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S. P. Rigaud
Feb. 21. 1827

ELEMENTS

OF

TRIGONOMETRY,

PLAIN and SPHERICAL;

Applied to the moſt uſeful PROBLEMS in HEIGHTS and DISTANCES, ASTRONOMY, and NAVIGATION:

For the USE of LEARNERS.

By WILLIAM PAYNE.

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MDCCLXXII.

TO THE

RIGHT HONOURABLE

William Henry Nassau de Zuleistein, Earl of *Rochford*, Viscount *Tunbridge*, and one of His Majesty's Principal Secretaries of State.

My Lord,

THE great and various advantages derived from the mathematicks in general, are universally acknowledged by the intelligent.

Among the several branches, the excellence of Trigonometry, and its usefulness to the commerce of mankind,

are

are ſo apparent; that it is not unworthy the attention of perſons in the moſt elevated ſtations.

This conſideration added to my long experience of your Lordſhip's goodneſs and condeſcenſion, emboldens me to addreſs this treatiſe to your Lordſhip; and I ſhall think myſelf happy, if my endeavours to eſtabliſh the elementary principles of the ſcience, obtain the ſanction of your Lordſhip's approbation.

I have the honour to be,

With the greateſt reſpect,

My Lord,

Your Lordſhip's much obliged,

And moſt obedient Servant,

WILLIAM PAYNE.

PREFACE.

THE following elements were composed for the Author's private use, when employed in teaching mathematicks; and are now published for the instruction of such young gentlemen, whose curiosity, or professions, may lead them to the study of these most agreeable and useful parts of knowledge.

The work is formed of materials which lie in common, and are open to all; for new theorems to excel and supercede the old ones, are little to be desired, and still less to be expected.

Al-

Although the Author has not the honour of introducing new discoveries, yet he claims the merit of many new demonstrations; and hopes to find his system prove easy to the learner, acceptable to the intelligent, and sufficiently extensive for a book of elements.

To these elements are added, the construction of the common tables, the doctrine of the sphere, and the principles of navigation, so far as was consistent with the plan; and all the cases in spherical triangles, are applied to astronomical problems.

All numbers marked with (p), refer to articles in Payne's Geometry; but all other references are to preceding articles in this book.

CON-

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BOOK I.

PLAIN TRIGONOMETRY.

BOOK II.

Of SPHERICAL TRIGONOMETRY.

PLAIN TRIGONOMETRY.

CHAP. I.

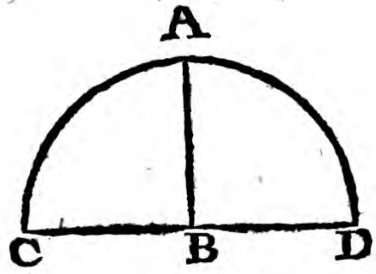
Containing the first Principles.

1. *A Point is that which has no parts, or magnitude.*

2. *A strait line is that which lies evenly between its extremes, or ends.*

3. *A plain angle is an opening or corner, made by two strait lines meeting one another.*

4. *When a strait line* AB *standing upon another* CD, *makes angles* ABC, ABD, *on each side equal to one another, each of those equal angles is called a right angle; and the line* AB *is said to be perpendicular to the line* CD.

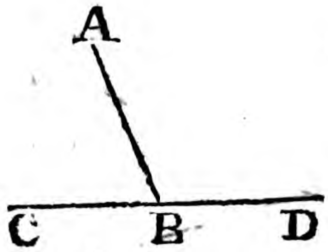


Note. *An angle is usually expressed by three letters, that placed at the angular point being always written in the middle.*

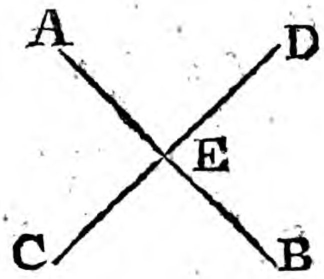
5. *An obtuse angle is one that is greater than a right angle.*

6. *An acute angle is one that is less than a right angle.*

7. *A line* AB *standing any how upon another line* CD, *makes two angles* CBA, ABD, *which taken together are equal to two right angles.* 67 p.



8. *A line* AB *cutting another line* CD *in* E, *will make the opposite angles equal; namely, the angle* AEC = BED, *and* AED = BEC - 70 *p.*



9. *A plain triangle is a figure made by three strait lines meeting one another.*

10. *Trigonometry is the art of finding the required parts of triangles; (namely, sides or angles), from other parts already known or given.*

11. *An equilateral triangle, is one that has three equal sides.*

12. *An isosceles triangle, is one that has only two equal sides.*

13. *A right angled triangle, is one that has a right angle.*

14. *An oblique angled triangle, is one that has no right angle.*

15. *In the same triangle, opposite to equal sides are equal angles, and opposite to equal angles are equal sides* - - - 63, 65 *p.*

16. *In the same triangle, opposite to the greater side, is the greater angle, and opposite to the greater angle, is the greater side* - - 72, 73 *p.*

17. *If any ſide of a plain triangle be produced, the outward angle will be equal to both the inward remote angles* - - - 86 *p.*

18. *The three angles of any plain triangle taken together, are equal to two right angles.* 86 *p.*

19. *A circle is a figure bounded by a curve line, called the circumference, every part whereof is equally diſtant from the middle point or center.*

20. *Any part of the circumference of a circle is called an arch.*

21. *The circumference of every circle is ſuppoſed to be divided into* 360 *equal parts, called degrees; each degree into* 60 *equal parts, called minutes; and each minute into* 60 *equal parts, called ſeconds, &c.*

22. *A quadrant of a circle will therefore contain* 90 *degrees, being a fourth part of* 360.

23. *Equal angles at the centers of all circles, will intercept equal numbers of degrees, minutes, &c. in their circumferences* - - 148 *p.*

24. *Hence every plain angle is meaſured by an arch of a circle, whoſe center is the angular point, and is ſaid to be of ſo many degrees, minutes, &c. as are contained in its meaſuring arch.*

25. *All right angles therefore are of* 90 *degrees, or contain* 90 *degrees, becauſe their meaſure is a quadrant.*

26. *The*

26. *The three angles of every plain triangle taken together contain* 180 *degrees, being equal to two right angles* - - - - 86 *p.*

27. *In a right angled plain triangle, the ſum of its two acute angles is* 90 *degrees* - 88 *p.*

28. *The complement of an arch, or of an angle, is its difference from a quadrant, or a right angle.*

29. *The ſupplement of an arch, or of an angle, is its difference from a ſemi-circle, or two right angles.*

30. *The magnitudes of arches and angles, are determined by certain ſtrait lines appertaining to a circle, called chords, ſines, tangents, &c.*

31. *The radius of a circle, is a line proceeding from the center to the circumference.*

32. *The chord of an arch is a ſtrait line joining its extreme points.*

33. *The ſine of an arch is a line drawn from either end of it perpendicular to a diameter meeting the other end.*

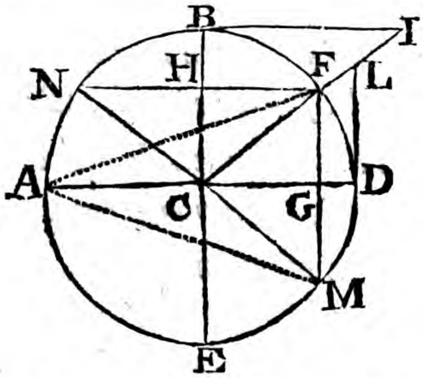
34. *The verſed ſine of an arch is that part of the diameter intercepted between the ſine and the end of the ſaid arch.*

35. *The tangent of an arch, is a line proceeding from either end, perpendicular to the radius joining it; and its length is limited by a line drawn from the center through the other end.*

36. *The secant of an arch, is the line proceeding from the center, and limiting the tangent of the same arch.*

37. *The cosine, cotangent, &c. of any arch, is the sine, tangent, &c. of its complement.*

38. *About the center* C, *at any distance* CA, *describe the circle* ABDE, *draw the diameters* AD, BE, *at right angles to each other; take the arch* DF *less than* DB, *draw* FGM, FHN, *perpendicular to* AD, BE, *produce the line* CF *to* I, *and erect* DL, BI *perpendicular to* AD, BE.



Then according to the preceding definitions, and the construction above.

The arches AB, BD, DE, EA *are quadrants*	22
The arch DF *measures the angle* DCF - -	24
Each of the lines CA, CB, CD, CE, *is a radius of the circle* - - - - - -	31
FM *is the chord of the arch* FDM - -	32
FG *is the sine of the arches* ABF *and* FD -	33
AG, GD, *are the versed sines of the arches* ABF, *and* FD - - - -	34

DL *is*

Remarks.

39. *All the lines described above, belong equally to an arch, and the angle measured by it.*

40. *In equal circles, the chords, sines, tangents, &c. of equal arches, or of equal angles, are equal to each other respectively* - - - 152 *p.*

41. *The greatest sine is that of 90 degrees (or of a quadrant) it being the radius of the circle: thus the radius* BC, *is the sine of the arch* DB, *or angle* BCD, *and is greater than* FG.

42. *The sine of an arch, is half the chord of double the same arch; or the sine of half an arch, is half the chord of the whole arch.*

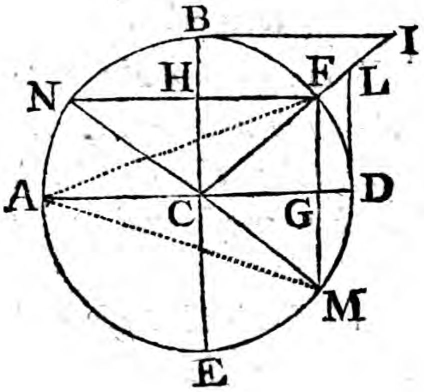
For if the arches DF, DM, *be equal, the angles* FCD, DCM, *will be equal* (150 *p.*) *and* CD *will bisect* FM *at right angles* - - 62 *p.*

43 *The sine of an angle at the circumference, is half the chord of the arch upon which it is constituted.*

For the angle FAM = (*half* FCM *by* 142 *p.* =) FCD, *therefore* FG *is the sine of the angle* FAM. 40

44. *The cosine of an arch, is equal to that part of the diameter, which is intercepted between the center of the circle, and sine of the said arch.*

For CGFH *is a rectangular figure by construction; therefore* CG = HF, *the cosine of the arch* DF, *or cosine of the angle* DCF. - - - 37



45. *The square of the radius, is equal to both the squares of the sine and cosine of any arch; namely,* $CF^2 = FG^2 + CG^2$, *because* FGC *is a right angle.* 100 *p.*

46. *As the cosine of any arch to its sine, so is the radius to the tangent of the same arch. For the triangles* CGF, CDL, *are equiangular, and therefore* CG : GF :: CD : DL - - 193 *p.*

47. *The radius is a mean proportional between the tangent and cotangent of the same arch. For the triangles* LDC, CBI, *are equiangular, and therefore* DL : DC :: CB : BI, *or* DC : DL :: BI : BC (193 *p.*).

48. *The*

48. *The tangents of any two arches are reciprocally proportional to their cotangents.*

Suppose T, *t*, *the tangents of two arches, and* C, *c*, *their respective cotangents, to the radius* R; *then* $T : R :: R : C$, *and* $t : R :: R : c$ (46). *Therefore* $T \times C = (R \times R =)\ t \times c$ (186 *p.*) *Therefore* $T : t :: c : C$ - - 187 *p.*

49. *The radius is a mean proportional between the cosine and secant of the same arch.*

For $CG : CF :: CD : CL$, *by similar triangles, therefore* $CG : CD :: CD : CL$; *putting* CD *for its equal* CF.

50. *The chord of* 60 *degrees, and the tangent of* 45 *degrees, are each equal to the radius.*

For if the angle FCM *is of* 60 *degrees, the equal angles* CFM, CMF, *will each contain* 60 *degrees* (26) *and the triangle* CFM, *will be equilateral* (15). *Again, if the angles* LCD, DLC, *be equal, each will contain* 45 *degrees, and* DL *will be equal to* CD - 15

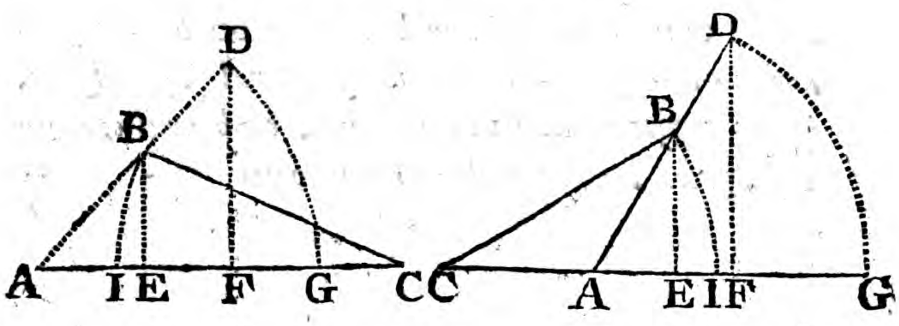
CHAP.

CHAP. II.

Containing theorems, and solutions of all the various cases in plain triangles.

Theorem I.

51. *In every plain triangle, the sides are proportional to the sines of their opposite angles.*



Let ABC be a plain triangle, then the side AB is to the sine of the angle C; as the side BC to the sine of the angle A.

Produce AB, make AD equal to CB, about the centers A and C, describe the arches DG, BI, and let fall the perpendiculars BE, DF, upon CA; produced in fig. 2. - - - 299 p.

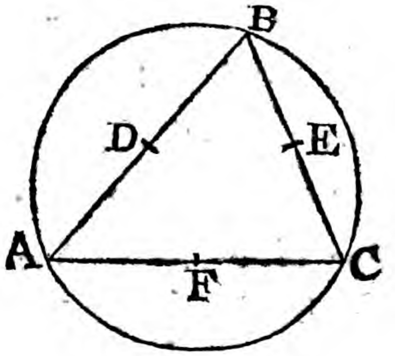
Now

Now AEB, AFD are right angles - con.
The angles ABE, ADF, are equal - 83 p.
And the angle DAF is common
Therefore the triangles ABE, ADF are equiangular - - - - - - 27 p.
Therefore AB : BE :: AD : DF - 193 p.
But BE, DF are fines of the angles C and A, to the fame radius CB or AD - 33
Th. AB : fine of C :: BC : fine of A. 56 p.
which was to be demonftrated.

52. Again.

52. Again.

The sides of any plain triangle are proportional to the sines of their opposite angles.



In the plain triangle ABC, the sine of the angle C, is to the side AB; as the sine of the angle A, to the side BC.

Bisect the sides in D, E, F, and describe a circle about the triangle - - 297, 330 p.

Now	AD	: BE	::	2AD	: 2BE		-	174 p.
Or	AD	: BE	::	AB	:	BC	-	174 p.
Th.	AD	: AB	::	BE	:	BC	-	175 p.

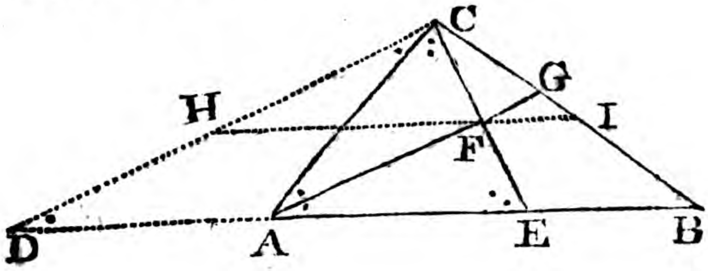
But AD, BE, are the sines of the angles C and A, at the circumference of the circle. 42

Therefore, the sine of C, is to the side AB; as the sine of A, to the side BC; which was to be demonstrated.

53. In

Theorem 3.

56. *In a plain triangle, the ſum of two unequal ſides, is to their difference; as the tangent complement of half the contained angle, to the tangent of the difference between the ſaid complement and each oppoſite angle.*



Let ABC be a plain triangle, having the ſide AB greater than the ſide AC; produce BA to D, make both AD and AE equal to AC, join DC, CE, draw AFG biſecting the angle CAB, and HFI parallel to DB.

Now the angle AFC = AFE, and CF = FE 62 p

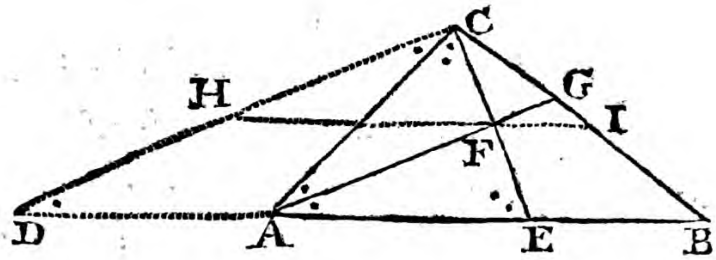
Therefore AFC, AFE, are right angles 9 p

And CI = IB - - - - - - - - 101 p

The angle ACF, or AEF, is the complement of half CAB - - - - - - - - - - 28

And the angle FCG, or ECB, is the difference of the ſaid complement from each of the angles ACB, ABC - - - - - - - - - 86 p

Again,

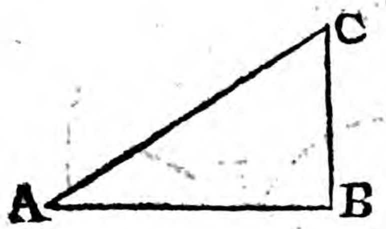


Again, the angle ADC = ACD - - 63 p
And the angle BAC = ADC + ACD - 86 p
Th. the angle BAG = ADC - - 51 p
And AG is parallel to DC - - - - - 80 p
Th. DB : EB :: HI : FI - - - - 195 p
:: CI : GI - - - - - - 193 p
:: BI : IG - - - - - 56 p
:: AF : FG - - - - 189 p
But DB = AB + AC, EB = AB — AC, and AF, FG, are tangents of the angles ACF, FCG, to the radius CF - - - - - - - - - - - 38
Th. AB + AC : AB — AC :: tan. ACF : tan. FCG - - - - - - - - - - - 56 p
which was to be demonſtrated.

Otherwiſe

53. Corollary.

In a right angled plain triangle, the hypothenuse is to the radius of a circle, as either of the sides to the sine of its opposite angle.



Let the plain triangle ABC have a right angle at B, then AC is the hypothenuse, and AB, BC, are the sides or legs.

Now AC : sine of B :: BC : sine of A - 51
And AC : sine of B :: AB : sine of C - 51
But the angle B contains 90 degrees (25) and its sine is a radius of the circle - - 40
Therefore AC : radius :: BC : sine of A. 56 p.
And AC : radius :: AB : sine of C. 56 p.
which was to be demonstrated.

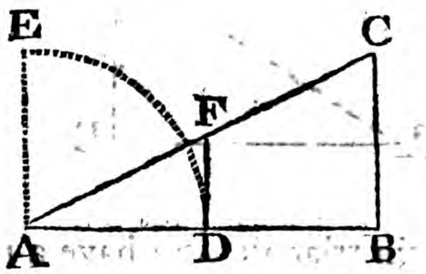
Remark.

Hence, radius : AC :: *sine* A : BC - 52
And radius : AC :: *sine* C : AB - 52

Theorem

Theorem 2.

54. *In a right angled plain triangle, the radius of a circle is to the tangent of either acute angle, as the adjacent side to the opposite side; and as either side to the radius, so is the other side to the tangent of its opposite angle.*



Let ABC be a plain triangle, having a right angle at B, about the center A, with any radius AD, describe the quadrant DE, and erect DF at right angles to AB.

Because ADF, ABC are right angles, and the angle BAC is common, the triangles ADF, ABC, are equiangular - - - - 87 p.

Therefore AD : DF :: AB : BC - - 193 p.

And - AB : AD :: BC : DF - - 193 p.

But AD is the radius of the circle, and DF is the tangent of the angle BAC - - 35

Therefore, radius : tan. A :: AB : BC 56 p.

And - AB : radius :: BC : tan. A 56 p.

55. Corollary.

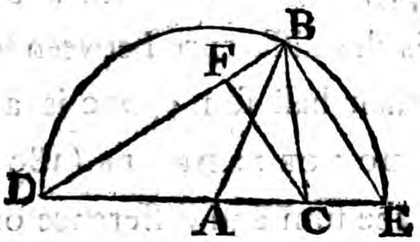
Hence AB : BC :: *radius* : *tan.* A - 175 *p.*

And $AB^2 : BC^2 :: \overline{radius}^2 : \overline{tan.}^2\,A$ - 207 *p.*

Theorem

Theorem 3.

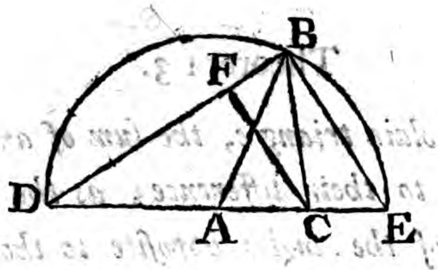
56. *In a plain triangle, the ſum of any two unequal ſides, is to their difference; as the tangent of half the ſum of the angles oppoſite to thoſe ſides, to the tangent of the difference between each and their half ſum.*



Let ABC be a plain triangle, having the ſide AB greater than the ſide AC; produce AC both ways, about the center A, with the radius AB, deſcribe the ſemicircle DBE, join DB, BE, and draw CF parallel to EB.

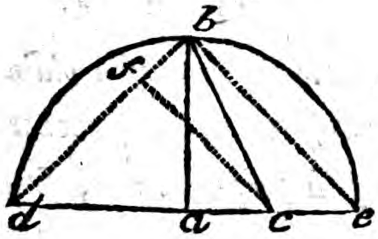
1. The angle BAD, is the ſum of BCA, ABC (86 p.) therefore the angle DEB, is half the ſum of BCA, ABC, ſince it is equal to half the angle BAD (142 p.). Again, the angles ABE, DCF, are each equal to DEB or AEB (63, 83 p.) therefore each of the angles ABE, DCF, is half the ſum of BCA, ABC, the oppoſite angles.

2. The



2. The angles BCF, DFC, are equal to CBE, DBE, each to each (82, 83 p.) and DBE is a right angle (153 p.). Therefore either of the angles BCF, CBE, is the difference between each opposite angle, and their half sum; DFC is a right angle, (54 p.) and DC : CE :: DF : FB (189 p.) Lastly, DC, CE, are the sum and difference of AB, AC, and DF, FB, are tangent of the angles DCF, BCF, to the radius CF (35.) Therefore, AB + AC : AB — AC :: tangent of half the sum of BCA, ABC : tangent of the difference between each and the half sum; which was to be demonstrated.

57. Remark.



Since the ſame two ſides may contain any angle whatſoever, it follows by equality of ratio, (173 p.) that the tangents of all half ſums, and differences, of angles oppoſite to the ſame ſides, will be proportional; namely, tangent of *dcf*: tangent of *fcb* :: tangent of DCF : tangent of FCB (in the laſt figure) ſuppoſing *ab*, *ac*, equal to AB, AC reſpectively; and when the contained angle is 90°, the half ſum of the oppoſite angles (*dcf*) is 45°, and the difference *fcb*, will be the exceſs of the greater acute angle above 45°. Hence ariſes another way (very uſeful when the logarithms of the ſides are given) to find the difference between half the ſum of the oppoſite angles, and each of them, by two proportions.

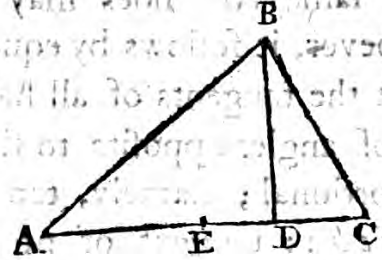
Thus,

1. As the leſſer ſide *ac*, to the greater ſide *ab*; ſo is radius, to the tangent of an angle (*acb*); from which take 45°.

2. As the tangent of 45°, to the tangent of the remainder (*fcb*); ſo is the tangent of half the oppoſite angles (DCF) to the tangent of the difference required FCB.

Theorem 4.

58. *As double the base of a plain triangle, to the sum of its sides; so is the difference of the sides, to the distance of a perpendicular, from the middle of the base.*



Let ABC be a plain triangle, having the side AB, greater than the side BC; and a perpendicular BD, falling upon the base AC, whose middle point is E.

Then $2AC : AB + BC :: AB - BC : ED$.

For $\overline{AB+BC} \times \overline{AB-BC} = 2AC \times ED$. 122 p.

Th. $2AC : AB + BC :: AB - BC : ED$. 187 p.

59. Corollary.

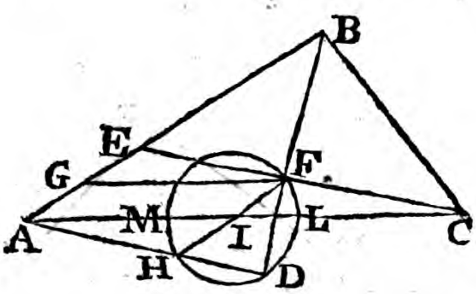
Hence $AD = \frac{1}{2}AC + ED$, *the greater segment.*

And $DC = \frac{1}{2}AC - ED$, *the lesser segment.*

Theorem

Theorem 6.

60. *In a plain triangle, the rectangle contained by any two sides, is to the square of radius; as a rectangle contained by the half sum, and half remainder, of the base and difference of those sides, to the square of the sine of half the vertical angle.*

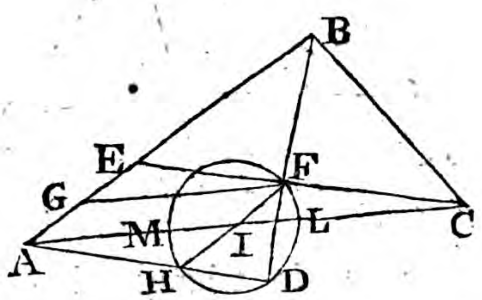


Let ABC be a plain triangle, having the side AB, greater than the side BC; make BE equal to BC; join EC; draw BD, bisecting the angle ABC and cutting EC in F; also draw AD, FG, FH, parallel to EC, CA, BA; and about the center I, with the radius IF, describe the circle FLM.

Now EC is bisected at right angles, by the line BD (62 p); therefore BDA is a right angle, being equal to BFE (83 p), and AE, AC, are bisected in G and I (101 p). Again, AG, AE, EF, are equal to IF, HF, AH, each to each (91 p); therefore HF is bisected in I, since AG, or IF, is the half of AE, or HF; and AG, IL, IM, are e-

 qual

equal (48 p.); therefore AL $= (\frac{1}{2}$AC $+ \frac{1}{2}$AE $=)$ $\frac{AC + AE}{2}$, and AM $= (\frac{1}{2}$AC $- \frac{1}{2}$AE $=)$ $\frac{AC - AE}{2}$. Thirdly, FH being a diameter of the circle, its circumference will paſs thro' the point D (153 p.) therefore AD $\times$ AH $=$ AL $\times$ AM (217 p.) therefore AD $\times$ EF $= \frac{AC + AE}{2} \times \frac{AC - AE}{2}$ (56 p.)

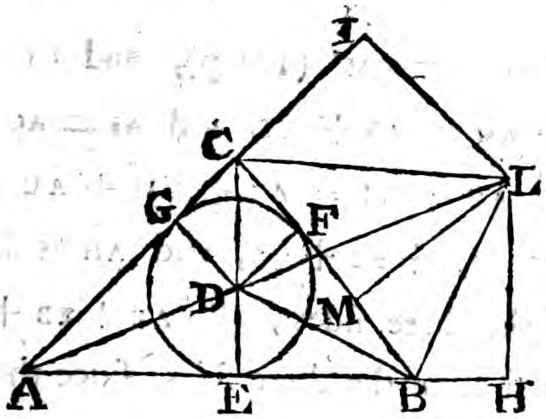


Laſtly, AB : radius :: AD : ſine of $\frac{1}{2}$ABC - 53
And BE : radius :: EF : ſine of $\frac{1}{2}$ABC - 53
Th. AB$\times$BE : $\overline{\text{radius}}^2$ :: AD$\times$EF : $\overline{\text{ſine}}^2$ of $\frac{1}{2}$ABC, 206p.
Th. AB$\times$BC : $\overline{\text{radius}}^2$ :: $\frac{AC + AE}{2} \times \frac{AC - AE}{2}$: $\overline{\text{ſine}}^2$, of half the angle ABC (56 p.); which was to be demonſtrated.

Theorem

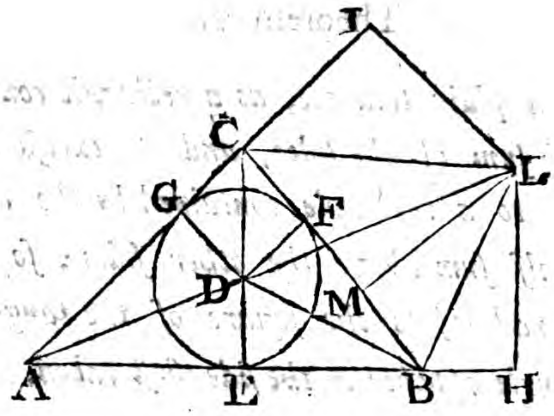
Theorem 7.

61. *In a plain triangle, as a rectangle contained by the half sum of the sides, and its excess above either side, to a rectangle contained by the excesses of that half sum above the other sides; so is the square of radius, to the square of the tangent of half the angle opposite to the side first taken.*



In the plain triangle ABC, find D the center of a circle touching the sides at E, F, G, (333 p.) continue AB, AC, AD, to H, I, L; draw BL bisecting the angle CBH; let fall the perpendiculars LH, LI, LM, and compleat the figure.

1. The angles HAL, LHA = IAL, LIA, each to each, and AL is common; therefore AH = AI, and LH = LI (77 p); for the same reason BH = BM, and LH = LM, therefore LI = LM



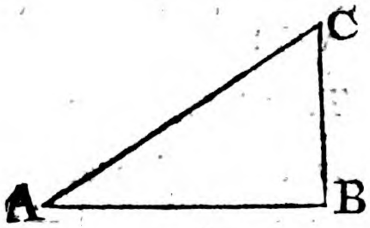
therefore $CI^2 = CM^2$ (100 p), and $CI = CM$; therefore $AH = AB + BM$, and $AI = AC + CM$; therefore $AH + AI = AB + BM + AC + CM$; or $2AH = AB + BC + CA$, and AH is half the ſum of the three ſides; alſo $AE + EB + CF$, is half the ſum of the three ſides, ſince theſe parts are equal to AG, BF, CG, each to each (77 p); therefore $AH = AE + EB + CF$; therefore $BH = CF$, and AE, EB, BH, are the exceſſes.

2. Becauſe E and F are right angles, the ſum of EDF, FBE will be two right angles (89 p); therefore the angles EDF, FBH are equal, and their halves EDB, HBL, are equal.

3. The

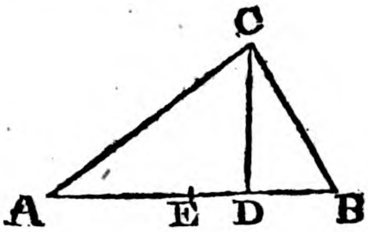
3. The triangle DEB, is equiangular to BHL, and AED, to AHL, by what appears above. Therefore DE : EB :: BH : HL, and AE : AH :: DE : LH. Therefore $DE \times HL = EB \times BH$ (185 p); and $AE^2 : AH \times AE :: DE^2 : DE \times HL$ (184 p). Therefore $AE^2 : DE^2 :: AH \times AE : EB \times BH$, by alternation and substitution; but $AE^2 : DE^2 :: \overline{\text{radius}}^2 : \overline{\text{tangent}}^2$ of DAE (55) Therefore $AH \times AE : EB \times BH :: \overline{\text{radius}}^2 : \overline{\text{tangent}}^2$ of half the angle BAC (173 p): which was to be demonstrated.

62. *Solutions to ſeven queſtions concerning a plain triangle* ABC, *having a right angle at* B.



Art.	Caſe.	Given	Req.	Proportions.	Art.
63	1	AC A	BC	radius : AC :: ſine A : BC	53
64	2	AC BC	A C	AC : radius :: BC : ſine A AC : radius :: BC : coſine C	53 53
65	3	AC BC	AB	AC : radius :: BC : coſine C radius : AC :: ſine C : AB	53 53
66	4	A or C AB	AC	ſine C : AB :: radius : AC	53
67	5	A or C AB	BC	ſine C : AB :: ſine A : BC radius : tan. A :: AB : BC	51 54
68	6	AB BC	A C	AB : radius :: BC : tan. A BC : radius :: AB : tan. C	54 54
69	7	AB BC	AC	AB : radius :: BC : tan. A ſine A : BC :: radius : AC	54 53

70. *Solutions of the four cases in oblique angled plain triangles* ABC, *having* AB *greater than* AC, *and a perpendicular* CD.



Art.	Case	Given.	Req.	Proportions.	Art
71	1	A, B, AB	AC CB	sine C : AB :: sine B : AC sine C : AB :: sine A : CB	51
72	2	AB, BC, A	C AC	BC : sine A :: AB : sine C sine A : BC :: sine B : AC	51
73	3	AB, AC, A	C B	AB + AC : AB — AC :: tan. of $\frac{1}{2}\ \overline{B+C}$: tant. of the dif.	56
74	4	AB, AC, CB	A	2AB : AB + AC :: AB — AC : ED - - - AC : radius :: AD : cos. A	58 53
75	again,	AB, AC, CB	A	$AB \times AC : \overline{\text{radius}}^2 :: \frac{BC+E}{2} \times \frac{BC-E}{2} : \overline{\text{sine}}^2,\ \frac{1}{2}A,$ if E = AB — AC.	60
76	Thirdly	AB, AC, CB	A	$\frac{1}{2}S \times \overline{\frac{1}{2}S - BC} : \overline{\frac{1}{2}S - AB} \times \overline{\frac{1}{2}S - AC} :: R^2 : T^2,\ \frac{1}{2}A.$	61

Remarks

Remarks.

77. *The second case is ambiguous, for the angle opposite to the side required, may be either obtuse or acute; and must be determined by the nature of the question to which the case is applied.*

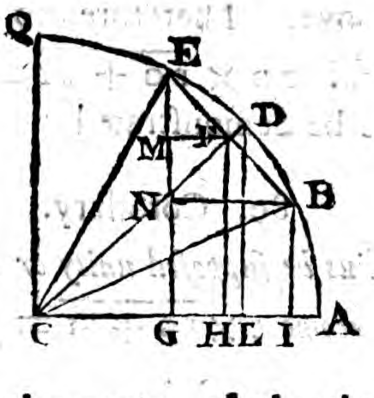
78. *Although I have given seven examples in right angled triangles, (which, by some, are called seven cases); yet, in reality, there are but four cases in plain triangles; for all the proportions used in solving the four cases above, are equally applicable to right angled triangles.*

CHAP. III.

Contains a method of making sines, tangents, &c.

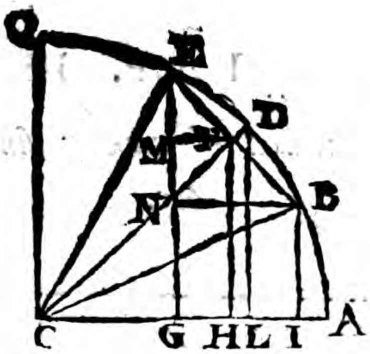
Theorem I.

79. *If three arches of a circle* AB, ABD, ADE, *have a common difference* BD, *or* DE, *the rectangle contained by the radius, and sum of the sines of the extremes; will be equal to a rectangle contained by the sine of the second arch, and twice the cosine of the common difference.*



Let C be the center of the circle, and join CA, CB, CD, CE, EB, and CD will bisect EB at right

right angles in F, becauſe the angles ECF, FCB, are equal (150 p.): Let fall EG, FH, DL, BI, perpendicular to CA, and FM, BN, perpendicular to EG.



Now CD is the radius, DL is the ſine of the ſecond arch AD, CF is the coſine of BD, or DE, the common difference; 2FH or 2MG, is the ſum of EG, BI, the ſines of the extremes, ſince EN is biſected in M (101 p.) and the triangles CDL, CFL, are evidently equiangular. Therefore CD : DL :: (CF : FH ::) 2CF : 2FH; but 2FH = EG + BI, as ſhewn above. Therefore CD : DL :: 2CF : EG + BI, and $CD \times \overline{EG + BI} = DL \times 2CF$; which was to be demonſtrated.

80. Corollary.

If the radius be ſuppoſed unity or 1, the laſt expreſſion will become $1 \times \overline{EG + BI} = DL \times 2CF$; *or* $EG + BI = DL \times 2CF$, *and therefore* $EG = DL \times 2CF - BI$, *in that caſe; which gives the following rule, to find the ſine of the greater extreme arch.*

The

The Rule.

Multiply the ſine of the ſecond arch, by twice the coſine of the common difference; from the product ſubſtract the ſine of the leſſer extreme, and the remainder will be the ſine of the greater extreme arch; the radius being 1.

Theorem 2.

81. *The ſame things being ſuppoſed as in article* 79; *the rectangle contained by the radius, and difference between the ſines of the extremes, will be equal to a rectangle contained by the ſine of the common difference, and twice the coſine of the ſecond arch.*

For CLD, EMF, *are right angles, and the angle* $DCL = (CFM =) FEM$, *therefore the triangles* DCL, FEM, *are equiangular, and* $CD : EF :: (CL : EM ::) 2CL : 2EM$; *therefore* $CD \times 2EM = EF \times 2CL$, *but* $EN = 2EM$, *ſince* EB *is biſected in* F (101 *p.*). *Therefore* $CD \times EN = EF \times 2CL$; *laſtly,* $EN = \overline{EG - BI}$; *therefore* $CD \times \overline{EG - BI} = EF \times 2CL$; *which was to be demonſtrated.*

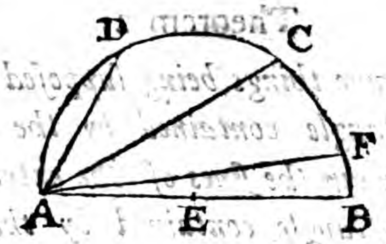
82. Corollary.

When the arch AD *is* 60°, *the line* CL = *ſine of* 30° = *half the chord of* 60° = $\frac{1}{2}$CD (42. 50) *and the equation above will become* $CD \times \overline{EG - BI} = EF \times CD$; *therefore* $EG - BI = EF$, *and* $EG = EF + BI$, *in that caſe.*

Problem

Problem I.

83. *To find the chord of a very small arch, that shall be a determined part of the circumference of a circle, whose radius is unity, or* I.



In the semicircle ABCD, apply the line AD, equal to the radius AE, and draw AC bisecting the angle BAD.

Now AD is the side of a regular hexagon (335 p.) and the arches BC, CD, are equal (148 p.) therefore each is $\frac{1}{6}$ of the compleat circumference, and the line AC is given (220 p); also by the same theorem, a series of bisecting lines will be given approaching to the diameter AB, and every intercepted arch (BC) will be a known part of the circumference. Having thus determined an arch BF, sufficiently minute for the purpose, by joining BF, the triangle ABF, is right

right angled at F (153 p.) and the chord BF is given (100 p.).

Operation.

No.	AC^2	AC	arch BF
1	3,0000000000	1,7320508075	$\frac{1}{6}$
2	3,7320508075	1,9318516525	$\frac{1}{12}$
3	3,9318516525	1,9828897227	$\frac{1}{24}$
4	3,9828897227	1,9957178465	$\frac{1}{48}$
5	3,9957178465	1,9989291743	$\frac{1}{96}$
6	3,9989291743	1,9997322757	$\frac{1}{192}$
7	3,9997322757	1,9999330678	$\frac{1}{384}$
8	3,9999330678	BF^2=0,0000669322	$\frac{1}{768}$

And consequently BF, the chord of $\frac{1}{768}$ part of the whole circumference is 0,00818121, very nearly.

Problem 2.

84. *To find the sine of one minute, the radius of the circle being* 1.

Since the whole circumference of a circle is divided into 360 degrees, it will contain 21600 minutes (21) and therefore $\frac{21600}{768}$ expresses the number of minutes in the arch BF, as determined in the preceding problem.

Again,

Again, the chords of very ſmall arches, are ſo nearly equal to the arches themſelves, that ſuch arches and their chords may be aſſumed for proportionals without any material error, as will appear by comparing the correſponding ſides of ſimilar inſcribed and circumſcribed polygons (353 p). Therefore, as $\frac{21600}{768}$ minutes:0,00818121 its chord :: 2 minutes : $\frac{0{,}00818121 \times 2 \times 768}{21600} = \frac{0{,}00818121 \times 768}{10800} = 0{,}000581775$, the chord of two minutes; and conſequently its half 0,000290888, is the ſine of 1 minute, very nearly (42).

Problem 3.

85. *To find the ſquare of the coſine of one minute, to the radius* 1.

The ſine of one minute is 0,000290888, by the preceding problem, and its ſquare is 0,00000008461,&c.therefore $1^2 - 0{,}00000008461 = 0{,}99999991539$, is the ſquare of the coſine of one minute (45). Therefore $\sqrt{0{,}99999991539} = 0{,}9999999577$, is the coſine of the ſame, and 1,9999999154 is twice the coſine of one minute, which was required; put $2c = 1{,}9999999154$.

Problem

Problem 4.

86. *To find the ſine correſponding to every minute of the quadrant.*

Rule.

Make the arch of one minute the common difference of three arches continually; and calculate the ſine of every minute up to 60 *degrees, by article* 80.

Thus.

Sine of $\overset{m}{1} \times 2c$ — ſine $\overset{m}{0}$ = ſine of $\overset{m}{2} = 0{,}00058176$
- - $2 \times 2c$ — ſine 1 = - $3 = 0{,}00087266$
- - $3 \times 2c$ — ſine 2 = - $4 = 0{,}00116355$
- - $4 \times 2c$ — ſine 3 = - $5 = 0{,}00145444$
&c. &c.

and after the very ſame manner we may proceed to the ſine of 60 degrees.

Remarks.

87. *The firſt arch being indefinitely ſmall, its ſine is* 0, *and the ſine of the ſecond arch is* 0,00029088, *the ſine of one minute, in that caſe.*

88. *The ſines being found to* 60 *degrees, all the ſines from thence to* 89° : 59′, *may be found by addition only; for the ſine of any arch above* 60°, *is equal to the ſine of an arch as much below* 60° + *the ſine of their common difference* - - - 82

Thus the ſine of 60° : 1′ = *ſine of* 59° : 59′ + *ſine of* 1 *minute, the ſine of* 70° = *ſine of* 50° + *ſine of* 10°, &c.

CHAP. IV.

Of LOGARITHMS.

92. *Logarithms are the indices of powers having the same root.*

Thus a^{-3}, a^{-2}, a^{-1}, a^{0}, a^{1}, a^{2}, a^{3}, a^{4}, &c. are powers of the number represented by a, and the indices -3, -2, -1, 0, 1, 2, 3, 4, &c. are the logarithms of those powers to which they belong.

So -3, -2, -1, 0, 1, 2, 3, 4, &c. are logs. of a^{-3}, a^{-2}, a^{-1}, a^{0}, a^{1}, a^{2}, a^{3}, a^{4}, &c. respectively.

93. *Let* $a = 10$.

Then -3, -2, -1, 0, 1, 2, 3, 4, &c. are logs of $\frac{1}{1000}$, $\frac{1}{100}$, $\frac{1}{10}$, 1, 10, 100, 1000, 10000, &c. respectively, of the kind in common use; but the logarithms of the intermediate numbers must be found by calculation.

94. *All*

94. *All intermediate logarithms consist of one of the indices, o, 1, 2, 3, &c. with a decimal fraction annexed.*

95. *The logarithm of* 1 *is* 0, *for* $a^0 = 1$.

96. *The logarithms of all numbers greater than* 1, *are positive, and of all less than* 1, *are negative.*

97. *The sum of two logarithms, is the logarithm of the product of their corresponding numbers, and their difference is the logarithm of the quotient.*

98. *Twice the logarithm of a number is the logarithm of its square, and half is the logarithm of its square root.*

99. *The logarithms of all powers, are found by multiplying the logarithm of the given number, by the given index.*

100. *The decimal part of all logarithms is positive, and it is the index only that is negative in the logarithms of decimal fractions.*

Problem 1.

101. *To find the logarithm of a number represented by* $1 + x$.

For the logarithm of $1 + x$, assume the series $ax + bx^2 + cx^3 + dx^4$, &c. of indeterminate quantities only, and the logarithm of $\overline{1+x}^2$ will be $= 2ax + 2bx^2 + 2cx^3 + 2dx^4$, &c. - - 98
Again $\overline{1+x}^2 = \overline{1+x} \times \overline{1+x} = 1 + \overline{2x + xx}$. Therefore its logarithm may be $a \times \overline{2x + xx} + b \times \overline{2x+xx}^2 + c \times \overline{2x+xx}^3 + d \times \overline{2x+xx}^4$, &c. according to the first assumption; this latter being also an indeterminate series, which expanded and properly ordered will be $= 2ax + \overline{a+4b} \times x^2 + \overline{4b+8c} \times x^3 + \overline{b+12c+16d} \times x^4$, &c. Therefore we have $2ax + 2bx^2 + 2cx^3$, &c. $= 2ax + \overline{a+4b} \times x^2 + \overline{4b+8c} \times x^3$, &c. each series expressing the log. of $\overline{1+x}^2$.

Th. $2a = 2a$ - - - -	Th. $a = + a$
$2b = a + 4b$ - - -	$b = -\frac{1}{2} a$
$2c = 4b + 8c$ - -	$c = +\frac{1}{3} a$
$2d = b + 12c + 16d$ -	$d = -\frac{1}{4} a$
&c. &c. - - -	&c. &c.

Th. the log. of $1 + x = ax - \frac{1}{2} ax^2 + \frac{1}{3} ax^3 - \frac{1}{4} ax^4$, &c.

Problem 2.

102. *To find the logarithm of the number* $1 - x$.

Let the log of $1 - x = a \times -x + b \times x^2 + c \times -x^3 + d \times x^4$, &c.

Then the log. of $\overline{1-x}^2 = 2a \times -x + 2b \times x^2 + 2c \times -x^3 + 2d \times x^4$, &c. by article 98.

Again $\overline{1-x}^2 = \overline{1-x} \times \overline{1-x} = 1 - 2x + xx$. Therefore the log. of $\overline{1-x}^2$, may be $a \times \overline{-2x+xx} + b \times \overline{-2x+xx}^2 + c \times \overline{-2x+xx}^3 + d \times \overline{-2x+xx}^4$, &c. after the manner of the first assumption. Or $2a \times -x + \overline{a+4b} \times x^2 + \overline{4b+8c} \times -x^3 + \overline{b+12c+16d} \times x^4$, &c. by expanding the series. Therefore $2a \times -x + 2b \times x^2 + 2c \times -x^3 + 2d \times x^4$, &c. $= 2a \times -x + \overline{a+4b} \times x^2 + \overline{4b+8c} \times -x^3 + \overline{b+12c+16d} \times x^4$, &c. each series expressing the log. of $\overline{1-x}^2$.

Th. $2a = 2a$	- - - and	$a = +a$
$2b = a + 4b$	- -	$b = -\frac{a}{2}$
$2c = 4b + 8c$	- - -	$c = +\frac{a}{3}$
$2d = b + 12c + 16d$	-	$d = -\frac{a}{4}$
&c.	&c. - -	&c. &c.

Th. the log. of $1 - x = -ax - \frac{1}{2}ax^2 - \frac{1}{3}ax^3 - \frac{1}{4}ax^4$, &c.

Problem

Problem 3.

103. *To find the logarithm of a number expressed by the fraction* $\frac{1+x}{1-x}$.

Solution.

Log. of $1+x=+ax-\frac{1}{2}ax^2+\frac{1}{3}ax^3-\frac{1}{4}ax^4$, &c. 101

Log. of $1-x=-ax-\frac{1}{2}ax^2-\frac{1}{3}ax^3-\frac{1}{4}ax^4$, &c. 102

Log. of $\frac{1+x}{1-x}=2ax+\frac{2}{3}ax^3+\frac{2}{5}ax^5+\frac{2}{7}ax^7$, &c. 97

Or L. of $\frac{1+x}{1-x}=\overline{x+\frac{1}{3}x^3+\frac{1}{5}x^5+\frac{1}{7}x^7, \&c.}\times 2a$.

Problem 4.

104. *To find the logarithm of a given number represented by n.*

Solution.

Put $\frac{1+x}{1-x}=n$, then $1+x=n-nx$, $nx+x=n-1$, and $x=\frac{n-1}{n+1}$, a given number.

Therefore the log. of $n=\overline{x+\frac{1}{3}x^3+\frac{1}{5}x^5+\frac{1}{7}x^7, \&c.}$
$\times 2a$ - - - - - - 103
which was required.

Problem 5.

105. *The logarithm of any number* (n) *being given, it is required to find the logarithm of* $n+1$.

Put $\frac{1+x}{1-x}=\frac{n+1}{n}$.

Then $n+nx=n+1-nx-x$, by multiplication.

Th. $2nx+x=1$.

Th. - $x=\frac{1}{2n+1}$.

Therefore x is given, and the logarithm of $\frac{1+x}{1-x}$ or $\frac{n+1}{n}$ is given(103); but the log. of $\frac{n+1}{n}$ + log. of n = log. of $\frac{n+1}{n}\times n$ (97) and is therefore given. Laftly, $\frac{n+1}{n}\times n=n+1$. Therefore the logarithm of $n+1$ is given, and is found by the following

Rule.

Put $x=\frac{1}{2n+1}$, and find the number expreffed by $\overline{x+\frac{1}{3}x^3+\frac{1}{5}x^5+\frac{1}{7}x^7, \&c.}\times 2a$, which is the logarithm of $\frac{1+x}{1-x}$, or $\frac{n+1}{n}$ (103). To this number add the log. of n, and the fum will be the log. of $n+1$; which was required.

Problem

Problem 6.

106. *To find the logarithm of* 2.

Now according to article 104, $n = 2$.

Th. $x = \frac{1}{3}$, $x^2 = \frac{1}{9}$, $x^3 = x \times \frac{1}{9}$, $x^5 = x^3 \times \frac{1}{9}$, &c.

Therefore.

x	$= 0,33333333$	$\frac{1}{1}$	$= 0,33333333$, &c.
x^3	$= 0,03703704$	$\frac{1}{3}$	$= 0,01234568$
x^5	$= 0,00411523$	$\frac{1}{5}$	$= 0,00082305$
x^7	$= 0,00045725$	$\frac{1}{7}$	$= 0,00006532$
x^9	$= 0,00005080$	$\frac{1}{9}$	$= 0,00000564$
x^{11}	$= 0,00000564$	$\frac{1}{11}$	$= 0,00000051$
x^{13}	$= 0,00000063$	$\frac{1}{13}$	$= 0,00000005$

The sum - - $= 0,34657358$, &c.
Multiply by - - - $2a$

And the logarithm of $2 = 0,69314716 \times a$ by article 104.

Problem

Problem 7.

107. *Having the logarithm of* 2, *it is required from thence to find the logarithm of* 3.

Now by article 105, we have $n = 2$, and $x = \frac{1}{5}$.

Th. $x^2 = \frac{1}{25}$, $x^3 = x \times \frac{1}{25}$, $x^5 = x^3 \times \frac{1}{25}$, &c.

Therefore,

$x = 0,20000000$	$\frac{1}{1} = 0,20000000$, &c.
$x^3 = 0,00800000$	$\frac{1}{3} = 0,00266666$
$x^5 = 0,00032000$	$\frac{1}{5} = 0,00006400$
$x^7 = 0,00001280$	$\frac{1}{7} = 0,00000183$
$x^9 = 0,00000051$	$\frac{1}{9} = 0,00000006$

The sum - -	$= 0,20273255$
Multiply by - - -	$2a$
The logarithm of $\frac{3}{2}$	$= 0,40546510 \times a$
The logarithm of 2	$= 0,69314716 \times a$
The logarithm of 3	$= 1,09861226 \times a.$

By article 105.

Problem

Problem 8.

108. *To find the logarithm of* 4, *from the logarithm of* 2, *already found* - - - 106

Now the logarithm of 2 - $= 0{,}693147 16a$
Multiply by - - - 2

And the logarithm of 4 - $= 1{,}38629432a$
By article - - - - - 98

Problem 9.

109. *To find the logarithm of* 5, *from the logarithm of* 4, *found above.*

Now by article 105, $n = 4$, and $x = \frac{1}{9}$.
Th. $x^2 = \frac{1}{81}$, $x^3 = x \times \frac{1}{81}$, $x^5 = x^3 \times \frac{1}{81}$, &c.

Therefore,

$x = 0{,}11111111$	$\frac{1}{1} = 0{,}11111111$, &c.
$x^3 = 0{,}00137176$	$\frac{1}{3} = 0{,}00045725$
$x^5 = 0{,}00001693$	$\frac{1}{5} = 0{,}00000339$
$x^7 = 0{,}00000021$	$\frac{1}{7} = 0{,}00000003$

Sum - - - $= 0{,}11157178$
Multiply by - - $2a$

Product is the log. of $\frac{5}{4} = 0{,}22314356a$
Add the logarithm of $4 = 1{,}38629432a$

The sum is the log. of $5 = 1{,}60943788a$

Problem

Problem 10.

110. *To find the logarithm of* 6, *from those of* 2 *and* 3, *already found.*

Now the log. of 2	- -	$= 0{,}693147 16a$
And the log. of 3	- -	$= 1{,}098612 26a$
Th. the log. of 6	- -	$= 1{,}791759 42a$
By article	- - - - -	97

Problem 11.

111. *To find the logarithm of* 7, *from the logarithm of* 6, *found above.*

Here $n = 6$, and $x = \frac{1}{13}$, by article 105.

