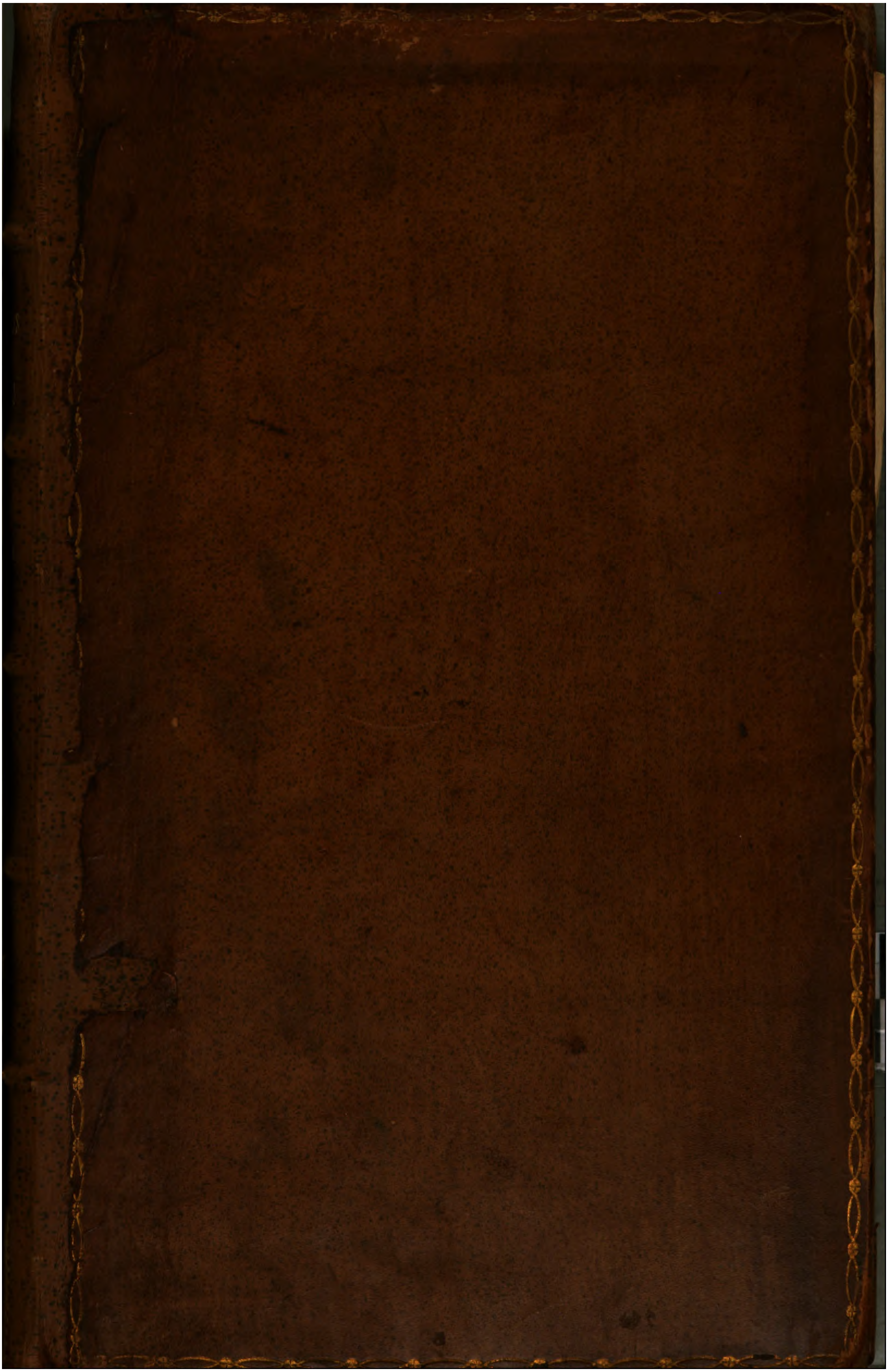

S. P. Rigau
Sept. 30. 183

Milnes

NEW
ELEMENTS of OPTICS;
OR, THE
THEORY of the ABERRATIONS,
DISSIPATION, and COLOURS
OF
LIGHT:
OF THE
General and Specific Refractive
POWERS and DENSITIES
OF
MEDIUMS;
THE
PROPERTIES of *Single* and *Compound*
LENSES:
AND
The NATURE, CONSTRUCTION, and USE
OF
Refracting and *Reflecting*
TELESCOPES and MICROSCOPES
Of every Sort hitherto publiſhed.

By *B. MARTIN.*

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The gift of the Author

THE

PREFACE.

IT is now about 20 Years ſince I publiſh'd a *Compendious* SYSTEM OF OPTICS; the general Reception it has been favour'd with by the Public, is one Reaſon why I have preſumed once more to offer them my farther Thoughts on this intereſting Subject. The Doctrine of *Refractions*, the Baſis of this Science, I had not then thoroughly conſider'd; I took it for granted, that there was nothing more to be known of it than what *Kepler*, *Des Cartes*, *Hugenius*, *Halley*, and Sir *Iſaac Newton* had delivered. I did not know from their Writings, that there was a two-fold Refraction, *viz.* a *general* and a *ſpecific Refraction* of Rays; and tho' I was always converſant with priſmatic Experiments, yet I never adverted to the different Dimenſions of the *colour'd Image* of the Sun, made by Priſms of different refractive Powers, nor obſerved what Proportion the Angle which thoſe Images ſubtended bore to the Angle of total Refraction, ſuppoſing it to be conſtantly the ſame in Priſms of every Sort of Glaſs.

And this ſeems to have been the general Caſe of other Opticians, till about the Year 1740. They all acquieſcing in what Sir *Iſaac Newton* had ſaid in his Optics——*That Light in paſſing thro' different refracting Mediums, as often as by contrary Refractions it is ſo corrected, that it emergeth in Lines parallel to thoſe in which it was incident, will ever appear white. But if the emergent Rays be inclined to the Incident, they will become* *tinged with Colours.*——But theſe Poſitions are not univerſally true; for unleſs the Ratio of the Difference of Refraction in the moſt and leaſt refrangible Rays to that of the whole Refraction of the Beam were conſtantly the ſame, there will be ſome Caſes where *the emergent Rays may be parallel to the Incident, and yet be colour'd*; and on the other Hand, *the emergent Rays may be inclined to the incident ones, and yet may be white or colourleſs.*

And becauſe this is extremely obvious in Experiments with the Priſms, it is amazing how it ſhould eſcape the Notice of Sir *Iſaac Newton*, who for near 70 Years made them his Amuſement, and all other Opticians for many Years after; and ſtill the more ſo, as it might eaſily be deduced from the Reaſon of Things.——For ſince whatever Degree of Refraction and Diſſipation of Rays are produced by a given Incidence of Rays on a Priſm of a given refracting Angle and Medium, the ſame can be corrected by another equal Priſm of the ſame Kind, inverted; and that if the equal Priſm be not of the ſame Medium, the ſaid Refraction and Diſſipation cannot be corrected, but there will be a Difference in both; and this Difference will be equal to the Refraction and Diſſipation proper to a certain Incidence of Rays, and refracting Angle of the ſame heterogeneous Priſm; and conſequently if this latter be added to the former, it will correct the reſidual Refraction and Diſſipation; and the Rays will emerge from the laſt Priſm white or with Colours, but at the ſame Time will make an Angle with the incident Rays.

And from thence it will follow, that if Lenſes analogous to ſuch Priſms were properly conſociated, they would have the ſame Effect; and ſo a compound Lens of a Teleſcope may be contrived that ſhall refract parallel Rays to the ſame Part of the Axis, or without Diſſipation and Colours, as in common refracting Teleſcopes.

Nor are we obliged to uſe Priſms or Lenſes of a different Medium for this Purpoſe; Lenſes of the ſame Sort of Glaſs will do, if we can by any Artifice contrive to have a different Degree of Refraction than what is natural in a given Incidence of Rays and Radius of Curvature in a Lens propoſed. And tho' this Method be leſs obvious than the former, yet it was the firſt, and only one that has yet been made public.

All Writers on Optics have the common Complaint of refracting Teleſcopes, but are at the ſame Time all ſilent concerning any Method of remedying the ſame, at leaſt any that have been genuine and effectual. Dr. HOOK contrived to *lengthen the focal Diſtance* of an Object-Glaſs, by combining a Lens of Water with one of Glaſs. Sir ISAAC NEWTON alſo propoſes to correct the Errors of Refraction from the Figure of a Lens, by making a compound Lens with two Glaſſes and Water between them. But Dr. *Gregory* is the firſt I find mentioning any Method of correcting Errors from the different Refrangibility of heterogeneous Rays, by *an Object Lens of a different Medium*; but this he does in ſuch a Manner, as plainly ſhews he had not a right Underſtanding of the Nature and Uſe of the Method he propoſed, for he recommended this in Imitation of Nature, who, in the FABRIC of the EYE, has (ſays he) placed the *cryſtalline Humour* between the *aqueous* and *vitreous Humours*, (which are in a ſmall Degree leſs refractive) in order that the Image may be painted as diſtinct as poſſibleon the Bottom of the Eye.——But as the cryſtalline Humour is *convex*, and not *concave*, it cannot prevent the Diſſipation of Rays in the Humours of the Eye; which Nature, by other Means, has moſt effectually provided againſt.

Other Expedients have been propoſed by other Authors, but I have ſeen none that were properly adapted to anſwer the Purpoſe, till that of Mr. CALEB SMITH appear'd, pear'd, which was preſented to the Royal Society, and publiſh'd in the 456th Number of the Tranſactions for *January* 1740, under the Title of, *A new Method of improving and perfecting catadioptrical Teleſcopes, by forming the Speculums of Glaſs inſtead of Metal.*

Mr. SMITH (among many other Things) aſſures us, this was his own Invention, in the follwing Words: — *Now, for this principal Defect* (*viz.* the Errors from the different Refrangibility of Rays) *no one, that we know of, has propoſed any Remedy.——And as the Removal of ſo great an Impediment, is of the greateſt Importance to the Science of* DIOPTRICS, *we have thought it worthy of a careful Examination, whether it might not be poſſible, in ſome Caſes, for contrary Refractions ſo to correct each others Inequalities as to make their Difference regular; and if this could be conveniently effected, Sir* ISAAC NEWTON *has acknowledged there would be no farther Difficulty**.—— *Now upon due Conſideration of this Subject, we have found it poſſible, by proper Methods and Expedients, to rectify thoſe Errors which proceed from the different Refrangibility of Rays.——And our preſent Deſign is to ſhew what Advantage this will yield towards improving and perfecting catadioptric Teleſcopes,* &c.

The Author's Reaſoning is founded on the well known and eſtabliſh'd Principle,——*That the Sines of Refraction of Rays differently refrangible are to one another in a given Proportion, when their Sines of Incidence are equal.* Whence it is plain, this new Invention requires nothing more than the common Principles of Optics, properly applied, for its Explication.

He then proceeds to explain the ARTIFICE of his *Invention*, which, as to Theory, is equally ingenious and perfect. It conſiſts in this; he ſhews how a *meniſcus*

Lens

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* *Philoſ Tranſ.* No. 88.

Lens may be uſed as a *Speculum*; and that the ſecond Refraction in the LENS, which encreaſes the Error of the firſt, is, by the Mediation of a reflecting Surface, made to correct the ſaid Error in the *Speculum* in a very great Degree, and the reſidual Error he corrects entirely by a concave Lens properly placed in the Axis; but as the Author has not given us the Proceſs of his Demonſtration, I have here ſupplied it, for the Satisfaction of inquiſitive Readers.

As by this Contrivance the Refractions are made at concave Surfaces in the Speculum, it will moreover have this Advantage, that the ſame convave Lens which corrects the Error of *different Rays*, may at the ſame Time be made to correct the Error of the *Figure of the Glaſſes*, ſo far, at leaſt, as to render it altogether inſenſible.

Therefore this Method is in itſelf quite perfect, and the only one that is ſo. For the other Method, which corrects the Errors of Rays differently refrangible by a convex and concave Lens, is extremely imperfect, as it makes the Errors from the Figure vaſtly greater than it is by the ſingle Object Lens. And it is no Wonder if Mr. *Smith*, who conſider'd this Affair principally in Theory, ſhould give the Preference to the *catadioptric* Method; for he expreſsly mentions *other Methods*, by which the Errors of Refraction in common Teleſcopes may be remedied, and there is Reaſon to think he means the *different refractive Powers* of Glaſs. In ſhort, the two Methods are eſſentially the ſame, only the Mode or Way of applying the ſame Principle in Practice, is more perfect in one than in the other.

Now as the Impediment of refracting Teleſcopes has ſo greatly alarm'd the World of late Years, and as it proceeds from two Cauſes which are in general but little known; and laſtly, as no Methods have been propoſed, but this of Mr. *Smith*, for remedying the ſame, I have thought

thought it would be acceptable to the Public to have the whole Doctrine explain'd from its firſt Principles, and applied in each particular Method as far as the Nature of the Thing required. I have endeavour'd to exhibit the Theory of each Sort of Aberration in the moſt natural and perſpicuous Manner; and hope the Means of rectifying or remedying the ſame will be found in ſo clear and fair a Light, that every Reader will be able to form a Judgment of them according to their Merit; and probably many will conclude with myſelf, *That one Method is too difficult for Practice; and the other too imperfect to deſerve much Regard from the judicious Part of the Public.*

Beſides this I have given the Conſtruction of all Kinds of TELESCOPES and MICROSCOPES, that have been made to the preſent Time, and illuſtrated the ſame by a great Number of Diagrams, to be found in no other Treatiſe of Optics; by which means Gentlemen may at once underſtand the Difference of Conſtruction, as well as the *Rationale* and Merit of each particular Form. They will alſo be convinced, that there is Room enough for Improvement in refracting and reflecting Teleſcopes in many Reſpects, beſides what relates to the *Aberration of Rays.*

To conclude, as I intend this for a Supplement to my SYSTEM OF OPTICS, in order to render the Theory of this Science compleat, I have not troubled the Reader with the Demonſtration of the original Theorems over again, as they are there contain'd at large; but I have here given them a new Dreſs, and adapted them to expreſs univerſally all the modern Improvements relative to all the Problems of optical Science. But the Demonſtrations of the Theorems relating to Aberration of Rays, occaſion'd by the Figure of *Specula* and *Lenſes*, are to be found in my *Philoſophia Britannica*; where the Reader will alſo find thoſe above mentioned. Many Years have laps'd ſince this Treatiſe was firſt began, in all which Time I have ſeen nothing on theſe Subjects; and as I was obliged to proceed in a trackleſs Field, and many and difficult Experiments neceſſarily occur'd, my Progreſs was but ſlow; and it has been with much more Labour than any one would imagine, that I have at length finiſhed this ſmall Treatiſe, by which the Reader will be fully certified, that all the former and latter Improvements in Optics are owing (not to *Strangers* and *Foreigners*) but to *Engliſhmen* only, whoſe peculiar Glory it is to have an acknowledged Superiority, in regard to INVENTION, *over all the World beſides.*

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CONTENTS.

THE CONSTRUCTION and USE OF REFRACTING TELESCOPES and MICROSCOPES.

PART I.

Containing an Explanation of the Nature of Viſion by Lenſes, and the Structure and Properties of all the uſual Forms of Teleſcopes; *viz.* the *Aſtronomical*, *Galilean*, and *Common Four-Glaſs Teleſcope*; *and alſo of the* Refracting Microſcope.

CHAP. I.

Of the different Sorts of Rays *of* Light, *and Forms of* Lenſes *uſed in* Teleſcopes, *with the Method of finding their* Focal Diſtances *for* Parallel Rays.

1. THE Word Teleſcope is of *Greek* Original, and ſignifies the *Perfection of Viſion*; and it is very juſtly ſo denominated, as it renders our View of diſtant Objects much more perfect than they appear to the naked Eye, and gives us a more certain Idea of what they are.

2. This noble Effect is producible two Ways. (1.) By *Refraction of Light* through *Lenſes*; and (2.) by *Reflection* of Rays of Light from the poliſh'd Surfaces of *Mirrours*.

3. The Rays of Light are of three Forms, *viz.* (1.) *Parallel Rays*, which proceed equally diſtant from each other; as thoſe of the Sun, repreſented by ABCD. (2.) *Converging Rays*, which all Fig. I. tend in their Progreſs towards one Point; as the Rays of the Sun collected by the Glaſs BC, are made to converge towards the Point F. (3.) *Diverging Rays*, which proceed from any Point F, diverging or going wider of each other towards the Parts EG.

4. The Point F, where the Rays of the Sun are collected into a ſmall round Spot, is called the *Focus*, that is, the *Burning Point*; becauſe the Rays being there condenſed, the Heat is augmented, and they burn proportionably.

5. A LENS is any tranſparent Subſtance of a regular Form, through which Light may be tranſmitted, or refracted; thoſe in common Uſe are of Glaſs ground and poliſhed to a proper Figure, on which Account there are divers Sorts; as (1.) a *Plain Lens*, both whoſe Surfaces are parallel to each other. (2.) A *Plano-Convex*, conſiſting of one *Plain*, and the other a *Convex*, or *Spherical Surface*. (3.) A *Double-Convex*, both whoſe Surfaces are of a Convex, or ſpherically round Form. (4.) A *Plano-Concave*, with one Plain, the other a ſpherically hollow or *Concave Surface*. (5.) A *Double-Concave*, both whoſe Surfaces are ſpherically *Concave*. (6.) A *Meniſcus*, which has one Surface *Convex*, the other *Concave*. Of which, and their Properties, in Order.

Fig. II. 6. RAYS OF LIGHT paſſing through a Plain Lens BC ſuffer no Alteration in reſpect of their Direction; for parallel Rays AB having paſſed through

through the Lens, proceed on parallel towards D: And the ſame is to be ſaid of the other Sorts of Rays.

7. If *Parallel Rays* AB, GH, ED, fall on the Fig. III. Convex Side of a Plano-Convex Lens BD, they will be converged to a Focus F, at nearly the Diſtance of the Diameter HF of the Sphere, of which the Lens is a Segment; or at the Diſtance of *twice the Radius* HC of the Sphere.

8. If Parallel Rays fall on a Double and equally Fig. IV. Convex Lens BD, they will be conven'd to a Focus F, which will be in the Center C of the Sphere of Convexity, or diſtant in the firſt Surface H by the Radius HC of the Sphere, very nearly.

9. If the Lens be not equally convex, parallel Rays will be converged to a Focus, whoſe Diſtance is eaſily aſſigned by the common Theory of Optics.

10. If *Parallel Rays* fall on a Plano-Concave Fig. V. Lens BD, they will be made to diverge afterwards towards IKL, as if they came from a Point F, nearly at the Diſtance of a Diameter of the Sphere of Concavity FH; which Point F is called the *Virtual Focus* of the Concave Lens.

11. *Parallel Rays* are made to diverge much Fig. VI. more by paſſing through a Double Concave Lens; which, if it be equally concave on both Sides, will cauſe them ſo to diverge, as if they came nearly from the Center C, of the Sphere of Concavity.

12. A MENISCUS LENS participates of the Properties, as well as the Forms of both the Convex and Concave Lenſes. For if C be the Center of the Convexity BHD; then (1.) if the other Surface BID had been of the ſame Convexity the

Focus F of parallel Rays would have been at the ſaid Center C, by *Art.* 8. (2.) If the ſaid Surface had been plain, the Focus would have been at L, by *Art.* 7. (3.) But ſince, in the preſent Caſe, it is concave, the Focus will be removed farther to a Point F. (4.) If the ſaid Surface BID were parallel to the other BHD, that is, if the Convexity and Concavity were equal, the Focus would be at an infinite Diſtance, or the Rays would proceed parallel after Refraction. (5.) If the Radius of Concavity were leſs than that of Convexity, the Rays would be made to diverge after paſſing thro' the Glaſs as in Concave Lenſes.

13. From this Account of the Nature and Effect of Lenſes, it appears, that if we look on F as a Radiant Point, or that from whence Rays of Light do proceed, then thoſe Rays, if collected by any of the aforeſaid Convex Lenſes, placed at their reſpective focal Diſtances FH, will proceed parallel after they have paſſed through the ſaid Lenſes.

14. Alſo if Concave Lenſes be made to receive converging Rays I, K, L, tending towards a Point F, which is at the Diſtance of the *Virtual Focus* of thoſe Glaſſes from them, then thoſe Rays, after having paſſed through the Lenſes, will likewiſe proceed parallel to each other.

15. The moſt ready Way to find the *focal Diſtance* of any Convex or Meniſcus Lens, is to hold it in the Sun-Beams, ſo that the Rays may fall perpendicularly upon it; then meaſure the Diſtance at which the Rays are collected in a white round Spot, well defined; for that is the Image of the Sun, Burning Point, or what we properly call the *Solar Focus* of the Lens.

16. To

16. To find the *Virtual Focus*, or Center of Concavity in any Concave Lens, proceed thus; cut a round Hole in a Piece of black Paper, *&c.* and lay it on the Concave Lens with a little Gum-Water, ſo that the Center of the Hole may be in the Middle of the Glaſs; then ſtrike a Circle on another Piece of Paſt-Board, *&c.* whoſe Diameter is juſt twice the Length of that of the Hole on the Glaſs; then hold the Lens in the Sun Beams, and move the Paper backward and forwards, till the Rays diverge ſo much as juſt to fill the ſaid largeſt Circle; then will the Diſtance between the Lens, and the ſaid Paper be equal to the Diſtance of the Virtual Focus from the Lens.

CHAP. II.

Of the Focal Diſtances *of* Converging *and* Diverging Rays. *Of forming the* IMAGE *of an* Object *in the* Focus *of a* Lens, *and on the* Retina *in the* EYE; *of the* Poſition, *Magnitude, and Diſtance of the Image*; *and of the* Viſible Area, *or* Field of View.

17. SINCE (as we have obſerved *Art.* 7 and 8) *Parallel Rays* are converged to the Focus of a Concave Lens; it will be eaſy to conceive, that if the Incident Rays are not *Parallel*, but *Converging*, or *Diverging*, they will, in the former Caſe, be made to unite in a Point

nearer to the Lens than the *Solar Focus*; and in the latter Caſe, they will be all conven'd in a Point beyond the ſaid Solar Focus in the Axis of the Lens.

18. And becauſe the *Radiant Point* muſt be at an *Infinite Diſtance*, in order that the incident Rays may be truly *parallel*; it muſt be at *more than an infinite Diſtance*, or be poſited *negatively*, on the other Side of the Lens, that the incident Rays may be conſidered as Converging; and laſtly, if the Diſtance of the Radiant be *leſs than infinite*; that is, if it be at any *finite Diſtance*, then the incident Rays will be *Diverging*, and have their *proper Focus* on the other Side the Lens, beyond the *Solar Focus*, as was ſaid before.

19. And hence it follows, that the nearer the Diſtance of the Radiant is to Infinite, or the greater its Diſtance from the Lens, the leſs will be the Diſtance of the *Proper Focus* from the *Solar Focus*, and conſequently from the Lens itſelf.

20. Thus as the Radiant Point approaches the Lens, the *Proper Focus* will recede from it; and, *vice verſa*; ſo that if DEG be a Double and equally Convex Lens, the Centers of whoſe Convex Surfaces are *a* and *b* reſpectively; then if we conſider ABC as an Object placed before it, and take therein the Point B in the Axis of the Lens, for the *Radiant Point*, or that from which we ſuppoſe Rays iſſue and fall diverging on the Lens; they will be collected all in the Point F on the other Side the Lens at a Diſtance EF, which is ſomewhat greater than the Diſtance of the Solar Focus E*a*.

21. In

21. In this Caſe, the Diſtance between the Solar Focus *a*, and the Proper Focus F is very ſmall, becauſe the Diſtance of the Object or Radiant Point B is remote from the Focus of the ſaid Lens *b*.

22. But on the other hand, if we conſider HFI as an Object placed near the Focus *a* of of the Lens, and F the Radiant Point; then becauſe the Rays proceeding from thence to the Lens, muſt fall on it more diverging; therefore the proper Focus to which they will be converged on the other Side the Lens will be at B, at a great Diſtance from *b* the Solar Focus, becauſe the Radiant Point is ſo near it.

23. If the Lens be a *Double and equally convex* One, and you know the Radius of Convexity, and Diſtance of the Radiant Point, you will find the Focal Diſtance of the Rays after Refraction through the Lens by the following eaſy Rule—*Multiply the Diſtance of the Radiant by the Radius of Convexity; divide the Product by the Difference between the ſaid Diſtance and Radius; the Quotient will be the Focal Diſtance acquired.* Note, the Rule is the ſame for a Plano-Convex, if for *Radius* you take *twice the Radius.* (See *Art.* 7 and 8.) If the Lens be not equally convex, it will be difficult to find the Length of each Radius; and, when found, the Rule is too intricate for finding the proper Focus; which thoſe who deſire to know, may find in Page 96, of my *Syſtem of Optics*; or my *Philoſophia Britannica.*

24. After the ſame Manner, if in the Object ABC we take either of the extreme Points A or C for the Radiant Point, then the Rays which proceed from thence to the Lens DEG, will by

it be converged to their reſpective Focus in the Points I or H, ſo that HFI ſhall be nearly in a Right Line, and equi-diſtant for the Lens DG.

25. It is further to be obſerved, that the Rays which proceed from the Points ABC in the Object, do in ſome wondrous and ineffable Manner convey or carry with them the Likeneſs or perfect Reſemblance of thoſe Parts of the Object to their focal Points HFI, where they appear as viſible to the Eye as they do in the Object itſelf.

