
ANT NUMERANTUR AVORUM
Silvester sc. 27 Strand London

THE

SYSTEM OF THE WORLD.

BY

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TRANSLATED FROM THE FRENCH

BY

J. POND, F.R.S.

IN TWO VOLUMES.

VOL. I.

LONDON:

PRINTED FOR RICHARD PHILLIPS, BRIDGE STREET, BLACKFRIARS.

1809.

W. Flint, Printer, Old Bailey.

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VOLUME THE FIRST.

BOOK II.

ADVERTISEMENT.

In this work I shall adopt the division of the quadrant into a hundred degrees, each degree being divided into a hundred minutes, and each minute into an hundred seconds. Temperature will be referred to the mercurial thermometer divided into one hundred degrees from the freezing to the boiling point of water, under a pressure equivalent to the weight of a column of mercury of the height of sixty-six centimetres. And I shall refer all linear measures to the metre, determined by the arc of the terrestrial meridian included between Dunkirk and Barcelona.

NOTE—BY THE TRANSLATOR.

The reader will find the angular measures and measures of time used by the author reduced in the margin to the sexigesimal system adopted in this country; this was thought better than altering the text of an original work of such importance.

ERRATA.

Vol. I. p. 126, *for* sexigonal, *read* sexigesimal.

Vol. II. p. 93, line 2, *for* 120^{g} *read* 2° and its reduction $1^{\circ}48'$.

Table of French Measures.

	English inches
Millimetre - -	**0.03937**
Centimetre - -	**9.39371**
Decimetre - -	**3.93710**
Metre, 3.281 feet - -	**39.37100**
Decametre - -	**393.71000**
Hecatometre - -	**3937.10000**
Chiliometre - -	**39371.00000**
Myrcometre - -	**393710.00000**
The old French foot -	**12.78933**

THE

SYSTEM OF THE WORLD.

> Me vero primum dulces ante omnia musæ
> Quarum sacra fero, ingenti perculsus amore
> Accipiant, cœlique vias, et sidera monstrent.

Of all the natural sciences, Astronomy is that which presents the longest series of discoveries. There is an immense distance from the first view of the heavens, to that general view by which, at the present day, we comprehend the past and future state of the system of the world. To arrive at this it was necessary to observe the heavenly bodies during a long succession of ages, to recognize from their appearances the real motion of the Earth, to develope the laws of the planetary motions, and from these laws to derive the principles of universal gravitation, and to redescend from this principle to the complete investigation of all the celestial phenomena, even in their minutest details. This is what the human understanding has accomplished in astronomy. The exposition of these discoveries, and of the most simple manner in which they may arise one from the other, would have the double advantage of presenting a great assemblage of important truths, and the true method which should be followed in investigating the laws of nature. This is the object I propose in the following work.

BOOK THE FIRST.

OF THE APPARENT MOTIONS OF THE HEAVENLY BODIES.

CHAP. I.

Of the diurnal Motion of the Heavens.

If in a fine night, and in a place where the horizon is uninterrupted, we follow with attention the appearance of the heavens, it will be seen to vary at every instant. Some stars are rising above, others setting below the horizon; some begin to appear in the east, others disappear towards the west; several, as the pole star and the stars of the Great Bear, never reach the horizon. In these various motions, their respective positions to each other remain unchanged, and they describe circles, so much the less as they are nearer a point, which seems to be immoveable. Thus the heavens appear to revolve round two fixed points, called from this circumstance, the POLES of the world, and in this motion is included the whole system of stars. The pole elevated above our horizon, is the north pole; the opposite pole, which we imagine beneath the horizon, is the south pole. Already several interesting questions present themselves to be resolved. What becomes during the day of the stars which we have seen in the night? From whence come those which begin to appear? Where are those gone, which have departed from our view? An attentive examination of these phenomena will afford a simple answer to these questions. In the morning, the brightness of the stars grows fainter as the dawning light increases, in the evening they become more brilliant as the twilight diminishes; it is not therefore because they cease to shine, but because they are effaced by the more vivid light of the sun, that we are unable to see them. The fortunate discovery of the telescope has enabled us to verify this explanation by shewing us the stars, even when the sun is at its greatest elevation above the horizon. Those which are near enough the pole never to reach the horizon appear constantly above it.

CHAP. II.

Of the Sun and of its proper Motion.

ALL the heavenly bodies participate in the diurnal motion of the celestial sphere ; but many have proper motions of their own, which it is interesting to follow, because they alone can conduct us to the knowledge of the system of the world. In the same manner as in measuring the distance of an object, we observe it in two different positions, so, to discover the laws of nature, we must consider her under different points of view, and observe the development of those laws in the change of appearance which she presents to us. Upon the earth we vary these phenomena by experiments ; in the heavens, we determine with care, all those which the celestial motions present to us. In thus interrogating nature, and submitting her answers to analysis, we can, by a train of reasoning and induction skilfully managed, arrive at the causes of these phenomena; that is to say, we can reduce them to general laws, from which the phenomena are derived. It is to discover these laws and to reduce them to the least possible number, that all our efforts should tend; for the first causes, and the intimate nature of beings, will be to us eternally unknown.

Of all the heavenly bodies which appear to have a motion of their own, the most remarkable is the Sun. Its proper motion in a contrary direction to the diurnal motion, manifests itself by the appearance of the heavens during the night, which changes and is renewed with the seasons. The stars situated in the path of the Sun and which set a little time after him, soon are lost in his light, and at length reappear before his rising; the Sun therefore advances towards them in a direction contrary to his diurnal motion; it is thus that for a long time his proper motion was examined, but at present this motion is determined with great precision by observing every day the meridian altitude of the sun, and the interval of time which elapses between his passage and that of the stars over the meridian. We have thus the motion of the Sun in the direction of the meridian and likewise in the direction of the parallels; the resulting motion obtained by the combination of these two is its true motion. In this manner it has been found that the Sun moves in an orbit which at the commencement of 1750 was inclined to the equator * 26° 0796, which orbit has been named the Ecliptic. It is by the combination of the proper motion of the Sun with its diurnal motion that the changes of the seasons are produced. The points of intersection with the ecliptic and equator are called the equinoxes, for in fact the Sun in these two points describing the equator by its diurnal motion and this circle being divided into two equal parts by all the horizons, the day is then equal to the night in every

* 23° 28′ 18″.

part of the earth. In proportion as the Sun, in leaving the equinox of spring, advances in its orbit, its meridian altitudes above our horizon increase more and more ; the visible arc of the parallels it describes every day, continually augments, and increases the length of the days till the Sun has attained its greatest altitude. At this epoch the days are the longest in the year, and because near this *maximum* the variations of the meridian altitude of the Sun are insensible, the Sun (considering only that altitude on which the length of the days depend) appears stationary, for which reason this point of the *maximum* has been named the summer *solstice*. The parallels which the the Sun describes on that day is called the *tropic* of summer. It then redescends towards the equator which it traverses again at the autumnal equinox, and from thence it arrives at its minimum of altitude, or at the winter *solstice*. The parallel then described by the sun, is the winter *tropic*, and the day is the shortest of the year ;

arrived at this term the Sun again ascends and returns to the vernal equinox. Such is the constant progress of the Sun and of the seasons. The spring is the season comprised between the vernal equinox and the summer solstice; the interval from this solstice to the autumnal equinox forms the summer. Autumn is the interval from this equinox to the winter solstice; and the interval from this solstice to the vernal equinox, is winter.

The presence of the Sun above the horizon being the cause of heat, it might be imagined that the temperature would be the same in summer as in spring, and in the autumn as in the winter, but the temperature is not the instantaneous effect of the presence of the Sun, but the result of its long continued action. It does not in a day produce its maximum of effect till some time after his greatest altitude above the horizon, nor in a year till the solstitial altitude is passed.

The different altitudes of the pole in different climates produce in the seasons those remarkable varieties which we will now examine, from the equator to the poles. At the equator the poles are in the horizon, which there cuts all the parallels in two equal parts, the day is therefore always equal to the night, and at the equinoxes the Sun passes through the zenith of the place. The meridian altitudes of the Sun at the solstices are the least, and equal to the complement of the inclination of the ecliptic to the equator. The solar shadows in these two positions of the Sun are diametrically opposite, a circumstance which never occurs in our climates, where at noon they are always directed towards the north.

At the equator, therefore, properly speaking, there are two summers and two winters every year. The same thing takes place in every country, where the height of the pole is less than the obliquity of the ecliptic. Beyond this limit there is only one summer and one winter in every year, the Sun never reaches the zenith, the longest day in summer augments, and the shortest in winter diminishes, as we approach the pole; and when the zenith is only distant from it by a quantity equal to the obliquity of the ecliptic to the equator, the Sun never sets on the day of the Summer solstice, nor rises at the winter solstice; still nearer the poles, the time of its presence and of its absence on the horizon exceeds several days and even months. Finally under the pole the horizon being the equator itself, the Sun is always above the horizon when on the same side of the equator, and always below it when on the opposite side; so that there is but one day and one night throughout the year.

The intervals which separate the equinoxes and the solstices are not equal: that from the vernal equinox to the autumnal is about seven days longer, than from the autumnal to the vernal; the proper motion of the Sun, therefore, is not uniform;—accurate and multiplied observations have taught us, that it is the most rapid in a certain point in the solar orbit, situated near the winter solstice, and slowest in an opposite point situated near the summer solstice; The Sun describes in a day * 1°1327, in the first point, and only † 1°0591 in the second; thus during the course of a year the Sun's daily motion varies from the greatest to the least, by three hundred and thirty-six thousandths of its mean value.

To obtain the law of this variation, and in general that of all the periodical inequalities, the following consideration has been made use of, since the sines and cosines of angles become the same at every circumference to which they arrive, they are proper to represent these inequalities; in this manner, therefore, all the inequalities of the celestial motions have been expressed, and it only remains to separate these inequalities from each other, and to determine the angles on which they depend. In this manner it has been

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* 1° 1′ 9″9. † 57′ 11″ 5.

found that the variation of the angular velocity of the Sun is very nearly proportional to the cosine of its mean angular distance from the point where its velocity is the greatest.

It is natural to think that the distance of the Sun from the Earth varies, with its angular velocity, and it has been proved to do so by the measures of its apparent diameter. This augments and diminishes in the same time and according to the same law as this velocity, but in a ratio only half as great; when the velocity is greatest the apparent diameter is * 6035″,7; when the velocity is least it is only † 5836″,3, thus its mean diameter is ‡ 5936″; this quantity should be diminished a few seconds, to allow for the effect of irradiation which dilates a little the apparent diameters of luminous bodies.

The distance of the Sun from the Earth, being reciprocally as its apparent diameter, its increase follows the same law as the

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* 32′ 35″ 5 † 28′ 49″ ‡ 30. 42,25.

diminution of its diameter. We name *Perigee* the point of the orbit in which the Sun is nearest to the Earth, and *Apogee* the opposite point in which it is the most remote. It is in the first of these points that the Sun has both the greatest apparent diameter, and the greatest velocity; at the second point both the diameter and the velocity are at a minimum. To explain the diminution of the apparent distance of the Sun it is sufficient to suppose it farther from the Earth, but if the variation in its motion arose from this cause alone, and if the real velocity of the Earth in its orbit was constant, its apparent velocity should be diminished in the same proportion as its apparent diameter, but it diminishes in a ratio twice as great; there is therefore an actual retardation in the motion of the Sun, as it recedes from the Earth. By the combined effect of this retardation of the velocity and augmentation of the distance, the angular motion in a day diminishes as the square of the distance increases, so that its product by this square is very nearly constant.

All the measures of the apparent diameter of the Sun, compared with the observations of its daily motion, confirm this result.

Let us imagine a straight line joining the centres of the earth and Sun, and call it the RADIUS VECTOR of the Sun, it is easy to see that the small sector, or area traced in a day by this radius round the Earth, is proportional to the product of the square of this area by the daily motion of the Sun: thus this area is constant, and the whole area traced by the radius vector, reckoning from a fixed radius, increases as the number of days elapsed from the epoch on which the Sun was upon this radius. From hence results this remarkable law in the motion of the Sun; namely that, *The area described by its radius vector are proportional to the times.*

If, from these data, we every day set down the position and length of the radius vector of the solar orbit, and then make a curve pass through the extremities of these radii, we shall perceive that this curve is not exactly circular, but that it is somewhat elongated in the direction of the straight line which, passing through the centre of the Earth, joins the points of the greatest and least distance of the Sun. The resemblance of this curve to an ellipse, having led to a comparison with it, their identity has been recognised, from whence this conclusion has been established, *that the solar orbit is an ellipse, of which the centre of the earth occupies one of the foci.* The ellipse is one of those celebrated curves, both in ancient and modern geometry, which being formed by the section of a surface of the cone by a plane, have been called *conic sections.* It is easy to describe it by fixing the extremities of a thread upon two immovable points, which thread being stretched upon a plane by a point which slides along it, the curve traced by the point in this motion is an ellipse, it is evidently elongated in the direction of the right line which joins the foci, and which being prolonged on each side till it meets the curve forms thegreater axis,whose length is equal to that of the thread. The greater axis divides the ellipse into two equal and similar parts,the lesser axis is the straight line drawn through the centre and prolonged each way till it meets the curve; the distance from the centre to one of the foci, is the *excentricity* of the ellipse; when the two foci become united in one point the ellipse becomesa circle; by separating them the ellipse gradually lengthens, and when the distance of the foci becomes infinite, the distance of the focus to the nearest summit of the curve remains finite, and the ellipse becomes a parabola.

The solar ellipse differs but little from a circle, for its excentricity is evidently the excess of the greatest above the mean distance of the Sun from the Earth; which excess we have seen is equal to one hundred and sixty-eight ten-thousandths of this distance. Observations seem to indicate in this excentricity, a very slow diminution, and scarcely perceptible in a century.

To have a just idea of the elliptic motion of the Sun, let us conceive a point moved uniformly in the circumference of a circle, of which the centre is that of the Earth, and whose radius is equal to the distance perigee of the Sun.

Suppose, moreover, that this point and the Sun, set off together from the perigee, and that the angular motion of the point is equal to the mean angular motion of the Sun. Whilst the radius vector of the point revolves uniformly round the Earth, the radius vector of the Sun moves in an unequal manner, by forming always with the distance perigee, and the arcs of the ellipse, sectors proportional to the times. At first it will precede the radius vector of the point, and make an angle with it, which after having augmented to a certain limit will diminish and become nothing; when the Sun is at the apogee, then the two radii will coincide with the greater axis. In the second half of the ellipse the radius vector of the point will precede the radius vector of the Sun and form with it angles, which are exactly the same as in the first half, at the same distance from the perigee, where it will again coincide with the radius vector of the Sun and the greater axis of the ellipse. The angle by which the radius vector of the Sun precedes the radius vector of the point is called the *equation of the centre*; its maximum, or greatest equation of the centre, was in 1750 equal * 2°1409. The angular motion of the point round the earth is concluded from the length of an entire revolution of the Sun in its orbit; by applying to this the equation of the centre, we obtain the angular motion of the Sun. The investigation of this equation is an interesting problem in analysis, which cannot be solved but by approximation; but the small excentricity of the solar orbit, leads to series, which converge rapidly, and are easily reduced to the form of tables.

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* 1 55′ 36″ 4.

The position of the greater axis of the solar ellipse is not constant. The angular distance of the perigee, to the equinox of spring reckoned in the direction of the Sun's motion was in the beginning of 1750 = * 309° 5790, but it has relatively to the fixed stars an annual motion of about † 36″7, in the same direction as that of the Sun.

The solar orbit approaches by insensible degrees to the equator; the secular diminution of its obliquity to the plane of this great circle may be estimated at about ‡ 154″ 3.

The elliptic motion of the Sun, will not exactly represent modern observations; their extreme precision has discovered small inequalities of which it would have been almost impossible by observation alone to have developed the laws. The investigation of these inequalities belongs to that branch of astronomy, which re-descends from

* 278° 37′ 16″. † 11″ 9. ‡ 50″.

causes to phenomena, and which will form the object of the fourth book.

The distance of the Sun to the Earth has interested astronomers at every period. They have endeavoured to measure it by all the means which astronomy has successively indicated. The most natural and the most simple is that which geometricians apply to measure the distance of terrestrial objects. From the two extremities of a known base they observe the angles which it makes with the visual rays of the object, deducting their sumfrom two right angles; they obtain the angles which these rays make at their point of union; this angle is what is called the parallax of the object, whose distance from the extremities of the base is by this means easily deduced.

In applying this method to the Sun, it is desirable to choose a base of the greatest extent that can be taken on the surface of the Earth. Let us imagine two observers placed under the same meridian and observing at the same instant the meridian altitude of the centre of the Sun and its distance from the same pole; the difference of these two observed distances, will be the angle, which the straight line joining the observers would subtend if seen from the centre of the Sun. The position of the observers gives this straight line, in parts of the terrestrial radius; it will therefore be easy to conclude from these observations the angle which the semi-diameter of the earth would subtend as seen from the centre of the Sun. This angle is called the parallax of the Sun, but it is too small to be determined by this method, which can only enable us to judge that its distance is at least six thousand diameters of the earth. We shall see hereafter, that astronomical discoveries furnish other methods much more exact of determining the parallax of the Sun, which we now know to be very nearly * 27″,2, at the mean distance of the

* 8″ 8.

Sun from the Earth, from whence it follows that this distance is 23405 times the radius of the earth. The smallness of the parallax of the Sun proves its immense magnitude; we are certain that at the same distance at which this luminary is seen under an angle of * 5936″, the earth would not appear under an angle of † 100 seconds, and the volumes of spherical bodies being as the cubes of their diameters, the volume of the Sun is at least two hundred thousand times greater than that of the Earth; it is about thirteen hundred thousand times bigger if, as the observations seem to indicate, the parallax is only ‡ 27″2.

Black spots are observed at the surface of the Sun of an irregular form, their number, position and magnitude are very variable—they are often very numerous and of considerable extent; some have been observed whose diameter exceeded four or five times that of the Earth; sometimes,

* 32′ 3″ 2. † 32″ 4. ‡ 9″.

though rarely, the Sun has appeared pure and withoutspots for several years together. The solar spots are almost always surrounded by a penumbra, which is inclosed in a cloud of light, more brilliant than the rest of the Sun, and in the midst of which the spots are seen to form and disappear. All this indicates that at the surface of this enormous mass of fire, vivid effervescences take place, of which our volcanoes form but a feeble representation. But whatever be the nature of the solar spots, they have made us acquainted with a remarkable phenomenon—the rotation of the Sun.

Amidst all their variations we can discover regular motions, which are exactly the same as the corresponding points of the surface of the Sun, if we suppose it to have a motion of rotation on an axis almost perpendicular to the ecliptic in the direction of its motion round the earth. From the continued observation of these spots the rotation of the Sun has been estimated at about twenty-five days and a half; that the solar axis is inclined * 8°⅓, to the plane of the ecliptic, and that the points of this equator when they ascend by their motion of rotation above this plane (i.e. the ecliptic) towards the north pole, intersect it in a point which, seen from the centre of the Sun, was † 86°20 distant from the vernal equinox in 1750.

The spots on the Sun are almost always comprised in a zone of its surface, whose breadth measured on the solar meridian extends only ‡ 33° or 34° on each side of the equator;—they have sometimes however been seen as far as § 44°. Bouguer found, by some curious experiments on the intensity of light on different parts of the Sun's disk, that this light was more intense at the centre than near the limb. Yet the same portion of the disk being carried round

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* 7° 28′ 12″. † 77° 34′ 48″. ‡ 29° 42′ or 30° 36′. § 39° 36′.

from the centre towards the limb, presents itself to us under a smaller angle; its light therefore should be more intense: it must therefore in some measure be extinguished, which can only be explained by supposing it surrounded by a dense atmosphere, which, being obliquely traversed by the rays emanating from the limb, weakens them more than those which proceeding from the centre pass it perpendicularly. Thus a solar atmosphere is indicated by this phenomenon with considerable probability.

It is the general opinion that it is this atmosphere which reflects to us that faint light which is visible, particularly about the vernal equinox, a little before the rising or after the setting of the Sun, and to which has been given the name of zodiacal light, (*lumiere zodiacale.*) The fluid which transmits it to us is extremely rare, since the stars are visible through it; its colour is white, and its apparent figure that of a cone whose base is applied to the Sun; such would be the appearance of an ellipsoid of revolution extremely flattened, whose centre and plane of its equator should be the same with those of the Sun. The length of the zodiacal light sometimes subtends an angle of more than * 100°; we shall see in the course of this work that the atmosphere of the Sun does not extend to so great a distance, it is not therefore this atmosphere that reflects the zodiacal light. Dominique Cassini, who was the first person who described this light correctly, remarked that it was weakened when the Sun had few spots, from which he conjectured that both the spots and the light arose from the same source, by the expansive force of the Sun which throws to its surface the dense matter of the spots, and which projects to a distance the rare and transparent matter of the zodiacal light. But the true cause of this light is still unknown.

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* 90°.

CHAP. III.

Of Time and its Measure.

Time relatively to us, is the impression which a series of objects leaves upon the memory, and of which we are certain the existence has been successive. Motion is a proper measure of time; for since a body cannot be in several places at the same time, it can only move from one place to another by passing through all the intermediate points. If we are assured that at every point of the line it describes, it is animated by the same force, it will describe it with an uniform motion, and the portions of this line will measure the time employed to describe them. When a pendulum at the end of every oscillation is in exactly the same position, the lengths of its oscillations are the same, and time may be measured by their number. We may employ likewise for this purpose the successive revolutions of the celestial sphere which appear perfectly uniform. But mankind have universally agreed for this purpose to make use of the motion of the Sun.

In civil life, the day is the interval of time which elapses between the rising and setting of the Sun; the night is the time during which the Sun remains below the horizon. The *astronomical day* comprehends the period of its diurnal revolution; it is the interval of time between two consecutive noons or midnights. It surpasses the length of a revolution of the heavens, which forms a *siderial day;* for if the Sun passes the meridian at the same instant as a star, the day after it will pass later in consequence of its proper motion by which it advances from west to east, and in the space of a year it will pass the meridian once less than the star; it is found that taking for unity the mean astronomical day, the sidereal day is *0.997269722. The

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* $23^h\ 56'\ 4''\ 1$.

astronomical days are not equal: two causes combine to produce their difference, the inequality of the proper motion of the Sun, and of the obliquity of the ecliptic. The effect of the first cause is sensible: thus at the summer solstice, near which the motion of the Sun is the slowest, the astronomical day approaches more to the siderial day than the winter solstice, where this motion is more rapid.

To conceive the effect of the second cause it must be noticed, that the excess of the astronomical day above the siderial day is only due to the proper motion of the Sun reduced to the equator. If we imagine two great circles of the celestial sphere to pass through the extremities of the small arc which the Sun describes in the ecliptic in one day, and likewise through the poles of the world, the arc of the equator which they intercept is the daily motion of the Sun reduced to the equator; and the time this arc employs to traverse the meridian is the excess of the astronomical above the siderial day; now it is evident that at the equinoxes the arc of the equator is less than the corresponding arc of the ecliptic, in the ratio of the cosine of the obliquity of the ecliptic to radius: at the solstices it is greater in the ratio of radius to the cosine of the same obliquity; the astronomical day, therefore, is diminished in the first case and augmented in the second. To obtain a mean day, independent of these causes, we imagine a second sun moved uniformly on the ecliptic, and always passing the greater axis of the solar orbit at the same instant as the true sun, which will cause the inequality arising from the proper motion of the Sun to disappear. The effect arising from the obliquity of the ecliptic is then made to disappear by imagining a third sun to pass the equinox at the same instant as the second sun, and to move along the equator in such a manner that the angular distances of these two suns from the vernal equinox may always be equal to each other. The interval of time between two consecutive transits of this third sun over the meridian forms the *mean astronomical day*. *Mean time* is measured by the number of these transits; *true time* or *solar time* by the number of transits of the real sun over the meridian. The arc of the equator intercepted by two meridians drawn through the centres of the real sun, and the third sun reduced into time, in the proportion of the whole circumference to one day, is that which is called the *equation of time*.

The returns of the Sun to the same equinox mark the years in the same manner as its returns to the meridian mark the days. In consequence of its annual motion it employs *365.242222 to return to the vernal equinox. This period forms the *tropical year*. Observation shews us that he employs a longer time to return to the same stars. The siderial year is the interval between two of these consecutive returns; it is greater than a tropical year

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* $365^d\ 5^h\ 48'\ 4''$.

by * 0, 01462 of a day. Thus the equinoxes have a retrograde motion on the ecliptic, or in a direction contrary to that of the Sun, by which they describe every year an arc equal to the mean motion of the Sun in the interval of 0,01462 or 155″09.†

For the uses of society different periods have been imagined to measure the portions of this revolution. Nature offers two very remarkable ones in the returns of the Sun to the meridian, and to the same equinox; but both require to be divided into smaller periods. The division of the day into ten hours, the hour into a hundred minutes, and the minute into a hundred seconds, is the most simple; it is natural to make the astronomical day commence at midnight, that it may comprehend in its duration all the time of the presence of the Sun above the horizon. It is convenient to fix the origin of the year at the vernal equinox, the period in which nature begins

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* 21′ 3″ 1. † 50″ 2.

to revive ; the seasons divide the year into four parts, which are each again subdivided into three months of thirty days. Each month has been divided into three periods of ten days or *decades*. In this manner the civil year would be divided into 360 days, but we have seen that it really consists of above 365 ; these days of excess are called complementary. Notwithstanding in this system of dividing the year, the order of things relative to the days of the decade will be something deranged by the complementary days; the correspondence of the days of the decade with the days of the month, and of the decadary festivals with that of the seasons, will render it preferable to the smaller division into weeks which are independent of the months.

If the length of the year was fixed at 365 days, its commencement would continually anticipate that of the tropical year, and the months would pass through all the different seasons with a retrograde motion in a period of about 1320 years. This method, formerly in use in Egypt, deprives the calendar of the advantage of fixing the months and festivals at the seasons, and of rendering them useful epochs for the purposes of agriculture. This great advantage may be preserved to the inhabitants of the country, by considering the origin of the year as an astronomical phenomenon, which should be fixed by observation and calculation at the moment of midnight, which precedes the vernal equinox; but then the years would lose the advantage of being regular periods of time, easy to subdivide into days, and introduce some confusion into history and chronology, and sometimes render even the origin of the year uncertain, which we always require should be known in advance. To obviate these conveniences, and to fix the same months and festivals at the same seasons, intercalations have been contrived. The most simple of all is that which the Persians adopted in the eleventh century; it consists in adding seven times following six complementary days instead of five every four years; and only to make this addition the eighth time every fifth year. The years thus augmented were called sextile, to distinguish them from the others, called common. According to this mode of intercalation, in thirty-three years, eight are sextile and twenty-five are common. This supposes the length of the 365 $\frac{8}{33}$ days greater by * 0.000202 of a day than the tropical year determined by observations; but a great number of centuries must elapse before the origin of the year would be so far removed from the equinox as to be sensible to the agriculturist.

It is much to be wished that all nations would adopt one common æra, not depending on moral revolutions, but determined by astronomical phenomena alone. We might fix its origin in the year in which the apogee of the solar orbit coincided with the summer solstice, which happened

* 174.

about the year 1250; we should take for this origin the moment of the mean vernal equinox, which in that year answered to the 15th March,* 5^{h} 3676 at Paris. The universal meridian, from which terrestrial longitudes should be reckoned, should he that of the place at which it was midnight at that instant, and which is to the east of Paris † 185°2960. If after a long succession of ages, the origin of the æra should become uncertain, it would be difficult to recover it exactly by the motion of the apogee, considering the slowness and the irregularity of this motion; but we should have no uncertainty as to this origin, or as to the position of the universal meridian, upon calling to mind that at the moment of the mean equinox the mean longitude of the moon was ‡ 143°7714. Thus whatever is arbitrary in the origin of time, and of terrestrial longitudes might be made

* 12^{h} 52′ 56″ 0. † 166° 51′ 59″ 0.
‡ 129° 23′ 39″ 3.

to disappear. By afterwards adopting the preceding intercalation and division of the year, month, and day, we should obtain the most natural and simple calendar that can be suited to the inhabitants of this side of the equator.

One hundred years form a century, which is the longest period hitherto employed in the division of time.

CHAP. IV.

Of the Motion of the Moon, its Phases and Eclipses.

