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AN ESSAY ON PERSPECTIVE.

BOOKS *Printed for* J. SENEX, W. TAYLOR, W. *and* J. INNYS, *and* J. OSBORNE.

MAthematical Elements of Natural Philosophy confirm'd by Experiments; or, An Introduction to Sir *Isaac Newton*'s Philosophy. Written in *Latin* by *William-James 's Gravesande*, Doctor of Laws and Philosophy, Professor of Mathematicks and Astronomy at *Leyden*, and F. R. S. of *London*. Translated into *English* by *J. T. Desaguliers*, L. L. D. F. R. S. and Chaplain to his Grace the Duke of *Chandos*.

Physico-Mechanical Experiments on various Subjects. Containing an Account of several surprizing Phænomena touching Light and Electricity, producible on the Attrition of Bodies: With many other remarkable Appearances, not before observ'd. Togther with the Explanations of all the Machines, (the Figures of which curiously engrav'd on Copper) and other Apparatus us'd in making the Experiments. To which is added a Supplement, containing several New Experiments not in the former Edition. The 2d *Edit*. By *F. Hawksbee*, F. R. S.

A Treatise of the Motion of Water, and other Fluids: With the Origin of Fountains and Springs, and the Cause of Winds. In which Treatise, the Manner of Levelling and Conducting Rivers, in order to make them Navigable; the making of Aqueducts, for the Supply of Gentlemens Seats; the whole Art of contriving and making Jetts of Water for Fountains; and the Manner how to proportion the Strength of Pipes, for conveying Water from any Height, are plainly demonstrated from the Laws of Hydrostaticks, by above 100 Curious Experiments. Written originally in *French*, by the Learned *M. Marriotte*, Member of the Royal Academy at *Paris*. And translated into *English*, (with several Annotations for Explaining the doubtful Places) by *J. T. Desaguliers*, L. L. D. F. R. S. Chaplain to his Grace the D. of *Chandos*.

The Elements of *Euclid*, with select Theorems out of *Archimedes*; by the Learned *Andrew Tacquet*. To which are added, Practical Corollaries, shewing the Uses of many of the Propositions. The whole abridg'd, and (in this 4th Edition) publish'd in *English*, by *W. Whiston*, M. A. Mr. *Lucas*'s Professor of the Mathematicks in the University of *Cambridge*.

Sir *Isaac Newton*'s Mathematick Philosophy more easily demonstrated; with Dr. *Halley*'s Account of Comets, illustrated. Being 40 Lectures read in the Publick Schools at *Cambridge*; by *W. Whiston*, M. A. For the Use of the young Students there. In this Edition, the whole is corrected, and improv'd by the Author.

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ESSAY

ON

PERSPECTIVE.

Written in *French* by
WILLIAM-JAMES 'S GRAVESANDE,
Doctor of Laws and Philosophy; Professor of Mathematicks and Astronomy at *Leyden*, and Fellow of the Royal Society at *London*.

And now Translated into *English*.
by by Ed. Stone

LONDON:

Printed for J. SENEX, in *Fleetstreet*; W. TAYLOR, in *Pater-Noster-Row*; W. and J. INNYS, in *Ludgate-street*; J. OSBORNE, in *Lombard-street*; and E. SYMON, in *Cornhill*. MDCCXXIV.

TO

Mr. *William Kent.*

SIR,

IN Dedicating this Tranſlation to you, I have deſignedly deviated from the general Cuſtom obſerv'd by almoſt all Dedicators, who make choice of ſuch Patrons that are Great and Rich, not at all conſidering their Merit, or whether they underſtand any thing of what is offer'd to them; ſince I have inſcrib'd this Treatiſe of *Perſpective* to one, whoſe daily Practice is the very Art it ſelf, and whoſe

Merit is undoubtedly excellent, as evidently appears from your own Works.

I ſhall likewiſe be particular with regard to the Manner of the Offering ; being perſuaded that Flattery, or even due Praiſe, which are the common Topicks handled in Dedications, muſt needs be offenſive to an Ingenious Perſon ; and ſo I ſhall be ſilent on theſe Heads ; and only crave your Acceptance and Protection of what is here offer'd by

Your Humble Servant,

E. STONE.

The AUTHOR'S PREFACE.

THE Reader will wonder, perhaps, to find me entring into a Path, which ſeems to have been too much trodden already; and eſteem as uſeleſs a New Eſſay, on an Art, whoſe Subject (one would think) ſhould have been long before this Time exhauſted; ſince there have been ſo many Perſons, who have written on the ſame.

The Name of Perſpective *now ſeems to ſound unpleaſant in the Ears of the Publick Enemies of Repetition; and it may be look'd upon as a Piece of Inadvertency, to venture to treat again on that ſame Subject. Yet, notwithſtanding this, I deſire the Reader to ſuſpend his Cenſure, until he has heard the*

 Rea-

Reasons that induc'd me to publish the following Work.

Having some Years ago busied my self in drawing Figures by the common Methods, I found out several Compendiums; which, by diligent working, naturally enough fall in one's way, without being entirely beholden to the Industry of others: And these first Successes made me hope for others more considerable; and so I thought that a more narrow Inspection into the Theory of Perspective, *might furnish me with Rules more general, for making the Practice thereof easy.*

I then thought upon several Methods to this Purpose; but, being suspicious that they were not so easy as they appear'd, I have try'd their Goodness, by exactly applying them to different Subjects; and have nicely examin'd all the Cases, and order'd it so as not to be deceiv'd by certain Operations, which at first seem easy, but, when put in Practice, are quite otherwise. Moreover, at convenient Times, I look'd over the best Part of the Authors of this kind, (whose Number is increas'd very much, without any manner of Necessity) some of which being advantagiously distinguish'd among the Crowd, have been very useful to me: But I dare affirm, there are but a very few that give a new Turn to the practical Part of Perspective.

Some content themſelves with the bare Explication of the Theory, and have left to the Reader the Trouble of applying the ſame to Practice; or elſe have given only ſome of the common Operations, and entertain us with general Reflections on Painting; which are indeed curious, but foreign to my Purpoſe: For I intend not to make a Man a Painter, but to render the Uſe and Exerciſe of Perſpective *eaſy to him.*

Other Authors, which (according to the Bulk of their Works) might be thought to have more carefully treated of the practical Part of Perſpective, *do indeed at firſt lay down ſome general Rules, common to them all; but are nothing the eaſier for having paſs'd thro' ſo many Hands; and that, indeed, becauſe they have not endeavour'd to make them ſo. They thought that all Objects might be thrown into Perſpective by theſe Rules, and therefore it would be uſeleſs to ſearch after others; and judg'd it more neceſſary to ſhew Painters the Application of them to an infinite Number of particular Examples; tho' that Application, at moſt, is but repeating over again the Uſe of the Rules already preſcrib'd. But what Advantage can Painters gain from hence, if they do not well underſtand general Operations? And if they do, I cannot conceive of what*

what Use such an excessive Variety of Examples will be to them.

I believ'd then, that I might be able to treat of this Art after another Manner: And altho' I know my self to be much inferior to several of those who have written on this Subject; yet I am of Opinion, that if Perspective *should lose any thing by me, on account of my want of Judgment; yet that may be regain'd, perhaps, (and with Interest too) by my great Diligence in this Business.*

I have consider'd, moreover, that the tedious Particulars, inherent to the Subject on which I have chosen to write, will always hinder Genius's capable of great Matters, from undertaking a Subject so little worthy their Endeavours, and so barren of great Discoveries.

Thus, hoping, on one hand, to give a new Turn to the Practice of Perspective, *and make it easier; and being persuaded, on the other, that more learned Persons than my self will not take this Trouble upon them; I venture to publish this small Work, and expose it to the Taste of the Learned World; from whom I expect no other Praise, but what may reasonably be claim'd by an assiduous Application.*

The

The Practice of Perspective *may be made easy, by the Three following Things in this Treatise :* Viz. 1. *In giving several new and easier Ways (than those commonly used) of solving the most general Problems upon which the whole Practice is founded : And the Reason why we have laid down several Solutions, is, because the same Way is not always equally convenient in all Cases ; whence it is necessary to have several, that so we may chuse one best suiting our Purpose.* 2. *The general Methods, which have been us'd hitherto, not being practicable on some particular Occasions ; to remedy this, we have added others to them ; which are indeed more difficult, but (in some Cases) there is an absolute Necessity for them.* 3. *When it is very difficult to resolve a particular Problem, by means of general ones ; then we have thought it convenient to give a particular Solution thereof.*

By this means, the Study of Perspective *becomes indeed more difficult ; but the Disadvantage is well recompensed by the Facility of the Practice, which we have entirely had in view. It is true, that a few general Rules do not so much burthen the Memory ; but when one has several general ones, and also particular ones, by them we can abridge Matters. And this Method being*

ing pursued at first, tho' it requires a little more Application, does afterwards save a great many Hours Study, in an Art that always appears difficult enough.

A Painter, in a short Time, may learn this Work, and make the Rules thereof familiar to him: And if this Study be repeated from time to time, for a few Days, he will find the Benefit thereof, in diminishing his Labour and Trouble.

But, that any one himself may see what I promise in this Essay ; *take the following short Abstract thereof. It is divided into Nine Chapters: The First, being as an Introduction to the rest, shews the Usefulness of* Perspective, *and gives you the Definitions of the Terms necessary for understanding this Treatise.*

The whole Theory is contain'd in the Second Chapter: Where, what has been found most useful in that Matter, is therein reduced to Three general Theorems; viz. *the first, second, and fourth: All the rest is deduced from them, by way of Corollary. To these Theorems, already known, are added some new ones, serving for the Demonstration of some necessary Propositions. Perhaps it might be wish'd, that I had shewn the Way that led me to the Truths which I discover:*

This

This I have done ſometimes; but it often would have been very long and troubleſome. In Geometry, the eaſieſt and ſhorteſt Way, is not always that which leads to Diſcoveries.

In the following Chapter, the Practice of Perſpective *upon the perſpective Plane, or Picture, conſider'd as upright, is explain'd: Wherein, among the different Ways laid down for the Solution of general Problems, you will find ſome effected by a Ruler only; ſo that after ſome Preparations, all Kinds of Objects may be drawn without Compaſſes, and that eaſier than by the common Operations. In that Problem, to find the Appearance of a Point out of the Geometrical Plane, it is commonly conſider'd as the Extremity of a Perpendicular, whoſe Repreſentation muſt firſt be found, before that of the Point can be had. But here we avoid this round about Way, and ſhew how to find the Appearance of the Point given, without being obliged to find the Perſpective of its Seat.*

As to the Appearance of a Cone and Cylinder, we determine the viſible Portions of the Baſe, and by this means avoid the uſeleſs Operations which the common Way is ſubject to. It is very difficult, if not impoſſible, to throw a Sphere into Perſpective, by means of general Problems; and in the Repreſentation of the Torus of a Column, it is ſtill

ſtill difficulter: Whence we are obliged to give particular Methods for the Reſolution of theſe two Problems.

The reſt of the Third Chapter is concerning Inclin'd Lines, and how to find their Appearance by the Accidental Point.

The Fourth Chapter ſhews the Manner of working on a perſpective Plane, to be view'd afar off, very obliquely, or which muſt ſtand in an high Place. Theſe different Situations require new Rules: For if the common Methods were to be uſed here, the perſpective Plane muſt be ſo large, as that it would be impoſſible to work upon it.

In the Two following Chapters, we treat of the perſpective Plane, conſider'd as Horizontal, or Inclin'd: Where there are laid down ſeveral general Ways of working; which, together with thoſe of the foregoing Chapters, will ſuffice (in my Opinion) for throwing any Object whatſoever into Perſpective, with Eaſe enough.

In the Seventh Chapter, which treats of Shadows, there is nothing particular, but what may be ſeen elſewhere: But that little we have ſaid concerning this Matter, is enough for giving an Idea of them, which the Reading of what goes before will make eaſy.

In the Eighth Chapter, are laid down ſome Mechanical ways for making the Uſe of Perſpective *eaſy, by means of Rulers and Threads, (eaſily to be gotten by any body, and not difficult to be put in practice) they being eaſier to uſe than any of the Inſtruments that have hitherto been invented for this Purpoſe.*

The laſt Chapter ſhews the Uſefulneſs of Perſpective *in Dialling.*

Such is the Plan of this ſmall Work; wherein I have not ſo much endeavour'd to advance Curioſities, as Things of real Uſe; hoping that, without making a Shew of Skill ill beſtow'd, I ſhall make my Book good enough, if by its Uſe I make it neceſſary. For which Reaſon, I have endeavour'd to lay down the whole, ſo as to be underſtood by thoſe who have only read the Elements of Euclid. *And tho' I have deviated from this Rule in ſome few Places; they are printed in* Italick, *that ſo they may be paſs'd over without any Hindrance to the Learner.*

Here I muſt not forget to mention, that in Reviſing this Eſſay, *I had the Happineſs of meeting with an able Painter; who has ſeriouſly conſider'd every Thing of his Profeſſion, neceſſary to be known, among which,*

Per-

Perspective *was not neglected. He has carried the Matter farther than could have been reasonably expected from one ignorant of Mathematicks ; and I am indebted to him for several Observations, which I my self should perhaps have never thought on.*

ERRATA.

Page 74. l. 17. for *x*, r. *in* X.
P. 83. l. 28. for *that*, r. *that for*.
P. 88. l. 8. for *Tube*, r. *Table*.

AN

AN ESSAY ON PERSPECTIVE.

CHAP. I.

DEFINITIONS.

1. *PERSPECTIVE* teaches us the Manner of Delineating by Mathematical Rules; that is, it ſhews us how to draw geometrically upon a Plane, the Repreſentations of Objects according to their Dimenſions and different Situations; in ſuch manner, that the ſaid Repreſentations produce the ſame Effects upon our Eyes, as the Objects whereof they are the Pictures.

In order to underſtand well how Mathematicks may be apply'd to Drawing; let us ſuppoſe a Man *A*, viewing an Object; and between him and the Object he looks at, let us imagine a tranſparent Plane *C*. Suppoſe moreover, that Lines be drawn upon this Plane, as in *D*, which cover the Bounds of the Object *B* in reſpect of the Spectator *A*, and each Part that he ſees thereof. Now, ſince all Objects are ſeen by the Rays of Light coming from every of their Points, and terminating at the Eye, and not otherwiſe; and ſince that here all the Rays proceeding from the Object *B*, likewiſe paſs thro' every Point of the Repreſentation *D*; it is manifeſt, that this Repreſentation will have the ſame Effect upon the Spectator's Eye, as the ſaid Object *B* hath. Now, by means of Geometry, we can find the Points of the Figure *D*, on the Plane *C*, placed in a given Situation, thro' which the Rays coming from the Object *B* to the Eye of the Spectator *A*, do paſs; and theſe Points are the Interſections of the Rays and the Plane. Alſo, (as others have very well obſerv'd) a *Perſpective* Plane, or Picture in Painting, may be conceiv'd as a Window, upon which the Objects ſeen thro' it are repreſented.

Fig. 1.

Now, without Mathematicks, this Repreſentation cannot be well found: For when Objects are drawn by only viewing, or looking at them; their true Repreſentations after this way, will be very often miſs'd on; whereas, by Geometry, we can always obtain them.

This Obſervation only, is ſufficient to eſtabliſh the Neceſſity of *Perſpective*: Tho' there are ſome Painters, who (according to the common Maxim) affirm, That what they do not know of this Art, is not worth the Pains of learning.

Hither-

Hitherto I have endeavour'd to give an Idea of *Perspective* in general: But there is yet another particular Signification of this Word, which it is necessary should be explain'd, as well as the other Terms of the Art, which are laid down in the following *Definitions*; and which every one, that intends to understand this *Treatise*, ought to be well acquainted with.

2. *The Perspective, Representation, or Appearance of an Object*, (for these Three Words are synonimous) * *is the Figure which the Rays, by which an Object is perceiv'd, form in passing thro' the transparent Plane: And the Perspective of a Point, is the Intersection of a Ray proceeding from that Point, and the transparent Plane.* Which Intersection is a Point: As the Figure *D* in the transparent Plane *C*, is the *Perspective* of the Object *B*; and the Point *e*, in the same Plane, is the *Perspective* of the Point *E*, in that Object. Def. 1. *Fig.* 1.

The Plane parallel to the Horizon, upon which the Spectator [stands, or] *is placed, as likewise the Objects that he views, is call'd the Geometrical Plane.* As *A B C D*. Def. 2. *Fig.* 2.

A Perspective Plane [or Picture] *is that which is placed between the Spectator and the Object, upon which the Objects are drawn*: As *F G R T*. This is commonly perpendicular to the Geometrical Plane, and consequently to the Horizon; because Pictures have generally this Situation: But yet it may be sometimes inclin'd, and even parallel to the Geometrical Plane, according as one would dispose the Design, or Picture that we are working. And for this Reason, in the following Chapter, we have laid down General *Theorems*, and their *Corollaries*, agreeing to all these Def. 3.

different Situations of the *Perſpective* Plane, [or Picture] which ought to be well obſerv'd.

Def. 4. *The Interſection of the Perſpective Plane and the Geometrical Plane, is call'd the Baſe-Line :* As *F G.*

The different Situation of the Eye, alters the Repreſentation of Objects in the perſpective Plane; for the Rays proceeding from the Object, and concurring in ſome other Point, will likewiſe fall upon the *Perſpective* Plane in different Places. And for determining this Situation of the Eye, in reſpect to the *Perſpective* Plane, we ſuppoſe,

Def. 5. *A Plane parallel to the Horizon, paſſing thro' the Eye, and every way extending it ſelf ; and this is call'd the Horizontal Plane :* As O M V N L.

Def. 6. *The Interſection of this Plane and the Perſpective Plane, is the Horizontal Line.* As M V N.

Def. 7. *The Perpendicular drawn from the Eye to the Horizontal Line, is the principal Ray.* As O V.

Def. 8. *The Point* V, *wherein the ſaid Perpendicular meets the Horizontal Line, is the Point of Sight, or principal Point.*

Note, There is a Perpendicular, let fall from the Eye upon the Geometrical Plane, meaſuring the Height of the Eye.

Def. 9. *The Point* S, *wherein the ſaid Perpendicular meets the Geometrical Plane, is the Station-Point.*

Def. 10. *The Plane paſſing thro' the aforeſaid Perpendicular, and the principal Ray, is call'd the Vertical Plane.* As S O L I.

The

The Interſection V H *of this Plane, and the Perſpective Plane, is the Vertical Line.* Def. 11.

And S H I, *the Interſection of it, and the Geometrical Plane, is the Station Line.* Def. 12.

Points of Diſtance, are two Points in the Horizontal Line, each way diſtant from the Point of Sight by the length of the principal Ray; *as* M N. Def. 13.

The Geometrical Line, is a Line, that paſſes through the Station Point, and is parallel to the baſe Line, as A B. Def. 14.

The Seat of an Object, is the Concurrence of Perpendiculars let fall from every of its Points upon the Geometrical Plane, and the ſaid Plane. Def. 15.

The Direction of a Line inclined to the Geometrical Plane, is the Interſection of the ſaid Plane, and another Plane perpendicular thereto, paſſing through the ſaid inclined Line. Def. 16.

CHAP. II.

The Theory of Perſpective.

LEMMA.

3. THE Perſpective, or Appearance of a Right Line, as *A B*, which being continued, does not paſs through the Eye *O*, is likewiſe a right Line: For the Rays, by which the Line *A B* is perceived, form a Plane cutting the perſpective Plane; and the common Section of theſe two Planes is a right Line, as *a b*. *Fig.* 3.

Theorem I.

Fig. 3. 4. *The Representation of a Line Parallel to the perspective Plane, is parallel to the Line whereof it is the Representation.*

Let *A B* be a Line Parallel to the perspective Plane; we are to prove that *a b* its Representation is Parallel thereto.

These two Lines *A B* and *a b*, will never meet each other, because *a b* is in the perspective Plane, and *A B* is supposed parallel to the said Plane. But they are also in one and the same Plane, because *a b* is the Interfection of the perspective Plane, and the Plane *O A B*, passing through the Eye and the Line *A B*; and therefore they are parallel between themselves: *Which was to be demonstrated.*

Corollary I.

5. *The Appearance of a Line, parallel to the base Line, is also parallel to the said base Line.*

For the base Line, and the Representation being parallel to the same Line, are parallel to one another.

Corollary II.

6. *The Representation of a Line parallel to the vertical Line, is parallel to the said vertical Line, and consequently perpendicular to the base Line.* This is demonstrated as in the last Corollary.

Corollary III.

7. *The Appearances of Lines parallel to the perspective Plane, and equally inclin'd the same Way upon the*

the Geometrical Plane, make Angles with the base Line, equal to those Angles that the Lines whereof they are the Appearances, make with the Parallels to the base Line, which cut them; and consequently the said Appearances are parallel between themselves.

This is evident, because the Appearances of Lines parallel to the base Line, are parallel to the said Line; and the Appearances of the inclined Lines are parallel to these Lines.

THEOREM II.

8, 9. *The Representation of a Figure, parallel to the perspective Plane, is similar to the said Figure; and the Sides of the said Figure are to their Representations, as the Distance of the Eye from the Plane of the Figure, to the Distance of the Eye from the perspective Plane.*

The given Figure is *A B C D*. We are first to prove, that its Representation *a b c d*, is similar thereto; that is, that the corresponding Angles of these two Figures *A B C D*, *a b c d*, are equal, and their Sides proportional. *Fig.* 4.

I. The Angles are equal, because * the Lines of which the two Figures consist, are parallel between themselves. * 4.

II. In the similar Triangles *A D O*, and *a d o*, we have

$$AD : ad :: OD : Od.$$

And in the similar Triangles *O D C*, and *O d c*, we have

$$DC : dc :: OD : Od.$$

then

$$AD : ad :: Dc : dc,$$

altern,

$$AD : DC :: ad : dc.$$

And consequently the Sides *A D*, and *D C* of the Figure *A B C D*, are Proportional to the

Sides *a d* and *d c* of the Figure *a b c d*. The ſame may be demonſtrated of the other Sides; and therefore the Figures are ſimilar.

Now to prove the other Part of the Theorem: If a perpendicular be ſuppoſed to be let fall from the Eye upon the Plane of the Figure, and continued as is neceſſary; it is evident, that *O D*, will be to *O d*, as this Perpendicular, which meaſures the Diſtance from the Eye to the Plane of the Figure, is to the Diſtance of the Eye from the Perſpective Plane, which is meaſur'd by the Part of the perpendicular, contain'd between the Eye and the perſpective Plane. Now this before was manifeſt; *viz.* that

$$OD : Od :: AD : ad :$$

Whence there is the ſame Proportion between *A d* one of the Sides of the Figure, and *A D* its Appearance, as the Theorem expreſſes. The ſame may be demonſtrated of the other Sides of the Figure. Which was to be demonſtrated.

Corollary I.

10. *If from a Point in the Geometrical Plane, three right Lines proceed, which are equal between themſelves, and parallel to the perſpective Plane; the firſt of which is in the Geometrical Plane, the ſecond elevated Perpendicular to the firſt, and the third inclined to it; the Appearances of theſe three right Lines are equal.*

This will appear clear enough in conſidering the Lines as a Figure parallel to the perſpective Plane; and ſo conſequently they will have the ſame Proportion as their Appearances.

Note, The firſt of the aforeſaid Lines is always parallel to the baſe Line; and the ſecond, when the perſpective Plane is perpendicular or upright,

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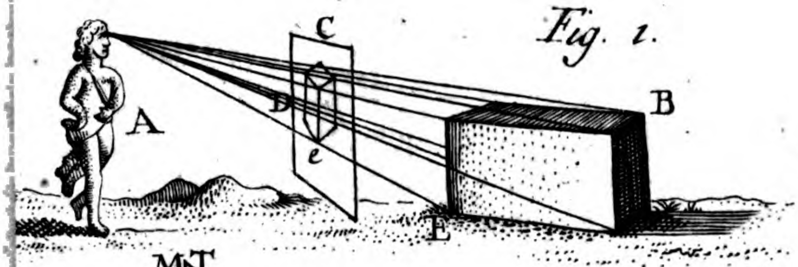


Fig. 1.

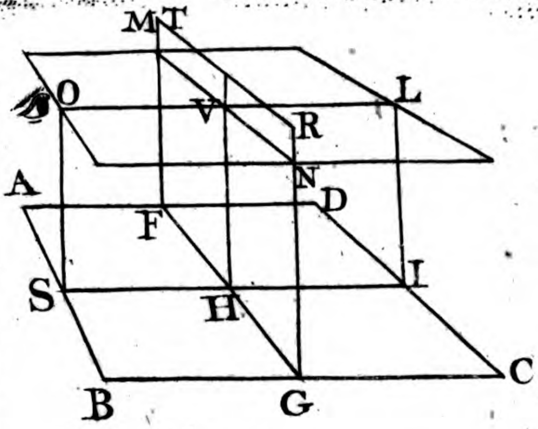


Fig. 2.

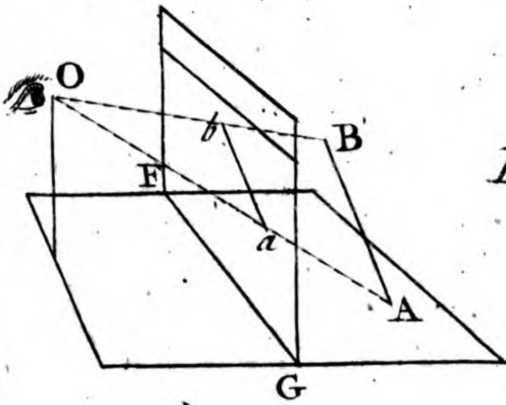


Fig. 3.

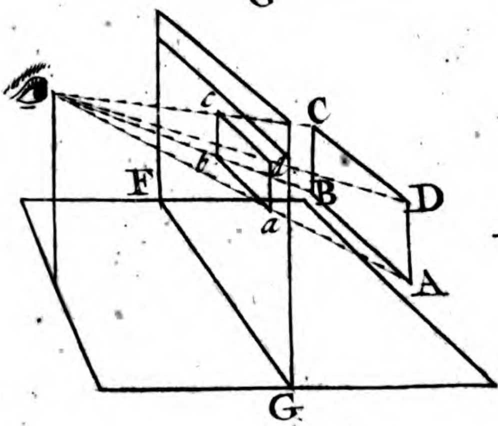


Fig. 4.

right, is alſo perpendicular to the Geometrical Plane, and the third is then in the Direction of the firſt.

COROLLARY II.

11. *If two right Lines, equal between themſelves, and parallel to the perſpective Planes, be equally diſtant from the perſpective Plane, their Appearances will be equal.*

For, becauſe they are in a Plane, parallel to the perſpective Plane, they will have the ſame Proportion to each other, as their Repreſentations.

THEOREM III.

12. *If a Line parallel to the Perſpective Plane, be view'd by two Eyes, both being in a Plane, parallel to the perſpective Plane, the Repreſentations of the ſaid Line will be equal.*

If we ſuppoſe a Plane, parallel to the Perſpective Plane, to paſs through the propoſed Line, this Proportion will be had ;* *viz.* As the Diſtance of the Eyes from this Plane, is to their Diſtance from the Perſpective Plane, ſo is the given Line to the Repreſentation thereof. But the three firſt Terms of this Proportion are the ſame for each of the Eyes, which are in one and the ſame Plane parallel to the Perſpective Plane : Therefore, the fourth Term of the Proportion will likewiſe be the ſame in both Caſes: Which was to be demonſtrated. *9.

THEOREM IV.

13. *If a right Line, being continued, meets the perſpective Plane in one Point, the Appearance thereof will be a Part of the Line drawn from the ſaid Point in the perſpective Plane, to another Point, whereat*

a right

a right Line drawn from the Eye parallel to the proposed Line, terminates.

Fig. 5. The Line *C D* being continued, will meet the perſpective Plane in the Point *E*. We are to prove, that its Appearance is a Part of the Line *E H*, drawn from the Point *E*, to the Point *H*, whereat the Line *O H* proceeding from the Eye parallel to the given Line *C D*, terminates.

The Interſection of the perſpective Plane, and the Plane *O D C*, is the Repreſentation of the given Line. Now the Plane *O D C*, is a Part of the Plane paſſing through the parallels *O H* and *E C*.

Therefore, this Repreſentation is a Part of the Interſection of the laſt mentioned Plane, and perſpective Plane; which Interſection is *E H*.

COROLLARY I.

14. *All Lines parallel between themſelves, and being produced, do fall upon the perſpective Plane, have Repreſentations, which being produced, will all concur in one Point.*

This is evident, becauſe but only one Line *O H* can be drawn from the Eye *O*, to the perſpective Plane, parallel to the ſaid Parallels, and becauſe all their Repreſentations are Parts of Lines concurring in the Point *H*.

Def. 17. *And this Point is called the accidental Point of the ſaid Parallels.*

COROLLARY II.

15. *Two or more parallel Lines, which being produced, do fall on the perſpective Plane, parallel to the Geometrical Plane, have their accidental Point in the Horizontal Line.*

For

For the Horizontal Plane, is parallel to the Geometrical Plane.

COROLLARY III.

16. *The Representations of all Lines parallel to the station Line, concur in the Point of Sight.*

This follows, because the principal Ray is parallel to the said Lines.

COROLLARY IV.

17. *Two or more equal Lines being perpendicular, or equally inclined the same Way, to the same Line parallel to the Station Line, have their Representations concurring in the principal Point.*

Because all these Lines are parallel and equal, the Line passing through their Vertices, is parallel to that passing through their Bases, and this being parallel to the Station Line, it follows, * that the Appearances of the said equal and parallel Lines concur in the principal Point. * 16.

THEOREM V.

18. *The Appearance of an indefinite Line does not alter, when the Eye moves in a Line parallel to a proposed Line.*

The Representation of this Line, is the Intersection of the perspective Plane, and a Plane passing through the Eye and the said Line. Now the Eye remains in the same Plane, when it moves in a Line parallel to the proposed Line; and consequently the Appearance of this last Line, will not be changed by that Motion.

Note, This Demonstration doth not extend to any particular Part of the given Line, but only to the Line in general.

Theorem VI.

19. *Let* AC *be a Line inclined to the Geometrical Plane, and* OD *another Line drawn parallel to*
Fig. 6. A C, *from the Eye to the perspective Plane. Now if* B A *be drawn in the Geometrical Plane, parallel to the base Line, and likewise* D E, *in the perspective Plane, parallel to the said Line, so that* B A *be to* A C, *as* E d *to* D O. *I say, the Appearance of the Line* B C, *passing through the Point* B, *and the Extremity of the Line* A C, *being continued, will meet the Point* E.

Now to prove this; it is evident, * that we
* 13. need but demonstrate, that *O E* is parallel to *B C:* And this may be done in the following Manner:

A B is parallel to *E D*, and *A C* to *O D*; whence the Angle (*E D O*) of the Triangle *O E D*, is equal to the Angle (*B A C*) of the Triangle *A C B*: And so these two Triangles are similar; because they have also their Sides Proportional. But since these two similar Triangles, have two of their Sides parallel, the third *B C* is also parallel to *O E*; *which was to be demonstrated.*

Corollary.

20. If *A B* be made equal to *A C*, and *E D* to *D O*, the Appearance of *B C* will pass thro' the Point *E*.

CHAP. III.

The Practice of Perspective upon the Perspective Plane, supposed to be perpendicular, or upright.

IN order to give a distinct Idea of the Theory, I have hitherto consider'd the Geometrical Plane, as it were the Ground upon which the Spectator and the Objects ſtand; and the Perſpective Plane, as a Window between the Spectator and the Objects, in which the Objects are requir'd to be repreſented. But, in Practice, this Matter muſt be quite otherwiſe conceiv'd; which I ſhall now endeavour to explain as clear as poſſible.