26. And ſince the ſame thing is to be underſtood of all the Points between A and C, it follows that there muſt be an adequate Repreſentation of the Object ABC in the focal Points between H and I; and this we call the *Image* of the Object.

27. All thoſe Rays which proceed from any ſingle Point A, B, or C, in an Object, to the whole Surface of the Lens DG, is called (from its Figure) *a Cone, or Pencil of Rays*; of which, that which is in the Middle, is called the Axis of the Pencil, and paſſes through the Center E of the Lens, to the ſeveral Points in the Image IFH.

28. Hence it is evident, that the Axes of all the Pencils of Rays, iſſuing from every Point in the Object, meet and croſs each other in the Center of the Lens; and conſequently, that Pencil of Rays which comes from the Right-hand extreme Point C in the Object, will be all determined to the Point H in the Left-hand Extreme of the Image; and thoſe which come from A will be made to repreſent the Point in

the

the Image at I; from whence it is very evident, the *Poſition of the Image muſt be inverted, or contrary to that of the Object.*

29. 'Tis farther evident, that ſince the Axes of the Extreme Pencils of Rays AEI, and CEH, terminate the Length of the Object and Image, and croſs each other in the Center of the Lens E, the Angles AEC and HEI will be equal; *and ſo the Object* AC, *and its Image* IH *will be ſeen under equal Angles from the Center of the Lens.*

30. For the ſame Reaſon alſo, we have *the Length of the Object* AC *to the Length of the Image* IH *in the ſame Proportion as the Diſtance of the Object* BE *to the Diſtance of the Image* EF. If the Object be a *Surface*, its *Area will be to that of the Image, as the Square of* BE *to the Square of* EF; and if it be a *Solid*, then the *Solidity will be to that of the Image, as the Cube of thoſe Lines* BE *and* EF, reſpectively.

31. From what has been premiſed, it evidently follows, that if HI be the Object, then will AC be the Image formed thereof; ſo that the Object and Image are reciprocal, in a contrary Poſition, and proportional to the Diſtances from the Lens.

32. Hence alſo it is evident, that Objects AC, though large in themſelves, if placed at a remote Diſtance from the Lens, will be repreſented in their Images IH very ſmall, and very near the Focus *a* of the Lens; and therefore

33. If the focal Length E*a* of the Lens, be increaſed or diminiſhed, the Images of Objects

will be proportionably increaſed or diminiſhed. Or, in other Words, the *greater the focal Length of the Lens*, the *larger is the Image* formed thereby.

34. All the Pencils of Rays which proceed from an Object AC, placed in the Focus of a Convex Lens DG, do, after Refraction through it, proceed parallel (by *Art.* 14.) and if in this Caſe they are received by the Eye EFK, ſo as to paſs through the Pupil PQ and the ſeveral Humours, they will by them be convened to their reſpective Focus's at the Bottom of the Eye, (*Art.* 13.) and there paint a lively Image IKH on the Retina; and thus *cauſe a diſtinct and perfect Viſion of the Object* ABC. Fig. IX.

35. If the two external Rays AD, CG, are conceived to proceed parallel to the Lens, they will be ſo refracted as to croſs each other in the Focus of the Lens, which will be here near the Middle of the Pupil, at O, and contain an Angle DOG, which is called the *Optic Angle of the Lens*, and determine the Diameter of the *Viſible circular Area*, or *Field of View*, as it is commonly called.

36. Since the Rays AD, CG, are ſuppoſed parallel, 'tis evident the viſible Part AC of any Object, will be greater or leſſer as the Diameter, or Aperture of the Lens, next the Eye, is ſo. Or, in other Words, the greater the Area of the Lens, the greater will be the *Viſible Part*, or *Field of View*, in any Inſtrument that procures Viſion by Means of a Convex Lens placed before the Eye.

37. The extreme Rays BD, BG, of any Pencil coming from any radiant Point B remotely ſituated, will, when they enter the Pupil of the Eye, or Lens DEG, be nearly co-incident with parallel Rays as AD, CG; and will therefore be fit for producing diſtinct Viſion in the Eye, as having the Focus F nearly coincident with the Focus *a* of parallel Rays on the Retina at the Bottom of the Eye; as is plain to be ſeen in the Figure. Fig. X.

CHAP. III.

Of the Structure of the Aſtronomical TELESCOPE, *its Magnifying Power, and Uſe.*

38. WE have premiſed ſuch Principles, as are neceſſary to a right Underſtanding of the Nature, Conſtruction, and Effects of *Refracting Teleſcopes*; which we ſhall next proceed to deſcribe according to their ſeveral Kinds, beginning with thoſe which are moſt ſimple and uncompounded; ſhew their ſeveral Properties, Powers of Magnifying, Methods of Improving them, *&c.*

39. The moſt ſimple Structure of a Teleſcope, is that which conſiſts only of one Object Glaſs DEG, and one Eye Glaſs KLM; and fewer than theſe there cannot be for magnifying any diſtant Object ABC. For the Object Lens only ſerves to form the Fig. I. Plate II.

Image HFI of the ſaid Object, but does not magnify it, ſince the Eye placed at the Center E of the ſaid Lens, views the Object and Image under equal Angles AEC and HEI (by *Art.* 29.) and conſequently they muſt appear of equal Magnitude.

40. And ſince in this Teleſcope, it is the Image HI which we view, and not the Object AC itſelf, if we can contrive to view this Image under a larger Angle than that which the Object ſubtends, it will appear larger than the Object; and ſo the Object is in that Caſe ſaid to be magnified. And this is eaſily effected, by fixing a Convex Lens KLM before the Eye, in ſuch Manner, that the Image HI ſhall be exactly in the Focus of the ſaid Lens. For then if we ſuppoſe the Rays IK, and IM to proceed parallel from the Image to the Lens KM, they will be convened to a Point O at the Pupil of the Eye, which ſhall be in the other Focus of the Lens (by Art. 7 and 8.) And ſince OL is equal to LF, and KM equal to HI, it is evident, the Angle KOM is equal to the Angle HLI, and is that under which the Image is viewed by the Eye through the Eye-Glaſs KM.

41. Now it is plain, the Angle HLI (or KOM) is to the Angle HEI, as EF is to FL; becauſe the Angle of apparent Magnitude of one and the ſame Object muſt always increaſe, as the Diſtance from the Eye decreaſes. Wherefore the Magnitude of the Image HI ſeen by an Eye at E, will be to its Magnitude ſeen from L as the Diſtance FL to the Diſtance FE. Or,

the

the Power of Magnifying in this Teleſcope is in the Proportion of the Focal Length of the Object Glaſs FE *to the Focal Length of the Eye-Glaſs* FL.

42. For Example: Let the focal Length of the Object Glaſs EF be 5 Feet or 60 Inches, and that of the Eye-Glaſs LF equal 2 Inches; then will the Magnitude of the Object ſeen by the naked Eye, be that of its Image ſeen by the Eye-Glaſs, as 2 to 60, that is, as 1 to 30; ſo that ſuch a Teleſcope magnifies the Length of an Object 30 Times, or makes it appear as large as it would do, were it placed 30 Times nearer to the Eye.

43. And ſince the *Surfaces, or Areas* of Bodies, encreaſe in Proportion to the Squares of the *Lengths*, and the *Solidity* in Proportion to the Cubes thereof; it follows that the *Superficies* of that Object is magnified (30 Times 30, or) 900 Times; and the *Solidity* or *Bulk* 30 Times 30 Times 30, or) 27000 Times; which is a very great Degree of Magnifying for an Inſtrument of 6 Feet Length. And this Power of Magnifying is encreaſed in Proportion as we encreaſe the focal Length of the Object Lens, as we ſhall ſhew more at large hereafter.

44. The *Object will appear inverted* in this Teleſcope (by *Art.* 29.) becauſe it is the Image HI we view, which has a contrary Poſition to that of the Object AC. Alſo the *Viſible Area*, or *Field of View*, is proportional to the Area of the Eye-Glaſs KM *(Art.* 36, 37.*)* And the Object (or, rather, the Image) will appear *more or leſs bright or luminous*, in Proportion as the Aperture, or Hole before the Object-Glaſs, is greater or leſ-

ſer,

ſer, becauſe the Rays of Light, which enter thereby to form the Image, will encreaſe or decreaſe in that Proportion.

45. Since the Object, in this Structure of the Teleſcope, appears inverted, it is therefore unfit for viewing terreſtrial Objects, which it turns topſy-turvy, and ſo makes the Sight of them unnatural, and conſequently unpleaſant. This Form or Structure is therefore wholly appropriated to aſtronomical Uſes, in viewing the heavenly Bodies, whoſe Poſition we do not regard, ſince *Globes*, or *Circular Spaces*, appear to the ſame Advantage one Way as the other. In this Reſpect therefore it is called the ASTRONOMICAL TELESCOPE.

46. Since all the heavenly Bodies, except the Sun, are beſt viewed in the Night, the Uſe of the Tube (which in Part is to make every thing dark about the Image in the Day-time) is now unneceſſary; and ſince moſt of the celeſtial Objects require a great Degree of Magnifying to render them very diſtinct and viſible; therefore Object-Glaſſes of a very long focal Diſtance (above 100 Feet) have ſometimes been uſed for this Purpoſe; which, as there was no Occaſion for the Tube, were uſually fixed on the Top of a very high Pole, &c. and an Eye-Glaſs properly adjuſted thereto in the Axis; and both being thus fixed, they conſtitute what is commonly called the AERIAL TELESCOPE. But they are now out of Uſe ſince the ſame Powers of Magnifying are to be had in *Reflecting Teleſcopes* of ſhort Lengths, and very eaſiy applied to uſe.

CHAP.

CHAP. IV.

Of the Galilean TELESCOPE, *and* Opera-Glaſs; *their Conſtruction and Properties explained.*

Plate II. Fig. II.

47. IN order to render the aforeſaid Refracting *Teleſcope* fit for viewing terreſtrial Objects, that is, for viewing them in an *erect Poſition*, a different Combination of Glaſſes has been introduced in two ſeveral Methods, *viz.* (1.) By uſing a *Concave Eye-Glaſs*; and (2.) by the *Addition of two Convex Eye-Glaſſes* to the former; ſo that a Teleſcope of this later Form, conſiſts of an Object Glaſs, and three Convex Eye-Glaſſes.

48. The firſt Structure of theſe *Terreſtrial Teleſcopes* is moſt ſimple; and, from its Inventor GALILEO, is called the *Galilean* TELESCOPE. The Effect of which Inſtrument is as follows: DEG is the Object-Glaſs, as in the other, but KLM is here a *Concave Eye-Glaſs* inſtead of the *Convex One*, uſed before. Now, all Rays AD, BE, CG, coming from an Object AC at a great Diſtance; and therefore parallel among themſelves, or nearly ſo, will proceed (after Refraction thro' the Object Glaſs DG) converging towards its Focus, ſuppoſe at O, (by *Art.* 7 and 8.) But if thoſe Rays, inſtead of going to the ſaid Focus O, ſhould be intercepted by a concave Lens KM, placed at its focal Diſtance from the Point O, they will (after Refraction thro' it) paſs *parallel*

rallel to each other, (by *Art.* 15.) and ſo are fit to produce diſtinct Viſion (by *Art.* 37.)

49. Therefore the Object ABC will be *diſtinctly ſeen* by Means of the Concave Eye-Glaſs KM; and it will be ſeen *erect*, or in its *natural Poſition*; becauſe the Rays, which come from the extreme Parts of the Object do not here croſs each other, as before with the Convex Eye-Glaſs; by which Croſſing of the Rays, the Inverſion of the Object can only be effected, as has been already explained.

50. The Object will likewiſe appear *magnified*; for thoſe Rays *a*QG and *b*QD, which croſs each other in the Axis of the Object-Glaſs, at the Focus Q, will after paſſing the Lens, go parallel to the Concave Eye-Glaſs KM, by which they will be made to diverge ſo as if they came each from the Virtual Focus F, and will contain the Optic Angle KFM, under which the viſible Part of the Object now appears to the Eye: But the Angle, under which the ſame viſible Part (*viz.* ab) appears to the naked Eye, is the Angle aQb, which is equal to the Angle DQG; and ſince KM is equal to DG, (becauſe the Rays DK and GM are parallel) therefore the Angle KFM will be as much greater than the Angle DQG as QE is greater than FL; and conſequently, in the ſame *Proportion will the Object be as ſeen by this Teleſcope to its magnitude as ſeen by the naked Eye.*

51. Hence we obſerve, that if the Object Lens remain the ſame, and the Sphere of Concavity be equal to that of Convexity in the Eye-Glaſs,

Glaſs, the magnifying Power will be the ſame, or equal in both (*Art.* 42.) And conſequently, in this Caſe, the *Galilean* Teleſcope will be ſhorter than the other with a Convex Eye-Glaſs, by *double the focal Diſtance of the ſaid Eye-Glaſs.* Or, on the other hand, if thoſe two Teleſcopes are of equal Lengths, then *the Galilean will have the greateſt Power of Magnifying*, as admitting an Object Glaſs whoſe focal Diſtance is longer than that of the other by double the focal Diſtance of the Eye-Glaſs, ſuppoſed the ſame in both.

52. Since only that Part of the Object will be viſible which is terminated by thoſe Rays which enter the Pupil of the Eye, 'tis evident, the *Viſible Area*, or *Field of View*, in this Teleſcope, will be always proportional to the Area of the Pupil; and therefore muſt be much leſs than in the other Teleſcope, where the ſaid *Viſible Area* is proportional to the *Area of the Eye-Glaſs* (*Art.* 36, 37, 45.) And this is the great Reaſon why this is not the Teleſcope of moſt common Uſe.

53. For ſince it takes in ſo ſmall a View, it proves very unſatisfactory, unleſs in ſhort Lengths, which take in a greater Compaſs of Objects than larger Ones can do; for the ſhorter the focal Diſtance EQ, the greater will be the Optic Angle DQG to the naked Eye, if the Diameter of the Object Glaſs DG continue the ſame. Conſequently the *ſhorter the Teleſcope*, the larger the *Field of View*, but the leſs will be the *Power of Magnifying*. And in this ſhort Length they

are called OPERA GLASSES for their being uſed in the *Opera Houſes*.

54. In regard to this Teleſcope, I ſhall only obſerve further, that the nearer the Eye is to the Lens KM, the greater Quantity of Rays it will take in, and conſequently the greater will be the Field of View, and the more bright and luminous the Objects will appear; becauſe the Rays of the ſeveral Pencils diverge, from the Eye-Glaſs, into a wider Space; and being denſer near the Glaſs, will there enter the Eye in greateſt Abundance, and produce the Effects above-mentioned.

55. In the laſt Place, as this Glaſs is very thin in the Middle, the Rays are leſs liable to be diſturbed by the Irregularities in the Subſtance of the Glaſs, in paſſing through it; and alſo, ſince here we view not the Image of Objects, but the Objects themſelves, by Rays coming directly from them, without croſſing, or interfering with each other; it follows, that on both theſe Accounts, we have a clear and moſt diſtinct View of an Object by this Teleſcope. And hence it is, that we can ſometimes ſee *Jupiter's Satellites* very plain, in a Teleſcope of this Sort, not above 20 Inches or two Feet long, when one of 4 or 5 Feet of the common Sort with Convex Eye-Glaſſes will ſcarcely render them viſible.

CHAP. V.

The Conſtruction *of the Common* Four-Glaſs TELESCOPE, *and its Uſes explained.*

56. WE have ſhewn a Method of rectifying the Appearance of an Object by a Concave Eye-Glaſs, in the laſt Chapter; but as that is not accommodated for Uſe in the beſt Manner; we ſhall here treat of another, which makes the Structure of our common Refracting Teleſcopes, by the Addition of two Convex Eye-Glaſſes to that in the Teleſcope of *Art.* 40, and is as follows.

Plate. II. Fig. III.

57. Since the Image HI is in the Focus of the firſt Eye-Glaſs KM, the Rays of every Pencil which come from every Point HFI therein, will, after Refraction through it, paſs on parallel among themſelves; and conſequently, if they were intercepted by another Convex Lens OP, they would thereby be made to converge and unite in the Focus thereof, and there paint a ſecond Image of the Object as QRS.

58. Now this *ſecondary Image* is to be formed in an *erect Poſition*, *viz.* the ſame with that of the Object; and, in order to that, the Diſtance of the two Lenſes KM, and OP, ought to be equal to the Sum of the Focal Diſtance of both Glaſſes: For then the Axis of each Pencil of Rays HK and IM, which come from the extreme Points of the inverted Image HI, will

croſs each other in N, the Focus of the Lens KM, and by that Means form an Image in a *contrary Poſition* in the Focus R of the Second Lens OP; which Image will therefore be *erect.*

59. As a Second Glaſs was neceſſary to rectify the Image, ſo a third Lens TV muſt be added to view the ſaid Image thus rectified; and in order to have diſtinct Viſion thereof, this third Glaſs muſt be placed at its focal Diſtance therefrom, becauſe then the Rays coming from every Point in the Image QRS, will paſs from the Glaſs parallel among themſelves, and ſo enter the pupil of the Eye at W.

60. Theſe parallel Rays are by Means of the ſeveral Humours of the Eye (but principally, the *Cryſtalline* X) convened to a Focus on the *Retina*, at the Bottom of the Eye, where a third Image YZ is formed of the Object ACB, and in an inverted Poſition; and thus diſtinct Viſion of the Object is effected by Means of three Eye-Glaſſes. See *Art.* 35. and preceeding.

61. And ſuch is the Structure of the Refracting Teleſcope in common Uſe, in which all the three Eye-Glaſſes, KM, OP, TV, are of the ſame Convexity and Size, and therefore the focal Diſtance the ſame in all; whence the Diſtance between each, is double the ſaid focal Diſtance; but it is not neceſſary that the Eye-Glaſſes ſhould be of the ſame Convexity, or focal Length; for were they different in this Reſpect, yet if they were placed as above directed, *viz.* ſo that the Focus of each might fall upon the ſame Point N and R, the Viſion

Viſion would ſtill be diſtinct, and the magnifying Power of the Teleſcope only would be altered thereby.

62. If all the Eye-Glaſſes be of the ſame Sphere or Convexity, as they commonly are, then will the *magnifying Power*, or *Field of View*, be the ſame as it would be with only one of them, KM, as is eaſy to conceive from a View of the Scheme. For the Images QRS and HFI are equal to each other, and viewed by Glaſſes of an equal magnifying Power, and of an equal Area or Size; and therefore the Appearance of the Object muſt be every way the ſame by three as by one of them, the *Poſition* only excepted.

63. And this is the moſt commodious Structure of a Refracting Teleſcope, where the Objects to be viewed are large and well enlightened, as the *Sun* and *Moon* among the heavenly Bodies; and the *People*, *Houſes*, *Trees*, and *Fields*, (if not at too great a Diſtance) among terreſtrial Objects. But very ſmall Objects, as the Planets, and eſpecially their Secondaries or Moons, require too nice and diſtinct a Viſion to be effected by three Eye-Glaſſes, (in ſhort Teleſcopes) ſince each Glaſs reflects a great deal of Light which is thereby loſt, and the Refraction through three Glaſſes cauſes the Viſion to be much leſs perfect and diſtinct than it would be through one only.

64. But two Things are here to be obſerved for the greater Perſpicuity and Diſtinctneſs of Viſion in this Sort of Teleſcopes, *viz.* (1.) That the Image QS or HI, which ever is ſeen, be duly circumſcribed with a proper Diaphragm, that is with a Piece of Wood with a Hole in the Middle,

Middle, to exclude or cut off all extraneous or uſeleſs Rays, ſuch as are refracted from the Sides or Edges of the Lenſes. (2.) That the Eye be placed at W, ſo that the focal Point of the Eye-Glaſs may fall as near as poſſible on the Pupil of the Eye; for elſe the Area or Field of View will not be ſo great, and what is ſtill worſe, the Object will appear coloured and confuſed towards the extreme Parts.

* * *

CHAP. VI.

The CONSTRUCTION *and* Rationale *of the* REFRACTING MICROSCOPE *and* MEGALASCOPE, Single *and* Compound.

Plate I. Fig. XI. 65. THE Nature and Conſtruction of a *Refracting* MICROSCOPE being ſimilar to that of a Teleſcope, will require but little to be ſaid to explain them. They are indeed eſſentially the ſame Thing, and differ only in *Mode* and *Accidents*. The ſame Thing is alſo to be obſerved of that Inſtrument we properly call a MEGALASCOPE, as being of a middle Nature between both the former.

Fig. VIII. 66. We have already obſerved, that if ABC be regarded as a diſtinct Object, then IFH will be the Image thereof formed near the Focus *a* of the Glaſs *g e d*, which in that Caſe was conſidered as the Object-Glaſs of the Teleſcope, (*Art.* 39.) But on the other Hand, if IFH be a ſmall Object, placed near to the Focus (*a*) of a Lens *g d*, whoſe focal Diſtance *e a* is very ſmall; then will an Image AC be formed thereby of the Object IH, which will be larger than the Object in the Proportion of *e*B to *e*F; and in this Caſe the Glaſs *g d* is the Object-Lens of a MICROSCOPE.

67. But this Lens may be applied two Ways to magnify the Appearance of an Object, *viz.* (1.) By itſelf, and then it is called a *Single Microſcope*; or, (2.) in Conſtruction with another, and then it is called a *Compound Microſcope*.

Plate I.

Fig. {III IV}

68. It is evident from what we have ſaid, (*Art.* 13.) that an Object placed in the Focus F of any Lens BD, will be ſeen diſtinctly by the Eye placed near to it, becauſe the Rays from every Point of that Object will be refracted parallel to the Eye, which is all that is neceſſary for diſtinct Viſion (37.)

69. Now it is known by Experience, that no Object can be ſeen by the naked Eye at a leſs Diſtance than *ſix Inches*, generally; therefore as much as the focal Diſtance FH of a ſmall Lens is leſs than ſix Inches, ſo many Times will the Object be magnified, that is, it will appear under an Angle ſo much larger than what it ſubtends to the naked Eye; ſince it is a well known Propoſition in Perſpective, *that the apparent lineal Dimenſions of Objects are as the Optic Angles under which they are ſeen by the Eye.*

70. Hence an Object placed in the Focus of a Lens at the Diſtance of *one Inch*, will appear through it ſix Times *longer and broader* than

it

it can be viewed by the Eye, and therefore 36 Times larger in *Surface*, and 216 Times in *Bulk or Solidity*.

71. Again, ſuppoſe the focal Diſtance of a *Spherule*, or very ſmall Lens, were only $\frac{1}{10}$ of an Inch; then is the Object view'd by it at a Diſtance 60 Times leſs than ſix Inches, and conſequently will appear magnified 60 Times in *Length and Breadth*, 3600 Times in *Surface*, and 21600 Times in *Bulk*, or ſolid Content.

72. When the focal Diſtance is ſmaller, the magnifying Power is as much greater in Proportion, and ſome Spherules have been made, whoſe focal Diſtances have not exceeded the 40th or 50th Part of an Inch. Such Glaſſes diſcover exceedingly ſmall Objects, and yet ſome of them appear but as animated Points, and require yet greater Powers of Magnifying to ſhew their Parts diſtinctly.

73. When the focal Diſtances of Glaſſes are leſs than *an Inch*, they are properly called MICROSCOPES, as they are fitted to render the *ſmalleſt Objects viſible*.

74. But Glaſſes whoſe focal Lengths are from one Inch to ſix, may be eſteem'd MEGALASCOPES in different Degrees, as they magnify all the *larger Sort of ſmall Objects*, and by this Means afford a much more beautiful and ſatisfactory View of ſuch Objects than can be had by the Eye alone.

Plote I.
Fig. XI.

75. In the ſecond Way of magnifying ſmall Objects, we uſe two Glaſſes, *viz.* an Object-Glaſs d e f, which makes a large Image ABC of a ſmall Object a c b placed near its Focus (*f*). This Image AC is viewed diſtinctly through the Eye-

Eye-Glaſs DEF placed at its focal Diſtance EB, and under the Angle DIF by the Eye at I; whence it is evident, that this Compound Microſcope in *Fig*. XI, is nothing more than the the two *Fig*. VIII and IX put together. And that whereas in a *Single Microſcope* we view the Object itſelf immediately; in the Compound one, we view the Object ab only in its magnified Image AC.

76. The Power of Magnifying in a Compound Microſcope, is thus eaſily underſtood; let the Diſtance of the Object ec from the Lens df, be *one Inch*, and the Diſtance of the Image AC be eB ſix Inches; then if we view the Image AC with the bare Eye, we ſhall ſee the Object therein *ſix Times magnified*: But if we view this Image through the Lens DF placed at its focal Diſtance EB one Inch, then will itſelf appear ſix Times larger than before to the Eye alone, and conſequently the Object will appear ſix Times ſix, or 36 Times magnified. Or the Angle DIF (equal to CEA) will be 36 Times bigger than that which the Object ab alone ſubtends to the naked Eye at the Diſtance of ſix Inches.

* * *

PART II.

Containing the THEORY of the *Aberration* of *Rays* ariſing from the Figure of the Glaſſes; the *Indiſtinctneſs* of VISION occaſioned thereby; the Means of correcting the ſame by a proper *Combination of Lenſes*; and of the NEW FORMS of TELESCOPES derived from thence, conſiſting of *three*, *four*, *five*, and *ſix Glaſſes*, with their *moſt advantageous Poſitions*.

CHAP. I.