After the Sun, the Moon of all the heavenly bodies is that which interests us the most; its phases afford us a measure of time so remarkable that it has been primitively in use among all people. The Moon, like the Sun, has a proper motion from west to east, the length of its siderial revolution was * 27°32166118036, at the commencement of 1700; it is not the same for every century. The comparison of modern with ancient observations shews incontestably an acceleration in the mean motion of the Moon. This acceleration, though but little sensible since the most ancient recorded eclipse, will be developed in progress of time. But will it for ever continue

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* $27^{d}\ 7^{h}\ 43'\ 11''\ 5$.

to increase, or will it stop and be changed into a retardation? this cannot be determined by observations, except after an immense number of ages; fortunately the discovery of its cause has anticipated them and shewn that it is periodical. The Moon moves in an elliptic orbit, of which the Earth occupies one of the foci. Its radius vector describes about this point equal areas in equal times. The mean distance of the Moon from the Earth being taken as unity, the excentricity of its ellipse is 0.0550368, which gives for the greatest equation of its centre * 7°0099. The lunar perigee has a direct motion, that is, in the same direction as the motion of the Sun, and the length of its sidereal revolution is † 3232,46643 days.

These laws, analogous to those of the solar motion, are very far from being sufficient to represent the observations; the lunar motion is subject to a great many other irregularities, which are evidently con-

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* $6^{\circ}\ 18'\ 32''$. † $9^{y}\ 312^{d}\ 11^{h}\ 11'\ 29''\ 5$.

nected with the position of the Sun. We shall indicate the three principal. The most considerable of all, and that which was first recognized, is what we now call the evection; this inequality, which amounts to * 1°4902 is proportional to the sine of double the mean angular distance of the Moon from the Sun, *minus* the mean angular distance of the Moon from the perigee of its orbit. In the oppositions and conjunctions of the Moon with the Sun, it is confounded with the equation of the center, which it constantly diminishes, and for this reason the ancient astronomers who only determined the elements of the lunar theory by the means of eclipses, and with a view of predicting these phenomena, always found the equation of the centre less than the truth by the whole quantity of the evection. We observe likewise in the lunar motions a great inequality, which disappears in the conjunctions and oppositions, and also in those points where

* 1° 20′ 28 2.

the Sun and Moon are distant from each other * 100°. It is at its maximum and amounts to † 0°6608 when their mutual distance is ‡ 50°; from whence it is inferred that it is proportional to the sine of double the mean distance of the Moon from the Sun: this inequality, which is called the *variation,* disappears in eclipses, and therefore could not have been discovered by the observation of those phenomena.

Lastly, the motion of the Moon is accelerated when that of the Sun is retarded, and reciprocally from which another inequality arises known by the name of the annual equation, and of which the law is exactly the same with that of the equation of the centre of the Sun, but with a contrary sign. This inequality, which at its maximum is § 0°2064, in eclipses becomes confounded with the equation of the centre of the Sun, and in calculating the instant of these phenomena it is indifferent whether we consider separately these two equations, or sup-

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* 90°. † 35′ 41″. ‡ 45°. § 11′ 8″ 7.

press the annual equation of the lunar theory, to augment the equation of the centre of the Sun. This is one of the principal reasons why the ancient astronomers gave too great a value to this last equation, at the same time that they assigned one too small to the equation of the center of the Moon, affected by the evection. The lunar orbit is inclined to the plane of the ecliptic* 5°7188; its points of intersection with it, called the *nodes,* are not fixed in the heavens, they have motion contrary to that of the Sun; which it is easy to recognize by the succession of stars the Moon meets with in traversing the ecliptic. The length of the sidereal revolution of these nodes is † 6793^d 3009. The ascending node is that in which the Moon rises above the ecliptic to advance towards the north pole; the *descending node* is that in which the Moon is descending towards the south. Their motion is subject to several inequalities, of which the greatest is proportional to the sine of

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* 5° 8′ 49″. † 18^y 223^d 7^h 13′ 17″ 7.

double the angular distance of the Sun from the ascending node of the lunar orbit, and amounts to * 1° 8105 at its maximum. The inclination of the orbit is equally variable, its greatest inequality amounting to† 0°1631, is proportional to the cosine of the same angle, on which the inequality of the motion of the nodes depends.

The lunar orbit, and the orbits of the other heavenly bodies, have no more a real existence than the parabolas described by the projectiles at the Earth's surface. To represent the motion of a body in space we imagine a line passing through all the successive positions of its centre; this line is its orbit, whose plane is that which passes through two consecutive positions of the body, and at the same time through the centre, round which the motion takes place. Instead of conceiving the motion of a body in this manner, we may in imagination suppose it projected on a fixed plane and de-

*1° 37′ 46″ †8′ 48″ 4.

termine the curve of projection and the height of the body above this plane. These different methods have their peculiar advantages, which, according to circumstances, may render one preferable to the other.

The apparent diameter varies in a manner analogous to the lunar motions, it is * 5438″, at its greatest distance, and † 6207 at its least.

The same means which were insufficient to determine the parallax of the Sun, on account of its smallness, have given that of the Moon equal to ‡ 10676″ at its distance from the Earth, which is an arithmetical mean between the two extremes; thus at the same distance at which the Moon appears to us to subtend an angle of § 5823″ the earth would be seen under an angle of ‖ 21352″, their diameters then are in the ratio of these numbers, or very nearly as 3 to 11, and the volume of the lunar globe

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* 29′ 22″ 2. † 33′ 31″ ‡ 57′ 39″ § 31′ 26″ 6.
‖ 1° 55′ 18″

is 49 times less than the volume of the Earth.

The phases of the Moon are among the most striking phenomena of the heavens. In disengaging itself in the evening from the rays of the Sun it re-appears with a slender crescent, which increases with its distance, and becomes an entire circle of light, when it is in opposition with the Sun. When it afterwards approaches to it, this circle is changed into a crescent which diminishes according to the same degrees by which it had increased,till in the morning it becomes immersed in the solar rays. The lunar crescent, always turned towards the Sun, evidently indicates that it is from the Sun it receives its light; and the law of the variation of its phases, which increase nearly as the versed sine of the angular distance of the Moon from the Sun, proves to us that it is spherical.

These phases are renewed at every conjunction,their return depends on the excess of the synodical motion of the Moon above that of the Sun, which excess is called the synodical motion of the Moon. The length of the synodic revolution of the Moon, or the period of its mean conjunction is * 29^{d}. 530588, it is to the tropical year nearly in the ratio of 19 to 235, that is 19 solar years form about 235 lunar months.

The *sysygies* are the points of the Moon's orbit, in which the Moon is in conjunction or opposition with the Sun; in the first point, the Moon is new, in the second full. The quadratures are the points of the orbit in which it is distant from the Sun † 100 or 300, reckoning in the direction of its proper motion. In those points which are called the first and second quarters of the Moon we see nearly half of its enlightened hemisphere, strictly speaking we see a little more, for when the exact half is presented to us the angular distance of the Moon from the Sun is a little less than ‡ 100 degrees. At this instant, which we recog-

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* 9^{d} 12^{h} 44′ 2″. † 90° or 270°. ‡ 90°.

nise, because the enlightened is separated from the obscure part of the Moon by a straight line, the radius drawn from the observer to the centre of the Moon is perpendicular to that which joins the centres of the Moon and Sun.

Thus in the triangle formed by the straight lines which join these centres and the eye of the observer, the angle at the Moon is a right one; the distance therefore of the Earth to the Sun may be determined in parts of that of the Moon from the Earth. The difficulty of fixing with precision the instant in which we see the half of the lunar disk enlightened, renders this method not very exact; we owe to it nevertheless the first just notions that were formed of the immense magnitude of the Sun and of its distance from the Earth.

The explanation of the phases of the Moon leads us to that of eclipses, objects of terror to men in the times of ignorance, but of curiosity to philosophers in all ages. The Moon can only be eclipsed by the interposition of some opake body which deprives it of the light of the Sun, it is evident that this opake body is the Earth, since an eclipse of the Moon never happens but at the oppositions, or when the Earth is between it and the Sun. The terrestrial globe projects behind it relatively to the Sun, a conical shadow, the axis of which is the straight line which joins the centres of the Sun and the Earth, and terminates in a point where the diameters of these two bodies are the same. These diameters seen from the centre of the Moon in opposition and at its mean distance are nearly * 5920″ for the Sun and † 21352″ for the Earth: thus the cone of the terrestrial shadow is at least three times longer than the distance of the Moon from the Earth; and its breadth at the points where it is traversed by the Moon is more than double the lunar diameter. The Moon, therefore, would be eclips-

* 31′ 58″. † 1° 55′ 18″.

ed every time she is in opposition with the Sun, if the plane of her orbit coincided with the ecliptic; but in consequence of the mutual inclination of these planes, the Moon in the oppositions is often elevated above, or depressed below the lunar shadow, and does not enter it but when she is near the nodes. If the whole of the disk is immersed in the shadow, the eclipse of the Moon is said to be total; it is partial if only a portion of the disk is obscured; and we may conceive that the greater or less proximity of the Moon to the nodes at the moment of opposition may produce all the varieties which are observed in these eclipses.

The mean period of the revolution of the Sun is, with respect to the node of the Moon's orbit, * 346.61963 days; it is to the synodical revolution of the Moon nearly in the ratio of 223to 19: thus after a period of 223 lunar months, the Sun and Moon return again to nearly the same posi-

* 346^{d} 14^{h} 52′ 16″.

tion relatively tothe nodes of the lunar orbit. As the eclipse should return again in the same order, this circumstance affords an easy manner of predicting them; but the inequalities of the motions of the Sun and the Moon produce very sensible differences, and besides the return of these two bodies to the same position with respect to the node in 223 months is not rigorously exact, the deviations which result, change at length the order of the eclipses that have been observed during one of these periods.

It is only in the conjunctions of the Sun and Moon that we can observe a solar eclipse, when the Moon interposed between the Earth and the Sun intercepts its rays. Although the Moon is incomparably smaller than the Sun yet by a remarkable circumstance, on account of its vicinity, its diameter differs but little from that of the Sun, it even happens from the variations in these diameters that they surpass alternately one another. Let us imagine the Sun and Moon in the same straight line with the eye of the observer, he will see the Sun eclipsed, and if the apparent diameter of the Moon surpasses that of the Sun, the eclipse will be total,but if the diameter be less, he will see a luminous ring formed by that part of the Sun which extends beyond the disk of the Moon and the eclipse will be *annular ;* if the Moon is not on the straight line which joins the centre of the Sun, and the observer, the Moon may then only conceal a part of the solar disk, and the eclipse will be partial Thus the difference in the distances of the Sun and Moon from the centre of the Earth, and of the proximity of the Moon to the node, must produce a great variety in the circumstances of a solar eclipse ; to this may be added the elevation of the Moon above the horizon, which changes the angle of its apparent diameter, and which by the effect of the lunar parallax, may so augment or diminish the apparent distances of the Sun and Moon, that of two observers, one may see an eclipse of the Sun, which will be totally invisible to the other. In this respect eclipses of the Sun differ from those of the Moon, which are the same to all places on the Earth.

We often see the shadow of a cloud transported by the winds rapidly pass over the hills and valleys, depriving those spectators which it reaches of the light of the Sun which others are enjoying; this is the exact image of a total eclipse of the Sun; a profound night which under favourable circumstances may last about five minutes accompanies these eclipses; the sudden disapparition of the Sun, with the sudden darkness that succeeds, fills all animals with dread; the stars which had been effaced by the light of day, show themselves in their full lustre, and the heaven resembles the most profound night: round the lunar disk, a crown of pale light has been perceived, which is probably the solar atmosphere, for its extent cannot accord with that of the Moon, as we are assured by eclipses of the Sun and stars, that the lunar atmosphere is nearly insensible. The atmosphere which we may suppose to surround the Moon, inflects the luminous rays towards its centre, and if (as should be the case) the atmospherical strata are rarer in proportion as they are removed from the surface, these rays in penetrating into it, will be inflected more and more, and will describe a curve concave towards its centre. An observer in the Moon will not cease to see a star till it is depressed below the horizon an angle, called the *horizontal refraction*. The rays emanating from this star, seen at the horizon after having just touched the surface of the Moon, will continue their route by describing a curve similar to that by which they arrived; thus an observer placed behind the moon, relatively to the star, will see it in consequence of the inflection of the lunar atmosphere. The diameter of the Moon is not sensibly augmented by the refraction of its atmosphere; therefore a star eclipsed by the Moon would be so, later than if this atmosphere did not exist; and for the same reason would sooner cease to be eclipsed: thus the effect of a lunar atmosphere would be principally perceived in the length of the eclipses of the Sun and stars by the Moon. Very exact and numerous observations have scarcely indicated a suspicion of this influence, and we may be assured that at the surface of the Moon, the horizontal refraction does not exceed * 5″. We shall see that at the surface of the Earth, this refraction is at least one thousand times greater; the lunar atmosphere, therefore, if any exist, must be of an extreme rarity, and even superior to that which we produce in our best-constructed air-pumps. From hence we may conclude that no terrestrial animal could live or respire at the moon, and that if it is inhabited, it must be by animals of another nature, and fluids being so

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* 1″ 6.

little compressed by an atmosphere so rare, would soon be reduced to vapours; there is then reason to think that all is solid at the surface of the moon, and this seems to be confirmed by the appearances which are seen in a good telescope, which shows it to us as an arid mass, on which some have thought they perceived the effects and even the explosion of volcanoes.

Bouguer has found by experiment that the light of the full Moon is about three hundred times fainter than that of the Sun. This is the reason why its light, collected in the focus of our largest mirrors, produces no sensible effect on the thermometer.

The Moon does not disappear entirely in its eclipses, but is still enlightened by a very faint light which comes to it by the Sun's rays inflected through the terrestrial atmosphere, and but for the great absorption of these rays by our atmosphere its brightness would be more vivid than when at the full moon. It is evident that this light should be more considerable in the eclipse's apogee than perigee; the vapours and clouds may weaken it so much as to render the Moon in these eclipses sometimes invisible; and the history of astronomy affords us several examples, though rare, of the total disapparition of the Moon.

We may sometimes distinguish, particularly about the time of new Moon, that part of the lunar disk which is not enlightened by the Sun; this feeble light has been called *lumière cendrée,* and is the effect of the light which the illuminated hemisphere of the Earth reflects upon the Moon; what proves it is, that it is most sensible at the new moon, when the greatest part of this hemisphere is directed to the Moon; for it is clear that to a spectator in the Moon, the Earth will present a succession of phases, similar to that which the Moon presents to us, but accompanied by a much more intense light from the greater extent of the terrestrial surface.

The lunar disk contains a great number of invariable spots which have been observed and accurately described. They shew us, that this body always presents to us very nearly the same hemisphere; it turns then upon itself in a period equal to its rotation round the Earth, for if we imagine an observer placed at the centre of the Moon, supposed transparent, he will see the Earth and its visual ray revolve about him, and since this ray always intersects nearly the same point of the lunar surface, it is evident that this point must revolve round the spectator in the same time, and in the same direction as the Earth.

Nevertheless, continued observation of the lunar disk has discovered some small diversity in these appearances, the spots are perceived to approach to and recede alternately from the limb; those that are very near the extremity, appear and disappear successively by periodical oscillations, which have been distinguished by the name of the *libration of the moon.*

To form a just idea of the principal causes of this phenomenon, we should consider that the disk of the Moon, seen from the centre of the Earth, is terminated by a great circle of the lunar globe, perpendicular to the radius drawn from this centre to that of the globe. It is upon the plane of this great circle, that the hemisphere of the Moon is projected directed towards the Earth, and its appearances arise from its motion of rotation, relatively to its radius vector: if it was without a motion of rotation, this radius vector would trace, at every lunar revolution, the circumference of a great circle upon its surface, every point of which would be successively turned to us; but at the same time that the radius vector traces this circumference, the lunar globe by its revolution brings always very nearly the same point of its surface to this radius, and consequently the same hemisphere towards the Earth. The inequalities of the motion of the Moon produce some small variations in these appearances, for the motion of rotation not partaking in a sensible manner of these inequalities, it is variable with respect to its radius vector, which thus intersects its surface at different points. The lunar globe therefore makes, with respect to this radius, oscillations corresponding to the inequalities of its motion, which causes some part of its surface to be alternately concealed and exposed to our observation.

Moreover, the axis of rotation of the Moon is not exactly perpendicular to the plane of its orbit. In supposing it nearly fixed during a whole revolution of the lunar globe, it inclines more or less upon the radius vector of the Moon, and the angle formed by these two lines is acute during one half of the revolution, and obtuse during the other half; a spectator at the Earth sees, therefore, alternately one and the other pole of rotation and the parts of the surface which are near it.

And lastly, the observer is not at the centre of the Earth, but at the surface; it is the visual ray drawn from his eye to the centre of the Moon, which determines the middle of the visible hemisphere, and it is clear that from the effect of the lunar parallax, this radius cuts the surface of the Moon at different points, according to the height of the Moon above the horizon.

All these causes produce only an apparent libration of the lunar globe, they are mere optical illusions, and do not affect its real motion of rotation; it is nevertheless true that this rotation may be subject to some small irregularities, but they have not yet been detected by observation.

It is not the same with the inequalities of the lunar equator. In endeavouring to determine its position by observations on the spots of the Moon, Dominique Cassini was led to this remarkable result, which contains all the astronomical theory of the real libration of the Moon. If we conceive a plane to pass through the centre of the Moon, perpendicular to its axis of rotation, a plane which will be coincident with that of its equator, if moreover, we imagine a second plane, parallel to that of the ecliptic, and a third plane, which is the mean plane of the lunar orbit, these three planes will always have a common intersection. The second plane, situated between the two others, forms with the first an angle of about * 1°,67; by and with the second, an angle of † 5°.7188. Thus the intersections of the lunar orbit, with the ecliptic, or its nodes, always coincide with the mean nodes of the lunar orbit, and like them, have a retrograde motion, whose period is ‡ 6793^{d}.3009. In this

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* 1° 30′ 10″ 8. † 5° 8′ 49″.

‡ 18^{y} 223^{d} 7^{h} 13′ 17″ 7.

interval, the two poles of the equator and of the lunar orbit, describe small circles parallel to the ecliptic, comprehending between them the poles of the ecliptic in such a manner, that these three poles are constantly situated on a great circle of the heavenly sphere.

Mountains of a very great height, rise up from the surface of the Moon; their shadows, projected on the plains, form spots which vary with the position of the Sun; upon the edge of the enlightened disk we see these mountains forming an indented border, extending beyond the line of light by a quantity which, being measured, indicates that their height is at least ten or twelve thousand feet. We recognize likewise by the direction of the shadows, that the surface is broken by cavities, nearly resembling the basons of our seas. Lastly, the lunar surface seems to shew the traces of volcanos; several observers have seen upon the unenlightened part, a vivid light, which they have attributed to a volcanic eruption. We may likewise attribute to this cause, the formation of several new lunar spots.

CHAP. V.

Of the Planets, and in particular of Mercury and Venus.

In the midst of the infinite number of shining bodies, which are spread over the surface of the heavenly firmament, and which preserve among themselves a position nearly constant, we may perceive six stars moving in regulated periods and according to complicated laws, the investigation of which is one of the principal objects of astronomy. These stars, to which have been given the name of PLANETS, are Mercury, Venus, Mars, Jupiter, Saturn and Uranus; the two first never recede from the Sun beyond certain limits, the others are occasionally separated from him by all the angular distances possible. The motions of all these bodies are comprehended in a zone of the celestial sphere called the zodiac, the breadth of which is about * 20°, and divided into two equal parts by the ecliptic.

The distance of Mercury from the Sun never exceeds † 32°; when it begins to appear in the evening it is with difficulty distinguished in the rays of twilight; it disengages itself more and more in the following days, and after arriving at about ‡ 25° from the Sun, it returns towards him again. In this interval the motion of Mercury with respect to the fixed stars is direct, but when in returning it comes within the distance of § 20° of the Sun, it seems stationary, after which its motion appears retrograde, it continues to approach the Sun, and is again in the evening lost in his rays. After continuing some time invisible, it is seen again in the morning, disen-

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* 18°. † 28° 48′. ‡ 22° 30′. § 18°.

gaging itself from the Sun's rays and departing from the Sun, its motion is still retrograde as before its disapparition; arrived at the distance of 20° it is again stationary, then resumes its direct motion, its distance increases to 25°, it then returns, and disappearing in the morning in the light of the dawn, is soon after seen again in the evening, producing the same phenomena as before.

The extent of the greatest digressions of Mercury, or its greatest separation from the Sun on each side varies from * 18 to † 32 degrees.

The length of its entire oscillation or return to the same position relatively to the Sun, varies likewise from one hundred and six, to one hundred and thirty days, the mean arc of its retrogradation is about ‡ 15°, and its mean duration 23 days, but there is a great difference in these quantities in different retrogradations.

* 16° 12'.   † 29.   ‡ 13° 30'.

In general the laws of the motion of Mercury are extremely complicated, they do not take place exactly in the plane of the ecliptic; sometimes the planet departs * 5° from it.

A long series of observations was no doubt necessary to recognise the identity of the two stars, which were alternately seen in the morning and in the evening to depart from and return to the Sun; but as one never shewed itself till the other disappeared, it was at last suspected to be the same planet which thus oscillated on each side of the Sun.

The apparent diameter of Mercury is very variable, and its changes are evidently connected with its relative position to the Sun and the direction of his motion. It is a minimum either when the planet in a morning immerges into the solar rays, or when in the evening it disengages itself

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* 4° 30′.

from them; it is at its maximum when it immerges into the Sun's light in an evening, or when it again becomes visible in the morning. Its mean apparent diameter is * 21″,3.

Sometimes, in the interval of its disapparition in the evening and its re-apparition in the morning, it is seen projected in the form of a black spot, upon the disk of the Sun on which it describes a chord.

It is known to be the same, by its position, its apparent diameter, and its retrograde motion being the same as it ought to be; these transits of Mercury are real annular eclipses of the Sun, from which we discover that the light of the planets is borrowed from it; when viewed in a good telescope it presents to us phases similar to those of the Moon, directed in the same manner towards the Sun, and of which the variable extent according to its re-

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* 7″.

lative position with respect to it and following the direction of its motion, throws a great light on the nature of its orbit.

The planet Venus offers the same phenomena as Mercury, with this difference that its phases are much more sensible, its oscillations more extensive and their period more considerable. The greatest digressions of Venus vary from * 50° to † 53° and the mean length of its entire oscillations is 584^d. The retrogradations commence or finish when the planet approaching the Sun in the evening or receding from it in the morning is distant from it ‡32°. The mean arc of its retrogradation is about § 18°, and its mean duration forty-two days. Venus does not move exactly in the plane of the ecliptic, but sometimes deviates from it several degrees.

Like Mercury, Venus appears sometimes to describe a chord of the disk of the

* 45. † 47° 42'. ‡ 27° 48. § 16° 12.

Sun. The lengths of these transits over the Sun observed at different places on the Earth, remote from each other, are very sensibly different—thisarises from the parallax of Venus, in virtue of which different observers refer it to different points of the solar disk, and see it describe different chords in its passage over it. In the transit which took place in 1769, the difference of its duration, as observed at Otaheite, in the South Sea, and at Lajanebourg in Swedish Lapland, amounted to more than * 15 minutes. As these durations can be calculated with great precision, their differences give very exactly the parallax of Venus, and consequently its distance from the Earth at the moment of conjunction. A remarkable law which we shall explain, at the end of the discoveries which made it known, connects this parallax with that of the Sun and planets; thus the observation of these

* 8′ 6″.

transits is of great importance in astronomy. After succeeding each other in the interval of eight years, they do not occur again for more than a century, when they succeed each other again, during an interval of eight years, and thus they continue.

The two last transits happened in 1761 and 1769. Astronomers were sent to different countries, where the observations could be made under the most favourable circumstances, and it is from the result of their observations that the parallax of the Sun has been concluded to be equal to * 27″2, at its mean distance from the Earth.

The great variations in the diameter of Venus prove that its distance is continually changing. The distance is the least at the moment of her transit over the disk of the Sun, its apparent diameter being then about † 177″. Its mean diameter is ‡ 51″ 54.

* 8″8. † 57″ 3. ‡ 16″ 6.

The motion of some spots upon its surface instructed Dominique Cassini that its rotation on its axis is performed in the interval of rather less than a day. Schroeter by continued observation of the variation of its horns and of some luminous points towards the edges of the unenlightened parts, has confirmed this result, concerning which some doubts had arisen. He has fixed the duration of its rotatory motion at † 0,973 days, and has found, like Cassini, that the equator of Venus makes a considerable angle with the ecliptic. Finally, he has concluded the existence of high mountains on its surface from his observations, and from the law by which its light gradually varies from its enlightened to its unenlightened side; he supposes the planet surrounded with an extensive atmosphere, the refracting power of which differs but little from that of the terrestrial atmosphere.

The extreme difficulty of seeing these spots even in the best telescopes, renders

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* 34^h 15′ 43″2.

these observations almost impossible in our climates, but they merit the attention of observers, who, situated in a more southern latitude, enjoy a more favourable climate.

Venus surpasses in brightness all the other stars and planets ; it is sometimes so brilliant as to be seen in full day and by the naked eye. This phenomenon, which is not unfrequent, never fails to excite surprise, and the credulous ignorance of the vulgar, suppose it connected with the remarkable events of the same time.

CHAP. VI.

Of Mars.

THE two planets which we have just considered seemto accompany the Sun like satellites,and their mean motion round the Earth is the same as that of the Sun. The other planets recede from the Sun to all possible angular distances, but their motions have such a connection with its position, as can leave no doubt of his influence on them.

Mars seems to move from west to east round the Earth; the mean length of a sidereal revolution is * $686^{d}979579$. Its motion is very unequal; when it begins to be visible in the morning it is direct and most rapid; it becomes gradually slower,and the planet when it arrives at about † 152° from the Sun is stationary; the motion then be-

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* $1^{y}\ 321^{d}\ 23^{h}\ 30'\ 35''6$. † 136° 48'.

comes retrograde, increasing in velocity till the moment of opposition of the planet with the Sun. This velocity then becomes a *maximum*, diminishes, and again becomes nothing, when Mars, approaching the Sun, is distant from it 152°. Its motion then becomes again direct, after having been retrograde during 73 days, and in this interval the planet describes an arc of retrogradation of about * 18°, continuing to approach the Sun it finishes by immerging in the evening into its rays. These singular phenomena are renewed at every opposition of Mars, but with a considerable difference as to the extent and duration of these retrogradations.

Mars does not move exactly in the plane of the ecliptic, but deviates occasionally several degrees. The variations in its apparent diameter are very great. It is about † 30″, in its mean state, and augments to

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* 16° 12′. † 13″ 3.

* 90″, as the planet approaches its opposition. At this time the parallax of Mars becomes sensible, and is nearly double that of the Sun. The same law which exists between the parallaxes of the Sun and Venus exists likewise between the Sun and Mars, and the observation of this last parallax had given a very near approximation of the solar parallax before the transit of Venus had ascertained it with greater precision.

The disk of Mars changes its form and becomes sensibly oval, according to the relative position of the Sun. These phases show that it is from the Sun it receives its light. From the observation of spots distinctly seen on its surface, it is inferred that it moves on itself from west to east in a period of † 1^{d} 02723, and on an axis inclined ‡ 66°. 33, to the ecliptic.

* 29″ 1. † $1^{d}\,0^{h}$ 44″ 45″ 3. ‡ 59° 41′ 49″ 8.

CHAP. VII.

Of Jupiter and its Satellites.

JUPITER moves from west to east in a period of * 4332^{d} 602208. It is subject to inequalities similar to those of Mars previous to its opposition, and when it is nearly † 128 distant from it, its motion becomes retrograde, its velocity augments till the moment of opposition, then diminishes, and recovers its usual direction, when the planet, in its approach towards the Sun is only 128° distant from it. The duration of this retrograde motion is about 121^{d} and the arc of retrogradation is ‡ 11°. But there are perceptible differences in the extent and duration of the retrograde

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* 11^{y} 318^{d} 14^{h} 27′ 10″ 7. † 115° 12′. ‡ 9° 54.′

motions of Jupiter. The path of this planet is not always confined to the ecliptic, it deviates from it occasionally *3 or 4 degrees.

Several obscure belts or stripes may be observed on the surface of Jupiter, evidently parallel among themselves, and also to the ecliptic; there are likewise other spots, the motion of which has demonstrated the rotation of this planet from west to east, upon an axis nearly perpendicular to the plane of the ecliptic, and in a period of †$0^{d}41377$. The variations of some of these spots, and the sensible difference in the period of rotation, deduced from their motions, induce the opinion that they are not attached to Jupiter. They appear to be clouds which the winds transport with various velocities, in an extremely agitated atmosphere.

