and thence have their colour: and so of other bodies. Every body reflects the rays of its own colour more copiously than the rest, and from their excess and predominance in the reflected light has its colour.

For if in the homogeneous lights obtained by the 2d experiment, you place bodies of several colours, you will find as I have done, that every body looks more splendid and luminous in the light of its own colour. Cinnaber in the homogeneous red is most resplendent, in the green light it is manifestly less resplendent, in the blue light still less. Indigo in the violet blue light is most resplendent, and its splendor is gradually diminished as it is removed thence by degrees through the green and yellow light to the red. By a leek the green light, and next that the blue and yellow which compound green, are more strongly reflected than the other colours red and violet, and so of the rest. But to make these experiments the more manifest, such bodies ought to be chosen as have the fullest and most vivid colours, and two of those bodies are to be compared together. Thus for instance, if cinnaber and ultra-marine blue, or some other full blue be held together in the red homogeneous light, they will both appear red; but the cinnaber will appear of strongly luminous and resplendent red, and the ultra-marine blue of a faint obscure and dark red. And if they be held together in the blue homogeneous light, they will both appear blue; but the ultra-marine will appear of a strongly luminous and resplendent blue, and the cinnaber of a faint and dark blue. Which puts it out of dispute that the cinnaber reflects the red light much more copiously than the ultra-marine doth, and the ultra-marine reflects the blue light much more copiously than the cinnaber doth. The same experiment may be tried successively with red and indigo or with any other two coloured bodies, if due allowance be made for the different strength or weakness of their colour and light.

And that this is not only a true reason of their colours, but even the

The permanent colours of natural bodies explained.

VIII. Experiment. Newt. Opt. P. 157.

the only reason, may appear farther from this consideration; that the colour of homogeneal light cannot be changed by the reflection of natural bodies^a. For if bodies by reflection cannot in the least change the colour of any one sort of rays, they cannot appear coloured by any other means, than by reflecting those which either are of their own colour, or by mixture must produce it.

^a Art. 30.

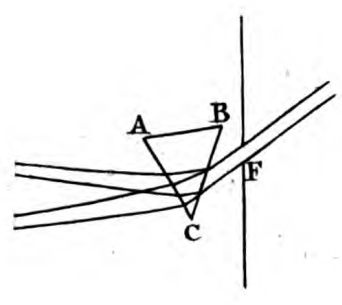
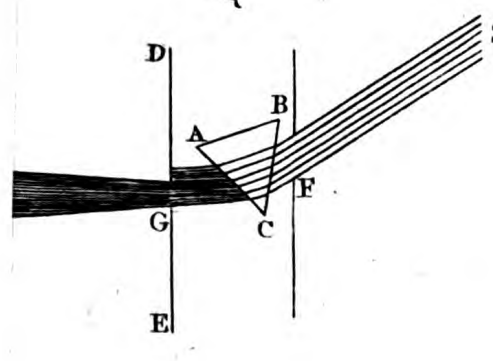
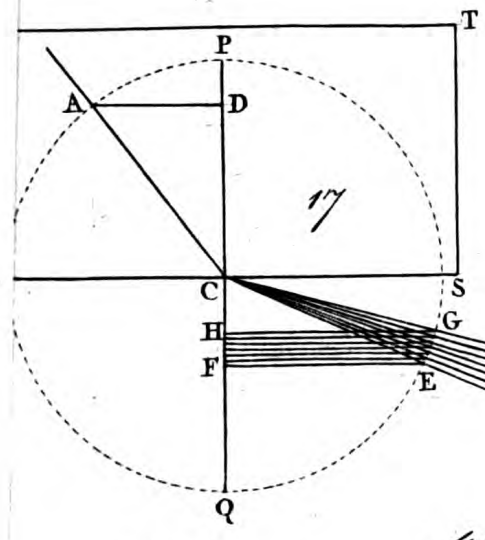
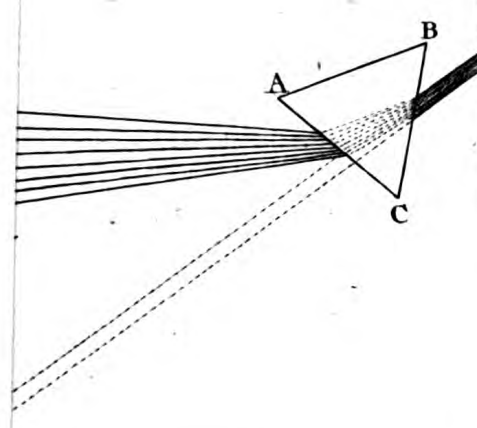
CHAP. III.

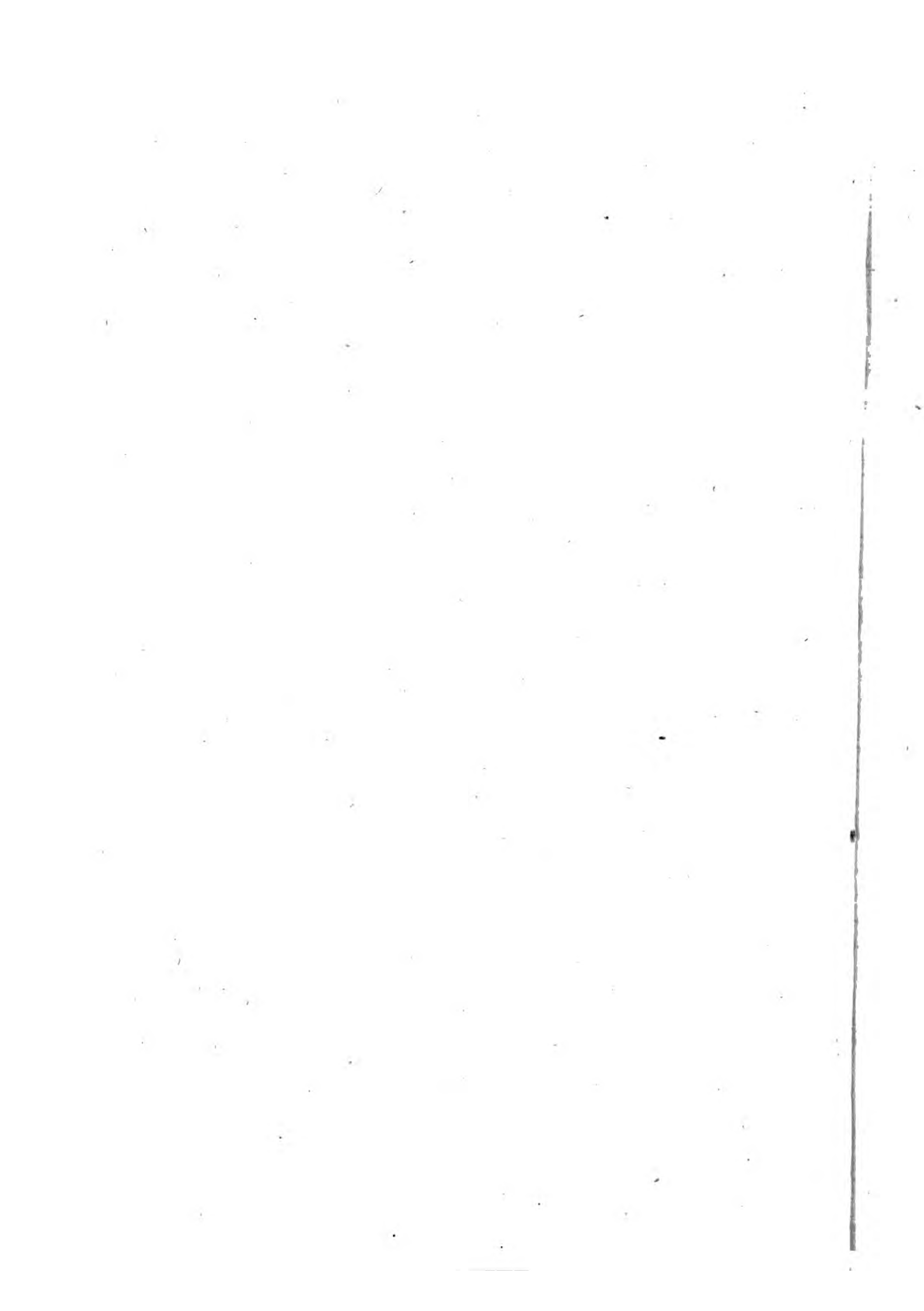
CONCERNING THE CAUSE OF REFRACTION, REFLECTION, INFLECTION AND EMISSION OF LIGHT, AND CONCERNING TRANSPARENCY, OPACITY, AND COLOURS IN BODIES.

Reflexion not caused by the impinging of light upon the medium. Newt. Opt. p. 237.

35. **T**HAT the cause of reflection is not the impinging of light on the solid or impervious parts of bodies, as is commonly believed, will appear by the following considerations. First, that in the passage of light out of glass into air there is a reflection as strong as in its passage out of air into glass, or rather a little stronger, and by many degrees stronger than in its passage out of glass into water. And it seems not probable that air should have more reflecting parts than water or glass. But if that should possibly be supposed, yet it will avail nothing; for the reflection is as strong or stronger when the air is drawn away from the glass, by an air-pump, as when it is adjacent to it. Secondly, if light in its passage out of glass into air be incident more obliquely than an angle of 40 or 41 degrees, it is wholly reflected, if less obliquely it is in a great measure transmitted^b. Now it is not to be imagined that light at one degree of obliquity should meet with pores enough in the air to transmit the greatest part of it, and at another degree of obliquity should meet with nothing but parts to reflect it wholly: especially considering that in its passage out of air into glass, how oblique soever be its incidence, it finds pores enough in the glass to transmit a great part of it. If any man supposes that it is not reflected by the air, but by the outmost superficial parts of the glass, there is still the same difficulty: besides that such a supposition is unintelligible, and will appear to be false by applying water behind some part of the glass instead of air. For so in a convenient obliquity of the rays suppose of 45 or 46 degrees, at which they are all reflected where air is adjacent to the glass, they shall be in a great measure transmitted where water is adjacent to it. Which argues that their reflection or transmission

^b Art. 17.





mission depends on the constitution of air and water behind the glass, and not on the striking of the rays on the parts of the glass. Thirdly, if the colours made by a prism, placed at the entrance of a beam of light into a darkened room, be successively cast on a second prism placed at a great distance from the former, in such manner that they are all alike incident upon it, (as they will be when transmitted through the holes in the two boards made use of in the 2d experiment,) the second prism may be so inclined to the incident rays, that those which are of a blue colour shall be all reflected by it, and yet those of a red pretty copiously transmitted. Now if reflection be caused by the parts of air or glass, I would ask why at the same obliquity of incidence, the blue should wholly impinge on those parts, so as to be all reflected, and yet the red find pores enough to be in a great measure transmitted? Lastly were the rays of light reflected by impinging on the solid parts of bodies, their reflections from polished bodies could not be so regular as they are. For in polishing glass with sand, putty or tripoli, it cannot be imagined that those substances can by grating and fretting the glass bring all its least particles to an accurate polish; so that all their surfaces shall be truly plane or truly spherical, and look all the same way, so as together to compose one even surface. This manner of polishing with powders can do no more than bring the roughness of the glass to a very fine grain, so that the scratches and frettings of the surface become too small to be visible. And therefore if light were reflected by impinging upon the solid parts of the glass, it would be scattered as much by the most polished glass as by the roughest. So then it remains a problem how glass polished by fretting substances can reflect light so regularly as it does.

36. And this problem is scarce otherwise to be solved than by saying that the reflection of a ray is effected not by a single point of the reflecting body, but by some power of the body which is evenly diffused all over its surface, and by which it acts upon a ray without immediate contact. For that the parts of bodies do act upon light at a distance, will appear by the following experiments.

37. The sun shining into my chamber through a hole a quarter of an inch broad, I placed at the distance of two or three feet from the hole a sheet of pastboard, which was blacked all over on both sides, and in the middle of it had a hole about three quarters of an inch square for the light to pass through. And behind the hole I fastened to the pastboard with pitch the blade of a sharp knife, to intercept some part of the light which passed through the hole. The planes of the pastboard and of the knife were parallel to one another

But by an active power diffused over its surface.

IX. Experiment. This power acts upon light at a distance by attracting and repelling it. Newt. Opt. p. 300.

other and perpendicular to the rays. And when they were so placed that none of the sun's light fell upon the pastboard, but all of it passed through the hole to the knife, and there part of it fell upon the blade of the knife, and part of it passed by its edge; I let this part of the light, which passed by, fall on a white paper two or three feet beyond the knife, and there saw two streams of faint light shoot out both ways from the beam of light into the shadow, like the tails of comets. But because the sun's direct light by its brightness upon the paper obscured these faint streams so that I could scarce see them, I made a little hole in the midst of the paper for that light to pass through, and fall upon a black cloth behind it, and then I saw the two streams plainly. They were like one another, and pretty nearly equal in length and breadth, and quantity of light. Their light at that end next the sun's direct light was pretty strong for the space of about a quarter of an inch or half an inch, and in all its progress from that direct light decreased gradually till it became insensible. The whole length of either of these streams measured upon the paper, at the distance of three feet from the knife, was about six or eight inches; so that it subtended an angle at the edge of the knife of 10 or 12, or at most 14 degrees.

X.
Experiment.

I placed another knife by this, so that their edges might be parallel and look towards one another, and that the beam of light might fall upon both knives and some part of it pass between their edges. And when the distance of their edges was about the 400th part of an inch, the stream parted in the middle and left a shadow between the two parts. This shadow was so black and dark that all the light which passed between the two knives seemed to be bent and to be turned aside to the one hand and to the other. And as the knives still approached one another the shadow grew broader, and the streams shorter at their inward ends next the shadow, until upon the contact of the knives the whole light vanished and left its place to the shadow. And hence I gather that the light which is least bent, and goes to the inward ends of the streams, passes by the edges of the knives at the greatest distance, and this distance when the shadow begins to appear between the streams is about the 800th part of an inch. And the light which passes by the edges of the knives at distances still less and less is more and more bent, and goes to those parts of the streams which are farther and farther from the direct light. Because when the knives approach one another till they touch, those parts of the streams vanish last which are farthest from the direct light.

Our author has made it appear from these and some other experiments,

ments, that bodies act upon light in some circumstances by an attractive and in others by a repulsive power. For he found that the shadows of hairs, threads, pins, straws, and such like slender substances, placed in a slender beam of light let into a dark room, were considerably broader than they ought to be, if the rays of light passed on by these bodies in right lines. Particularly he found that the shadow of a hair of a man's head, at the distance of 10 feet from the hair, was 35 times broader than the hair itself^a.

^a Newt. Opt, p. 293.

38. That this power which acts upon light is infinitely stronger than the power of gravity will appear by the following argument. Sir *Isaac Newton* has demonstrated that all bodies attract one another by the force of gravity, and that the attractive forces of two homogeneous spheres, upon particles of matter placed very near their surfaces, are to each other in proportion as the diameters of the spheres¹. That is to say, if a refracting medium be spherical and of the same density as the earth, the earth's force of attraction near its surface, will exceed the medium's force near its surface, as much as the diameter of the earth exceeds the diameter of the medium; or almost infinitely with respect to human conceptions. Yet we observe that a cannon-ball, just shot from the mouth of the cannon, is scarce sensibly deflected towards the earth by its attraction; and the least particle of the ball, if it was separate from the rest, would be no more deflected than the whole; because gravity makes bodies of all sorts and sizes descend with the same swiftness, by affecting them alike whether joined or separated. Therefore a particle of light which moves, I may say, infinitely quicker than a cannon-ball, would be infinitely less bent than the particle of the ball by the attraction of the whole earth, and still infinitely less, than this last bending, by the attraction of the spherical medium, which was shewn to be infinitely weaker than that of the earth. But in fact we find it is very sensibly bent or refracted; and therefore it must be affected by some other power of the medium, which near its surface is infinitely stronger than the power of gravity.

And is infinitely stronger than the power of gravity.

39. It is difficult to determine the exact law of this refractive power, or the degrees of its force at given distances from the refracting surface. However since we find that the effects of gravity, which decrease as the squares of the distances from the center increase, are very sensible at great distances, we may conclude that the refractive power of a medium, which at its surface we find is infinitely stronger than gravity, and yet vanishes at a very small distance

And decreases much quicker.

¹ Princip. lib. 1. prop. 74. cor. 2. & lib. 3. prop. 8.

^a Art. 37.

from it^a, decreases much quicker or in a greater proportion than gravity does.

This one power both refracts and reflects light. Newt. Opt. p. 244.
^b Art. 17.

40. It is reasonable to conclude that bodies reflect and refract light by one and the same power variously exercised in various circumstances; because when light goes out of glass into air as obliquely as it can possibly do, if its incidence be made still more oblique, it becomes totally reflected^b: (for the power of the glass after it has reflected the light as obliquely as is possible, if the incidence be still made more oblique becomes too strong to let any of its rays go through and by consequence causes total reflections:) And for this other reason, that those surfaces of transparent bodies which have the greatest refracting power, reflect the greatest quantity of light, as will be shewn in the 47th article.

Its forces in different bodies are as their densities nearly. Newt. Opt. p. 245.

41. From the different ratios of the sines of incidence and refraction in a great many different bodies, our author has also collected that the forces of bodies to reflect and refract light are very nearly proportionable to their densities, excepting that unctuous and sulphureous bodies refract more than others of the same density. Whence, he says, it seems rational to attribute the refractive power of all bodies chiefly, if not wholly to the sulphureous, oily particles with which they abound. For it is probable that all bodies abound more or less with sulphurs. And as light congregated by a burning glass acts most upon sulphureous bodies to turn them into fire and flame, so since all action is mutual, sulphurs ought to act most upon light. For that the action between light and bodies is mutual, may appear from this consideration; that the densest bodies which refract and reflect light most strongly grow hottest in the summer sun, by the action of the refracted or reflected light. If bodies be conceived to have certain densities exactly proportionable to their refractive powers, these may be called their refractive densities.

Refractive densities what.

This force acts in lines perpendicular to the refracting surface.

42. The direction of the refractive force of a medium, acting upon particles of light, is every where perpendicular to the refracting surface. For whether this force be a real attraction, or whether it be an impulse upon light, caused by the spring or elastick power of a subtil fluid which pervades the medium, and being gradually denser without than within it, may impel the light towards the medium by its greater elasticity without than within^c; be this as you please, yet if the medium be uniform in all its parts, its immediate power upon the light it self, or upon the subtil fluid which acts upon it, will be equally strong in every point of a plane drawn parallel to the refracting surface; though its strength may be different in the next parallel plane, and still different in the next, and so on

^c Newt. Opt. p. 323, &c.

as far as that power is extended on each side of the surface of the medium. The extent of this power will therefore be terminated by two planes, parallel to one another and to the refracting surface; and the space between them may be called the space of activity, whether the power attracts or repels. This being premised, I say the force of the medium will act upon light, either in attracting or repelling it, in lines perpendicular to its surface. For let p be a particle of light acted upon by any uniform power in the line de parallel to the refracting surface AB , pc a line perpendicular to those parallels, cutting de in c ; it is evident that the force of the power at c will move the particle p in the line pc ; and taking any two points d, e at equal distances on each side of c , the powers at d and e being equal and acting at equal distances, pd, pe , equally inclined to pc , cannot move p in any direction but that of pc ; and what has been said of the equal powers in the line de is applicable to the powers in every line drawn parallel to AB , that is to the whole power of the refracting medium.

43. Now when a ray of light falls perpendicularly upon the space of activity its particles will be accelerated or retarded in the same perpendicular direction, according as the power of the medium acts with or against the course of their motion; and when the particles are got through that space they will proceed with an uniform velocity. But if a ray op or sr falls obliquely upon the space of activity $klmn$, the force of the medium now acting sideways or obliquely upon the particles, will bend their course into a curve pqr , during their passage through that space. For as light has this property in common with all other bodies, of moving straight forwards, while its motion is not disturbed by any oblique force, so when it is disturbed, we may reasonably conclude, it will follow those other laws of motion, to which all other bodies are equally subject. The force of the medium acting sideways upon its oblique course, will therefore draw it perpetually out of one direction into another. But having passed through the space of activity, it will then proceed straight forwards; for being attracted or impelled every way alike, or else not all if it be in empty space, it will have the same freedom of motion in both cases: just as an animal surrounded with air, though violently pressed on every side, feels no constraint, but has an equal facility of moving in any direction. Thus we see that the refraction of light is performed in the same manner as if a stone was thrown in the direction op , and its course was bent into a curve pqr by its gravity; or being thrown the contrary way in the direction sr , it was bent into the curve rqp in ascending: and supposing the

Space of activity what.

Fig. 19, 20.

The manner of its operation in causing refractions and reflections. Fig. 21.

Fig. 22.

the attraction of the earth to reach no higher than the line kl the stone would from thence proceed in a straight line po . Now the gravity of the stone may be so great, or the force of projection so weak, or the direction of the motion so oblique to the horizontal line kl , that it cannot ascend so high as this line. In this case the stone will descend from the highest point of its course by the same degrees of curvity with which it ascended; and if its gravity be supposed to cease in all places below the line mn , the stone will go on in the direction of the last particle of the curve produced. This is a parallel case to that of reflections from the farther surface of dense mediums, when the incident ray is so much inclined to that surface as to be pulled back into the same medium. Hitherto I have supposed the refracting medium to be contiguous to empty space; but the manner of reflection and refraction is the same at the common surface of any two mediums. For since the separate forces of the mediums act in the same lines, perpendicular to their common surface^a, and in contrary directions; the light will be affected with the difference of those forces in the same manner as before. And if the mediums have equal forces they will balance each other, without causing any reflection or refraction at all. It has been observed already that the perpendicular breadth of the space of activity is exceeding small, and consequently in physical experiments the incurvation of the ray may still be considered as performed in a physical point.

^a Art. 42.

And in causing the different refrangibility of rays.
Newt. Opt. p. 347.

Fig. 23.

44. According to this theory nothing more is requisite for producing all the variety of colours and degrees of refrangibility, than that the rays of light be bodies of different sizes; the least of which may make violet, the weakest and darkest of the colours, and be more easily diverted by refracting surfaces from its right course; and the rest, as they are bigger and bigger, may make the stronger and more lucid colours, blue, green, yellow and red; and be more and more difficultly diverted. For particles of different sizes, that fall upon the space of activity $klmn$ in the line op , having different forces, may describe different curves, as pa , pb , pc , and consequently will emerge from that space in different angles.

And in causing the angles of reflection to be the same in all sorts of rays.
Fig. 23.

45. Thus may heterogeneous particles diverge from one another by refraction, though not by reflection. For if the line of their incidence op be so oblique to the space of attraction $klmn$, that all the particles are pulled back into the same medium, they will return in parallel lines rs , tv , xy , &c. inclined to that space in the same angles as the line of incidence op is inclined to it. Just as several balls of different sizes, shot with different forces out of a canon

non

non op in any fixt position, will describe different curves, as pdr , pet , pfx , &c. yet in returning to the ground they will all strike upon it in equal angles, at r , t , x , &c. every one being equal to the angle of elevation at p . Now since the space of attraction is exceeding thin, the parallel lines rs , tv , &c. will be so close together that the sense cannot perceive a distinct sensation of the separated particles, and consequently the reflected and incident light will appear of the same colour. And when the incident light consists of several rays, though the particles in each ray may be a little separated after reflection, and proceed in different lines, yet those several lines will be mixt together, and consequently the reflected light will appear white or of the same colour as the incident light.

46. Sir *Isaac Newton's* notion of the cause and manner of reflection from opake bodies, and from the first surface of transparent bodies, seems to be this that follows. Let the attractive power of the dense medium $ABCD$ end at the line kl , and there let the repulsive power begin^a, and let it end at the parallel line bi ; and when a ray op falls from air upon the space of repulsion $bikl$, it will be perpetually diverted from one direction into another by the opposition of the repulsive force, and so will describe a curve pqr , till it emerges from that space in the same angle at r with which it immersed at p , and then it will proceed in a right line rs . This will be the course of the ray if its progressive force be but weak, or the repulsive force be so strong as to hinder it from entering the space of attraction $klmn$. For if it enters this space, instead of being reflected, it will be refracted into the dense medium. And in reality some part of the incident light is always reflected and some refracted at all transparent surfaces; the cause of which our author has also considered^b.

And in causing reflections from opake bodies and from the first surface of transparent ones.

Fig. 24.
a Art. 37.

b Newt. Opt. p. 253.

Hence it seems to follow that the repulsive power of a dense medium is less extended or else weaker than the attractive. For if the bending of a ray by the repulsive power, was not less than the contrary bending made by the attractive, the refraction into a dense medium could not always be made towards the perpendicular, as it always is. We may also observe that a refracted ray, in its passage through the surface of a transparent medium, is bent backward and forward with a motion like that of an eel; and our author takes notice of the same sort of motion in its passage by the edges and sides of bodies. It follows also that the repulsive power does not extend to a sensible distance from the medium; for if it did, it would be discovered by a sensible incurvation of the ray throughout that extent; contrary to experience.

47. Those

Stronger and weaker reflections how caused. Newt. Opt. p. 220. ^a Art. 41.

^b Art. 17.

47. Those superficies of transparent bodies reflect the greatest quantity of light which have the greatest refractive power; that is which intercede mediums that differ most in their refractive densities ^a: and in the confines of equally refracting mediums there is no reflection. The analogy between refraction and reflection will appear by considering, that when light passes obliquely out of one medium into another which refracts from the perpendicular, the greater is the difference of the refracting densities, the less obliquity of incidence is requisite to cause a total reflection ^b. Those superficies therefore which refract most, do soonest reflect all the light which is incident upon them, and so must be allowed most strongly reflective. But the truth of this proposition will farther appear by observing, that in the superficies interceding two transparent mediums (such as are air, water, oil, common glass, crystal, metalline glass, island glasses, white transparent arsenick, diamonds, &c.) the reflection is stronger or weaker accordingly as the superficies hath a greater or less refractive power. For in the confine of air and sal-gem it is stronger than in the confine of air and water, and still stronger in the confine of air and common glass or crystal, and stronger in the confine of air and a diamond. If any of these and such like transparent solids, be immersed in water, its reflection becomes much weaker than before, and still weaker if they be immersed in the more strongly refracting liquors of well rectified oil of vitriol or spirit of turpentine. If water be distinguished into two parts by an imaginary surface, the reflection in the confine of these two parts is none at all; in the confine of water and ice it is very little; and in that of water and oil it is something greater; in that of water and sal-gem still greater, and in that of water and glass or crystal or other denser substances still greater, accordingly as those mediums differ more or less in their refracting powers. Hence in the confine of common glass and crystal there ought to be a weak reflection, and a stronger reflection in the confine of common and metalline glass, though I have not yet tried this. But in the confine of two glasses of equal densities, as of two object-glasses of long telescopes pressed gently together, there is not any sensible reflection. For objects may be seen by rays obliquely transmitted through a round black spot where the glasses touch one another, but not through other places where the light is reflected at the interval between the glasses. And the same may be understood of the superficies interceding two crystals, or two liquors, in which no reflection is caused. So then the reason why uniform pellucid mediums, such as water, glass, or crystal, have no sensible reflection, but in their external superficies,

perfacies, where they are adjacent to other mediums of a different density, is because all their contiguous parts have one and the same degree of density: or this uniform density of their contiguous parts is a necessary condition of the transparency of the whole.

48. The least parts of almost all natural bodies are in some measure transparent: and the opacity of those bodies ariseth from the multitude of reflections caused in their internal parts. That this is so hath been observed by others, and will easily be granted by them that have been conversant with microscopes. And it may also be tried by applying any substance to a hole, through which some light is immitted into a dark room. For how opaque soever that substance may seem in the open air, it will by that means appear very manifestly transparent, if it be of a sufficient thinness. Only white metalline bodies must be excepted, which by reason of their excessive density seem to reflect almost all the light incident on their first superficies; unless by solution in menstruums they be reduced into very small particles, and then they become transparent.

Opacity caused by a multitude of internal reflections. Newt. Opt. p. 222.

49. Between the parts of opaque and coloured bodies are many spaces, either empty or replenished with mediums of other densities; as water between the tinging corpuscles wherewith a liquor is impregnated; air between the aqueous globules that constitute clouds or mists; and for the most part spaces void of both air and water, but yet perhaps not wholly void of all substance, between the parts of hard bodies. The truth of this is evident by the two preceding articles. For by the latter article there are many reflections made by the internal parts of bodies, which by the former article would not happen if the parts of these bodies were continued, without any such interstices between them: because reflections are caused only in superficies which intercede mediums of a different density by article 47.

The constitution of opaque and coloured bodies what. Newt. Opt. p. 223.

But farther, that this discontinuity of parts is the principal cause of the opacity of bodies, will appear by considering, that opaque substances become transparent by filling their pores with any substance of equal or almost equal densities with their parts. Thus paper dipped in water or oil, the *oculus mundi* stone steeped in water, linen cloth oiled or varnished, and many other substances soaked in such liquors as will intimately pervade their little pores, or separating parts, become by that means more transparent than otherwise. So on the contrary, the most transparent substances may by evacuating their pores, or separating parts, be rendered sufficiently opaque; as salts or wet paper or the *oculus mundi* stone by being dried; horn by being scraped; glass by being reduced to powder or otherwise

flawed; turpentine by being stirred about with water till they mix imperfectly; and water by being formed into many small bubbles, either alone in the form of froth, or by shaking it together with oil of turpentine or oil of olive or with some other convenient liquor, with which it will not perfectly incorporate.

The constitu-
tion of trans-
parent bodies
what.
Newt. Opt.
p. 225.

50. The parts of bodies and their interstices must not be less than of some definite bigness, to render them opaque and coloured. For the opakest bodies, if their parts be subtilly divided, (as metals by being dissolved in acid menstruums, &c.) become perfectly transparent; and at the superficies of the object-glasses, mentioned in the 47th article, where they were very near to one another though they did not absolutely touch, there was no sensible reflection. And likewise if a bubble be blown with water first made tenacious by dissolving a little soap in it, and be covered with a clear glass, to defend it from being agitated by the external air, and be suffered to rest a while, till by the continual subsiding of the water it becomes very thin; at the top where it is thinnest, there will grow a round, black, spot (like that between the object-glasses) which will continually dilate it self more and more till the bubble breaks; now this spot appears black and transparent for want of a sensible reflection, whereas the sides of the bubble which are thicker than the top appear coloured and opaque by a strong reflection.

On these grounds I perceive it is that water, salt, glass, stones and such like substances are transparent. For upon diverse considerations they seem to be as full of pores or interstices between their parts as other bodies are, but yet their parts and interstices to be too small to cause reflections in their common surfaces.

CHAP. IV.

CONCERNING THE REFRACTIONS OF A SINGLE RAY OF LIGHT
IN ITS PASSAGE THROUGH A PRISM, GLOBE, OR LENS.

51. **A** RAY of light EF falling obliquely upon a flat piece of glass, or any medium terminated by two parallel planes represented by the lines AB, CD , will emerge from it after both refractions at F and G in a line GH parallel to the incident ray EF . For since any line FG which the ray describes in passing between the parallel planes, is equally inclined to them both^a, it will be bent as much at G in going forward, as it would be at F in going backward^b; and these equal bendings being made contrary ways, the incident and emergent rays EF and GH are therefore parallel.

Refraction of a ray through parallel plane surfaces. Fig. 25.

^a Eucl. I. 29. ^b Art. 11.

52. The lines described by the incident and emergent rays EF and GH , being produced are closer together when the glass is thinner, and also when the ray falls less obliquely upon it; because the bendings at F and G are then less^c: and in these cases if the glass be not flat but bent a little as represented in the 26th figure by two parallel arches AB, CD , the line EF, GH will still be nearly parallel. For the bended surfaces refract the ray $EFGH$ just as much as two planes would do supposing they touched the surfaces at F and G ^d: and these planes will be nearly parallel when the line FG is but little inclined to the surfaces; being exactly so when it stands perpendicular to them both.

Refraction of a ray through parallel spherical surfaces. Art. 16.

^d Art. 19.

53. A thin piece of glass or of any transparent substance bounded on one side by a polished plane surface, represented by the line EF , and on the other side by a small portion of a polished spherical surface, represented by the arch ACB ; or bounded on both sides by spherical surfaces ACB, EDF , is called a lens or simply a glass; and is conceived by mathematicians to be generated or described by turning the figure $ACBFDE$ round about the line CD , drawn through the middle of it perpendicularly to both its sides. This line CD produced is therefore called the axis of the lens; and it passes through G and H , the centers of its surfaces. The points C, D where it cuts the surfaces are called the vertexes of the lens, and the middle point between them is called its center. The 27th figure represents a plano-convex glass, the 28th a plano-concave, the 29th a double-convex, the 30th a double-concave, and the 31st and

A lens what. Fig. 27. to 32.

32d two concavo-convex glasses, whereof the first is called a meniscus, because it resembles a little moon. It must be remembered once for all, that the thickness CD of all these glasses is generally so small, that it seldom need be considered ^a.

^a Art. 61.

Refraction of
a single ray
through a
prism.

Fig. 33, 34,
35.

54. When a ray of light $EFGH$ is refracted at F and G in passing through the sides, AB, BC , of a prism, the course of the emergent ray, GH , always deviates from, EF , the course of the incident ray, towards the thicker part of the prism, more or less, as the refracting angle ABC is greater or smaller. And if the refracting angle be given (or invariable) and the refractions be but small, the quantity of deviation will also be given, though the position of the incident ray be varied at pleasure.

Fig. 33.

For supposing at first that the ray FG , within the prism, is equally inclined to its sides AB, BC , as in fig. 33, it is evident from the position of the perpendiculars to those sides at the points F and G , that both the refractions are made from the edge B towards the opposite side AC ^b.

^b Art. 12.

Fig. 34.

Now let FG become unequally inclined to the sides AB, BC , by turning it gradually into the position fg ; and while it becomes less and less oblique to one side, suppose AB , it will become more and more oblique to the other side BC . Consequently supposing a ray to go both ways along this variable line fg , it will be more and more bent in going through the side BC and less and less in going back through the side AB ; so that the total bending of the ray, consisting of both its bendings, or angles efg and fgb , taken together, will continue to be much the same in all its positions. The circulation of the line fg , may be farther continued till it becomes perpendicular to the side AB ; and then the bending at this side is nothing: it may also be continued still farther till the bending at f is made the contrary way; which still takes off from the perpetual increase of the greater bending at g and keeps the total bending invariable.

Fig. 35.

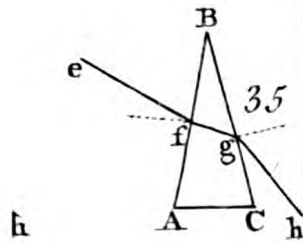
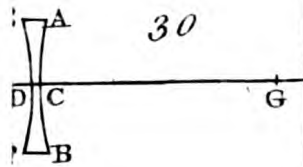
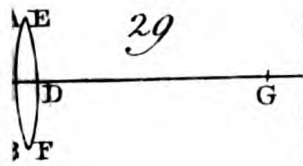
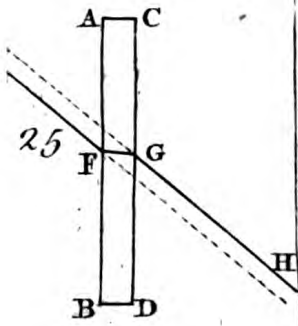
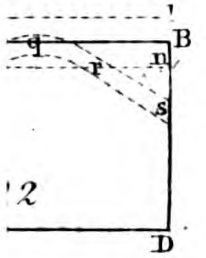
Fig. 34.

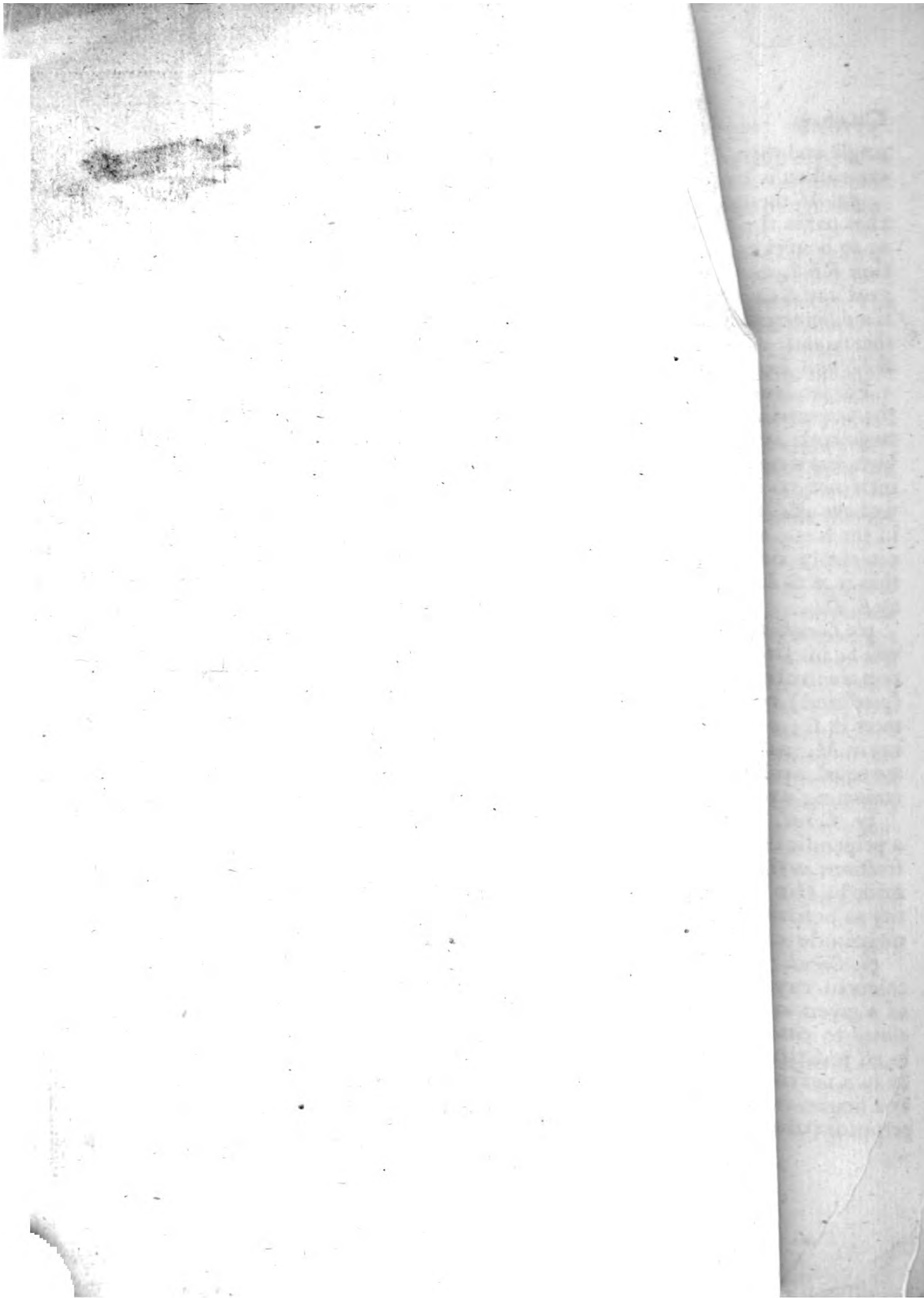
When fg is perpendicular to AB , let the latter plane BC be turned gradually towards the former BA , upon the edge B , and the ray that comes along fg will gradually fall less obliquely upon it; and consequently the bending at g will be gradually diminished^c; and reduced to nothing when the refracting angle ABC vanishes. Lastly if several homogeneous rays be supposed to come parallel to one another they will all emerge parallel to one another^d. Therefore the quantity of deviation of a ray does not at all depend upon its passage through a thicker or thinner part of the prism, nor upon its inclinations to the sides of the prism, but is proportioned to the quantity of the refracting angle ABC ; and the more exactly as this

^c Art. 14, 15.

^d Art. 23.

angle





angle and the refractions at its sides are smaller. The truth of this conclusion is proved mathematically in the next article.

55. If the angles of incidence and refraction of a ray, $\angle ACS$, Fig. 36, 37. that passes through a very small angle of a prism, AIC , be so little as to be reckoned proportionable to their sines; the angle of deviation RFS , contained under the incident ray $\angle AFR$ and the emergent ray $SCFT$ produced, will be to the refracting angle AIC , as the difference of the sines of incidence and refraction to the lesser of them; and consequently the magnitude of the angle of deviation RFS will be invariable in all positions of the ray.