The THEORY *of the* ABERRATION *of* RAYS *cauſed by the* FIGURE *of* Glaſſes, *and the Indiſtinctneſs of* VISION *ariſing from thence, explained.*

77. THE Teleſcopes hitherto deſcribed for viewing Objects erect, conſiſt of four Glaſſes only; and we find no other in Uſe to the Times of Mr. *William Molyneux* here, and *Hugenius* abroad. But neither of theſe Authors have given us a Print or Diagram of a Teleſcope with any more than four Glaſſes, tho' each of them has conſidered the Nature of Viſion by two Convex Lenſes inſtead of one; and *Hugens* only has mentioned the Advantage of ſuch a Combination. What *Molyneux* did was only to find the focal Diſtance of two ſuch Glaſſes

Glaſſes which he received from Mr. *Flamſteed*, in a Letter dated *Jan.* 17, 168$\frac{6}{7}$.

Plate II.

Fig. IV.

78. The Uſe of two Glaſſes is to correct or diminiſh the Error of one ſingle Glaſs ariſing from its Spherical Figure, therefore it will be firſt neceſſary to explain what this Error is, and then how it is in a great Meaſure corrected by two Glaſſes. The firſt who gave the Rationale thereof was Sir *Iſaac Newton*, whoſe Method of explaining this Matter I have ſhewn at large in my *Philoſ. Britannica*, from the following Diagram, where NBM is the Spherical Surface of a Plano-Convex Lens NGMB; C the Center; CB the Radius or Semi-diameter taken in the Axis; AN an incident Ray, and NK the ſame refracted, cutting the Axis produced in the Point K. Alſo let F be the Focus of parallel Rays which paſs through the Glaſs infinitely near to the Axis; let FD be a Perpendicular to the Axis in the Point F, then will KF be the *Error*, or *Difference of the Focal Diſtance of parallel Rays which are incident near the Axis and at the Diſtance* GN, the *Semi-aperture of the Lens*. This is called the *Aberration of the extreme Ray in Longitude*.

79. Again, let any Ray (*an*) be incident on the other Side the Lens, at the Diſtance *b*G, the refracted Part of this Ray *nd* will interſect the other refracted Ray ND in the Point Q, at the Perpendicular Diſtance QO from the Axis; this is called the *Lateral Error*, or the *Aberration* in *Latitude*.

80. It is evident from the Figure, that as the Ray (*an*) approaches the extreme Ray *A*M,

the Point of Interſection Q will approach the Axis; and when *a n* coincides with *A*M, the Point Q will coincide with the Point K in the Axis; and it is as obvious that Point Q will coincide with F, when the Ray (*a n*) approaching the Axis aB, at laſt becomes coincident with it: Therefore there is *one Poſition* of the Ray (*a n*) in which it will cut the Ray ND in a Point Q, which will make QO a *Maximum*, or the greateſt of all.

81. If we take the Arch B*m*=B*n*, and BM=BN, the Rays incident on *m* and M will interſect in the Point P on the other Side, and ſo make PQ=2QO; and it is alſo plain (1.) That all the Rays which fall on the Lens between N and M are refracted through the Space PQ. (2.) That PQ is the Diameter of the leaſt Circular Space poſſible, in which all the Rays can be congregated, becauſe there will be ſome Ray (*a n*) that will meet the extreme Ray ND, at the Diſtance QO=$\frac{1}{2}$QP from the Axis. (3.) Hence it follows, that this circular Space is the *Focus* or *Place of the Image* of an Object belonging to parallel Rays incident on the Lens NM.

82. Further, by reaſon of ſimilar Triangles KOQ, KFD, and NGK, we have QO : KO :: DF : KF :: NG : GK. But it is demonſtrated, ‡ that when QO is greateſt, then KO=$\frac{1}{4}$KF, and alſo that KF is always $\frac{9}{2}$ of GB, the Thickneſs of the Lens; ſo then KO=$\frac{9}{8}$GB, and conſequently GK : GN :: $\frac{9}{8}$GB : QO, whence $\frac{9\text{GB}\times\text{GN}}{8\text{GK}}$=OQ; whence PQ, the Diameter

of

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‡ Philoſ. Britannica.

of the Circle of Aberration, is known for any given Lens.

83. It is alſo demonſtrated,† that the Errour PQ will always be proportioned to $\frac{NG^3}{BC^2}$; ſo that when the Radius is given, *the Error will be as the Cube of the Aperture directly:* And when the Aperture is given, *the ſaid Error will be as the Square of the Radius inverſely.*

84. It is alſo to be demonſtrated, that when the Convex Side of the Lens NBM is turned towards parallel Rays, the Error KF will be but $\frac{7}{6}$ of the Thickneſs of the Lens GB, and *therefore near four Times leſs than in the other Caſe*; for $\frac{9}{2}$GB : $\frac{7}{6}$GB : : 54 : 14, which is almoſt as 4 to 1.

85. If the Side NGM, inſtead of being a Plane, were a Spherical Surface deſcribed, with a Radius equal to ſix Times the Radius CB, then the Error KF would ſtill be leſs; for then it would be $\frac{15}{14}$ of the Thickneſs of the Lens. But if this Lens were turned with the leaſt convex Side to the diſtant Radiant, the Error would be $\frac{145}{42}$ of the Thickneſs. In the firſt Poſition it is the beſt Glaſs, and in the laſt, the worſt of all others; yet in the beſt Poſition, it differs ſo little from the *Plano-Convex*, as not to deſerve notice.

86. If the Side NGM were Concave, ſo that the Lens became a *Meniſcus*, there is no Proportion of the Radii, or Poſition of the Lens with regard to the Radiant, but what will give the Aberration KF greater than in the Plano-Convex in its beſt Poſition. And ſince this was firſt obſerved by Opticians, the *Meniſcus* began to looſe

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† Ibid.

looſe Ground in the Conſtruction of Optical Inſtruments, and is now quite rejected, tho' formerly in ſuch high Eſtimation.

87. Laſtly, it is demonſtrated, that the Aberration PQ *is as the Square of the Sine of Refraction* (the *Sine of Incidence* being *Unity*) in all *Mediums* of different refractive Powers: Thus if a Lens of the ſame focal Diſtance and Aperture were made of *Glaſs and Water*, and ſuppoſe thoſe Sines in Glaſs to be as R to 1, and in Water as *r* to 1; then will PQ in the Glaſs Lens be to the ſame in the Water Lens as R^2 to r^2, or the Area of the Circles of Aberration, and of Courſe the Indiſtinctneſs of the Object *will be as the Refractions* R and *r of the Mediums.*

88. Whatever has been obſerv'd with regard to *Convex* and *Plano-Convex Lenſes*, will hold good in *Concave and Plano-Concave ones.* And in both Sorts, it is ſuppoſed that all of them have the *ſame focal Diſtances*, *Apertures*, *and Thickneſſes*, while we are comparing their reſpective Aberrations.

Fig. V. 89. After the ſame Manner Sir *Iſaac Newton* has ſhewn how to eſtimate the Aberration ariſing from the Figure of the Speculum NBM, by reflected Light. Thus let CB = Radius, FB = $\frac{1}{2}$CB = Diſtance of the principal or ſolar Focus F; and AN, *an*, two incident Rays parallel to the Axis, and ND, *nd*, the ſame reflected, interſecting each other in Q, and the Axis in K and *k*, as by refraction before. Alſo let NG be the Sine, and GB the verſed Sine of the Angle of Incidence ANK or NKB.

90. Then it is demonſtrated,* *That the Longitudinal Aberration* FK$=\frac{1}{2}$GB *nearly*; and therefore always *proportion'd to the verſed Sine of Incidence* GB, *or to the Square of the Sine* NG, becauſe GB is as NG^2 conſtantly.

91. Again, when QO is a Maximum, then KO$=\frac{1}{4}$KF; and becauſe KG is nearly equal to BF, therefore $GK:GN :: KO : QO=\frac{GN\times KO}{GK}$; whence *the Value of* PQ (the *lateral Error*) *is known*, becauſe KO$=\frac{1}{4}$KF$=\frac{1}{8}$GB.

92. Laſtly, it is ſhewn that this lateral Error PQ is always *as the Cube of* NG *the Semi-aperture of the Speculum*, when the Radius is given; and *as the Square of Radius inverſely*, when the Aperture of the Mirrour is given: That is, univerſally, PQ is as $\frac{NG^3}{CB^2}$ (as at *Art.* 83.)

Fig. VI.

93. From what has been ſaid, it is very evident, that if Rays proceed from any Point, as (*a*) at an infinite Diſtance to a Lens NM, the Image of that Point will not be *a Point*, but the *Area of a Circle*, whoſe Diameter is PQ; and therefore that Point (*a*) *cannot be diſtinctly repreſented*; but will be *render'd indiſtinct and confuſed in Proportion to the Area of the ſaid Circle of Aberration* in the Lens or Speculum, as it is the Image of this Circle (or *dilated Point*) that is impreſs'd on the *Retina*, and excites the Idea of the Point (*a*) in the Mind.

94. Hence it appears, that the Points in the Surfaces or Subſtances of Bodies *cannot be perfectly and diſtinctly ſeen*, as each of them will be dilated into a ſenſible Area; and ſuch as are con-

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* Philoſ. Britannica.

contiguous (as *a*, *b*, *c*,) will have their confus'd Images all blended together nearly in the ſame Space, *viz.* the *Circle of Aberration*, whoſe Diameter is PQ.

95. Hence the Stars, which, as to Senſe, are only lucid Points; will appear to have ſome Magnitude (and not as Points) in the Focus of the beſt Sort of Teleſcopes, even ſuppoſing there were no other Cauſe of Confuſion or indiſtinct Viſion beſides what reſulted from the *Spherical Figure of the Lens or Speculum.*

CHAP. II.

The Method of correcting the Errors *of this* Aberration *by a proper* COMBINATION *of Convex* GLASSES; *with a general* THEORY *for that Purpoſe.*

96. AS the indiſtinct Viſion of Objects is ſo ſenſible and conſiderable in ſingle Lenſes, and increaſes in proportion as the *Square of the Radius* decreaſes, (83) it is no Wonder if the Learned in Opticks have employ'd their Skill to correct it as much as poſſible, in order to render Dioptric Inſtruments more perfect. Several Ways have been eſſay'd for this Purpoſe, but none ſo ſucceſsfully as that by uſing a Compoſition of *two Lenſes* inſtead of *one*; and the firſt I find deſcribing this Method is the celebrated *Hugenius* in his *Dioptrica Nova*, who has ſaid but very little on the Subject; nor do I find

at all conſider'd by any late Writer in Optics; and, to the beſt of my Remembrance, I have ſeen no Diagrams of Teleſcopes where this Contrivance has been applied, but that one of the above-named Author, and what has been copied from it.

97. But as much depends on this Doctrine, I ſhall here deliver the general Theory, and then apply it in the Conſtruction of Teleſcopes: Suppoſe parallel Rays AN, BM, fall on a Lens MN, and are refracted to a Focus at F; if at any Diſtance leſs than CF another Convex-Lens GH be placed to receive the ſaid refracted Rays, it will converge them ſooner to ſome Point f, which is call'd the *Compound Focus* of both the Lenſes. The Optic Angle G f H is equal to that form'd by ſome ſingle Lens EF, whoſe focal Diſtance is Q f. Fig. VII.

98. Let
$$\left\{\begin{array}{l} \left.\begin{array}{l} CF=F \\ OP=y \\ Qf=x \end{array}\right\} \text{be the focal Diſtance of the Lens} \left\{\begin{array}{l} NM \\ GH \\ EF \end{array}\right. \\ Of=f, \text{ the Compound Focal Diſtance.} \\ CO=D, \text{ the Diſtance of the Lenſes NM and GH.} \end{array}\right.$$

99. The Rays from the firſt Lens NM fall *converging* on the ſecond GH, tending to the Point F; if now we put $OF=d$, we ſhall have by common Optics $d=\frac{yf}{y-f}=F-D$; from which Equation we get $F+y-D : F-D :: y : f$. And thus the *compound focal Diſtance* Of is known.

100. After the ſame Manner the parallel Rays LG, SH are refracted by the Lens GH

to the Lens NM tending to a Point y; but are converged by it to a Point f, which is the *compound Focus* on that Side; and now it is $Cy=\frac{\mathrm{F\,f}}{\mathrm{F-f}}=y-\mathrm{D}$; whence $\mathrm{F}+y-\mathrm{D}:y-\mathrm{D}::\mathrm{F}:\mathrm{f}=\mathrm{C\,f}$, which therefore is known alſo.

101. Becauſe of the ſimilar Triangles FNC, FGO, and fEQ, fGO; and EQ=NC; we have CF : OF :: NC (=EQ) : GO :: Qf : Of; that is, $\mathrm{F}:\mathrm{F-D}::x:f$. Whence $\frac{\mathrm{F}\,f}{\mathrm{F-D}}=x$.

102. But we had above (99.) $f=\frac{\overline{\mathrm{F-D}}\times y}{\mathrm{F}+y-\mathrm{D}}$; which ſubſtituted for f in the laſt Article will give $x=\frac{\mathrm{F}\,y}{\mathrm{F}+y-\mathrm{D}}$; from which Theorem we have a Solution of the following *Problem*, *viz.*

103. *Given the focal Lengths of two Lenſes, and the Diſtance between them; to find the focal Length of a ſingle Lens that has the ſame magnifying Power, or Optic Angle, with the two combined ones.*

By this RULE,

Divide the *Product* of the two focal Lengths of the given Lenſes, by their *Sum* leſſen'd by their *Diſtance*, and the Quotient will be the focal Length of the ſingle Lens as required.

104. From that general Theorem, the Analyſt will eaſily deduce the following Solutions, *viz.*

105. Given F, x, and D, to find y? The Theorem is $y=\frac{\mathrm{F}\,x-x\,\mathrm{D}}{\mathrm{F}-x}$.

106. Alſo,

106. Alſo, Given y, x, and D, to find F? The Theorem is $F=\frac{xD-xy}{x-y}$.

107. Laſtly, Given F, y, and x, to find D? The Rule is $D=F+y-\frac{Fy}{x}$.

108. From theſe Theorems, and others to be deduced from them, by ſuppoſing $D=o$, $F=y$, $F-D=y$; or ſuppoſing F, y, or f, to be negative, *&c.* any thing relating to a Compariſon between a Combination of two Lenſes and a ſingle one in reſpect of the magnifying Power, Diſtinctneſs of Viſion, *&c.* may eaſily be determined, whether for Eye-Glaſſes or Object-Glaſſes, or both, as we ſhall ſee further on.

109. We may alſo obſerve from the foregoing Analogies (99, 101.) that we have the Ratio of the *compound focal Diſtances* Of and Cf, thus $f : \mathrm{f} :: \overline{F-D}\times y : \overline{y-D}\times F$; and therefore when $f=\mathrm{f}$, then $F=y$, or the ſaid focal Diſtances can never be equal but when the Lenſes are ſo.

110. Laſtly, we may obſerve, that ſince the parallel Rays LG SH, refracted thro' both the combined Lenſes, interſect the Axis in the ſame Point f, as it would do if it were refracted by the ſingle Lens EF, as is evident by continuing it to R; therefore ſince GO=RQ it will follow, that the Diameter IK of the principal Pencil of Rays KfI diverging from the Focus f will be the ſame as it would be if it proceeded directly to the ſingle Lens EF; and conſequently this Combination of Lenſes makes no Alteration in that Reſpect.

Scholium.

111. From what has been ſaid of a Combination of Lenſes, it appears that if we conſider GH as a *Conſtant Lens*, and NM as a *variable one*, then (1.) if F be the Focus of *parallel Rays* falling on GH, thoſe Rays will all be collected into that Point (7.) (2.) The Addition of the Lens NM, *if Convex*, will cauſe the *parallel Rays* AN, BM, to become *converging* ones NG, MH, which therefore by the Lens GH will be refracted to a Focus *f*, ſo that the focal Diſtance O*f* will be leſs than OF. (3.) If NM be *Concave*, the *parallel Rays* will be refracted thro' it diverging on GH, by which Means they will be refracted beyond the Point F to ſome Focus P; all which is evident from common Optics. (4.) In each Caſe the *Aberration from the principal Focus will be as the Square of* OF *to the Square of* O*f or* OP *reſpectively*; as is plainly demonſtrated by *Hugenius* in his Diop. Pag. 247. but would be too tedious here to tranſcribe.

CHAP. III.

The foregoing THEORY *continued for a* Computation *of the Degree of* Diſtinctneſs *and* Indiſtinctneſs of Viſion *in a propoſed* Combination *of Glaſſes.*

112. WE have ſeen the Effects of a Combination of two Glaſſes in regard to the *magnifying Power*, we ſhall now examine according to the Theory what the Caſe will be with reſpect

reſpect to the *Diſtinctneſs of Viſion*, that is to ſay, what the Diſtinctneſs of the View will be thro' *two Lenſes* compared with that thro' *one* of the *ſame magnifying Power.*

113. In order to this it muſt be conſider'd, that ſince the *Aberration*, or *Indiſtinctneſs of Viſion*, is as the *Square of Radius inverſely*, (83.) the *Diſtinctneſs of Viſion* will be as the *Square of Radius directly*; and therefore if by Means of *two Glaſſes we can get the View of an Object*, where the *Radii* of the Glaſſes bear a greater Proportion to their reſpective Apertures, than the Radius of a ſingle Glaſs of equal magnifying Power does to its Aperture, it is evident the Diſtinctneſs of that View will be promoted in Proportion to the Square of that Ratio.

114. For Example: Suppoſe $F-D=y$, or OF to be the focal Diſtance of the Lens GH, ſo that the Focus of each of the Lenſes NM and GH falls on the ſame Point F; then by the Theorem (102) we have $x=\frac{1}{2}F$, or $Qf=\frac{1}{2}CF$: Alſo, ſince in this Caſe we have $F:y::x:f$ (101) therefore $f=\frac{1}{2}y$, or $Of=\frac{1}{2}OF$. Now ſince we have the *ſame Optic Angle* GfO by both the Glaſſes, as by the ſingle one EF, the Ratio of the Radius OF to the Aperture GO, or of the Radius CF to the Aperture NC, is double the Ratio of Of to OG, or of the Radius Qf to the Aperture EQ, and therefore the *Diſtinctneſs of Viſion* by both the Lenſes is *four Times greater than that by the ſingle Lens* EF.

115. The ſame Thing may be demonſtrated from the Conſideration that the Aberration PQ

is in the ſame Glaſs always proportion'd to the *Cube of the Semi-aperture* EQ, or *Sine* of *half the Optic Angle* E*f*Q, (ſee *Art.* 83.) and that in ſmall Angles (as in the Object Glaſſes of Teleſcopes, *&c.*) the Sine EQ is nearly as the Angle E*f*Q. The Aberration therefore being as the Cube of that Angle, it is plain if we make the ſame Angle by *two Refractions inſtead of one*, the Quantity of the Aberration will be greatly leſſen'd, ſince *the Sum of the Cubes of the Parts will be much leſs than the Cube of the whole*; and when the Parts are *equal*, *the Sum of Cubes of each will be but a fourth Part of the Cube of the whole.* Thus if the whole Angle E*f*Q be as 1, the Cube thereof is 1; but the Half is $\frac{1}{2}$, the Cube of which is $\frac{1}{8}$, and twice that $\frac{2}{8}=\frac{1}{4}$, which is as the Aberration ariſing from the *two Halves*, and is therefore but *a fourth Part of the whole.*

116. This is evidently the Caſe when the Optic Angle G*f*O (=E*f*Q) is made by two Refractions, by the two Lenſes NM and GH, ſo poſited that the Focus of each may fall on the ſame Point F; for then the Angle G*f*O=LG*f*, which is compoſed of the two Angles LGF, and FG*f*, the firſt of which LGF=TNF=GFO, (by reaſon of the parallel Lines TN, LG, and FC) which is the Part made by the Lens NM. Alſo the Angle FG*f* is the Refraction of the Ray NG, or ſecond Refraction of the Ray AN; and ſince in the preſent Caſe, O*f*=*f*F (as we have ſhewn 114.) and O*f* in ſmall Angles is equal to G*f* nearly; therefore the Angle GFO is equal to the Angle Fig. VII.

FG*f*

FGf very nearly, thoſe Angles being in the ſame Ratio with the equal Lines Gf and fF, when they are not large; and the Optic Angle $GfO = GFO + FGf$; conſequently the *Aberration* PQ *is but* $\frac{1}{4}$ *Part ſo great by the two Lenſes* NM *and* GH *together, as it is by Lens* EF *alone.*

117. But to render this Theory general for any Poſition or Form of the Lenſes NM and GH, it is evident, ſince the Aberration is leſſen'd by dividing the Optic Angle, the Diſtinctneſs of Viſion will be thereby promoted; and becauſe each of the Angles contribute thereto in Proportion to its Magnitude, *the joint Effect of both Parts, or Angles* GFf and fGF, *will will be as the Product or Rectangle under both*, or *as the Rectangle of the Lines* Of and fF; but according to our former Notation (98.) $Ff = F - D - f$; and $Of = f$; conſequently $Ff - Df - ff$ will be every where as the *Diſtinctneſs of Viſion by the two Lenſes above that of a ſingle Lens, of the ſame magnifying Power.*

118. Let the Degree of Diſtinctneſs thus obtained be repreſented by $G = Ff - Df - ff$; when this is a *Maximum*, or the greateſt poſſible, the Fluxions thereof will be nothing, *viz.* $F\dot{f} - D\dot{f} - 2f\dot{f} = 0$, whence $F - D = 2f$; or $FO = 2Of$, that is $Of = fF$, or the Angle $GFf = fGF$, in the beſt Poſition of the Lenſes, as before demonſtrated.

119. Conſequently ſince in that Poſition we have ſhewn the Diſtinctneſs of Viſion is 4 Times as great as by a ſingle Lens (116.) that will be the whole Effect of a Combination of *two Glaſſes*, and it may be ſhewn that 3 Glaſſes will produce 9 Times the Diſtinctneſs; and ſo on in Propor-

tion to the *Square of the Number of Glaſſes*; but then if we conſider the Evil to be remedied is but ſmall, and the Damage we ſuſtain in Loſs of Light and Irregularity of Refraction thro' ſo many Lenſes, we may ſoon make the Remedy worſe than the Diſeaſe; and every thing conſider'd, it will appear *that two Lenſes are better than a greater Number*.

120. We ſhall now apply this Theory in ſome practical Caſes, and illuſtrate them by Example. In order to this, let the focal Lengths of the Lens NM be $F=3$, that of the Lens GH be $y=2$, and then in their beſt Poſition we have alſo $F-D=y=2$; whence the focal Length of the equal Lens GH will be (102.) $x=\frac{Fy}{F\times y-D}=\frac{Fy}{2y}=\frac{1}{2}F=1,5=Q\,f$.

121. In the Example above, the Diſtance of the Lenſes $CO=F-y=D=1$; if this Diſtance be increaſed, both the magnifying Power and Diſtinctneſs will be diminiſhed; thus ſuppoſe $D=2$, then $x=2$: But as the focal Length of the Lens EF encreaſes, the magnifying Power of the Teleſcope decreaſes. Alſo $f=\frac{Fx-Dx}{F}$ (99.) $=\frac{2}{3}$; therefore $\overline{F-D-f}\times f=G=\frac{2}{9}$, which is the Degree of Diſtinctneſs, and is leſs than $\frac{2}{8}$ or $\frac{1}{4}$ (when $D=1$) in the Proportion of 8 to 9.

122. Again, ſuppoſe $D=3$, then $F-D=0$ and $x=F$; but in this Caſe the compound focal Diſtance $f=\frac{\overline{F-D}\times x}{F}=0$; conſequently the Diſtinctneſs $\overline{F-D-f}\times f=G=0$; ſo that the Advantage gain'd

gain'd by the Combination of two Lenſes here, ceaſes in that Reſpect alſo.

123. On the other Hand, if both the Lenſes are placed together, ſo that $D=0$; then the focal Diſtance of the *equal Lens* will be $x=\frac{Fy}{F+y}=1,2$; the *compound focal Diſtance* will be $f=x=2$: Alſo the Diſtinctneſs will be as $F-f\times f=G$ (117.) but $F-f:f::\frac{3}{5}:\frac{2}{5}$ (taking $F-D=F=1$) therefore $F-f\times f=\frac{3}{5}\times\frac{2}{5}=\frac{6}{25}=G$, the Diſtinctneſs in this Poſition, which is not much leſs than in the beſt, where $D=1$, and the Diſtinctneſs as $\frac{1}{4}=\frac{6}{24}$. Wherefore the Ratio in this Caſe and that, is as 24 to 25. But 24 is nearer to 25 than 8 is to 9, (as in (121.) where $D=2$) and conſequently this Poſition is better than that.

124. Again it is evident, from the Theorem $x=\frac{Fy}{F+y-D}$, that however you vary the Magnitude of y, (provided it does not exceed F) neither the magnifying Power or Diſtinctneſs of Viſion will be alter'd thereby in the beſt Poſition of thoſe Glaſſes, becauſe ſince then $F-D=y$, it will ever be $x=\frac{1}{2}F$, a conſtant Quantity; and becauſe in this Caſe y or OF will ever be biſected by the compound Focus f, the Diſtinctneſs will ever be the ſame, and a *Maximum*, (119.)

125. In what has been ſaid, Regard has been had to the Viſion of an Object RS placed in the compound Focus f exterior to the Glaſſes; but it makes no odds, if that Focus be ſuppoſed *negative*, or the Image *between* the Glaſſes, which will always be the Caſe when D is greater than

y. Thus let $F=3$, $D=2$, $y=1$, then $x=1,5$ as before, and the Degree of Diſtinctneſs alſo the ſame, ſince $F-D=y$.