Therefore,

$x = 0{,}07692307$	$\frac{1}{1} = 0{,}07692307$, &c.
$x^3 = 0{,}00045516$	$\frac{1}{3} = 0{,}00015172$
$x^5 = 0{,}00000269$	$\frac{1}{5} = 0{,}00000054$
$x^7 = 0{,}00000001$	$\frac{1}{7} = 0{,}00000000$

The sum	- -	$= 0{,}07707533$
Multiply by	- -	$2a$
The log. of $\frac{7}{6}$	-	$= 0{,}15415066a$
Add the log. of 6	-	$= 1{,}79175942a$
And the log. of 7	-	$= 1{,}94591008a$

Problem 12.

112. *To find the logarithm of* 8, *from those of* 4 *and* 2, *by article* 97.

Now the log. of 4	- -	= 1,386294324a
And the log. of 2	- -	= 0,693147216a
Th. the log. of 8	- -	= 2,079441480a

Problem 13.

113. *To find the logarithm of* 9, *from that of* 3, *by article* 98.

The log. of 3	- -	= 1,098612260a
Multiplied by	- - -	2
Gives the log. of 9	- -	= 2,197224520a

Problem 14.

114. *To find the logarithm of* 10, *from those of* 5 *and* 2, *by article* 97.

Here the log. of 5	- -	= 1,609437880a
And the log. of 2	- -	= 0,693147216a
Th. the log. of 10	- -	= 2,302585040a

Problem

Problem 15.

115. *To find the logarithms in common uſe, from thoſe before inveſtigated.*

Now the logarithm of 10, is univerſally expreſſed by $2,30258504 \times a$ - - 114

And the logarithm of 10 by the common tables, or by article 93 is 1.

Therefore $2,30258504 \times a = 1$; by the problem.

Therefore $a = \frac{1}{2,30258504} = 0,43429$, &c. which is the multiplicator for reducing the logarithms of the kind before inveſtigated, to others of the common ſort.

Remarks.

116. *Since the number (a) may be taken at pleaſure, there may be an infinite variety of logarithms; and when a is unity or 1, they will be the ſame as were firſt of all publiſhed by Lord Neper; but the logarithms in common uſe are ſaid to be an improvement by Mr. Briggs.*

117. *By*

117. *By taking $a=1$, we have already found Neper's logarithms of the first ten numbers, and multiplying each of them by* 0,43429,&c. *the products which arise will be the corresponding logarithms in Briggs's form, as in the table below.*

Numbers.	Nepers's log.	Briggs's log.
1	0,000000	0,000000
2	0,693147	0,301030
3	0,098612	0,477121
4	1,386294	0,602060
5	1,609438	0,698970
6	1,791759	0,778151
7	1,945910	0,845098
8	2,079441	0,903090
9	2,197225	0,954242
10	2,302646	1,000000

And after the same manner, the whole logarithmic table may be constructed.

118. *The*

118. *The decimal part of every logarithm belongs equally to a whole number, a mixt number, and a decimal fraction that are expressed by the same figures in the same order; but the index will vary according to the value of the expression.*

Thus if a number is expressed by 7854, *then according to its value, we shall give its decimal part, and compleat logarithm, in the following table.*

Number.	Decim. part.	Compleat log.
7854	0,895091	3,895091
785,4	0,895091	2,895091
78,54	0,895091	1,895091
7,854	0,895091	0,895091
0,7854	0,895091	$\bar{1}$,895091
0,07854	0,895091	$\bar{2}$,895091
&c.	&c.	&c.

From hence it will be easy to find the logarithm of a given number, and the number corresponding to a given log. by the tables.

CHAP. IV.

Containing the Practice of plain Trigonometry.

119. *If any three parts of a plain triangle be given, any required part may be found, both by construction and calculation.*

120. *If two angles of a plain triangle are known in degrees, minutes, &c. the third angle is found by substracting their sum from* 180 *degrees.* 26

121. *In a right angled plain triangle, if either acute angle (in degrees) be taken from* 90 *degrees, the remainder will express the other acute angle* - - - - - - - 27

122. *When the sine of an obtuse angle is required, substract such obtuse angle from* 180 *degrees, and take the sine of the remainder, or supplement* - - - - - 29, 38

123. *If two sides of a triangle are equal, a line bisecting the contained angle will be perpendicular to the remaining side, and divide it equally* 62 p

124. *Before the required side of a triangle can be found by calculation, its opposite angle must first be given, or found.*

125. *The required part of a triangle must be the last term of four proportionals written in order under one another; whereof the three first are given, or known.*

126. *If the logarithm of a given number* (n) *be taken from the log. of* 1 *with a borrowed index* (x). *The remainder is called its complement; and is only the log of* $\frac{1}{n}$ *with an artificial index. For the log. of* 1—*log. of* $n = x$, 000000 — *log. of* $n =$ *log. of* $\frac{1}{n}$ - - - - 97

127. *The complement of any logarithm, sine, or tangent in the common tables, is its difference from the radius* 10,000000, *or its double* 20,000000.

128. *In*

128. *In four proportional quantities, either of them may be made the last term; thus let* A, B, C, D *be proportional quantities. Then*

1. *As first to second, so is third to fourth* A : B :: C : D
2. *As second to first, so is fourth to third* B : A :: D : C
3. *As third to fourth, so is first to second* C : D :: A : B
4. *As fourth to third, so is second to first* D : C :: B : A

Because proportion is an equality of ratios 160 *p*

129. *Against the three first terms of every proportion (or stating), must be written their respective values, taken from the proper tables.*

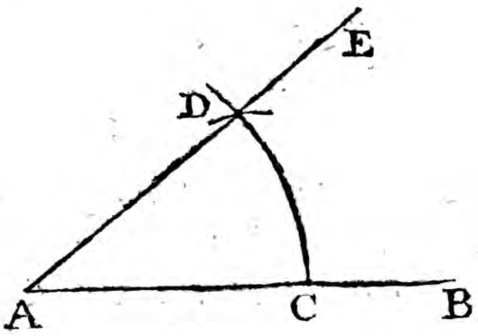
130. *If the value of the first term be taken from the sum of the second and third, the remainder will be the value of the fourth term or thing required; because the addition and substraction of logarithms, corresponds with the multiplication and division of natural numbers.* - - 97

131. *If to the complement of the first value be added, the second and third values; the sum (rejecting the borrowed index) will be the tabular number, expressing the thing required.*

N. B. This method is generally practiced when the radius falls not among the proportionals.

Problem I.

132. *At a given point* A, *with the strait line* AB, *to make an angle that shall contain a given number of degrees, supposing* 33° : 28′.



Construction by the plain scale.

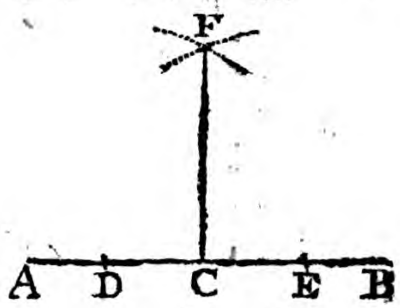
Take the chord of sixty degrees by the compasses, and with that extent about the center A, describe a circle meeting AB in C; again, with the chord of the given number of degrees 33° : 28′, about the center C, describe an arch cutting the former in D, and draw ADE.

For the chord of 60° is equal to the radius. 50
Th. the circle described is equal to the circle divided - - - - 55 P
Also the chord of CD is equal to the chord of 33° : 28′ - - - con.
Th. CD is equal to an arch of 33° : 28′. 151 P
Th. BAE contains - 33° : 28. 24

Problem

Problem 2.

133. *To erect a perpendicular to a given strait line* AB, *at a given point* C *therein.*



Construction.

In the line AB, take the equal parts CD, CE, of any magnitude; about the points D and E as centers, with an extent of the compasses greater than DC or CE, describe arches cutting each other in F, and draw CF.

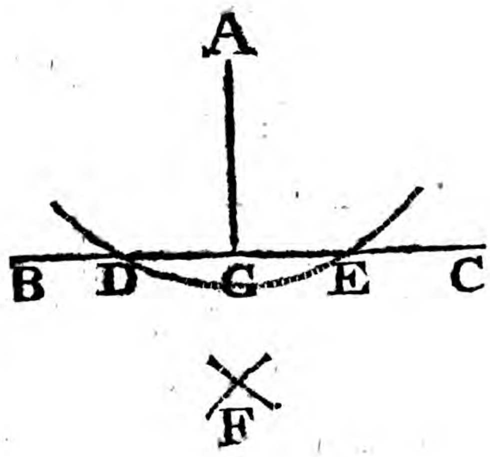
Then CF will be perpendicular to AB. 298 p

Otherwise.

At the point C, with the line AB, make the angle ACF, or BCF to contain 90 degrees, (by the proceeding problem) and the thing is done, for each will be a right angle - - 25

Problem 3.

134. *From a given point* A, *to let fall a perpendicular upon an infinite ſtrait line* BC.



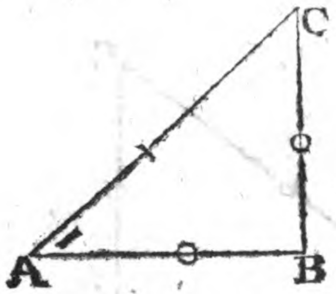
Conſtruction.

About the center A, deſcribe a circle cutting the given line in D and E, and about the centers D and E, with the ſame radius, deſcribe arches cutting each other in F, then lay a ruler to the points A, F, and draw A G, the perpendicular required - - - 299 p

Problem

Problem 4.

135. *In the triangle* ABC *is given a right angle at* B, *the angle* A *of* 42° 45′, *and the hypothenuse* AC 76 *yards long; to find the length of the base* AB, *and perpendicular* BC.



Draw AB at pleasure, make the angle A to contain 42° : 45 by article 132, upon AC lay 76 equal parts, and let fall the perpendicular C B, by article 134; then ABC will represent the triangle.

Angle B	=	90°	: 00
- - A	=	42	: 45
- - C	=	47	: 15

Proportion for A B. art. 53.

As the radius - - -		10,000000
To the hypoth. AC = 76 -	log.	1,880813
So is the sine of C = 47° : 15′		9,865887
To the base AB = 55, 81 -		1,746700

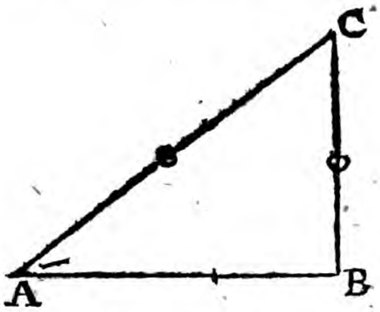
Proportion for BC art. 53.

As the radius - - -		10,000000
To the hypoth. AC = 76 -	log.	1,880813
So is the sine A = 42° : 45′ -		9,831742
The perpend. BC = 51, 59 -		1,712555

Problem

Problem 5.

136. *In the triangle* ABC, *is given a right angle at* B, *the angle* A 39° : 56′, *and the ſide* AB 547 *meaſures of any kind; to find* AC *and* BC *in the ſame.*



Draw AB, and make it of 547 equal parts, erect BC perpendicular to AB, make the angle A of 39° : 56′; and meaſure AC, BC, by the line of equal parts.

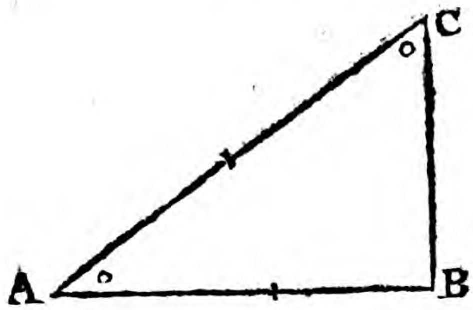
Angle B = 90° : 00′
- - A = 39 : 56
- - C = 50 : 04

Proportion for AC.			art. 53.
As the ſine of C	= 50° : 04 comp.		0,115323
To AB - -	= 547	-	2,737987
So is radius -	-	-	10,000000
To AC - -	= 713,4	-	2,853310
Proportion for BC.			art. 54.
As radius -	-	-	10,000000
To the tan. of A	= 39° : 56′		9,922787
So is AB - -	= 547	-	2,737987
To BC - -	= 457,9	- -	2,660774

Problem

Problem 6.

137. *In the triangle* ABC, *is given a right angle at* B, *the base* AB 5637, *and the hypothenuse* AC 6828 ; *to find the angles* A *and* C.



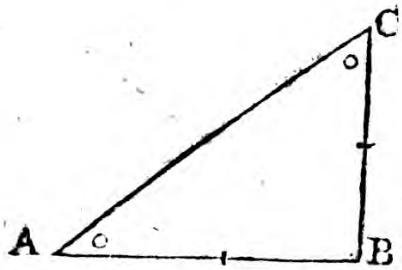
Draw AB, and make it 5637 equal parts, erect the perpendicular BC, about the center A, with a radius of 6828 equal parts, describe a circle cutting BC in C, draw AC, and measure the angles by a line of chords, as in article 132.

Proportion for the angle A.		art. 53.
As AC - -	= 6828	3,834294
To radius - - -		10,000000
So is AB - -	= 5637	3,751048
To the cosine of A	= 34° : 21	9,916754

Proportion for the angle C.		art. 53.
As AC - -	= 6828	3,834294
To radius - - -		10,000008
So is AB - -	= 5637	3,751040
To the sine of C -	= 55° : 39′	9,9 6754

Problem 7.

138. *In the triangle* ABC, *is given a right angle at* B, *the base* AB 547,6, *and the perpendicular* 456,7; *to find the angles* A *and* C.



Draw AB, and make it 547,6; erect BC at right angles at AB, and make it 456,7; draw AC, and measure the angle A and C, by a line of chords.

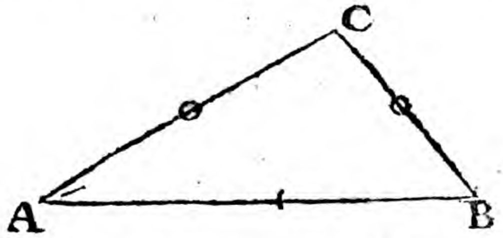
Proportion for the angle A.		art. 54.
As AB - -	= 547,6	2,738463
To radius - - -		10,000000
So is BC - -	= 456,7	2,659630
To the tangent of A	= 39° : 50′	9,921167

Proportion for the angle C		art. 54.
As BC - -	= 456,7	2,659630
To radius - - -		10,000000
So is AB - -	= 547,6	2,738463
To the tangent of C	= 50° : 10′	10,078833

Problem

Problem 8.

139. *In the trangle* ABC *is given, the base* AB 7854 *feet, the angle* A 30° : 36′, *and the angle* B 52° : 28′; *to find the sides* AC *and* BC.



Draw AB and make it equal to 7854 equal parts, make the angle BAC of 30° : 36′, the angle ABC of 52° : 28′; and measure AC, CB by the line of equal parts.

Angle A =	30°	: 36′
- - B =	52	: 28
Sum - =	83	: 04
Sub. fr.	180	: 00
Angle C =	96	: 56

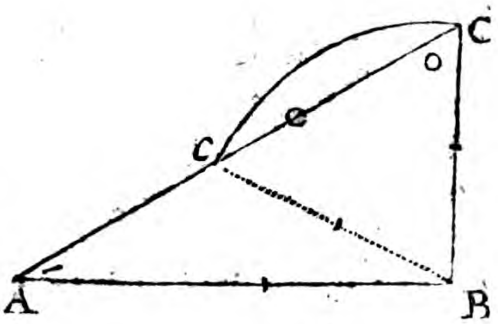
Proportion for AC		art. 52.
As the sine of C = 96° : 56′		comp. 0,003188
To the side A B = 7854	-	3,895091
So is the sine of B = 52° : 28	-	9,899273
To the side A C = 6274	- -	3,797552

Proportion for BC		art. 52.
As the sine of C = 96° : 56′		comp. 0,003188
To the side AB = 7854	- -	3,895091
So is the sine of A = 30° : 36′	-	9,706753
To the side BC = 4028	- -	3,605032

Problem

Problem 9.

140. *In the triangle* ABC *is given, the ſide* AB 7855, *the ſide* BC 4976, *and the angle* A 30° : 36′, *to find the ſide* AC *when the angle* C *is acute.*



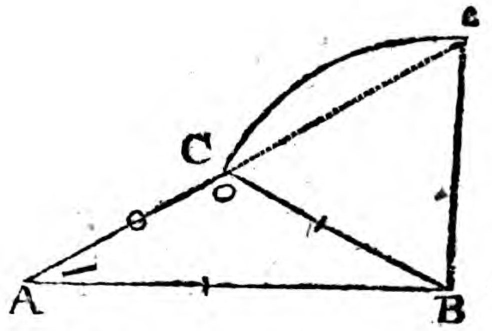
Make the baſe AB 7855 equal parts, and the angle BAC 30° : 36′, about the center B, with a radius of 4976 equal parts deſcribe the arch *c* C, and draw BC.

Proportion for the angle ACB.		art. 52.
As the ſide BC = 4976	comp.	6,303120
To the ſine of A = 30° : 36′	- -	9,706753
So is the ſide AB = 7855	- - -	3,895146
To the ſine of C = 53 : 28	- - -	9,905019
Angle A + C = 30° : 36′ + 53° : 28′		= 84 : 04
Subſtract from	- - -	180 : 00
Remains the angle B	- -	= 95 : 56
Proportion for the ſide AC		art. 52.
As the ſine of A = 30° : 36′	comp.	0,293247
To the ſide BC = 4976	- - -	3,696880
So is the ſine of B = 95 : 56	- -	9,997667
To the ſide AC = 9723	- - -	3,987794

Pro-

Problem 10.

141. *In the triangle* ABC *is given the ſide* AB 7855, *the ſide* BC 4976, *and the angle* A 30° : 36′, *to find the ſide* AC *when the angle* C *is obtuſe.*



Make the baſe AB 7855 equal parts, and the angle BAC 30° : 36′, about the center B with a radius of 4976 equal parts deſcribe the arch C *c*, and draw BC.

Proportion for the angle ACB.

As the ſide BC	= 4276	comp.	6,303120
To the ſine of A	= 30° : 36′	--	9,706753
So is the ſide AB	= 7855	- --	3,895146
To the ſine of C	= 126° : 32′	--	9,905019

Angle A+C = 30° : 36′ + 126 : 32′ = 157° : 08′
Subſtract from - - - 180 : 00
Remains the Angle B - - = 22 : 52

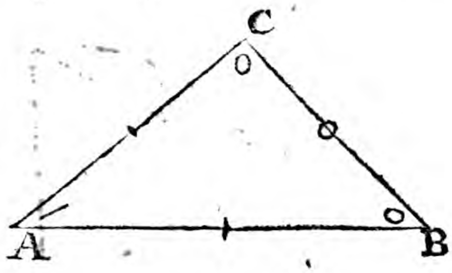
Proportion for the ſide AC.

As the ſine of A	= 30° : 36′	comp.	0,293247
To the ſide BC	= 4976	- - -	3,696880
So is the ſine of B	= 22 : 52	- -	9,589489
To the ſide AC	= 3798,5	- -	3,579616

Pro-

Problem 11.

142. *In the triangle* ABC *is given, the base* AB 785, *the side* AC 627, *and the contained angle* A 30° : 36′; *to find the angles* B *and* C, *and the side* BC.



Make the angle BAC to contain 30° : 36′, the sides AB, AC of 785 and 627 equal parts respectively, and draw CB,

AB	=	785	All the angles	=	180° : 00
AC	=	627	The angle A	=	30 : 36
Sum =		1412	Angle B+C	=	149 : 24
Diff. =		158	Half B+C	=	74 : 42

Proportion for the difference between either unknown angle and their half sum art. 56.

As the sum of AB, AC =	1412	comp.	6,850164
To the diff. of AB, AC =	158	- -	2,198657
So tan. of ½ the angles =	74° : 42′		10,562933
To tang. of the diff. =	22 : 15		9,611754

Hence the angle C = 96 : 57
And the angle B = 52 : 27

Proportion for BC. art. 52.

As the sine of B	= 52° : 27	comp.	0,100825
To AC - - -	= 627	- -	2,797268
So is the sine of A	= 30 : 36	-	9,706753
To BC - - -	= 402,5	- -	2,604846

Again.

143. *Let the plain triangle* ABC, *have the baſe* AB $= 758$, *ſide* AC $= 627$, *and the angle* A $= 30^\circ : 36'$; *to find the angles* B *and* C.

Solution by article 57.

As the leſſer ſide AC $= 627$	-	2,797268
To the greater ſide AB $= 785$	-	2,894869
So is radius - -	-	10,000000
To tan. of an angle $= 51^\circ : 23'$		10,097601
From which ſubſtract 45 : 00		
There remains an ang. $=$ 6 : 23		

All the ang.	$=$	180 : 00
Angle A -	$=$	30 : 36
B + C -	$=$	149 : 24
Half -	$=$	74 : 42

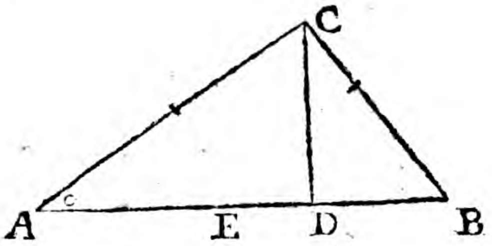
As the tangent of -	$45^\circ : 00'$	10,000000
To the tangent of -	6 : 23	9,048727
So is the tangent of -	74 : 42	10,562933
To the tangent of -	22 : 15	9,611660

Th. the angle C $=$ 96 : 57
And the angle B $=$ 52 : 27, as before determined.

Problem

Problem 12.

143. *In the triangle* ABC *is given, the base* AB 785, *the side* AC 627, *and the side* BC 403; *to find either of the angles; suppose it* A.



Make the line AB of 785 equal parts, on the centers A and B, with the distances of 627 and 403 equal parts respectively; describe circles intersecting at C, and draw AC, CB.

First method.

Let fall the perpendicular CD, and let E be the middle point of the base AB.

Proportion for ED.			art. 58.
As twice AB -	= 1570	comp.	6,804101
To sum of AC, CB	= 1030	-	3,012857
So diff. of AC, CB	= 224	-	2,350258
To the part ED	= 146,9	-	2,167196

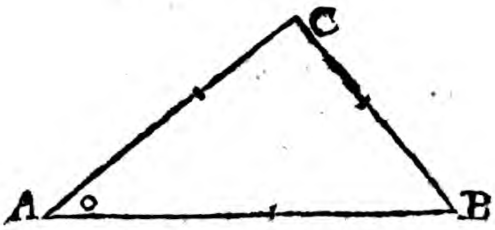
AE = 392,5
ED = 146,9

AD = 539,4
DB = 245,6

Proportion for the angle A.			art. 53.
As AC -	= 627	comp.	7,202732
To radius -	-	-	10,000000
So is AD -	= 539,4	-	2,731911
To the cosine of A	= 30° : 39′	-	9,934643

Problem

144. *Second method of finding the angle* A, *by theorem* 6, *article* 60.



AB	-	= 785.	BC = 403
AC	-	= 627.	diff. = 158
diff.	-	= 158.	sum = 561, its half is 280,5
			Rem. = 245, its half is 122,5

AB -	= 785	-	comp. 7,105131
AC -	= 627	-	comp. 7,202732
Half sum	= 280,5	-	log. 2,447932
Half rem.	= 122,5	-	log. 2,088136
Sum of the four logarithms		-	= 18,843931

Half is the sine of $15^{\circ} : 19' : 14''$ = 9,421965
Th. the angle A is 30 : 38 : 28, which was required.

 145. *Third*

145. *Third Method of finding the angle* A, *by theorem* 7, *article* 61.

BC	-	= 403
AB	-	= 785
AC	-	= 627
Sum	-	= 1815

Half ſum	= 907,5	- comp.	7,042153
1ſt exceſs	= 504,5	- comp.	7,297139
2d exceſs	= 122,5	- log.	2,088136
3d exceſs	= 280,5	- log.	2.447932
Sum of the four logarithms		- =	18,875360
Half is the tang. of $15^{\circ} : 19' : 14''$		=	9,437680

Th. the angle A is 30 : 38 : 28, as before determined

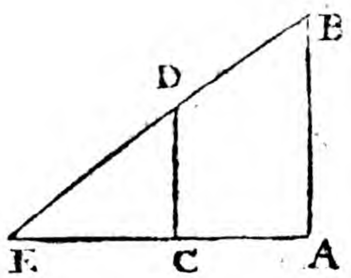
C H A P.

CHAP. V.

Of meaſuring Heights and Diſtances.

Problem I.

146. *To find the height of an acceſſible object,* A B, *that is perpendicular to the plain of the horizon.*

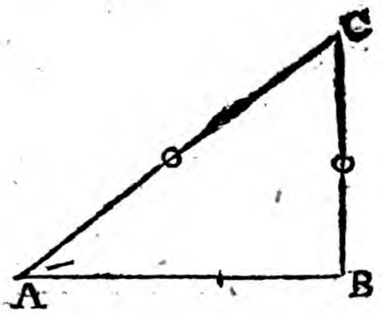


Meaſure the ſhadow of the object AB, and the ſhadow of a given ſtaff C D, erected perpendicular to the horizon, both terminating at E, and make the following proportion - 193 p

As the ſhadow of the ſtaff EC.
To the height of the ſtaff CD.
So is the ſhadow of the object EA.
To the height of the object AB.

Second method.

147. *Of measuring the height of an accessible object* BC, *that is perpendicular to the horizon.*



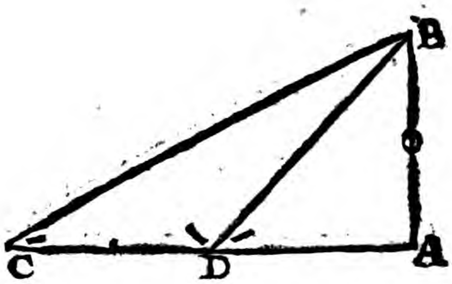
Chuse any convenient station A, take the angle of elevation BAC, measure the horizontal distance AB, and make the following proportion - - - 54

As radius
To the tangent of elevation BAC.
So is the horizontal distance AB.
To the height above the eye BC.

Problem

Problem 2.

148. *To meaſure the height of an inacceſſible object* AB.



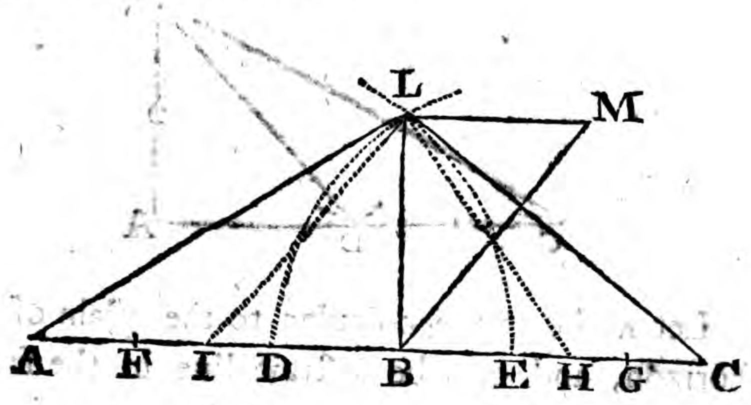
Let AB be a perpendicular to the plain of the horizon, and CDA be a ſtrait line in the ſame; at the ſtations C and D take the angles of elevation BCA, BDA, and meaſure the baſe CD.

Then to find DB. - - 51.
As the ſine of CBD.
To - - CD.
So is the ſine of BCD.
To - - DB.

Secondly, to find AB. - 53.
As radius,
To - - - DB.
So is the ſine of BDA.
To - - - AB.

Problem 3.

149. *To find the height of an inaccessible object, by means of three stations.*



At the stations A, B, C, in a strait line, observe the angles of elevation, and let their natural cotangents be x, y, z, make AD to DB as x to y, and BE to EC as y to z (305 p); take DF = DB, EG = EB, find DH a fourth proportional to AF, FD, AD, and EI a fourth proportional to CG, GE, CE; about the centers H and I, at the distances HD, IE describe circles intersecting at L, join BL, draw LM perpendicular to BL, make the angle LBM equal to the observed angle at B,

and

and meaſure LM the height required. The method of conſtruction ariſes from - 219, 319 p

Method of calculation.

1. Make $x+y : x :: AB : AD$; $y+z : z :: BC : CE$; and AD, CE will be found. - - 176 p

2. Make AF : FD :: AD : DH; CG : GE :: CE : EI; and the ſemidiameters HL, IL will be found - - - - - 219 p

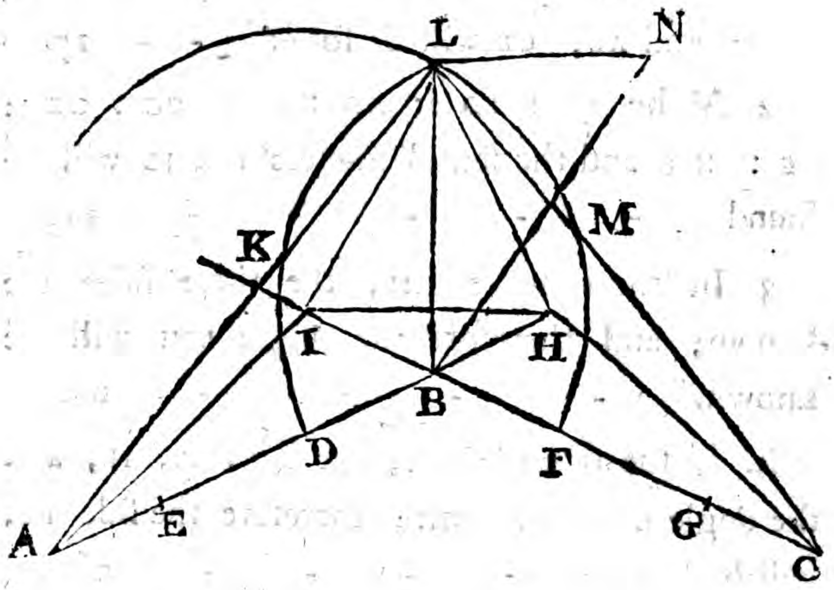
3. In the triangle IHL, the three ſides are known; and therefore the angle LIH will be known. - - - - - 60, 61

4. In the triangle IBL, the ſides IB, IL, and the angle LIB are known; therefore the ſide BL, will be known - - - - 56, 51

Laſtly, In the triangle BLM, the diſtance BL is known, BLM is a right angle, and LBM is the angle of elevation at B; therefore LM is known, (54); which was required.

Problem 3.

150. *Case* 2. *when the station lines form a given angle* ABC.



Suppose A, B, C the stations, and x, y, z, the natural cotangents of the angles of elevation of the object.

Make AD : DB :: x : y; BF : FC :: y : z (305 p), take DE = DB, FG = BF, produce AB, CB, make AE : ED :: AD : DH, CG : GF :: CF : FI, about the centers H and I, describe the circles DKL, FML; draw BL, and LN perpendicular to

BL,

BL, make the angle LBN equal to the observed angle at B, and measure LN, the height required.

The construction arises from 219 p.

Method of calculation.

1. $x+y:y$ :: AB : DB, and $y+z$: y :: BC : BF; whence DB, BF, are given. - - 176 p

2. AE : ED :: AD : DH, and CG : CF :: CF : FI; therefore DH, FI are given. 219 p

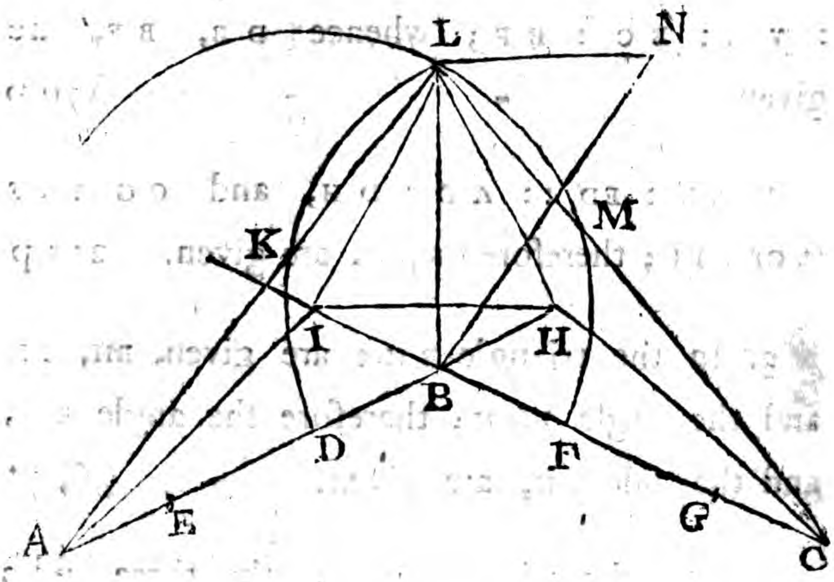
3. In the triangle BHI are given, BH, BI, and the angle IBH; therefore the angle BIH, and the side IH, are given. - 56, 51

4. In the triangle IHL, the three sides are given, and therefore the angle HIL is given - - - 60, 61

5. In the triangle BIL are given, the sides BI, IL, and the angle BIL; therefore the distance BL is given - - 56, 51

Lastly,

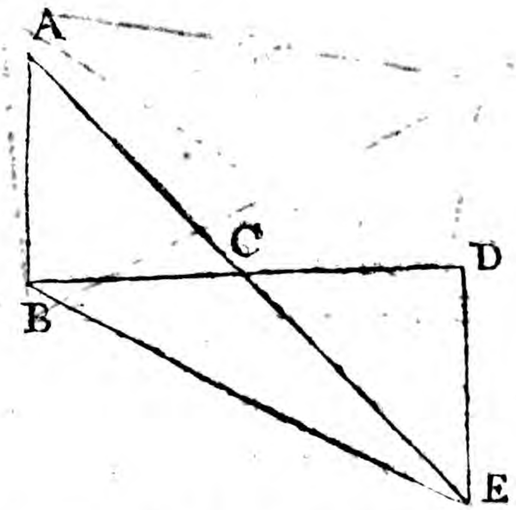
Lastly, In the triangle BLN, is given a right angle at L, the angle of elevation at B, and the base BL. Therefore LN is given (54); which was required.



Problem

Problem 4.

151. *To find the distance of an inaccessible object* A.



From a convenient place at B, trace the line BC at right angles to BA, of any convenient length, continue BC, make CD equal to BC, trace DE, at right angles to BD, till ECA is a strait line, and measure DE.

For if the lines are in the same plain, DE is equal to BA. - - - - 77 *p.*

Otherwise.

At the stations B and E, observe the angles BEA, EBA, and measure the base BE.

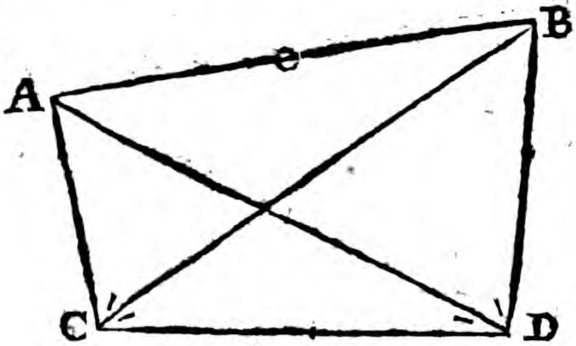
Then for BA. 51

As the sine of BAE.
To - - - - BE.
So is the sine of BEA.
To - - - BA.

Problem

Problem 5.

152. *To find the distance between two inaccessible objects* A *and* B.



At two convenient stations C and D, observe the angles CDA, ADB, DCB, BCA, and measure the base CD.

Method of calculation.

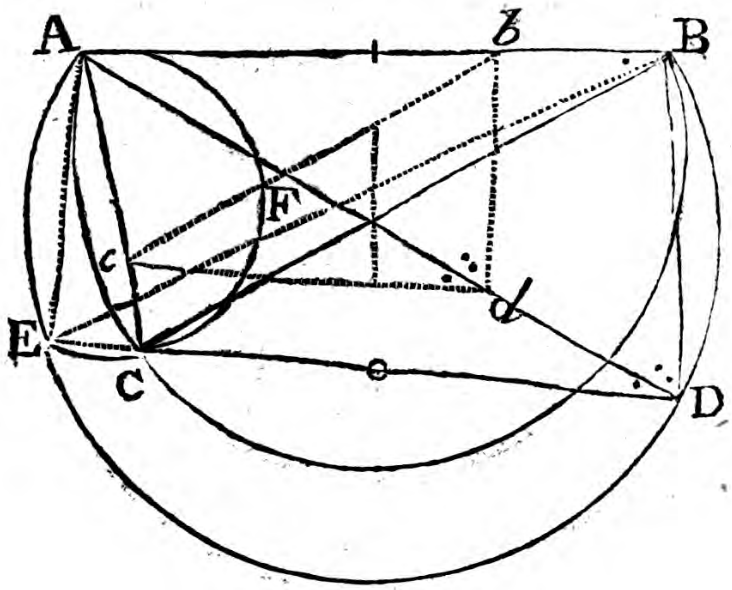
1. In the triangle CDA are given, the base CD, and all the angles; whence CA is given - 51
2. In the triangle CDB are given, the base CD, and all the angles; whence CB is given - 51
3. In the triangle ABC are given, the sides AC, CB, and the contained angle ACB; whence the angles BAC, CBA, are given - - 56

Lastly, In the same triangle ABC are given, the sides AC, CB, and all the angles, and consequently AB will be determined - - 51

Problem

Problem 6.

153. *Having the distance of two objects* A *and* B, *with the angles observed at the stations* C *and* D; *it is required to find the distance between the stations* C *and* D.



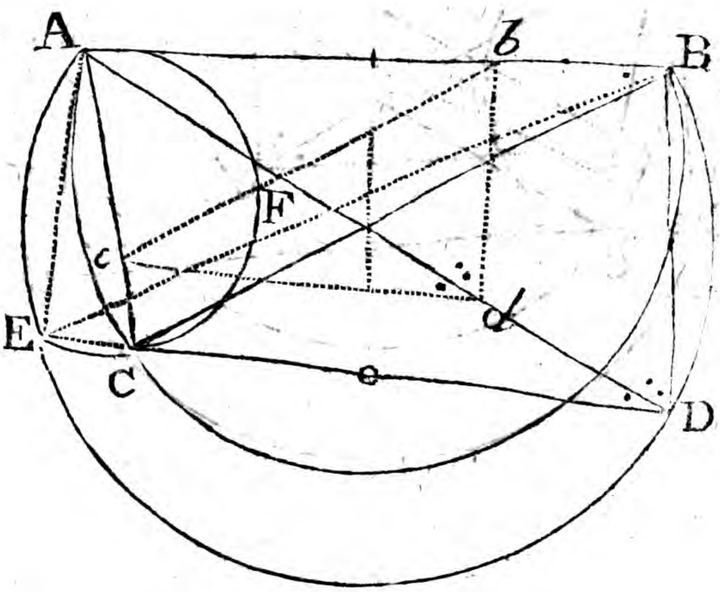
Construction.

Upon AB describe segments of circles (by 315 p) that will contain the given angles ACB, ADB, make the angle ABE = ADC, draw AE, upon AE describe a segment of a circle AFCE, that will contain the supplement of the angle ACD, cutting the circle ACB in C, draw ECD, and the thing is evidently done.

Otherwise.

Otherwife.

Affume *cd* for the line of ftations, make the angles *cd*A, A*db*, *dcb*, *bc*A, of the given magnitude, join A*b* and continue it to B, make AB equal to the given diftance, produce A*c*, A*d* to C and D, and draw BD, DC parallel to *bd*, *dc*.



Method of calculation.

1. In the triangle *cd*A are given all the angles, and by taking *cd* at pleafure (fuppofe 1000) we may find *c*A - - 51

2. In the triangle *cdb* are given, all the angles, and the fide *cd*, whence *cb* is given. 51

3. In

3. In the triangle A*bc* are given, the ſides A*c*, *cb*, and the contained angle A*cb*, whence the angles *b*A*c*, *cb*A are given - - 56

4. In the ſame triangle A*bc* are given, the ſides A*c*, *cb*, and all the angles, whence A*b* is given - - - 51

Laſtly, The figures A*bdc*, ABDC, are ſimilar by conſtruction, therefore A*b* : AB :: *cd* : CD, and conſequently CD is determined. 165 p

BOOK

BOOK II.

Of SPHERICAL TRIGONOMETRY.

CHAP. I.

Contains the first Principles, in relation to Spherical Triangles.

154. *A ſphere or globe is a ſolid perfectly round, in which all ſtrait lines drawn from the center to the ſuperficies are equal.*

155 *Great circles of the ſphere, are thoſe whoſe plains paſs through the center of that ſphere.*

Hence, the center of a ſphere is the common center of all its great circles, and they are conſequently equal. - - - - - 55 P.

156. *Small circles, or lesser circles of the sphere, are those whose plains pass beside the center of that sphere.*

157. *The pole of any circle on the sphere, is that point in the superficies from which all strait lines drawn to the circumference of the circle are equal to one another.*

158. *A spherical angle is that made on the superficies of a sphere, by two arches of great circles that meet each other.*

159. *A spherical triangle is a figure made on the superficies of a sphere, by three arches of great circles.*

160. *Spherical trigonometry is the art of finding the required parts (sides or angles) of a spherical triangle, from other parts already known or given.*

Axioms.

161. *A spherical angle is the same with the angle made by tangents to the arches at the point of contact or intersection, and is consequently equal to the inclination of the plains of those circles forming it; because the angle made by the tangents is the inclination of the plains* (231 *p*).

162. *One arch of a great circle meeting another, makes with it two angles, which, taken together, are equal to two right angles.*

163. *One great circle cutting another, makes the opposite angles equal to each other.*

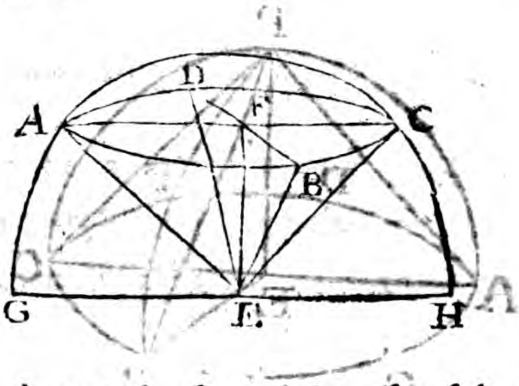
164. *A great circle may be described, that ſhall cut another great circle in a given angle, at a given point in the ſame.*

165. *Spherical triangles that have two ſides of the one equal to two ſides of the other each to each, and the contained angles equal; have likewiſe their remaining parts equal to one another each to each, namely, the parts oppoſite to equal things.*

Theorem

Theorem I.

166. *Every section of a sphere made by a plain will be a circle.*



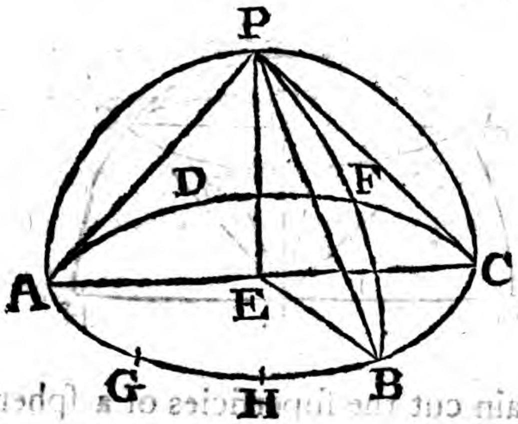
Let a plain cut the superficies of a sphere thro' the points A, B, C, D; then the section ABCD is a circle.

If the plain passes thro' E the center of the sphere, the section is evidently a circle; and if otherwise, let EF be drawn perpendicular to the plain of the section, and join AF, FB, BE, EA.

Now AFE, EFB are right angles (228 p); therefore $EA^2 = EF^2 + FA^2$, $EB^2 = EF^2 + FB^2$ (100 p); but EA and EB are equal (154), therefore $FA^2 = FB^2$, and AF, FB are equal; for the same reason, all strait lines drawn from the point F to the circumference of the section, are equal to one another, and consequently ABCD is a circle. - - - - - - - 30 p.

Theorem 2.

167. *Great circles bisect one another.*



Let the great circles ABC, APC cut each other in A and C; then ABC, APC are semicircles.

For the ſtrait line AC is the common ſection of their plains (249 p), therefore AC will paſs thro' (E) the common center of the circles, and the ſphere (155). Therefore AC is a diameter of each circle, and conſequently ABC, APC are ſemicircles. - - 33 p.

Theorem

Theorem 3.

168. *The arch of a great circle intercepted between the pole and circumference of any other great circle, is a quadrant. See the figure of article* 167.

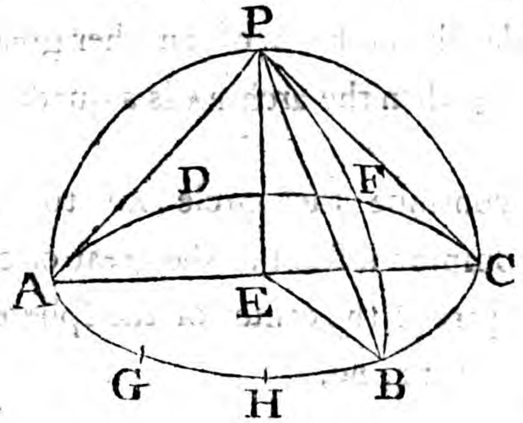
Let ABCD be a great circle, whoſe pole is P, and let the arch PA of another great circle be deſcribed; then the arch PA is a quadrant.

For continue the circle AP to C, let AC be the common ſection of the great circles (which will paſs thro' E the center of the ſphere by 155), and join AP, PE, PC.

Now EA, AP, PE = EC, CP, PE each to each, by what preceeds; therefore the angles AEP, PEC are equal (66 p), and the arches AP, PC are equal (148 p). Again, AC is a ſtrait line (249 p), therefore APC is a ſemicircle, and conſequently the arch PA is a quadrant. - - 38

Theorem 4.

169. *If two quadrants* PB, PC, *be drawn from any point* P, *on the ſuperficies of a ſphere; the point* P *will be the pole of a great circle* ABCD, *paſſing through their extremities.*



Let E be the center of the ſphere, and join PE, PA, PB, PC; EA, EB, EC.

Becauſe the arches PB, PC are quadrants, the angles PEB, PEC are right angles (22), therefore PE is perpendicular to the plain of the circle ABCD (250 p), therefore PEA is a right angle (228 p), therefore PA, PB, PC, &c. are equal, and P is the pole of ABCD (157).

Theorem

Theorem 5.

170. *A strait line joining the pole and center of any circle, will be perpendicular to the plain of the same circle.*

Let ABCD be any circle of the sphere, whose pole is P and center is E, and join PE; then the strait line PE is perpendicular to the plain of the circle ABCD.

Draw AC a diameter of the circle, and join EB, PA, PB, PC.

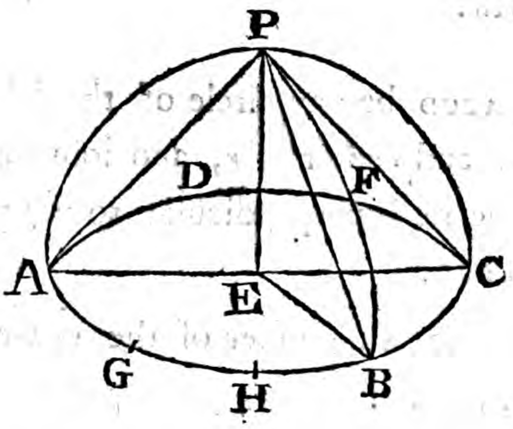
Now EA, EB, EC are equal; PA, PB, PC are equal (157), and PE is common; therefore the angles PEA, PEB, PEC are equal; therefore PEA, PEC are right angles (4), and consequently PEB is a right angle; therefore PE is perpendicular to the plain ABCD (250 p).

Corollary.

171. *The poles and centers of all parallel circles, are in the same diameter of the sphere.* - 256 p.

Theorem 6.

172. *If one great circle passes thro' the pole of another, it will meet that other circle at right angles.*



Suppose P the pole of the great circle ABC, and let the arch PB of a great circle be described; then PB falls at right angles to ABC.

Let E be the center of the sphere, and join PE, EB.

Now the center of the sphere, is the common center of all its great circles, and therefore E is the center of both the circles ABC and PB (155), therefore the line PE is in the plain of the circle PB (6 p), and the line PE is perpendicular to the plain of the circle ABC (170), therefore the plain of the circle PB is perpendicular to the plain of the circle ABC (260 p), therefore the arch PB falls at right angles to the arch ABC (161).

Theorem

Theorem 7.

173. *If one great circle cuts another at right angles, it will paſs thro' the pole of the ſame circle.*

Let the great circle BFP, cut the great circle ABCD at right angles; then the circle BFP will paſs through the pole of ABCD.

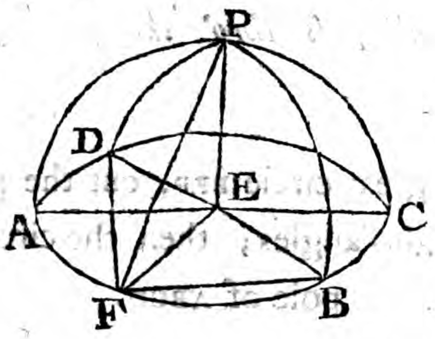
Let APC be a ſection thro' E the center of the ſphere, perpendicular to the plain of the circle ABCD; and join PE, EB.

Now the plain of the circle BFP is perpendicular to the plain of ABCD (161), and the plain of the ſection APC is likewiſe perpendicular to ABCD (by conſtr.) therefore PE their common ſection, is perpendicular to the plain ABCD (262 p); Th. PEA, PEB, PEC, are right angles (228 p), Th. PA, PB, PC, &c. are equal (62 p), and conſequently P is the pole of ABCD (157).

Theorem

Theorem 8.

174. *If one great circle be described thro' the pole of another, the pole of the circle described will fall in the circle given.*



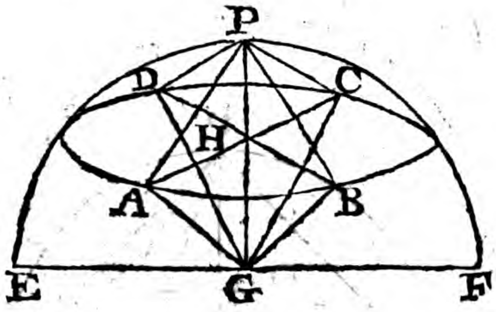
Let P be the pole of the great circle ABCD, and thro' P let the great circle DPB be described, then the pole of DPB will fall in the circle ABCD.

Suppose E the center of the sphere, and DEB the common section of the plains of the circles; in the plain ABCD draw EF at right angles to DB, and join PE, PF, DF, FB.

Now PE is perpendicular to the plain ABCD (170), therefore PEF is a right angle (228 p); also DEF, FEB are right angles by construction; therefore FE is perpendicular to the plain of the circle DPB (250 p). Again, ED, EP, EF, EB are equal, and the contained angles FED, FEP, FEB are equal, being right angles; therefore FD, FP, FB are equal; for the same reason all strait lines joining the point F, and the circumference of the circle DPB are equal; therefore F is the pole of the circle DPB. - - 157

Theorem 9.

175. *If a great circle passes thro' the pole of a small circle, their plains will be perpendicular to each other.*



Suppose ABCD a small circle whose pole is P, and through P let the great circle EPF be described; then the plains of those circles are perpendicular to each other.

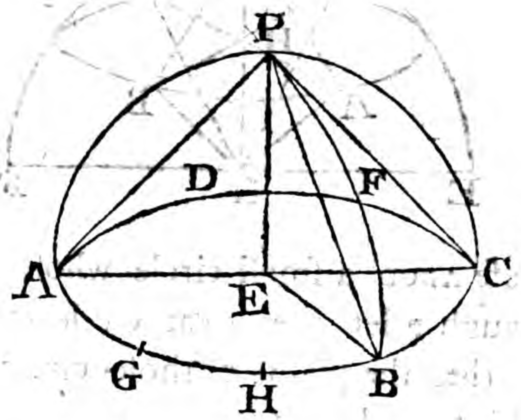
Let G be the center of the sphere, join PG cutting the plain of the small circle in H, through H draw AC, DB, and compleat the figure.

Now GA, GB, GC, GD, are equal (154), also PA, PB, PC, PD, are equal (157), and PG is common; therefore the angles GPA, GPB, GPC, GPD are equal (66 p); therefore the angles PHA, PHB, PHC, PHD are equal (62 p), and consequently are all right angles (9p); therefore PH or PG, is perpendicular to the plain ABCD (228 p); therefore the plain of EPF is perpendicular to the plain of ABCD (260p), and consequently the plain of the small circle ABCD, is perpendicular to the plain of the great circle EPF.

Theorem

Theorem 10.

176. *A spherical angle* BPC, *is measured by an arch of a great circle* BC; *whose pole is the angular point* P.



Let E be the center of the sphere, and join PE, EB, EC.

Now PE is perpendicular to the plain of the circle ABC (170), therefore PEB, PEC are right angles (228 p), therefore the angle BEC is the inclination of the plains PEB, PEC (231 p), for PE is their common section; therefore the angle BEC is equal to the spherical angle BPC (161). Again, the arch BC measures the angle BEC, therefore the arch BC measures the spherical angle BPC.

Corollary

Corollary.

177. *The angle made by two great circles, is measured by an arch of a great circle intercepted between their poles.*

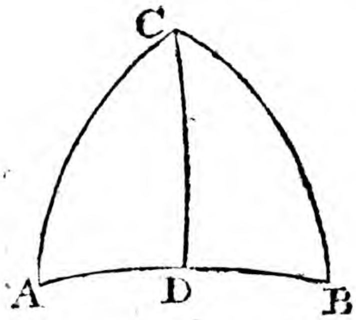
For if the points G and H, in the great circle AGBC, are poles of the great circles PB, PC; the arches GB, HC will be quadrants (168), and therefore GH is equal to BC, the measure of the ſpherical angle BPC.