Suppoſe then, that a Painter has a mind to draw upon his Perſpective Plane, or Picture, (whoſe Bigneſs is as he thinks fit) a Proſpect of a Country, wherein are Trees, Houſes, Rivers, &c. Now, from what has been ſaid, this Country will be his Geometrical Plane; and he ought to conſider his Perſpective Plane as a Window, upon which the Points thro' which the Rays coming from all the Points of the Objects towards the Eye, muſt be found. But theſe Interſections of the Rays and the Window cannot be determin'd, unleſs by Lines being drawn in the Geometrical Plane to the Baſe Line.

Now, it is impoſſible for Painters to draw Lines of this Nature on the Ground; wherefore they uſe another more convenient Geometrical Plane thus. At the Foot of their Perſpective Plane, they place a Plane, upon which are drawn in Minature the Baſes of Houſes and Trees, which are in the Country to be repreſented; and the Seats of the Points which, in the Objects, are elevated above the Country; always obſerving, that there be the ſame Diſpoſition between the Objects and their different Parts, upon this new Geometrical Plane, as the Objects truly have in the Country to be repreſented.

Now, to determine the Magnitude of the Space the Figures muſt take up upon this Geometrical Plane, a Painter muſt firſt chuſe the Diſpoſition of his Eye in reſpect to the Perſpective Plane; and then (from the Station Point, thro' the Extremities of the Perspective Plane) he must draw right Lines; which will limit the Space wherein the Figures must be placed; since the Rays of Figures, without those Lines coming towards the Eye, will not pass thro' the Perspective Plane.

21. The Figures being thus drawn on the Geometrical Plane, the next Thing is to find their Appearance upon the Perspective Plane. Now, these Figures are made up of either straight Lines, or crooked ones. To find the Representation of a straight Line, its Extremes need only be sought: And to have the Appearance of a crooked Line, several Points thereof need only be found. Since all this is equally applicable to Figures, as well in the Geometrical Plane, as those above it; it follows, that the whole Business of Perspective consists in only finding the Representation of a Point.

And to find this Representation in the following *Problems*, we only use certain Lines drawn in the Geometrical and Horizontal Planes; which, by their Intersection with the Base and Horizontal Lines, shew the manner of drawing new Lines upon the Perspective Plane, which determine the propos'd Appearances. Now, it is plain, that in finding the said Intersections, it is not necessary to place the Perspective Plane perpendicular to the Geometrical and Horizontal Planes; which would render the Work extreamly laborious: Whence the Perspective and Horizontal Planes may be consider'd as lying upon the Geometrical Plane, and so coinciding therewith.

The Perspective Plane may lye upon the Geometrical Plane two ways; *viz.* Either upon the Face respecting the Objects, or upon that next to the Eye. Now, as in the latter Situation, Repreſentatious are drawn upon the Face of the Perſpective Plane next to the Object, the Perſpective Plane lying down upon its other Face; what ought to be on the Right Hand, appears on the Left; and that on the Left, appears on the Right; producing exactly the ſame Effect, as looking thro' the Back-ſide of a Paper, at a Picture drawn thereon.

Yet, notwithſtanding this Deficiency, we prefer the latter way of the Perſpective Plane's lying down to the former, for the following Reaſons.

1. When the Perſpective Plane lies down in the former manner, it lies upon the Part of the Geometrical Plane wherein Figures have been drawn; which, together with the new Lines that muſt be drawn, cauſes a very great Confuſion, and always obliges one to copy his Work. An Inconveniency which the latter Method is ſeldom ſubject to.

2. We work with much more Eaſe in the manner I have choſen.

Finally, The Default we have obſerv'd, may ſeveral ways be remedied. For, in drawing upon the Geometrical Plane, we need but place that on the Right Hand which we have a mind ſhould appear on the Left; or if the Geometrical Plane be drawn upon Paper, it may be oil'd, or dipp'd in Varniſh, which will render it tranſparent; and then the Back-ſide of the Paper may be thrown into Perſpective.

If all this be not found convenient, the ſaid Default may be eaſily corrected geometrically, in copying the Work after the Drawings are finiſhed. And this may be yet eaſier done, if the Figures are expoſed before a Looking-glaſs;

for then, what is on the Right, will appear on the Left.

Therefore, I lay my Perſpective Plane upon the Geometrical Plane; ſo that it be between the Horizontal Plane, and the Figures requir'd to be thrown into Perſpective.

PROBLEM I.

22. *To find the Appearance of a Point, which is in the Geometrical Plane.*

Fig. 7. Let *Z* be the Geometrical Plane, *I E* the Baſe Line, *D V* the Horizontal Line, *V* the Point of Sight, *D* one of the Points of Diſtance, and *A* the given Point.

OPERATION.

From the Point *A*, let fall the Perpendicular *A B* upon the Baſe Line; and from the Point of Concurrence *B*, draw the Line *B V* to the Point of Sight; then aſſume *B E* in the Baſe Line equal to *B A*, and from the Point *E* draw the Line *E D* to the Point of Diſtance *D*: And the Point (*a*), the Interſection of *B V* and *E D*, is the Repreſentation ſought.

DEMONSTRATION.

23. The Appearance of the Line *AB*, is * a Part
* 16. of the Line *B V*. Now, if we conceive a Line iſſuing from the Eye towards the Point *D*, and another from the Point *A* towards the Point *E*; theſe two Lines will be parallel, becauſe they are in parallel Planes, aud each make half a right Angle with the Perſpective Plane; whence the

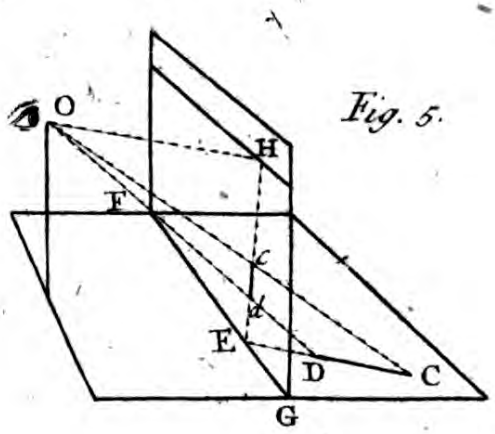


Fig. 5.

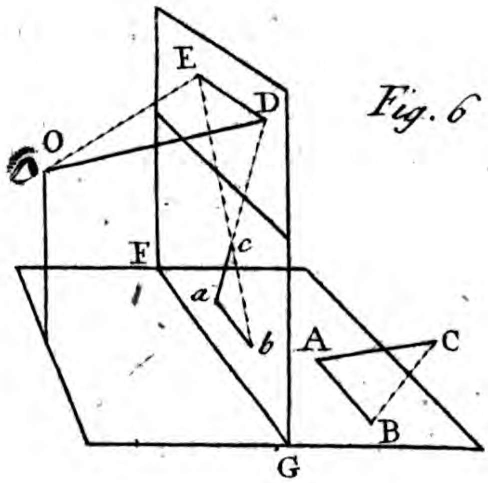


Fig. 6

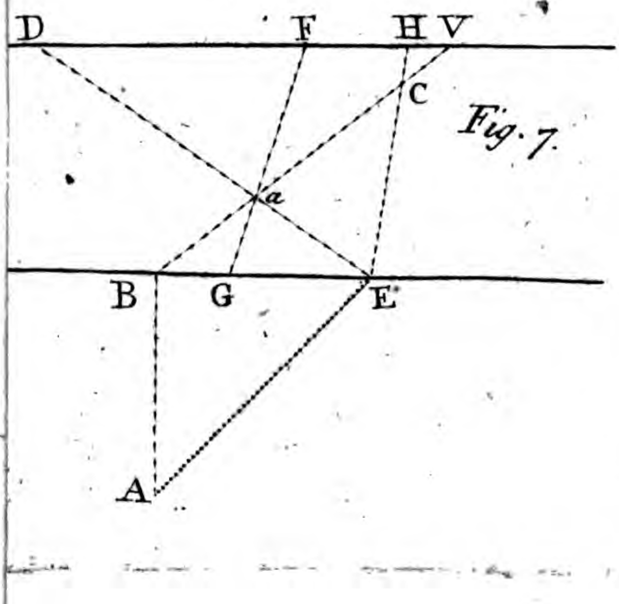


Fig. 7.

the Appearance of the Line *A E*, is * a Part of the Line *E D*. Now, ſince the Point *A* is in the two Lines *A B*, *A E*; the Appearance of the ſaid Point will likewiſe be in the Appearances of the aforeſaid two Lines, and conſequently is in the Point *a*, the common Section of *B V* and *E D*. * 13.

REMARKS.

24. If the Diſtance of the Eye be ſo great, that one of the Points of Diſtance cannot be denoted upon the horizontal Line; another Point, *F*, muſt be uſed, diſtant from the Point of Sight by about one third, or fourth Part of the Diſtance of the Eye. But then, a correſpondent Part of the Perpendicular *A B* muſt be likewiſe taken, and laid off from *B* to *G*, in the Baſe Line.

25. And in this manner may the Repreſentation of a very diſtant Point be found, if its Diſtance from the Perſpective Plane be known, together with the Place wherein a Perpendicular drawn from that Point cuts the Baſe Line. For, having firſt drawn a Line, as *B V*, from the ſaid Point of Concurrence to the Point of Sight, then *B E* muſt be aſſum'd in the Baſe Line; for Example, equal to the tenth Part of the Diſtance of the Point whoſe Repreſentation is ſought; and *V H* in the Horizontal Line, likewiſe equal to the tenth Part of the Eye's Diſtance. Then *C*, the Interſection of *B V* and *E H*, will be the Appearance ſought.

Note, By this Method may be found the Deepnings in Pictures.

The Appearance of the Point *A* may yet be otherwiſe found, without drawing the Line *B V* from the Point *A*, in taking *B I* equal to *B A*, and drawing a Line from the Point *I* to the other Point of Distance; which, by its Intersection with *E D*, will give the Appearance of the Point *A*.

Method II.

26. *Y* is the Horizontal Plane, *X* the Perspective Plane, *Z* the Geometrical Plane, *O* the Eye, *D C* the Horizontal Line, *B E* the Base Line, and
Fig. 8. *A* the given Point.

Operation.

Draw a Line from the Point *A*, to the Eye *O*, cutting the Base Line in the Point *B*, and the Horizontal Line in the Point *C*: Then assume *BE* in the Base Line, equal to *B A*; and *C D* in the Horizontal Line, equal to *CO*; and join the Points *E* and *D*, by a Line cutting the Line *A O* in the Point *a*; which will be the Appearance sought.

Demonstration.

27. The Triangle *O D C* in the Horizontal Plane, is similar to the Triangle *A B E* in the Geometrical Plane; and consequently *AB* is parallel to *OC*, and *AE* to *O D*. But the Appear-
* 13. ance of *A* must * be in the Lines *B C*, and *ED*; and therefore it will be in *a*, their Intersection.

Remarks.

28. If the Place wherein the Eye ought to be in the Horizontal Plane be not known, but the Point of Sight is; then, to find the Place of the Eye, a Perpendicular must be rais'd from the Point of Sight

Sight to the Horizontal Line, equal in Length to the principal Ray; and the Extremity of this Perpendicular will be the Point ſought.

If nothing is determin'd, the Place of the Eye may be taken at pleaſure in the Horizontal Plane.

METHOD III.

29. The ſame Things being given as in the precedent Method, about the Eye *O*, as a Center; deſcribe the Arc of a Circle *I H*, touching the Horizontal Line. *Fig.* 9.

OPERATION.

About the given Point *A*, as a Center, deſcribe the Arc of a Circle *L C*, touching the Baſe Line: Then draw two Lines, *C H* and *L I*, touching the two Circles *L C* and *H I*; and the Point *a*, the Interſection of the ſaid two Lines, will be the Appearance ſought.

DEMONSTRATION.

30. To demonſtrate this, draw the Line *A B* perpendicular to the Baſe Line; *O V* perpendicular to the Horizontal Line; and *A C*, *O H* perpendicular to the Tangent *H C*. All theſe Perpendiculars will cut the Lines to which they are perpendicular, in the Points wherein theſe laſt touch the Circle *L B C*, or *H V I*. Likewiſe draw the Line *A E* from the given Point *A*, to the Point *E*, wherein the Line *H C* cuts the Baſe Line. Finally, draw *O D*, from the Eye *O* to the Point *D*, wherein the ſaid Line *H C* cuts the Horizontal Line.

*27. Now, it is evident, * that to prove the Appearance of A is in the Line CH, we need but demonstrate that OD is parallel to AE; which may be done thus;

Because the Triangles OGV, and ABF, are similar.

$$AF : AB :: OG : OV :$$

altern.

$$AF : OG :: AB : OV :$$

Divid. and altern. the first Proportion.

$$AF - AB\ (=CF) : OG - OV = HG :: AB : OV.$$

But because the Triangles ECF, HGD are similar.

$$CF : HG :: EF : GD.$$

Now, by observing the two last Proportions of the other two Triangles,

$$EF : GD :: AF : OG,$$

And the Angle AFE, being equal to the Angle OGD, the Triangles AEF and ODG are similar; and therefore AE is parallel to OD: Which was to be demonstrated.

After the same manner we prove, that the Appearance of the Point A is in the Line LI, and consequently is in the Intersection of this Line and HC.

REMARK.

Altho' this Method appears more difficult than the precedent one, as to the Geometrical Consideration thereof, yet the Operation is easier, if the Points are not too far distant from the Base Line: For Lines may well enough be drawn by Guess, or Sight only, to touch Circles, and Circles to touch Lines.

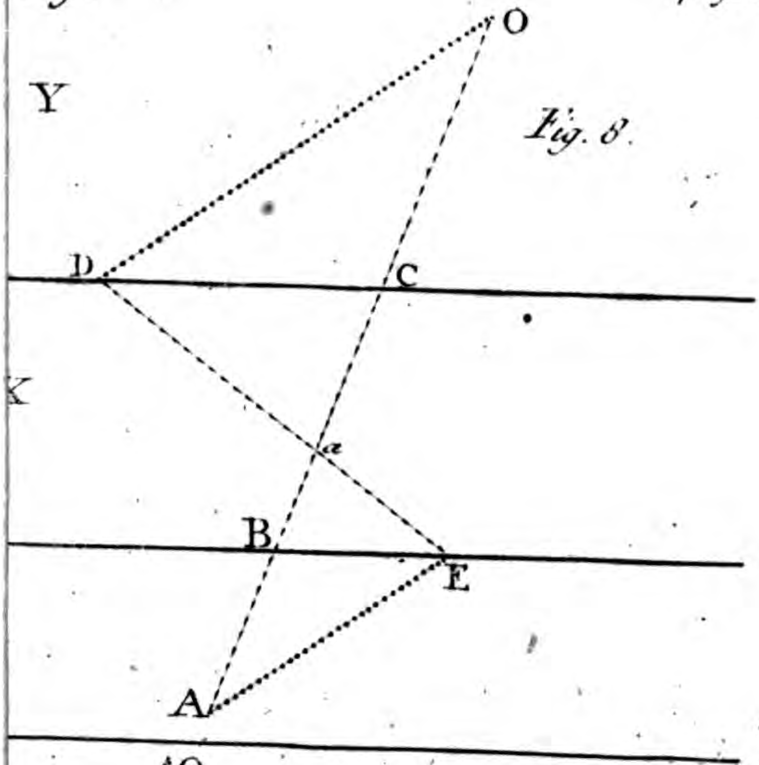


Fig. 8.

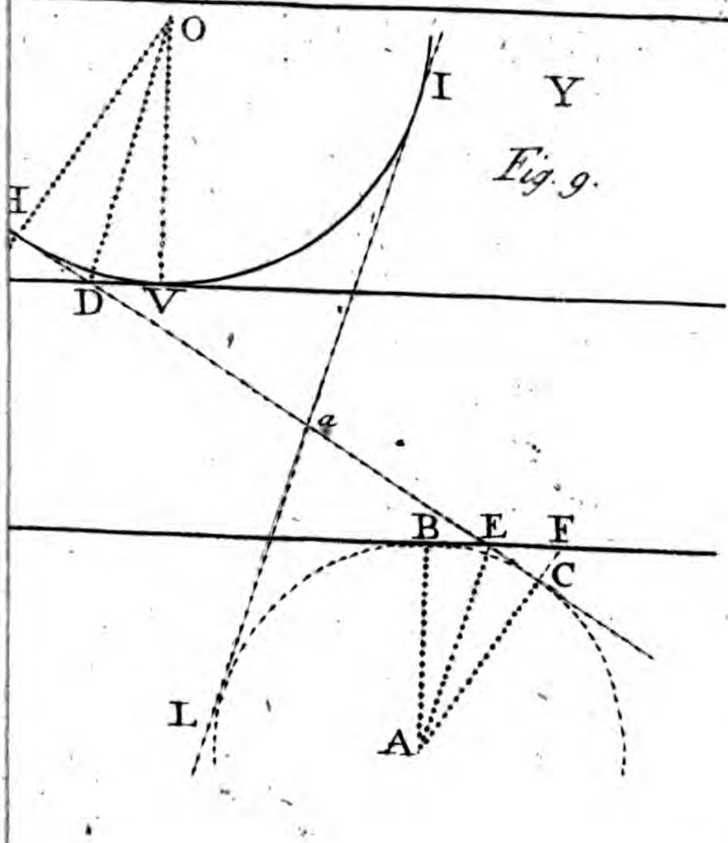


Fig. 9.

METHOD. IV.

31. Draw the Line *FOG* through the Eye *O*, parallel to the Base Line, then assume *FO* in this Line, equal to the Height of the Eye, and *O G* equal to the Length of the principal Ray. *A* is the given Point. *Fig.* 10.

OPERATION,
Without Compasses.

From the given Point *A*, draw the Lines *AO*, *A F*, to the Points *O* and *F*, and from the Point *E*, wherein *A F* cuts the Base Line, draw the Line *E G* to the Point *G*; then the Point *a*, the Interfection of *A O*, and *E G*, is the Representation sought.

DEMONSTRATION.

Let fall the perpendicular *G M* from the Point *G*, upon the Base Line, and through the Eye *O*, draw the Line *O D* to the Point *D*, the Interfection of the Horizontal Line, and the Line *GE*.

Then because the Triangles *GDL*, *GEM* are fimilar,

$$GD : GE :: GL : GM.$$

But *G O* is equal to *G L*, and *G F* to *L M*.

whence

$$GD : GE :: GO : GF.$$

And confequenely the Triangles *G O D*: and *G F E*, are fimilar, and the Lines *O D*, and *A E F*, are parallel between themfelves; and therefore * the Appearance of *A E*, is a Part of the Line *E D G*. It has alfo been prov'd *, that the Reprefentation of the Point *A*, is in the Line *A O*; therefore it is in *a* the Interfection *13 *27.

 of

of this laſt Line, and the Line *EDG*, which was to be demonſtrated.

REMARKS.

33. By this Demonſtration it appears, that there is no Neceſſity in taking *GO* exactly equal to the Eye's Diſtance, and *OF* equal to its Height: But it is ſufficient if they have the ſame Proportion, as the aforeſaid Diſtance has to the Height. Likewiſe there is no Neceſſity in aſſuming the Points *G* and *F*, in a Line parallel to the Baſe Line; for any other Line paſſing through the Eye *O* may be uſed at Pleaſure. For Example, let *gOf* be a Line any how drawn through the Eye *O*, and take the Point *g* at Pleaſure in this Line, through which draw alſo the Line *gNI* at Pleaſure, cutting the Horizontal Line in *N*, and the Baſe Line in *I*; and draw the Line *ON*, and through the Point *I*, draw the Line *If* parallel thereto, cutting the Line *gOf* in *f*.

This being done, the Points *g* and *f* may be uſed inſtead of *G* and *F*: for among all the Lines that can be drawn (as *GNI*) it is manifeſt, that *gN* will always be to *gI* :: *gO* : *gf*, which is ſufficient for the Demonſtration.

If the Point *f* be firſt determin'd, the Point *g* muſt be found by an Operation quite contrary to that we have laid down.

34. When nothing is determinate, we may (a Baſe Line being firſt drawn) take at Pleaſure, in another Line any how drawn, the three Points *gOf*; ſo that in this Caſe, there is no Manner of Neceſſity to uſe Compaſſes, in throwing any Figure whatſoever, which is on the Geometrical Plane, into Perſpective. But if after having thus work'd, the Point of Sight, Height and Diſtance of the Eye be requir'd, the Perpen-

Perpendiculars $f P$, $O H$, must be let fall from the Points f and O, on the Base Line, and the Line $P g$ drawn; then the Point V, wherein it cuts the Perpendicular $O H$, is the Point of Sight sought, and the Parts $O V$, and $V H$ determine the Height and Distance of the Eye.

METHOD V.

35. *When the Appearance of a Point is known,*
Let A be a Point in the Geometrical Plane, and a its Representation in the perspective Plane, it is requir'd to find the Appearance of the Point B. *Fig.* 11.

OPERATION,
Without Compasses.

Draw a Line from the Point B to the Eye O, and another from the Point E, wherein the said Line continued, cuts the Base Line, to the Point A; then draw the Line $E a$, and where it cuts $B O$, is the Point b sought.

DEMONSTRATION.

The Point E is its own Representation; and because the Point a is the Representation of A, the Line $E a$ is that of $E A$. Now since the Point B is in the Line $E A$, the Appearance of this Point will be likewise in $E a$, as also * in $B O$; therefore it is in b the Intersection of the Lines $E a$, and $B O$. * 27.

REMARK.

37. If the Point A be in the Line $B O$, or the Line $B A$ be parallel, or a very little inclined to the Base Line, we cannot then use this Me-

thod, unless by Means of the Point *A*, the Appearance of some other Point taken at Pleasure upon the Geometrical Plane be found, by Means of which, the Appearance of the Point *B* may be afterwards gotten; but in these Cases, the shortest Way, is to use some one of the precedent Methods.

COROLLARY.

38. It appears from this Method, that when the Representations of two Points are found, the Appearance of any third Point whatsoever may be had, without having any Regard to the Situation of the Eye; because two Lines as *E a* may be drawn, whose Intersection will be the Point sought.

METHOD VI.

Fig. 12. 39. The same things being given, as in the second Method, let *FC* be the Geometrical Line.

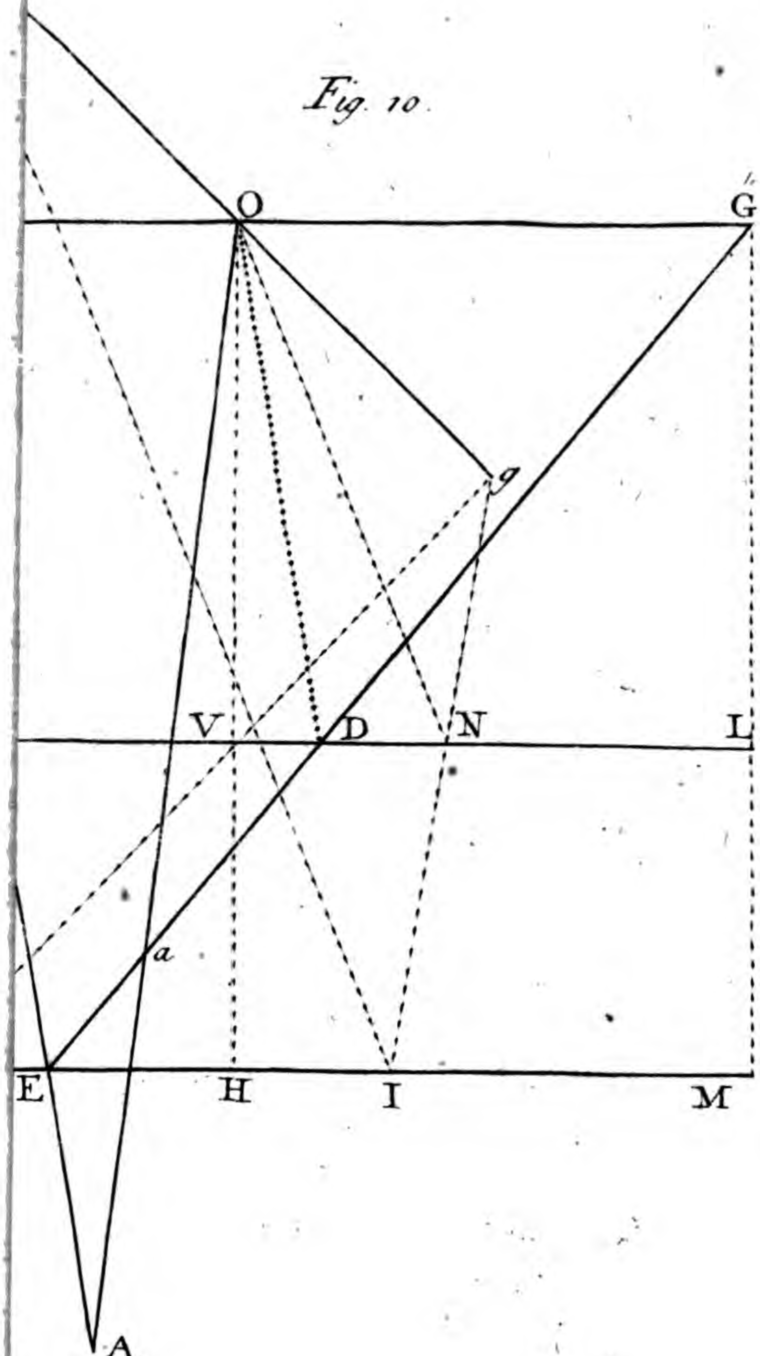
OPERATION.

Draw two Lines *AF* and *AC* from the given Point *A* at Pleasure, cutting the Base Line in the Points *E* and *B*, and intersecting the Geometrical Line in the Points *F* and *C*. From these two last Points draw the Lines *FO* and *CO* to the Eye; then draw *E a* through the Point *E*, parallel to *FO*, and *B a* through the Point *B*, parallel to *CO*, and the Point *a* the Intersection of these two Lines will be that sought.

Note, We might first have drawn the Lines *OF* and *OC* at Pleasure, and then have drawn the Lines *AC* and *AF* through their Concurrence

Plate 4. page 24.

Fig. 10.



rence with the Geometrical Line; which would come to the ſame thing.

DEMONSTRATION.

Firſt continue the Line *E a*, until it meets the Horizontal Line in *D*, and draw a Line from *D* to the Eye, and another through the Eye parallel to the Baſe Line.

Then the Parallels *O M* and *F C* are at the ſame Diſtance from each other, as *L D* is from *E B*; whence it follows, that *F O* is equal to *E D*, and therefore *O D* is parallel to *A F*. Whence * the Appearance of *E A*, is a Part of *E D*. And after the ſame Manner we prove, that the Repreſentation of *B A* is a Part of *B a*. * 13:

REMARKS.

40. When there are no Lines drawn, and we would uſe this Method, the Horizontal Line may be laid aſide; and then having firſt drawn the Geometrical Line, whoſe Diſtance from the Baſe Line is equal to the Length of the Ray, we aſſume the Diſtance from the Eye to the Geometrical Line, equal to the Height of the Eye.

Although this Method appears uſeleſs, as being more difficult than the precedent ones, yet in the Eighth Chapter we have ſhewn the Uſe that may be drawn from it.

COROLLARY.

41. It follows from this Demonſtration, that the Appearances of Lines paſſing through the Station Point, are all perpendicular to the Baſe Line; for if the Perpendicular *O S* be let fall from

from the Eye *O* upon the Baſe Line, the Appearances of all Lines paſſing through *S*, will be perpendicular to the Baſe Line; but the ſaid Point *S* is the Station Point. Whence, *&c.*

PROBLEM II.

42. *To throw a Line in the Geometrical Plane into Perſpective.*

* 21. It has been ſhewn *, that to have the Repreſentation of a right Line, the Perſpective of the Extremes of the ſaid Line, need only be
* 22. found; and although it is difficult to find * the Repreſentation of two Points, nevertheleſs I ſhall ſhew here how to find more eaſy the Repreſentation of a Line in ſome Caſes.

Fig. 13. 1. Let *AB* be a Line parallel to the Baſe Line: To draw the Repreſentation of which, having firſt found the Point *a* the Appearance of *A*, one of the Ends of the given Line; afterwards through that Appearance, draw a Parallel to the Baſe Line; then the Line *BO*, drawn from *B* to the Eye *O*, will cut the ſaid parallel in the Point *b*, and *ba* will be the Repreſentation ſought.

43. 2. Let *CG* be a Line, which continued out, cuts the baſe Line in *E*. Now to draw the Appearance thereof; through the Eye *O*, draw a Line parallel thereto, cutting the Horizontal Line in *D*, and joyn the Points *E* and *D*, by the Line *ED*, which cut in the Points *c* and *g*, by Lines drawn from *C* and *G* to the Eye; then the Part *cg* of the Line *ED*, is the Appearance ſought.

REMARK.

If the Lines *GO* and *CO* cut *ED* very obliquely, and ſo their Interſection cannot be exactly

actly determined, this Method ought not then to be used.

PROBLEM III.

44. *To find the Appearance of the Divisions of a Line in the Geometrical Plane.*

Let *AB* be a Line, whose Appearance is *a b*. Fig. 14. Now to find the Representation of the Divisions of this Line, there must be Lines drawn from the Divisions of the Line to the Eye, whose Intersections with *a b* will give the Points sought.

Note, When these Lines very obliquely cut *a b*, the following Way ought to used.

METHOD. II.

45. To find the Representations of the Divisions of the Line *GC*, make choice of the Point *D* at Pleasure without this Line, and find * the Representation *d* thereof; then draw Lines through the proposed Divisions to the Point *D*; and from the Points wherein these Lines continued out cut the Base Line, draw other Lines through the Representation *d*, which will cut *cg* the Representation of *CG* in the Points sought. Fig. 14. * 22.

PROBLEM IV.

46. *To throw a Polygon, or any other regular Figure on the Geometrical Plane into Perspective.*

The Representation of any Kinds of Figures may be found * by any one of the Methods of Problem I. the fourth in general is the easiest; and may be first used in finding * the Representations of Points, or sometimes of one only; and then the fifth Method serves * for finding * 21. * 22. 31.

ſinding the reſt. But yet the Work may be ſhorten'd by the two precedent Problems, as we ſhall ſhew in the following Examples.

Example I.

To throw a Pentagon having one Side parallel to the Baſe Line, into Perſpective.

Fig. 15. Let *ABCDE*, be the propoſed Pentagon, wherein draw the Line *BD* which will be parallel to *AE*, becauſe the Pentagon is a regular one.

* 42. Now find * the Repreſentation of the ſaid two Lines *AE* and *BD*, and you will have the Repreſentation of four of the Corners of the Pentagon; and to determine the Repreſentation of
* 43. the fifth Corner, find * the Appearance of a Line drawn from *C* to *E*, which in this Example is parallel to *AB*, which is ſuppoſed parallel to the Baſe Line.

Example II.

To throw a Parallelogram, divided into ſeveral other Parallelograms into Perſpective.

Fig. 17. Let *ABCD* be a Parallelogram, divided into ſeveral other Parallelograms.

Draw the Line *OG* thro' the Eye *O* parallel to the Side *AD*, cutting the Horizontal Line in *G*; likewiſe draw *OF* parallel to the Side *AB* cutting the Horizontal Line in *F*, and produce the Sides of the Parallelogram, and the Lines dividing it, to the Baſe Line; then from the Points wherein *AD* and *CB*, and the Lines parallel thereto cut the Baſe Line, draw Lines to the Point *G*. Alſo from the Points wherein *AB* and *CD* and their parallels cut the ſaid Line, draw Lines to the Point *F*, whoſe Interſections with theſe

draw-

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page 28.

Fig. 16

drawn to the Point *G*, will give the Appearance ſought.