For let the perpendicular AB , to the first surface AI , cross CD , the perpendicular to the second, in E ; and supposing the ray AC to go both ways out of the prism, the angle of incidence ACD will be to the angle of emergence DCT , in the given ratio of the sine of incidence to the sine of refraction, that is of i to r ; and disjointly, we have ACD to ACT as i to $r-i$; and the angle CAB is to CAR , in the same ratio, supposing the ray to go backward along CA ; and conjointly or disjointly we have $ACD \pm CAB$ to $ACT \pm CAR$, that is AED or AIC to RFS in the same given ratio of i to $r-i$. $\angle E. D.$

56. *Corol. 1.* Hence any two homogeneal emergent rays produced, will be inclined to one another in the same angle as the two incident rays are inclined to one another. For let the two incident rays $\angle F, qf$ Fig. 38. (produced) meet in K ; and let the emergent rays SF, sf (produced) meet in L ; and let one of the incident rays cross the other emergent ray in M ; and since in the triangles KMF, LMf , the angles at M are equal and also those at F and f by article 55, it follows that the remaining angles at K and L are also equal.

57. *Corol. 2.* When the ray AC within the prism, coincides with Fig. 39. a perpendicular to either of the planes, as with AB ; one of the refractions will vanish at A ; and then the angle of deviation RFS made by the other single refraction, will continue the same in quantity as before, when it was made by two refractions; because the magnitude of the angle of deviation is invariable by article 55.

58. *Corol. 3.* Therefore when an heterogeneous ray is separated into coloured rays, by small refractions through a small refracting angle of a given quantity, the emergent rays of given colours will be inclined to one another and to the incident ray in certain given angles, in all positions of the incident ray. Because these inclinations made by two refractions, are every where equal to the inclinations made by a single refraction at the second plane, when the incident ray falls perpendicular upon the first plane.

59. When.

Refraction of a single ray through the edge of a lens, or the sides of a globe.

Fig. 40. to 47.

^a Art. 19.

^b Art. 54.

59. When a ray of light $EFGH$ passes through the edge of a convex or concave lens, or the sides of a globe, its emergent part GH always deviates from the course of the incident part EF towards the thicker part of the glass, if the medium in which they are placed, is rarer than the globe or lens. For the refractions at F and G are the same as if they were made by two planes FA, GC , that touch the spherical surface at F and G ^a; and so the sides of the glass may be considered as inclined to each other like the sides of a prism. But the course of the ray is bent towards the thicker part of the prism^b, and therefore towards the thicker part of the globe or lens.

The 44th and 59th figures represent the refractions of a ray passing through a sphere placed within a medium denser than itself; the bending of which ray may, in like manner, be proved to be towards the thinner part of the sphere.

Refractions of a single ray through the middle of a lens.

Fig. 48. to 55.

This ray is considered as straight, and is called the axis of a pencil.

^c Art. 60.

60. From the same method of reasoning it follows, that the deviation of the course of the emergent ray from that of the incident ray is gradually diminished as the ray goes nearer and nearer to the middle of the glass; till, when it goes through the middle, its emergent and incident parts are either parallel to each other, or else are one continued line, when the ray coincides with the axis of the glass. For the angle made by the touching planes, FA, GC , is gradually diminished as the ray FG approaches to the middle: till at last it vanishes when they become parallel, as in the 51st article*.

61. When a pencil of rays falls upon any glass, that ray which passes through its center, or middle point, is called the axis of the pencil. And because its incident and emergent parts EF and GH , are either one continued line or two parallel lines^c, its whole course in optical experiments may be always taken for one straight, physi-

^a Art. 55.

* The author's demonstration of this article being applicable to those cases only, in which the angle at the vertex of the touching prism and the angles of incidence are very small^a, the following demonstration of the manner in which a ray is refracted by the sides of a globe is inserted from Dr. Barrow's Opt. Lect.

PROPOSITION. *The farther a ray is distant from the center of a globe through which it passes, the greater will be its deviation.*

Fig. 60.

Let $NRSP$ represent a globe, which differs in density from the medium in which it is placed; RS, NP , two rays passing through it, of which RS is the more remote from the center C , than NP is. Draw the radii CN, CP, CR, CS , and the angle RCS will be less than the angle NCP ^b. Therefore the sum of the angles CRS, CSR is greater than the sum of the angles CNP, CPN ^c; and each of the angles CRS, CSR is greater than each of the angles CNP, CPN ^d. But the angle of refraction at R exceeding that at N , the angle of incidence at R will also be the greater^e. Since therefore the angles of incidence at R and S are greater than those at N and P , the deviation of RS is greater than that of NP ^f. Q. E. D.

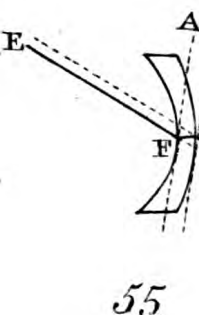
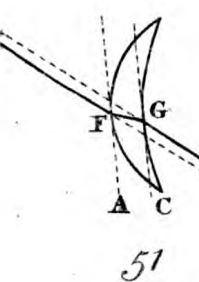
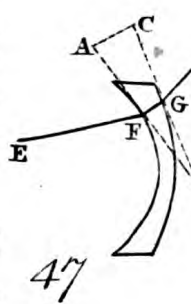
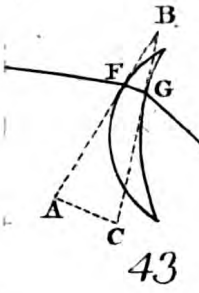
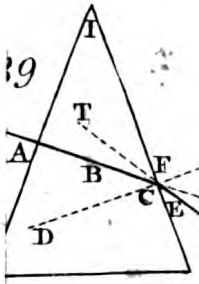
^b Euc. III. 15. & I. 25.

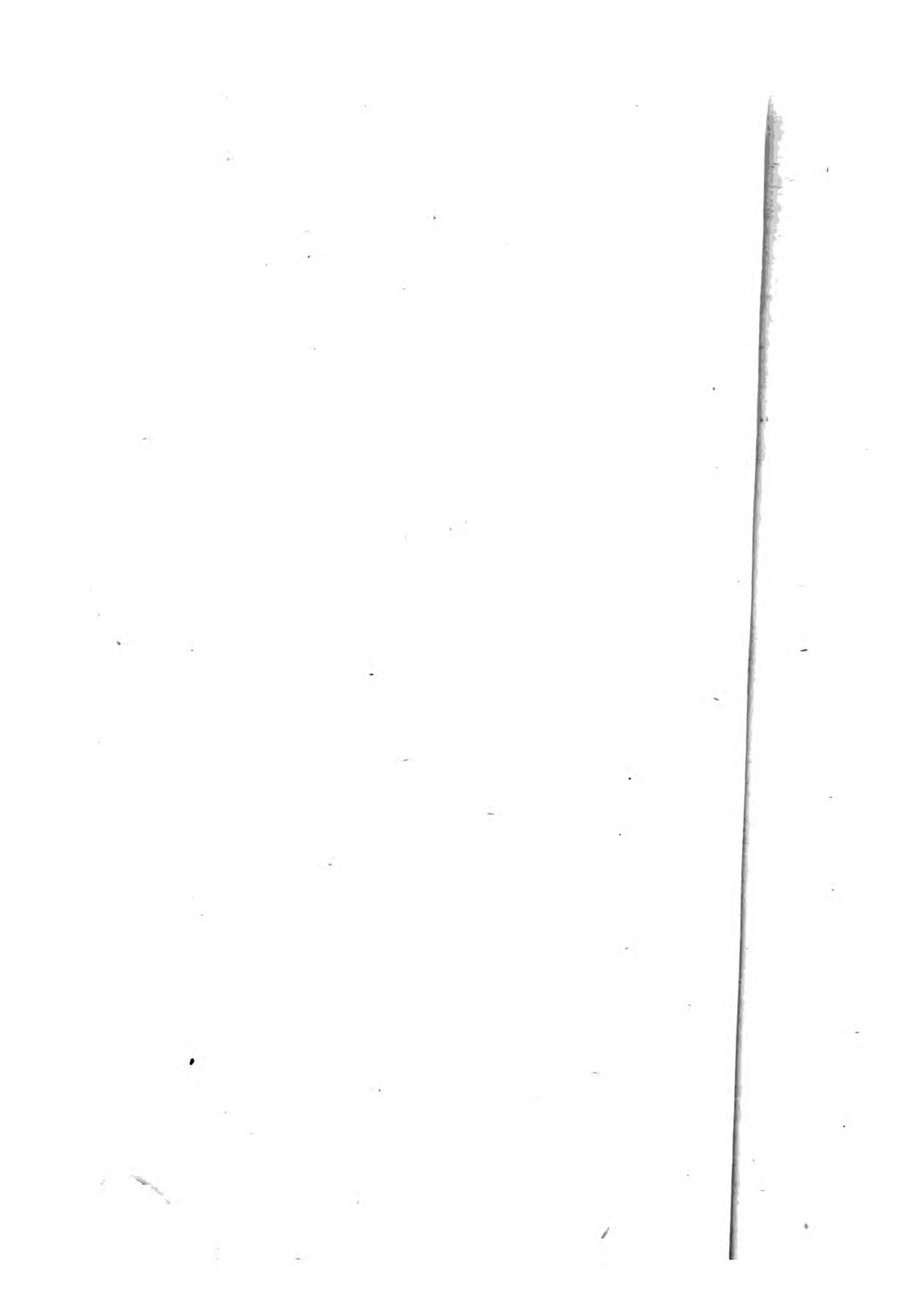
^c Euc. I. 32.

^d Euc. I. 5.

^e Art. 13.

^f Art. 16.





cal line: from which it differs insensibly when the thickness of the glass is small, and when the pencil falls not too obliquely upon it. Because the parallel lines EF and GH produced, go closer together in proportion as the line FG is shorter, and as the bendings at F and G are smaller.

62. When two homogeneal rays are refracted through the same point of any lens, whose thickness is inconsiderable, the angle contained under their incident parts is equal to the angle under their emergent parts. For the thickness of the lens being very small, if the rays have a common point of incidence, their points of emergence will be very near one another; or if the point of emergence be common to both rays, their points of incidence will be very near one another; and still nearer if the rays cross one another within the lens; consequently the refractions through the lens will be nearly the same as through two planes, that touch its surfaces at two given points, near the points of incidence and emergence, and contain a given angle with one another^a.

Refractions of two homogeneal rays through the same point of any lens.

Fig. 57, 58.

^a Art. 56.

63. An heterogeneal ray, refracted through a given point in a lens, has the same property as in the prism; that is, the emergent rays of given colours are inclined to one another and to the incident ray in given angles in all positions; for the reasons mentioned in the last article and in article 58.

Refractions of an heterogeneal ray through the same point in any lens.

64. All rays, as $EF GH$ and $efgb$, which cross each other in a refracting globe, and pass through it at equal distances from its center, so as to touch a concentric globe, are equally bent. For in this case the chords FG , fg being equal, their obliquities to the surface of the globe are also equal, and consequently the bendings of the ray $EF GH$ at F and G , both severally and together, are equal to the bendings of the ray $efgb$ at f and g : as is evident by conceiving the rays to go both ways along the chords FG , fg . Therefore the angle made by the incident and emergent parts of one ray, produced till they meet, will be equal to the angle made by the incident and emergent parts of the other ray produced till they meet; which is what I mean when I say the rays are equally bent.

Rays are equally bent which pass at equal distances from the center of a globe.

Fig. 56. to 59.

65. All rays, as $EF GH$, $efgb$, which cross each other at any given point of a lens, or which pass through it at equal distances from its center, are equally bent, provided they do not fall very obliquely upon it. Imagine a line FG within the glass, at first to be equally inclined to its sides, and then to be turned a little about any point of it, till it comes into the position fg ; and while it becomes more and more oblique to one side of the glass, suppose Ff , it will become

And from the center of a lens.

Fig. 57. 58.

become less and less oblique to the other side Gg . Consequently if a ray be supposed to go both ways along this variable line fg it will be more and more bent in going through the side Ff , and less and less bent in going through the other side Gg ^a: so that the total bending of the ray consisting of both its bendings, or angles efg , fgb , taken together, will continue to be much the same in all its positions. The circulation of the line fg , about the given point, may be farther continued till the bending at g is diminished to nothing; and still farther till it be made the contrary way; (as was explained in the 54th article;) which still takes off from the perpetual increase of the greater bending at f and keeps the total invariable. To keep the same bending it is only necessary that the rays FG , fg should keep at equal distance from the axis of the lens as near as possible: and nothing alters the total bending but the alteration of that distance^b; because the inclination of the tangent planes, like the refracting angle of a prism, will then only be altered.

^a Art. 16.

^b Art. 54.

CHAP. V.

TO FIND THE FOCUS OF RAYS REFLECTED FROM ANY GIVEN SURFACE.

PROPOSITION I.

Fig. 61. 66. **L**ET ACB be a reflecting plane, and Q the focus of the incident rays, and QC a perpendicular to that plane; and if this perpendicular be produced to q , so that qC be equal to QC , the point q shall be the focus of the reflected rays.

For let QA be any incident ray; draw qA and produce it towards O , and CA towards D . Then because Cq is made equal to CQ , the triangles CAq , CAQ will be equal^a. And consequently the angle DAO , which is equal to the opposite angle CAq , is also equal to the angle CAQ . Therefore AO is the reflected ray^b. Q, E, D .

^a 8th Ax. Euc.

^b Art. 9.

67. *Corol.* Hence all the rays that flow towards q , will flow to Q after reflection^c.

^c Art. 11.

LEMMA.

68. *Quantities and their proportions, which so approach to a state of equality as to become equal at last, may be taken for equal in a state immediately*

diately preceding the last; and also in a state somewhat remote from the last without sensible error in physical subjects: and the same may be said of figures which continually approach to a state of similitude; especially if these errors, when computed, are found inconsiderable.

The meaning of the lemma will appear very plain when applied to the following propositions.

PROPOSITION II.

69. When parallel rays as DA, EC fall almost perpendicularly upon a spherical surface ACB , the focus, T , of the reflected rays will bisect that semidiameter EC , which is parallel to the incident rays. Fig. 62, 63.

For drawing EA , it will be perpendicular to the spherical surface at A , and since EC is in the same plane as the angle of incidence DAE , the reflected ray Aq (produced backwards in fig. 63.) will meet EC somewhere in q^a ; so that the angle of reflection EAq may ^a Art. 7. be equal ^b to the angle of incidence EAD , or to the alternate angle ^b Art. 8. AEq . The two sides Aq, Eq of the triangle AqE are therefore equal to each other ^c; and consequently each of them greater than ^c Euc. I. 6. half the third side EA , or than ET by construction. But as the point of incidence at A approaches towards C , the lines Eq, ET , continually approach towards equality, and become equal when the triangle AEq is vanishing: and so the focus of rays falling almost perpendicularly on the surface, or the nearest to the point C , is to be reckoned at T^d . $Q. E. D.$ ^d Art. 68.

70. *Corol.* Hence if T be the focus of the incident rays, the reflected ones will go parallel to the line TE^e . ^e Art. 11.

The point T is called the principal focus of the reflecter ACB , and TE its focal distance. And in general, when the rays come parallel to each other, that point to which they converge or from which they diverge after reflection or refraction, is called the *principal focus* of the reflecter or refracter. And the distance of that point from the center of the reflecter or refracter is called the *focal distance* of the reflecter or refracter, and by some authors its *focal length*; in figures 89, 90, 95, &c. to 100, F is the principal focus, and FE the focal distance.

PROPOSITION III.

71. Let ACB be a reflecting surface of any sphere whose center is E . Fig. 64, 65. Bisect any radius thereof, suppose EC , in T ; and if in this radius, on the same side of the point T , you take the points Q and q , so that TQ, TE and Tq be continual proportionals; and the point Q be the focus of the incident rays, the point q shall be the focus of the reflected ones.

E

Let

Let $\mathcal{Q}A$ and Aq be an incident and reflected ray (produced) making equal angles with the perpendicular AE ; and the reflected ray Aq (produced) will cut $\mathcal{Q}E$ (produced) somewhere in q , as being in the plane of incidence $EA\mathcal{Q}$ ^a. Draw EG parallel to Aq , and let it meet $A\mathcal{Q}$ in G ; and also Eg parallel to $A\mathcal{Q}$, meeting Aq in g : then because the angles EAG, EAq are equal^b, it follows that the triangles EAG, EAq are equiangular at their common base AE ^c, and therefore equicrural^d; and also equal to each other; and consequently each side of the equilateral figure $AGEg$, in its vanishing state when A comes to C , will be equal to half its diagonal AE ^e, or by construction to ET . Now because the triangles $G\mathcal{Q}E, gEq$ are equiangular^f, it will be as $G\mathcal{Q}$ to GE , so gE to gq ^g; that is, when the point A is coinciding with C , and consequently the points G, g with T , as $T\mathcal{Q}$ to TE so TE to Tq ^h. $\mathcal{Q}.E.D.$

72. *Corol. 1.* If q be the focus of incident rays, \mathcal{Q} will be the focus of the reflected onesⁱ. And if either of these focuses recedes from T , the other will approach towards it. For the middle term TE of the continual proportionals $T\mathcal{Q}, TE, Tq$ ^k, being invariable in the same reflecter, the rectangle under the extremes is also invariable^l; and therefore $T\mathcal{Q}$ varies as Tq inversely. These focuses will meet each other at the center E , and at the surface C . For at these points, the three terms in the proportion are equal^m.

73. *Corol. 2.* The rays that belong to \mathcal{Q} may be reckoned parallel when the distance $T\mathcal{Q}$ is infinite, and then by this proposition its reciprocal Tq becomes nothing; which is the second proposition.

Fig. 65. 74. *Corol. 3.* Hence also we may deduce the first proposition; for supposing \mathcal{Q} the focus of incident rays upon the convex surface AB ; since $T\mathcal{Q}, TC, Tq$ are continual proportionals, it is well known that their differences $C\mathcal{Q}, Cq$ must become equal when the lines themselves are infinitely great; that is when the surface becomes a plane by removing its center to an infinite distance.

The figures serve for the cases of a convex surface supposing the incident rays to go backwards in the same lines produced through the surface.

75. By the demonstrations of the two last propositions, it appears that the focus of reflected rays there determined, is nothing else in strictness of geometry, but the intersection of the axis of the surface, that is of the ray passing through its center, and of the nearest rays to it: and also that other rays intersect the axis in different points farther and farther from that focus, as they fall farther and farther from the vertex of the surface. So that a spherical surface cannot possibly reflect all the incident rays to a single point.

point *. Nevertheless when these aberrations of the remoter rays from the geometrical focus shall be considered, it will appear hereafter, that the density of their interfections, near that focus, is immensely greater than their density at any considerable distance from it. So that in physical things, the focus of all the rays, that fall almost perpendicular upon a spherical surface, may be considered as a physical point. And the same is to be understood of the focus of refracted rays, as will appear by the like sort of demonstrations ^a. ^a Chap. XI.

76. Hence it appears that the focus of rays reflected from any curved surface whatever, must be reckoned the same as if they were reflected from a spherical surface of an equal curvity to that surface about the points of incidence. As if CD be any curve whatever, C Fig. 66. the point of incidence, $CE \perp$ perpendicular to the curve, or to its tangent at C , CE the radius of a circle ACB of the same degree of curvity at C ; the rays coming parallel to CE , will be reflected to the same focus T from either of the surfaces; and also the rays that flow from any point Q , will be reflected by either surface to the same focus q . Because we consider the focus of those rays only, that fall

* All the incident rays belonging to the same pencil may be reflected to a single point by means of surfaces, which are generated by the revolution of some conic section round its transverse axis. For

1st, Let AC represent a parabola, the axis of which is CQ and focus T ; describe a Fig. 67. paraboloid ACB by turning AC round its axis; draw EF touching its surface in any point A ; draw also the diameter dAD , join AT and produce it to t . Now the angles DAF , TAE are equal ^a, and therefore the angle dAE equals tAF . Since then the in- ^a Hamilton's cident and reflected rays are equally inclined to the reflecting plane ^b, if DA and dA be Conics, II. 15. incident rays, AT and At will be their respective reflected rays; or if TA , tA be inci- ^b Art. 9. dent rays, AD , Ad will be their respective reflected rays.

Hence, all the rays which are incident on the concave side of this paraboloid parallel to its axis will be reflected converging to the focus: and all the rays, which diverge from the focus will be reflected parallel to the axis. The convex surface will make parallel rays diverge from a single point, and converging rays go parallel, after reflection.

2dly, Let ACQ represent an ellipse, and ACB an hyperbola, the transverse axis of Fig. 68, 69. which is QC , and T, D , the foci; let a spheroid be generated by turning the ellipse QAC round QC , and an hyperboloid by the motion of AC round its transverse axis; draw EF touching the surface in any point A ; join TA, DA , and produce them to t and d . Since the angles, which TAt, DAd , make with FE are equal ^c, and since the ^c Hamilton's incident and reflected rays are equally inclined to the reflecting plane ^b, it is evident that, Conics, II. 16. if TA or tA be an incident ray, Ad or AD will be the direction in which they are re- & 17. flected.

Hence all the rays, which are incident diverging from one focus of the spheroid, will be reflected converging to the other focus; and all the rays, which are incident upon the convex side of the spheroid converging to one focus, will be reflected diverging from the other. In like manner, the convex side of the hyperboloid will make a pencil of diverging rays diverge from, and a pencil of converging rays converge to, a single point after reflection. It will make diverging rays diverge more, and converging rays converge less. And the concave side of this reflector increases the convergency, and lessens the divergency, of a pencil of rays.

upon the common points of both curves about C , all the rest being dispersed much thinner into other places.

77. In all these propositions when the focuses \mathcal{Q} , q lye on the same side of the reflecting surface, if the incident rays flow from \mathcal{Q} the reflected ones will flow towards q ; and if the incident rays flow towards \mathcal{Q} , the reflected ones will flow from q ; and the contrary happens when \mathcal{Q} and q are on contrary sides of the surface. Because the incident and reflected rays go contrary ways.

CHAP. VI.

TO DETERMINE THE PLACE, MAGNITUDE AND SITUATION OF IMAGES FORMED BY REFLECTED RAYS.

PROPOSITION I.

78. **I**MAGES formed by reflections from a plane surface are similar and equal to the objects; and their parts have the same situation with respect to the backside of the plane as the parts of the object have with respect to its fore-side.

Fig. 70, 71.

* Art. 66.

From any number of points P , \mathcal{Q} , R of an object in any situation, draw the perpendiculars PA , $\mathcal{Q}C$, RB to the plane ACB , and produce them through it to the points p , q , r , each as far behind the plane as P , \mathcal{Q} , R are before it. The points p , q , r being the respective focuses of the rays that belonged to P , \mathcal{Q} , R , and being evidently in the same order, together with infinite others, will constitute an image of the object, equal to it in the whole and in every corresponding part, and alike situated: as will appear by conceiving the surface of the object, and of its image, divided into corresponding lines, such as $P\mathcal{Q}R$, pqr , by planes such as $PprR$ perpendicular to the reflecting plane; and by folding up or doubling each plane in its line of intersection, AB , with the reflecting plane. For the parts of each plane on each side of AB will exactly cover each other, as appears by the construction. \mathcal{Q} , E , D .

PROPOSITION II.

Fig. 72. to 75. 79. If an arch of a circle PQR , concentrick to a concave or convex spherical surface AB , be considered as an object, its image pqr will also be a similar concentrick arch, whose length will be to the length of the object, in the ratio of their distances from the common center E ; and its situation will be erect or inverted, according as it is on the same or the opposite side of the center to the object.

For

For as the focus \mathcal{Q} was found by making $T\mathcal{Q}$, TE , Tq continual proportionals in the line $\mathcal{Q}E$ drawn through the center^a; so the focus p , of rays that belong to any other point P , is found by drawing PEA , and bisecting EA in S , and by making SP , SE , Sp continual proportionals. The two first terms of one proportion are severally equal to the two first of the other; and consequently the third terms Tq , Sp are equal; and thence Ep and Eq are equal. The same being true of every point of the circular object $P\mathcal{Q}R$, shews that its image pqr is a concentrick arch, similar to it, both being terminated by the same lines EPp , ERr ; and consequently their lengths are in the same ratio as their semidiameters $E\mathcal{Q}$, Eq . Lastly, according as the corresponding extremities P and p , of the object and image, are on the same or opposite sides of the center E , they are also on the same or opposite sides of their middle points \mathcal{Q} , q ; that is, the image is accordingly erect or inverted. $\mathcal{Q}.E.D.$

80. *Corol.* The smaller the circular object is with respect to its radius or distance from the center, the nearer it approaches in shape to a straight line, and so does its similar image. Consequently a small straight object, placed at a good distance from the center of the glass, may be reckoned to have a straight image very nearly: though in strictness of geometry it is an arch of a conick section.

81. The images of all sorts of objects may be determined, by finding the images of their out-lines, by the foregoing propositions. For instance, if the plane of the figures PER , pEr be turned round their common diameter $\mathcal{Q}Eq$, the circular surface generated by pqr will be the image of the circular object generated by $P\mathcal{Q}R$: and if the same figures PER , pEr be moved a little about an axis EF , situated in their own plane, and perpendicular to the diameter $\mathcal{Q}Eq$, the curvilinear figure generated by this motion of pqr , will be the image of a similar figure generated by $P\mathcal{Q}R$. Because the reflecting arch ACB generates the reflecting spherical surface at the same time.

82. But if the whole figure $PERrp$ be moved parallel to itself in a direction EF , now perpendicular to its own plane, so that the arch ACB may generate a portion of a cylindrical surface, the figure described by this motion of pqr , will still be the image of that described by $P\mathcal{Q}R$; but will not be similar to it, except when they are placed at equal distances on each side the center E , and consequently are equal to each other: and their dissimilitude will be so much the greater as the disproportion between Eq and $E\mathcal{Q}$, or between their lengths pr , PR , is greater; their breadths, described by the motion aforesaid, being always equal to each other.

CHAP. VII.

TO FIND THE FOCUS OF RAYS FALLING ALMOST PERPENDICULARLY UPON ANY REFRACTING SURFACE, SPHERE OR LENS.

DEFINITION.

Fig. 76, 77. 83. **T**HE sine of an angle ABC , or of an arch AC that measures that angle, is a line AD drawn from the extremity of one of the femidiameters, AB , BC , perpendicular to the other, produced if the angle be obtuse. And therefore an angle ABC and its complement ABE , to two right angles, have each the same sine AD ; and when the sines of several angles are compared together, they are always understood to belong to the same or to equal circles.

84. The sines of very small angles, and of their complements, become at last insensibly different from the arches that measure them; and consequently are proportionable to the angles themselves.

LEMMA.

Fig. 78. 85. *The sines of the angles of any triangle are proportionable to the opposite sides: as in the triangle ABC , the sine of the angle ABC is to the sine of the angle BCA , as CA to AB .*

For the perpendiculars CD , BE upon those sides AB , AC produced, are the sines of the angles ABC , BCA or BCE with respect to circles whose radius is BC^a . And since the triangles CAD , BAE are equiangular^b, we have CD to BE as CA to AB . *Q. E. D.*

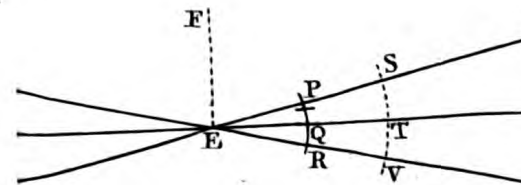
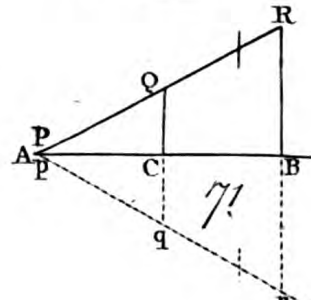
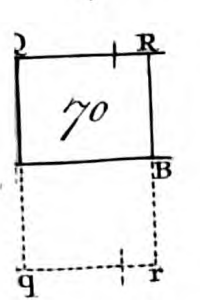
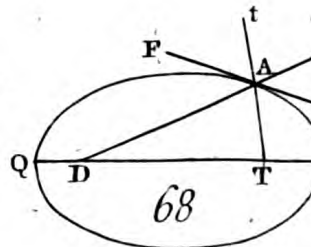
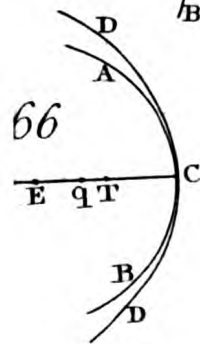
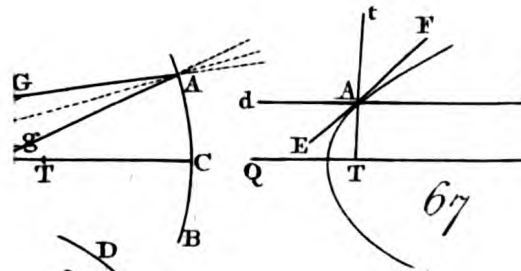
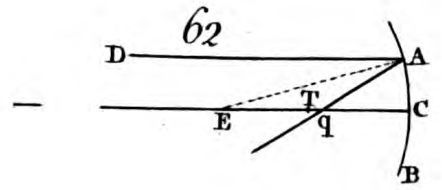
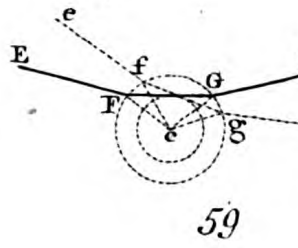
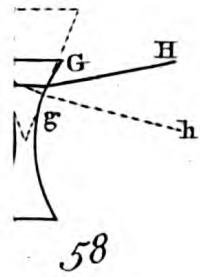
86. *Corol.* Small angles, as BAC , BCE , subtended by the same perpendicular BE , are reciprocally as their legs BA , BC or EA , EC . For the angle BAC is to BCE , when very small, as the sine of BAC to the sine of BCE ^c, or as BC to BA ^d, or as EC to EA ^e.

^a Art. 83.
^b Euc. I. 32.
Cor. 2.
^c Art. 84.
^d Art. 85.
^e Art. 68.

PROPOSITION I.

Fig. 79. to 82. 87. *Let ACB be a refracting plane, and Q the focus of the incident rays, and QC a perpendicular to that plane; and if qC be taken in this perpendicular, on the same side of the plane as QC , and in proportion to QC , as the sine of incidence to the sine of refraction, the point q shall be the focus of the refracted rays.*

For





Let the lines QA and Aq , produced as in the figures, represent an incident and a refracted ray, cutting QC in any point q ever; and since a perpendicular to the plane at A is parallel to the angle AQC will be equal to the angle of incidence, and to the angle of refraction. Therefore since equal angles have equal sines, the sine of incidence is to the sine of refraction (as the sine of the angle AQC to the sine of AqC , or as Aq to AQ^a , that is when the ray QA is almost perpendicular to the plane AB) as QC to Qq^b . *Q. E. D.*

^a Art. 85.

^b Art. 68.

89. *Cor. 1.* If the surface ACB is glass, qQ is half of QC , and a third of qC . For qC is to QC as 3 to 2^c; and therefore qQ is to QC as 1 to 2^d. In like manner, if ACB is water, qQ is a third of QC , and a fourth of qC .

^c Art. 13.

^d Euc. V. Def. 16.

90. *Cor. 2.* Hence the refractions of a pencil of rays through a prism may be determined. Let Q be the focus of incident rays upon its first side AB , and QC perpendicular to AB . To QC take QT , equal to half QC ; and T will be the focus of the rays QA , QB , &c. after refraction at the surface AB^c ; and being also the focus of incident rays at a and b upon the second surface ab , from T drawn perpendicular to ab , take away Tq equal to a third part of Tc^c ; and q will be the focus of the emergent rays qa , qb &c. produced.

Fig. 83.

^c Art. 88.

90. *Cor. 3.* Hence the focuses of incident and emergent rays at a prism always very nearly at equal distances from it; provided the refractions and the refracting angle be but small. For then the perpendiculars TC , Tc are nearly equal; and in glass QC and qC are two thirds of them respectively.

91. *Cor. 4.* By proceeding in the same manner as in the 2d *Cor.* it is found that, when the planes AB , ab are parallel, and TC , Tc coincide, Qq is one third of Cc , the thickness of the glass *.

Fig. 83, 84.

* But the distance Cc , between the parallel planes, and the ratio between the sines of incidence and refraction at the first surface, I and R , being given, the distance Qq may be found more readily in all cases by means of the following proposition.

When a pencil of converging or diverging rays passes through two parallel planes, the distance between the planes is to the distance between the foci of incident and emergent rays, as the sine of incidence on the first surface is to the difference between the sines of incidence and refraction.

Let AB , ab , be the refracting planes; Q the focus of incident rays; QCc a perpendicular to the planes; T the point in this perpendicular where the directions of the refracted rays Aa , Bb , meet it; q the focus of emergent rays. And QA being parallel to the base of the triangle Tqa^a ,

Fig. 84.

^a Art. 51.

^b Euc. VI. 2.

And by alt. and inver.

$$TQ : Qq :: TA : Aa^b, \text{ that is, as } TC : Cc^b$$

But

$$TC : TQ :: Cc : Qq$$

And by conver.

$$TC : QC :: I : R^c$$

^c Art. 87.

Therefore

$$Cc : Qq :: I : I - R. \quad Q. E. D.$$

PROPOSITION II.

Fig. 85. to 88. 92. Let ACB represent a refracting spherical surface whose center is E , and let the incident rays as DA come parallel to any semidiameter CE , in which produced forward or backward, according as the denser medium is convex or concave, take CT to CE as the sine of incidence to the difference of the sines, and T will be the focus of the refracted rays.

For let the refracted ray AT (produced) cut the semidiameter CE produced, in any point T whatever; and since the semidiameter EA is perpendicular to the refracting surface at A , the angle of incidence will be equal to the angle AEC , and the angle of refraction, or its complement to two right ones, will be EAT ; consequently the sine of incidence is to the sine of refraction, (as the sine of the angle AEC , to the sine of EAT , or as AT to TE ^a, that is, when A comes nearest to C , and so the incident rays are almost perpendicular to the surface,) as CT to TE ^b; and disjointly^c the sine of incidence is to the difference of the sines as CT to CE . *Q. E. D.*

^a Art. 85.

^b Art. 68.

^c Euc. V. 17.

93. *Corol. 1.* CT is to TE as the sine of incidence to the sine of refraction.

94. *Corol. 2.* If this point T be the focus of incident rays, the refracted rays will go parallel to TE ^d.

^d Art. 11.

PROPOSITION III.

Fig. 89, 90. 95. When parallel rays fall upon a sphere, either denser or rarer than the ambient medium, in the diameter CD produced, which is parallel to the incident rays, as QA , let T be their focus after their first refraction at AC ; and the point F which bisects TD shall be their focus after their second refraction at DG .

For let the incident and emergent rays, QA , FG produced, meet in H , and since the refractions at A and G are equal, as appears by supposing a ray to go both ways along the chord AG , the triangle AHG is equiangular at its base AG , and therefore equicrural^e; and so is the similar triangle GFT , the lines AH , FT being parallel. Therefore when A approaches toward C till G is coinciding with D and the triangle GFT is vanishing, the leg GF will become equal to half the base GT ; that is, DF will become equal to half DT ^f. *Q. E. D.*

^e Euc. I. 6.

^f Art. 68.

LEMMA.

Fig. 91. to 94. 96. There is a certain point E within every double convex or double concave lens, through which every ray that passes, will have its incident and emergent

emergent parts QA, aq parallel to each other: but in a plano-convex or plano-concave lens that point E is removed to the vertex of the concave or convex surface; and in a meniscus and in that other concavo-convex lens, it is removed a little way out of them, and lyes next to the surface which has the greatest curvity.

For let REr be the axis of the lens joining the centers R, r of its surfaces A, a . Draw any two of their femidiameters RA, ra parallel to each other, and join the points A, a , and the line Aa will cut the axis in the point E above described. For the triangles REA, rEa being equiangular, RE will be to Er in the given ratio of the femidiameters RA, ra ; and consequently the point E is invariable in the same lens. Now supposing a ray to pass both ways along the line Aa , it being equally inclined to the perpendiculars to the surfaces, will be equally bent and contrary ways in going out of the lens; so that its emergent parts AQ, aq will be parallel. Now any of these lenses will become plano-convex or plano-concave, by conceiving one of the femidiameters RA, ra to become infinite, and consequently to become parallel to the axis of the lens, and then the other femidiameter will coincide with the axis; and so the points A, E or a, E will coincide. $Q. E. D.$

97. *Corol.* Hence when a pencil of rays falls almost perpendicularly upon any lens, whose thickness is inconsiderable, the course of the ray which passes through E , above described, may be taken for a straight line passing through the center of the lens, without sensible error in sensible things. For it is manifest from the length of Aa and from the quantity of the refractions at its extremities, that the perpendicular distance of AQ, aq when produced, will be diminished both as the thickness of the lens and the obliquity of the ray is diminished.

DEFINITION. The point E is called the center of the lens.

PROPOSITION IV.

98. *To find the focus of parallel rays falling almost perpendicularly upon any given lens.* Fig. 95. to 100.

Let E be the center of the lens, R and r the centers of its surfaces, Rr its axis, gEG a line parallel to the incident rays upon the surface B , whose center is R . Parallel to gE draw a femidiameter BR , in which produced let V be the focus of the rays after their first refraction at the surface B , and joining Vr let it cut gE produced in G , and G will be the focus of the rays that emerge from the lens.

For since V is also the focus of the rays incident upon the second

F

surface

surface A , the emergent rays must have their focus in some point of that ray which passes straight through this surface; that is in the line Vr , drawn through its center r : and since the whole course of another ray is reckoned a straight line gEG^a , its intersection G with Vr determines the focus of them all. *Q. E. D.*

^a Art. 97.

99. *Corol. 1.* When the incident rays are parallel to the axis rR , the focal distance EF is equal to EG . For let the incident rays that were parallel to gE be gradually more inclined to the axis till they become parallel to it; and their first and second focuses V and G will describe circular arches VT and GF whose centers are R and E . For the line RV is invariable; being in proportion to RB in a given ratio of the lesser of the sines of incidence and refraction to their difference^b; consequently the line EG is also invariable, being in proportion to the given line RV in the given ratio of rE to rR , because the triangles EGr , RVr are equiangular.

^b Art. 92.

100. *Corol. 2.* The last proportion gives the following rule for finding the focal distance of any thin lens. As Rr , the interval between the centers of the surfaces, is to rE , the semidiameter of the second surface, so is RV or RT , the continuation of the first semidiameter to the first focus, to EG or EF , the focal distance of the lens. Which according as the lens is thicker or thinner in the middle than at its edges, must lye on the same side as the emergent rays or the opposite side.