CHAP. II.

Contains properties of ſpherical Triangles.

Theorem I.

178. *If the vertical angle of an iſoſceles ſpherical triangle is biſected by a great circle, the ſaid circle will alſo biſect the baſe at right angles.*



Let ABC be a ſpherical triangle, having the ſide BC equal to the ſide CA, and let the arch CD of a great circle biſect the angle

ACB;

ACB; then AD is equal to DB, and ADC, CDB are right angles.

For AC being equal to BC, and CD common, there are two ſides AC, CD, equal to two ſides BC, CD, each to each, and the contained angles ACD, BCD are likewiſe equal (con.) therefore AD = DB, and the angle ADC = CDB. - - - - - - 165

Th. ADC, CDB, are right angles - - 162

Corollary.

179. *In the ſame ſpherical triangle, oppoſite to equal ſides are equal angles.*

For AC, CD = BC, CD *each to each* - - *theo.*

And the angle ACD = BCD - - - *con.*

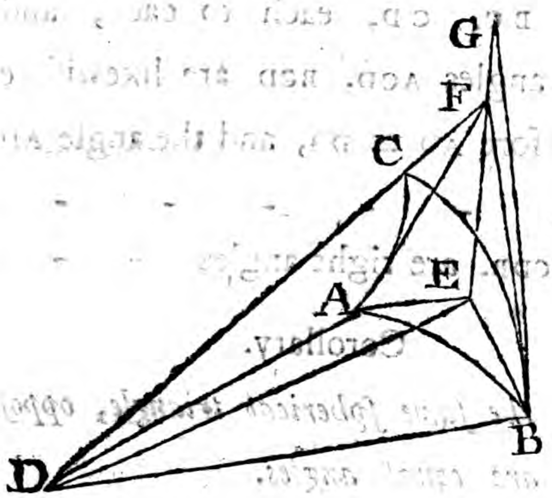
Th. the angle CAD = CBD - - - - 165

Or the angle CAB = CBA

Again,

Again.

* 179. *In the same spherical triangle, opposite to equal sides are equal angles.*



Let the spherical triangle ABC, have the side BC equal to the side CA; then the angle A is equal to the angle B.

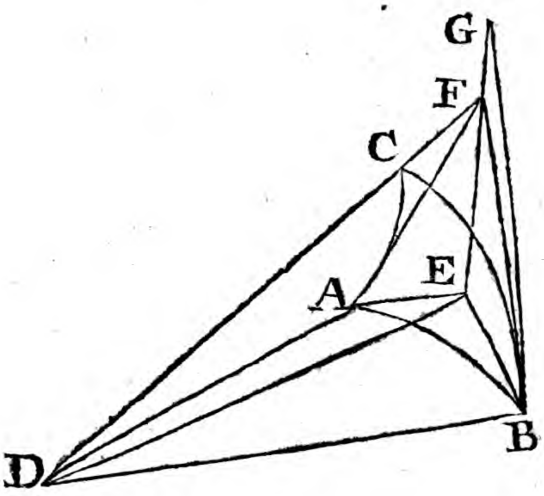
Let D be the center of the sphere, join DA, DB, DC, draw BE, AE, AF touching the arches AB, AC, produce DC to F, and join EF, FB.

Now AD, DF are equal to BD, DF each to each, and the contained angles ADF, BDF are equal

equal (150 p), therefore AF, BF are equal, and the angles DAF, DBF are equal (62 p); but DAF is a right angle by conſtruction, therefore DBF is a right angle, and BF is a tangent to the arch BC (35). Again, DAE, DBE, are right angles by conſtruction; therefore $DA^2 + AE^2 = (DE^2) = DB^2 + BE^2$ (100 p), but DA, DB are equal, and conſequently AE, EB are alſo equal. Laſtly, EA, AF, FE = EB, BF, FE each to each, as ſhewn above; therefore the angles EAF, EBF are equal (66 p), and are made by tangents to the arches at the points of interſection; therefore the ſpherical angles CAB, CBA are likewiſe equal (161).

Theorem 2.

180. *In the ſame ſpherical triangle, oppoſite to equal angles are equal ſides.*



Let the ſpherical triangle ABC have the angle B equal to the angle A; then the ſides AC, CB are likewiſe equal.

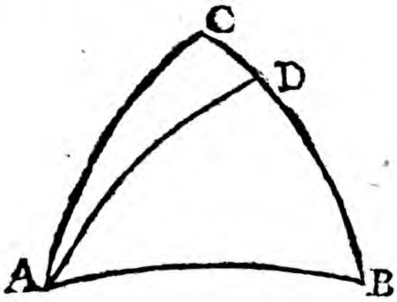
Suppoſe D the center of the ſphere, join DA, DB, DC; draw AE, AF, touching the arches AB, AC; and BE, BG, touching the arches BA, BC; conceive plains paſſing through the tangents AE, AF, and BE, BG, to cut each other in the line EFG; and join DE, DF, FB.

Now

Now DA, DB, are perpendicular to the plains paſſing thro' EA, AF, and EB, BF (250 p); therefore DAF, DBF, are right angles (228 p); therefore $DA^2 + AF^2 = (DF^2 =) DB^2 + BF^2$; but DA is equal to DB, therefore AF is equal to BF; and for the ſame reaſon AE is equal to BE; therefore EA, AF, FE = EB, BF, FE, each to each; therefore the angles EAF, EBF are equal (66 p); but the angles EAF, EBG are equal, being the ſame with the ſpherical angles A and B (161); therefore the angles EBF, EBG are equal, and the point G falls in F; therefore the points D, C, F, are all in the interſection of the plains of the circles AC, BC; and conſequently DCF is a ſtrait line (249) p. Laſtly, AD, DF, FA = BD, DF, FB each to each; therefore the angles ADC, BDC, are equal; and the arches AC, CB, are conſequently equal (148 p).

Otherwife.

* 180. *In the fame fpherical triangle, oppofite to equal angles are equal fides.*



Let a fpherical triangle ABC, have the angle B equal to the angle A; then the fide AC is equal to the fide BC.

For if the fides AC, BC, are not equal; conceive a great circle to be defcribed, making the fide AD equal to the fide BD.

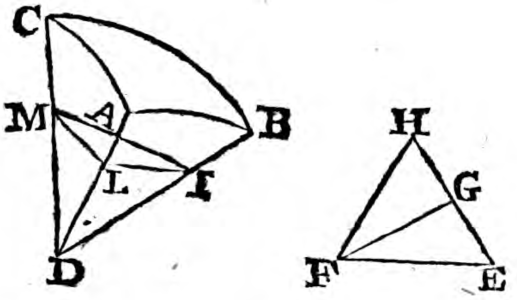
Now AD is equal to BD,	- -	con.
Th. the angle ABD = BAD	- -	179
But the angle ABD = BAC	- - -	theo.
Th. the angle BAD = BAC	- -	48 p.
Which is impoffible	- - -	52 p.

Therefore AD, BD, cannot be equal; and confequently, AC muft be equal to BC.

Theorem

Theorem 3.

181. *Any two ſides of a ſpherical triangle, taken together, are greater than the remaining ſide.*



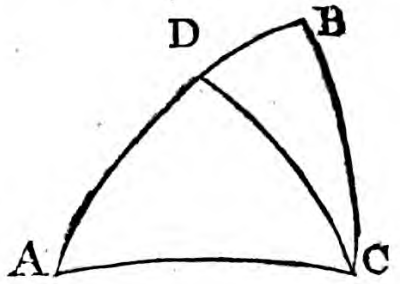
Let ABC be a ſpherical triangle, then BA + AC is greater than CB.

Let D be the center of the ſphere, join DA, DB, DC; make the angle EFG equal to the angle BDA, and the angle GFH equal to the angle ADC, take FE, FH at pleaſure, ſo that each be leſs than the radius of the ſphere, join HE, make DI, DL, DM, equal to FE, FG, FH, each to each, and draw IL, LM, MI.

Now ID, DL, are equal to EF, FG, each to each, and the contained angles IDL, EFG are equal, by conſtruction, therefore IL, EG, are equal (62 p); for the ſame reaſon LM, GH are equal; therefore IL + LM is equal to EH; but IL + LM is greater than MI, therefore HE is greater than MI; again, EF, FH, are equal to ID, DM, each to each, and the baſe HE is greater than the baſe MI (above), therefore the angle EFH is greater than IDM (76 p); but the angle EFH = IDA + ADM (con.), therefore the angle IDA + ADM is greater than IDM: Laſtly, the arches BA, AC, CB, are meaſures of the angles IDA, ADM, IDM (24); therefore BA + AC is greater than CB (56 p).

Theorem 4.

182. *In the same spherical triangle, opposite to the greater angle is the greater side.*



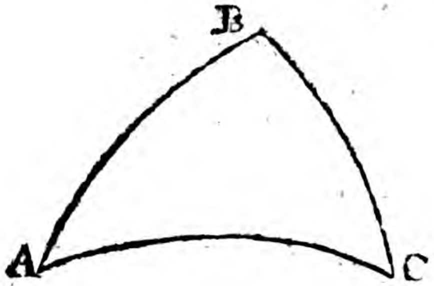
Let the spherical triangle ABC, have angle BCA greater than the angle BAC; then the side AB is greater than the side BC.

For let the arch CD of a great circle be described, making the angle ACD equal to the angle BAC. - - - - 164

Now the angle ACD $=$ CAD - - con-
Th. - - - AD $=$ CD - - 180
Th. - AD $+$ DB $=$ CD $+$ DB - - 49 P
Or - - - AB $=$ CD $+$ DB
But CD $+$ DB is greater than BC - - - 181
Therefore AB is greater than BC - - - 56 P

Theorem 5.

183. *In the ſame ſpherical triangle, oppoſite to the greater ſide is the greater angle.*



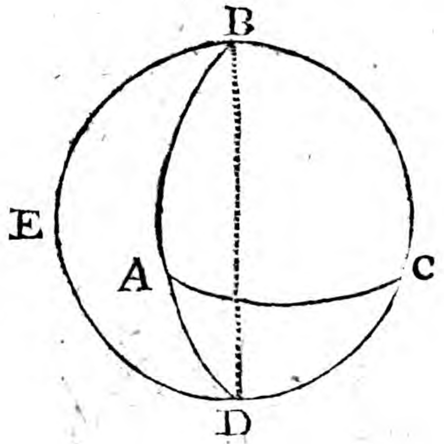
Let the ſpherical triangle ABC, have the ſide AB greater than the ſide BC; then the angle C is greater than the angle A.

For if the angle C was equal to the angle A, the ſides AB, BC would be equal (180), which they are not (theo.); therefore the angle C is not equal to the angle A. Again, if the angle C was leſs than the angle A, the ſide AB would be leſs than the ſide BC (182), but it is otherwiſe (theo.); therefore the angle C is not leſs than the angle A, and conſequently the angle C muſt be greater than the angle A.

Theorem

Theorem 6.

184. *The three sides of any spherical triangle taken together; are less than a great circle.*



Let ABC be a spherical triangle, then AB + BC + CA, is less than the great circle BCDE.

Let the arches BA, BC meet again in D.

Now BAD, BCD are semicircles. - - - 167

Th. BAD + BCD = BCDE - - - - 49 p.

Or AB + BC + AD + DC = BCDE. - - 49 p.

But CA is less than AD + DC. - - - - - 181

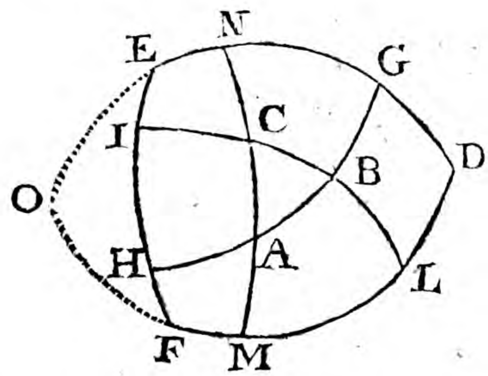
Th. AB + BC + CA is less than BCDE - 49 p.

Therefore the three sides of the spherical triangle ABC, are less than the great circle BCDE.

Theorem

Theorem 7.

185. *If the angular points of one ſpherical triangle, are poles to the ſides of another, the meaſures of the angles in the one will be ſupplemental to the ſides of the other.*



Let ABC be a ſpherical triangle, whoſe angular points are poles of the great circles forming the triangle DEF; then the meaſures of the angles A, B, C, are ſupplements to the ſides DE, EF, FD; and the meaſures of the angles D, E, F, are ſupplements to the ſides AC, AB, BC.

Produce the ſides of ABC both ways, to meet the ſides of DEF; at G, H, I, &c.

Since

Since A is the pole of ED (con.) the pole of AB will fall in the circle ED (175), and becauſe B is the pole of EF (con.) the pole of BA will fall in the circle EF (175); therefore the point E is the pole of AB; for the ſame reaſon F is the pole of BC, and D is the pole of AC.

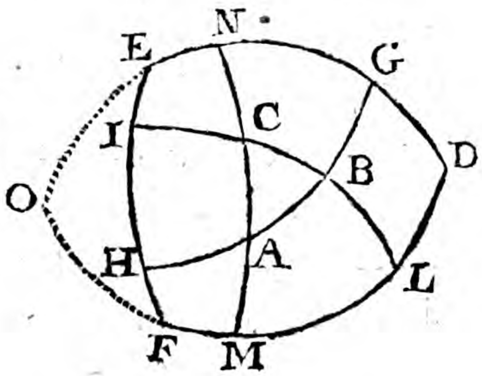
Secondly, EG, ND are quadrants (168); therefore a ſemicircle = (EG + ND = EN + NG + NG + GD =) ED + NG; therefore NG is the ſupplement of ED (29), and conſequently IH, ML, are ſupplements to EF, FD.

Thirdly, NG is the meaſure of A (171), and NG is the ſupplement of ED (above), therefore the meaſure of the angle A is the ſupplement of the ſide ED; and IH, ML, the meaſures of the angles B and C, are the ſupplements of EF, FD; and after the ſame manner it is proved, that the meaſures of the angles D, E, F, are ſupplements of the ſides AC, AB, BC.

Theorem

Theorem 8.

186. *The three angles of a ſpherical triangle taken together, are greater than two right angles, and leſs than ſix right angles.*



Let ABC be the triangle, produce its ſides both ways, and let DE, EF, FD, be arches of great circles deſcribed about its angular points as poles.

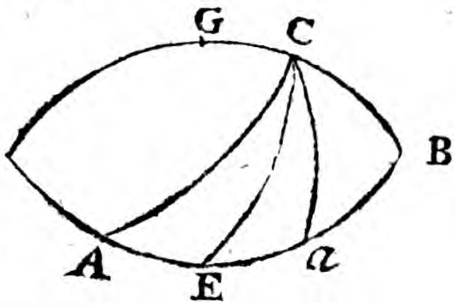
Then the meaſures of the angles A, B, C, together with the three ſides of the ſupplemental triangle DEF, are equal to three ſemicircles (185); but the three ſides of the triangle DEF are leſs than two ſemicircles (184); therefore the meaſures of the angles A, B, C are greater than one ſemicircle, or two right angles.

Secondly, All the outward and inward angles of the triangle are equal to ſix right angles (162); therefore all the inward angles are together, leſs than ſix right angles.

Theorem

Theorem 9.

187. *In a right angled ſpherical triangle, the oblique angles are of the ſame kind with their oppoſite ſides; that is, if a ſide be greater or leſs than a quadrant, the oppoſite angle is accordingly greater or leſs than a right angle.*



Let ABC be a ſpherical triangle, right angled at B, then the angle ACB is of the ſame kind with its oppoſite ſide AB.

1. If AB is greater than a quadrant, make BE a quadrant, and deſcribe a great circle thro' C and E.

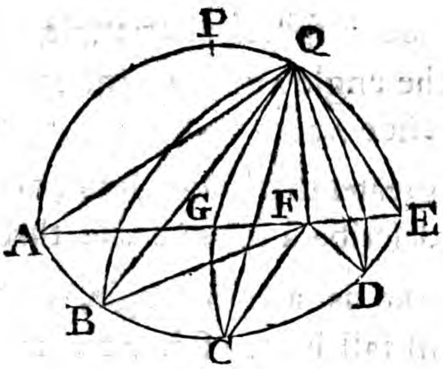
Becauſe ABC is a right angle, the pole of the circle BC will fall in the ſide AB (173) therefore E is the pole of BC (168) and BCE is a right angle (172), but the angle BCA is greater than BCE, therefore the angle BCA is greater than a right angle.

2. If *a*B is leſs than a quadrant, make BE a quadrant, and deſcribe the arch CE.

Then as before, BCE is a right angle, but the angle BC*a* is leſs than BCE, therefore the angle BC*a* is leſs than a right angle.

Theorem 10.

188. *If from any point* Q *on the superficies of a sphere, beside the pole of a great circle* ABCDE; *there be drawn to that circle many arches of great circles* QA, QB, QC, *&c. the greatest of them is* QA *passing through* P *the pole of* ABCDE; *of other arches* QB, QC, QD, *the nearer to* QA *is the greater, and* QE *the remainder of* APQ *is the least of all.*



Let AE be the intersection of the plains of the great circles APQE, ABCE, draw QF perpendicular to AE, join QA, QB, QC, QD, QE, FB, FC, FD, and let G be the center of the sphere.

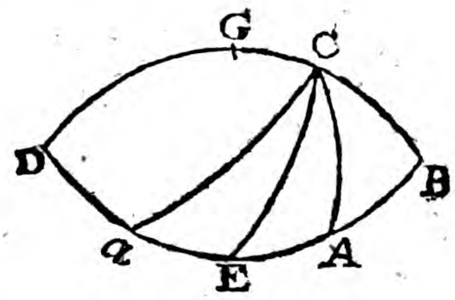
Now

Now of the lines drawn from F, to the circumference of ABCE, the greateſt is FGA, the leaſt FE, and FB is greater than FC (133 p). Therefore FA^2, is greater than FB^2, and FA^2+FQ^2 is greater than FB^2+FQ^2, but the circles ABCE, APQE, meet at right angles (172), their plains are perpendicular (161), and QF is perpendicular to the plain of the circle ABCE (229 p). Therefore $QA^2=FA^2+FQ^2$, and $QB^2=FB^2+FQ^2$, (100 p), Therefore QA^2 is greater than QB^2, QA is greater than QB, and the arch QA is greater than the arch QB (152p). For the ſame reaſon the arch QB, is greater than QC, the arch QC is greater than the arch QD, and the arch QE is the leaſt of all.

Theorem

Theorem 11.

189. *If the sides of a right angled spherical triangle be of the same kind, the hypothenuse is less than a quadrant, and if they are of a different kind, the hypothenuse is greater than the quadrant.*



In a spherical triangle ABC, right angled at B, if the sides A B, B C, are of the same, or of a different kind; the hypothenuse AC, is accordingly less, or greater than a quadrant.

1. If AB, BC, be each less than a quadrant, and BE, BG, be taken quadrants, then G and E are the poles of AB, BC (173, 168) and CE is a quadrant (168), but CE is greater than CA (188), therefore CA is less than a quadrant; and after the same manner it is proved in the triangle

DAC

DAC, when the ſides DA, DC, are greater than quadrants, that AC is leſs than a quadrant.

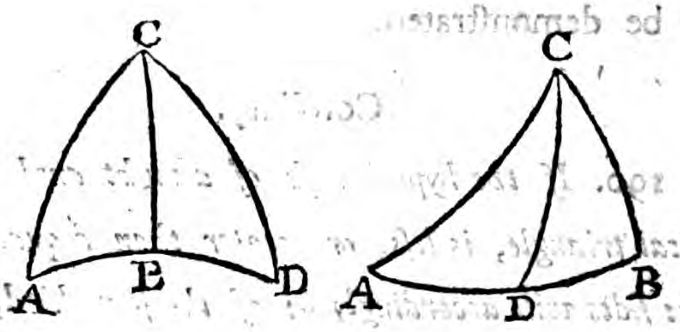
2. If BC is leſs, and B*a* is greater than a quadrant, then *a*C will be greater than a quadrant. For if BE be made a quadrant, E will be the pole of BC, and CE will be a quadrant, as ſhewn above; but *a*C is greater than CE (188), therefore *a*C is greater than a quadrant; which was to be demonſtrated.

Corollary.

190. *If the hypothenuſe of a right angled ſpherical triangle, is leſs, or greater than a quadrant; the ſides will accordingly be of the ſame kind, or of a different kind.*

Theorem 12.

191. *In an oblique angled ſpherical triangle* ACD, *if the perpendicular* CB *falls within the triangle, the angles at the baſe* A *and* D, *will be of the ſame kind; but if the perpendicular falls without the triangle, the angles* A *and* CDA, *will be of a different kind.*



1. If CB falls within the triangle, then becauſe ABC, CBD are right angles, the angles A and D, are each of the ſame kind with CB. - - - - - - - 187

2. If CB falls without the triangle, the angles A and CDB, are each of the ſame kind with CB (187).

(187). But the angles CDB, CDA, are of a different kind (162). Therefore the angle CDA is of a different kind from CB, and consequently the angles A, and CDA, are of a different kind.

Corollary.

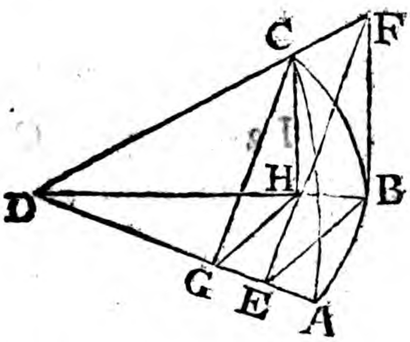
192. *When the angles at the base of a triangle are of the same kind, a perpendicular will fall within; and when the angles at the base are of a different kind, the perpendicular will fall without the triangle; upon the base continued.*

CHAP. III.

Contains Theorems, and Solutions of the ſeveral Caſes in Spherical Triangles.

Theorem 1.

193. *In right angled ſpherical triangles. The ſine of either ſide, is to the radius; as the tangent of the other ſide, to the tangent of its oppoſite angle.*



Let ABC be a ſpherical triangle, right angled at B, then the ſine of AB, is to radius; as the tangent of BC, to the tangent of the angle A.

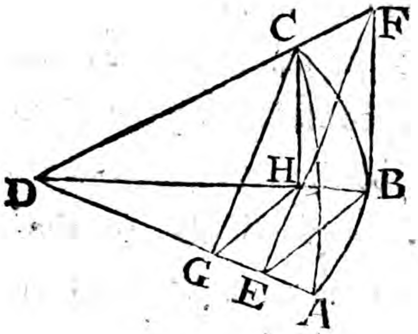
Sup-

Suppofe D the center of the fphere, join DA, DB, DC; let fall BE perpendicular to DA, in the plain of DAC erect EF perpendicular to DA, produce DC to F, and draw BF.

Now DE is at right angles to both BE and EF (con.). Therefore DE is perpendicular to the plain of BEF (250 p). Therefore the plain of DBA, is perpendicular to the plain of EBF (260), and confequently the plain of EBF is perpendicular to the plain of DBA; Alfo, the plain of DBC is perpendicular to the plain of DBA, becaufe ABC is a right angle (161); Therefore the interfection BF is perpendicular to the plain of DBA (262 p), and FBD, FBE, are right angles (228 p). Therefore EB : radius :: BF : tangent of BEF (54), but EB is the fine of AB; BF is the tangent of BC (38); the angle BEF is the inclination of the plains DAB, DAC (231 p), and is equal to the fpherical angle BAC (161). Therefore, the fine of AB, is to radius; as the tangent of BC, to the tangent of the angle BAC.

Theorem 2.

194. *In right angled ſpherical triangles, the ſine of the hypothenuſe, is to the radius; as the ſine of either ſide, to the ſine of its oppoſite angle.*



Let the ſpherical triangle ABC, be right angled at B; then the ſine of AC, is to radius; as the ſine of BC, to the ſine of the angle BAC.

Suppoſe D the center of the ſphere, join DA, DB, DC, let fall CG perpendicular to DA; in the plain of DAB erect GH perpendicular to DA, and draw CH.

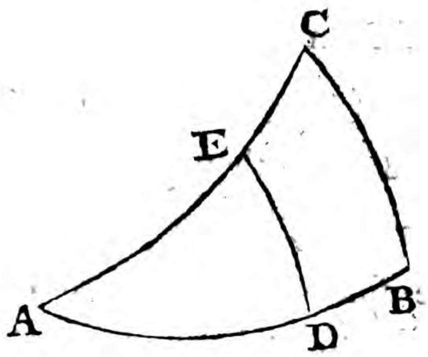
Now DG is at right angles to both CG and GH (con.) therefore DG is perpendicular to the plain

of

of GHC (250 p); therefore the plain of DBA is perpendicular to the plain of GHC (260), and consequently the plain of GHC is perpendicular to the plain of DBA; Also, the plain of DBC, is perpendicular to the plain of DBA, because ABC is a right angle (161); therefore the common section CH, is perpendicular to the plain of DBA (262 p), and CHD, CHG, are right angles (228 p.); therefore C G : radius :: C H : sine of CGH (53); but CG is the sine of AC, CH is the sine of BC (38), and the angle CGH being the inclination of the plains DAC, DAB (231 p), is equal to the spherical angle B A C (161). Therefore, as the sine of A C, to radius; so is the sine of B C, to the sine of the angle B A C.

Theorem 3.

195. *In right angled spherical triangles that have the same acute angle at the base, the sines of the bases, are as the tangents of the perpendiculars; and the sines of the hypothenuses, are as the sines of the perpendiculars.*



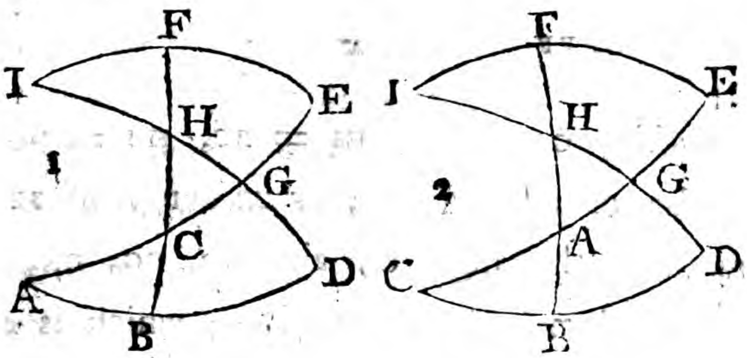
Let the spherical triangles ABC, ADE, be right angled at B and D, and have a common acute angle at A.

Then	*s*, AB : *t*, BC :: R : *t*, A	-	193
And	*s*, AD : *t*, DE :: R : *t*, A	-	193
Th.	*s*, AB : *t*, BC :: *s*, AD : *t*, DE	-	173P
Th.	*s*, AB : *s*, AD :: *t*, BC : *t*, DE	-	175P
Again,	*s*, AC : *s*, CB :: R : *s*, A	-	194
And	*s*, AE : *s*, ED :: R : *s*, A		194
Th.	*s*, AC : *s*, CB :: *s*, AE : *s*, ED	-	173P
Th.	*s*, AC : *s*, AE :: *s*, CB : *s*, ED	-	175P

Theorem

Theorem 4.

196. *If the sides of a right angled spherical triangle* ABC, *(whose angles* A *and* C *are acute) be continued to* D, E, F, *and about the angular points* A *and* C *as poles, the great circles* DGI, EFI, *be described; we shall obtain four pairs of right angled triangles, each pair having the same acute angle at the base, and the parts which compose them will be equal to the parts of the triangle* ABC, *or to their complements.*



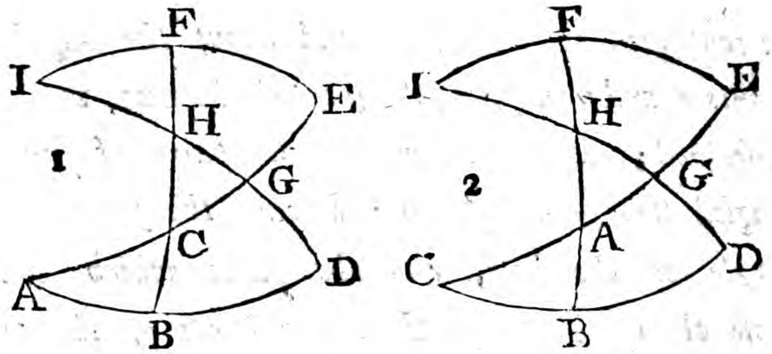
Thus in fig. 1.

1. The angles at D, G, E, F, are right angles - - - - - - 172

2. Since B, D, G, E, are right angles, the points H and I, are poles of the circles BD, GE respectively - - - - 173

3. The

3. The arches AD, AG, HB, HD, IE, IG, are quadrants - - - - - - 168



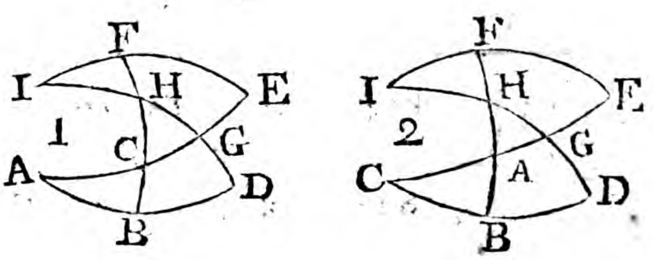
4. The angles A and C, are measured by the arches DG, EF - - - - 176

Laſtly, GE = AC, HF = BC, HI = DG = meaſure of the angle A; IF = comp. of FE = comp. of the angle C; alſo BD, CG, CH, are the complements of AB, AC, BC; which is evident by what appears above, and the ſame things are to be underſtood of fig. 2.

Remark

Remark.

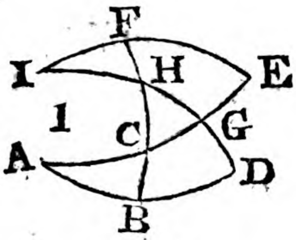
197. *From the preceding article, arises an easy method of solving all the cases of right angled spherical triangles.*

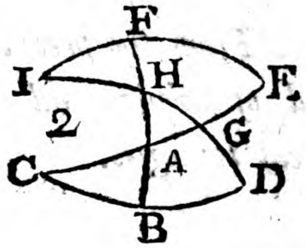


For if any two parts of the spherical triangle ABC, beside the right angle at B be given, and the angles at A and C are acute, by constructing a figure according to the preceding article, we shall find a pair of triangles having the same acute angle at the base (either A, C, H, or I) in which three sides will be given; namely, the measures or complements of the two given parts, and a quadrant; and a fourth side may be found by one of the proportions in article 195, that will measure a required part of the triangle ABC, or the complement of such required part (196), which is exemplified in the following solutions.

198. *Solu-*

198. *Solutions of the* 16 *cases of right angled spherical triangles* ABC, *right angled at* B.





Art.	Case.	Given.	Req.	Proportion by article 195.	fig.
199	1	AC and A	BC	s, AG : s,AC :: s,DG : s,BC, or, R : s,AC :: s, A : s,BC.	1
200	2	AC and A	AB	s,IE : s, IF :: t,EG : t, FH, or, R : cos. A :: t,AC : t, AB.	2
201	3	AC and A	C	s,AE : t,EF :: s,AG : t, GH, or, R : t, A :: cos.AC : cot. C.	2
202	4	AC and AB	BC	s,BD : s,CG :: s,HB : s,HC, or, cos.AB : cos. AC :: R : cos. BC.	1
203	5	AC and AB	C	s,GE : s,IG :: s,HF : s,IH, or, s,AC : R :: s,AB : s, C.	2
204	6	AC and AB	A	t, EG : t, FH :: s, IE : s, IF, or, t, AC : t, AB :: R : cos. A.	2
205	7	AB and A	BC	s, AD : s, AB :: t, DG : t, BC, or, R : s, AB :: t, A : t, BC.	1
206	8	AB and A	AC	s, IF : s, IE :: t, FH : t EG, or, cos. A : R :: t, AB : t, AC.	2

The table of solutions continued.

Art.	Case	Given.	Req.	Proportion by article 195.	fig.
207	9	A B and A	C	s,AF : s,FE :: s, AH : s,HG; or, R : s, A :: cos. AB : cos. C.	2
208	10	B C and A	AB	t,DG : t,BC :: s,AD : s,AB ; or, t, A : t,BC :: R : s,AB.	1
209	11	B C and A	AC	s,DG : s,BC :: s,AG : s,AC ; or, s, A : s,BC :: R : s,AC.	1
210	12	B C and A	C	s, CH : s,HG :: s,CF : s,FE ; or, cos. BC: cos. A:: R : s,C.	1
211	13	A B and B C	AC	s,HB : s, BD :: s, HC : s,CG;or, R : cos. AB:: cos. BC: cos. AC.	1
212	14	A B and BC	A	s,AB : s,AD :: t,BC : t,DG ; or, s,AB : R :: t,BC : t,A.	1
213	15	A and C	AB	s,EF : s,FA :: s,GH : s,HA ; or, s, A : R :: cos. C : cos. AB.	2
214	16	A and C	AC	t,EF : s,AE :: t,GH : s,AG ; or, t, A : R :: cot. C : cos. AC.	2

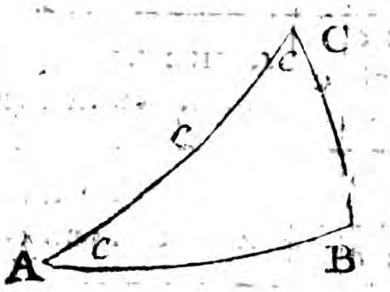
Remark.

The 10th, 11th, and 12th cases are ambiguous, for each admits of two answers; and must be determined by the nature of the questions, to whose solutions those cases are applied.

215. Of

215. *Of Lord Neper's rule by circular parts.*

The circular parts of a right angled spherical triangle, are the sides containing the right angle, the complement of the hypothenuse, and the complements of the two oblique angles; the right angle not being permitted to interrupt the conceived circular order of the things above mentioned.



If the situation of any three circular parts be considered, that is called the middle part which is either contiguous to both the extremes, or is seperated from both by a circular part.

Thus in the spherical triangle ABC, right angled at B, if the complement of A, AB, and

BC

BC be propoſed; then AB is the middle part, and the extremes A and BC are contiguous, but if the complement of A, AB, and complement of C be propoſed; then C will be the middle part, and the extremes A and AB, are both ſeparated from C.

All the caſes in right angled ſpherical triangles, are contained in the two following theorems, which Lord Neper dicovered by comparing the preceding ſolutions among themſelves.

Neper's

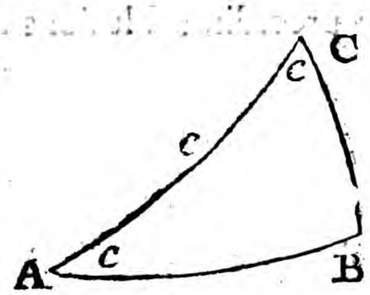
Neper's first theorem.

216. *The rectangle contained by the radius and sine of the middle part, is equal to the rectangle contained by the tangents of the contiguous extremes.*

Neper's second theorem.

217. *The rectangle contained by the radius and sine of the middle part, is equal to the rectangle contained by the cosines of the separated extremes.*

The truth of these two theorems is easily proved from the preceding solutions, for if the side AB is the middle part.

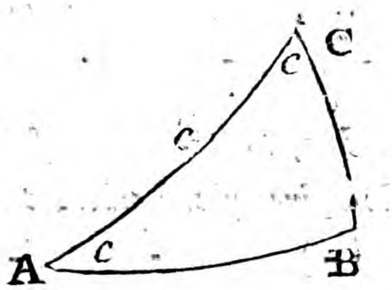


Then,	R : t, A :: s, AB : t, BC - - -	205
But,	R : t, A :: cot. A : R - - - -	46
Th.	s, AB : t, BC :: cot. A : R - -	173 p.
Th.	R $\times$ s, AB $=$ t, BC $\times$ cot. A - -	185 p.
Ag.	s, AC : R :: s, AB : s, C - - -	203
Th.	R $\times$ s, AB $=$ s, AC $\times$ s, C - -	185 p.

Note. The cosine of a complement is the sine of the part itself.

Secondly

Secondly, If the complement of AC *is the middle part.*



Then,	R : t, A :: cos. AC : cot. C. -	- 201
But,	R : t, A :: cot. A : R -	- 47
Th.	cos. AC : cot. C :: cot. A : R -	173 P
Th.	R × cos. AC = cot. A × cot. C	185 P
Ag.	R : cos. AB :: cos. BC : cos. AC -	211
Th.	R × cos. AC = cos. AB × cos. BC	185 P

Thirdly, If the complement of the angle A, *is the middle part.*

Then,	R : t, AC :: cos. A : t, AB -	- 200
But,	R : t, AC :: cot. AC : R -	- 47
Th.	cos. A : t, AB :: cot. AC : R -	173 P
Th.	R × cos. A = t, AB × cot. AC. -	185 P
Ag.	cos. BC : cos. A :: R : s, C -	- 210
Th.	R × cos. A = cos. BC × s, C -	185 P

Hence, If the middle part is required, the proportion must begin with radius; but if an extreme part be required, the proportion must begin with the other extreme part.

K *Thus,*

Thus,

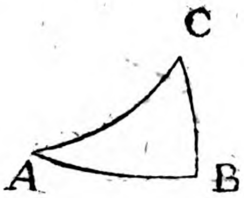
218. *As radius, to the tangent of either contiguous part; so is the tangent of the other, to the sine of the middle part* - - - - 187 p

219. *As radius, to the cosine of either seperated part; so is the cosine of the other, to the sine of the middle part* - - - 187 p

220. *As the tangent of either contiguous part, to radius; so is the sine of the middle part, to the tangent of the other part* - - - 187 p

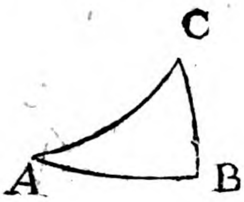
221. *As the cosine of either seperated part, to radius; so is the sine of the middle part, to the cosine of the other part* - - - - 187 p

222. *Solutions of the 16 cases of right angled spherical triangles* ABC, *right angled at* B; *by Neper's rules.*



Art.	Case.	Given.	Req.	Middle part and proportion.	Art
223	1	AC and A	BC	BC is the middle part, AC and C separated - - - R : *s*, AC :: *s*, A : *s*, BC.	219
224	2	AC and A	AB	A is the middle part, AC and AB contiguous - - cot. AC : R :: cos. A : *t*, AB.	220
225	3	AC and A	C	AC is the middle part, A and C contiguous - - cot. A : R :: cos. AC : cot. C.	220
226	4	AC and AB	BC	AC is the middle part, AB and BC separated . - cos. AB : R :: cos. AC : cos. BC.	221
227	5	AC and AB	C	AB is the middle part, AC and C separated - - *s*, AC : R :: *s*, AB : *s*, C.	221
228	6	AC and AB	A	A is the middle part, AC and AB contiguous - - R : *t*, AB :: cot. AC : cos. A.	218
229	7	AB and A	BC	AB is the middle part, A and BC contiguous - - cot. A : R :: *s*, AB : *t*, BC.	220
230	8	AB and A	AC	A is the middle part, AC and AB contiguous - - *t*, AB : R :: cos. A : cot. AC.	220

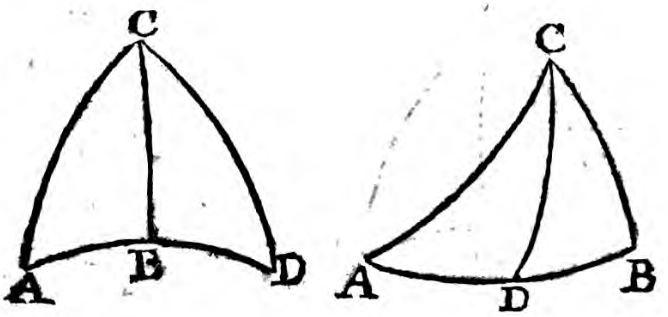
The ſolutions by Neper's rules continued.



Art.	Caſe.	Given.	Req.	Middle part and proportion.	Art
231	9	AB and A	C	C is the middle part, AB and A ſeparated. - - - R : s, A :: coſ. AB : coſ. C.	219
232	10	BC and A	AB	AB is the middle part, BC and A contig. : - - R : t, BC :: cot. A : s, AB.	218
233	11	BC and A	AC	BC is the middle part, A and AC ſeparated - - - s, A : R :: s, BC : s, AC.	221
234	12	B C and A	C	A is the middle part, BC and C ſeparated - - - coſ. BC : R :: coſ, A : s, C.	221
235	13	A B and B C	AC	AC is the middle part, AB and BC ſep. - - R : coſ. AB :: coſ. BC : coſ. AC	219
236	14	A B and B C	A	AB is the middle part, BC and A contiguous - - - t, BC : R :: s, AB : cot. A.	220
237	15	A and C	AB	C is the middle part, A and AB ſeparated - - - s, A : R :: coſ. C : coſ. AB.	221
238	16	A and C	AC	AC is the middle part, A and C contiguous - - - R : cot. A :: cot. C : coſ. AC.	218

Theorem 5.

239. *In all spherical triangles, the sines of the sides are proportional to the sines of their opposite angles.*



Let ACD be an oblique angled spherical triangle. I say, the sine of the angle A, is to the sine of the side CD; as the sine of the angle D, to the sine of the side AC.

Let the arch CB be perpendicular to the side AD, continued in fig. 2. - - 192

Now $R : s,AC :: s,A : s,BC$ - - 199

And $R : s,CD :: s,D : s,BC$ - - 199

Th. $R \times s,BC = s,AC \times s,A$ - - 185 p

And $R \times s,BC = s,CD \times s,D$ - - 185 p

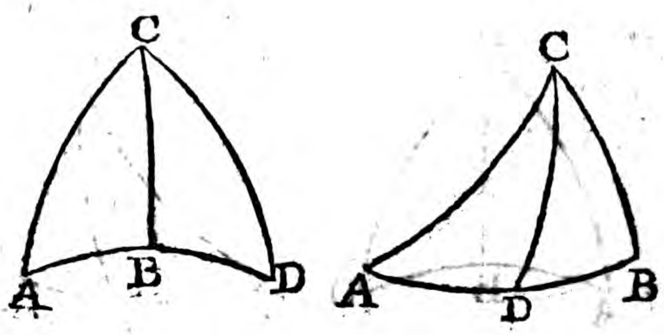
Th. $s,AC \times s,A = s,CD \times s,D$ - 48 p.

Th. $s,A : s,CD :: s,D : s,AC$ - - 187 p

 Theorem

Theorem 6.

240. *In oblique angled spherical triangles* ACD, *having a perpendicular* CB *falling upon the base. The cosines of the sides, are as the cosines of the segments of the base.*



For, cos. AB : cos. AC :: R : cos. BC - 202
And cos. BD : cos. DC :: R : cos. BC - 202
Th. cos. AB : cos. AC :: cos. BD : cos. DC. 173 p

Theorem 31.

241. *The cosines of the angles at the vertex, are as the cotangents of the sides.*

For, R : *t*, BC :: cot. AC : cos. ACB - - 228
And, R : *t*, BC :: cot. DC : cos. BCD - - 228
Th. cot. AC : cos. ACB :: cot. DC : cos. BCD 173p
Th. cot. AC : cot. DC :: cos. ACB : cos. BCD. 175p

Corollary.

242. *The cosines of the angles at the vertex, are reciprocally as the tangents of the sides.*

For, cot. AC : *cot.* DC :: *cos.* ACB : *cos.* BCD 241
And, cot. AC : *cot.* DC :: *t*, DC : *t*, AC 48
Th. t, DC : *t*, AC :: *cos.* ACB : *cos.* BCD. 173 *p*

Theorem

Theorem 7.

243. *In oblique angled spherical triangles* ACD, *having a perpendicular* CB *falling upon the base. The sines of the angles at the vertex, are as the cosines of the angles at the base.*

For,	cos. BC : R :: cos. A : s, ACB -	210
And,	cos. BC : R :: cos. D : s, BCD - -	210
Th.	cos. A : cos. D :: s, ACB : s, BCD.	173 p

Theorem 8.

244. *The tangents of the angles at the vertex, are as the tangents of the segments of the base.*

For,	R : s, CB :: t, ACB, t, AB - -	193
And,	R : s, CB :: t, BCD, t, BD - -	193
Th.	t, ACB : t, BCD :: t, AB : t, BD. -	173 p

Theorem 9.

245. *The tangents of the angles at the base, are reciprocally as the sines of the segments of the base.*

For,	R : s, AB :: t, A : t, BC - -	205
And,	R : s, BD :: t, D : t, BC - -	205
Th.	s, AB × t, A = (R × t, BC =) s, BD × t, D. - - - - - -	185 p.
Th.	s, AB : s, BD :: t, D : t, A. -	187 p

Corollary.

246. *The sines of the segments of the base, are as the cotangents of the angles at the base.*

For, s, AB : s, BD :: t, D : t, A - - 245
And cot. A : cot. D :: t, D : t, A - - 48
Th. s, AB : s, BD :: cot. A : cot. D - 173 P

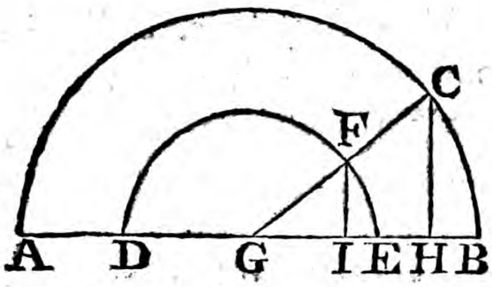
Remark.

247. *In the practice of spherical trigonometry, the perpendicular must fall from the end of a given side, and opposite to a given angle.*

Theorem

Theorem 10.

248. *The versed sines of equal angles, are proportional to the semidiameters of the circles.*



Let the semicircles ABC, DEF, have a common center at G, draw GC at pleasure, and upon the diameter AB, let fall the perpendiculars CH, FI.

Now AH, HB, are the versed sines of the arches AC, CB; also DI, IE, are the versed sines of the arches DF, FE, to the common angles AGC, CGB (34); and the triangles GCH, GFI are equiangular. - - - - con.

Th.	GC : GH :: GF : GI - - -	193 p
Or,	GB : GH :: GE : GI - -	56 p
Th.	GB : AH :: GE : DI - - -	176 p
And	GB : HB :: GE : IE - -	177 p
Th.	GB : GE :: AH : DI - -	175 p
And	GB : GE :: HB : IE - -	175 p

Theorem

Theorem 11.

249. *The square of the sine of half an arch, is equal to a rectangle contained by the radius and half the versed sine of that whole arch; plate* 1. *fig.* 1.

Let FED be a semicircle, and H its center; take the point E at pleasure, join FE, ED, upon DE, EF, let fall the perpendiculars HO, H*o*, and upon the diameter DF, the perpendiculars OS, E*p*, *os*.

Now DE, EF, are bisected in O and *o* - 127 p
Th. D*p*, *p*F are bisected in S and *s* - 101 p
Th. DO, F*o* are sines of half the arches DE, EF - - - - - - 42
And DS, F*s*, are half the versed sines of DE, EF; for D*p*, F*p* are the versed sines themselves. 34
Again, the triangles DHO, FH*o* have right angles at O and *o*, with perpendiculars to their bases OS, *os*.

Th. HD : DO :: DO : DS - - 199 p
And HF : F*o* :: F*o* : F*s* - - - 199 p
Th. $DO^2 = HD \times DS$ - - 185 p
And $Fo^2 = HF \times Fs$ - - 185 p

Theorem

Theorem 12.

250. *The rectangle contained by the sines of half the sum and half the difference of two arches; is equal to the rectangle contained by the radius, and half the difference of the versed sines of those arches; plate* 1. *fig.* 1.

In the great circle BCDE, whose center is H, take the arch CM equal to CL, and CI less than CM, join HL, HC, HI, LM, LI, IM, draw HN, IY perpendicular to LI, HC, and NQ, IR, parallel to LM, CH, respectively.

Now LI is bisected in N (127 p), therefore IM, IR, are besected in Q and T (101 p), therefore NI is the sine of half CL+CI, and IQ is the sine of half $\overline{\text{CL}}$—$\overline{\text{CI}}$, or half $\overline{\text{CM}-\text{CI}}$ (42). Again, CW, CY, are versed sines of the arches LC, CI (34) their difference is YW, or IR its equal, and IT is half their difference. Thirdly, the angles at N, T, R, are right angles by construction, and the angle IQT=(IML= half IHL=) IHN (142p). Therefore the triangles NIH, TIQ, are equiangular (87 p) and NI : IH :: TI : IQ (193 p), or NI : HD :: IT : IQ, therefore NI × IQ = IT × HD (185 p).

Theorem

Theorem 13.

251. *In a spherical triangle, the rectangle contained by the sines of the sides, is to the square of radius; as the excess of the versed sine of the base, above the versed sine of the difference of the sides, to the versed sine of the angle opposite to the base;* *plate* 1. *fig.* 1.

Let ABC be a spherical triangle, having the side AB greater than the side BC; in the great circle FBCM, take BG, BI each equal to BA, and CL, CM, each equal to CA; then CI = BA — BC. Draw GI, LM, intersecting at *a*, which will be diameters of small circles, whose poles are the points B and C (157), and the plains of those small circles will be perpendicular to the plain of the great circle FBCM (175). Therefore their common section A*a* is perpendicular to the same plain (262 p); and *a*I is the versed sine of the angle ABC, to the radius VI (248). Join HB, HC, LI, IM, draw HN bisecting LI, and NQ, IR, parallel to LM, CH; then HB, HC, will cut GI, LM, at right angles (126 p), and IM,

IR,

IR, will be divided equally in Q and T, (101 p). upon HB, HD, let fall the perpendiculars CX, IY, draw the diameter FD, let the arch DE meaſure the angle CBA, and PD will be its verſed ſine (34).

Now the angles at X, V, W, R, being right angles, and the angle HCX = (HKW = aKV =) aIR, the triangles HCX, aIR, are equiangular, and XC : CH or HD :: IR : aI - 193 p.
But VI : HD :: aI : PD - - 248
Th. VI × XC : HD² :: IR × aI : PD × aI. 206 p.
Th. VI × XC : HD² :: IR : PD - 184 p.

But VI, XC, are ſines of the ſides AB, BC; HD is the radius; IR is the exceſs of the verſed ſine CW, above the verſed ſine CY; PD is the verſed ſine of the angle ABC, oppoſite to the baſe; and conſequently the theorem is demonſtrated.

Theorem

Theorem 14.

252. *In any ſpherical triangle, the rectangle contained by the ſines of two ſides, is to the ſquare of radius; as the rectangle contained by the ſines of half the ſum, and half the remainder of the baſe and difference of the ſides, to the ſquare of the ſine of half the contained angle; plate* 1. *fig.* 1.

Retaining the laſt conſtruction, join ED, biſect ED in O, draw HO, and to the diameter FD, let fall the perpendiculars EP, OS.

Now $VI \times XC : HD^2 :: IR : PD$ - - 251
Alſo $IT : SD :: IR : PD$ - - 174 p
Th. $VI \times XC : HD^2 :: IT : DS$ - - 173 p
Ag. $IT : DS :: IT \times HD : DS \times HD$ - 184 p
Th. $VI \times XC : HD^2 :: IT \times HD : DS \times HD$ 173 p
But $NI \times IQ = IT \times HD$ (250), and $DO^2 = DS \times HD$ - - - - 249
Th. $VI \times XC : HD^2 :: NI \times IQ : DO^2$ 56 p

Laſtly, VI, XC, are the ſines of the arches AB, BC; HD is the radius; NI, IQ, are the ſines of half the arches $\overline{CA + CI}$, $\overline{CA - CI}$; and DO is the ſine of half the arch DE, which meaſures the angle ABC (42, 176). Therefore, ſine of AB × ſine of BC : ſquare of radius :: ſine of $\frac{CA+CI}{2}$ × ſine of $\frac{CA-CI}{2}$: ſquare of the ſine of half the angle ABC.

* 252. Otherwise. Plate I. fig. 2.

Suppose ABC a spherical triangle, having the side BA, greater than the side BC, and D the center of the sphere. In the great circle BCN, take BE, BF, each equal to BA, and CG, CH, each equal to CA; draw EF, GH, intersecting in *a*; join DB, DC, which will cut them at right angles in S and T, join likewise FH, draw the diameter IL parallel to EF, continue the arch BA to a quadrant at M; let a perpendicular fall from M, on the superficies of the sphere, to *m* in the diameter IL; draw *m*N at right angles to IL, join IN, NL, draw DO, DP, at right angles to IN, NL, and OQ, PR, parallel to N*m*.

Now SF : DL :: *a*F : *m*L - - 248

And *s,a* : S,H :: FH : *a*F - - plain trig.

Th. SF × *s, a* : DL × *s*, H :: *a*F × FH : *m*L × *a*F - - - - 206 p

Th. SF × *s, a* : DL × *s*, H :: FH : *m*L 184 p

Th. SF × *s, a* : DL × *s*, H :: $\frac{1}{2}$ FH : $\frac{1}{2}$*m*L 174 p

Th. SF × *s, a* × $\frac{1}{2}$ *m*L = DL × *s*, H × $\frac{1}{2}$ FH 185 p

But

But NP = PL (127 p). Th. RL = $\frac{1}{2}$ mL - 101 p

Th. SF × s,a × RL = DL × s, H × $\frac{1}{2}$ FH - 56 p

Th. SF × s,a : s,H × $\frac{1}{2}$ FH :: DL : RL - 187 p

Th. SF × s,a : s,H × $\frac{1}{2}$ FH :: DL^2 : RL × DL 184 p

Ag. PL^2 = RL × DL - - cor. to 199 p

Th. SF × s, a : s,H × $\frac{1}{2}$ FH :: DL^2 : PL^2 - 56 p

Laftly, SF is the fine of BF, or BA; the angle FaH = (BDC =) the arch BC; the angle FHa = ($\frac{1}{2}$ GCF =) $\frac{CA + \overline{BA - BC}}{2}$; $\frac{1}{2}$ FH = (fine of $\frac{1}{2}$ the arch FH =) fine of $\frac{CA - \overline{BA - BC}}{2}$; DL is the radius; and PL is the fine of $\frac{1}{2}$ the arch NL, or of $\frac{1}{2}$ the arch ML, which meafures the angle ABC; and confequently the theorem is again demonftrated.