REMARKS.

When this Method cannot be us'd, the Perſpective of the Diviſions dividing the Sides of the Parallelogram, muſt be found *. And we are often oblig'd to have recourſe to this Expedient, notwithſtanding the accidental Points, *G* and *F*, being had. And this happens, when the Parallelogram is ſo far diſtant from the perſpective Plane, that its Sides being produc'd, cannot meet the Baſe Line. * 44.

47. Note, moreover, that this one Example is ſufficient to ſhew how to throw any Kinds of Figures in the Geometrical Plane into Perſpective. To effect which, we circumſcribe any Parallelogram about the Figures, which we divide into ſeveral others: Then we throw this Parallelogram (thus divided) into Perſpective, and transfer the given Figure therein, ſo that it may have the ſame Situation with reſpect to the little Parallelograms in the perſpective Plane, as it had in regard to the ſmall Parallelograms in the Geometrical Plane.

EXAMPLE III.

48. *To throw a Circle into Perſpective.*

The Repreſentation of ſeveral Points of a Circle, or any other Curve Line requir'd to be thrown into perſpective, muſt be * found. This may be well enough done, by drawing ſeveral Chords in the Circle, or Curve, parallel between themſelves, the Repreſentations of which muſt be found *; then the Extremities of thoſe Repreſen- *Fig.* 16. * 21. * 43.

presentations being join'd, will give the Perspective sought. The same may be done, in drawing the Chords thro' a Point, whose Representation is known.

Remarks.

Fig. 16. 49. *Let* GI *be the Geometrical Line; and thro' the Center* P *of the Circle, whose Perspective is sought, let fall the Perpendicular* PF *upon the said Line* GI, *which bisect in the Point* R. *About* R, *as a Center, and with the Radius* RP, *describe an Arc of a Circle* MPN, *cutting the given Circle in the Points* M *and* N. *Now, if the Perspective of* LH *and* NM *be found, the two Conjugate Diameters of an Ellipsis, which is the Representation of the given Circle, will be had. And, an Ellipsis may be drawn by some one of the Methods laid down by those who have treated of Conick Sections.*

I shall not spend Time here in demonstrating the Truth of this. See Prop. 10. lib. 2. *of the great* Latin *Treatise of Conic Sections, written by M.* de la Hire; *the Demonstration of which may be here apply'd. If we consider,* 1. *That Lines drawn from the Points* M *and* N *to the Point* F, *will touch the Circle in the said Points* M *and* N. 2. *That the visual Rays, going from the Eye towards all the Parts of the Circumference of the Circle, form a Cone,* 3. *That the Appearance of the Circle, is the Section of a Cone, made by the perspective Plane. Finally, That the Line* GI *ought to be conceiv'd, as being the Intersection of the Geometrical Plane, and a Plane passing thro' the Eye parallel to the perspective Plane.*

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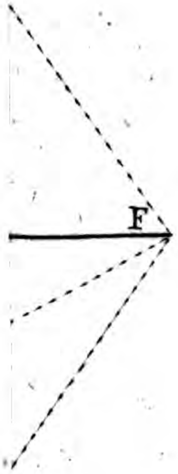

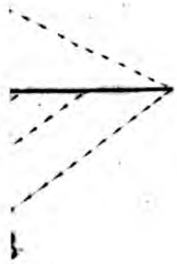

PROB. V.

50. *To find the Representation of a Point, elevated above the Geometrical Plane.*

Let *G S* be the Geometrical Line, and *S* the Station Point: Make *S F*, in the Geometrical Line, equal to the Height of the Eye; and let *A* be the Seat of the given Line. *Fig.* 18.

OPERATION.

Assume *FC* in the Geometrical Line, equal to the Height of the Eye, above the Geometrical Plane: Then draw Lines from the Point *A* to the Points *S* and *C*, and on the Point *B*, the Intersection of the Line *AS* and the Base Line, raise the Perpendicular *B I* to the Base Line, equal to *EB*, *plus FC*; and the Point *I* will be the Perspective sought.

DEMONSTRATION.

51. Let us suppose a Plane to pass thro' the given Point, and the Eye perpendicular to the Geometrical Plane; then it is manifest, that the Intersection of these two Planes is the Line *A B S*, and the Intersection of the said suppos'd Plane and the perspective Plane, is *B I*. Now, let *X* be this suppos'd Plane; *a*, *b*, *s*, the Point mark'd with the same Letters in the precedent Figure, *b i* the Intersection of this Plane and the perspective Plane; *O* the Eye, and *D* the propos'd Point: We are to prove, that if *O D* be drawn, the Line *B I* of the precedent Figure will be equal to *b i* in this Figure. *Fig.* 19.

To

To demonstrate which, draw the Line DLM thro' the Point D, parallel to abs. Then, because the Triangles DMO and DLi are similar, we have,

$DM=as : DL=ab :: MO : Li$. Again, in the precedent Figure, the Triangles ASC and ABE are similar: Whence,

$$AS : AB :: CS : EB.$$

The three first Terms of these two Progressions are the same: For CS is equal to MO, since they are each the Difference of the Height of the Eye, and that of the given Point; and consequently, EB is equal to Li: But BI was made equal to BE, *plus* FC the Height of the given Point above the Geometrical Plane; and bi is equal to Li, *plus* bL; which being equal to aD, is likewise the Height of the given Point above the Geometrical Plane; whence the Lines BI and bi are equal. Which was to be demonstrated.

Note, When the Height of the given Point is greater than the Height of the Eye, EB must be taken from that first Height, to have the Magnitude of BI.

PROB. VI.

52. *To throw a Pyramid, or Cone, into Perspective.*

Fig. 20. Now, to throw a Pyramid into perspective, the Appearance of its Base * and Center must be found *: After which, Lines must be drawn from the Representation of the Vertex, to the Appearance of those Angles of the Base that are visible; and then the Perspective sought will be had.

* 46.
* 50.

Fig. 21. And to throw a Cone into perspective, the Representation of its Base * and Vertex must be first

* 46.

firſt found *; and then if Lines be drawn from the Repreſentation of the Vertex touching the Repreſentation of the Baſe, the Repreſentation of the Cone will be had. * 47.

But ſince, according to this Manner, we are obliged to find the Perſpective of all the Baſe; whereas it often cannot be all ſeen; we may determine, by the following Method, what Part of the Baſe is viſible, and ſo only find the Repreſentation thereof. And then, to compleat the Cone, we draw Lines from the Extremities of the viſible Part of the Baſe, to the Repreſentation of the Vertex.

53. *To determine the viſible Part of the Baſe of a Cone.*

Let the Circle *LIF* be the Baſe of a Cone in the Geometrical Plane, and *A* the Center thereof. Fig. 21.

OPERATION.

Aſſume *PQ* ſomewhere in the Baſe Line, equal to the Semidiameter of the Circle *LF*; and from the Point *P*, raiſe *PDG* perpendicular to the Baſe Line, meeting the Horizontal Line in *G*; and in this Perpendicular, make *PD* equal to the Height of the Cone; and draw the Line *QDH*, meeting the Horizontal Line in *H*. Then, about the Point *A* as a Center, and with the Radius *GH*, draw the Circle *BCE*; and from the ſaid Point *A*, draw a Line to the Station Point *S*: Biſect *AS* in *R*; and about *R*, as a Center, with the Radius *RA*, deſcribe the Circular Arc *BAC*, cutting the Circle *BEC* in the Points *B* and *C*. Draw the Lines *BAF*, and *CAL*; and the viſible Portion, (*LIF*,) of the Circular Base of the Cone will be determin'd.

DEMONSTRATION.

To prove this, draw the Lines BC *and* LF, *cutting the Line* AS *in the Points* N *and* M; *and make the Line* Gn *equal to* AN, *and draw the Line* nDm. *It is now manifest, that if the Cone be continued out above its Vertex, (that is, if the opposite Cone be form'd) it will cut the Horizontal Plane in a Circle equal to* BEC, *whose Seat will be* BEC: *So that the Point* S, *in respect of* BEC, *is in the same Situation as the Eye hath, with respect to the Circle form'd in the Horizontal Plane, by the Continuation of the Cone. Whence it follows, that* BC *is the Seat of the visible Portion of that Circle. For, by Construction,* B *and* C *are the Points of Contact of the Tangents to the Circle* BEC, *which pass thro' the Point* S; *because the Angle* ABS, *which is in a Semicircle, is a right one.*

Now, if a Plane be conceiv'd, as passing thro' some Points in the Horizontal Plane, whose Seats are B *and* C, *and which cuts the two opposite Cones thro' their Vertex; it is evident, that this Plane continued, will cut the Geometrical Plane in a Line parallel to* BNC; *and that this Line upon the said Plane, will determine the visible Part of the Cone's Base. So, since* Gn *was made equal to* AN, *we have only to prove, that* Pm *is equal to* AM: *For, it follows from thence, that* LMF *is the Common Section of the Geometrical Plane, and the Plane which we have here imagin'd.*

The Triangles DQP *and* GHD *are similar, whence*
DG : DP :: GH : PQ.

And

And the Triangles D P m *and* D G n *are ſimilar :*
Wherefore
DG : DP : Gn : : Pm.
And
GH : PQ : : Gn : Pm.
The Triangles B A N *and* L A M *are ſimilar :*
Therefore,
BA : AL : : AN : AM.
But the three firſt Terms of the two laſt Proportions, are equal between themſelves ; whence P m *is alſo equal to* A M. Which was to be demonſtrated.

REMARKS.

54. When the Height of the Cone is greater than the Height of the Eye, the Points, *G* and *H*, will fall below the Point *D* ; in which Caſe, the Lines *AB* and *AC* muſt be produc'd, till they cut the Circle in the Points *l* and *f*, oppoſite to *L* and *F* : Then *l I f* will be the viſible Part of the Baſe.

When the Cone is inclin'd, ſo that *T* (for Example) is the Seat of its Vertex ; *AT* muſt be drawn : And then having aſſum'd *PD* equal to the perpendicular Height of the Cone, and *Pt* equal to *AT* ; the Line *t D x* muſt be drawn ; and the Part *TX*, taken in *AT*, equal to *Gx*. Alſo, *XS* muſt be drawn, and *As*, equal and parallel thereto.

This being done ; the ſame Method muſt be apply'd here, that I have laid down for the upright Cone ; with this Difference only, that the Point *s* muſt be us'd inſtead of the Station Point *S*. But when the Height of the Cone is greater than the Height of the Eye, the Point *X* muſt be aſſum'd in the Line *TA*, between the Points *T* and *A*.

The Reason of this is evident, from the Demonstration of the upright Cone: For, it is manifest, that X *is the Seat of the Center of the Circle, which the Cone continued forms in the Horizontal Plane; and consequently, the Point* s, *in regard to the Circle* BED, *is in the same Situation as the Eye is, in respect of the Intersection of the continued Cone, and the Horizontal Plane.*

Note, moreover, that a Cone can scarcely ever be thrown into Perspective, by the common Method, so exact as by this.

PROBLEM VII.

55. *To find the Perspective of a Line, perpendicular to the Geometrical Plane.*

Fig. 22. It is requir'd to find the Appearance of a Line equal to *BC*, and perpendicular to the Geometrical Plane, in the Point *A*.

OPERATION.

Assume *ED*, any where in the Base Line, equal to *BC*; and from the Points *D* and *E*, draw *DF* and *EF* to some Point *F*, taken at pleasure in the Horizontal Line. Then having
* 22. found * *a*, the Representation of the Point *A*; draw *aH* parallel to the Base Line, and *aI* perpendicular thereto: And if *aI* be made equal to *GH*, the said *aI* will be the Perspective sought.

DEMONSTRATION.

* 6. 56. The Appearance of the said Line, is * per-
* 10. pendicular to the Base Line, and equal * to the Perspective of the Line *AL*, drawn from the Point

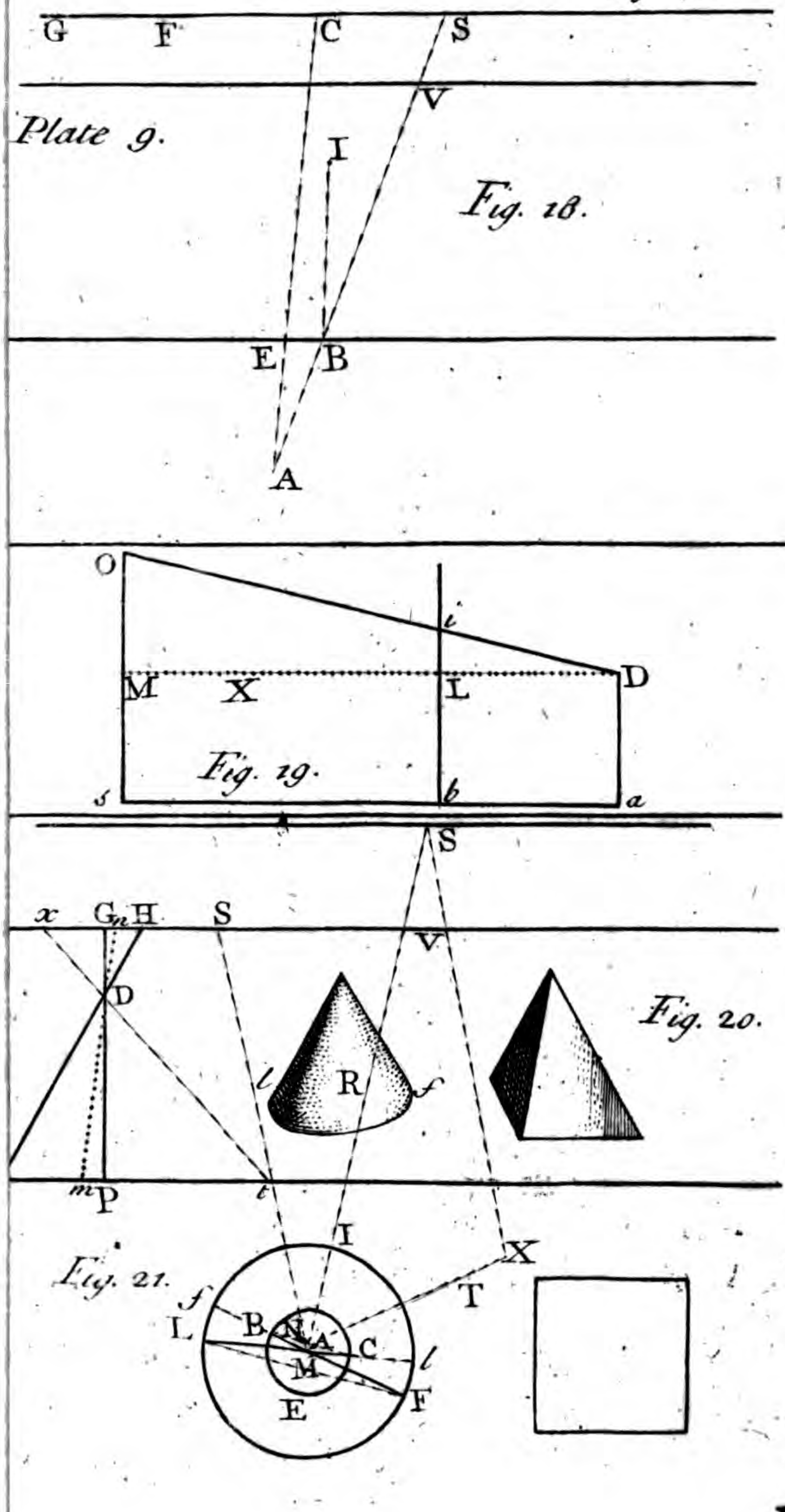
Plate 9.
Fig. 18.
G F C S V I E B A
Fig. 19.
O i D M X L s b a
Fig. 20.
S x G n H S V D l R f m P t
Fig. 21.
I X T f L B N A C l M E F

Point *A* parallel to the Base Line, and made equal to *BC*. Now if from the Extremities of the said Line *AL*, Perpendiculars are let fall, meeting the Base Line in the Points *P* and *M*, and from these Points, Lines are drawn to the Point of Sight *V*; then *a N* will likewise be * the Perspective of *AL*; and since *PM* is equal to *DE*, *a N* will be likewise equal to *GH*, and consequently *a N* will be likewise equal to *a I*, which is equal to *GH*. * 5, 16.

METHOD II.

57. The same Things being given, as in the precedent Method, about the Point *A*, as a Center, and with the Radius *BC*, describe the Arc of a Circle *LM*, and draw the Line *OL* from the Eye touching it; then about *a*, (which is the Representation of *A*) as a Center describe the Circular Arc *GI* touching the Line *LO*, and cutting another Line drawn through *a* Perpendicular to the Base Line in the Point *I*: I say the Point *I* is the Extremity of the Representation sought. *Fig.* 23.

DEMONSTRATION.

To prove this, let fall the Perpendiculars *AL* and *aG* upon the Line *OL*, which will meet the said Line in the Points wherein it touches the circular Arcs *ML* and *GI*.

Also assume *DE* in the Base Line equal to *BC* or *AL*, and draw the Line *DF*; then through *a*, draw *aH* parallel to the Base Line.

Now let us consider the Figure *X*, which represents a Plane passing through the Eye and the Point *A* of the foregoing Figure, wherein *Of* here, represents *OF* there; *fe* here, *FE* there; *Fig.* 24.

there; and finally *e A* here, *E A* in that Figure.

* 27. This being ſuppoſed, *o f* is parallel * to *e A*, and conſequently the Triangle *o f a* is ſimilar to the Triangle *a e A*, and therefore we have this Proportion.

$$of : fa :: Ae : eo.$$

Comp.

$$of + af : fa :: Ae + ea : ea.$$

Altern.

$$of + af : Ae + ea :: fa : ea.$$

Comp. and Perm.

$$of + fa + Ae + ea : of + fa :: fa + ea : fa.$$

This laſt Proportion being reduced to the precedent Figure, we ſhall have this,

$$OA : oa :: FD : Fa.$$

Again, becauſe the Triangles *O A L* and *O a G* are ſimilar, we ſhall have

$$OA : Oa :: AL : aG.$$

And ſince the Triangle *FED* and *FaH* are ſimilar;

$$FE : Fa :: DE : Ha.$$

And ſo if theſe three laſt Propoſitions be conſider'd, we ſhall have

$$AL : aG :: DE : Ha.$$

But *D E* was made equal to *A L*, and therefore *a G* or *a I* is alſo equal to *a H*, which is * equal to the Repreſentation ſought. Which was to be demonſtrated. * 56.

METHOD III.

Fig. 25. 58. Near one of the Sides of the perſpective Plane, raiſe the Perpendicular *CB* to the Baſe Line, equal to the Height of the Eye, in which take *B L* equal in length to twice the Perpendicular, whoſe Perſpective is requir'd. Let *S* be the

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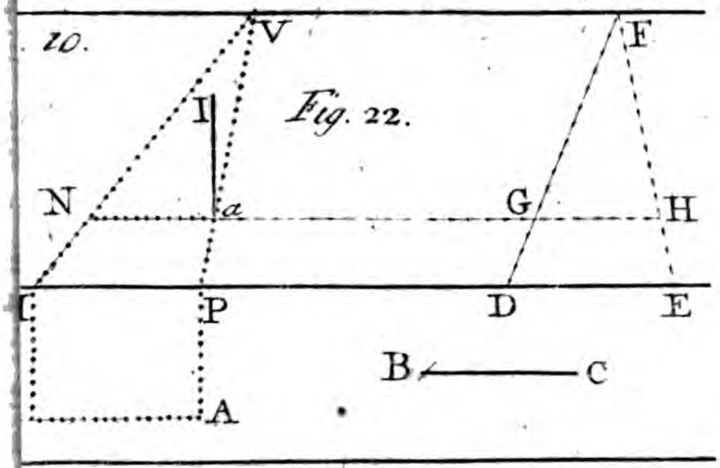


Fig. 22.

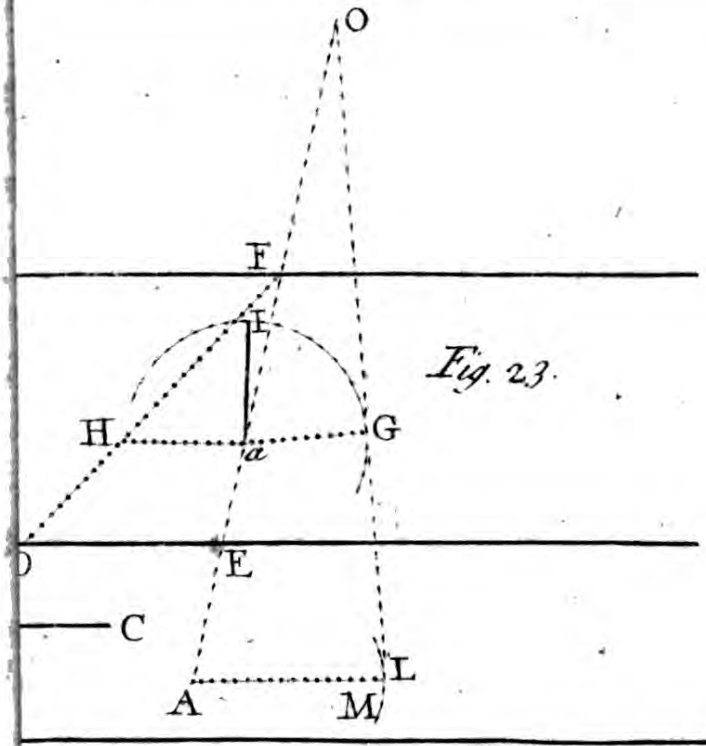
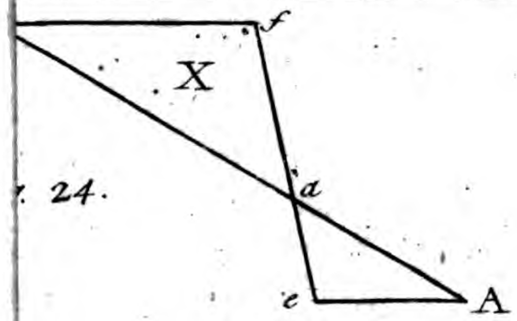


Fig. 23.



. 24.

the Station Point, and *A* the Point wherein the Perpendicular meets the Geometrical Plane.

OPERATION, *Without Compaſſes.*

Having firſt found * the Perſpective *a* of the Point *A*, draw the Line *A S* cutting the Baſe Line in *E*, through which Point *E* draw the Line *E a*; then from the Point *B* draw a Line *B a* to the Point *a*, cutting the Horizontal Line in F. Again through *F* draw a Line to the Point *L*, cutting *E a* in *I*; and *a I* is the Repreſentation ſought. * 31.

DEMONSTRATION.

To prove this, let *G N* be a Perpendicular to the Baſe Line drawn from the Point G, wherein the ſaid Baſe Line is cut by the Line *B F*; alſo let *G D* be equal to the Perpendicular whoſe Appearance is ſought, and *a H* parallel to the Baſe Line.

It is plain that the Perſpective of *E A* is *E a*: But *E A* paſſes through the Station Point; and conſequently * its Repreſentation is perpendicular to the Baſe Line; therefore * we are only to prove, that *a I* is equal to *a H*. * 41. * 56.

Now the Triangles *B G C* and *B F M* are ſimilar; and ſo

$$BC : BM :: BG : BF.$$

But *B M* by Conſtruction is the double of *B C*; whence *B F* is alſo the double of *B G*, which, conſequently, is equal to *G F*.

Becauſe the Triangles F *G N* and *F B L* are ſimilar, therefore

$$FG : FB :: GN : BL.$$

Now we have proved, that FG is the half of FB, therefore GN is likewiſe equal to the half of BL, and conſequently equal to the Height of the ſuppoſed Perpendicular.

Again, the ſimilar Triangles FGN and FaI give

$$FG : Fa :: GN : aI.$$

But $FG : Fa :: GD : aH$; becauſe the Triangles FGD and FaH are ſimilar.

Whence

$$GN : aI :: GD : aH.$$

Now becauſe GN has been proved to be equal to the Perpendicular, whoſe Perſpective is requir'd, and DG is ſuppoſed equal to that Perpendicular; it follows, that GN and GD are equal; and therefore aI and aH are alſo equal. *Q. E. D.*

SCHOLIUM.

I might have aſſumed CP equal to the Perpendicular, and uſed the Points C and P inſtead of B and L. But uſing the ſaid Points B and L is better: For when the Points C and P are uſed, the Horizontal Line muſt almoſt always be continued, that ſo a Line drawn through the Points c and a may cut it; moreover this Interſection will ſometimes be at an infinite Diſtance; whereas in uſing the Point B, MN can never be greater than thrice the Breadth of the Deſign to be drawn.

COROLLARY.

The ſixth Problem may be ſolv'd by this; for a Point elevated above the Geometrical Plane, may be conceived as the Extremity of a Perpendicular to the Geometrical Plane.

PRO-

PROBLEM VIII.

59. *To throw a Priſm or Cylinder into Perſpective, both of them being Perpendicular to the Geometrical Plane.* *Fig.* 26.

Let *G H I L M N* be the Baſe of the Priſm in the Geometrical Plane, and the viſible Part thereof upon the perſpective Plane, let be *n g h i*; then to compleat the Repreſentation of the Priſm, draw Perpendiculars from the Points *n g h* and *i* to the Baſe Line, whoſe Length let be * ſuch that they may repreſent Perpendiculars to the Geometrical Plane, equal to the Height of the Priſm, and find * the Perſpective of the other Angular Points of the upper Surface of the Priſm, in conſidering them as Points elevated above the Geometrical Plane: This being done, if the Repreſentations of all the ſaid Angular Points be joyn'd, the whole Priſm will be thrown into Perſpective. * 55. * 50.

Now to throw a Cylinder into Perſpective, the Repreſentation of its Baſe and upper Surface muſt firſt be had, by finding * the Appearance of ſeveral Points of the Periphery of its upper Surface, and then two Perpendiculars muſt be ſo drawn to the Baſe Line, that they may touch the Appearances of the two circular Ends of the Cylinder, and the Appearance of the Cylinder will be had. But to avoid uſeleſs Operations, the viſible Part of the Baſe of the Cylinder may be thus determin'd. Draw the Line *A S* from the Point *A* to the Station Point *S*, then this Line muſt be biſected in the Point *R*, about which, as a Centre, and with the Radius *R A*, the Circular Arc *B A C*, muſt be deſcrib'd cutting the Baſe of the Cylinder in the Points * 50.

B and

B and *C*, which will be the two moſt extreme ones that can be ſeen.

To do this another Way.

Fig. 26, 27. 61. If the upper Face of the Cylinder or Priſm be otherwiſe requir'd to be found, the ſame Things being given as in the foregoing Method, we draw the Line *PQ* in the perſpective Plane, parallel to the Baſe Line, whoſe Diſtance therefrom we make equal to the Height of the Priſm or Cylinder, whoſe Perſpective is requir'd. Then we change its Geometrical Plane, ſo that the Baſe Line coincides with *PQ*, and that in this Tranſpoſition a Perpendicular to the Baſe Line coin-
*46. cides with this ſame Perpendicular continued towards *PQ*. Finally we find * the Perſpective of the Baſe of the Priſm or Cylinder, thus changed in Situation by uſing *PQ* for a Baſe Line, and the ſaid Perſpective is the Repreſentation of their upper Faces.

Demonstration.

If we ſuppoſe the Plane of the upper Surface of the Priſm to be continued, it will meet the Perſpective Plane in *PQ*; and the upper Face in this Plane continued, will have the ſame Situation in Reſpect to *PQ*, as the Baſe hath on the Geometrical Plane with Regard to the Baſe Line. If then the ſaid continued Plane be conceived to lye on the perſpective Plane, the upper Faces of the Priſm or Cylinder, will be as the Baſes changed in the Manner aforeſaid; therefore the Appearance of the ſaid Baſes changed, will be that of the upper Surfaces.

Note, By folding the Paper it is eaſy to tranſpoſe Figures, and when the Height of the Priſm

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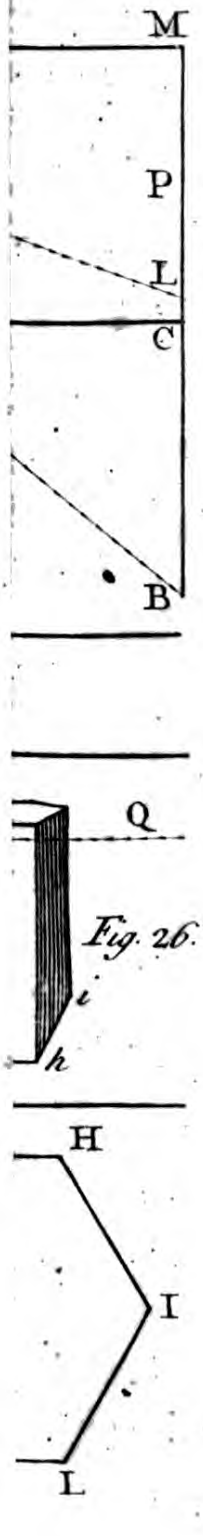


Fig. 26.

Prism is greater than the Height of the Eye, the precedent Method is the shortest.

PROBLEM IX.

62. *To throw a Concave Body into Perspective.* *Fig.* 28.

Having first found the Perspective of the said Body, afterwards find the Appearance of its Cavity, in considering the Cavity as a new Body.

PROBLEM X.

63. *To throw a Sphere into Perspective.* *Fig.* 29.

Let *A* be the Seat of the Centre of the Sphere; then the Point *I* the Perspective of the Centre must be found, * and the Line *IV* drawn to the Point of Sight *V*. This being done, raise *VF* perpendicular to *VI*, which make equal to the Distance from the Eye to the perspective Plane; and in this Perpendicular continued, take *VP* equal to the Distance from the Centre of the Sphere to the perspective Plane. Through the Point *P* draw *PQ* parallel to *VI* cutting a Line drawn from *F* through *I*, in *Q*; and about *Q* as a Centre, with the Semidiameter of the Sphere, draw the Circle *CB*, to which from the Point *F*, draw the Tangents *FC* and *FB*, cutting the Line *IV* in the Points *G* and *E*. On the Line *GE* describe the semicircle *EDTG*, wherein draw the Line *GD* perpendicular to *FI*, which bisect in *H*, and about *H*, as a Centre with the Radius *HD*, describe the Arc of a Circle, *LDR*, cutting the Line *FI* in the Points *L* and *R*. Take the Chord *GT* in the Semicircle *EDTG* equal to *RL*, and describe a Semicircle *TmG* upon GT; in which Semicircle draw several Lines, as *mn* Perpendicular to *GT*; and cutting the Line

* 50.

Line *GE*, in the Points *p*, from every of which raiſe Perpendiculars *p q*, each of which muſt be continued on each Side the Line *G E*, equal to *m n* the Part of the correſpondent Line *p m*. Now if a great Number of the Points *q* be thus found, and they are joyn'd by an even Hand, you will have a Curve Line which will be the Repreſentation ſought.

Demonstration.