126. If the focal Diſtance of one Lens be ſuppoſed infinite, as F, then the Lens NM becomes a plain Glaſs, and the Compoſition ceaſes, and conſequently the Diſtinctneſs of Viſion occaſioned thereby now vaniſhes alſo.

127. If one of the Lenſes be concave as NM, then will its focal Diſtance be negative, or $-F$; and the Theorem (102.) becomes $x=\frac{-Fy}{y-F-D}$, for the Focus of the *equal Lens* EF, which will always be *poſitive* while $F+D$ exceeds y; but otherwiſe *negative*, unleſs $y=F+D$, then it becomes *infinite*, or the Rays are refracted parallel to each other from the Lens GH.

128. The *compound focal Diſtance* is in this Caſe $f=\frac{-Fx-Dx}{-F}$, (99.) and conſequently muſt always be *affirmative* while x is ſo. And when $D=0$, then $f=x$.

129. The Expreſſion for the Diſtinctneſs of Viſion (118.) now becomes wholly negative or $-Ff-Df-ff=G$, or changing all the Signs, $Ff+Df-ff=-G$; but *negative Diſtinctneſs* is *Indiſtinctneſs*, therefore *Objects are viewed to a Diſadvantage by Convex and Concave Lenſes combined together*.

130. For Example: Let the Concave NM have a negative focal Diſtance $-F=3$, alſo let $y=2$, and their Diſtance $D=1$; then will the focal Diſtance of the *equal Lens* EF be $x=\frac{-6}{2-4}=3$, and the compound focal Diſtance

$f=$

$f=\frac{-12}{-3}=4$. Whence in this Caſe $FO=2fO$, which is juſt the Reverſe of the Caſe where F is poſitive, or the Lens NM convex, (118.)

131. If $-F=y$, and $D=0$, that is, if a Concave and Convex Lens be placed together, and their focal Diſtance equal, then x becomes infinite, or the Rays after Refraction will be parallel as before.

132. But if two ſuch Lenſes are placed at a Diſtance from each other, they will have a magnifying Power equal to that of ſome Lens EF.

For Example: Let $-F=3$, $y=3$, and $D=1$; then the focal Diſtance $x=\frac{-9}{-1}=9$; and the compound focal Length is $f=\frac{-18}{-3}=6$.

133. A *Concave and Convex Lens* placed together of different focal Lengths, will have a magnifying Power. Thus, ſuppoſe $-F=3$, $y=2$, and $D=0$; then $x=\frac{-6}{-1}=6$. And in this Caſe $f=x=6$ alſo.

134. From theſe Inſtances it appears, that a Convex and Concave Lens combined together, always magnify leſs than the Convex Lens alone, and ever diminiſhes the Diſtinctneſs, and therefore ought by no Means to be uſed but when ſome particular Circumſtances render ſuch a Combination neceſſary.

Scholium.

135. From what has been ſaid it evidently appears, that the *compound focal Diſtance* of the combined Lenſes is nothing more than the focal Diſ-

Diſtance f found by the common Theorem of Optics $\frac{d\,r}{d-r}=f$, adapted to the conſtant Lens GH, where $OF=r$, and $OP=f$, when the Rays are *diverging*; or $Of=-f\left(=\frac{-d\,r}{-d-r}\right)$ when the Rays fall converging on the Lens GH.

136. Since the Aberration is ever as OF^2 to Of^2, or OP^2; (111.) therefore when $F=3$, and $y=2$, and $D=1$, we have OF^2 to Of^2, as 4 to 1; ſo the Aberration in the Focus f is 4 Times leſs than in F, agreeable to (116.)

137. But if it be $-F=3$, $y=2$, and $D=o$; then $OF : OP :: 2 : 6$; and therefore $OF^2 : OP^2 :: 4 : 36 :: 1 : 9$. Whence the Aberration in the compound Focus f will be to that in the Focus P, as 1 to 36; *or the Aberration will be 36 Times greater with the Concave, than with the Convex combined with the Lens* GH, in the Caſes now ſpecified.

CHAP. IV.

The beſt DISPOSITION *of Glaſſes in* Dioptric Inſtruments, *according to the foregoing* THEORY; *and the different* FORMS *of* TELESCOPES *deduced from thence.*

138. AFTER having thus particularly ſtated and illuſtrated the Theory of diſtinct and indiſtinct Viſion, ſo far as it depends on the Figure of the Glaſſes, and is to be remedied by a Com-

a *Combination* or rather *Duplicature* of them; there will be no Occaſion to ſpend much Time in the Application of it to the Improvement of our common refracting Teleſcopes.

139. For with regard to the Glaſſes themſelves, it is evident this Theory may be applied as well to *Object-Glaſſes* as *Eye-Glaſſes*; for when we ſuppoſed $F=3$, and $y=2$, and $D=1$, *&c.* if theſe Numbers denote Inches, they are applicable to *Eye-Glaſſes*; but if they denote *Feet*, then they will have Relation to *Object-Glaſſes*, for in this Caſe $F=36$, $y=24$, and $D=12$ Inches. And therefore if the Teleſcope be conſtructed with two Object-Glaſſes, whoſe focal Lengths are 36 and 24 Inches, and placed at the Diſtance of 12 Inches, they will magnify as much as a ſingle Glaſs whoſe focal Length is $x=15$ Inches, or Half that of the greateſt alone, (124.)

140. Then with regard to the Poſition of the Glaſſes of either Sort, we have ſhewn that it is indifferent whether they are placed together or aſunder, provided they are ſo placed that their Focus's coincide or fall on the ſame Point; for in all ſuch Caſes the magnifying Power will not be alter'd; and the Diſtinctneſs of Viſion will be the greateſt poſſible,(118.) *viz.* 4 Times as great as by the *ſingle Lens* of the ſame *magnifying Power*.

141. Where there are but three Glaſſes in a Teleſcope, if the Image be between the two Glaſſes next the Eye, and their Diſtance D be equal to, or exceeds F the focal Diſtance of the largeſt, then the ſaid largeſt Lens is to be conſider'd as com-

combined with the Object-Glaſs; in all other Caſes it is combin'd with the Glaſs next the Eye, and this is a Circumſtance neceſſary to be adverted to, as it makes a conſiderable Difference in the magnifying Power, as will appear by the following Example.

142. Let the focal Diſtance of the Object-Glaſs NM be =15 Inches, that of the middle Glaſs GH =3; the Diſtance D=12; then will the Image be equal to that made by a ſingle Lens of 7,5 Inches focal Diſtance: And if the Eye-Glaſs AB be 1,5 Inch, the Power of magnifying will be as 5 to 1. Whereas if the middle Glaſs be combined with the Glaſs near the Eye in the beſt Manner, the magnifying Power will then be as 15 to 1,5, or as 10 to 1, which is double of the former.

Plate III. Fig. I. 143. But if the two Glaſſes next the Eye, *viz.* AB and GH, be ſo poſited that their Focuſes may coincide, and the Diſtance between them exceed the focal Diſtance of the leſſer Glaſs AB, then will the Glaſs GH contribute to form the Image IK more perfectly than could be done by the Object Glaſs NM alone; and alſo, to correct the Error in viewing it by a ſingle Eye-Glaſs AB. Therefore this muſt be the beſt Diſpoſition of Glaſſes, for an *Aſtronomical Teleſcope.*

144. To form the Teleſcope for viewing *Terreſtrial Objects* in the beſt Manner, that is, to ſhew them upright and as free from the Aberration of the Figure of the Glaſſes as poſſible, to the three Glaſſes NM, GH, and IQ of the Aſtronomical Teleſcope, (143.) we muſt add three others, *viz.* TR, OP, and AB; the firſt of theſe TR is to

Fig. II.

be of the ſame Size and focal Length with LQ, and placed at twice the Diſtance of the compound Focus S from the ſaid Glaſs; and what that Diſtance is, we have ſhewn how to find. By this Means a ſecond Image VW will be formed of the firſt in a contrary Poſition to it, and therefore the ſame with that of the Object itſelf, as is evident from (58.)

145. This Image VW is view'd to the greateſt Advantage alſo by two Glaſſes O P and AB, as we have ſo often ſhewn from the Theory. And by the two Glaſſes extraordinary GH and OP, the *Field of View* is conſiderably enlarged, and the Uſe of the Inſtrument made ſo much the more agreeable. For the Lens GH contracts the Image of each particular Object, and therefore brings many more into the Field, which are view'd more perfectly by the Glaſs OP in conjunction with AB, than by the Lens AB alone. All which is evident from the ſame Principles.

146. But ſince this Diſpoſition of Glaſſes is very different from that of the *Six-Glaſs Teleſcope* commonly ſold in the Shops, it will be neceſſary to ſhew the Reaſon why we place them in this Manner, and this it will be eaſy to comprehend from the following Diagram.

147. For let I M be the Image formed by the Object-Glaſs, and I A a Ray incident on the firſt Eye-Glaſs A B; this Ray will be one of thoſe belonging to a Pencil proceeding from the Point I, and whoſe Axis is the Ray IG paſſing thro' the Center F of the Lens AB. Plate III. Fig. III.

148. Now let the Point I be the Focus of Rays parallel to the Axis IG, and infinitely near to it; and let Ad be another Ray parallel to the ſaid Axis, but incident on the Glaſs at the Diſtance AF, then by the Theory it appears (78.) that the Focus of ſuch a Ray will be ſome Point (a) in the Axis nearer to the Glaſs than the Point I; ſo that I a will be the Aberration of ſuch a Ray.

149. Therefore ſome Ray DA inclined to the Axis I G and diverging from it, will fall on the Glaſs AB in the Point A, and be refracted to the Point I. Therefore, *vice verſa*, the Ray IA will be refracted to its proper Focus in its own Axis IG produced; the Diſtance of which from the Glaſs AB may be eaſily found by the Theorem $\frac{d\,r}{d-r}=f$ in common Optics, becauſe a I will always be equal to $\frac{5}{3}$ of the Thickneſs of the Lens AB, if it be equally convex, that is I a $=\frac{5}{3}$ FR. But FE the geometrical focal Diſtance of the Lens is known, and therefore FI $=d$, and conſequently Fa $=r$, the focal Diſtance of parallel Rays dA; whence $f=$ focal Diſtance belonging to the Ray IA after Refraction is known of Courſe.

150. The Ray AD in its Progreſs falls on the ſecond Lens CD, which we ſuppoſe to be equally convex, or the ſame with the Lens AB, and placed at double the focal Diſtance (HO $=$ OR) from it. Let QH be drawn thro' H (the Center of the Lens CD) parallel to IG, then ſhall this Line continued be the Axis proper to the refracted Ray DL, which it will interſect in L

151.

151. Now becauſe LQ is parallel to IG, and the Ray AD is equally inclined to both, it muſt at its Incidence at A and D ſuffer an equal Refraction in all Reſpects, and therefore DL will be equal and parallel to AI: Alſo HD is equal to AF, and PL equal to EI. Conſequently the Point I in the Image IM is repreſented at L in the ſame Situation and Diſtance, with reſpect to the Lens CD and its Axis PH. And becauſe what we ſaid with regard to the Ray IM, and the Point I, is equally demonſtrable of every other Point in the Image IM, it follows, that the ſecond Image L K is *exactly ſimilar*, and *every way equal to the firſt Image*, but in an *inverted Poſition*.

152. Hence it is plain, that the Error of Refraction from the Figure of the Glaſs AB is adequately corrected by the ſecond Glaſs CD, and that the Image LK is, ſo far, a perfect Repreſentation of IM. And it is not in the Power of Art by Glaſſes to make an Image *more perfect than the Object*, or to give a more diſtinct View of an Object in its Image, than we have of itſelf when view'd under the ſame Angle, even ſuppoſing there were nothing in the Glaſs to hinder the moſt perfect Refraction of Light.

153. If now we ſuppoſe the Glaſs CD not ſimilar, and equally convex with the Glaſs AB, it will follow, that the Ray AD will not be alike and equally refracted by it; and that the Aberration occaſion'd by the Glaſs AB cannot be adequately corrected by the Lens CD in a con-

trary Refraction, and therefore that the ſecond Image LK cannot be equal to, nor equally perfect with the firſt IM.

CHAP. V.

Of the Improvement *and* New Conſtruction *of* Compound MICROSCOPES, *by the ſame Theory.*

154. AS the Aberration from the Figure of the Glaſſes required a Correction in the *refracting Teleſcope*, ſo does it alſo in the *compound Microſcope*, and in a much greater Degree, ſince the more Convex a Lens is, and the greater the Diſtance between the geometrical and and proper Focus, the greater will this Kind of Error prove; and therefore in the Microſcope, where both theſe Points are in an extreme Degree, there will be required the utmoſt Aſſiſtance from Art to correct it.

Plate III. Fig. IV. 155. This appear'd ſo ſenſibly the Caſe, even to former Opticians, that they were ſoon directed to add to the ſingle Eye-Glaſs AB another and larger Lens EF, by which Means the Viſion of the Object OQ was render'd more perfect in the Image IM, and the Field of View at the ſame Time enlarged, as in the Teleſcope.

156. But

156. But notwithſtanding this Improvement by the Addition of a third Lens to the Microſcope is ſo very obvious both in Practice and Theory, yet *Hugenius*, who has conſider'd this Sort of Aberration in Microſcopes more particularly than any one, has not given any Iconiſm of this Inſtrument with more than two Glaſſes. The ſame may be ſaid of Mr. *Molyneux*, Dr. *Gregory*, and moſt other Authors, who ſeem wholly unapprized of the Advantage of this Sort of Conſtruction, in Point of *Diſtinctneſs of Viſion*, at leaſt they ſay nothing at all of the Matter.

157. It is not only evident from the Theory of this Aberration, that the Image of any Point is render'd leſs confuſed by Refraction thro' *two Lenſes*, than by an equal Refraction thro' one; but alſo it follows from the ſame Principle, that the ſame Point has its Image ſtill leſs confuſed when form'd by Rays refracted thro' *three Lenſes*, than by an equal Refraction thro' two; and therefore a third Lens GH added to the former two, will contribute to make the Image more diſtinct, and conſequently the Inſtrument more compleat.

158. At the ſame Time the Field of View is amplified, and the Uſe of the Microſcope render'd more agreeable by the Addition of the Glaſs GH, and that very ſenſibly above what is to be found in the *common Conſtruction* of a *Three-Glaſs Microſcope*.

159. Since the Diſtinctneſs of Viſion is thus promoted by the Glaſs GH, we may allow a ſomewhat larger Aperture to the Object Lens

KI; and thereby encreaſe the Brightneſs of Objects, and greatly heighten the Pleaſure of viewing them.

160. And tho' each of theſe Particulars is eaſily confirm'd by Experiments, as well as demonſtrable from Theory, yet are all the Optic Writers totally ſilent with reſpect to a *Four-Glaſs Microſcope*. And as there is no mention of ſuch a Thing, there is no *Iconiſm* of it of Courſe; and therefore this which I have here given, will prove at leaſt a Novelty to the Reader in Print.

161. For I do not pretend to ſay no Microſcope with four Glaſſes has been made, I know they have, and it is more than 30 Years ſince I bought one of the ſame Conſtruction nearly with what I have here given; and it excell'd by much any Three-Glaſs Microſcope I ever ſaw; but could not learn who the Maker was, it being an old Inſtrument when I firſt had it. It had its Origin therefore from Experience, and not from Theory; as in all Probability it was made before *Hugenius*'s Dioptrics was publiſh'd, which is the firſt Book that contains the Theory of the Aberration from the Figure of Glaſſes that I have heard of.

162. As to the beſt Manner in which theſe four Glaſſes are to be placed in the Microſcope, there can be no Difficulty to thoſe who underſtand the Theory; and thoſe who do not, it does not at all concern. But both in the Teleſcope and Microſcope there will always be *once particular Diſpoſition or Conſtruction* of a given Number of Glaſſes that will be the beſt of all others, and it is the Excellency of a *Theory*, that by it alone ſuch an intereſting Point can be determined.

PART III.

P A R T III.

Containing the THEORY of the *Aberration* of RAYS ariſing from their *different Refrangibility*; and the Method of removing the ſame by a proper COMBINATION of LENSES of *different Figures* and *refractive Powers*, demonſtrated from the Nature of the *priſmatic Refraction* of Light. With Theorems for finding the focal Diſtances of *double* or *triple* COMPOUND LENSES; and their Application to TELESCOPES and MICROSCOPES.

C H A P. I.

The general Doctrine *of* REFRACTION *of* Light *explain'd*; *alſo the* different Refrangibility *and* colorific Quality *of* heterogeneal RAYS.

163. WE are now to conſider the *ſecond Source* or Cauſe of indiſtinct or imperfect Viſion in *dioptric Inſtruments*, *viz.* that which reſults from the *different Refrangibility* of the Rays of Light, in their Paſſage thro' *Mediums* of different refractive Powers; and of the proper Methods by which the *Aberration* of Rays, and the Imperfection of dioptric Viſion occaſion'd

ſion'd thereby may be remedied, or rather totally removed. This, it is true, was conſider'd in ſuch a Light by the great Author of this Part of Pholoſophy, as by him to be thought impracticable; but Experience often convinces us that the wiſeſt Men may be miſtaken, and the Annihilation of this Sort of Aberration or Error is ſo far from being impoſſible or deſperate, that it may be effected more Ways than one, as has been diſcover'd more than 20 Years ſince; and which we ſhall now proceed to explain, having firſt premiſed the following Principles, by way of *Data*, as they are confirm'd by the moſt certain and obvious Experiments.

164. *Firſt*, That the Particles of all Matter are affected with certain Powers, by which they are *attracted and repelled.*

165. *Secondly*, That theſe Powers *are of greater Energy* in the Particles of ſome Sorts of Matter or Bodies than in others.

166. *Thirdly*, That this Power acts with the greateſt Force *upon the Surface of Bodies*, and *decreaſes* not ſimply as the *Diſtance increaſes*, but according to *ſome Power of* the Diſtance.

167. *Fourthly*, That it is the ſame, or acts with the ſame Force, at equal Diſtances from the Surface.

168. *Fifthly*, That a *Ray of Light* is *real Matter*, or that the Particles of Light are only Particles of Matter attenuated or ſubtilized to an extreme Degree.

169. *Sixthly*, That the Particles of Light and all other Matter *mutually act upon each* other *by Attraction and Repulſion.*

170. *Seventhly*, That the Force of any Body is (*cæteris paribus*) *proportion'd to the Number of Particles* it contains, or to its *Denſity*.

171. *Eighthly*, That this Force is (*cæteris paribus*) porportionate in any Body to the *intenſive or peculiar Force of its Particles*.

172. *Ninthly*, That therefore the abſolute Force of any Body is proportional to the *Denſity* thereof, and to the *ſpecific Force* of its Particles conjointly.

173. *Tenthly*, That the ſame Particles of Matter in ſome (if not in all) Bodies are affected with *different Forces on their different or oppoſite Sides*. For Satisfaction in this Particular, ſee Sir *Iſaac Newton*'s Optical Queries relating to *Iſland Cryſtal*.

174. Theſe Principles, as I ſaid, are indiſputably true by different and numerous Experiments; on theſe, therefore, we now proceed to explain the various Phænomena of refracted Light and Viſion occaſioned thereby; obſerving the Conſequences, and making the neceſſary Deductions as we go along, for explaining what relates to the *Diſtinctneſs* and *Indiſtinctneſs of Viſion* by *dioptric Inſtruments*, and particularly *Teleſcopes*, and how far they are capable of Improvement from this Doctrine.

175. Let ABC be a Section of the *Blade* of a *Knife*, held with its Edge a little below a Beam of the Sun's Light, *a b c*; and it appears by Experiment, that the Rays A*a* which paſs neareſt the Edge of the Knife, are inflected towards the Side thereof to (*d*); and thoſe which are farther from the Edge, are leſs bent out of their proper Direc- **Fig. V. Plate III.**

Directions, till at laſt the attracting Force ceaſing at the Diſtance of the Ray (be) that Ray paſſes ſtrait on: But immediately beyond, the Rays are repell'd by the Knife, and from the Direction cg, they are inflected the other way towards f. So that by both theſe Powers the Beam of parallel Rays is made to ſpread and diverge after it has paſs'd by the Edge of the Knife. See much more on this Head in Sir *Iſaac Newton*'s Optics, and *Graveſande*'s Elements, &c.

Fig. VI. 176. Let there be two contiguous Mediums zt and sy ſeparated by a plain Surface st; and ſuppoſe sy the denſeſt Medium of the two: Let the Diſtance, to which the attracting Power of its Particles extends, be denoted by sq; and take $sv=sq$; and thro' q and v draw qr and vw parallel to st; then is $qr\ wv$ called the *Space of Attraction*.

177. Now ſuppoſe a Ray of Light were incident from the Medium zt on the Medium sy in the Direction ad; then the Particles of this Ray, when they arrive at the Space of Attraction at d, will be attracted by the new Force of the Medium sy in a Direction dg perpendicular to the Surface st; and the Particles there being urged in two different Directions, ad and dg, can't poſſibly proceed in either of them but in one that is compounded of both, which will be inclined towards the Perpendicular dg.

178. But becauſe the attracting Force increaſes according to ſome Power of the decreaſing Diſtance from the Surface st on each Side (166.) therefore the new Direction of the Ray cannot

be

be a ſtrait Line but a Curve, as it is continually urged in a perpendicular Direction by a Force uniformly accelerating the ſame. Wherefore during the Time of its being within the Space of Attraction *q w*, it muſt deſcribe ſome Curve from *d* to *k*.

179. When the Particles arrive at *k* in the Limit of the Space of Attraction *v w*, they will then be again affected with an equal Force on every Side, and their Motion will now be again uniform, and they will proceed in the right Line *kp*, which is a Tangent to the Curve *d k* in its loweſt Point *k*: Conſequently the Ray *k p* muſt make Angle with, or be inclined to the incident Ray *a d*, which is alſo a Tangent to the Curve in its firſt Point *d*. All this follows from the Principles of *Mechanics*.

180. And ſince this is the Caſe of any other Rays *b e*, *c f*, &c. it is evident the whole incident Beam of Light *a c f d* will paſs, or be refracted in a curvilineal Direction thro' the Space of Attraction into the Beam *k p m n*, whoſe Velocity will now be greater than in the firſt Medium, by all that it acquired from the *accelerating Force* of this new *Medium*.

181. This Space of Attraction is however ſo very ſmall, as not to be ſenſible in phyſical Matters; and may therefore be repreſented by a Right Line interceding the two Mediums.

182. Let therefore AB be the Surface of any denſe *Medium*, as Glaſs, Water, &c. and ſuppoſe IC a Ray of Light incident upon it from the *Air* in the Point C. On that Point, as a Center, let a Circle be deſcribed, and let HE paſs thro' Fig. VII.

the ſame Point C perpendicular to the Surface AB. Then is the Angle ICH called the *Angle of Incidence*, of which ab is the Sine.

183. The attracting Power of the Medium ABDG being greater than that of Air, will cauſe the Ray IC to be bent out of its firſt Direction ID into another CF, making a leſs Angle ECF with the Perpendicular EC; the Sine of which Angle is em, and is call'd the *Sine of Refraction.*

184. The Angle FCD meaſures the Quantity of the Refraction; and which we ſhall call the *Total Refraction.*

185. It is found by Experience, that the Sine of Incidence ab is to the Sine of Refraction em always in one conſtant Ratio, let the Angle of Incidence be what it will; which ſuppoſe to be as m to n: That is, it will always be as $ab : em :: m : n$.

186. If FC be a Ray paſſing out of a denſe *Medium* into Air of a leſs refractive Power, it will then be refracted from the Perpendicular into the Direction CI; and the Sine of Incidence em will be to the Sine of Refraction ab as n to m.

187. If the Rays were all of *one Kind*, or equally affected by the attracting Force of Bodies, there would be an equal Refraction of all the Parts of a Beam of Light; but this is not the Caſe, for Experiments ſhew that *ſome* Fig. VIII. *Rays of the Beam will always be more refracted than others.* Thus, for inſtance, if IC be a Beam incident from the Air on any more powerful Medium ABDG, then Part of it will be refracted

to *r*, making the Angle *r*C*b* of the *leaſt Quantity*, and C*r* is of Courſe the *leaſt refrangible Ray* of that Beam.

188. Again, another Part of the Beam will be refracted to *v*, making the Angle *v*C*b* of the *greateſt Quantity*; and therefore C*v* is the *moſt refrangible Ray*. The Ray C *m*, which equally divides the Angles *v* C *r*, is the *mean refrangible Ray*. And *m* C *b* is the Quantity of mean *Refraction*, and what for the future will be called *General Refraction* of the Medium; as the Angle *v*C*r* will be called the *Angle of Diſſipation*, as being that into which the entire Beam IC is diſſipated by Refraction.

189. The Rays of the diſſipated Beam *v*C*r* are generally reckon'd of *ſeven Sorts*, becauſe each Sort has a different Effect on the Fibres of the Optic Nerve, and excites a peculiar Idea of Colour in the Mind; and the diſſipated Beam itſelf, if view'd as it comes from a Priſm, appears variouſly colour'd from *r* to *v*. The Colours are in the followidg Order, *Red*, *Orange*, *Yellow*, *Green*, *Blue*, *Indigo*, and *Violet*.

190. Alſo *d r*, *e m*, *f v*, are the Sines of the *leaſt refrangible* or *red-making* Rays C *r*, of the mean or green Rays C *m*, and of the leaſt refrangible or violet colour'd Rays C *v*; and when the Angle of Incidence ICH is ſmall, the Angle of Refraction EC *m* is ſmaller; and in Caſe of ſuch Angles as are leſs than 20 or 15 Degrees, *the Proportion of the Sines and of the Angles themſelves may be reckon'd the ſame*, without any ſenſible Error.