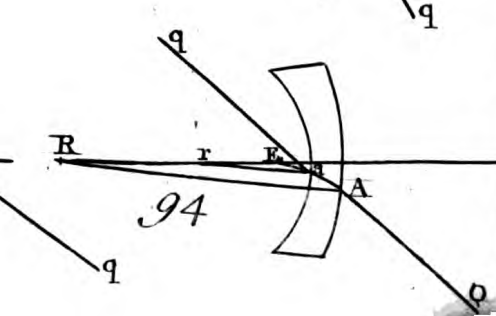
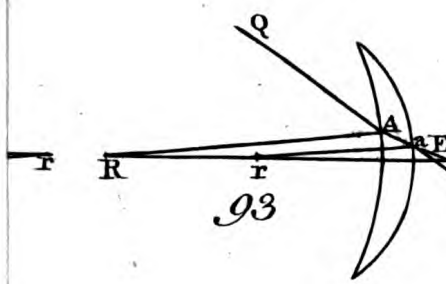
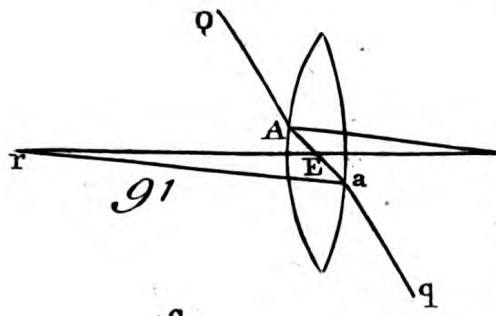
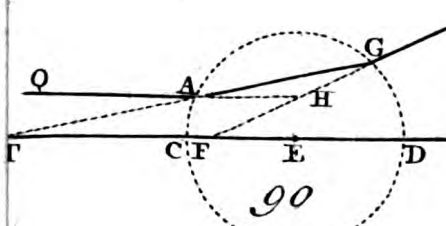
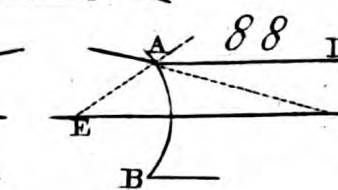
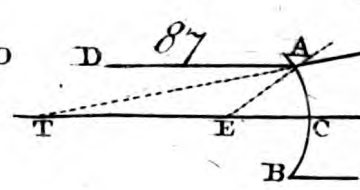
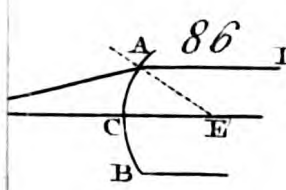
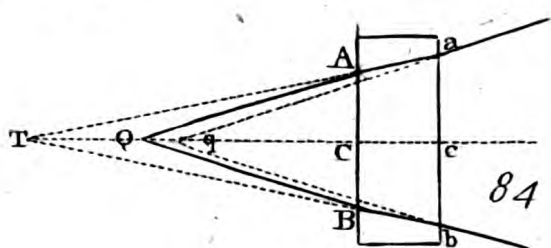
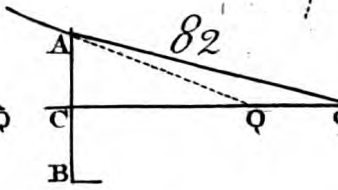
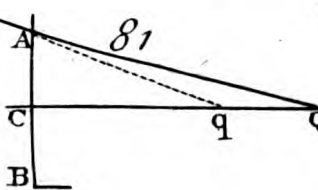
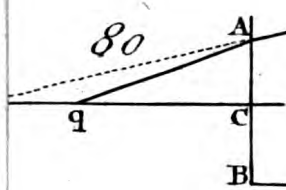
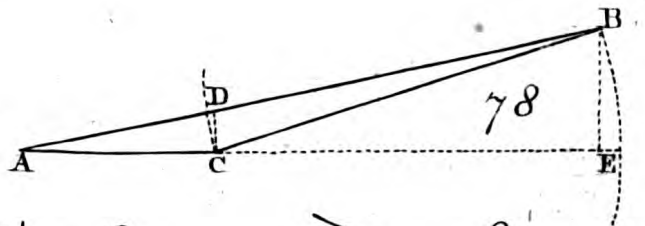
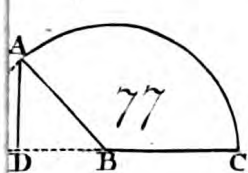
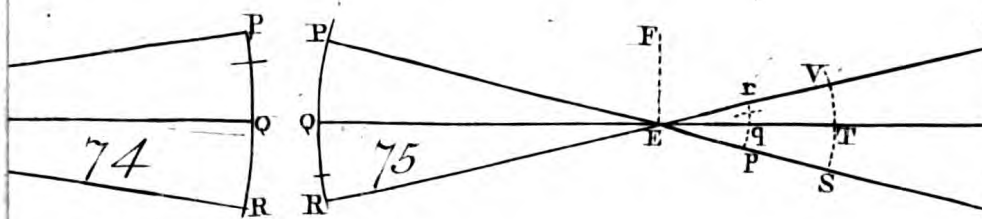
101. *Corol. 3.* Hence when rays fall parallel on both sides of any lens, the focal distances EF , Ef are equal. For let rt be the continuation of the semidiameter Er to the first focus t of rays falling parallel upon the surface A ; and the same rule that gave rR to rE as RT to EF , gives also rR to RE as rt to Ef . Whence Ef and EF are equal, because the rectangles under rE , RT and also under RE , rt are equal. For rE is to rt and also RE to RT in the same given ratio^c.

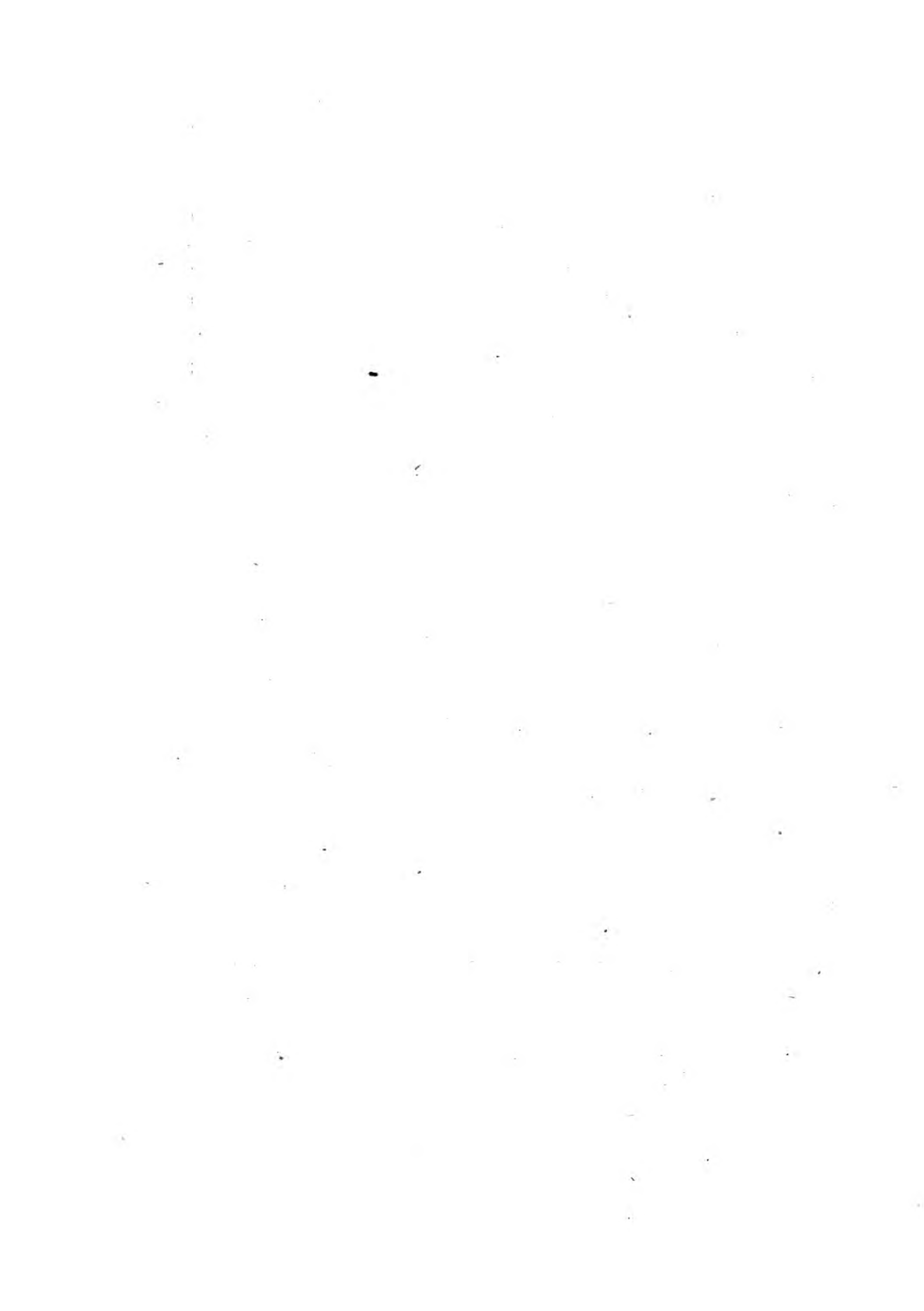
^c Art. 92.

102. *Corol. 4.* Hence in particular in a double-convex or double-concave lens made of glass, it is as the sum of their semidiameters (or in a meniscus as their difference) to either of them, so is double the other, to the focal distance of the glass. For the continuations RT , rt are severally double their semidiameters: because in glass

^d Art. 93. 13. ET is to TR and also Et to tr as 3 to 2^d.

103. *Corol. 5.* Hence if the semidiameters of the surfaces of the glass be equal, its focal distance is equal to one of them; and is equal to the focal distance of a plano-convex or plano-concave glass whose semidiameter is as short again. For considering the plane surface as having an infinite semidiameter, the first ratio of the last mentioned proportion may be reckoned a ratio of equality.





PROPOSITION V.

104. *The focus of incident rays upon a single surface, sphere or lens being given, it is required to find the focus of the emergent rays.* Fig. 101. to 106.

Let any point \mathcal{Q} be the focus of incident rays upon a spherical surface, lens, or sphere, whose center is E ; and let other rays come parallel to the line $\mathcal{Q}Eg$ the contrary way to the given rays, and after refraction let them belong to a focus F ; then taking Ef equal to EF in the lens or sphere, but equal to CF in the single surface, say as $\mathcal{Q}F$ to FE so Ef to fq , and placing fq the contrary way from f to that of $F\mathcal{Q}$ from F , the point q will be the focus of the refracted rays, without sensible error; provided the point \mathcal{Q} be not so remote from the axis, nor the surfaces so broad as to cause any of the rays to fall too obliquely upon them.

For with the center E and semidiameters EF and Ef describe two arches FG, fg cutting any ray $\mathcal{Q}Aaq$ in G and g , and draw EG and Eg . Then supposing G to be a focus of incident rays, (as GA ,) the emergent rays (as agq) will be parallel to GE ^a; and on the other hand supposing g another focus of incident rays (as ga ,) the emergent rays (as $AG\mathcal{Q}$,) will be parallel to gE . Therefore the triangles $\mathcal{Q}GE, Egq$ are equiangular, and consequently $\mathcal{Q}G$ is to GE as Eg to gq ; that is, when the ray $\mathcal{Q}Aaq$ is the nearest to $\mathcal{Q}Eq$, $\mathcal{Q}F$ is to FE as Ef to fq ^b. Now when \mathcal{Q} accedes to F and coincides with it, the emergent rays become parallel, that is q recedes to an infinite distance; and consequently when \mathcal{Q} passes to the other side of F , the focus q will also pass through an infinite space from one side of f to the other side of it. $\mathcal{Q}. E. D.$ Art. 94. 99. 101.

105. *Corol. 1.* In refractions at a spherical surface AC , the focus q may also be found by this rule, as $\mathcal{Q}F$ to FC so Cf to fq ; because FC and Ef and also FE and Cf are equal^c.

106. *Corol. 2.* It may also be found by this rule, as $\mathcal{Q}F$ to $\mathcal{Q}E$ so $\mathcal{Q}C$ to $\mathcal{Q}q$; placing $\mathcal{Q}q$ so that all the four distances from \mathcal{Q} may lye on one side of it, or else two on each. For the triangles $\mathcal{Q}GE, \mathcal{Q}Aq$ being equiangular we have $\mathcal{Q}G$ to $\mathcal{Q}E$ as $\mathcal{Q}A$ to $\mathcal{Q}q$. Art. 93.

107. *Corol. 3.* In a sphere or lens the focus q may be found by this rule, as $\mathcal{Q}F$ to $\mathcal{Q}E$ so $\mathcal{Q}E$ to $\mathcal{Q}q$, to be placed the same way from \mathcal{Q} as $\mathcal{Q}F$ lyes from \mathcal{Q} . For let the incident and emergent ray $\mathcal{Q}A, qa$ be produced till they meet in e ; and the triangles $\mathcal{Q}GE, \mathcal{Q}eq$ being equiangular, we have $\mathcal{Q}G$ to $\mathcal{Q}E$ as $\mathcal{Q}e$ to $\mathcal{Q}q$; and when the angles of these triangles are vanishing, the point e will coincide with E ; because in the sphere the triangle Aea is equiangular at the base Aa , and consequently Ae and ae will at last be-

come semidiameters of the sphere. In a lens the thickness Aa is inconsiderable.

108. *Corol. 4.* In all cases the distance $f q$ varies reciprocally as $F Q$ does; and they lye contrary ways from f and F ; because the rectangle or the square under EF and Ef , the middle terms in the foregoing proportions, is invariable. Hence if either of the corresponding focuses Q, q be put in motion along the axis of the pencil, the other focus will move the same way: and therefore if these focuses be on contrary sides of the glass, while one moves towards it the other will move from it; but if they be both on the same side of the glass, they will both move from it or both towards it; and will come nearer to each other as they come nearer to the glass, till when one coincides with its surface the other will do so too, in the single surface accurately ^a, and in the lens very nearly, provided the glass be very thin, and the distance of the ray from its axis be very small ^b. But these focuses cannot coincide at the surface of a globe; for QF and QE being finite, Qq cannot vanish ^c. They will coincide at the center of the single surface, because the rays fall perpendicularly, and therefore pass through without suffering any refraction ^d.

^a Art. 105.

^b Art. 104.

^c Art. 107.

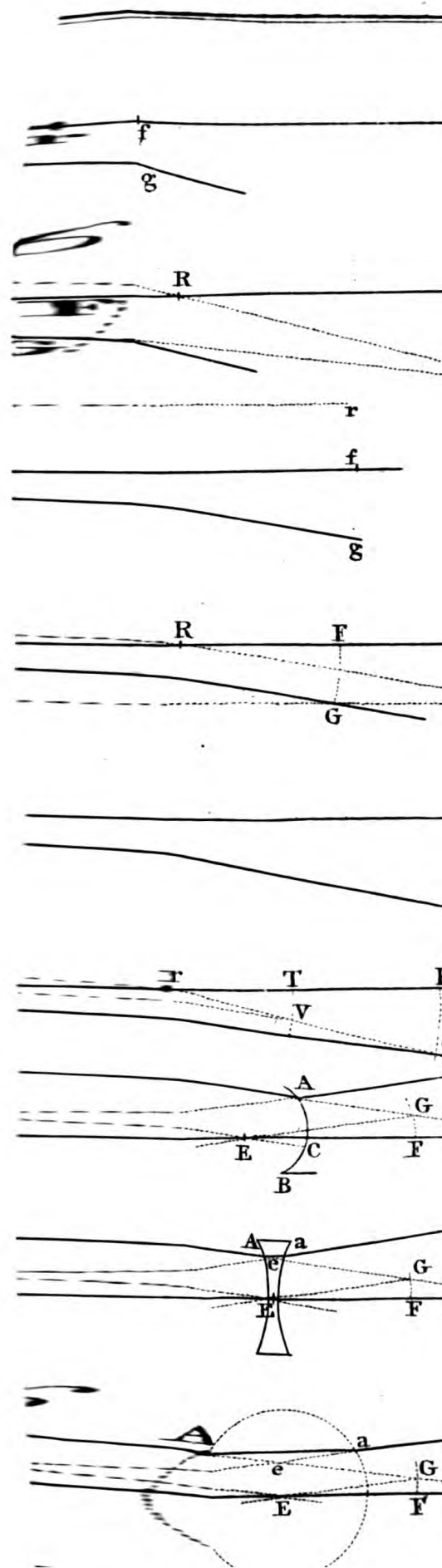
^d Art. 15.

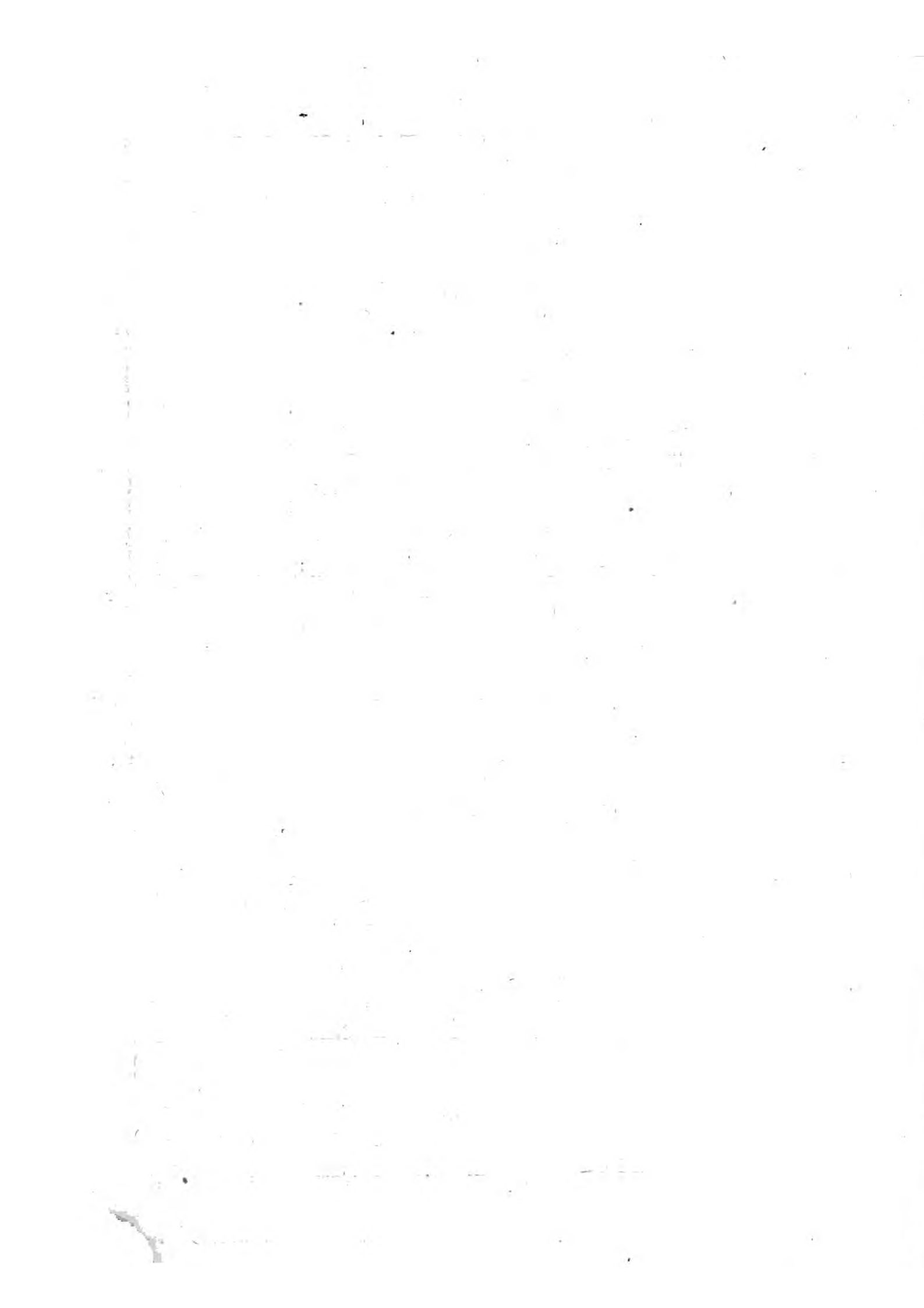
109. *Corol. 5.* Convex lenses of different shapes that have equal focal distances, when put into each others places, have equal powers upon any pencil of rays to refract them to the same focus. Because the rules abovementioned depend only upon the focal distance of the lens, and not upon the proportion of the semidiameters of its surfaces.

110. *Corol. 6.* The rule that was given for a sphere of an uniform density, will serve also for finding the focus of a pencil of rays refracted through any number of concentrick surfaces, which intercede uniform mediums of any different densities. For when rays come parallel to any line drawn through the common center of these mediums, and are refracted through them all, the distance of their focus from that center is invariable, as in an uniform sphere.

111. *Corol. 7.* When the focuses Q, q lye on the same side of the refracting surfaces, if the incident rays flow from Q , the refracted rays will also flow from q ; and if the incident rays flow towards Q , the refracted will also flow towards q : and the contrary will happen when Q and q are on contrary sides of the refracting surfaces. Because the rays are continually going forwards.

The 75th and 76th articles are applicable to refractions as well as reflections.





After Des Cartes had discovered the true law of refraction^a, it^a Art. 13. recently suggested to him the impossibility of collecting all the rays of the same pencil into a single point by means of spherical surfaces only. And not being aware that the aberration of rays from the geometrical foci of the optical glasses then in use was owing to any other cause than the *sphericalness* of their figures, he immediately applied himself to investigate what curves could make all the rays belonging to the same pencil converge after refraction to a single point. The seeming importance of this problem induced other mathematicians after him to prosecute the solution of it: and among other surfaces, it was found that those, which are generated by the revolution of *conic sections* round their axes, would produce the effect. The limits of an abstract will not permit us to relate all their discoveries upon this subject; in the following problems and their corollaries are contained the principal discoveries of Des Cartes and his demonstrations of the more difficult cases. *

“ P R O B L E M I.

“ *To collect every ray belonging to a pencil of parallel rays into one point, when they are incident upon the refracting surface of a denser medium.*

“ Take DK to HI as the sine of incidence to the sine of refraction; with the axis major DK and foci H, I , describe an ellipse DBK ; describe also a spheroid by the revolution of this ellipse round its transverse axis: and all the rays, which are incident upon the convex surface DBb , parallel to DK , will converge to the farther focus I . Fig. 107.

“ P R O B L E M II.

“ *To collect every ray belonging to a pencil of parallel rays into one point, when they are incident upon the refracting surface of a rarer medium.*

“ Take DK to HI as the sine of incidence to the sine of refraction; with the transverse axis DK and foci H, I , describe an hyperbola; describe also an hyperboloid by the revolution of this hyperbola round its transverse axis: and all the rays, which are incident upon the concave surface BDY , parallel to DK , will converge to the farther focus I . Fig. 108.

“ DE-

* Des Cartes's Dioptr. Ch. 8.

Fig 107, 108.

“ DEMONSTRATION. Let AB, MD be two parallel rays, of
 “ which MD is perpendicular to the surface: join HB, IB ; through
 “ B draw two straight lines CBE, LBG , which are at right angles
 “ to each other, and one of which CBE is a tangent to the curve
 “ at the point of incidence B : through H draw HO parallel to LG ,
 “ meeting IB in the hyperbola, or IB produced in the ellipse, in O ,
 “ and the tangent in C . And the angles OBC, CBH being
 “ equal^a, and HO cutting BC at right angles, the triangles OBC ,
 “ CBH are equal; and therefore OI is equal to the sum or differ-
 “ ence of HB and BI ; that is, OI equals DK ^b. Now LG
 “ being perpendicular to the surface, the angle ABN is either the
 “ angle of incidence or its complement to two right angles; and
 “ therefore the sine of the angle BNI , which is alternate to ABN ,
 “ is equal to the sine of incidence^c. And the sine of BNI is
 “ to the sine of NBI as BI to NI ^d; that is, as OI to HI ^e, or as
 “ DK to HI ; that is, as the sine of incidence to the sine of
 “ refraction^f. Wherefore the sine of NBI must be equal to
 “ the sine of refraction, and BI is the course of the refracted ray.
 “ Q. E. D.

^a Hamil.

Con. Sect.

B. II. p. 16, 17.

^b Hamil.

Con. Sect.

B. II. p. 14.

^c Art. 83.^d Art. 85.^e Euc. VI. 2.^f By construction.

Fig 109.

“ *Corol.* 1. If, therefore, with the center I and any radius IB , less
 “ than ID , a circular arch BQ be described, and the figure BQD
 “ revolve round its axis DQ , it will determine the shape of a piece
 “ of glass, which, placed in air, will collect into one point I all the
 “ rays that fall upon its convex surface parallel to DI . For the first
 “ surface BDB of this solid refracts them all to I ; and the radii of
 “ a circle being perpendicular to its circumference, the course of
 “ these rays will not be changed by the second surface BQB .
 “ Likewise, if from any point B in the hyperbola the right line BQ
 “ be drawn perpendicular to the axis, the figure BQD by revolving
 “ round DQ will generate a solid piece of glass, which, placed in
 “ air, will have the same effect upon all rays that are incident upon
 “ its plane surface parallel to the transverse axis.

Fig. 110.

“ *Corol.* 2. Each of the glasses described in the preceding corollary
 “ will make all the rays, which are incident diverging from the same
 “ point, move parallel after refraction. For, if I be made the focus
 “ of incident rays, the refracted rays will be parallel to ID by ar-
 “ ticle. 11.

Fig. 111.

“ *Corol.* 3. Let an ellipse be constructed as before; and draw the
 “ circular arch RO with the center I and any radius RI greater than
 “ DI ;

“ *DI*; assume also any point *B* in the ellipse, which is not farther
 “ from *D* than from *K*, draw the right line *OB* tending to *I*; and
 “ the glass, which is generated by the revolution of the figure
 “ *BDRO* about its axis *DR*, will make all rays, that are incident
 “ upon the concave side parallel to its axis, diverge from the point
 “ *I* after refraction. For the mediums continuing the same, it is
 “ evident that the ray *PB* is as much bent out of its course by the
 “ concave side of the refracter *DB*, as the ray *AB* is by the convex
 “ side of the same refracter^a. Since therefore *AB* and *PB* are in ^{a Art. 11. 17.}
 “ the same straight line, and *AB* moves in the line *BI* after refraction,
 “ *BO*, which is in the same direction with *BI*, must be the
 “ course of the ray *PB* after refraction.

“ Parallel rays may also be refracted diverging accurately from
 “ the same point by means of a glass constructed in the following
 “ manner. Describe an hyperbola *BDB* similar to that which is ^{Fig. 112.}
 “ described in the second problem; assume any point *O*, from which
 “ two straight lines can be drawn in such a manner that one of them
 “ *OB* shall be parallel to the axis and cut the curve in some point *B*,
 “ and that the other *OR* shall be perpendicular to the axis and not
 “ cut the curve. Let the figure *OBDR* be turned round its axis,
 “ and it will generate the solid required.

“ *Corol. 4.* Each of the glasses constructed in the preceding corollary
 “ will make all rays, that are incident converging to the same
 “ point, move parallel after refraction; for the reason given in
 “ corol. 2d.

“ *Corol. 5.* If two glasses be constructed, each of which is similar ^{Fig. 113.}
 “ to the elliptic refracter described in the first corollary, and be placed
 “ in air, so that their axes are in the same straight line, and that their
 “ convex surfaces are turned towards each other, all the rays, which
 “ are incident diverging from the same point *I*, will after refraction
 “ converge to the same point *i*.

“ Or two hyperbolic refracters, each of which is similar to the ^{Fig. 114.}
 “ glass described in the first corollary, and placed in such a manner
 “ that their axes are parallel, and that their plane surfaces are turned
 “ towards each other, will produce the same effect. And since the ^{Fig. 115.}
 “ refractions at the curved surfaces will be the same, when the plane
 “ surfaces coincide as when they are asunder, a single convex glass,
 “ which is formed by turning the hyperbolic arches *DB, db* round
 “ their axes, will collect into one point *i* all the rays which diverge
 “ from *I*.

“ *Corol.*

Fig. 116, 117. “*Corol. 6.* If it be required to make a pencil of rays, which are incident diverging from a single point *I*, diverge after refraction from a different point *i*, use two elliptic glasses, the figures and situations of which are represented in fig. 116, or the single hyperbolic refracter, which is represented in fig. 117. The same glasses will also make converging rays converge to a different point after refraction.

Fig. 118, 119. “*Corol. 7.* Lastly, a pencil of converging rays will be made to diverge accurately from a single point after refraction, either by means of two elliptic glasses, which are represented in fig. 118, or by the single hyperbolic glass represented in fig. 119.

“ This apparent superiority of conical surfaces induced not only the mechanicks but the most eminent mathematicians to contrive engines for grinding and polishing these surfaces; till Sir Isaac Newton discovered the different refrangibility of the rays of light ^a. He then saw that, however accurately rays of the same colour might be collected into a single point, rays of different colours, though they belong to the same incident beam, must move towards different focuses after refraction; and proved that the aberration arising from the unequal refrangibility of different kinds of light was 5449 times greater than the aberration, which is caused by the spherical figure of a glass ^b. He observed besides, that, though a conical lens could refract such rays as are parallel to its axis more nearly to a single point than a spherical surface, yet the latter would be preferable to the former in respect of rays, which fall with some degree of obliquity; since the curvature of a sphere is uniform in every part, whereas different parts of an ellipse or hyperbola have a different curvature. For these reasons and on account of the insuperable difficulty that attended the construction of conical surfaces, all further attempts to correct the aberrations in refractions by means of such surfaces have long been discontinued.”

^a Art. 28.

^b Art. 220.

Plate 6

B



CHAP. VIII.

TO DETERMINE THE PLACE, MAGNITUDE AND SITUATION OF IMAGES FORMED BY REFRACTED RAYS.

PROPOSITION I.

112. **I**MAGES formed by refractions at plane surfaces are similar to the objects, and are always erect, or in a similar situation to the objects, and on the same side of the planes*.

Let PQR be an object radiating upon a refracting plane ACB ; Fig. 120, 121. to which draw the perpendiculars PA, QC, RB , &c. and in these perpendiculars take Ap to AP , Cq to CQ , Br to BR in the given ratio of the sine of incidence to the sine of refraction^a; and the focuses p, q, r , &c. will constitute a similar image in a similar situation to the object; the parts pq, qr being in the same ratio as PQ, QR . This is self-evident when the object is parallel to the refracting plane; and when it is inclined, produce it till it cuts the plane in D ; and the produced image will also cut it in the same point D . For supposing the perpendicular BrR to move towards D , the lines BR, Br being in a given ratio will vanish both together: and because the triangle pDP is cut by parallel lines qQ, rR , it will be as pq to PQ (so qD to QD) so qr to QR ^b; and alternately pq to qr as PQ to QR . Art. 87. Euc. VI. 2. In like manner if the rays that belong to the focuses p, q, r be refracted again by another plane, either parallel or inclined to AB , their second focuses will constitute a second image, similar to the first image, and consequently similar to the object; and so on. $Q. E. D.$

PROPOSITION II.

113. An image pqr , formed by a flat piece of glass $ABba$ is upright, parallel and equal to the object PQR , and lies on the same side of the glass as the object; but nearer to it by a third part of the thickness of the glass.

Let PA, QC, RB be drawn perpendiculars to the surface ACB , Fig. 122, 123. and the focuses p, q, r of the several pencils that flow from P, Q, R ,

* This proposition is only true, when the object is a straight line or a plane superficies.

- ^a Art. 87. lying in these perpendiculars^a, the image must be upright. But we have also shewn, that each of the focuses p, q, r , lyes nearer to ACB , than the points P, Q, R , by a third part of Cc ^b; therefore the image must be so much nearer than the object, and parallel to it. $Q. E. D.$
- ^b Art. 91.

PROPOSITION III.

114. *An image formed by a prism is always upright, and equal to the object, and lies on the same side of the prism, and at the same distance from it as the object itself; provided the refracting angle of the prism, and the refractions made by it, be but small.*

Fig. 124.

- Take two rays, PE, QE , which coming from the extremities of the object, pass through a point E , so near to the angular point of the refracting angle, that the distances between their points of incidence and emergence need not be mentioned. And since the total bendings of the rays PEN, QEO are equal^c, they will cross each other, so as that the angle PEQ will be equal to the angle NEO or to pEq , made by the emergent rays produced backwards: and because the distance Ep of the focus p , of the pencil that flowed from P , is equal to EP ^d, and in like manner the focal distance Eq equal to EQ ; the image pq will be upright and equal to the object, and at an equal distance on the same side of the prism. $Q. E. D.$
- ^c Art. 55.
- ^d Art. 90.

PROPOSITION IV.

Fig. 125. to 128.

115. *If an arch of a circle PQR described upon the center E , of a spherical surface, sphere or lens, be considered as an object, its image pqr will be a similar concentrick arch; whose length will be to the length of the object in the ratio of their distances from the common center E ; and the image will be erect or inverted, with respect to the object, according as they lye on the same side of the center or on contrary sides.*

- The proposition is evident by inspection of the 125th figure in all cases of refractions made by concentrick surfaces; because the parts of these surfaces are alike exposed to the parts of the concentrick object. And in a lens the focuses of all the pencils of parallel rays lye also in a concentrick arch GFH ; whence Pp and Qq being third proportionals to two pair of equal distances PG and PE, QF and QE ^e, are also equal; and so the image pqr is also a concentrick arch. Now since the axes of the pencils are reckoned straight lines passing through E ^f, the angles pEr, PER are equal; and therefore the ratio of the image to the object, is the same as of their distances from
- ^e Art. 107.
- ^f Art. 97.

from the center E . And according as their corresponding extremities P, p are on the same or on contrary sides of E , they lye on the same or contrary sides of their middle points Q, q ; that is, the image is accordingly erect or inverted. Q, E, D .

116. *Corol.* The smaller the circular object is with respect to its radius or distance from the center E , the nearer it approaches in shape to a straight line, and so does its similar image. Consequently a small straight object placed at a good distance from the center of the glass may be reckoned to have a straight image very nearly ^a: ^a Art. 68. though in strictness of geometry it is an arch of a conic section. And by these propositions the images of all objects may be determined, by finding the images of their out-lines.

CHAP. IX.

CONCERNING THE EYE AND MANNER OF VISION.

117. **C**ONSIDERING what has been said in the 92d and 115th articles, one might contrive a tolerable eye in this manner, by placing a pellucid hemisphere ABC to serve for the fore part, and another concentrick one DqE , opposite to the former, to serve for its bottom or back part; making the semidiameter, Oq , of the latter triple the semidiameter, OB , of the former; and then by filling the whole cavity of both with water. By this means rays of light flowing from the points P, Q, R , &c, of remote objects, after refraction at the surface ABC , will be collected to as many other points p, q, r , of the cavity DqE , and paint an image upon it. And because a spherical surface does not accurately refract all the rays of a large pencil to a single point ^a, but only those that go pretty near its axis; this imperfection might be remedied by covering the base AC , of the lesser hemisphere, all but a moderate hole about the center O ; which would answer the purpose much better than if the surface itself was covered, all but a hole in the middle about B . For in this latter case the surface ABC would not receive rays from the lateral points P, R , so directly as those from the middle of the object, to all which it is exposed alike when the hole is left open at the center O .

118. Though this construction of the eye appears not amiss at first sight, yet we shall see presently that the author of nature has

^a A fictitious eye described by *Hugens* *.
Fig. 129.

^a See the latter part of chap. 7.

And compared to the natural eye.

* *Opuscula Posthuma*. p. 112.

wisely varied some things for the better, and added others absolutely necessary; though in every thing we cannot perceive his designs. In the first place he would not make use of an entire hemisphere ABC , but retaining the middle part, has taken off pretty much from the sides, and yet without contracting the compass of objects taken in at one view. The reason of this was to bend inwards the edges of the larger hemisphere about D and E , thereby reducing the shape of the eye to a rounder figure, for the convenience of its motion every way in the cavity that contains it. He has therefore given it such a shape, as is expressed in this other figure, representing an human eye dissected through its axis, all the parts being twice as big as in the life to render them more conspicuous.

Fig. 130.

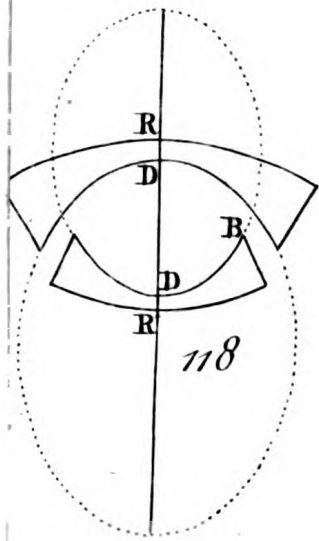
An human eye described.

119. Here the transparent parts of the coat called the cornea is ABC ; the remainder $ATYC$ being opaque, and a portion of a larger sphere. Within this outward coat anatomists distinguish two others; the innermost of which is called the retina, being like a fine net composed of the fibres of the optick nerve YVT woven together, and is white about the parts p, q, r , at the bottom of the eye. The cavity of the eye is not filled with one liquor, but with three of different sorts. That contained in the outward space $ABCOEGFDO$ is called the aqueous humor, being perfectly fluid like water; the other contained in the inward space $E.pqrDFG$ is a little thicker like the white of an egg, and is called the vitreous humor; the third humor FG is shaped like a lens of unequal convexities, lying between the two former, and fixed to the side coats by filaments or threads extended all round it, and is called the crystalline humor, being hard like the white of an egg boiled, but as clear as the other two, and differs from them in a greater degree of refractive power; whereby the rays that came from the points P, Q, R , having received a degree of convergence by the refraction of the cornea ABC , are made to converge a little more by other refractions at the surfaces of the crystalline FG ; so that uniting in as many other points p, q, r , upon the retina, they represent the points of the object P, Q, R , from whence they came. And perhaps the rays are so directed by these secondary refractions at the crystalline, as to fit the cavity pqr intended to receive them; which otherwise must have been a portion of a larger sphere^a, according to the fictitious design in the former figure.

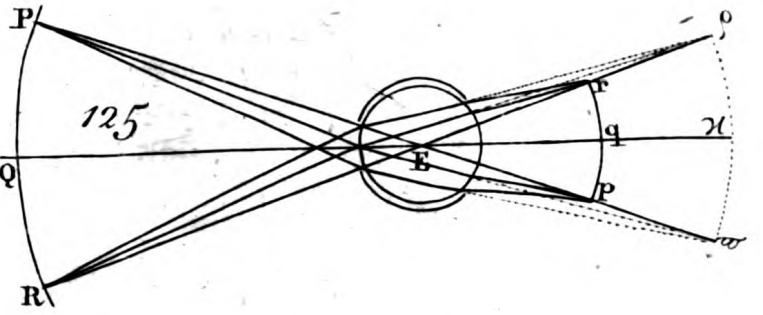
^a Art. 111. latter part.

The crystalline makes all pictures distinct.