Jupiter is, next to Venus, the most bril-

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* $2^{\circ}\ 42'$ or $3^{\circ}\ 36'$. † $9^{h}\ 55'\ 49''\ 7$.

liant of the planets, and sometimes even surpasses it in brightness. Its apparent diameter is the greatest possible; in its opposition, when it is *149″ its mean diameter in the direction of the equator, is †120″. But it is not equal in every direction. The planet is evidently flattened at the poles of its axis, and it has been found by very accurate measurement that its diameter in the direction of its poles is to that of its equator, nearly in the proportion of 13 to 14.

It is observed that there are four small stars round Jupiter, which incessantly accompany it. Their relative situation to each other changes every instant; they oscillate on each side of the planet, and it is by the extent of these oscillations that the rank of these satellites is determined, that being called the first satellite whose oscillation is the least. They are sometimes

* 48″2. † 38″ 8.

seen to pass over Jupiter's disk, and project a shadow, which then describes a chord of this disk. Jupiter and his satellites therefore are opaque bodies enlightened by the Sun, and when they interpose between the Sun and Jupiter, they produce real solar eclipses precisely similar to those which the Moon occasions on the Earth.

This phenomenon leads to the explanation of another which the satellites present. They are often observed to disappear, though at some distance from the disk of the planet; the third and fourth reappear sometimes, on the same side of the disk.

The shadow which Jupiter projects behind it (relatively to the Sun), is the only cause that can explain these disappearances, which are perfectly similar to eclipses of the Moon.

The circumstances which accompany them leave no doubt of the reality of this cause. The satellites are always observed to disappear on that side of the disk opposite to the Sun, and consequently on the same side to which the conical shadow is projected. They are eclipsed nearest to the disk when the planet is nearest to its opposition. Finally, the duration of these eclipses answers precisely to the time which they should employ to traverse the cone of the shadow of Jupiter.

Thus these satellites move from west to east in returning orbits round this planet. Observations of their eclipses are the most exact means of determining their motions. Their mean sidereal and synodical revolutions as seen from the center of Jupiter, are very accurately determined by comparing eclipses at long intervals from each other, and observed near the opposition of the planet.

It is thus discovered that the motion of the satellites of Jupiter is nearly circular and uniform, since this hypothesis corresponds very nearly with those eclipses which happen when we see this planet in the same position relatively to the Sun.

Therefore the positions of the satellites at every instant, as seen from the centre of Jupiter, may be determined. From hence results a simple and sufficiently exact method of comparing with each other the distances of Jupiter and of the Sun from the Earth. A method which the ancient astronomers did not possess. For the parallax of Jupiter is insensible even to the precision of modern observations when it is nearest to us. They only judged of its distance by the time of its revolution, as they estimated those planets to be the farthest from us whose period of revolution was the longest.

Let us suppose that the total duration of an eclipse of the third satellite has been observed. At the middle of the eclipse the satellite seen from the centre of Jupiter, is nearly in opposition to the Sun. Its sidereal position, observed from this centre, and which it is easy to conclude, from its mean motion, is therefore the same as that of the centre of Jupiter seen from that of the Sun.

Direct observation, or the known motion of the Sun, gives the position of the Earth as seen from its centre. Thus, imagining a triangle formed by the right lines which join the centres of the Sun; the Earth, and Jupiter, we have in this triangle the angle of the Sun, observation will give that of the Earth, and we shall get at the instant of the middle of the eclipse the rectilinear distance from Jupiter to the Earth and to the Sun, in parts of the distance from the Sun to the Earth.

It is found by these means that Jupiter is at least five times farther from us than the Sun, when its apparent diameter is *120″. The diameter of the Earth at the same distance would not subtend an angle of †11″; the volume of Jupiter is therefore at least a thousand times greater than that of the Earth.

The apparent diameters of these satellites being insensible, their magnitudes can-

* 38′ 3. † 3″ 5.

not be exactly measured. The attempt has been made to appreciate it by the time they take to penetrate the shadow of the planet. But there is a great discordance in the observations, which have been made to ascertain this circumstance; this arises from the various powers of telescopes, the different degrees of perfection in the sight of the observer, the state of the atmosphere, the altitude of the satellites above the horizon, their apparent distance from Jupiter, and the change of the hemisphere presented to us. The comparative brightness of the satellites is independent of the four first causes, which only alter their proportional light, and ought therefore to afford information concerning the rotatory motion of these bodies. Dr. Herschel who is occupied in this delicate investigation, has observed that they surpass each other alternately in brilliance, a circumstance that enables us tojudge of their respective light. The relation of the maximum and minimum of their light with their mutual positions, has persuaded him that they revolve upon themselves like a moon in the period equal to the duration of their revolution round Jupiter. A result which Maraldi had already concluded for the fourth satellite, from the returns of the same spot observed on its disk, in its passage over the planet. The great distance of the celestial bodies weakens the phenomena which their surfaces present, till they are reduced to slight variations of light, which escape the first view, and are only rendered sensible by a long course of observations. But this means of supplying the imperfection of our organs ought to be used with the greatest circumspection, to avoid being deceived concerning the causes on which these varieties depend.

CHAP. VIII.

Of Saturn, of its Satellites, and its Ring.

THE period of the sidereal motion of Saturn round the Earth, is *10759^{d}.077215. This motion, which is from west to east, and nearly in the plane of the ecliptic, is subject to inequalities similar to those of Jupiter and Mars. It commences and finishes its retrograde motion when the planet before and after its opposition is about †121° distant from the Sun. The duration of this retrogradation is nearly 131 days, and the arc of retrogradation about ‡7°. At the moment of opposition, its diameter is a maximum, its mean magnitude is § 54″4. Saturn presents a phenomenon *unique* in the system of the universe. Two small bodies are always

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* 29^{y} 174^{d} 0^{h} 18′ 31″ 8. † 108° 54. ‡ 6° 18. § 17″ 6.

observed on each side of it, which appear to adhere to it, and of which the form and magnitude are extremely variable, sometimes they even disappear, and then Saturn appears round like the other planets. By following these singular appearances with care, and combining them with the positions of Saturn relatively to the Sun and the Earth, Huygens has discovered that they are produced by a large thin ring which surrounds the globe of Saturn and which is every where separated from it. This ring being inclined *54°8 to the plane of the ecliptic, always presents itself obliquely to the Earth, under the form of an ellipse, whose breadth, when at a maximum, is nearly the half of its length. In this position its shorter axis exceeds in diameter the disk of the planet. The ellipse becomes narrower in proportion as the visual ray, proceeding from Saturn to the Earth, becomes less inclined to the plane

* 49° 19′ 12″.

of the ring, the farthest arc of which is at length hid behind the planet. And the anterior arc is confounded with it. But its shadow is projected upon the disk, and forms an obscure band which can be seen in very powerful telescopes, which proves that Saturn and its ring are opake bodies enlightened by the Sun. No part of the ring can then be distinguished except that which projects on each side of Saturn ; this diminishes by degrees, and finally disappears when the Earth in consequence of the motion of Saturn, is in the plane of the ring, the thickness of which is not perceptible. The ring disappears also when the Sun being opposite to it only enlightens its thickness. It continues invisible as long as it remains between the Sun and Earth, and reappears when both are on the same side the planet from the respective motions of Saturn and the Sun.

The plane of the ring meeting the solar orbit at every semi-revolution of Saturn, the phenomena of its disappearance and re-appearance, return every fifteen years, but frequently with very different circumstances, two disappearances, and two re-appearances may occur in the same year, but never more. When the ring disappears its thickness reflects the light of the Sun to us, but in too small a quantity to be perceptible. Nevertheless it may be conceived, that it might be seen by augmenting the power of the telescope. Dr. Herschell found this in the last disappearance of Saturn's ring, which continued visible to him through the whole period, when it had ceased to be so to any other observer.

The inclination of the ring to the ecliptic is measured by the largest opening which the ellipse presents to us. The position of its nodes may be determined by the apparent situation of Saturn, when the ring disappears or reappears, the Earth being in its plane. All the disappearances and appearances from which the same sidereal positions of the nodes of the ring result, take place when its plane meets the Earth. The others when the same plane meets the Sun. It may therefore be known by the situation of Saturn when the ring disappears or reappears, whether this phenomenon is produced by its plane meeting the Sun or the Earth. When this plane passes through the Sun, the position of its nodes gives that of Saturn, as seen from the centre of the Sun, and the rectilinear distance of Saturn from the Earth may be determined as that distance of Jupiter is by the eclipses of its satellites. It is thus found that Saturn is about nine times and a half farther from us than the Sun, when its apparent diameter is * 54″ 4. The apparent breadth of the ring is nearly equal to its distance from the surface of Saturn, both appear to be one-third the diameter of the planet. But on account of the irradiation its real breadth should be smaller.

Its surface is not uniform, a black band

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* 17″ 6.

which is concentric with it, divides it into two parts, appearing to form two distinct rings. Observations of some bright points of this ring have proved to Dr. Herschel its rotation from west to east in a period of *0^d 437, round an axis perpendicular to its plane, and passing through the centre of Saturn.

Seven satellites have been observed in motion round this planet, from west to east in orbits nearly circular. The six first move nearly in the plane of the ring. The orbit of the seventh approaches more to the plane of the ecliptic. When this satellite is to the east of Saturn, its light becomes weaker till it is scarcely perceptible, which can only happen from the spots which cover the hemisphere which it presents to us. But for this phenomenon to occur always in the same position, it is requisite that this satellite (similar in that respect to our Moon and to the satellites of Jupiter) should revolve

* 10^h 29′ 16″ 8.

upon its own axis, in a period equal to that of its revolution round Saturn. Thus equality in the period of rotation and revolution appears to be a general law of the motion of satellites.

The diameters of Saturn, are not equal among themselves. That which is perpendicular to the plane of the ring, is less by at least one-eleventh than that situated in the plane of the ring. If this ellipticity be compared with that of Jupiter it may be concluded with great probability that Saturn turns rapidly round its shorter axis, and that the ring moves in the plane of the equator. Dr. Herschel has confirmed this result, by direct observation, he has found the duration of Saturn's rotation to be *0^{d} 428. This rotation is like all the motions of the planetary system from west to east. Dr. Herschel has likewise observed on the surface of the planet five belts nearly parallel to its equator.

+ 10^{h} 16′ 19″ 5.

CHAP. IX.

Of Uranus and his Satellites.

THE five planets that we have hitherto considered have been known from the most remote antiquity. The planet Uranus had escaped the observation of ancient astronomers from its minuteness. Flamstead at the end of the last century, and Mayer and Le Monnier in this, had observed it as a small star. But it was not till 1781 that Dr. Herschel discovered its motion, and soon after, by following this star carefully, it has been ascertained to be a true planet. Like Mars, Jupiter, and Saturn, Uranus moves from west to east round the Earth. The duration of its siderial revolution is *30689^d. Its mo-

* 84^y 29^d.

tion, which is nearly in the plane of the ecliptic, begins to be retrograde when, previous to the opposition, the planet is * 115° distant from the Sun. It ceases to be retrograde when, after the opposition, the planet in its approach to the Sun is only 115° distant from it. The duration of its retrogradation is about 151 days, and its arc of retrogradation, † 4 degrees. If the distance of Uranus were to be estimated by the slowness of its motion, it should be on the confines of the planetary system. Its apparent diameter is very small and hardly amounts to ‡ 12″. Dr. Herschel by means of a very powerful telescope, has discovered six satellites moving round this planet, in orbits almost circular and nearly perpendicular to the plane of the ecliptic.

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* 103°30′. † 3° 36″. ‡ 3″ 8.

CHAP. X.

Of Comets.

STARS are often observed, which, at first scarcely perceptible, augment in size and in velocity, then diminish, and finally disappear. These stars, which are called comets, appear almost always accompanied with a nebulosity, which increasing terminates sometimes in a train of considerable length, which is composed of some substance extremely thin, since the stars are seen through it. The appearance of comets, followed by these long trains of light, has for a long period terrified mankind, always agitated by extraordinary events of which the causes are unknown. The light of science has dissipated the vain terrors which comets, eclipses, and many other phenomena inspired in the ages of ignorance. The phase observed in the comet of 1744, of which only half the disk was enlightened, proves that these stars are opaque bodies which borrow their light from the Sun. Comets participate like all the other stars in the diurnal motion of the heavens, and this combined with the smallness of their parallax proves that they are not meteors engendered in our atmosphere. Their own motions are very complicated, and they are not confined like planets, in one direction, nor to planes little inclined to the ecliptic.

CHAP. XI.

Of the fixed Stars and their Motions.

The parallax of the fixed stars is insensible; when examined by the most powerful telescopes, their disks are reduced to luminous points. It is in this circumstance that these celestial bodies differ from planets, the apparent diameter of which is increased by telescopes. The smallness of the apparent diameter of the fixed stars is proved particularly by their rapid disappearance in their occultations by the Moon, the time of which not amounting to one second, proves that their diameter is less than* five seconds of a degree.

* 1′ 6.

The vivacity of the light of the most brilliant stars, compared with the smallness of their apparent disk, leads us to believe that they are much farther from us than the planets, and that they do not like them borrow their light from the Sun, but are themselves luminous; and as the smallest stars are subject to the same motions as the most brilliant, and constantly preserve the same position relatively to each other, it is extremely probable that all these celestial objects are of the same nature, that they are so many luminous bodies of different magnitudes, and placed more or less distant from the planetary system.

Periodical variations have been observed in the intensity of the light of many of these stars, which have been called on that account changeable. Sometimes stars have been known to appear suddenly, and vanish after they have shone with the most brilliant splendour. Such was the famous star observed in 1572, in the constellation of Cassiopeia; in a short time it surpassed the brilliance of the most beautiful stars, and even of Jupiter itself. Its light afterwards diminished, and it disappeared entirely sixteen months after its discovery, without having changed its place in the heavens; its colour suffered considerable alterations, it was at first of the most dazzling white, then of a reddish yellow, and finally of a lead-coloured white. What is the cause of these phenomena? The extensive spots which these fixed stars present to us periodically in turning on their own axis nearly in the same manner as the last satellite of Saturn, and the interposition of large opake bodies which circulate round them, are sufficient to explain the periodical variations of changeable stars. As to those stars which suddenly appear with a most brilliant light, and then vanish, it may be supposed with probability, that great conflagrations occasioned by extraordinary causes, take place on their surface, and this supposition is confirmed by their change of colour, analogous to that which is presented to us on the earth by bodies which are consumed by fire.

A white light, of an irregular form, and to which the name of the Milky Way has been given, surrounds the heavens in the form of a zone. So great a number of small stars have been discovered in it by means of telescopes, that it is highly probable that the Milky Way is nothing more than an assemblage of these stars, which are so close together as to appear one continued band of light. Small white spots of light are also observed in different part of the heavens, which seem to be of the same nature as the Milky Way. Many of them viewed in a telescope equally display the union of several small fixed stars; others only present a white and continued light, perhaps on account of their great distance, which confounds the light of the stars of which they are composed. These white spots are called nebulæ.

The constant position of the fixed stars with respect to each other, has determined astronomers to refer to them, as to so many fixed points, the motions proper to the other celestial bodies. But for this purpose it was necessary to class them, that they might be recognized; and it was with this design that the heavens have been divided into various groupes of stars called constellations. It was likewise necessary to ascertain with precision the position of the fixed stars on the celestial sphere, and it was thus accomplished: Let a great circle be conceived to pass through the two poles of the world, and through the centres or any celestial body; this circle is called the *circle of declination,* and cuts the equator perpendicularly: the arc of this circle, comprised between the equator, and the celestial object, measures its declination, which is *north* or *south* accord ing to the denomination of that pole to which it is the nearest.

As all the heavenly bodies situated on the same parallel of the equator would have the same declination, it was necessary to assume a new element to determine their situation. The arc of the equator, contained between the circle of declination and the vernal equinox, has been chosen for this purpose. This arc, reckoned from the equinox in the direction of the motion proper to the Sun, that is, from west to east, is called the *right ascension,* thus the position of any celestial object is determined by its right ascension and declination. The meridian altitude of a celestial object compared with the latitude, gives its distance from the equator or its declination. The determination of its right ascension presented greater difficulties to the ancient astronomers on account of the impossibility of comparing directly the fixed stars with the Sun; as the Moon might be compared by day with the Sun, and by night with the stars, they employed it as a medium to measure the difference of right ascension of the Sun and the fixed stars; keeping in consideration the motions proper to the Moon and Sun during the interval of the observations. The theory of the Sun afterwards giving its right ascension, they obtained from it that of several of the principal fixed stars, to which they compared the others. It was by this means that Hipparchus formed the first catalogue of fixed stars of which we have any knowledge. Some time after, they gave greater precision to this method, by employing instead of the Moon the planet Venus, which can sometimes be perceived at noon, and the motion of which during a short interval of time is slower and more equal than the lunar motion.

At present, when the important application of the pendulum to clocks furnishes a very precise measure of time, we can determine directly, and with an accuracy extremely superior to that of the ancient astronomers, the difference of right ascension between the Sun and any other celestial object, by the interval that elapses between their meridian transits. The position of the heavenly bodies may be referred in a similar manner to the ecliptic, which is principally useful in the theory of the Moon.

A great circle is supposed to pass through the centre of the object perpendicular to the plane of the ecliptic, and which is called a circle of latitude.

The arc of this circle included between the ecliptic and the star, measures its latitude, which is either north or south, according to the denomination of the pole situated on the same side of the ecliptic.

The arc of the ecliptic contained between the circle of latitude, and the vernal equinox, and reckoned from this equinox towards the east and west, is called the *longitude* of a celestial object, the position of which is thus determined by its longitude and latitude. It may be easily conceived that the inclination of the ecliptic to the equator being known, the longitude and latitude of a star may be deduced from its observed right ascension and declination.

An interval of only a few years was necessary to observe the variation of the fixed stars in right ascension and declination. It was soon remarked that in changing their position relatively to the equator, they preserved the same latitude with regard to the ecliptic, and from hence it was concluded that their variations in right ascension and declination were owing to the motion of the celestial sphere round the poles of the ecliptic. These variations might also be represented by supposing the fixed stars immovable, and making the poles of the equator move round those of the ecliptic. In this motion the inclination of the equator to the ecliptic remains the same, and its nodes or the equinoxes retrograde uniformly about * 154″ 63 annually. It has been shewn previously that this retrogradation of the equinoxes renders the tropical year a little shorter than the sidereal year.

Thus the difference of the tropical and sidereal years, and the variation of the fixed stars in right ascension and declination, depends on this motion by which the pole of the equator describes annually an arc of 154″ 63, of a small circle of the celestial sphere parallel to the ecliptic. It is in this that the phenomenon consists, known by the name of the precession of the equinoxes.

The precision of modern astronomy, for which it is principally indebted to the application of telescopes, to astronomical instruments, and that of the pendulum to clocks, has rendered minute periodical inequalities perceptible in the inclination

* 50′ 1.

of the equator to the ecliptic, and in the precession of the equinoxes. Bradley who discovered them, and who has followed them with great attention during many years, has discovered their law, which may be represented in the following manner:—Let the pole of the equator be supposed moved upon the circumference of a small ellipse, tangent to the celestial sphere, whose centre, which may be regarded as the mean pole of the equator, describes uniformly every year 154″63 of the parallel of the ecliptic on which it is situated. The greater axis of this ellipse, always tangent to the circle of latitude, and in the plane of this great circle, subtends an angle of about * 62″ 2, and the smaller axis subtends an angle of 46″ 3. The situation of the real pole of the equator upon this ellipse is determined thus:—there is supposed to be a small circle on the plane of the ellipse, concentric with it, and the diameter of

* 20″ 1. † 14″ 9.

which is equal to its greater axis; conceive a radius of this circle, moved uniformly with a retrograde motion, so that this radius coincides with that half of the greater axis nearest to the ecliptic, every time that the mean ascending node of the lunar orbit coincides with the vernal equinox. Finally, from the extremity of this moveable radius, suppose a perpendicular let fall upon the greater axis of the ellipse. The point where this perpendicular cuts the circumference of this ellipse, is the place of the real pole of the equator; this motion of the pole is called its nutation. The fixed stars, in consequence of the motions we have described, preserve among themselves a constant position. But the illustrious observer to whom we owe the discovery of the nutation, has found in all the fixed stars, a general and periodical motion, which alters a little their respective positions; to represent this motion, it must be imagined that each star describes annually a small circumference parallel to the ecliptic, the centre of which is the mean position of the fixed star, and the diameter of which, seen from the Earth, subtends an angle of * 125″; and that it moves upon this circumference, like the Sun in its orbit, in such a manner that the Sun may be always more advanced than the star by † 100°; this circumference, projected upon the surface of the heavens, appears under the form of an ellipse, more or less flattened, according to the height of the star above the ecliptic; the lesser axis of this ellipse being to the greater axis as the sine of this altitude is to radius; from hence arise all the varieties of this periodical motion of the fixed stars, which is called aberration.

Independent of these general motions, many fixed stars have movements peculiar to themselves; very slow, but which the

* 40″ 5. † 90°.

lapse of time has rendered sensible, and which give us reason to believe that all the fixed stars have similar motions, which will develope themselves in succeeding ages. It has been hitherto principally remarkable in Syrius and Arcturus, two of the most brilliant of the fixed stars.

CHAP. XII.

Of the Figure of the Earth, and of the Variation of Gravity on its Surface; and of the Decimal System of Weights and Measures.

LET us now descend from the heavens to the earth, and see what can be derived from observations relative to its dimensions and figure. It has been already seen that the earth is nearly spherical; gravity every where directed towards its centre, retains bodies on its surface; though in places directly opposite to each other, they are in contrary positions. The sky and stars always appear above the earth, for elevation and depression are only relative terms regarding the direction of gravity.

From the moment that man recognized

the spherical form of the globe which he inhabits, his curiosity would lead him to measure its dimensions. It is then extremely probable that the first attempts to attain this object were made at a period much anterior to those of which history has preserved the record, and that they have been lost in the physical and moral changes which the earth has experienced; the relation which many measures of the most remote antiquity have to each other, and to terrestrial circumference, strengthens this conjecture, and seems to indicate not only that this measure was very accurately known at an extremely ancient period, but that it has served as the base of a complete system of measures, the vestiges of which have been found in Egypt and Asia. Be that as it may, the first precise measure of the earth, of which we have any certain knowledge is that which Picard executed in France, towards the end of the last century, and which has been many times verified ; it is easy to imagine this operation. As we advance towards the north, the pole seems to be more and more elevated, the meridian altitude of stars situated towards the north augments, and that of the southern stars diminishes, some even become invisible. The first idea of the curvature of the earth was owing no doubt to the observation of these phenomena, which could not fail to attract the attention of mankind in those first ages of society when the return of the seasons was only distinguished by the rising and setting of the principal stars, compared with that of the Sun. The elevation or depression of the stars, gives the angle which verticals elevated at the extremity of the arc passed over, form at their point of contact ; for this angle is evidently equal to the difference of the meridian altitudes of the same star, minus the angle which the arc described, would subtend at the centre of the star ; and we are certain that this is insensible. It was then only requisite to measure this arc. It would be a long and tedious operation to apply our measures to so great an extent; it is a much more simple process to connect its extremities by a chain of triangles, to those of a base of 12 or 15,000 feet, and considering the precision with which the angles of these triangles may be determined, its length can be obtained very exactly. It is thus that the arc of the terrestrial meridian which crosses France from Dunkirk to Montjoui near Barcelona has been measured; that part of this arc, whose amplitude is equal to the hundredth part of a right angle, and whose central point corresponds to * $51°\frac{1}{3}$ of latitude, is equal to 100,179 metres.

Of all the re-entering figures, the spherical one is the most simple, since it only depends on a single element, the size of its radius. The natural inclination of the human mind, to attribute that form to bodies

* 46° 10′ 12″.

which it comprehends with the greatest facility, disposed it to give the Earth a spherical form. But the simplicity of nature should not always be measured by that of our conceptions. Infinitely varied in her effects, nature is only simple in her causes, and her œconomy consists in producing a great number of phenomena, often very complicated, by means of a small number of general laws.

The figure of the Earth is the result of these laws, which, modified by a thousand circumstances, might alter it sensibly from a sphere. Small variations observed in the length of the degrees in France indicate these alterations. But the inevitable errors of observation left doubts on this interesting phenomenon, and the academy of sciences, in which this great question was anxiously agitated, judged with reason that the difference of the terrestrial degree, if it really existed, would be principally manifested in the comparison of the degrees at the equator, and towards the poles. And academicians were sent even to the equator, where they found the degree of the meridian equal to 99552^{me}. 3, less by 465^{me}. 6 than the degree corresponding to the mean parallel. Other academicians travelled towards the north to about *73°,7 of latitude, and the degree of the meridian was observed to be 100696^{me}. 0 greater by 1143^{me}. 7 than at the equator; thus the increase of the meridional degrees from the equator to the poles was incontestably proved by these measures, and it was discovered that the earth was not exactly spherical.

These celebrated journies of the French academicians, having directed the attention of astronomers towards this object, new degrees of the meridian were measured in Italy, Germany, Africa and Pennsylvania. All these measures concurred to indicate an increase of the degrees from the equator towards the poles. The ellipse being, next to the circle, the most simple of

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* 66° 19′ 48″.

the re-entering curves, the earth was considered as a solid formed by the revolution of an ellipse about its shorter axis. Its compression in the direction of the poles is a necessary inference from the observed increase of the meridional degrees from the equator to the poles.

The radii of these degrees, being in the direction of gravity, are, by the law of the equilibrium of fluids, perpendicular to the surface of the sea, with which the earth is in a great measure covered. They do not tend as in a sphere to the centre of the ellipsoid. They have neither the same direction nor the same length as radii drawn from this centre to the surface, and which cut it obliquely every where but at the equator and at the poles. The point of contact of two adjoining verticals, is the centre of the small terrestrial arc which they comprise between them; if this arc was a straight line, these verticals would be parallel, or would only meet at an infinite distance, but in proportion as they are curved, they meet at a distance so much the shorter as they are more curved. Thus the extremity of the shorter axis being the point where the ellipse approaches most to a straight line, the radius of a degree at the pole, and consequently the degree itself, is of its greatest length. It is the contrary at the extremity of the greater axis of the ellipse. At the equator where the curvature is the greatest, the degree in the direction of the meridian is the shortest. Passing from the second to the first of these extremes the degrees augment, and if the ellipse is but little flattened their increase is very nearly proportional to the square of the sine of the latitude.

The measure of two degrees in the direction of the meridian is sufficient to determine the two axes of the generating ellipse, and consequently the figure of the Earth, supposing it elliptic. If this is the hypothesis of nature the same proportion should be found very nearly between the two axes, comparing two by two the degrees of France, of the north, and of the equator;

but their comparison gives in this respect differences which it is difficult to attribute to errors of observation alone. The excess of the axis of the equator above that of the pole taken as unity is called the compression or ellipticity of the elliptic spheroid; now the degrees of the north and of France give $\frac{1}{146}$ for the ellipticity of the Earth, which the degrees of France and at the equator give $\frac{1}{314}$, it appears then that the Earth differs sensibly from an ellipsoid. There is even reason to believe that it is not a solid of revolution, and that its two hemispheres are not equal on each side of the equator. The degree measured by La Caille at the Cape of Good Hope in the southern latitude of *37° 01 has been found to be 100050^{me}. 5, which surpasses the degree in France in the latitude of †51°$\frac{1}{3}$, and is greater than that which has been measured in Pennsylvania, in the latitude of ‡ 43° 56, the length of which was only 99789^{me}. 1.

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* 33° 18′ 32″ 9. † 46° 11′ 49. ‡ 39° 12′ 14″ 4.

The degree at the Cape is yet greater than the degree measured in Italy in the latitude of * 47° 80, and which has been found to be 99948^{me}. 7. Yet it ought to be smaller than all these degrees if the earth was a regular solid of revolution, formed of two similar hemispheres. Every thing leads us to believe that this is not the case. Let us see what then is the nature of the terrestrial meridians.

The celestial meridian as determined by astronomical observations, is formed by a plane which passes through the centre of the earth and the zenith of the observer, because this plane bisects the arcs of the parallels to the equator, which the stars describe above the horizon.