CHAP. II.

The THEORY *of the* ABERRATION *or* DISSIPATION *of* heterogeneous LIGHT, *explain'd and illuſtrated.*

191. THE next Step in our Buſineſs is to raiſe a few eaſy Theorems for aſſertaining the Quantity of the *total and general Refraction*, and of the *Angle of Diſſipation*, under all the Circumſtances of *different Angles of Incidence*, and *Mediums* of *various Denſities* and *ſpecific Powers* of the Particles.

In order to this let the Quantities be repreſented by the following *Symbols*, *viz.*

$m =$ The Sine of Incidence.
$n =$ The Sine of Refraction.
$D =$ Denſity of the Medium.
$S =$ Specific Power of its Particles.
$R =$ Refractive Power of the Medium.
$T =$ Total Refraction.
$A =$ Angle of Diſſipation.
$I =$ Angle of Incidence.
$F =$ Angle of Refraction.

192. Then in ſmall Angles, (190.) when the Refraction is made out of a leſs into a more powerful Medium, we ſhall have $m : n :: I : F$; and

and therefore $m : m-n :: I : I-F$; conſequently $\frac{m-n}{m} \times I = T$, the Total Refraction, or Angle FCD, (*Fig*. VII.)

193. But if the Refraction be made out of a greater into a leſs refractive Medium, then $n : m :: I : F$, and $n : m-n :: I : F-I = T$; therefore $\frac{m-n}{n} \times I = T$, the total Refraction or Angle vCa.

194. Hence it appears, ſince n is ever as I, the Refraction T will always be as $m-n$, let the Ray be refracted which way it will. Alſo when I is given, $\frac{m-n}{n} = T$; or $\frac{m-n}{m} = T$, (192.)

195. Moreover ſince the refractive Power R of any Medium is as D when S is given, and as S when D is given, (170, 171) it will in general be as DS ; that is, we ſhall always have $R : DS$.

196. It is alſo evident, ſince Effects are always proportion'd to their Cauſes, that therefore, in a given Angle of Incidence, the Refraction T will be as the Cauſe R which produces it, *viz.* $T : R$; whence $\frac{m-n}{n} : DS : R : T$.

197. Laſtly, it is plain that the Quantity or Angle of Diſſipation vCr is ſome Part of the Refraction mCb ; or that $A = \frac{1}{p} \times T$, or $pA = T = \frac{m-n}{n} \times I$.

Fig. VIII.

198. Let us now compare theſe Quantities in any two different Mediums ABD, which we will denote by X, and ABF, which let be call'd Y; and then ſuppoſe the Quantities denoted by the Roman

Roman Capitals above belong to X; let m, n, d, s, r, t, a, p, i, in ſmall Italics ſignify the ſame Things in the Medium Y. And becauſe we ſhall more generally conſider the Refraction as made out of a greater X into a leſs refractive Medium Y, we ſhall uſe the Expreſſion $\frac{m-n}{n}$ in our Compariſons. (See 194.)

199. And therefore we have for the firſt Analogy $\frac{\mathrm{m-n}}{\mathrm{n}} \times \mathrm{I} : \frac{m-n}{n} \times i :: \mathrm{T} : t$.

200. And when I and i are given, we have $\frac{\mathrm{m-n}}{\mathrm{n}} : \frac{m-n}{n} :: \mathrm{T} : t :: \mathrm{R} : r :: \mathrm{DS} : ds$, (195.)

201. But to ſhorten the Work, let $\frac{\mathrm{m-n}}{\mathrm{n}} = \mathrm{b}$, and $\frac{m-n}{n} = b$. Then $\mathrm{pA} : : pa :: \mathrm{T} : t :: \mathrm{bI} : bi$, (199.)

202. Theſe Analogies will afford us Theorems ſufficient for our Purpoſe, after we have firſt determined the true Value of theſe ſeveral Quantities in ſuch Mediums as will be neceſſary to conſider, in regard to Viſion, by refracted Light in dioptric Inſtruments, and particularly TELESCOPES and MICROSCOPES.

* * *

CHAP. III.

Of the General *and* Specific REFRACTION, *and* DENSITY *of* Mediums; *with the* ANGLES *of* DISSIPATION, *and* THEOREMS *for computing the ſame.*

203. THE Quantities to be determined, and their Values in different Mediums aſcertain'd, are the following: (1.) The Ratio of *m* to *n*, or of the Sine of Incidence to that of Refraction. (2.) The refractive Power of the Medium. (3.) The Denſity of the Medium. (4.) The ſpecific Power of its Particles. (5.) And the Angle of Diſſipation.

204. The *Firſt* of theſe is to be done very accurately, either by a *Priſm* or a *Lens*, or many other Ways well known. The *Second* is determined by meaſuring nicely the Quantity of Refraction made in equal Angles of Incidence in ſeveral Mediums, The *Third* is known from the ſpecific Gravity of the Bodies or Mediums taken by the *Hydroſtatic Ballance.* The *Fourth* is found by dividing the refractive Power by the Denſity: And the *Fifth* is eaſily known by meaſuring accurately the Length of the colour'd Image of the Sun form'd in the Focus of a Lens placed by the Priſm, for then having the focal Diſtance of the Lens, the Angle of Diſſipation is known, and from thence alſo what Part it makes of the whole Refraction.

205. Moſt of theſe Quantities have been determined by Sir *Iſaac Newton* for many different Mediums and Bodies, which I have tranſcribed from him as in the following Table.

The refracting Bodies.	The Proportion of the Sines of Incidence and Refraction of yellow Light.	The refracting Force of the Body.	The Denſity and ſpecifick Gravity of the Body.	The ſpecific refractive Power of the Body.
	$m : n$	DS. : R	D	S
A Pſeudo-Topaz.	23 to 14	1,699	4,27	0,3979
Air.	3201 to 3200	0,000625	0,0012	0,5208
Glaſs of Antimony.	17 to 9	2,568	5,28	0,4864
A Selentis.	61 to 41	1,213	2,252	0,5386
Glaſs vulgar.	31 to 20	1,4025	2,58	0,5436
Cryſtal of the Rock.	25 to 16	1,445	2,65	0,5450
Iſland Cryſtal.	5 to 3	1,778	2,72	0,6536
Sal Gemmæ.	17 to 11	1,388	2,143	0,6477
Alum.	35 to 24	1,1267	1,714	0,6570
Borax.	22 to 15	1,1511	1,714	0,6716
Niter.	32 to 21	1,345	1,9	0,7079
Dantzic Vitriol.	303 to 200	1,295	1,715	0,7551
Oil of Vitriol.	10 to 7	1,041	1,7	0,6124
Rain Water.	529 to 396	0,7845	1,	0,7845
Gum Arabick.	31 to 21	1,179	1,375	0,8574
Spirit of Wine well rectified.	100 to 73	0,8765	0,866	1,0121
Camphire.	3 to 2	1,25	0,996	1,2551
Oil Olive.	22 to 15	1,1511	0,913	1,2607
Lintſeed Oli.	40 to 27	1,1948	0,932	1,2819
Spirit of Turpentine	25 to 17	1,1626	0,874	1,3222
Amber.	14 to 9	1,42	1,04	1,3654
A Diamond.	100 to 41	4,949	3,4	1,4556

206. In this Table the Ratio of *m* to *n* are as in the *firſt Column*; thus, for the Medium of *Water*, we have $m : n :: 529 : 396$; for *Glaſs*, (ſuch as he uſed) $m : n :: 31 : 20$; and ſo of the reſt.

207. The *ſecond Column* ſhews the Ratio of the refractive Powers R and r for different Mediums; thus, for *Glaſs and Water*, we have $R : r :: 1,4025 : 0,7845 :: DS : ds$; and, with reſpect to *Glaſs and Diamond*, it is $1,4025 : 4,949 : DS : ds$.

208. The *third Column* contains the Ratio of the Denſities D, d, of thoſe Bodies. Thus, for *Glaſs and Water*, $D : d :: 2,58 : 1$. Of *Glaſs and Diamond*, $D : d :: 2,58 : 3,4$.

209. In the *fourth Column* are contain'd Numbers expreſſing the ſpecific Attraction S, s, of the Particles of thoſe Bodies; as, for *Glaſs and Water*, we have $0,5436 : 0,7845 :: S : s : \frac{R}{D} : \frac{r}{d}$.

From this Table we may obſerve,

210. Firſt, That the *denſeſt Medium has not always the greateſt refractive Power*. Thus the Denſity of *Water* 1 is greater than that of *Spirit of Turpentine* 0,874; but its refractivePower 0,7845 is leſs than that of the Spirit 1,1026: And hence it will follow, that a Ray of Light IC paſſing out of a denſe Medium X into a rarer Y, may be refracted from C to *b towards the Perpendicular* CH. This is eaſily confirm'd by Experiments. Plate III. Fig. IX.

211. Secondly, *That Light may paſs out of one Medium into another of different Denſity, and yet ſuffer no Refraction.* For thoſe *Media* may have the ſame refractive Powers, as *Borax* and *Olive Oil* have each of them 1,1511, but different Denſities 1,714 and 0,913. Conſequently a Beam of Light IC paſſing out of a Priſm of *Borax* X

into another of *Oil Olive* Y, will not be refracted but paſs ſtrait on to *b*.

212. Thirdly, *Light may be refracted in paſſing out of one Medium into another, tho' the Denſity of each be the ſame*; for one may exceed the other in the ſpecific Refraction of its Particles. Thus X may be a Priſm of *Dantzic Vitriol*, and Y a Priſm of *Alum*, whoſe Denſities are the ſame, but their ſpecific Refractions as 0,7551 and 0,657, and therefore a Beam of Light IC paſſing out of X into Y will be refracted from the Perpendicular CH into the Beam C *a*.

213. Fourthly, *A Beam of Light may paſs thro' two different Mediums, and be refracted from the laſt* without any Diſſipation *of its Rays*; *and yet the Direction of the refracted Ray ſhall not be* parallel to, *but ſhall make* an Angle *with the Direction of the incident one*. This follows from the ſame Principles. But as it is the *fundamental Article* on which the Buſineſs of preventing the Imperfection of Viſion, cauſed by the Error ariſing from the *Diſſipation* or *different Refrangibility of Light* in dioptric Inſtruments entirely depends, it will be neceſſary to give a more ample and particular Elucidation thereof in the next Chapter.

⁂

CHAP.

CHAP. IV.

The DENSITIES, *and* general *and* ſpecific REFRACTIVE POWERS *of different Sorts of* GLASS, CRYSTAL, &c. *determin'd by* EXPERIMENTS.

214. THE Refraction of Light thro' any other Mediums than thoſe which are uſed in dioptric Inſtruments, will be of no Uſe to conſider; and ſince only the different Sorts of *Glaſs* and *Cryſtals* are fit for ſuch Purpoſes, we muſt next enquire, what Effect they will have in refracting the Rays of Light, both *ſingly* and *combined* under all the different Circumſtances of *Denſity*, *refractive Power*, *ſpecific Refraction*, and *Diſſipation of the Rays*; firſt in the Form of PRISMS; and ſecondly, in the Form of Analogous LENSES.

215. For this Purpoſe it became neceſſary in the firſt Place to determine the Ratio of m to n, the ſeveral Mediums of Glaſs and Cryſtal as are in common Uſe, which I did by the following Method. In reſpect of any given Lens, double and equally convex, if we put $r=$ Radius; $d=$ Diſtance of the Radiant, or Object; $f=$ to its proper focal Diſtance, and $b=\frac{m-n}{n}$; then by a Theorem demonſtrated in my *Philoſ. Britannica*, we have this other Theorem, *viz.* $\frac{dr+rf}{2df}=b$.

216. To get the Values of theſe Quantities, I cauſed Lenſes of the ſeveral Sorts of Glaſs and Cryſtal to be ground on a Tool, whoſe Radius was juſt 21,5 Inches, ſo that in all of them $r=21,5$.

217: Then at the Diſtance of 436 Inches from a luminous Object, I placed a Screen in a dark Place, to receive the Image thereof, by the Lens moved forwards and backwards on a Rule divided into Inches and Tenths, till the Image became perfect; and then by Inſpection the Value of f was known for the ſeveral Lenſes, which ſubducted from the whole Diſtance 436, left the Value of d, or the Diſtance of the Radiant from the Lenſes ſeverally.

218. Thus in a Lens of *white Flint-Glaſs* $r=21,5$, $f=18,75$, and $d=417,25$; therefore by the Theorem (215.) $\frac{dr+rf}{2\,df}=b=0,5998=\frac{m-n}{n}$; whence by putting $n=1$, we have $m-1=0,5998$; and ſo $m=1,5998$; therefore $m:n :: 1,5998 : 1 :: 1,6 : 1$ nearly.

219. If the Radiant be the Sun, then d is infinite, and the Theorem becomes $\frac{r}{2b}=17{,}923$ for the *ſolar focal Diſtance*, which therefore is leſs than the Radius by 3,577 Inches.

220. In a Lens of *common Plate* or *Coach Glaſs*, I found $f=19,65$, and $d=416,35$, and $r=21,5$ as before; then $\frac{dr+rf}{2\,df}=0,573$: whence ($d$ being infinite) $\frac{21,5}{2\times 0,573}=18,76$ the Diſtance

of

of the ſolar Focus. And in this Caſe $m : n :: 1,573 : 1$.

221. In a Lens of *Crown-Glaſs* I found $f=21,25$, $d=414,75$, and $r=21,5$; therefore $\frac{dr+rf}{2df}=0,5318$; the Diſtance of the ſolar Focus $=20,214$; and the Ratio $m : n :: 1,5318 : 1$.

222. The *yellow Plate*, or *Venetian Glaſs*, as alſo *Brazil Pebble*, have ſo nearly the ſame Numbers for the focal Diſtance f, and the Ratio of m to n with the *Crown-Glaſs*, that the Difference is ſcarce diſcernable by Experiment, and therefore not worth Notice.

223. The next Thing to be done was to determine the *Denſities* of thoſe ſeveral Mediums or Lenſes, which I did by taking their *ſpecific Gravities* by the hydroſtatic Balance, and found them as below.

The ſpecific Gravity of	White Flint — —	1 to 3,29
	Common Plate Glaſs	1 — 2,76
	Crown Glaſs — —	1 — 2,52
	Yellow Plate ——	1 — 2,52
	Brazil Pebble — —	1 — 2,62

224. Hence it appears, that the refracting Powers of the ſeveral Lenſes were very nearly as their ſpecific Gravities, which are nearly equal in *Crown Glaſs*, *yellow Glaſs*, and *Braſil Pebbles*; as alſo in that Glaſs call'd *Dutch White*, and in *Corniſh Diamond*.

225. As *white Flint* and *Crown Glaſs* are in the Extremes, and are moſt eaſy to be had for Optical Uſes, I ſhall chuſe them for a Compariſon.

And in theſe we have found $R : r :: DS : ds :: 0,5998 : 0,5318$ (ſee 196, 218, 221.) and alſo $D : d :: 3,29 : 2,52$; whence we have $S : s :: 0,5998 \times 2,52 : 0,5318 \times 3,29 :: 1,5115 : 1,7496$. Whence it appears, that tho' the *Denſity* of *Crown Glaſs* be leſs than that of *white Flint*, yet the *ſpecific Attraction* of its *Particles* is conſiderably greater.

226. Having thus determined the *refractive Powers*, *Denſities*, and *ſpecific Attraction* of theſe two Sorts of Glaſs in general, we ſhall proceed to conſider the *Phænomena* of Light refracted thro' them in the Form of Priſms, in regard to the Quantity of *Refraction*, *Diſſipation*, and *Direction* of the incident and emergent Rays.

CHAP. V.

Of the Priſmatic REFRACTION *of* LIGHT. THEOREMS *determining the Angles of* Total REFRACTION *and* DISSIPATION *of* RAYS; *with the* METHODS *of rendering the* refracted Beam Colourleſs, *tho' inclined to the* incident One *in any* given Angle.

Plate III. Fig. IX.

227. IN treating of the Refraction of Light thro' a Priſm, as BAD, it muſt be obſerved, that the Angle ICE contain'd between the Perpendiculars IC and EC to the Sides of the Priſm, is always equal to the vertical Angle A contain'd under thoſe Sides reſpectively; and this will always be the Caſe, whatever be the Quantity

of the Angle at D. But IC is here the incident Ray on the Side AB, and therefore the Angle of Incidence is equal to the Angle of the Priſm at A, which is call'd the *refracting Angle of the Priſm*, becauſe the Refraction occaſion'd by the Priſm is proportion'd to that Angle, in as much as it is in Proportion to the Angle of Incidence. For ſince the Ratio of m to n is conſtant, the greater m is, the greater n will be; and alſo the greater will be their Difference $m-n$, which is the Refraction we ſpeak of.

228. Wherefore let ABC be a Priſm of *Crown Glaſs*, a Beam of Light sp parallel to the Baſe AC, will be refracted thro' it to q, and from them a ſecond Time refracted and diſſipated into the Beam rqv, the mean Ray of which gq contains an Angle gqh, which is the total Refraction of the Priſm, and which is expreſs'd by the Theorem $t=pa=bi$, (201.) Plate IV. Fig. I.

229. Suppoſe BCD an equal Priſm of *white Flint Glaſs* in an inverted Poſition with reſpect to the other, and SP a Beam of Light incident on it in the ſame Direction as before, it will be refracted from P to Q, and from Q it will be refracted and diſſipated into the Beam RQV, the mean Ray of which will make an Angle GQH $=T=Ap=bI$, (201.) Fig. II.

230. Let QF be drawn parallel to gq, and the Angle GQF will be equal to the Difference of Refraction in the two Priſms; and therefore when they are both placed together, (as in *Fig.* III.) the ſame incident Beam sp refracted thro' the firſt from p to q, and thro' the Second from q to $\mathcal{Q}$ will be from thence refracted and diſſipated into the Beam *VQR*, the mean Ray of which

which *GQ* will contain an Angle *GDH* with the incident Ray *sH*, equal to the above-mentioned Angle *GQH*, or Difference of Refraction in the two Priſms.

231. Whence in this Combination of the two Priſms, ſince the refracting Angles at C and B are equal, we have $\mathrm{I}=i$; and therefore $\mathrm{pA} : pa :: \mathrm{b} : b$ (201.) Therefore $\mathrm{A} : \frac{\mathrm{b}}{\mathrm{p}}$, and $a : \frac{b}{p}$; and conſequently $\mathrm{A}-a : \frac{\mathrm{b}}{\mathrm{p}} - \frac{b}{p}$, which therefore will be as the Angle *GQH* of Diſſipation.

232. Alſo we have $\mathrm{p} : p :: \frac{\mathrm{b}}{\mathrm{A}} : \frac{b}{a}$, and therefore if the Angles of Diſſipation *rqv* and RQV be exactly meaſured in the equal Priſms, the Quantities p and p will be known. Let the Values of A and a in this one Caſe be denoted by Z and z; then in the general Equation $\mathrm{pA}bi = ap\mathrm{bI}$, when $\mathrm{I}=i$ (as in the preſent Caſe) we ſubſtitute Z and z for A and a, and it becomes $\mathrm{p} : p :: z\mathrm{b} : \mathrm{Z}b$.

233. If we ſuppoſe the Angle ABC encreaſed, or the Angle BCD diminiſhed to a proper Degree, there will be no compound Refraction, but the emergent Ray *QH* will be parallel to the incident one *sp*. In this Caſe $\mathrm{T}=t$, and $\mathrm{pA}=pa$, (201.) and ſo $\mathrm{A} : a :: p : \mathrm{p}$; and conſequently there will be a Diſſipation of the emergent Beam, and the Object will appear *colour'd* tho' in its proper Place. In this Caſe $\mathrm{bI}=bi$, and ſo $\mathrm{I} : i :: b : \mathrm{b} :: 0{,}5318 : 0{,}5998$. Or *the refracting Angles will be inverſly as the refracting Powers of the Priſms.*

Fig. III.

234. The Angle DCB may be ſo far diminiſhed, or the Angle ABC ſo much encreaſed, that the Angle of Diſſipation *RQV* ſhall quite vaniſh, or the Rays of the refracted Beam ſhall be all parallel among themſelves: Since in the general Theorem $\mathrm{p\,A}\,b\,i = a\,p\,\mathrm{b\,I}$; if we put $\mathrm{A} = a$, we have $\mathrm{I} : i :: \mathrm{p}\,b : p\,\mathrm{b} :: \mathrm{z\,b}\,b : \mathrm{Z\,b}\,b :: z : \mathrm{Z}$. Conſequently when the refracting Angles of the Priſms are inverſely as the Values of Z and *z*, (232.) there will be no Diſſipation of Rays, *and the Object will be ſeen by refracted Light without Colours.*

235. But in this very Caſe there will be a Refraction of the Beam from the Direction of its Incidence; for ſince $\mathrm{p\,A} : p\,a :: \mathrm{T} : t$, when $\mathrm{A} = a$, or $\mathrm{A} - a = 0$; then $\mathrm{T} : t :: \mathrm{p} : p$. And ſo $t - \mathrm{T}$, or the compound Refraction by both the Priſms will be as $p - \mathrm{p}$, a conſtant Quantity.

Fig. III.

236. This may be effected two Ways, *viz.* by a Combination of *two Priſms*, or *three*; for if with a Priſm of *Crown Glaſs* ABC you combine one of *white Flint* ECB, whoſe Angle ECB is to ABC as *z* is to Z, it will anſwer the Purpoſe.

Fig. IV. V.

237. Alſo, if while the Priſms ABC and DCB remain together with equal refracting Angles, a third Priſm of Crown Glaſs be added to them, as C D E, whoſe refracting Angle D, together with B ſhall be to the Angle of the Flint Priſm C as Z to *z* the ſame Thing is effected; for then the incident Beam *sp* will be refracted thro' the *three Priſms* in ſuch a Manner, that the refracted Beam PS will make an Angle SPH with the incident Beam *sp* continued, and with-

Fig. V

out any Diſſipation of the Rays, ſo that the Object or Radiant will now appear without Colours at G elevated above its true Place by the Angle *s*PG.

238. There are other Ways of ſhewing how this Effect is produced from ſuch a Combination of two or three Priſms, but what I have ſaid is ſufficient to give the intelligent Reader an Idea of it, as it may moſt naturally be ſhewn by Priſms: I now proceed to conſider the ſame Thing in a *Combination of Lenſes* in *Teleſcopes.*

CHAP. VI.

Of the DOCTRINE *of* REFRACTIONS *by* LENSES *of different* Mediums, *and the* CONSTRUCTION *of* TELESCOPES *with* compound LENSES *for preventing the* DISSIPATION *of Rays producing* colour'd Light.

Plate IV. Fig. VI. 239. WITH regard to the Refraction of Light thro' Priſms and Lenſes, it will appear to be the ſame in both when the following Particulars are conſider'd. (1.) That a *Plano-Convex Lens* is only a Segment BLDC of a Sphere DBE, whoſe Radius is NL or NB.
240. (2.) That a Ray of Light SB, parallel to the Axis of the Lens NG incident upon it in the extreme Part B, will be refracted to ſome Part of
241. the Axis. (3.) Let AB touch the Point B; then (4.) ABC will be a *Priſm*, whoſe Baſe AC coincides with the Axis of the analogous Lens.

(5.)

(5.) Since the Ray of Light SB falls on a Point 242.
B, which is the ſame in the Lens and the Priſm,
it will be refracted in the ſame Manner from
both; therefore (6.) the Ray will be not only 243.
refracted, but *diſſipated* by the Lens to ſeveral
Parts of the Axis between V, G, R. (7.) The 244.
refracting Angle of the Priſm ABC is equal to
the Angle of Incidence SBN, ſince they are ſeverally
equal to the Semi-angle of the Lens
BNL. (8.) Continue NB at Pleaſure, and in the 245.
Focus G of the mean refrangible Ray B G let
a Perpendicular G I be raiſed to interſect NB
produced in I, and S B continued in H. (9.) 246.
Then I B H is the *Angle* of *Incidence*, and (10.) 247.
then alſo H F, H G, H O are as the Quantities
of the *leaſt*, *mean*, *and greateſt Refraction*
of Rays in the Beam S B. (11.) The 248.
focal Diſtance LG will be leſs as the Angle of
the Priſm ABC, or Semi-aperture of the Lens
LB, is inlarged. (12.) Let the Angle of Diſſipation 249.
FBO be denoted by Z, and belong to a
Lens of *white Flint*. (13.) Let f B o be the 250.
Angle of *Diſſipation* belonging to a Lens of
Crown Glaſs, and be denoted by z. (14.) The 251.
Sine of Incidence is to that of Refraction, as
BG to NG; or BG : NG : : $n : m$ :: 1 : m :: 1 :
1,5998. Alſo Bg : Ng : : 1 : 1,5318. (15.) 252.
FO is the Diameter of a Circle, into which all
the Rays of a diſſipated Beam or Pencil will be
diffuſed, and therefore may be call'd the *Circle
of Diſſipation* or *Aberration of Rays*, ariſing from
the different Refrangibility. (16.) When LG 253.
is large, we have IH : IG :: $n : m$, whence HG =
0,5998 = b. (17.) Whence FG:HG::$\frac{1}{2}$Z:0,5998, 254.

or FO : (2 HG =) B D : : Z : 1,1996 : : FBO :
BGD. And z : 1,0636 :: f Bo : BgD :: fo : BD.
255. (18.) When B is extremely near to L, then LG
: NG :: 1 : 1,5998 1 : 1,6 (nearly) :: $f : r+f$;
whence $1{,}6f = r+f$, and ſo $0{,}6f = bf = r$, there-
fore $f = \frac{r}{b} =$ focal Diſtance of parallel Rays near
256. the Axis in a Plano-Convex Lens. (19.) If the
Lens be a double and equally Convex one as
BPDL, then the Priſm ABQ, being double of
ABC, the Refraction will be HBG twice as much
as before, conſequently the focal Diſtance LG
but half LG; that is, in this Caſe $f = \frac{r}{2b}$ as be-
fore was ſhewn from the general Theorem, (219.)
257. (20.) Since while BC remains the ſame, if the
Angle ABC of the Priſm becomes *greater or leſſer*
the Perpendicular NB will be *leſſer or greater*, it
is plain that the ſaid Perpendicular of the Priſm
or *Radius of the correſponding Lens will ever be in-*
258. *verſely as the refracting Angle* of the Priſm. (21.)
As a *Convex Lens* BD anſwers in its Effects to two
Priſms ABQ and ADQ placed together by their
common Baſe AQ, ſo a *Concave Lens* anſwers to
two equal Priſms put together on their *Vertices*
or angular Points.