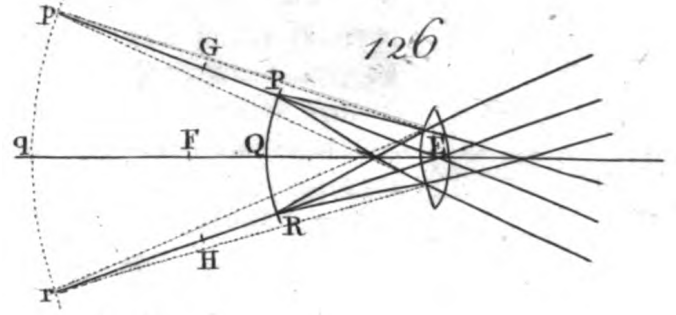
120. Besides this there was greater need of the lens FG upon another account; namely, to help the eye to conform itself for seeing objects distinctly at all distances, which was wanting in the fictitious eye. There are two ways of doing it by the help of this lens FG , in order



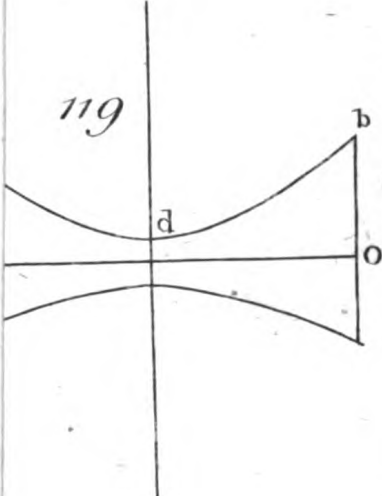
118



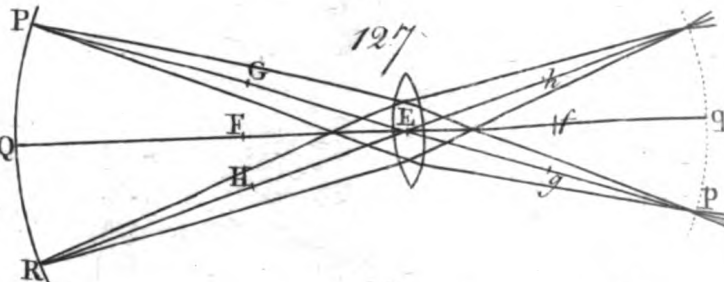
125



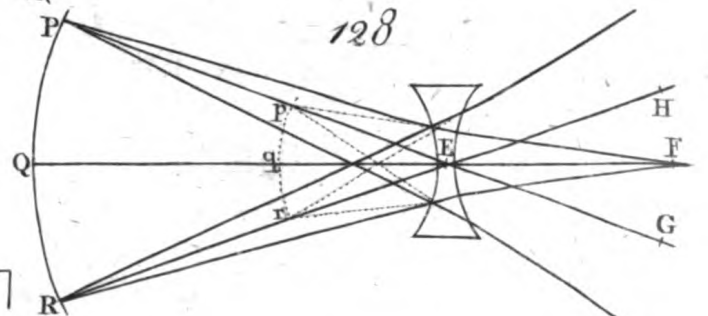
126



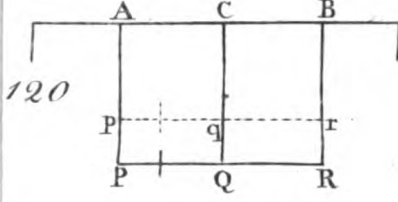
119



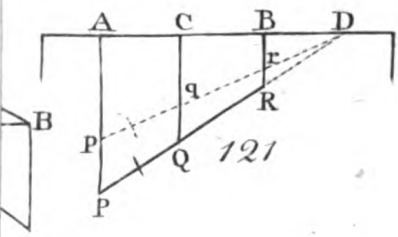
127



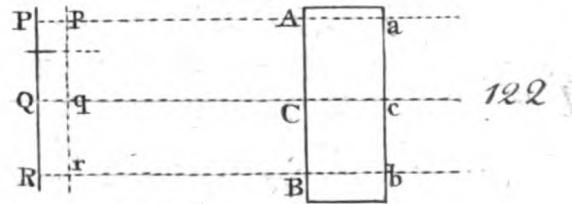
128



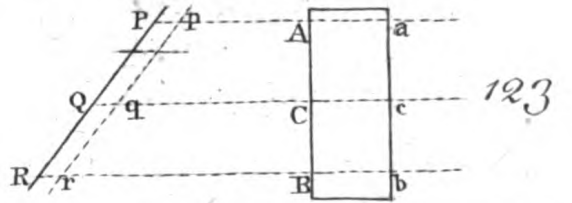
120



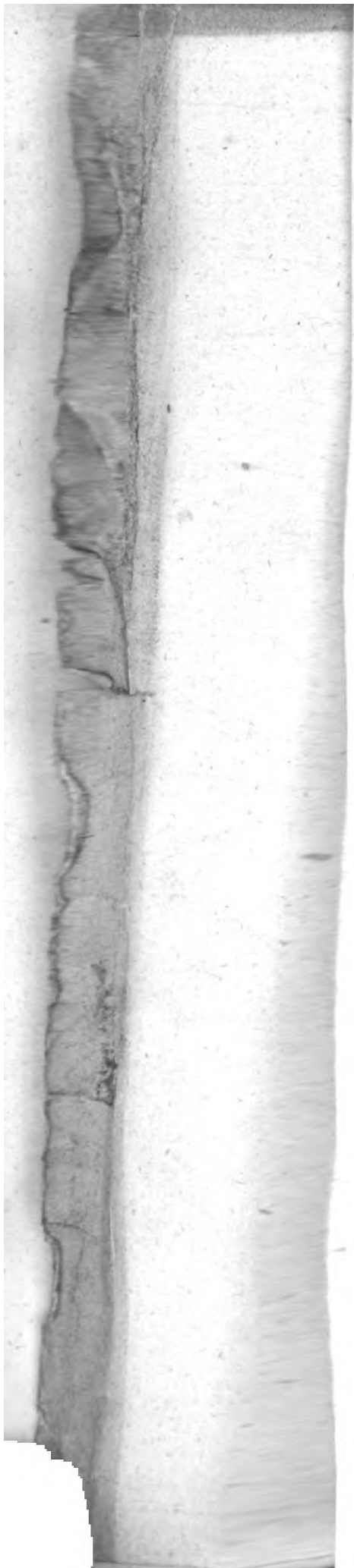
121



122



123



order to see things near at hand; either by moving it nearer to the outward cornea, or by increasing its convexity, or perhaps by doing both at once. If it is moved towards the cornea, this may be effected by the pressure of the muscles against the sides of the eye, and consequently against the vitreous humor; but if the crystalline alters its figure and becomes rounder for seeing near objects, the filaments *DF, EG*, whose greater tension helps to flatten it, may perhaps be slackened by the lateral pressure aforesaid; and possibly both these alterations are made at the same time. The hole or pupil *O* is not placed in the center of the cornea *ABC*, as in the fictitious eye, but somewhat nearer to its front. The reason is uncertain, unless this also may contribute to make the images coincide with the cavity of the retina, (in all their parts,) which otherwise must have been shaped according to a larger sphere ^a.

^a Art. 111. latter part.

121. The diameter *AY* of the sphere of the eye is about an inch of the Rheinland foot, which is much the same as the old Roman foot; and the diameter of the outward cornea is about three fifths of an inch; the breadth of the pupil, *O*, has no fixt measure, being greater or smaller, as any one may try, according as less or more light shines upon the eye; it also contracts at the near approach of any small object, when we endeavour to view it distinctly. Its fabrick is admirable in this respect that while it alters its size it preserves its roundness ¹. So far Mr. *Hugens* ², to which I add something from Sir *Isaac Newton* ³.

Some dimensions of an human eye.

122. This account of the eye and of the cause of vision is farther confirmed by these arguments; that anatomists when they have taken off from the bottom of the eye that outward and thickest coat called the *dura mater*, can see through the thinner coats the pictures of objects lively painted thereon. And these pictures propagated by motion along the fibres of the optick nerves into the brain, are the cause of vision. For according as these pictures are perfect or imperfect, the object is seen perfectly or imperfectly. If the eye be tinged with any colour (as in the disease of the jaundice) so as to tinge the pictures in the bottom of the eye with that colour, then all objects appear tinged with the same colour.

Pictures on the retina the cause of vision.

123. If the humors of the eye decay by old age so as by shrinking to make the cornea and coat of the crystalline humor grow flatter than before, the light will not be refracted enough, and for want of a sufficient refraction will not converge to the bottom of the eye,

Confused pictures in old men's eyes how caused, and mended by convex glasses. Fig. 131.

¹ See this accounted for in Mr. *Cheffelden's Anatomy*, p. 319. 3d Ed.

² *Dioptrica prop.* 31.

³ *Opticks pag.* 12.

but

Fig. 132.

Confused pictures in short-sighted eyes, how caused and mended by concave glasses.
Fig. 133.
Art. 108.

Fig. 134.

Glasses for defective eyes determined.

Fig. 135.

but to some place beyond it; and by consequence will paint in the bottom of the eye a confused picture; and according to the indistinctness of the picture the object will appear confused. This is the reason of the decay of sight in old men, and shews why their sight is mended by spectacles. For the convex glasses supply the defect of plumpness in the eye, and by increasing the refractions make the rays converge sooner, so as to convene distinctly at the bottom of the eye, if the glass has a due degree of convexity.

124. And the contrary happens in short-sighted men, whose eyes are too plump. For the refraction being now too great, the rays converge and convene in these eyes before they come at the bottom; and therefore the picture made in the bottom and the vision caused thereby will not be distinct, unless the object be brought so near the eye as that the place where the converging rays convene may be removed to the bottom^a; or that the plumpness of the eye be taken off and the refraction diminished by a concave glass, of a due degree of concavity, as is represented in fig. 134; or lastly that by age the eye grows flatter till it comes to a due figure. For short-sighted men see remote objects best in old age, and therefore they are accounted to have the most lasting eyes. So far Sir *Isaac Newton*.

125. In order to determine the properest glasses for defective eyes, the limits of confused and distinct vision, or the distances of those places from the eye, where an object begins to appear confused, may be found by measuring the least distance from which a long-sighted person can see a pretty large print distinctly and read it readily: and likewise by measuring the greatest and the least distances, from which a short-sighted person can see a small print distinctly and read it readily: or still more exactly by placing the end of a long ruler close to the eye, or rather a little under it, and by observing the greatest and least distances at which the lines drawn lengthways along the ruler, begin to appear confused. I shall call those glasses the properest for defective eyes, which are the least concave or the least convex of any that will answer the purpose of distinct vision; for a reason to be mentioned hereafter.

126. Let Eg be the least distance from which any small objects appear distinct to the eye of a long-sighted person; and $E\mathcal{Q}$ the least distance from which he wants to see them distinctly. Towards g take $\mathcal{Q}F$ to $\mathcal{Q}E$ as $\mathcal{Q}E$ to $\mathcal{Q}g$, and EF will be the focal distance of a convex lens, which being put close to his eye, will make him see an object distinctly at any place between \mathcal{Q} and F , and possibly beyond F . For the rays that flow from \mathcal{Q} will emerge from the glass and will enter the eye as if they had come directly from g , to the naked eye;

eye^a; and supposing \mathcal{Q} to recede from the eye, q will also recede^a Art. 107. from it to infinity through places where the naked eye can see distinctly^b. And therefore the refracted rays, diverging as from these^b Art. 108. places, will also produce distinct vision of the object \mathcal{Q} as far as to F ; and still farther if the person can see distinctly by converging rays.

127. Therefore if he wants to see distinctly from no less a distance than half Eq , that is, only as near again as with his naked eye, a convex lens whose focal distance is Eq will be the properest; and will make him see an object distinctly at any distance not less than half Eq . For supposing $\mathcal{Q}q$ and $\mathcal{Q}E$ to be equal, the point F will fall upon q by the foregoing proportion.

128. Let EF be the greatest distance from which an object at F appears distinct to the eye of a short-sighted person; and it will also be the focal distance of a concave lens, which being put close to his eye at E , will be the properest for seeing remote objects distinctly. Because the rays of a pencil, which come from any remote object, and consequently fall parallel upon the lens, will emerge from it to the eye, as if they had come directly to the naked eye from an object at F . And consequently the picture of the remote object formed upon the retina by rays refracted through the lens, will be as distinct as the picture of an object at F seen by unrefracted rays. Glasses for short sighted persons. Fig. 136.

129. Let Eq be the least distance from which the same person can see an object distinctly with his naked eye; then say as $\mathcal{Q}F$ to $\mathcal{Q}E$ so $\mathcal{Q}E$ to $\mathcal{Q}q$, and placing $\mathcal{Q}q$ towards F , the point q will be the nearest point, which he will be able to see distinctly through the lens abovementioned. For by art. 107, the rays of a pencil which fall upon the lens converging towards \mathcal{Q} , will after refraction converge to q ; and on the contrary, the rays which flow from q will emerge from the lens diverging from \mathcal{Q} ; and supposing the point q to recede from the eye, the point \mathcal{Q} will also recede from it to such places where the naked eye can see distinctly^b. But if the point q approaches towards the eye, the point \mathcal{Q} will also approach towards it, to such places where the naked eye cannot see distinctly, by the supposition. Fig. 137. Art. 108.

130. Consequently if $\mathcal{Q}F$, the space between the limits of confused vision, be not less than $\mathcal{Q}E$, that one glass whose focal distance is EF , will make all objects appear to him distinct which are any where placed beyond F , the reach of his naked eye. For in this case $\mathcal{Q}q$ cannot be greater than $\mathcal{Q}F$, as is manifest by the proportion above.

131. But if he wants a pair of concave spectacles to read or write with, let the distance Eq be no greater than what is convenient for that purpose, and let $\mathcal{Q}F$ be the limits of confused vision as before; and Fig. 138.

and towards q take FG to FE as FE to Fq , and a concave lens whose focal distance is EG will be the properest for this purpose. For by art. 107, the rays of a pencil which fall on this glass converging towards F will converge to q after refraction, and on the contrary; and therefore he will see an object distinctly as far off as q ; and also nearer than F , if QF be but half of EF . For supposing rays to fall on the lens converging towards Q , say as QG to QE so QE to QH , and the refracted rays will converge to H and consequently H will be the nearest point that can be seen distinctly through this glass. But if Q bisects EF it is manifest that QH is less than QF ; because QG, QF, QH are now continual proportionals.

Directions for
the choice of
convex and
concave
spectacles.

132. Thus any person may be fitted with the properest glasses though he lives at a distance from the shops where they are sold, by sending the workman their focal distances computed by the foregoing rules. But if choice of glasses be at hand, he may be better fitted by trial; observing only to use those glasses which are the least concave or the least convex of any that will fit the eye. These are the glasses which I have computed and called the properest. For since they cannot be put quite close to the eye, the less any glass is concave, the less it diminishes the pictures of objects upon the retina. It will also accustom the eye to that conformation of its coats and humors, which is proper for seeing objects as far off as it can; and consequently may prevent the eye from growing more and more short-sighted. On the other hand, the less any glass is convex, the less it magnifies the pictures of objects upon the retina; and also obliges the eye to that conformation, which is requisite for seeing objects as near as it can. Both which may prevent the eye in some measure from growing more and more long-sighted. For when the picture upon the retina is very large, it need not be quite so distinct, as when it is smaller, to convey an idea of the same number of parts of an object; and consequently the eye will be more at liberty to recede from that conformation, which is proper for the glass; and to relapse into that to which it inclines, and which is only proper for seeing remote objects.

Equal parts
of a small ob-
ject subtend
equal angles
at the eye.
Fig. 139.

133. When the perpendicular subtense BC of a small angle BAC is divided into any number of equal parts BH, HI, IC , the lines, HA, IA , drawn from the points of division to A , will divide the angle BAC into the same number of parts, which will be nearly equal among themselves. For they would be so exactly if the line BC was an arch of a circle, drawn upon the center A ; from which it differs so much the less as the angle at A is smaller; and so the proposition is exactest in the smallest angles.

134. When

134. When the distance AB is double or treble of Ab , the subtense BC will be double or treble of the subtense bc of the same angle at A^a . Divide BC into its parts BH, HI, IC , each equal to bc , and the rays HA, IA , will divide the angle BAC into as many equal parts^b. Therefore when two angles bAc, BAH are subtended by the same or by equal lines bc, BH , the magnitude of the first angle bAc , will be to the magnitude of the second BAH , as the second distance BA to the first distance bA .

Small angles subtended by the same perpendicular are reciprocally as its distances from the angular points.
^a Art. 24.
^b Art. 133.

135. In determining the magnitude of pictures upon the retina, only one ray in each pencil need be considered; because when the picture is distinct, all the rays in any one pencil are collected to one and the same point of the retina. Or, which is much the same, we may suppose the pupil of the eye contracted to a point: and, for greater simplicity and ease of the imagination that this point O is a little hole at the center of a dark, hollow hemisphere DqE , admitting only single rays straight through it without any refraction at all. For then the lengths of these pictures pqr will increase and decrease as the angle pOr does, or as the angle POR does; which I am going to shew to be the property of the natural eye^c: and if the semi-diameter Oq , of this hollow hemisphere, be about $\frac{5}{8}$ of an inch, or of the axis of an human eye, the pictures of the same objects will always have the same bigness in both sorts of eyes^c, very nearly.

The pupil of the eye may be considered as a point.

Fig. 129.

^c Art. 136.

^d Art. 142.

136. The diameters or lengths of the pictures of objects upon the retina are measured by, or proportionable to, the angles which the rays that come from the extremities of the object do make in falling on the eye; provided those angles be but small. For let two or more objects PQ and $\pi\kappa$, either parallel or oblique to each other, subtend the same angle POQ or $\pi O\kappa$ at O ; and because the particles of light flowing from P and π describe the same line $P\pi O$, they will be refracted to the same point p upon the retina; and in like manner those that flow from Q and κ will be refracted to the same point q ; and so the pictures pq of the objects $PQ, \pi\kappa$, which subtend the same angle at O , are the same in magnitude; which was the first thing to be proved.

Diameters of pictures on the retina are as the angles subtended by the object at the eye.
 Fig. 140.

Now the pictures of objects painted upon the retina of a dead eye are found by experience to be perfectly well shaped and proportioned in their parts^e; that is, the proportion of the parts pq, qr , of the whole picture pqr , is the same as that of the parts PQ, QR , of the whole object PQR , and this latter proportion is very nearly the same as that of the angles POQ, QOR subtended by the parts PQ, QR ^f; and so the proposition is proved when the objects PQ, QR are both at the same distance from the eye. And since it was shewn just before,

^e Art. 122.

^f Art. 133.

fore, that the objects PQ and $\omega\kappa$ have the same picture pq , it follows that the proportion of the pictures of the objects $\omega\kappa$ and QR is the same as that of the angles $\omega O\kappa$, QOR , subtended by them at the eye.

Are reciprocally as the distances of the object from the eye.

^a Art. 136.

^b Art. 134.

Brightness of pictures not altered by the distance of the object from the eye.

^c Art. 25.

Faintness of pictures of remote objects how caused.

Their degrees of brightness by day-light and moon-light compared.

137. When an object approaches towards the eye, the diameter of its picture upon the retina increases in the same proportion as the distance between the eye and the object decreases; and on the contrary, it decreases in the same proportion as that distance increases. For the diameter of its picture increases in the same proportion as the angle increases, which the object subtends at the eye^a; and this angle, when small, increases in the same proportion as the distance between the eye and object decreases^b.

138. The degree of brightness of the picture of an object painted upon the retina continues the same, at all distances between the eye and the object; provided none of the rays be stopt by the way, and that the pupil does not alter its aperture. For instance, when the eye approaches as near again to the object, the picture upon the retina becomes double in length and double in breadth, and consequently quadruple in surface; for the surface would be double, if its length alone or breadth alone was double. The quantity of rays received through the same aperture of the pupil, at half the distance from the object, is also quadruple^c: and being equally spread over four times the quantity of surface of the retina, they are just as dense as before when the object was at twice the distance.

139. It follows then that the faint appearance of remote objects is owing to the opacity of the atmosphere, which hinders part of their light from coming to the eye. Accordingly we find that the sun, moon, and stars appear very faint when near the horizon, and brighter continually as they rise higher; because the tract of vapours, which lies in the way of the rays, is longest and thickest near the horizon; and becomes thinner and shorter as the objects rise higher, and consequently does less obstruct the passage of the rays.

140. The sensibility of the eye, or its power to discern objects, without inconvenience, by different quantities of light, is vastly extensive. For instance, the disproportion in the quantities of light, cast upon the horizon by the sun and moon, at any equal altitudes, I find is no less than 90 thousand to 1, when the moon is full; or no less than 180 thousand to 1, when the moon is in the quarters. And the proportion between those parts of the lights of the sun and moon, whatever they be, which are reflected to our eyes from the same object by day and by night, can hardly be different from the proportion of the whole lights. Allowing then that the aperture of the pupil

pupil may possibly be 8 or 9 times less by day than by night, (that is about 3 times less in diameter,) yet the proportion in the quantities of day-light and moon-light, received by the eye from the same object, to illuminate a picture of the same bigness, will be no less than 20 thousand to 1, when the nights have a middle degree of moon-light; I say no less, because the numbers here given are deduced from a rule, which is built upon this principle; that the moon reflects all the light received from the sun; which cannot be true, by reason of the appearance of very large obscure places in her body; and in all probability a great part of the incident light is buried and lost even in the brightest places.

The rule I mentioned is this, day-light is to moon-light as the surface of an hemisphere, whose center is at the eye, to the part of that surface which appears to be possessed by the enlightened part of the moon: so that the whole heavens covered with moons would only make day-light. This will be evident enough from the following considerations, though I invented it another way. Day-light is made by innumerable reflections of the sun's rays from all sorts of bodies till at last they come to our eyes: for if this were not so, we could see nothing in the world, even in the day time, but the sun and stars and self-shining substances^a. Accordingly we find that day-light is^a Art. 2. much the same, whether the sun shines out or not, in the place we are in; because his light is reflected to us from a vast quantity of earth, air and clouds extended all round us, perhaps to a hundred miles or more. So that the absence of the sun's rays from a particular place scarce alters day-light. Another thing is that the moon by day appears like a cloud in the air of a middle degree of brightness; some appearing duller and some brighter than the moon itself. The rays of the sun being therefore intercepted in the night from all the visible clouds, and being reflected to us by the moon only, it follows that day-light is to moon-light, as the apparent surfaces of all the visible clouds, to the apparent surface of the visible part of the moon, considered as the only cloud which remains enlightened. And these two lights, whatever be the distances of the moon and clouds, are just the same as if those bodies were all placed at any equal distances from us, and composed the surface of an hemisphere^b; whose parts are the true measures of the parts of the^b Art. 138. light which comes to us.

141. A vast disproportion between the lights of the sun and moon appears also by experiments made with burning-glasses; either by refraction of the rays through very broad lenses, or by reflection from very broad concave-glasses or metals: which by collecting the

And confirmed by experiments with burning-glasses.

rays of the sun into a small round image at the focus, do excite a more violent heat and burn quicker than the hottest wind-furnaces: as appears by their melting and calcining the hardest metals, and by vitrifying bricks and stones, in much less portions of time than a minute ¹. Yet the rays of the moon being collected by the same glasses, do not excite the least sensible heat; nor do they sensibly affect the nicest thermometer, when cast upon the ball of it ², though the brightness of the light be very sensibly increased. By measuring the breadth of the round image at the focus, and by comparing it with the breadth of the glass itself, it appears that some of these burning-glasses collect the incident rays into a space about 2 thousand times less than they possessed at their incidence. But by the preceding calculation, the light of the full moon must be condensed about 90 thousand times ³, to make it as dense and as warm as the direct rays of the sun. It is no wonder then that the heat of the moon's rays is not sensible in the focus of the glass, being then even 40 or 50 times thinner than the direct rays of the sun. For it is found by experiments made with these glasses that the degrees of heat are proportionable to the densities of the rays: which being compared with a scale of the degrees of heat and warmth of several natural bodies, determined by Sir *Isaac Newton*, in the philosophical transactions ³, it appears there is a vast disproportion between the degrees of light which the eye can bear and be sensible of, and the degrees of its heat which the touch can bear and be sensible of.

² Art. 140.

Vision limited both by magnitude and distance.

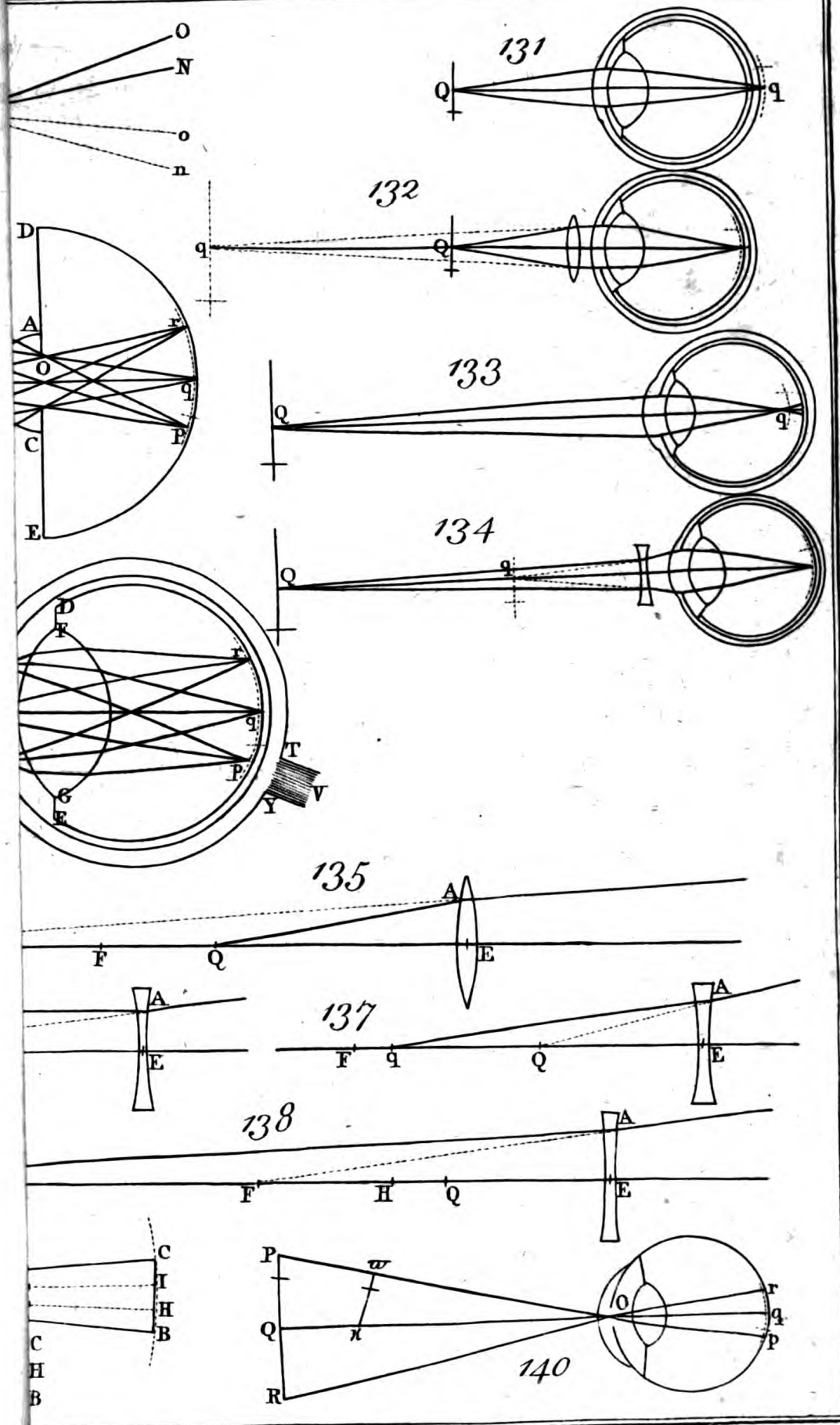
142. Dr. *Hook* assures us that the sharpest eye cannot well distinguish any distance in the heavens, suppose a spot of the moon's body, or the distance of two stars, which subtends a less angle at the eye than half a minute; and that hardly one of a hundred can distinguish it when it subtends a minute ⁴. If the angle be not bigger than this, the two stars appear to the naked eye, as if they were but one. I have been present at making the experiment, when a friend of mine, who had the best eyes of all the company, could scarce perceive a white circle upon a black ground, or a black circle upon a white ground, or against the sky-light, when it subtended a less angle at the eye than two thirds of a minute; or which is the same thing, when its distance from the eye exceeded 5156 times its own diameter: which agrees well enough with Dr. *Hook's* observation.

¹ Phil. Trans. abr. by *Lowth.* Vol. 1. p. 211. and by *Jones* Vol. 4. p. 190.

² Ibid. *Lowth.* p. 213. and Mem. de l'Acad. Roy. des Scien. ann. 1705. p. 455. 8°.

³ N° 270. or *Jones's* abr. Vol. 4. part. 2. p. 1. It. Mem. de l'Acad. an. 1703. p. 233.

⁴ Animadversions on Hevelii machina cœlestis p. 8.





From whence and from a rule I invented, it appears, that the diameter of the picture of that circle upon the retina was but the 8000th part of an inch at most; and this may be called a sensible point of the retina. That this point is very small any one may perceive from hence that the breadth of the finest hair is visible at the length of one's arm.

143. The apparent magnitude of an object is a quantity of visible extension, measured by, or proportionable to, the angle which the two ^a rays that come from the extremities of the object do make in falling on the eye. For the extremities of the object are seen in the directions of these rays; and in proportion as they make a greater or smaller angle at the eye, the magnitude of the picture upon the retina is longer or shorter ^b; and consequently causes a ^b sensation of a greater or smaller visible extension; consisting of a greater or smaller number of visible points, answering to the number of sensible points of the retina ^c, of what magnitude soever these points are supposed to be. Apparent magnitude to the naked eye defined. ^a Art. 135.

144. The apparent magnitude of any given object is reciprocally as its distance from the eye: that is to say, when the object approaches to the eye, its apparent magnitude increases (in proportion) as its real distance decreases; and on the contrary, it decreases (in proportion) as that distance increases. For the apparent magnitude of an object was defined to be a quantity of visible extension proportionable to the angle which the object subtends at the eye ^d, and this angle increases very nearly in proportion as the real distance between the eye and object decreases ^e. How it varies. ^d Art. 143. ^e Art. 134.

145. The apparent magnitude of an object seen by the naked eye, in opposition to its apparent magnitude seen through glasses, and for shortness of expression, is often called its true magnitude. And in speaking of the apparent magnitude of an object I always mean the apparent magnitude of its diameter, or of its length or breadth, or of any principal line of it, and not of its surface, or solidity, unless it be particularly specified. When called true magnitude.

CHAP. X.

CONCERNING VISION WITH GLASSES.

Apparent di-
rections of vi-
sible points
defined.

Art. 18.
Fig. 4.

Fig. 141. to
152.

Fig. 153.

Art. 54.

146. ANY small object or point of an object, seen by refracted or reflected rays, appears somewhere in the direction of that line, which the visual ray describes after its last refraction or reflection in falling upon the eye.

In the experiments to prove the laws of reflection and refraction^a, the pin at *B*, seen by a ray reflected from the water, appeared somewhere in the line *AC* produced, which the visual ray *BCA* described after reflection at *C*, when it advanced to the eye. And as the whole line *CE* appeared lifted up by the refraction at the water, as if it had been a continuation of the line *AC* straight on, so if a straight oar be in part immersed obliquely in water, it appears crooked, as if the part immersed was broken at the surface, and lifted higher. For this part of the oar is seen in the direction of rays which are bent downwards by refraction at their emergence from the water, and consequently advance to the eye as if they came from a place in the water which is higher than the real place of the oar. In like manner any point *P* of an object seen by the ray *PAO* twice refracted, either by passing through the edge of a prism, or of a concave or convex lens, or through the sides of a globe or decanter, or of a drinking glass filled with any transparent liquor; or seen by a ray *PAO* reflected from a plane or spherical looking-glass, appears to the eye at *O*, somewhere in the direction of the last refracted or reflected ray *AO*. Lastly, an object *P* viewed by the eye at *O*, through a multiplying glass, appears at one view in as many different places, *p*, *p* 1, *p* 2, situated in as many different directions *OA*, *OB*, *OC* of the last refracted rays produced, as the glass has different surfaces *DE*, *EF*, *FG* differently inclined to the opposite surface *DH*. For these surfaces, like so many different prisms, give the visual rays *PIAO*, *PKBO*, *PLCO*, so many different bendings at *I* and *A*, *K* and *B*, *L* and *C*, and make them fall upon the eye at *O* in as many different directions *AO*, *BO*, *CO*^b. And in all these instances when the reflecting or refracting surfaces of the water or glasses are shaken by the wind, or otherwise, the objects seen by reflection or refraction appear to shake and tremble; because the last directions of the visual rays are shaken and varied by those motions.

Now

Now the reason why an object or point of an object appears always in the direction of the last refracted or reflected ray, is, because the place of its picture upon the retina is the same as it would be if the object was really removed from its proper place into the direction of that ray, and was seen by direct rays. And having no sensation of the previous reflections or refractions of the rays at the glasses, but only of their action upon a certain place of the retina, we form the same judgement of the place of the object as we used to do in the more common cases of direct vision.

It is manifest then, that any point P of an object PQ seen by re-
And determined. Fig. 141. to 153.
 fractions or reflections, appears somewhere in the line pO , drawn from the corresponding point p of its last image to the eye at any place O . Because all the rays which flowed from P do after the last refraction or reflection flow from or towards the corresponding point p of the last image. The reason why I say the last image will be mentioned in the 155th article.

147. It is also manifest why an object seen by refracted or reflected rays appears sometimes upright and sometimes inverted. For
Their apparent situation determined. Art. 146.
 when the refracted or reflected rays AO , CO , have the same situation with respect to each other, as two rays that come directly from the same points of the object to the eye, these points will appear in the same situation with respect to each other in both cases ^a. But if the rays that come from these points shall have crossed each other before they arrive at the eye, they will then have a contrary situation to that of two rays coming directly from the same points to the eye; and consequently these two points will appear in the glass in a contrary situation ^b. And one may add that in the former case, the picture upon the retina of the eye will have the same position, though not the same magnitude, as if the glass was removed, and will have a contrary position in the latter case. Art. 146.

148. The apparent magnitude of an object, PQ , seen by refracted or reflected rays either upright or inverted, is a quantity of visible extension, measured by the angle, AOC , which the two rays, AO , CO , that came from its extremities, P , Q , do make, after their last refraction or reflection, in falling on the eye. Or in other words, the object appears greater or smaller in proportion as that angle AOC is greater or smaller. Because its extremities appear in the directions of the last refracted or reflected rays OA , OC ^d; and also because its picture upon the retina is greater or smaller in proportion as these rays constitute a greater or smaller angle at the eye ^e. Apparent magnitude in glasses defined. Art. 135. Art. 146. Art. 136.

149. Therefore the apparent magnitude of an object, PQ , is also measured by the angle pOq which its last image pq subtends at
And determined.
 the

the eye. For the lines AO , pO are but one line continued, and so are CO , qO , and therefore the angles AOC , pOq are the same when the image lies before the eye, and are equal when it lies behind it.

How it varies.

^a Art. 144.

^b Art. 134.

When invariable.

Fig. 154. to 157.

^c Art. 104, 70.

Fig. 158, 159.

^d Art. 148.

Compared to the true magnitude.

^e Art. 149.

^f Art. 143.

When equal to the true.

^g Art. 108.

150. Hence the apparent magnitude of an object increases and decreases in proportion as the eye approaches to or recedes from its last image, (just as if it was a real object ^a;) placed either before or behind the eye. For when the image is fixed, the angle pOq , when small, increases in the same proportion as Oq decreases, and on the contrary ^b.

151. Hence if the last image be removed to an infinite distance, that is, if the object be placed in the principal focus of a lens, sphere, or concave looking-glass ^c, its apparent magnitude to the eye at any place whatever will be invariably the same; and equal to its apparent magnitude seen by the naked eye, supposing it put into the place of the center of the sphere, lens, or reflecting concave. For since all the rays of any one pencil, are parallel to its axis PE , the angle COA , which measures the apparent magnitude at any point O , is every where equal to the angle QEP at the center E .

The apparent magnitude of the object will also be invariable where-ever it be placed, when the eye is fixed at the principal focus of any glass which makes parallel rays converge to the eye. For conceiving them to flow back again from the eye to the object, they will fall upon the same points of the object from whence they came while it is moved in any place along the axis of the glass: and no other rays but these can return from the same points of the object to the eye in that place: therefore the several parts of the object will always be seen under the same angles, and consequently will appear of the same magnitudes ^d.

152. The apparent magnitude of an object seen by reflected or refracted rays being measured by the angle which its last image subtends at the eye ^e, and its apparent magnitude to the naked eye in any place being measured by the angle which the object itself subtends at the eye in that place ^f, it follows that the former apparent magnitude is to the latter, as the former angle to the latter angle. For the measures of things and the things measured by them are proportionable.

153. Consequently the apparent magnitude of an object seen in a glass, will be equal to its apparent magnitude to the naked eye in the same place, if the glass was removed. First, when the object touches any thin lens, or any single surface. For the image is then equal to the object and coincides with it ^g. Secondly, when the eye touches any thin lens or any reflecting surface. For then the ray

PAO

PAO will pass from the object to the eye through the middle of the lens very nearly, and therefore being almost straight^a will make^a Art. 60. nearly the same angle with the axis as an unrefracted ray would do: and when the point of incidence, *A*, coincides with *C* at any reflecting surface, the incident and reflected rays *PA*, *AO*, produced, will also make equal angles with the axis or perpendicular $\mathcal{Q}C^b$; and so^b Art. 8. the object will appear under the same angle as it would do to the naked eye turned about. Thirdly, when the eye is at the center of a reflecting concave. For then the incident and reflected rays *PA*, *AO* will coincide with the direct ray PE^c , and consequently will^c Art. 10. make the same angles with the axis. Fourthly, when the object is at the center of the reflecting concave. For then the reflected image is also at the center and is equal to the object^d. Fifthly, when a ray^d Art. 72. 79. coming directly from *P* to *O*, would make an angle with the axis equal Fig. 146. 148. to the angle *AOC*, which the refracted or reflected ray *PAO* makes^e 152. with it on the other side.

154. These cases being excepted the apparent magnitude of an object seen through a concave lens is always less than the true; and when it is seen upright through a convex lens, or a globe, it is greater than the true. For the ray *PAO*, coming from the extremity of the object to the eye, is bent by the concave lens from its axis, and therefore makes a less angle with it at the eye than a ray coming directly from that extremity to the eye. But the same ray is bent by the convex lens towards its axis, and therefore makes a greater angle at the eye than the direct ray: and the apparent magnitudes are measured by these angles. Less than the true through a concave and greater through a convex glass.

155. What has hitherto been demonstrated concerning the apparent magnitude of an object *PQ*, will continue in force if you suppose the object *PQ* to be an image formed by another glass or other glasses. For the rays diverge from either of them in the same manner, and for this reason I have always called *pq* the last image of the object. The whole applied to vision through any number of glasses.

156. The place of the eye at *O* being given, to determine what part of an object is visible in a given portion or aperture *AC* of any refracting or reflecting glass, draw *OA* to the edge of the aperture and produce it till it cuts the image in *p*, and through the center of the glass draw *pE* cutting the object in *P*; and *PQ* will be the part in view in the aperture *AC*. For the whole pencil of rays flowing from *P* will belong to *p* after refraction or reflection^e, and consequently some one of those rays will advance to the eye in the line *AO* drawn through *p*. If the image be at an infinite distance all the rays that belonged to *p* will be parallel to the axis of the pencil; What part of an object is visible in any glass. ^e Art. 61. Fig. 154. to 157.

therefore PQ is now determined by drawing EP parallel to OA . In a plane looking-glass, pP must be drawn from p parallel to qQ , or perpendicular^a to the glass, to cut off the part PQ visible in the aperture AC . For this glass may be considered as having a center at an infinite distance from it.

Fig. 149.

^a Art. 66. 78.

How it varies.

157. Hence if the glass and object be fixed, the part in view in a given aperture will decrease perpetually while the eye recedes from the glass; unless the image be behind the eye. For then it will decrease only till the eye arrives at the image, and after the eye has passed by the image it will increase perpetually. The reason is because the object and image, being fixed in their places, do both increase or both decrease together, being both terminated by two lines Pp , Qq that meet or cross in E the center of the glass.

When greatest and least.

158. Therefore the part in view is greatest when the eye is close to the glass, and least when close to the image; and, in this latter case, it appears infinitely magnified. For conceiving the distance Oq infinitely diminished, the parts pq , PQ cut off by the lines AOp , pEP will both be infinitely diminished; but the magnitude of the angle at O , subtended by pq or by AC , continues finite while the angle subtended by PQ at O is infinitely diminished: and so the disproportion between these angles, that is, between the apparent and true magnitudes of the particle PQ ^b is infinitely great. It appears also infinitely confused, when the pupil is open, for the reason given in the following articles.

^b Art. 152.

The size of a looking-glass sufficient to see all ones own body.

Fig. 149.

^c Art. 66.