Theorem 15.

253. *In a spherical triangle: The rectangle contained by the sines of any two sides, is to the rectangle contained by the sines of half the sum of the three sides, and its excess above the base; as the square of radius, to the square of the cosine of half the angle opposite to the base; plate* 1. *fig.* 2.

For SE : DI :: E*a* : I*m* - - 248
And *s*,*a* : *s*,G :: EG : E*a* - - plain trig.
Th. SE × *s*, *a* : DI × *s*, G :: E*a* × EG : I *m* × E*a* - - - - 206 p
Th. SE × *s*, *a* : DI × *s*, G :: EG : I*m* 184 p
Th. SE × *s*, *a* : DI × *s*, G :: $\frac{1}{2}$ EG : $\frac{1}{2}$ I*m* 174 p
But IQ = $\frac{1}{2}$ I*m* (because IO = $\frac{1}{2}$ IN) - 101 p
Th. SE × *s*,*a* : DI × *s*, G :: $\frac{1}{2}$ EG : IQ - subs.
Th. SE × *s*,*a* × IQ = DI × *s*,G × $\frac{1}{2}$ EG - 185 p
Th. SE × *s*,*a* : *s*,G × $\frac{1}{2}$ EG :: DI : IQ - 187 p
Th. SE × *s*,*a* : *s*,G × $\frac{1}{2}$ EG :: DI^2 : DI × IQ 184 p
Again, IO^2 = DI × IQ - cor. to 199 p
Th. SE × *s*,*a* : *s*,G × $\frac{1}{2}$ EG :: DI^2 : IO^2 - subs.

Laftly, SE is the fine of BE or BA, the angle $a = (sav = VDT =)$ BC; the fine of G = fine of $\frac{1}{2}$ ENH, or $\frac{1}{2}$ EBH, or $\frac{AB + BC + CA}{2}$; $\frac{1}{2}$ EG = fine of $\frac{AB + BC - CA}{2}$, or $\frac{AB + BC + CA}{2} - CA$; DI is radius, and IO is the cofine of half the angle ABC, by the conftruction. Therefore, $s, AB \times s, BC : s, H \times s, \overline{H - AC} :: \overline{radius}^2 : \overline{cofine}^2, \frac{1}{2} ABC$, if H = half the fum of the three fides.

Theorem.

Theorem 16.

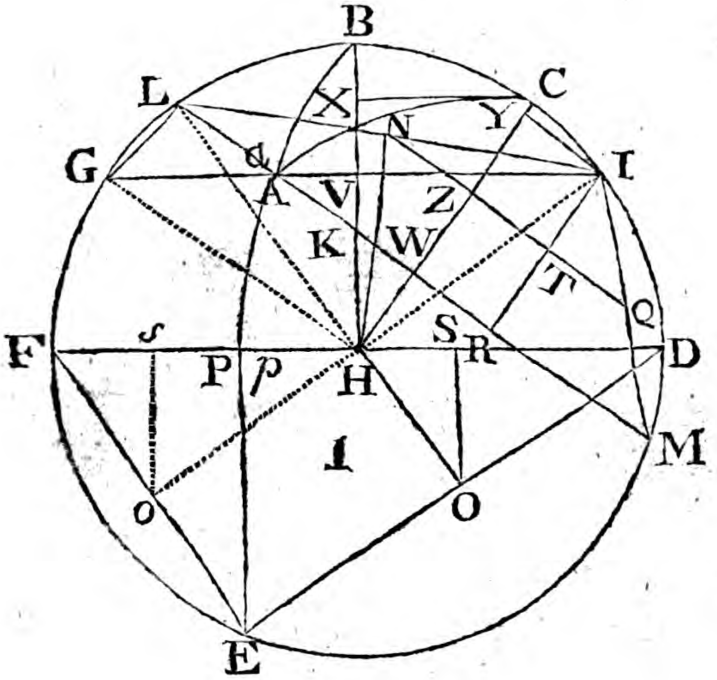
254. *In a spherical triangle. The rectangle contained by the sines of half the sum of the three sides, and its excess above either side, is to the rectangle contained by the sines of the excesses of the half sum above each of the other sides; as the square of radius, to the square of the tangent of half the angle opposite to the side first taken. Plate 1. fig 2.*

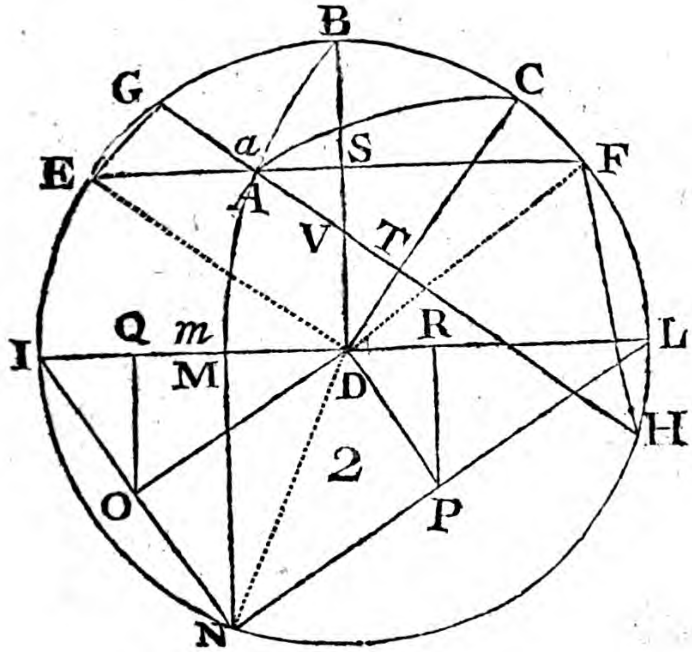
For $SE \times s,a : DI^2 :: s,G \times \frac{1}{2} EG : IO^2$ - 253
And $DL = DI$, $DP = (ON =)$ IO - con.
Th. $SE \times s,a : DL^2 :: s,G \times \frac{1}{2} EG : DP^2$ - subs.
Also, $SE \times s,a : DL^2 :: s,H \times \frac{1}{2} FH : PL^2$ - *252
Th. $s, G \times \frac{1}{2} EG : DP^2 :: s, H \times \frac{1}{2} FH : PL^2$ 173 p
Th. $s, G \times \frac{1}{2} EG : s, H \frac{1}{2} FH :: DP^2 : PL^2$ - 175 p
But $\overline{\text{radius}}^2 : \overline{\text{tan.}}^2$ of ang. PDL $:: DP^2 : PL^2$ 55
Th. $s, G \times \frac{1}{2} EG : s, H \times \frac{1}{2} FH :: \overline{\text{radius}}^2 : \overline{\text{tan.}}^2$ of PDL. - - - 173 p

Laſtly, the ſine of G = ſine of ½ ENH, or ½ EBH, or of $\frac{BA+BC+CA}{2}$; ½ EG = ſine of half the arch EG = ſine of $\frac{BA+BC-CA}{2}$, or $\overline{\frac{BA+BC+CA}{2}-CA}$; the ſine of H = ſine of ½ GCF, or $\frac{CA+BA-BC}{2}$ or of $\frac{CA+BA+BC}{2}-BC$; ½ FH = ſine of half the arch FH, or of $\frac{CA-\overline{BA-BC}}{2}$, or $\frac{CA-BA+BC}{2}$, or of $\frac{CA+BA+BC}{2}-BA$; and the angle PDL is meaſured by half the arch NL, that meaſures the angle ABC.

Th. $s,H \times s,\overline{H-AC} : s,\overline{H-AB} \times s,\overline{H-BC} :: \overline{\text{radius}}^2 : \overline{\text{tangent}}^2$ of half the angle ABC; putting H for half the ſum of the three ſides.

Plate I.

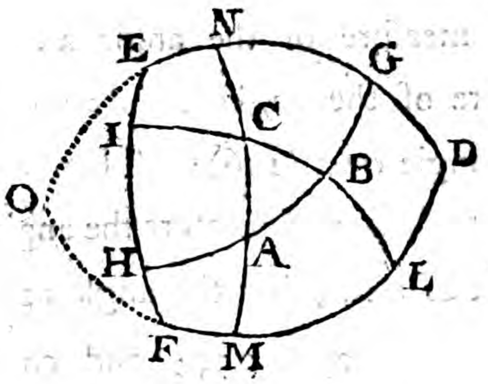




To be placed opposite page 148.

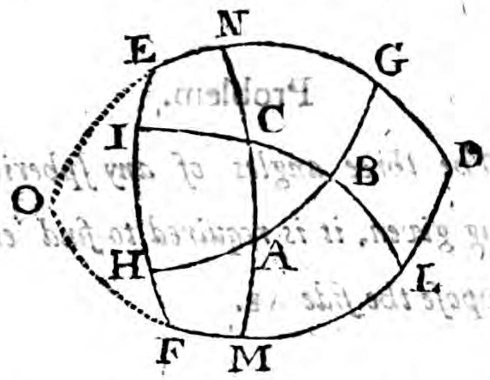
Problem.

255. *The three angles of any spherical triangle* ABC, *being given, it is required to find either of the sides; suppose the side* AB.



Produce the sides both ways, and let great circles be described about the angular points as poles, forming the triangles DEF, FEO.

Now GN, HI, ML, measure the angles, A, B, C, and DE, EF, FD, are their supplements (185). Also EO, FO, are supplements of DE, DF, since DEO, DFO, are semicircles (167). Therefore EO, FO, are equal to GN, ML each to each, therefore the sides of the triangle FEO are given; namely, FE is the supplement of the angle B;



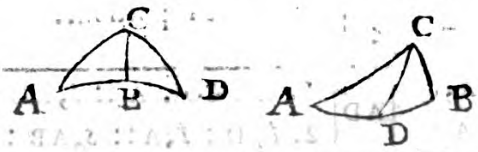
EO is the meaſure of the angle A; and FO is the meaſure of the angle C. Again, GH meaſures the angle GEH (176); Therefore AB, the ſupplement of GH, will meaſure the angle FEO, the ſupplement of GEH; but the angle FEO is given by articles 252, 253, 254; and conſequently any ſide (AB) is given by the following

Rule.

256. *Change the greater angle adjacent to the ſide required into its ſupplement, then proceed with thoſe angles after the very ſame manner, as if they were ſides given to find an angle; and there will ariſe an expreſſion for half the ſide required.*

Solutions

Solutions of the 12 caſes of oblique ſpherical triangles.



Art.	Caſe.	Given.	Req.	Proportions.	Art
257	1	AC, CD A	D	ſ, CD : ſ, A :: ſ, AC : D -	239
258	2	AC, CD A	C	1. R : t, A :: coſ. AC : cot. ACB 2. cot. AC : coſ. ACB :: cot. DC : coſ. BCD - -	201 241
259	3	AC, CD A	AD	1. R : coſ. A :: t, AC : t, AB. 2. coſ. AC : coſ. AB :: coſ. CD : coſ. BD - -	200 240
260	4	AC, AD A	CD	1. R : coſ. A :: t, AC : t, AB 2. coſ. AB : coſ. AC :: coſ. BD : coſ. DC - -	200 240
261	5	AC, AD A	D	1. R : coſ. A :: t, AC : t, AB. 2. ſ, BD : ſ, AB :: t, A : t, BDC - -	200 245
262	6	A, C AC	D	1. R : t, A :: coſ. AC : cot. ACB 2. ſ, ACB : coſ. A :: ſ, BCD : coſ. D - -	201 243
263	7	A, C AC	CD	1. R : t, A :: coſ. AC : cot. ACB 2 coſ. ACB : cot. AC :: coſ. BCD : cot. CD - -	201 241
264	8	A, D AC	CD	ſD : ſ, AC :: ſ, A : ſ, CD.	239

Solutions of oblique triangles continued.

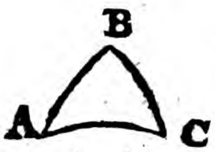
Art.	Case	Given.	Req.	Proportions.	Art
265	9	A, D AC	AD	1. R : cos. A :: t, AC : t, AB. 2. t, D : t, A :: s, AB : s, BD.	200 245
266	10	A, D AC	C	1. cot. A : R :: cos. AC : cot. ACB - - 2. cos. A : s, ACB :: cos. D : s, BCD - -	225 243
267	11	AB, BC CA	A	s, AB $\times$ s, AC : $\overline{\frac{BC+D}{2}} \times \overline{\frac{BC-D}{2}}$ :: R^2 : $\overline{\text{sine}}^2$, $\frac{1}{2}$ A. If D = AC — AB. - -	252
268	11	Ditto	A	s, AB $\times$ s, AC : s, H $\times$ s, $\overline{\text{H}-\text{BC}}$ :: R^2 : $\overline{\text{cos.}}^2$ $\frac{1}{2}$ A; if H = $\frac{1}{2}$ sum of the sides -	253
269	11	Ditto	A	s, H $\times$ s, $\overline{\text{H}-\text{BC}}$: s, $\overline{\text{H}-\text{AB}}$ $\times$ s, $\overline{\text{H}-\text{AC}}$ :: R^2 : $\overline{\text{tan.}}^2$ of $\frac{1}{2}$ A - - -	254
270	12	A, B C	AB	s, A $\times$ s, b : s, $\overline{\frac{C+D}{2}}$ $\times$ s, $\overline{\frac{C-D}{2}}$ :: R^2 : $\overline{\text{sine}}^2$, $\frac{1}{2}$ AB; if D = A — b, or b — A - -	256 252
271	12	Ditto	AB	s, A $\times$ s, b : s, H $\times$ s, $\overline{\text{H}-\text{C}}$ :: R^2 : $\overline{\text{cosine}}^2$, $\frac{1}{2}$ AB -	256 253
272	12	Ditto	AB	s, H $\times$ s, $\overline{\text{H}-\text{C}}$: s, $\overline{\text{H}-\text{A}}$ $\times$ s, $\overline{\text{H}-b}$ :: R^2 : $\overline{\text{tangent}}^2$, $\frac{1}{2}$ AB - - -	256 254

Figures to cases 9 and 10.

Remark 1.

The 1st, 2d, and 3d cases are ambiguous, and depend upon the question.

Figure to cases 11 and 12.



Remark 2.

In case 12, b represents the supplement of B, *and* H *the half sum of the three angles, according to the rule* - - - 256

CHAP.

CHAP. IV.

Of the Celestial Sphere.

273. *The celestial sphere is that in whose superficies all objects in the heavens (sun, stars, &c.) appear to be situated.*

274. *The axis of the celestial sphere is a strait line passing through the center of the earth, about which all objects in the heavens appear daily to revolve.*

275. *The poles of the celestial sphere, are the extreme points of its axis, one called the north, the other the south pole.*

276. *The equinoctial is a great circle of the celestial sphere, equally distant from both its poles; therefore the poles of the equinoctial circle are the same with the poles of the celestial sphere* (157) *and the axis is perpendicular to the equinoctial plain* - - - 170

277. *The equinoctial divides the heavens into two equal parts, called the northern, and the ſouthern hemiſpheres.*

278. *Meridians are great circles paſſing through the poles of the equinoctial, and therefore muſt cut the ſaid circle at right angles* - - 172

279. *The horizon is that great circle which ſeparates the viſible half of the heavens from the inviſible; its cardinal points are north, ſouth, eaſt, weſt.*

280. *The zenith is a point in the celeſtial ſphere, directly over head, or it is a point in the viſible hemiſphere equally diſtant from every part of the horizon.*

281. *The nadir is a point in the celeſtial ſphere diametrically oppoſite to the zenith, and is therefore directly under foot.*

282. *The zenith and nadir, are poles of the horizon* - - - - 157

283. *Azimuth circles (or vertical circles) are great circles paſſing thro' the zenith and nadir, and they cut the horizon at right angles* - 172

284. *The prime vertical, is the azimuth circle which paſſes thro' the eaſt and weſt points of the horizon.*

285. *The meridian of any place, coincides with the azimuth circle; that paſſes thro' the north and ſouth points of the horizon.*

286. *The*

286. *The ecliptick is that great circle in which the sun makes his apparent annual progress; it cuts the equinoctial in the angle of* 23° : 28′, *and the points of intersection are called aries aud libra.*

287. *The ecliptick is divided into twelve equal parts, called signs, and consequently each sign contains* 30 *degrees; their names and marks are as follows.*

Aries ♈, *taurus* ♉, *gemini* ♊, *cancer* ♋, *leo* ♌, *virgo* ♍, *libra* ♎, *scorpio* ♏, *sagitarius* ♐, *capricorn* ♑, *aquarius* ♒, *pisces* ♓.

288. *The poles of the ecliptick are* 23° : 28′ *distant from the poles of the equinoctial* - 177

289. *Circles of longitude are great circles passing thro' the poles of the ecliptick, and they intersect it at right angles* - - - 172

290. *Parallels of declination, are lesser circles parallel to the equinoctial.*

291. *The tropicks are those two parallels of declination, which touch the ecliptick in the opposite points of cancer and capricorn; and consequently are limits of the sun's progress, to the north or south of the equinoctial.*

292. *Parallels of altitude, are lesser circles parallel to the horizon.*

293. *The longitude of any object in the heavens, is an arch of the ecliptick contained between the*

first

first point of aries, and the circle of longitude passing thro' the center of that object.

294. *The latitude of any object in the heavens, is an arch of a circle of longitude contained between the center of that object and the ecliptick.*

295. *The declination of any object in the heavens, is an arch of the meridian, contained between the center of that object and the equinoctial.*

296. *The right ascension of any object in the heavens, is an arch of the equinoctial, contained between the point aries and the meridian, passing thro' the center of that object.*

297. *The oblique ascension of any object in the heavens, is that point of the equinoctial which rises with the center of that object.*

298. *The ascensional difference, is an arch of the equinoctial, contained between the right and oblique ascension of the same object.*

299. *The azimuth of any object in the heavens, is an arch of the horizon contained between the north or south points, and the azimuth circle, passing thro' the center of that object.*

300. *The amplitude of any object in the heavens, is an arch of the horizon, contained between the east or west points, and the center of that object at its rising or setting.*

301. *The altitude of any object in the heavens, is the arch of an azimuth circle, contained between its center and the horizon.*

302. *The*

302. *The zenith distance of any object in the heavens, is the arch of an azimuth circle, contained between its center and the zenith.*

303. *The polar distance of any object in the heavens, is an arch of the meridian, contained between its center and either pole of the equinoctial.*

304. *The astronomical latitude of a place, is an arch of the meridian, contained between its zenith and the equinoctial, or it is the elevation of the pole above the horizon.*

305. *The celestial sphere is represented in plate 2, the north pole being elevated in the first figure, and the south pole in the second, the eye is supposed to view them from the east point of the horizon, and according to the preceding definitions; in plate 2.*

PS is the axis of the sphere - -	274
P is the north, and *s* the south pole -	275
EQ is the equinoctial - - -	276
PES, P☉S are meridians - -	278
HR is the horizon - - -	279
Z is the zenith - -	280
N is the nadir - - -	281
ZHN, Z☉N, are azimuth circles -	283
ZCN is the prime vertical, C is both the east and west point of the horizon - -	284
PZHR is the meridian of the place, R is the north, and H the south point of the horizon -	285
KL is the ecliptick - - -	286

The

The equinoctial points, aries and libra, in these figures are at c, and the solstitial points, cancer and capricorn, are at K and L - 286

K*d*, *a*L are parallels of declination - 290

K*d* is the tropick of cancer, and *a*L is the tropick of capricorn - - 291

ad is a parallel of altitude - 292

EZ, or RP, is the latitude of a place in the northern hemisphere; and QZ, or HS, is the latitude of a place in the southern hemisphere. 304

CHAP.

CHAP. V.

Contains examples to all the cases in spherical triangles, applied to astronomical problems.

306. *Example to cases* 1, 2, 3, *in right angled spherical triangles; plate* 2. *fig.* 1.

In the spherical triangle c*xy* is given, a right angle at *y*, the sun's longitude c*x* 57° : 30′, and the angle *x*c*y* made by the ecliptick and equinoctial 23° : 28′; to find the sun's declination *xy*, his right ascension c*y*, and the angle c*xy* made by the ecliptick and meridian.

1. For the declination *xy*		art. 109
As radius - - -		10,000000
To the sine of c*x*	= 57° : 30′	9,926029
So is the sine of *x*c*y*	= 23 : 28 -	9,600118
To the sine of *xy*	= 19 : 37 -	9,526147

2. For

2.	For the right ascension cy.		art. 200
As radius	- - -		10,000000
To the cosine of xcy	$= 23° : 28'$		9,962507
So is the tangent of cx	$= 57 : 30$		10,195813
To the tangent of cy	$= 55 : 13$		10,158320

3.	For the angle cxy.		art. 201
As radius	- - -		10,000000
To the tangent of xcy	$= 23 : 28$		9,637610
So is the cosine of cx	$= 57 : 30$		9,730216
To the cotangent of cxy	$= 76 : 52$		9,367826

307. *Example to cases 4, 5, 6, in right angled spherical triangles; plate 2. fig. 1.*

In the spherical triangle PR⊙ is given, a right angle at R, the polar distance P⊙ 66° : 32′, and the elevation of the pole RP 47° : 15′; to find the amplitude from the north ⊙R, the angle of the meridian and horizon P⊙R, and the hour angle ⊙PR.

1.	For the amplitude ⊙R.		art. 226.
As the cosine of	RP = 47°	: 15′	9,831742
To radius	- -	-	10,000000
So is the cosine of	P⊙ = 66	: 32	9,600118
To the cosine of	⊙R = 54	: 05	9,768376
2.	For the angle P⊙R.		art. 194.
As the sine of	P⊙ = 66	: 32	9,962507
To radius	- -	-	10,000000
So is the sine of	RP = 47	: 15	9,865887
To the sine of	P⊙R = 53	: 11	9,903380
3.	For the hour angle ⊙PR.		art. 228.
As radius	- -	-	10,000000
To the tangent of	RP = 47	: 15	10,034144
So is the cotan. of	⊙P = 66	: 32	9,637610
To the cosine of	⊙PR = 62	: 00	9,671754

308. *Example*

308. *Example to cases 7, 8, 9, in right angled spherical triangles; plate 2. fig. 1.*

In the spherical triangle c*mn*, is given a right angle at *m*, the sun's azimuth from the east or west at six c*m* 16° : 25′; and the latitude of the place, or measure of the angle *mcn* 47° : 15′; to find the sun's altitude at six *mn*, the sun's declination c*n*, and the angle c*nm*.

1. For the altitude at six *mn*. art. 205.

As radius	- - -	10,000000
To the sine of c*m*	= 16° : 25′	9,441203
So is the tangent of *mcn*	= 47 : 15	10,034144
To the tangent of *mn*	= 16 : 38	9,475347

2. For the sun's declination c*n*. art. 230.

As the tangent of c*m*	= 16 : 25	9,469380
To radius	- - -	10,000000
So is the cosine of *mcn*	= 47 : 15	9,831742
To the cotangent of c*n*	=23 : 28	10,362362

3. For the angle c*nm*. art. 231.

As radius	- - -	10,000000
To the sine of *mcn*	= 47 : 15	9,865887
So is the cosine of c*m*	= 16 : 25	9,981923
To the cosine of c*nm*	= 45 : 13	9,847810

309. *Example to caſes* 10, 11, 12, *in right angled ſpherical triangles; plate* 2. *fig.* 1.

In the ſpherical triangle CD☉, is given a right angle at D, the declination of the ſun D☉ 23° : 28′, and the angle DC☉ 42° : 45′, the co-latitude; to find the aſcentional difference CD, the amplitude C☉, and the angle C☉D.

1. For the aſcentional difference CD. art. 232.

As radius - - -			10,000000
To the cotang. of ☉CD	= 42°	: 45′	10,034144
So is the tangent of D☉	= 23	: 28	9,637610
To the ſine of CD -	= 28	: 00	9,671754

2. For the amplitude C☉. art. 209.

As the ſine of ☉CD -	= 42	: 45	9,831742
To radius - - -			10,000000
So is the ſine of D☉	= 23	: 28	9,600118
To the ſine of C☉ -	= 35	: 55	9,768376

3. For the angle C☉D. art. 210.

As the coſine of D☉ -	= 23	: 28	9,962507
To the coſine of ☉CD	= 42	: 45	9,865887
So is radius - - -			10,000000
To the ſine of C☉D -	= 53	: 11	9,903380

310. *Example to cases* 13, 14, *in right angled spherical triangles*; *plate* 2. *fig.* 1.

In the spherical triangle *cmn*, is given a right angle at *m*, the sun's azimuth from the east or west at six c*m* 16° : 25′, and his altitude at six 17° : 00′; to find the declination c*n*, and the latitude of the place, or angle *m*c*n*.

1.	For the declination c*n*.		art. 211.
As radius	- - -		10,000000
To the cosine of c*m*	= 16° : 25′		9,981923
So is the cosine of *mn*	= 17 : 00		9,980596
To the cosine of c*n*	= 23 : 28		9,962519
2.	For the angle *m*c*n*.		art. 212.
As the sine of c*m*	= 16 : 25		9,451203
To radius	- - -		10,000000
So is the tangent of *mn*	= 17 : 20		9,485339
To the tangent of *m*c*n*	= 47 : 15		10,034136

 311. *Example*

311. *Example to cases 15, 16, in right angled spherical triangles.*

In the spherical triangle ⊙RP, is given a right angle at R, the angle ⊙PR 62° : 00′, and the angle P⊙R 53° : 11′; to find the elevation RP, and the sun's distance from the north pole P⊙; *plate 2. fig. 1.*

1. For the elevation of the pole RP. art. 213.

As the sine of ⊙PR	= 62°	: 00′	9,945935
To radius	-	-	10,000000
So is the cosine of P⊙R	= 53	: 11	9,777631
To the cosine of RP	= 47	: 15	9,831696

2. For the polar distance P⊙. art. 238.

As the radius	-	-	10,000000
To the cotan. of ⊙PR	= 62	: 00	9,725674
So is the cotan. of P⊙R	= 53	: 11	9,874220
To the cosine of P⊙	= 66	: 32	9,599894

312. *Example*

312. *Example to cases* 1, 2, 3, *in oblique spherical triangles, when the perpendicular falls within the triangle*; *plate* 2 *fig.* 2.

In the spherical triangle sz⊙ are given, the complement of the latitude sz 42° : 30′, the zenith distance z⊙ 29° : 11′, and the hour angle zs⊙ 20 : 45′; to find the angle of position s⊙z, the angle of azimuth sz⊙ (for it is measured by the arch HC⊙), and the polar distance s⊙.

1. For the angle of position s⊙z. by art. 239.

As the sine of z⊙	= 29° : 11′	*co.* 0,311931
To the sine of zs⊙	= 20 : 45	9,549360
So is the sine of sz	= 42 : 30	9,829683
To the sine of s⊙z	= 29 : 24	9,690974

2. Suppose zF a perpendicular to s⊙ (247) and find the angle szF. by art. 201.

As radius - - - -		10,000000
To the tangent of zs⊙	= 20° : 45′	9,578486
So is the cosine of sz	= 42 : 30	9,867631
To the cotan. of szF	= 74 : 24	9,446117

3. For the angle FZ⊙ by art. 242.

As the tangent of Z⊙	$=29^\circ$	: 11′	*co.* 0,252977
To the tangent of ZS	$=42$	: 30	9,962052
So is the cosine of SZF	$=74$	: 24	9,429623
To the cosine of FZ⊙	$=63$	: 49	9,644652
Th. the angle SZ⊙	$=138$	: 13	

4. For the arch SF by art. 200.

As radius - - - - - - - -			10,000000
To the cosine of ZSF	$=20^\circ$	: 45′	9,970874
So is the tangent of SZ	$=42$	: 30	9,962052
To the tangent of SF	$=40$	: 36	9,932926

5. For the arch F⊙ by art. 240.

As the cosine of SZ	$=42^\circ$	: 30′	*co.* 0,132369
To the cosine of Z⊙	$=28$	: 58	9,941059
So is the cosine of SF	$=40$	: 36	9,880397
To the cosine of F⊙	$=25$	: 57	9,953825
Therefore S⊙	$=66$	: 33	

313. *Ex-*

*313. *Example to cases* 1, 2, 3, *in oblique spherical triangles, when the perpendicular falls without the triangle; plate* 2. *fig.* 2.

In the spherical triangle sz⊙ is given, the polar distance s⊙ 66 : 32, the zenith distane z⊙ 29° : 11′, and the hour angle zs⊙ 20° : 45′; to find the angle of azimuth sz⊙, the angle of position s⊙z, and the co-latitude sz.

1. For the angle sz⊙ by art. 239.

As the sine of z⊙	= 29° : 11′	*co.* 0,311931
To the sine of zs⊙	= 20 : 45	9,549360
So is the sine of s⊙	= 66 : 32	9,962507
To the sine of sz⊙	= 138 : 13	9,823798

2. Suppose ⊙L a perpendicular to the arch szQ (247), and find the angle s⊙L. by art. 201.

As radius - - - - - - - -		10,000000
To the tangent of Ls⊙	= 20° : 45′	9,578486
So is the cosine of s⊙	= 66 : 32	9,600118
To the cotan. of s⊙L	= 81 : 25	9,178604

3. For

3. For the angle z⊙l. by art. 241.

As the cotangent of s⊙	= 66° : 32′	co. 10,362389
To the cotangent of z⊙	= 29 : 11	10,252977
So is the cosine of s⊙l	= 81 : 25	9,173908
To the cosine of z⊙l	= 52 : 01	9,789274
Th. the angle s⊙z	= 29 : 24	

4. For the arch sl. by art. 200.

As radius - - - - - - -		10,000000
To the cosine of ls⊙	= 20° : 45′	9,970874
So is the tangent of s⊙	= 66 : 32	10,362389
To the tangent of sl	= 65 : 06	10,333263

5. For the arch zl. by art. 240.

As the cosine of s⊙	= 66° : 32′	co. 0,399882
To the cosine of z⊙	= 29 : 11	9,941046
So is the cosine of sl	= 65 : 06	9,624319
To the cosine of zl	= 22 : 37	9,965247
Th. the colat. sz	= 42 : 29	

Again.

Again.

313. *In the spherical triangle* ⊙PN *is given, the polar distance* ⊙P 113° : 28′, *the nadir distance* ⊙N 150° : 49′, *and the hour angle* NP⊙ 159° : 15′ ; *to find that arch* NP, *which is equal to the complement of the latitude* SZ ; *plate* 2. *fig.* 2.

Here the triangle SZ⊙ being supplemental to ⊙PN, the sides S⊙, Z⊙, and the angle ZS⊙ are given (29). Whence the computation may be the very same as in article *312 ; but if it be required to calculate in the given triangle ⊙PN, let ⊙L be a perpendicular to PQZ (247).

1.	For the arch PL.		by art. 200.
As radius	- - - - - - -		10,000000
To the cosine of ⊙PL	= 20°	: 45′	9,970874
So is the tan. of P⊙	= 113	: 28	10,362389
To the tan. of PL	= 114	: 54	10,333263

2.	For the arch NPL.		by art. 240.
As the cosine of P⊙	= 113°	: 28′	*co.*0,399882
To the cosine of N⊙	= 150	: 49	9,941046
So is the cosine of PL	= 114	: 54	9,624319
To the cosine of NL	= 157	: 23	9,965247
Therefore NP	= 42	: 29	

314. *Ex-*

314. *Example to case 4 and 5, in oblique spherical triangles; plate 2. fig. 2.*

In the spherical triangle sz⊙ is given, the complement of the latitude sz 42° : 30′, the polar distance s⊙ 66° : 32′, and the hour angle zs⊙ 20° : 45′; to find the zenith distance z⊙, and the angle of position z⊙s.

1. Let zf be perpendicular to s⊙, and find sf by the proportion in article 200.

As radius - - - - - - - -		10,000000
To the cosine of zs⊙	= 20° : 45′	9,970874
So is the tan. of sz	= 42 : 30	9,962052
To the tan. of sf	= 40 : 36	9,932926
Therefore f⊙	= 25 : 56	

2. For the zenith distance z⊙. by art. 240.

As the cosine of sf	= 40° : 36′	*co.*0,119603
To the cosine of sz	= 42 : 30	9,867631
So is the cosine of f⊙	= 25 : 56	9,953906
To the cosine of z⊙	= 29 : 10	9,941140

3. For the angle of position z⊙s. by art. 245.

As the sine of f⊙	= 25° : 56′	*co.*0,359196
To the sine of sf	= 40 : 36	9,813430
So is the tan. of zs⊙	= 20 : 45	9,578486
To the tangent of z⊙s	= 29 : 25	9,751112

315. *Ex-*

315. *Example to cases 6 and 7, in oblique spherical triangles; plate 2. fig. 2.*

In the spherical triangle sz☉ is given, the hour angle zs☉ 20° : 45′, the azimuth angle sz☉ 138° : 12′, and the complement of latitude sz 42° : 30′; to find the angle of position z☉s, and the zenith distance z☉.

1. Let zf be perpendicular to s☉ (247) and find the angle szf, by article 201.

As radius	- - - - - - -		10,000000
To the tangent of z☉s	= 20°	: 45′	9,578486
So is the cosine of sz	= 42	: 30	9,867631
To the cotangent of szf	= 74	: 24	9,446117

Theref. the angle fz☉ = 63 : 48.

2. For the angle of position z☉s. by art. 243.

As the sine of szf	= 74°	: 24′	co.0,016301
To the cosine of zsf	= 20	: 45	9,970874
So is the sine of fz☉	= 63	: 48	9,952917
To the cosine of z☉s	= 29	: 25	9,940092

3. For the zenith distance z☉. by art. 241.

As the cosine of szf	= 74°	: 24′	co.0,570377
To the cotan. of sz	= 42	: 30	10,037947
So is the cosine of fz☉	= 63	: 48	9,644936
To the cotan. of z☉	= 29	: 10	10,253260

*315. *Again,*

*315. *Again, for caſes 6 and 7.*

In the ſpherical triangle sz⊙ is given, the hour angle zs⊙ 20° : 45′, the angle of poſition z⊙s 29° : 24′, and the polar diſtance s⊙ 66° : 32′; to find the azimuth sz⊙, and the zenith diſtance z⊙; plate 2. fig. 2.

1. Suppoſe ⊙L perpendicular to szQ (247), and find the angle s⊙L by article 201.

As radius - - - - - - -			10,000000
To the tangent of zs⊙	= 20°	: 45′	9,578486
So is the coſine of s⊙	= 66	: 32	9,600118
To the cotangent of s⊙L	= 81	: 25	9,178604
Therefore the angle z⊙L	= 52	: 01	

2. For the angle ⊙zL, the ſupplement of sz⊙, by article 243.

As the ſine of s⊙L	= 81°	: 25′	*co.* 0,004892
To the coſine of ⊙sL	= 20	: 45	9,970874
So is the ſine of z⊙L	= 52	: 01	9,896631
To the coſine of ⊙zL	= 41	: 48	9,872397
Th. the angle sz⊙	= 138	: 12	

3. For the zenith diſtance z⊙. by art. 241.

As the coſine of s⊙L	= 81°	: 25′	*co.* 0,826092
To the cotangent of s⊙	= 66	: 32	9,637610
So is the coſine of z⊙L	= 52	: 01	9,789180
To the cotangent of z⊙	= 29	: 11	10,252882

316. *Ex-*

316. *Example to cases* 8, 9, 10, *in oblique spherical triangles; plate* 2. *fig*. 2.

In the spherical triangle sz⊙ is given, the hour angle zs⊙ 20° : 45′, the angle of position s⊙z 29° : 24′, and the complement of latitude sz 42° : 30′; to find the zenith distance z⊙, the polar distance s⊙, and the azimuth angle sz⊙.

1. For the zenith distance z⊙. by art. 240.

As the sine of s⊙z	= 29°	: 24′	*co*.0,309004
To the sine of sz	= 42	: 30	9,829683
So is the sine of zs⊙	= 20	: 45	9,549360
To the sine of z⊙	= 29	: 11	9,688047

2. Let zf be perpendicular to s⊙ (247) and find the part sf by article 200.

As radius - - - - - - - -			10,000000
To the cosine of zsf	= 20°	: 45′	9,970874
So is the tangent of sz	= 42	: 30	9,962052
To the tangent of sf	= 40	: 36	0,932926

3. For

3. For the part F☉ by art. 245.

As the tangent of s☉z	$= 29^{\circ}$	: 24′	co.10,249128
To the tangent of zsf	= 20	: 45	9,578486
So is the sine of sf	= 40	: 36	9,813430
To the sine of f☉	= 25	: 57	9,641044
Therefore s☉	= 66	: 33	

4. For the angle szf by art. 201.

As radius - - - - - - -			10,000000
To the tangent of zs☉	$= 20^{\circ}$	: 45′	9,578486
So is the cosine of sz	= 42	: 30	9,867631
To the cotangent of szf	= 74	: 24	9,446117

5. For the angle fz☉. by art. 243.

As the cosine of zsf	$= 20^{\circ}$	: 45′	co.0,029126
To the cosine of f☉z	= 29	: 24	9,940125
So is the sine of szf	= 74	: 24	9,983699
To the sine of fz☉	= 63	: 48	9,952950
Th. the angle sz☉	= 138	: 12	

317. Ex-

317. *Example to case* 11, *in oblique spherical triangles*; *plate* 2. *fig.* 1.

In the spherical triangle PZ☉ are given, the co-latitude PZ 42° : 30′, the polar distance P☉ 66° : 32′, and the zenith distance Z☉ 29° : 10′; to find the hour angle ZP☉.

First method, by art. 267.

The base Z☉	= 29° : 10,	P☉	= 66° : 32
And P☉—PZ	= 24 : 2	PZ	= 42 : 30
The sum	= 53° : 12′	its half	= 26° : 36′
The rem.	= 5 : 8	its half	= 2° : 34

Sine of P☉	= 66° : 32′	*co*. 0,037493
Sine of PZ	= 42 : 30	*co*. 0,170317
Sine of half sum	= 26 : 36	9,651044
Sine of half diff.	= 02 : 34	8,651102
Their sum - - - - - -		= 18,509956
Its half = sine of ½ ZP☉	= 10 : 22	9,254978
Th. the angle ZP☉	= 20 : 44	

N Second

Second method. by art. 268.

Now P⊙	= 66° : 32 ſine - -	co.0,037493
PZ	= 42 : 30 ſine - -	co.0,170317
Z⊙	= 29 : 10 the baſe -	0,000000
The ſum	= 138 : 12 - - -	0,000000
Th. H	= 69 : 6 ſine - -	9,970442
H — Z⊙	= 39 : 56 ſine - -	9,807464
Sum of the four ſines is - -		19,985716
Its half = coſ. of ½ ZP⊙ = 10° : 22′		9,992858

Th. the angle ZP⊙ = 20 : 44.

Third method. by art. 269.

Here Z⊙ = 29° : 10′ the oppoſite ſide.
P⊙ = 66 : 32 a containing ſide.
PZ = 42 : 30 a containing ſide.

The ſum	= 138 : 12	
Th. H	= 69 : 06 ſine - -	co.0,029558
H — Z⊙	= 39 : 56 ſine - -	co.0,192536
H — P⊙	= 02 : 34 ſine - -	8,651102
H — PZ	= 26 : 36 ſine -	9,651044
Sum of the ſines - -		18,524240
Its half = tan. of ½ ZP⊙ = 10° : 22		9,262120

Therefore the angle ZP⊙ = 20 : 44.

Again.

Again.

If it was required to find the angle NS⊙ in the fupplemental triangle to PZ⊙, in which is given SN 42° : 30′, S⊙ 113° : 28′, and ⊙N 150° : 50′; it may be done by either of the three methods.

Thus by the firft method.

The arch ⊙N	= 150° : 50′	SN =	113° : 28′
S⊙—SN	= 70 : 58	SN =	42 : 30
Sum	= 221° : 48′	its half =	110° : 54′
Rem.	= 79 : 52	its half =	39° : 56

Sine of S⊙ =	113° : 28	-	co.0,037493
Sine of SN =	42 : 30	-	co.0,170317
Sine ½ fum =	110 : 54	-	9,970442
Sine ½ rem. =	39 : 56	-	9,807464
Sum of the fines		-	19,985716
Its half = fine of ½ NS⊙ =	79° : 38		999.858

Therefore the angle NS⊙ = 159 : 16.

By the second method. art. 268.

The arch s⊙ =	113° : 28′	its sine	*co*.0,037493
The arch sN =	42 : 30	its sine	*co*.0,170317
The base ⊙N =	150 : 50	- - -	0,000000
The sum =	306 : 48	- -	0,000000
Th. H =	153 : 24	its sine	9,651044
Th. H—⊙N =	2 : 34	its sine	8,651102
Sum of the sines	- - - - -		18,509956
Its half = cos. of ½ NS⊙ =	79° : 38′		9,254978
Therefore the angle NS⊙ =	159 : 16		

By the third method. art. 269.

Now ⊙N =	150° : 50	the opposite side.
sN =	42 : 30	a containing side.
s⊙ =	113 : 28	a containing side.
The sum =	306 : 48	

Th. H =	153 : 24	its sine -	*co*.0,348956
H—⊙N =	2 : 34	its sine -	*co*.1,348898
H—sN =	110 : 54	its sine -	9,970442
H—s⊙ =	39 : 56	its sine -	9,807464
Sum of the sines	- - - - -		21,475760
Its half = tang. of ½ NS⊙ =	79° : 38′		10,737880
Therefore the angle NS⊙ =	159 : 16		

318. Ex-

318. *Example to case 12, in oblique spherical triangles; plate 1. fig. 1.*

In the spherical triangle ZP☉, are given the hour angle ZP☉ $20^\circ : 45'$, the azimuth angle ☉ZP $138^\circ : 13'$, and the angle of position Z☉P $29^\circ : 24$; to find the co-latitude ZP.

First method. art. 270.

Put $z = (180^\circ : 00' - 138^\circ : 13' =)\ 41^\circ : 47'$, the supplement of the angle ☉ZP.

The angle Z☉P	$= 29^\circ : 24'$ the opposite angle.			
And Z — ZP☉	$= 21 : 02$ the difference.			
Their sum -	$= 50 : 26$	half $= 25^\circ : 13$		
And their diff.	$= 8 : 22$	half $= 4 : 11$		
The sine of Z	$= 41 : 17$	-	*co.*	0,176320
The sine of P	$= 20 : 45$	-	*co.*	0,450640
The sine of $\frac{1}{2}$ sum	$= 25 : 13$	-	-	9,629453
The sine of $\frac{1}{2}$ diff.	$= 4 : 11$	-	-	8,863014
Sum of the four sines		-	-	19,119427
Half = sine of $\frac{1}{2}$ ZP	$= 21^\circ\ 16'$	-		9,559713
Therefore - -	ZP $= 42 : 32$.			

 Second

Second method. art. 271.

Put z = supplement of ⊙zp = 41° : 47′.

Now the angle z	= 41° : 47′	its sine *co.*	0,176320
The angle zp⊙	= 20 : 45	its sine *co.*	0,450640
The angle z⊙p	= 29 : 24		
Their sum	= 91 : 56		
Therefore h	= 45 : 58	its sine	9,856690
And h—z ⊙p	= 16 : 34	its sine	9,455044
Sum of the four sines			19,938694
Half is cos. of ½ zp	= 21° : 16		9,169347
Therefore zp	= 42 : 32.		

Third method. art. 272.

Here. z⊙p = 29° : 24′ the opposite angle.
zp⊙ = 20 : 45 a containing angle,
And z = 41 : 47 a containing angle.

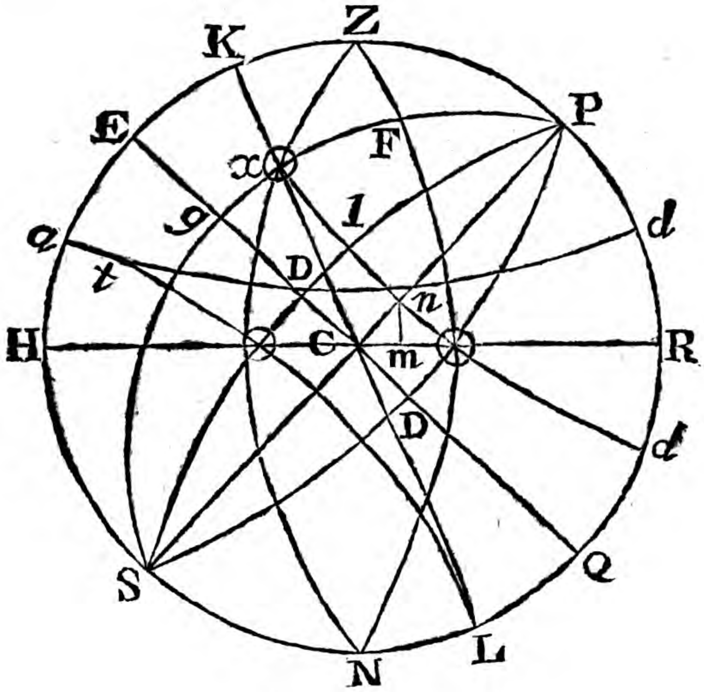
Their sum = 91 : 56

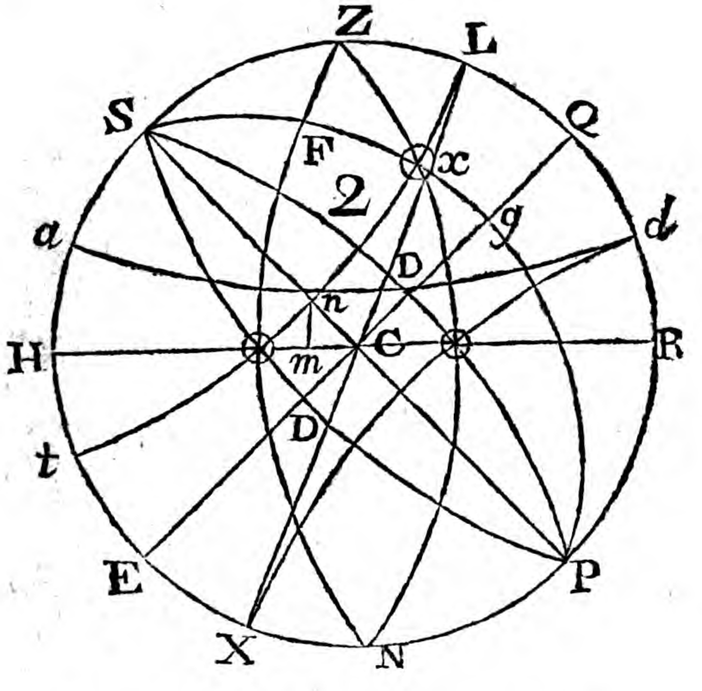
Th. h	= 45 : 58	its sine	*co.* 0,143310
h—z⊙p	= 16 : 34	its sine	*co.* 0,544956
h—zp⊙	= 25 : 13	its sine	9,629453
h—z	= 4 : 11	its sine	8,863014
The sum of the sines			19,180733
Half = tangent of ½ zp	= 21° : 16′		9,590366

Therefore the co-lat. zp = 42 : 32.

BOOK

Plate II.





To be placed opposite Page 182.

BOOK III.

Of NAVIGATION.

CHAP. I.

Contains the first principles, and theorems in plain sailing.

319. *NAvigation is the art of finding the place of a ship at sea, with her course, and distance from thence to any other place.*

320. *The terrestrial sphere, is the globe, or planet that we inhabit, and call the earth.*

321. *The axis of the earth is that diameter about which it revolves, or turns round towards the east.*

322. *The rotation of the earth towards the east, must cause all objects in the heavens to have an apparent motion towards the west.*

323. *The equator is a great circle of the terrestrial sphere, equally distant from each extremity of the axis.*

324. *The extreme points of the axis are poles of the equator* (157), *and the axis is perpendicular to its plain* - - - - - - - - - - - 170

325. *The poles of the equator, are also the poles of the terrestrial sphere.*

326. *The equator is generally (by seamen) called the line, and the two equal parts into which it divides the earth, are called the northern and southern hemispheres.*

327. *Meridians are great circles passing thro' the poles, and consequently they cut the equator at right angles* - - - - - 172

328. *Parallels of latitude are lesser circles parallel to the equator.*

329. *The terrestial and celestial spheres, have a common center, the equator of the first corresponds with the equinoctial of the second, and parallels of latitude on the earth correspond with parallels of declination in the heavens.*

330. *Every point upon the surface of the earth is supposed to have a meridian and parallel of latitude.*

331. *The latitude of a place, is an arch of the meridian contained between it and the equator; or it is the distance of a place from the equator, estimated by degrees of the meridian.*

332. *It is necessary for the purposes of geography and navigation, to call the meridian of some remarkable place the first, and to estimate the longitudes of all other places from that meridian.*

333. *The longitude of a place is an arch of the equator, contained between the first meridian and the meridian of that place.*

334. *The longitudes of places are denominated east or west, as their meridians fall to the east or west side of the first meridian.*

335. *The greatest latitude is 90 degrees, and the greatest longitude is 180 degrees; the first being a quadrant, and the second a semicircle.*

336. *The difference of latitude between two places, is an arch of any meridian intercepted between their parallels.*

337. *The difference of longitude between two places,*

places, is an arch of the equator intercepted between their meridians.

338. *Since every meridian returns to the center of the ſun (by the rotation of the earth about its axis) in* 24 *hours, the difference of longitude between all places may be expreſſed by intervals of time; for each hour will correſpond with* 15 *degrees of the equator, each minute of time with* 15 *minutes of a degree, each ſecond of time with* 15 *ſeconds of a degree, &c.*

339. *A rhumb line is a curve upon the ſurface of the ſphere, cutting all the meridians in equal angles.*

340. *The mariners compaſs is a repreſentation of the horizon, which is the circle, that ſeparates the viſible part of the heavens from the inviſible.*

341. *If a ſhip be ſteered due north or ſouth her track or courſe is upon ſome meridian, and her diſtance ſailed, and difference of latitude is the ſame* - - - - - - - 336

342. *If a ſhip be ſteered towards any point of the horizon beſide north or ſouth, her courſe will be a rhumb line.* - - - - 339

343. *If a ſhip is ſteered due eaſt or weſt, her courſe will be upon ſome parallel of latitude, or*

upon

upon the equator; for it being at right angles to every meridian, there can be no deviation towards either north or ſouth, or no change of latitude.

344 *Hence parallels of latitude cut the meridians at right angles.*

345. *The angle of the courſe, is the angle in which the track (or rhumb) cuts the meridians.*

346. *The diſtance of two places, is the length of the rhumb line contained between them; and is eſtimated by geographical miles, or minutes of a great circle.*

347. *The bearing of places upon the ſame meridian is north and ſouth, upon the ſame parallel is eaſt and weſt; but of all places otherwiſe ſituated, it is the angle of the courſe from the one to the other.*

348. *In ſailing from the equator the latitude increaſes, but in ſailing towards the equator the latitude decreaſes.*

Theorem

Theorem 1.

349. *In sailing upon a rhumb line, the differences of latitude will be proportional to the distances; plate 3. fig. 1.*

Suppose AEQI to represent a portion of the earths superficies, EQ the equator, EA, BC, DF, &c. meridians, LMNO, &c. a rhumb line, RM, SN, TO, &c. parallels of latitude, and the elementary triangles LRM, MSN, NTO, &c. so little, as to be deemed plain triangles.

Now the angles RLM, SMN, TNO, &c. are equal 339
And the angles LRM, MSN, NTO, &c. are equal 344
Th. the triangles LRM, MSN, &c. are equiangular 87 p
Th. LM : LR :: MN : MS :: NO : NT, &c. 193 p
Th. LM : LR :: LO : LR+MS+NT, &c. 179 p
Or LM : LR :: LI : LA - - - 179 p

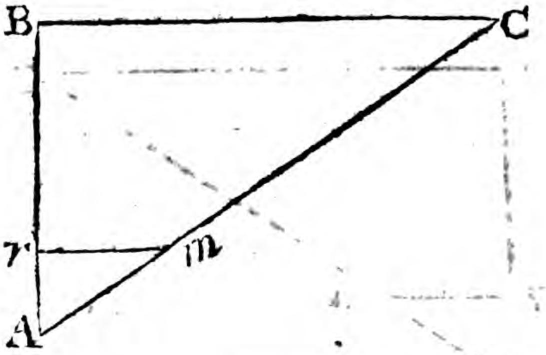
But LM, LI, are distances, and LR, LA, are the corresponding differences of latitude 346, 336

Therefore the differences of latitude, are proportional to the distances.

Theorem

Theorem 2.

350. *Strait lines equal in length to the distance and difference of latitude, each to each; will constitute the hypothenuse and base of a right angled plain triangle.*



Suppose the angle BAC equal to the angle of the course RLM, the strait lines AC, AB, equal to LI, LA the distance and difference of latitude each to each; A*m*, A*r*, equal to LM, LR each to each, in fig. 1. plate 3. and join BC, *rm*.

Now LM : LR :: LI : LA - - -	349
Th. A*m* : A*r* :: AC : AB - -	56 p
Th. the angle ABC = A*rm* - -	198 p
But A*rm* = LRM = a right angle - -	344
Th. ABC is a right angle, - -	54 p

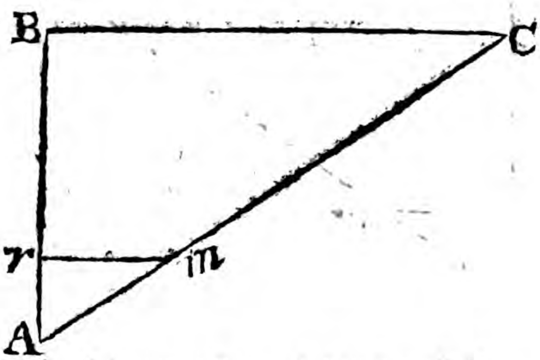
Remark.