359. Theſe Particulars connect the *ſimilar Nature* of *Priſms and Lenſes*, and plainly indicate what Lenſes muſt be choſen for a Combination, that ſhall produce the ſame Effects as a given Combination of Priſms, and therefore what they muſt be that ſhall anſwer to that particular Combination of Priſms, (in *Fig.* V.) which tranſmits the Rays without Diſſipation, and conſequently ſuch

ſuch Lenſes as will form the Images of Objects in the *compound Focus*, without the *Colours* as uſual in Refraction by a *ſingle Lens*.

Fig. V.

260. If the Angle B of the Priſm ABC of *Crown Glaſs* be equal to the Angle C of the Priſm BCD of *white Flint*, then it is known by Experiment that the Angle D of the Priſm CDE of Crown Glaſs muſt be very nearly equal to Half the Angle B of the firſt Priſm, that the three Priſms together may refract the Rays of Light without Diſſipation; and therefore the Sum of the two refracting Angles B + D is to the Angle C, as 3 to 2. Or $B + D : C :: Z : z :: 3 : 2.$

Fig. VII.

261. Therefore if we take a double Convex Lens ABCG of *Crown Glaſs*, a double and equally Concave Lens CGAFD of *white Flint*, and a *Plano Convex Lens* DGFE of *Crown Glaſs*, and in a Poſition ſimilar to that of the three Priſms, they will together make a *compound Object-Glaſs* of a Teleſcope, which will form the Images of Objects *without Colours*, and as diſtinctly as the Nature of ſuch compound Refraction will admit of.

262. If the Teleſcope be ſhort as thoſe of the *Galilean* Sort, uſually call'd *Opera-Glaſſes*, then the Convexity of the Lenſes will be conſiderable, and in ſuch Caſes it is convenient to uſe three Lenſes as above deſcribed, and as is repreſented in *Fig*. VII.

Fig. III.

263. But in Teleſcopes of any Length exceeding 9 or 12 Inches, two Lenſes are ſufficient. For if the refracting Angle of the Priſm ABC of

of *Crown Glaſs*, conjoin'd with a Priſm ECB of *white Flint*, whoſe Angle $C=\frac{2}{3}B$, will refract the Rays without Diſſipation (235.) it is plain, if we take any double and equally Convex Lens ABCG of *Crown Glaſs*, and another double Concave
Fig. VIII. EBCGH of *white Flint*, theſe two Lenſes put together will do the ſame Thing, that is, they will refract the Rays to their Axis without *Diſſipation*, provided the Radii of the Concave have their *proper Proportion* to each other.

Fig. III. 264. To determine which, let CG be a Perpendicular on the Baſe EB of the Priſm BCE; then will it divide the ſaid Baſe EB, or Angle E C B into two Parts ECG and G C B, which will be to each other as 1 to 3; conſequently the Radius of the external Concavity in the Lens CD muſt be to that of the internal Concavity in the inverſe Proportion, *viz.* of 3 to 1, (257.) But ſince the Angle $GCB=\frac{1}{2}ABC$, the Radius of the internal Cavity is the ſame as the Radius of Convexity in the Lens AB.

265. And here I muſt obſerve, that the Priſm ECB muſt have the ſame Poſition (or be contiguous) to the Priſm ABC, as repreſented in the Figure, to produce the ſame Effect; for if it be placed ſo that either of its Sides be perpendicular to the Baſe AC, tho' the two Priſms are in contact at C, there will be a *Diſſipation of the Rays*, and the Object will appear *colour'd*.

Fig. VIII. 266. Hence appears the Reaſon why this Lens ECH ſhould be a *double Concave* of unequal *Radii*: And conſequently we ſee the true Form of the *compound Object-Glaſs* in a Teleſcope made on this Principle, and conſtructed according to the *Theory*, and which is different from any Thing we have yet ſeen made.

267. What relates to the Number and Diſpoſition of the Eye-Glaſſes, is the ſame as has been already obſerved (144.) in others; for here all the Care has been to form the Image IM *without Colours*, or *Aberration of Rays* from their *different Refrangibility*.

CHAP. VII.

THEOREMS *for finding the* focal Diſtances *of* compound LENSES, *for any* determined Lengths *of* TELESCOPES, OPERA'S, *&c.*

268. IT will be neceſſary to direct the young *Optical Analyſt* how he is to proceed in finding the focal Diſtances of compound Lenſes, or ſuch as conſiſt of two or three ſingle ones of different Forms and refractive Powers; and to illuſtrate the Method by a Proceſs for ſome of the moſt uſeful Caſes.

269. And firſt let the compound Lens conſiſt of three ſingle ones, which we ſhall denote by the Letters A, B, and C; and the refractive Powers of the Mediums of which they are made be expreſs'd by the ſmall Letters a, b, c. Then for a double Convex Lens of unequal Radii r, r, and for any given Diſtance d of the Radiant,

we find the focal Diſtance f by this Theorem $\frac{p\,d\,\mathrm{r}\,r}{d\,\mathrm{r}+d\,r-p\,\mathrm{r}\,r}=f.$ *

270. Here $p=\frac{n}{m-n}$ and the refracting Power of any *Medium* being as $\frac{m-n}{n}$, or as $m-1$, (when $n=1$) as we have ſhewn (192); therefore a, b, and c, will be as $\frac{m-n}{n}$ in their reſpective *Media*, and ſo p will be as $\frac{1}{a}$, $\frac{1}{b}$, $\frac{1}{c}$; and the above general Theorem will be for a Lens of each Medium thus expreſs'd:

For A, $\frac{d\,\mathrm{r}\,r}{a\,d\,\mathrm{r}+a\,d\,r-\mathrm{r}\,r}=f.$ For B, $\frac{d\,\mathrm{r}\,r}{b\,d\,\mathrm{r}+b\,d\,r-\mathrm{r}\,r}=f.$ For C, $\frac{d\,\mathrm{r}\,r}{c\,d\,\mathrm{r}+c\,d\,r-\mathrm{r}\,r}=f.$

271. Now it has been ſhewn, that in the triple compound Lens, the firſt ſingle Lens A is a double and equally convex one, and conſequently the Theorem will (ſince $\mathrm{r}=r$) ſtand thus for diverging Rays, $\frac{d\,r}{2\,a\,d-r}=f.$

272. If one of the Radii (r) be infinite, then the ſaid Lens becomes a *Plano-Convex*, and the Theorem for it is $\frac{d\,r}{a\,d-r}=f.$ And ſuch is the other outſide Glaſs C of the triple Lens, as ſhewn (260.)

273. If the Radii (r, r,) be both Negative; the Lens becomes a double and unequally Concave one, *viz.* $\frac{d\,\mathrm{r}\,r}{-a\,d\,\mathrm{r}-a\,d\,r-\mathrm{r}\,r}=f.$

274.

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* See my *Syſtem of Optics*, Page 60 and 76. Alſo *Philoſophia Britannica*, ſecond Edition; and the Table of *Theorems* which hereafter follow.

274. If one Radius (r) be infinite, then the Lens is a *Plano-Concave*, and the Theorem is $\frac{dr}{-ad-r}=f$.

275. When both the Radii are equal, (*viz.* r=*r*) then the Lens is double and equally Concave, and its Theorem is $\frac{dr}{-2adr-r}=f$. And this we have ſhewn (261.) is the Form of the Middle Glaſs B in the triple compound Lens.

276. Since the Object Lens D is a Plano-Convex, its focal Diſtance will be $\frac{dr}{cd-r}=f$, and this will expreſs alſo the Radiant Diſtance (d) with reſpect to the middle Lens B, whoſe Theorem in the preſent Caſe being $\frac{dr}{-2bd-r}=f$, for diverging Rays; it will be for converging Rays $\frac{-dr}{2bd-r}=f$, for ſo the Rays will fall on the ſecond Lens, and therefore (d) will be negative.

277. Hence we have $d=\frac{fr}{2bf+r}$; and therefore $\frac{dr}{cd-r}=\frac{fr}{2bf+r}$ which will give $\frac{dr}{cd-2bd-r}=f$, the compound focal Diſtance of both the Glaſſes C and B together.

278. Now ſince the middle Glaſs B is ſuppoſed to have the greateſt refractive Power, or (*b*) greater than (*c*), therefore *f* laſt found will be negative; and as that is the radiant Diſtance, with reſpect to the third Lens A, or that next the Eye, the Rays will proceed from the ſecond Lens B *diverging* upon it; and therefore the

Theorem (in *Art.* 270.) will give $\frac{rf}{2af-r}$ = Radiant Diſtance = $\frac{dr}{gd-r}$, by putting $c-2b=g$.

(277.) And from hence we get $\frac{dr}{2ad-gd+r}=f$, the compound focal Diſtance of all the three Lenſes A, B, C, as required.

279. I have ſuppoſed the Lens C a Plano-Convex, as it is the only genuine Form, (by 260.) and as the Theorems are more ſimple on that Account; but if we had ſuppoſed it a double and unequally Convex, we ſhould in a more tedious Manner have arrived at the ſame Concluſion or *Formula* for the triple Lens; for then we ſhould have had this Theorem, $\frac{drR}{2adR+2bdR+rR-dcR-dcr}=f$; where by ſuppoſing $2b-c=-g$, and R infinite, we get $\frac{dr}{2ad-gd+r}=f$, the ſame as before.

280. If this compound Lens be expoſed to parallel Rays, then d is infinite, and the Theorem becomes $\frac{r}{2a-g}=f$, which is the Caſe of a Teleſcope made with ſuch a triple compound Object-Glaſs.

281. When we ſuppoſe the Lens C to be a *Plano-Convex* one, it is to adapt the Theorem to the preſent Conſtruction, where all the three Lenſes are Glaſs, and two of them A and C of the ſame Sort of Glaſs; and conſequently in this Caſe $a=c$, and the Theorem $\frac{r}{2a-g}=\frac{r}{2a-2b+a}=\frac{r}{3a-2b}=f$.

282. We have ſhewn (218, 221.) that in white Flint and Crown Glaſs we have $a=0{,}5328$, and $b=0{,}5998$; but we may take $a=0{,}53$, and $b=0{,}6$, and be near enough the Truth for Uſe. Thus for Example; let $r=10$, then $\frac{r}{3a-2b}=\frac{10}{1{,}59-1{,}2}=\frac{10}{0{,}39}=2{,}82=f$, the focal Diſtance of ſuch a compound Object-Glaſs.

283. Suppoſe the compound Lens were to conſiſt of two Glaſſes only, A and B; then A will be a double and equally Convex Lens of *Crown Glaſs*, *yellow Plate*, or *Pebble*, (224.) and B a double and unequal Concave, as we have ſhewn (263.)

284. The above Theorem for ſuch a Lens becomes $\frac{d\mathrm{r}r}{-bd\mathrm{r}-bdr-\mathrm{r}r}=f$; and ſuppoſing ($d$) infinite, then $\frac{\mathrm{r}r}{-\mathrm{r}b-rb}=f$, the focal Diſtance of parallel Rays.

285. Now this is the Radiant Diſtance with regard to the interior Lens A; and therefore we have $\frac{\mathrm{r}r}{-\mathrm{r}b-rb}=d=\frac{\mathrm{r}f}{2af-r}$; and putting $\mathrm{r}+r=g$, we get $\frac{\mathrm{r}r}{2a\mathrm{r}-gb}=f$, the compound focal Diſtance of thoſe two Lenſes A and B.

286. But it has been ſhewn (264.) that in caſe of two ſuch Lenſes for taking away Colours, we ſhall have the Proportion of the Radii as 3 to 1; that is, $\mathrm{r}:r::3:1$; conſequently $\mathrm{r}+r=4=g$, hence the Theorem becomes $\frac{3}{6a-4b}=f$

=

$= \frac{3}{3,18-2,4} = \frac{3}{0,78} = 3,84$ the focal Diſtance of both, when the Radius (r) of the double Convex A is 1.

287. By theſe Theorems the focal Diſtances for all compound Lenſes of two or three different Mediums may be found, and thereby the Method of adapting them to Teleſcopes and Microſcopes render'd univerſal. I might enlarge much on this Subject, if it were worth while, but what I have ſaid is more than ſufficient for the intelligent Optician.

CHAP. VIII.

Remarks on the Nature, Conſtruction, and Effects of the foregoing Achromatic * TELESCOPE.

288. THE Teleſcope in *Fig.* VIII. is ſuch, as I think provides againſt all the Errors of Viſion by refracted Light, in the beſt Manner that any one of this Kind can do; but it is at the ſame Time a *Memento*, that we ought to expect *no abſolute Perfection in the Works of Art*; that is the ſole Prerogative of the divine Being, and to be ſought for in the *Works of Nature* only.

Fig. VIII. 289. There has been no Teleſcope made of the Conſtruction here repreſented, that I know of: The

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[*] From the Greek α *privative*, and χρῶμα, *Colour*. But to ſay the Truth, no Teleſcope can ſhew an Object *without Colours*, if the Object be viewed with a *Lens*; for the ocular Lens only will cauſe a colour'd Image on the *Retina* in the Fund of the Eye.

The *Opera-Glaſs* in *Fig.* VII. has been made with a triple Object-Glaſs, but they are put together in a Manner different from that in which they here ſtand, and which we have ſhewn to be analogous to the three Priſms.

290. A Teleſcope, has alſo been made with a double Object-Glaſs, conſiſting of a *double Convex Lens*, and a *Plano-Concave*, with its plain Side *outwards*; and five Glaſſes at the Eye-End all differently placed from thoſe in any of the Teleſcopes here repreſented in theſe Plates.

291. Now with regard to this or any Sort of refracting Teleſcopes, there are many Imperfections inſeparable from them in the Nature of Things, and which therefore all our Art will be in vain applied to elude: For

292. *Firſt*, There is no ſuch Thing as a perfect Medium to be found, or ſuch as will refract the Light with a Regularity neceſſary for perfect Viſion. *Glaſs* of any Kind (being an *artificial Mixture)* is far from being ſuch a Medium; nay *Cryſtal* itſelf, the moſt perfect and natural of all tranſparent Subſtances, will admit of no perfect Refraction on many Accounts.

293. *Secondly*, If the Medium were perfect, there is yet no ſuch Thing as a perfect *Politure* of Surface to be effected by Art, which yet is neceſſary for a true Refraction of Light, and perfect Viſion by it.

294. *Thirdly*, The Light incident on a Lens is reflected externally and internally from both the Surfaces, and many Times internally from one Surface to the other, which muſt neceſſarily occaſion a Confuſion in the Refraction in

in ſome Degree; for this internal Reflection of Light in a Lens is ſo conſiderable, as to occaſion a *ſecondary Focus* in the Lens, in which Objects may be magnified much more than in the *common Focus*, tho' this has paſs'd hitherto unobſerved.*

295. *Fourthly*, The greater the Number of Glaſſes, the greater the Confuſion in Refraction, and *Obſcurity* in the Object, by Loſs of Light at each Surface by Reflection and Refraction.

296. *Fifthly*, There will ever be an Error from the Figure of the Lenſes, as we have ſhewn, and we can proceed only *one Step* in correcting it, ſo that there will ever remain an Imperfection of Viſion on that Account.

297. *Sixthly*, The Error ariſing from the different Refrangibility of Light admits only of a *palliative Cure*; for I believe there are but few People who will pretend to make a Teleſcope *in* FACT *as perfect as it is in* THEORY.

298. *Seventhly*, Beſides, unluckily for this Teleſcope, that promiſes ſo much Perfection, it *cannot mend one Fault without making another*, for to annihilate the Colours it muſt be aſſociated with a *Concave Lens*; and ſuch a Conjunction of diſſimilar Lenſes very much encreaſes the Error from the Figure of them, as we have amply ſhewn, (from 127 to 137.)

299. *Eighthly*, The Aberration of Rays from the Figure is ſtill farther encreaſed, by our being obliged to have a Concave Lens of ſuch a Form and Poſition as the Theory forbids in regard to that

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* I have often magnified Objects this new Way, by a Lens 3 or 4 Inches Diameter, and 12 or 18 Inches common focal Diſtance, but the ſecondary Focus not more than 2, or 2½.

that very Point. A *Plano-Concave* being the beſt Form, and its Concave Side placed *outwards*, the beſt Poſition, (84—88.) But neither of theſe Particulars can be admitted in this Teleſcope, (266.)

300. *Ninthly*, The Image IM ſhould be reverſed, or the ſecond Image OP form'd, by two Glaſſes only, *viz.* EF and GH, as we have ſhewn, (144.) A Lens interpoſed between the Image IM and double Object-Glaſs, appears to me abſurd; for if the ſaid Image be free from the *Errors of the Figure* and *different Refrangibility both*, (as we are told) then it is certain it can only do Harm *. Yet ſuch a Glaſs I find in all the Teleſcopes I have ſeen of this Conſtruction. Fig. VIII.

301. *Tenthly*, The Image OP ſhould be view'd with two Eye-Glaſſes IK, LN, as we have before ſhewn (140.) which however it is not in thoſe I have ſeen, tho' conſtructed with *ſeven Lenſes*.

302. From theſe and many other Conſiderations diſadvantageous to refracting Teleſcopes, which the very beſt Conſtructions are liable to, it plainly appears they are far enough from Perfection, or from affording that clear and diſtinct Viſion of Objects which the World has been perſuaded to believe, and expect; but this will be farther evinced, when we have firſt ſhewn the Nature and Conſtruction of a *reflecting Teleſcope*.

CHAP.

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* If it be ſaid—it *amplifies the Field of View*, it is granted; but then at the ſame Time, it will unequally refract the Rays, and, in ſome Degree, it regenerates the Colours that were before deſtroy'd.

CHAP. IX.

The NATURE, CONSTRUCTION, *and* IMPROVEMENT *of the* CATADIOPTRIC *or common* REFLECTING TELESCOPE *explain'd.*

303. THE Nature and Conſtruction of a *Catadioptric* or REFLECTING TELESCOPE I have largely explain'd in my *Syſtem* of OPTICS, as alſo in my *Philoſophia Britannica*; the principal Particulars and Properties on which it depends are as follow.

304. *Firſt, That the Angle of Incidence is equal to the Angle of Reflection* in every Sort of Light.

305. *Secondly*, That the Image of an Object is form'd by Rays reflected from a *Speculum* in *one and the ſame Point of its Axis*, provided thoſe Rays are incident *very near the Vertex* or *Axis* of the *Speculum*,

306. *Thirdly*, That therefore ſuch an Image is entirely free from that Confuſion and Colour of Rays, by which it is impair'd in the Focus of a common Lens.

307. *Fourthly*, That with regard to thoſe Rays which fall at a Diſtance from the *Vertex* on the Speculum, there will be an *Aberration of ſuch Rays*, or they will not be all reflected to the ſame Point of the Axis 'tis true, but then the Error here by Reflection is but as 1 to 2½, or as 2 to 5, nearly compared with that by *Refraction*, when the *Radius of the Sphere is the ſame.*

308. *Fifthly*, Therefore the Area of a *Circle of Aberration*, (93.) or confuſed Repreſentation of each Point in the Object, will ſcarcely be a *ſixth Part* ſo great *by Reflection as by Refraction*, the Ratio being as 1 to 5,76.

309. *Sixthly*, When the *focal Diſtance* of the *Speculum* is equal to that of the Lens, the Error PQ by reflected Light, will be that by Refraction but as 1 to 16.

310. *Seventhly*, That therefore the *Area* of the *Circle of Aberration*, or Confuſion of the Image, is here by the *Speculum* but a 256th Part of what it is by a refracting Lens.

311. *Eighthly*, That upon the whole the Error is every way ſo ſmall by reflected Light, that it is altogether inſenſible in Experiments, and in the Conſtruction of Inſtruments, particularly the *reflecting Teleſcope**.

312. Theſe Conſiderations firſt gave Riſe to that noble Invention, for it was ſoon obſerv'd, that ſince the Image was form'd ſo very perfect by reflected Light, it might not only be inverted as in a Refractor, but alſo be magnified at the ſame Time by a ſmall reflecting Speculum properly placed, and yet ſo as to ſhew the Object very diſtinctly.

213. Hence a double Power of magnifying an Object is acquired in the *reflecting Teleſcope*, and conſequently the *great Length* of Refractors neceſſary for a great Power of magnifying, is thereby avoided; and alſo the Conveniency of viewing Objects in general, and eſpecially the celeſtial Bodies is thereby render'd very great.

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* *All theſe Particulars are but ſo many Corollaries from the Propoſitions in* 83 *and* 92.

Plate IV. Fig. IX. 314. For Illuſtration, ſuppoſe AB, CD are parallel Rays which fall on the large poliſh'd *Speculum* BD, and by it reflected to its Focus F, where an Image LM is firſt form'd of the diſtant Object. This Image is ſo perfect, that being plac'd a little farther than the Focus (f) of the ſmall Speculum E from it, the Rays of Light which conſtitute this Image will fall upon, and be ſo reflected from the ſmall *Speculum* as to form a *ſecond Image JM* in its proper Focus *F* belong- to the Diſtance EF of the firſt Image; and this ſecondary Image being erect, is view'd by the Eye thro' the Plano-Convex HK.

315. This is the moſt ſimple Structure of the common *reflecting Teleſcope*, and its Power of magnifying is thus computed: Suppoſe BF=9 Inches, the *focal Diſtance* of the large Speculum BD; and F f=1½ Inch, of the ſmall Speculum EG. Then if EG were placed juſt at its focal Diſtance F f from the Image IM, it would reflect the Rays parallel to the Eye; and the Image would appear larger than the Object in proportion of 6 to 1, that is to ſay, the Object is magnified *ſix Times by the large Speculum* only.

316. But now let the Image IM be conſider'd as a ſmall Object to be magnify'd by the ſmall Speculum EG, (in the Manner of a MICROSCOPE) then the ſmall Speculum being removed farther from IM than its focal Diſtance, *viz.* to the Diſtance E F, an Image will be form'd thereof at *J M* as much larger than IM, as the Diſtance E *F* is greater than the Diſtance HF, which let be as 8 to 1. Then is the Object magnified in the Ratio of 6×8, or 48 to 1; or it will

will appear 48 *Times nearer or larger in Diameter thro' the Teleſcope than to the naked Eye.* But this Power of magnifying by Refraction would require a Length of Tube not leſs than 8 or 10 Feet in a Refractor.

317. I need not mention that only ſo much of the ſecond Image *J M* can be ſeen as falls within the Tube or Part next the Eye, *viz.* a b, and that is to be duly circumſcribed by a Hole in a Piece of Braſs, call'd a *Diaphragm*, placed in the Focus of the Eye-Glaſs HK. Alſo at the End of the *Eye-piece* at (e) there ought to be a ſmall Hole of a determinate Bigneſs, to ſhew the Object in its Image *J M* diſtinctly; if this Hole be too big or too little, the Teleſcope will be vitiated in proportion.

318. But this is not the Conſtruction of the reflecting Teleſcope in common Uſe; for in that you always find a Glaſs Lens (c d) placed juſt at the End of the Eye-Piece where it ſcrews on to the large Tube. This Glaſs reduces the ſecondary Image *J M* to a leſſer one *i m*, (in the Manner as has been ſhewn in the Refractor (97, *&c.*) and ſo this Image *i m*, which we view in the common Refractor, is made partly by the Speculum EG, and partly by the Lens c d; and ſo is neither one Thing nor the other; that is to ſay, it is not form'd by *Refraction* or *Reflection* alone, but by both together; and makes the Inſtrument what it is truly call'd, a CATADIOPTRIC, inſtead of a CATOPTRIC TELESCOPE, as it ought to be.

319. In the *Catoptric Teleſcope* above deſcribed the Images are both form'd *entirely by refracted*

Light, and therefore not affected by the different Refrangibility of the Rays, conſequently the Viſion is as perfect as Art and Nature can make it by Inſtruments.

320. But in the common *Catadioptric Teleſcope*, the firſt Image only is form'd by Reflection, but the ſecond Image is made by *Reflection and Refraction conjointly*; for the Rays reflected from the ſmall Speculum EG fall converging on the Lens c d, and by it are refracted to *f*, ſhort of the proper Focus *F* to which they tended before.

321. Now the Rays paſſing thro' the Lens c d muſt neceſſarily ſuffer a different Refraction, and conſequently the Image *i m* muſt be thereby render'd imperfect by the *Aberration of Light*, and the Colours and Confuſion of Rays (by which it is form'd) conſequent thereupon. Whence we may conclude, that this Glaſs being aſſociated with the ſmall Mirror does much Harm, and *leſſens the Effect and Value of a reflecting Teleſcope.*

322. Beſides it will appear that this Glaſs is not only injurious in itſelf in that Situation, but that it is injudiciouſly placed there, ſince the Image *J M* (form'd by *Reflection* only in the *Catoptric* or *true reflecting Teleſcope*) is to be view'd with the ſame Degree of Diſtinctneſs in the compound Focus of two Eye-Glaſſes placed together, as we have before directed, (140.) and as they are repreſented in the Figure by HK, and LN. And when the Glaſſes are thus diſpoſed, I ſee nothing more at preſent that can add to the Perfection of Viſion by ſuch an Inſtrument *in the Form they are now made.*

CHAP. X.