The Rays by which we perceive a Sphere, do form an upright Cone, whoſe Axis paſſes through the Center of the Sphere, and whoſe Section made by the Perſpective Plane, is the Repreſentation ſought : from whence it follows, that I *is the Point in the Perſpective Plane, through which the Cone's Axis paſſes. But when an upright Cone is ſo cut by a Plane, that the Section is an Ellipſis, as in this Caſe, the tranſverſe Diameter of this Ellipſis, will paſs through the Point of Concurrence of the ſaid Plane, and Axis of the Cone, and that Point wherein a Perpendicular drawn from the Vertex of the Cone, cuts the ſaid Plane. This will appear evident enough to any one of but mean Knowledge in Conick Sections. Therefore the tranſverſe Axis of the Ellipſis, which is the Repreſentation of the Sphere, is ſome Part of* V I ; *for the Eye is the Vertex of the Cone formed by the viſual Rays of the Sphere.*

Now let us conceive a Plane to paſs through the Eye, and the Line I V ; *this will paſs through the Center of the Sphere : And if a Perpendicular be let fall from the Center upon the principal Ray continued, that Part of the ſaid Ray included between the Point of Sight, and the Point wherein this Perpendicular falls, which is always parallel to the Perſpective Plane, will be equal to the Diſtance from the Center of the Sphere to the Perſpective Plane,*

and

and consequently to V P. *Therefore if the before-mentioned Plane be supposed to revolve upon the Line* V I, *as an Axis, until it coincides with the Perspective Plane; the Center of the Sphere will meet the Perspective Plane in* Q, *and the Eye in* F; *whence the Part* G E *of the Line* I V *is the transverse Diameter of the Ellipsis.*

Again let G D E *in Figure* 30, *and* g e f, *in Figure* 31 *represent the Points denoted with the same Letters in the foregoing Figure. Now if the Cone, whose Profile is denoted by the Lines* f g *and* f e *be supposed to be compleated, and to be cut by a Plane passing through the Line* g e *perpendicular to the Plane of the Figure; we shall have an Ellipsis* g 4 e 3 *similar to that which is the sought Representation of the Sphere. Further if the said Cone be conceived to be cut by a Plane* l 4 m 3 *parallel to its Base, and bisecting* g e *in* n, *it is manifest, that* 3 4, *the common Section of the Circle* l 4 m 3, *and the Ellipsis* g 4 e 3, *is the conjugate Axis of the Ellipsis. And therefore this conjugate Axis is equal to the Line* 3 4, *Perpendicular in the Point* n *to the Diameter* l m *of the Circle* l 4 m 3. *Now draw the Lines* E O *and* G Y *in Figure* 30, *parallel to* L M, *then the Triangles* E G Y *and* E N M *are similar, whence* Fig. 30, 31.

E G : E N :: G Y : N M.

But E G *is twice* E N; *wherefore* G Y *is also the double of* N M, *and so* N M *equal to* G Z. *After the same manner we demonstrate, that* L N *is equal to* X E; *whence it follows, that* G D *is equal to* L M, *and is so cut in* z *as* L M *is in* N; *and therefore* R L *or* G T *of Figure* 29, *is equal to* 3 4 *in Figure* 31; *and consequently equal to the conjugate Axis of the Ellipsis to be drawn. On the other Hand, it is manifest by Construction, that some one of the Perpendiculars* m n, *Figure* 29, viz. *that which passes through the Center of the semicircle*

4 G m T,

G m T, *bisects the Axis* G E: *For if a Line be drawn from* T *to* E, *it will be perpendicular to* G T, *and consequently parallel to* m n: *Whence the conjugate Axis of the Curve* G q E, *is equal to the conjugate Axis of the Ellipsis to be drawn: And therefore we are only to prove, that the Curve passing through the Points* q, *is an Ellipsis. Which may be shewn thus.*

The Parts G n *of the Line* G T, *are Proportional to the Parts* G p *of the Line* G E: *Whence the Rectangles under* G p *and* p E, *are Proportional to the Rectangles under* G n *and* n T; *but these last Rectangles are equal to the Squares of the Ordinates* n m, *which Squares are equal to the Squares of the Ordinates* p q; *therefore these last Squares are Proportional to the Rectangles under* G p *and* p E, *which is a Property of the Ellipsis.*

DEFINITION.

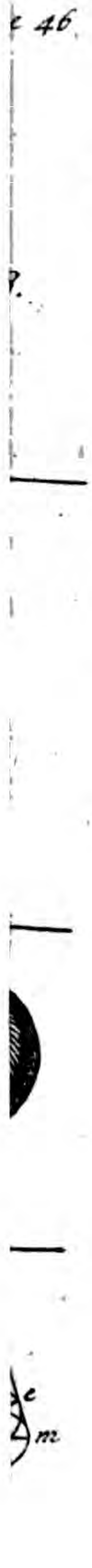
Fig. 33. The semicircular Part *h m* of a Column, encompassing the same like a Ring, is called the *Torus.*

PROBLEM XI.

64. *To throw the Torus of a Column into Perspective.*

Fig. 32. Let *BNC* be the Base of the Column in the Geometrical Plane; draw a Line from the Center *A* to the Station Point *S*, which bisect in the Point *R*, and describe the Arc of a Circle *BAC* about the Point *R*, as a Center with the Radius *RA*.

Fig. 33. Let *X* be the Profile of the Column, in which draw the Line *z* 36, through the Center of the semicircle *hm*, parallel to the Base of the Column; and in the Line *sa*, which goes through the Center of the Column, parallel to its Sides, take



take the Part 2 *s*, equal to the Height of the Eye above the Point 2, which is in the Base of the Column; likewise assume *s a* in the said Line equal to *SA* of the precedent Figure, and from the Point *a* draw to *a s* the indefinite Perpendicular *a T*. These General Preparations being made, take at Pleasure the small equal Parts 6 *i* and 6 9 in the Line *s a*; draw the Lines *i b* and 9 *m* Parallel to 63 *z*, and from the Point *b* draw the Line *b* 3 4, thro' the Center 3 of the Semicircle *b m*; assume *a* 5 in *a T* equal to *i* 4, and draw the Line 5 *s* cutting *i b* in *g*, and 9 *m* in *q*. And, (in Figure 32.) about the Point *A*, as a Center, with the Radius *i b* or 9 *m*, which are equal, describe the Circle *F L M H*, cutting the Arc *B A C* in the Points *D* and *E*; then draw the Line *D E* cutting the Line *A S* in *I*; assume *I G* equal to *i g*, and *I Q* equal to 9 *q*; and thro' the Points *Q* and G, draw *F H* and *L M*, parallel to the Line *E D*, cutting the Circle *D M E F* in the Points *L*, *M*, *F*, and *H*. Now if the Representations of Four Points, whereof *L M* F and *H*, are the Seats, and the two first of which is equal to 2 9, and of the two others 2 *i*, be found*; the Representation of the said four Points will be so many Points of Appearance sought. And by drawing two other Lines, as *i b* and 9 *m*, and proceeding as before, the Representation of so many more Points will be had.

Note, Because a part of the *Torus* is hid by the Column, therefore to avoid useless Operations, a Circle must be described about the Center *A*, with the Radius 36, cutting the Arc *B C A* in the Points *T* and *O*, and the Lines *STT* and *SOZ* must be drawn; then all the Points as *F* and *H*, falling between the Lines *T T* and *O Z* are useless, and *L* and *M* not coming under this Observation must only be used; Note also, that there is no necessity to determine Geometrically (which *Fig.* 32.

might

might be done) the Point on the Semicircle *h z m*, as far as the Parallels (as 9 *m*) are useful: For when these Parallels are useless, the Point *q* will fall beyond the Point *m*: But then the Perspective of the *Torus* is entirely drawn already, if those Parallels were first begun to be drawn near to 6 3 *z*, and the others continually going from it.

In order to demonstrate this *Problem*, the following *Lemma* is necessary.

LEMMA.

65. If two Circles *CDHE* and *DEFL* cut
Fig. 34. each other, thro' whose Centers *C* and *B* the Line *CL* passes, and *DE* joyns their Intersections; then, if the Radius *AC* or *AH* be called *a*, and *BF* or *BL*, *b*, and the Distance *AB* between the two Centers *c*, I say *AG* is equal to $\frac{bb-aa}{2c}-\frac{1}{2}c$.

DEMONSTRATION.

Let us call AG, x, *and* GD *or* GE, y. *Then by the Property of the Circle, if* y *be conceiv'd as an Ordinate of the Circle,* CDH; $yy=aa-xx$. *And if it be likewise consider'd as an Ordinate of the Circle* FDL, $yy=bb-cc-2cx-xx$: *Whence* $aa-xx=bb-cc-2cx-xx$, *and so* $2cx=bb-aa-cc$; *and dividing each Side of this last Equation by* 2c, *we have* $a\,x=\frac{bb-aa}{2c}-\frac{1}{2}c$. *Which was to be Demonstrated.*

The

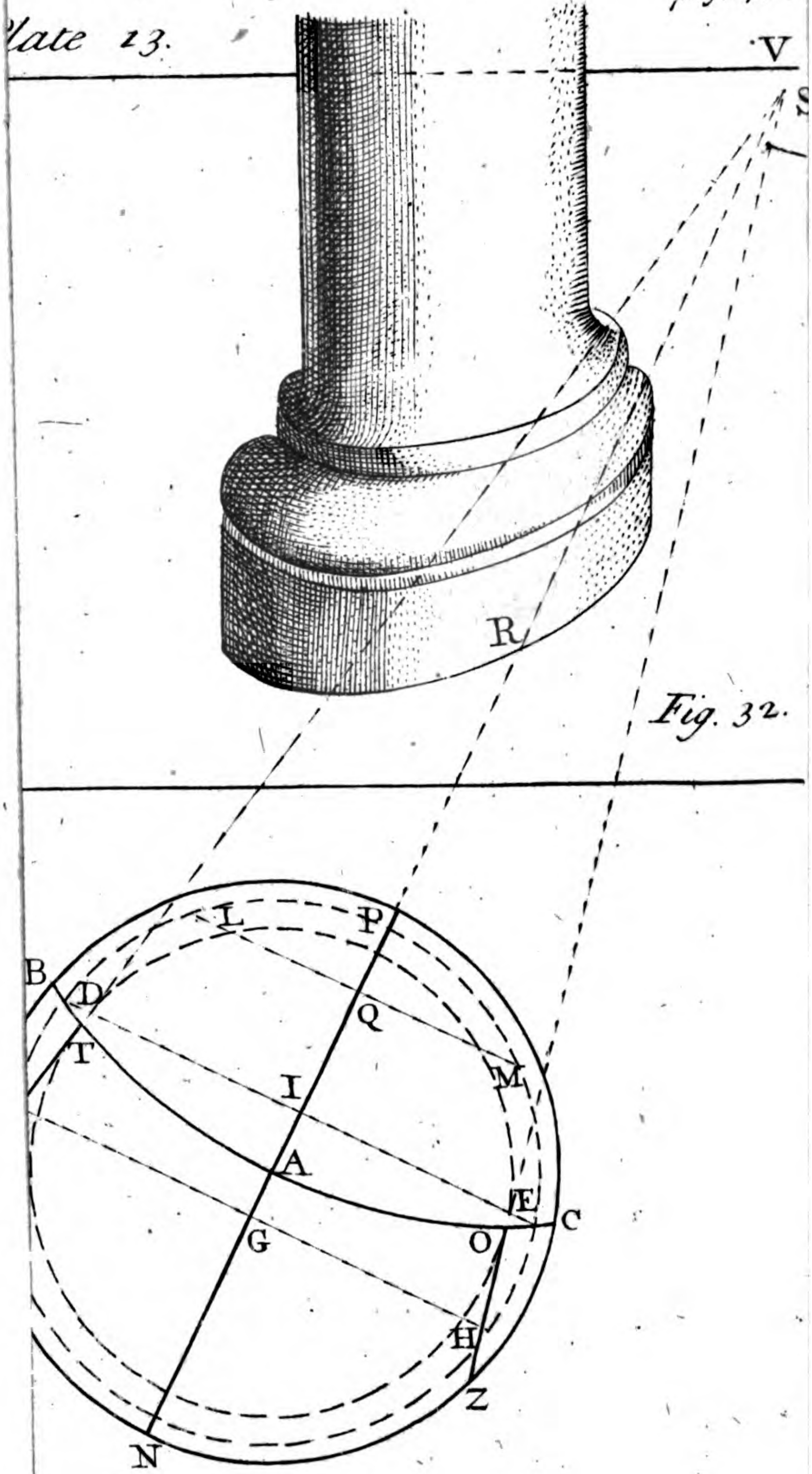


Fig. 32.

The Demonstration *of the* PROBLEM.

66. The Torus *of the Column must be conceiv'd as made up of an Infinite Number of Circular Planes, lying one upon another. And it is evident that the Reason why each of those Circles cannot be wholly seen, is because that which is immediately under it hides a Part thereof; from whence it follows, that if the Plane of one of these Circles be every way continu'd, and the Circle immediately under it, be thrown * into Perspective upon it, (which Perspective is also a Circle) the two Points of Intersection of this Representation, and the Circle in the Plane, will determine the visible Part of the said Representation; and consequently if the Representation of these two Points of Intersection be found upon the Perspective Plane, we shall have two Points of the Perspective of the* Torus *of the proposed Column. This is what I have done in the Solution of the Problem, as we shall now Analytically demonstrate.* * 8.

Let O *be the Eye,* A M *a part of the* Torus *of the Column,* A P *a Perpendicular to the Base passing through the Center of the Column, and* A B *a Parallel to the Base, drawn thro' the Center* B *of the Semicircle Concavity of the* Torus. *Let* M P *be a Semidiameter of one of the Circles spoken of in the the beginning of this Demonstration. Then if the Line* m p *be drawn parallel and infinitely near* C M P *and the Lines* m O *and* p O *are drawn cutting* M P *in* D *and* T, *it is evident that* D T, *which is in the Plane of the Circle passing thro'* M P, *will be the Semidiameter of the Perspective of the Circle immediately underneath.* Fig. 35.

Now let fall the Perpendicular O S *from the Eye to the Line* A B, *and continue the Lines* M P *and* m p, *till they meet the said Perpendicular in the Points* Q *and* q. *Moreover, continue the Line* M P *to the*

Point R, *wherein it is cut by the Line* m R *perpendicular to* m p. *Assume* AS $=$ e, OQ $= x$, *and* MP $= y$. *Then in the similar Triangles* O q m, *and* m R D, *we have,*

$$\mathrm{O q}\,(x) : \mathrm{q m}\,(e+y) :: \mathrm{m R}\,(dx)\ \mathrm{R D} : \left(\frac{edx+ydx}{x}\right)$$

The similar Triangles O p q *and* p T D, *give,*

$$\mathrm{oq}\,(x) : \mathrm{q p}\,(e) :: \mathrm{p P}\,(dx) : \mathrm{P T}\ \left(\frac{edx}{x}\right)$$

P R *is equal to* $y+dy$, *and if* P T $\left(\frac{edx}{x}\right)$ *be added to it, and then from the Aggregate be taken* R D $\left(\frac{edx+ydx}{x}\right)$ *we shall have* T D $= y+dy, -\frac{ydx}{x}$

Now to find the Points of Intersection of the two Circles, whose Radii are T D *and* P M, *and Centers distant from each other, by the Space* T P, *the Square*
* 65. *of* T D *less the Square of* P M *must* * *be divided by Twice* P T, *and then half of* P T *must be taken therefrom, which may be here neglected, because it is infinitely small in comparison of the rest; and we shall have* $\frac{xydy}{dex} - \frac{yy}{e}$ *for the Part of the Line* PM *included between* P *and the Point wherein this Line is cut by a Line joyning the two Points of Intersection of the two Circles.*

Now before what I have here demonstrated be apply'd to the Problem, we must observe, that if from the Point M, *a Line be drawn thro' the Center* B, *the Triangles* M P C *and* m R M *will be similar; for the Angle* m M P *is the Exterior Angle of the Triangle* m R M, *and the Angle* m M C *is a right one. And consequently*

$$\mathrm{m R}\,(dx) : \mathrm{R M}\,(dy) :: \mathrm{M P}\,(y) : \mathrm{P C}\left(\frac{ydx}{dx}\right).$$

Fig. 32, 33. *Now if* S A $=$ s a *in the* 32*d and* 33*d Figures be represented by* e *in this Computation; as likewise* s i *by* x, *and* i h *be* y; *it is manifest, that* i 4 = a 5 *being Algebraially Expressed, will be* $\frac{ydy}{dx}$.

Again, the similar Triangles, s a 5 *and* s i g *give* s a (e): a 5 $\left(\frac{ydy}{dx}\right)$: : s i (x): i g $\left(\frac{xydx}{edx}\right)$ *Also by the Construction of Figure* 32,

$$AS\,(e) : AP = ih\,(y) :: AP\,(y) : AI\left(\frac{yy}{e}\right);$$

Whence it follows, since I G = i g, *that* A G = (I G—A I) = $\frac{xyd}{cdx} - \frac{yy}{e}$. *And consequently,* H *and* F *are the Seats of the two Points whose Perspective is required, and those Points are both in a Plane parallel to the Geometrical Plane, which is the height of* 2 i *above the Geometrical Plane.*

If the precedent Calculation be apply'd to the Lower Part of the Torus, *the Expression* $\frac{xydy}{edx} - \frac{yy}{e}$, *will be chang'd into this,* $-\frac{xydy}{edx} - \frac{yy}{e}$; *which shews that these two Quantities must be assumed on the same Side of* A, viz. *towards* S. *Moreover* 9 q, *in the Line* 9 m, *is equal to* $\frac{xydy}{edx}$; *for* 9 8 $\left(\frac{ydy}{e}\right)$ = i 4. *Which shews that* M *and* L *are also the Seats of two Points whose Perspective must be found, and which are both in a Plane parallel to the Geometrical Plane, and above it the Height of* 29.

REMARKS.

67. This Problem may be likewise solved in considering the *Torus* of a Column as made up of an infinite Number of Bases of Cones, whose Altitudes are determin'd by the concurrence of the Tangents of the Semicircular Concavity of the Axis of the Column; and then determining * the visible

* 53.

visible Portions of the said Bases. Note, This Method may be demonstrated without *Algebra*, but it would be very long.

PROBLEM IX.

68. *To find the Accidental Point of several parallel Lines, which are inclin'd to the Geometrical Plane.*

Fig. 36. Let *AB* be the Direction of one of the Lines, whose accidental Point is sought; and *ECP*, the Angle that the said Lines make with the Geometrical Plane.

OPERATION.

Draw a Line, *OD*, thro' the Eye *O*, parallel to *AB*, and thro' the Point *D*, wherein it cuts the Horizontal Line, and which is the accidental Point of the Directions of the given Lines, draw *DF* perpendicular to the said Horizontal Line; in which assume *DG*, equal to *DO*. Finally, thro' the Point *G*, draw the Line *GF*, making an Angle with the Horizontal Line, equal to *ECP*; and then the Point *F*, (the Intersection of this Line) and the Perpendicular *DF*, is the accidental Point sought.

Note, When the Lines are inclin'd towards the perspective Plane, *DF* and *GF* must be drawn below the Horizontal Line: And, contrariwise, when the said Lines are inclin'd towards the opposite Part of the perspective Plane, the aforesaid Lines must be drawn above the said Horizontal Line, as is done here.

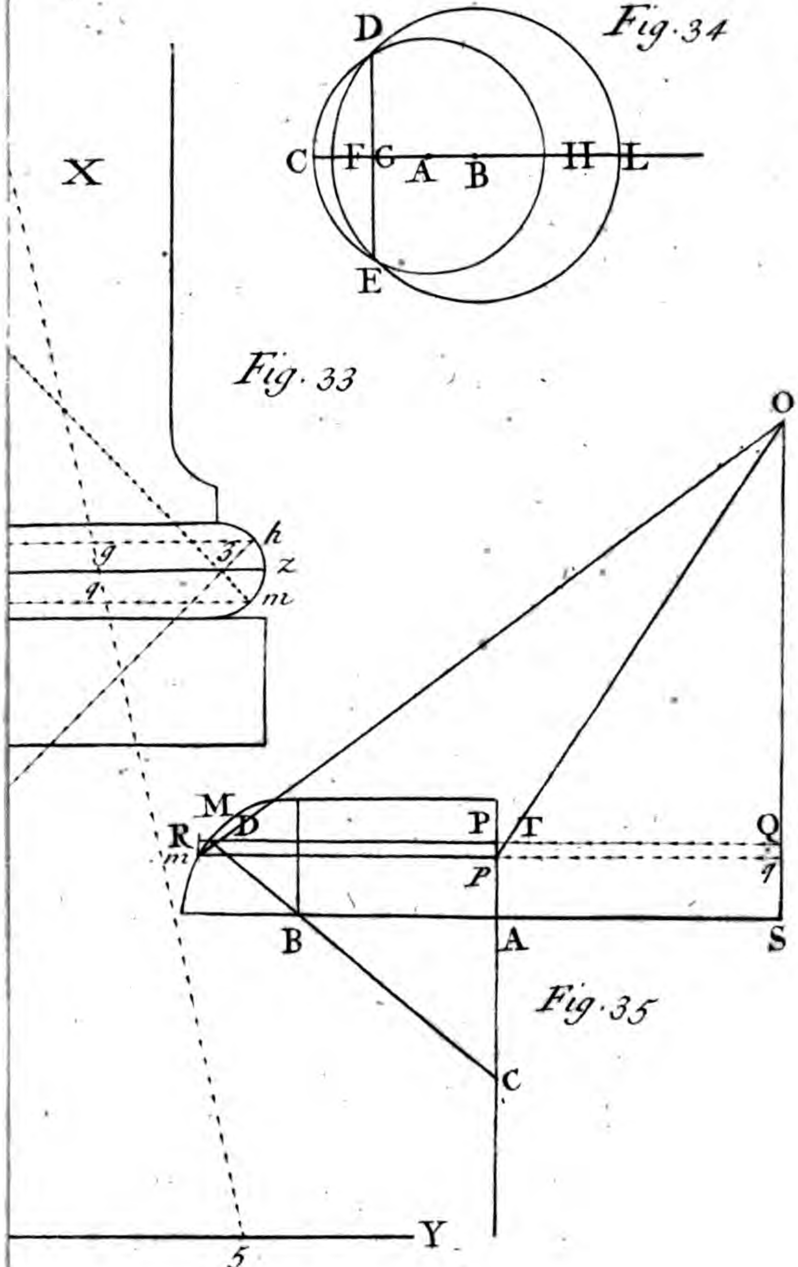


Fig. 34

Fig. 33

Fig. 35

DEMONSTRATION.

If a Plane be conceiv'd to paſs thro' the Eye, perpendicular to the Geometrical Plane, and parallel to the given Lines; it is evident, that the ſaid Plane will cut the Horizontal Plane in the Line *O D*, and the perſpective Plane in *D F*. It is, moreover, manifeſt, that a Line drawn thro' the Eye, parallel to the given Line, is in the ſaid Plane, and (with the Line *OD*) makes an Angle, equal to the Angle *E C P*, below the Horizontal Plane, if the Lines be inclin'd towards the perſpective Plane, and above it, if they incline to the oppoſite ſide; whence this laſt Line makes a right-angled Triangle with *O D* and *D F*, whoſe Angle at the Point *O*, is equal to the Angle *C E P*. But *DGF* is likewiſe a right-angled Triangle, as having the Angle at the Point *G*, equal to *ECP*; therefore theſe two Triangles are ſimilar. And ſince the Side *D G* is equal to the Side *D O*, the Triangles are alſo equal: Therefore the Line *D F*, being common to theſe two Triangles; the Point *F*, is the Point wherein the Line, paſſing thro' the Eye parallel to the given Line, meets the Perſpective Plane: And this Point is * the accidental one ſought.

Note, This Demonſtration as well regards inclin'd Lines entirely ſeparate from the Geometrical Plane, as thoſe that meet it in one of their Extremes only. * 13, 14.

PROBLEM X.

69. *To find the Repreſentation of one or more Lines, inclin'd to the Geometrical Plane.*

Fig. 36.

Let *A* be a Point given in the Geometrical Plane; whereon ſtands a Line, whoſe Length, Direction, and Angle of Inclination is known.

Operation.

In ſome ſeparate Place, draw the Lines *CE* and *CP*, making an Angle with each other equal to the Angle of Inclination of the given Line; and in one of theſe Lines, aſſume *CE* equal to the given Line, and let fall the Perpendicular *EP*, from the Point *E* upon the other Line. Then aſſume *AB*, in the Direction of the propos'd Line, equal to *CP*; and after having found *a*, the Perſpective of *A*, and the
* 50. Point *T**, the Perſpective of a Point elevated above *B*, the Height of *PE*; join the Points *a* and *T* by a right Line; and the ſought Appearance will be had.

Demonstration.

If from the Extremity of the inclin'd Line, a Perpendicular be let fall upon the Geometrical Plane, the ſaid Perpendicular will meet this Plane in the Point *B*, and will be equal to *PE*; as is evident by the Conſtruction of the Figure *CPE*. But the Point *T* is the Repreſentation of the Extremity of this Perpendicular; and therefore it is alſo the Extremity of the inclin'd Line. Which was to be demonſtrated.

Remarks.

There are ſome Caſes of this Propoſition, that may be ſhorten'd. As, 1. When there are ſeveral Lines of this Kind parallel between themſelves, and whoſe accidental Point can be
* 68. found *: And, 2. When an inclin'd Line is parallel to the perſpective Plane. The Manner of making

making these Abbreviations, will be laid down in the following Methods.

METHOD II.

70. *By the accidental Point of inclin'd Lines.*

Thro' *F*, the accidental Point of the inclin'd parallel Lines, draw *FH*, parallel to the Base Line, and equal to *FG*. And let *A* be the Point, wherein one of the inclin'd Lines meets the Geometrical Plane. *Fig.* 36.

OPERATION.

Assume *RQ* in the Base Line, equal to the inclin'd Line; and draw Lines from the Points *R* and *Q*, to the Point *Z*, taken at pleasure in the Horizontal Plane.

Thro' *a*, the Perspective of *A*, draw *aN* parallel to the Base Line; in which assume *aL*, equal to *MN*; and draw a Line from the Point *a*, to the Point *F*; and from the Point *L*, draw another to the Point *H*. Then *aT* will be the Perspective sought.

DEMONSTRATION.

By * the Nature of the accidental Point, the Perspective sought is a Part of the Line *aF*; and therefore, we are only to demonstrate, that the Extremity of the said Perpendicular is in the Line *LH*. Which may be thus done. * 14.

Let us suppose a Line, *AI*, to pass thro' the Point *A*, parallel to the Base Line, and equal to the inclin'd Line. It is then manifest *, that *L* is the Perspective of *I*; and consequently, *LH* * is the Appearance of a Line passing * 56. * 20.

through *I*, and the Extremity of the propoſed Line; and therefore the Perſpective of this Extremity is in the Line *L H*; which was to be demonſtrated.

Note, if *F H* had been aſſumed, the one half, or one third, &c. of what it is; then it is mani-
* 19. feſt * that *R Q* muſt alſo have been taken equal to the one half, or third Part, &c. of *C E*.

METHOD. III.

71. *For inclined Lines not meeting the Geometrical Plane.*

Fig. 37. Let *A* and *B* be the Seats of the Extremities of the given Line. Let *X* repreſent a Plane paſſing through the given Line perpendicular to the Geometrical Plane. Likewiſe let *M N* in this Plane, repreſent the Line whoſe Perſpective is requir'd; and let *C N* and *P M* be perpendicular to the Geometrical Plane: Whence *P C* repreſents *A B*, and conſequently is equal thereto.

OPERATION.

* 50. Find the Point *I* *, the Perſpective of a Point, above the Point *A*, the Height of *C N*; and draw the Line *B S*, from the Point *B*, to the Station Point *I*, cutting the Baſe Line in *E*; and from the Point *I*, draw a Line to the accidental Point *F*; which cut by a Perpendicular to the Baſe Line, raiſed at the Point *E*; and then *I T* will be the Appearance ſought.

METHOD IV.

72. *For inclined Lines parallel to the perſpective Plane.*

The

e. 15

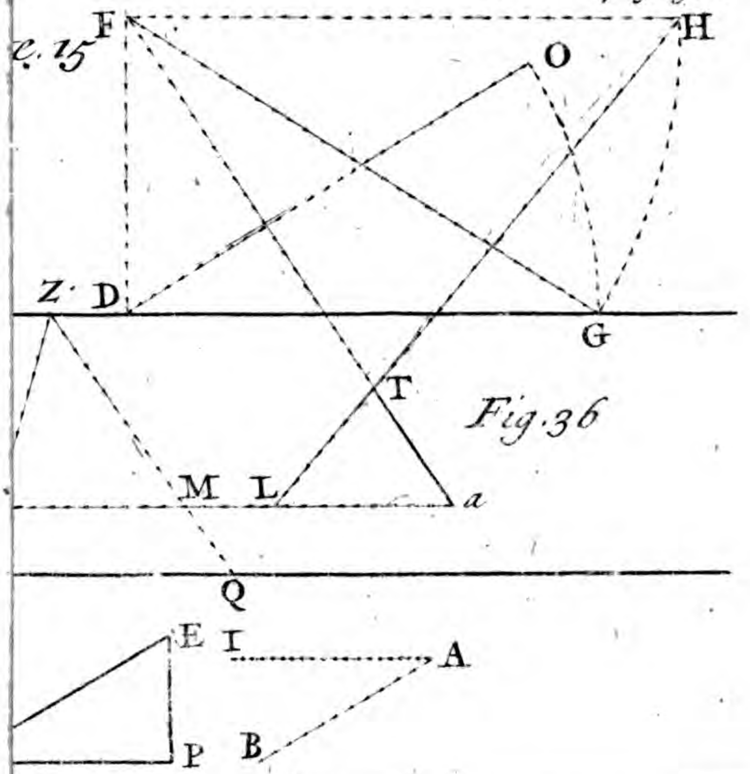


Fig. 36

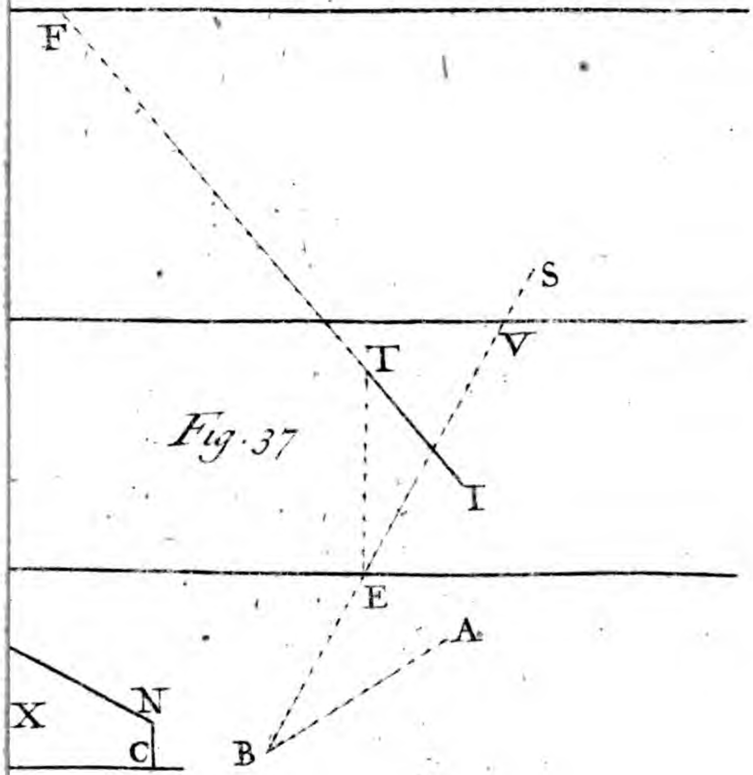


Fig. 37

The Operation of *Prob.* VII. muſt be uſed here, but with this Difference (*ſee Fig. of the ſaid Prob.*) that whereas *a I* in the ſaid Problem is perpendicular to the Baſe Line, here it muſt make an Angle with the Baſe Line, equal to the Angle of Inclination of the given Lines.

For the Demonſtration of this, *ſee* n. 7, and 10.

PROB. XIV.

73. *To throw a Body into Perſpective, having ſome one or all of its Sides inclined to the Geometrical Plane.*

The Appearances of the Lines forming the Angles of the propoſed Body muſt be found: And this may be eaſily done by *Prob.* 10. * which * 69.
takes in all the Caſes. And in this Manner the Appearance of a Pyramid, an inclined Priſm, &c. may be found. But nevertheleſs, it happens ſometimes, that the Operations of the precedent Problem may be abbreviated; as when the Extremity of ſeveral Lines are found in one and the ſame Line, or when inclined Lines, that have difficult accidental Points, interſect one another, and ſo mutually determine each other. This will appear manifeſt by the following Examples.