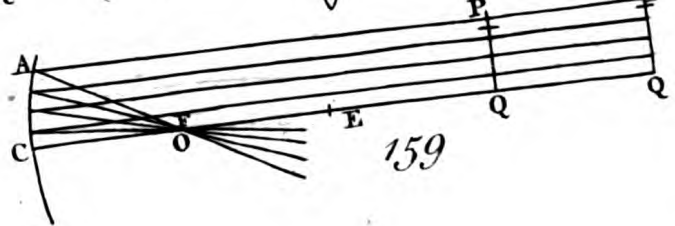
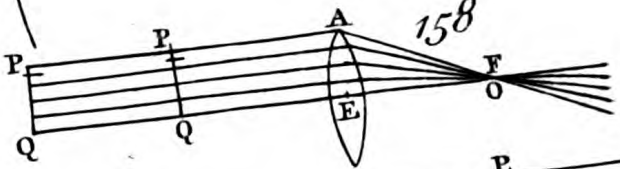
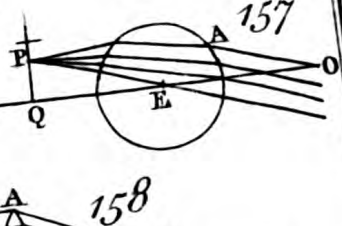
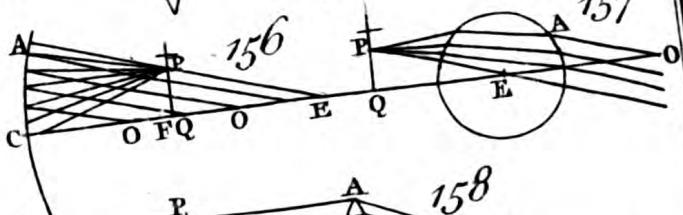
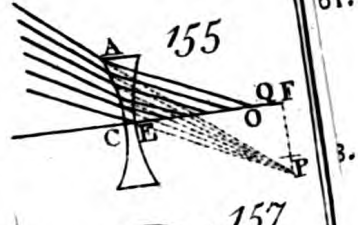
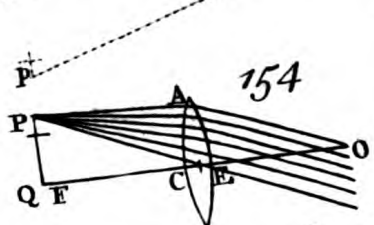
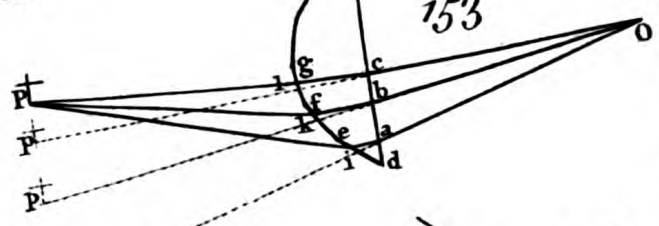
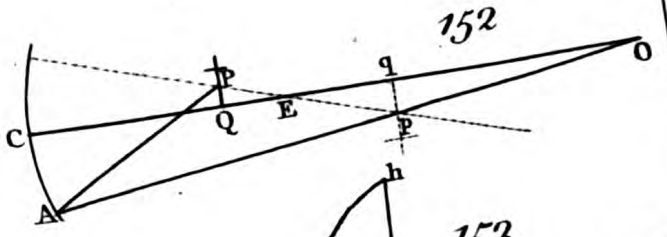
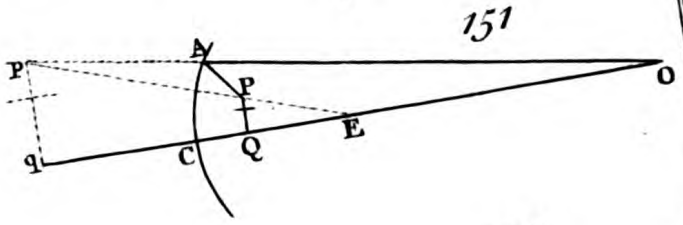
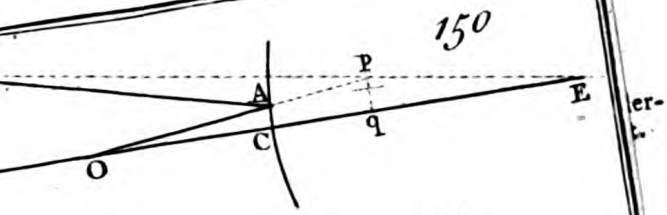
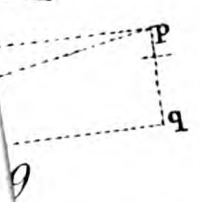
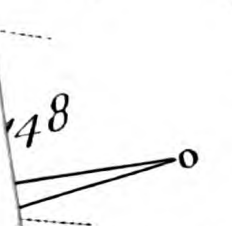
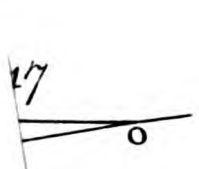
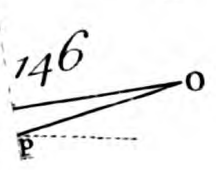
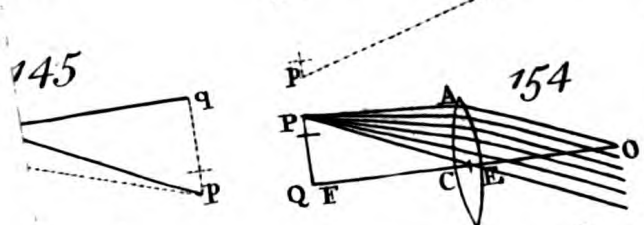
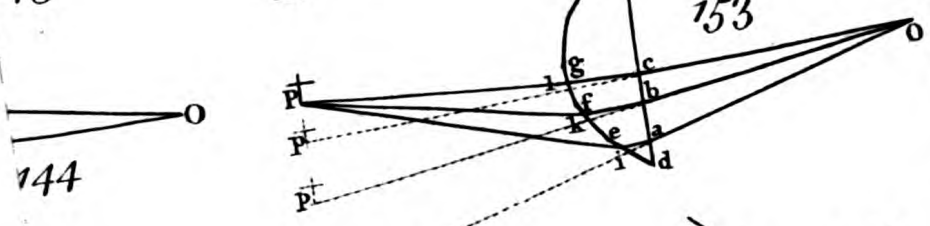
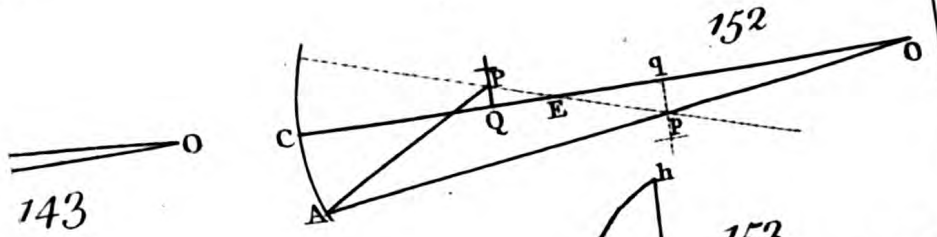
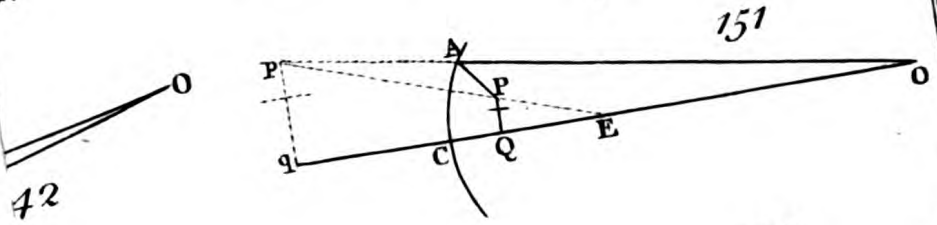
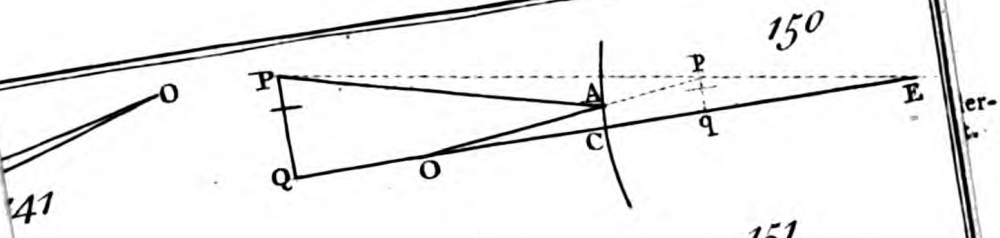
^d Art. 24.

Vision when confused by glasses.

^e Art. 135.

159. When a person views himself in a plane looking-glass he appears to himself to fill the same part of the glass wherever he stands: and the length and breadth of this part is always half the length and breadth of the corresponding part of his own body. For when O and Q coincide, OC is half of Oq or Qq ^c, and therefore AC is half of pq ^d or PQ .

160. Hitherto I have considered the pupil of the eye as no bigger than a point, admitting but a single ray from every point of the object^e; by which means the picture upon the retina would be distinct in all cases. But when the pupil is open, if the image formed by the glass be nearer to the eye than the least distance at which we can see objects distinctly with the naked eye, the appearance through the glass will be confused. Because the rays diverge too much from so near an image to be reduced by the eye to a distinct picture upon the retina. On the other hand, when the rays converge to an image behind the eye, they will be collected to a distinct picture before they arrive at the retina, because the eye is not naturally used to conform itself to converging rays; and so the vision will



mi-
low
lag-
61.

3.
8.
hat

Fig. 1
• Art.

How
rics.

Whe
est an

• Ar

The
look
suffic
see a
own
Fig.
• Ar
• Ar
Vifi
conf
glaf
• Ar

will be confus'd in both cafes, but may be rendered distinct as followeth.

161. Things which appear confus'd when seen by direct, refract-^{How render-}ed or reflected rays may be rendered distinct either by looking^{ed distinct.} through a little hole in a thin plate or bit of paper, or through a convex or concave glass of a proper degree of convexity or concavity; and provided the hole or glass be put close to the eye, the apparent magnitude and situation of the object will be the same in both cafes. For if the hole be so small as to admit but a single ray from every distinct point of the object, these rays will fall upon the retina in as many other distinct points, and will make a distinct picture. And when the pencils of rays fall upon a thin lens, their axes go straight through the middle of it^a, and consequently will proceed to the same^a Art. 61. points upon the retina as when they pass'd through the hole. Now supposing the lens to have such a figure that the rays of every pencil shall be refracted by it, and by the eye together, to those very points of their axes, which touch the retina, the picture will still be distinct: and will be the same in magnitude and position as before: and the only difference in the effects of the hole and lens will be in the degree of brightness of the picture upon the retina.

162. A single microscope is only a very small globule of glass, or a small double convex glass, whose focal distance is very short. A^{A single mi-} minute object pq seen distinctly through a small glass AE by the^{crofscope, how} eye put close to it, appears so much greater than it would to the^{much it mag-} naked eye, placed at the least distance qL from whence it appears suf-^{nifies.} ficiently distinct, as this latter distance qL is greater than the former^{Fig. 160, 161.} qE . For having put your eye close to the glass EA , in order to see as much of the object as possible at one view^b, remove the object pq ^{Art. 158.} to and fro till it appears most distinctly, suppose at the distance Eq . Then conceiving the glass AE to be removed, and a thin plate, with a pin-hole in it, to be put in its place, the object will appear distinct, and as large as before^c, when seen through the glass, only not so^{Art. 161.} bright. And in this latter case, it appears so much greater than it does to the naked eye, at the distance qL , either with the pin-hole or without it, as the angle pEq is greater than the angle pLq ^d, or as^{Art. 143.} the latter distance qL is greater than the former qE ^{148.}^c Art. 134.

163. Since the interposition of the glass has no other effect than to render the appearance distinct, by helping the eye to increase the refraction of the rays in each pencil, it is plain that the greater apparent magnitude is intirely owing to a nearer view than could be taken by the naked eye. If the eye be so perfect as to see distinctly by pencils of parallel rays falling upon it, the distance Eq , of the

object from the glass, is then the focal distance of the glass. Now if the glass be a small round globule whose diameter is $\frac{1}{3}$ of an inch, such as are easily made, its focal distance Eg being three quarters of its diameter^a, is $\frac{1}{20}$ of an inch; and if qL be 8 inches, the usual distance at which we view minute objects, this globule will magnify at the rate of 8 to $\frac{1}{20}$ or of 160 to 1.

^a Art 95:13.

Astronomical telescope, how much it magnifies, and why. Fig. 162.

164. An astronomical telescope is composed of two convex glasses in the following manner. PQ represents the semidiameter of a remote object, and pq its picture formed by the convex lens L , which being next to the object is called the object-glass. In the axis of this glass, QLq produced, EA represents another glass more convex than L , so placed, that as qL is the focal distance of the glass L , so qE is the focal distance of the glass E ; and EL the sum of their focal distances. In this situation of the glasses, I say the object will appear to the eye at any point O , distinct, inverted and magnified at the rate of qL to qE , that is of the focal distance of the object glass to the focal distance of the eye-glass.

For the rays which diverge from the point q of the picture pq , being refracted by the eye-glass, will emerge upon the eye at O in lines parallel to the axis qEO ; because qE is supposed to be the focal distance of the eye-glass; and for the same reason, the rays which diverge from any collateral point p , of that picture pq , will emerge from the eye-glass, after refractions at A , in lines parallel to the line or ray pE ; this line being the axis of an oblique pencil of rays, part of which diverge from p upon the glass. An eye therefore which can see distinctly by pencils of parallel rays being placed anywhere at O^* , among the intersections of these pencils, will see the points of the object distinctly.

^b Art. 148.

Now to the eye at O the apparent magnitude of the picture pq , or object PQ , is measured by the angle EOA^b , or by the equal an-

Fig. 162.

* PROPOSITION. *The point O , where the axes LB, LA of the extreme pencils cross the axis LE , is a little beyond the principal focus of the eye-glass.*

For we may suppose all the axes of the several pencils to proceed from the point L as from a focus of incidence; and their focus after refraction may be found by article 104. But the first term Lq , in the rule there laid down, is considerably greater than the second qE , by construction: wherefore the fourth term, that is, the distance of O from the other principal focus is but small, when compared with the focal length of the eye-glass. Q, E, D .

N. B. The point O , thus determined, is the place from which an eye can see the most possible of an object, through a given telescope.

The situation of common eyes in a double microscope is determined after the same manner.

Hugens's Dioptr.

gle

gle qEp ; but to the naked eye at L , if the glass was removed, the apparent magnitude of the object is measured by the angle $\angle L P$, or by the equal angle qLp ; the oblique axis PLp being straight ^a. ^a Art. 61. Therefore the former apparent magnitude is to the latter, as the angle qEp , to the angle qLp ; and consequently as the latter distance qL , to the former qE ^b.

^b Art. 134.

165. The object which appeared inverted in the former telescope ^c, will appear upright and distinct through two more convex eye-glasses subjoined to it; at a distance from each other, equal to the sum of their focal distances; and when their focal distances are equal to each other, the object will be magnified just as much as it was before. For the pencils of parallel rays EOF , AOB , &c, which are continued to the glass FB , will be formed by it into a second image $\omega\kappa$; and the focus ω , of any oblique pencil OB , will be determined by the intersection of the line $\omega\kappa$, perpendicular to the common axis of the glasses, and of the oblique axis $F\omega$, drawn parallel to the incident rays OB ^d. This point ω being the focus of incident rays on the last glass GC , the emergent rays CD will be parallel to their oblique axis ωG ; because the rays that flow from κ are supposed to emerge parallel to the direct axis. Therefore to the eye at D , where these emergent pencils cross, the object will appear distinct, and upright ^e. And when the glasses F and G are exactly equal, the image $\omega\kappa$ will be exactly in the middle between them; and so the triangles $\omega F\kappa$, $\omega G\kappa$ will be exactly equal. Consequently the angle CDG , which now measures the apparent magnitude to the eye at D , will be equal to the angle $\omega G\kappa$ or $\omega F\kappa$ or BOF or AOE , which measured it before to the eye at O .

^c A telescope made of four convex glasses considered.

Fig. 163.

^e Art. 147.

^d Art. 116.

^e Art. 147.

166. In a telescope of a given length the quantity of objects taken in at one view, depends upon the breadth of the eye-glass. For as AE is greater or smaller, the angle ALE or its equal PLQ is also greater or smaller; and this angle takes in all the objects that can be seen at one view on one side of the axis of the telescope.

How much they take in at one view. Fig. 162, 163.

167. The difference between the astronomical telescope and Galileo's telescope or a common perspective-glass is this; instead of the convex eye-glass placed behind the image to make the rays of each pencil go parallel to the eye, there is placed a concave eye-glass AE as much before it; which opens the rays of each pencil that converged to q and p , and makes them emerge parallel upon the eye; as is evident by conceiving the rays to go back again through the eye-glass, whose focal distance we supposed was Eg . The eye must be put close to the glass to receive as many pencils as possible; and then, supposing an emerging ray of an oblique pencil produced backward along

Galileo's telescope considered.

Fig. 164.

along AO , the apparent magnitude of the object is measured by the angle AOE^a or its equal qEp , which is to the angle qLp (or QLP , the measure of the true magnitude,) as qL to qE , as before in the other telescope. It is manifest, by the 147th article, that objects in this telescope appear upright.

^a Art. 148.
This takes in less than the former.

168. The quantity of objects taken in at one view in this telescope does not depend upon the breadth of the eye-glass, as in the astronomical telescope, but upon the breadth of the pupil of the eye. Because the pupil is less than the eye-glass, and the lateral pencils do not now converge to, but diverge from the axis of the glasses. Upon this account the view being narrower is not so pleasant as in the former telescope.

See *If. Newton's* reflecting telescope. Fig. 165.

169. Sir *Isaac Newton's* reflecting telescope magnifies the diameter of a remote object in the proportion of the focal distance of the reflecting concave to the focal distance of the convex eye-glass, and shews it inverted. Let ST be an image of a remote object PQ formed by reflections from a large concave surface AC , and terminated by the lines $PESA$, $QETC$ drawn through its center E . Now because this image cannot be viewed through an eye-glass placed directly before it (for then the spectator would intercept the rays that are coming to the concave) therefore let the several pencils of rays which converge towards it in coming from the broad concave AC , be reflected sideways from a small polished plane, represented by ac ; and then the second image st , formed by this plane, will be equal to the first image ST^b . Let tl be the focal distance of a small convex eye-glass kl and the rays which flow from any point s will be refracted through this glass, to the eye at o , in the lines ko drawn parallel to the oblique axis sl ; and so the apparent magnitude of the object, PQ , to the eye at o , will be measured by the angle kol or slt^c : but to the naked eye at E , it is measured by the angle PEQ or SET . Therefore the former apparent magnitude is to the latter, as the angle slt to the angle SET or, (because their subtenses st , ST are equal,) as ET to lt^d or as CT to lt , when the object is remote^e. Note that the plane acb is much too broad in comparison to the concave ACB , which could not be helped in so small a draught. That the appearance of the object is inverted or turned from right to left, is evident by the 147th article.

^b Art. 66. 78.

^c Art. 148.

^d Art. 134.

^e Art. 69.

Why so much shorter than others.

170. Dioptrick telescopes which magnify much being very long and troublesome to be managed, Sir *Isaac Newton* proposed this method to shorten telescopes^f; which answers to admiration; as appears by a table in the 12th chapter, of the lengths of both sorts of

telescopes which magnify equally with equal distinctness. The reason why dioptrick telescopes cannot be shortened as much as these, and still magnify as much, by diminishing the focal distances of the eye-glasses^a, in short is this. The images made by refractions^a through the convex object-glasses, being much more imperfect than those which are made by reflections from concave surfaces, will not bear to be magnified so much by so small eye-glasses^b, without appearing confused; and the chief cause of those imperfections in the pictures is the unequal refrangibility of rays of different colours^c.

^a Art. 164.

^b Art. 162.

^c Art. 220.

171. The following description of Mr. *Gregorie's* reflecting telescope differs from the author's chiefly in this; that he directs his larger reflecting concave to be made of a parabolick figure, and his lesser of an elliptical one, instead of the spherical surfaces now used; which are the only figures that can be polished without insuperable difficulties.

Mr. Gregorie's reflecting telescope described. Fig. 166.

It is proposed to make a reflecting telescope with two concave metals and a convex eye-glass and to shew its effects. Let the given focal distances of the lesser and the larger concave and of the convex eye-glass, be equal respectively to the lines t, T, q ; and in a given line $ctqCl$, designed for their common axis, take in one and the same direction, $ct = t$, $tq = T$, $qC = \frac{t \times t}{T}$ and $ql = q$; and place the eye-glass at l , the lesser concave at c , and the larger at C ; so that their concavities may respect each other; and let the incident rays, as QA, QB , be reflected from the larger to the lesser concave, and from thence to the larger again, where let them pass through a moderate hole made in the middle of it at C , and then be refracted through the eye-glass kl to the eye at o ; I say a remote object will appear distinct and upright and magnified in the ratio of $T \times T$ to $t \times q$; that is, of the square of the focal distance of the larger concave, to the rectangle under the focal distances of the lesser concave and of the eye-glass.

172. For a pencil of rays QA, QB coming parallel to the common axis, will be reflected from the larger concave ACB to its principal focus T ; where crossing one another, and falling upon the lesser concave acb , they will be reflected from it to the point q . For since the focal distance $TC = T = tq$ by construction; by taking away the common part Tq , we have $tT = qC = \frac{t \times t}{T}$ by construction; that is, we have tT, tc, tq continual proportionals, as they should be^d; and since pl is the focal distance of the eye-glass kl , the

^d Art. 71.

rays

rays that flow from q will emerge from it in parallel lines, and therefore will produce a distinct appearance of the remote point \mathcal{Q} from which they came.

Fig. 167.

173. Let ST be the image of the object $P\mathcal{Q}$ formed by reflection from the large concave; and it will be terminated by the line PES , drawn through E , the center of this concave, parallel to the rays PA, PA that flow from P^a . Again, the rays that flow from this image ST , will be reflected from the lesser concave and form a second image pq ; which will be terminated by the line $Se\phi$ drawn through the center e of this concave^b; and the rays that diverge from ϕ will emerge from the eye-glass kl in the lines ko parallel to the line ϕl , drawn through the center of the eye-glass^c. Therefore the object $P\mathcal{Q}$ will appear upright, because the rays ko lye on the same side of the common axis $\mathcal{Q}l$ as the point P from which they came.

^a Art. 79.

^b Art. 79.

^c Art. 98.

174. In the second image pq take a line qs equal to the first image TS ; and if the image pq was equal to qs , the object would appear through the eye-glass under an angle equal to q/s ^d; which is to the angle $PE\mathcal{Q}$ or SET , under which it appears to the naked eye at E , as TE or TC to ql ^e; and so the object would be magnified in the same ratio as in Sir *Isaac Newton's* telescope. But since the triangles epq, eST are similar; and since we had tq to te (as te to tT ^f, and disjointly as eq to eT , that is,) as pq to ST or qs ; it appears that pq is bigger than qs , and also the visual angle kol or ϕlq bigger than q/s , in the said ratio of tq to te . And so the object being farther magnified in this ratio of tq to te or, by construction, of TC to tc , is magnified in the whole in the compound ratio of TC to tc , and of TC to ql , that is in the ratio of TC square, to the rectangle under tc and ql .

^d Art. 151.

155.

^e Art. 134.

^f Art. 172.

To adapt it to a near object.

^g Art. 72.

^h Art. 172.

And to short-sighted eyes.

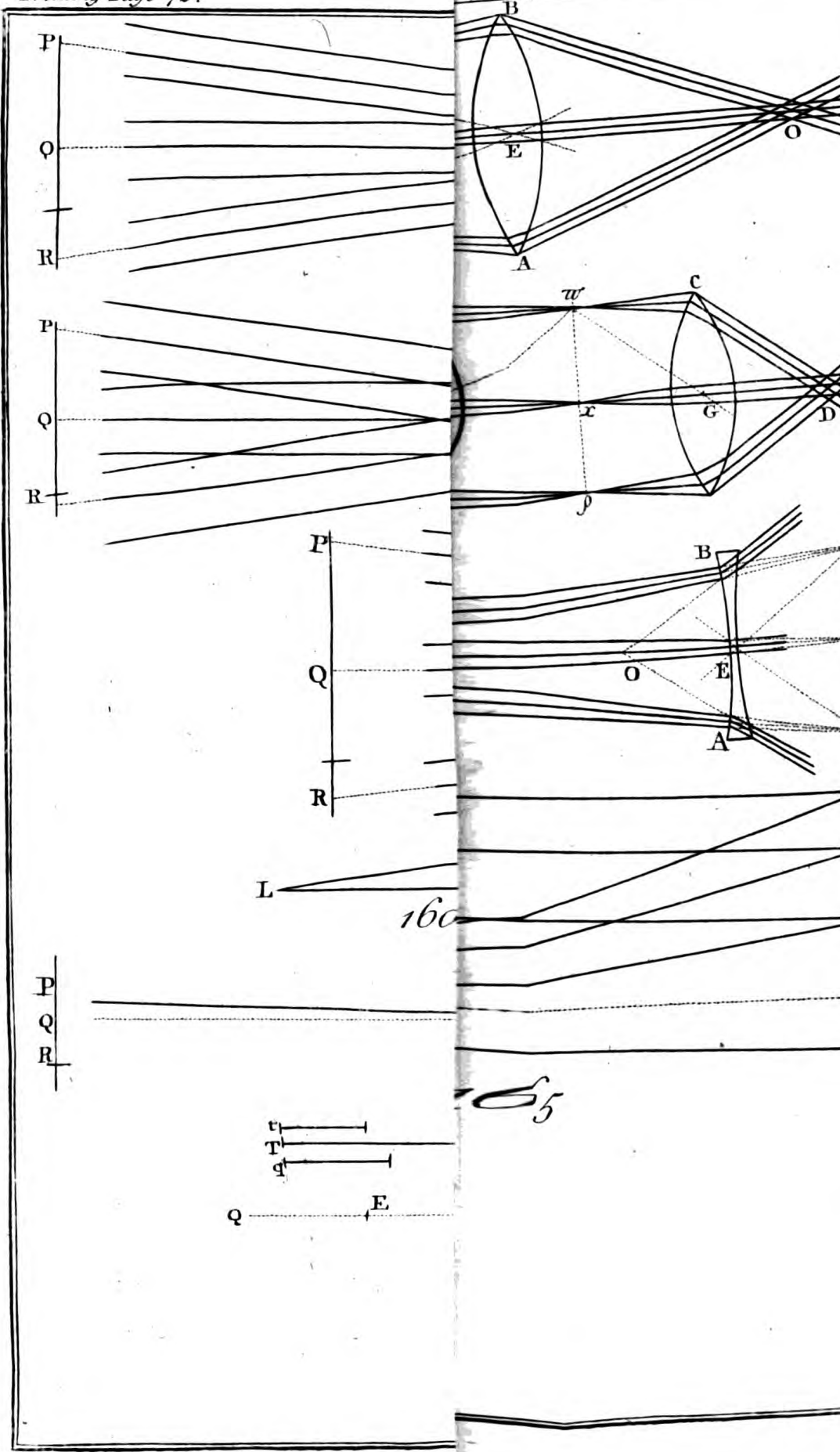
175. For viewing near objects the little concave must be removed a little from the large one. Because while a remote object approaches, its image TS will also approach towards t ^g; and while tT is diminished, its reciprocal tq will be increased^h.

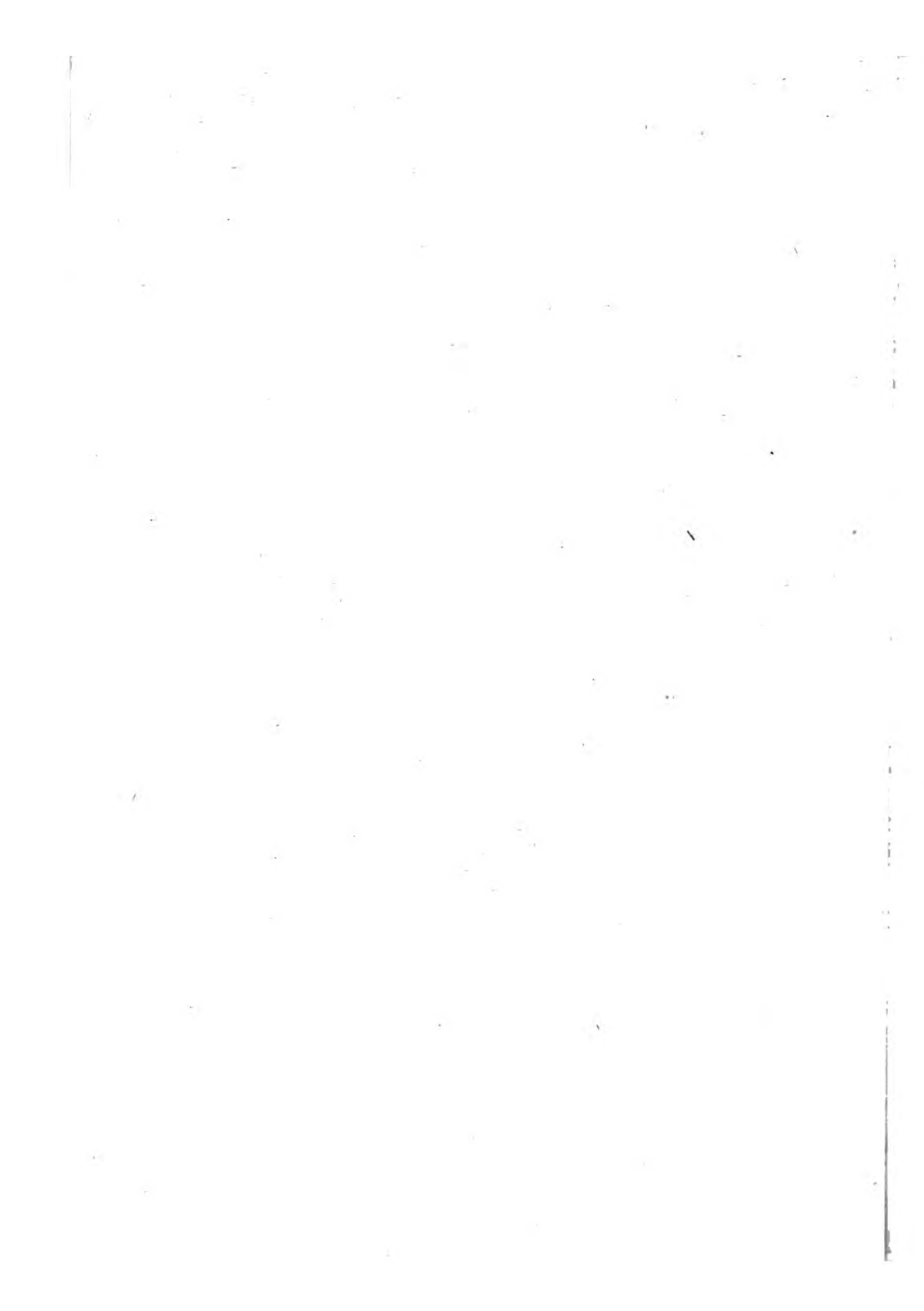
176. Therefore to fit this telescope for a short-sighted person, since the eye-glass is usually fixt, the little speculum must be moved somewhat nearer to the large one. For then the interval tT will also be diminished and its reciprocal tq will be increased; and so the rays will fall upon the eye-glass diverging from a nearer point than its focal distance; and consequently will emerge from it diverging upon the eye.

A more general rule for the magnifying power.

177. By a farther contraction of the interval between the concaves, the image pq may be projected through the hole in the large concave,

Fronting Page 72.





concave, to any given place behind it; and by removing the eye-glass to the same distance from the images as before, the vision would become distinct again; and the object would be more magnified than before, as much as the ratio of tq to te or tc is made bigger than the ratio of TC to tc ; as appears by the demonstration^a. But by^a Art. 174. enlarging the image pq , it becomes more obscure and imperfect, and consequently the appearance of the object less bright and distinct. Besides, as the image becomes larger, the less of it, and of the object, can be seen at one view through a given eye-glass.

178. All things being fixt in their places, the diameter of an object taken in at one view, is proportionable to the breadth of the eye-glass, if the hole in the large concave does not limit it. For the angle of reflection pce , at the middle point of the lesser concave, being equal to the angle of incidence $eccS$; it appears, that while pq and kl are increased or diminished in any ratio, the image ST and the object PQ will also be increased or diminished in the same ratio. The visible area is as the breadth of the eye-glass.

179. Now if an eye-glass of a given focal distance and convexity, be made very broad, it will become too thick; and so the rays will fall too obliquely upon one or both its surfaces near the margin of it; and this obliquity will cause too many of them to be reflected, and the rest that are transmitted, to be too much refracted, in comparison to those pencils that pass through the middle of the said lens. Therefore to increase the visible area of the object, it is necessary to project the image pq two or three inches beyond the hole in the large concave, and to intercept the rays that are tending towards it, with a thinner and broader convex glass fg put close to the backside of this concave; which glass will cause the rays to converge quicker than before, and to form an image vx nearer to it, and smaller, than pq ; both being terminated by a line pvg drawn through the center of this glass. And then the rays of each pencil diverging from this new image vx , must be received by another convex eye-glass bi , that shall make them emerge towards the eye in parallel lines. A meniscus glass, whose convex side is placed towards the converging pencils fvb , is fittest for this purpose; because the rays will pass through its edges less obliquely, than through a glass of any other shape. Is enlarged by two eye-glasses. Fig. 168, 169.

180. To prevent collateral rays, that pass by the sides of the smaller concave, through the hole in the larger, and those also which are reflected from the imperfect margins of them both, from entering into the eye; it is necessary to place a thin plate with a proper hole in it to circumscribe the image at x , and also another very small

K

hole

hole at o , where all the pencils cross one another immediately before they enter the eye. The breadth of this latter hole must be no bigger than that of the principal pencil at o , and the places of them both must be exactly adjusted; otherwise the telescope can have no good effect.

The little concave speculum may be changed for a convex one. Fig. 170.

^a Art. 71.

^b Art. 79.

181. Telescopes of this kind are sometimes made with a little convex speculum instead of the concave one. If their focal distances be equal, and the vertex of the convex de , be placed at e , where the center of the concave was, the telescope will magnify in the same ratio as before; but will shew the object inverted; unless it be set upright by three convex eye-glasses, as in a dioptrick telescope. For a pencil of rays converging from the large concave towards its focus T , being intercepted by the little convex de , will be reflected by it to the same point q as before by the little concave bc . For the point t being the principal focus of both these little speculums, we have tT, te (or tc) and tq continual proportionals as before^a. Through any point S of the first image ST and through the center e of the little concave, draw Sep terminating the image pq formed by this concave^b; in like manner through c the center of the little convex de , and through the same point S , draw cSr terminating the image qr formed by this convex. These images pq, qr lye on contrary sides of the axis, and therefore the object appears in contrary positions. But these images are equal, and of consequence the object appears equally magnified. For we have $tq : te :: te : tT :: tq = te : te = tT$, that is $:: eq : eT :: cq : cT$. And the triangles peq, TeS being similar, and also qcr, Tcs , we have $pq : ST (:: eq : eT :: cq : cT ::) qr : ST$; therefore pq is equal to qr .

Double microscope considered. Fig. 171.

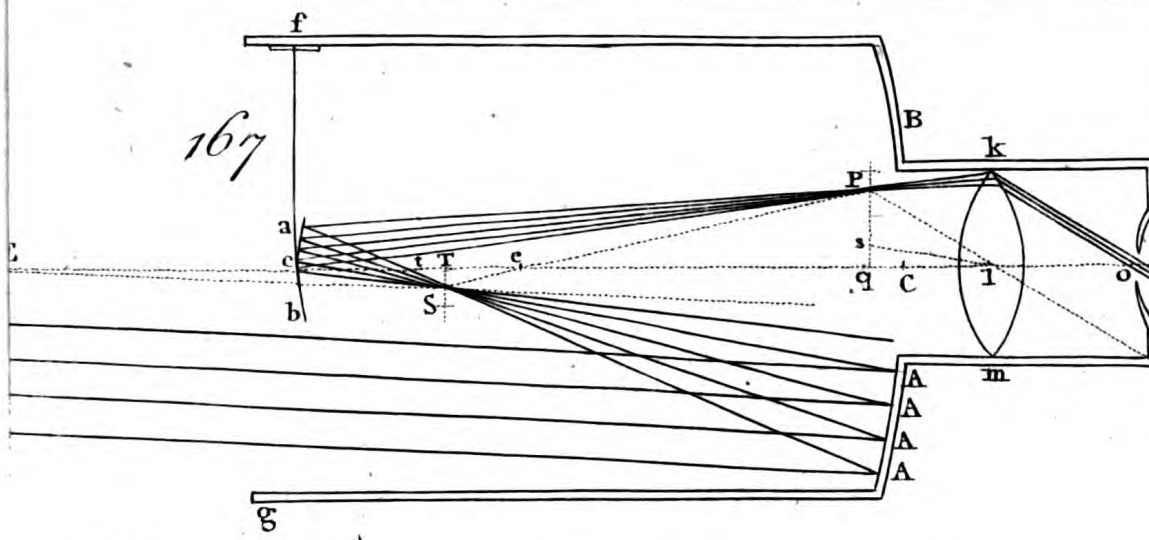
^c Art. 108.

^d Art. 115.

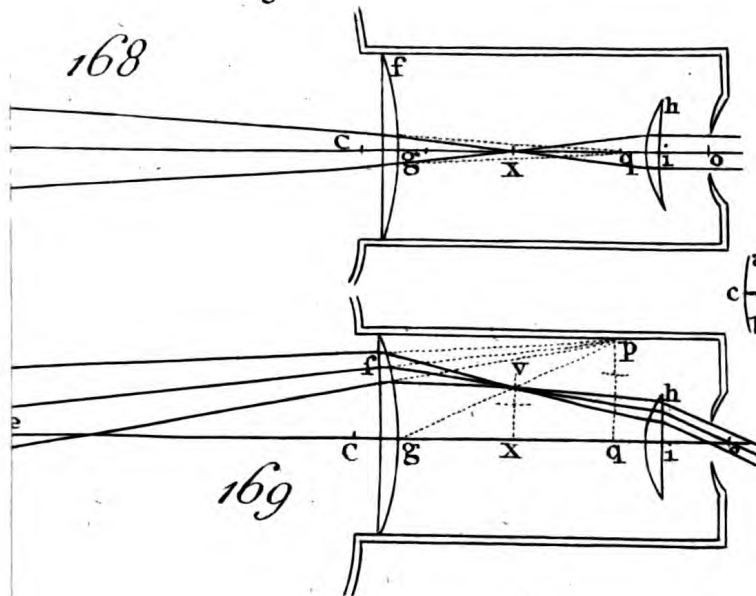
^e Art. 115.

182. A double microscope is composed of two convex glasses placed at E and L . The glass L next the object PQ is very small and very much convex, and consequently its focal distance LF is very short; the distance LQ of the small object PQ is but a little greater than LF ; so that the image pq may be formed at a great distance from the glass^c, and consequently may be much greater than the object itself^d. This picture pq being viewed through a convex eye-glass AE , whose focal distance is qE , appears distinct as in a telescope. Now the object appears magnified upon two accounts; first because if we viewed its picture pq , with the naked eye, it would appear as much greater than the object, at the same distance, as it really is greater than the object, or as much as Lq is greater than LQ ^e; and secondly because this picture appears magnified through the eye-glass as much as the least distance at which it can be seen distinctly with the naked eye, is greater than qE , the focal

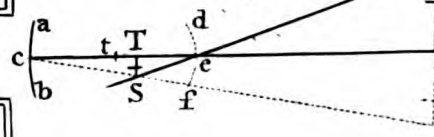
167



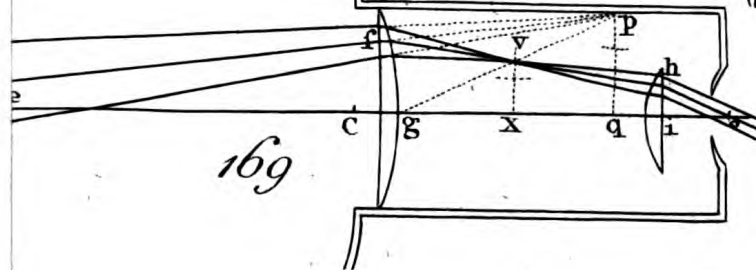
168



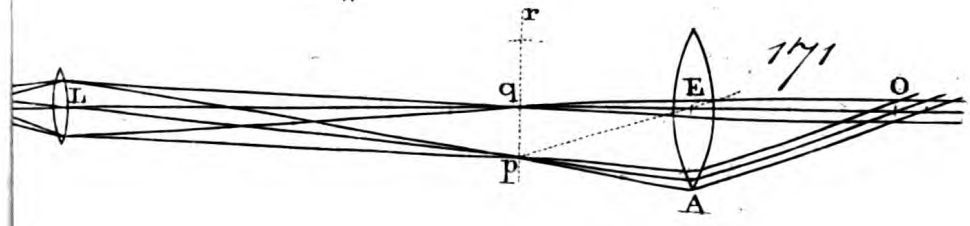
170



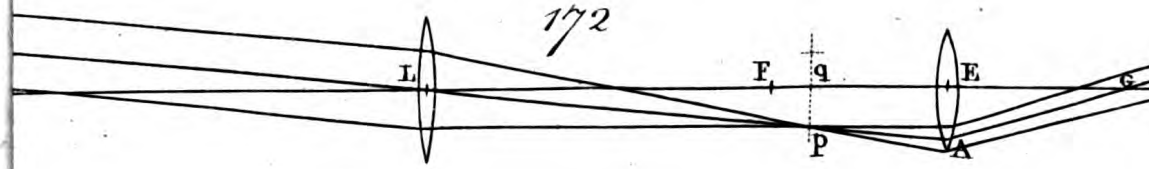
169



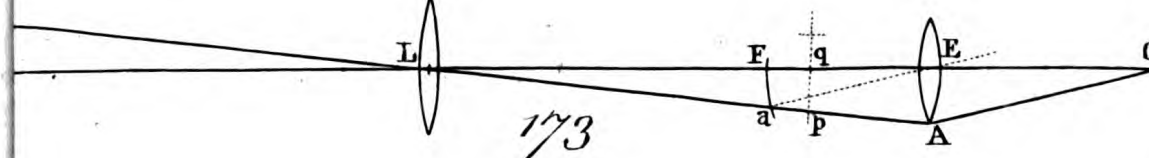
171



172



173



focal distance of the eye-glass ^a. For example, if this latter ratio be ^a Art. 162. 5 to 1, and the former ratio of Lq to LQ be 20 to 1, then upon both accounts the object will appear 5 times 20, or 100 times greater than to the naked eye.