All the places on the Earth which have their zeniths in the circumference of this meridian form the corresponding terrestrial meridian. Considering the immense distance of the stars, verticals elevated from

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* 43° 37′ 12″.

each of these places may be considered parallel to the plane of the celestial meridian. The terrestrial meridian may then be defined a curve formed by the junction of the bases of all the verticals parallel to the plane of the terrestrial meridian. This curve lies altogether in this plane when the Earth is considered as a solid of revolution, in every other case it departs from it, and generally is one of those lines which geometricians call *curves of double curvature*.

The terrestrial meridian is not exactly the line which determines trigonometrical measurements in the direction of the celestial meridian. The first side of the line measured is a tangent to the surface of the earth, and parallel to the plane of the celestial meridian.

If this side be prolonged till it meets a vertical infinitely near it, and this prolongation be bent to the base of the vertical, the second side of the curve will be formed, and thus with all the others. The line thus traced is the shortest that can be drawn on the surface of the earth between any two points taken on this line. It is not in the plane of the celestial meridian, and is not confounded with the terrestrial meridian, except in the case where the Earth is a solid of revolution; but the difference between the length of this line, and that of the corresponding arc of the terrestrial meridian is so small that it may be neglected without any sensible error.

The figure of the Earth being extremely complicated, it is important to multiply the measures of it in every direction, and in as many places as possible.

We may always, at every point of its surface, suppose an osculatory ellipse, which sensibly coincides with it, to a small extent round the point of contact.

Terrestrial arcs measured in the direction of the meridian, and perpendicular to it, compared with observations of latitudes, and of the angles which the direction of the extremities of these arcs form with their respective meridians, will inform us of the nature and position of this ellipsoid, which may not be a solid of revolution, and which varies sensibly at great distances.

The operations which Delambre and Mechain have executed in France to obtain the length of the metre, determine very nearly the osculatory ellipse of that part of the terrestrial surface. They have observed the latitude not only at the extremities of the arc, but at three intermediate points.

The astronomical and trigonometrical observations were made with a repeating circle, which gives great precision in the measure of angles.

Two bases of above twelve thousand metres have been measured, the one near Melun, the other near Perpignan. And what confirms the correctness of all these observations is, that the base at Perpignan deduced from that at Melun, by the chain of triangles which unites them, does not differ by the third of a metre from its measurement, though the distance which separates the two bases is above nine hundred thousand metres.

The principal results of this important operation are shewn in the following table:

Observed Latitudes.

	Decimal.	Sexigonal.
Montjoui -	45°. 958281	41° 21′ 45″
Carcassone -	48. 016790	41 12 54
Evaux -	51. 309414	46 10 42
Pantheon at Paris	54. 274614	48 50 49.7
Dunkirk -	56. 706944	51 29 10

Arc of the terrestrial meridian comprised between Montjoui and

Carcassone - -	205621^{me} 3
Evaux - - -	534714 5
Pantheon - -	831536 4
Dunkirk - -	1075058 5

The comparison of these results evidently indicates a diminution in the terrestrial degrees from the pole to the equator, but the law of this diminution seems very irregular. Notwithstanding if the ellipsoid which satisfies these measures nearer than any other be required, we find that to represent them in this hypothesis, it is sufficient to alter the observed latitudes about four seconds and a half.

The compression is then $\frac{1}{150}$, the semi-axis of the pole parallel to that of the Earth, is 6344011^{me}, and the degree corresponding to the mean parallel is 99983^{me}. 7. An error of four seconds and a half, though very small, is not admissible, considering the great precision of the observations; but we may at least consider this ellipsoid as osculatory to the surface of the earth in France, at * 51° of latitude, and suppose that it coincides with it to an extent of 5 or 6 degrees round the point of osculation. It gives 100716^{me}. 9, for the degree perpendicular to the meridian, at † 56° 3144 latitude, and by a very exact operation lately made in England it has been found to be 100700^{me}. 5.

* 45° 54′. † 50° 40′ 58″.

This agreement proves that the action of the Pyrenees and other mountains in the south of France has had very little influence on the latitudes observed at Evaux, Carcassone and Montjoui, and that the great compression of the osculatory ellipse depends on attractions much more extended, the effect of which is felt in the north as well as the south of France, and even in England, Italy, and Austria; for the degrees which have been carefully measured are very nearly the same as on this ellipsoid. There is reason then to presume that if the arc measured from Dunkirk to Montjoui was extended to the island of Cabrera, the corresponding degree of the mean parallel which would result from this measure would not exceed one hundred thousand metres. The total arc included between this island and Dunkirk, being divided into two nearly equal parts by this parallel; the length of the quarter of the meridian concluded from this arc, becomes totally independant of any hypothesis of the ellipticity of the Earth.

Mechain had consequently proposed to join Cabrera to Montjoui, and had already prepared every thing for this new measure; but events have not suffered him to perform it: let us hope that favorable circumstances will soon permit it to be resumed.

It appears by the observed directions of the sides of the arc measured from Dunkirk to Montjoui, that the osculatory ellipse is not exactly a solid of revolution. But more certain information would be obtained on this subject, if, as is much to be desired, a perpendicular to the meridian of the observatory was to be measured in the broadest part of France with the same means that were employed in the measure of the meridian, and if the latitude and direction of its sides relative to their respective meridians were to be determined with precision on different points of this perpendicular.

Whatever be the nature of the terrestrial meridians, it is evident as their degrees diminish from the poles to the equator, that the Earth is flattened in the direction of its poles, that is to say, that the axis of the poles is less than the diameter of the equator. To explain this, let us suppose the Earth a solid of revolution, and let us represent the radius of the degree at the north pole, and the series of all these radii from the pole to the equator, which radii by the supposition continually diminish. It is visible that these radii form by their consecutive intersections a curve which at first touches the axis of the pole, it afterwards separates from it, its convexity being constantly turned towards it, and raises itself towards the pole till the radius of the meridional degree takes a direction perpendicular to the first: it will then be in the plane of the equator. If this radius of the polar degree be supposed flexible, and that it involves successively the arc of the curve which we have considered, its extremity will describe the terrestrial meridian, and the part of it intercepted between the meridian and the curve, will be the corresponding radius of the degree of the meridian. This curve is what geometricians call the evolute of the meridian. Let us consider at present, the intersection of the diameter of the equator with the axis of the pole, as the centre of the Earth. The sum of the two tangents to the evolute of the meridian drawn from this centre, the one following the axis of the pole and the other the diameter of the equator, will be greater than the arc of the evolute which they include between them. Now the radius drawn from the centre of the Earth to the north pole, is equal to the radius of the polar degree, *minus* the first tangent; the semi-diameter of the equator is equal to the radius of the degree of the meridian at the equator, *plus* the second tangent. The excess of the semi-diameter of the equator above the terrestrial radius of the pole is then equal to the sum of these two tangents *minus* the excess of the radius of the polar degree, above the radius of the degree of the meridian at the equator; this last excess is the arc itself of the evolute, which is less than the sum of the extreme tangents. The excess then of the semi-diameter of the equator above the radius drawn from the centre of the Earth to the north pole is positive. It can be proved in the same manner, that the excess of the semi-diameter of the equator above the radius drawn from the centres of the earth to the south pole is positive. The whole axis of the poles thus is less than the diameter of the equator, or, what comes to the same thing, the Earth is flattened in the direction of the poles.

Considering every portion of the meridian as a very small arc of its osculatory circumference, it is easy to see that the radius drawn from the centre of the Earth to that extremity of the arc which is nearest to the pole, is less than the radius drawn from the same centre to the other extremity. From whence it follows, that the terrestrial radii increase from the poles to the equator, if, as all observations indicate, the degrees of the meridian augment from the equator to the poles. The difference of the radii of the degrees of the meridian from the pole to the equator is equal to the difference of the corresponding terrestrial radii, *plus* the excess of twice the evolute above the sum of the two extreme tangents, which excess is evidently positive: thus the degrees of the meridian increase from the equator to the poles in a greater proportion than the diminution of the terrestrial radii. It is evident that these demonstrations equally apply, if the two northern and southern hemispheres were not equal and similar, and it is easy to extend them to the supposition of the earth's not being a solid of revolution.

But it is remarkable that the observations made in the northern hemisphere give the evolute of the meridian from * forty-three to † seventy-three degrees of latitude very little different from that of an ellipsoid of $\frac{1}{150}$ compression, and of which the mean degree is $99983^{me.}$ 7. For this

* 38° 42'. † 65° 42'.

ellipsoid nearly satisfies the measures lately made in France, the degrees measured in Italy and Lapland, and that which has been measured in England perpendicular to the meridian.

It also represents the degree of the meridian measured in Austria at * 53° of latitude, and which Liesganig has found to be 100114$^{me.}$ 2. Finally it agrees with the degree of the longitude measured in France at † 48° 4 latitude, and of which Cassini and La Caille have fixed the length at 72003 $^{me.}$ 5.

Curves have been constructed at the principal places in France on the line which has been considered as the meridian of the observatory of Paris, traced in the same manner as this line, with this difference only that the first side, always a tangent to the surface of the Earth, instead of being parallel to the plane of the celestial meridian of Paris, is perpendicular to it.

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* 47° 47′ 24″ † 43° 33′ 36″

It is by the length of these curves and by the distance from the observatory to the points where they meet the meridian, that the position of these places is determined. This labour, the most useful to geography which has yet been performed, is a model which every enlightened nation will no doubt, hasten to imitate.

To connect objects thus, it is requisite that they should be but little distant from each other. To fix the respective positions of places separated by great distances or by the sea, recourse must be had to celestial observations. The knowledge of these positions is one of the greatest advantages which astronomy has produced. To arrive at it, the method has been followed that was made use of to form the catalogue of stars, conceiving on the terrestrial surface circles similar to those which had been imagined on the surface of the heavens.

Thus the axis of the celestial equator intersects the surface of the earth in two points diametrically opposite, which have each one of the poles of the world for their zenith, and which may be considered as the poles of the earth.

The intersection of the plane of the celestial equator with this surface, is a circumference which may be regarded as the terrestrial equator. The intersection of all the planes of the celestial meridian with the same surface, form so many curved lines, which unite at the poles, and which are the corresponding terrestrial meridians, if the earth is a solid of revolution, which may be supposed in geography without any sensible error. Finally, small circles traced from the equator to the poles upon the earth parallel to the equator, are terrestrial parallels, and that of any place whatever, corresponds to the celestial parallel which passes through its zenith.

The position of a place upon the earth is determined by its distance from the equator, or the arc comprised between its parallel and the equator ; and by the angle which its meridian forms with a principal meridian, the position of which is arbitrary, and to which all the others are referred. The distance from the equator depends on the angle included between the zenith and the celestial equator, and this angle is evidently equal to the altitude of the pole above the horizon. This altitude, is called latitude in geography. Longitude is the angle which the meridian of a place makes with the principal meridian; it is the arc of the equator contained between these two meridians, it is east or west according as the place is east or west of the principal meridian. An observation of the altitude of the pole gives the latitude. The longitude is determined by means of any celestial phenomenon observed at the same time on the meridians of which the respective position is required. The instant of noon is not the same on these meridians ; if that from which the longitude is reckoned is to the east of that of which the longitude is required, the Sun will arrive sooner at the celestial meridian.

If, for example, the angle formed by the terrestrial meridians be a quarter of the circumference, the difference between the instants of noon on these meridians will be a quarter of a day. Suppose then, that a phenomenon is observed on each of them, which occurs at the same physical moment to every place on the Earth, such as the beginning or end of an eclipse of the moon or of the satellites of Jupiter. The difference of time which the observers will reckon at the moment of the phenomenon will be to the whole day as the angle formed by the two meridians is to the circumference.

Eclipses of the Sun, and occultations of stars by the Moon, furnish the most exact means of finding longitudes, from the precision with which the beginning and end of these phenomena may be observed. They do not in fact occur at the same physical instant to the whole earth, but the elements of the lunar motions are sufficiently known to enable us to keep an exact account of this difference. To determine the longitude of a place, it is not necessary that the celestial phenomenon should be observed at the same time on the principal meridian; it is sufficient if it is observed on one whose position with the principal meridian is known. It is thus that by connecting the meridians with each other the position of the most distant points on the surface of the Earth is ascertained. The longitudes and latitudes of a great number of places on the Earth are already known by astronomical observation. Great errors in the position and extent of countries long known have been corrected. And the situation of new countries which the interest of commerce and the love of science have caused to be discovered, is fixed.

But though voyages undertaken in these later periods have added considerably to our geographical knowledge, much yet remains to be discovered. The interior of Africa and America include vast countries which are totally unknown. We have only uncertain and frequently contradictory relations concerning many others, of which, geography, hitherto abandoned to the hazard of conjecture, only waits for new information from astronomy, to determine the positions irrevocably.

It is principally to the navigator, when in the midst of the ocean with no other guide then the stars and his compass, that it is important to know his own position, that of the places for which his course is bound, and of the rocks and quicksands which may occur in his passage.

He may easily know his latitude, by observations of the stars; but the sphere, in consequence of its diurnal motion, presenting itself daily in nearly the same manner to all the points of his parallel, it is difficult for the navigator to fix the point on which he is. To supply the deficiency of celestial observations, he measures his velocity and the direction of his motion, he estimates his passage in the direction of the parallels, and comparing it with his observed latitude, determines his longitude relat ive to the place of his departure. The inaccuracy of this method exposes him to errors, which might become fatal when he abandons himself during the night to the winds, near shores or banks, which from estimation he believed himself far from. It is to secure him from these dangers that as soon as the progress of arts and astronomy led to the hope that methods might be found to obtain the longitude at sea, commercial nations have hastened to direct the views of scientific men and artists to this important object by powerful encouragements.

Their wishes have been fulfilled by the invention of chronometers, and by the accuracy to which the tables of the lunar motions have been carried. Two methods, good in themselves, and which are yet improved by lending each other a mutual support.

A chronometer, well regulated in a

port the situation of which is known, and preserving the same rate, when carried on board a vessel, would indicate at every instant the time which was reckoned in this port. Comparing it with that observed at sea, the relation of the difference of these hours to the whole day, would be, as has been already seen, that of the corresponding difference of longitude to the whole circumference. But it was difficult to obtain such watches, the irregular motion of the vessel, the variations of temperature, and the inevitable friction which is very sensible in so delicate a machine, were so many obstacles which opposed their accuracy.

These have been fortunately overcome, and chronometers are now made, which, during many months, preserve a rate nearly uniform, and thus afford the most simple method of finding the longitude at sea; and as this method is so much the more precise as the time is short, during which these chronometers are employed without verifying their rate, they are very useful to determine the respective position of places very near to each other. They have in this respect, some advantage over astronomical observations, the precision of which is not increased by the smallness of the observer's distance.

The eclipses of the satellites of Jupiter, which occur frequently, offer to a navigator an easy method of knowing his longitude, if he could observe them at sea; but the endeavours that have been made to overcome the difficulties arising from the motion of the vessel, which oppose this species of observation, have hitherto been fruitless. Navigation and geography have however derived great advantages from these eclipses, and chiefly from those of the first satellite, of which the commencement and end can be observed with precision. The navigator employs them with success when he can land. It is necessary indeed to know the hour at which the same eclipse that he observes would be seen upon a known meridian, since the difference of time which is reckoned on these two meridians, gives the difference of longitude ; but the tables of the first satellite of Jupiter, considerably improved in our time, give the instants of these eclipses, with a precision almost equal to observation.

The extreme difficulty of observing these eclipses at sea, has compelled us to have recourse to other celestial phenomena, among which the lunar motions are the only ones which can be made subservient to the determination of terrestrial longitudes. The position of the Moon, such as it would be observed at the centre of the earth, may be easily concluded from the measure of its angular distances from the Sun and stars. The tables of its motion then give the hour at the principal meridian, when the same phenomenon is observed, and the navigator, comparing the time which he reckons on board his vessel, at the moment of observation, determines his longitude by the difference of time.

To appreciate the accuracy of this method, it should be considered that from the errors of observation, the Moon's place, as determined by the observer, does not correspond exactly to the hour pointed out by his chronometer, and that from the errors of the tables, this same place does not relate to the corresponding hour, which they indicate on the first meridian; the difference of these hours then would not be that given by an observation and tables rigorously correct. Let us suppose that the error produced by this difference is a minute. In this interval forty minutes of the equator pass the meridian; this is the corresponding error in the longitude of the vessel, and which, upon the equator, is about forty thousand metres, but it is less on the parallels: besides, it may be diminished by multiplying the observations of the lunar distances from the Sun and stars, and repeating them during many days, to compensate and destroy by each other, the errors of the observations and tables.

It is obvious that the errors of longitude corresponding to those of the tables and of observation, are so much the less as the motion of the celestial body is more rapid; thus the observations of the Moon in perigee, are in this respect preferable to those in apogee. If the motion of the Sun be employed, which is about thirteen times slower than that of the Moon, the errors of longitude will be about thirteen times as great: from whence it follows, that of all the celestial bodies, the Moon is the only one whose motion is sufficiently rapid to be employed for the determination of longitudes at sea. It is evident therefore how important it is to render these tables perfect.

A very remarkable phenomenon, the knowledge of which we owe to astronomical voyages, is the variation of gravity at the surface of the earth. This singular power acts in the same spot, upon all bodies proportionally to their masses, and tends to impress on them equal velocities in equal times. It is impossible, by means of a balance, to ascertain these variations, because they affect equally the body which is weighed, and the weight to which it is compared ; but observations of the pendulum are those most proper to discover them, since it is clear that these oscillations should be slower in those places where gravity is least powerful. This instrument, the application of which to clocks is one of the principal causes of the progress of modern astronomy, consists of a body suspended at the end of a thread or a rod, moveable round a fixed point, placed at the other extremity. The instrument is drawn a little from its vertical position, then abandoning it to the action of gravity, it makes small oscillations, which are very nearly of the same duration, notwithstanding the difference of the arcs described. This duration depends on the size and figure of the suspended body, on the mass and length of the rod; but geometricians have found general rules to determine, by the observed oscillations of a compound pendulum of any figure whatever, the length of a pendulum, whose oscillation shall have a certain duration, and in which the mass of the rod shall be supposed nothing relative to that of the body, considered as an infinitely dense point. It is by this imaginary pendulum, called the *simple pendulum*, that all the experiments of the pendulum, made in different parts of the Earth, are referred.

Richer, sent in 1672, to Cayenne, by the academy of sciences, to make astronomical observations there, found that his clocks, regulated at Paris to mean time, lost a perceptible quantity every day.

This interesting observation gave the first direct proof of the diminution of gravity at the equator. It has been repeated with care, in a great number of places, keeping account of the resistance and temperature of the air.

The result of all the observed measures of a pendulum vibrating seconds is, that it increases from the equator to the poles, like the degrees of the meridian, and that its increase, which, at the pole itself, is equal to five hundred and fifty-five hundred thousandths of its gravity at the equator, is proportional to the square of the sine of the latitude.

Borda, by a very exact experiment, has found recently that the length of the pendulum vibrating seconds at the Observatory at Paris, and reduced to a vacuum, is 0[me].741887 ; it results that its length in France, under the parallel of * 50° is equal 0[me].741606, and that thus the simple pendulum of the length of the metre would make 86116.5 oscillations in a day.

This length, which is very exact, and the measure of the degree of the meri-

* 45°.

dian, corresponding to the same parallel, will serve to recover our weights and measures, if by the lapse of time they should alter.

There has likewise been remarked, by means of the pendulum, a minute diminution of gravity on the summits of high mountains. Bouguer has made a great number of experiments on this subject at Peru. He has found that the force of gravity at the equator, and at the level of the sea being expressed by unity, is 0.999249 at Quito, elevated 2857 metres above this level; and 0.998816 on the summit of Pichenca, at 4744^{me} in height. This diminution of gravity at altitudes always very small relatively to the terrestrial radius, gives us reason to suspect that this force diminishes considerably at great distances from the centre of the Earth.

I ought, when speaking of the observations of the pendulum, to call the attention of natural philosophers to the two following circumstances: One is, the slight resistance which bodies, in changing their temperature, appear to me to oppose to their change of volume, nearly as water resists its conversion into ice, and can retain its form at several degrees below zero. It is then sufficient to agitate it to render it solid. In the numerous experiments on the dilatation of bodies, which I made with Lavoisier, we were sometimes obliged to give them a slight concussion to make them take the form proper to their temperature. The second object relates to the invariable pendulums which are used to determine the difference of gravity on various places of the earth. If the rod of the pendulum is of steel, its to be feared that the effect of terrestrial magnetism should become complicated with that of gravity; and as the object is to appreciate very small quantities in these experiments, it is important to be assured that this effect is insensible.

Observations of the pendulum vibrating seconds, furnishing a length which is invariable and easily recoverable at any period, have given rise to the idea of employing an universal measure.

The prodigious number of measures in use, not only among different people, but in the same nation; their whimsical divisions, inconvenient for calculation, and the difficulty of knowing and comparing them; finally, the embarrassment and frauds which result from them in commerce, cannot be observed without acknowledging that the adoption of a system of measures, of which the uniform divisions are most easily subjected to calculation, and which are derived in a manner the least arbitrary, from a fundamental measure, indicated by nature itself, would be one of the greatest services which science and government can render to the human race. A people who should adopt such a system of measures, would unite to the advantage of gathering the first fruits of it, that of seeing their example followed by other nations, of which it would thus become the benefactor. For the slow but irresistible empire of reason, at length overcomes all the national jealousies and obstacles which oppose themselves to an advantage that would be universally felt.

Such were the reasons that determined the constituent assembly to charge the academy of sciences with this important object. The new system of weights and measures is the result of the labors of a commission appointed by them, and aided by the zeal and abilities of many members of the national representation.

The identity of the decimal calculus with that of integral numbers, leaves no doubt of the advantages of dividing every kind of measure into decimal parts.

To be convinced of this, it is sufficient to compare the difficulty of complicated multiplication and division, with the facility of the same operations on integral numbers, which facility may be still increased by logarithms, the use of which might be made extremely popular by means of instruments very simple, and of little expense. The academy, therefore, did not hesitate to adopt the decimal division; and to render the whole system of measures uniform, it was resolved that they should all be derived from the same linear measure, and its decimal divisions. The question was thus reduced to the choice of this universal measure, to which the name of *metre* was given.

The length of the pendulum, and that of the meridian, are the two principal means which nature offers to fix the unity of linear measures. Both independent of moral revolutions, cannot experience a sensible alteration, but by great changes in the physical state of the world. The first expedient, easy in use, has this inconvenience, of making the measure of distance depend on two heterogeneous elements, gravity and time, the measure of which is besides arbitrary. The second measure was therefore chosen, which appeared to have been employed in the most remote antiquity: so natural is it to man to compare itinerary measures even to the dimensions of the globe which he inhabits, that in travelling he may know by the mere denomination of the space he has passed over, the relation of that space to the whole circuit of the earth. There is likewise the advantage of making nautical and celestial measures correspond. It is often necessary to the navigator to determine one by the other, the distance he has traversed, and the celestial arc included between the places of his departure and arrival. It is then interesting that one of these measures should be the expression of the other by nearly the difference of their unities. But for this purpose the fundamental unity of linear measures should be an aliquot part of the terrestrial meridian, which corresponds to one of the divisions of its circumference; thus the choice of the metre was reduced to that of the unity of angles.

The right angle is the limit of the inclination of a line to a plane, and of the altitude of objects above the horizon. Besides, it is in the first quarter of the circle that sines are formed, and, generally, all the lines employed in trigonometry, and the proportion of which to the radius, are reduced to tables. It is then natural to take the right angle for the unity of angles, and the quarter of the circumference for the unity of their measures. It is divided into decimal measures, and that the measures on the earth might correspond, the quarter of the terrestrial meridian is divided into the same parts, which had been done at a very ancient period; for the measure of the earth, mentioned by Aristotle, and the origin of which is unknown, gives a hundred thousand stadia to the quarter of the meridian. It was then only requisite to obtain its exact length. Here many questions present themselves, which the ignorance we are in respecting the true figure of the earth will not permit us to resolve. Is the earth a spheroid of revolution? Are its two hemispheres equal and similar on each side of the equator? What is the proportion of an arc of the meridian, measured at a given latitude, to the whole meridian? In the most natural hypothesis of the formation of the terrestrial spheroid, the difference of the meridians is insensible; and the decimal degree, bisected by the mean parallel between the north pole and the equator, is the hundredth part of the quarter of the meridian. The error of these hypotheses, if any exists, cannot influence geographical distances, where it is of no importance. The length of the quarter of the meridian may then be concluded from that of the arc which crosses France, from Dunkirk to the Pyrenees, and which has been carefully measured in 1740, by the French academicians. But as a new measure of a greater arc, in which still more precise methods were employed, would inspire an interest in favor of the new system of measures, likely to extend the use of it, it was resolved to measure the arc of the terrestrial meridian contained between Dunkirk and Barcelona.

The operations which Delambre and Mechain have lately performed, have given this arc, the amplitude of which is * 10° 748663, equal to fifty-five million one hundred and fifty eight thousand four hundred and seventy hundredths of the iron toise used at the equator at the temperature, † 16°$\frac{1}{4}$. The middle of the arc being ‡ one degree and a third, more north than the mean parallel, the quarter of the meridian could not be determined by this measure, without adopting an hypothesis of the ellipticity of the Earth: that which results from the arc measured in France compared with that in Peru, appeared to deserve the preference, from the length and distance of the two arcs, and from the care and reputation of the observers. The

* 6° 40′ 26″ 2. † 14° 37′ 35″. ‡ 1° 11′ 24″.

quarter of the meridian was thus found to be equal to 5130740 toises.

The ten thousandth part of this length was taken for the metre, or unity of the linear measures. The decimal above this was too large, that below too small; and the metre, the length of which is 0,513074 toises, supplies the place advantageously of the toise and ell, the two of our measures in most common use.

To preserve the length of the metre, the national convention has decreed, that a standard executed from the experiments and observation of the committee charged with its determination, should be preserved in the care of the legislative body.

This standard, which was presented by the national institute, can only represent the metre, at one determined degree of temperature. That of melting ice has been chosen as the one most fixed and independent of the modifications of the atmosphere. To recover the metre at any period without being obliged to have recourse to the measure of the great arc from which it was derived, it was important to fix very precisely its proportion to the length of the pendulum vibrating seconds; and it is with this view that Borda has again determined this length at the observatory at Paris.

All the measures are derived from the metre in the most simple manner; the linear measures are decimal multiples and submultiples of it.

The unity of the superficial measures of land is a square the side of which is ten metres, it is called *are*.

A measure equal to a cubic metre and usually employed to measure fire wood, is called *stere*.

The unity of cubic measure is the cube of the tenth of a metre, it is called *litre*.

The unity of weight which is called *gramme*, is the absolute weight of a cube of the hundredth part of a metre of distilled water, and considered at its *maximum* of density.

By a remarkable singularity this *max-*

imum does not correspond to the freezing point, but is above it at about four degrees of the thermometer. Water in falling below this temperature again dilates, and thus prepares itself for that increase of volume which it receives in its passage from the fluid to the solid state. Water was chosen as one of the most homogenous substances, and which can be most easily reduced to a state of purity. Le Fevre Gineau has determined the *gramme* by a long series of delicate experiments on the specific gravity of a hollow cylinder of brass, the volume of which he has measured with extreme care. The result is, that the livre, supposed the twenty-fifth part of the pile of fifty *marcs,* which is preserved at the mint of Paris, is to the *gramme* in the proportion of 489,5058 to unity.

All these measures being constantly compared with the *livre* in money it was particularly important to divide it into decimal parts. It is now called, *the silver* *franc,* its tenth part,*decime,* its hundredth, *centime.*

To facilitate the calculation of the fine gold and silver contained in pieces of money, the alloy has been fixed at a tenth of their weight, and that of the *franc* has been made equal to five *grammes.* Thus the pieces of money are exact multiples of the unity of weight, which is extremely useful in commerce.

Finally, the uniformity of the whole system of weights and measures seemed to require that the day should be divided into ten hours, the hour into one hundred minutes, the minutes into a hundred seconds, &c. This division of the day which will become necessary to astronomers,is less advantageous in civil life, where there is little occasion to employ time as a multiplier or divisor. The difficulty of adapting it to clocks and watches, and our commercial connexions with foreigners in the sale of watches have suspended its use indefinitely. We may however believe that at length the decimal division of the day will supersede its actual division, which differs too much from the division of other measures not to be abandoned.