351. *The strait line* (BC) *forming a right angled plain triangle, with the distance and difference of latitude, is called departure.*

Theorem

Theorem 3.

352. *The departure corresponding to any course and distance, is equal to the sum of all the intermediate departures.*

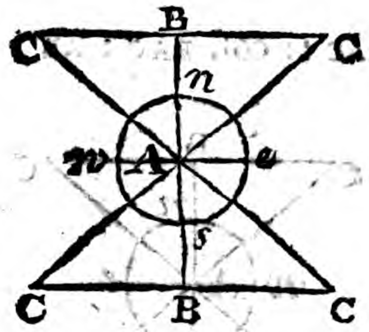


The little triangle LRM, MSN, NTO, &c. in fig. 1, plate 3, are equiangular; as was shewn in article - - - - - - 349
Th. LM : RM :: MN : SN :: NO : TO, &c. 193 p
Th. LM : RM :: LI : RM+SN+TO, &c. 179 p
But *Am* = LM, and *rm* = RM; by the construction of article 350, and - - - - 62 p
Th. *Am* : *rm* :: LI : RM+SN+TO, &c. 56 p
Also *Am* : *rm* :: AC : BC - - - - - - 193 p
Th. AC : BC :: LI : RM+SN+TO, &c. 173 p
Again, AC = LI - - - - - - - con.
Th. BC = RM+SN+TO, &c. - - - 167 p

Theorem

Theorem 4.

353. *As the distance run, to radius; so is the difference of latitude, to the cosine of the course.*



Here AC represents the distance, AB the difference of latitude, the angle BAC the course, BC the departure, and ABC a right angle 351
Th. AC : radius :: AB : cosine BAC 53

Theorem 5.

354. *As the cosine of the course, to the difference of latitude; so is radius, to the distance.*
For cosine BAC : AB :: radius : AC . 53

Theorem 6.

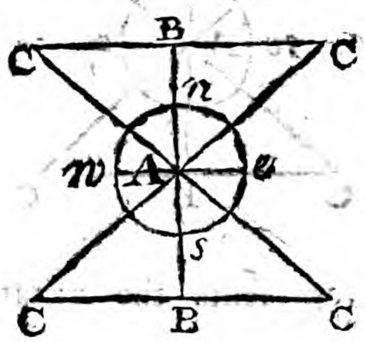
355. *As the sine of the course, to the departure; so is radius, to the distance.*
For sine BAC : BC :: radius : AC . . 53

Theorem

Theorem 7.

356. *As radius, to the distance; so is the cosine of the course, to the difference of latitude.*

For radius : AC :: cos. BAC : AB - - 53



Theorem 8.

357. *As radius, to the distance; so is the sine of the course, to the departure.*

For radius : AC :: sine of BAC : BC - 53

Theorem 9.

358. *As the difference of latitude, to the departure; so is radius, to the tangent of the course.*

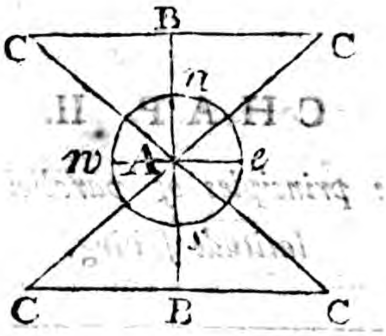
For AB : BC :: radius : tan. BAC - - 54

Theorem

Theorem 10.

359. *As radius, to the tangent of the course; so is the difference of latitude to the departure.*

For radius, tan. BAC :: AB : BC • 54



Theorem 11.

360 *In sailing due east or west, the departure is equal to the distance run.*

For when AB becomes infinitely little, or vanishes; AC will coincide with BC.

Remark.

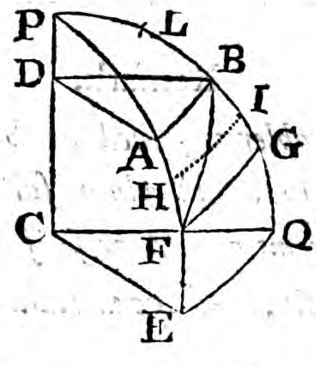
361. *All problems solved by the proceeding theorems are said to be in plain sailing; because the very same theorems would arise if the surface of the earth was a plain, and the meridians parallels.*

CHAP. II.

Contains the principles of parallel and middle latitude ſailing.

Theorem I.

362. *In ſailing upon a parallel, the coſine of the latitude, is to radius; as the diſtance run, to the difference of longitude.*



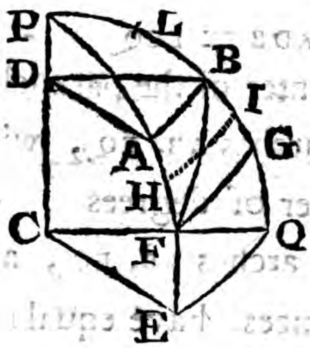
Suppoſe C the center of the ſphere, P the north pole, EQ an arch of the equator, PAE, PBQ meridians, and AB the diſtance run, or the

the arch described; draw the semidiameter CP cutting the plain of the parallel in D, and join DA, DB, CE, CQ.

Now the intersections DA, DB, are parallel to CE, CQ, each to each - - 258 p
Th. the angle ADB = ECQ - - 255 p
And D is the center of the parallel AB - 171
Therefore the arches AB, EQ, will each contain the same number of degrees - - 23
Therefore, the arches AB, EQ, and their respective circumferences, have equal ratios 159 p
Th. AB : paral. :: EQ : equat. - 160 p
Th. AB : EQ :: paral. : equat. - - 175 p
Also, DB : CQ :: paral. : equat. - - 225 p
Th. DB : CQ :: AB : EQ - - 173 p
But DB is the sine of PB, or cosine of the latitude QB; CQ is the radius; and EQ is the difference of longitude.
Th cos. lat. : radius :: dist. : dif. long.

Theorem 2.

363. *As radius, to the cosine of latitude; so is the difference of longitude, to the length of the corresponding arch of the parallel.*



For DB : CQ :: AB : EQ - - - - 361
Th. CQ : DB :: EQ : AB - - - 128

Theorem 3.

364. *The corresponding arches of parallels, are proportional to the cosines of their respective latitudes.*

For CQ : DB :: EQ : AB - - 362
Th. CQ : EQ :: DB : AB - - 175 P
Or, CQ : EQ :: cos. QB : AB
Th. CQ : EQ :: cos. QG : FG for the same reason
Th. cos. QB : AB :: cos. QG : FG - 173 P
Th. cos. QB : cos. QG : AB : FG - 175 P

Theorem

Theorem 4.

365. *As the cosine of middle latitude, to radius; so is the departure, to the difference of longitude, nearly.*

The last construction remaining, let FB represent the rhumb line, or course between F and B; make BL equal to QG, bisect the arch BG in I, and describe the parallel HI.

In sailing from F to B, or from B to F, the departure is greater than AB, and less than FG (352); and will therefore be nearly equal to the arch HI, which bisects BG.

And, cos. QI : radius :: HI : EQ - - - 362

But $QI = (IL = \frac{1}{2}\overline{QB+BL} =) \frac{1}{2}\overline{QB+QG}$, by construction, which is called the middle latitude; and EQ is the difference of longitude 337

Therefore, as the cosine of middle latitude, to radius; so is the departure, to the difference longitude, nearly.

Theorem 5.

366. *As the difference of latitude, to the difference of longitude; so is the cosine of middle latitude, to the tangent of the course, nearly.*

For, dif. of lat. : radius :: dep. : tan. course 358
And, cos. mid. lat. : radius :: dep. : dif. longit. 365
Th. dif. of lat.×tan. course = radius×dep. 185 p
And cos. mid. lat.×dif. long. = radius×dep. 185 p
Th. dif. of lat.×tan. course = cos. mid. lat.× dif. long. - - - - - - - - - - - 48 p
Th. dif. lat. : dif. long. :: cos. mid. lat. : tan. course - - - - - - - - 187 p

Corollary.

367. *As the cosine of middle latitude, to the tangent of the course; so is the difference of latitude, to the difference of longitude, nearly.*

For dif. lat. : dif. long. :: cos. mid. lat. : tan. co. 366
Th. cos. mid. lat. : tan. course :: dif. lat. : dif. long.

Remark.

When the latitudes are of one kind, half their sum is the middle latitude; but if they are of a different kind, the method fails.

CHAP.

CHAP. III.

Of MERCATOR'S SAILING.

368. *Suppose eqia Mercator's chart, constructed to represent the spherical superficies* EQIA *(plate 3); in which eq is the equator; ea, bc, df, &c. are meridians; lmni is a rhumb line; and rm, sn, to, ai, are parallels of latitude; each strait line in the chart representing its corresponding arch upon the globe.*

The meridians in this projection being made parallels, *rm* is equal to *eb* (91 p); therefore *rm* is the difference of longitude corresponding with RM - - - - - 337
Therefore, cos. ER : radius :: RM : *rm* . 362

Now in order to have the little triangle *lrm* in the chart, equiangular to its original LRM upon the globe, the difference of latitude LR, muſt be increaſed in the ſame ratio with the departure RM (198 p); therefore coſ. ER : radius :: LR : *lr*; but by article 49, as coſ. ER : radius :: radius : ſec. ER; therefore, radius : ſec. ER :: LR : *lr* - - 173 p

Again. If the radius and difference of latitude LR, be each ſuppoſed one minute of the meridian, the laſt proportion will become 1 : ſec. ER :: 1 : *lr*; and conſequently the increaſed minute *lr*, is equal to the ſecant of the latitude, in that caſe - - - - 167 p

Remarks.

369. *The meridian line of Mercator's chart, is a ſcale of ſecants to every minute of latitude contained in it. Thus ſuppoſing the difference of latitude* LA *to be four minutes, then the enlarged meridian la is equal to lr + ms + nt + oh; which are ſecants of the latitudes el, hm, dn, go, reſpectively* - - - - - 368

370. *Mr. Wright made the firſt table of meridional parts, by continually adding the ſecants of each minute of the quadrant, namely,* 1′, 2′, 3′, 4′, &c.

371. *As*

371. *As the cosine of middle latitude, to radius; so is a greater difference of latitude, to the corresponding increased difference of latitude, nearly.*

372. *The increased, or enlarged difference of latitude, is generally called meridional difference of latitude.*

373. *A plain triangle formed by the distance, difference of latitude, and departure; will be similar to a triangle formed upon the chart by the enlarged distance, meridional difference of latitude, and difference of longitude; for each is equiangular to the elementary triangle* LMR. 350 *and* 368.

374. *In Mercator's chart, the rhumbs are all strait lines, the latitudes, longitudes, and bearings of all places are truly represented; but the distances of places are distorted.*

375. *The meridional difference of latitude between places is found by tables of meridional parts, or logarithmic tangents. The curious reader may see the latter method elegantly demonstrated in Robertson's navigation.*

Theorem

Theorem 1.

376. *As the meridional difference of latitude, to the difference of longitude; so is radius, to the tangent of the course; plate* 3.

Suppose the places are at L and I upon the globe, and their projections at *l* and *i* upon the chart; then will *ali* represent the angle of the course, *la* the meridional difference of latitude ($= lr + ms + nt$, &c.), *ai* the difference of longitude, and *lai* is a right angle - - - 368

Th. *la* : *ai* :: radius : tan. *ali* - - - 54

Theorem 2.

377. *As radius, to the tangent of the course; so is the meridional difference of latitude, to the difference of longitude; plate* 3.

For *la* : *ai* :: radius : tan. *ali* - - - 376

Th. radius : tan. *ali* :: *la* : *ai* - - 128

Theorem

Theorem 3.

378. *As the proper difference of latitude, to the departure; so is the meridional difference of latitude; to the difference of longitude.*

For, radius : tan. course :: dif. lat. : dep. 359
And, radius : tan. course :: mer. dif. lat. : dif. long. - - - - - - - 377
Th. dif. lat. : dep. :: mer. dif. lat. : dif. long. 173 p

379. Remark.

Since cos. mid. lat. : radius :: dif. lat. : la. (nearly) 371
Th. radius : la. :: cos. mid. lat. : dif. lat. 128
Ag. radius : tan. ali :: la : dif. long. - - 377
Th. radius : la. :: tan. ali. : dif. long. 175p
Th. cos. mid. lat. : dif. lat. :: tan. ali : dif. long. 173 p
Th. cos. mid. lat. : tan. ali. :: dif. lat. : dif. long. 175p
Or, the cosine of middle latitude, is to the tangent of the course; as the difference of latitude, to the difference of longitude, as before investigated - 367

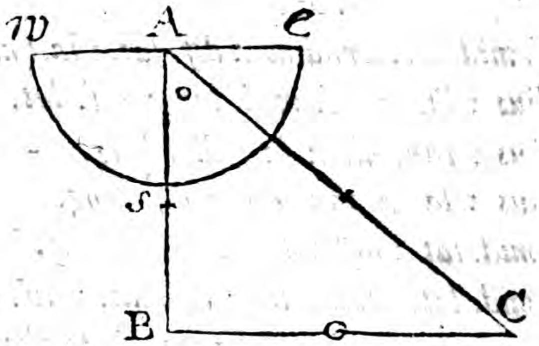
CHAP.

CHAP. IV.

Contains Questions in Navigation.

Quest. 1.

380. *A ship sailed 68 miles between south and east, and made 43 miles difference of latitude, her course and departure is required?*

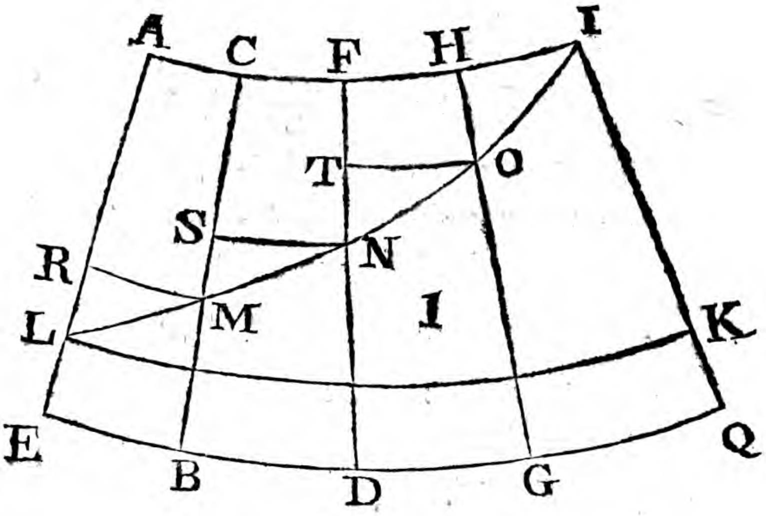


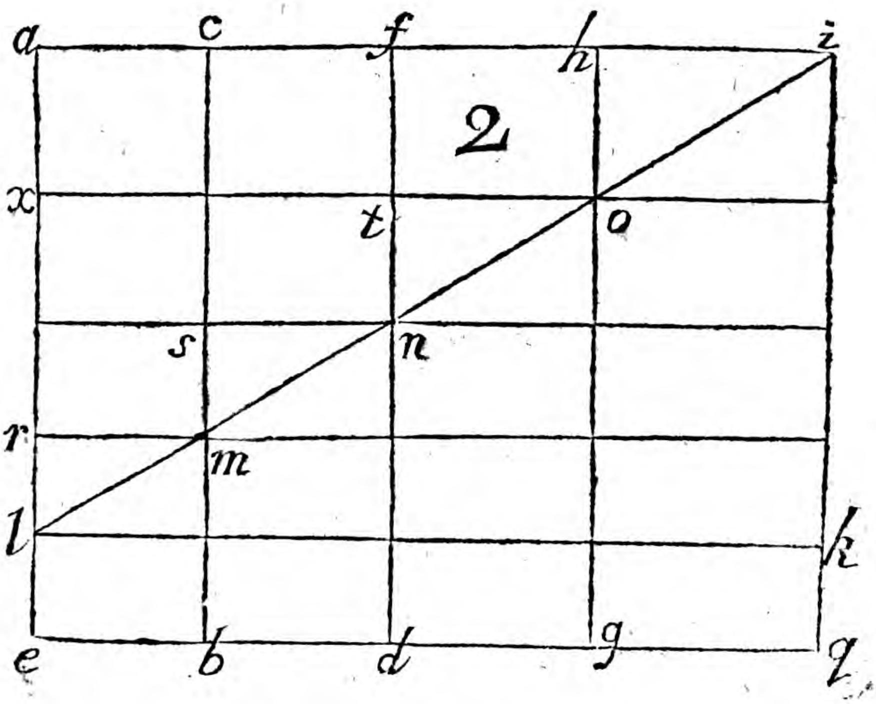
1.	For the course BAC		art. 353.
As the distance AC		= 68	1,832509
To radius - - - - - - - -			10,000000
So is the diff. of lat. AB		= 43	1,633468
To the cos. course BAC		= 5° : 47′	9,800959

2.	For the departure BC		art. 357.
As radius - - - - - - -			10,000000
To the distance AC		= 68	1,832509
So is sine of course BAC		= 50° : 47′	9,889167
To the departure BC		= 52,6	1,721676

Quest.

Plate III.

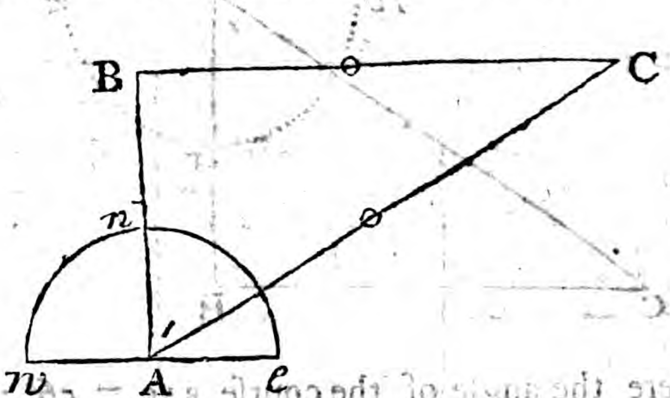




To be placed opposite Page 204.

Queſt. 2.

381. *If a ſhip be ſteered* N 60° E, *'till her difference of latitude is one degree, or 60 miles; what is her diſtance ſailed, and departure made?*



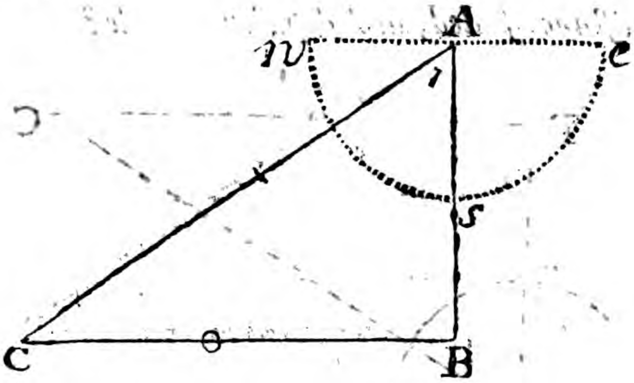
1.	For the diſtance AC	art. 354.
As the coſ. courſe BAC = 60° : 00′		9,698970
To the diff. of lat. AB = 60		1,778151
So is radius - - - - - - - -		10,000000
To the diſtance AC = 120		2,079181

2.	For the departure BC	art. 359.
As radius - - - - - - -		10,000000
To the tan. courſe BAC = 60° : 00′		10,238561
So is the dif. of lat. AB = 60		1,778151
To the departure BC = 103,9		2,016712

Queſt.

Queſt. 8.

382. *If a ſhip ſails* s. w. by w. 360 *miles, what difference of latitude and departure will ſhe make?*



Here the angle of the courſe BAC = 56° : 15′ being five points from the meridian.

1. For the difference of latitude AB		art. 356.
As radius - - - -		10,000000
To the diſtance AC	= 360	2,556303
So is the coſ. courſe BAC	= 56° : 15′	9,744739
To the dif. of lat. AB	= 200	2,301042

2. For the departure BC		art. 357.
As radius - - - -		10,000000
To the diſtance AC	= 360	2,556303
So is ſine courſe BAC	= 56° : 15′	9,919846
To the departure BC	= 299,3	2,476149

Queſt.

Queſt. 4.

383. *A ſhip in chace ſailed* W 8 *miles,* E N E 14 *miles,* W N W ½ W 19 *miles, and* N 5 *miles; what courſe and diſtance did ſhe make good, upon the whole traverſe?*

N°	Courſe	Diſt.	Diff. Lat.		Departure.	
C	Deg.	Miles	N	S	E	W
1	90 : 00	8	—	—	—	8,0
2	67 : 30	14	5,3	—	12,9	—
3	73 : 7	19	5,5	—	—	18,2
4	0 : 0	5	5,0	—	—	—
Sums - - - -			15,8	0	12,9	26,2

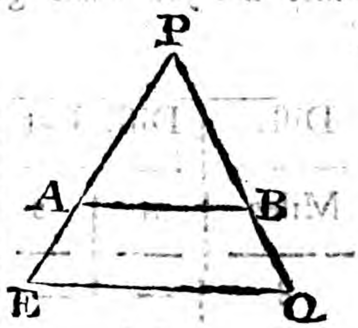
Diff. of lat. = 15,8 depart. = 13,3

1. For the courſe		art. 358.
As the diff. of lat.	= 15,8	1,198657
To the departure	= 13,3	1,123852
So is radius - - - - - -		10,000000
So is tan. courſe	= 40° : 05′	9,925195
2. For the diſtance		art. 355.
As ſine of the courſe	= 40° : 05′	9,808119
To the departure	= 13,3	1,123852
So is radius - -	= - - -	10,000000
To the diſtance	= 20,6	1,315033

Queſt.

Queft. 5.

384. *If a ſhip runs 200 miles in the parallel of 49° : 30′, what difference of longitude will ſhe make?*



For the difference of longitude EQ art. 362.

As the coſ. of lat. EA	= 49° : 30′	9,812544
To radius - - - - - - -		10,000000
So is the diſtance AB	= 200	2,301029
To the dif. of lon. EQ	= 308	2,488485

Queſt. 6.

385. *What is the diſtance between two places in the latitude of 60° : 00′? whoſe difference of longitude is 1°, or 60 miles.*

For the diſtance AB. art. 363.

As radius - - - -		10,000000
To the coſine of lat. EA	= 60° : 00′	9,698970
So is the diff. of long. EQ	= 60	1,778151
To the diſtance AB	= 30	1,477121

Queſt.

Queſt. 7.

386. *Suppoſing a ſhip from the parallel of 43° : 33′* N, *to ſail* N 56° : 15′ E, *'till ſhe arrives in latitude* 49° : 30′ N; *what difference of longitude will ſhe make?*

1.	For the middle latitude		art. 367.
The latitude in	= 49° : 30′		= 49° : 30′
The latitude from	= 43 : 33		= 43 : 33
The ſum	= 93 : 03,	dif.=	5 : 57
The middle lat.	= 46 : 31.	dif.=	357

2. For diff. of long.		art. 367.
As the coſine of mid. lat.	= 46° : 31′	*co.*0,162320
To the tan. of the courſe	= 56 : 15	10,175108
So is the dif. of lat.	= 357	2,552890
To the dif. of long.	= 777	2,890318

P 2. By

By Mercator.		art. 375.
The latitude in	= 49° : 30′ *m. p.*	= 3429
The latitude from	= 43 : 33 *m. p.*	= 2910
The meridional dif. of lat.	-	= 519

For the dif. of longitude		art. 377.
As radius	- - - - - -	10,000000
To the tan. courſe	= 56° : 15′	10,175108
So is the mer. dif. lat.	= 519	2,715167
To the dif. of long.	= 777	2,890275

Queſt.

Quest. 8.

387. *Supposing a ship from latitude* 59° : 30′ s, *to sail between* N *and* W, *'till she arrives in latitude* 54° : 30′ s, *with* 360 *miles departure; what difference of longitude will she make?*

By the middle latitude.

The latitude departed from	= 59° : 30′	
The latitude arrived in	= 54 : 30	
The sum - - -	= 114 : 00	
Half is the middle latitude	= 57 : 00	367

For the difference of longitude. art. 365.

As the cosine of middle lat.	= 57° : 00′	9,736109
To radius - - - - - - - -		10,000000
So is the departure	= 360	2,556303
To the difference of long.	= 661	2,820194

2. By Mercator. art. 375.

The latitude from	$= 59° : 30'$ *m. p.*	$= 4468$
The latitude in	$= 54 : 30$ *m. p.*	$= 3916$
Differences of latitude	$= 5 : 00$ and,	552

For the difference of longitude. art. 378.

As the proper dif. lat. $5° : 00'$	$= 300$	*co.*7,522879
To the departure - -	$= 360$	2,556303
So is the merid. dif. lat.	$= 552$	2,741939
To the dif. of longitude	$= 662$	2,821121

Queft.

Queft. 9.

388. *What is the courfe and diftance from London to Conftantinople?*

Lond. lat.	51° : 32′ M. P. =	3620	long. 00° : 00′
Conf. lat.	41 : 00 M. P. =	2702	long. 28 : 58, E
Differences	10 : 32	918	28 : 58
In miles	632	918	1738

Sum of lat. 92 : 32 middle latitude = 46 : 16

For the courfe by mid. lat. art. 366.

As the dif. of lat.	= 632	co. 7,199283
To the dif. of long.	= 1738	3,240049
So is cof. mid. lat.	= 46° : 16′	9,839668
To tan. of the courfe	= 62 : 15	10,279000

For the courfe by Mercator art. 376.

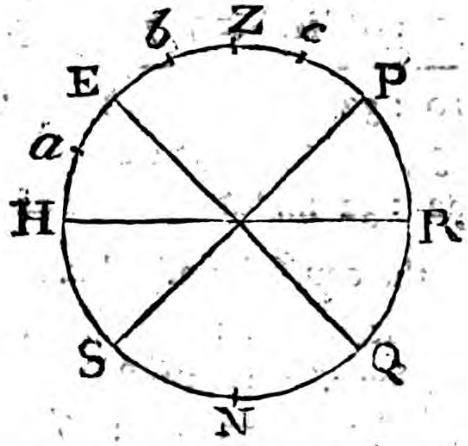
As the mer. dif. lat.	= 918	co. 6,962842
To the dif. of long.	= 1738	3,240049
So is radius	- - -	10,000000
To tan. of the courfe	= 62° : 10′	10,277207

For the diftance art. 354.

As cof. of the courfe	= 62° : 10′	9,669225
To the dif. of lat.	= 632	2,800717
So is radius	- - -	10,000000
To the diftance	= 1353	3,131492

Problem.

389. *The meridian zenith distance, and declination of any object in the heavens being given; it is required to find the latitude of the place.*



Here the circle HZRN represents the meridian of the place, EQ the equinoctial, Z the zenith, EZ the latitude, and *a*, *b*, *c*, objects upon the meridian.

Case 1.

If the equinoctial is between the zenith and the object, the difference between the zenith distance and declination is the latitude, of a contrary name to the declination; for $Za - Ea = EZ$, the latitude.

Case 2.

If the object is between the zenith and equinoctial, the sum of the zenith distance and declination is the latitude, of the same name with the declination; for $Zb + Eb = EZ$.

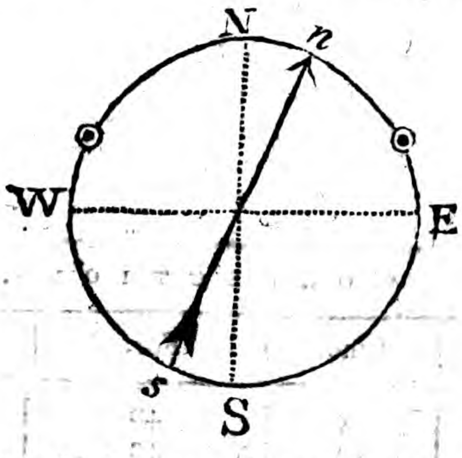
Case 3.

If the zenith is between the equinoctial and the object, the difference between the declination and zenith distance is the latitude, of the same name with the declination; for, $Ec - Zc = EZ$.

Problem

Problem.

390. *To find the variation (or error) of the campass.*



Here the circle represents the horizon, N its north points, and ⊙ the sun at his rising or setting:

Rule.

1. Observe what number of degrees the sun rises or sets, from the north point of the compass *n*, represented by the arch of the horizon *n*⊙; which is called the magnetical amplitude.

2. Find by calculation, the suns true amplitude from the north N⊙.

3. The difference of the degrees thus found is the variation N*n*.

Example.

Suppose at sun rising, the true amplitude N⊙ = 76° : 30′, and the magnetical *n*⊙ = 64° : 30′

Operation.

From N⊙ - - - -	=	76° : 30,
Substract *n*⊙ - - -	=	64 : 30
And the variation N*n* - -	=	12 : 00 East

F I N I S.

CORRECTIONS.

Page.	Line.	For,	Read.
9	5	46	47
12	13	42	43
13	11	40	41
16	8	tangent	tangents
28	9	CFL	CFH

A TABLE OF LOGARITHMS,

From ONE to TEN THOUSAND.

N.	Log.	N.	Log.	N.	Log.
1	0.000000	34	1.531479	67	1.826075
2	0.301030	35	1.544068	68	1.832509
3	0.477121	36	1.556303	69	1.838849
4	0.602060	37	1.568202	70	1.845098
5	0.698970	38	1.579783	71	1.851258
6	0.778151	39	1.591064	72	1.857332
7	0.845098	40	1.602060	73	1.863323
8	0.903090	41	1.612784	74	1.869232
9	0.954242	42	1.623249	75	1.875061
10	1.000000	43	1.633468	76	1.880813
11	1.041393	44	1.643452	77	1.886491
12	1.079181	45	1.653212	78	1.892094
13	1.113943	46	1.662758	79	1.897627
14	1.146128	47	1.672098	80	1.903090
15	1.176091	48	1.681241	81	1.908485
16	1.204120	49	1.690196	82	1.913814
17	1.230449	50	1.698970	83	1.919078
18	1.255272	51	1.707570	84	1.924279
19	1.278753	52	1.716003	85	1.929419
20	1.301030	53	1.724276	86	1.934498
21	1.322219	54	1.732394	87	1.939519
22	1.342422	55	1.740362	88	1.944482
23	1.361728	56	1.748188	89	1.949390
24	1.380211	57	1.755875	90	1.954242
25	1.397940	58	2.763428	91	1.959041
26	1.414973	59	1.770852	92	1.963788
27	1.431364	60	1.778151	93	1.968483
28	1.447158	61	1.785330	94	1.973128
29	1.462398	62	1.792391	95	1.977723
30	1.477121	63	1.799340	96	1 982271
31	1.491361	64	1.806180	97	1.986772
32	1.505150	65	1.812913	98	1.991226
33	1.518514	66	1.819544	99	1.995635

No.	0	1	2	3	4
100	000000	000434	000868	001301	001734
101	004321	004751	005181	005609	006038
102	008600	009026	009451	009876	010299
103	012837	013259	013679	014100	014521
104	017033	017451	017868	018284	018700
105	021189	021603	022016	022428	022841
106	025306	025715	026125	026533	026942
107	029384	029789	030195	030599	031004
108	033424	033826	034227	034628	035029
109	037426	037825	038223	038620	039017
110	041393	041787	042182	042576	042969
111	045323	045714	046105	046495	046885
112	049218	049606	049993	050379	050766
113	053078	053463	053846	054229	054613
114	056905	057286	057666	058046	058426
115	060698	061075	061452	061829	062206
116	064458	064832	065206	065579	065953
117	068186	068557	068928	069298	069668
118	071882	072249	072617	072985	073352
119	075547	075912	076276	076640	077004
120	079181	079543	079904	080266	080626
121	082785	083144	083503	083861	084219
122	086359	086716	087071	087426	087781
123	089905	090258	090610	090963	091315
124	093422	093772	094122	094471	094820
125	096910	097257	097604	097951	098298
126	100371	100715	101059	101403	101747
127	103804	104146	104487	104828	105169
128	107210	107549	107888	108227	108565
129	110589	110926	111263	111599	111934

5	6	7	8	9	D
002166	002598	003029	003461	003891	432
006466	006894	007321	007748	008174	428
010724	011147	011570	011993	012415	424
014940	015359	015779	016197	016616	419
019116	019532	019947	020361	020775	416
023252	023664	024075	024486	024896	412
027349	027757	028164	028571	028978	408
031408	031812	032216	032619	033021	404
035429	035829	036229	036629	037028	400
039414	039811	040207	040602	040998	496
043362	043755	044148	044539	044932	393
047275	047664	048053	048442	048830	389
051153	051538	051924	052309	052694	386
054996	055378	055700	056142	056524	382
058805	059185	059563	059942	060320	379
062582	062958	063333	063709	064083	376
066326	066699	067071	067443	067815	372
070038	070407	070776	071145	071514	369
073718	074085	074451	074816	075182	366
077368	077731	078094	078457	078819	363
080987	081347	081707	082067	082426	360
084576	084934	085291	085647	086004	357
088136	088490	088845	089198	089552	355
091667	092018	092369	092721	093071	351
095169	095518	095866	096215	096562	349
098644	098989	099335	099681	100026	316
102091	102434	102777	103119	103462	343
105510	105851	106191	106531	106871	340
108903	109241	109579	109916	110253	338
112269	112605	112940	113275	113609	335

N	0	1	2	3	4
130	113943	114277	114611	114944	115278
131	117271	117603	117934	118265	118595
132	120574	120903	121231	121559	121888
133	123852	124178	124504	124830	125156
134	127105	127429	127753	128076	128399
135	130334	130655	130977	131298	131619
136	133539	133858	134177	134496	134814
137	136721	137037	137354	137671	137987
138	139879	140194	140508	140822	141136
139	143015	143327	143639	143951	144263
140	146128	146438	146748	147058	147367
141	149219	149527	149835	150142	150449
142	152288	152594	152899	153205	153509
143	155336	155639	155943	156246	156549
144	158362	158664	158965	159266	159567
145	161368	161667	161967	162266	162564
146	164352	164650	164947	165244	165541
147	167317	167613	167908	168203	168497
148	170262	170555	170848	171141	171434
149	173186	173478	173769	174059	174351
150	176091	176381	176669	176959	177248
151	178977	179264	179552	179829	180126
152	181844	182129	182415	182699	182985
153	184691	184975	185259	185542	185825
154	187521	187803	188084	188316	188647
155	190332	190612	190892	191171	191451
156	193125	193403	193681	193959	194237
157	195899	196176	196453	196729	197005
158	198657	198932	199206	199481	199755
159	201397	201670	201943	202216	202488

5	6	7	8	9		D
115611	115943	116276	116608	116939		333
118926	119256	119586	119915	120245		330
122216	122544	122871	123198	123525		328
125481	125806	126131	126456	126781		325
128722	129045	129368	129689	130012		323
131939	132259	132579	132899	133219		321
135133	135451	135769	136086	136403		318
138303	138618	138934	139249	139564		315
141449	141763	142076	142389	142702		314
144574	144885	145196	145507	145818		311
147676	147985	148294	148603	148911		309
150756	151063	151369	151676	151982		307
153815	154119	154423	154728	155032		305
156852	157154	157457	157759	158061		303
159868	160168	160469	160769	161068		301
162863	163161	163459	163758	164055		299
165838	166134	166430	166726	167022		297
168792	169086	169380	169674	169968		295
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907	957607	957655	957703	957751	957799
908	958086	958134	958181	958229	958277
909	958564	958612	958659	958707	958755

5	6	7	8	9	D
944729	944779	944828	944877	944927	49
945222	945272	945321	945370	945419	49
945715	955764	945813	945862	945912	49
946207	946256	946305	946354	946403	49
946698	946747	946796	946845	946894	49
947189	947238	947287	947336	947385	49
947679	947728	947777	947826	947875	49
948168	948217	948266	948315	948364	49
948657	948706	948755	948804	948853	49
949146	949195	949244	949292	949341	49
949634	949683	949731	949780	949829	49
950121	950170	950219	950267	950316	49
950608	950657	950706	950754	950803	49
951095	951143	951192	951240	951289	49
951580	951629	951677	951726	951775	49
952066	952114	952163	952211	952259	48
952550	952599	952647	952696	952744	48
953034	953083	953131	953179	953228	48
953518	953566	953615	953663	953711	48
954001	954049	954098	954146	954194	48
954484	954532	954580	954628	954677	48
954966	955014	955062	955110	955158	48
955447	955495	955543	955592	955639	48
955928	955976	956024	956072	956120	48
956409	956457	956505	956553	956601	48
956888	956936	956984	957032	957080	48
957368	957416	947464	957512	957559	48
957847	957894	957942	957990	958038	48
958325	958373	958421	958468	958516	48
958803	958850	958898	958946	958994	48

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910	959041	959089	959137	959185	959232
911	959518	959566	959614	959661	959709
912	959995	960042	960090	960138	960185
913	960471	960518	960566	960613	960661
914	960946	960994	961041	961089	961136
915	961421	961469	961516	961563	961611
916	961895	961943	961990	962038	962085
917	962369	962417	962464	962511	962559
918	962842	962889	962937	962985	963032
919	963315	963363	963410	963457	963504
920	963788	963835	963882	963929	963977
921	964259	964307	964354	964401	964448
922	964731	964778	964825	964872	964919
923	965202	965249	965296	965343	965389
924	965672	965719	965766	965813	965859
925	966142	966189	966236	966283	966329
926	966611	966658	966705	966752	966799
927	967079	967127	967173	967220	967267
928	967548	967595	967642	967688	967735
929	968016	968062	968109	968156	968202
930	968483	968529	968576	968623	968669
931	968949	968996	969043	969089	969136
932	969416	969463	969509	969556	969602
933	969882	969928	969975	970021	970068
934	970347	970393	970439	970486	970533
935	970812	970858	970904	970951	970997
936	971276	971322	971369	971415	971461
937	971739	971786	971832	971879	971925
938	972203	972249	972295	972342	972388
939	972666	972712	972758	972804	972851

5	6	7	8	9	D
959279	959328	959375	959423	959471	48
959757	959804	959852	959899	959947	48
960233	960280	960328	960376	960423	48
960709	960756	960804	960851	960899	48
961184	961231	961279	961326	961374	47
961658	961706	961753	961801	961848	47
962132	962179	962227	962275	962322	47
962606	962653	962701	962748	962795	47
963079	963126	963174	963221	963268	47
963552	963599	963646	963693	963741	47
964024	964071	964118	964165	964212	47
964495	964542	964589	964637	964684	47
964966	965013	965061	965108	965155	47
965437	965484	965531	965578	965625	47
965907	965954	966001	966048	966095	47
966376	966423	966470	966517	966564	47
966845	966892	966939	966986	967033	47
967314	967361	967408	967454	967501	47
967782	967829	967875	967922	967969	47
968249	968296	968343	968389	968436	47
968716	968763	968809	968856	968903	47
969183	969229	969276	969323	969369	47
969649	969695	969742	969789	969835	47
970114	970161	970207	970254	970300	47
970579	970626	970672	970719	970765	46
971044	971090	971137	971183	971229	46
971508	971554	971601	971647	971693	46
971971	972018	972064	972110	972157	46
972434	972481	972527	972573	972619	46
972897	972943	972989	973035	973082	46

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940	973128	973174	973220	973266	973313
941	973589	973636	973682	973728	973774
942	974051	974097	974143	974189	974235
943	974512	974558	974604	974649	974696
944	954972	975018	975064	975109	975156
945	975432	975478	975524	975569	975616
946	975891	975937	975983	976029	976075
947	976349	976396	976442	976488	976533
948	976808	976854	976899	976946	976992
949	977266	977312	977358	977403	977449
950	977724	977769	977815	977861	977906
951	978181	978226	978272	78317	978363
952	978637	978683	978728	978774	978819
953	979093	979138	979184	979229	979275
954	979548	979594	979639	979685	979730
955	980003	980049	980094	980139	980185
956	980458	9805[illegible]3	980549	980594	980639
957	980912	9809[illegible]7	981003	981048	981093
958	981366	981411	981456	981501	981547
959	981819	981864	981909	981954	981999
960	982271	982316	982362	982407	982452
961	982723	982769	982814	982859	982904
962	983175	983220	983265	983310	983356
963	983626	983671	983716	983762	983807
964	984077	984122	984167	984212	984257
965	984527	984572	984617	984662	984707
966	984977	985022	985067	985112	985157
967	985426	985471	985516	985561	985606
968	985875	985920	985965	986009	986055
969	986324	986369	986413	986458	986503

5	6	7	8	9	D
973359	973405	973451	973497	973543	46
973820	973866	973913	973959	974005	46
974281	974327	974374	974419	974466	46
974742	974788	974834	974880	974926	46
975202	975248	975294	975339	975386	46
975662	975707	975753	975799	975845	46
976121	976167	976212	976258	976304	46
976579	976625	976671	976717	976763	46
977037	977083	977129	977175	977220	46
977495	977541	977586	977632	977678	46
977952	977998	978044	978089	978135	46
978409	978454	978500	978545	978591	46
978865	978911	978956	979002	979047	46
979321	979366	979412	979457	979503	46
979776	979821	979867	979912	979958	46
980231	980276	980322	980367	980412	45
980685	980730	980776	980821	980867	45
981139	981184	981229	981275	981320	45
981592	981637	981683	981728	981773	45
982045	982090	982135	982181	982226	45
982497	982543	982588	982033	982678	45
982949	982994	983039	983085	983129	45
983401	983446	983491	983536	983581	45
983852	983897	983942	983987	984032	45
984302	984347	984392	984437	984482	45
984752	984797	984842	984887	984932	45
985202	985247	985292	985337	985382	45
985651	985696	985741	985786	985830	45
986099	986144	986189	9 6234	986279	45
986548	986593	986637	986682	986727	45

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970	986772	986817	986861	986906	986951
971	987219	987264	987309	987353	987398
972	987666	987711	987756	987800	987845
973	988113	988157	988202	988247	988291
974	988559	988604	988648	988693	988737
975	989005	989049	989094	989138	989183
976	989449	989494	989539	989584	989628
977	989895	989939	989983	990028	990072
978	990339	990383	990428	690472	990516
979	990783	990827	990871	990916	990960
980	991226	991270	991315	991359	991403
981	991669	991713	991758	991802	991846
982	992111	992156	992199	992244	992288
983	992554	992598	992642	992686	992730
984	992995	993039	973083	993127	993172
985	993436	993480	993524	993568	993613
986	993877	993921	993965	994009	994053
987	994317	994361	994405	994449	994493
988	994756	994801	994845	994889	994933
989	995196	995240	995284	995328	995372
990	995635	995679	995723	995767	995811
991	996074	996117	996161	996205	996249
992	996512	996555	996599	996643	996687
993	996949	996993	997037	997080	997124
994	997386	997430	997474	997517	997561
995	997823	997867	997910	997954	997998
996	998259	998303	998347	998390	998434
997	998695	998739	998782	998826	998869
998	999130	999174	999218	999261	999305
999	999565	999609	999652	999696	999739

5	6	7	8	9	D
986996	987040	987085	987129	987175	45
987443	987488	987532	987577	987622	45
987889	987934	987979	988024	988068	45
988336	988381	988425	988469	988514	45
988782	988826	988871	988916	988960	45
989227	989272	989316	989361	989405	45
989672	989717	989761	989806	989850	44
990117	990161	990206	990250	990294	44
990561	990605	990649	990694	990738	44
991004	991049	991093	991137	991182	44
991448	991492	991536	991580	991625	44
991890	991935	991979	992023	992067	44
992333	992377	992421	992465	992509	44
992774	992819	992863	992907	992951	44
993216	993259	993304	993348	993392	44
993657	993701	993745	993789	993833	44
994097	994141	994185	994229	994273	44
994537	994581	994625	994669	994713	44
994977	995021	995065	995108	995[illegible]52	44
995416	995459	995504	995547	[illegible]95591	44
995854	995898	995942	995986	996029	44
996293	996337	996380	996424	996468	44
996731	996774	996818	996862	996906	44
997168	997212	997255	997299	997343	44
997605	997648	997692	997736	997779	44
998041	998085	998129	998172	998216	44
998477	998521	998564	998608	998652	44
998913	998956	998999	999043	999087	44
999348	999392	999435	999479	999522	44
999783	999826	999869	999913	999957	43

A

TABLE

OF

ARTIFICIAL SINES

AND

TANGENTS

TO EVERY

DEGREE and MINUTE

OF THE

QUADRANT.

The Radius being 10,000000.

LONDON:

Printed in the Year M.DCC.LXXII.

M	Sine	Co-sine	Tangent	Co-tan.	
0	0.000000	10.000000	0.000000	*Infinite.*	60
1	6.463726	9.999999	6.463726	13.536274	59
2	6.764756	9.999999	6.764756	13.235244	58
3	6.940847	9.999999	6.940847	13.059153	57
4	7.065786	9.999999	7.065786	13.934214	56
5	7.162696	9.999999	7.162696	12.837304	55
6	7.241877	9.999999	7.241878	12.758122	54
7	7.308824	9.999999	7.308825	12.691175	53
8	7.366816	9.999999	7.366817	12.633183	52
9	7.417968	9.999999	7.417970	12.582030	51
10	7.463726	9.999998	7.463727	12.536273	50
11	7.505118	9.999998	7.505120	12.494880	49
12	7.542906	9.999997	7.542909	12.457091	48
13	7.577668	9.999997	7.577671	12.422328	47
14	7.609853	9.999996	7.609857	12.390143	46
15	7.639816	9.999996	7.639820	12.360180	45
16	7.667844	9.999995	7.667849	13.332151	44
17	7.694173	9.999995	7.694179	12.305821	43
18	7.718997	9.999994	7.719003	12.280997	42
19	7.742477	9.999993	7.742484	12.257516	41
20	7.764754	9.999993	7.764761	12.235239	40
21	7.785943	9.999992	7.785951	12.214049	39
22	7.806146	9.999991	7.806155	12.193845	38
23	7.825451	9.999990	7.825460	12.174540	37
24	7.843934	9.999989	7.843944	12.156056	36
25	7.861662	9.999989	7.861674	12.138326	35
26	7.878695	9.999988	7.878708	12.121292	34
27	7.895085	9.999987	7.895099	12.104901	33
28	7.910879	9.999986	7.910894	12.089106	32
29	7.926119	9.999985	7.926134	12.073866	31
30	7.940842	9.999983	7.940858	12.059142	30
	Co sine	Sine	*Co-tan.*	Tangent	M

89 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
30	7.940842	9.999983	7.940858	12.059142	30
31	7.955082	9.999982	7.955100	12.044900	29
32	7.968870	9.999981	7.968889	12.031111	28
33	7.982233	9.999980	7.982253	12.017747	27
34	7.995198	9.999978	7.995219	12.004781	26
35	8.007787	9.999978	8.007810	11.992191	25
36	8.020021	9.999976	8.020044	11.979956	24
37	8.031919	9.999975	8.031945	11.968055	23
38	8.043501	9.999973	8.043527	11.956473	22
39	8.054781	9.999972	8.054809	11.945191	21
40	8.065776	9.999971	8.065806	11.934194	20
41	8.076500	9.999969	8.076531	11.923469	19
42	8.086965	9.999967	8.086997	11.913003	18
43	8.097183	9.999966	8.097217	11.902783	17
44	8.107167	9.999964	8.107203	11.892797	16
45	8.116926	9.999963	8.116963	11.883037	15
46	8.126471	9.999961	8.126510	11.873490	14
47	8.135810	9.999959	8.135851	11.864149	13
48	8.144953	9.999958	8.144996	11.855004	12
49	8.153907	9.999956	8.153952	11.846048	11
50	8.162681	9.999954	8.162727	11.837273	10
51	8.171280	9.999952	8.171328	11.828672	9
52	8.179713	9.999950	8.179763	11.820237	8
53	8.187985	9.999948	8.188036	11.811964	7
54	8.196102	9.999946	8.196156	11.803844	6
55	8.204070	9.999944	8.204126	11.795874	5
56	8.211895	9.999942	8.211953	11.788047	4
57	8.219581	9.999940	8.219641	11.780359	3
58	8.227134	9.999938	8.227195	11.772805	2
59	8.234557	9.999936	8.234621	11.765379	1
60	8.241855	9.999934	8.241921	11.758079	0
	Co-sine	Sine	Co-tan.	Tangent	M

89 DEG.

M	Sine	Co-fine	Tangent	Co-tan.	
0	8.241855	9.999934	8.241921	11.758079	60
1	8.249033	9.999932	8.249102	11.750898	59
2	8.256094	9.999929	8.256165	11.743835	58
3	8.263042	9.999927	8.263115	11.736885	57
4	8.269881	9.999925	8.269956	11.730044	56
5	8.276614	9.999922	8.276691	11.723309	55
6	8.283243	9.999920	8.283323	11.716677	54
7	8.289773	9.999918	8.289856	11.710144	53
8	8.296207	9.999915	8.296292	11.703708	52
9	8.302546	9.999913	8.302634	11.697367	51
10	8.308794	9.999910	8.308884	11.691116	50
11	8.314954	9.999907	8.315046	11.684954	49
12	8.321027	9.999905	8.321122	11.678878	48
13	8.327016	9.999902	8.327114	11.672886	47
14	8.332924	9.999899	8.333025	11.666975	46
15	8.338753	9.999897	8.338856	11.661144	45
16	8.344504	9.999894	8.344610	11.655390	44
17	8.350180	9.999891	8.350289	11.649711	43
18	8.355783	9.999888	8.355895	11.644105	42
19	8.361315	9.999885	8.361430	11.638570	41
20	8.366777	9.999882	8.366895	11.633105	40
21	8.372171	9.999879	8.372292	11.627708	39
22	8.377499	9.999876	8.377622	11.622378	38
23	8.382762	9.999873	8.382889	11.617111	37
24	8.387962	9.999870	8.388092	11.611908	36
25	8.393101	9.999867	8.393234	11.606766	35
26	8.398179	9.999864	8.398315	11.601685	34
27	8.403199	9.999861	8.403338	11.596662	33
28	8.408161	9.999858	8.408304	11.591696	32
29	8.413068	9.999854	8.413213	11.586787	31
30	8.417919	9.999851	8.418068	11.581932	30
	Co-fine	Sine	Co-tan.	Tangent	M

M	Sine	Co-sine	Tangent	Cotan.	
30	8.417919	9.999851	8.418068	11.581932	30
31	8.422717	9.999848	8.422869	11.577131	29
32	8.427462	9.999844	8.427618	11.572382	28
33	8.432156	9.999841	8.432315	11.567685	27
34	8.436800	9.999838	8.436962	11.563038	26
35	8.441394	9.999834	8.441560	11.558440	25
36	8.445941	9.999831	8.446110	11.553890	24
37	8.450440	9.999827	8.450613	11.549387	23
38	8.454893	9.999824	8.455070	11.544930	22
39	8.459301	9.999820	8.459481	11.540519	21
40	8.463665	9.999816	8.463849	11.536151	20
41	8.467985	9.999812	8.468172	11.531828	19
42	8.472263	9.999809	8.472454	11.527546	18
43	8.476498	9.999805	8.476693	11.523307	17
44	8.480693	9.999801	8.480892	11.519108	16
45	8.484848	9.999797	8.485050	11.514949	15
46	8.488963	9.999794	8.489170	11.510830	14
47	8.493040	9.999790	8.493250	11.506750	13
48	8.497078	9.999786	8.497293	11.502707	12
49	8.501080	9.999782	8.501298	11.498702	11
50	8.505045	9.999778	8.505267	11.494733	10
51	8.508974	9.999774	8.509200	11.490800	9
52	8.512867	9.999769	8.513098	11.486902	8
53	8.516726	9.999765	8.516961	11.483039	7
54	8.520551	9.999761	8.520790	11.479210	6
55	8.524343	9.999757	8.524586	11.475414	5
56	8.528102	9.999753	8.528349	11.471651	4
57	8.531828	9.999748	8.532080	11.467920	3
58	8.535523	9.999744	8.535779	11.464221	2
59	8.539186	9.999740	8.539447	11.460553	1
60	8.542819	9.999735	8.543084	11.456916	0
	Co-sine	Sine	Co-tan.	Tangent	M