EXAMPLE I.

To throw ſeveral parallel Shores which ſtrengthen a Wall, into Perſpective.

I ſuppoſe here that the Baſes of theſe Shores, *Fig.* 38:
which are the Places where they meet the Surface of the Ground, are all in a right Line, parallel to the Side of the Wall; and then the ſaid Shores may be thrown into Perſpective in the following Manner: Having firſt found * * 68.
their

their accidental Point *F*, afterwards find the Representation of their Bases: This being done, denote the Appearances of the Lines wherein the Shores meet the Wall, upon the Perspective of the Wall; the Appearances here are the Lines *p t*, *r s*, which represent Lines parallel to the Geometrical Plane, from the Supposition, that the Shores are parallel between themselves, and their Bases equally distant from the Wall. Finally, draw Lines from the Angles of the Representations 1 2 3 4, to the Point *F*, which will be terminated by their Intersections with *p t* and *r s*, and will give the Appearances sought, as you see in the Figure.

EXAMPLE II.

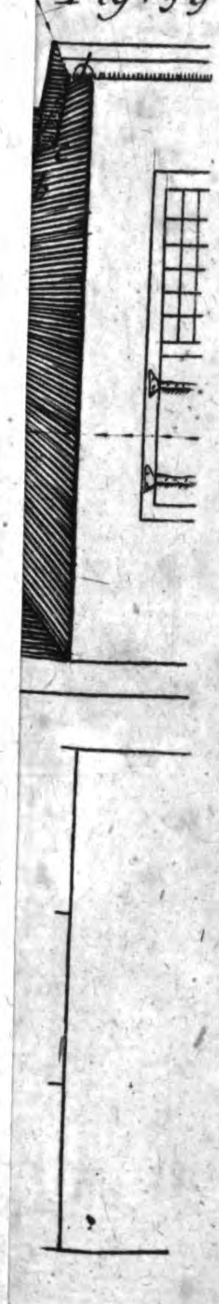
To throw several parallel Roofs of a House into Perspective.

Fig. 39. Having found the accidental Points *G* and *Q* of the said Roofs, in the Representation of the Wall sustaining them, denote the Points *a b c d*, wherein the said Roofs meet the Wall: Then from the Point G draw Lines through the Points *a b c*; and from the Point *Q* others to the Points *b c d*; these Lines by their mutual Intersection will determine each other, and give the Representations sought.

CONCLUSION.

74. From what has been already said, it will not be difficult to throw any Objects whatsoever into Perspective. But since it is very difficult, and indeed impossible for a Painter to make a Design entirely according to the Rules we have prescribed; the Number of Points to be found being almost infinite: therefore the Figures drawn,

Fig. 39



drawn upon the Geometrical Plane, and the principal Points of the Objects without the said Plane, need only be thrown into Perspective. Which being once obtained, he may make use of these Appearances so found, as a Rule whereby the rest may be compleated by the Eye, without running the Risque of committing some considerable Fault, which by this Means may be avoided.

CHAP. IV.

Of the Practice of Perspective upon the Perspective Plane still consider'd as being upright.

IT often happens that Painters offend all Rules of true Appearance when they paint Pictures to stand aloft, to be seen Sideways, or at a considerable Distance. Their Custom is to paint Pictures to be view'd, after the same Manner as they themselves look at them when they are working; whence in the following Cases, this Practice of theirs will be useless; and so to avoid enormous Faults, they are necessitated to have recourse to Perspective But what has been said in the last Chapter, does not reach these particular Cases; therefore we shall here add some new Problems, which together with the former ones, will take in all Cases.

PROBLEM I.

75. *To throw Figures which are in the Geometrical Plane into Perspective, when the Eye is at so great a Distance that it cannot be denoted in the Horizontal*

tal Plane, or one of the Points of Distance on the Horizontal Line.

The Representation of two Points of these
* 24. Figures must be first found *; and then by Means of these two Points the Appearances of others
* 38. may be had *.

Example.

Fig. 40. Let *ABCDE*, be a Pentagon, whose Appearance is requir'd; *V* the Point of Sight; and *VF* the sixth Part of the Eye's Distance from
* 24. the perspective Plane. Now find * *b* and *e* the Appearance of *B* and *E*, by means of which,
* 38. the Appearance of the Point *A* will be had *. In like Manner, by means of the Representation of *A* and *E*, will that of *D* be had; and by using *B* and *A*, the Perspective of *C* may be found.

76. *Note*, the Perspective of Lines perpendicu-
* 55. lar to the Geometrical Plane *; as also of Lines inclined thereto *, may be found by the Methods
* 69. of the precedent Chapter.

Problem II.

77. *To throw Figures, which are in the Geometrical Plane into Perspective, when the Eye is so oblique that it cannot be marked in the Horizontal Plane, or the Point of Sight in the Horizontal Line.*

We must proceed here according to the Directions of the precedent Problem, after having found the Perspective of several Points of the given Figures.

At any Point *C*, taken at Pleasure in the Base
Fig. 41. Line, draw the Perpendicular *CD* to the said Line, and likewise draw the Line *CE* from the same Point in such manner, that if it could be con-

age 60.

continued, it would cut the Horizontal Line in the Point of Sight.

This is done in aſſuming *C H* equal to $\frac{1}{3}$, or $\frac{1}{4}$ Part, &c. of the Diſtance from the Point *C*, to the Foot of the vertical Line; and in raiſing the Perpendicular *H E*, in the Point *H*, equal to $\frac{1}{3}$ or $\frac{1}{4}$ Part, &c. of the Height of the Eye. Now *A* is a given Point, whoſe Appearance is ſought.

OPERATION.

Draw a Parallel *A B*, through the Point *A*, to the Baſe Line, meeting the Line *C D* in the Point *B*, and let a ſecond Eye be ſuppoſed at the ſame Height and Diſtance as the firſt; then find * *F G* the Repreſentation of *A B* for this ſecond Eye, which continue until it meets the Line *C E* in *b*, and in this Continuation aſſume *b a* equal to *F G*; then *a* will be the Perſpective ſought.

* 42.

DEMONSTRATION.

Becauſe the Height and Diſtance of the ſecond Eye, is equal to the Height and Diſtance of the firſt; the ſaid two Eyes are both in one parallel Line *A B*; and conſequently *, the Perſpective of *A B* muſt be a Part of *F G* continued, and alſo equal to *F G*: And therefore becauſe * the Perſpective of *B* is in the Line *C E*, *a b* is the Perſpective of *A B*; and *a* *, that of *A*. Which was to be demonſtrated.

* 18. * 12. * 16.

78. *Note*, as to Lines perpendicular, and inclined to the Geometrical Plane, *ſee* n. 76. This is ſcarcely uſeful, unleſs for the Decorations of a Theatre.

PROB.

PROB. III.

79. *To find the Representation of a Figure in the Geometrical Plane, when the Perspective Plane is placed above the Eye.*

When the perspective Plane is situated above the Eye, we suppose the Geometrical Plane to pass through the Top of the perspective Plane; upon which Geometrical Plane are drawn the Figures of Objects meeting it; as also the Seats of those Objects that are underneath it, by Means of Perpendiculars; and the Height of the Eye is here measur'd by a Perpendicular drawn from the Eye to the Geometrical Plane; whence the perspective Plane, elevated in respect to the Eye, is the same thing, as an Eye elevated in regard to the perspective Plane.

Fig. 42. Let *I L* be the Base Line, and *H* the Foot of the vertical Line; then in the Base Line assume the Points *I* and *L* at Pleasure, towards the Sides of the perspective Plane. Make *I S* equal to $\frac{1}{3}$ or $\frac{1}{4}$ Part of *I H*, and raise the Perpendicular *S X*, in the Point *S*, to the Base Line, equal to a correspondent Part of the Height and Distance of the Eye taken together; draw the Line *X I G*, and moreover *Y L Q*, in assuming *L T* equal to $\frac{1}{3}$ or $\frac{1}{4}$ &c. of *L H*. Again draw the Line *G Q* in the Geometrical Plane, parallel to the Base Line, and distant therefrom (for Example) a third Part of the Height of the Eye; draw also *F P* in the perspective Plane, parallel to the Base Line, and distant therefrom, a fourth Part of the Eye's Distance; these two Lines will cut *X I* in *G* and *F*, and *Y L* in *Q* and *P*. *Note*, if the Distance of *G Q* from the Base Line, had been assumed equal to a fourth Part of the Eye's Distance; then *F P* must have been

been drawn from the Base Line, equal to a fifth Part of the Eye's Distance, and so on. Now *A* is a Point whose Representation is requir'd.

OPERATION.

Draw the Lines *AF* and *AP*, from the Point *A* to the Points *F* and *P*, cutting the Base Line in the Points *E* and *B*; then draw the Lines *EG* and *BQ*, which continue till they intersect each other in *a*, which is the Representation sought.

DEMONSTRATION.

Let us suppose the perspective Plane continued, *CD* the Horizontal Line, and *O* the Eye denoted in the Horizontal Plane. It is evident * by Construction, that the Line *GF* continued, passes through the Eye *O*; produce the Line *GSa*, until it meets the Horizontal Line in *D*, and draw the Line *OD*. Let fall the Perpendicular *GNR*, from the Point *G* upon the Horizontal Line, which intersect in *R*, by the Line *OR*, passing through the Eye parallel to the Horizontal Line. Now by Construction, *GM* is $\frac{1}{3}$ of *MN*; and consequently it is $\frac{1}{4}$ of *GN*; *MZ* is likewise $\frac{1}{4}$ of *NR*: Therefore

$$GM : MZ :: GN : NR.$$

Compon. and Altern.

$$GM : GN :: GM + MZ = GZ : GN + NR = GR.$$

Because the Triangles *GMI* and *GNC* are similar, we have

$$GM : GN :: GI : GC.$$

The Triangles *GZF* and *GRO* being also similar,

$$GZ : GR :: GF : GO.$$

Whence

* 77.

Whence

$GI : GC :: GF : GO.$

Again, becauſe the Triangles *GIE* and *GCD* are ſimilar, we have

$GI : GC :: GE : GD.$

And conſequently

$GF : GO :: GE : GD.$

And ſo the Triangles *GFE*, and *GOD* are ſimilar; and the Line *FEA* is parallel to *OD*:
* 13. Whence it follows *, that the Perſpective of *EA*, is a Part of *EaD*. We demonſtrate in the ſame Manner, that *Ba* is the Perſpective of *BA*, and ſo the Perſpective of the Point *A*, the common Section of *EA* and *BA*, is *a*, the Interſection of the Appearances of the ſaid two Lines.

PROB. IV.

80. *To find the Repreſentation of a Line, perpendicular to the Geometrical Plane, when the perſpective Plane is above the Eye.*

Fig. 43. In the Baſe Line *BE*, aſſume the Line *ED*, equal in Length to the propoſed Perpendicular; and draw *CL*, parallel to the Baſe Line, and diſtant therefrom (for Example) $\frac{1}{4}$ of the Height of the Eye; make *FL* equal to $\frac{3}{4}$ of *DE*, and draw the Lines *EL* and *DF*. *Note*, if the Diſtance from *CL* to *BE*, had been aſſumed equal to a fifth Part of the Height of the Eye, *FL* muſt have been aſſumed equal to $\frac{4}{5}$ Parts of *ED*. Now let *a* be the Perſpective of the Foot of the propoſed Perpendicular; through which draw *aH* parallel to the Baſe Line, and *aI* perpendicular to the ſaid Line; then make *aI* equal to *GH*, and the propoſed Perſpective will be had. The Demonſtration of this Operation is
* 56. manifeſt *, in conſidering that *DF* and *EL*

4 being

Plate. 18.

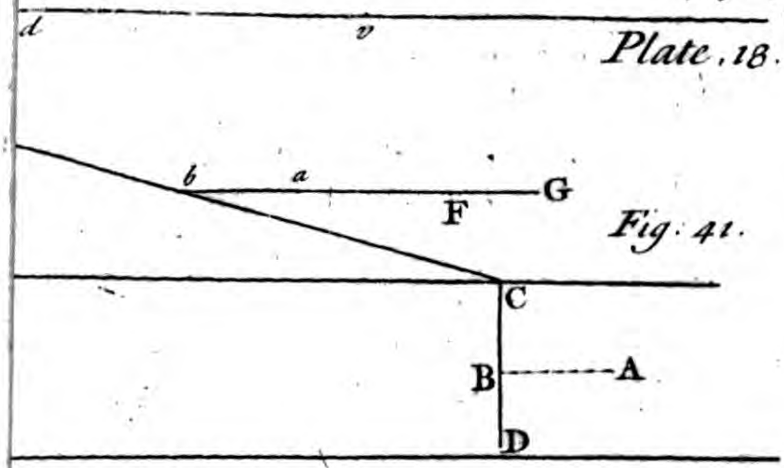


Fig. 41.

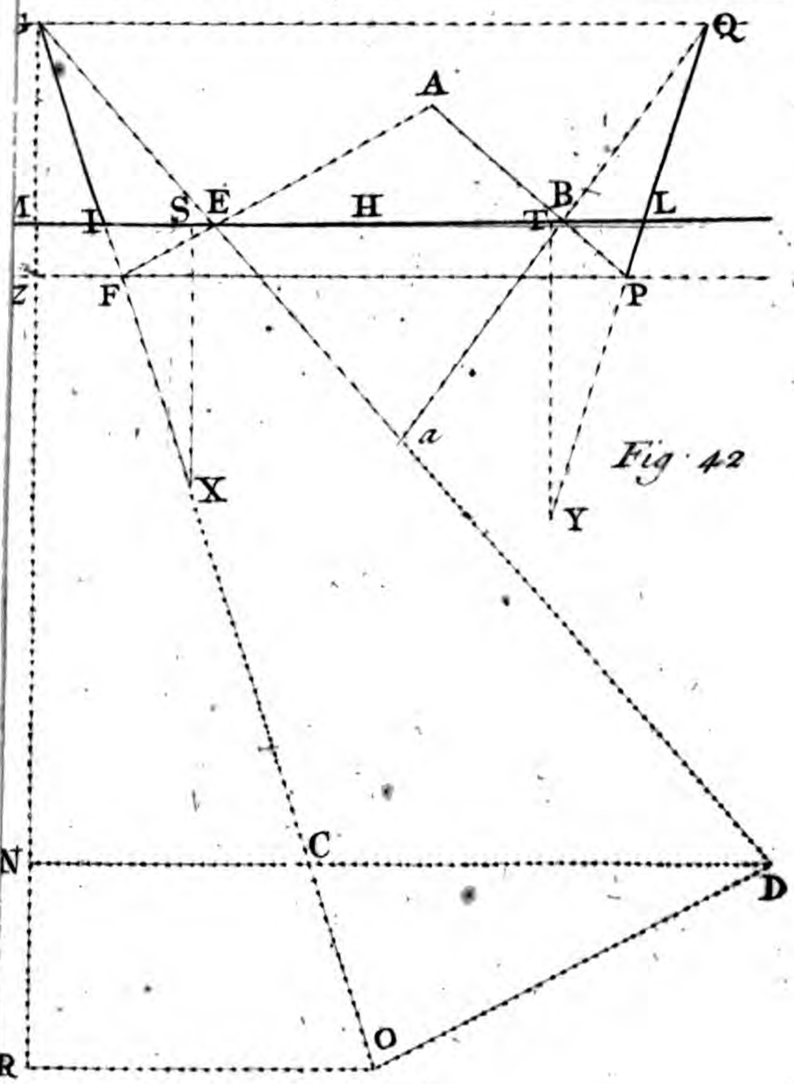


Fig. 42

being produced, will meet each other in the Horizontal Line.

CHAP. V.

Of throwing Figures into Perſpective, when the Perſpective Plane is conſider'd as being inclined.

PROBLEM I.

81. TO *find the Perſpective of a Figure in the Geometrical Plane.* Fig. 44.

Let *X* be the Vertical Plane; *S I* the Station Line, *S* the Station Point, and *H* the Interſection of the Station Line and Baſe Line. Now draw the Vertical Line *H V* through the Point *H*, making an Angle with *S I*, equal to the Angle of Inclination of the perſpective Plane; then raiſe the Perpendicular *I O* to *S I*, in the Station Point *S*, equal to the Height of the Eye; and through the Extremity of the ſaid Perpendicular, draw the principal Ray *O V*, parallel to *S I*, and cutting *H V* in the Point of Sight *V*.

Now it is evident, that *O V* determines the Length of the principal Ray, and *H V* the Diſtance from the Baſe Line to the Horizontal Line; and ſince the Demonſtration of the Problems in the aforegoing Chapters regarding the Geometrical Plane, have alſo Relation to the perſpective Plane being inclined, the ſaid Problems may be here uſed; and conſequently, this inclined perſpective Plane is reduced to a Perpendicular one, view'd by an Eye, whoſe Height is *H V*, and Diſtance *O V*.

Problem II.

82. *To find the Appearance of a Point above the Geometrical Plane.*

Fig. 45. Let *H C* be the Base Line: And let *T* be the accidental Point of the Lines perpendicular to the Geometrical Plane. This Point will
* 13. be * in that Place of the Vertical Line, wherein it is cut by the Prolongation of the Line measuring the Height of the Eye; for this last Line is parallel to the said Perpendiculars. And so likewise the aforesaid Point is the same as the Point *T* of *Fig.* 44: Let *V* be the Point of Sight, *S* the Station Point, and *Q* the Station Point of the upright perspective Plane, to which
* 81. the inclined perspective Plane is reduced *. And lastly, let *A* be the Seat of the given Point.

Operation.

Draw two Lines *M P* and *P E* separately, making a right Angle with each other; in one of which, assume *P E*, equal to the Height of the given Point, whose Perspective is sought; and draw the Line *E M*, making an Angle with *M P*, equal to the Angle of Inclination of the perspective Plane. Again let fall the Perpendicular *A D* from the Point *A* to the Base Line, in which assume *A L* equal to *P M*, towards the Base Line, when the perspective Plane is inclined towards the Objects (as we have here supposed) but on the other Side of *A*, when the perspective Plane inclines towards the Eye. Then from the Point *A*, draw a Line to the Point *S*, cutting the Base Line in *B*, and joyn the Points *L* and *Q*, by a Line cutting the Base Line in *C*. This being done, draw the Line

Line TBX; which interſect in the Point X, by a Perpendicular to the Baſe Line, in the Point C; and then the Point X is the Appearance ſought.

DEMONSTRATION.

In *Fig.* 44. where V, S, T, and H, repreſent the ſame Points as thoſe that are denoted with the ſame Letters in this Figure; we have,

$$TH : HS :: TV : VO.$$

Compon. and altern.

$$TH : TV :: TH + HS : TV + VO.$$

This being apply'd to *Fig.* 45. and it will be,

$$TH : TV :: TS : TV + VO.$$

If now TX be continued, till it cuts the Horizontal Line in F; we ſhall have,

$$TH : TV :: TB : TF.$$

And conſequently,

$$TB : TF :: TS : TV + VO.$$

Whence it follows, that if a Line be drawn from the Eye, to the Point F, it will be parallel to SBA. Therefore * the Perſpective of BA, is a Part of BX; and ſo the Repreſentation of A is in the ſaid Line. The Perſpective of a Line perpendicular to the Geometrical Plane, in the Point A, paſſes thro' the Perſpective of the Point A, and thro' the Point T*; therefore it is a Part of TX. But the given Point is in the aforeſaid Perpendicular: And ſo its Perſpective is in TX.

*13. *13,14.

Again; it is otherwiſe manifeſt, that the Perſpective of CL, is a * Part of CX; and conſequently, the Appearance of L is in this Line. Now, if a Line be ſuppos'd to be drawn from the Point L, thro' the propos'd Point, it will be parallel to the Vertical Line; and ſo its * Perſpective is parallel to the Baſe Line. And ſince this

*41. *6.

this Appearance passes thro' that of the Point *L*, it will be a Part of *CX*. But because that Line, drawn from the Point *L*, passes thro' the propos'd Point; the Representation of the said Point is also in *CX*; and so in *X*, the common Intersection of *CX*, and *TX*.

REMARK.

83. If the Point *T* should be at too great a Distance; or if *TBX*, or *CX*, should too obliquely cut each other; the perspective Plane
* 81. must then be suppos'd to be reduc'd * to a perpendicular, or upright one; and the Representation of a Point, above the Geometrical Plane, (whose Seat is *L*, and Height *ME*) must be
* 50. found *.

PROBLEM III.

84. *To find the Representation of a Line, perpendicular to the Geometrical Plane.*

Fig. 45. The Appearance of the Extremity of the Per-
* 82. pendicular must be found *, in considering the said Extremity as a Point above the Geometrical Plane, by the Height of the propos'd Perpendicular: Then if a Line be drawn from the Point
* 16. *D*, to the Point of Sight; its Intersection * with *TX*, will give the Appearance *a* of the Seat of the Perpendicular propos'd.

Note, When there is a Necessity of having recourse to the Remarks of the foregoing Problem, in order to find the Point *X*; then the Point *a* may be found, in drawing *AS* and *DV*, and afterwards joining the Points *B* and *X* by a Line. And when *BX* and *DV* cut each other too

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Plate. 19

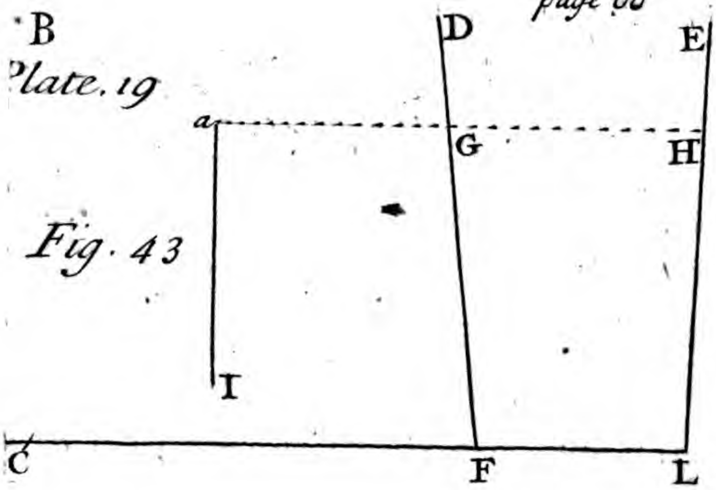


Fig. 43

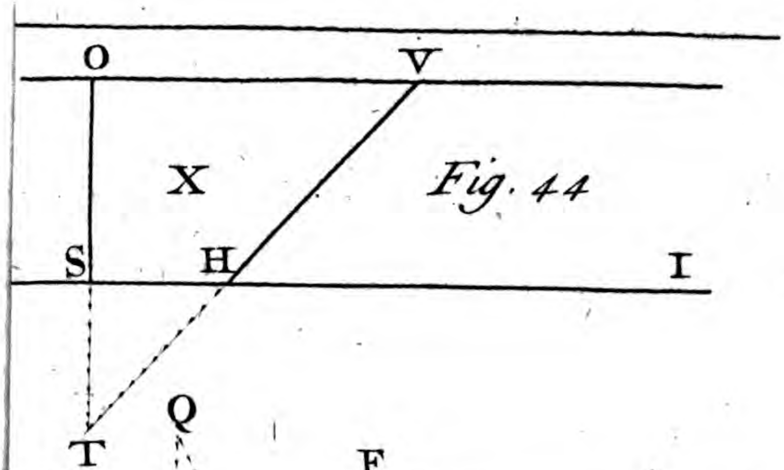


Fig. 44

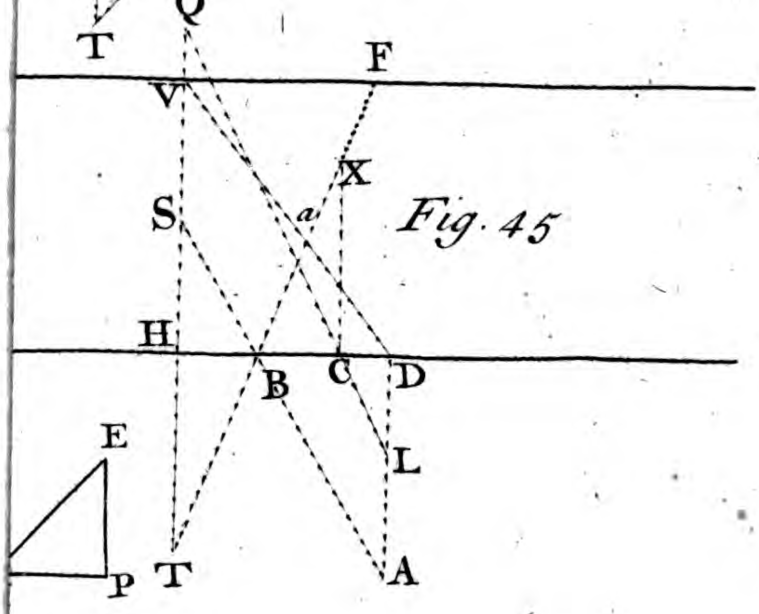


Fig. 45

too obliquely, recourſe muſt be had to *Problem* I.*, *81.
to find the Appearance of *a*.

METHOD II.

85. *A* is the Foot of the Perpendicular: The *Fig.* 46.
Triangle, *EPM*, is drawn * as directed: And *82.
T is the accidental Point of the Perpendiculars, to the Geometrical Plane.

OPERATION.

Thro' the Point *a*, the Appearance of *A*, draw a Perpendicular to the Baſe Line; which make equal * in Repreſentation to the Line *55.
ME; in conſidering this laſt Line, as being parallel to the Vertical Line. Then, from the Extremity *I* of this Perſpective, to the Point of Sight *V*, draw a Line cutting the Line *Ta*, in the Point *X*; which will be the Repreſentation of the Extremity of the propos'd Line.

DEMONSTRATION.

Let us ſuppoſe a Line paſſing thro' the Point *A*, equal to *ME*, and parallel to the Vertical Line. Suppoſe, moreover, that another Line is drawn thro' the Extremity of this Line, and that of the propos'd Perpendicular; then this laſt Line, by the Conſtruction of the Figure *MEP*, will be parallel to the Station Line;
and conſequently, its Repreſentation * will paſs *16.
thro' the Point of Sight; and its Interſection with *Ta*, will be the Extremity of the Repreſentation ſought. But *aI* is * the Perſpective *56.
of the firſt Line, made equal to *EM*; and conſequently, *VI* is that of the ſecond. Which was to be demonſtrated.

Note, When VI and Ta cut each other very obliquely, recourſe muſt be had to the Obſervation at the End of the aforegoing Method, or the following Way may be uſed.

METHOD III.

Fig. 47. 86. Let T be the accidental Point of the Lines perpendicular to the Geometrical Plane, thro' which Point draw a parallel to the Baſe Line, in which aſſume TR equal to OT of *Fig.* 44.

OPERATION.

Aſſume DN ſomewhere in the Baſe Line, equal to the propoſed Line, and draw the Lines DF and NF to the Point F, taken at Pleaſure in the Horizontal Line; then through the Point a, the Appearance of A, draw the parallel aH, to the Baſe Line, in which aſſume aQ, equal to GH. Then if the Lines Ta, and RQ be drawn, and continued, till they cut each other in the Point X, aX will be the Appearance ſought.

DEMONSTRATION.

* 56. The Part aQ of the Line aH, is * the Appearance of a Line proceeding from A in the Geometrical Plane, and which is equal to the propoſed Line, and parallel to the Baſe Line;
* 20. and conſequently *, the Line RQ paſſes through the Perſpective of the Extremity of the propoſed Line: And therefore X the Interſection of RQ and Ta, is the Appearance of the ſaid Extremity.

RE-

REMARK.

It is manifeſt *, that TR may be aſſumed equal to $\frac{1}{2}$ or $\frac{1}{3}$ &c. of what it is taken here, provided likewiſe that then DN be aſſumed equal to a correſpondent Part of the propoſed Line. * 19.

PROBLEM IV.

87. *To throw a Sphere into Perſpective.*

The Method of ſolving this Problem before laid down *, muſt be uſed here, but with this Difference; that inſtead of uſing the Point of Sight, the Point wherein a Perpendicular drawn from the Eye to the perſpective Plane, meets the ſaid Plane, muſt be uſed. And you muſt obſerve, that this Perpendicular meaſures the Eye's Diſtance from the perſpective Plane. * 63.

PROBLEM V.

88. *To find the accidental Point of any Number of Lines inclined to the Geometrical Plane.* *Fig.* 48.

Let AB be the Direction of one of the inclined Lines, O the Eye in the Horizontal Plane, and S the Station Point.

OPERATION.

Draw the Line OD, thro' the Eye O parallel to AB, meeting the Horizontal Line in D, which will be * the Accidental Point of the Directions of the given Line; and thro' the Station Point S, * 13, 14.

 draw

draw $S N$ perpendicular to the ſaid Line $A B$, cutting the Baſe Line in N, and draw the Line $N D$. Then about the Point D, as a Center, and with the Radius $D O$, deſcribe the Circular Arc $O o$: And about N, as a Center, with the Radius NS, draw the Circular Arc $S s$. This being done, draw the Line $s o$ touching the ſaid two Arcs, and the Line $D o$ perpendicular to $s o$. Then draw $o F$, making an Angle with $o D$, equal to the Angle of the Inclination of the Lines given, and cutting $N D$ continued in F: Now I ſay F is the Accidental Point ſought when the Lines do not incline towards the Perſpective Plane: But if they do, $o F$ muſt be drawn below $o D$.

DEMONSTRATION.