Such is the new system of weights and measures which the men of science have offered to the national convention, which has hastened to sanction them. This system, founded on the measure of terrestrial meridians, is equally suitable to every nation. It has no other relation with France than by the arc of the meridian which crosses it; but the position of this arc bisected by the mean parallel, and the extremities of which are bounded by the two oceans, is so advantageous that the learned of every nation, united to fix an universal measure, would have chosen it.

To multiply the advantages of this system, and to render it useful to the whole world, the French government has invited foreign powers to take part in an object of such general interest. Many have sent eminent men of science to Paris, who united with the committee of the national institute, have determined the fundamental unities of weight and length, by the discussion of observations and experiments; so that the determination of these unities ought to be considered as a work common to the learned who have assembled there, and the people whom they represent. It is then permitted to hope that this system, which reduces all measures and their calculation to the scale and to the most simple operations of decimal arithmetic, will be as generally adopted as the system of numeration to which it is analogous, and which without doubt had to oppose the same obstacles, which prejudices and habit oppose to the introduction of the new measures.

CHAP. XIII.

Of the Flux and Reflux of the Ocean.

ALTHOUGH the earth and the fluids which cover it must long since have assumed the state which is suitable to the equibrium of the forces which animate them, nevertheless the figure of the ocean changes every instant of the day, by regular and periodical oscillations, which are known by the name of tides. It is a circumstance truly astonishing, to behold even in calm and serene weather, the intense agitation of this great fluid mass, whose waves constantly break with impetuosity on the shore.

This phenomenon gives rise to reflection, and excites a strong desire to penetrate the cause. But not to be bewildered in vague hypothesis, we should first know the laws of this singular appearance and follow them in their details.

At the commencement of this century, by the request of the Academy of Sciences, a great number of observations were made in our ports of the flux and reflux of the ocean.

They were continued every day at Brest during six consecutive years, and they form from their number, and from the magnitude and regularity of the tides at this port, the most valuable and complete collection of observations of this kind that we possess. Since a thousand accidental causes may alter the course of nature in these phenomena, it is necessary to consider at once a great number of observations, that the partial causes, compensating each other, the mean results may show only the effects, which are regular and constant. It is likewise necessary, by a happy combination of observations, to make those phenomena evident which we wish to determine, and by separating them from the rest, to make them better known. It is from thus investigating these observations, that I arrived at the following result, of the truth of which there can remain no doubt.

The ocean rises and falls twice in every interval of time comprehended between two consecutive returns of the Moon to the superior meridian. The mean interval of these returns is* 1,035050 days; thus the mean interval of two consecutive high tides is † 0, 51525 of a day, so that there are some solar days in which we may observe but one single tide. The moment of low water divides this interval nearly into equal parts, nevertheless at Brest the tide employs ‡ nine or ten minutes more to descend than to rise. Similar to all magnitudes which are susceptible of a maximum and a minimum, the increase and diminution of the tides near the limits are proportional to the squares of the time elaps-

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* $1^d\ 0^h\ 49'\ 16''\ 2$. † $12^h\ 15'\ 14'\ 1$.
‡ $12'\ 57''\ 6$ or $14'\ 24''$.

ed since the moments of high and low water.

The elevation of the water at high tide is not constantly the same ; it varies every day and its variation is evidently connected with the phases of the Moon. It is greatest about the time of the new and full moons, then it diminishes and becomes least about the quadratures. At Brest the highest tide does not take place exactly on the day of the sysygy, but a day and a half later ; so that if the sysygy happens at the moment of high water the greatest tide is the third that follows. In the same manner if the quadrature happens at the moment of high tide, the third tide which follows is the least. This phenomenon is observed to be nearly the same in all the ports of France, but the hours of high and low water are very different. The greater the elevation of the water at high tide, the greater the depression at the low tide which succeeds it. We call a *total tide* the half sum of the heights of two consecutive high tides above the level of the intermediate low tide. The mean value of this total tide at Brest is $5^{me.}$ 888 at its maximum about the sysygies, and $2^{me.}$ 789 at its minimum about the quadratures.

The distance of the Moon from the Earth influences in a very perceptible manner, the heights of the tides. All other circumstances being the same, they augment and diminish with the lunar parallax but in a greater ratio. If this parallax increases $\frac{1}{18}$, the total tide increases $\frac{1}{8}$ in the sysygies, and about $\frac{1}{4}$ in the quadratures; and as the tide is nearly twice as great in the first as in the second case, its increase in the two cases is the same.

The greatest variation in the diameter of the Moon being (either more or less,) about $\frac{1}{15}$ of the whole, the corresponding variation of the total tide in the sysygies is $\frac{1}{20}$ of its mean height, or about $0^{me.}$ 88,3: thus the entire effect of the change of distance between the Moon and the Earth is $1^{me.}$ 766, in the total tides.

The variation in the distance of the Sun from the Earth, influences the tides in a similar manner, but in a much less degree. All other circumstances being the same; in the winter when the Sun is nearest to us, the tides of the sysygy are the greatest, and those of the quadrature less than in summer, when the Sun is farthest from the Earth.

The declinations of the Sun and Moon have a remarkable influence on the tides, they diminish the total tides of the sysygies, and the tides at Brest are about $0^{me.}8$ less in the solstices than at the equinoxes; the total tides of the quadratures are likewise less by the same quantity in the equinoxes than at the solstices.

It is principally about the maxima and minima of the total tides, that it is interesting to know the law of the variation. We have seen that the moment of the maximum at Brest follows the sysygy a day and a half. The diminution of those total tides that are near it, is proportional to the squares of the times elapsed from that instant to the time of the intermediate low tide, to which the total tide is referred, it is $0^{me.}1064$, when this interval is a lunar day. Near the instant of the minimum which follows the quadrature one day and a half, it is proportional to the square of the time elapsed since that instant; it is nearly double the diminution of the tides near their maximum.

The declinations of the Sun and of the Moon influence very sensibly these variations. The diminution of the tides at the sysigies of the solstices is not above three fifths of the corresponding diminution of the sysygies of the equinoxes. A small difference has likewise been observed between the tides of the morning and the evening, which depends on the declinations of the Sun and Moon, and which disappears when these bodies are in the equator. This variation may be perceived, by comparing the tides of the first and second day after the sysygy or the quadrature, because the tides being then very near their maximum or minimum, vary very little from one day to another, and the more easily admit the difference to be perceived, between two tides in the same day. Thus it is found at Brest that at the sysygy of the summer solstice the morning tides the first and second day after it, are less than the evening tides by $0^{me.}183$. They are on the contrary greater by the same quantity in the sysygy of the winter solstice. In like manner in the quadratures of the autumnal equinox the morning tides the first and second day after the quadrature surpass the evening tides by $0^{me.}131$; they are less by the same quantity in the quadratures of the vernal equinox.

Such are in general the phenomena that are observed in our ports relatively to the height of our tides. Their intervals offer other phenomena which we are now to develope. When the high tide happens at Brest the moment of the sysygy, it follows the instant of mid-night, or mid-day * $0^{d}.14822$, according as it happens in the morning or in the evening: this interval which is very different in harbours that are even

* $3^{h}\ 33'\ 33''\ 6$.

very near each other, is called the hour of the port, because it is from that hour that we reckon in determining the times of high water relatively to the phases of the Moon, as will be explained in treating of the causes of the tides. The high tide that takes place at Brest the moment of the quadrature follows the instant of mid-night or mid-day *0^{d} 35464. The tide of the sysygy advances or retards †264″ for every hour that it precedes or follows the sysygy; the tide of the quadrature advances or retards ‡ 416″ for every hour that it precedes or follows the quadrature. The hours of high water at the sysygies and the quadratures vary with the distances of the Sun and Moon from the Earth and principally with the distances of the Moon. In the sysygies, every minute of increase or diminution in the apparent semi-diameter of the Moon advances or retards the hour of high water by §354″. This phenomenon likewise takes place at the quadratures but is three times less.

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* 9^{h} 61′ 40″ 8. † 1′ 26″ 1. ‡ 2′ 20″7. § 1′54″6.

In a similar manner the declinations of the Sun and Moon influence the time of high water at the sysygies and the quadratures. In the sysygies of the solstices the time of high water advances about * 2 minutes, it is retarded by the same quantity in the sysygies of the equinoxes; on the contrary on the quadratures of the equinoxes the time of high water advances about † 8 minutes, and retards about the same quantity in the quadratures of the solstices.

We have seen that the retardation of the tides from one day to another is ‡ $0^d.03505$ in its mean state; so that if the time of high water happens at $0^d.1$, it will happen the next morning at § 0.13505. But this retardation varies with the phases of the Moon; it is the smallest possible about the sysygies, when the total tides are at their maximum, and then it is only ¶ $0^d.02705$; when the tides are at their minimum it is the greatest possible, and amounts to ‖ $0^d.05207$. Thus the difference of the times of high

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* 2′ 52″ 8. † 11′ 31″ 2. ‡ 50′ 28″ 2. § 2^h 24″. ¶ 38′ 57″ 1. ‖ 1^h 14″ 58″ 8.

water corresponding to the moment of the sysygy and quadrature (and which is shewn above to be *$0^{d}\cdot20542$) increases for the tides which follow in the same manner these two phases, and become nearly equal to a quarter of a day relatively to the maximum and minimum of the tides.

The variations in the distances of the Sun and Moon, and principally of the latter, influence the retardation of the tides from one day to the other. Every minute of increase or of diminution in the apparent semi-diameter of the Moon, augments or diminishes this retardation † 258″ about the sysygies. This phenomenon takes place likewise in the quadratures, but is three times less.

The daily retardation of the tides varies likewise with the declinations of the Sun and Moon. In the sysygies of the solstices it is about ‡ 155″ greater than in its mean state; it is less by the same quantity in the sysygies of the equinoxes: on the contrary

* 5^{h} 21′ 14″ 6. † 3′ 42″ 9. ‡ 2′ 49″.

in the quadratures of the equinoxes, it surpasses the mean by * 543″, and is surpassed by the same quantity in the quadratures of the solstices. Thus the inequalities in the heights and in the intervals of the tides, have very different periods, some of half a day, and a day, others of half a month, a month, a half year, and of a year; others again have the same periods with the revolutions of the nodes, and of the perigee of the lunar orbit; the position of which influences the tides by the effect of the declinations of the Moon, and of its distances from the Earth.

The height, and generally all the phenomena of the tides, appear to have been the same in the new as in the full Moon.

These phenomena equally take place, in all the harbours and coasts along the shores of the sea; but the local circumstances, without changing at all the above laws of the tides, influence very materially the heights of the tides, and the hour of high water for a given port.

* 7′ 49.

CHAP. XIV.

Of the Terrestrial Atmosphere and of Astronomical Refractions.

A THIN, transparent, and elastic fluid envelopes the Earth, and extends itself to a considerable altitude; it gravitates like all other bodies, and its weight balances that of the mercury in the barometer.

At the temperature of melting ice and at the mean height of the barometer at the level of the sea, which height is nearly seventy-six *centimètres*, the weight of air is to that of a similar column of mercury in the proportion of unity to 10283. Therefore at this temperature to lower the mercury a centimetre when its height is seventy-six, it is sufficient to raise the barometer 102.83 metres, and if the density of the atmosphere was every where the same, its height would be 7815 metres. But the air is compressible very nearly in the ratio of the weight which presses on it; from whence it follows that at equal temperatures its density is proportional to the height of the barometer; its inferior strata are therefore more dense than the superior ones, being compressed by their weight.

They become rarer in proportion as we elevate ourselves in the atmosphere; and if their temperature were the same a very simple calculation would shew us, that their altitude, increasing in arithmetical progression, their density would diminish in geometrical progression.

The cold which exists in the elevated regions of the atmosphere augments the density of the higher strata: for air, like all other bodies, contracts by cold, and expands by heat; and it has been observed that towards the temperature of melting ice the increase of a degree in its temperature augments its volume about a 250th part.

We have taken advantage of these data to measure the heights of mountains by means of the barometer. If at every reriod and throughout its whole extent, the temperature of the atmosphere was every where equal to that of melting ice, the result would be, that multiplying by 17972$^{me.}$ 1, the tabular logarithm of the proportion of the height of the barometer observed at any two stations, we should obtain the height of one of the stations above the other. But this height requires a correction relative to the erroneous hypothesis of a uniform heat and temperature equal to zero. It is obvious that if the mean temperature of the stratum of air contained between the two stations is greater than zero, its density diminishes and we must ascend, to make the barometer fall the same quantity. The multiplier 17972$^{me.}$ 1, must then be augmented by as many times its 250th part, as there are degrees in this mean temperature, which is the same as observing the degrees of the thermometer at the two stations, and multiplying their sum by 35$^{me.}$ 944, to add the product to 1792$^{me.}$ 1. A slight correction should likewise be applied to the height of the barometer on account of the difference of temperature at the two stations. The density of the mercury is not the same in both.

Now its dilatation relative to the increase of a degree in its temperature is a 5412th part of its volume; the height of the barometer in the coldest station must then be augmented by its 5412th part, taken as many times as there are degrees of difference in the temperature of the two stations.

By means of this rule we shall obtain very nearly the difference of their heights, if they separate but little from the same vertical.

The air is invisible in small masses, but the rays of light reflected by all the strata of the atmosphere produce a sensible impression; they give it a blue shade, which spreads a tint of the same colour over all distant objects and which forms the celestial azure. This blue vault to which the stars appear to be attached, is then very near us, it is only the terrestrial atmosphere, beyond which these bodies are placed at immense distances. The solar rays, which its particles transmit to us in abundance before the rising and after the setting of the Sun, produce the dawn and twilight, which extending to above * 20° distance from that luminary, proves to us that the extreme particles of the atmosphere are elevated at least sixty thousand metres.

If the eye could distinguish and refer to their true place the points of the exterior surface of the atmosphere, we should see the heavens like the segment of a sphere formed by the portion of this surface which would be cut off by a plane tangent to the surface of the Earth. And as the height of the atmosphere is very small in proportion to the terrestrial radius, the sky would appear under the form of a flattened vault. But though we cannot distinguish the limits of the atmosphere, yet as the rays which it transmits come from a greater depth of the horizon than the zenith, we ought to consider it more extended in the first direction. To this cause is joined the

* 18°.

interposition of objects in the horizon, which contributes to augment the apparent distance of that part of the heavens which we refer to it, the sky, therefore, should appear to us very much flattened, like a small portion of a large sphere.

A star at * 26° of altitude, seems to divide into two equal parts the length of the curve, which the section of the surface of the heavens by a vertical plane, forms from the horizon to the zenith. From whence it follows that the horizontal radius of this apparent celestial vault is to its vertical radius nearly as three and a quarter is to unity, but this proportion varies with the causes of this illusion.

The magnitude of the Sun and Moon being proportional to the angle under which they are seen, and to the apparent distance of that point of the heavens to which they are referred, they appear larger at the horizon, though they are seen under a smaller angle.

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* 23° 24′.

The rays of light do not move in a straight line through the atmosphere, they inflect continually towards the Earth. The observer, who perceives objects in the direction of the tangent of the curve which they describe, sees them more elevated than they really are, and the stars appear above the horizon even when they are sunk below it. Thus the atmosphere, by reflecting the rays of the Sun, lengthens our enjoyment of its presence, and augments the duration of the days, which are still further prolonged by the dawn and twilight.

It is extremely important to astronomers to determine the laws and quantity of refraction, to obtain the true position of the stars. But before I present the result of their researches on this object, I will explain in a few words the principal properties of light.

A luminous ray in passing from one transparent medium to another approaches or recedes from the perpendicular to the surface which separates them. The law of refraction is such that, *the sines of the* *angles which its directions form with this perpendicular, the one before, the other after its entrance into the new medium, are in a constant ratio whatever the angles may be.* But light when refracted presents a remarkable phenomenon which has led to the discovery of its nature. A ray of white light, received in a dark chamber, after its passage through a prism, forms an oblong image variously coloured, this ray is a pencil of an infinite number of rays differently coloured, which the prism separates in consequence of their various refrangibility. The most refrangible ray is the violet, then the indigo, blue, green, yellow, orange and red: but though we only distinguish seven rays, the continuation of the image proves that there exists an infinity which approach each other by insensible shades of refrangibility and colour. All these rays, collected by means of a lens, produce the white colour, which is therefore only a mixture of all the simple or homogeneous colours, in determined proportions.

When a ray of a perfectly homogeneous

colour is well separated from the others, it does not change either its refrangibility or colour, whatever reflections and refractions it may undergo.

It is not then due to the modification which light receives in the media which it traverses, but it is a natural property. Nevertheless a similitude of colour does not prove similitude of light; by mixing together several differently coloured rays of the solar image decomposed by the prism, we may form a colour perfectly similar to one of the simple colours of this image; thus the mixture of homogeneous red and yellow, produces an orange, similar in appearance to the homogeneous orange, but the refraction of the combined rays through another prism separates them, and makes the component colours reappear, while the rays of the homogeneous orange remain unaltered.

Rays of light are reflected if they fall on a mirror, making with the perpendicular to its surface, angles of reflection equal to the angles of incidence.

The refractions and reflections which rays of light undergo in drops of rain produce the rainbow, the explanation of which, founded on a rigorous calculation which satisfies exactly all the details of this curious phenomenon, is one of the most beautiful results of natural philosophy.

Most bodies decompose the light which they receive, they absorb one part and reflect the other in every direction; they appear blue, red, green, &c. according to the colour of the rays which they receive in the greatest abundance. Thus the white light of the Sun diffusing itself over all nature is decomposed and reflects to our eyes an infinite variety of colours.

After this short digression on light I return to astronomical refractions. Very precise experiments have ascertained that at the same temperature the refractive power of air increases or diminishes as its density. But at equal densities does this force vary as the temperature? What is the influence of the hygrometrical state of the air, and of the proportion in which the two gases azote and oxygen are combined in the atmosphere, upon refraction? It is this which is unknown, and which, considering the importance of the object, deserves to be investigated.

Till the present time it has been supposed that the refractive power of the atmosphere only depends on the density of its strata; so that to determine the path of light which traverses it, it was sufficient to know the law of their temperature. But this law is unknown to us, and besides it varies at every instant. The temperature of the atmosphere being supposed every where the same, and equal to that of melting ice, the density of its strata diminishes in geometrical progression, and the refraction is *74′ in the horizon, it would only be † 56′, if the density of the atmospherical strata diminished in arithmetical progression, and became nothing at its surface. The horizontal refraction which is observed to

* 39′ 57″ 6. † 30′ 14″ 4.

be about * 64′½, is a mean between these limits. The law of the diminution of the density of the strata is then nearly the medium between the arithmetical and geometrical progressions, which agrees with the observations of the barometer and thermometer. In general all these observations and those of astronomical refraction may be reconciled by means of very probable hypotheses of the diminution of heat from ascending in the atmosphere without having recourse, as some natural philosophers have done, to a particular fluid which, mingled with the air, produces refraction.

When the apparent altitude of a star exceeds † 12°, the refraction only depends sensibly on the state of the barometer and thermometer in the place of the observer; and it is nearly proportional to the tangent of the apparent distance of the star from the zenith diminished by four times the refraction. It has been discovered by different

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* 34′ 49″ 8. † 10° 48′.

means that at the temperature of melting ice, when the height of the barometer is 76 centimetres, the co-efficient, which multiplied by this tangent gives the astronomical refraction is * 185″.9, but it varies as the density of the air in the place of observation : this density varies a 250th part for every degree of the thermometer ; this co-efficient must therefore be increased or diminished as many times its 250th part, as the thermometer of the observer indicates degrees above or below zero; the density of the air at unequal temperatures being proportional to the height of the barometer, the co-efficient thus corrected must be varied in the proportion of the observed altitude of the barometer, to 76 centimetres.

By means of these data, a table of refractions may be constructed from 12° of apparent altitude to the zenith, in which interval almost all astronomical observations are made.

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* 1.5

This table would have the advantage of being independent of all hypotheses of the constitution of the atmosphere, and might be as useful at the summits of the highest mountains as at the level of the sea.

The atmosphere weakens the light of the stars, particularly at the horizon where their rays traverse it through a greater extent.

It follows from the experiments of Bouguer, that the barometer being at 76 centimetres in height, if we take the intensity of the light of a star at its entrance into the atmosphere as unity, the intensity of a star in the zenith when it reaches the observer is reduced to 0,8125.

The height of the atmosphere reduced throughout its whole extent to the density of the air corresponding to zero of temperature, and to the pressure of a column of mercury $0^{me.}$ 76 in height, would be $7815^{me.}$ Now it is natural to suppose that the extinction of a ray of light which traverses it is the same as in this hypothesis, since it meets the same number of aerial particles: thus a stratum of air of the preceding density and of 7815$^{me.}$ thickness, reduces the force of light to 0 8123.

It is easy to conclude from hence the diminution of light, in a stratum of air of equal density and of any given thickness; for it is obvious that if the intensity of light be reduced to one quarter in traversing a given thickness, an equal thickness will reduce this quarter to a sixteenth of its original value; from whence we see that as the thickness increases in arithmetical progression, the intensity of the light diminishes in geometrical progression: its logarithms then follow the ratio of the thickness: thus, to obtain the tabular logarithm of the intensity of light, when it crosses a stratum of air of a given thickness, we must multiply 0.0902835, the logarithm of 0.8123, by the proportion of this thickness to 7815$^{me.}$ and if the density of the air is greater or less than the preceding, this logarithm must be augmented or diminished in the same proportion.

To determine the diminution of the light of stars relating to their apparent altitude we may imagine a luminous ray moving in a channel every where of the same size, and we may reduce the air in this channel to the preceding density. The length of the column of air thus reduced will determine the diminution of the light of a star thus considered. Now we may suppose from the zenith to about 12° of apparent altitude, the strata of the atmosphere, sensibly plane and parallel, and the passage of light rectiliniar, then the thickness of every stratum in the direction of the luminous ray is to its depth in a vertical direction as the secant of the apparent distance of the star from the zenith is to radius; multiplying then this secant by 0.0902835, and by the proportion of the height of the barometer to $0^{me.}$ 76, we shall get the logarithm of the intensity of the stars' light. This very simple rule will give the diminution of the light of the stars, at the summits of mountains and at the level of the sea, which may be useful, both to correct the eclipses of Jupiter's satellites, and to estimate the intensity of the solar light at the focus of burning glasses. We ought, however, to observe that the vapors floating in the air influence considerably the extinction of the light of stars; the serenity of the heavens in southern climates makes their light in general much more brilliant there; and if we were to transport our great telescopes to the top of the high mountains of Peru, there is no doubt but we should discover many celestial phenomena, which a thicker and less transparent atmosphere renders invisible in our climates.

The intensity of light at small altitudes depends on the constitution of the atmosphere as well as refraction. If its temperature was every where the same the logarithms of the intensity of light would be proportional to the astronomical refractions, divided by the cosines of the apparent altitude, and then this intensity at the horizon would be reduced to the four thousandth part of its primitive value: it is on this account that the Sun, whose lustre at noon is too dazzling to be borne, can be regarded without pain in the horizon.

It is natural to think that every particle of the Sun's surface sends in every direction the same quantity of light. Two equal and very small portions of this surface, seen from the Earth, one at the centre of the disk and the other towards its edge, appear to occupy different spaces, which are to each other as the radius is to the cosine of the arc of the great circle of the solar surface which separates these two parts, therefore the intensity of the light is in an inverse ratio. Notwithstanding this Bouguer has found by experiments that the light of the Sun is more brilliant at the centre than towards the edges. In comparing that of the centre, with the light of a point distant from the limb by a quarter of the semidiameter, the intensities of these two lights appear to be in the proportion of 48 to 35. This difference indicating a thick atmosphere round the Sun which weakens its light, it follows from the preceding results, and from the experiment of Bouguer, that the intensity of the light of a star seen from the surface of the Sun, at the zenith, is reduced to 0.24065, and that the Sun deprived of its atmosphere, would appear twelve times and a third more luminous. A horizontal stratum of air, at the temperature of zero, and under the pressure of a column of mercury $0^{me.}76$ ought to have $53548^{me.}$ of thickness to weaken light in the same degree as the atmosphere of the Sun.

This then would be the height of this atmosphere reduced to the density of this aerial stratum, if at equal densities its transparency was the same as that of the air, but of this we are ignorant. However, these results are subordinate to the correctness of Bouguer's experiment, which deserves to be repeated with care, in the different aspects of the solar disk. The vibrations of the air produce sounds which, according to the rapidity or slowness of the vibrations, are sharp or grave. But whatever their nature may be, the rapidity of their propagation is the same, and sound strong or weak, grave or sharp, traverses $2914^{me.}$ in a second.

The winds, from the zephyr to the most

impetuous whirlwinds, are produced by the air being displaced with more or less violence. In the most violent tempests this velocity is about thirty metres in a second. It is only about a third of this in ordinary winds. Without doubt, the cause which raises regularly the waters of the sea, and which appears to reside in the Sun and Moon, disturbs equally the equilibrium of the atmosphere, which it must penetrate to act upon the ocean. But the periodical winds which result from it are too weak to be observed amidst the agitations which the atmosphere experiences from a great number of other causes.

It is in the atmosphere that cloud, storms, the aurora borealis, and all meteors are formed. The air dissolves water, and this dissolving property varies with its density and its heat.

Thus, water is dissolved and precipitated alternately in the atmosphere, in consequence of all the causes which vary the temperature and density of the air. The water of the sea in dissolving, abandons the salt which it contains. It descends under the form of dew, snow, rain, or hail, a part of which, collected by mountains and elevated places, is filtrated by the earth it passes through, and forms springs and rivers which return to the sea.

Electricity obtains a passage through the atmosphere with difficulty: its different strata are habitually electrified, and this appears to increase in proportion to their height. The clouds formed in the higher strata are then more electrified than the lower strata into which they descend. But whatever be the cause of the electricity of clouds, it is asserted that thunder is an electric explosion between the clouds and the earth.

Air is not a homogeneous substance; experiments have proved that it is composed of three parts of a gas called azote, and on part of oxygen gas, a gas peculiarly respirable, in which bodies exhibit a brilliant light during combustion. It is this gas alone which is necessary to combustion and to the respiration of animals, which is known to be a slow combustion and the principal source of animal heat.

Other aëriform fluids mingle with the atmosphere and rise in it, in proportion to their specific lightness. The lightest of these fluids is that which is called hydrogen gas, it is 15 or 16 times lighter in its state of purity than the atmospheric air. Combined with oxygen gas in nearly the proportion of one to six, it forms water, which far from being an element,as was believed for a long time, may be composed and decomposed at pleasure. The decomposition of bodies in marshes and stagnant water, developes a great quantity of hydrogen gas which is carried to the confines of the atmosphere,where being enflamed by natural electricity, it produces falling stars, globes of fire, and those trains of light which are observed in great heats, and which being seen sometimes at the same instant, at great distances, indicate that their height is at least one hundred thousand metres.

When contained in some light substance, hydrogen gas rises with the bodies which are attached to it till it meets with a stratum of the atmosphere, sufficiently rare for it to rest in equilibrium.

By these means, the fortunate discovery of which we owe to the French philosophers, man has extended his domain and his power. He may launch into the air, traverse the clouds, and interrogate nature in the exalted regions of the atmosphere formerly inaccessible.

The atmosphere transmits the light of the Sun freely, and heat with difficulty. It increases therefore the heat at the surface of the Earth; and perhaps without the resistance which it opposes to the diffusion of the solar heat, we should experience excessive cold at the equator itself.

It is to heat that the aëriform state of the atmosphere is owing; it is the pressure of the atmosphere and heat that retains the ocean in its fluid state. To establish these truths let us present in a few words one of the principal late discoveries concerning heat.

Whatever its nature may be, heat dilates bodies. It changes solids into fluids, and fluids into vapour. These changes of form are marked by singular phenomena which we will trace from ice. Let us consider a volume of snow or pounded ice in an open vessel, submitted to the action of great heat. If the temperature of ice is below that of melting ice, it will augment up to zero of temperature. Arrived at this degree the ice will melt successively by new additions of heat. But if care is taken to agitate it till it is all melted, the water will remain at the temperature of zero, the heat communicated by the vessel will not be sensible to the thermometer immersed in it, it will be entirely occupied in rendering the ice fluid. Afterwards the additional heat will raise the temperature of the water and the thermometer till the moment of ebullition. The thermometer will then again become stationary, and all the heat communicated by the vessel will be employed to reduce the water to steam, which will be at the same temperature as the boiling water.