88 DEG.

M	Sine	*Co-sine*	Tangent	*Co-tan.*	
0	3.542819	9.999735	8.543084	11.456916	60
1	8.546422	9.999731	8.546691	11.453309	59
2	8.549995	9.999726	8.550268	11.449732	58
3	8.553538	9.999722	8.553817	11.446183	57
4	8.557054	9.999717	8.557336	11.442664	56
5	8.560540	9.999713	8.560827	11.439172	55
6	8.563999	9.999708	8.564291	11.435709	54
7	8.567431	9.999703	8.567727	11.432272	53
8	8.570836	9.999699	8.571137	11.428863	52
9	8.574214	9.999694	8.574520	11.425480	51
10	8.577566	9.999689	8.577877	11.422123	50
11	8.580892	9.999585	8.581208	11.418792	49
12	8.584193	9.999680	8.584514	11.415486	48
13	8.587469	9.999675	8.587795	11.412205	47
14	8.590721	9.999670	8.591051	11.408949	46
15	8.593948	9.999665	8.594283	11.405717	45
16	8.597152	9.999660	8.597492	11.402508	44
17	8.600332	9.999655	8.600677	11.399323	43
18	8.603488	9.999650	8.603838	11.396161	42
19	8.606622	9.999645	8.606978	11.393022	41
20	8.609734	9.999640	8.610094	11.389906	40
21	8.612823	9.999635	8.613189	11.386811	39
22	8.615891	9.999629	8.616262	11.383738	38
23	8.618937	9.999624	8.619313	11.380687	37
24	8.621962	9.999619	8.622343	11.377657	36
25	8.624965	9.999614	8.625352	11.374648	35
26	8.627948	9.999608	8.628340	11.371660	34
27	8.630911	9.999605	8.631308	11.368692	33
28	8.633854	9.999597	8.634256	11.365744	32
29	8.636776	9.9 9592	8.637184	11.362816	31
30	8.639679	9.999586	8.640093	11.359907	30
	Co-sine	Sine	*Co-tan.*	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	M
30	8.639679	9.999586	8.640093	11.359907	30
31	8.642563	9.999581	8.642982	11.357017	29
32	8.645428	9.999575	8.645853	11.354147	28
33	8.648274	9.999570	8.648704	11.351296	27
34	8.651102	9.999564	8.651538	11.348463	26
35	8.653911	9.999558	8.654352	11.345648	25
36	8.656702	9.999553	8.657149	11.342851	24
37	8.659475	9.999547	8.659928	11.340072	23
38	8.662230	9.999541	8.662689	11.337311	22
39	8.664968	9.999535	8.665433	11.334567	21
40	8.667689	9.999529	8.668160	11.331840	20
41	8.670393	9.999523	8.670869	11.329130	19
42	8.673080	9.999518	8.673563	11.326437	18
43	8.675751	9.999512	8.676239	11.323761	17
44	8.678405	9.999506	8.678899	11.321100	16
45	8.681043	9.999499	8.681544	11.318456	15
46	8.683665	9.999493	8.684172	11.315828	14
47	8.686272	9.999487	8.686784	11.313216	13
48	8.688862	9.999481	8.689381	11.310619	12
49	8.691438	9.999475	8.691963	11.308037	11
50	8.693998	9.999469	8.694529	11.305471	10
51	8.696543	9.999462	8.697081	11.302919	9
52	8.699073	9.999456	8.699617	11.300383	8
53	8.701589	9.999450	8.702139	11.297851	7
54	8.704090	9.999443	8.704646	11.295354	6
55	8.706576	9.999437	8.707139	11.292860	5
56	8.709049	9.999431	8.709618	11.290381	4
57	8.711507	9.999424	8.712083	11.287917	3
58	8.713952	9.999418	8.714534	11.285466	2
59	8.716383	9.999411	8.716972	11.283028	1
60	8.718800	9.999404	8.719396	11.280604	0
	Co-sine	Sine	Co-tan.	Tangent	M

87 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
0	8.718800	9.999404	8.719396	11.280604	60
1	8.721204	9.999398	8.721806	11.278194	59
2	8.723595	9.999391	8.724204	11.275796	58
3	8.725972	9.999384	8.726588	11.273412	57
4	8.728336	9.999378	8.728959	11.271041	56
5	8.730688	9.999371	8.731317	11.268683	55
6	8.733027	9.999364	8.733663	11.266337	54
7	8.735354	9.999357	8.735996	11.264004	53
8	8.737667	9.999350	8.738317	11.261683	52
9	8.739969	9.999343	8.740626	11.259374	51
10	8.742259	9.999336	8.742922	11.257078	50
11	8.744536	9.999329	8.745207	11.254793	49
12	8.746801	9.999322	8.747479	11.252521	48
13	8.749055	9.999315	8.749740	11.250260	47
14	8.751297	9.999308	8.751989	11.248011	46
15	8.753528	9.999301	8.754227	11.245773	45
16	8.755747	9.999294	8.756453	11.243547	44
17	8.757955	9.999286	8.758668	11.241332	43
18	8.760151	9.999279	8.760872	11.239128	42
19	8.762337	9.999272	8.763065	11.236935	41
20	8.764511	9.999265	8.765246	11.234754	40
21	8.766675	9.999257	8.767417	11.232583	39
22	8.768828	9.999250	8.769578	11.230422	38
23	8.770970	9.999242	8.771727	11.228273	37
24	8.773101	9.999235	8.773866	11.226134	36
25	8.775223	9.999227	8.775995	11.224005	35
26	8.777333	9.999220	8.778114	11.221886	34
27	8.779434	9.999212	8.780222	11.219778	33
28	8.781524	9.999204	8.782320	11.217680	32
29	8.783605	9.999197	8.784408	11.215592	31
30	8.785675	9.999189	8.786486	11.213514	30
	Co-sine	Sine	Co-tan.	Tangent	M

86 DEG.

M	Sine	Co-*sine*	Tangent	Co-*tan.*	
30	8.785675	9.999189	8.786486	11.213514	30
31	8.787736	9.999181	8.788554	11.211446	29
32	8.789787	9.999174	8.790613	11.209387	28
33	8.791828	9.999166	8.792662	11.207338	27
34	8.793859	9.999158	8.794701	11.205299	26
35	8.795881	9.999150	8.796731	11.203269	25
36	8.797894	9.999142	8.798752	11.201248	24
37	8.799897	9.999134	8.800763	11.199237	23
38	8.801891	9.999126	8.802765	11.197235	22
39	8.803876	9.999118	8.804758	11.195242	21
40	8.805852	9.999110	8.806742	11.193258	20
41	8.807819	9.999102	8.808717	11.191283	19
42	8.809777	9.999094	8.810683	11.189317	18
43	8.811726	9.999086	8.812641	11.187359	17
44	8.813667	9.999077	8.814589	11.185411	16
45	8.815598	9.999069	8.816529	11.183471	15
46	8.817522	9.999061	8.818461	11.181539	14
47	8.819436	9.999052	8.820384	11.179616	13
48	8.821342	9.999044	8.822298	11.177702	12
49	8.823240	9.999036	8.824205	11.175795	11
50	8.825130	9.999027	8.826103	11.173897	10
51	8.827011	9.999019	8.827992	11.172008	9
52	8.828884	9.999010	8.829874	11.170126	8
53	8.830749	9.999002	8.831748	11.168252	7
54	8.832606	9.998993	8.833613	11.166387	6
55	8.834456	9.998984	8.835471	11.164529	5
56	8.836297	9.998976	8.837321	11.162679	4
57	8.838130	9.998967	8.839163	11.160837	3
58	8.839956	9.998958	8.840998	11.159002	2
59	8.841774	9.998950	8.842825	11.157175	1
60	8.843585	9.998941	8.844644	11.155356	0
	Co-*sine*	Sine	Co-*tan.*	Tangent	M

86 DEG.

M	Sine	*Co-sine*	Tangent	*Co-tan.*	
0	8.843584	9.998941	8.844644	11.155356	60
1	8.845387	9.998932	8.846455	11.153545	59
2	8.847183	9.998923	8.848260	11.151740	58
3	8.848971	9.998914	8.850057	11.149943	57
4	8.850751	9.998905	8.851846	11.148154	56
5	8.852525	9.998896	8.853628	11.146372	55
6	8.854291	9.998887	8.855403	11.144597	54
7	8.856049	9.998878	8.857171	11.142829	53
8	8.857801	9.998869	8.858932	11.141068	52
9	8.859546	9.998860	8.860686	11.139314	51
10	8.861283	9.998851	8.862433	11.137567	50
11	8.863014	9.998841	8.864173	11.135827	49
12	8.864738	9.998832	8.865906	11.134094	48
13	8.866454	9.998823	8.867632	11.132368	47
14	8.868165	9.998813	8.869351	11.130649	46
15	8.869868	9.998804	8.871064	11.128936	45
16	8.871565	9.998795	8.872770	11.127230	44
17	8.873255	9.998785	8.874469	11.125531	43
18	8.874938	9.998776	8.876162	11.123838	42
19	8.876615	9.998766	8.877849	11.122151	41
20	8.878285	9.998757	8.879529	11.120471	40
21	8.879949	9.998747	8.881202	11.118798	39
22	8.881607	9.998738	8.882869	11.117131	38
23	8.883258	9.998728	8.884530	11.115470	37
24	8.884903	9.998718	8.886185	11.113815	36
25	8.886542	9.998708	8.887833	11.112167	35
26	8.888174	9.998699	8.889476	11.110524	34
27	8.889801	9.998689	8.891112	11.108888	33
28	8.891421	9.998679	8.892742	11.107258	32
29	8.893035	9.998669	8.894366	11.105634	31
30	8.894643	9.998659	8.895984	11.104016	30
	Co-sine	Sine	*Co-tan.*	Tangent	M

85 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
30	8.894643	9.998659	8.895984	11.104016	30
31	8.896246	9.998649	8.897596	11.102404	29
32	8.897842	9.998639	8.899203	11.100797	28
33	8.899432	9.998629	8.900803	11.099197	27
34	8.901017	9.998619	8.902398	11.097602	26
35	8.902596	9.998609	8.903987	11.096013	25
36	8.904169	9.998599	8.905570	11.094430	24
37	8.905736	9.998589	8.907147	11.092853	23
38	8.907297	9.998578	8.908719	11.091281	22
39	8.908853	9.998568	8.910285	11.089715	21
40	8.910404	9.998558	8.911846	11.088154	20
41	8.911949	9.998548	8.913401	11.086599	19
42	8.913488	9.998537	8.914951	11.085049	18
43	8.915022	9.998527	8.916495	11.083505	17
44	8.916550	9.998516	8.918034	11.081966	16
45	8.918073	9.998506	8.919568	11.080432	15
46	8.919591	9.998495	8.921096	11.078904	14
47	8.921103	9.998485	8.922619	11.077381	13
48	8.922610	9.998474	8.924136	11.075864	12
49	8.924112	9.998464	8.925649	11.074351	11
50	8.925609	9.998453	8.927156	11.072844	10
51	8.927100	9.998442	8.928658	11.071342	9
52	8.928587	9.998431	8.930155	11.069845	8
53	8.930068	9.998421	8.931647	11.068353	7
54	8.931544	9.998410	8.933134	11.066866	6
55	8.933015	9.998399	8.934616	11.065384	5
56	8.934481	9.998388	8.936093	11.063907	4
57	8.9359[illegible]2	9.998377	8.937565	11.062435	3
58	8.937398	9.998366	8.939032	11.060968	2
59	8.938850	9.998355	8.940494	11.059506	1
60	8.940296	9.998344	8.941952	11.058048	0
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	
0	8.940296	9.998344	8.941952	11.058048	60
1	8.941738	9.998333	8.943404	11.056596	59
2	8.943174	9.998322	8.944852	11.055148	58
3	8.944606	9.998311	8.946295	11.053705	57
4	8.946034	9.998300	8.947734	11.052266	56
5	8.947456	9.998289	8.949168	11.050832	55
6	8.948874	9.998277	8.950597	11.049403	54
7	8.950287	9.998266	8.952021	11.047979	53
8	8.951696	9.998255	8.953441	11.046559	52
9	8.953099	9.998243	8.954856	11.045144	51
10	8.954499	9.998232	8.956267	11.043733	50
11	8.955894	9.998220	8.957674	11.042326	49
12	8.957284	9.998209	8.959075	11.040925	48
13	8.958670	9.998197	8.960473	11.039527	47
14	8.960052	9.998186	8.961866	11.038134	46
15	8.961429	9.998174	8.963254	11.036746	45
16	8.962801	9.998163	8.964639	11.035361	44
17	8.964170	9.998151	8.966019	11.033981	43
18	8.965534	9.998139	8.967394	11.032606	42
19	8.966893	9.998128	8.968766	11.031234	41
20	8.968249	9.998116	8.970133	11.029867	40
21	8.969600	9.998104	8.971495	11.028504	39
22	8.970947	9.998092	8.972855	11.027145	38
23	8.972289	9.998080	8.974209	11.025791	37
24	8.973628	9.998068	8.975560	11.024440	36
25	8.974962	9.998056	8.976906	11.023094	35
26	8.976293	9.998044	8.978248	11.021752	34
27	8.977619	9.998032	8.979586	11.020414	33
28	8.978941	9.998020	8.980920	11.019079	32
29	8.980259	9.998008	8.982251	11.017749	31
30	8.981573	9.997996	8.983577	11.016423	30
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	*Co-sine*	Tangent	*Co-tan.*	
30	8.981573	9.997996	8.983577	11.016423	30
31	8.982883	9.997984	8.984899	11.015101	29
32	8.984189	9.997971	8.986217	11.013783	28
33	8.985491	9.997959	8.987532	11.012468	27
34	8.986789	9.997947	8.988842	11.011158	26
35	8.988083	9.997935	8.990149	11.009851	25
36	8.989374	9.997922	8.991451	11.008549	24
37	8.990660	9.997910	8.992750	11.007250	23
38	8.991943	9.997897	8.994045	11.005955	22
39	8.993222	9.997885	8.995337	11.004663	21
40	8.994497	9.997873	8.996624	11.003376	20
41	8.995768	9.997860	8.997908	11.002092	19
42	8.997036	9.997847	8.999188	11.000812	18
43	8.998299	9.997835	9.000465	11.999534	17
44	8.999560	9.997822	9.001738	10.998262	16
45	8.000816	9.997809	9.003007	10.996993	15
46	8.002069	9-997797	9.004272	10.995728	14
47	8.003318	9.997784	9.005534	10.994466	13
48	8.004563	9.997771	9.006792	10.993208	12
49	8.005805	9.997758	9.008047	10.991953	11
50	8.007044	9.997745	9.009298	10.990702	10
51	9.008278	9.997732	9.010546	10.989454	9
52	9.009510	9.997719	9.011790	10.988210	8
53	9.010737	9.997706	9.013031	10.986969	7
54	9.011962	9.997693	9.014268	10.985732	6
55	9.013182	9.997680	9.015502	10.984498	5
56	9.014399	9.997667	9.016732	10.983268	4
57	9.015613	9.997654	9.017959	10.982041	3
58	9.016824	9.997641	9.019183	10.980817	2
59	9.018031	9.997628	9.020403	10.979597	1
60	9.019235	9.997614	9.021620	10.978380	0
	Co-sine	Sine	*Co-tan.*	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	
0	9.019235	9.997614	9.021620	10.978380	60
1	9.020435	9.997601	9.022834	10.977166	59
2	9.021632	9.997588	9.024044	10.975956	58
3	9.022825	9.997574	9.025251	10 974749	57
4	9.024016	9 997561	9.026455	10.973545	56
5	9.025203	9.997548	9.027655	10.972345	55
6	9.026386	9.997534	9.028852	10.971148	54
7	9.027567	9.997520	9.030046	10.969954	53
8	9.028744	9.997507	9.031237	10.968763	52
9	9.029918	9.997493	9.032425	10.967575	51
10	9.031089	9.997480	9.033609	10.966391	50
11	9.032257	9.997466	9.034791	0.965209	49
12	9.033421	9.997452	9.035969	0.964031	48
13	9 034582	9.997439	9.037[illegible]44	10.962856	47
14	9.035741	9.997425	9.038316	10.96[illegible]684	46
15	9.036896	9.997411	9.039485	10.960515	45
16	9.038048	9.997397	9.040651	10.959349	44
17	9.039197	9.997383	9.041813	10.958187	43
18	9.040342	9.997369	9.042973	10.957027	42
19	9.041485	9.997355	9.044130	10.955870	41
20	9.042625	9.997341	9.045284	10.954716	40
21	9.043762	9.997327	9.046434	10 953566	39
22	9.044895	9.997313	9.047582	10.952418	38
23	9.046026	9.997299	9.048727	10.951273	37
24	9.047154	9.997285	9.049869	10.950131	36
25	9.048279	9.997271	9.051008	10.948992	35
26	9.049400	9.997256	9.052144	10.947856	34
27	9.050519	9.997242	9.053277	10.946723	33
28	9.051635	9.997228	9.054408	10.945592	32
29	9.052749	9.997214	9.055535	10.944465	31
30	9.053859	9.997199	9.056660	10.943340	30
	Co-sine	Sine	Co-tan.	Tangent	M

83 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
30	9.053859	9.997199	9.056660	10.943340	30
31	9.054966	9.997185	9.057781	10.942219	29
32	9.056071	9.997170	9.058900	10.941100	28
33	9.057172	9.997156	9.060016	10.939984	27
34	9.058271	9.997141	9.061130	10.938870	26
35	9.059367	9.997127	9.062240	10.937760	25
36	9.060460	9.997112	9.063348	10.936652	24
37	9.061551	9.997098	9.064453	10.935547	23
38	9.062638	9.997083	9.065556	10.934444	22
39	9.063723	9.997068	9.066655	10.933345	21
40	9.064806	9.997053	9.067752	10.932248	20
41	9.065885	9.997039	9.068847	10.931153	19
42	9.066962	9.997024	9.069938	10.930062	18
43	9.068036	9.997009	9.071027	10.928973	17
44	9.069107	9.996994	9.072113	10.927887	16
45	9.070176	9.996979	9.073197	10.926803	15
46	9.071242	9.996964	9.074278	10.925722	14
47	9.072306	9.996949	9.075356	10.924644	13
48	9.073366	9.996934	9.076432	10.923568	12
49	9.074424	9.996919	9.077505	10.922495	11
50	9.075480	9.996904	9.078576	10.921424	10
51	9.076533	9.996889	9.079644	10.920356	9
52	9.077583	9.996874	9.080710	10.919290	8
53	9.078631	9.996858	9.081773	10.918227	7
54	9.079676	9.996843	9.082833	10.917167	6
55	9.080719	9.996828	9.083891	10.916109	5
56	9.081759	9.996812	9.084947	10.915053	4
57	9.082797	9.996797	9.085999	10.914000	3
58	9.083832	9.996782	9.087050	10.912950	2
59	9.084864	9.996766	9.088098	10.911902	1
60	9.085894	9.996751	9.089144	10.910856	0
	Co-sine	Sine	Co tan.	Tangent	M

83 DEG.

M	Sine	*Co-sine*	Tangent	*Co-tan.*	
0	9.085894	9.996751	9.089144	10.910856	60
1	9.086922	9.996735	9.090187	10.909813	59
2	9.087947	9.996720	9.091228	10.908772	58
3	9.088970	9.996704	9.092266	10.907734	57
4	9.089990	9.996688	9.093302	10.906698	56
5	9.091008	9.996673	9.094336	10.905664	55
6	9.092024	9.996657	9.095367	10.904633	54
7	9.093037	9.996641	9.096395	10.903604	53
8	9.094047	9.996625	9.097422	10.902578	52
9	9.095056	9.996610	9.098446	10.901554	51
10	9.096062	9.996594	9.099468	10.900532	50
11	9.097065	9.996578	9.100487	10.899513	49
12	9.098066	9.996562	9.101504	10.898496	48
13	9.099 65	9.996546	9.102519	10.897481	47
14	9.100062	9.996530	9.103532	10.896468	46
15	9.101056	9.996514	9.104542	10.895458	45
16	9.102048	9.996498	9.105550	10.894450	44
17	9.103037	9.996482	9.106556	10.893444	43
18	9.104025	9.996465	9. 07559	10.892441	42
19	9.105010	9.996449	9.108560	10.891440	41
20	9.105992	9.996433	9.109559	10.890441	40
21	9.106973	9.996417	9.110556	10.889444	39
22	9.107951	9.996400	9.111551	10.888449	38
23	9.108927	9.996384	9.112543	10.887457	37
24	9.109901	9.996368	9.113533	10.886467	36
25	9.110873	9.996351	9.114521	10.885478	35
26	9.111842	9.996335	9.115507	10.884493	34
27	9.112809	9.996318	9.116491	10.883509	33
28	9.113774	9.996302	9.117472	10.882528	32
29	9.114737	9.996285	9.118452	10.881548	31
30	9,115698	9.996269	9.119429	10.880571	30
	Co-sine	Sine	*Co-tan.*	Tangent	M

82 DEG.

M	Sine	*Co-sine*	Tangent	*Co-tan.*	
30	9.115698	9.996269	9.119429	10.880571	30
31	9.116656	9.996252	9.120404	10.879596	29
32	9.117612	9.996235	9.121377	10.878623	28
33	9.118567	9.996218	9.122348	10.877652	27
34	9.119519	9.996202	9.123317	10.876683	26
35	9.120469	9.996185	9.124284	10.875716	25
36	9.121417	9.996168	9.125249	10.874751	24
37	9.122362	9.996151	9.126211	10.873789	23
38	9.123306	9.996134	9.127172	10.872828	22
39	9.124248	9.996117	9.128130	10.871870	21
40	9.125187	9.996100	9.129087	10.870913	20
41	9.126125	9.996083	9.130041	10.869959	19
42	9.127060	9.996066	9.130994	10.869006	18
43	9.127993	9.996049	9.131944	10.868056	17
44	9.128925	9.996032	9.132893	10.867107	16
45	9.129854	9.996015	9.133839	10.866161	15
46	9.130781	9.995998	9.134784	10.865216	14
47	9.131706	9.995980	9.135726	10.864274	13
48	9.132630	9.995963	9.136666	10.863334	12
49	9.133551	9.995946	9.137605	10.862395	11
50	9.134470	9.995928	9.138542	10.861458	10
51	9.135388	9.995911	9.139476	10.860524	9
52	9.136303	9.995894	9.140409	10.859591	8
53	9.137216	9.995876	9.141340	10.858660	7
54	9.138127	9.995859	9.142269	10.857731	6
55	9.139037	9.995841	9.143196	10.856804	5
56	9.139944	9.995824	9.144121	10.855879	4
57	9.140850	9.995806	9.145044	10.854956	3
58	9.141754	9.995788	9.145966	10.854035	2
59	9.142655	9.995770	9.146885	10.853115	1
60	9.143555	9.995753	9.147803	10.852198	0
	Co-sine	Sine	*Co-tan.*	Tangent	

82 DEG.

M	Sine	Co-fine	Tangent	Co-tan.	
0	9,143555	9,995753	9.147803	10,852198	60
1	9,144453	9,995735	9,148718	10,851282	59
2	9,145349	9,995717	9,149632	10,850368	58
3	9,146244	9,995699	9,150544	10,849456	57
4	9,147136	9,995682	9,151454	10,848546	56
5	9,148026	9,995664	9,152363	10,847637	55
6	9,148915	9,995646	9,153269	10,846731	54
7	9,149802	9,995628	9,154174	10,845826	53
8	9,150686	9,995610	9,155077	10,844923	52
9	9,151569	9,995592	9,155978	10,844022	51
10	9,152451	9,995573	9,156877	10,843123	50
11	9,153330	9,995555	9,157775	10,842225	49
12	9,154208	9,995537	9,158671	10,841329	48
13	9,155083	9.995519	9,159565	10,840435	47
14	9,155957	9,995501	9,160457	10,839543	46
15	9,156830	9,995482	9,161347	10,838653	45
16	9,157700	9,995464	9,162236	10,837764	44
17	9,158569	9,995446	9,163123	10,836877	43
18	9,159435	9,995427	9,164008	10,835992	42
19	9,160301	9,995409	9,164892	10,835108	41
20	9,161164	9,995390	9,165774	10,834226	40
21	9,162025	9,995372	9,166654	10,833346	39
22	9,162885	9,995353	9,167532	10,832468	38
23	9,163743	9,995335	9,168409	10,831591	37
24	9,164600	9,995316	9,169284	10,830716	36
25	9,165454	9,995297	9,170157	10,829843	35
26	9,166307	9,995279	9,171029	10,828971	34
27	9,167159	9,995260	9,171899	10,828101	33
28	9,168008	9,995241	9,172767	10,827233	32
29	9,168856	9,995222	9,173634	10,826366	31
30	9,169702	9,995203	9.174499	10.825501	30
	Co-fine	Sine	Co-tan.	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	
30	9.169702	9.995203	9.174499	10.825501	30
31	9.170547	9.995184	9.175362	10.824638	29
32	9.171389	9.995165	9.176224	10.823776	28
33	9.172230	9.995146	9.177084	10.822916	27
34	9.173070	9.995127	9.177942	10.822058	26
35	9.173908	9.995108	9.178799	10.821201	25
36	9.174744	9.995089	9.179655	10.820345	24
37	9.175578	9.995070	9.180508	10.819492	23
38	9.176411	9.995051	9.181360	10.818640	22
39	9.177243	9.995032	9.182211	10.817789	21
40	9.178072	9.995013	9.183060	10.816940	20
41	9.178900	9.994993	9.183907	10.816093	19
42	9.179727	9.994974	9.184752	10.815248	18
43	9.180551	9.994955	9.185597	10.814403	17
44	9.181374	9.994935	9.186439	10.813561	16
45	9.182196	9.994916	9.187280	10.812720	15
46	9.183016	9.994896	9.188120	10.811880	14
47	9.183834	9.994877	9.188957	10.811042	13
48	9.184651	9.994857	9.189794	10.810206	12
49	9.185467	9.994838	9.190629	10.809371	11
50	9.186280	9.994818	9.191462	10.808538	10
51	9.187092	9.994799	9.192294	10.807706	9
52	9.187903	9.994779	9.193124	10.806876	8
53	9.188712	9.994759	9.193953	10.806047	7
54	9.189519	9.994739	9.194780	10.805220	6
55	9.190325	9.994719	9.195606	10.804394	5
56	9.191130	9.994700	9.196430	10.803569	4
57	9.191933	9.994680	9.197253	10.802747	3
58	9.192734	9.994660	9.198074	10.801926	2
59	9.193534	9.994640	9.198894	10.801106	1
60	9.194332	9.994620	9.199712	10.800287	0
	Co-sine	Sine	Co-tan.	Tangent	

81 DEG.

M	Sine	Co-sine		Tangent	Co-tan.	
0	9.194332	9.994620		9.199712	10.800287	60
1	9.195129	9.994600		9.200529	10.799471	59
2	9.195925	9.994580		9.201345	10.798655	58
3	9.196719	9.994560		9.202159	10.797841	57
4	9.197511	9.994540		9.202971	10.797029	56
5	9.198302	9.994519		9.203782	10.796218	55
6	9.199091	9.994499		9.204592	10.795408	54
7	9.199879	9.994479		9.205400	10.794600	53
8	9.200666	9.994459		9.206207	10.793793	52
9	9.201451	9.994438		9.207013	10.792987	51
10	9.202234	9.994418		9.207817	10.792183	50
11	9.203017	9.994398		9.208619	10.791381	49
12	9.203797	9.994377		9.209420	10.790580	48
13	9.204577	9.994357		9.210220	10.789780	47
14	9.205354	9.994336		9.211018	10.788982	46
15	9.206131	9.994316		9.211815	10.788185	45
16	9.206906	9.994295		9.212611	10.787389	44
17	9.207679	9.994274		9.213405	10.786595	43
18	9.208452	9.994254		9.214198	10.785802	42
19	9.209222	9.994233		9.214989	10.785011	41
20	9.209992	9.994212		9.215780	10.784220	40
21	9.210760	9.994191		9.216568	10.783432	39
22	9.211526	9.994171		9.217356	10.782644	38
23	9.212291	9.994150		9.218142	10.781858	37
24	9.213055	9.994129		9.218926	10.781074	36
25	9.213818	9.994108		9.219710	10.780290	35
26	9.214579	9.994087		9.220492	10.779508	34
27	9.215338	9.994066		9.221272	10.778728	33
28	9.216097	9.994045		9.222052	10.777948	32
29	9.216854	9.994024		9.222830	10.777170	31
30	9.217609	9.994003		9.223607	10.776393	30
	Co-sine	Sine		Co-tan.	Tangent	M

80 D E G.

M	Sine	Co-fine	Tangent	Co-tan.	
30	9,217609	9,994003	9,223607	10,776393	30
31	9,218363	9,993982	9,224382	10,775618	29
32	9,219116	9,993960	9,225156	10,774844	28
33	9,219868	9,993939	9,225929	10,774071	27
34	9,220618	9,993918	9,226700	10,773300	26
35	9,221367	9,993897	9,227471	10,772529	25
36	9,222115	9,993875	9,228240	10,771760	24
37	9,222861	9,993854	9,229007	10,770993	23
38	9,223606	9,993832	9,229774	10,770226	22
39	9,224349	9,993811	9,230539	10,769461	21
40	9,225092	9,993789	9,231302	10,768698	20
41	9,225833	9,993768	9,232065	10,767935	19
42	9,226573	9,993746	9,232826	10,767174	18
43	9,227311	9,993725	9,233586	10,766414	17
44	9,228048	9,993703	9,234345	10,765655	16
45	9,228784	9,993681	9,235103	10,764897	15
46	9,229518	9,993660	9,235859	10,764141	14
47	9,230252	9,993638	9,236614	10,763386	13
48	9,230984	9,993616	9,237368	10,762632	12
49	9,231715	9,993594	9,238120	10,761880	11
50	9,232444	9,993572	9,238872	10,761128	10
51	9,233172	9,993550	9,239622	10,760378	9
52	9,233899	9,993528	9,240371	10,759629	8
53	9,234625	9,993506	9,241118	10,758882	7
54	9,235349	9,993484	9,241865	10,758135	6
55	9,236073	9,993462	9,242610	10,757390	5
56	9,236795	9,993440	9,243354	10,756646	4
57	9,237515	9,993418	9,244097	10,755903	3
58	9,238235	9,993396	9,244839	10,755161	2
59	9,238953	9,993374	9,245579	10,754421	1
60	9,239670	9,993351	9,246319	10,753681	0
	Co-fine	Sine	Co-tan.	Targent	M

80 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
0	9,239670	9,993351	9,246319	10,753681	60
1	9,240386	9,993329	9,247057	10,752943	59
2	9,241101	9,993307	9,247794	10,752206	58
3	9,241814	9,993284	9,248530	10,751470	57
4	9,242526	9,993262	9,249264	10,750736	56
5	9,243237	9,993240	9,249998	10,750002	55
6	9,243947	9,993217	9,250730	10,749270	54
7	9,244656	9,993195	9,251461	10,748539	53
8	9,245363	9,993172	9,252191	10,747809	52
9	9,246070	9,993149	9,252920	10,747080	51
10	9,246775	9,993127	9,253648	10,746352	50
11	9,247478	9,993104	9,254374	10,745626	49
12	9,248181	9,993081	9,255100	10,744900	48
13	9,248883	9,993059	9,255824	10,744176	47
14	9,249583	9,993036	9,256547	10,743453	46
15	9,250282	9,993013	9,257269	10,742731	45
16	9,250980	9,992990	9,257990	10,742010	44
17	9,251677	9,992967	9,258710	10,741290	43
18	9,252373	9,992944	9,259429	10,740571	42
19	9,253067	9,992921	9,260146	10,739854	41
20	9,253761	9,992898	9,260863	10,739137	40
21	9,254453	9,992875	9,261578	10,738422	39
22	9,255144	9,992852	9,262292	10,737708	38
23	9,255834	9,992829	9,263005	10,736995	37
24	9,256523	9,992806	9,263717	10,736283	36
25	9,257211	9,992783	9,264428	10,735572	35
26	9,257898	9,992759	9,265138	10,734862	34
27	9,258583	9,992736	9,265847	10,734153	33
28	9,259268	9,992713	9,266555	10,733445	32
29	9,259951	9,992690	9,267261	10,732739	31
30	9,260633	9,992666	9,267967	10,732033	30
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	*Co-sine*	Tangent	*Co-tan.*	
30	9,260633	9,992666	9,267967	10,732033	30
31	9,261314	9,992643	9,268671	10,731329	29
32	9,261994	9,992619	9,269375	10,730625	28
33	9,262673	9,992596	9,270077	10,729923	27
34	9,263351	9,992572	9,270779	10,729221	26
35	9,264027	9,992549	9,271479	10,728521	25
36	9,264703	9,992525	9,272178	10,727822	24
37	9,265378	9,992501	9,272876	10,727124	23
38	9,266051	9,992478	9,273573	10,726427	22
39	9,266723	9,992454	9,274269	10,725731	21
40	9,267395	9,992430	9,274964	10,725036	20
41	9,268065	9,992406	9,275658	10,724342	19
42	9,268734	9,992382	9,276351	10,723649	18
43	9,269402	9,992358	9,277043	10,722957	17
44	9,270069	9,992335	9,277734	10,722266	16
45	9,270735	9,992311	9,278424	10,721576	15
46	9,271400	9,992287	9,279113	10,720887	14
47	9,272063	9,992263	9,279801	10,720199	13
48	9,272726	9,992239	9,280488	10,719512	12
49	9,273388	9,992214	9,281174	10,718826	11
50	9,274049	9,992190	9,281858	10,718142	10
51	9,274708	9,992166	9,282542	10,717458	9
52	9,275367	9,992142	9,283225	10,716775	8
53	9,276025	9,992118	9,283907	10,716093	7
54	9,276681	9,992093	9,284588	10,715412	6
55	9,277337	9,992069	9,285268	10,714732	5
56	9,277991	9,992045	9,285947	10,714053	4
57	9,278645	9,992020	9,286624	10,713376	3
58	9,279297	9,991996	9,287301	10,712699	2
59	9,279948	9,991971	9,287977	10,712023	1
60	9,280599	9,991947	9,288652	10,711348	0
	Co-sine	Sine	Tangent	*Co-tan.*	M

79 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
0	9,280599	9,991947	9,288652	10,711348	60
1	9,281248	9,991922	9,289326	10,710674	59
2	9,281897	9,991897	9,289999	10,710001	58
3	9,282544	9,991873	9,290671	10,709329	57
4	9,283190	9,991848	9,291342	10,708658	56
5	9,283836	9,991823	9,292013	10 707987	55
6	9,284480	9,991799	9,292682	10,707318	54
7	9,285124	9,991774	9,293350	10,706650	53
8	9,285766	9,991749	9,294017	10,705983	52
9	9,286408	9,991724	9,294684	10,705316	51
10	9,287048	9.991699	9,295349	10,704651	50
11	9,287688	9,991674	9,296013	10,703987	49
12	9,288326	9,991649	9,296677	10,703323	48
13	9,288964	9,991624	9,297339	10,702661	47
14	9,289600	9,991599	9,298001	10,701999	46
15	9,290236	9,991574	9,298662	10,701338	45
16	9,290870	9,991549	9,299322	10,700678	44
17	9,291504	9,991524	9,299980	10,700020	43
18	9,292137	9,991498	9,300638	10,699362	42
19	9,292768	9,991473	9,301295	10,698705	41
20	9,293399	9,991448	9.301951	10,698049	40
21	9,294029	9,991422	9,302607	10,697393	39
22	9,294658	9,991397	9,303261	10,696739	38
23	9,295286	9,991372	9,303914	10,696086	37
24	9,295913	9,991346	9,304567	10,695433	36
25	9,296539	9,991321	9,305218	10,694782	35
26	9,297164	9,991295	9,305869	10,694131	34
27	9,297788	9,991270	9,306519	10,693481	33
28	9,298412	9,991244	9,307168	10,692832	32
29	9,299034	9,991218	9,307816	10,692184	31
30	9,299655	9,991193	9,308463	10,691537	30
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	
30	9,299655	9,991193	9,308463	10,691537	30
31	9,300276	9,991167	9,309109	10,690891	29
32	9,300895	9,991141	9,309754	10,690246	28
33	9,301514	9,991115	9,310399	10,689601	27
34	9,302132	9,991090	9,311042	10,688958	26
35	9,302749	9,991064	9,311685	10,688315	25
36	9,303364	9,991038	9,312327	10,687673	24
37	9,303979	9,991012	9,312968	10,687032	23
38	9,304593	9,990986	9,313608	10,686392	22
39	9,305207	9,990960	9,314247	10,685753	21
40	9,305819	9,990934	9,314885	10,685115	20
41	9,306430	9,990908	9,315523	10,684477	19
42	9,307041	9,990882	9,316159	10,683841	18
43	9,307650	9,990855	9,316795	10,683205	17
44	9,308259	9,990829	9,317430	10,682570	16
45	9,308867	9,990803	9,318064	10,681936	15
46	9,309474	9,990777	9,318697	10,681303	14
47	9,310080	9,990750	9,319330	10,680670	13
48	9,310685	9,990724	9,319961	10,680039	12
49	9,311289	9,990697	9,320592	10,679408	11
50	9,311893	9,990671	9,321222	10,678778	10
51	9,312495	9,990645	9,321851	10,678149	9
52	9,313097	9,990618	9,322479	10,677521	8
53	9,313698	9,990591	9,323106	10,676894	7
54	9,314297	9,990565	9,323733	10,676267	6
55	9,314897	9,990538	9,324358	10,675642	5
56	9,315495	9,990512	9,324983	10,675017	4
57	9,316092	9,990485	9,325607	10,674393	3
58	9,316689	9,990458	9,326231	10,673769	2
59	9,317284	9,990431	9,326853	10,673147	1
60	9,317879	9,990404	9,327475	10,672525	0
	Co-sine	Sine	Co-tan.	Tangent	M

78 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
0	9,317879	9,990404	9,327475	10,672525	60
1	9,318473	9,990377	9,328095	10,671905	59
2	9,319066	9,990351	9,328715	10,671285	58
3	9,319658	9,990324	9,329334	10,670666	57
4	9,320250	9,990297	9,329953	10,670047	56
5	9,320840	9,990270	9,330570	10,669430	55
6	9,321430	9,990243	9,331187	10,668813	54
7	9,322019	9,990215	9,331803	10,668197	53
8	9,322607	9,990188	9,332418	10,667582	52
9	9,323194	9,990161	9,333033	10,666967	51
10	9,323780	9,990134	9,333646	10,666354	50
11	9,324366	9,990107	9,334259	10,665741	49
12	9,324950	9,990079	9,334871	10,665129	48
13	9,325534	9,990052	9,335482	10,664518	47
14	9,326117	9,990025	9,336093	10,663907	46
15	9,326700	9,989997	9,336702	10,663298	45
16	9,327281	9,989970	9,337311	10,662689	44
17	9,327862	9,989942	9,337919	10,662081	43
18	9,328442	9,989915	9,338527	10,661473	42
19	9,329021	9,989887	9,339133	10,660867	41
20	9,329599	9,989860	9,339739	10,660261	40
21	9,330176	9,989832	9,340344	10,659656	39
22	9,330753	9,989804	9,340948	10,659052	38
23	9,331328	9,989777	9,341552	10,658448	37
24	9,331903	9,989749	9,342155	10,657845	36
25	9,332478	9,989721	9,342757	10,657243	35
26	9,333051	9,989693	9,343358	10,656642	34
27	9,333624	9,989665	9,343958	10,656042	33
28	9,334195	9,989637	9,344558	10,655442	32
29	9,334766	9,989609	9,345157	10,654843	31
30	9,335337	9,989581	9,345755	10,654245	30
	Co-sine	Sine	Co-tan.	Tangent	M

77 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
30	9,335337	9,989581	9,345755	10,654245	30
31	9,335906	9,989553	9,346353	10,653647	29
32	9,336475	9,989525	9,346949	10,653051	28
33	9,337043	9,989497	9,347545	10,652455	27
34	9,337610	9,989469	9,348141	10,651859	26
35	9,338176	9,989441	9,348735	10,651265	25
36	9,338742	9,989413	9,349329	10,650671	24
37	9,339306	9,989384	9,349922	10,650078	23
38	9,339870	9,989356	9,350514	10,649486	22
39	9,340434	9,989328	9,351106	10,648894	21
40	9,340996	9,989299	9,351697	10,648303	20
41	9,341558	9,989271	9,352287	10,647713	19
42	9,342119	9,989243	9,352876	10,647124	18
43	9,342679	9,989214	9,353465	10,646535	17
44	9,343239	9,989186	9,354053	10,645947	16
45	9,343797	9,989157	9,354640	10,645360	15
46	9,344355	9,989128	9,355227	10,644773	14
47	9,344912	9,989100	9,355813	10.644187	13
48	9,345469	9,989071	9,356398	10,643602	12
49	9,346024	9,989042	9,356982	10,643018	11
50	9,346579	9,989014	9,357566	10,642434	10
51	9,347134	9,988985	9,358149	10,641851	9
52	9,347687	9,988956	9,358731	10,641269	8
53	9,348240	9,988927	9,359313	10,640687	7
54	9,348792	9,988898	9,359893	10,640107	6
55	9,349343	9,988869	9,360474	10,639526	5
56	9,349893	9,988840	9,361053	10,638947	4
57	9,350443	9,988811	9,361632	10,638368	3
58	9,350992	9,988782	9,362210	10,637790	2
59	9,351540	9,988753	9,362787	10,637213	1
60	9,352088	9,988724	9,363364	10,636636	0
	Co-sine	Sine	Co-tan.	Tangent	M

77 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
0	9.352088	9.988724	9.363364	10.636636	60
1	9.352635	9.988695	9.363940	10.636060	59
2	9.353181	9.988666	9.364515	10.635485	58
3	9.353726	9.988636	9.365090	10.634910	57
4	9.354271	9.988607	9.365664	10.634336	56
5	9.354815	9.988578	9.366237	10.633763	55
6	9.355358	9.988548	9.366810	10.633190	54
7	9.355901	9.988519	9.367382	10.632618	53
8	9.356443	9.988489	9.367953	10.632047	52
9	9.356984	9.988460	9.368524	10.631476	51
10	9.357524	9.988430	9.369094	10.630906	50
11	9.358064	9.988401	9.369663	10.630337	49
12	9.358603	9.988371	9.370232	10.629768	48
13	9.359141	9.988341	9.370799	10.629201	47
14	9.359679	9.988312	9.371367	10.628633	46
15	9.360215	9.988282	9.371933	10.628067	45
16	9.360752	9.988252	9.372499	10.627501	44
17	9.361287	9.988223	9.373064	10.626936	43
18	9.361822	9.988193	9.373629	10.626371	42
19	9.362356	9.988163	9.374193	10.625807	41
20	9.362889	9.988133	9.374756	10.625244	40
21	9.363422	9.988103	9.375319	10.624681	39
22	9.363954	9.988073	9.375881	10.624119	38
23	9.364485	9.988043	9.376442	10.623558	37
24	9.365016	9.988013	9.377003	10.622997	36
25	9.365546	9.987983	9.377563	10.622437	35
26	9.366075	9.987953	9.378122	10.621878	34
27	9.366604	9.987922	9.378681	10.621319	33
28	9.367132	9.987892	9.379239	10.620761	32
29	9.367659	9.987862	9.379797	10.620203	31
30	9.368185	9.987832	9.380354	10.619646	30
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	
30	9.368185	9.987832	9.380354	10.619646	30
31	9.368711	9.987801	9.380910	10.619090	29
32	9.369236	9.987771	9.381466	10.618534	28
33	9.369761	9.987740	9.382021	10.617980	27
34	9.370285	9.987710	9.382575	10.617425	26
35	9.370808	9.987679	9.383129	10.616871	25
36	9.371330	9.987649	9.383682	10.616318	24
37	9.371852	9.987618	9.384234	10.615766	23
38	9.372373	9.987588	9.384786	10.615214	22
39	9.372894	9.987557	9.385337	10.614663	21
40	9.373414	9.987526	9.385888	10.614112	20
41	9.373933	9.987496	9.386438	10.613562	19
42	9.374452	9.987465	9.386987	10.613013	18
43	9.374970	9.987434	9.387536	10.612464	17
44	9.375487	9.987403	9.388084	10.611916	16
45	9.376003	9.987372	9.388631	10.611369	15
46	9.376519	9.987341	9.389178	10.610822	14
47	9.377035	9.987310	9.389724	10.610276	13
48	9.377549	9.987279	9.390270	10.609730	12
49	9.378063	9.987248	9.390815	10.609185	11
50	9.378577	9.987217	9.391360	10.608640	10
51	9.379089	9.987186	9.391903	10.608097	9
52	9.379601	9.987155	9.392447	10.607553	8
53	9.380113	9.987124	9.392989	10.607011	7
54	9.380624	9.987092	9.393531	10.606469	6
55	9.381134	9.987061	9.394073	10.605927	5
56	9.381643	9.987030	9.394614	10.605386	4
57	9.382152	9.986998	9.395154	10.604846	3
58	9.382661	9.986967	9.395694	10.604306	2
59	9.383168	9.986936	9.396233	10.603767	1
60	9.383675	9.986904	9.396771	10.603229	0
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	
0	9.383675	9.986904	9.396771	10.603229	60
1	9.384181	9.986873	9.397309	10.602691	59
2	9.384687	9.986841	9.397846	10.602154	58
3	9.385192	9.986809	9.398383	10.601617	57
4	9.385697	9.986778	9.398919	10.601081	56
5	9.386201	9.986746	9.399455	10.600545	55
6	9.386704	9.986714	9.399990	10.600010	54
7	9.387207	9.986683	9.400524	10.599476	53
8	9.387709	9.986651	9.401058	10.598942	52
9	9.388210	9.986619	9.401591	10.598409	51
10	9.388711	9.986587	9.402124	10.597876	50
11	9.389211	9.986555	9.402656	10.597344	49
12	9.389711	9.986523	9.403187	10.596813	48
13	9.390210	9.986491	9.403718	10.596282	47
14	9.390708	9.986459	9.404249	10.595751	46
15	9.391206	9.986427	9.404778	10.595222	45
16	9.391703	9.986395	9.405308	10.594692	44
17	9.392199	9.986363	9.405836	10.594164	43
18	9.392695	9.986331	9.406364	10.593636	42
19	9.393190	9.986299	9.406892	10.593108	41
20	9.393685	9.986266	9.407419	10.592581	40
21	9.394179	9.986234	9.407945	10.592055	39
22	9.394673	9.986202	9.408471	10.591529	38
23	9.395166	9.986169	9.408996	10.591003	37
24	9.395658	9.986137	9.409521	10.590479	36
25	9.396150	9.986104	9.410045	10.589955	35
26	9.396641	9.986072	9.410569	10.589431	34
27	9.397131	9.986039	9.411092	10.588908	33
28	9.397621	9.986007	9.411615	10.588385	32
29	9.398111	9.985974	9.412137	10.587863	31
30	9.398600	9.985942	9.412658	10.587342	30
	Co-sine	Sine	Co-tan.	Tangent	M

75 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
30	9.398600	9.985942	9.412658	10.587342	
31	9.399088	9.985909	9.413179	10.586821	29
32	9.399575	9.985876	9.413699	10.586301	28
33	9.400062	9.985843	9.414219	10.585781	27
34	9.400549	9.985811	9.414738	10.585262	26
35	9.401035	9.985778	9.415257	10.584743	25
36	9.401520	9.985745	9.415775	10.584225	24
37	9.402005	9.985712	9.416293	10.583707	23
38	9.402489	9.985679	9.416810	10.583190	22
39	9.402972	9.985646	9.417326	10.582674	21
40	9.403455	9.985613	9.417842	10.582157	20
41	9.403938	9.985580	9.418358	10.581642	19
42	9.404420	9.985547	9.418873	10.581127	18
43	9.404901	9.985513	9.419387	10.580613	17
44	9.405382	9.985480	9.419901	10.580099	16
45	9.405862	9.985447	9.420415	10.579585	15
46	9.406341	9.985414	9.420927	10.579072	14
47	9.406820	9.985380	9.421440	10.578560	13
48	9.407299	9.985347	9.421951	10.578048	12
49	9.407777	9.985314	9.422463	10.577537	11
50	9.408254	9.985280	9.422973	10.577026	10
51	9.408731	9.985247	9.423484	10.576516	9
52	9.409207	9.985213	9.423993	10.576007	8
53	9.409682	9.985180	9.424503	10.575497	7
54	9.410157	9.985146	9.425011	10.574989	6
55	9.410632	9.985112	9.425519	10.574481	5
56	9.411106	9.985079	9.426027	10.573973	4
57	9.411579	9.985045	9.426534	10.573466	3
58	9.412052	9.985011	9.427041	10.572959	2
59	9.412524	9.984977	9.427547	10.572453	1
60	9.412996	9.984944	9.428054	10.571947	0
	Co-sine	Sine	Co-tan.	Tangent	M

75 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
0	9.412996	9.984944	9.428052	10.571947	60
1	9.413467	9.984910	9.428557	10.571442	59
2	9.413938	9.984876	9.429062	10.570938	58
3	9.414408	9.984842	9.429566	10.570434	57
4	9.414878	9.984808	9.430070	10.569930	56
5	9.415347	9.984774	9.430573	10.569427	55
6	9.415815	9.984740	9.431075	10.568925	54
7	9.416283	9.984706	9.431577	10.568423	53
8	9.416751	9.984672	9.432079	10.567921	52
9	9.417217	9.984637	9.432580	10.567420	51
10	9.417684	9.984603	9.433080	10.566920	50
11	9.418149	9.984569	9.433580	10.566419	49
12	9.418615	9.984535	9.434080	10.565920	48
13	9.419079	9.984500	9.434579	10.565421	47
14	9.419544	9.984466	9.435078	10.564922	46
15	9.420007	9.984432	9.435576	10.564424	45
16	9.420470	9.984397	9.436073	10.563927	44
17	9.420933	9.984363	9.436570	10.563430	43
18	9.421395	9.984328	9.437067	10.562933	42
19	9.421857	9.984293	9.437563	10.562437	41
20	9.422318	9.984259	9.438059	10.561941	40
21	9.422778	9.984224	9.438554	10.561446	39
22	9.423238	9.984189	9.439048	10.560952	38
23	9.423697	9.984155	9.439543	10.560457	37
24	9.424156	9.984120	9.440036	10.559964	36
25	9.424615	9.984085	9.440529	10.559471	35
26	9.425073	9.984050	9.441022	10.558978	34
27	9.425530	9.984015	9.441514	10.558486	33
28	9.425987	9.983980	9.442006	10.557994	32
29	9.426443	9.983945	9.442497	10.557503	31
30	9.426899	9.983910	9.442988	10.557012	30
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	Co-fine	Tangent	Co-tan.	
30	9.426899	9.983910	9.442988	10.557012	30
31	9.427354	9.983875	9.443479	10.556521	29
32	9.427809	9.983840	9.443968	10.556031	28
33	9.428263	9.983805	9.444458	10.555542	27
34	9.428717	9.983770	9.444947	10.555053	26
35	9.429170	9.983735	9.445435	10.554565	25
36	9.429623	9.983699	9.445923	10.554077	24
37	9.430075	9.983664	9.446411	10.553589	23
38	9.430527	9.983629	9.446898	10.553102	22
39	9.430978	9.983594	9.447384	10.552616	21
40	9.431429	9.983558	9.447870	10.552129	20
41	9.431879	9.983523	9.448356	10.551644	19
42	9.432328	9.983487	9.448841	10.551159	18
43	9.432778	9.983452	9.449326	10.550674	17
44	9.433226	9.983416	9.449810	10.550190	16
45	9.433675	9.983380	9.450294	10.549706	15
46	9.434122	9.983345	9.450777	10.549223	14
47	9.434569	9.983309	9.451260	10.548740	13
48	9.435016	9.983273	9.451743	10.548257	12
49	9.435462	9.983238	9.452225	10.547775	11
50	9.435908	9.983202	9.452706	10.547294	10
51	9.436353	9.983166	9.453187	10.546813	9
52	9.436798	9.983130	9.453668	10.546332	8
53	9.437242	9.983094	9.454148	10.545852	7
54	9.437686	9.983058	9.454628	10.545372	6
55	9.438129	9.983022	9.455107	10.544893	5
56	9.438572	9.982986	9.455586	10.544414	4
57	9.439014	9.982950	9.456064	10.543936	3
58	9.439456	9.982914	9.456542	10.543458	2
59	9.439897	9.982878	9.457019	10.542980	1
60	9.440338	9.982842	9.457496	10.542503	0
	Co-fine	Sine	Co-tan.	Tangent	M

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M	Sine	Co-sine	Tangent	Co-tan.	
0	9.440338	9.982842	9.457496	10.542503	60
1	9.440778	9.982805	9.457973	10.542027	59
2	9.441218	9.982769	9.458449	10.541551	58
3	9.441658	9.982733	9.458925	10.541075	57
4	9.442096	9.982696	9.459400	10.540600	56
5	9.442535	9.982660	9.459875	10.540125	55
6	9.442973	9.982624	9.460349	10.539651	54
7	9.443410	9.982587	9.460823	10.539177	53
8	9.443847	9.982550	9.461297	10.538703	52
9	9.444284	9.982514	9.461770	10.538230	51
10	9.444720	9.982477	9.462242	10.537758	50
11	9.445155	9.982441	9.462714	10.537285	49
12	9.445590	9.982404	9.463186	10.536814	48
13	9.446025	9.982367	9.463658	10.536342	47
14	9.446459	9.982330	9.464129	10.535871	46
15	9.446893	9.982294	9.464599	10.535401	45
16	9.447326	9.982257	9.465069	10.534931	44
17	9.447759	9.982220	9.465539	10.534461	43
18	9.448191	9.982183	9.466008	10.533992	42
19	9.448623	9.982146	9.466476	10.533523	41
20	9.449054	9.982109	9.466945	10.533055	40
21	9.449485	9.982072	9.467413	10.532587	39
22	9.449915	9.982035	9.467880	10.532120	38
23	9.450345	9.981998	9.468347	10.531653	37
24	9.450775	9.981961	9.468814	10.531186	36
25	9.451204	9.981923	9.469280	10.530720	35
26	9.451632	9.981886	9.469746	10.530254	34
27	9.452060	9.981849	9.470211	10.529789	33
28	9.452488	9.981812	9.470676	10.529324	32
29	9.452915	9.981774	9.471141	10.528859	31
30	9.453342	9.981737	9.471605	10.528395	30
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	
30	9.453342	9.981737	9.471605	10.528395	30
31	9.453768	9.981699	9.472068	10.527931	29
32	9.454194	9.981662	9.472532	10.527468	28
33	9.454619	9.981624	9.472995	10.527005	27
34	9.455044	9.981587	9.473457	10.526543	26
35	9.455469	9.981549	9.473919	10.526081	25
36	9.455892	9.981512	9.474381	10.525619	24
37	9.456316	9.981474	9.474842	10.525158	23
38	9.456739	9.981436	9.475303	10.524697	22
39	9.457162	9.981398	9.475763	10.524237	21
40	9.457584	9.981361	9.476223	10.523777	20
41	9.458006	9.981323	9.476683	10.523317	19
42	9.458427	9.981285	9.477142	10.522858	18
43	9.458848	9.981247	9.477601	10.522399	17
44	9.459268	9.981209	9.478059	10.521941	16
45	9.459688	9.981171	9.478517	10.521483	15
46	9.460108	9.981133	9.478975	10.521025	14
47	9.460527	9.981095	9.479432	10.520568	13
48	9.460946	9 981057	9.479889	10.520111	12
49	9 461364	9.981019	9.480345	10.519655	11
50	9.461782	9.980980	9.480801	10.519199	10
51	9.462199	9.980942	9.481257	10.518743	9
52	9.462616	9.980904	9.481712	10.518288	8
53	9.463032	9.980866	9.482167	10.517833	7
54	9.463448	9.980827	9.482621	10.517379	6
55	9.463864	9.980789	9.483075	10 516925	5
56	9.464279	9.980750	9 483528	10.516471	4
57	9.464694	9.980712	9.483982	10.516018	3
58	9.465108	9.980673	9.484434	10.515565	2
59	9.465522	9.980635	9.484887	10.515113	1
60	9.465935	9.980596	9.485339	10.514661	0
	Co-sine	Sine	Co-tan.	Tangent	M