The Imperfection *of* VISION *by the* Achromatic TELESCOPE *ſhewn by many* EXPERIMENTS, *and the great* SUPERIORITY *of the* REFLECTING TELESCOPE *evinced.*

323. FROM what has been ſaid of the Nature of *refracting and reflecting Teleſcopes*, it evidently appears, that if any refracting Teleſcope could truly correct the Errors ariſing from the *Figure of the Glaſſes*, and alſo from the *different Refrangibility of Rays*, it would be in its own Nature more perfect than the *reflecting Teleſcope*, becauſe this latter is not free from the *Errors of the Figure* of its *Specula* and Lenſes.

324. And ſince we find the Reflector with this Imperfection ſo capable of a *double magnifying Power*, how much more might we expect it in that Refractor which pretends to be without any at all? If the *Reflector*, notwithſtanding the Errors of the Figure, Loſs of Light in the Metals, and other Diſadvantages, will yet give a clear and diſtinct View of an Object by means of a *ſecond Image*, who would not expect this noble Effect in a much higher Degree in a *Refractor* ſubject to no Sort of Errors, ſuſtaining little Loſs of Light, and having all the Perfections that the Science of Dioptrics can impart?

325. Any one would readily conclude ſuch a Conſequence muſt neceſſarily follow; and that now we ſhould have *double Refractors* equally good,

good, if not ſuperior to Reflectors, that ſhould be much eaſier made and applied, and be purchaſed at a much leſs Expence. But how much muſt be abated of theſe great Expectations, will appear by attending to the following Experiments.

Fig. X. 326. I took the double Object-Glaſs of a Three-feet Refractor, A B E H G, whoſe focal Diſtance A F was about 30 Inches, and combined it with the triple Object Lens a b d e g of an *Opera*, whoſe focal Diſtance e f was 6 Inches: Theſe two compound Lenſes were ſo adjuſted as to make of the firſt ſmall Image L M a large one *J M*, after the ſame Manner by Refraction, as the ſame two Images were before form'd by Reflection.

327. This ſecond Image I view'd in the beſt Manner by the two Eye-Glaſſes H K, LN, as in the Reflector of the ſame magnifying Power, but found a very diſſimilar Effect. The Object in the double Refractor, inſtead of being bright and diſtinct as in the Reflector, was dark and confus'd; the Title Page of a Book at a Diſtance, clear and eaſily legible in one, was very obſcure and unintelligible in the other.

328. It appears by this, which I look upon as an *experimentum Crucis*, that both the firſt and ſecond Images in the *double Refractor* were far leſs perfect than in the *Reflector*; and tho' I gave it all the Advantages I could think of, it was to no Purpoſe, no Object could be view'd by it with any tolerable Degree of Diſtinctneſs.

329. As I knew the Glaſſes I had taken from the Teleſcope and Opera (306) were not preciſely of the ſame Figure and Poſition the Theory required, I took the Trouble of making others that were ſo, but ſtill found it an unſucceſsful Attempt; and ſince, by Experience, I found ſo great a Difference in the Effects of a *double magnifying Power*, I was determined to try what I could learn from a Compariſon of the ſingle magnifying Powers of theſe two Inſtruments, which I peruſed in the following Manner.

330. I took the compound Object-Glaſs of the Opera, (306.) and form'd it into an *aſtronomical Teleſcope*, with a triple compound Eye-Glaſs, (ſee *Art.* 39, 40, *&c.*) The Inſtrument thus conſtructed magnified about ſix Times, and was truly an *achromatic Teleſcope*, or free from Colours. I then took *a Reflector* whoſe focal Length of the *larger Speculum* was *ſix Times* that of the ſmall one, and therefore an Object viewed by theſe two Speculums only was magnified juſt as much as in the Refractor, and alſo without Colours; but there was no Compariſon between the Diſtinctneſs of Viſion in one and in the other. The Letters in the Title Page of a Book placed at a proper Diſtance, were extremely clear and well defined in one, that ſeem'd as it were in a miſty Atmoſphere, or whitiſh Medium in the other. I could eaſily read a Paragraph in the News Paper at a certain Diſtance with the *two Speculums*, but ſcarcely one Word of it with the *two compound Lenſes* could be diſtinctly diſcern'd.

331. Having found ſo great a Difference in the Appearance of Images formed in the ſolar Focus of a compound Lens and Speculum, I thought it would be neceſſary to compare the Appearance of the Images form'd in the proper or conjugate Focuſſes of a ſmall triple Lens and and Speculum, applied as an Object-Glaſs in a compound Microſcope, (ſee *Art.* 75. *&c.*) which I did by giving the ſame Length to the Images, and viewing them with the ſame Eye-Glaſſes; and conſequently the ſmall Objects were equally magnified in the refracting and reflecting Microſcope, but with how great a Difference in Point of Clearneſs and Perfection in all the Circumſtances of Viſion, thoſe only will be able to conceive who ſhall try the Experiment themſelves.

332. I reſolved further to ſee what Difference there would appear in a Landſcape form'd by a double *achromatic* Lens, and a Speculum of the ſame focal Diſtances; and having applied both in a *camera Obſcura*, the Succeſs was what I now expected. The Images of Objects in the Picture of the Lens were hazy, and towards the Margin indiſtinct; whereas thoſe of the Speculum were exceſſively clear, luminous, vivid, and diſtinct, almoſt equally to the Borders of the Field, of equal Extent with the other.

333. There remain'd yet another Method of ſhewing the very great Difference in the Perfection of Images form'd by thoſe compound heterogeneous Lenſes, and the ſimple Reflection of Mirrours; and that was to apply them ſeverally as SOLAR MEGALASCOPES: The Speculum and com-

compound Lens were of the ſame focal Length, and the Object equally illumined and magnified in each; but the Image of the former compound with that of the latter, was ſufficient to ſhew how much the ſimple Operations of Nature excel thoſe which are more compound, forced, and artificial.

334. I frequently obſerved in trying to find the Focus of a compound Lens, that it was attended with unuſual Difficulty and Uncertainty; I could ſcarcely tell when I had got it, or when not, ſo dubious was this Point, till after I had practiſed with them a conſiderable Time. And even now, unleſs the Sun be very clear, its Image is ſo indiſtinct in the *compound Focus*, that I cannot eaſily determine where it is moſt perfect; whereas in a *ſingle Lens* of the ſame focal Diſtance, the Image of the Sun at once appears diſtinct, in one Place only, and ever ſo little a Diſtance on one Side or the other, induces a ſenſible Confuſion.

335. Another Thing diſcover'd itſelf in theſe Experiments about the Focuſſes of compound and ſingle Lenſes, which at firſt raiſed ſome Admiration; it was this, the Subject on which I took the Focus of the Lenſes, or Image of the Sun, was a Piece of Cork fix'd on to the End of a long Rule divided into Inches and Tenths; the Surface of the Cork was made black with Ink; and the Moment I had the Focus of the ſingle Lens on the Cork, it began to ſmoak intenſely; but when the Focus of the compound Lens was received on the ſame Cork, no Smoak appear'd in that ſudden Manner as before, nor

till

till after it had remain'd a ſenſible Time on the Cork, and then the Smoak and Burning were not comparable to that produced by a ſingle Lens.

336. It appears by the two laſt (and ſome other) Experiments, that the ſolar Rays are more perfectly united in the Focus of a *ſingle*, than in that of a *compound Lens*, and that their Force and Action is much greater. But ſince in the ſolar Focus of the ſame Diameter made by a Speculum, there can be no Error from the different Refrangibility of Rays; and that the Error of the Figure is here but a ſixteenth Part ſo great as in the Lens, it muſt evidently follow, that the Perfection of the ſolar Image of the Speculum muſt greatly exceed that of the ſingle Lens in all Reſpects, and much more that of the compound one.

337. Upon the Whole, therefore it appears, both by Theory and Experiments, *that a Combination of Lenſes is far from rendering Viſion perfect in refracting Teleſcopes; and that it is in every reſpect much inferior to the Effect of a* Speculum *in a* REFLECTING TELESCOPE *for exhibiting the beſt and moſt pleaſing View of Objects.*

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CHAP. XI.

The common THEOREMS *of* OPTICS tabulated, *and* rectified *to expreſs univerſally the* focal Diſtance *of* RAYS *reflected from* MIRRORS, *or refracted thro'* LENSES *of any* Medium.

338. AS I know by too much Experience, how eaſy it is to make a Miſtake amidſt ſo great a Variety of Characters, affected with ſuch Contrariety and Alternation of Signs, in the vaſt Multiplicity of *complex Theorems* neceſſary in all Optical Enquiries and Inveſtigation; and that when any ſuch Miſtake is made, how widely it makes us ſtray from the Truth we are ſeeking, and often occaſions our taking up with erroneous Poſitions, or totally baffling our moſt laborious and pertinacious Purſuits; I preſume it will be very acceptable to the young Optician to have all the uſeful Theorems placed together in one View, as in the following ſynoptic Tables, where he will immediately find the Theorem that ſuits his Purpoſe in any optical Enquiry. Obſerving only that the Thickneſs of the Speculum or Lens is ſuppoſed to be inconſiderable in compariſon of the Radius of Curvature; and then in CATOPTRICS we have the following Theorems for the different Sorts of Rays.

339. SPECULUMS.

Rays.	Convex.	Concave.
Diverging	$\frac{dr}{2d+r}=f.$	$\frac{-dr}{2d-r}=f.$
Parallel	$\frac{r}{2}=f.$	$\frac{-r}{2}=f.$
Converging	$\frac{-dr}{-2d+r}=f.$	$\frac{dr}{-2d-r}=f.$

340. In DIOPTRICS the Theorems for determining the *Foci* of Rays by *ſimple Refractions* out of *Air into Glaſs*, whether *Convex* or *Concave*, and *vice verſa*, are as in the Table annexed.

I. Out of AIR into GLASS.

Rays.	Convex.	Concave.
Diverging	$\frac{mdr}{md-nd-nr}=f.$	$\frac{-mdr}{md-nd+nr}=f.$
Parallel	$\frac{mr}{m-n}=f.$	$\frac{-mr}{m-n}=f.$
Converging	$\frac{mdr}{md-nd+nr}=f.$	$\frac{-mdr}{md-nd-nr}=f.$

341. II. Out of GLASS into AIR.

Rays.	Convex.	Concave.
Diverging.	$\frac{-ndr}{md-nd+mr}=f.$	$\frac{ndr}{md-nd-mr}=f.$
Parallel	$\frac{-nr}{m-n}=f.$	$\frac{nr}{m-n}=f.$
Converging	$\frac{-ndr}{md-nd-mr}=f.$	$\frac{ndr}{md-nd+mr}=f.$

342. As every Lens has two refracting Surfaces, there will be two *Radii* belonging to each, *viz.* r, r; and becauſe of a double Refraction, the Theorems for Lenſes will be very complex, but may be abbreviated by putting $\frac{m-n}{n}=a$, or if $n=1$, then $a=m-n$. Whence the Theorems will be for the ſeveral Forms of Lenſes as in the following Table.

343. I. A LENS with unequal *Radii*.

Rays.	Convex.	Concave.
Diverging	$\frac{d\mathrm{r}r}{adr+ad\mathrm{r}-r\mathrm{r}}=f$	$\frac{d\mathrm{r}r}{-adr-ad\mathrm{r}r-\mathrm{r}r}=f.$
Parallel	$\frac{r\mathrm{r}}{ar+a\mathrm{r}}=f.$	$\frac{r\mathrm{r}}{-ar-a\mathrm{r}}=f.$
Converging	$\frac{-dr\mathrm{r}}{-adr-ad\mathrm{r}-r\mathrm{r}}=f.$	$\frac{-dr\mathrm{r}}{adr+ad\mathrm{r}-r\mathrm{r}}=f.$

344. II. A LENS with equal *Radii.*

Rays.	Convex.	Concave.
Diverging	$\frac{dr}{2ad-r}=f.$	$\frac{-dr}{2ad+r}=f.$
Parallel	$\frac{r}{2a}=f.$	$\frac{-r}{2a}=f.$
Converging	$\frac{-dr}{-2ad-r}=f.$	$\frac{dr}{-2ad+r}=f.$

345. III. A LENS with one Radius (r) infinite.

Diverging	$\frac{dr}{ad-r}=f.$	$\frac{-dr}{ad+r}=f.$
Parallel	$\frac{r}{a}=f.$	$\frac{-r}{a}=f.$
Converging	$\frac{-dr}{-ad-r}=f.$	$\frac{-dr}{ad-r}=f.$

346. IV. A LENS with one Radius (r) negative.

	Unequal.	Equal.
Diverging	$\frac{-dr\mathrm{r}}{adr-ad\mathrm{r}+r\mathrm{r}}=f$	$-d=f.$
Parallel	$\frac{-r\mathrm{r}}{ar-a\mathrm{r}}=f.$	$\frac{-r\mathrm{r}}{0}=f.$
Converging	$\frac{dr\mathrm{r}}{ad\mathrm{r}-adr+r\mathrm{r}}=f$	$d=f.$

347. Theſe are all the uſeful Theorems in Dioptrics for finding the *focal Diſtance* of any *Speculum* or *Lens*. And the Reader will obſerve they here ſtand in a different Form from what he finds them in other Books of Optics; the Reaſon is, becauſe in vulgar *Optics*, and in practical Caſes, it has been thought ſufficient to take the Ratio of the Sines of Incidence and Refraction out of Air into Glaſs to be that of 3 to 2; and if that were true, we ſhould have $\frac{m-n}{n}=\frac{1}{2}=a$, or $2a=1$; then, for Inſtance, in an equally Convex Lens the focal Diſtance would be $\frac{dr}{d-r}$; and for parallel Rays, $f=r$, as uſually repreſented, and not as you find them in *Art.* (344.)

348. But this Poſition is very erroneous, and requires as much to be corrected, if not more, than thoſe we have already conſider'd: Becauſe, in the *firſt* Place, 'tis an *Opprobrium* to the *Science* of OPTICS to have ſuch a vulgar Error continued in it without Notice. *Secondly*, it ſuppoſes all the Sorts of Glaſs we uſe to have the ſame refractive Power, which is falſe and abſurd: And *thirdly*, many late Inventions in Optics require a due Conſideration of the real refractive Power and focal Diſtances of Lenſes, and therefore make it neceſſary to have proper Theorems to repreſent and compute them by, and ſuch I have here ſupplied in the above Tables.

349. And in order to fit them to every Sort of tranſparent Medium that can be of Uſe in Optics, I have nicely examined, and by Experiments determined the Ratios of the Sines of Incidence

Incidence and Refraction in Water, Glaſs, Pebble, *&c.* with reſpect to the *leaſt*, *mean*, and *moſt refrangible Rays*, and have expreſs'd them in the following Numbers.

350. In *New River Water* I find the Ratio of $m : n :: 1{,}3233 : 1$; which is ſo near the common Ratio 1,3333 to 1, that the Difference is not worth regarding. So that for the ſeveral Rays we have

$$\left.\begin{array}{r}\text{For Red, } m : n :: 1{,}32727 : 1\\ \text{Green, } m : n :: 1{,}33333 : 1\\ \text{Violet, } m : n :: 1{,}33939 : 1\end{array}\right\}\ \begin{array}{r}\text{and } a = 0{,}32727\\ a = 0{,}33333\\ a = 0{,}33939\end{array}$$

351. In Pebble, Cryſtal, yellow Glaſs, and Crown Glaſs, I find the Ratios to be as follow.

$$\left.\begin{array}{r}\text{For Red, } m : n :: 1{,}52213 : 1\\ \text{Green, } m : n :: 1{,}53180 : 1\\ \text{Violet, } m : n :: 1{,}54147 : 1\end{array}\right\}\ \begin{array}{r}\text{and } a = 0{,}52213\\ a = 0{,}53180\\ a = 0{,}54147\end{array}$$

352. The Glaſs hitherto moſt in Uſe (though the worſt of all on many Accounts) is *common Plate Glaſs*; we have in this the following Analogies.

$$\left.\begin{array}{r}\text{For Red, } m : n :: 1{,}563 : 1\\ \text{Green, } m : n :: 1{,}573 : 1\\ \text{Violet, } m : n :: 1{,}583 : 1\end{array}\right\}\ \begin{array}{r}\text{and } a = 0{,}563\\ a = 0{,}573\\ a = 0{,}583\end{array}$$

353. Laſtly, in *white Flint Glaſs*, from a Trial of many Sorts, and taking the Mean of all, the Numbers came out thus;

$$\left.\begin{array}{r}\text{For Red, } m : n :: 1{,}5889 : 1\\ \text{Green, } m : n :: 1{,}5998 : 1\\ \text{Violet, } m : n :: 1{,}6107 : 1\end{array}\right\}\ \begin{array}{r}\text{and } a = 0{,}5889\\ a = 0{,}5998\\ a = 0{,}6107\end{array}$$

354. And thus the Values of *m*, *n*, and *a*, are known for LENSES, MIRROURS, and RAYS of every Sort; and by which any of the foregoing Theorems may be applied in optical Inveſtigations, to expreſs the Truth to the utmoſt Exactneſs that can be required. And it is very notorious, how faulty all our Computations muſt be that are made by Theorems not thus rectified; ſuch Inaccuracies are inconſiſtent with the Preciſion of Truth, and the Honour of Science. The *Mechanical Optician* takes it for granted, *that the Diſtance of the ſolar Focus is equal to the Radius*, not knowing that in a Lens of 20 Inches only it is leſs by $3\frac{1}{2}$ Inches (219.) if it be of *white Flint*; and in the moſt common or *Plate Glaſs*, the Difference is near 3 Inches in 20, (220.)

355. Let the Radius of a Lens be of any Quantity, it appears from the foregoing Tables, (344.) that the focal Diſtance of parallel Rays is to the Radius as $r : \frac{r}{2a} :: 2a : 1$.

Or, R : F :: 1,1996 : 1 in a Lens of *white Flint*.
R : F :: 1,146 : 1 ——— of *Plate Glaſs*.
R : F :: 1,0636 : 1 ——— of *Crown ditto*.

356. Hence it appears, that if the Object-Glaſs of a Teleſcope be *Plate*, whoſe Radius of Convexity is $114\frac{1}{2}$ Inches, the Diſtance of the *ſolar Focus* will be but 100 Inches. Therefore in *meaſuring the Diſtance of an Object by a Teleſcope, or the Dimenſions of the heavenly Bodies by the new Micrometer*, or

in the Solution of any *Problem depending on the Radius or geometrical Focus of the Object Lens*, very great Errors muſt enſue, if ſtrict Regard be not had to thoſe important Particulars, and how much this concerns not only the *Optician*, but the *Aſtronomer*, and every other *Artiſt* to whom the Science of Optics can be of Uſe, I may hereafter take another Opportunity to ſhew more at large.

CHAP.

CHAP. XII.

An Illuſtration of the METHOD *invented by Mr.* CALEB SMITH, *for correcting the* ERRORS *ariſing from the* different Refrangibility *of* RAYS, *by Means of a* Catadioptric SPECULUM *and* Concave LENS.

357. THE firſt Diſcovery of the Art of correcting the Errors from the different Refrangibility of Rays, was exemplified in thoſe Refractions which are made by a catadioptric SPECULUM and a LENS, properly aſſociated together in their common Axis. And tho' the Principles on which it depends, are the ſame with thoſe in the Method by Lenſes already explain'd; yet as the Proceſs is different, we ſhall, for the Sake of Variety, give the following brief Explication of it.

358. Let ABCD be a Glaſs Speculum, foliated on the convex Side CD; put r = Radius of the concave Surface AB, and *r* = Radius of the convex one CD. Suppoſe EAGC a Beam of Light parallel to the Axis BN, and incident on the *Speculum* in A, it will there be refracted to the ſecond Surface at C, from whence it will be reflected to the firſt Surface again at O; and there be a ſecond Time refracted to the Axis, which the Plate III. Fig. X.

the *moſt refrangible* Rays will interfect in F, and the *leaſt Refrangible* in H, and FH will be the Aberration or Error occaſion'd by the Diſſipation of the Beam in the two Refractions.

359. Let n be the Sine of Incidence out of Glaſs into Air; and m, m, the Sines of Refraction of the moſt and leaſt refrangible Rays; then ſince the Beam firſt paſſes out of Air into *Concave Glaſs*, we ſhall have the focal Diſtance $f=\frac{-\mathrm{m}\mathrm{r}}{\mathrm{m}-n}$, for *Violet* Rays, (340.) put $\mathrm{m}-n=a$, then $f=\frac{-\mathrm{m}\,\mathrm{r}}{a}$.

360. The Beam of Rays is now diſſipated and refracted to the Surface C D at the Part C, where the Rays will now be reflected as from a concave Speculum; for which the Theorem (in vulgar Optics) is $\frac{-dr}{2-dr}=f$; but here $d=f$, in the foregoing Article, therefore we ſhall have $\frac{-\mathrm{m}\,\mathrm{r}\,r}{2\,\mathrm{m}\,\mathrm{r}-a\,r}=f$, the focal Diſtance of the reflected violet Rays.

361. Theſe Rays are now to be conſider'd as *converging*, and incident on the convex Surface of Air, OB, in the Point O. The focal Diſtance of which will be $f=\frac{-n\,d\,\mathrm{r}}{\mathrm{m}\,d-n\,d-\mathrm{m}\,\mathrm{r}}=\frac{-n\,d\,\mathrm{r}}{a\,d-\mathrm{m}\,\mathrm{r}}$, (341.) whence we get $d=\frac{\mathrm{m}\,\mathrm{r}\,f}{a\,f+n\,r}=\frac{\mathrm{m}\,\mathrm{r}\,r}{2\,\mathrm{m}\mathrm{r}-a\,r}$; and from thence, $f=\frac{n\,\mathrm{r}\,r}{2\,\mathrm{m}\,\mathrm{r}-2\,\mathrm{m}\,r+2\,n\,r}=\mathrm{B\,F}$, by reſtoring the Value of (a).

362. In like Manner the focal Diſtance of the

the red Rays will be $\frac{n\mathrm{r}r}{2m\mathrm{r}-2mr+2nr}=\mathrm{BH}$: Therefore the Value of the Aberration $\mathrm{BH}-\mathrm{BF}=\mathrm{FH}=\frac{n\mathrm{r}r}{2m\mathrm{r}-2mr+2nr}-\frac{n\mathrm{r}r}{2\,\mathrm{mr}-\mathrm{m}r+2\,nr}$; this when reduced, and abbreviated, (by putting $\mathrm{r}-r=g$, and $\mathrm{m}-m=q$) will ſtand thus $\frac{gqn\mathrm{r}r}{2\mathrm{m}mg^2+2\mathrm{m}ngr+2mnrg+2n^2r^2}=\mathrm{FH}$.

363. If this Aberration be compared with the focal Diſtance of the Speculum, we ſhall have $\mathrm{FH}:\mathrm{FB}::gq:gm+nr$; and, putting $\mathrm{FH}=\mathrm{A}$, we have $\mathrm{A}=\frac{fgq}{gm+nr}$.

364. By ſuppoſing $\mathrm{r}=r$, we find $f=\frac{1}{2}r$, as in common Optics; and then $\mathrm{A}=0$, or the Aberration is nothing in a Speculum, whoſe two Surfaces are parallel to each other.

365. If r be ſuppoſed infinite, or AB a plane Surface; then the focal Diſtance is $f=\frac{nr}{2m}$; and the Aberration $\mathrm{A}=\frac{\mathrm{m}-m}{m}\times f$; which is in this Caſe a *Maximum*.

366. In this Reſpect the *Speculum* is only a *Plano-Convex Lens* foliated on its Convex Surface; by which Contrivance or Artifice, a much ſhorter focal Diſtance and *leſs Diſſipation* of Rays are produc'd by one refracting Surface, than could have been done in the ſame Glaſs uſed as a Lens; for in the Lens the focal Diſtance for parallel Rays is $\frac{nr}{m-n}=f$, which is nearly twice the Radius; whence in the Speculum $\frac{nr}{2m}$ $(=f,)$ is but

about $\frac{1}{3}$ of the ſame Radius of Convexity ; and therefore the focal Diſtance of the Speculum is but about $\frac{1}{6}$ Part ſo large as that of the Lens.

367. Then as to the *Aberration* of the moſt different heterogeneous Rays, *viz.* the *Red* and *Violet*; for the Red, we have $f=\frac{nr}{m-n}$, and for the Violet, $f=\frac{nr}{\mathrm{m}-n}$, whence the Aberration $\frac{nr}{m-n}-\frac{nr}{\mathrm{m}-n}=\frac{\overline{\mathrm{m}-m}\times nr}{\overline{\mathrm{m}-n}\times\overline{m-n}}$ in the Lens, is very large if compared with that in the Speculum $\frac{\mathrm{m}-m}{m}\times f$, as will appear by reducing them both to Numbers.

368. Mr. SMITH uſes Sir ISAAC NEWTON's Numbers, *viz.* $n=100$, $\mathrm{m}=156$, and $m=154$; therefore $\mathrm{m}-m=q=2$, $\mathrm{m}-n=56$, and $m-n=54$; whence the Aberration in the Speculum $A=\frac{2f}{154}=\frac{2r}{462}$, but that of the Lens is $A=\frac{200r}{3024}=\frac{1}{15}r$ nearly; whence the two Errors are as $\frac{1}{15}$ to $\frac{1}{231}$, or as 231 to 15.