The water produced by the melting of the ice, and the vapours to which the boiling water is changed, absorb at the moment of their formation a great quantity of heat, which re-appears in the return of the aqueous vapours to the state of water, and of water to the state of ice: for steam, when condensing on a cold body, communicates much more heat than would be received from an equal weight of boiling water; and besides it is known that water will preserve its fluid state though its temperature may be many degrees below zero; if then it is a little agitated it transforms itself into ice, and the thermometer plunged into it, mounts to zero from the heat given out during this change.

Without the pressure of the atmosphere melted ice would itself be converted into steam. But this pressure restrains the repulsive force which heat communicates to fluid particles, and retains melted ice under the form of water till the heat is sufficiently great for the repulsive force to overcome the pressure of the atmosphere.

At this instant water boils and becomes steam: the degree of the temperature of boiling water, varies therefore with the pressure of the atmosphere, it is less at the summit of mountains than at the level of the sea, and in a receiver in which the air may be rarified and condensed at pleasure, the heat of boiling water may be increased or diminished at pleasure. Thus heat renders the sea fluid, and the pressure of the atmosphere prevents its being reduced to vapour.

All bodies which can be made to pass from a solid to a fluid state present similar phenomena. But the temperature at which their fusion commences is very different in each of them. Mercury, for example, becomes solid at forty degrees below zero, as is found by experience. It begins to melt at this degree of temperature, it boils at 376, and under the pressure of a column of Mercury of 0.76 metres: so that at this pressure of the atmosphere, the interval of temperature included between fusion and ebullition which is for water 100°, increases to 416° for mercury.

There exist bodies which cannot become fluid by the greatest heat we can induce. There are others which the greatest cold experienced on the earth, cannot reduce to a solid form. Such are the fluids which form our atmosphere,. and which, notwithstanding the pressure and the cold to which they have been submitted, have still maintained themselves in the state of vapour. But their analogy with aëriform fluids, to which we reduce a great number of substances by heat, and their condensation by pressure and cold, leaves no doubt but that these atmospheric fluids are extremely volatile bodies, which an intense cold would reduce to a solid state. To make them enter this state, it would be sufficient to remove the Earth from the Sun, as it would be sufficient to bring it nearer, to induce water, and many other bodies, to enter into our atmosphere. These great vicissitudes take place on comets, and principally on those which in their perihelion approach very near the Sun. The nebulosities which surround them, and the long trains which follow them, are the result of the evaporation of the fluids at their surface. The cold, which is the consequence of this, should temper the excessive heat produced by their proximity to the Sun. And the condensation of the same fluids, when they depart from it, repairs partly the diminution of heat which this separation should produce; so that the double effect of the evaporation and condensation of the fluids, approximates considerably the greatest heat and cold which comets experience at each of their revolutions.

THE

SYSTEM OF THE WORLD.

BOOK II.

On the real Motions of the Heavenly Bodies.

——Provehemur portiu terræque urbesque recedunt.
VIRG. Eneid. Lib. III.

IF man had confined himself to collecting facts alone, science would only have presented a steril nomenclature, and he would never have attained the knowledge of the great laws of nature. It is by comparing phenomena together, and by endeavouring to trace their connection with each other, that he has succeeded in discovering these laws, the existence of which may be perceived even in the most complicated of their effects. Then it is, that nature in discovering herself, has shewn how from a small number of general causes she has produced the infinite variety of phenomena which have been observed; and thus enabled us to determine those, which successive circumstances will bring to light, and being assured that nothing will interpose between these causes and their effects, we venture to extend our views into futurity, and contemplate the series of events, which time alone can develope. It is only in the theory of the system of the world, that the human understanding has attained to this state of perfection. Let us examine the path that has been followed in this investigation.

CHAP. I.

Of the Motion of Rotation of the Earth.

When we reflect on the diurnal motion to which all the heavenly bodies are subject, we cannot but recognize the existence of one general cause which moves them, or which seems to move them round the axis of the Earth. If we consider that these bodies are insulated, with respect to each other, and placed at very different distances from the Earth, that the Sun and the Stars are at a much greater distance from it than the Moon, and that the variations in the apparent diameters of the planets, indicate great alterations in their distances; and moreover, that the comets traverse the heavens freely in all directions, it will be difficult to conceive that it is the same cause which impresses on all these bodies a common motion of rotation. But since the heavenly bodies present the same appearances to us, whether the firmament carries them round the Earth, considered as immovable, or whether the Earth itself revolves in a contrary direction; it seems much more natural to admit this latter motion, and to regard that of the heavens as only apparent.

The Earth is a globe whose radius is only about four thousand English miles; the Sun, as we have seen, is incomparably larger. If its centre coincided with that of the Earth, its volume would embrace the orbit of the Moon, and extend as far again, from which we may judge of its immense magnitude, besides, it distance from us is twenty-three thousand times the semi-diameter of the Earth. Is it not infinitely more simple to attribute to the globe we inhabit, a motion of rotation on its own axis, than to suppose, in a mass so considerable, and so remote, as the Sun, such an extreme rapid motion as would be requisite to revolve in one day round the Earth? What immense power would it not require to contain it, and counterbalance its centrifugal force? Every one of the stars presents similar difficulties, which are all removed by the rotation of the Earth.

We have seen that the pole of the equator seems to move slowly round that of the ecliptic, from whence results the precession of the equinoxes. If the Earth is immoveable, the pole of the equator is equally so, since it always corresponds to the same point of the terrestrial surface; the ecliptic therefore moves round these poles, and in this motion carries all the heavenly bodies with it. Thus the whole system, composed of so many bodies, differing from each other so much in their magnitude, motions, and distances, would be again subject to a general motion, which disappears, and is reduced to a simple appearance, if we suppose the terrestrial axis to move round the poles of the ecliptic.

Carried on with a velocity which is common to every thing that surrounds us, we are in the case of a spectator placed in a ship that is in motion. He fancies himself at rest, and the shore, the hills, and all the objects placed out of the vessel, appear to him to move. But on comparing the extent of the shore, the plains, and the height of the mountains, with the smallness of his vessel, he recognizes that the apparent motion of these objects, arises from the real motion of himself. The numberless stars which fill the celestial regions, are relatively to the Earth, what the shores and the hills are to the vessel; and the same reasons which convince the navigator of the reality of his own motion, prove to us the motion of the Earth.

These arguments are likewise strengthened by analogy. A rotatory motion has been observed in several planets, and always from west to east, similar to that which the diurnal motion of the heavens seems to indicate in the Earth. Jupiter, greatly exceeding the Earth in magnitude, moves round its axis in less than twelve hours. An observer on its surface would see the heavens revolve round him in that time; yet that motion would only be apparent. Is it not therefore reasonable to think that it is the same with that which we observe on the Earth? What confirms, in a very striking manner, this analogy, is, that both the Earth and Jupiter, are flattened at the poles. We comprehend, in fact, that the centrifugal force which tends to remove every particle of a body from its axis of rotation, should flatten the Earth at its poles, and elevate it at the equator. This force should likewise diminish that of gravity at the equator; and that this diminution does take place, is proved by experiments which have been made on the lengths of pendulums. Every thing then leads us to conclude, that the Earth has really a motion of rotation, and the diurnal motion of the heavens is merely an illusion which is produced by it;—an illusion similar to that which represents the heavens as a blue vault, to which all the Stars are fixed, and the Earth as a plane, on which it rests.

Thus astronomy has surmounted the illusions of the senses, and it is not till they have been dissipated by a great number of observations and calculations, that man has at last recognized the motion of the globe which he inhabits, and its true position in the universe.

CHAP. II.

Of the Motion of the Planets round the Sun.

We shall consider at present the phenomena of the motions proper to the planets, and first let us follow the motion of Venus, its apparent diameter, and its phases. In the morning, when it begins to disengage itself from the rays of the Sun, it is seen previous to the rising of this luminary, under the form of a crescent, and its apparent diameter is at its maximum. She is then nearer to us than the Sun, and almost in conjunction with it. Its crescent augments, and its apparent diameter diminishes, in proportion as it recedes from the Sun. Arrived at about * fifty degrees distance from the Sun, it returns towards it, discovering more and more of its enlight-

* 45°.

ened hemisphere to us. Its apparent diameter continues to diminish till the moment when it replunges in the morning into the solar rays.

At this instant Venus appears to us full, and its apparent diameter is a minimum. It is then, in this position, farther from us than the Sun. After disappearing for some time, this planet re-appears in the evening, and produces, in an inverse order, the phenomena it had displayed, previous to its disappearance. Its enlightened hemisphere turns more and more from the Earth, its crescent diminishes, and at the same time its apparent diameter augments, as it becomes more distant from the Sun. When it is about fifty degrees, she returns towards it, its crescent continues to diminish, and its diameter to augment, till it again immerges into the rays of the Sun. Sometimes, during the interval which separates its disappearance in the evening, from its re-appearance in the morning, it is seen under the form of a spot, moving on the Sun's disk, It is evident, from all these phenomena, that the Sun is nearly in the centre of the orbit of Venus, and that in its motion round the Earth, it carries with it this orbit.

This result, obtained by observations of the phases and apparent diameter of Venus, explains so naturally its alternate, direct, and retrograde motion in longitude, and its whimsical and complicated motion in latitude, that it is impossible to relapse into doubt concerning it. Mercury offers us the same appearances as Venus, therefore the Sun is nearly in the centre of its orbit. These two planets accompany it in its revolution about the Earth, without departing from it above certain limits, which depend on the angles under which their orbits are seen. Those planets which leave the Sun at the greatest possible angular distance, present to us other phenomena. Their diameters are at a maximum in the opposition; they diminish in proportion as they approach the Sun; therefore the Earth is not in the centre of motion of these planets. Previous to the opposition, this motion, from direct, becomes retrograde ; after the opposition, it resumes its direct motion, when the planet, in its approach to the Sun, is about as far distant from it, as at the commencement of its retrogradation ; and it is at the moment of opposition, that its retrograde velocity is the greatest. This evidently indicates that the observed motion of these planets, is the result of two motions, alternately in the same and contrary directions, and of which one is governed by that of the Sun. Such are the motions of Mercury and Venus, which, revolving round the Sun, are carried with it round the Earth. It is natural to extend the same law to the other planets, with only this difference, that the Earth, placed without the orbits of Venus and Mercury, is within those of Mars, Jupiter, Saturn, and Uranus. All the appearances of the motions and diameters of these planets, proceed so naturally from this hypothesis, that the mechanism of nature cannot be mistaken in it. That the motion of the planets round the Sun is almost circular is proved in case of Jupiter, by the eclipses of its satellites.

It has been previously shewn, that these phenomena give the distances of these planets from the Sun, in parts of the mean distance of the Sun from the Earth; and it has been thus found, that these distances vary little in the course of one revolution, and that the motions of these planets are nearly uniform. We are then conducted by the comparison of these phenomena, to place the Sun in the centre of the orbits of all the planets which move round it, while it moves, or appears to move, round the Earth.

CHAP. III.

Of the Motion of the Earth round the Sun.

SHALL we now suppose the Sun accompanied by the planets and satellites in motion round the Earth, or shall we imagine the Earth and other planets to revolve round the Sun? The appearances of the heavenly motions are the same in the two hypotheses, but the second should be preferred, for the following considerations.

The masses of the Sun and of several of the planets, are considerably greater than that of the Earth; it is much more simple to make the latter revolve round the Sun than to put the whole solar system in motion round the Earth. What a complication in the heavenly motions, would the immobility of the Earth suppose! What a rapidity of motion, must be given to Jupiter, to Saturn, which is ten times farther from the Sun than we are, and to Uranus which is still more remote, to make them every year revolve round us, at the same time they are revolving round the Sun.

This complication and this rapidity of motion disappear by transferring the motion to the Earth: a motion conformable to the general law, by which the small celestial bodies revolve round the large ones which are placed in their vicinity.

The analogy of the Earth with the planets confirms this hypothesis; like Jupiter it revolves on its axis and is accompanied by a satellite. An observer on the surface of Jupiter, would conclude that the solar system was in motion round him, and the magnitude of that planet would render this illusion less improbable than for the Earth. Is it not therefore reasonable to imagine that the motion of the solar system round us, is likewise only an illusion. Let us transport ourselves in imagination to the surface of the Sun, and from thence let us consider the Earth and the planets. All these bodies will appear to move from west to east; already this identity in the direction indicates a motion of the Earth, but that which demonstrates it evidently is the law which exists between the times of the revolutions of the planets, and their distance from the Sun. They revolve round it slower as their distances are greater, and in such a manner that the squares of the periodic time are in proportion to the cubes of their mean distances. According to this remarkable law, the length of a revolution of the Earth, supposing it in motion round the Sun, should be exactly a sidereal year. Is not this an incontestable proof that the Earth moves like the other planets and is subject to the same laws? For would it not be very strange to suppose the terrestrial globe which hardly subtends a visible angle at the Sun, immovable amidst the other planets which are revolving round it, and that the Sun should be carried with them about the Earth?

Ought not the force which balances the centrifugal force and retains the planets in their respective orbits, likewise to act upon the Earth ? and must not the Earth oppose to this action the same centrifugal force ? Thus the consideration of the celestial motions as observed from the Sun, leaves no doubt of the real motion of the Earth. An observer on the surface of the Earth has another evident proof of its motion in the phenomenon of the aberration, which is a necessary consequence of it, as we shall now explain.—About the end of the seventeenth century, Roemer observed that the eclipses of the satellites of Jupiter happened sooner about the oppositions of this planet, and later towards the conjunctions ; this led him to suspect that light was not transmitted instantaneously from those bodies to the Earth, but that it employed a perceptible interval of time to traverse the diameter of the orbit of the Sun. In fact, Jupiter being in his oppositions nearer to us than in the conjunctions, by a quantity equal to this diameter, the eclipses ought to happen sooner to us in the first case than in the latter, by the time which the light takes to traverse this orbit. The law of retardation observed in these eclipses, answers so exactly to this hypothesis, that it is impossible to refuse assent to it. It appears that light employs * 571″ in coming from the Sun to the Earth.

Now an observer at rest would see the stars according to the directions of their rays, but this will not be the case on the supposition that he partakes of the motion of the Earth. To reduce this case to that of the observer at rest, it is sufficient to assign in a contrary direction, both to the stars, to the light, and to the observer himself, a motion equal to that by which he is impelled, which would not change the apparent position of the stars; for it is a general law of optics that if all the bodies of a system are impelled by a common motion, there will result no

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* 8′ 5″.

change in their respective situations. Let us imagine, then, that a motion equal and contrary to that of the observer, be given to the rays of light, and generally to all the other bodies, and let us see what phenomena should result in the apparent position of the stars. We may leave out of the question the diurnal motion of the Earth, which is not even at the equator $\frac{1}{60}$th part of that in its orbit round the Sun. We may here suppose also without sensible error that all the rays which each point of the disk of a heavenly body transmits to us are parallel to each other, and to that ray which would come from the centre of the star to the centre of the Earth if it were transparent. Thus the phenomena which the stars would present to an observer placed at the centre of the Earth, and which depend on the motion of light combined with that of the Earth, are nearly the same for every observer on its surface. Moreover we may neglect the small excentricity of the terrestrial orbit.

In the interval of * 571″ that light employs to traverse the terrestrial orbit, the Earth describes a small arc of this orbit equal to † 62″ 5; now it follows from the law of the composition of motion, that if through the centre of a star we imagine a small circle parallel to the ecliptic, and whose diameter subtends in the heavens an angle of ‡ 125″, the direction of the motion of light combined with the motion of the Earth, and applied in a contrary direction, will meet this circumference in a point where it is intersected by a plane drawn through the centre of the star tangentially to the terrestrial orbit. The star therefore should move upon this circumference, and describe it every year in such a manner that it should constantly be less advanced by § 100° than the Sun in its apparent orbit.

This phenomenon is exactly that which we have explained in Chap. XI. Book I. from the observations of Bradley, to whom

* 8′ 5″. † 20″ 2. ‡ 40″ 5. § 90°.

we are indebted both for the discovery and cause. To reduce these stars to their true position, it is sufficient to place them in the centre of the small circle they appear to us to describe.

Their annual motion, therefore, is only an illusion produced by the combination of the motion of light with that of the Earth. The relation of this motion with the position of the Sun, would lead to a suspicion that it was only apparent, but the foregoing explanation proves it beyond a doubt. It affords a sensible demonstration of the motion of the Earth round the Sun, in the same manner as the increase of degrees and of the force of gravity in going from the equator to the poles, proves its revolution on its axis.

The aberration of light affects the positions of the Sun, the planets and their satellites, and comets, but in a different manner according to their particular motions. To divest them of this, and to obtain the true position of the stars, let us suppose at every instant, a motion impressed on all these bodies equal and contrary to that of the Earth, which thus may be supposed at rest, this, as observed above, neither alters their respective positions nor appearances. Then it is evident that a heavenly body, at the moment we observe it, is no longer in the direction of the luminous ray which strikes our sight; it has left it in consequence of its real motion combined with that of the Earth which we supposed impressed in a contrary direction. The combination of these two motions, as seen from the Earth, forms the apparent or, as it is termed, the geocentric motion. We shall have then the true position of the object, by adding to the observed geocentric longitude and latitude, its geocentric motion in longitude and latitude, for the interval of time which light employs to come from the heavenly body to the Earth. Thus the centre of the Sun seems constantly less advanced by * 62″ 5, in its orbit, than if its

* 20″ 2.

light was transmitted to us instantaneously.

The aberration of light changes the relations of celestial phenomena, both as to space and as to duration. At the moment we see them they no longer exist. We do not see the termination of the eclipses of Jupiter's satellites till *25′ or 30′ after they have recovered their light; and the variations of light of some of the changeable stars precede by many years the instant of their observations. But the cause of these illusions being well understood, we can always refer the phenomena of the solar system to their true place and epoch.

The consideration of the celestial motions leads us, then, to displace the Earth from the centre of the world where we had placed it, deceived by appearances, and by the natural propensity of man to regard himself as the principal object in nature. The globe which we inhabit is a planet in motion on itself and round the Sun.

* 36′ or 42′.

In considering it in this point of view, all the phenomena are explained in the most simple manner, all the celestial motions become uniform, and the analogies are preserved. Like Jupiter, Saturn, and Uranus, the Earth is accompanied by a satellite, it turns on itself like Venus, Mars, Jupiter, and Saturn, and probably the other planets; it like them borrows its light from the Sun, and moves round it in the same direction, and according to the same laws. Finally the hypothesis of the Earth's motion unites in its favour simplicity, analogy, and every thing which characterizes the true system of nature. We shall see that following it in its consequences, the celestial phenomena are brought, even in their minutest details, to one single law, of which they are only the necessary developments. The motion of the Earth will thus acquire all the certainty of which physical truths are susceptible. And it may result either from the great number and variety of phenomena which it explains, or from the simplicity of the laws on which it is made to depend. None of the branches of natural knowledge unite these advantages in a higher degree than the theory of the system of the world founded on the motion of the Earth. This motion ennobles our conceptions of the universe, by affording for a measure of the distances of the heavenly bodies, an immense base—the diameter of the terrestrial orbit. By this we have accurately determined the dimensions of the planetary orbs.

Thus the motion of the Earth, after having, by illusions of which it is itself the cause, retarded our knowledge of the planetary motion for a great length of time, at last conducted us to them, and that in a more accurate manner than if we had been placed in the centre of their system.

Nevertheless the annual parallax of the stars, or the angle which the diameter of the terrestrial orbit would subtend at this centre, is insensible, and does not amount to * 6″ even relatively to those stars which from their extreme brilliance appear to be nearest to us. They are, therefore at least a hundred thousand times farther from us than the Sun. Their prodigious brightness at so immense a distance, proves to us that they do not, like the planets and their satellites, borrow their light from the Sun, but that they shine with their own light ; so that they are so many suns scattered in the immensity of space, and which, similarly to ours, may be the foci of so many planetary systems. It would in fact be sufficient to place ourselves upon the nearest of these stars in order to see the Sun only as a luminous object, the diameter of which would be less than the thirtieth of a second.

It results from the immense distance of the stars, that their motion in right ascension and declination, are only appearances produced by the motion of the Earth's

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* 1″ 9.

axis of rotation. But some stars appear to have motions proper to themselves, and it is probable that they are all in motion as well as the Sun which carries with it the whole system of planets in space, in the same manner as each planet draws its satellites in its motion round the Sun.

CHAP. IV.

Of the Phenomena which arise from the Motion of the Earth.

FROM the point of view in which the comparison of the celestial observations has placed us, let us consider the heavenly bodies, and show the perfect identity of their appearances with those which we observe. Whether the heavens revolve round the axis of the world, or the Earth moves itself in a contrary direction to the heavens, supposed at rest, it is clear that the stars will present themselves to us in the same manner. There will be no other difference but that in the first case they will come and place themselves successively over the different terrestrial meridians, which in the second will place themselves under the stars. The motion of the Earth being common to all bodies situated on its surface, and to the fluids which cover it, their relative motions are the same as if the Earth was at rest. Thus in a vessel whose motion is uniform, every thing moves as if the vessel were at rest. A projectile thrown directly upwards falls on the same spot from which it was projected, it seems to describe a vertical line to those in the vessel, but seen from the shore it really describes a parabolic curve. Thus the rotation of the Earth cannot be sensible on its surface, except by the effects of the centrifugal force which flattens the terrestrial spheroid at the poles, and diminishes the force of gravity at the equator; two phenomena with which the measure of the degrees at the meridian, and of the pendulum, have made us acquainted.

In the revolution of the Earth round the Sun, its centre and all the points of its axis of rotation, being moved with velocities equal and parallel, this axis remains always parallel to itself: in impressing then at every instant to all the parts of the Earth a motion equal and contrary to that of its centre, it would rest immovable like its axis of rotation; this impressed motion does not change at all the appearances of that of the Sun, it only transports in a contrary direction to the Sun, the real motion of the Earth. The appearances are consequently the same in the hypothesis of the Earth at rest, and in that of its motion round the Sun. To follow more particularly these appearances, let us imagine a radius drawn from the centre of the Sun to that of the Earth: this radius will be perpendicular to the plane which separates the hemisphere which is enlightened from that which is in obscurity. A spectator at the point where this intersects the terrestrial surface, will see the Sun perpendicularly above him, and every point of the terrestrial parallel through which this radius successively passes in consequence of its diurnal motion, will have at noon the Sun in its zenith. Thus whether the Sun turns round the Earth, or the Earth round the Sun and on its own axis, this axis preserving a parallel position, it is evident that this radius will trace the same curve on the surface of the Earth, it will in each case cut the same parallels to the equator when the Sun has the same apparent longitude: this luminary will be equally elevated above the horizon, and the days will be of the same length. Thus the seasons and the days are the same in the hypothesis of the immobility of the Sun, or of its motion round the Earth: and the explanation of the seasons will be equally intelligible by either hypothesis.

The planets all move in the same direction round the Sun, but with different velocities, but the length of their revolutions increase in a greater ratio than their distances from the Sun. Jupiter, for instance, employs nearly twelve years to perform its revolution, but the radius of his orbit is only five times greater than that of the Earth; its real velocity is therefore less than that of the Earth; this diminution of velocity in the planets as they recede from the Sun applies generally to all of them, from Mercury, which is the nearest, to Uranus, which is the most remote; and it results from the laws which we shall hereafter demonstrate, that the mean velocities of the planets are reciprocally as the square roots of their mean distances from the Sun.

Let us consider a planet whose orbit is surrounded by that of the Earth, and follow it from its superior to its inferior conjunction; its apparent or geocentric motion is the result of its real motion combined with that of the Earth considered as moving in a contrary direction. In the superior conjunction, the real motion of the planet is contrary to that of the Earth, its geocentric motion is then the sum of the two motions, and it has then the same direction as the geocentric motion of the Sun, which results from the real motion of the Earth transferred in a contrary direction to the Sun, and thus the apparent motion of the planet is direct.

In the inferior conjunction the motion of the planet has the same direction as that of the Earth; and as it is greater, the geocentric motion preserves the same direction, but it is only the excess of the real motion of the planet above that of the Earth, it has therefore a motion contrary to that of the Sun, and consequently it is retrograde.

It is easy to conceive that in the interval from the direct to the retrograde motion, the planet should appear without motion, or stationary; and that this will happen between the greatest elongation and the inferior conjunction, when the geocentric motion of the planet, resulting from its real motion and that of the Earth applied in a contrary direction, is directed in the same line as the visual ray to the planet. These phenomena are entirely conformable to the motions that are observed to take place in the planets Mercury and Venus.

The motion of the planets whose orbits comprehend that of the Earth, has the same direction in their oppositions as the motion of the Earth, but it is less, and being combined with this last motion applied in a contrary direction, it takes a motion contrary to its primitive direction, the geocentric motion of the planet is then retrograde, in the conjunctions it is direct, the same as in the superior conjunctions of Mercury and Venus.

In transferring to the stars (but in a contrary direction) the motion of the Earth, they should describe every year a circumference equal and parallel to the terrestrial orbit, and of which the diameter should subtend an angle equal to that which this orbit subtends at the distance of the star. This apparent motion has a great resemblance to that which results from the combination of that of the Earth with that of light, and by which the stars seem annually to describe a circle parallel to the ecliptic, the diameter of which subtends an angle of *125″, but it differs in this, that these stars have the same position as the Sun on the first circumference, whereas in the second they are less advanced than the Sun by †100°. By this circumstance the two motions may be distinguished from each other, and it appears that the first is insensible. The immense distance of the stars from us rendering insensible the angle which the terrestrial orb subtends when viewed from them.

The axis of the world being nothing more than the prolongation of the Earth's axis of rotation, we should refer to this axis the motion of the poles of the celestial equator, indicated by the phenomena of *precession* and *nutation*, which see in Chap. XI. of the first Book. Thus at the same time that the Earth moves on its own axis, and round the Sun, its axis of rotation moves very slowly round the poles of the ecliptic: but subject to small

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* 40″ 5. † 90°

oscillations of which the period is the same as that of the motion of the nodes of the lunar orbit. But this motion is not peculiar to the Earth alone, for we have seen in the fourth Chapter of the first Book, that the axis of the Moon moves in the same period round the poles of the ecliptic.

CHAP. V.

Of the Figure of the planetary Orbits, and of the Law of their Motions round the Sun.

Nothing would be more easy than to calculate the position of the planets at any given moment, from the preceding data, if their motions round the Sun were circular and uniform. But they are subject to very perceptible inequalities, the laws of which form one of the most important objects of astronomy, and are the only clew that can conduct us to the general principles of the celestial motions.

To recognize these laws in those appearances which the planets offer to us, it is requisite to deprive their movements of the effects of the Earth's motion, and to refer to the Sun their position observed from different points of the terrestrial orbit. It is then necessary, first of all to determine the dimensions of this orbit and the law of the Earth's motion.

It has been shewn in the second Chapter of the first Book, that the apparent orbit of the Sun is an ellipse, of which the centre of the earth occupies one of the foci; but as the Sun is really immovable it must be placed in the focus of the ellipse, and the Earth at the circumference. The apparent motion of the Sun would be the same, and to obtain the position of the Earth, as seen from the centre of the Sun, it would be sufficient to augment by two right angles the position of that body. It has been also seen that the Sun seems to move in its orbit in such a manner that the radius vector which joins its centre to that of the Earth, appears to trace round it areas proportionate to the times in which they are described: but in reality these areas are traced round the Sun. In general, all that has been said in the chapter already cited, upon the excentricity of the solar orbit and its variations on the position and motion of its perigee, ought to be applied to the terrestrial orbit, observing only that the Earth's perigee is at the distance of two right angles from that of the Sun. The figure of the terrestrial orbit being thus known, let us see how the knowledge of the orbits of the other planets has been arrived at. Let us take for example, the planet Mars, which by the great excentricity of its orbit, and its proximity to the Earth, is very well calculated to discover to us the laws of the planetary motions.