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M	Sine	*Co-sine*	Tangent	*Co-tan.*	
0	9.465935	9.980596	9.485339	10.514661	60
1	9.466348	9.980558	9.485791	10.514209	59
2	9.466761	9.980519	9.486242	10.513758	58
3	9.467173	9.980480	9.486693	10.513307	57
4	9.467585	9.980441	9.487143	10.512857	56
5	9.467996	9.980403	9.487593	10.512407	55
6	9.468407	9.980364	9.488043	10.511957	54
7	9.468817	9.980325	9.488492	10.511507	53
8	9.469227	9.980286	9.488941	10.511059	52
9	9.469637	9.980247	9.489390	10.510610	51
10	9.470046	9.980208	9.489838	10.510162	50
11	9.470455	9.980169	9.490286	10.509714	49
12	9.470863	9.980130	9.490733	10.509267	48
13	9.471271	9.980091	9.491180	10.508820	47
14	9.471678	9.980052	9.491627	10.508373	46
15	9.472086	9.980012	9.492073	10.507927	45
16	9.472492	9.979973	9.492519	10.507481	44
17	9.472898	9.979934	9.492964	10.507035	43
18	9.473304	9.979894	9.493410	10.506590	42
19	9.473710	9.979855	9.493854	10.506145	41
20	9.474115	9.979816	9.494299	10.505701	40
21	9.474519	9.979776	9.494743	10.505257	39
22	9.474923	9.979737	9.495186	10.504813	38
23	9.475327	9.979697	9.495630	10.504370	37
24	9.475730	9.979658	9.496073	10.503927	36
25	9.476133	9.979618	9.496515	10.503485	35
26	9.476536	9.979578	9.496957	10.503043	34
27	9.476938	9.979539	9.497399	10.502601	33
28	9.477340	9.979499	9.497840	10.502159	32
29	9.477741	9.979459	9.498282	10.501718	31
30	9.478142	9.979419	9.498722	10.501278	30
	Co-sine	Sine	*Co-tan.*	Tangent	M

M	Sine	Co-fine	Tangent	Co-tan.	
30	9.478142	9.979419	9.498722	10.501278	30
31	9.478542	9.979380	9.499163	10.500837	29
32	9.478942	9.979340	9.499602	10.500397	28
33	9.479342	9.979300	9.500042	10.499958	27
34	9.479741	9.979260	9.500481	10.499519	26
35	9.480140	9.979220	9.500920	10.499080	25
36	9.480538	9.979180	9.501359	10.498641	24
37	9.480936	9.979140	9.501797	10.498203	23
38	9.481334	9.979099	9.502235	10.497765	22
39	9.481731	9.979059	9.502672	10.497328	21
40	9.482128	9.979019	9.503109	10.496891	20
41	9.482525	9.978979	9.503546	10.496454	19
42	9.482921	9.978939	9.503982	10.496018	18
43	9.483316	9.978898	9.504418	10.495582	17
44	9.483712	9.978858	9.504854	10.495146	16
45	9.484106	9.978817	9.505289	10.494711	15
46	9.484501	9.978777	9.505724	10.494276	14
47	9.484895	9.978736	9.506158	10.493841	13
48	9.485289	9.978696	9.506593	10.493407	12
49	9.485682	9.978655	9.507027	10.492973	11
50	9.486075	9.978615	9.507460	10.492540	10
51	9.486467	9.978574	9.507893	10.492107	9
52	9.486859	9.978533	9.508326	10.491674	8
53	9.487251	9.978493	9.508759	10.491241	7
54	9.487642	9.978452	9.509191	10.490809	6
55	9.488033	9.978411	9.509622	10.490377	5
56	9.488424	9.978370	9.510054	10.489946	4
57	9.488814	9.978329	9.510485	10.489515	3
58	9.489204	9.978288	9.510916	10.489084	2
59	9.489593	9.978247	9.511346	10.488654	1
60	9.489982	9.978206	9.511776	10.488224	0
	Co-fine	Sine	Co-tan.	Tangent	M

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M	Sine	Co-sine	Tangent	Co-tan.	
0	9.489982	9.978206	9.511776	10.488224	60
1	9.490371	9.978165	9.512206	10.487794	59
2	9.490759	9.978124	9.512635	10.487365	58
3	9.491147	9.978083	9.513064	10.486936	57
4	9.491534	9.978042	9.513493	10.486507	56
5	9.491922	9.978000	9.513921	10.486079	55
6	9.492308	9.977959	9.514349	10.485651	54
7	9.492695	9.977918	9.514777	10.485223	33
8	9.493080	9.977877	9.515204	10.484796	52
9	9.493466	9.977835	9.515631	10.484369	51
10	9.493851	9.977794	9.516057	10.483942	50
11	9.494236	9.977752	9.516484	10.483516	49
12	9.494620	9.977711	9.516910	10.483090	48
13	9.495005	9.977669	9.517335	10.482665	47
14	9.495388	9.977628	9.517761	10.482239	46
15	9.495771	9.977586	9.518185	10.481814	45
16	9.496154	9.977544	9.518610	10.481390	44
17	9.496537	9.977503	9.519034	10.480966	43
18	9.496919	9.977461	9.519458	10.480542	42
19	9.497301	9.977419	9.519882	10.480118	41
20	9.497682	9.977377	9.520305	10.479695	40
21	9.498063	9.977335	9.520728	10.479272	39
22	9.498444	9.977293	9.521151	10.478849	38
23	9.498824	9.977251	9.521573	10.478427	37
24	9.499204	9.977209	9.521995	10.478005	36
25	9.499584	9.977167	9.522417	10.477583	35
26	9.499963	9.977125	9.522838	10.477162	34
27	9.500342	9.977083	9.523259	10.476741	33
28	9.500720	9.977041	9.523679	10.476320	32
29	9.501099	9.976999	9.524100	10.475900	31
30	9.501476	9.976956	9.524520	10.475480	30
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	*Co-sine*	Tangent	*Co-tan.*	
30	9.501476	9.976956	9.524520	10.475480	30
31	9.501854	9.976914	9.524939	10.475060	29
32	9.502231	9.976872	9.525359	10.474641	28
33	9.502607	9.976830	9.525778	10.474222	27
34	9.502984	9.976787	9.526197	10.473803	26
35	9.503360	9.976745	9.526615	10.473385	25
36	9.503735	9.976702	9.527033	10.472967	24
37	9.504110	9.976660	9.527451	10.472549	23
38	9.504485	9.976617	9.527868	10.472132	22
39	9.504860	9.976574	9.528285	10.471715	21
40	9.505234	9.976532	9.528702	10.471298	20
41	9.505608	9.976489	9.529118	10.470881	19
42	9.505981	9.976446	9.529535	10.470465	18
43	9.506354	9.976404	9.529950	10.470049	17
44	9.506727	9.976361	9.530366	10.469634	16
45	9.507099	9.976318	9.530781	10.469219	15
46	9.507471	9.976275	9.531196	10.468804	14
47	9.507843	9.976232	9.531611	10.468389	13
48	9.508214	9.976189	9.532025	10.467975	12
49	9.508585	9.976146	9.532439	10.467561	11
50	9.508955	9.976103	9.532852	10.467147	10
51	9.509326	9.976060	9.533266	10.466734	9
52	9.509696	9.976017	9.533679	10.466321	8
53	9.510065	9.975974	9.534092	10.465908	7
54	9.510434	9.975930	9.534504	10.465496	6
55	9.510803	9.975887	9.534916	10.465084	5
56	9.511171	9.975844	9.535328	10.464672	4
57	9.511540	9.975800	9.535739	10.464261	3
58	9.511907	9.975757	9.536150	10.463849	2
59	9.512275	9.975713	9.536561	10.463439	1
60	9.512642	9.975670	9.536972	10.463028	0
	Co-sine	Sine	*Co-tan.*	Tangent	M

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M	Sine	Co-sine	Tangent	Co-tan.	
0	9.512642	9.975670	9.536972	10.463028	60
1	9.513009	9.975626	9.537382	10.462618	59
2	9.513375	9.975583	9.537792	10.462208	58
3	9.513741	9.975539	9.538202	10.461798	57
4	9.514107	9.975496	9.538611	10.461389	56
5	9.514472	9.975452	9.539020	10.460980	55
6	9,514837	9.975408	9.539429	10.460571	54
7	9,515202	9.975364	9.539837	10,460163	53
8	9.515566	9.975321	9.540245	10.459755	52
9	9.515930	9.975277	9.540653	10.459347	51
10	9.516294	9.975233	9.541061	10.458939	50
11	9.516657	9.975189	9.541468	10.458532	49
12	9.517020	9.975145	9.541875	10.458125	48
13	9.517382	9.975101	9.542281	10,457719	47
14	9.517745	9.975057	9.542688	10.457312	46
15	9.518107	9.975013	9.543094	10.456906	45
16	9.518468	9.974969	9.543499	10.456501	44
17	9.518829	9.974925	9.543905	10.456095	43
18	9.519190	9.974880	9 544310	10.455690	42
19	9.519551	9.974836	9.544715	10.455285	41
20	9.519911	9.974792	9.545119	10.454881	40
21	9.520271	9.974747	9.545524	10.454476	39
22	9.520631	9.974703	9.545927	10,454072	38
23	9.520990	9.974659	9.546331	10.453669	37
24	9.521349	9.974614	9.546735	10.453265	36
25	9.521707	9.974570	9.547138	10.452862	35
26	9.522065	9.974525	9.547540	10.452459	34
27	9.522423	9.974480	9.547943	10.452057	33
28	9.522781	9.974436	9.548345	10.451655	32
29	9.523138	9.974391	9.548747	10.451253	31
30	9.523495	9.974346	9.549149	10.450851	30
	Co-sine	Sine	Co-tan.	Tangent.	M

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M	Sine	Co-sine	Tangent	Co-tan.	
30	9.523495	9.974346	9.549149	10.450851	30
31	9.523852	9.974302	9.549550	10.450450	29
32	9.524208	9.974257	9.549951	10.450049	28
33	9.524564	9.974212	9.550352	10.449648	27
34	9.524920	9.974167	9.550752	10.449248	26
35	9.525275	9.974122	9.551152	10.448848	25
36	9.525630	9.974077	9.551552	10.448448	24
37	9.525984	9.974032	9.551952	10.448048	23
38	9.526339	9.973987	9.552351	10.447649	22
39	9.526693	9.973942	9.552750	10.447250	21
40	9.527046	9.973897	9.553149	10.446851	20
41	9.527400	9.973852	9.553548	10.446452	19
42	9.527753	9.973807	9.553946	10.446054	18
43	9.528105	9.973761	9.554344	10.445656	17
44	9.528458	9.973716	9.554741	10.445259	16
45	9.528810	9.973671	9.555139	10.444861	15
46	9.529161	9.973625	9.555536	10.444464	14
47	9.529513	9.973580	9.555933	10.444067	13
48	9.529864	9.973535	9.556329	10.443671	12
49	9.530214	9.973489	9.556725	10.443275	11
50	9.530565	9.973443	9.557121	10.442879	10
51	9.530915	9.973398	9.557517	10.442483	9
52	9.531265	9.973352	9.557912	10.442088	8
53	9.531614	9.973307	9.558308	10.441692	7
54	9.531963	9.973261	9.558702	10.441298	6
55	9.532312	9.973215	9.559097	10.440903	5
56	9.532661	9.973169	9.559491	10.440509	4
57	9.533009	9.973123	9.559885	10.440115	3
58	9.533357	9.973078	9.560279	10.439721	2
59	9.533704	9.973032	9.560673	10.439327	1
60	9.534052	9.972986	9.561066	10.438934	0
	Co-sine	Sine	Co-tan.	Tangent	M

70 DEG.

M	Sine	Co-ſine	Tangent	Co-tan.	
0	9.534052	9.972986	9.561066	10.438934	60
1	9.534399	9.972940	9.561459	10.438541	59
2	9.534745	9.972894	9.561851	10.438148	58
3	9.535091	9.972848	9.562244	10.437756	57
4	9.535437	9.972801	9.562636	10.437364	56
5	9.535783	9.972755	9.563028	10.436972	55
6	9.536129	9.972709	9.563419	10.436581	54
7	9.536474	9.972663	9.563811	10.436189	53
8	9.536818	9.972617	9.564202	10.435798	52
9	9.537163	9.972570	9.564592	10.435407	51
10	9.537507	9.972524	9.564983	10.435017	50
11	9.537851	9.972477	9.565373	10.434627	49
12	9.538194	9.972431	9.565763	10.434237	48
13	9.538537	9.972384	9.566153	10.433847	47
14	9.538880	9.972338	9.566542	10.433457	46
15	9.539223	9.972291	9.566932	10.433068	45
16	9.539565	9.972245	9.567320	10.432679	44
17	9.539907	9.972198	9.567709	10.432291	43
18	9.540249	9.972151	9.568097	10.431902	42
19	9.540590	9.972105	9.568486	10.431514	41
20	9.540931	9.972058	9.568873	10.431126	40
21	9.541272	9.972011	9.569261	10.430739	39
22	9.541612	9.971964	9.569648	10.430351	38
23	9.541953	9.971917	9.570035	10.429964	37
24	9.542292	9.971870	9.570422	10.429578	36
25	9.542632	9.971823	9.570809	10.429191	35
26	9.542971	9.971776	9.571195	10.428805	34
27	9.543310	9.971729	9.571581	10.428419	33
28	9.543649	9.971682	9.571967	10.428033	32
29	9.543987	9.971635	9.572352	10.427648	31
30	9.544325	9.971588	9.572738	10.427262	30
	Co-ſine	Sine	Co-tan.	Tangent	M

69 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
30	9.544325	9.971588	9.572738	10.427262	30
31	9.544663	9.971540	9.573123	10.426877	29
32	9.545000	9.971493	9.573507	10.426492	28
33	9.545338	9.971446	9.573892	10.426108	27
34	9.545674	9.971398	9.574276	10.425724	26
35	9.546011	9.971351	9.574660	10.425340	25
36	9.546347	9.971303	9.575044	10.424956	24
37	9.546683	9.971256	9.575427	10.424573	23
38	9.547019	9.971208	9.575810	10.424189	22
39	9.547354	9.971161	9.576193	10.423807	21
40	9.547689	9.971113	9.576576	10.423424	20
41	9.548024	9.971065	9.576958	10.423041	19
42	9.548358	9.971018	9.577341	10.422659	18
43	9.548693	9.970970	9.577723	10.422277	17
44	9.549026	9.970922	9.578104	10.421896	16
45	9.549360	9.970874	9.578486	10.421514	15
46	9.549693	9.970826	9.578867	10.421133	14
47	9.550026	9.970779	9.579248	10.420752	13
48	9.550359	9.970731	9.579628	10.420371	12
49	9.550692	9.970683	9.580009	10.419991	11
50	9.551024	9.970634	9.580389	10.419611	10
51	9.551355	9.970586	9.580769	10.419231	9
52	9.551687	9.970538	9.581149	10.418851	8
53	9.552018	9.970490	9.581528	10.418472	7
54	9.552349	9.970442	9.581907	10.418092	6
55	9.552680	9.970394	9.582286	10.417714	5
56	9.553010	9.970345	9.582665	10.417335	4
57	9.553340	9.970297	9.583043	10.416956	3
58	9.553670	9.970249	9.583422	10.416578	2
59	9.554000	9.970200	9.583800	10.416200	1
60	9.554329	9.970152	9.584177	10.415823	0
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	
0	9,554329	9,970152	9.584177	10,[illegible]822	60
1	9,554658	9,970103	9,584555	10,415445	59
2	9,554987	9,970055	9,584932	10,415068	58
3	9,555315	9,970006	9,585309	10,414691	57
4	9,555643	9,969957	9,585686	10,414314	56
5	9,555971	9,969909	9,586062	10,413938	55
6	9,556299	9,969860	9,586439	10,413561	54
7	9,556626	9,969811	9,586815	10,413185	53
8	9,556953	9,969762	9,587190	10,412810	52
9	9,557279	9,969713	9,587566	10,412434	51
10	9,557606	9,969665	9,587941	10,412059	50
11	9,557932	9,969616	9,588316	10,411684	49
12	9,558258	9,969567	9,588691	10,411309	48
13	9,558583	9,969518	9,589066	10,410934	47
14	9,558909	9,969469	9,589440	10,410560	46
15	9,559234	9,969419	9,589814	10,410186	45
16	9,559558	9,969370	9,590188	10,409812	44
17	9,559883	9,969321	9,590562	10,409438	43
18	9,560207	9,969272	9,590935	10,409065	42
19	9,560531	9,969223	9,591308	10,408692	41
20	9,560855	9,969173	9,591681	10,408319	40
21	9,561178	9,969124	9,592054	10.407946	39
22	9,561501	9,969075	9,592426	10.407574	38
23	9,561824	9,969025	9,592798	10.407201	37
24	9,562146	9,968976	9,593170	10.406829	36
25	9,562468	9,968926	9,593542	10.40[illegible]458	35
26	9,562790	9,968877	9,593914	10,406086	34
27	9,563112	9,968827	9,594285	10,405715	33
28	9,563433	9,968777	9,594656	10,405344	32
29	9,563754	9,968728	9,595027	10,404973	31
30	9,564075	9,968678	9,595397	10,404602	30
	Co-sine	Sine	Co tan.	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	
30	9,564075	9,968678	9,595397	10,404602	30
31	9,564396	9,968628	9,595768	10,404232	29
32	9,564716	9,968578	9,596138	10,403862	28
33	9,565036	9,968528	9,596508	10,403492	27
34	9,565356	9,968478	9,596878	10,403122	26
35	9,565675	9,968428	9,597247	10,402753	25
36	9,565995	9,968378	9,597616	10,402384	24
37	9,566314	9,968328	9,597985	10,402015	23
38	9,566632	9,968278	9,598354	10,401646	22
39	9,566951	9,968228	9,598722	10,401277	21
40	9,567269	9,968178	9,599091	10,400909	20
41	9,567587	9,968128	9,599459	10,400541	19
42	9,567904	9,968078	9,599827	10,400173	18
43	9,568222	9,968027	9,600194	10,399806	17
44	9,568539	9,967977	9,600562	10,399438	16
45	9,568855	9,967927	9,600929	10,399071	15
46	9,569172	9,967876	9,601296	10,398704	14
47	9,569488	9,967826	9,601662	10,398337	13
48	9,569804	9,967775	9,602029	10,397971	12
49	9,570120	9,967725	9,602395	10,397605	11
50	9,570435	9,967674	9,602761	10,397239	10
51	9,570751	9,967623	9,603127	10,396873	9
52	9,571065	9,967573	9,603493	10,396507	8
53	9,571380	9,967522	9,603858	10,396142	7
54	9,571695	9,967471	9,604223	10,395777	6
55	9,572009	9,967420	9,604588	10,395412	5
56	9,572322	9,967370	9,604953	10,395047	4
57	9,572636	9,967319	9,605317	10,394683	3
58	9,572949	9,967268	9,605682	10,394318	2
59	9,573263	9,967217	9,606046	10,393954	1
60	9,573575	9,967166	9,606409	10,393590	0
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	
0	9,573575	9,967166	9,606409	10,393590	60
1	9,573888	9,967115	9,606773	10,393227	59
2	9,574200	9,967064	9,607136	10,392863	58
3	9,574512	9,967012	9,607500	10,392500	57
4	9,574824	9,966961	9,607863	10,392137	56
5	9,575135	9,966910	9,608225	10,391774	55
6	9,575447	9,966859	9,608588	10,391412	54
7	9,575758	9,966807	9,608950	10,391050	53
8	9,576068	9,966756	9,609312	10,390688	52
9	9,576379	9,966705	9,609674	10,390326	51
10	9,576689	9,966653	9,610036	10,389964	50
11	9,576999	9,966602	9,610397	10,389603	49
12	9,577309	9,966550	9,610758	10,389241	48
13	9,577618	9,966499	9,611119	10,388880	47
14	9,577927	9,966447	9,611480	10,388520	46
15	9,578236	9,966395	9,611841	10,388159	45
16	9,578545	9,966344	9,612201	10,387799	44
17	9,578853	9,966292	9,612561	10,387438	43
18	9,579162	9,966240	9,612921	10,387078	42
19	9,579469	9,966188	9,613281	10,386719	41
20	9,579777	9,966136	9,613641	10,386359	40
21	9,580084	9,966084	9,614000	10,386000	39
22	9,580392	9,966032	9,614359	10,385641	38
23	9,580698	9,965980	9,614718	10,385282	37
24	9,581005	9,965928	9,615077	10,384923	36
25	9,581311	9,965876	9,615435	10,384565	35
26	9,581618	9,965824	9,615793	10,384207	34
27	9,581923	9,965772	9,616151	10,383848	33
28	9,582229	9,965720	9,616509	10,383491	32
29	9,582534	9,965668	9,616867	10,383133	31
30	9,582840	9,965615	9,617224	10,382776	30
	Co-sine	Sine	Co-tan.	Tangent	M

67 DEG.

M	Sine	Co-ſine	Tangent	Co-tan.	
30	9,582840	9,965615	9,617224	10,382776	30
31	9,583144	9,965563	9,617581	10,382418	29
32	9,583449	9,965511	9,617938	10,382061	28
33	9,583753	9,965458	9,618295	10,381705	27
34	9,584058	9,965406	9,618652	10,381348	26
35	9,584361	9,965353	9,619008	10,380992	25
36	9,584665	9,965301	9,619364	10,380635	24
37	9,584968	9,965248	9,619720	10,380279	23
38	9,585271	9,965195	9,620076	10,379924	22
39	9,585574	9,965143	9,620432	10,379568	21
40	9,585877	9,965090	9,620787	10,379213	20
41	9,586179	9,965037	9,621142	10,378858	19
42	9,586481	9,964984	9,621497	10,378503	18
43	9,586783	9,964931	9,621852	10,378148	17
44	9,587085	9,964878	9,622206	10,377793	16
45	9,587386	9,964825	9,622561	10,377439	15
46	9,587687	9,964772	9,622915	10,377085	14
47	9,587988	9,964719	9,623269	10,376731	13
48	9,588289	9,964666	9,623623	10,376377	12
49	9,588589	9,964613	9,623976	10,376024	11
50	9,588890	9,964560	9,624330	10,375670	10
51	9,589190	9,964507	9,624683	10,375317	9
52	9,589489	9,964454	9,625036	10,374964	8
53	9,589789	9,964400	9,625388	10,374612	7
54	9,590088	9,964347	9,625741	10,374259	6
55	9,590387	9,964294	9,626093	10,373907	5
56	9,590686	9,964240	9,626445	10,373555	4
57	9,590984	9,964187	9,626797	10,373203	3
58	9,591282	9,964133	9,627149	10,372851	2
59	9,591580	9,964080	9,627501	10,372499	1
60	9,591878	9,964026	9,627852	10,372148	0
	Co-ſine	Sine	Co-tan.	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	
0	9,591878	9,964026	9,627852	10,372148	60
1	9,592175	9,963972	9,628203	10,371797	59
2	9,592473	9,963919	9,628554	10,371446	58
3	9,592770	9,963865	9,628905	10,371095	57
4	9,593067	9,963811	9,629255	10,370745	56
5	9,593363	9,963757	9,629606	10,370394	55
6	9,593659	9,963703	9,629956	10,370044	54
7	9,593955	9,963650	9,630306	10,369694	53
8	9,594251	9,963596	9,630655	10,369344	52
9	9,594547	9,963542	9,631005	10,368995	51
10	9,594842	9,963488	9,631354	10,368645	50
11	9,595137	9,963433	9,631704	10,368296	49
12	9,595432	9,963379	9,632053	10,367947	48
13	9,595727	9,963325	9,632401	10,367598	47
14	9,596021	9,963271	9,632750	10,367250	46
15	9,596315	9,963217	9,633098	10,366901	45
16	9,596609	9,963162	9,633447	10,366553	44
17	9,596903	9,963108	9,633795	10,366205	43
18	9,597196	9,963054	9,634143	10,365857	42
19	9,597490	9,962999	9,634490	10,365510	41
20	9,597783	9,962945	9,634838	10,365162	40
21	9,598075	9,962890	9,635185	10,364815	39
22	9,598368	9,962836	9,635532	10,364468	38
23	9,598660	9,962781	9,635879	10,364121	37
24	9,598952	9,962726	9,636226	10,363774	36
25	9,599244	9,962672	9,636572	10,363428	35
26	9,599536	9,962617	9,636918	10,363081	34
27	9,599827	9,962562	9,637265	10,362735	33
28	9,600118	9,962507	9,637610	10,362389	32
29	9,600409	9,962453	9,637956	10,362044	31
30	9,600700	9,962398	9,638302	10,361698	30
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	
30	9.600700	9.962398	9.638302	10.361698	30
31	9.600990	9.962343	9.638647	10.361353	29
32	9.601280	9.962288	9.638992	10.361007	28
33	9.601570	9.962233	9.639337	10.360662	27
34	9.601860	9.962178	9.639682	10.360318	26
35	9.602149	9.962122	9.640027	10.359973	25
36	9.602439	9.962067	9.640371	10.359629	24
37	9.602728	9.962012	9.640716	10.359284	23
38	9.603017	9.961957	9.641060	10.358940	22
39	9.603305	9.961902	9.641404	10.358596	21
40	9.603594	9.961846	9.641747	10.358253	20
41	9.603882	9.961791	9.642091	10.357909	19
42	9.604170	9.961735	9.642434	10.357566	18
43	9.604457	9.961680	9.642777	10.357223	17
44	9.604745	9.961624	9.643120	10.356880	16
45	9.605032	9.961569	9.643463	10.356537	15
46	9.605319	9.961513	9.643806	10.356194	14
47	9.605606	9.961458	9.644148	10.355852	13
48	9.605892	9.961402	9.644490	10.355510	12
49	9.606179	9.961346	9.644832	10.355168	11
50	9.606465	9.961290	9.645174	10.354826	10
51	9.606750	9.961235	9.645516	10.354484	9
52	9.607036	9.961179	9.645857	10.354142	8
53	9.607322	9.961123	9.646199	10.353801	7
54	9.607607	9.961067	9.646540	10.353460	6
55	9.607892	9.961011	9.646881	10.353119	5
56	9.608176	9.960955	9.647222	10.352778	4
57	9.608461	9.960899	9.647562	10.352438	3
58	9.608745	9.960842	9.647903	10.352097	2
59	9.609029	9.960786	9.648243	10.351757	1
60	9.609313	9.960730	9.648583	10.351417	0
	Co-sine	Sine	Co-tan.	Tangent	M

66 D E G.

M	Sine	Co-sine	Tangent	Co-tan.	
0	9.609313	9.960730	9.648583	10.351417	60
1	9.609597	9.960674	9.648923	10.351077	59
2	9.609880	9.960617	9.649263	10.350737	58
3	9.610163	9.960561	9.649602	10.350398	57
4	9.610446	9.960505	9.649942	10.350058	56
5	9.610729	9.960448	9.650281	10.349719	55
6	9.611012	9.960392	9.650620	10.349380	54
7	9.611294	9.960335	9.650959	10.349041	53
8	9.611576	9.960279	9.651297	10.348703	52
9	9.611858	9.960222	9.651636	10.348364	51
10	9.612140	9.960165	9.651974	10.348026	50
11	9.612421	9.960109	9.652312	10.347688	49
12	9.612702	9.960052	9.652650	10.347350	48
13	9.612983	9.959995	9.652988	10.347012	47
14	9.613264	9.959938	9.653326	10.346674	46
15	9.613545	9.959881	9.653663	10.346337	45
16	9.613825	9.959824	9.654000	10.345999	44
17	9.614105	9.959768	9.654337	10.345662	43
18	9.614385	9.959710	9.654674	10.345325	42
19	9.614665	9.959653	9.655011	10.344989	41
20	9.614944	9.959596	9.655348	10.344652	40
21	9.615223	9.959539	9.655684	10.344316	39
22	9.615502	9.959482	9.656020	10.343980	38
23	9.615781	9.959425	9.656356	10.343643	37
24	9.616060	9.959367	9.656692	10.343308	36
25	9.616338	9.959310	9.657028	10.342972	35
26	9.616616	9.959253	9.657363	10.342636	34
27	9.616894	9.959195	9.657699	10.342301	33
28	9.617172	9.959138	9.658034	10.341966	32
29	9.617450	9.959080	9.658369	10.341631	31
30	9.617727	9.959023	9.658704	10.341296	30
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	
30	9.617727	9.959023	9.658704	10.341296	30
31	9.618004	9.958965	9.659039	10.340961	29
32	9.618281	9.958908	9.659373	10.340627	28
33	9.618558	9.958850	9.659708	10.340292	27
34	9.618834	9.958792	9.660042	10.339958	26
35	9.619110	9.958734	9.660376	10.339624	25
36	9.619386	9.958677	9.660710	10.339290	24
37	9.619662	9.958619	9.661043	10.338957	23
38	9.619938	9.958561	9.661377	10.338623	22
39	9.620213	9.958503	9.661710	10.338290	21
40	9.620488	9.958445	9.662043	10.337956	20
41	9.620763	9.958387	9.662376	10.337623	19
42	9.621038	9.958329	9.662709	10.337291	18
43	9.621313	9.958271	9.663042	10.336958	17
44	9.621587	9.958212	9.663374	10.336625	16
45	9.621861	9.958154	9.663707	10.336293	15
46	9.622135	9.958096	9.664039	10.335961	14
47	9.622409	9.958038	9.664371	10.335629	13
48	9.622682	9.957979	9.664703	10.335297	12
49	9.622956	9.957921	9.665035	10.334965	11
50	9.623229	9.957862	9.665366	10.334634	10
51	9.623502	9.957804	9.665697	10.334302	9
52	9.623774	9.957745	9.666029	10.333971	8
53	9.624047	9.957687	9.666360	10.333640	7
54	9.624319	9.957628	9.666691	10.333309	6
55	9.624591	9.957570	9.667021	10.332979	5
56	9.624863	9.957511	9.667352	10.332648	4
57	9.625134	9.957452	9.667682	10.332318	3
58	9.625406	9.957393	9.668012	10.331987	2
59	9.625677	9.957334	9.668343	10.331657	1
60	9.625948	9.957276	9.668672	10.331327	0
	Co-sine	Sine	Co-tan.	Tangent	M

65 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
0	9.625948	9.957276	9.668672	10.331327	60
1	9.626219	9.957217	9.669002	10.330998	59
2	9.626490	9.957158	9.669332	10.330668	58
3	9.626760	9.957099	9.669661	10.330339	57
4	9.627030	9.957040	9.669990	10.330009	56
5	9.627300	9.956981	9.670320	10.329680	55
6	9.627570	9.956922	9.670649	10.329351	54
7	9.627840	9.956862	9.670977	10.329022	53
8	9.628109	9.956803	9.671306	10.328694	52
9	9.628378	9.956744	9.671634	10.328365	51
10	9.628647	9.956684	9.671963	10.328037	50
11	9.628916	9.956625	9.672291	10.327709	49
12	9.629184	9.956565	9.672619	10.327381	48
13	9.629453	9.956506	9.672947	10.327053	47
14	9.629721	9.956447	9.673274	10.326725	46
15	9.629989	9.956387	9.673602	10.326398	45
16	9.630257	9.956327	9.673929	10.326070	44
17	9.630524	9.956268	9.674256	10.325743	43
18	9.630792	9.956208	9.674584	10.325416	42
19	9.631059	9.956148	9.674910	10.325089	41
20	9.631326	9.956088	9.675237	10.324763	40
21	9.631592	9.956029	9.675564	10.324436	39
22	9.631859	9.955969	9.675890	10.324110	38
23	9.632125	9.955909	9.676216	10.323783	37
24	9.632392	9.955849	9.676543	10.32345[illegible]	36
25	9.632657	9.955789	9.676869	10.323[illegible]31	35
26	9.632923	9.955729	9.677194	10.322805	34
27	9.633189	9.955669	9.677520	10.322480	33
28	9.633454	9.955609	9.677845	10.322154	32
29	9.633719	9.955548	9.678171	10.321829	31
30	9.633984	9.955488	9.678496	10.321504	30
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	*Co sine*	Tangent	*Co tan.*	
30	9,633984	9,955488	9,678496	10,321504	30
31	9,634249	9,955428	9,678821	10,321179	29
32	9,634514	9,955367	9,679146	10,320854	28
33	9,634778	9,955307	9,679471	10,320529	27
34	9,635042	9,955247	9,679795	10,320205	26
35	9,635306	9,955186	9,680120	10,319880	25
36	9,635570	9,955126	9,680444	10,319556	24
37	9,635833	9,955065	9,680768	10,319232	23
38	9,636097	9,955005	9,681092	10,318908	22
39	9,636360	9,954944	9,681416	10,318584	21
40	9,636623	9,954883	9,681740	10,318260	20
41	9,636886	9,954823	9,682063	10,317937	19
42	9,637148	9,954762	9,682386	10,317613	18
43	9,637411	9,954701	9,682710	10,317290	17
44	9,637673	9,954640	9,683033	10,316967	16
45	9,637935	9,954579	9,683356	10,316644	15
46	9,638197	9,954518	9,683678	10,316321	14
47	9,638458	9,954457	9,684001	10,315999	13
48	9,638720	9,954396	9,684324	10,315676	12
49	9,638981	9,954335	9,684646	10,315354	11
50	9,639242	9,954274	9,684968	10,315032	10
51	9,639503	9,954213	9,685290	10,314710	9
52	9,639764	9,954152	9,685612	10,314388	8
53	9,640024	9,954090	9,685934	10,314066	7
54	9,640284	9,954029	9,686255	10,313745	6
55	9,640544	9,953968	9,686577	10,313423	5
56	9,640804	9,953906	9,686898	10,313102	4
57	9,641064	9,953845	9,687219	10,312781	3
58	9,641323	9,953783	9,687540	10,312460	2
59	9,641583	9,953722	9,687861	10,312139	1
60	9,641842	9,953660	9,688182	10,311818	0
	Co sine	Sine	*Co-tan.*	Tangent	M

64 DEG.

M	Sine	Co-fine	Tangent	Co-tan.	
0	9.641842	9.953660	9.688182	10.311818	60
1	9,642101	9,953598	9,688502	10,311498	59
2	9,642360	9,953537	9,688823	10,311177	58
3	9,642618	9,953475	9,689143	10,310857	57
4	9,642876	9,953413	9,689463	10,310537	56
5	9,643135	9,953351	9,689783	10,310217	55
6	9,643393	9,953290	9,690103	10,309897	54
7	9,643650	9,953228	9,690423	10,309577	53
8	9,643908	9,953166	9,690742	10,309258	52
9	9,644165	9,953104	9,691062	10,308938	51
10	9,644423	9,953042	9,691381	10,308619	50
11	9,644680	9,952980	9,691700	10,308300	49
12	9,644936	9,952917	9,692019	10,307981	48
13	9,645193	9,952855	9,692338	10,307662	47
14	9,645449	9,952793	9,692656	10,307343	46
15	9,645706	9,952731	9,692975	10,307025	45
16	9,645962	9,952668	9,693293	10,306706	44
17	9,646218	9,952606	9,693612	10,306388	43
18	9,646473	9,952544	9,693930	10,306070	42
19	9,646729	9,952481	9,694248	10,305752	41
20	9,646984	9,952419	9,694566	10,305434	40
21	9,647239	9,952356	9,694883	10,305117	39
22	9,647494	9,952294	9,695201	10,304799	38
23	9,647749	9,952231	9,695518	10,304482	37
24	9,648004	9,952168	9,695835	10,304164	36
25	9,648258	9,952105	9,696153	10,303847	35
26	9,648512	9,952043	9,696470	10,303530	34
27	9,648766	9,951980	9,696786	10,303213	33
28	9,649020	9,951917	9,697103	10,302897	32
29	9,649274	9,951854	9,697420	10,302580	31
30	9,649527	9,951791	9,697736	10,302264	30
	Co-fine	Sine	Co-tan.	Tangent	M

63 DEG.

M	Sine	Co-sine	Tangent	Co tan.	
30	9,649527	9,951791	9,697736	10,302264	30
31	9.649781	9.951728	9.698052	10.301947	29
32	9.650034	9.951665	9.698369	10.301631	28
33	9.650287	9.951602	9.698685	10.301315	27
34	9.650539	9.951539	9.699001	10.300999	26
35	9.650792	9.951476	9.699316	10.300684	25
36	9.651044	9.951412	9.699632	10.300368	24
37	9.651296	9.951349	9.699947	10.300052	23
38	9.651548	9.951286	9.700263	10.299737	22
39	9.651800	9.951222	9.700578	10.299422	21
40	9.652052	9.951159	9.700893	10.299107	20
41	9.652303	9.951095	9.701208	10.298792	19
42	9.652555	9.951032	9.701523	10.298477	18
43	9.652806	9.950968	9.701837	10.298163	17
44	9.653057	9.950905	9.702152	10.297848	16
45	9.653307	9.950841	9.702466	10.297534	15
46	9.653558	9.950777	9.702780	10.297219	14
47	9.653808	9.950714	9.703095	10.296905	13
48	9.654059	9.950650	9.703409	10.296591	12
49	9.654309	9.950586	9.703722	10.296277	11
50	9.654558	9.950522	9.704036	10.295964	10
51	9.654808	9.950458	9.704350	10.295650	9
52	9.655057	9.950394	9.704663	10.295337	8
53	9.655307	9.950330	9.704976	10.295023	7
54	9.655556	9.950266	9.705290	10.294710	6
55	9.655805	9.950202	9.705603	10.294397	5
56	9.656053	9.950138	9.705915	10.294084	4
57	9.656302	9.950074	9.706228	10.293771	3
58	9.656550	9.950009	9.706541	10.293459	2
59	9.656799	9.949945	9.706853	10.293146	1
60	9.657047	9.949881	9.707166	10.292834	0
	Co-sine	Sine	Co-tan.	Tangent	M

63 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
0	9,657047	9,949881	9,707166	10,292834	60
1	9.657295	9.949816	9.707478	10.292523	59
2	9.657542	9.949752	9.707790	10.292210	58
3	9.657790	9.949687	9.708102	10.291898	57
4	9.658037	9.949623	9.708414	10.291586	56
5	9.658284	9.949558	9.708726	10.291274	55
6	9.658531	9.949494	9.709037	10.290962	54
7	9.658778	9.049429	9.709349	10.290651	53
8	9.659024	9.949364	9.709660	10.290340	52
9	9.659271	9.949300	9.709971	10.290029	51
10	9.659517	9.949235	9.710282	10.289718	50
11	9.659763	9.949170	9.710593	10.289407	49
12	9.660009	9.949105	9.710904	10.289096	48
13	9.660255	9.949040	9.711215	10.288785	47
14	9.660500	9.948975	9.711525	10.288475	46
15	9.660746	9.948910	9.711836	10.288164	45
16	9.660991	9.948845	9.712146	10.287854	44
17	9.661236	9.948780	9.712456	10.287544	43
18	9.661481	9.948715	9.712766	10.287234	42
19	9.661726	9.948650	9.713076	10.286924	41
20	9.661970	9.948584	9.713386	10.286614	40
21	9.662214	9.948519	9.713695	10.286304	39
22	9.662459	9.948453	9.714005	10.285995	38
23	9.662702	9.948388	9.714314	10.285686	37
24	9.662946	9.948323	9.714624	10.285376	36
25	9.663190	9.948257	9.714933	10.285067	35
26	9.663433	9.948191	9.715242	10.284758	34
27	9.663677	9.948126	9.715551	10.284449	33
28	9.663920	9.948060	9.715859	10.284140	32
29	9.664163	9.947995	9.716168	10.283832	31
30	9.664406	9.947929	9.716477	10.283523	30
	Co-sine	Sine	Co-tan.	Tangent	M

62 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
30	9.664406	9.947929	9.716477	10.283523	30
31	9.664648	9.947863	9.716785	10.283215	29
32	9.664891	9.947797	9.717093	10.282907	28
33	9.665133	9.947731	9.717401	10.282598	27
34	9.665375	9.947665	9.717709	10.282290	26
35	9.665617	9.947599	9.718017	10.281983	25
36	9.665858	9.947533	9.718325	10.281675	24
37	9.666100	9.947467	9.718633	10.281367	23
38	9.666341	9.947401	9.718940	10.281060	22
39	9.666583	9.947335	9.719248	10.280752	21
40	9.666824	9.947269	9.719555	10.280445	20
41	9.667065	9.947203	9.719862	10.280138	19
42	9.667305	9.947136	9.720169	10.279831	18
43	9.667546	9.947070	9.720476	10.279524	17
44	9.667786	9.947004	9.720783	10.279217	16
45	9.668026	9.946937	9.721089	10.278911	15
46	9.668266	9.946871	9.721396	10.278604	14
47	9.668506	9.946804	9.721702	10.278298	13
48	9.668746	9.946738	9.722008	10.277991	12
49	9.668986	9.946671	9.722315	10.277685	11
50	9.669225	9.946604	9.722621	10.277379	10
51	9.669464	9.946537	9.722927	10.277073	9
52	9.669703	9.946471	9.723232	10.276768	8
53	9.669942	9.946404	9.723538	10.276462	7
54	9.670181	9.946337	9.723843	10.276156	6
55	9.670419	9.946270	9.724149	10.275851	5
56	9.670657	9.946203	9.724454	10.275546	4
57	9.670896	9.946136	9.724759	10.275240	3
58	9.671134	9.946069	9.725065	10.274935	2
59	9.671372	9.946002	9.725369	10.274630	1
60	9.671609	9.945935	9.725674	10.274326	0
	Co-sine	Sine	Co tan.	Tangent	M

62 DEG.

M	Sine	Co-fine	Tangent	Co-tan.	
0	9.671609	9.945935	9.725674	10.274326	60
1	9.671847	9.945868	9.725979	10.274021	59
2	9.672084	9.945800	9.726284	10.273716	58
3	9.672321	9.945733	9.726588	10.273412	57
4	9.672558	9.945666	9.726892	10.273107	56
5	9.672795	9.945598	9.727197	10.272803	55
6	9.673032	9.945531	9.727501	10.272499	54
7	9.673268	9.945463	9.727805	10.272195	53
8	9.673505	9.945396	9.728109	10.271891	52
9	9.673741	9.945328	9.728412	10.271587	51
10	9.673977	9.945261	9.728716	10.271284	50
11	9.674213	9.945193	9.729020	10.270980	49
12	9.674448	9.945125	9.729323	10.270677	48
13	9.674684	9.945058	9.729626	10.270374	47
14	9.674919	9.944990	9.729929	10.270070	46
15	9.675154	9.944922	9.730232	10.269767	45
16	9.675389	9.944854	9.730535	10.269464	44
17	9.675624	9.944786	9.730838	10.269162	43
18	9.675859	9.944718	9.731141	10.268859	42
19	9.676094	9.944650	9.731443	10.268556	41
20	9.676328	9.944582	9.731746	10.268254	40
21	9.676562	9.944514	9.732048	10.267952	39
22	9.676796	9.944446	9.732351	10.267649	38
23	9.677030	9.944377	9.732653	10.267347	37
24	9.677264	9.944309	9.732955	10.267045	36
25	9.677497	9.944241	9.733257	10.266743	35
26	9.677731	9.944172	9.733558	10.266441	34
27	9.677964	9.944104	9.733860	10.266140	33
28	9.678197	9.944036	9.734162	10.265838	32
29	9.678430	9.943967	9.734463	10.265537	31
30	9.678663	9.943898	9.734764	10.265236	30
	Sine	Co-fine	Co-tan.	Tangent	M

61 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
30	9,678663	9,943898	9,734764	10,265236	60
31	9,678895	9,943830	9,735066	10,264934	29
32	9,679128	9,943761	9,735367	10,264633	28
33	9,679360	9,943692	9,735668	10,264332	27
34	9,679592	9,943624	9,735968	10,264031	26
35	9,679824	9,943555	9,736269	10,263731	25
36	9,680056	9,943486	9,736570	10,263430	24
37	9,680288	9,943417	9,736870	10,263130	23
38	9,680519	9,943348	9,737171	10,262829	22
39	9,680750	9,943279	9,737471	10,262529	21
40	9,680982	9,943210	9,737771	10,262229	20
41	9,681213	9,943141	9,738071	10,261929	19
42	9,681443	9,943072	9,738371	10,261629	18
43	9,681674	9,943003	9,738671	10,261329	17
44	9,681904	9,942933	9,738971	10,261029	16
45	9,682135	9,942864	9,739271	10,260729	15
46	9,682365	9,942795	9,739570	10,260430	14
47	9,682595	9,942725	9,739870	10,260130	13
48	9,682825	9,942656	9,740169	10,259831	12
49	9,683055	9,942587	9,740468	10,259532	11
50	9,683284	9,942517	9,740767	10,259233	10
51	9,683514	9,942448	9,741066	10,258934	9
52	9,683743	9,942378	9,741365	10,258635	8
53	9,683972	9,942308	9,741664	10,258336	7
54	9,684201	9,942239	9,741962	10,258038	6
55	9,684430	9,942169	9,742261	10,257739	5
56	9,684658	9,942099	9,742559	10,257441	4
57	9,684887	9,942029	9,742858	10,257142	3
58	9,685115	9,941959	9,743156	10,256844	2
59	9,685343	9,941889	9,743454	10,256546	1
60	9,685571	9,941819	9,743752	10,256248	0
	Co-sine	Sine	Co-tan.	Tangent	M

61 DEG.

M	Sine	*Co-sine*	Tangent	*Co-tan.*	
0	9.685571	9.941819	9.743752	10.256248	60
1	9.685799	9.941749	9.744050	10.255950	59
2	9.686027	9.941679	9.744348	10.255652	58
3	9.686254	9.941609	9.744645	10.255355	57
4	9.686481	9.941539	9.744943	10.255057	56
5	9.686709	9.941468	9.745240	10.254760	55
6	9.686936	9.941398	9.745538	10.254462	54
7	9.687163	9.941328	9.745835	10.254165	53
8	9.687389	9.941257	9.746132	10.253868	52
9	9.687616	9.941187	9.746429	10.253571	51
10	9.687842	9.941116	9.746726	10.253274	50
11	9.688069	9.941046	9.747023	10.252977	49
12	9.688295	9.940975	9.747319	10.252680	48
13	9.688521	9.940905	9.747616	10.252384	47
14	9.688747	9.940834	9.747912	10.252087	46
15	9.688972	9.940763	9.748209	10.251791	45
16	9.689198	9.940693	9.748505	10.251495	44
17	9.689423	9.940622	9.748801	10.251199	43
18	9.689648	9.940551	9.749097	10.250902	42
19	9.689873	9.940480	9.749393	10.250607	41
20	9.690098	9.940409	9.749689	10.250311	40
21	9.690323	9.940338	9.749985	10.250015	39
22	9.690548	9.940267	9.750281	10.249719	38
23	9.690772	9.940196	9.750576	10.249424	37
24	9.690996	9.940125	9.750872	10.249128	36
25	9.691220	9.940053	9.751167	10.248833	35
26	9.691444	9.939982	9.751462	10.248538	34
27	9.691668	9.939911	9.751757	10.248243	33
28	9.691892	9.939840	9.752052	10.247948	32
29	9.692115	9.939768	9.752347	10.247653	31
30	9.692339	9.939697	9.752642	10.247358	30
	Co-sine	Sine	*Co-tan.*	Tangent	M

60 DEG.

M	Sine	Co-fine	Tangent	Co-tan.	
30	9.692339	9.939697	9.752642	10.247358	30
31	9.692562	9.939625	9.752937	10.247063	29
32	9.692785	9.939554	9.753231	10.246769	28
33	9.693008	9.939482	9.753526	10.246474	27
34	9.[illegible]93[illegible]31	9.939410	9.753820	10.246180	26
35	9.693453	9.939339	9.754115	10.245885	25
36	9.693676	9.939267	9.754409	10.245591	24
37	9.693898	9.939195	9.754703	10.245297	23
38	9.694120	9.939123	9.754997	10.245003	22
39	9.694342	9.939051	9.755291	10.244709	21
40	9.694564	9.938980	9.755584	10.244415	20
41	9.694786	9.938908	9.755878	10.244122	19
42	9.695007	9.938835	9.756172	10.243828	18
43	9.695229	9.938763	9.756465	10.243535	17
44	9.695450	9.938691	9.756759	10.243241	16
45	9.695671	9.938619	9.757052	10.242948	15
46	9.695892	9.938547	9.757345	10.242655	14
47	9.696113	9.938475	9.757638	10.242362	13
48	9.696334	9.938402	9.757931	10.242069	12
49	9.696554	9.938330	9.758224	10.241776	11
50	9.696774	9.938257	9.758517	10.241[illegible]83	10
51	9.696995	9.938185	9.758810	10.241190	9
52	9.697215	9.938112	9.759102	10.240898	8
53	9.697435	9.938040	9.759395	10.240605	7
54	9.697654	9.937967	9.759687	10.240313	6
55	9.697874	9.937895	9.759979	10.240021	5
56	9.698093	9.937822	9.760271	10.239728	4
57	9.698313	9.937749	9.760564	10.239436	3
58	9.698532	9.937676	9.760856	10.239144	2
59	9.698751	9.937603	9.[illegible]61147	10.238852	1
60	9.6989[illegible]0	9.937531	9.761439	10.238561	0
	Co-fine	Sine	Co-tan.	Tangent	M

60 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
0	9.698970	9.937531	9.761439	10.238561	60
1	9,699189	9,937458	9,761731	10,238269	59
2	9,699407	9,937385	9,762023	10,237977	58
3	9,699626	9,937312	9,762314	10,237686	57
4	9,699844	9,937238	9,762606	10,237394	56
5	9,700062	9,937165	9,762897	10,237103	55
6	9,700280	9,937092	9,763188	10,236812	54
7	9,700498	9,937019	9,763479	10,236521	53
8	9,700716	9,936945	9,763770	10,236230	52
9	9,700933	9,936872	9,764061	10,235939	51
10	9,701151	9,936799	9,764352	10,235648	50
11	9,701368	9,936725	9,764643	10,235357	49
12	9,701585	9,936652	9,764933	10,235067	48
13	9,701802	9,936578	9,765224	10,234776	47
14	9,702019	9,936505	9,765514	10,234486	46
15	9,702236	9,936431	9,765805	10,234195	45
16	9,702452	9,936357	9,766095	10,233905	44
17	9,702669	9,936284	9,766385	10,233615	43
18	9,702885	9,936210	9,766675	10,233325	42
19	9,703101	9,936136	9,766965	10,233035	41
20	9,703317	9,936062	9,767255	10,232745	40
21	9,703533	9,935988	9,767545	10,232455	39
22	9,703748	9,935914	9,767834	10,232166	38
23	9,703964	9,935840	9,768124	10,231876	37
24	9,704179	9,935766	9,768413	10,231587	36
25	9,704395	9,935692	9,768703	10,231297	35
26	9,704610	9,935618	9,768992	10,231008	34
27	9,704825	9,935543	9,769281	10,230719	33
28	9,705040	9,935469	9,769570	10,230430	32
29	9,705254	9,935395	9,769859	10,230140	31
30	9,705469	9,935320	9,770148	10,229852	30
	Co-sine	Sine	Co-tan.	Tangent	M

59 DEG.

M	Sine	Co-*sine*	Tangent	Co-*tan.*	
30	9.705469	9.935320	9.770148	10.229852	30
31	9,705683	9,935246	9,770437	10,229563	29
32	9,705897	9,935171	9,770726	10,229274	28
33	9,706112	9,935097	9,771015	10,228985	27
34	9,706326	9,935022	9,771303	10,228697	26
35	9,706539	9,934948	9,771592	10,228408	25
36	9,706753	9,934873	9,771880	10,228120	24
37	9,706967	9,934798	9.772168	10,227832	23
38	9,707180	9,934723	9,772456	10,227543	22
39	9.707393	9 934649	9,772745	10,227255	21
40	9,707606	9,934574	9,773033	10,226967	20
41	9,707819	9,934499	9,773321	10,226679	19
42	9,708032	9,934424	9,773608	10,226391	18
43	9,708245	9,934349	9,773896	10,226104	17
44	9,708457	9,934274	9,774184	10,225816	16
45	9,708670	9,934199	9,774471	10,225529	15
46	9,708882	9,934123	9,774759	10,225241	14
47	9,709094	9,934048	9,775046	10,224954	13
48	9,709306	9,933973	9,775333	10,224666	12
49	9,709518	9,933897	9,775621	10,224379	11
50	9,709730	9,933822	9,775908	10,224092	10
51	9,709941	9,933747	9,776195	10,223805	9
52	9,710153	9,933671	9,776482	10,223518	8
53	9,710364	9,933596	9,776768	10,223232	7
54	9,710575	9,933520	9,777055	10,222945	6
55	9,710786	9,933444	9,777342	10,222658	5
56	9,710997	9,933369	9,777628	10,222372	4
57	9,711208	9,933293	9,777915	10,222085	3
58	9,711418	9,933217	9,778201	10,221799	2
59	9,711629	9,933141	9,778487	10,221513	1
60	9,711839	9,933066	9,778774	10,221226	0
	Co-*sine*	Sine	Co-*tan.*	Tangent	