If a Plane be ſuppoſed to paſs thro' the Eye parallel to theinclin'd Lines; the common Sections of this Plane, and the Horizontal and Geometrical Planes, will be $O D$ and $S N$. It is now manifeſt, that if a Line be drawn in the ſaid Plane, below the Horizontal Plane, when the Lines incline towards the perſpective Plane, and above it when they incline the other way, making an Angle with $O D$ equal to the Angle of Inclination of the propoſed Lines; I ſay it is manifeſt, that the ſaid Line will be parallel to
* 13, 14. the propoſed Lines, and will meet * the perſpective Plane in the Accidental Point ſought. If now the before ſuppoſed Plane be conceiv'd to turn about the Line $N D$, the Eye, and the Station Point in the ſaid Plain, will then meet the perſpective Plane in the Points o and s; for the Lines $D o$ and $N s$ are equal to $D O$ and $N S$, and form right Angles with the Line $s o$ joyning their Extremities. Now the two Points s and o anſwer to the Situation of the Eye and Station

Point

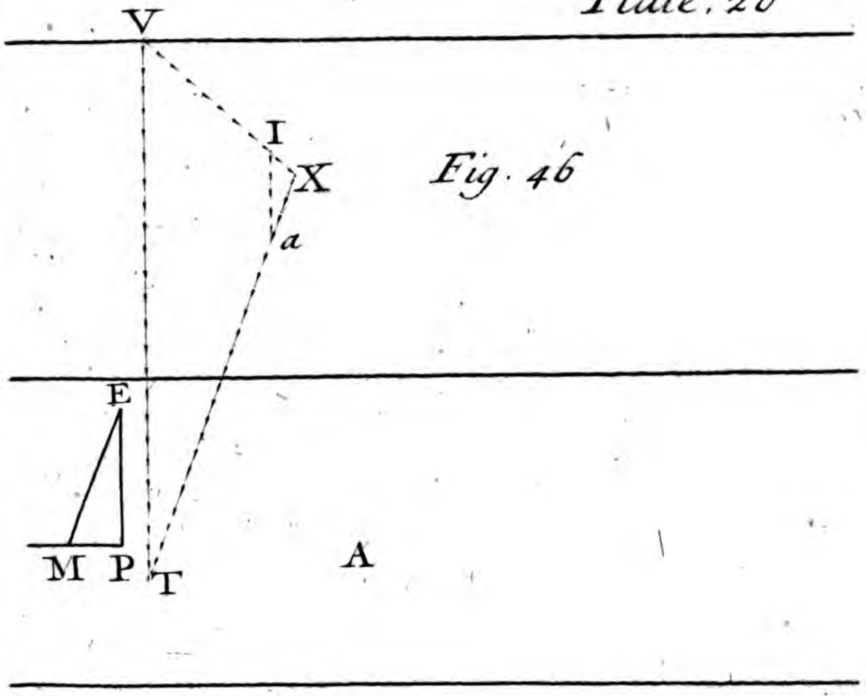


Fig. 46

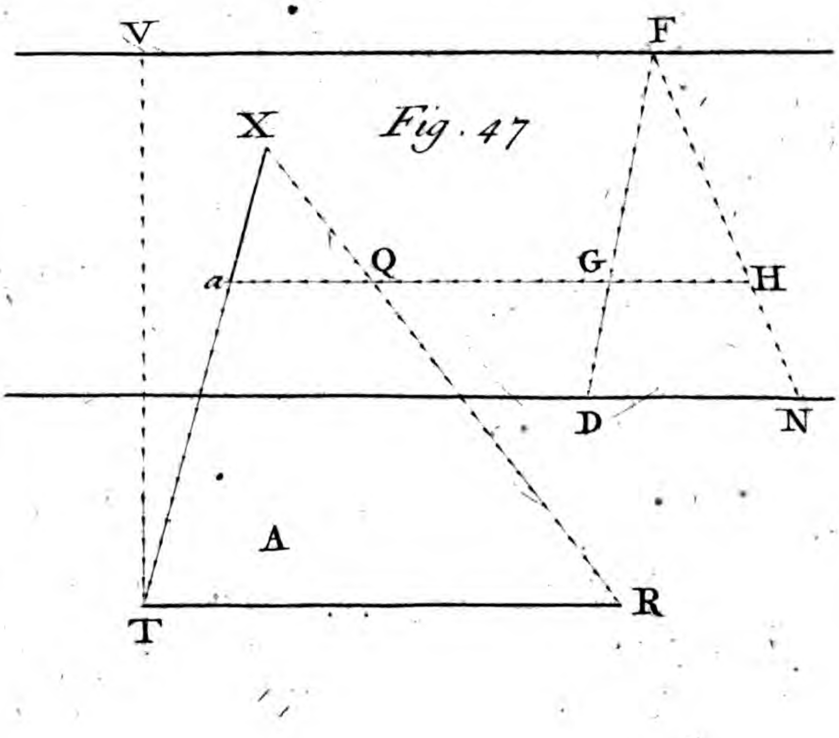


Fig. 47

Point in respect to each other, in the before supposed Plane. Therefore the Line *o F* answers likewise to the Line in the said Plane imagined to be parallel to the proposed Lines; and consequently the Point *F*, is that wherein the said Parallel meets the Perspective Plane; and therefore it is the accidental Point sought.

Note, If the accidental Point *T* of Perpendiculars to the Geometrical Plane be found, the Operation of this Problem may be shorten'd, in drawing the Line *T D*, which will necessarily pass thro' the Point *N*, and then the Point *o* will be found by the Intersection of the Arc *O o*, and a Semi-circle, whose Diameter is *T D*.

PROBLEM VI.

89. *To find the Perspective of one or more Lines inclin'd to the Geometrical Plane.*

Let *A* be the Foot of a Line inclin'd to the Geometrical Plane, and *a* its Representation. *Fig.* 48.
Now determine, by Means of the Triangle *C P E* according to the Manner lay'd down * for the Perspective Plane when supposed perpendicular, * 69.
the Length *A B* of the Direction of the proposed Line. This being done, find the Point * *X* the Perspective of a Point above the Geometrical Plane * 82.
by the Length of *PE*; and then *a X* will be the Perspective sought.

METHOD II.

90. *To solve this Problem by the Accidental Points of inclined Lines, and their Directions.*

Let *AB* be the Direction of an inclin'd Line; *D* the Accidental Point of the Directions, & *F* that of *Fig.* 48.
the

the Lines themſelves, and *T* the Accidental Point of Perpendiculars to the Geometrical Plane.

OPERATION.

Continue the Line *A B* until it meets the Baſe Line in *G*, and draw the Line G *D*, which cut in *a* and *b* by Lines drawn from *A* and *B* to the Eye. Then draw the Lines *a F* and *T b* Interſecting each other in *X*; and *a x* will be the Appearance ſought.

DEMONSTRATION.

* 43. Becauſe *a* b is * the Repreſentation of *A B*, the Appearance of the inclin'd Line is one Part of *a F*. But the Extremity of the inclin'd Line is in a Perpendicular to the Geometrical Plane raiſed at the Point *B*; therefore the Repreſentation of the ſaid Extremity is in *T b*, and conſequently × in the Interſection of this Line and *a F*.

METHOD III.

91. Draw *F H* thro' the Accidental Point of the inclin'd Lines, parallel to the Baſe Line, *Fig.* 49. and equal to *o F* in *Fig.* 48. then *a* is the Perpendicular of the Foot of the inclin'd Line whoſe Perſpective *a* × may be found as directed *n.* 70.

CHAP.

Page 74.

CHAP. VI.

Of throwing Figures into Perſpective, when the Perſpective Plane is conſider'd as being parallel to the Geometrical Plane.

PROB. I.

92. TO *find the Perſpective of a Figure, which is in the Geometrical Plane.*

When the perſpective Plane is parallel to the Horizon, we commonly conſider it, as being it ſelf the Geometrical Plane, in which Caſe, the Problem is fully reſolved: But when it happens, that another Geometrical Plane is ſuppoſed, either above or below the perſpective Plane, upon which the Figures upon the Geometrical Plane are requir'd to be drawn; we muſt draw geometrically thereon, Figures ſimilar to thoſe in the Geometrical Plane; ſo that the Lines on the perſpective Plane, be to their correſpondent ones on the Geometrical Plane, as the Eye's Diſtance from the perſpective Plane, is to its Diſtance from the Geometrical Plane.

The Demonſtration of this is evident from *n.* 8, and 9.

PROB. II.

93. *To find the Perſpective of a Line, perpendicular to the Geometrical Plane.*

Draw the Line *O S*, in which aſſume *O R* equal to the Eye's Diſtance from the perſpective Plane; *Fig.* 50.

Plane; and at the Points *R* and *S*, raiſe the indefinite perpendiculars *RG* and *SM*; and aſſume the Point *M* at Pleaſure on *SM*; from which raiſe the Perpendicular *MN*, equal to the given Line, and draw the Lines *MO* and *NO*, cutting
Fig. 41. the Line *RG* in the Points *E* and *G*. Then having drawn a Line at Pleaſure in the perſpective Plane through the Point *T*, which is that wherein a Perpendicular falling from the Eye on the perſpective Plane meets it, aſſume *TH* in the ſaid Line, equal to *RE*, and *TI* equal to *RG*; draw the Lines *Ta*, *Ha*, through the Point *a*, the Perſpective of the Foot of the given Perpendicular, and through the Point *I*, the Line *IX*, parallel to *Ha*, and cutting *Ta* in *X*, then *aX* will be the Appearance ſought.

Demonstration.

* 13, 14. It is manifeſt *, that the Point *T*, is the accidental Point of Lines perpendicular to the Geometrical Plane; and conſequently the Perſpective ſought is a Part of *Ta*. Moreover, it
* 4. is manifeſt *, that if the Feet and Extremities of two equal right Lines, perpendicular to the Geometrical Plane be joyn'd by Lines, theſe Lines of Junction will have parallel Repreſentations; becauſe they are parallel to each other, as likewiſe to the perſpective Plane. And conſequently, ſince *HI*, by Conſtruction, is the Perſpective of a Line perpendicular to the Geometrical Plane, and equal to the given Line, and *Ha* paſſes through the Appearances of the Foot of the ſaid Perpendicular, and the given Perpendicular; I ſay, that *XI*, which is parallel to *Ha*, and paſſes through the Extremity of the Appeaarance *HI*, likewiſe paſſes through the Extremity of the given Line; and therefore

fore the Point *X* is the Representation of the said Extremity.

COROLLARY.

94. It is manifest from hence, that when the Perspective of a Line, perpendicular to the Geometrical Plane is once found, it is easy afterwards to find the Representations of any Perpendiculars of the same Length as that.

METHOD II.

The Perspective Plane being consider'd as the Geometrical Plane.

95. Let *T* (as in *Fig.* 51.) be the accidental Point of perpendicular Lines to the Geometrical Plane; *HI* the Arc of a Circle, whose Center is *T*, and semidiameter the Eye's Distance from the perspective Plane: Also let *a* be the Point where the Perpendicular, whose Appearance is sought, meets the perspective Plane, and *BC* the Length of this Perpendicular. *Fig.* 53.

OPERATION,

About the Point *a*, as a Center, and with the Semidiameter *BC*, describe the Circle *LF*, and draw the Line *IL*, or *HF*, touching each of the Circles *HI*, and *FL*; and then *aX* or *ax*, is the Appearance sought, *viz.* *aX*, when the Perpendicular is above that Surface of the perspective Plane next to the Eye, and *ax*, when the Perpendicular is on the opposite Side.

DEMONSTRATION.

Draw the Radii *aF*, *aL*, *TI*, *TH*, to the Points of Contact *F*, *L*, *I*, and *H*. Then because

cause the Triangles THX and aFX are similar,

$$TH - aF : aF :: Ta : aX.$$

And because the Triangles TIx and axL, are also similar, we have

$$TI + aL :: aL : Ta : ax.$$

Fig. 54. Now let $PMNR$ be the perspective Plane, O the Eye, AQ the Perpendicular, whose Perspective is requir'd, and Ot a perpendicular let fall from the Eye upon the perspective Plane, and so t will be the same, as the Point T in the aforegoing Figure. Now if the Lines OQ be drawn, it is manifest that Ax, or AX, is the Perspective of AQ, according as this Line is above or below the perspective Plane in respect to the Eye. Then because the Triangles Otx and QAx are similar, we have

$$Ot - AQ : AQ :: tA : Ax.$$

And since the Triangles OtX and XAQ are similar,

$$Ot + AQ : AQ :: tA : AX.$$

Now Ot is equal to TH or TI of the aforegoing Figure, and AQ to aF or aL of the same Figure; as likewise At, Ta: Therefore if these two last Proportions be compared with the two precedent ones, we shall find $Ax = aX$, and $AX = ax$; which was to be demonstrated.

Remarks.

96. When the two Circles intersect each other, or fall within one another, and so this Way becomes useless; a Line must be drawn at Pleasure, through the Point T, equal to the Distance of the Eye from the perspective Plane; and then a parallel equal to the given Perpendicular must be drawn to the said Line through the Point a, either towards L or F, according as the Perpendicular

ıв
2

dicular is on one Side or other of the perſpective Plane with reſpect to the Eye. And the Line paſſing through the Extremities of the ſaid Parallels, will determine the Repreſentation ſought, by its interſecting the Line Ta, as is evident by what is demonſtrated.

METHOD III.

97. *To find the Repreſentation of ſeveral Perpendiculars equal in Length to ſome one, whoſe Perſpective is already drawn.*

Let HI, be the Perſpective of a Perpendicular to the Geometrical or Perſpective Plane. Now about the accidental Point T, as a Center, and with the Radius TH, deſcribe the circular Arc HG, whoſe Chord let be equal to HI, and draw the indefinite Line TGC, and let a and b, repreſent the Feet of the Perpendiculars, whoſe Repreſentations are requir'd. *Fig.* 52.

OPERATION.

Deſcribe about the Center T, the Arcs bFE, and aDC, paſſing through the Points a and b, and draw the Lines Tb and Ta; in each of which aſſume bL equal to EF, and aX equal to CD; and the ſought Repreſentations will be had.

DEMONSTRATION.

If HI, and aX repreſent Perpendiculars of the ſame Length, it follows from the Demonſtration of the precedent Method, that IH is to HT, and aX to aT as the Difference of the ſaid Perpendiculars, and Height of the Eye, is

4 to

to the Length of the ſaid Perpendiculars: And therefore

$$HI : TH :: aX : aT.$$

But in the Conſtruction of this Problem, becauſe the Triangles TCD and THG, are ſimilar;

$$HG = HI : TH :: CD = aX : TD = aT;$$

and conſequently HI and aX, repreſent Perpendiculars of the ſame Length. Which was to be demonſtrated.

PROB. III.

98. *To find the accidental Point of any Number of parallel Lines inclined to the Geometrical Plane.*

Fig. 55. Let ab be the Perſpective of the Direction of one of the given Lines.

OPERATION.

Draw the Line FTL, parallel to ab, through the accidental Point T of the Lines perpendicular to the Geometrical Plane; and at the Point T, raiſe the Perpendicular TG, which make equal to the Diſtance of the Eye from the perſpective Plane; then draw the Line GL, or GF, ſo that the Angle TLG, or TFG, be equal to the Angle of the Inclination of the given Lines; and the Point L, will be the Accidental Point ſought, if the given Lines incline towards b; but if they incline towards a, F will be the Accidental Point.

DEMONSTRATION.

It is manifeſt by Conſtruction, that if TG be ſuppoſed to be raiſed perpendicularly to the Geometrical Plane, GL or GF, will be parallel to the

the given Lines; and consequently * *L*, or *F*, will be the Accidental Point sought. * 13, 14.

PROB. IV.

99. *To find the Representation of one or more Lines inclined to the Geometrical Plane.*

Let *a b* be the Perspective of the Direction of the given Line: Now the Length of its Direction may be determin'd, by Means of the Triangle *E C P*, according to the Directions of *n. 69.* This being done, the Line *b X* must be drawn through the Point *b*, equal to *E P*, and this represents a Perpendicular to the Geometrical Plane; then *a X* being drawn, will be the Representation sought. *Fig.* 56.

METHOD II.

100. *To solve the same Problem by Means of the accidental Points* F *and* T, *the one being that of the Lines proposed; and the other, that of the Perpendiculars to the Geometrical Plane.* *Fig.* 56.

OPERATION.

Draw a Line from the Point *F* through the Point *a*, which intersect in the Point X, by another Line drawn from the Point *T* through *b*: and then *a X* will be the Perspective sought.

METHOD III.

101. The same Things being given as in the aforegoing Method, draw *a I* through the Point *a*, equal to *E P*, representing a Line perpendicular to the Geometrical Plane. Then draw a parallel to *F T* through the Point *I*, whose Inter- *Fig.* 101.

section with $F a$, will determine $a X$ the Representation sought.

REMARKS.

Although we suppose the Eye in all the Problems in this Chapter to be above the perspective Plane, yet it may likewise be under the said Plane; in which Case, the Geometrical Plane is supposed above the Objects, as we have
* 79. already done * on another Occasion.

CHAP. VII.

Of Shadows.

FIRST we must observe, with those who have already treated on this Subject, that when a luminous Body is equal to an opaque Body it enlightens, the Shadow of the said Body is contain'd between parallel Lines, and consequently, it is equal upon all parallel Lines placed at any Distance whatsoever beyond the opaque Body. And when the luminous Body is lesser than the opaque Body, the Shadow thereof, increases, and is infinitely augmented. And on the contrary, when an opaque Body is less than the luminous Body, the Shadow thereof decreases and terminates in a Point.

Now because the Sun is vastly greater than any of the Bodies on the Earth's Surface it enlightens, and is at so great a Distance therefrom, therefore its Rays may be consider'd as being parallel; and consequently, the Bodies it shines upon as enclosed between parallels: And this is the first Kind of Shadows I shall here explain; after

after which, I ſhall touch upon thoſe continually increaſing. What I ſhall ſay on this Matter, is ſufficient for deſigning the Shadows of right-lin'd Bodies; as to the Shadows of other Bodies, it is ſo difficult to determine them Geometrically that it is much better to examine thoſe which are daily obſerved, and ſo imitate them.

I ſhall not ſay any thing concerning Shadows terminating in a Point, becauſe their Variety is ſo great, that they cannot be geometrically determin'd. Beſides, Painters ſcarcely ever ſuppoſe their perſpective Planes or Pictures enlightned after this third Manner, unleſs only when they have a Mind to repreſent a Chamber, wherein the Light enters through the Windows; but then the Number of Windows, their Situation, and the different Reflections that the Light ſuffers in the Chamber, produce ſo many different Alterations, that a Painter had better imitate them, than have recourſe to Rules that do not take in all Caſes. I ſhall likewiſe be ſilent as to the *clair-obſcure*, for a ſmall Attention to daily Experience will better illuſtrate this Matter than a long Diſcourſe thereon. Beſides, it is impoſſible to furniſh general Rules on this Subject; and likewiſe the vaſt Number of Figures, will not permit us to ſeparately examine them; add to all this, that a Painter to draw the *clair-obſcure*, he ought to have, not only regard to the Figures of Objects, but likewiſe to their Colour and Matter.

Of Solar Shadows.

PROBLEM I.

103. *To find the Perſpective of the Shadow of a Point above the Geometrical Plane, whoſe Height and Seat is given.*

Fig. 57. Let Z be the Geometrical Plane; A the Seat of the given Point; and AB the Direction of the Sun's Rays.

Operation.

Draw two right Lines, making a right Angle with each other; in one of which, assume PE, equal to the Height of the given Point, above the Geometrical Plane: Then draw the Line EC thro' the Point E; making an Angle with CP, equal to the Sun's Altitude; and make AB equal to CP. Find the Appearance of the Point B; and the Representation sought, will be had.

Note, This Operation, as likewise all the others in this Chapter, regard all the Situations of the perspective Plane; and is so evident, that there is no need of demonstrating it.

Problem II.

104. *To find the Representation of an elevated Point, whose Appearance, as also that of its Seat, is given, without using the Geometrical Plane.*

Fig. 58. * 68, 88, 98. Find * F, the accidental Point of the Sun's Rays, and D, that of their Direction: Then draw a Line from the Point D, thro' a, the Perspective of the Seat of the given Point; and another from F, thro' I, the Perspective of the given Point: And then b, the Intersection of the said two Lines, will be the Point sought; as is manifest.

Remarks.

In order to find the accidental Point of any Number of inclin'd Lines, we have suppos'd

Plate. 23

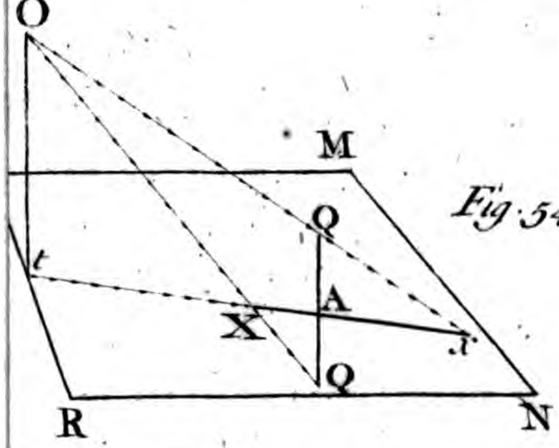


Fig. 54

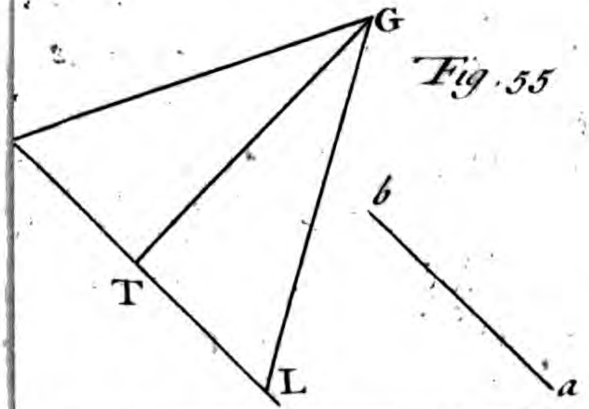
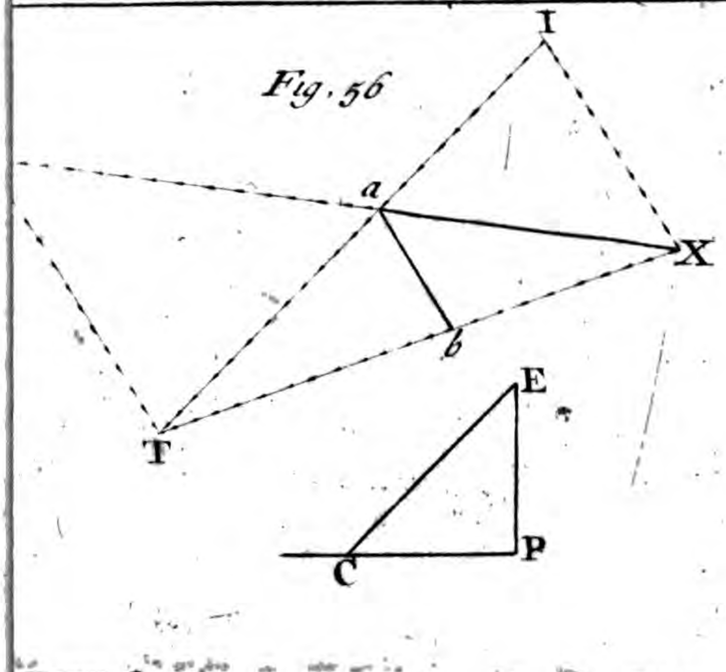


Fig. 55

Fig. 56



* one of the Directions drawn upon the Geome- *68, 88.
trical Plane; but it is sufficient here, that the Angle the said Directions make with the Base Line, be only known: And so, as the Problem expresses it, the Geometrical Plane may be entirely laid aside.

105. When the perspective Plane is parallel, the Sun's Rays will have no accidental Point; for their Representations are then parallel; in which case, one of the Parallels must be drawn thro' the Point *a*, instead of the Line *Da*. Moreover, when the perspective Plane is perpendicular, or inclin'd, and the Sun's Rays are parallel thereto; a Line must be drawn thro' the Point *a*, parallel to the Base Line; as likewise, another Line thro' the Point *I*, parallel to the Sun's Rays; cutting the first Line in the Point sought.

PROBLEM III.

106. *To find the Perspective of the Shadow of an elevated Point, when there is some Body hindring its falling upon the Geometrical Plane.*

The Perspective of the Section of the Body made by a Plane passing through the given Point perpendicular to the Geometrical Plane, and parallel to the Sun's Rays, must be found: And then the Intersection of the said Perspective, and a Line drawn from the Appearance of the given Point to the Representation of its Shadow, is the Representation sought.

Of the Shadows of a small Light.

PROB. IV.

107. *To find the Perspective of the Shadow of a Point, whose Seat, and Height, above the Geometrical Plane is known.*

Fig. 59. Let Z be the Geometrical Plane, *A* the Seat of the given Point; and C that of the Light: Draw the indefinite Line *CAB*; and about *C*, as a Center, with a Semidiameter equal to the Height of the Light above the Geometrical Plane, describe the Arc *F*: Again, about the Point *A*, with a Semidiameter equal to the Height of the given Point, describe the Arc *E*. This being done, draw the Line *E* F touching the said two Arcs, and cutting the Line C *A* in *B*. Then if the Perspective of *B* be found, the Perspective of the Shadow of the Point will be had.

PROBLEM V.

108. *To find the Perspective of the Shadow of an elevated Point, the Representation of which, as also of its Seat being known, without using the Geometrical Plane.*

* 104. The Operation of *Prob.* 3. * for Solar Shadows must be used here, with this Difference; that instead of the accidental Point of the Sun's Rays, the Perspective of the small Sight must be used; and in the Room of the accidental Points of the Directions of the said Rays, the Perspective of the Seat of the Light must be assumed.

Plate . 24

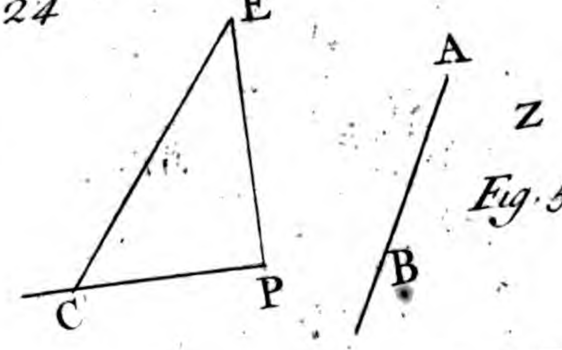


Fig. 57

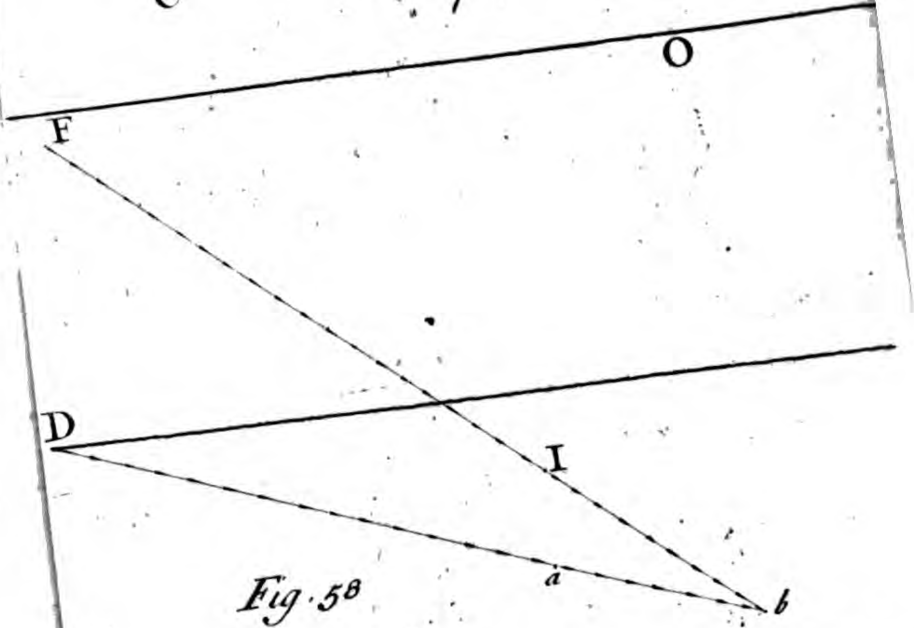
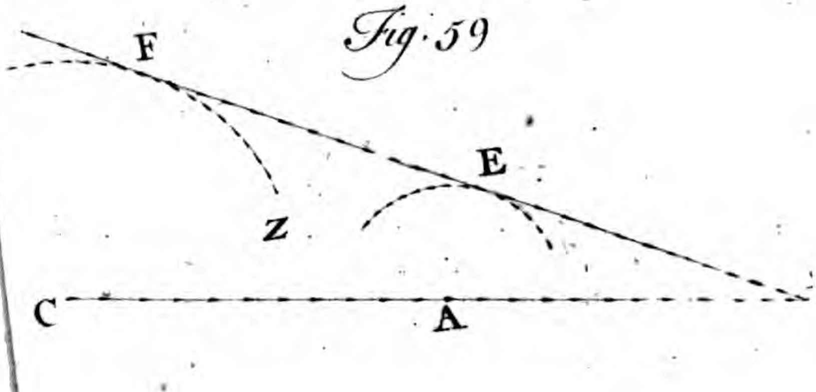


Fig. 58

Fig. 59



REMARKS.

What hath been obſerved * on the Solar Shadows, hath no Regard to thoſe mentioned here: For it matters not in this *Problem*, whether the perſpective Plane be perpendicular, inclined, or parallel; becauſe in theſe different Situations, the two Points that are uſed may always be found. * 105.

Moreover, it muſt be obſerved, that the third *Problem* * takes in the Shadows of a ſmall Sight, as well as thoſe of the Sun; but not without this Difference, that the Plane which in the third *Problem* is ſuppoſed parallel to the Sun's Rays, ought here to be ſuppoſed to paſs through the Light whoſe Shadows are ſought. * 106.

CHAP. VIII.

Of mechanically ſhortning the Operations of Perſpective.

I. WHEN the perſpective Plane is ſuppos'd perpendicular or upright.

PROBLEM I.

109. *To find the Repreſentations of Figures in the Geometrical Plane.*

Let *O* be the Eye, *R H* the Baſe Line; *F* and *G* the Points denoted with the ſame Letters in *Fig.* 10. * Fig. 60

* 31.

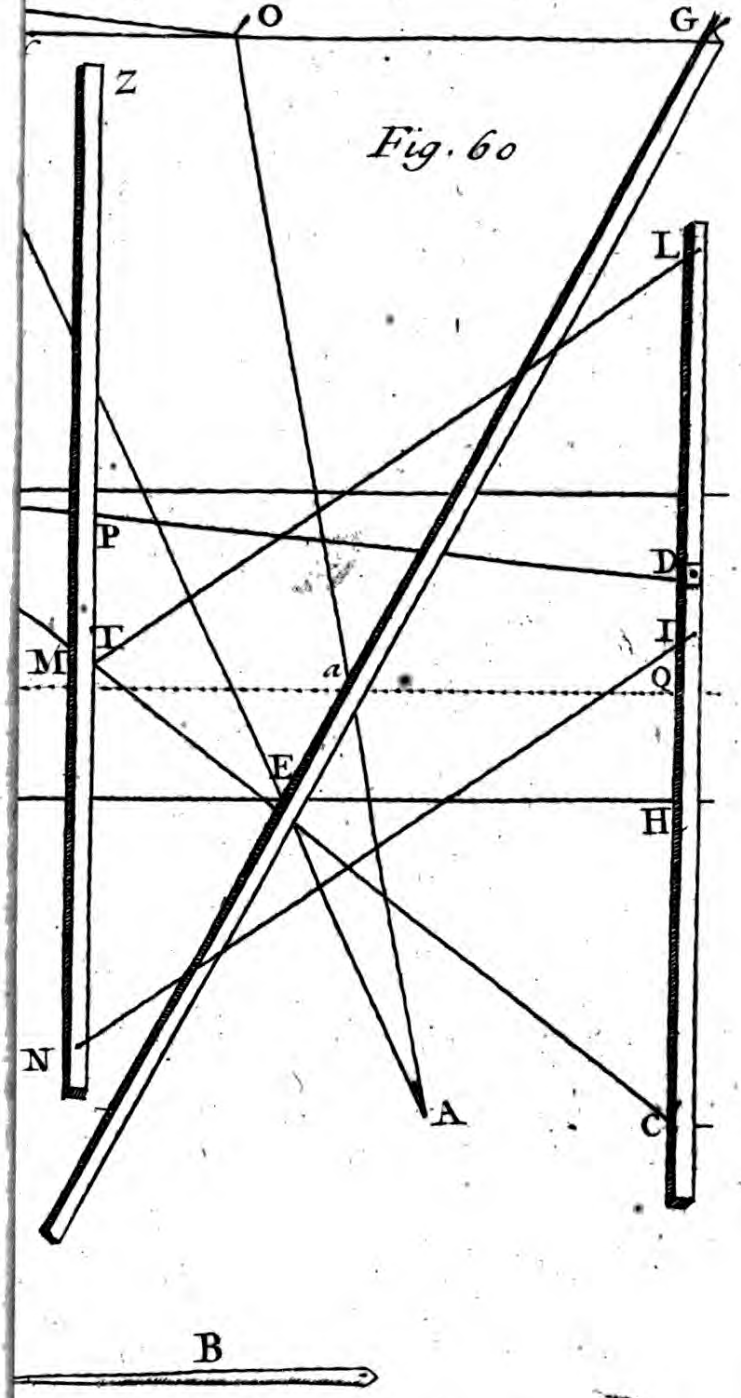
Then take a Ruler and faſten to the Point *G*, ſo that it may turn about the ſame in ſuch manner, that right Lines drawn along one of its Sides, may paſs through the Point *G*. This being done, faſten one End of a Thread, put through the Eye of a Needle *B*, in the Point *F*; and then put the ſaid Thread about a Pin faſten'd in the Point *O*; ſo that when the Thread is uſing, it may be always kept tight by means of a Plummet fix'd to its other End, and freely hanging under the Tube. *Note*, the aforeſaid Needle ought to be Braſs or Silver, ſharp at both Ends, and having its Eye pretty near one of them.