The motion of Mars round the Sun and its orbit would be known, if we could obtain for any instant the angle which its radius vector makes with an invariable straight line passing through the Sun's centre, and the length of this radius. To simplify this problem, we choose those positions of Mars in which one of these quantities can be found separately; and it is this which takes place very nearly in the oppositions in which this planet is seen opposite the same points in the ecliptic, to which it might be referred from the centre of the Sun. The difference of the motion of Mars and the Earth makes the planet correspond to different points of the heavens, in its successive oppositions; by comparing then a great number of observed oppositions, the law which exists between the time and the angular motions of Mars round the Sun may be discovered; this motion is called heliocentric. Analysis offers for this purpose several methods, which become simple in the present case from the consideration that as the principal inequalities of Mars become the same at each of its sidereal revolutions, their sum may be expressed by a rapidly-converging series of the sines of the angles multiplied by its mean motion, the co-efficients of which series it is easy to obtain, by means of some select observation. The law of the radius vector of Mars may be obtained by comparing the observations of this planet towards the quadratures, or when, being about * 100° of the Sun, this radius subtends the greatest angle. In the triangle formed by the right lines which join the centres of the Earth, the Sun and Mars, the angle at the

* 90°.

law of the heliocentric motion of Mars gives the angle at the Sun, and the radius vector of Mars may be concluded in parts of that of the Earth, which is itself given in parts of the mean distance of the Earth from the Sun. The comparison of a great number of the radii vectores thus determined, will give the law of their variations, corresponding to angles which they form with an invariable right line, and the figure of the orbit may be traced.

It was by a similar method that Kepler discovered the elongation and excentricity of the orbit of Mars: he conceived the fortunate idea of comparing its figure with that of the ellipse, placing the Sun in one of the foci, and the numerous observations of Tycho, exactly represented in the hypothesis of an elliptic orbit, left no doubt concerning the truth of this hypothesis.

That extremity of the greater axis which is nearest to the Sun is called the perihelion, and that which is farthest the aphelion.

Earth is given directly by observation, the

It is in the perihelion that the angular velocity of Mars round the Sun is the greatest; it diminishes as the radius vector augments, and is the least at the aphelion: comparing this velocity with the powers of the radius vector, Kepler found that it is proportionate to its square: so that, the product of the daily heliocentric motion of Mars by the square of its radius vector is always the same.

This product is the double of the small sector which this radius describes every day about the Sun: the area then which it describes, departing from an invariable line passing through the centre of the Sun, increases as the number of days passed since the epoch when the planet was upon this line. This is what Kepler announced, in establishing that the areas described by the radius vector of Mars are proportional to the time.

These laws of the motion of Mars are the same with those of the apparent motion of the Sun, which have been developed in the second Chapter of the first Book; they therefore apply equally to the Earth. It was natural to extend them to the other planets.

Kepler then established as the fundamental laws of the motions of these bodies, the two following ones, which all observations have confirmed.

The planetary orbits are ellipses, of which the centre of the Sun occupies one of the foci.

The areas described round this centre by the radii vectores of the planets are proportionate to the time employed to describe them.

These laws are sufficient to determine the motions of the planets round the Sun. But it is necessary to know for each of them, seven quantities, which are called *elements of the elliptic motion.* Five of these elements, relative to the motion of the ellipse, are, first, the duration of the sidereal revolution; second, half the greater axis of the orbit, or the mean distance of the planet from the Sun; third, the excentricity from which the greatest equation of the centre is derived; fourth, the mean longitude of the planet at any given epoch; fifth, the longitude of the perihelion at the same epoch. The two other elements relate to the position of the orbit, and are, first, the longitude at a given epoch, the nodes of the orbit, or the points of intersection with a plane, which is usually supposed to be that of the ecliptic; second, the inclination of the orbit to this plane. There are then forty-nine elements to determine for the entire system of the known planets. The following table presents all these elements for the beginning of 1750.

The inspection of this table shews us, that the duration of the planetary revolutions, increase with their mean distances from the Sun: this made Kepler suspect that they were related to these distances by some proportion, which he proposed to discover. After a great number of trials continued through seventeen years, he at last found that *the squares of the times of the planetary revolutions, are to each other as the cubes of the greater axes of their orbits.*

Such are the laws of the motion of the planets; fundamental laws which have given a new aspect to astronomy, and have led to the discovery of universal gravitation.

The planetary ellipses are not unalterable. Their greater axes appear to be always the same; but their excentricities, their inclinations to a fixed plane, the position of their nodes and of their perihelia are subject to variations, which appear up to the present time to have increased proportional to the times. These variations only becoming sensible through the lapse of ages, have been called *secular inequalities*. There is no doubt of their existence, but modern observations not being sufficiently distant from each other, and ancient observations not being sufficiently exact, to ascertain them with precision, there remains yet a great incertitude regarding their quantity. The following tables offer values which appear to satisfy most correctly the mass of these observations.

Periodical inequalities have been likewise remarked, which disturb the elliptic motions of the planets: that of the Sun is a little altered, as has been seen in the preceding Book, but these inequalities are principally sensible in the two largest planets, Jupiter and Saturn. Comparing modern observations with the ancient, astronomers have remarked a diminution in the duration of Jupiter's revolution, and an increase in that of Saturn. Modern observations compared with each other give a contrary result, which seems to indicate great inequalities in the motions of these planets, the periods of which are very long. Even in this century the duration of Saturn's revolution has appeared different, following the points of the orbit from which the departure of the planet has been fixed; its returns to the vernal equinox have been more rapid than to the autumnal. Finally, Jupiter and Saturn experience inequalities which amount to many minutes, and which appear to depend on the situation of these planets, whether among themselves or with relation to their perihelia. Thus every thing announces that in the planetary system, independent of the principal cause which makes the planets move round the Sun in elliptic orbits, there exist particular causes which disturb their movements, and which alter at length the elements of their ellipses.

TABLE

OF THE ELLIPTIC MOTIONS OF THE PLANETS.

Duration of their sidereal revolution.

	Days.	yrs.	days.	hrs.	min.	sec.	
Mercury .	87.969255	0	87	23	15	44	0
Venus .	224.700817	0	224	16	49	11	0
The Earth	365.256384	1	0	6	9	8	0
Mars .	686.979579	1	321	23	30	35	6
Jupiter .	4332.602208	11	317	14	27	10	7
Saturn .	10759.077213	29	174	1	51	11	2
Uranus .	30689.000000	84	29	0	0	0	0

Semi major axes of their orbits, or their mean distances.

Mercury . .	0.387100
Venus . . .	0.723332
The Earth . .	1.000000
Mars . .	1.523693
Jupiter . .	5.202592
Saturn . .	9.540724
Uranus . .	19.183620

Proportion of the excentricity of the

semi major axes, for the beginning of the year 1750.

Mercury . .	0.205513
Venus . .	0.006885
The Earth . .	0.016814
Mars . .	0.093808
Jupiter . .	0.048877
Saturn . .	0.056223
Uranus . .	0.046683

The secular variations of this propor tion. (The sign — indicates a diminution.)

Mercury . .	0.000003369
Venus . .	—0.000062905
The Earth . .	—0.000045572
Mars . .	0.000090685
Jupiter . .	0.000134245
Saturn . .	—0.000261553
Uranus . .	—0.000026228

The mean longitudes at the commencement of 1750. These longitudes are reckoned from the mean vernal equinox at the epoch of the 31st December, 1749, at noon, mean time at Paris.

	deg.	deg.	min.	sec.	
Mercury .	281.3194	253	11	14	8
Venus .	31.4963	46	20	48	0

	deg.	deg.	min.	sec.	
The Earth .	311.1218	280	0	34	5
Mars .	24.4219	21	58	49	9
Jupiter .	4.1201	3	42	29	0
Saturn .	257.0438	231	20	21	9
Uranus .	353.9610	228	33	53	6

Longitudes of the perihelion at the beginning of 1750.

	deg.	deg.	min.	sec.	
Mercury .	81.7401	73	33	57	9
Venus .	141.9759	127	46	41	9
The Earth .	309.5790	278	37	15	9
Mars .	368.3005	331	28	13	6
Jupiter .	11.5012	10	21	3	8
Saturn .	97 9466	88	9	6	9
Uranus .	185.1262	166	36	48	8

The sidereal and secular motion of the perihelion, (the sign — indicates a retrograde motion.)

	sec.	min.	sec.	
Mercury .	1735.50	9	22	3
Venus .	—699.07	3	46	4
The Earth .	3671.63	19	49	6
Mars .	4834.57	26	6	4
Jupiter .	2030.25	10	57	8
Saturn .	4967.64	26	49	5
Uranus .	759.85	4	6	1

The inclination of the orbit to the ecliptic at the beginning of 1750.

		deg.	deg.	min.	sec.
Mercury	.	7.7778	7	0	0
Venus	.	3.7701	3	23	35
The Earth	.	0.0000	0	0	0
Mars	.	2.0556	1	51	0
Jupiter	.	1.4636	1	19	2
Saturn	.	2.7762	2	29	54
Uranus	.	0.8599	0	46	26

The secular variation of the inclination to the true ecliptic.

			sec.	sec.	
Mercury	.	.	55 09	17	50
Venus	.	.	13.80	4	47
The Earth	.	.	0.00	0	0
Mars	.	.	—4.45	—1	4
Jupiter	.	.	—67.40	—21	8
Saturn .	.	.	—47.87	—15	5
Uranus	.	.	9.38	3	0

Longitude of the ascending node upon the ecliptic at the beginning of 1750.

		deg.	deg.	min.	sec.	
Mercury	.	50.3836	45	20	42	8
Venus	.	12.7093	74	26	18	0

	deg.	deg.	min.	sec.	
The Earth .	0.0000	0	0	0	0
Mars .	52.9377	47	38	38	0
Jupiter .	108.8062	97	55	32	0
Saturn .	123.9327	111	32	21	9
Uranus .	80.7015	72	37	52	8

The sidereal and secular motion of the node upon the true ecliptic.

	sec.	min.	sec.	
Mercury .	—2332.90	12	35	8
Venus .	—5673.60	30	38	2
The Earth .	0.00	0	0	0
Mars .	—7027.41	37	58	0
Jupiter .	—4509.50	24	21	0
Saturn .	—5781.54	31	13	2
Uranus .	—10608.00	57	16	9

CHAP. VI.

Of the Figures of the Orbits of Comets, and of the Laws of their Motion round the Sun.

The Sun being the focus of the planetary orbits, it is natural to suppose it equally that of the orbits of comets. But these celestial bodies disappear after having been visible some months; their orbits instead of being nearly circular, like those of the planets, are very excentric, and the Sun is extremely near to that part in which they are visible. The ellipse, by the infinite varieties which it admits of from the circle to the parabola may suit these different orbits. Analogy leads us then to suppose that comets move in elliptic orbits of which the Sun occupies one of the foci, and to consider them as following the same laws with the planets, so that the areas described by their radii vectores are equal in equal times.

It is almost impossible to know the duration of the revolution of a comet, and consequently the greater axis of its orbit, by the observation of only one of its appearances; the area which its radius vector describes in a given time cannot then be determined rigorously. But it should be considered that the small portion of the ellipse described by the comet during its appearance may be supposed to coincide with a parabola, and thus its motion during a short interval may be calculated as if it was parabolical.

According to the laws of Kepler, the sectors described in the same time by two planets, are to each other as the areas of their ellipses divided by the squares of the times of the revolution, and these squares are as the cubes of their semi-major axes. It is easy to conclude that if we imagine a planet moving in a circular orbit, of which the radius is equal to the perihelion distance of a comet, the sector described by the radius vector of the comet will be to the corresponding sector described by the radius vector of the planet, as the square root of the aphelion distance of the comet is to the square root of the semi-major axis of its orbit, a relation which, when the ellipse changes to a parabola, becomes that of the square root of two to unity.

The relation of the sector of the comet to that of the imaginary planet is thus obtained, and it is easy by what has been already said, to get the proportion of this last sector, to that which the radius vector of the Earth describes in the same time. The area described by the radius vector of the comet may then be determined for any instant whatever, setting out from the moment of its passage through the perihelion, and its position may be fixed in the parabola which it is supposed to describe.

Nothing more is necessary but to deduce from observation the elements of the parabolic motion, that is to say, the perihelion distance of the comet, the position of the perihelion, the instant of its passage through the perihelion, the inclination of its orbit to the ecliptic, and the position of its nodes. The investigation of these five elements presents much greater difficulties than that of the elements of the planets, which being always visible, and having been observed during a long succession of years may be compared when in the most favourable position for determining these elements, instead of which comets only appear for a very short time, and frequently in circumstances where their apparent motion is rendered very complicated, by the real motion of the Earth, which always carries us in a contrary direction.

Notwithstanding all these difficulties, it is possible to determine the elements of the orbits of comets by different methods. Three complete observations are more than sufficient for this object; others only serve to confirm the accuracy of these elements, and the truth of the theory which we have just explained. Above four and twenty comets, the numerous observations of which are exactly represented by this theory, have confirmed it beyond all doubt.

It appears therefore that comets which have been considered as meteors for many years, are of the same nature as planets; their motions and their returns are regulated by the same laws as planetary motions.

Let us observe here how the true system of nature, as it is developed, becomes more and more confirmed.

The simplicity of the celestial phenomena on the supposition of the Earth's motion, compared to their extreme complication, on that of its being stationary, renders the first of these suppositions extremely probable. The laws of elliptic motion then common to the planets and the Earth, augment this probability, which becomes yet greater from the motion of comets, subjected by this hypothesis to the same laws.

Comets do not always move in the same direction like the planets. The real motion of some is direct, of others retrograde. The inclination of their orbits is not confined within a narrow zone, like that of the planetary orbits; they present every variety of inclination, from an orbit coincident with the plane of the ecliptic, to that perpendicular to it.

A comet is recognized when it reappears by the identity of the elements of its orbit with those of the orbit of a comet already observed. If its perihelion distance, the position of its perihelion, its nodes, and the inclination of its orbit are very nearly the same, it is probable that the comet which appears is that which had been observed before, and which, having receded to such a distance as to be invisible, returns to that part of its orbit nearest to the Sun. The duration of the revolution of comets being very long, and having been observed with very little care, till within about two centuries; the period of the revolution of one comet only is known with certainty, that of 1682, which had been already observed in 1607 and 1531, and which has reappeared in 1759. This comet takes about 76 years to return to its perihelion; therefore taking the mean distance of the Sun from the Earth as unity, the greater axis of its orbit is 35.9, and as its perihelion distance is only 0.58, it recedes from the Sun at least thirty-five times more than the Earth, describing a very excentric ellipse. Its return to the perihelion has been longer by thirteen months from 1531 to 1607, than from 1607 to 1682; it has been eighteen months shorter from 1607 to 1682, than from 1682 to 1759. It appears then that causes similar to those which alter the elliptic motion of the planets, disturb that of comets in a yet more perceptible manner.

The return of some other comets has been suspected: the most probable of these returns was that of the comet of 1532, which has been believed to be the same with that of 1661, and the revolution of which was fixed at 129 years; but this comet not having reappeared in 1790, as was expected, there is great reason to believe that these two comets were not the same. This ought to render us very circumspect in pronouncing on the identity of two observed comets. Let us try to calculate the probability of this identity when the elements are a little different.

Suppose the difference to be only a degree in the inclination of the orbit, and in the places of the ascending node and perihelion, and a hundredth of a degree in the perihelion distance, the mean distance of the Sun and Earth being taken as unity: suppose again, that the errors of the elements deduced from observation, and the alteration which these elements might experience in the interval of two appearances of the comet be included within these limits, so that nothing should hinder us from considering the two comets as the same.

The inclination of the orbit of a new comet to the ecliptic may vary from zero to the semi circumference but above one hundred* degrees of inclination the motion changes its direction; thus, whether the motion be direct or retrograde, may be indicated by the inclination alone; the probability that the inclination of the orbit of a new comet should not separate more than a degree from the orbit of a preceding one, is then equal to $\frac{1}{100}$. The position of the ascending node of a comet may vary from zero to † 400°, the probability then that it should not differ more than a degree from that of the node of a comet observed before, is therefore $\frac{1}{200}$°.

In a similar manner the probability that the position of the perihelion of a new comet should not differ more than a degree from that of a former one, is $\frac{1}{200}$. We shall suppose that the perihelion distance may equally vary in the interval comprised between zero and 1.5. Comets have in fact been seen, the perihelion distance of

* 90°. † 360°.

which has surpassed 1.5; but these cases are sufficiently rare for us to dispense with any attention to them in this trial of calculation, as a greater perihelion distance almost always renders the comet invisible. The probability that the perihelion distance of a new comet should not differ more than one hundredth of a degree from the perihelion distance of a comet formerly observed, will then be nearly $\frac{4}{300}$. Thus the probability that the elements of a new comet should not differ from those of a preceding comet beyond these limits, will be the product of the four numbers, $\frac{1}{100}$, $\frac{1}{200}$, $\frac{1}{200}$, $\frac{1}{300}$, and consequently will be equal to a fraction, the numerator of which being unity, the denominator is equal three hundred millions.

The theory of chances gives the following rule to know the probability of a new comet being the same with one observed before: multiply this fraction by the number of comets visible, and not yet observed, augmented by unity; divide unity by this product *plus* one, the quotient will be the probability sought.

If the limits of the errors of the elements deduced from observation are greater than the preceding, instead of the fraction one divided by three hundred millions, the product of this fraction by that of four numbers, which express how often each limit contains the preceding supposed limit, must be employed.

The number of comets visible and not yet observed being unknown, it is impossible to calculate the probability we are now considering, nevertheless we may believe that they do not exceed a million. Supposing it equal to this quantity, there are 300 to one, that a comet, the elements of which do not differ from those of one formerly observed more than the preceding quantities, is the same. Comparing thus the elements of the comets of 1607 and 1682, Halley was able to announce, with a probability equal to $\frac{1200}{1201}$, that they were the same, and that it would reappear towards the middle of this century. The fear of being deceived, though very small, almost vanished when he recognized nearly the elements of this comet in that observed in 1531, and it has totally disappeared to us who have seen the comet in 1759.

But it is not thus with the comet of 1532; its elements have been determined upon the observation of Appian and Frucastor, and their observations are so inaccurate that they leave an uncertainty of *41° upon the situation of the nodes, of †10° upon its inclination, of ‡22° on the position of its perihelion, and of §0°.225 upon its perihelion distance. It is necessary therefore to multiply the fraction unity divided by three hundred millions, by the product 41.10.22.17, which reduces it to 0.000517; supposing then that there are yet a thousand comets visible and not yet observed, which is not improbable, the probability that the two comets of 1532 and 1661 are the same would be $\frac{2}{3}$, a probability much too small to pronounce their identity. Therefore we ought not

* 36° 54'. † 9°. ‡ 19° 48'.
§ 15'.

to be surprised that this comet has not reappeared in these last years.

The nebulosity with which these comets are almost always surrounded, seems to be formed by the vapours which the solar heat raises on their surface. It is imagined that the great heat which they experience towards their perihelion, rarifies the particles which have been congealed by the excessive cold of the aphelion.

It appears also that the trains of comets are only these vapours elevated to a considerable height by this rarefaction, combined, either with the solar rays, or with the dissolution of those vapours in the fluid which reflects the zodiacal light to us. This seems to result from the direction of their trains, which are always beyond the comet, relatively to the Sun, and which only becoming visible near their perihelion, are not at a maximum till after their passage through this point, when the heat communicated to the comet by the Sun, is encreased by its duration, and by the proximity to this luminary.

CHAP. VII.

Of the Laws of the Motion of Satellites round their Planets.

We have explained in the Sixth Chapter of the preceding Book, the laws of the motion of the Earth's satellite and of its principal irregularities. It now remains to consider those of the motion of the satellites of Jupiter and Saturn.

If the semidiameter of the equator of Jupiter, supposed *60″.185 at the mean distance of the planet from the Sun, be taken as unity, the mean distance of its satellites from its centre will be nearly:

I. Satellite . .	5.697300
II. Satellite . .	9.065898
III. Satellite . .	14.461628
IV. Satellite . .	25.436000

* 19.″5

The duration of their siderial revolutions are :

	J.	Days.	hrs.	min.	sec.
I. Satellite	1.769137787069931	1	18	27	30
II. Satellite	3.551181016734509	3	13	13	43
III. Satellite	7.154552807541524	7	3	42	28
IV. Satellite	16.689019396008634	16	16	32	9

The synodical revolutions of these satellites, or the intervals of the return of their mean conjunctions with Jupiter, may be easily concluded from the duration of their sidereal revolutions, and that of the sidereal revolution of Jupiter.

At the beginning of 1700, the mean longitudes of the satellites were :

	Deg.	Deg.	min.	sec.
I. Satellite .	85.8491	77	15	51
II. Satellite .	83.5827	75	18	17.9
III. Satellite .	182.4495	164	12	16.3
IV. Satellite .	253.1545	23	50	20.5

Comparing the distances of the four satellites of Jupiter, with the duration of their revolution, the same beautiful proportion has been observed between these quantities, that exists between the mean distances of the planets from the Sun, and the duration of their revolutions, that is to say, *the squares of the times of the sidereal revolutions of the satellites are to each other, as the cubes of their mean distances from Jupiter.*

The frequent eclipses of the satellites have furnished astronomers with the means of ascertaining their motions with a precision that could not have been expected from the observation of their distance from Jupiter. They have produced the following results :

The ellipticity of the orbit of the first satellite is insensible, its plane coincides nearly with that of the equator of Jupiter, the inclination of which to the orbit of the planet is *4.°4444.

The ellipticity of the orbit of the second satellite is equally insensible. Its inclination to the orbit of Jupiter varies as well

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* 3° 33′ 20″.

as the position of its nodes. All these variations are represented nearly by supposing the orbit of the satellite inclined about * 5182″ to the equator of Jupiter, and giving to its nodes a retrograde motion on this plane, the period of which is about thirty Julian years.

A slight ellipticity is observable in the orbit of the third satellite. The extremity of its greater axis nearest to Jupiter, and which is called perijove, has a direct motion, and the excentricity of the orbit appears subject to very perceptible alterations. Towards the end of the last century, the equation of the centre was at its maximum, which amounts to nearly † 2661″, it afterwards diminished, and towards 1775 was at its minimum, about ‡ 759″. The inclination of the orbit of this satellite to that of Jupiter, and the position of its nodes, are variable. These variations may be nearly represented by

* 27′ 58″ 9. † 14′ 22″ 1. ‡ 4′ 5″ 9.

supposing the orbit inclined about *2244″ to the equator of Jupiter, and assigning to its nodes a retrograde motion on the plane of this equator, in a period of 137 years.

The orbit of the fourth satellite has a very sensible ellipticity, its perigee has a direct motion of about †7852″. This ‡272″ to the orbit of Jupiter. It is in consequence of this inclination that the fourth satellite frequently passes behind the planet relatively to the Sun without being eclipsed. Since the discovery of the satellites, till 1760, its inclination has appeared constant, but it has augmented by a sensible quantity in these later years. We intend to return to these irregularities, when we shall explain their cause.

Independent of these variations the motions of the satellites of Jupiter are subject to inequalities which disturb their elliptic motions, and which render their

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* 12′ 7″. † 42′ 24″. ‡ 1′ 28″ 1.

theory extremely complicated. These are principally sensible in the three first satellites, the motions of which present very remarkable relations.

Their mean motions are such that the motion of the first satellite plus twice that of the third, is nearly equal to three times the mean motion of the second satellite. The same proportion exists between their mean synodical motions; for the synodical motion being nothing more than the excess of the sidereal motion of a satellite above that of Jupiter, if we substitute the synodical instead of the mean motion, in the preceding equality, the mean motion of Jupiter disappears, and the equality remains the same.

The mean longitudes, whether synodical or sidereal, of the three first satellites seen from the centre of Jupiter, are such that the motion of the first satellite *minus* three times that of the second, *plus* twice that of the third, is nearly equal to the semicircumference. This equality is so near, that we are tempted to consider it as rigorous, and to reject as errors of observation, those very small quantities by which they differ from it. We may at least be assured that it will continue through a long succession of ages, from which it results that from this time to an immense number of years, the three first satellites of Jupiter cannot be eclipsed at the same time.

The periods and the laws of the principal inequalities of these satellites are the same. The inequality of the first advances or retards its eclipses *233″ in time, at its maximum. Comparing it motion with the respective positions of the two first satellites, it has been found that it disappears when these satellites, seen from the centre of the planet, are in opposition to the Sun at the same time; that it afterwards increases, and is the greatest possible when the first satellite, at the moment of its opposition, is † 50° more advanced than the second:

1′ 15″. †45°.

that it is again annihilated when it is more advanced by *100°; beyond this, it takes a contrary sign, and retards the eclipses, it augments to †150° distance between the satellites, where it is at its negative maximum; it then diminishes and disappears at ‡200 distance: and that finally in the second half of its circumference it follows the same laws as in the first. It has been concluded from this that there exists in the motion of the first satellite of Jupiter an inequality of § 5258″ at its maximum, and proportional to the sine of double the excess of the mean longitude of the first satellite above that of the second; an excess equal to the difference of the mean synodical longitudes of the two satellites. The period of this inequality is not four days. But how, in the eclipses of the first satellite, does it transform itself into a period of ‖ 437^{j} 75? This is what we are going to explain.

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* 90°. † 135°. ‡ 180°. § 28′ 2″. ‖ 437^{d} 18^{h}.

Let us suppose that the two first satellites set out together, from their mean oppositions to the Sun. At every circumference which the first satellite describes in consequence of its mean synodical motion, it will be in the mean opposition. If we imagine a fictitious star, whose angular motion may be equal to the excess of the mean synodical motion of the first satellite, above twice that of the second. Then twice the difference of the mean synodical motions of the two satellites will be, in the eclipses of the first, equal to the multiple of its circumference *plus* the motion of the fictitious star. The sine of this last motion will be then proportional to the inequality of the first satellite in its eclipses and may represent it. Its period is equal to the duration of the motion of the fictitious star, a duration which from the mean synodical motions of the two satellites is 437.ʲ75. It is thus determined with greater precision than by direct observation.

The inequality of the second satellite

follows a law similar to that of the first, with this difference, that it has always a contrary sign. It accelerates or retards the eclipses * 1059″ in time at its maximum. Comparing it with the respective positions of the two first satellites, it is observed that it disappears when they are at the same time in opposition to the Sun. That it then retards more and more the eclipses of the second satellite, till the two satellites are separated 100° from each other at the instant of these phenomena; that this retardation diminishes and becomes nothing when the mutual distance of the two satellites is 200°: finally, that beyond this term, the eclipses advance in the same manner as they before retarded. It has been concluded from these observations that an inequality of * 11923″ at its maximum exists in the motion of the second satellite, proportional (but affected by a contrary sign), to the sine of the excess of the mean longitude

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* 5′ 43″.   + 1° 4′ 23″.

of the first satellite above that of the second, an excess equal to the difference of the mean synodical motions of the two satellites.

If the two satellites set out together, from their mean opposition to the Sun, the second satellite will be in its mean opposition at each circumference which it will describe in consequence of its mean synodical motion. If we conceive, as before, a star whose angular motion may be equal to the excess of the mean synodical motion of the first satellite above twice that of the second, then the difference of the synodical motion of the two satellites will be, in the eclipses of the second, equal to a multiple of its circumference plus the motion of the fictitious star. The inequality of the second satellite, in its eclipses, will be then proportional to the sine of the motion of this imaginary star. We thus see the reason why the period and the law of this irregularity are the same as those of the inequality of the first satellite.

The influence of the first satellite upon the inequality of the second, is very probable: but if the third satellite produced in the motion of the second, an inequality similar to that which the second seems to produce in the motion of the first, that is to say, proportional to the sine of double the difference of the mean longitudes of the second and third satellites; this new inequality would confound itself with that which is due to the first satellite; for in consequence of the relation which the longitudes of the three first satellites have to each other, and which we have described before, the difference of the mean longitudes of the two first satellites is equal to the semi-circumference plus double the difference of the mean longitudes of the second and third satellites; so that the sine of the first difference is the same with the sine of double the second difference with a contrary sign. The inequality produced by the third satellite in the motion of the second would therefore have the same sign, and follow the same law, as the inequality observed in this motion; it is therefore very probable that this inequality is the result of two inequalities dependent on the first and third satellites. If by the succession of ages the relation between the mean longitudes of these three satellites should cease to exist, these two inequalities, at present confounded, would separate, and their respective value might be known. But according to observation this relation should subsist for a very long period, and we shall see in the fourth Book that it is rigorously exact.

Finally the inequality relating to the third satellite in its eclipses, compared with the respective positions of the second and third satellites, offers the same proportion as the inequality of the second compared with the respective positions of the two first satellites. There exists then in the motion of the third satellite, an inequality proportional to the sine of the excess of the mean longitude of the second satellite above that of the third, which inequality at its maximum is *827″.

* 4′ 27″ 9.

If we suppose a star whose angular motion may be equal to the excess of the mean synodical motion of the second satellite above twice the mean synodical motion of the third, the inequality of the third satellite will be, in its eclipses, proportional to the sine of the motion of this fictitious star. Now in consequence of the proportion which exists between the mean longitudes, the sine of this motion is, exclusive of the sign, the same with the motion of the first fictitious star which we considered. Thus the inequality of the third satellite, in its eclipses, has the same period, and follows the same laws, as the inequalities of the two first satellites.