183. To fit these telescopes and microscopes to short-sighted eyes, To fit tele-
the glasses E and L must be placed a little nearer together; so that scopes and
the rays of each pencil may not emerge parallel but may fall di- microscopes
verging upon the eye ^b; and then the apparent magnitude will be ^b eyes.
altered a little but scarce sensibly; as is demonstrated in the next ^c Art. 104.
article. ^d 108. 111.

184. Suppose the interval LE between the two convex glasses to ^a A more gene-
be greater or less than the sum of their focal lengths, and let EF be ral demon-
the focal distance of the eye-glass, and Lq that of the object-glass; stration of
I say the apparent magnitude will be to the true as LF to FE , that telescopes.
is, as the interval of the glasses diminished by the focal distance of Fig. 172, 173,
the eye-glass, to the focal distance of the eye-glass. For the axes of 174, 175.
all the pencils which pass through L , as PLA , will be refracted by
the eye-glass to a focus G , where the eye being placed will see the
whole object PQ , though the aperture of the pupil and of the object-
glass be never so small ^c; and the object PQ will appear under the ^c See note to
angle AGE . But L being a focus of incident rays upon the eye- Art. 164.
glass, we have $LF : LE :: LE : LG$ ^d, and disjointly $LF : FE ::$ ^d Art. 107.
 $(LE : EG ::$ the angle EGA , to the angle ELA ^e or PLQ ^f) as the ^e Art. 134.
apparent magnitude to the true. ^f Art. 61.

185. Hence according as the interval of the glasses is greater or less than the sum of their focal distances, the apparent magnitude is to the true, in a greater or less proportion than that of the focal distances.

186. The brightness of the appearance through a given telescope ^a The apparent
or microscope is more or less in proportion to the aperture of the brightness
object-glass. For supposing it covered with paper, all but a small through
hole in the middle, the magnitudes of the pictures pq in the focus them.
of the glasses, and of that upon the retina would not be altered; but
the hole at L being smaller than before, there are fewer rays in every
pencil, and consequently in every point of those pictures, and so they
appear more obscure. If the aperture and object-glass remain the
same, things appear brighter or fainter according as the focal di-
stance of the eye-glass is longer or shorter; that is, according as the
telescope or microscope magnifies less or more ^b. For the same ^b Art. 164.
quantity of light spread over a smaller or larger picture or part of 182.
the retina will make it brighter or duller.

All appearances the same when the eye is out of the axis of the glass or glasses.
Fig. 176.

187. Hitherto I have supposed the eye to be always placed at some point O in the common axis of the refracting or reflecting surfaces. Now let it be placed at any point o in a line Oo perpendicular to the axis Qq ; I say that all the appearances will be the same or at least not sensibly different from what they were before. For let $p q$ be the last image of an object, and $P Q$ the last but one, or the object itself; draw the lines po, qo meeting the next surface in a and c ; and the points P and Q will appear to the eye at o in the directions of those lines oa, oc . Whence drawing pO meeting the surface in A ; since the directions OA, oa , in which P is seen, lye the same way from the directions OC, oc , in which Q is seen, it is evident that the apparent situation of the extremities P, Q is the same at both places of the eye; and also the apparent magnitude, which is measured by the angle aoc^a or poq or pOq or AOC . For the small angles poq, pOq , being subtended by the same image $p q$, very nearly at equal distances po, qO from o and O , are very nearly equal. The apparent brightness of the object is also the same; because the density of the rays, that enter the pupil, at any part of the perpendicular plane represented by Oo , is nearly the same^b. For the rays flow from or towards the last image $p q$ just as if it was a luminous body. And lastly the degree of apparent distinctness or confusion is the same also, because the angles which the pupil, placed at O or at o , subtends at p and q , or the mutual inclinations of the rays in each pencil are very nearly equal.

^a Art. 148.

^b Art. 25.

A general observation upon vision.

^c Art. 160.

^d Art. 148.

^e Art. 147.

The portable camera obscura.
Fig 177.

188. This general observation upon vision is worth remembering. That the apparent distinctness and confusion of an object depends upon the mutual inclination of the rays to each other in any one pencil when they fall upon the eye^c; the apparent magnitude, upon the inclination of the rays of different pencils to each other when they fall upon the eye^d; the apparent situation, upon the real situation of the extrem pencils when they fall upon the eye^e; and the apparent brightness and obscurity, upon the quantity of rays in every pencil.

189. The portable *camera obscura*, or dark chamber, commonly sold in the shops, will require but a short description. The theory is this; the rays that come from the object PQR after passing the lens E are tending to form an image pqr ; but being reflected upwards by the looking-glass ABC , they form an horizontal image $w x y$ upon a glass plane, whose unpolished side lyes uppermost; upon which a copy of the picture may be sketched out with a black lead pencil; and to the spectator facing the object, the picture appears upright.

upright. This figure represents a section of the machine through the axis of the tube that holds the lens, and through the middle of the square box and the looking-glass within it. The section of the side opposite to the tube is not here represented, it being a door that opens sideways; the edges of the rough glass at the top are placed in two grooves upon the sides of the box; and being taken off, it is placed in a drawer *ef* at the bottom of the box; the looking-glass *ABC* may also be drawn out of the grooves in the sides of the box and lodged in the same drawer. The square wooden tube consists of 3 parts; the innermost that carries the lens, draws outwards or inwards to make the pictures distinct. The parts *gb* and *ik*, being fixt together and to the box with small bolts, may be taken asunder and put into the box; then the lid *at* at the top, and the door at the end, being both shut and fixt, the machine becomes more commodious for carriage. The inside of the lid whose section is *at*, has two wings, that open to right angles on each side of it, and rest upon the sides of the box, to shade the image upon the rough glass.

190. The construction of the magick lantern is briefly this; *ABCD* is a tin lantern, from whose side there proceeds a square or round arm or tube *bnkclm*, consisting of two parts; the outermost whereof *nklm* slides over the other, so as that the whole tube may be lengthened or shortened thereby. In the end of the arm *nklm* is fixt a convex glass *kl*: about *de* there is a contrivance for admitting and placing an object *de* painted in dilute and transparent colours on a plane thin glass; which object is there to be placed inverted. This is usually some ludicrous or frightful representation, the more to divert the spectators: *bbc* is a deep convex glass, so placed in the other end of the prominent tube, that it may strongly cast the light of the flame *a* on the picture *de* painted on the plane thin glass. And here it is to be noted, that the glass *bbc* is only designed for the strong illumination of the picture *de*, and has nothing to do in the representation; and therefore in some of these lanterns, instead of the glass *bbc*, we shall find a concave speculum so placed, that it may strongly cast the light of the flame *a* on the picture at *de*; and sometimes both are used.

Description
of the magick
lantern by
Mr. Molyneux.
Fig. 178.

191. Wherefore let us now consider the picture *de* as a very lightsome object of distinct colour and parts. And let us conceive *de* more remote from the glass *kl* than its focus. It is then manifest that the distinct image of the object *de*, shall be projected by the glass *kl* on the opposite white wall *FH* at *fg*; and here it shall be represented erect. For now the whole chamber *EFGH* is dark, the lantern

lantern $ABCD$ inclosing all the light; so that in effect this appearance of the magick lantern is no more than what we are told concerning the representation of outward objects in a dark room by a convex glass; and here we may observe, that if the tube be contracted, and thereby the glass kl brought nigher the object de , the representation fg shall be projected so much the larger; and so much the more distant from the glass kl ; according to the rules before laid down^a. So that the smallest picture at de may be projected at fg in any greater proportion required, within due limits. From whence the name of *Lanterna Megalographica*. And consequently, protracting the tube and drawing the glass kl more distant from the object de , will diminish the representation fg , and project it nigher the glass kl .

^a Art. 104.
208. 115.

A comparison
of different
ways of illu-
minating mi-
croscopical
objects, pi-
ctures in a
magick lan-
tern, &c.
Fig. 179, 180,
181.

192. Of a luminous object QR let qr be the image formed by reflection from a concave surface, or by refraction through a convex lens, or sphere AC ; whose center is E , principal focus F , axis QEF , and semiaperture AC ; and let a perpendicular FG , to the axis, cut the outermost ray QA in G ; I say the brightness of the several pictures qr , will be very nearly as \overline{FG}^2 directly and \overline{FE}^2 inversely.

For, not regarding the small losses of light by the several reflections and refractions, the quantity collected to the point q is very nearly as $\frac{\overline{AC}^2}{CQ^2}$ ^b, and consequently the quantity in the area of the

^b Art. 25.

whole picture qr , as $\frac{\overline{AC}^2}{CQ^2} \times \overline{QR}^2$ or $\frac{\overline{FG}^2}{FQ^2} \times \overline{QR}^2$. But the area of

the picture is as $qr^2 = \frac{\overline{Eq}^2}{EQ^2} \times \overline{QR}^2 = \frac{\overline{FE}^2}{FQ^2} \times \overline{QR}^2$. Because, in the

^c Art. 71.

reflecting concave, we have $Fq : FE :: FE : FQ$ ^c; and consequently $Eq : EQ :: FE : FQ$; and in the lens and sphere we have $Qq : QE :: QE : QF$ ^d, and consequently $Eq : EQ :: FE : FQ$. Therefore

^d Art. 107.

the brightness of the picture, or the density of the rays in its area, being as their quantity directly and the area inversely, is as $\frac{\overline{FG}^2}{\overline{FE}^2}$

very nearly; and the more exactly as the aperture is smaller and the object farther off.

193. *Corol. 1.* In a given speculum, lens or sphere, the brightness of the picture of a given object, is as \overline{FG}^2 ; and therefore increases continually with the distance of the luminous object from the focus F .

194. *Corol.*

194. *Corol. 2.* If the luminous object be very remote, and the apertures of several specula, lenses and spheres be equal to one another, the degrees of brightness of the several pictures formed by them are reciprocally as the squares of their respective focal distances very nearly.

195. *Corol. 3.* Consequently if the several apertures be equal portions of equal spheres, the degrees of brightness of the several pictures formed by a concave speculum, a double-convex glass, a glass sphere and a plano-convex glass, are respectively as the squares of the decreasing musical progression 12, 6, 4, 3. Because the respective focal distances are $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$ of the diameter of the given sphere, by Art. 69, 103, 95; and the reciprocals of an arithmetical progression are called a musical progression.

196. *Corol. 4.* Therefore the concave speculum has greatly the advantage of the sphere and lenses, for illuminating microscopical objects, and also for burning things in the sun-shine, though not in so great a proportion, as will appear by the next corollary.

A comparison of the burning powers of glasses and speculums,

197. *Corol. 5.* Though the rays in two pictures of the sun formed by similar specula, be equally dense^a, yet the picture formed by the larger speculum, being proportionably larger, will burn things more vehemently than the smaller; because the burning particles of matter communicate and propagate their heat to one another. And when the specula are similar, the aberrations of the rays from the peripheries of the pictures, are also similar^b.

^a Art. 192.

^b Art. 213.

C H A P. XI.

TO DETERMINE THE ABERRATIONS OF RAYS, FROM THE GEOMETRICAL FOCUS, CAUSED BY THEIR UNEQUAL REFRACTIBILITY, AND ALSO BY THE SPHERICALNESS OF THE FIGURE OF REFLECTING AND REFRACTING SURFACES.

P R O P O S I T I O N I.

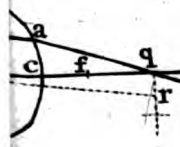
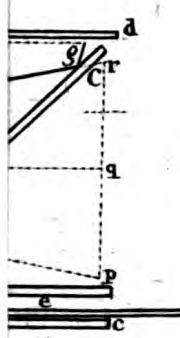
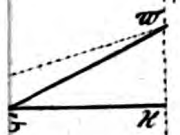
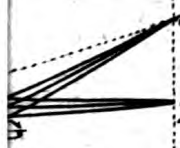
198. **L**ET the common sine of incidence be to the sine of refraction of the least refrangible rays as I to R , and to the sine of refraction of the most refrangible rays as I to S ; and the diameter of the least circular space into which heterogeneal parallel rays can be collected, by a spherical surface or by a plano-convex lens, will be to the diameter of its aperture, in the constant ratio of $S - R$ to $S + R - 2I$.

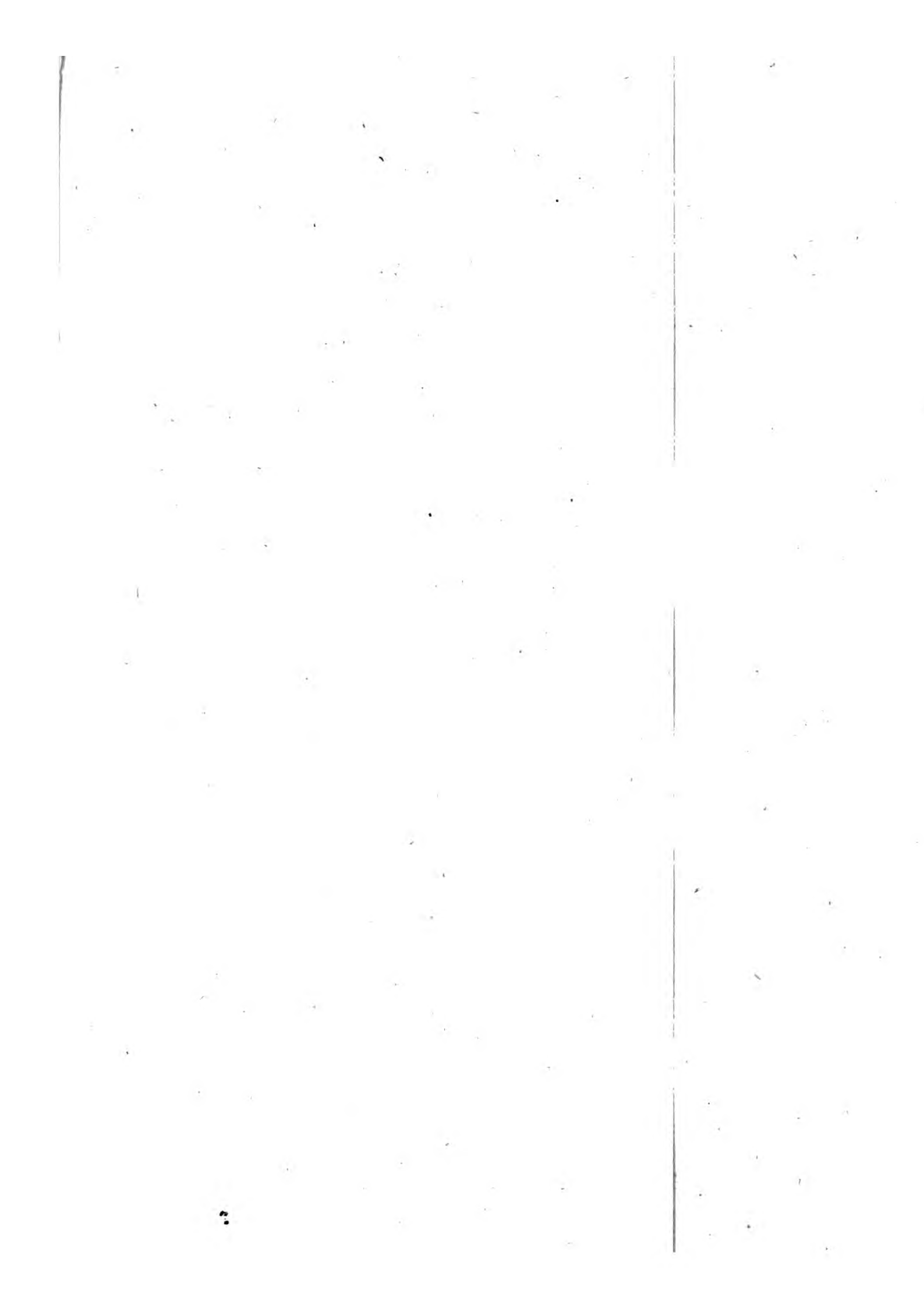
Fig. 182.

For let an heterogeneal ray PA fall upon a spherical surface ACB , and let it be separated by refraction into the rays AF , Af , cutting the axis EC , drawn parallel to PA , in F and f . Take the arch CB equal to CA , and let another heterogeneal ray PB , coming parallel to PA be refracted into the lines BF , Bf , cutting the two former rays in R and S . Join RS and produce it till it meets the incident rays produced in I and K , and the perpendiculars EA , EB to the refracting surface at the points A , B , in H and L . And when AB , the breadth of the aperture or of the pencil, is but moderate, and consequently the refractions at A , B but small, the angles of incidence and refraction, HAI , HAR , HAS , or the arches that measure them, or their perpendicular subtenses HI , HR , HS , will be to each other very nearly in the same given ratios as those of the sines I , R , S of those angles^a. And disjointly the differences of those subtenses will be proportionable to the differences of these sines: that is, the line $RS : RI :: S - R : R - I$, and doubling the consequents, $RS : 2RI$ or $IK - RS :: S - R : 2R - 2I$; and conjointly $RS : IK$ or $AB :: S - R : S + R - 2I$. From this given ratio of RS to AB in which they increase or decrease together, it appears that all the intermediate rays which fall upon AB will pass through RS . And when parallel rays fall perpendicularly upon the plane side of a plano-convex lens, they are refracted only at their emergence from its convex surface; and so the aberrations are the same in both cases. *Q. E. D.*

^a Art. 68. 84.

199. *Corol.*





199. *Corol. 1.* Hence the diameter RS , of the circle of aberrations that contains all the incident rays, is a 55th part of the diameter AB of the aperture of a plano-convex glass, whatever be its focal distance. For supposing AR and AS to be the outmost red and indigo rays, their sines of incidence and refractions I, R, S are to each other as 50, 77, 78^a. Whence $S - R$ is to $S + R - 2I$ as^a Art. 31. 1 to 55.

200. *Corol. 2.* The diameter of the least circle that can receive the rays of any single colour or of several contiguous colours is also determinable from the proportions of their sines. Thus all the orange and yellow is contained in a circle whose breadth is the 260th part of the breadth of the aperture of the plano-convex glass; the sines of the outmost orange AR and yellow AS being to the common sine of incidence as $77\frac{1}{3}$ and $77\frac{1}{3}$ to 50^b. ^b Art. 31.

201. *Corol. 3.* In different surfaces, or plano-convex glasses, the angles of aberration RAS are as the breadths of the apertures AB directly and as the focal distances CF inversely; because any angle, as RAS , is as its subtense RS directly and as its radius AR or CF inversely.

202. *Corol. 4.* If several glasses of several sorts or shapes have the same focal distance, and the same aperture; the diameter of the circle of aberrations of heterogeneous parallel rays from their principal focus, will be the same in them all; being the same as in a plano-convex glass when its plane side is turned to the incident rays^c; and^c Art. 57. 58. may therefore be determined by art. 198. And when the rays in the Fig. 182. incident pencil are either parallel or inclined to the axis of the lens, the diameter of the circle of aberrations is as its distance from the lens; because the angle RAS is invariable.

203. *Corol. 5.* Therefore with respect to these aberrations by colours separately considered, it is indifferent which side of the lens is turned to the incident rays; because its focal distance is the same in both positions^d. ^d Art. 101.

LEMMA.

204. *The versed sines AB, AC of very small arches BD, CD , of Fig. 183. 184. unequal circles BDG, CDH , that have the same right sine AD , are reciprocally proportionable to their diameters BG, CH very nearly; that is $AB : AC :: CH : BG$.*

For since the rectangles under BAG and CAH are each equal to the square of AD ^e, and consequently to each other; their sides are^e Euc. VI. reciprocally proportionable^f, that is AB is to AC as AH to AG or^f Euc. VI. AS ¹⁴.

L

as CH to BG very nearly, when the versed sines are incomparably less than the diameters themselves^a. $\mathcal{Q}, E. D.$

^a Art. 68.

PROPOSITION II.

Fig. 185.

205. When homogeneous parallel rays NA, EC fall upon a spherical surface AC whose center is E , the longitudinal aberration FT of any refracted ray AT from F the focus of the pencil, is to the versed sine of the arch AC intercepted between the point of incidence and the axis ECF , in the given ratio of the square of the sine of refraction, to the rectangle under the sine of incidence and the difference of the sines very nearly; and the aberration is the same when the rays fall perpendicularly upon the plane side of a plano-convex lens.

For when the refraction is made in the passage of a ray NA from a denser to a rarer medium, the intersection T of the refracted ray AT with the axis ECF , lies between the refracting surface and its focus F . With the center T and semidiameter TA having described the arch AD cutting the axis in D , draw the sine AP of the arches AC, AD , and also EN and EM the sines of incidence and refraction, for which put n and m ; then because the triangles ETM, ATP are similar, it will be as $ET : TA$ or $TD :: (EM : AP$ or $EN ::) EF : FC$ ^b; and disjointly $TF : EF :: (FC - TD$ or) $TF - CD : FC$; and alternately $TF : TF - CD :: EF : FC$; and disjointly $TF : CD :: (EF : EC ::) m : m - n$ ^c. Again since $(PD : PC :: CE : DT$ ^d or FC ^e, and conjointly) $CD : CP :: (EF : FC ::) m : n$; by compounding this and the foregoing proportion, it will be as $TF : CP :: mm : m - n, n$. $\mathcal{Q}, E. D.$

^b Art. 93.

^c Art. 92.

^d Art. 204.

^e Art. 68.

206. *Corol. 1.* The segment $ACBPA$ may be considered as a plano-convex lens; and when rays fall parallel upon its plane side, the longitudinal aberration of the extreme ray falling upon A is equal to $\frac{2}{3}$ of its thickness PC , as appears by putting 3 and 2 for m and n respectively.

207. *Corol. 2.* Also this aberration $FT = \frac{mm}{m-n, n} \times \frac{AP^2}{2EC} =$

^f Euc. VI. 13. $\frac{mm}{m-n} \times \frac{AP^2}{2CF}$. For $PC = \frac{AP^2}{2EC}$ very nearly^f, and $EC = \frac{m-n}{n}$
^g Art. 92. $\times CF$ ^g.

208. *Corol. 3.* Let the refracted ray ATG produced, cut the line FG , perpendicular to the axis, in G , and the lateral aberration

FG

$$FG = \frac{mm}{nn} \times \frac{AP^3}{2EC^2} = \frac{mm}{m-n|^2} \times \frac{AP^3}{2CF^2}. \text{ For } FG : TF :: AP :$$

$$TP \text{ or } CF \text{ or } \frac{n}{m-n} \times CE.$$

209. *Corol. 4.* When the semidiameter of the convexity or the focal distance is given, the longitudinal aberrations are as the squares, and the lateral aberrations as the cubes, of the linear apertures of a plano-convex lens.

PROPOSITION III.

210. *When parallel rays QA, EC are reflected from a spherical concave ACB whose center is E and whose aperture ACB is but small, the longitudinal aberration TF of the extrem ray AT from the geometrical focus F, is equal to half the versed sine CP of the semiaperture AC very nearly.* Fig. 186.

In fig. 185. imagine *EM*, the sine of refraction to be diminished to nothing, and then to become negative and equal to *EN* the sine of incidence, and the refraction of the ray to be changed to reflection as in fig. 186; and by the former proposition it will be as *TF* : *CP* :: *mm* : $-\overline{m-n}, n$:: *nn* : $-2nn$:: 1 : -2.

But the particular proof is this. By the last lemma the versed sine *CP* nearly equals $\frac{1}{2}$ the versed sine *PD* of the arch *AD* whose center is *T* and semidiameter *TA* or *TE* or $\frac{1}{2}$ the semidiameter of the arch *AC*^a very nearly. But $2TF = 2TE - 2EF = ED - EC$ ^a Art. 69. = *CD* exactly or *CP* nearly. Therefore *TF* = $\frac{1}{2}CP$ nearly.

211. *Corol. 1.* We had $2TF = CD$ exactly; which is the excess of the secant *ED* of the arch *AC* above its radius *EA*. For joining *AD* the angle *DAE* in the semicircle *DAE* is a right one.

212. *Corol. 2.* The longitudinal aberration $TF = \frac{AP^2}{4CE}$. For $CP = \frac{AP^2}{2CE}$ nearly^b.

^b Euc. VI. 13.

213. *Corol. 3.* The lateral aberration $FG = \frac{AP^3}{2CE^2}$. For *FG* : *FT* :: *AP* : *PT* or $\frac{1}{2}CE$ nearly.

214. *Corol. 4.* When the diameter of the concave, or its focal distance, is given, the longitudinal aberrations are as the squares, and the lateral ones as the cubes of the diameters of the apertures.

PROPOSITION IV.

215. When parallel rays of any one sort are refracted by a plano-convex object-glass, or when rays of all sorts are reflected by a spherical concave, the diameter of each circle of aberrations caused by the sphericity of the figures, is equal to $\frac{1}{2}$ the lateral aberration of the extrem ray in each; and therefore is given by the former propositions.

Fig. 187, 188. Let $\alpha Y\tau$ be any refracted or reflected ray cutting the axis ECT in τ , and the extrem ray ATG , that comes from the contrary side of the axis, in Y . Draw YX perpendicular to the axis, and supposing the line ATG immoveable, as the point of incidence α moves from the vertex C , the perpendicular XY will first increase, because the angle $C\tau\alpha$ continually increases, and afterwards will decrease, because the line $T\tau$ continually decreases; and when XY is the greatest, it is evident that all the rays, incident upon the same side of the axis as itself, will pass through it. To find its greatest quantity, let the incident ray $q\alpha$ cut the chord APB in β , and supposing the variable aperture $P\beta = v$, the variable $TX = x$ and the given lines $PA = a$, $PT = f$, $TF = b$; by cor. 4. prop. 2 and 3, the aberration $F\tau$ is to the aberration FT (b) as $\pi\alpha^2$ or $P\beta^2$ (vv) to PA^2 (aa). Wherefore $F\tau = \frac{vv}{aa}b$ and thence $TF - F\tau = T\tau = \frac{b}{aa} \times \overline{aa - vv}$. Again PT (f) : PA (a) :: TX (x) : $XY = \frac{ax}{f}$; also $\pi\alpha$ (v) : $\pi\tau$ or PT (f) :: XY ($\frac{ax}{f}$) : $X\tau = \frac{ax}{v}$. Hence again $T\tau$, or $X\tau + XT = \frac{ax}{v} + x = \frac{b}{aa} \times \overline{aa - vv}$ found before; or $\frac{x}{v} \times \overline{a + v} = \frac{b}{aa} \times \overline{a + v} \times \overline{a - v}$. Whence $x = \frac{b}{aa} v \times \overline{a - v}$, and therefore x or TX is the greatest possible when the rectangle $v \times a - v$, or $P\beta \times \beta B$ is greatest, that is when its sides ^a $P\beta$, βB are equal, or when $v = \frac{1}{2}a$. Substitute this value for v in the last equation and it gives the greatest value of $x = \frac{1}{4}b$ or the greatest $TX = \frac{1}{4}TF$, and therefore the greatest $XY = \frac{1}{4}FG$, because $TX : XY :: TF : FG$, and this XY turned about the axis PX describes the circle of aberrations through which all the rays falling up AB will just pass. Q. E. D.

^a Euc. VI. 13.

PROPOSITION V.

216. *If the density of the reflected rays in the circle of aberrations be* Fig. 188. *uniform, it is to the density of the incident rays falling perpendicularly upon a plane AP, as the whole surface of the sphere of which the speculum is a portion, to the area of a circle whose diameter is the versed sine PC of the small arch AC very nearly, and the more exactly as this arch is smaller; supposing also that all the incident rays are reflected.*

For since the very same rays pass through two circles described by the lines AP and XY turned about EC; their densities in these circles are reciprocally as the circles themselves; that is, the density of the reflected rays, is to the density of the incident rays, as AP^2 to XY^2 ^a, or $\frac{1}{16} FG^2$ ^b, or $\frac{AP^6}{16 \times 4 CE^4}$ ^c; that is, putting D for ^a Euc. XII. 2. $2CE$, as $4D^4$ to AP^4 ; that is, as $4D^2$ to PC^2 , (because D, AP, PC are very nearly continual proportionals^d;) that is, as the area of ^d Euc. VI. 8. 4 great circles of the sphere, or the whole surface of the sphere^e, to ^e Cor. the area of a circle whose diameter is PC very nearly. ^e Archim. de Sph. & Cyl.

217. *Cor. 1.* Therefore the greatest density of the reflected rays is at the focus F , considered as a physical point; and is immensely greater than the density of the incident rays. For the proposition above becomes geometrically exact when AP is infinitely diminished, and XY comes to its limit at F ; and the density at F is always the same whether a slender pencil falls upon the speculum or a large one, because the outward rays are reflected wide of the focus F .

218. *Corol. 2.* In like manner when rays fall parallel upon the Fig. 187. plane side of a plano-convex lens, (putting m to n for the ratio of majority of the sines of incidence and refraction) their greatest density at their focus F , is to the density of the incident rays, as the whole surface of a sphere whereof the lens is a portion, to the area of a circle whose diameter is $\frac{m m}{n n} PC$, or in glass $\frac{2}{3}$ of the versed sine of the smallest aperture of the lens; that is immensely great. It follows from art. 208.

219. *Cor. 3.* Therefore the density of reflected or refracted rays in the several points of an image of a very remote object, is also immensely greater than the density of the incident rays of any one pencil. For it would be immensely great, if all the rays of every pencil were rejected, except a few that go near to their axes, and those outward rays being scattered upon points collateral to each point.

point of the image, help to increase the density of the rays in the whole image.

PROPOSITION VI.

220. *The circle of aberrations caused by the sphericity of the figure of the object-glass of a telescope, compared with the circle of aberrations caused by the unequal refrangibility of rays, is altogether inconsiderable.*

Newt. Opt.
p. 83.

For if the object-glass be plano-convex and the plane side be turned towards the object, and the diameter of a sphere whereof this glass is a segment be called D , and the semidiameter of the aperture of the glass be called S , and the sine of incidence out of glass into air be to the sine of refraction as n to m ; the rays which come parallel to the axis of the glass shall in the place where the image of the object is most distinctly made, be scattered all over a little circle

whose diameter is $\frac{m m}{n n} \times \frac{S^3}{D D}$ very nearly, if they were all equally refrangible by article 215 and 208. As for instance, if the sine of incidence n be to the sine of refraction m as 20 to 31, and if D , the diameter of the sphere to which the convex side of the glass is ground, be 100 foot or 1200 inches, and consequently the telescope

Art. 92.

about 100 foot long^a, and S the semidiameter of the aperture be

2 inches; the diameter of this circle of aberrations, that is $\frac{m m}{n n} \times$

$\frac{S^3}{D D}$, will be $\frac{31 \times 31 \times 8}{20 \times 20 \times 1200 \times 1200}$ or $\frac{961}{72000000}$ parts of an inch.

But the diameter of the little circle through which these rays are scattered by unequal refrangibility, will be about the 55th part of the breadth of the aperture of the object-glass^a, which is here 4 inches. And therefore the aberration arising from the spherical figure of the glass, is to the aberration arising from the different re-

Art. 199.

frangibility, as $\frac{961}{72000000}$ to $\frac{4}{55}$, that is as 1 to 5449; and therefore

being in comparison so very little, deserves not to be considered in the theory of telescopes. If we suppose the little circle of aberrations arising from unequal refrangibility, to be 250 times narrower than the circular aperture of the object-glass, it would contain all the orange and yellow, and would permit the other fainter and darker colours to pass by it^c, which perhaps may scarce affect the sense; yet even in this case the aberration caused by the spherical figure, would be to the aberration caused by the unequal refrangibility,

Art. 200.
Newt. Opt.
p. 88.

lity,

lity, in a 100 foot telescope, but as $\frac{961}{72000000}$ to $\frac{4}{250}$, or only as 1 to 1200, which sufficiently proves the proposition. *Q. E. D.*

221. *Corol. 1.* If the focal distances and apertures of a reflecting concave and a plano-convex glass be both the same, the diameter of the circle of aberrations, caused by their figures, will be above 30 times less in the reflecter than in the refracter. For these diameters

are $\frac{AP^3}{16CF^2}$ and $\frac{mm}{m-n|^2} \times \frac{AP^3}{4CF^2}$ by art. 215, 213, and 208; which

are as $\frac{1}{4}$ to $\frac{mm}{m-n|^2}$ or $\frac{31 \times 31}{11 \times 11}$. Hence if the length of each telescope be 100 foot, the lateral aberrations in the reflecter would be 30×5449 or 163470 times less than the lateral aberrations caused by unequal refrangibility in the refracter.

222. *Corol. 2.* The number of pencils, some of whose rays are mixed together in every point of a confused picture, is as the area of the circle of aberrations of the rays in any one pencil; and consequently the mixture of the rays of different pencils, caused by the sphericallness of the figure of an object-glass, if they were all alike refrangible, would be to their mixture caused by their unequal refrangibility, as 1 to 5449×5449 or 29691601 in the present instance. For conceiving any point in the confused picture to be a center of a circle of aberrations, it is manifest that all other equal circles of aberrations, whose centers fall upon the first mentioned circle will cover its center; that is some rays of as many pencils will be mixed in this center as there are points in the circle itself; or, which is the same thing, the number of pencils mixed in this center is as the area of the circle of aberrations.

CHAP. XII.

A REFRACTING OR REFLECTING TELESCOPE BEING GIVEN, WHOSE APERTURE AND EYE-GLASS ARE ADJUSTED BY EXPERIENCE, TO DETERMINE THE LENGTH, APERTURE AND EYE-GLASS OF ANOTHER TELESCOPE, THROUGH WHICH AN OBJECT SHALL APPEAR AS BRIGHT AND DISTINCT AS IN THE GIVEN ONE, AND MAGNIFIED AS MUCH AS SHALL BE REQUIRED.

PROPOSITION I.

223. **I**N all sorts of telescopes and double microscopes, the apparent indistinctness of a given object, is as the area of a circle of aberrations in the focus of the object-glass directly, and as the square of the focal distance of the eye-glass inversely.

For in vision with the naked eye or with glasses, the apparent indistinctness of a given object, is as the area of a circle of aberrations in its picture painted upon the retina. Because any one sensible point of the retina, being the center of a circle of aberrations, will at once be affected by a mixture of the rays of as many distinct pencils, as there are sensible points in the area of that circle^a; and so will at once convey to the mind a mixt or confused sensation of the same number of visible points in the object, from whence those pencils flowed; and this number of points is as the magnitude of the area of a circle of aberrations, whatever be the magnitude of a sensible point of the retina. Now in vision with telescopes, the diameter of a circle of aberrations in the picture upon the retina, is as the apparent magnitude of the diameter of the corresponding circle of aberrations in the common focus of the glasses^b, that is as the angle subtended by this diameter at the center of the eye-glass^c; that is as the diameter itself directly, and the focal distance of the eye-glass inversely^d. And so the area of that circle of aberrations upon the retina, is as the area of the corresponding circle of aberrations in the focus of the object-glass directly, and as the square of the focal distance of the eye-glass inversely^e.

224. *Corol.* In all sorts of telescopes and double microscopes a given object appears equally distinct, when the focal distances of the eye-

^a Art. 222.

^b Art. 136.

^c Art. 164.

^d Art. 86.

^e Euc. XII. 2.

eye-glasses are as the diameters of the circles of aberrations in the focus of the object-glasses.

225. The alteration in the confusion which may arise from aberrations caused by the eye-glasses, is not here regarded, as being inconsiderable. We only consider the confusion of those points in the image which lye very near the axis of the telescope, as of the point *g* in fig. 162. Now if this point was perfectly distinct the rays going from it would emerge from the eye-glass in parallel lines without sensible error; because the breadth of this cylinder of rays is exceeding small compared to the breadth of the eye-glass, being in proportion to the breadth of the aperture of the object-glass as their focal distances; and the refractions at so small a distance from the axis are sufficiently true and regular. It is the largeness of the aperture of the object-glass and of its focal distance, which causes the irregularity in its refractions. Add to this that the differently refrangible rays cannot be separated sensibly in going so short a distance as between the eye-glass and the eye. Besides this we find by experience that objects and images distinct in themselves, appear sufficiently distinct through very small eye-glasses when their apertures are small.

PROPOSITION II.

226. *In refracting telescopes the apparent indistinctness of a given object, is directly as the area of the aperture of the object-glass, and inversely as the square of the focal distance of the eye-glass.*

This appears from prop. 1, because the area of the circle of aberrations at the focus of the object-glass is as the area of its aperture^a; and because the aberrations arising from the eye-glass^b, and from the sphericity of the figure of them both are inconsiderable^c.

^a Art. 198.

^b Art. 225.

^c Art. 220.

227. *Corol.* In refracting telescopes a given object appears equally distinct, when the diameters of the apertures of their object-glasses, are as the focal distances of their eye-glasses.

PROPOSITION III.

228. *In all sorts of telescopes and double microscopes the apparent brightness of a given object is as the square of their linear apertures directly and as the square of their linear amplifications inversely.*

For if the squares of the linear amplifications, that is if the areas of the pictures upon the retina were the same, their brightness

M

would

would be as the quantities of light coming through the areas of the apertures, that is as the squares of the linear apertures; and if the apertures or quantities of light were the same, the brightness of the pictures would be as their areas inversely or as the squares of the linear amplifications inversely. Therefore when neither the apertures nor the amplifications are the same, the brightness is as the square of the linear apertures directly, and as the square of the linear amplifications inversely. *Q. E. D.*

229. *Corol. 1.* Hence in refracting and reflecting telescopes a given object appears equally bright, when their linear apertures are as their linear amplifications, that is as the focal distances of the object-glasses directly and as the focal distances of the eye-glasses inversely.

^a Art. 227.

^b Art. 164.

230. *Corol. 2.* If the breadth of the aperture of a given object-glass and the focal distance of the eye-glass be each increased in any given ratio, the distinctness will remain the same as before^a; and the linear amplification will be diminished in the same ratio^b; but the apparent brightness will be increased in a ratio quadruplicate of the former ratio by this proposition; and on the contrary.

Dioptr. p.
215.

231. *Hugens* observes that the same degrees of distinctness here demonstrated do not exactly agree with experience, as he found by looking at the same object through different telescopes, or through the same telescope with different apertures; and that through the larger aperture the object appeared not quite so distinct as through the smaller. He found also that in viewing objects of different brightness through the same aperture, the apparent indistinctness of the brighter object was a little greater than that of the duller: and therefore the aperture adjusted for the duller planets may be somewhat larger than for the brighter.

PROPOSITION IV.

232. *In reflecting telescopes the apparent indistinctness of a given object is as the sixth power of the diameter of the aperture of the object-metal directly, and as the fourth power of its focal distance inversely, and also as the square of the focal distance of the eye-glass inversely.*

^c Art. 215.
213.

For the area of a circle of aberrations in the focus of the object-metal is as the sixth power of its linear aperture directly and as the fourth power of its focal distance inversely^c; and therefore the apparent indistinctness of the object, is as the sixth power of the linear aperture

aperture directly, as the fourth power of the focal distance of the object-metal inversely, and as the square of the focal distance of the eye-glass inversely ^a. *Q. E. D.*

^a Art. 223.

233. *Corol.* In reflecting telescopes a given object appears equally distinct when the cubes of the linear apertures of the object-metals, are as the solids whose bases are the squares of the focal distances of the object-metals, and heights are the focal distances of the eye-glasses: or when the focal distances of the eye-glasses are as the cubes of the linear apertures of the object-metals, applied to the squares of their focal distances.