369. Now it is known from the Doctrine of Refractions before explain'd, any Degree of Refraction or Aberration produced by a given Incidence and Radius of Sphericity in a convex or concave Lens, will require a concave or convex Lens of the ſame Radius and Angle of Incidence to correct that Aberration, and in that Caſe the Rays incident on the firſt, and thoſe which emerge from the ſecond Lens, will have the ſame Direction; and conſequently an Aſſociation of two ſuch Lenſes can be of no Uſe in Optics, as we have ſhewn.

370.

370. But if in any Lens we can by any Expedient produce, from a given Angle of Incidence, a Degree of Refraction or Aberration different from what is peculiar to the ſame Lens by itſelf, or any other Lens of the ſame Radius and Angle of Incidence; then two Lenſes of a contrary Figure may be found with different Radii, and Angles of Incidence that ſhall have the ſame Aberration; and conſequently that the Error of one may be corrected by the other; and at the ſame Time the incident and emergent Rays ſhall have a different Direction, or make a given Angle with each other. (See *Chap.* V.)

371. This we have ſhewn how to perform with two Lenſes of a *different ſpecific Refraction*; and here we have ſhewn, that by Means of Reflection, a Lens ſhall have a different Refraction and Aberration from that which is proper to it as a Lens only, with a given Incidence of Rays. Therefore there is ſome *concave Lens* I K L M, whoſe Aberration will be equal to that of the Speculum ABCD, and will correct it, or converge the Rays AH, OF to one Point N in the Axis.

372. Suppoſe the Radius of the Surface I K be denoted by x, and that of the Surface L M be called y: Then the general Theorem for a double concave Lens $\frac{ndxy}{-mdx+ndx-mdy+ndy-nxy}=f$ (342, 343.) And if we put $y=f$, the Theorem will be abridged to this Form $\frac{mdx}{nd-md-nx}=f=y=$ K N; but as the Lens is here placed, it receives the Rays in a converging State, and therefore the Diſtance $d=$ F K is negative, and the Theorem becomes

becomes $\frac{mdx}{nd+nx-md}=f=$ K H, for the leaſt refrangible or red-making Rays O H; and for the Violet O F, the Theorem is $\frac{\mathrm{m}dx}{nd+nx-\mathrm{m}d}=$ K F, whence K H — K F $=\frac{mdx}{nd+nx-md}-\frac{\mathrm{m}dx}{nd+nx-\mathrm{m}d}=$ FH, will be the Aberration of this concave Lens with regard to thoſe incident Rays.

373. Now ſince the Aberration of this Lens muſt in this Caſe be equal to that of the Speculum, that ſo it may correct it, we muſt make an Equation of each, which, when properly reduced and order'd, will give $x=\frac{\overline{dd+d\mathrm{A}}\times\overline{\mathrm{m}-m}}{n\mathrm{A}-\overline{\mathrm{m}-m}\times d}$ the Radius of the Surface I K next the Mirrour.

374. In this Equation the Quantities A, m, and n, are given; and d may be aſſumed at Pleaſure of any Value between $\frac{\mathrm{m}-n}{\mathrm{m}-m}\times$ A and $\frac{m\mathrm{A}}{\mathrm{m}-m}$ (the Reaſon of which Limitation appears from the Proceſs at large), and therefore x being known, the other Radius y is alſo known from the Equation in Article (372.)

375. The Point N thus determined is the compound Focus of the Speculum and Lens conjointly where the Images of diſtinct Objects are all form'd without Colours or Confuſion, and may be view'd very diſtinctly by an Eye-Glaſs, if it could be conveniently applied for that Purpoſe.

376. But there lies the Difficulty; the Focus N muſt either be thrown by a ſmall reflecting Plane

Plane to a Hole in the Side of the Tube or Teleſcope, as in the *Newtonian Form*; or elſe reflected back thro' a Hole in the central Part of the large Glaſe Speculum, by a ſmall *Concave*, as in the *Gregorian* or common Form of a Reflector. But as the Trouble and Difficulty will be every way not a little, we believe it will not be attempted in Practice, eſpecially as the chief End propoſed is only to ſeparate the two Images form'd by the firſt and ſecond Surfaces of the Speculum farther aſunder, than they are in the *Newtonian* Reflector; for there is no Diſſipation of Rays in the Reflection of Light from a Speculum, whoſe two Surfaces are parallel, (364.) and conſequently excepting the Vicinity of the two Images, there can be nothing to hinder the Perfection of Viſion by the *Newtonian* Teleſcope, and that is nothing compared to the Trouble of viewing Objects thro' the Side, which is the chief Reaſon of its being ſo little in Uſe.

377. Beſides, were it worth while, we have a Theorem in Dr. *Gregory*'s *Dioptrics*, by which the two Surfaces AB, C D, may be ſuch as ſhall form both the Images in the ſame Part of the Axis; but then there will be the common Error from the Diſſipation of Rays, and we loſe more than we get by this Artifice.

378. Upon the whole it appears, that the REFLECTING TELESCOPE, truly made, is by far the moſt perfect of all dioptric Inſtruments; and is in its own Nature capable of every Improvement from Art, and Conveniency of Application, for viewing diſtant Objects in the moſt ſatisfactory and advantageous Manner.

379. As the double Refraction in the Speculum is made at a concave Surface of Glaſs AB, the *Error ariſing from the Figure* is here alſo capable of Correction ſo far as to render it inſenſible; ſo that by this Contrivance both the Errors are provided for, (one wholly removed, the other ſufficiently corrected) by reaſon the refracting Surfaces of the Speculum and Lens are of the *ſame Kind*; whereas in two Lenſes of a different Figure, tho' one Error be taken away, the other is greatly encreaſed; ſo that we may eſteem this Method of the *Speculum and Lens* as the *moſt perfect that can be in Point of Theory*, for removing and correcting of Errors in *dioptric Viſion*.

* * *

CHAP. XIII.

LENSES *made according to the Figures of the* CONIC SECTIONS, *or any other Curves, are ſubject to the ſame* ERRORS *from* REFRACTION *as thoſe of a* Spherical Form.

380. AS we have given a ſufficient Demonſtration that no *refracting Teleſcope* will admit of perfect and diſtinct Viſion by *ſpherical Lenſes*; let us next enquire what may be expected from other Forms; and here the CONIC SECTIONS next offer themſelves.

381. The Theorem $\frac{\frac{1}{2}pmd}{md-nd-\frac{1}{2}pn}=f$, is general for finding the focal Diſtance of a Lens made with the Curvature of the *Ellipſes*, *Parabola*, or *Hyperbola*, ſuppoſing $p=$ Parameter of each reſpectively, as I have elſewhere ſhewn *. But here it muſt be underſtood to relate to thoſe Rays only, which fall extremely near the Axis.

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* *Philoſophia Britannica.*

382. For none of thoſe Rays which diverge from a Point at a finite Diſtance *d*, and are incident at any ſenſible Diſtance from the Axis, can by thoſe figured Lenſes be united again in a Point; nothing but a *mechanical Sort of Oval* being ſufficient for this Purpoſe, of which *Deſcartes* has deſcribed four different Forms in his Geometry; and Sir *Iſaac Newton* (by a general Conſtruction) in *Prob.* XXXIV. of his optical Lectures. But thoſe Forms are ſo irregular, that nothing can poſſibly be done with them in Practice.

383. There is indeed one Caſe, *viz.* where *d* is *infinite*, or the Rays *parallel*, that will admit of their being united into one Point in the Axis; and then the Theorem becomes $\frac{\frac{1}{2}pm}{m-n}=f$; where we obſerve the Theorem is the ſame as in a SPHERICAL Lens, whoſe Radius $r=\frac{1}{2}p$, the Semi-parameter of the conic Section (340.)

384. But theſe parallel Rays are to be underſtood to be alſo *parallel to Axis* of the Section or Lens that is made thereof; for thoſe that flow from a Point at an infinite Diſtance, but out of the Axis, will not be united into a Point by ſuch a *conic Lens*; and therefore after *Deſcartes* had himſelf made many compound Lenſes of theſe Forms, and applied them in every different Way, he at laſt concludes,—“ That the Inequa-
“ lity of Curvature in theſe Lines (or Sections)
“ which

“ which compoſe the Figure of theſe Lenſes, “ will ever be the Cauſe why Rays which are “ referred to divers Points, or parallel Rays “ proceeding from divers Parts, can never be “ ſo equally and exactly refracted as thoſe are “ which reſpect one Point only, or come from “ one and the ſame Part parallel upon the “ Lens *.”—Dr. BARROW, who publiſhed his optical Lectures ſoon after, declared he had nothing to add on this Head after *Deſcartes*; and the ſame is intimated alſo by Sir *Iſaac Newton.*

385. If therefore no equal and uniform Refraction of Rays, which come from divers Points, can be effected by *conic Lenſes*, there muſt be an *Aberration* of thoſe Rays, and ſuch as cannot be corrected by any Sort of *Combination*. The Image therefore of an Object will neceſſarily be imperfect in the Focus of ſuch a Lens, ſingle or compound, and conſequently no refracting Teleſcope can admit of perfect or diſtinct Viſion, if conſtructed with ſuch Lenſes.

386. But ſuppoſing the Figure of thoſe Lenſes would form a perfect Image, yet that would ſtand us in little ſtead, while the Means of attaining to thoſe Figures are inſuperably difficult, if not altogether impoſſible; for ſuppoſing the focal Diſtance of the convex Object-Glaſs were only 9 Inches, then in a Lens of the *elliptic*, *parabolic*, *hyperbolic*, and *ſpherical* Form, the Radius

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* Dioptrics, *Page* 122.

dius of Curvature at the Vertex of each is the ſame, *viz.* 3 Inches, equal to the Semi-parameter of thoſe Curves; and at that ſmall Diſtance on each Side the Axis, as is equal to the Semi-diameter of the Object-Glaſs, (*viz.* $\frac{1}{2}$ an Inch) all thoſe Curves would ſo nearly coincide, as by no Means to admit of any ſenſible Difference; and conſequently leave us without any Poſſibility of making Tools of ſuch an exact and determinate Figure as is neceſſary to produce the Lenſes under Conſideration. In this Sort of *Enchireſis* a Man may poſſibly equal, but can never hope to exceed *Deſcartes*, who after all his tedious and laborious Endeavours, was obliged to deſiſt, finding the Laws of Refraction as irreſiſtible as thoſe of Fate itſelf.

Since then it appears that Lenſes formed according to the *conic Sections*, or any other Curve whatſoever, are ſubject to the ſame Kind of Refraction from the Figure as ſpherical Lenſes are, it is evident, it muſt be corrected after the ſame Manner likewiſe, *viz.* by making the optic Angle by two Refractions inſtead of one; and conſequently both the Refractions muſt be the ſame Way, or made with two convex or two concave Lenſes; for if the Refractions are contrary, or made by a *convex or concave Lens* conjointly, inſtead of correcting they muſt encreaſe the Error greatly, as was ſhewn to be the

Caſe

Of CONIC LENSSES *not to be* annihilated. Caſe of thoſe of a ſpherical Figure. If any Doctrine be advanced to the contrary of this, the *Public has a Right to demand a Demonſtration of the Truth of it*; till then, they may think the achromatic Teleſcope itſelf a ſufficient Proof that no *refracting Teleſcope* can be ſo *diſtinct* as a *Reflector*. They will alſo moſt of them be ſo prudent, as to remember the Motto, *Nullius in Verba*, &c.

FINIS.

Addenda, Corrigenda, &c.

It may be proper to remark, that whereas there has been of late Years great Talk of a *Night Teleſcope*, as if it were a modern Invention, the Reader is to know that this Teleſcope is in reality no other than the *aſtronomical Teleſcope with three Glaſſes*, the Invention of which Conſtruction was before the Time of Mr. *William Molyneaux*, who in his Dioptrics, ſpeaking of a Combination of two Lenſes, has this Remark, —*This Problem* (*viz.* of finding the Focus of the Glaſſes ſo combined) *is of conſiderable Uſe in Dioptrics, being the Foundation of an excellent Sort of Teleſcope much uſed in* England *for the Night.* Page 73. This Book was dedicated in the Year 1690. So that this *Night Teleſcope* has been in Uſe at leaſt 70 or 80 Years. See alſo in *Huygens*'s *Dioptrics* the Conſtruction of the ſame Teleſcope. *Page* 184.

ERRATA.

Page 41, Line 7, for $x=2$ read $x=1,2$.
ib. —— 9 —— 1 —— 1,8.
42 —— 24 —— $-ff$ —— $+ff$.

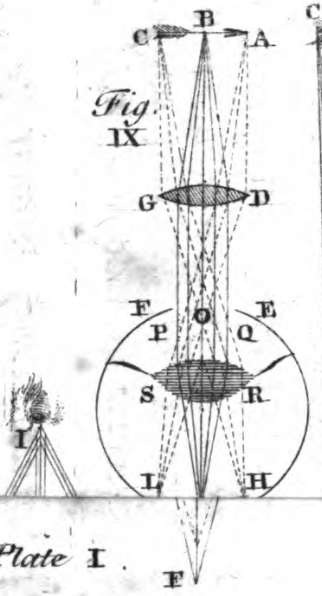
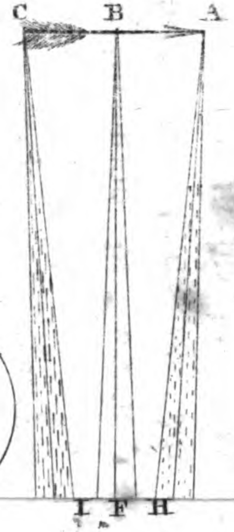
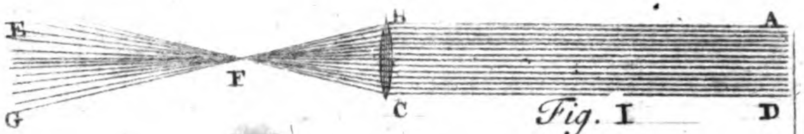


Fig. IX

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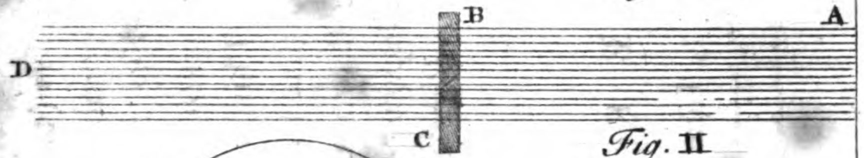


Fig. II

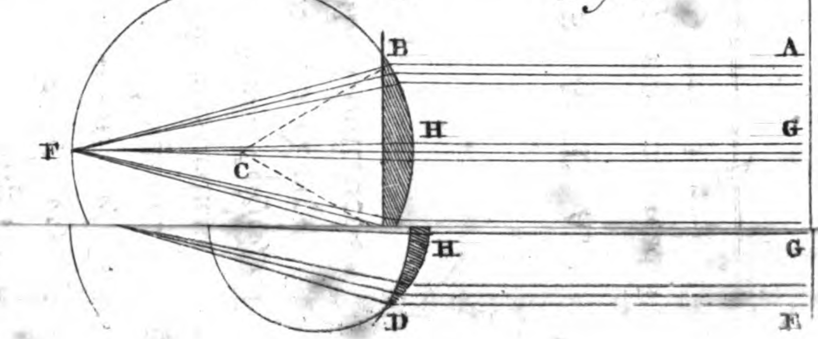


Plate I.

Plate II.

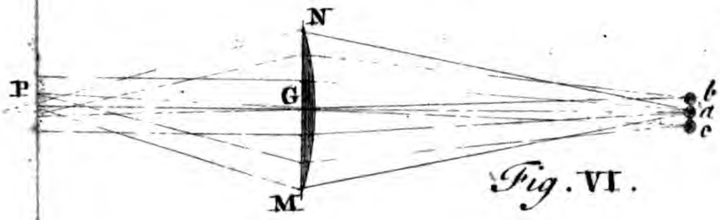
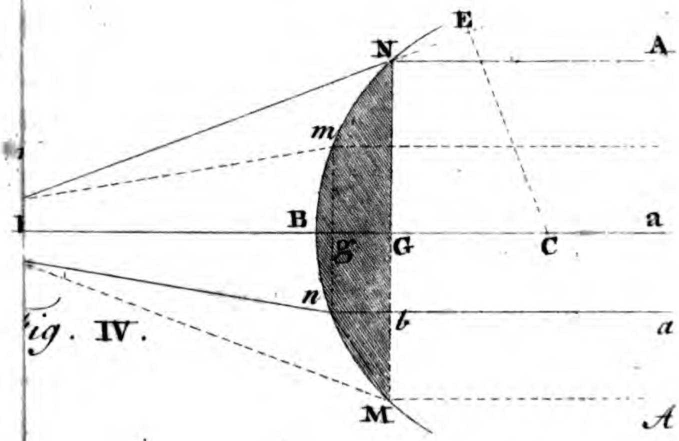


Fig. VI.



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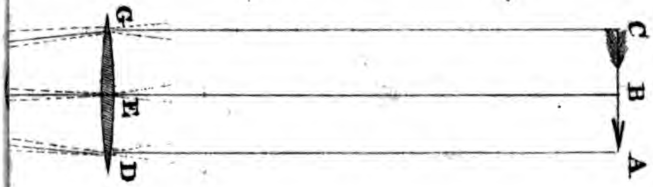
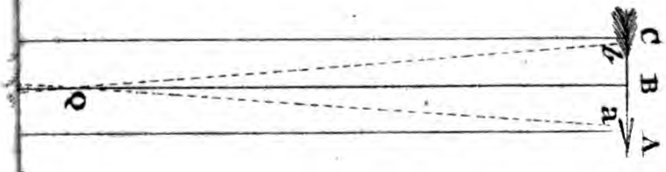




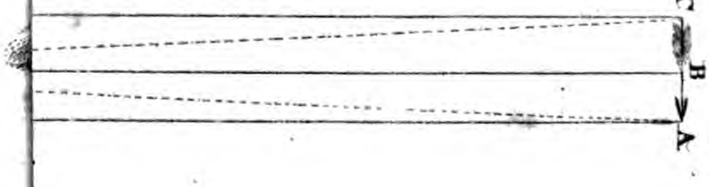


Fig. V.

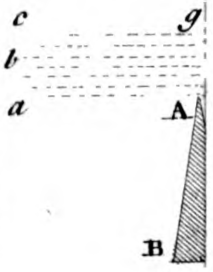
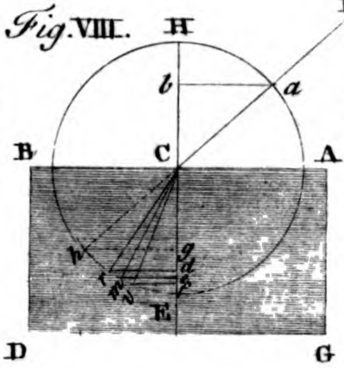


Fig. VIII.



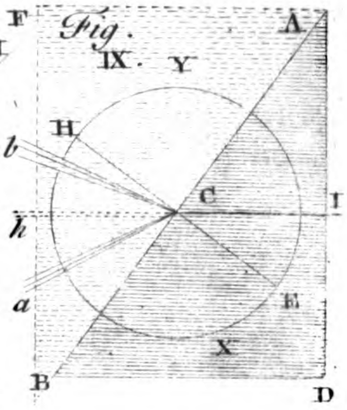
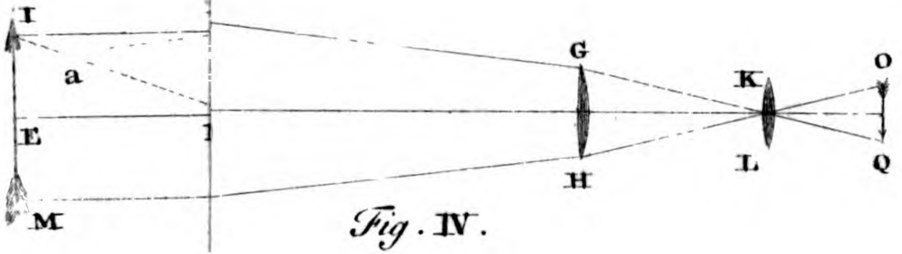


Fig. IX.



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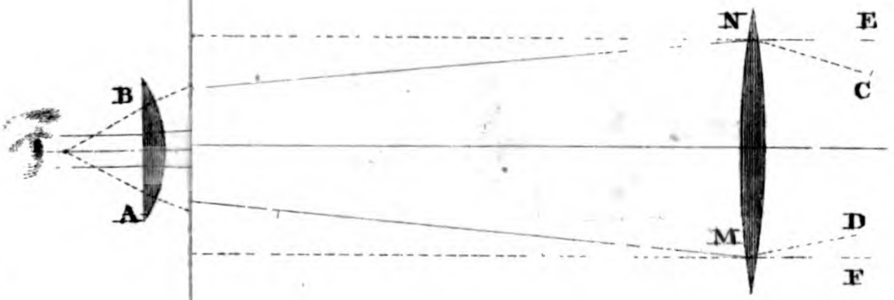


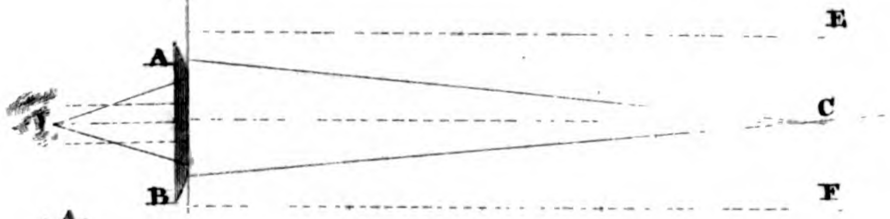
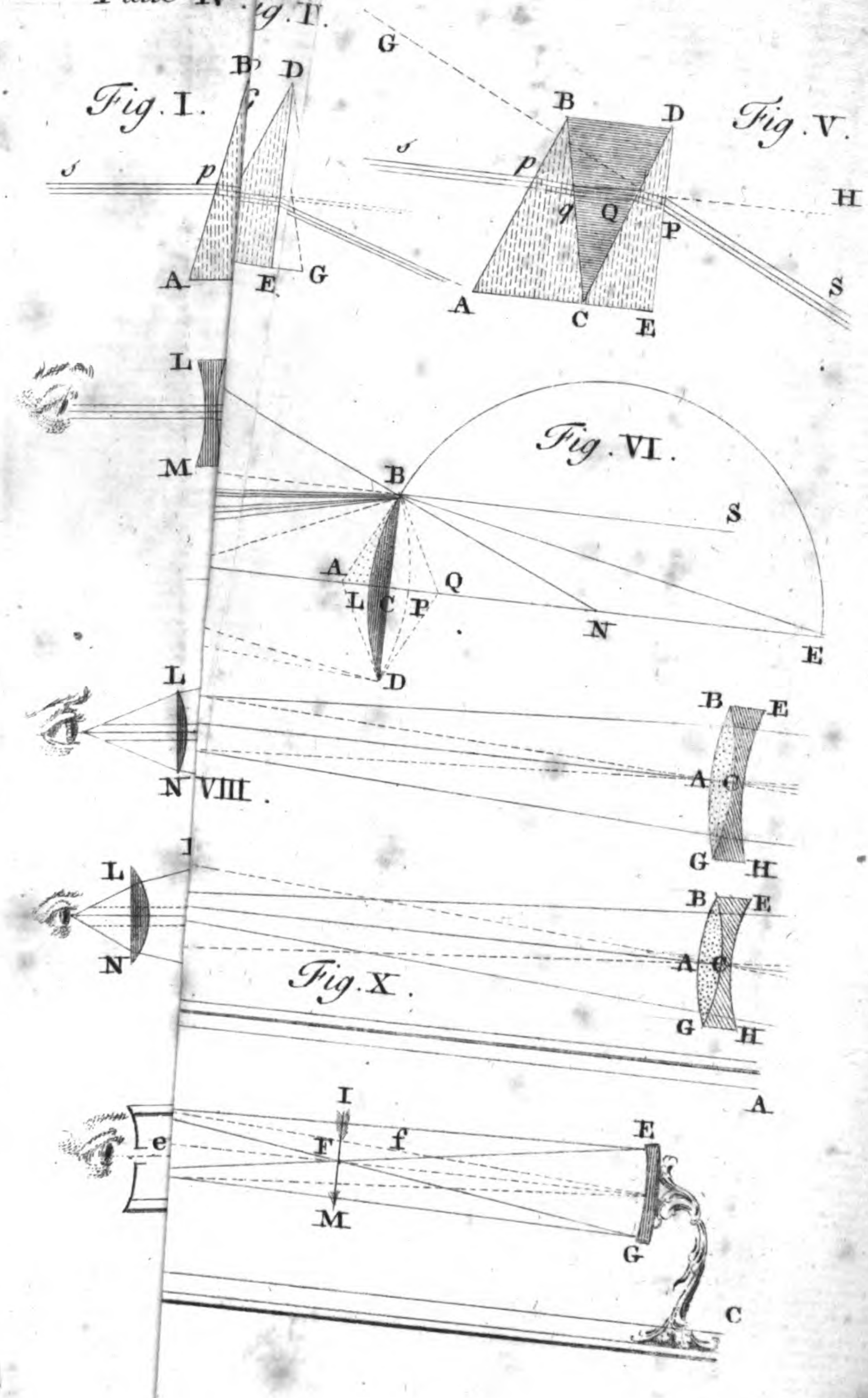


Plate III.

Plate IV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