OPERATION.

Let *A* be a Point of the Figure to be thrown into Perſpective. Now place that of the two Points of the Needle, which is nigheſt to the Eye thereof, upon the ſaid Point *A*, and move the Ruler *G E*, until it cuts the Thread *A F*, in the Point *E*, wherein the ſaid Thread cuts the Baſe Line; and then the Point *a*, wherein the Ruler cuts the Thread *A O*, is the Point ſought, and it may be marked with that Point of the Needle, fartheſt diſtant from its Eye, having firſt preſſed the Ruler down upon the Paper and Thread, that ſo the Plummet may not make the Thread ſlip. And in this manner may any Number of Points be found. This is demonſtrated in *N.* 32. Sometimes it is more convenient to uſe the following Method.

METHOD II.

Fig. 61. 110. Let *O* be the Eye, *H E* the Baſe Line, and *F I* the Geometrical Line. Take a Ruler *M N*, having two Threads equal in Length faſten'd to it, and about *O* as a Center, and with the


O
G
Z
Fig. 60
L
P
D
T
I
M
a
Q
E
H
N
A
C
B

the Distance of the two Points whereat the Thread is fasten'd on the Ruler, describe an Arc cutting the Geometrical Line in *F*; then fasten the Extremity of one of the aforesaid Threads in the said Point *F*, and the Extremity of the other in the Point *O*: Take moreover another Thread, put through the Eye of a Needle, as in the aforegoing Method; one End of which, fasten in *F*, and afterwards put it about a Pin placed in *O*. Then the only Difference between this Way and the precedent one, is, that the Ruler *M N* must be used, by always keeping the Threads *M F*, and *N O* tight, instead of one turning about a Point.

For the Demonstration of this, *vide n.* 39.

PROB. II.

111. *To find the Perspective of one or more Lines perpendicular to the Geometrical Plane.*

Take two Rulers *L C* and *N Z*, having the four Ends of two Threads, or rather four Brass or Steel Wires of equal Length fixed on them at the Places *L*, *I*, *N* and *M*, so that *L I* be equal to *M N*. Then fix one of these Rulers upon the Edge of the pespective Plane, perpendicular to the Base Line. Now take a Thread, put thro' the Eye of a Needle, hang a Plummet at one End, for the same Use as in *Problem* I*, and fasten the other End to the Slider or Cursor *D*, freely moveable on the Ruler *L C*; put this Thread about a Pin, set up against the Ruler *C L* in *c*; so that *CH* be equal to the Height of the Eye. *Fig.* 60.

* 109.

OPERATION.

Let *T* be the Representation of the Foot of a Perpendicular. Move the Cursor *D* along *C L*, until

until *CD* be equal to twice the Length of the given Perpendicular. This being done, move the Needle along the Horizontal Line, suppose to *R*, until the Point of the Thread *RS* passes through the Point *T*; then keeping the Thread tight in this Manner, if the Ruler *MN* be mov'd, till its Edge also passes through the Point *T*, and *P* is the Point wherein the Edge of the said Ruler crosses the Part of the Thread *RD*, the Line *TP* will be the Representation sought.

The Demonstration of this is evident from what is said in *n.* 59.

METHOD II.

112. *When all the Perpendiculars have the same Length.*

Fig. 60. Let *FG* be parallel to the Base Line, and *FO* equal to the Height of the Eye; assume *Ff*, equal to the Length of either of the given Perpendiculars, and fasten the Thread fixed in *F*, in the Point *f*. Then raise *RS* perpendicular to the Base Line, which make equal to *Ff*, and draw *SQ* parallel to the Base Line. This being
* 60. done, transpose * the Figures in the Geometrical Plane, in such Manner, that the Point *R* coincides with *S*, and *RH* with *SQ*. Then if *SQ* be taken for a Base Line, and the Appearances
* 109. * of the Feet of the Perpendiculars be found, the Representations of their Extremities will be had.

METHOD III.

Fig. 61. 113. *For Perpendiculars of the same Length.*
* 112. The Figures in the Geometrical Plane being transposed in the Manner aforesaid *, assume *Tt*, in

in the Perpendicular *RS*, continued equal to *RS*, and draw *fi* parallel to the Base Line, in which take the Point *f* in *EI*, the same as *F* in *FI* *; then if the Threads which before were fasten'd in *F*, being placed in *f*, and by using them thus fasten'd, as likewise *SQ* for a Base Line, the Representations of the Feet of the Perpendiculars be found *, you will have the Representations of their Extremities.

* 110.

* 110.

The Demonstration of the two last Ways.

114. If a Plane be imagined to pass through the Extremities of the equal Perpendiculars, it will be parallel to the Geometrical Plane, and will meet the perspective Plane in *SQ*; because *RS* is equal to the said Perpendiculars: Moreover, the Extremities of these Perpendiculars form a Figure in this supposed Plane, similar to that which their Feet form in the Geometrical Plane; and the said Figure hath the same Situation with regard to the Line *QS*, as that on the Geometrical Plane hath in respect of *HR*: And consequently, if the Figure in the Geometrical Plane be so raised up, that it hath the same Respect to *QS*, as it had to *HR*, and if the Appearances of the Feet of the proposed Perpendiculars be found, the Representations of their Extremties will be had. But the before supposed Transposition of the Figure in the Geometrical Plane, gives it the requisite Situation with regard to *QS*, and the Representation of the Figure consider'd in this new Geometrical Plane is found *; because *SQ*, is taken for the Base Line, *Of* (*Fig.* 60.) equal to the Height of the Eye above this Plane, and *fi* (*Fig.* 61.) is the Geometrical Line in the said Plane.

Fig. 60, *and* 61.

* 32, 39.

II. *When*

II. *When the Perſpective Plane is inclined.*

PROB. III.

115. *To find the Repreſentation of Figures which are in the Geometrical Plane.*

* 109, 110. The Operations laid down * for the perſpective Plane being perpendicular, may be uſed here, because the inclined perſpective Plane may be
* 81: changed * into a perpendicular or upright one.

PROB. IV.

116. *To find the Appearances of any Number of Lines of the ſame Length, which are perpendicular to the Geometrical Plane.*

Fig. 62. Raiſe a Perpendicular *R C*, in ſome Point on the Baſe Line, in which aſſume *R L* equal in Length to the given Lines; and draw the Line *L P*, through the Point *L*, ſo that the Angle *L P R*, be equal to the Angle of Inclination of the perſpective Plane; then having made *R S* equal to *P L*, and *S C* equal to *P R*, draw the Lines *S Q* and *C D*, parallel to the Baſe Line. This being done, raiſe up the Figures of the Geometrical Plane, ſo that the Point *R* coincides with *c*, and the Line *R H*, with *C D*; then
*112, 113. proceed as is directed * for the perpendicular perſpective Plane, in uſing *S Q* for the Baſe Line.

This is demonſtrated in *n.* 114.

REMARKS.

The Point C muſt be aſſumed below the Point *S*, when the perſpective Plane is inclined towards the

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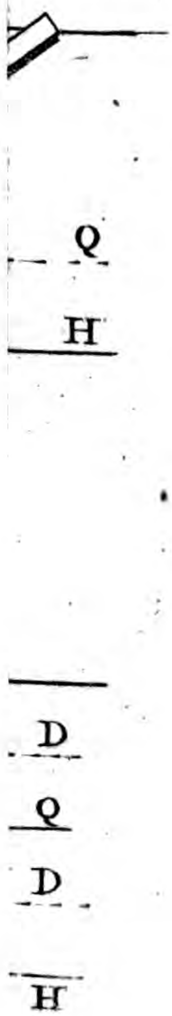


the Eye, and above it, when the said Plane is inclined towards the Objects. Observe likewise, that *Ff* of *Fig.* 60, must here be assumed equal to *RS*, and the Line *Tt*, *Fig.* 61, must be here a Part of the Line *ZC* continued, and made equal to *RS*.

III. *When the Perspective Plane is Parallel or Horizontal.*

PROB. V.

117. *To throw Figures which are in the Geometrical Plane into Perspective.*

Draw the Line *CF* at Pleasure, in which assume the Point *I*, and make *IH* and *IG*, equal to the Eye's Distance from the perspective Plane: Moreover, make *IC*, and *IF* equal to the Eye's Distance from the Geometrical Plane, or at least, let *IG* and *IH*, be to *IF* and *IC*, as the Eye's Distance from the perspective Plane, is to its Distance from the Geometrical Plane. This being done, raise two Perpendiculars to the Line *CF*, in the Points *G* and *H*, and take two Rulers, each of which has two equal Threads so fasten'd to them, that the Distances *PQ*, and *NM* be equal: Then about *F* and *C* as Centers, with the Semidiameters *MN* or *PQ*, describe two Arcs cutting the Perpendiculars raised at the Points *G* and *H*, in the Points *C* and *D*, and fix the Extremities of the two Threads of one Ruler, in the Points *C* and *D*, and the Threads of the other, in the Points *F* and *E*. *Fig.* 63.

OPERATION.

Let *Z* be the Geometrical Plane, and *A* a Point of the given Figures. Move the two Rulers,

Rulers, keeping all the Threads tight, so that the two Threads fix'd in the Points C and F, cross each other in the Point *A*; and then the Point *a*, wherein the two other Threads cross each other, is the Perspective sought. And in this Manner may the Representations of any Number of Points be found.

Demonstration.

118. The Triangle *D a E*, is similar to the Triangle *C A F*: And because all the Triangles formed for finding the Representations of different Points, have the same Bases *D E* and *C F*, which are between themselves, as the Eye's Distance from the perspective Plane, to its Distance from the Geometrical Plane; whence their Vertices form similar Figures, whose correspondent Lines are in the same Proportion; and which
* 8, 9. consequently *, are the Appearances sought.

Remarks.

It will be convenient to have the two Threads *PE*, and *M D* of one Colour, and the two Threads *Q F*, and *C N* of another.

Prob. VI.

119. *To find the Representations of any Number of Lines, equal and perpendicular to the Geometrical Plane.*

Let *C D E F G I H*, be the Points denoted
Fig. 64. with the same Letters in the precedent Figure, as also let *P Q* and *M N* be the Rulers: Moreover, let *B* be the Point, wherein a Perpendicular

lar drawn from the Eye to the Geometrical Plane, meets the ſaid Plane; and *T* the Perſpective of that Point, found by the aforegoing *Problem.* Now make *F L* and *C R*, equal to the Length of the given Lines; and about the Points *R* and *S*, as Centers, with the Radius *M N* or *P Q*, deſcribe two Arcs cutting the Perpendiculars *H D* and *G H*, in the Points *X* and *S*: Then fix the Extremities of the Threads, which before were faſten'd to the Points *F* and *E*, to the Points *L* and *S*; and alſo the Extremities of thoſe two Threads, which were before faſten'd to the Points *C* and *D*, in the Points *R* and *X*: Then moving the two Rulers, until the Threads *S P* and *X M*, croſs each other in the Point *T*, mark the Point *O*, wherein the two other Threads croſs one another, through which, and the Point *B*, draw the indefinite Line *B O V*: this being done, tranſpoſe the Lines of the Geometrical Plane, ſo that the Point *B*, coincides with the Point *O*, and the Line *B O*, with *O V*. And by the precedent Problem, find the Appearances of the Feet of the Perpendiculars, and you will have the Repreſentations of their Extremities.

DEMONSTRATION.

If a Plane be ſuppoſed to paſs through the Extremities of the Perpendiculars, it will be parallel to the Geometrical Plane, and conſequently, likewiſe to the perſpective Plane, becauſe all the Perpendiculars are ſuppoſed equal. But the Figure formed by the Extremities of the Perpendiculars in the ſaid Plane, is ſimilar and equal to that form'd by their Feet in the Geometrical Plane: and therefore, the Repreſenta-

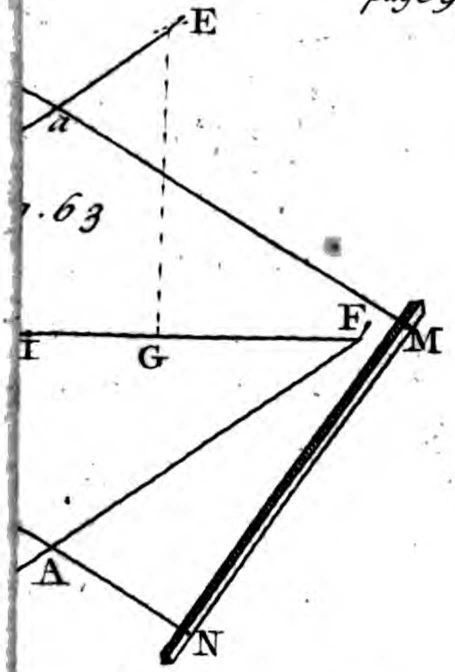
sentation of the Figure which is in the second Plane, is likewise similar to the Figure which is in the Geometrical Plane, and the Lines composing this Appearance to the correspondent Lines in the second Plane, as the Eye's Distance from the perspective Plane, is to its Distance from the before supposed Plane. But by means of the Threads fastned in the Manner aforesaid, we find the Representation of a Figure, whose
* 118. Lines are * in the aforenamed Proportion: Whence this Figure is the Perspective sought, and is situated as it ought to be, with regard to the Representations of the Pigures in the Geometrical Plane; because these Figures are so slid, that the Appearance of the Perpendicular in the Point *B*, is only a Point. The said Appearances are also in their proper Positions, because the Line *B Q*, is made to coincide with *O N*.

120. *For Solar Shadows in all Situations of the Perspective Plane.*

Prob. VII.

6. *To find the Representations of the Shadows of any Number of Points, being at the same Height above the Geometrical Plane.*

* 103. Find * a Point in the Geometrical Plane, which is the Shadow of one of the given Points: Transpose the Figures in the Geometrical Plane, so that the Seat of the said given Point coincides with its Shadow; and the Line drawn through the said Seat, and the Shadow of the given Point, coincides wtth its Prolongation. Then if according to the Situation of the perspective Plane, the Representations of the Seats of the given Points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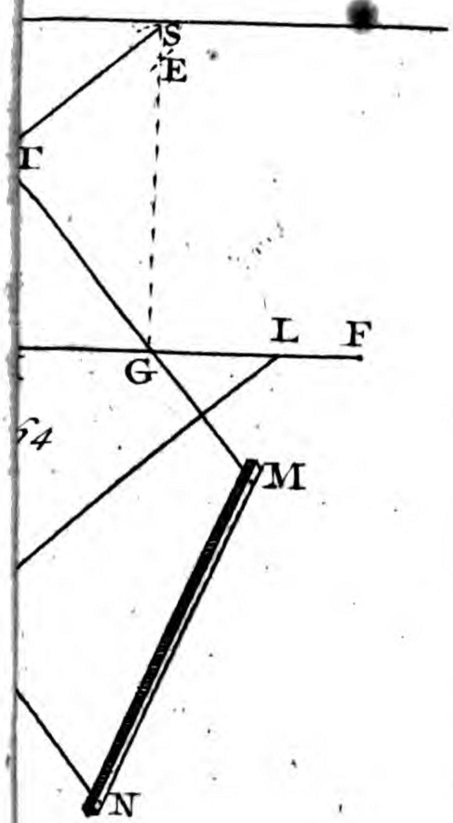
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Points be found *, the Representations of their Shadows will be had. * 109, 115, 117.

CHAP. IX.

The Use of Perspective in Dialling; shewing how to draw the Hour Lines upon any Kind of Plane, by means of an Horizontal Dial.

PERSPECTIVE is not only useful in drawing, but likewise in other Parts of Mathematicks, and principally in Dialling: For if the Extremity of the Style be conceived as the Eye, and the Sun's Rays as visual Rays, all possible Kinds of Dials may be drawn for the same Latitude, by Means of an Horizontal Dial, as we are now going to shew,

121. Let *A B C D* be an Horizontal Dial made for any given Latitude; *E F* its Style, and *H I M L* a Plane, upon which a Dial is to be drawn. Now if this Plane be so situated, that the Extremity of its Style *F G*, coincides with the Extremity of the Style of the Horizontal Dial; and if the Perspective of one of the Hour Lines of the Horizontal Dial *A B C D*, be found upon the Plane *H I M L*, in conceiving the Point *F* as the Eye, it is evident *, that the Shadow of the Point *F*, will fall upon the said Perspective, at the same Time that it falls upon the Hour Line, whereof it is the Perspective; and consequently, the said Shadow will shew the same Hour upon the Plane *H I M L*, that it shews upon the Horizontal Dial: There- *Fig.* 65. * 2.

fore the before-mention'd Perſpective will be an Hour-Line of a Dial, drawn upon the Plane *HLMI*, and whoſe Style is *GF*.

The ſame may be demonſtrated of the Repreſentations of other Hour-Lines, which form a Dial upon the Plane *HLMI*.

We now proceed to lay down the beſt way of determining the ſaid Repreſentations.

PROB. I.

122. *To draw Vertical Dials.*

Fig. 66. Draw the Line *EO*, thro' *E*, the Foot of the Style of the Horizontal Dial *ABFD*, equal to the Length of the Style of the Dial to be drawn, and making an Angle with the Meridian *C. XII*, equal to the Plane's Declination.

This Angle muſt be aſſum'd towards the Point *D*, when the Plane's Declination is *South-Eaſt*, as here; towards *F*, when the Declination is *Weſtward*; towards *A*, when it is *North-Eaſtwardly*; and towards *B*, when it is *North Weſtwardly*.

Now, thro' the Extremity *O* of the ſaid Line, draw the Line *IH*, perpendicular thereto; and the Line *CP*, thro' the Center of the Dial, parallel, and equal to *EO*; thro' whoſe Extremity, *P*, draw the Line *PS*, parallel to *HI*.

Fig. 67. Then, to make the Dial, draw the Line *hi*, in which prick down the Diviſions of the Line *HI*; and in the Point *o*, (which is the ſame as *O*) raiſe the Perpendicular *op*, equal to the Length of the Style of the Horizontal Dial *ABDF*. This being done; draw a Parallel, *hi*, thro' the Extremity of this Perpendicular; on which, prick down the Diviſions of the Line *PS*, Plate. 28

Fig. 65

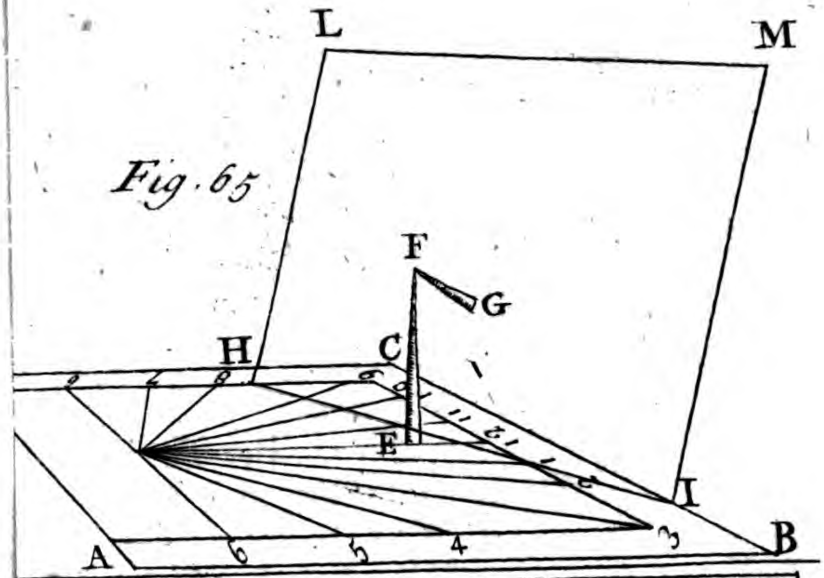
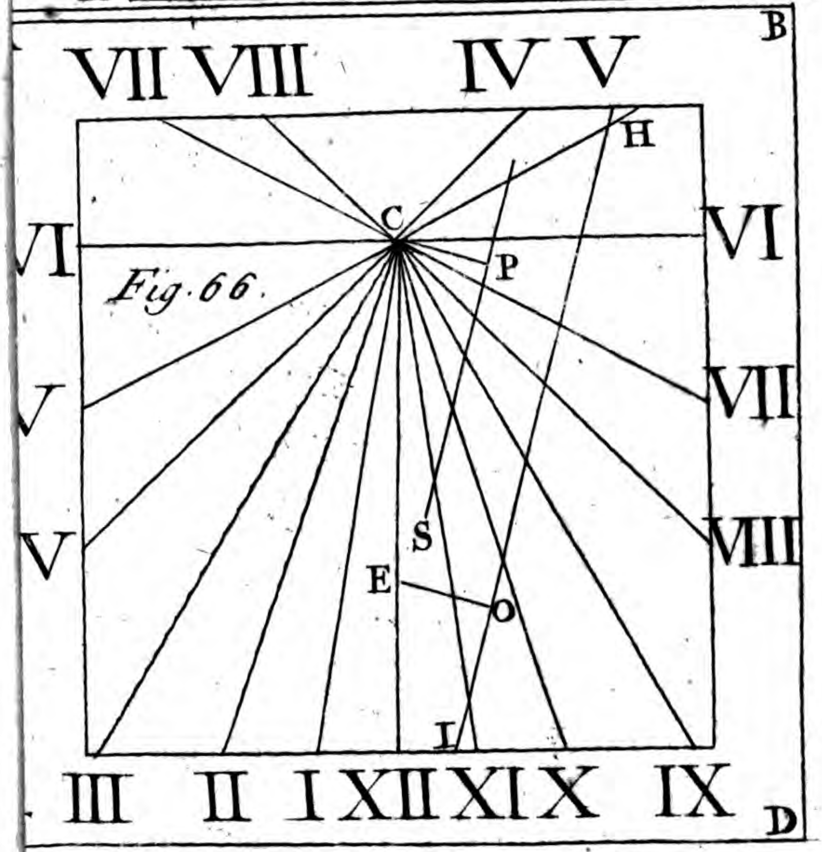


Fig. 66



PS, and making the Point *p* be the ſame as *P*. Then join each Diviſion of this Line, to the correſpondent Diviſion of the Line *bi*; and the Dial ſought, will be drawn: *p* being the Foot of the Style, and *ps* the Horizontal Line.

DEMONSTRATION.

The Baſe Line is *bi*; *ps* is the Horizontal Line; *p* the Point of Sight; and *EO*, or *CP*, of *Figure* 66. is the Length of the principal Ray. *Fig.* 66, 67.

Now, ſuppoſe the Plane *psbi*, to be ſet perpendicularly upon the Horizontal Dial, in ſuch manner, that the Line *bi* coincides with *HI*, and the Point *o* with *O*. Suppoſe, moreover, that thro' the Extremity of the Style, which we conſider as the Eye, Lines are drawn in the Horizontal Plane, parallel to the Hour-Lines of the Dial; it is evident, that theſe Lines will meet the Horizontal Line *ps*, in the Point already prick'd down; and conſequently *, the Appearances of the Hour-Lines, are the Lines joining the Diviſions of the Lines *bi* and *ps*. * 13.

REMARK.

If the Line *HI* happens to meet the Meridian; the common Method, by the Horizontal Dial, is eaſier than this.

PROB. II.

123. *To draw inclining Dials.*

Theſe Dials are drawn in the ſame manner as Vertical ones; the following Preparations being firſt made.

Fig. 68. Draw the Line *e c*, equal to the Length of the Style of the Horizontal Dial; and, at its two Extremities raiſe the Perpendiculars *e o* and *c p*: Then draw the Line *c G*, thro' the Point *c*, equal in Length to the Style of the Dial to be drawn; making an Angle with *c e*, equal to the Inclination of the Dial-Plane. After which, draw the Line *o G p*, thro' the Extremity *G* of the ſaid Line, perpendicular thereto. This Preparation being finiſh'd, we uſe the Operations of the precedent Problem, in making *EO* and *CP*, in the Horizontal Dial, equal to *eo* and *cp* of this Figure; and *op*, in the Dial to be drawn, equal to *op* of this Figure; in which, the Point *G* is the Foot of the Style.

Fig. 69. If it happens, in the aforeſaid Preparation, that the Line *p o* cuts the Line *e c*; *EO* muſt not then be aſſum'd in the ſame Line, in the Horizontal Dial, as otherwiſe it muſt have been; but, in that Line, continued on the other ſide of the Foot of the Style.

The Demonſtration of this Problem, is the ſame as that of the precedent one; in conſidering the Angle *p o Q*, equal to the Angle *G c e*, which is equal to the Inclination of the Plane.

We might have further ſhewn the Uſe of Perſpective, in facilitating the Operations of Dialling: But this would be deviating from our Subject; and ſo we ſhall content our ſelves with this Short Eſſay, touching the moſt common and uſeful Problem in Dialling.

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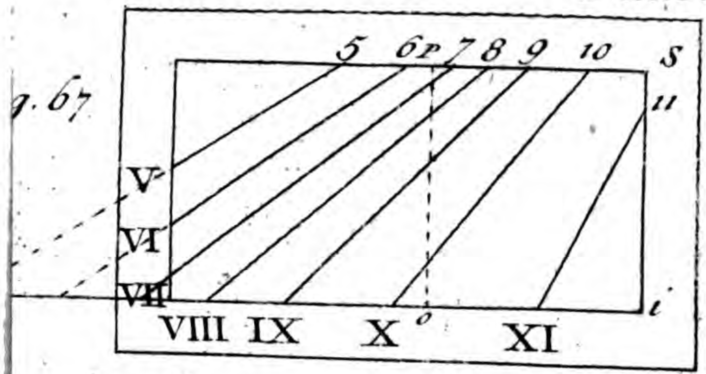
Plate. 29



g. 67

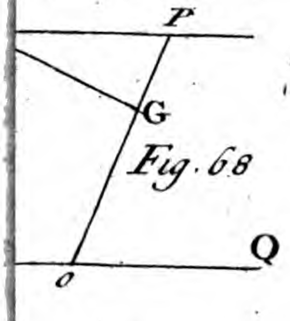


Fig. 68

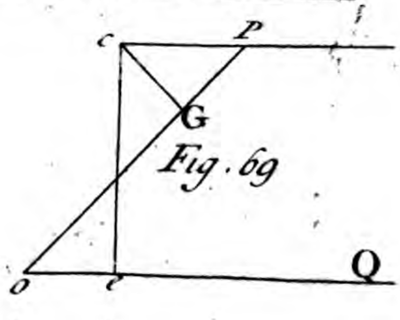


Fig. 69

The Use of the CAMERA OBSCURA *in Designing.*

ADVERTISEMENT.

EVery one knows how easy it is by one Convex Glass only, to represent outward Objects in any darken'd Place, according to their natural Appearances, where the Liveliness of Colours, and the Diversity of Motions, are wonderfully pleasant to behold; and it is so easy to make this Invention useful in Designing, that our treating of it so fully as we have done, is undoubtedly something necessary.

A few Hints and Observations seem to be sufficient for putting a diligent Person in the way of contriving himself some Machine to perform the Uses hereafter mentioned; whence we might have let him had the Pleasure of the Invention himself, after it was made easy to him. And this indeed is what I first resolv'd upon; but afterwards considering that in the Construction of a Machine for facilitating the Business of Designing, several Things cannot be foreknown till try'd; and that much Time may be spent in vain, and several Methods attempted, before one of these Machines can be made simple and useful, as I have found by Experience: Therefore, that others may not be at this Trouble, I shall here lay down the Description of two Machines (hoping it will not be unacceptable) which after several Alterations, in my Opinion, are now made convenient enough.

The first of these Machines is undoubtedly much preferable to the other; because it is firmer, and renders the Work easier and exacter; and Prints may be easier represented in it, than in the other. Add to all this, that with a small Alteration it may be made capable of a few Uses, which are peculiar to the second Machine; yet since this latter one is much simpler, of a much less Price, and is easier to be carry'd from Place to Place, I thought it convenient to lay down also its Description in this small Tract.

I shall not here take up the Reader's Time in enumerating the Advantages accruing to Painters from Machines of this Nature: But only add, that they are of great Use for reducing or lessening several separate Objects in the same Picture; which therefore may be copy'd after Nature in the most perfect manner possible. It is very difficult to give several Objects their true Bigness in a Picture, and to dispose them so as to have the same Point of Sight; but this is done extremely easy by means of Machines; for the Point of Sight in them will be the same always, as long as the Convex Glass has the same Disposition, and the Bigness of the Representations of Objects in the Machines, do depend upon the Objects Distances from them.

This Invention, by Industry, may be certainly improv'd, and the following Observations will be not a little conducing thereto. 1. *You must use but one Convex Glass; for when there are two or more, the true Representations of Objects will be lost; which is an Inconveniency one is also subject to, when a Concave Glass is any ways us'd in the Construction of the Machine.* 2. *When more than two Mirrours or Looking-glasses are us'd, the Rays, after having suffer'd a triple Reflection, are so weaken'd, that the Objects will not be well represented: And even when but two Mirrours are us'd, they must be well polish'd* 3. *The*

Mirrours

Mirrours must not be plac'd within the Machine; for in such a close Place, ones Breath will sully them; but not the Convex Glass, because it is enclosed in a Tube.

The Use of the CAMERA OBSCURA *in Designing.*

DEFINITION.

1. A Camera Obscura *is any dark Place, in which outward Objects exposed to Broad-Day-light, are represented upon Paper, or any other white Body.*

The Way to represent Objects in the *Camera Obscura,* is to make a small Hole in that side thereof next to the Objects, and place a Convex-Glass therein, then if a Sheet of Paper be extended in the Focus of the said Glass, the Objects will appear inverted upon the Paper.

THEOREM I.

2. *The* Camera Obscura *gives the true Representation of Objects.*

The Figures represented in the *Camera Obscura* are form'd (as is demonstrated in Dioptricks) by Rays which coming from all the Points of the Objects, pass through the Centre of the Glass: So that an Eye placed in the said Centre, would perceive the Objects by the said Rays, which consequently by their Intersection with a Plane, must give the true Representation of the Objects. But the Pyramid which the said Rays forms without the *Camera Obſcura*, is ſimilar to that which they form, after having paſſed through the Glaſs: Therefore the Rays which fall upon the Paper in the *Camera Obſcura*, likewiſe give the true Repreſentation of the Objects thereon. *Which was to be demonſtrated.*

Theſe Objects appear inverted, becauſe the Rays croſs each other in paſſing through the Glaſs; thoſe coming from above going below, &c.

THEOREM II.

3. *The Reflection which the Rays of Light ſuffer upon a plain Mirrour or Speculum, before they fall upon a Convex Glaſs, no-wiſe deforms the Repreſentation of Objects.*

This is evident: For the Speculum reflects the Rays in the ſame Order as it receives them.

Now to ſhew the Uſe that may be drawn from the *Camera Obſcura* in Deſigning, I ſhall here lay down the Deſcription and Uſe of Two Machines, which I uſe for this End.

The Deſcription of the Firſt Machine.

4. This Machine is ſomething in Figure of a Chair, (ſuch as People are carried in) the back Part of the Top is rounded, and its Foreſide *P Q* ſwells
Fig. 70. out in the middle: *Vide Fig.* 70. which repreſents the Machine, the Side whereof oppoſite to the Door, is ſuppoſed to be raiſed up, that ſo its Inſide may be ſeen.