PROPOSITION V.

234. *In refracting telescopes of various lengths a given object will appear equally bright and equally distinct, when their linear apertures and focal distances of their eye-glasses are severally in a subduplicate ratio of their lengths or focal distances of their object-glasses: and then also their linear amplifications will be in a subduplicate ratio of their lengths.*

For to shew the object equally bright, the rectangle under the linear aperture and the focal distance of the eye-glass must be as the length of the telescope ^b, and to shew it equally distinct the linear ^b Art. 229. aperture must be as the focal distance of the eye-glass ^c; and there- ^c Art. 227. fore to perform both things together, the square of the linear aperture, and also the square of the focal distance of the eye-glass, must be severally (as the rectangle under each, or) as the length of the telescope; and consequently the linear aperture, and also the focal distance of the eye-glass, as the square root of that length. Now the linear amplification was as the linear aperture ^d, or by this demon- ^d Art. 227. stration, as the square root of the length of the telescope. *Q. E. D.*

235. *Hugens's* standard telescope 30 foot long, or 360 inches, ^{Diop. p. 210.} bears an aperture whose breadth is 3 inches, and an eye-glass whose focal distance is 3 inches and 3 tenths. From whence he has given us the following table of apertures and eye-glasses for other telescopes ^e, computed by the following rule. ^e Art. 244.

Multiply the number of feet in the focal distance of any proposed object-glass by 3000, and the square root of the product will give the breadth of its aperture in hundredth parts of an inch. And the same breadth of the aperture, increased by a tenth part of itself, gives the focal distance of the eye-glass in hundredth parts of an inch. And the magnifying powers are as the breadths of the apertures.

For since the standard telescope has 30 foot focal distance of its
M 2
object-

object-glass, put F for the number of feet in any other focal distance, and say by the proposition as $\sqrt{30}$ to \sqrt{F} , so is the standard aperture 3 inches or 300 centesimals or $\sqrt{300 \times 300}$, to the aperture sought; which therefore is $\sqrt{3000F}$ in centesimals of an inch. The focal distance of the eye-glass of the standard telescope is $3\frac{3}{10}$ inches, that is a tenth part more than the breadth of the aperture of the object-glass; consequently the focal distance of the new eye-glass must be a tenth part more than the linear aperture of the new object-glass, by the last proposition.

Diop. p. 215. 236. He also adds the following directions how to suit these telescopes to all sorts of objects seen either by day or by night. They are proportioned in the following table for astronomical observations, and therefore will require more light when used in the day time. For when the eye is dazzled with the brightness of the day, objects will appear through them but obscure, which in the night are sufficiently bright. Therefore (says *Hugens*) when I used these telescopes to observe objects by day-light, by experience I found it requisite to change the eye-glasses for others whose focal distances were double the former. By this means the apparent brightness became quadruple, because the surfaces of the images in the bottom of the eye were diminished in the same proportion ^a. For as the aperture remains unaltered, so does the quantity of light, and therefore it illuminates a lesser space so much the more. Now if the aperture was increased without changing the eye-glass, the brightness would be increased too, but then the mist arising from greater aberrations would also be greater; and therefore this remedy must not be used.

^a Art. 164.

237. But one may ask this question, since by substituting an eye-glass of a longer focal distance, the apparent indistinctness hitherto examined is diminished, why may not the aperture of the object-glass be so far increased, till the same degree of indistinctness returns again as belongs to a telescope regulated by the table? For from hence more light is gained and the distinctness is not altered ^b. The answer is this, which I hinted before ^c, that the mist arising from *Newton's* aberration, though the same in quantity, becomes more sensible in proportion to the brightness of the image. For the brightness of the mist increases at the same time. And we find by experience, that as soon as the apertures of those day-light telescopes are increased, the mist arising from the aberrations of a brighter object begins to be troublesome. The apertures therefore must not be altered.

^b Art. 230.

^c Art. 231.

238. Again one may ask, if a telescope fitted for Saturn be applied to the Moon, which is 100 times brighter (I mean in each equal parts

parts, though not in the whole, as being 10 times nearer to the Sun;) one may ask I say whether the breadth of the aperture and the focal distance of the eye-glass may not both be lessened in the same proportion to make the regions of the moon no brighter than those of saturn, but much greater in appearance than before. For instance, in a 30 foot telescope, if 3 inches, the breadth of the aperture be reduced to $\sqrt{\frac{9}{100}}$ of an inch, which is somewhat less than ($\sqrt{\frac{9}{9}}$ or) a third part of the former, and also the focal distance of the eye-glass be shortened in the same proportion; the proportion of the apparent brightness in these two telescopes, the object being the same, would be quadruplicate of 3 to $\sqrt{\frac{9}{100}}$ ^a that is as 100 to 1; ^a Art. 230. and since the regions in the moon are 100 times brighter in themselves than those in saturn, the moon would appear in the darker telescope just as bright as saturn did in the lighter. But the apparent indistinctness hitherto considered would also be the same in both^b, and the amplification of the moon would be greater than^b Art. 230. that of saturn in the ratio of 3 to $\sqrt{\frac{9}{100}}$ ^c, which is more than triple.^c Art. 164. So that this reduction of the aperture and eye-glass seems very advantageous; but in reality it is quite otherwise; and that for two reasons. First because the minute parts of the moon may be better discerned when all the light remains in the telescope, than when it is reduced to an 100th part, though not in the same proportion. The other reason is that when the aperture is too much contracted, the out-lines that circumscribe the pictures in the eye become confused; which is carefully to be minded, and also what are the limits of this confusion. This is certain that as the aperture is contracted, the slender pencils or cylinders of rays, that emerge from the eye-glass into the eye, are also contracted in the same proportion. Now if the breadth of one of these pencils be less than $\frac{1}{5}$ or $\frac{1}{6}$ of a line, that is less than $\frac{1}{60}$ or $\frac{1}{72}$ part of an inch, the out-lines of the pictures are spoiled, for some unknown reason in the make of the eye, whether in the choroid, or in the retina, or in the humors it is uncertain. For by looking through an hole, in a thin plate, narrower than $\frac{1}{5}$ or $\frac{1}{6}$ of a line, the edges of objects begin to appear confused and so much the more as the hole is made narrower. Now it is easy to shew in the last mentioned telescope that the cylinder of rays is too slender. For by adding $\frac{1}{10}$ of the aperture to itself^d, the^d Art. 235. focal distance of the eye-glass becomes $\sqrt{\frac{9}{100}} + \frac{1}{10} \sqrt{\frac{9}{100}}$, that is $\frac{11}{10} \sqrt{\frac{9}{100}}$ of an inch; and by similar triangles subtended at the common focus *q* by the aperture and cylinder sought^e, it is as the focal di-^e Fig. 162. stance of the object-glass, to the focal distance of the eye-glass, so the breadth of the aperture, to the breadth of the cylinder; that is

as

as 30 feet or 360 inches to $\frac{1}{10}\sqrt{\frac{9}{10}}$ inches, so is $\sqrt{\frac{9}{10}}$ of an inch to $\frac{1}{4000}$ inch or almost $\frac{1}{30}$ of a line; which is much less than $\frac{1}{6}$. But in the telescope regulated in the table, it is as 360 to $3\frac{1}{10}$ so 3 to $\frac{1}{4000}$ of an inch or almost $\frac{1}{3}$ of a line for the breadth of that cylinder; which can possibly do no harm. Hence we learn that the breadth of the aperture and focal distance of the eye-glass cannot be contracted much more than $\frac{1}{3}$ of themselves; for even then the breadth of the cylinder at the eye will not much exceed $\frac{1}{3}$ of a line. The same is to be understood of telescopes of all lengths regulated as in the table, the breadth of the cylinder being the same in all. For by the proportion just mentioned it equals the breadth of the aperture multiplied into the focal distance of the eye-glass and divided by the focal distance of the object-glass, and consequently it is proportionable to the linear aperture directly and the linear amplification inversely; which two ratios must compound a ratio of equality to preserve the same apparent brightness, by art. 229.

239. Hence though we transferred one of these telescopes from Saturn to Venus which is 225 times brighter, being 15 times nearer to the Sun, yet the breadth of the aperture must not be contracted above $\frac{1}{3}$ part of the whole; and if too much light still remains, it must be diminished by darkening the eye-glass with the smoak of a candle. For a greater contraction of the aperture is hurtful for another reason, that all the little bubbles and veins in the eye-glass become more conspicuous by intercepting the whole or a greater part of those little cylinders above mentioned, and consequently the particles of the object they came from.

240. Upon the whole I conclude we may lengthen our telescopes at pleasure, according to the laws of the table, with good success; since not only the brightness and distinctness remain unaltered, but also the breadth of the pencils that enter the eye. Lastly to observe exceeding small stars and especially the Satellites of Jupiter and Saturn, the best way is to increase very much both the aperture and focal distance of the eye-glass. For since they appear like points even through the telescopes, there is nothing gained by endeavouring to increase their diameters; but their brightness must be increased as much as possible; and this is chiefly done by increasing the aperture. By doubling its breadth, the light received into it becomes quadruple, and then by doubling also the focal distance of the eye-glass, the distinctness returns to the same as at first^a. But still the brightness will not become 16 times greater, according to cor. 2. prop. 3, but only 4 times; because as I said the picture of the star upon the retina is but a sensible point, whose brightness cannot

^a Art. 227.

cannot therefore be increased by a diminution of its breadth, but only by an addition of new light. The case is different when we view the moon and primary planets through the same telescope, whose several parts receive 16 times more light than before. Thus by widening the apertures we very much increase the power of the telescope for finding out small stars and the sattelites of Saturn, so that perhaps with a 30 foot glass, whose aperture is 6 inches or double the usual one, as much may be done as with another of 120 foot whose aperture by the table is also 6 inches. So far from *Hugens*.

PROPOSITION VI.

241. *In reflecting telescopes of various lengths a given object will appear equally bright and equally distinct, when their linear apertures and also their linear amplifications are as the square-square roots of the cubes of their lengths: and consequently when the focal distances of their eye-glasses are also as the square-square roots of their lengths.*

Put A for the linear aperture of the reflecting concave, L for its focal distance or the length of the telescope, F for the focal distance of the eye-glass; and when the distinctness is given A^3 is as FLL^2 ; ^a Art. 233. and when the brightness is given the amplification or $\frac{L}{F}$ is as A^b , ^b Art. 229.

that is F is as $\frac{L}{A}$. Therefore when the distinctness and brightness

are both given, A^3 is as $\frac{L^3}{A}$; or A^4 as L^3 ; or A as $\sqrt[4]{L^3}$. The amplification $\frac{L}{F}$ was as A , that is as $\sqrt[4]{L^3}$; and therefore F is as

$$\frac{\sqrt[4]{L^4}}{\sqrt[4]{L^3}} \text{ or } \sqrt[4]{L}, \quad \text{Q. E. D.}$$

242. In the reflecting telescope made and described by *John Hadley*, Esq; F. R. S. in the Philosophical Transactions N^o. 376 and 378, $L = 62\frac{1}{2}$ inches, $F = \frac{1}{3}$ or $\frac{3}{10}$ or $\frac{1}{40}$ of an inch. For he uses 3 eye-glasses and as many apertures for the reflecter whose breadths are $4\frac{1}{2}$, 5, $5\frac{1}{2}$ inches. Hence the linear amplifications or $\frac{L}{F}$ are $187\frac{1}{2}$, $208\frac{1}{3}$, $227\frac{2}{3}$ respectively. Taking the middle eye-glass and aperture for a standard I computed the following table for telescopes of other lengths by this Rule. Call the number of inches in the length of

of any telescope L , and the focal distance of its eye-glass will be equal to $60 \sqrt[4]{10} L$ in thousandth parts of an inch. The quotient of L

^a Art. 169. divided by $60 \sqrt[4]{10} L$ or F gives the amplification^a, which multiplied by 24 will always give the linear aperture in thousandth parts of an inch. For by the proposition $\sqrt[4]{L}$ is as F ; that is $\sqrt[4]{62\frac{1}{2}}$ or $\sqrt[4]{\frac{125}{2}}$ or $\sqrt[4]{\frac{625}{10}}$ or $5 \sqrt[4]{\frac{1}{10}}$ is to $\sqrt[4]{L}$ as $\frac{3}{10}$ or 300 millesimals in the given eye-glass, to the millesimals in the correspondent eye-glass or in $F = 60 \sqrt[4]{10} L$. And the aperture being as the amplification by the proposition, say, as the amplification given or $208\frac{1}{3}$ is to $\frac{L}{F}$, the amplification found, so is 5 inches, the aperture given, to the

aperture sought $= \frac{5}{208\frac{1}{3}} \times \frac{L}{F} = \frac{24}{1000} \times \frac{L}{F}$ inches.

^b Art. 221. ^c Art. 209. ^{214.} 243. Were it not for the unequal refrangibility of rays, refracting telescopes, though not so short as these^b, would also be proportioned by this rule^c: which not agreeing with experience, shews again that the aberrations arising from the spherical figure are inconsiderable in comparison to the other aberrations arising from the unequal refrangibility of the rays.

Length of the telescope or focal dist. of the object-glasses.	Linear aperture of the object-glasses.	Focal dist. of the eye-glasses.	Linear amplification, or magnifying power.	Length of the telescope or focal dist. of the concave	Focal dist. of the eye-glasses.	Linear amplification, or magnifying power.	Linear aperture of the concave-metal.
Feet.	Inch & Dec.	Inch & Dec.		Feet.	Milles. Inch.		Milles. Inch.
1	0.55	0.61	20	$\frac{1}{2}$	0.167	36	0.864
2	0.77	0.85	28	1	0.199	60	1.440
3	0.95	1.05	34	2	0.230	102	2.448
4	1.09	1.20	40	3	0.261	138	3.312
5	1.23	1.35	44	4	0.281	171	4.104
6	1.34	1.47	49	5	0.297	202	4.848
7	1.45	1.60	53	6	0.311	232	5.568
8	1.55	1.71	56	7	0.323	260	6.240
9	1.64	1.80	60	8	0.334	287	6.888
10	1.73	1.90	63	9	0.344	314	7.536
13	1.97	2.17	72	10	0.353	340	8.160
15	2.12	2.32	77	11	0.362	365	8.760
20	2.45	2.70	89	12	0.367	390	9.360
25	2.74	3.01	100	13	0.377	414	9.936
30	3.00	3.30	109	14	0.384	437	10.488
35	3.24	3.56	118	15	0.391	460	11.040
40	3.46	3.81	126	16	0.397	483	11.592
45	3.67	4.04	133	17	0.403	506	12.143
50	3.87	4.26	141				
55	4.06	4.47	148				
60	4.24	4.66	154				
70	4.58	5.04	166				
80	4.90	5.39	178				
90	5.20	5.72	189				
100	5.48	6.03	199				
120	6.00	6.60	218				
140	6.48	7.13	235				
160	6.93	7.62	252				
180	7.35	8.09	267				
200	7.75	8.53	281				
220	8.12	8.93	295				
240	8.48	8.83	308				
260	8.83	9.71	321				
280	9.16	10.08	333				
300	9.49	10.44	345				
400	10.95	12.05	398				
500	12.25	13.47	445				
600	13.42	14.76	488				

245. These propositions, in *Hugens's* table for refracting telescopes, are measured by the Rheinland foot which is to the English foot as 139 to 135; so that taking their lengths of as many English feet, their apertures and eye-glasses and linear amplifications should be severally diminished in the subduplicate ratio of 139 to 135 by art. 234. that is nearly in the ratio of 139 to 137 or about $\frac{1}{65}$ or $\frac{1}{70}$ part of the whole.

CHAP. XIII.

CONCERNING THE RAIN-BOW.

LEMMA I.

Fig. 189. 246. **T**HE ratio of the tangents, CT, CV , of any two angles, CBD, CBE , is compounded of the ratio of their sines, CD, CE , taken directly, and of their cosines, BD, BE , taken inversely.

For the right angled triangles BCT, BDC are equiangular, and so are the right angled triangles BCV, BEC . Therefore the ratio of CT to CV , which is compounded of CT to CB and of CB to CV , or of CD to DB and of EB to EC , is the same as the ratio of the rectangle under CD, EB to the rectangle under DB, EC , which is compounded of the ratio of CD to CE and of EB to DB , that is of the sines directly and cosines inversely. $\mathcal{Q}. E. D.$

LEMMA II.

Fig. 190, 191. 247. The least increment of an angle of incidence, is to the contemporary increment of the angle of refraction, as the tangent of the angle of incidence, to the tangent of the angle of refraction.

Let two rays AB, aB , containing a very small angle ABa , be refracted at B along the lines BE, Be by a plane or by any curve-surface. From any point C , of the line BC perpendicular to that surface, draw CDd cutting the incident rays (produced) at right angles b in D and d ; and likewise CEe cutting the refracted rays (produced) at right angles in E and e . Then because CD is to CE and Cd to Ce in the same ratio of the sines, disjointly we have Dd to Ee as CD to CE . Now the ratio of the small angles (ABa or) DBd and EBe , which are the contemporary increments or decrements of the angles of incidence and refraction, being compounded of the ratio of Dd to Ee and of BE to BD , that is of CD to CE and of BE to BD , is the same as the ratio of the tangents of incidence and refraction d . $\mathcal{Q}. E. D.$

PROPOSITION I.

248. *When a ray of light is refracted into a circle, and successively reflected within it any given number of times before it emerges out of the circle by a second refraction; let the angle of refraction be multiplied by the number of successive reflections increased by an unite; and the excess of the resulting angle above the angle of incidence will be equal to half the angle contained under the incident and the emergent ray produced till they meet: that is, the excess abovementioned is equal to half that angle, under the incident and the emergent ray, in which the refracting circle lyes, when the number of reflections is odd; and is equal to half the other angle, under the same rays, which is the complement of the former to two right angles, when the number of reflections is even.*

For let $ABCDE$ be a great circle of a sphere whose center is O , Fig. 192. to
 and let an incident ray SA be refracted at A to B , and be reflected 195.
 from B to C ; and at C let it either go out by refraction to G , or be
 reflected to D^a ; where let it either go out by refraction to H , or be Art. 35, &c.
 reflected to E ; and so on. And when the number of reflections is
 odd, a line OR drawn through the center O and the middlemost point
 of reflection, will bisect the angle at R under the incident and
 the emergent ray produced: because the reflections and refractions
 on each side of the line OR are equal in number and magnitude;
 the chords AB, BC, CD, DE described by the reflected ray being
 equal to one another. And for the same reason when the number
 of reflections is even, a line OT , drawn through the center O per-
 pendicular to the chord that joins the two middlemost points of re-
 flection, will bisect one of the angles at T under the incident and
 the emergent ray produced; and a line TV , perpendicular to TO ,
 will bisect the other angle under them, which is the complement of
 the former to two right ones. Hence the line TV is parallel to the
 middlemost chord, because TO is perpendicular to them both. Draw
 a diameter POQ parallel to the incident ray SAM , and let it cut
 the reflected rays BC, CD, DE produced, in β, γ, δ , respectively.
 Join OA, OB and in fig. 192. the sums of the three angles in each
 of the triangles OAB, OAR , are equal to one another; take away
 the common angle AOB , and the sum of the equal angles $OAB,$
 OBA in the first triangle, will be equal to the sum of the angles
 OAR, ORA in the second triangle. And by subtracting the angle
 of incidence OAR or OAM from both sums, we have $2OAB -$
 $OAM = ORA = BOQ$. Hence in fig. 193. the angle STV or
 $P\beta C$, being an external angle of the triangle $OB\beta$, equals $OBC +$
N 2
BOQ

$BOQ = OAB + 2OAB - OAM = 3OAB - OAM$. Hence again in fig. 194. the angle SRO or POC , being an external angle of the triangle $OC\beta$, equals $OCB + P\beta C = OAB + 3OAB - OAM = 4OAB - OAM$. Hence again in fig. 195. the angle STV or $P\gamma D$, being an internal angle of the triangle $CO\gamma$, equals $OCD - CO\gamma = 5OAB - OAM$, throwing away two right angles. For $CO\gamma = 2$ right angles $- POC = 2$ right angles $- 4OAB + OAM$. And so forward continually. Therefore if the number of successive reflections increased by an unite be called m , it appears that $mOAB - OAM$ equals half the angle under the incident and emergent rays. *Q. E. D.*

PROPOSITION II.

249. *Things remaining as they were, let the angle of incidence increase from nothing till it becomes a right angle; and the angle under the incident and the emergent ray, after any given number of reflections called n , will first increase and then decrease again; and will be the greatest of all when the tangent of the angle of incidence, is to the tangent of the angle of refraction, as $n + 1$ to 1.*

Fig. 192. to
195.
a Art. 248.

For putting $m = n + 1$, we had half the angle under an incident and the emergent ray equal to the excess of $mOAB$ above OAM^a ; which excess, when the angles OAB, OAM are very small, will also be but small; and will increase so long as the successive increments of $mOAB$ shall exceed the contemporary increments of OAM ; and will decrease again when the successive increments of $mOAB$ are exceeded by the increments of OAM ; and consequently will be the greatest of all when m times the least increment of OAB is equal to once the contemporary increment of OAM ; that is when the least increment of the angle of incidence OAM is to the contemporary increment of the angle of refraction OAB , and consequently the tangent of incidence is to the tangent of refraction^b, as m to 1. *Q. E. D.*

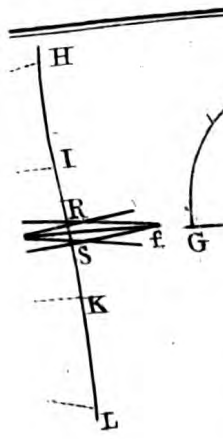
b Art. 247.

PROPOSITION III.

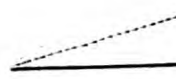
250. *It is proposed to find two angles, whose sines shall be in a given ratio of I to R, and whose tangents shall be in another given ratio of m to 1.*

Fig. 196.

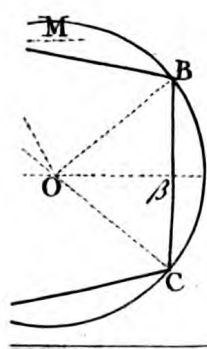
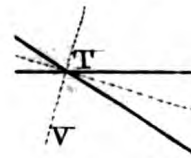
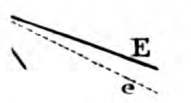
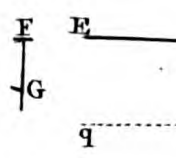
In any given line $CEDA$, let CA be to CD as I to R , and CA to CE as m to 1; with the center C and semidiameter CD describe an arch DB , cutting a circle ABE whose diameter is AE , in B ; draw ABF , and joining BC , the sine of the angle CBF will be to the sine

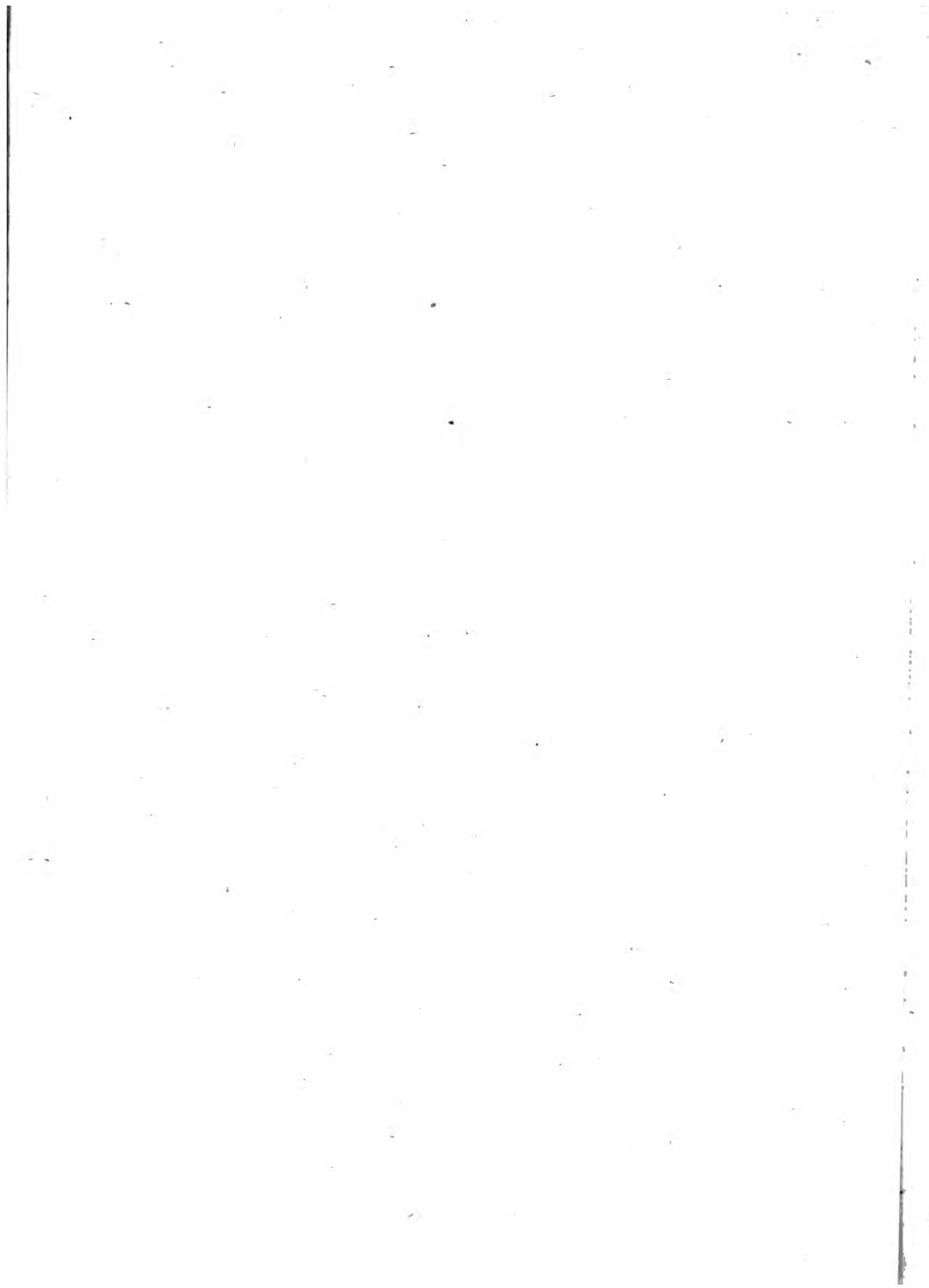


Q —————
186



Q —————





fine of CAF as I to R ; and the tangent of CBF to the tangent of CAF as m to 1 ; and consequently CBF , CAF are the angles required. For in the triangle CAB the fine of the angle CBA or CBF , is to the fine of CAF , as CA to CB^a or CD , as I to R by ^a Art. 85. construction. Join BE and compleat the parallelogram $CEBG$; and CG produced will cut ABF at right angles in F , because ABE is a right angle in the semicircle ABE . Therefore the lines FC , FG are tangents of the angles CBF , GBF or CAF to the radius BF ; and the tangent FC is to the tangent FG as FA to FB^b or ^b Euc. VI. 2. as CA to CE^b or as m to 1 by construction. Q. E. D.

251. *Corol. 1.* When parallel rays of the sun fall upon a spherical drop of rain, let the given ratio of I to R stand for the ratio of the fine of incidence to the fine of refraction; and let n be any given number of successive reflections made by every ray before it emerges out of the drop, and let $m = n + 1$; and by these propositions it appears, that half the greatest angle which any of the emergent rays can make with the incident rays, is equal to $m \times \text{ang. } CBF - CAF$. For CBF and CAF or GBF are angles whose fines are as I to R , and whose tangents are as m to 1 ; and consequently are the angles of incidence and refraction of that ray, whose incident and emergent parts produced contain the greatest angle.

252. *Corol. 2.* The foregoing construction for determining the angle CBF is Dr. *Halley's* ^c, and Sir *Isaac Newton's* rule for calculat- ^c Phil. Transf. No. 297.

ing it, is this that follows. As $\sqrt{mm - 1} \times RR$ is to $\sqrt{II - RR}$, so is the tabular radius to the cosine of the angle of incidence CBF . Whence this angle and its fine are given by the tables, and from thence by the ratio of I to R the tabular fine of the angle of refraction and the angle itself are also given. The rule may thus be demonstrated. We had $CA : CB :: I : R$ and $FA : FB :: m : 1$.

Hence $CA^2 = \frac{II}{RR} CB^2$, and $AF^2 = mm BF^2$; and so $\frac{II}{RR} CB^2 - mm BF^2 = (CA^2 - AF^2 = FC^2^d =) CB^2 - BF^2$. Hence ^d Euc. I. 47. $\frac{II}{RR} CB^2 - CB^2 = mm BF^2 - BF^2$, and $\overline{II - RR} \times CB^2 = \overline{mm - 1} \times RR \times BF^2$; and by resolving this equality into a proportion, and by extracting the roots, we have $\sqrt{mm - 1} \times RR : \sqrt{II - RR} :: CB : BF :: \text{radius} : \text{cosine ang. } CBF$.

PROPOSITION IV.

To explain the Phænomena of the Rain-bow.

Design.

253. Having premised such mathematical principles as are necessary for an exact computation of the apparent diameters and breadths of the Rain-bows, I will here subjoin Sir *Isaac Newton's* entire explication of the colours of the bows, and of the manner in which they are formed; taking the liberty here and there of making a few additions to it; for the sake of such readers as may not be so skilful as those that he generally writes to.

Newt. Opt.
P. 147.

254. This bow never appears but where it rains in the sun shine, and may be made artificially by spouting up water which may break aloft, and scatter into drops, and fall down like rain. For the sun shining upon these drops, certainly causes the bow to appear to a spectator standing in a due position to the rain and sun. And hence it is now agreed upon that this bow is made by refraction of the sun's light in drops of falling rain. This was understood by some of the ancients, and of late more fully discovered and explained by the famous *Antonius de Dominis* archbishop of *Spalato* in his book *de Radiis Visus & Lucis* published by his friend *Bartolus* at *Venice* in the year 1611, and written above 20 years before. For he teaches there how the interior bow is made in round drops of rain by two refractions of the sun's light, and one reflection between them; and the exterior bow by two refractions and two sorts of reflections between them in each drop of water; and proves his explications by experiments made with a phial full of water, and with globes of glass filled with water, and placed in the sun to make the colours of the two bows appear in them. The same explication *Des-Cartes* has pursued in his *Meteors* and mended that of the exterior bow. But while they understood not the true origin of colours, it is necessary to pursue it a little farther.

Fig. 197.

255. For understanding therefore how the bow is made, let a drop of rain or any other spherical transparent body be represented by the sphere *B N F G* described with the center *C* and semidiameter *C N*. And let *A N* be one of the sun's rays incident upon it at *N*, and be thence refracted to *F*, where let it either go out of the sphere by refraction toward *V*, or be reflected to *G*; and at *G* let it either go out by refraction to *R*, or be reflected to *H*; and at *H* let it go out by refraction towards *S*, cutting the incident ray in *Y*. Produce *A N* and *R G* till they meet in *X*, and upon *A X* and *N F* let fall the perpendiculars *C D* and *C E*, and produce *C D* till it falls upon

upon the circumference at L . Parallel to the incident ray AN draw the diameter BQ ; and let the sine of incidence out of air into water, be to the sine of refraction as I to R . Now if you suppose the point of incidence N to move from the point B continually till it comes to L , the arch QF will first increase and then decrease, and so will the angle AXR which the rays AN and GR contain; and the arch QF and angle AXR will be biggest when ND is to NC as $\sqrt{II-RR}$ to $\sqrt{3RR}$, in which case NE will be to ND as $2R$ to I . Also the angle AYS , which the rays AN and HS contain will first decrease and then increase; and grow least when ND is to NC as $\sqrt{II-RR}$ to $\sqrt{8RR}$, in which case NE will be to ND as $3R$ to I ; and so the angle which the next emergent ray (that is the emergent ray after three reflections) contains with the incident ray AN will come to its limit, when ND is to NC as $\sqrt{II-RR}$ to $\sqrt{15RR}$; in which case NE will be to ND as $4R$ to I . And the angle which the ray next after that emergent, (that is the ray emergent after four reflections) contains with the incident, will come to its limit, when ND is to NC as $\sqrt{II-RR}$ to $\sqrt{24RR}$; in which case NE will be to ND as $5R$ to I ; and so on infinitely, the numbers 3, 8, 15, 24, &c. being gathered by continual addition of the terms of the arithmetical progression 3, 5, 7, 9, &c.

256. Now it is to be observed, that as when the sun comes to his tropicks, days increase and decrease but a very little for a great while together; so when by increasing the distance CD , these angles come to their limits, they vary their quantity but very little for some time together; and therefore a far greater number of rays which fall upon all the points N in the quadrant BL shall emerge in the limits of these angles than in any other inclinations. Add to this, that of all the rays which fall upon the quadrant BL , those contiguous ones can only emerge parallel to one another, which emerge in the limits of these angles; and that all other contiguous rays emerge diverging from points either behind or before the drop; and consequently will fall much thinner upon the eye, at a great distance from the drop, than the parallel rays. For those rays which converge to points behind the eye, placed at a great distance from a small drop, are not sensibly different from parallel rays. This will appear by observing that while the arch BN is continually increasing from nothing, and the angle AXR , for example, is also increasing; the successively emergent rays, being continually less and less inclined to the incident ones or to the fixt line BQ , are also successively inclined in small angles to one another; and the same

Art. 252.

Art. 246.

249.

Fig. 198.

Fig. 197.

same thing is manifest while the angle AXR is decreasing; the successive rays being more and more inclined to BQ ; consequently in the limit between the increase and decrease of this angle the contiguous incident rays must emerge parallel to one another.

Fig. 199.

257. And farther it is to be observed, that the rays which differ in refrangibility will have different limits of their angles of emergence; and by consequence according to their different degrees of refrangibility, emerge most copiously in different angles; and being separated from one another appear each in their proper colours. Add to this that although the heterogeneous rays of any slender pencil whatever, as AN , will be separated by refractions at the drop into rays $NFGR$ of one colour, and $Nfgr$ of another, as by refractions through a prism; yet these emergent rays GR , gr will not affect the eye with their distinct colours, unless they be in the limits of the angles AXR Axr ; because every where within these greatest angles, an infinite number of such coloured pencils being variously inclined to one another are mixt together, and consequently appear white or without distinct colours. And the same may be said of the rays emerging any where within the greatest angle NYS . Fig. 197.

^a Art. 31.

258. Now what these angles are may be gathered first by computing the angles of incidence and refraction by art. 252, and then the angles AXG , AYS themselves by the 248th article. For in the least refrangible rays the sines I and R are 108 and 81^a, and thence by computation the greatest angle AXR will be found 42 degrees and 2 minutes; and the least angle AYS , 50 degrees and 57 minutes. And in the most refrangible rays the sines I and R are 109 and 81, and thence by computation the greatest angle AXR will be found 40 degrees and 17 minutes, and the least angle AYS , 54 degrees and 7 minutes.

Fig 200.

259. Suppose now that O is the spectator's eye, and OP a line drawn parallel to the sun's rays. Let POE , POF , POG , POH be angles of 40 degr. 17 min.; 42 deg. 2 min.; 50 deg. 57 min.; 54 deg. 7 min. respectively; and these angles turned about their common side OP , shall with their other sides OE , OF , and OG , OH describe the verges of two rain-bows $AFBE$ and $CHDG$. For if E , F , G , H be drops placed any where in the conical superficies described by OE , OF , OG , OH and be illuminated by the sun's rays SE , SF , SG , SH ; the angle SEO being equal to POE or 40 deg. 17 min. shall be the greatest angle in which the most refrangible rays can after one reflection be refracted to the eye; and therefore all the drops in the line OE shall send the most refrangible rays most copiously

copiously to the eye; and thereby strike the senses with the deepest violet colour in that region. In like manner the angle SFO being equal to the angle POF or 42 deg. 2 min. shall be the greatest in which the least refrangible rays after one reflection can emerge out of the drops; and therefore these rays shall come most copiously to the eye from the drops in the line OF , and strike the senses with the deepest red colour in that region. And by the same argument the rays which have intermediate degrees of refrangibility shall come most copiously from drops between E and F and strike the senses with the intermediate colours in the order which their degrees of refrangibility require; that is in the progress from E to F or from the inside of the bow to the outside in this order, violet, indigo, blue, green, yellow, orange, red. But the violet by the mixture of the white light of the clouds will appear faint and incline to purple. It may be farther observed, that all the rays but the violet in the line SE will emerge from E in a greater angle than SEO made by the violet, and consequently will pass below the eye; and that all the rays but the red in the line SF will emerge from F in a lesser angle than SFO made by the red, and consequently will pass above the eye; by which means only red will appear in the line SF and only violet in the line SE .

260. Again the angle SGO being equal to the angle POG or 50 deg. 57 min. shall be the least angle in which the least refrangible rays can after two reflections emerge out of the drops; and therefore the least refrangible rays shall come most copiously to the eye from the drops in the line OG , and strike the sense with the deepest red in that region. And the angle SHO being equal to the angle POH or 54 deg. 7 min. shall be the least angle in which the most refrangible rays after two reflections can emerge out of the drops; and therefore these rays shall come most copiously to the eye from the drops in the line OH , and strike the sense with the deepest violet in that region. And by the same argument the drops in the regions between G and H shall strike the sense with intermediate colours in the order which their degrees of refrangibility require; that is in the progress from G to H , or from the inside of the bow to the outside in this order; red, orange, yellow, green, blue, indigo, violet. And since these four lines OE , OF , OG , OH may be situated any where in the abovementioned conical surfaces, what is said of the drops and colours in these lines is to be understood of the drops and colours every where in those surfaces.

261. Thus shall there be made two bows of colours, an interior and stronger by one reflection in the drops, and an exterior and

fainter by two; for the light becomes fainter by every reflection. And their colours shall lye in a contrary order to one another; the red of both bows bordering upon the space GF , which is between the bows. The apparent breadth of the interior bow EOF , measured cross the colours, shall be 1 deg. 45 min. and the breadth of the exterior, GOH , shall be 3 deg. 10 min. and the apparent distance between them, GOF , shall be 8 deg. 55 min. the greatest semidiameter of the innermost, that is, the angle POF being 42 deg. 2 min. and the least semidiameter of the outermost, POG , being 50 deg. 57 min.

Fig. 201.

262. These are the measures of the bows as they would be were the sun but a point, for by the breadth of his body the breadths of the bows will be increased and their distance decreased by half a degree. And so the breadth of the interior Iris will be 2 deg. 15 min. that of the exterior 3 deg. 40 min., their distance 8 deg. 25 min., the greatest semidiameter of the interior bow 42 deg. 17 min., and the least of the exterior 50 deg. 42 min. For let SEO be the limit of all the angles under the rays of any one colour, which coming from the center of the sun are reflected from the drop at E to the eye at O . In the ray SE take any point S at pleasure, and let the angles ESM , ESN and also EOM , EON be severally equal to a quarter of a degree, that is to half the apparent breadth of the sun. And joining OS , since the sums of the angles at the base OS , of the several triangles OSM , OSE , OSN , are equal among themselves, their vertical angles at M , E , N are also equal among themselves. Consequently the angle SMO will be the limit of all the angles contained under the incident and emergent rays of the same colour as before, which came from m the highest point of the sun; and SNO the limit of all the angles contained under the incident and emergent rays of the same colour as before, which came from n the lowest point of the sun. Therefore if all the rays of the sun were of the same colour, or alike refrangible, the apparent breadth of the bow, measured by the angle MON , would be but half a degree or equal to the apparent breadth of the sun measured by the angle MSN or mSn . But since his rays are differently refrangible, conceive the drop E to be placed any where in the inward or outward verges of the bows above described, upon supposition that the sun was but a point; and then it is manifest that the angle EOM must be added to the inside, and EON to the outside of the angles which the breadths of those bows subtend at O , to obtain their apparent breadths. A rain-bow is therefore a circular image of the sun reflected to the eye from the farther surfaces of innumerable drops
of

of falling rain, and dilated in breadth by the unequal refrangibility of rays of different colours.