M	Sine	Co-sine	Tangent	Co-tan.	
0	9,711839	9,933066	9,778774	10,221226	60
1	9,712049	9,932990	9,779060	10,220940	59
2	9,712259	9,932914	9,779346	10,220654	58
3	9,712469	9,932838	9,779632	10,220368	57
4	9,712679	9,932761	9,779918	10,220082	56
5	9,712889	9,932685	9,780203	10,219796	55
6	9,713098	9,932609	9,780489	10,219511	54
7	9,713308	9,932533	9,780775	10,219225	53
8	9,713517	9,932457	9,781060	10,218940	52
9	9,713726	9,932380	9,781346	10,218654	51
10	9,713935	9,932304	9,781631	10,218369	50
11	9,714144	9,932227	9,781916	10,218084	49
12	9,714352	9,932151	9,782201	10,217799	48
13	9,714561	9,932074	9,782486	10,217514	47
14	9,714769	9,931998	9,782771	10,217229	46
15	9,714977	9,931921	9,783056	10,216944	45
16	9,715186	9,931845	9,783341	10,216659	44
17	9,715394	9,931768	9,783626	10,216374	43
18	9,715601	9,931691	9,783910	10,216090	42
19	9,715809	9,931614	9,784195	10,215805	41
20	9,716017	9,931537	9,784479	10,215520	40
21	9,716224	9,931460	9,784764	10,215236	39
22	9,716431	9,931383	9,785048	10,214952	38
23	9,716639	9,931306	9,785332	10,214668	37
24	9,716846	9,931229	9,785616	10,214384	36
25	9,717053	9,931152	9,785900	10,214099	35
26	9,717259	9,931075	9,786184	10,213816	34
27	9,717466	9,930998	9,786468	10,213532	33
28	9,717672	9,930920	9,786752	10,213248	32
29	9,717879	9,930843	9,787036	10,212964	31
30	9,718085	9,930766	9,787319	10,212681	30
	Co-sine	Sine	Co-tan.	Tangent	M

58 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
30	9,718085	9,930766	9,787319	10,212681	30
31	9,718291	9,930688	9,787603	10,212397	29
32	9,718497	9,930611	9,787886	10,212114	28
33	9,718703	9,930533	9,788170	10,211830	27
34	9,718909	9,930456	9,788453	10,211547	26
35	9,719114	9,930378	9,788736	10,211264	25
36	9,719320	9,930300	9,789019	10,210981	24
37	9,719525	9,930223	9,789302	10,210698	23
38	9,719730	9,930145	9,789585	10,210415	22
39	9,719935	9,930067	9,789868	10,210132	21
40	9,720140	9,929989	9,790151	10,209849	20
41	9,720345	9,929911	9,790433	10,209566	19
42	9,720549	9,929833	9,790716	10,209284	18
43	9,720754	9,929755	9,790999	10,209001	17
44	9,720958	9,929677	9,791281	10,208719	16
45	9,721162	9,929599	9,791563	10,208436	15
46	9,721366	9,929521	9,791846	10,208154	14
47	9,721570	9,929442	9,792128	10,207872	13
48	9,721774	9,929364	9,792410	10,207590	12
49	9,721978	9,929286	9,792692	10,207308	11
50	9,722181	9,929207	9,792974	10,207026	10
51	9,722385	9,929129	9,793256	10,206744	9
52	9,722588	9,929050	9,793538	10,206462	8
53	9,722791	9,928972	9,793819	10,206180	7
54	9,722994	9,928893	9,794101	10,205899	6
55	9,723197	9,928814	9,794383	10,205617	5
56	9,723400	9,928736	9,794664	10,205336	4
57	9,723603	9,928657	9,794945	10,205054	3
58	9,723805	9,928578	9,795227	10,204773	2
59	9,724007	9,928499	9,795508	10,204492	1
60	9,724210	9,928420	9,795789	10,204211	0
	Co-sine	Sine	Co-tan.	Tangent	M

58 DEG.

M	Sine	*Co-sine*	Tangent	*Co-tan.*	
0	9,724210	9,928420	9,795789	10,204211	60
1	9,724412	9,928341	9,796070	10,203930	59
2	9,724614	9,928262	9,796351	10,203649	58
3	9,724816	9,928183	9,796632	10,203368	57
4	9,725017	9,928104	9,796913	10,203087	56
5	9,725219	9,928025	9,797194	10,202806	55
6	9,725420	9,927946	9,797474	10,202525	54
7	9,725622	9,927867	9,797755	10,202245	53
8	9,725823	9,927787	9,798036	10,201964	52
9	9,726024	9,927708	9,798316	10,201684	51
10	9,726225	9,927628	9,798596	10,201404	50
11	9,726426	9,927549	9,798877	10,201123	49
12	9,726626	9,927469	9,799157	10,200843	48
13	9,726827	9,927390	9,799437	10,200563	47
14	9,727027	9,927310	9,799717	10,200283	46
15	9,727228	9,927231	9,799997	10,200003	45
16	9,727428	9,927151	9,800277	10,199723	44
17	9,727628	9,927071	9,800557	10,199443	43
18	9,727828	9,926991	9,800836	10,199163	42
19	9,728027	9,926911	9,801116	10,198884	41
20	9,728227	9,926831	9,801396	10,198604	40
21	9,728427	9,926751	9,801675	10,198325	39
22	9,728626	9,926671	9,801955	10,198045	38
23	9,728825	9,926591	9,802234	10,197766	37
24	9,729024	9,926511	9,802513	10,197487	36
25	9,729223	9,926431	9,802792	10,197207	35
26	9,729422	9,926351	9,803072	10,196928	34
27	9,729621	9,926270	9,803351	10,196649	33
28	9,729820	9,926190	9,803630	10,196370	32
29	9,730018	9,926110	9,803908	10,196091	31
30	9,730216	9,926029	9,804187	10,195813	30
	Co-sine	Sine	*Co-tan.*	Tangent	M

M	Sine	*Co-sine*	Tangent	*Co-tan.*	
30	9.730216	9.926029	9.804187	10.195813	30
31	9,730415	9,925949	9,804466	10,195534	29
32	9,730613	9,925868	9,804745	10,195255	28
33	9,730811	9,925787	9,805023	10,194977	27
34	9,731009	9,925707	9,805302	10,194698	26
35	9,731206	9,925626	9,805580	10,194420	25
36	9,731404	9,925545	9,805859	10,194141	24
37	9,731601	9,925464	9,806137	10,193863	23
38	9,731799	9,925384	9,806415	10,193585	22
39	9,731996	9,925303	9,806693	10,193307	21
40	9,732193	9,925222	9,806971	10,193028	20
41	9,732390	9,925141	9,807249	10,192751	19
42	9,732587	9,925060	9,807527	10,192473	18
43	9,732784	9,924978	9,807805	10,192195	17
44	9,732980	9,924897	9,808083	10,191917	16
45	9,733177	9,924816	9,808361	10,191639	15
46	9,733373	9,924735	9,808638	10,191362	14
47	9,733569	9,924653	9,808916	10,191084	13
48	9,733765	9,924572	9,809193	10,190807	12
49	9,733961	9,924491	9,809471	10,190529	11
50	9,734157	9,924409	9,809748	10,190252	10
51	9,734353	9,924328	9,810025	10,189975	9
52	9,734548	9,924246	9,810302	10,189697	8
53	9,734744	9,924164	9,810580	10,189420	7
54	9,734939	9,924083	9,810857	10,189143	6
55	9,735134	9,924001	9,811134	10,188866	5
56	9,735330	9,923919	9,811410	10,188589	4
57	9,735525	9,923837	9,811687	10,188313	3
58	9,735719	9,923755	9,811964	10,188036	2
59	9,735914	9,923673	9,812241	10,187759	1
60	9,736109	9,923591	9,812517	10,187483	0
	Co-sine	Sine	*Co-tan.*	Tangent	M

57 DEG.

33 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
0	9.736109	9.923591	9.812517	10.187483	60
1	9.736303	9.923509	9.812794	10.187206	59
2	9.736497	9.923427	9.813070	10.186930	58
3	9.736692	9.923345	9.813347	10.186653	57
4	9.736886	9.923263	9.813623	10.186377	56
5	9.737080	9.923180	9.813899	10.186101	55
6	9.737274	9.923098	9.814175	10.185824	54
7	9.737467	9.923016	9.814452	10.185548	53
8	9.737661	9.922933	9.814728	10.185272	52
9	9.737854	9.922851	9.815004	10.184996	51
10	9.738048	9.922768	9.815279	10.184720	50
11	9.738241	9.922686	9.815555	10.184445	49
12	9.738434	9.922603	9.815831	10.184169	48
13	9.738627	9.922520	9.816107	10.183893	47
14	9.738820	9.922438	9.816382	10.183617	46
15	9.739013	9.922355	9.816658	10.183342	45
16	9.739205	9.922272	9.816933	10.183066	44
17	9.739398	9.922189	9.817209	10.182791	43
18	9.739590	9.922106	9.817484	10.182516	42
19	9.739783	9.922023	9.817759	10.182240	41
20	9.739975	9.921940	9.818035	10.181965	40
21	9.740167	9.921857	9.818310	10.181690	39
22	9.740359	9.921774	9.818585	10.181415	38
23	9.740550	9.921691	9.818860	10.181140	37
24	9.740742	9.921607	9.819135	10.180865	36
25	9.740934	9.921524	9.819410	10.180590	35
26	9.741125	9.921441	9.819684	10.180315	34
27	9.741316	9.921357	9.819959	10.180041	33
28	9.741507	9.921274	9.820234	10.179766	32
29	9.741698	9.921190	9.820508	10.179492	31
30	9.741889	9.921107	9.820783	10.179217	30
	Co-sine	Sine	Co-tan.	Tangent	M

56 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
30	9,741889	9,921107	9,820783	10,179217	30
31	9.742080	9.921023	9.821057	10.178943	29
32	9.742271	9.920939	9.821332	10.178668	28
33	9.742461	9.920855	9.821606	10.178394	27
34	9.742652	9.920772	9.821880	10.178120	26
35	9.742842	9.920688	9.822154	10.177846	25
36	9.743032	9.920604	9.822429	10.177571	24
37	9.743223	9.920520	9.822703	10.177297	23
38	9.743412	9.920436	9.822977	10.177023	22
39	9.743602	9.920352	9.823250	10.176749	21
40	9.743792	9.920268	9.823524	10.176476	20
41	9.743982	9.920184	9.823798	10.176202	19
42	9.744171	9.920099	9.824072	10.175928	18
43	9.744361	9.920015	9.824345	10.175655	17
44	9.744550	9.919931	9.824619	10.175381	16
45	9.744739	9.919846	9.824892	10.175107	15
46	9.744928	9.919762	9.825166	10.174834	14
47	9.745117	9.919677	9.825439	10.174560	13
48	9.745306	9.919593	9.825713	10.174287	12
49	9.745494	9.919508	9.825986	10.174014	11
50	9.745683	9.919424	9.826259	10.173741	10
51	9.745871	9.919339	9.826532	10.173468	9
52	9.746059	9.919254	9.826805	10.173195	8
53	9.746248	9.919169	9.827078	10.172922	7
54	9.746436	9.919084	9.827351	10.172649	6
55	9.746624	9.918999	9.827624	10.172376	5
56	9.746811	9.918915	9.827897	10.172103	4
57	9.746999	9.918830	9.828170	10.171830	3
58	9.747187	9.918744	9.828442	10.171558	2
59	9.747374	9.918659	9.828715	10.171285	1
60	9.747562	9.918574	9.828987	10.171012	0
	Co-sine	Sine	Co-tan.	Tangent	M

56 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
0	9.747562	9.918574	9.828987	10.171012	60
1	9.747749	9.918489	9.829260	10.170740	59
2	9.747936	9.918404	9.829532	10.170468	58
3	9.748123	9.918318	9.829805	10.170195	57
4	9.748310	9.918233	9.830077	10.169923	56
5	9.748497	9.918147	9.830349	10.169651	55
6	9.748683	9.918062	9.830621	10.169379	54
7	9.748870	9.917976	9.830893	10.169106	53
8	9.749056	9.917891	9.831165	10.168834	52
9	9.749242	9.917805	9.831437	10.168563	51
10	9.749429	9.917719	9.831709	10.168291	50
11	9.749615	9.917634	9.831981	10.168019	49
12	9.749801	9.917548	9.832253	10.167747	48
13	9.749986	9.917462	9.832525	10.167475	47
14	9.750172	9.917376	9.832796	10.167204	46
15	9.750358	9.917290	9.833068	10.166932	45
16	9.750543	9.917204	9.833339	10.166660	44
17	9.750729	9.917118	9.833611	10.166389	43
18	9.750914	9.917032	9.833882	10.166118	42
19	9.751099	9.916945	9.834154	10.165846	41
20	9.751284	9.916859	9.834425	10.165575	40
21	9.751469	9.916773	9.834696	10.165304	39
22	9.751654	9.916686	9.834967	10.165033	38
23	9.751838	9.916600	9.835238	10.164762	37
24	9.752023	9.916514	9.835509	10.164491	36
25	9.752207	9.916427	9.835780	10.164220	35
26	9.752392	9.916340	9.836051	10.163949	34
27	9.752576	9.916254	9.836322	10.163678	33
28	9.752760	9.916167	9.836593	10.163407	32
29	9.752944	9.916080	9.836864	10.163136	31
30	9.753128	9.915994	9.837134	10.162866	30
	Sine	Co-sine	Co-tan.	Tangent	M

55 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
30	9,753128	9,915994	9.837134	10.162866	30
31	9,753312	9,915907	9,837405	10,162595	29
32	9,753495	9,915820	9,837675	10,162325	28
33	9,753679	9,915733	9,837946	10,162054	27
34	9,753862	9,915646	9,838216	10,161784	26
35	9,754046	9,915559	9,838487	10,161513	25
36	9,754229	9,915472	9,838757	10,161243	24
37	9,754412	9,915385	9,839027	10,160973	23
38	9,754595	9,915297	9,839297	10,160702	22
39	9,754778	9,915210	9,839568	10,160432	21
40	9,754960	9,915123	9,839838	10,160162	20
41	9,755143	9,915035	9,840108	10,159892	19
42	9,755325	9,914948	9,840378	10,159622	18
43	9,755508	9,914860	9,840647	10,159352	17
44	9,755690	9,914773	9,840917	10,159083	16
45	9,755872	9,914685	9,841187	10,158813	15
46	9,756054	9,914597	9,841457	10,158543	14
47	9,756236	9,914510	9,841726	10,158273	13
48	9,756418	9,914422	9,841996	10,158004	12
49	9,756600	9,914334	9,842266	10,157734	11
50	9,756781	9,914246	9,842535	10,157465	10
51	9,756963	9,914158	9,842804	10,157195	9
52	9,757144	9,914070	9,843074	10,156926	8
53	9,757326	9,913982	9,843343	10,156657	7
54	9,757507	9,913894	9,843612	10,156387	6
55	9,757688	9,913806	9,843882	10,156118	5
56	9,757869	9,913718	9,844151	10,155849	4
57	9,758049	9,913630	9,844420	10,155580	3
58	9,758230	9,913541	9,844689	10,155311	2
59	9,758411	9,913453	9,844958	10,155042	1
60	9,758591	9,913364	9,845227	10,154773	0
	Co-sine	Sine	Co-tan.	Tangent	M

55 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
0	9,758591	9,913364	9,845227	10,154773	60
1	9,758772	9,913276	9,845496	10,154504	59
2	9,758952	9,913187	9,845764	10,154236	58
3	9,759132	9,913099	9,846033	10,153967	57
4	9,759312	9,913010	9,846302	10,153698	56
5	9,759492	9,912921	9,846570	10,153429	55
6	9,759672	9,912833	9,846839	10,153161	54
7	9,759851	9,912744	9,847107	10,152892	53
8	9,760031	9,912655	9,847376	10,152624	52
9	9,760210	9,912566	9,847644	10,152356	51
10	9,760390	9,912477	9,847913	10,152087	50
11	9,760569	9,912388	9,848181	10,151819	49
12	9,760748	9,912299	9,848449	10,151551	48
13	9,760927	9,912210	9,848717	10,151283	47
14	9,761106	9,912121	9,848985	10,151015	46
15	9,761285	9,912031	9,849254	10,150746	45
16	9,761464	9,911942	9,849522	10,150478	44
17	9,761642	9,911853	9,849789	10,150210	43
18	9,761821	9,911763	9,850057	10,149943	42
19	9,761999	9,911674	9,850325	10,149675	41
20	9,762177	9,911584	9,850593	10,149407	40
21	9,762356	9,911495	9,850861	10,149139	39
22	9,762534	9,911405	9,851128	10,148872	38
23	9,762712	9,911315	9,851396	10,148604	37
24	9,762889	9,911226	9,851664	10,148336	36
25	9,763067	9,911136	9,851931	10,148069	35
26	9,763245	9,911046	9,852199	10,147801	34
27	9,763422	9,910956	9,852466	10,147534	33
28	9,763599	9,910866	9,852733	10,147267	32
29	9,763777	9,910776	9,853001	10,146999	31
30	9,763954	9,910686	9,853268	10,146732	30
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	Co-fine	Tangent	Co-tan.	
30	9.763954	9.910686	9.853268	10.146732	30
31	9.764131	9.910596	9.853535	10.146465	29
32	9.764308	9.910506	9.853802	10.146198	28
33	9.764485	9.910415	9.854069	10.145930	27
34	9.764662	9.910325	9.854336	10.145664	26
35	9.764838	9.910235	9.854603	10.145397	25
36	9.765015	9.910144	9.854870	10.145130	24
37	9.765191	9.910054	9.855137	10.144863	23
38	9.765367	9.909963	9.855404	10.144596	22
39	9.765544	9.909873	9.855671	10.144329	21
40	9.765720	9.909782	9.855937	10.144062	20
41	9.765896	9.909691	9.856204	10.143796	19
42	9.766071	9.909601	9.856471	10.143529	18
43	9.766247	9.909510	9.856737	10.143263	17
44	9.766423	9.909419	9.857004	10.142996	16
45	9.766598	9.909328	9.857270	10.142730	15
46	9.766774	9.909237	9.857537	10.142463	14
47	9.766949	9.909146	9.857803	10.142197	13
48	9.767124	9.909055	9.858069	10.141931	12
49	9.767299	9.908964	9.858336	10.141664	11
50	9.767474	9.908873	9.858602	10.141398	10
51	9.767649	9.908781	9.858868	10.141132	9
52	9.767824	9.908690	9.859134	10.140866	8
53	9.767999	9.908599	9.859400	10.140600	7
54	9.768173	9.908507	9.859666	10.140334	6
55	9.768348	9.908416	9.859932	10.140068	5
56	9.768522	9.908324	9.860198	10.139802	4
57	9.768696	9.908233	9.860464	10.139536	3
58	9.768871	9.908141	9.860730	10.139270	2
59	9.769045	9.908049	9.860995	10.139005	1
60	9.769219	9.907958	9.861261	10.138739	0
	Co-fine	Sine	Co-tan.	Tangent	M

54 DEG.

M	Sine	Co-fine	Tangent	Co-tan.	
0	9.769219	9.907958	9.861261	10.138739	60
1	9,769392	9,907866	9,861527	10,138473	59
2	9,769566	9,907774	9,861792	10,138208	58
3	9,769740	9,907682	9,862058	10,137942	57
4	9,769913	9,907590	9,862323	10,137677	56
5	9,770087	9,907498	9,862589	10,137411	55
6	9,770260	9,907406	9,862854	10,137146	54
7	9,770433	9,907314	9,863119	10,136880	53
8	9,770606	9,907221	9,863385	10,136615	52
9	9,770779	9,907129	9,863650	10,136350	51
10	9,770952	9,907037	9,863915	10,136085	50
11	9,771125	9,906945	9,864180	10,135820	49
12	9,771298	9,906852	9,864445	10,135554	48
13	9,771470	9,906760	9,864710	10,135289	47
14	9,771643	9,906667	9,864975	10,135024	46
15	9,771815	9,906574	9,865240	10,134759	45
16	9,771987	9,906482	9,865505	10,134495	44
17	9,772159	9,906389	9,865770	10,134230	43
18	9,772331	9,906296	9,866035	10,133965	42
19	9,772503	9,906203	9,866300	10,133700	41
20	9,772675	9,906111	9,866564	10,133436	40
21	9,772847	9,906018	9,866829	10,133171	39
22	9,773018	9,905925	9,867094	10,132906	38
23	9,773190	9,905832	9,867358	10,132642	37
24	9,773361	9,905738	9,867623	10,132377	36
25	9,773533	9,905645	9,867887	10,132113	35
26	9,773704	9,905552	9,868152	10,131848	34
27	9,773875	9,905459	9,868416	10,131584	33
28	9,774046	9,905365	9,868680	10,131320	32
29	9,774217	9,905272	9,868945	10,131055	31
30	9,774388	9,905179	9,869209	10,130791	30
	Co-fine	Sine	Co-tan.	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	
30	9,774388	9,905179	9,869209	10,130791	30
31	9,774558	9,905085	9,869473	10,130527	29
32	9,774729	9,904992	9,869737	10,130263	28
33	9,774899	9,904898	9,870001	10,129999	27
34	9,775070	9,904804	9,870265	10,129735	26
35	9,775240	9,904711	9,870529	10,129471	25
36	9,775410	9,904617	9,870793	10,129207	24
37	9,775580	9,904523	9,871057	10,128943	23
38	9,775750	9,904429	9,871321	10,128679	22
39	9,775920	9,904335	9,871585	10,128415	21
40	9,776090	9,904241	9,871849	10,128151	20
41	9,776259	9,904147	9,872112	10,127888	19
42	9,776429	9,904053	9,872376	10,127624	18
43	9,776598	9,903959	9,872640	10,127360	17
44	9,776768	9,903864	9,872903	10,127097	16
45	9,776937	9,903770	9,873167	10,126833	15
46	9,777106	9,903676	9,873430	10,126570	14
47	9,777275	9,903581	9,873694	10,126306	13
48	9,777444	9,903487	9,873957	10,126043	12
49	9,777613	9,903392	9,874220	10,125780	11
50	9,777781	9,903298	9,874484	10,125516	10
51	9,777950	9,903203	9,874747	10,125253	9
52	9,778119	9,903108	9,875010	10,124990	8
53	9,778287	9,903013	9,875273	10,124727	7
54	9,778455	9,902919	9,875536	10,124464	6
55	9,778623	9,902824	9,875799	10,124200	5
56	9,778792	9,902729	9,876063	10,123937	4
57	9,778960	9,902634	9,876326	10,123674	3
58	9,779127	9,902539	9,876589	10,123411	2
59	9,779295	9,902444	9,876851	10,123149	1
60	9,779463	9,902349	9,877114	10,122885	0
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	
0	9.779463	9.902349	9.877114	10.122885	60
1	9,779631	9,902253	9,877377	10,122623	59
2	9,779798	9,902158	9,877640	10,122360	58
3	9,779965	9,902063	9,877903	10,122097	57
4	9,780133	9,901967	9,878165	10,121834	56
5	9,780300	9,901872	9,878428	10,121572	55
6	9,780467	9,901776	9,878691	10,121309	54
7	9,780634	9,901681	9,878953	10,121047	53
8	9,780801	9,901585	9,879216	10,120784	52
9	9,780968	9,901489	9,879478	10,120522	51
10	9,781134	9,901394	9,879741	10,120259	50
11	9,781301	9,901298	9,880003	10,119997	49
12	9,781467	9,901202	9,880265	10,119734	48
13	9,781634	9,901106	9,880528	10,119472	47
14	9,781800	9,901010	9,880790	10,119210	46
15	9,781966	9,900914	9,881052	10,118948	45
16	9,782132	9,900818	9,881314	10,118686	44
17	9,782298	9,900722	9,881576	10,118424	43
18	9,782464	9,900626	9,881839	10,118161	42
19	9,782630	9,900529	9,882101	10,117899	41
20	9,782796	9,900433	9,882363	10,117637	40
21	9,782961	9,900337	9,882625	10,117375	39
22	9,783127	9,900240	9,882886	10,117113	38
23	9,783292	9,900144	9,883148	10,116852	37
24	9,783457	9,900047	9,883410	10,116590	36
25	9,783623	9,899951	9,883672	10,116328	35
26	9,783788	9,899854	9,883934	10,116066	34
27	9,783953	9,899757	9,884195	10,115804	33
28	9,784118	9,899660	9,884457	10,115543	32
29	9,784282	9,899563	9,884719	10,115281	31
30	9,784447	9,899467	9,884980	10,115020	30
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	
30	9.784447	9.899467	9.884980	10.115020	30
31	9.784612	9.899370	9.885242	10.114758	29
32	9.784776	9.899273	9.885503	10.114497	28
33	9.784941	9.899175	9.885765	10.114235	27
34	9.785105	9.899078	9.886026	10.113974	26
35	9.785269	9.898981	9.886288	10.113712	25
36	9.785433	9.898884	9.886549	10.113451	24
37	9.785597	9.898787	9.886810	10.113190	23
38	9.785761	9.898689	9.887072	10.112928	22
39	9.785925	9.898592	9.887333	10.112667	21
40	9.786088	9.898494	9.887594	10.112406	20
41	9.786252	9.898397	9.887855	10.112145	19
42	9.786416	9.898299	9.888116	10.111884	18
43	9.786579	9.898201	9.888377	10.111623	17
44	9.786742	9.898104	9.888638	10.111361	16
45	9.786906	9.898006	9.888899	10.111100	15
46	9.787069	9.897908	9.889160	10.110840	14
47	9.787232	9.897810	9.889421	10.110579	13
48	9.787395	9.897712	9.889682	10.110318	12
49	9.787557	9.897614	9.889943	10.110057	11
50	9.787720	9.897516	9.890204	10.109796	10
51	9.787883	9.897418	9.890465	10.109535	9
52	9.788045	9.897320	9.890725	10.109275	8
53	9.788208	9.897222	9.890986	10.109014	7
54	9.788370	9.897123	9.891247	10.108753	6
55	9.788532	9.897025	9.891507	10.108493	5
56	9.788694	9.896926	9.891768	10.108232	4
57	9.788856	9.896828	9.892028	10.107972	3
58	9.789018	9.896729	9.892289	10.107711	2
59	9.789180	9.896631	9.892549	10.107451	1
60	9.789342	9.896532	9.892810	10.107190	0
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	Co-sine	Tangent	Co-tan.	
0	9.789342	9.896532	9.892810	10.107190	60
1	9.789504	9.896433	9.893070	10.106930	59
2	9.789665	9.896335	9.893330	10.106669	58
3	9.789827	9.896236	9.893591	10.106409	57
4	9.789988	9.896137	9.893851	10.106149	56
5	9.790149	9.896038	9.894111	10.105889	55
6	9.790310	9.895939	9.894371	10.105628	54
7	9.790471	9.895840	9.894632	10.105368	53
8	9.790632	9.895741	9.894892	10.105108	52
9	9.790793	9.895641	9.895152	10.104848	51
10	9.790954	9.895542	9.895412	10.104588	50
11	9.791115	9.895443	9.895672	10.104328	49
12	9.791275	9.895343	9.895932	10.104068	48
13	9.791436	9.895244	9.896192	10.103808	47
14	9.791596	9.895144	9.896452	10.103548	46
15	9.791756	9.895045	9.896712	10.103288	45
16	9.791917	9.894945	9.896971	10.103028	44
17	9.792077	9.894846	9.897231	10.102769	43
18	9.792237	9.894746	9.897491	10.102509	42
19	9.792397	9.894646	9.897751	10.102249	41
20	9.792557	9.894546	9.898010	10.101990	40
21	9.792716	9.894446	9.898270	10.101730	39
22	9.792876	9.894346	9.898530	10.101470	38
23	9.793035	9.894246	9.898789	10.101211	37
24	9.793195	9.894146	9.899049	10.100951	36
25	9.793354	9.894046	9.899308	10.100692	35
26	9.793513	9.893946	9.899568	10.100432	34
27	9.793673	9.893845	9.899827	10.100173	33
28	9.793832	9.893745	9.900086	10.099913	32
29	9.793991	9.893645	9.900346	10.099654	31
30	9.794149	9.893544	9.900605	10.099395	30
	Co-sine	Sine	Co-tan.	Tangent	M

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M	Sine	Co-sine	Tangent	Co-tan.	
30	9.794149	9.893544	9.900605	10.099395	30
31	9.794308	9.893444	9.900864	10.099135	29
32	9.794467	9.893343	9.901124	10.098876	28
33	9.794626	9.893243	9.901383	10.098617	27
34	9.794784	9.893142	9.901642	10.098358	26
35	9.794942	9.893041	9.901901	10.098099	25
36	9.795101	9.892940	9.902160	10.097839	24
37	9.795259	9.892839	9.902419	10.097580	23
38	9.795417	9.892738	9.902678	10.097321	22
39	9.795575	9.892637	9.902937	10.097062	21
40	9.795733	9.892536	9.903196	10.096803	20
41	9.795891	9.892435	9.903455	10.096544	19
42	9.796049	9.892334	9.903714	10.096285	18
43	9.796206	9.892233	9.903973	10.096027	17
44	9.796364	9.892132	9.904232	10.095768	16
45	9.796521	9.892030	9.904491	10.095509	15
46	9.796678	9.891929	9.904750	10.095250	14
47	9.796836	9.891827	9.905008	10.094991	13
48	9.796993	9.891726	9.905267	10.094733	12
49	9.797150	9.891624	9.905526	10.094474	11
50	9.797307	9.891522	9.905784	10.094215	10
51	9.797464	9.891421	9.906043	10.093957	9
52	9.797621	9.891319	9.906302	10.093698	8
53	9.797777	9.891217	9.906560	10.093440	7
54	9.797934	9.891115	9.906819	10.093181	6
55	9.798091	9.891013	9.907077	10.092923	5
56	9.798247	9.890911	9.907336	10.092664	4
57	9.798403	9.890809	9.907594	10.092406	3
58	9.798560	9.890707	9.907852	10.092147	2
59	9.798716	9.890605	9.908111	10.091889	1
60	9.798872	9.890503	9.908369	10.091631	0
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	Co-fine		Tangent	Co-tan.	
0	9,798872	9,890503		9,908369	10,091631	60
1	9.799028	9.890400		9.908627	10.091373	59
2	9.799184	9.890298		9.908886	10.091114	58
3	9.799339	9.890195		9.909144	10.090856	57
4	9.799495	9.890093		9.909402	10.090598	56
5	9.799651	9.889990		9.909660	10.090340	55
6	9.799806	9.889888		9.909918	10.090081	54
7	9.799961	9.889785		9.910176	10.089823	53
8	9.800117	9.889682		9.910435	10.089565	52
9	9.800272	9.889579		9.910693	10.089307	51
10	9.800427	9.889476		9.910951	10.089049	50
11	9.800582	9.889374		9.911209	10.088791	49
12	9.800737	9.889271		9.911467	10.088533	48
13	9.800892	9.889167		9.911724	10.088275	47
14	9.801047	9.889064		9.911982	10.088017	46
15	9.801201	9.888961		9.912240	10.087760	45
16	9.801356	9.888858		9.912498	10.087502	44
17	9.801510	9.888755		9.912756	10.087244	43
18	9.801665	9.888651		9.913014	10.086986	42
19	9.801819	9.888548		9.913271	10.086729	41
20	9.801973	9.888444		9.913529	10.086471	40
21	9.802127	9.888341		9.913787	10.086213	39
22	9.802282	9.888237		9.914044	10.085956	38
23	9.802435	9.888133		9.914302	10.085698	37
24	9.802589	9.888030		9.914560	10.085440	36
25	9.802743	9.887926		9.914817	10.085183	35
26	9.802897	9.887822		9.915075	10.084925	34
27	9.803050	9.887718		9.915332	10.084668	33
28	9.803204	9.887614		9.915590	10.084410	32
29	9.803357	9.887510		9.915847	10.084153	31
30	9.803510	9.887406		9.916104	10.083895	30
	Co-fine	Sine		Co-tan.	Tangent	M

M	Sine	*Co-sine*	Tangent	*Co-tan.*	
30	9.803510	9.887406	9.916104	10.083895	30
31	9,803664	9,887302	9,916362	10,083638	29
32	9,803817	9,887198	9,916619	10,083381	28
33	9,803970	9,887093	9,916876	10,083123	27
34	9,804123	9,886989	9,917134	10,082866	26
35	9,804276	9,886884	9,917391	10,082609	25
36	9,804428	9,886780	9,917648	10,082352	24
37	9,804581	9,886675	9,917905	10,082094	23
38	9,804734	9,886571	9,918163	10,081837	22
39	9,804886	9,886466	9,918420	10,081580	21
40	9,805038	9,886361	9,918677	10,081323	20
41	9,805191	9,886257	9,918934	10,081066	19
42	9,805343	9,886152	9,919191	10,080809	18
43	9,805495	9,886047	9,919448	10,080552	17
44	9,805647	9,885942	9,919705	10,080295	16
45	9,805799	9,885837	9,919962	10,080038	15
46	9,805951	9,885732	9,920219	10,079781	14
47	9,806103	9,885627	9,920476	10,079524	13
48	9,806254	9,885521	9,920733	10,079267	12
49	9,806406	9,885416	9,920990	10,079010	11
50	9,806557	9,885311	9,921247	10,078753	10
51	9,806709	9,885205	9,921503	10,078496	9
52	9,806860	9,885100	9,921760	10,078240	8
53	9,807011	9,884994	9,922017	10,077983	7
54	9,807162	9,884889	9,922274	10,077726	6
55	9,807314	9,884783	9,922530	10,077469	5
56	9,807464	9,884677	9,922787	10,077213	4
57	9,807615	9,884572	9,923044	10,076956	3
58	9,807766	9,884466	9,923300	10,076699	2
59	9,807917	9,884360	9,923557	10,076443	1
60	9,808067	9,884254	9,923813	10,076186	0
	Co-sine	Sine	*Co-tan.*	Tangent	M

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M	Sine	*Co-sine*	Tangent	*Co-tan.*	
0	9,808067	9,884254	9,923813	10,076186	60
1	9.808218	9.884148	9.924070	10.075930	59
2	9.808368	9.884042	9.924327	10.075673	58
3	9.808519	9.883936	9.924583	10.075417	57
4	9.808669	9.883829	9.924839	10.075160	56
5	9.808819	9.883723	9.925096	10.074904	55
6	9.808969	9.883617	9.925352	10.074647	54
7	9.809119	9.883510	9.925609	10.074391	53
8	9.809269	9.883404	9.925865	10.074135	52
9	9.809419	9.883297	9.926121	10.073878	51
10	9.809569	9.883191	9.926378	10 073622	50
11	9.809718	9.883084	9.926634	10.073366	49
12	9.809868	9.882977	9.926890	10.073110	48
13	9.810017	9.882871	9.927147	10.072853	47
14	9.810166	9.882764	9.927403	10.072597	46
15	9.810316	9.882657	9.927659	10.072341	.5
16	9.810465	9.882550	9.927915	10.072085	44
17	9.810614	9.882443	9.928171	10.071829	43
18	9.810763	9.882336	9.928427	10.071573	42
19	9.810912	9.882228	9.928683	10.071317	41
20	9.811061	9.882121	9.928940	10.071060	40
21	9.811210	9.882014	9.929196	10.070804	39
22	9.811358	9.881907	9,929452	10.070548	38
23	9.811507	9.881799	9.929708	10.070292	37
24	9.811655	9.881692	9.929964	10.070036	36
25	9.811804	9.881584	9.930219	10.069781	35
26	9.811952	9.881477	9.930475	10.069525	34
27	9.812100	9.881369	9.930731	10.069269	33
28	9.812248	9.881261	9.930987	10.069013	32
29	9.812396	9.881153	9.931243	10.068757	31
30	9.812544	9.881045	9.931499	10.068501	30
	Co-sine	Sine	*Co-tan.*	Tangent	M

M	Sine	Co-fine	Tangent	Co-tan.	
30	9,812544	9,881045	9.931499	10,068501	30
31	9.812692	9.880937	9.931755	10.068245	29
32	9.812840	9.880829	9.932010	10.067989	28
33	9.812988	9.880721	9.932266	10.067734	27
34	9.813135	9.880613	9.932522	10.067478	26
35	9.813283	9.880505	9.932778	10.067222	25
36	9.813430	9.880397	9.933033	10.066967	24
37	9.813578	9.880289	9.933289	10.066711	23
38	9.813725	9.880180	9.933545	10.066455	22
39	9.813872	9.880072	9.933800	10.066200	21
40	9.814019	9.879963	9.934056	10.065944	20
41	9.814166	9.879855	9.934311	10.065688	19
42	9.814313	9.879746	9.934567	10.065433	18
43	9.814460	9.879637	9.934822	10.065177	17
44	9.814607	9.879529	9.935078	10.064922	16
45	9.814753	9.879420	9.935333	10.064666	15
46	9.814900	9.879311	9.935589	10.064411	14
47	9.815046	9.879202	9.935844	10.064156	13
48	9.815193	9.879093	9.936100	10.063900	12
49	9.815339	9.878984	9.936355	10.063645	11
50	9.815485	9.878875	9.936610	10.063389	10
51	9.815631	9.878766	9.936866	10.063134	9
52	9.815777	9.878656	9.937121	10.062879	8
53	9.815923	9.878547	9.937376	10.062623	7
54	9.816069	9.878438	9.937632	10.062368	6
55	9.816215	9.878328	9.937887	10.062113	5
56	9.816361	9.878219	9.938142	10.061858	4
57	9.816506	9.878109	9.938397	10.061602	3
58	9.816652	9.877999	9.938653	10.061347	2
59	9.816797	9.877890	9.938908	10.061092	1
60	9.816943	9.877780	9.939163	10.060837	0
	Co-fine	Sine	Co-tan.	Tangent	M

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M	Sine	Co-sine	Tangent	Co-tan.	
0	9,816943	9,877780	9,939163	10,060837	60
1	9,817088	9,877670	9,939418	10,060582	59
2	9,817233	9,877560	9,939673	10,060327	58
3	9,817378	9,877450	9,939928	10,060072	57
4	9,817523	9,877340	9,940183	10,059816	56
5	9,817668	9,877230	9,940438	10,059562	55
6	9,817813	9,877120	9,940693	10,059306	54
7	9,817958	9,877009	9,940948	10,059051	53
8	9,818103	9,876899	9,941203	10,058796	52
9	9,818247	9,876789	9,941458	10,058542	51
10	9,818392	9,876678	9,941713	10,058287	50
11	9,818536	9,876568	9,941968	10,058032	49
12	9,818681	9,876457	9,942223	10,057777	48
13	9,818825	9,876347	9,942478	10,057522	47
14	9,818969	9,876236	9,942733	10,057267	46
15	9,819113	9,876125	9,942988	10,057012	45
16	9,819257	9,876014	9,943243	10,056757	44
17	9,819401	9,875904	9,943498	10,056502	43
18	9,819545	9,875793	9,943752	10,056248	42
19	9,819689	9,875682	9,944007	10,055993	41
20	9,819832	9,875571	9,944262	10,055738	40
21	9,819976	9,875459	9,944517	10,055483	39
22	9,820119	9,875348	9,944771	10,055229	38
23	9,820263	9,875237	9,945026	10,054974	37
24	9,820406	9,875125	9,945281	10,054719	36
25	9,820549	9,875014	9,945535	10,054464	35
26	9,820693	9,874903	9,945790	10,054210	34
27	9,820836	9,874791	9,946045	10,053955	33
28	9,820979	9,874679	9,946299	10,053701	32
29	9,821122	9,874568	9,946554	10,053446	31
30	9,821264	9,874456	9,946808	10,053192	30
	Co-sine	Sine	Co-tan.	Tangent	M

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M	Sine	Co-fine	Tangent	Co-tan.	
30	9.821264	9.874456	9.946808	10.053192	30
31	9,821407	9,874344	9,947063	10,052937	29
32	9,821550	9,874232	9,947317	10,052682	28
33	9,821692	9,874120	9,947572	10,052428	27
34	9,821835	9,874008	9,947826	10,052173	26
35	9,821977	9,873896	9,948081	10,051919	25
36	9,822120	9,873784	9,948335	10,051664	24
37	9,822262	9,873672	9,948590	10,051410	23
38	9,822404	9,873560	9,948844	10,051156	22
39	9,822546	9,873447	9,949099	10,050901	21
40	9,822688	9,873335	9,949353	10,050647	20
41	9,822830	9,873223	9,949607	10,050393	19
42	9,822972	9,873110	9,949862	10,050138	18
43	9,823114	9,872998	9,950116	10,049884	17
44	9,823255	9,872885	9,950370	10,049630	16
45	9,823397	9,872772	9,950625	10,049375	15
46	9,823538	9,872659	9,950879	10,049121	14
47	9,823680	9,872546	9,951133	10,048867	13
48	9,823821	9,872434	9,951388	10,048612	12
49	9,823962	9,872321	9,951642	10,048358	11
50	9,824104	9,872208	9,951896	10,048104	10
51	9,824245	9,872094	9,952150	10,047850	9
52	9,824386	9,871[illegible]1	9,952404	10,047595	8
53	9,824527	9,871868	9,952659	10,047341	7
54	9,824667	9,871755	9,952913	10,047087	6
55	9,824808	9,871641	9,953167	10,046833	5
56	9,824949	9,871528	9,953421	10,046579	4
57	9,825090	9,871414	9,953675	10,046325	3
58	9,825230	9,871301	9,953929	10,046071	2
59	9,825370	9,871187	9,954183	10,045817	1
60	9,825511	9,871073	9,954437	10,045562	0
	Co-fine	Sine	Co-tan.	Tangent	M

M	Sine	Co-fine	Tangent	Co-tan.	
0	9.825511	9.871073	9.954437	10.045562	60
1	9.825651	9.870960	9.954691	10.045308	59
2	9.825791	9.870846	9.954945	10.045054	58
3	9.825931	9.870732	9.955199	10.044800	57
4	9.826071	9.870618	9.955453	10.044546	56
5	9.826211	9.870504	9.955707	10.044292	55
6	9.826351	9.870390	9.955961	10.044038	54
7	9.826491	9.870275	9.956215	10.043784	53
8	9.826631	9.870161	9.956469	10.043531	52
9	9.826770	9.870047	9.956723	10.043277	51
10	9.826910	9.869933	9.956977	10.043023	50
11	9.827049	9.869818	9.957231	10.042769	49
12	9.827189	9.869704	9.957485	10.042515	48
13	9.827328	9.869589	9.957739	10.042261	47
14	9.827467	9.869474	9.957993	10.042007	46
15	9.827606	9.869360	9.958246	10.041753	45
16	9.827745	9.869245	9.958500	10.041500	44
17	9.827884	9.869130	9.958754	10.041246	43
18	9.828023	9.869015	9.959008	10.040992	42
19	9.828162	9.868900	9.959262	10.040738	41
20	9.828301	9.868785	9.959515	10.040485	40
21	9.828439	9.868670	9.959769	10.040231	39
22	9.828578	9.868555	9.960023	10.039977	38
23	9.828716	9.868439	9.960277	10.039723	37
24	9.828855	9.868324	9.960530	10.039469	36
25	9.828993	9.868209	9.960784	10.039216	35
26	9.829131	9.868093	9.961038	10.038962	34
27	9.829269	9.867978	9.961291	10.038708	33
28	9.829407	9.867862	9.961545	10.038455	32
29	9.829545	9.867747	9.961799	10.038201	31
30	9.829683	9.867631	9.962052	10.037947	30
	Co-fine	Sine	Co-tan.	Tangent	M

M	Sine	*Co-sine*	Tangent	*Co-tan.*	
30	9,829683	9,867631	9,962052	10,037947	30
31	9.829821	9.867515	9.962306	10.037694	29
32	9.829959	9.867399	9.962560	10.037440	28
33	9.830096	9.867283	9.962813	10.037187	27
34	9.830234	9.867167	9.963067	10.036933	26
35	9.830372	9.867051	9.963320	10.036680	25
36	9.830509	9.866935	9.963574	10.036426	24
37	9.830646	9.866819	9.963827	10.036173	23
38	9.830784	9.866703	9.964081	10.035919	22
39	9.830921	9.866586	9.964335	10.035665	21
40	9.831058	9.866470	9.964588	10.035412	20
41	9.831195	9.866353	9.964842	10.035158	19
42	9.831332	9.866237	9.965095	10.034905	18
43	9.831469	9.866120	9.965348	10.034651	17
44	9.831606	9.866004	9.965602	10.034398	16
45	9.831742	9.865887	9.965855	10.034144	15
46	9.831879	9.865770	9.966109	10.033891	14
47	9.832015	9.865653	9.966362	10.033638	13
48	9.832152	9.865536	9.966616	10.033384	12
49	9.832288	9.865419	9.966869	10.033131	11
50	9.832425	9.865302	9.967122	10.032878	10
51	9.832561	9.865185	9.967376	10.032624	9
52	9.832697	9.865068	9.967629	10.032371	8
53	9.832833	9.864950	9.967883	10.032117	7
54	9.832969	9.864833	9.968136	10.031864	6
55	9.833105	9.864715	9.968389	10.031611	5
56	9.833241	9.864598	9.968643	10.031357	4
57	9.833376	9.864480	9.968896	10.031104	3
58	9.833512	9.864363	9.969149	10.030851	2
59	9.833648	9.864245	9.969403	10.030597	1
60	9.833783	9.864127	9.969656	10.030344	0
	Co-sine	Sine	*Co-tan.*	Tangent	M

47 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
0	9.833783	9.864127	9.969656	10.030344	60
1	9,833919	9,864010	9,969909	10,030091	59
2	9,834054	9,863892	9,970162	10,029838	58
3	9,834189	9,863774	9,970416	10,029584	57
4	9,834324	9,863656	9,970669	10,029331	56
5	9,834460	9,863537	9,970922	10,029078	55
6	9,834595	9,863419	9,971175	10,028825	54
7	9,834730	9,863301	9,971428	10,028571	53
8	9,834865	9,863183	9,971682	10,028318	52
9	9,834999	9,863064	9,971935	10,028065	51
10	9,835134	9,862946	9,972188	10,027812	50
11	9,835269	9,862827	9,972441	10,027559	49
12	9,835403	9,862709	9,972694	10,027306	48
13	9,835538	9,862590	9,972948	10,027052	47
14	9,835672	9,862471	9,973201	10,026799	46
15	9,835806	9,862353	9,973454	10,026546	45
16	9,835941	9,862234	9,973707	10,026293	44
17	9,836075	9,862115	9,973960	10,026040	43
18	9,836209	9,861996	9,974213	10,025787	42
19	9,836343	9,861877	9,974466	10,025533	41
20	9,836477	9,861757	9,974719	10,025280	40
21	9,836611	9,861638	9,974973	10,025027	39
22	9,836745	9,861519	9,975226	10,024774	38
23	9,836878	9,861399	9,975479	10,024521	37
24	9,837012	9,861280	9,975732	10,024268	36
25	9,837146	9,861161	9,975985	10,024015	35
26	9,837279	9,861041	9,976238	10,023762	34
27	9,837412	9,860921	9,976491	10,023509	33
28	9,837546	9,860802	9,976744	10,023256	32
29	9,837679	9,860682	9,976997	10,023003	31
30	9,837812	9,860562	9,977250	10,022750	30
	Co-sine	Sine	Co-tan.	Tangent	M

M	Sine	Co-fine	Tangent	Co-tan.	
30	9.837812	9.860562	9.977250	10.022750	30
31	9.837945	9.860442	9.977503	10.022497	29
32	9.838078	9.860322	9.977756	10.022244	28
33	9.838211	9.860202	9.978009	10.021991	27
34	9.838344	9.860082	9.978262	10.021738	26
35	9.838477	9.859962	9.978515	10.021485	25
36	9.838609	9.859842	9.978768	10.021232	24
37	9.838742	9.859721	9.979021	10.020979	23
38	9.838875	9.859601	9.979274	10.020726	22
39	9.839007	9.859480	9.979527	10.020473	21
40	9.839140	9.859360	9.979780	10.020220	20
41	9.839272	9.859239	9.980033	10.019967	19
42	9.839404	9.859118	9.980285	10.019714	18
43	9.839536	9.858998	9.980538	10.019461	17
44	9.839668	9.858877	9.980791	10.019209	16
45	9.839800	9.858756	9.981044	10.018956	15
46	9.839932	9.858635	9.981297	10.018703	14
47	9.840064	9.858514	9.981550	10.018450	13
48	9.840196	9.858393	9.981803	10.018197	12
49	9.840328	9.858272	9.982056	10.017944	11
50	9.840459	9.858150	9.982309	10.017691	10
51	9.840591	9.858029	9.982562	10.017438	9
52	9.840722	9.857908	9.982814	10.017185	8
53	9.840854	9.857786	9.983067	10.016933	7
54	9.840985	9.857665	9.983320	10.016680	6
55	9.841116	9.857543	9.983573	10.016427	5
56	9.841247	9.857421	9.983826	10.016174	4
57	9.841378	9.857300	9.984079	10.015921	3
58	9.841509	9.857178	9.984331	10.015668	2
59	9.841640	9.857056	9.984584	10.015416	1
60	9.841771	9.856934	9.984837	10.015163	0
	Co-fine	Sine	Co-tan.	Tangent	M

46 DEG.

M	Sine	*Co-sine*	Tangent	*Co-tan.*	
0	9,841771	9,856934	9,984837	10,015163	60
1	9.841902	9.856812	9.985090	10.014910	59
2	9.842033	9.856690	9.985343	10.014657	58
3	9.842163	9.856568	9.985596	10.014404	67
4	9.842294	9.856445	9.985848	10.014151	56
5	9.842424	9.856323	9.986101	10.013899	55
6	9.842555	9.856201	9.986354	10.013646	54
7	9.842685	9.856078	9.986607	10.013393	53
8	9.842815	9.855956	9.986859	10.013140	52
9	9.842945	9.855833	9.987112	10.012888	51
10	9.843076	9.855710	9.987365	10.012635	50
11	9.843206	9.855588	9.987618	10.012382	49
12	9.843336	9.855465	9.987871	10.012129	48
13	9.843465	9.855342	9.988123	10.011877	47
14	9.843595	9.855219	9.988376	10.011624	46
15	9.843725	9.855096	9.988629	10.011371	45
16	9.843855	9.854973	9.988882	10.011118	44
17	9.843984	9.854850	9.989134	10.010866	43
18	9.844114	9.854727	9.989387	10.010613	42
19	9.844243	9.854603	9.989640	10.010360	41
20	9.844372	9.854480	9.989893	10.010107	40
21	9.844502	9.854356	9.990145	10.009855	39
22	9.844631	9.854233	9.990398	10.009602	38
23	9.844760	9.854109	9.990651	10.009349	37
24	9.844889	9.853986	9.990903	10.009096	36
25	9.845018	9.853862	9.991156	10.008844	35
26	9.845147	9.853738	9.991409	10.008591	34
27	9.845276	9.853614	9.991662	10.008338	33
28	9.845404	9.853490	9.991914	10.008086	32
29	9.845533	9.853366	9.992167	10.007833	31
30	9.845662	9.853242	9.992420	10.007580	30
	Co-sine	Sine	*Co tan.*	Tangent	M

45 DEG.

M	Sine	Co-sine	Tangent	Co-tan.	
30	9.845662	9.853242	9.992420	10.007580	30
31	9,845790	9,853118	9,992672	10,007328	29
32	9,845919	9,852994	9,992925	10,007075	28
33	9,846047	9,852869	9,993178	10,006822	27
34	9,846175	9,852745	9,993430	10,006569	26
35	9,846304	9,852620	9,993683	10,006317	25
36	9,846432	9,852496	9,993936	10,006064	24
37	9,846560	9,852371	9,994189	10,005811	23
38	9,846688	9,852246	9,994441	10,005559	22
39	9,846816	9,852122	9,994694	10,005306	21
40	9,846944	9,851997	9,994947	10,005053	20
41	9,847071	9,851872	9,995199	10,004801	19
42	9,847199	9,851747	9,995452	10,004548	18
43	9,847327	9,851622	9,995705	10,004295	17
44	9,847454	9,851497	9,995957	10,004043	16
45	9,847582	9,851372	9,996210	10,003790	15
46	9,847709	9;851246	9,996463	10,003537	14
47	9,847836	9,851121	9,996715	10,003285	13
48	9,847964	9,850996	9,996968	10,003032	12
49	9,848091	9,850870	9,997221	10,002779	11
50	9,848218	9,850745	9,997473	10,002527	10
51	9,848345	9,850619	9,997726	10,002274	9
52	9,848472	9,850493	9,997979	10,002021	8
53	9,848599	9,850367	9,998231	10,001769	7
54	9,848726	9,850242	9,998484	10,001516	6
55	9,848852	9,850116	9,998737	10,001263	5
56	9,848979	9,849990	9,998989	10,001011	4
57	9,849106	9,849864	9,999242	10,000758	3
58	9.849232	9,849737	9,999495	10,000505	2
59	9,849359	9,849611	9,999747	10,000253	1
60	9,849485	9,849485	10,000000	10,000000	0
	Co-sine	Sine	Co-tan.	Tangent	M

Problem.

To find the logarithmic ſine of an angle (a) leſs than 1′, or 60″.

Rule.

As 60″ - - - - - - - - *log.* 1,778151

To a - - - - - - - - - *log.*

So is the ſine of 60″, or 1′ - - - - 6,463726

To the ſine of *a*″ . - - - -

Becauſe very ſmall arches are proportional to their ſines.

F I N I S.