5. The Board *A* within-ſide, ſerves as a Table, and turns upon two Iron-pins, going into the Wood of the Fore-ſide of the Machine, and is ſuſtain'd by two ſmall Chains, that ſo the ſaid Table may be lifted up; and therefore one may

more

more conveniently go in at the Door in the Side of the Machine.

6. There are two Tin-Tubes bent at each End, (one of which is repreſented in *Fig.* 76. becauſe they could not be ſhewn in the Figure of the Machine) placed in the Furniture, near the Back-ſide of the Machine, each having one End without the Machine. Theſe Tubes ſerve to give Air to Perſons ſhut up in the Machine, yet ſo, that no Light may enter through them.

7. At the Places *c*, *c*, *c*, *c*, on the Out-ſide of the Back-part of the Machine, are four Iron-Staples, in which ſlide two Wooden Rulers *DE*, *DE*, about 3 Inches broad, having two other thin Rulers going through holes made at their Tops, at the Places *D*, *D*, to which thin Rulers the Board *F* is fixed; and ſo by this Means the ſaid Board may be vertically moved backwards or forwards.

8. On the Top of the Machine, there is a Board about 15 Inches long, and 9 Inches broad, having an hole *PMOQ* quite through it, about 9 or 10 Inches long, and four Inches broad.

9. Upon the aforeſaid Board are fix'd two Dovetail'd Rulers, between which another Board ſlides of the ſame Length as that, and whoſe breadth is about 6 Inches. In the middle of this ſecond Board is a round Hole about three Inches Diameter, hollowed into a female Screw, in which is fitted a Cylinder about four Inches long, which carries the Convex Glaſs; Of which more hereafter.

10 The Figure *X* is a ſquare Box about 7 or 8 Inches broad, and 10 in height, which ſlides upon the plain Board mentioned in *Numb.* 8. the Side *B* ſerving as a Door, is next to the fore-ſide of the Machine, as it appears in the Figure; and the

the Back-ſide thereof hath in it a Square opening *N*, each Side being about four Inches in Length, which may be ſhut by the little Board *I* ſliding between two Rulers.

11. Over the ſaid Square opening, there is a Slit parallel to the Horizon, going along the whole Breadth of the Back-ſide of the Box; through which Slit, a little Mirrour or Looking-Glaſs is to be put into the Box, whoſe two Sides may ſlide between two Rulers ſo placed, that the poliſhed Side of the Glaſs being turn'd towards the Door *B*, may make an Angle of $112\frac{1}{2}$ Degrees, with the Horizon. Note, This Diſpoſition of the Looking-glaſs could not well be repreſented in the Figure.

12. The before-mention'd Mirrour hath a ſmall Iron Plate, on the Middle of that Side which is without the Box, (when the ſaid Looking-glaſs is in the Diſpoſition mention'd in *N.* 11.) being the Baſe of a Screw faſten'd to the Middle thereof, that ſo the Looking-glaſs may be fix'd upright (as appears *per* Figure) in any Place *H* upon the Top of the Machine, and vertically turn all ways; and this is done by putting the Screw through a Hole made in the plain Board of *N.* 9. and through a Slit made for this End in the plain Board of *N.* 8. and then fixing it with the Nut *R.* Now when the Mirrour is taken from this Situation, the ſaid Slit is ſhut by a ſmall Board ſliding between two little Rulers within the Machine. As to the Slit mention'd *N.* 11. it is partly ſhut by the little Board *I*, when the Aperture *N* is open'd, and the two Ends thereof remaining open'd, are ſhut by two ſmall Rulers.

13. There are two Iron Staples on one Side of
* 7. the Box, like thoſe which are * on the Back-ſide of the Machine; in which a Ruler going ſeveral Inches out behind the Box *X* may ſlide, having a Hole

a Hole at its End, through which the above-mention'd Screw may paſs, and ſo the Mirrour *H* be fixed to any Inclination before the Aperture *N*.

14. Beſides the Mirrour *H*, there is another leſſer one, *L*, faſten'd near its Middle to a Ruler going out through the Middle of the Top of the Box. This Ruler may Screw on, and ſerves to raiſe or lower the Mirrour faſten'd to it, ſo that it may be fixed to all Angles of Inclination.

REMARKS.

If the Tubes mention'd in *N. 6.* be not thought ſufficient for giving Air to the Machine, a ſmall Pair of Bellows may be put under the Seat, which may be blown by one's Foot. And by this means the Air within the Machine may be continually remov'd; for the Bellows driving the Air out of the Machine, obliges the external Air to enter through the Tubes therein.

Uſe of the Machine.

PROBLEM I.

15. *To repreſent Objects in their natural Diſpoſition.*

When Objects are to be repreſented within the Machine, we extend a Sheet of Paper upon the Table *A*; or, which is better, we lay a Sheet of Paper upon another Board, ſo that it ſpreads beyond the Edges of the Board; then we ſqueeze the ſaid Paper and Board into a Frame, ſo that it be fixed therein by means of two Dove-tail'd Rulers. This being done, we place a Convex Glaſs in the Cylinder *C*, * which ſcrews into the Top of the Machine, having its Focal Length nearly

Fig. 70.

* 9.

nearly equal to the Height of the Top of the Machine above the Table; Then we open the Aperture *N* at the Back-part of the Box upon the Machine; and incline the Mirrour *L*, ſo as to make an Angle of 45 deg. with the Horizon, when Objects are to be repreſented for the perpendicular Picture. Then, if the Mirrour *H* be taken away, as alſo the Board *F*, together with the two Rulers *D E* and *D E*, we ſhall perceive the Repreſentation upon the Sheet of Paper on the Table *A*, of all Objects, whoſe Rays falling upon the Looking-glaſs *L*, can be thereby reflected upon the Convex Glaſs; which Convex Glaſs muſt be rais'd or lower'd, by means of the Screw about the Cylinder carrying it, until the ſaid Objects appear entirely diſtinct.

16. When the ſame Objects are requir'd to be repreſented for the inclin'd Picture, the Looking-glaſs *L* muſt have half the Inclination we would give to the Picture.

17. When the ſaid Objects are to be repreſented upon the Picture being parallel, the Aperture *N* muſt be ſhut, and the Door *B* open'd; then the Mirrour *H* muſt be rais'd to the Top of the Box, in putting it in a Situation parallel to the Horizon. This Diſpoſition of the Machine may ſerve when one is upon a Balcony, or ſome other high Place; to deſign a *Parterre* underneath.

18. If we have a mind to deſign a Statue ſtanding in a Place ſomething elevated, and it is requir'd to be ſo repreſented, as to be painted againſt a Cieling; the Back-ſide of the Machine muſt be turned towards the Statue; and the Box *X* ſo turned, that the Door *B* may face the Statue; then, the Door being open'd, the Looking-glaſs *L* muſt be placed vertically, with its poliſh'd Side towards the Statue; and the Box mo-

ved

ved backwards or forwards, or elſe the Looking-glaſs raiſed or lower'd, until the Rays proceeding from the Statue may be reflected by the Mirrour upon the Convex Glaſs. When theſe Alterations of the Box, or Mirrour, are not ſufficient to throw the Rays upon the Convex Glaſs, the whole Machine muſt be removed backwards or forwards.

DEMONSTRATION.

Concerning the before-mention'd Inclination of the Mirrours.

19. In order to demonſtrate, that the Mirrour *L* hath been conveniently inclin'd, we need only prove, that the reflected Rays fall upon the Table *A* under the ſame Angle, as the direct Rays do upon a Plane, having the ſame Situation as one would give to the Picture.

Now let *A B* be a Ray falling from a Point of ſome Object upon the Mirrour *G H*, and from thence is reflected in the Point *a* upon the Table of the Machine: We are to demonſtrate, that if the Line *D I* be drawn, making an Angle with *FE* equal to the Inclination of the Picture; that is, * if the Angle *DIE* be the double of the Angle *D F I*; I ſay, we are to demonſtrate, that the Angle *B a f* is equal to the Angle *BCD*. *Fig.* 71. * 15, 16.

The Angle *DIE*, by Conſtruction, is the double of the Angle *DFI*; and conſequently this laſt Angle is equal to the Angle *I D F*; and ſince the Angle of Incidence *CBD* is equal to the Angle of Reflection *a B F*, the Triangle *BCD* is ſimilar to the Triangle *F a B*: Whence it follows, that the Angle *B a F* is equal to the Angle *B C D*. *Which was to be demonſtrated.*

20. Con-

20. Concerning what hath been ſaid of the Picture being parallel, it muſt be obſerv'd, that in the precedent Demonſtration, the Angle of Inclination of the Picture in this Demonſtration is meaſur'd next to the Objects; and if this Angle be diminiſh'd till it become nothing, we ſhall have a Picture parallel to the Horizon underneath the Eye. But, by the Demonſtration, the Inclination of the Mirrour being half the Inclination of the Picture, it follows, that the Inclination of the Mirrour is alſo equal to nothing, and conſequently it ought to be likewiſe parallel to the Horizon. In the ſame manner we demonſtrate, that the Looking-glaſs muſt be vertically ſituated, when we conſider the Picture parallel above the Eye: For to give this Situation to the Picture, the Angle of the Inclination of the Picture meaſur'd next to the Objects, muſt be augmented till it be 180 Degrees, whoſe half 90 Degrees is conſequently the Inclination of the Mirrour.

PROB. II.

22. *To repreſent Objects, ſo that what appears on the right Hand, ought to be on the left.*

Fig. 70. 23. Having placed the Box *X*, in the Situation as *per* Figure, the Door *B* muſt be opened, and the Aperture *N* ſhut; then putting the Mirrour *H* in the Diſpoſition mentioned in *Numb.* 11, raiſe up the Mirrour *L* towards the Top of the Box; and incline it towards the firſt Mirrour, in ſuch manner that it makes an Angle with the Horizon of $22\frac{1}{2}$ Degrees; that is, that the Top of the Machine, after a double Reflection, appears vercal in the Mirrour *H*.

24. Now if Objects are to be repreſented for the Picture inclin'd, the Mirrour *L* muſt make an

Angle with the Horizon, equal to half the Inclination of the Picture less $\frac{1}{4}$ of a right Angle. This Angle is found exactly enough for Practice, by inclining the Mirrour *L*, until the Representation of the Top of the Machine, after a double Reflection, appears in the other Mirrour under an Angle with the Horizon, equal to the Inclination one would give the Picture. Note, If the Inclination of the Picture be lesser than $\frac{1}{4}$ of 90 Degrees, the Looking-Glass *L* must not be inclin'd towards the other, as is directed, * but the contrary Way, in making the Angle of the Inclination of the Looking-Glass, equal to the Difference of the Inclination of the Picture, and $\frac{1}{4}$ of 90 Degrees. * 23.

25. When the Objects are to be represented for a parallel Picture, the Looking-Glass *L* must be placed in the Disposition of *Numb.* 15. and the Looking-Glass *H* in that mentioned, *Numb.* 13. by inclining it towards the Horizon, under an Angle of 45 Degrees; the polished Side thereof facing downwards, when the Picture is supposed underneath the Eye; and upwards, when it is supposed above the Eye.

26. This Disposition of the Machine may be likewise useful for inclin'd Pictures, making very small Angles with the Horizon; in which Case, the Inclination of one of the Looking-Glasses must be diminish'd, by half of the Inclination of the Picture.

27. *A Demonstration of the Inclination of the Mirrours.*

We have mentioned, * that for a perpendicular Picture, one of the Mirrours must make an Angle * of $112\frac{1}{2}$ Degrees with the Horizon; and the other, *L*, must be inclin'd towards the first, * 22. * 11.

and

and make an Angle of 22½ with the Horizon.
Fig. 72. Let *MN* and *GH* be two Mirrours in the before-mentioned Situation; we are to demonſtrate, that if the Ray *AB* is parallel to the Horizon, after being reflected in *B* and *C*, it ought to fall perpendicularly upon the Machine. The Angle
* 11. *ABN* is * 112½ Degrees; and conſequently the Angle *ABM*, and its equal, the Angle of Reflection *CBG*, are each 67½ Degrees. The Angle *BPQ*, is the Complement of the Angle
* 23. *NBA*, *plus* the Angle *PQB*, which is * 22½ Degrees; whence the Angle *BPQ* is 45 Degrees. Again, the Angle *PCB* is the Complement of the two Angles *CBP* and *BPC* to 180 Degrees; and conſequently it is 67½ Degrees, which is the ſame as its equal, the Angle *QCa* of Reflection. And reaſoning after the ſame Manner, the Angle *CRQ* of the Triangle *RCQ*, is a right one. *Which was to be demonſtrated.*

28. It is not abſolutely neceſſary to give the Mirrours the aforeſaid Inclinations; for the Angle *ABN* may be aſſumed at Pleaſure, from which muſt be taken an Angle of 135 Degrees, to have the Inclination of the Miror *GH*. Nevertheleſs, the Angles we have determin'd, are the moſt advantagious for a perpendicular Picture.

Fig. 73. 29. When a Picture is inclin'd, and makes the Angle *DIA* with the Horizon, the Mirrour
* 24. *MN* muſt * keep its Situation, and the Angle *CQR* is equal to half the Angle *DIA*, leſs ¼ of a right Angle: Then I ſay, the Angle *FaC*, or its equal *CRQ*, will be equal to the
* 11. Angle *BID*. Now the Angle *PBQ*, is * 112½ Degrees; whence the Angle *BPQ*, which is the Complement of *PBQ*, and *PQB* to two right
* 27. Angles, is * 90 Degrees, leſs the half of the Angle *DIA*: Wherefore becauſe *NBC* is 67½ the

An-

Angle *BCP*, and its equal *RCQ*, is 22 ½ *plus* ½ of *DIA*. Now if the Angle *RQC* be added to this Angle, their Sum will be equal to the Angle *DIA*; whence it follows, that the Angle *CRQ* is equal to *DIR*. *Which was to be demonſtrated.*

30. If the Angle *RBN* be alter'd, and it be called *a*, the Angle *DIA*, *b*, and the right Angle *d*; then the Angle $CQR = d + \frac{1}{2} b - a$.

31. When a Picture is parallel, it appears manifeſt, the Mirrours *GH* and *MN* being each inclin'd under an Angle of 45 Degrees, that a Ray, which is perpendicular to the Horizon, likewiſe falls, after a double Reflection, perpendicularly upon the Table *A*. *Fig.* 74.

PROP. III.

32. *To repreſent Objects which are roun'd about the Machine, and make them appear erect to the Perſon ſeated within the ſame.*

The Back-ſide of the Machine muſt be turned towards the Sun, and the Objects behind the ſame repreſented * by one Reflection only; then their Appearance will always be clearer, altho' they be in the Shade, than the Appearance of the Objects on the other Sides of the Machine, which cannot be perceiv'd unleſs by a double Reflection. * 15.

33. The Objects that are on the right and left of the Machine, may be repreſented by means of the Mirrour *H*, ſituated * as *per* Figure; but the ſaid Mirrour muſt be cover'd with a Paſtboard Caſe, having two Apertures therein; the one next to the Objects, and the other next to the Aperture *N*, of the Box *X*. The Reaſon of our uſing this Precaution is, becauſe when the Mirrour is not cover'd at all, it reflects the Rays of Light coming *Fig.* 70. * 12.

Side-ways upon the Mirrour *L*, which being again reflected by the said Mirrour *L*, and going through the Convex Glass, extremely weakens the Representation.

34. The Objects before the Machine are represented according to *N.* 22, and 28.

PROBLEM IV.

35. *To represent Pictures or Prints.*

Fig. 70. If we have a mind to represent Pictures and Prints, they must be fasten'd against the Board *F* on that Side, regarding the Back of the Machine, which must be so turned, that the Pictures be exposed to the Sun. Then they are represented in this Situation as * the other Objects, but with this Difference, that the Convex Glass in the Cylinder *C* must be changed: For if Prints are requir'd to have their true Bigness, the focal Distance of the Convex Glass must be equal to half the Height of the Machine above the Table; that is, equal to half *A C.* Again, if the said Pictures or Prints are requir'd to be represented greater than they really are, the focal Distance of the Convex Glass must still be lesser. And if, on the contrary, they are to be represented lesser than they really are, the focal Length of the Glass must be greater than the Length *A C.* Moreover, the proper Distance whereat the Pictures or Prints must be placed, may be found in sliding the Board *F* backwards or forwards, until they distinctly appear within the Machine. This Distance also may be determin'd by the following Proportion:

* 15.

As the Machine's Height above the Table, less the Glass's focal Length;

is to

The

The Height of the Machine above the Table;
So is
The Glasses focal Length,
to the
Distance of the Figure from the Glass.

Note, The said Distance of the Convex Glass from the Figure, is measured by a Ray, proceeding from the Figure parallel to the Horizon, which is perpendicularly reflected upon the Convex Glass, by the Mirrour. *Note*, Moreover, that when we have a Mind to place the Figures out beyond the Back-side of the Machine, they must be fastned against the Side *F* of the Board, which must be so turned, that the said Side be next to the Aperture *N*.

37 *Remarks concerning the Representation of Persons Faces.*

It is certainly very curious and useful to design Persons Faces to the Life; which by this Machine, may be very well done in Miniature: For if the Face of any known Person be thus represented, by only looking at the Appearance, we may very readily know whose Face it is, when at the same time the Appearance of the Whole Person will not take up half an Inch upon the Paper on the Table: But it is very difficult to represent a Face distinctly as big as the Life; for when we would represent a Face in its natural Bigness, such a Convex Glass as is mentioned in *Numb.* 35. must be used, and the Face placed where the Board *F* is *. But the said Face which then appears distinct enough, that so the Person whereof it is the Representation may thereby be known, hath not its Lineaments sufficiently enough represented, as to be followed by a Painter as they ought, in order to keep the true Re- * 35.

 semblance.

ſemblance. The Reaſon of which is, that the Lineaments appear lively and diſtinct within the Machine, when the Re-union of the Rays proceeding from a given Point in the Face, happens exactly upon the Paper in one Point only: But the leaſt Diſtance that one Point is more than another from the Convex Glaſs, (when the Diſtance of the Face from the Glaſs is ſo ſmall, as it muſt be to repreſent it in its natural Bigneſs) ſo alters the Place of the ſaid Re-union, that for different Parts of the Face, thoſe Places of Re-union will differ about two Inches and a half. Whence it is no wonder that all the Lineaments be not repreſented as could be wiſhed; ſince in all Diſtances choſen, there will be always a great many Rays, whoſe Re-union will fall above an Inch beſides the Paper. The Confuſion ariſing from this Diverſity, though not being very diſtinguiſhable by the Eye, yet is prejudicial, and hinders our getting the exact Reſemblance of the Face. We have obſerved this, in order to give an exact Idea of the Goodneſs of this Machine, in equally ſhewing wherein it may be really uſeful, and wherein its apparent Uſefulneſs is ſubject to an Error rather diſcovered by Experience than Reaſon.

38. We muſt not forget in all the precedent Problems, to examine the Aperture the Convex Glaſs ought to have; for although we cannot reduce this Aperture to a fixed Meaſure, yet it is proper to obſerve the following Remarks. 1. The Convex Glaſs may commonly have the ſame Aperture, as we would give a Perſpective Glaſs, having the ſaid Glaſs for its Object Glaſs. 2. When Objects are very much enlightned, the ſaid Aperture muſt be leſſened; and contrariwiſe, when they are expoſed to a weaker Light, it muſt be made greater; and when any Repreſentation is

to be copyed, the Convex Glaſs muſt have the leaſt Aperture poſſible; but yet with this Caution, that the Light coming into the Machine, muſt not be too much extenuated. From theſe Obſervations it is manifeſt, that we ought to be provided with ſeveral round Pieces of Tin or thin Braſs, having round Holes of different bigneſſes therein, in order to give a neceſſary Aperture to the Glaſs; or Holes of different bigneſſes may be made in a long thin Piece of Braſs which may ſlide upon the Convex Glaſs: Or elſe, we may uſe a round Plate, having Holes of different bigneſſes therein, which turning about its Centre, may bring any deſired Hole for the Glaſſes Aperture.

A Deſcription of the Second Machine.

39. This Machine is a kind of Box, the Side *ACGB* being open, whoſe breadth *BD*, and height *AB*, are equal, each being about 18 Inches: Its greateſt width *FB*, is 10 Inches, and the Side *EF* is ſloping, ſo that *AE* is but about 6 Inches. *Fig.* 78.

40. The Frame *G* ſlides at the Bottom of the ſaid Box, in which the Paper is * faſtned. * 15.

41. There is a round Hole in the Middle of the Top of the Box, in which the Cylinder carrying the Convex Glaſs ſcrews.* * 9.

42. The two Sticks *HI* and *LM*, ſlide in four little Iron Staples fixed to the Inſide of the Top of the Box, like thoſe mentioned in *Numb.* 7. Theſe Sticks come about two Feet without the Box, and the Diſtance of their Extremities *I* and *M*, is equal, or ſomething greater than the Length of the Box. Their Uſe is to hang a black Cloth upon, which is faſtned to the Three Sides, *BA*, *AC* and *CD*, of the opening of the Box, that ſo the Box may be darkned, when Objects

are

are to be reprefented upon the Paper in the Frame *G*.

43. There are two pieces of Wood, (one of which is reprefented in *Fig.* 77.) ferving to fuftain the Box upon its Support or Foot. One of thefe Pieces may be faftned to one Side of the Support, and the other to the other Side thereof, by means of four Iron Pins, two of which go thro' the Holes *N* and *P* in the Side of the Support, and the Holes *T* and *V* in the Piece *R*, and the other two in like manner, through Holes made in the other Side of the Support, and the other Piece, when we have Mind the Bottom of the Box fhould be parallel to the Horizon; but when the Box is to be a little inclin'd, that Pin going through the Hole *P*, muft be put through the Hole *O*, in the Piece *R*. Underftand the fame of the other Piece.

44. We are fometimes obliged to fet the Box forwarder on its Support; and this is done, in ufing the Holes *Q* and *S*, inftead of *N* and *P*. It is likewife fomething neceffary to incline the Box a little backwards; which may be done, by putting the Pin in *S*, into the Hole *X*, made in a Piece of Wood faftned to the Back-fide of the Box, and the correfpondent Pin on the other Side, into another Hole made on the other Side of the faid Piece.

45. The Box *T* flides upon the Top of the Machine, and is like that already defcribed,* but with this Difference only, that it is leffer. On the Top of the Box are two little Staples *Z Z*, in which a Ruler flides, having a Mirrour faftned to it, in the Manner as is mentioned *Numb.* 13. and fo by this Means the faid Mirrour may be put in the fame Situation, as that in the Figure of the firft Machine it hath in *H*.

* 10, 11, 13.

46. When

46. When we have a mind to remove this Machine from one place to another, we lay the Box *B E C* upon the croſs Pieces 2, 3, 4, 5, with its opening *A B C* upwards; then we put the little Box *Y*, the Ruler and Mirrour (mentioned *Numb.* 13.) the black Cloth, and the two Sticks *M L* and *IH*, all into the ſaid great Box; and afterwards partly cover it by the Frame *G*, * which is ſuſtained by two very thin Rulers, and then by another little Board, when the Frame is not big enough. The whole Machine thus taken to pieces, will take up no more room than the Support itſelf doth: and ſo it is very eaſy to remove from Place to Place. Now when Objects are to be repreſented in this Machine, it muſt be put together again, as *per* Figure; and the black Cloth, for a Perſon to put his Head under, hanging upon the Sticks, and faſtned to the Sides of the opening *A B*, *A C*, and *C D*. * 40.

The Uſe of this Machine.

47. The Uſe of this Second Machine is the ſame as that of the Firſt; but it ought to be obſerved, that when we incline * the Machine, the Angle of Inclination of the Mirrour and Horizon muſt be made leſs, by half the Inclination of the Bottom of the Box; and when the Machine is ſomewhat inclin'd backwards, * the ſaid Angle muſt be made greater by a like half. You muſt likewiſe obſerve, that when Objects are to be repreſented for a perpendicular Picture, the Machine muſt be placed according to the former Part of *Numb.* 44. Prints muſt be faſtned to a Board entirely ſeparated from the Machine, which Board muſt be ſet upon a Support, that may conveniently be moved backwards or forwards, according to Neceſſity.

* 43.

* 44.

A De-

A Demonstration of the Inclination of the Looking-Glass.

Fig. 75. 48. Let *A B* be a Ray, proceeding from some Point of an Object. We are to demonstrate, * if the Line *D I* hath the Inclination given to a Picture, and the Looking-Glass *G H* hath the Inclination we have prescribed, that the Angle *B a F*, will be equal to the Angle *BCD*. Now to prove this, draw the Line *FI* parallel to the Horizon, then the two Angles *I D F* and *DFI*, of the Triangle *IDF*, are together equal to the Angle *DIE*; but the Angle *DFI*, which is the Inclination of the Looking-Glass, is equal * to half the Angle *D I E*, less half the Angle *IFa*; and consequently it is less than the Angle *FD I*, by the Quantity of the whole Angle *IFa*: Therefore if the Angle *I Fa* be added to the Angle *D F I*, we shall have the Angle *D F a*, equal to the Angle *F D I*: Therefore the Angle *Fa B* will be * likewise equal to the Angle *BCD*. *Which was to be demonstrated.*

* 19.
* 16. 47.
* 19.

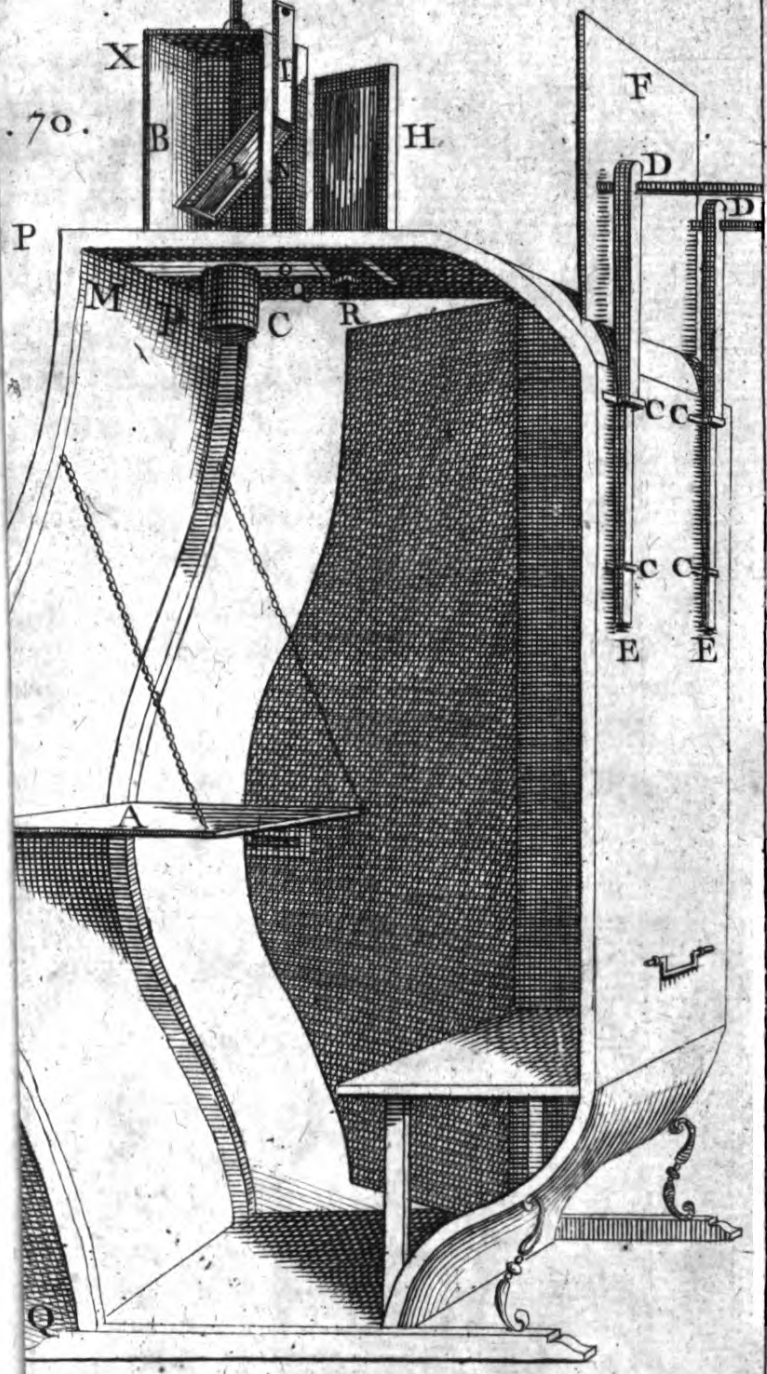
In reasoning nearly after the same Manner, we demonstrated what is mentioned * concerning the Inclination of the Mirrour, when the Box is inclin'd a little backwards.

* 47.

F I N I S.

ate .30.

.70.



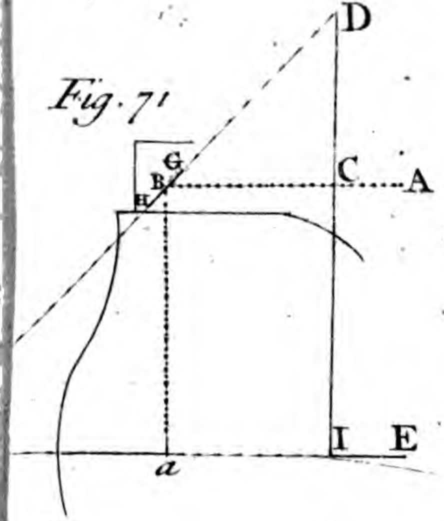


Fig. 71

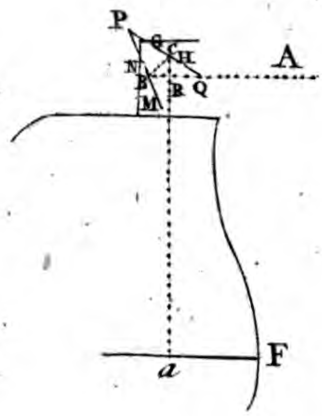


Fig. 72

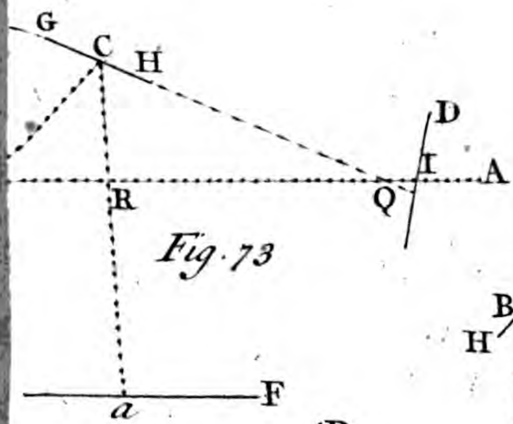


Fig. 73

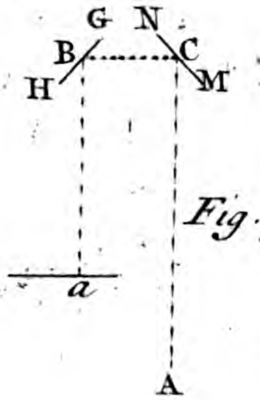


Fig. 74

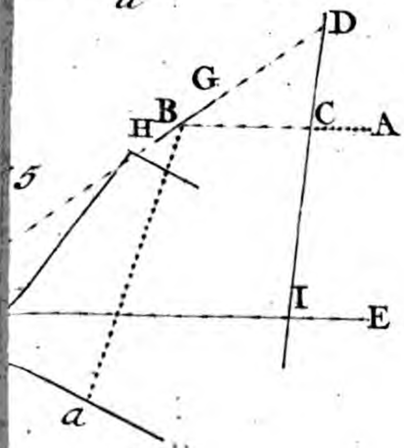


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Plate. 32.