263. And such are the dimensions of the bows in the heavens found to be very nearly, when their colours appear strong and perfect. For once by such means as I then had I measured the greatest semidiameter of the interior iris about 42 degrees, the breadth of the red, yellow, green in that iris 63 or 64 minutes, besides the outmost faint red obscured by the brightness of the clouds, for which we may allow 3 or 4 minutes more. The breadth of the blue was about 40 minutes more besides the violet, which was so much obscured by the brightness of the clouds that I could not measure its breadth. But supposing the breadth of the blue and violet together to equal that of the red, yellow and green together; the whole breadth of this iris will be about $2\frac{1}{4}$ degrees, as above. The least distance between this iris and the exterior iris was about 8 degrees and 30 minutes. The exterior iris was broader than the interior, but so faint, especially on the blue side, that I could not measure its breadth distinctly. At another time when both bows appeared more distinct I measured the breadth of the interior iris 2 deg. 10 min. and the breadth of the yellow and green in the exterior iris was to the breadth of the same colours in the interior as 3 to 2.

264. Whoever has a mind to repeat these observations after Sir *Isaac Newton* may observe, that the apparent semidiameter of the bow, (or of any ring of colours in either of the bows) is equal to the apparent altitude of its highest point added to the sun's altitude, and consequently may be measured by a common quadrant. For let SOP be the axis of the bows passing through the sun at S and the eye at O , GOH an horizontal line, E the highest point of any ring of either of the bows, whose apparent semidiameter EOP is required. It is manifest that the angle $EOP = EOH + HOP = EOH + SOG$. Fig. 202.

265. This explication of the rain-bow is yet farther confirmed by the known experiment (made by *Antonius de Dominis* and *Des Cartes*) of hanging up any where in the sun-shine a glass-globe filled with water, and viewing it in such a posture that the rays which come from the globe to the eye may contain with the sun's rays an angle of either 42 or 50 degrees. For if the angle be about 42 or 43 degrees the spectator supposed at O , shall see a full red colour in that side of the globe which is opposed to the sun; as it is represented at F : and if that angle be made less, suppose by depressing the globe to E , there will appear other colours yellow, green and blue successively in the same side of the globe. But if the angle be Fig. 200.

made about 50 degrees, suppose by lifting up the globe at *G*, there will appear a red colour in that side of the globe which lyes toward the sun: and if the angle be made greater, suppose by lifting up the globe to *H*, the red will turn successively to other colours, yellow, green and blue. The same thing I have tried by letting a globe rest, and by raising or depressing the eye, or otherwise moving it to make the angle of a just magnitude. So far Sir *Isaac Newton*.

LEMMA III.

266. *The tangent of the sum of two angles, is to the sum of their tangents, as the square of the radius, to the square of the radius diminished by the rectangle under the tangents: and the tangent of the difference of two angles, is to the difference of their tangents, as the square of the radius, to the square of the radius increased by the rectangle under the tangents.*

Fig. 203, 204. Let *RA* and *RB* be tangents of two angles *ROA*, *ROB*. Then as *AB*, the sum or difference of the tangents, is to *AO*, the secant of either of the angles, so let *AO* be to *AC*, to be placed from *A* towards *B*. Again as *RC* is to *RO*, so let *RO* be to *RD*; and *RD* will be the tangent of the sum or difference of the two angles *ROA*, *ROB*. For joining *CO*, by the first of these proportions the triangles *AOB*, *ACO* will be equiangular^a; and so the angle *AOB* is equal to *ACO*, or to *ROD* by the second proportion^b. Hence in

^a Euc. VI.6.

^b Euc. VI.8.

fig. 203, because $AC = \frac{AOq}{AB} = \frac{RAq + ROq}{RB + RA}$, we have $RC =$

$$(AC - AR =) \frac{RAq + ROq}{RB + RA} - RA = \frac{ROq - RB \times RA}{RB + RA}; \text{ whence}$$

$$RD = \frac{RB + RA}{ROq - RB \times RA} \times ROq. \text{ By a like process fig. 204, we}$$

$$\text{have } AC = \frac{RAq + ROq}{RB - RA}; \text{ whence } RD = \frac{RB - RA}{ROq + RB \times RA} \times$$

ROq. Q. E. D.

267. *Corol. 1.* Hence the tangent of the sum of any number of given angles, or the tangent of any multiple of a given angle, may be easily computed. Put $RO = r$, $RA = a$, $RB = b$, then the tangent of the sum of the angles whose tangents are a and b , that

$$\text{is } RD = \frac{b + a}{rr - ab} \times rr; \text{ call this tangent } x; \text{ then for the same}$$

reason, the tangent of the sum of this last angle and of a third angle, whose

whose tangent is c , is $\frac{x+c}{rr-xc} \times rr$ or (by substituting the value of

x) $\frac{rr \times a + b + c - abc}{rr - ab - ac - bc}$, the tangent of the sum of three angles

whose tangents are a, b, c ; and so on.

268. *Corol. 2.* Now put $a = b = c$; and for the tangent of a double angle we have $\frac{2a}{rr-aa} rr$; and for the tangent of a tre-

ble angle $\frac{3arr - a^3}{rr - 3aa}$; and so on.

PROPOSITION V.

269. *The apparent semidiameter of any rain-bow, or the greatest angle under an incident and emergent ray after any given number of successive reflections, being given; to find the ratio of refraction.*

Let m be the given number of successive reflections increased by an unite, and supposing the angles ABC, ABD to be the angles of incidence and refraction sought, let the angle $ABE = m \times ABD$, and the angle CBE , or $m \times ABD - ABC$, will be half the given angle under the incident and the emergent ray after $m - 1$ reflections^a. Put the common radius $AB = r$, the unknown tangent of refraction $AD = a$, and the tangent of incidence $AC = ma^b$, also $AE = x$, and t for the tangent of the given angle CBE answering to the radius r . Then by the lemma $t : x - ma :: rr : rr + xma$; whence $t = \frac{x - ma}{rr + xma} rr$.

Case 1. In the first rain-bow $m = 2$; whence $t = \frac{x - 2a}{rr + 2xa} rr$, and by art. 268, $x = \frac{2a}{rr - aa} rr$ the tangent of $2ABD$. Substitute this value for x in the former equation and by reduction it becomes $a^3 - \frac{3}{2}taa - \frac{1}{2}trr = 0$. By resolving this equation the tangent a of the angle of refraction will be given, and the tangent of the angle of incidence $AC = 2a$ by art. 251, whence the ratio of their sines is given by the tables.

Case 2. In the second rain-bow $m = 3$, whence $t = \frac{x - 3a}{rr + 3xa} rr$, and by art. 268, $x = \frac{3arr - a^3}{rr - 3aa}$, the tangent of $3ABD$. Substitute

stitute this value for x and you will find $a^4 + \frac{8}{3} \frac{rr}{t} a^3 - 2rraa^* - \frac{1}{3} r^4 = 0$;

or putting $T = \frac{rr}{t}$ the tangent of half the angle of this bow^a, $a^4 + \frac{8}{3} T a^3 - 2rraa^* - \frac{1}{3} r^4 = 0$. The same method serves for other bows to infinity.

^a Art. 248.

270. *Corol.* In the first case putting T for $2a$ or AC the tangent of the angle of incidence, and substituting $\frac{1}{2}T$ for a in the former equation $a^3 - \frac{3}{2}taa - \frac{1}{2}trr = 0$, it is changed to this $T^3 - 3tTT - 4rrt = 0$, the same as Dr. *Halley's*, who proposed this problem as an expeditious method for finding the ratio of refraction in any fluid, by observing (when the sun is low and shines very bright) the angle under an incident and the emergent ray from a drop of any fluid hanging at the end of a capillary tube. See his examples *Phil. Trans.* N^o. 267, and also the Rev^d. Dr. *Morgan's* Dissertation upon the Rain-bow among the Notes upon *Robault's* Physics. P. 3. Ch. 17.

CHAP. XIV.

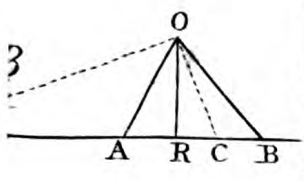
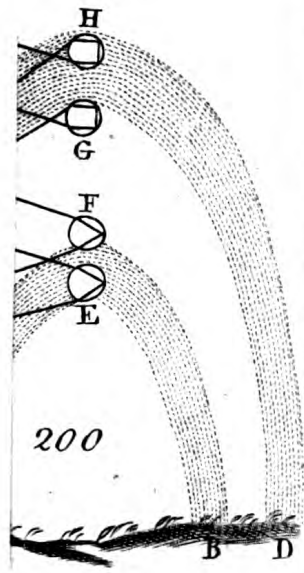
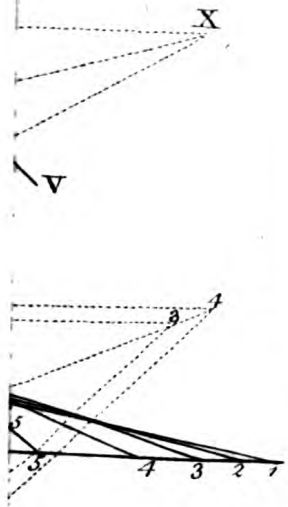
TELESCOPICAL DISCOVERIES IN THE FIXT STARS.

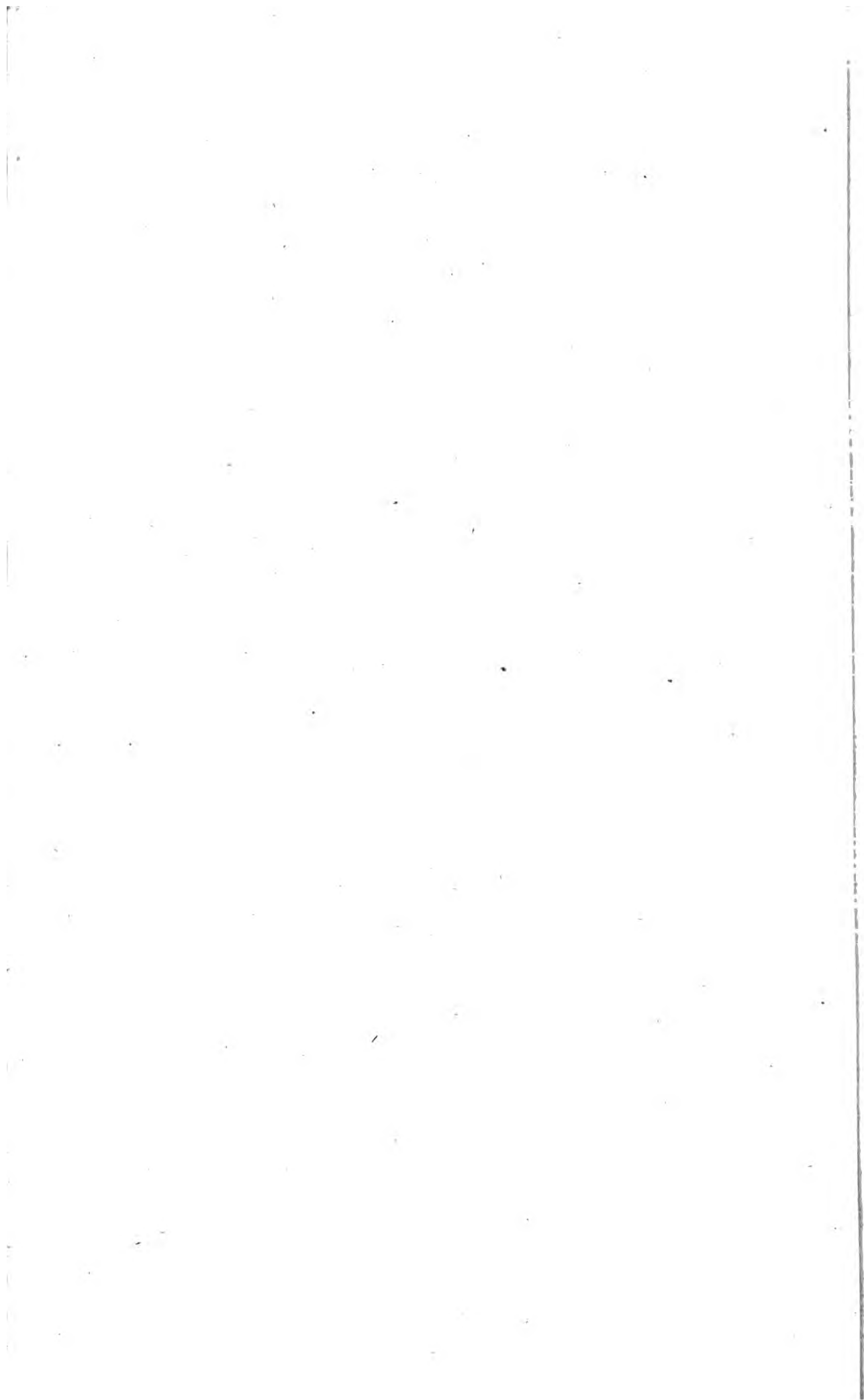
Multitudes of Telescopick Stars.

^a Art. 240.

271. **T**HAT the fixt stars have no sensible parallax, or, which is the same thing, that the earth's annual orbit (whose diameter a cannon ball could not describe in less than 50 years,) would appear of no sensible magnitude through a telescope placed at a fixt star, is such an amazing conclusion as could not be believed, were it not supported by undeniable evidence. But as this is the case, it is no longer a wonder why the best telescopes do not at all magnify the apparent diameters of the fixt stars, though they discover vast multitudes that are quite imperceptible to the naked eye; and the more of them as the aperture is more enlarged to take in more light, and the eye-glass made flatter to render it distinct^a. The Milky Way, which had puzzled the ancient philosophers for many ages, was found at last to be nothing else but a prodigious number of very minute stars, so close to one another that the naked eye can only perceive a whitish mixture of their faint lights. This was *Galileo's* discovery, who found also that those faint stars, which Astronomers call *Nebulosæ*, appeared through his telescope to be small clusters of very minute stars.

272. Hu-





272. *Hugenius* in the year 1656, looking by chance through a large telescope, at three small stars very close to one another in the middle of Orion's sword, saw several more as usual. But the three little stars very near one another (marked θ by *Bayer*), together with four more, shone out as it were through a whitish cloud, much brighter than the ambient sky: which being very black and serene caused that lucid part to appear like an aperture, that gave a prospect into a brighter region. He viewed it many times, and found it continued in the very same place, and of the same shape as the figure represents, and called it *Portentum cui certe simile aliud nusquam apud reliquas fixas potuit animadvertere*^a.

^a Lucid spots among the fixt Stars.

Fig. 206.

273. But in the Philosophical Transactions^b, there is an account of a later discovery of five more such lucid spots, though less considerable than this of *Hugenius*; the middle of which, we are there told, is at present in Π . 19° . $00'$. with south latitude 28° . $45'$; and that it sends forth a radiant beam into the south east, as another in the girdle of *Andromeda* seems to do into the north east. It is also there remarked, that "though these spots are in appearance but small, and most of them but a few minutes in diameter, yet since they are among the fixt stars, as having no annual parallax, they cannot fail to occupy spaces immensely great, and perhaps not less than our whole solar system; in all which spaces it should seem, that there is a perpetual uninterrupted day."

^a Systema Saturnium p. 8.

^b N. 347. Jones's abr. Vol. 4. p. 224.

274. It is to the Author of these reflections, if I mistake not, that we owe another curious account of what is principally remarkable in the new stars that have appeared and disappeared for 150 years last past^c. I will mention but one or two. "That in the chair of *Cassiopeia* was not seen by *Cornelius Gemma* on the eighth of November 1572, who says, he that night considered that part of the Heaven in a very serene sky, and saw it not: but that the next night, Novemb. 9, it appeared with a splendour exceeding all the fixt stars and scarce less bright than Venus. This was not seen by *Tycho Brahe* before the 11th of the same month; but from thence he assures us, that it gradually decreased and died away, so as in March 1574, after 16 months, to be no longer visible; and at this day not the least signs of it remain. The place thereof in the sphere of the fixt stars, by the accurate observations of the same *Tycho*, was 0° . $9'$. from the first star of Aries, with 53° . $45'$. north latitude." To this account Sir *Isaac Newton* adds^d; that in November, when it first appeared, it seemed equal to Venus in brightness, in December to Jupiter, in January 1573 less than Jupiter, but bigger than Sirius, and equal to him in February and March; in April and May equal to the

^c New Stars.

^c Phil. Transf. N. 346. Jones's abr. Vol. 1. p. 222.

^d Philosoph. Princip. p. 526.

the stars of the second magnitude, in June, July and August to those of the third, in September, October and November to those of the fourth, in December and January 1574 to those of the fifth, in February to those of the sixth, and in March it vanished. That its colour was at first clear, white and splendid, afterwards yellow, and in March 1573 red and fiery like Mars or Aldebaran, in May of a pale livid colour like that of Saturn, which grew fainter and fainter till it vanished.

275. "That such another star was seen and observed by the scholars of *Kepler* to begin to appear on Sept. 30. *St. Vet. Anno* 1604, which was not to be seen the day before; but it broke out at once with a lustre greater than that of Jupiter; and like the former, died away gradually, and in much about the same time disappeared totally, there remaining no footsteps thereof in January 1605. This star was near the ecliptick, following the right leg of *Serpentarius*; and by the observations of *Kepler* and others, was in $7^{\text{f.}} 20^{\circ} 00'$ from the first star of *Aries*, with north latitude $1^{\circ} 56'$.

276. Lastly, that the sudden eruption of such another star, shining out more than usual, engaged *Hipparchus* to make the first catalogue of the fixt stars; that posterity might know what changes might happen among them.

The Origine
of new Stars.

277. How minute soever the particles of light may be, the *perpetual* emission of them from the body of the sun must have caused, before this time, a sensible diminution in his magnitude, without some supply of new matter. But since a diminution of the sun's diameter has not yet been discovered by the most accurate observations, *Sir Isaac Newton* therefore imagined, that those comets which approach so near the sun as to pass through his atmosphere, may be so much resisted and retarded after several revolutions, as at last to fall upon the sun and so become a mean of keeping his magnitude nearly the same. And this opinion led him farther to conjecture, that the stars we have mentioned, which suddenly shine out with very great splendor and then decay gradually till they vanish out of sight, may now and then be stirred up and blaze out again by the shock of a comet falling down upon them. But those other new stars, which appear and disappear periodically, which increase by very slow degrees, and seldom exceed the stars of the third magnitude (several of which may be seen in the history I mentioned) he takes to be of another sort, or at least in another state; which revolving about their axes, like the sun, may expose their light and dark parts to us successively. For the fixt stars are undoubtedly self-shining bodies of the same kind as the sun, and therefore equally subject to large
dark

dark spots or crufts upon their fufaces. Beaufe the light of the fun propagated to thofe vaft diftances, and reflected back from opaque bodies of no fenfible apparent magnitudes, would be too much rarified to affect our fenfes; as *Galileo* collected from the fainter lights of the remoter planets from the fun, compared to the luftre of the fixt ftars.

278. After feveral attempts by Dr. *Hook*, Mr. *Flamftead*^a, and others^b, to determine the annual parallax of the fixt ftars, the honourable *Samuel Molyneux* Esq; in the year 1725, erected at *Kew* a very accurate inftrument, in order if poffible to arrive at fome degree of certainty in this difficult enquiry: in the profecution of which he followed Dr. *Hook* in fome refpects, as in taking the zenith diftances of the brighteft ftar in the Dragon's head at the times of its tranfits over the meridian, and alfo in the form of his inftrument, conftituted almoft upon the fame principles with the Doctor's, but executed to a degree of exactnefs vaftly greater, and chiefly owing to the care and contrivance of Mr. *George Graham*.

279. The Rev. Mr. *Bradley*, Profefor of Astronomy at Oxford, who all along affifted Mr. *Molyneux* in the profecution of this noble defign, has obliged the publick with a very accurate hiftory of it, in a letter to D. *Halley*^c; containing not only an account of feveral new and furprifing phænomena that attended the obfervations, (which he therefore continued and repeated after Mr. *Molyneux*'s deceafe,) but alfo a compleat difcovery of the true caufe of them; which at laft enabled him to fettle the point in queftion, and to draw from it fome admirable confequences relating to the propagation of light. As I look upon thefe difcoveries to be fome of the fineft that we have had fince the invention of telefcopes, I will endeavour to give the fubftance of them in as clear a manner as I can.

280. The refult of the obfervations upon the bright ftar in the dragon's head, marked γ by *Bayer*, was this;

Beginning from December 3, 1725, its diftance from the zenith being taken feveral days, at the time of its *tranfit* over the meridian, there appeared no material difference in the obfervations.

281. On Decem. 17, it paffed a little more foutherly from the zenith than before, and ftill more on the 20th; which was matter of furprize, both beaufe no fenfible alteration of parallax could fo foon be expected in this ftar at that time of the year, and beaufe it was the contrary way to what it would have been, had it proceeded from an annual parallax.

An enquiry into the annual parallax of the fixt ftars.

^aWallifii opera, Vol. 3. p. 701.

^bPhil. Tranf. N. 364.

^cAbridg. Vol. 6. p. 165.

^cPhil. Tranf. N. 406.

Abridg. Vol. 6. p. 167.

Some account of the obfervations and phænomena.

282. About the beginning of March 1726, the star was found to be 20" more southerly than at the time of the first observation, and seemed to have arrived at its utmost limit southwards.

283. By the middle of April it appeared to be returning back again towards the north, and about the beginning of June it passed at the same distance from the zenith as it had done in December when it was first observed.

284. From that time it continued to move northwards till September following, when it again became stationary, being then near 20" more northerly than in June, and no less than 39" more northerly than it was in March.

285. From September it returned towards the south, till it arrived in December at the same situation it was in at that time twelve months, allowing for the difference of declination on account of the precession of the equinox.

286. By the like observations made upon a small star almost opposite in right ascension to γ *Draconis*, and at about the same distance from the north pole of the equator, it appeared to change its declination 19", that is about half as much as γ *Draconis* did in the same time. Which plainly proved, as Mr. *Bradley* observes, that these apparent changes were not owing to a nutation of the earth's axis, since the changes on this account would have been nearly equal in these stars, as lying near the solstitial colure.

287. Upon comparing the observations with each other it was discovered in both these stars, that the apparent difference of declination, reckoned from the limits above mentioned, was always nearly proportionable to the versed sine of the sun's distance from the equinoctial points.

288. And that the whole difference of declination in these stars, was as the sine of the latitude of each respectively.

Mr. *Bradley's*
hypothesis to
solve these
phenomena.

289. After a year's observations upon many other stars, in different parts of the heavens, made with a new instrument set up at *Wansted* in 1727, Mr. *Bradley* found out some other properties of their apparent motions; and after examining and rejecting two or three hypotheses, by which he attempted to solve them, at last he conjectured that all these phenomena proceeded from the progressive motion of light and the earth's annual motion in her orbit. For he perceived, that if light was propagated in time, the apparent place of a fixed object would not be the same when the eye is at rest, as when it is moving in any other direction than that of a line passing through the eye and object; and that when the eye is moving

ing in different directions, the apparent place of the object would be different. I will first deduce some consequences from this hypothesis and then compare them with the phænomena.

290. If an eye moves uniformly in a straight line from a to b in the time that the light of a fixt star descends uniformly in a straight line from c to b , the star will appear in a direction constantly parallel to ac . Some consequences from the hypothesis. Fig. 207.

For conceiving the eye to carry the line ac parallel to itself, its intersection with the fixt line bc will move uniformly^a from c to b , and will therefore accompany a particle of light descending uniformly from c to b ; and because this intersection is a moving point, not only in the fixt line bc , but also in the moving line ac , it is plain that the particle which accompanies the intersection c , moves relatively in the moving line ac . In like manner a particle of every other ray, parallel to cb , which the moving line ac successively meets with, moves also in the moving line ac ; and thus a succession of these particles, moving along ac , constitute a visual ray in whose direction the star appears. ^a Euc. VI. 2.

291. Hence supposing the earth's center b to move uniformly in a circular orbit $\gamma \delta Ab$, round the sun in its center B ; if a line BC drawn towards a fixt star, supposed infinitely distant, you take a distance BC in proportion to Bb or BA , as the velocity of light to the velocity of the earth's center, an observer upon the earth at b , will constantly see that star in a direction very nearly parallel to a line AC , connecting the point C with a point A in the orbit constantly 90 degrees behind the earth. Fig. 208.

For drawing ba and bc parallel to BA and BC respectively, and tending the same ways from b and B , and also any line ac parallel to AC ; by the similar triangles bca , BCA , we have $bc : ba :: BC : BA$, as the velocity of light to the velocity of the earth. Consequently if an eye be supposed to move along the tangent ab with this latter velocity, it will see the star in a direction constantly parallel to ac ^b or AC . But the eye in the orbit moves with that velocity, and passes by the point b in the direction of that tangent, and therefore at that passage it saw the star in the same direction in which the other eye in the tangent sees it constantly. The earth's diurnal motion alters this conclusion so little that I need not here consider it. ^b Art. 290.

292. Therefore the apparent parallax of the star to an observer at b , is constantly measured by the angle ACB , if the point A be always 90 degrees behind the earth at b , and consequently 90 degrees before the sun's apparent place \odot in the ecliptick.

293. Hence the apparent latitude of any star, supposed infinitely distant, will be least of all when the sun's place in the ecliptick is 90 degrees forwarder than the star's; and from that time it will increase for half a year, and then decrease for the next half year; and its increment reckoned from these limits will be constantly as the versed sine of the sun's longitude reckoned from his place before mentioned.

Fig. 209.

For drawing CD perpendicular to the plane of the orbit, join AD , and draw DB cutting the orbit in L and M . The point L , nearest to D , is the star's place in the ecliptick, and the point \odot , opposite to b , is the sun's place therein. Now when the point \odot was at N , 90 degrees forwarder than L , the point A , being always 90 degrees forwarder than \odot^a , was at M , the farthest point from the perpendicular CD ; and consequently the star's apparent latitude, always measured by the angle CAD , was then the least possible.

^a Art. 292.

Draw AE perpendicular to LM , and joining CL , CE , CM , draw MF perpendicular to CE produced. Then conceiving the point A to move in the perpendicular AE towards E , the angle CAD will approach to a *maximum* CED , and therefore will increase very little, especially as the angular approach ACE is exceeding small. Therefore instead of the apparent latitude CAD we may take CED , and consequently the angle ECM for the increment of the least latitude CMD : now this small angle ECM is as its sine MF or (because the ratio of MF to ME varies very little) as ME the versed sine of the arch MA equal to $N\odot$, the sun's longitude from N , 90 degrees forwarder than the star's place L .

294. When a star is situated any where in the solstitial colure, the increments or decrements of its latitude and declination are the very same quantities; and therefore if the star be supposed infinitely distant, and its longitude be in the beginning of Capricorn, with north declination, its apparent declination will be the least at the time of the vernal equinox, and the greatest at the autumnal; and its increments and decrements reckoned from these limits, will be proportionable to the versed sine of the sun's longitude reckoned from the equinoctial points: which agrees with the phenomena in Art. 287.

Fig. 210.

295. The whole apparent parallax LPM of a star in the pole of the ecliptick, is to the whole apparent parallax LCM in the latitude of any other star, as the radius to the sine of the latitude CBL . For since BP equals BC , drawing BF and BG perpendiculars to CL and CM , the small angle BPL is to BCL as BL to BF , that is, as the radius to the sine of the angle BLF or of the latitude CBL^b . Again, the small angle BPM is to BCM as BM to BG , that is, as the

^b Art. 68.

the radius to the sine of BMG or of the latitude CBL , as before. Therefore the whole angle LPM is to LCM as the radius to the sine of the latitude CBL .

Hence, from the observed parallax in latitude, or in declination of such stars as lye in or near the solsticial colure, we have the parallax that would belong to a star in the pole of the ecliptick, which is plainly the greatest of all. Thus in γ *Draconis* whose latitude $CBL = 74^\circ. 58'. 20''$, the observed parallax LCM was $39''^a$, and thence the greatest parallax LPM comes out $40''.4$. Likewise in the little star above mentioned, whose distance from the north pole of the equator is $38^\circ. 28'. 35''^b$, and consequently its latitude a little above $28^\circ. 02'. 25''$, as being almost opposite in right ascension to γ *Draconis*, the observed parallactick angle LCM was $19''^c$, and thence LPM comes out $40''.4$ as before.

^a Art. 284.

^b The 36th Star *Camelopardi Hevelii* in *Flamsteed's* Catalogue.

^c Art. 286.

296. Mr. *Bradley* having applied his observations upon the parallax in declination of stars in any situation whatever, to his theory farther pursued, assures us they all conspire to prove, that the greatest parallax LPM is about 40 or 41 seconds, and thinks the *medium* $40''.\frac{1}{2}$ cannot differ so much as one second from the truth. The greatest apparent parallax.

297. Hence the velocity of star-light comes out 10210 greater than the velocity of the earth's mean motion round the sun. For the former velocity is to the latter as BP to BL or BM^d , that is, as the radius to the tangent of BPL or $BPM = 20''.\frac{1}{4}$ as above determined. The velocity of star-light.

^d Art. 290.

298. From what has been said Mr. *Bradley* infers, 1. That the lights of all those stars arrive at the earth with equal velocities. 2. That unless their distances from us are all equal, (which for other reasons besides that of their different lustre, is highly improbable) their lights are propagated uniformly to all distances from them. 3. That the velocity of star-light is such as carries it through a space equal to the sun's distance from us in $8'. 13''$, (this time being to the time in which the earth might describe that distance, with the velocity of her mean motion round the sun, as 1 to 10210, and this latter time, to half a year, as the diameter of a circle to its circumference.) 4. That the time so determined can scarce differ 5 or 10 seconds from the truth, which is such a degree of exactness as can never be expected from the eclipses of Jupiter's satellites. 5. That as this determination of the velocity of star-light, comes out a *medium* among several determinations of the velocity of the sun's light reflected from those satellites, we may reasonably conclude that the velocities of these lights are equal. And lastly, since it is highly probable that the velocity of the sun's emitted light is also equal to that of star-light, Some properties of light...

* Art. 277. light^a, it is equally probable that its velocity is not altered by reflection into the same medium.

299. From Art. 291, 295, &c. it follows plainly, that a star placed in the pole of the ecliptick would appear in a year's time, to describe about the pole a little circle whose apparent semidiameter is $20''\frac{1}{4}$; and that any other star will appear to describe, about its true place, an ellipsis whose long axis is at right angles to the circle of longitude passing through the star's true place, and equal to the diameter of the little circle just mentioned, and whose short axis is to the long one, as the sine of the star's latitude to the radius*.

Real parallax
of the stars
insensible.

300. Upon this theory farther pursued Mr. *Bradley* proceeds synthetically, by assuming the *maximum* of apparent parallax as determined above, and calculating tables of the differences in declination of γ *Draconis* situated near the solsticial colure, and of γ *Ursæ Majoris* nearer to the equinoctial than the solsticial colure; and by com-

* The following elegant proposition is reprinted from T. Simson's *Mathematical Essays*, as it affords an easy solution of the consequences mentioned in this article, and serves to confirm other parts of this Theory.

PROPOSITION. *To find the path which the progressive motion of light and the motion of the Earth in its orbit make a Star appear to describe in one entire annual revolution of the Earth.*

Fig. 211.

Let *ATBA* be the orbit of the earth; *S* the sun in one focus; *F* the other focus; *T* the earth moving in its orbit from *A* towards *B*; *DTn* a tangent at *T*; and *SD*, *FE* perpendiculars thereto: Let $\mathcal{Q}mKR\mathcal{Q}$ be part of an indefinite plane parallel to that of the ecliptick, passing through *R* the center of the given star; and take *Tn* to *TR*, as the velocity of the earth in its orbit at *T* to that of a particle of light coming from the said star: Let *Tm* be parallel to *nR*; *PnV* perpendicular to *AB*; and $\mathcal{Q}RK$ parallel to *PnV*: Then from the 290th article it is manifest, that a ray of light, coming from *R* to the earth at *T*, will appear as if it proceeded from *m*, where the line *Tm* produced, intersects the said parallel plane, and therefore because *Tm* is parallel to *Rn*, and any parallelogram, intersecting two parallel planes, cuts them alike in every respect, it is evident that *Rm* must be equal to *Tn*, and $\mathcal{Q}Rm$ to *VnD*; wherefore since *D* and *P* are equal to two right angles, *DSP* and *DnP* must be equal, also, to two right angles^a, and consequently $\mathcal{Q}Rm (= VnD^b) = DSP = AFE$. But *Tn* or *Rm*, expressing the celerity of the earth at *T*, is known to be inversely as *SD*^c; or because *SD* \times *FE* is every where the same^d, directly as *FE*; wherefore the angles *AFE*, $\mathcal{Q}Rm$ being every where equal, and *Rm* in a constant proportion to *FE*, the curve $\mathcal{Q}mK$ described by *m*, the apparent place of the star in the said parallel plane, will, it is manifest, be similar in all respects to *AEB* described by the point *E*: But this curve is known to be a circle^e; therefore $\mathcal{Q}mK$ must likewise be a circle, whose diameter $\mathcal{Q}RK$ is divided by *R*, the true place of the star, in the same proportion as the transverse axis of the earth's orbit is divided by either of its foci. Wherefore, forasmuch as a small part of the circumjacent heavens may, in this case, be considered as a plane passing perpendicular to a line joining the eye and star, it follows from the principles of orthographic projection, that the star will be seen in the heavens as describing an ellipsis, whose center (as the excentricity of the orbit is but small) nearly coincides with the true place of the star, except the said place be in the pole or plane of the ecliptick; in the former of which cases the star will appear to describe a circle, and in the latter an arch of a great circle of the sphere, which by reason of its smallness may be considered as a right line.

^a Eucl. I. 32.

Cor. 1.

^b Eucl. XI. 10.

^c Newt. prin-

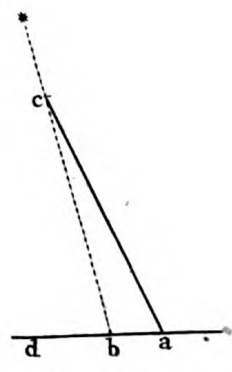
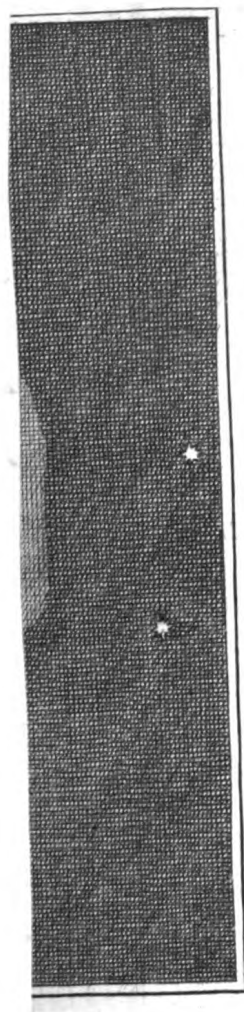
cip. I. 1. Cor.

^d Hamilton's

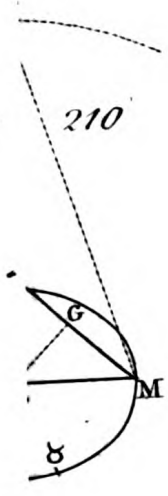
Conics, II. 21.

^e Hamilton's

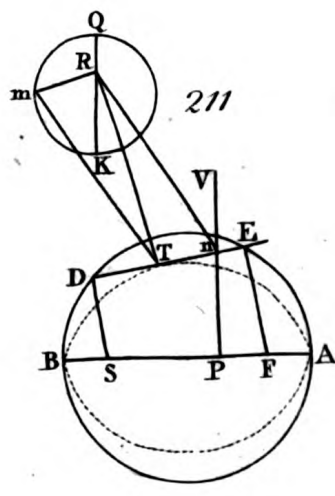
Conics, II. 20.



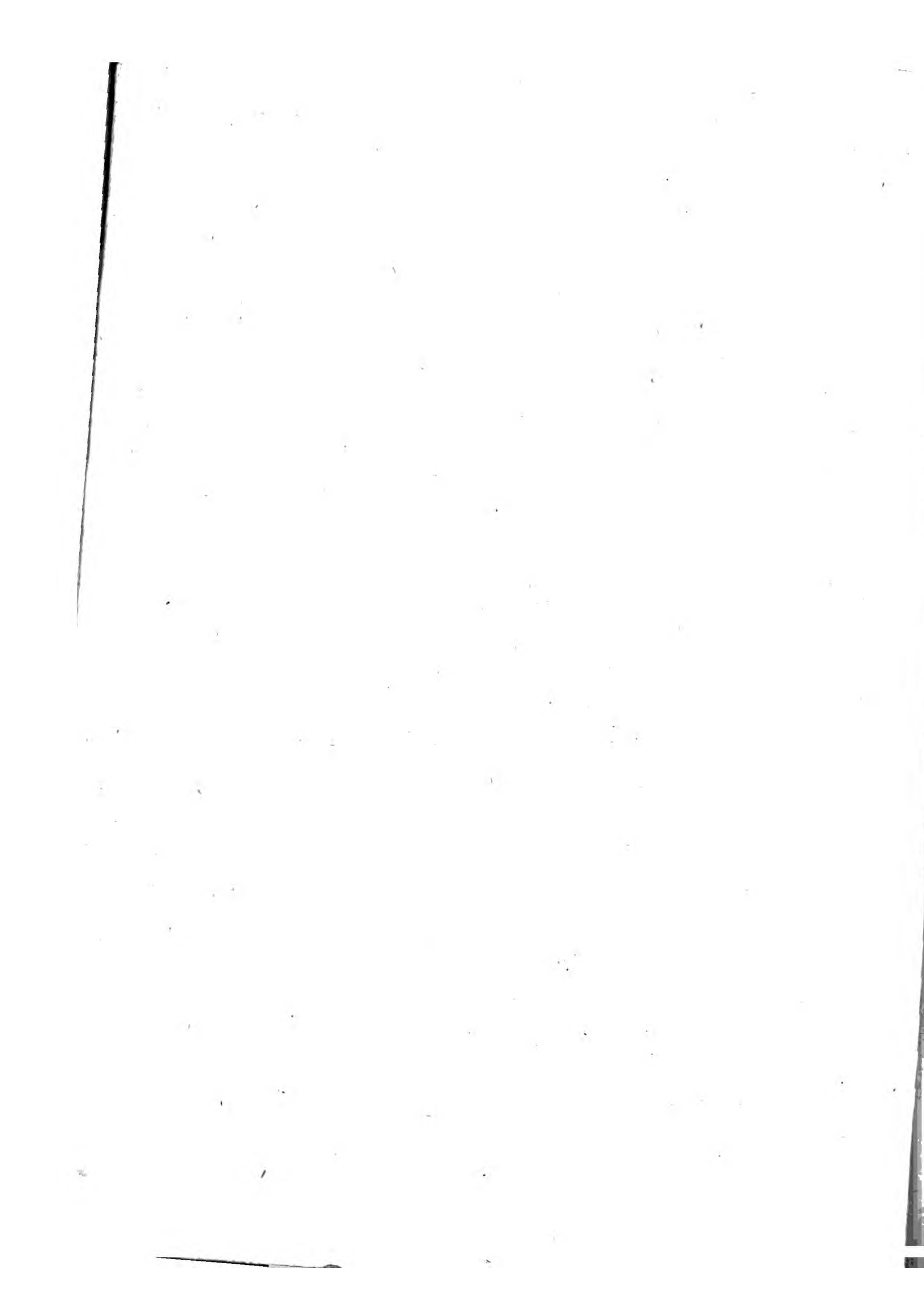
207



210



211



paring the tables with his observations, he found they agreed together throughout the year, sometimes in the very same number of seconds, and that in 50 or 60 observations of each star, they never differed so much as two seconds; allowing for the variation of declination caused by the regression of the equinoctial points: which amounts to a physical demonstration of the truth of his theory, and does in consequence afford a very satisfactory answer to the point in question, concerning the real parallax and distance of the fixt stars. As to which he believes he may venture to say, that the real parallax in either of the stars abovementioned does not amount to 2", being of opinion that if it were 1" he should have perceived it in the great number of observations that he made especially upon γ *Dragonis*; which agreeing with the theory, without allowing any thing for a real parallax^a, nearly as when the sun was in conjunction with,^a Art. 290, 291, &c. as in opposition to this star, it seemed to him very probable, that its real parallax is not so great as one single second; and consequently that it is above 400000 times farther from us than the sun.

F I N I S.

11
The first of these is the
fact that the
the first of these is the
fact that the
the first of these is the
fact that the