Such is the motion of the principal inequalities of the three first satellites of Jupiter, which Bradley seems to have suspected, and which Wargentin has since investigated with the greatest accuracy.

Their correspondence and that of their mean motions and longitudes, seem to form a separate system of these three bodies, animated by common forces, and united by common proportions.

Let us now consider the satellites of Saturn. If the semi-diameter of this planet seen at its mean distance from the Sun be taken as unity, the distances of the satellites from its centre will be as follows :

I.	-	3.080
II.	-	3.952
III.	-	4.893
IV.	-	6.268
V.	-	8.754
VI.	-	20.295
VII.	-	59.154

The durations of their sidereal revolutions are :

		J.	Days.	hrs.	min.	sec.
I.	-	0.94271	0	22	37	30
II.	-	1.37024	1	8	53	5.8
III.	-	1.88780	1	21	18	25.9
IV.	-	2.73948	2	17	41	36.6
V.	-	4.51749	4	12	25	11
VI.	-	15.9453	15	22	41	13.9
VII.	-	79.3296	19	7	54	27

Comparing the durations of the revolutions of these satellites, at their mean distances from the centre of Saturn, we again find the beautiful proportion discovered by Kepler, relative to the planets, and which we have seen exist in the satellites of Jupiter, that is to say, *that the squares of the times of the revolutions of Saturn's satellites are to each other, as the cubes of their mean distances from the centre of this planet.*

The great distance of the satellites of Saturn and the difficulty of observing their positions, has not permitted us to discover the ellipticity of their orbits, and still less the inequalities to which their motions are subject; nevertheless the ellipticity of the orbit of the sixth satellite is perceptible.

If we take for unity the semi-diameter of Uranus supposed * 6″ seen at the mean distance of the planet from the Sun, the distances of its satellites will be:

I.	-	13.120
II.	-	17.022

* 1″9.

III.	-	19.845
IV.	-	22.752
V.	-	45.507
VI.	-	91.008

The durations of their sidereal revolutions are:

		J.	Days.	hrs.	min.	sec.
I.	-	5.8926	5	21	25	20
II.	-	8.7068	8	16	57	47
III.	-	10.9611	10	23	2	47
IV.	-	13.4559	13	10	56	29
V.	-	38.0750	38	1	48	0
VI.	-	107.6944	107	16	39	56

These durations, with the exception of the second and fourth, have been concluded from the greatest observed elongations, and from the hypothesis that the squares of the sidereal revolutions of the satellites are as the cubes of their mean distances from the centre of the planet: an hypothesis which observation has confirmed regarding the second and fourth satellites of Uranus, so that it should be considered as a general law of the motion of a system of bodies round a common focus.

In the mean time what are the principal forces which retain the planets, satellites, and comets in their respective orbits? What particular forces disturb their elliptic motions? What cause produces the retrograde motion of the equinoxes? and moves the axes of rotation of the Earth and Moon? By what forces finally, are the waters of the ocean raised twice a day? The supposition of one sole principle on which all these effects depend, is worthy of the simplicity of nature.

The generality of the laws which the celestial motions present, seem to indicate its existence. Already even we may suspect this principle in the relation of these phenomena with the respective position of the bodies in the solar system. But to bring it forward with evidence the laws of the motion of matter must be known.

THE

SYSTEM OF THE WORLD.

BOOK III.

OF THE LAWS OF MOTION.

Surrounded by an infinite number of phenomena which continually succeed each other on the Earth, philosophers have succeeded in discovering the small number of general laws to which the motions of matter are subject. To them all nature is obedient; and every thing is derived from them as necessarily as the returns of the seasons; the curve described by the lightest atom that seems carried about by the winds as chance directs, is regulated by laws as certain as the planetary orbs.

The importance of these laws, on which we never cease to depend, ought to have excited the curiosity of mankind in all ages, but by the effect of indifference too common to the human mind, they were utterly unknown till the commencement of the last century, when Galileo first laid the foundation of the science of mechanics by his beautiful discoveries relative to the descent of falling bodies. Geometricians in following the steps of this great man, have finally reduced the whole science of mechanics to general formulæ, which leave nothing to be desired but to bring the art of analysis to perfection.

CHAP. I.

Of Forces and their Composition.

A BODY appears to us to be in motion when it changes its situation relatively to a system of bodies which we suppose to be in a state of repose. Thus in a vessel moving in a uniform manner bodies seem to us to move when they correspond successively to different parts of the vessel. This motion is only relative, for the vessel itself moves on the surface of the sea, which revolves round the axis of the Earth, the centre of which moves round the Sun, which is itself carried along the regions of space with the Earth and all the planets. To conceive a term to these motions, and to arrive at last at some fixed points from which we may reckon the absolute motion of bodies, we imagine a space without bounds, immovable and penetrable to matter. It is to parts of this space, real or imaginary, that we refer in imagination the position of bodies, and we conceive them in motion when they correspond successively to different places in this space.

The nature of this singular modification by virtue of which a body is transported from one place to another is, and always will be, to us unknown. It has been designated by the name of FORCE; its effects and the law of its action is all that we can possibly determine.

The effect of a force acting on a material point is to put it in motion if nothing opposes it. The direction of the force is the straight line which it tends to make it describe. It is evident that if two forces act in the same direction they will increase the effect of each other; but if they act in opposite directions, the point will only move in consequence of their difference, and it would remain at rest if the forces were equal. If the directions of the two forces make an angle with each other, the resulting force will take a mean direction, and it can be demonstrated by geometry alone, that if reckoning from the point of concourse of the two forces, we take on their directions, straight lines to represent them, and then form a parallelogram with these straight lines, its diagonal will represent both as to its direction and magnitude, their resulting force.

Thus the resulting force may be substituted for two composing forces, and reciprocally may be substituted for any force whatever, two others of which it is the resulting force. Any force, therefore, may be decomposed into two others parallel to two axes situated in the same plane, and perpendicular to each other. To do this it is sufficient to draw from the first extremity of the line representing the force, two lines parallel to these axis and to form with these lines a rectangle, the diagonal of which should be the force required to be decomposed. The two sides of this rectangle will represent the forces into which the given force may be decomposed parallel to these axes.

If the force is inclined to a plane given in position, then, taking to represent it, a line in its direction, the extremity of which is in the surface of the plane, the perpendicular which falls from the other extremity on the plane, will be the primitive force decomposed in a direction perpendicular to the plane. The straight line which in the plane joins the force (or the other extremity of the line) with the perpendicular, will represent the primitive force decomposed parallel to the plane. This second partial force may be itself decomposed again into two others parallel to two axes in the same plane perpendicular to each other. Thus every force may be decomposed into three others parallel to three axes perpendicular to one another.

Hence arises a very simple method of having the resulting force, of any number of forces which act upon a material point, for by decomposing every one of them into three others parallel to three axes given in position and perpendicular to each other, it is clear that all the forces parallel to the same axis will be reduced to one single force equal to the sum of the forces which act in one direction *minus* the sum of those which act in a contrary direction. Thus the point will be sollicited by three forces perpendicular one to the other; if then, three straight lines in each of their directions be taken to represent them, reckoning from the point of concourse, and on these three lines we form a rectangular parallelopipedon, the diagonal of this solid, will represent both the quantity and direction of the resulting force of all those which act upon the point.

CHAP. II.

Of the Motion of a Material Point.

A POINT in repose cannot give to itself any motion, since it contains not within itself any cause why it should move in one direction in preference to another. When sollicited by any force and then abandoned to itself, it will move constantly and uniformly in the direction of that force, if it meets with no resistance; that is at every instant the force and direction of its motion will be the same. This tendency of matter to persevere in a state of either motion or rest, is what is termed *inertia*, it is the first law of the motion of bodies.

The direction of motion in a straight line follows necessarily from this, that there is no reason why the point should deviate to the right, rather than to the left of its primitive direction, but the uniformity of its motion is not equally evident. The nature of the moving force being unknown, it is impossible to know *à priori* if this force should preserve itself or not. It is true that since a body is incapable of giving itself any motion, it seems equally impossible that it should be able to effect any change in that which it has received, so that the law of inertia is at least the most natural and most simple that can be imagined. It is likewise confirmed by experience, for in fact we observe on the Earth that motions are perpetuated for a longer time in proportion as the obstacles which oppose them are diminished, and this would lead us to suppose that without these obstacles they would continue for ever. But the inertia of matter is principally remarkable in the motions of the heavenly bodies, which for a great number of ages have not experienced any sensible alteration. For these reasons we shall consider inertia as a law of nature, and when we observe any alteration in the motion of a body, we shall conclude that it arises from the action of some foreign cause.

In uniform motion the spaces described are proportional to the times. But the times employed to describe a given space, are greater or less according to the magnitude of the moving force. From these differences has arisen the idea of velocity, which in uniform motion is the relation between the space and the time employed to describe it. To avoid comparing together, quantities so heterogeneous as space and time, we take an interval of time, a second for example, as an unity of time; and in like manner a portion of space, as a foot, for a unity of space. Time and space thus become abstract numbers, which express how many unities each contain of their own species, and thus they may be compared one with another. Velocity becomes the ratio of two abstract numbers, and its unity is the velocity of a body which describes one foot in one second. In reducing in this manner space, time, and velocity to abstract numbers, it appears that space is equal to the product of the velocity by the time, which latter is consequently equal to the space divided by the velocity.

Force being only known to us by the space which it causes to be described in a given time, it is natural to take this space for its measure, but this supposes, that several forces acting in the same direction, will cause to be described in a unity of time, a space equal to the sum of the spaces which each would have caused to be described separately, or in other words that the space is proportional to the velocity. This is what we cannot be assured of *à priori*, considering our ignorance of the nature of the moving force: upon this subject we must have recourse to experience, for whatever is not a necessary consequence of the few data we have given on the nature of things, is only to us the result of observation.

Force may be expressed by an infinity of functions of the velocity, without implying a contradiction. There is none in supposing it proportional to the square of the velocity. In this hypothesis it is easy to determine the motion of a point sollicited by any number of forces, whose velocities are known; for if we take upon the directions of these forces, straight lines representing their velocities, beginning at their point of concourse, and if upon these directions, and from the same point other lines be taken, which are to each other as the squares of the first, these lines will represent the forces themselves. By combining them according to the rules already given, we shall obtain their resulting force, and also the straight line which represents it, and which will be to the square of the corresponding velocity, as the straight line representing one of the composing forces is to the square of its velocity. By this it appears how the motion of a point may be determined, whatever be the function of the velocity which expresses the force. Among all the functions mathematically possible, let us examine which is that of nature.

It is observed upon the Earth that a body sollicited by any force, moves in the same manner, whatever be the angle which the direction of this force makes with the direction of the motion which is common to the body and to the part of the terrestrial surface to which it corresponds ; the same thing takes place in a vessel, whose motion is uniform ; a moveable body submitted to the action of a spring, or of gravity, or any other force, moves relatively to the parts of the ship in the same manner whatever be the velocity and direction of the vessel. It may then be established as a general law of terrestrial motions, that if in a system of bodies carried on by a common motion, any force be impressed on one of them, its apparent or relative motion will be the same, whatever be the general motion of the system, and the angle which its direction makes with the impelling force.

The proportionality of force to velocity, results from this law supposed rigorously exact; for if we suppose two bodies moving upon one straight line with equal velocities, and that by impressing on one of them a force, which encreases the primitive force, its relative velocity to the other body remains the same as if both of them had been primitively in a state of repose; it is evident that the space described by the body in consequence of its primitive force, and of that which is added to it, becomes equal to the sum of the spaces, which each of them would have caused it to describe in the same time; which supposes the force proportional to the velocity.

And reciprocally, if the force is proportional to the velocity, the relative motions of a system of bodies, animated by any forces whatever, are the same, whatever be their common motion, for this motion decomposed into three others, parallel to three fixed axes, only increases by the same quantity the partial velocities of each body, parallel to these axes, and since the relative velocities only depends on the difference of these partial velocities, it will be the same, whatever be the motion common to all the bodies. It is therefore impossible to judge of the absolute motion of a system of which we make a part, by any appearances that can be observed. Which circumstance characterizes this law, the ignorance of which has so long retarded our knowledge of the system of the world, by the difficulty of conceiving the relative motions of projectiles about the surface of the Earth, carried along by a double motion round its own axis and round the Sun.

But considering the smallness of the most considerable motions which we can impress on bodies on the Earth, compared with that which they share in common with the Earth, it is sufficient for the appearances of a system of bodies to be independant of the direction of this motion that a small augmentation of the force by which the earth is animated, be to the corresponding increase in its velocity, in the ratio of these quantities themselves. Thus our experiments only prove the reality of this proportion, which if it existed, whatever were the velocity of the Earth, would give the law of the velocity proportional to the force. It would give likewise this law, if the function of the velocity expressing the force, was only composed of a single term. If then the velocity be not proportional to the force we must suppose in nature the function of the velocity expressing the force to consist of several terms, which is not at all probable. We must moreover suppose the velocity of the Earth to be exactly such as suits the above proportion, which is likewise extremely improbable. Besides the velocity of the Earth varies in different seasons of the year, it is about one-thirtieth greater in winter than in summer: this variation is still more considerable, if, as every thing seems to indicate, the solar system itself is in motion in space, for according as this progressive motion conspires with that of the Earth, or is contrary to it, there should result great variations in the course of the year in the absolute motion of the Earth, and this should alter the proportion of which we are speaking, and the relation of the force impressed to the relative velocity resulting from it, if this proportion and this relation were not independant of the motion of the Earth. Nevertheless the most precise experiments have never shewn the slightest sensible variation.

All the celestial phenomena serve to corroborate these proofs. The velocity of light, determined by the eclipses of Jupiter's satellites, combines itself with that of the Earth, exactly according to the law of the proportionality of the force to the velocity, and all the motions of the solar system, calculated after this law, are entirely conformable to observation.

Hence we have two laws of motion, that is to say, inertia, and that of the force proportional to the velocity, given by observation; they are the most simple and most natural that can be imagined, and without doubt they are derived from the very nature of matter; but this nature being to us unknown, these laws are to us only observed facts. They are the only ones that the science of mechanics borrows from experience.

The velocity being proportional to the force, these two quantities may be represented one by the other, and by the preceding rules we can always determine the velocity of a point sollicited by any number of forces, whose velocities and directions are known.

If the point is sollicited by a number of forces which act in a continued manner, it will describe with a motion incessantly variable, a curve, the nature of which will depend on the forces by which the point is sollicited. To determine it we must consider the curve in its elements, examine how they arise one from the other, and ascend from the law of the augmentation of their ordinates to their finite expression. It is here that the infinitesmal calculus becomes indispensible, and that we become sensible of the utility of bringing to perfection this powerful engine of the human mind.

We have in the instance of gravity, a daily example of a force, which seems to act without interruption. It is true we are ignorant whether its successive actions are separated by intervals of time, whose duration is insensible, but the phenomena being nearly the same either in this hypothesis or that of a continued action, geometricians have adopted the former as the most simple and commodious. Let us investigate the laws of these phenomena. Gravity seems to act upon bodies in the same manner whether they are in a state of repose or motion. In the first instant a body abandoned to its action acquires a degree of velocity infinitely small, in the second instant a new degree of velocity is added to the first, and so continues, the velocity increasing in proportion to the time.

If we imagine a right-angled triangle,

and one side to represent the time and increase with it, the other side may represent the velocity. The element of the surface of the triangle, being equal to the product of the element of the time by the velocity, it will represent the element of the space which gravity causes to be described; and thus this space will be represented by the intire surface of the triangle, which increasing as the squares of one of its sides, shews us, that in motion accelerated by the action of gravity, the velocities increase as the times and the heights from which bodies fall fall from a state of rest, vary as the squares of the times, or as the squares of the velocities. In expressing thus by unity the space a body falls in one second, it will descend four unities in two seconds, nine in three seconds and so on; in such a manner that in each second it will describe spaces increasing as the odd numbers 1, 3, 5, 7, &c.

The space which a body will describe in consequence of the velocity acquired by its fall in a time equal to that of its descent, will be the product of this time by its velocity, but this product is the double of the surface of the triangle, therefore a body moving uniformly with its acquired velocity will describe in the time equal to that of its descent a space double to that through which it has fallen.

The ratio of the required velocity to the times is constant for the same accelerating force, it augments and diminishes as they are greater or less, it may therefore serve to express them. The double of the space described being the product of the time by the velocity, the accelerating force is equal to this double space divided by the square of the times. It is likewise equal to the square of the velocity divided by this double spacc. These three methods of expressing the accelerating force are useful on many occasions, they do not give the absolute values of these forces, but their ratios to each other or to one of them, taken as unity, which is all that is required in mechanics.

On an inclined plane, the action of gravity is decomposed into two, one perpendicular to the plane which is destroyed by its resistance, the other parallel to the plane which is to the primitive force of gravity as the height of the plane is to its length ; motion is therefore uniformly accelerated on inclined planes ; but the velocities, and the spaces described, are to the velocities and spaces described in the same time in the direction of the vertical, as the height of the plane to its length. From hence it follows that all the chords of a circle which terminate in one of the extremities of a vertical diameter, are described in the same time by the action of gravity as the diameter itself.

A body projected in the direction of any straight line whatever, continually deviates from this direction, describing a curve concave to the horizon, and of which this line is the first tangent. Its motion when referred to this line by vertical lines, is uniform, but it accelerates in the direction of these verticals according to the laws already explained. If, therefore, vertical lines are drawn from every part of the curve to this first tangent, they will be proportional to the squares of the coresponding parts of this tangent, a property which characterizes the parabola. If the force of projection is in the direction of the vertical itself, the parabola is confounded with this vertical line, and thus the formulæ for parabolic motion, give those for accelerated and retarded motion in the direction of the vertical.

Such are the laws of the descent of the heavy bodies, as discovered by Galileo. It appears now to us not difficult to have discovered them, but as they escaped the investigation of philosophers notwithstanding the phenomena which incessantly occurred it must have required an extraorgenius to have developed them.

We have seen in the first Book that a material point suspended at the extremity of a straight line without mass, and fixed at its other extremity, forms the simple pendulum. This pendulum when removed from its vertical position, tends by its gravity to return, and this tendency is very nearly proportional to its deviation, when that is not very considerable. Let us imagine two pendulums of the same length quitting the vertical position at the same instant with very small velocities; they will describe, in the first instant, arcs proportional to the velocities; at the beginning of the second instant equal to the first, the velocities will be retarded in proportion to the arcs described, and to the primitive velocities. The arcs described, therefore, in this instant, will be proportional to these velocities; it will be the same with the arcs described in the third and fourth instants, &c. Thus at every instant the velocities and the arcs measured from the vertical, will be proportional to the primitive velocities, the pendulums will arrive, therefore, at the same moments at a state of repose; they will return again to the vertical by a motion accelerated according to the same laws by which their velocities had been retarded, and will arrive at the same instant, and with their primitive velocity, they will oscillate in the same manner on the other side of the vertical, and would continue to oscillate for ever but for the resistance which they experience. It is evident that the extent of their oscillations is in proportion to their primitive velocities, but the length of their oscillations is the same and consequently independent of their extent. The accelerating force of the pendulum, not being exactly in proportion to the extent of the arc from the vertical, this isochronism is only approximative, relatively to a body performing very small oscillations in a circle. But it is rigoously exact in a curve, in which the force of gravity, decomposed parallel to the tangent, is proportional to the arc, reckoned from the lowest point of the curve, which immediately gives it differential equation. Huygens, to whom we are indebted for the application of pendulums to clocks, was interested in finding this curve, and in the manner of making pendulums describe it. He found that it was a cycloid placed vertically, so that its summit might be the lowest point, and that to cause a body suspended at the extremity of an inextensible thread to describe this curve, he found that it would be necessary to fix the other extremity at the common origin of two cycloids, equal to that to be described by the pendulum, and so placed that the thread in its vibrations may envelope alternately a portion of these curves. But whatever ingenuity may have been manifested in these investigations, experience has preferred the circular pendulum, as the most simple and sufficiently exact in practice, but the theory of evolutes which has been derived from them, is become very important by its application to the system of the world.

The length of the very small oscillations of a circular pendulum, is to the time which a body will employ in falling from a height equal to double the length of the pendulum, as the semi-circumference is to the diameter : and thus the time of the descent of a body through a small arc of a circle terminated in a vertical diameter, is to the time of the descent through the diameter, or what is the same thing, as the time required to describe the chord of this arc, as the quarter of the circumference is to the diameter, the straight line joining the two points is not therefore the line of swiftest descent from one point to another; the investigation of this line has excited the attention of mathematicians, and they have found that it is a cycloid whose origin is the most elevated point.

The length of a simple pendulum vibrating seconds, is to double the height which bodies describe by the force of gravity in one second, as the square of the diameter to the square of the circumference.

It has been observed in the First Book, that very exact experiments have given the length of the pendulum vibrating decimal seconds at Paris 2.28386, from which it results that bodies fall by the force of gravity $11^f.2704$ in the first second of time. The connection between the lengths of pendulums, which may be observed with great precision, and the time of descent of falling bodies was an ingenious discovery for which we are indebted to Huygens.

The times of the very small oscillations of pendulums of different lengths and animated by the same force of gravity, are as the square roots of these lengths. If the pendulums are of the same length, but animated by different forces, the times of their oscillations will be reciprocally as the square roots of these forces.

It is by means of these theorems that variation of the force of gravity at the surface of the Earth, and on the summits of mountains has been determined. The observations made on pendulums have demonstrated that gravity neither depends on the figure nor on the surface of bodies, but that it penetrates into all their component particles, and tends to impress on them equal velocities in equal times. To be assured of this, Newton made several bodies oscillate, of the same weight but different in their figure and matter, placing them in the interior of the same surface that the resistance in the air might be the same. Whatever precision he employed in the experiment, he was never able to perceive the smallest difference in the times of the vibrations of these bodies, from which it follows that were it not for the resistances which different bodies experience, their velocities acquired by the action of gravity, would always be the same in equal times.

We have likewise in circular motion an instance of force acting in a continued manner. The motion of matter abandoned to itself being uniform and rectilinear, it is evident that a body moving on the circumference of a circle tends at every instant to deviate from the centre in the direction of the tangent. The effort which it makes to this effect, is called the centrifugal force, and on the contrary every force directed to the centre is called a central or centripetal force; in a circular motion the centripetal force is equal and directly contrary to the centrifugal force; it incessantly tends to draw the body towards the centre, and in an extremely small interval of time, its effect may be measured by the versed sine of the arc described.

We may by this result compare the force of gravity with the centrifugal force arising from the rotation of the Earth; at the equator bodies describe, in consequence of this rotation, an arc of 40.″1095 of the circumference in one second of time, the radius of this equator being 19634778 feet very nearly. The versed sine of this arc is 0.0389704. The force of gravity causes bodies to descend, at the equator, 11.23585 in one second; thus the central force necessary to retain these bodies at the surface of the Earth, and consequently the centrifugal force due to its motion, is to the force of gravity at the equator as 1. to 288.3. The centrifugal force diminishes the force of gravity, and bodies descend to the Earth at the equator by the difference only between these two forces.

Calling then gravity the entire weight which would subsist without this diminution, the centrifugal force at the equator is $\frac{1}{289}$ of gravity. If the rotation of the Earth were seventeen times more rapid, the arc described at the equator in one second, would be seventeen times greater, and its versed sine would be 289 times more considerable, the centrifugal force would therefore be equal to that of gravity, and bodies at the equator would cease to gravitate towards the Earth.

In general the expression of a constant accelerating force acting always in the same direction, is equal to double the space it causes the body to describe, divided by the square of the time. Every accelerating force in an interval of time extremely short, may be considered as constant and acting in the same direction; moreover the space which is described by the action of the central force in circular motion, is the versed sine of the arc described, which is nearly equal to the square of the arc divided by the diameter, the expression of this force is therefore the square of the arc described divided by the square of the time and by the radius of the circle. But the arc divided by the time is the velocity of the body, the central and likewise the centrifugal force are each therefore equal to the square of the velocity divided by the radius.

Let us compare this result with that found above, according to which gravity is equal to the square of the velocity acquired, divided by twice the space described; we shall see that the centrifugal force is equal to the force of gravity, even when the velocity of the revolving body is the same as that acquired by a heavy body, which should fall from a height equal to half the radius of the circumference described.

The velocities of several bodies moving in circles, are to each other as the circumferences which they describe, divided by the times of their revolutions, the circumferences are as the radii, therefore the squares of the velocities are as the squares of the radii divided by the squares of the times. The centrifugal forces, therefore, are to each other as the radii of the circumferences divided by the squares of the times of revolution. Hence it follows that the centrifugal force, on the different terrestrial parallels, is proportional to the radii of these parallels. These beautiful theorems, discovered by Huygens, conducted Newton to the general theory of motion in curves, and to the law of universal gravitation.

A body describing a curve of any kind has a constant tendency to deviate in the direction of the tangent; now we may easily imagine a circle to pass through two adjacent elements of the curve, which is called the *circle of curvature* or the *osculatory circle* of that curve, in two consecutive instants the body may be conceived as moving in the circumference of this circle, its centrifugal force is therefore equal to the square of its velocity divided by the radius of this circle of curvature; but the position and magnitude of this circle is constantly varying.

If the curve be described by virtue of a force directed to a fixed point, this force may be decomposed into two, one according to the radius of curvature, the other according to the element of the curve. The first is in equilibrio with the centrifugal force, the second augments or diminishes the velocity of the body, this velocity therefore continually varies. But it always is such that *the areas described by the radius vector about the origin of the force, are proportional to the times; and reciprocally, if the areas traced by the radius vector about a fixed point are proportional to the times, the force which sollicits the body will always be directed to this point.* These fundamental propositions in the theory of the system of the world, are easily demonstrated in the following manner:

The accelerating force may be supposed to act only at the commencement of each instant during which the motion may be supposed to be uniform, the radius vector will thus describe a small triangle. If the force ceased to act in the following instant, the radius vector would trace in this new instant, a new triangle equal to the first, because the triangles having their vertex at the fixed point which is the origin of the force ; their bases situated on the same straight line will be equal, since they are described with the same velocity during two instants which are supposed equal, but at the commencement of this new instant, the accelerating force combining itself with the tangential force of the body, makes it describe the diagonal of a parallelogram whose sides represent these forces. The triangle which the radius vector describes in virtue of this combination of forces is equal to that which it would have described without the action of the accelerating force, for these triangles have for a common basis the radius vector at the end of the first instant, and the vertex of each is on a straight line parallel to this basis. The areas, therefore, described by the radius vector, in the two consecutive instants, are equal to each other, and consequently the sector described by the radius vector increases as the number of these instants, or as the times. It is evident that this is only true inasmuch as the accelerating force is directed to one fixed point, otherwise the triangles which we have considered, will not have the same base and the same height: thus the proportionality of the areas to the times, demonstrates that the accelerating force is constantly directed to the origin of the radius vector.

In this case if a very small sector be imagined to be described during an extremely short instant of time, and from the first extremity of this sector, a tangent be drawn to the curve, and that the radius vector drawn from the origin of the force to the other extremity of the sector, be prolonged till it meet this tangent, then the part of this radius intercepted between this curve and the tangent, will evidently be the space described by the central force.

In dividing the double of this space by the square of the time, we obtain an expression for this force; but the sector is proportional to the time, the central force, therefore, is, as the part of the radius vector intercepted by the curve and the tangent, divided by the squarc of the sector; strictly speaking, the central force in different points of the curve is not exactly equal to these quotients, but it approaches the nearer to them as we take the sectors very small, so that it is exactly equal to the limits of these quotients. The differential analysis gives this limit in a function of the radius vector, when the nature of the curve is known, and then that function of the distance is obtained to which the central force is proportional.

If the law of the force is given, the investigation of the curve described presents greater difficulties. But whatever be the nature of the forces by which a body is animated, the elementary variations of its motion may easily be determined in the following manner:

Let us imagine three fixed axes, perpendicular among themselves, the position of a body may be determined at any instant, by three co-ordinates parallel to these axes, in decomposing every one of the forces which act upon the point, into three others parallel to these axes, the product of the resulting force of all these forces, parallel to one of these co-ordinates, by the element of the time during which it acts, will express the increase of the velocity of the body, in the direction parallel to that co-ordinate; now this velocity, during this element, may be considered as uniform and equal to the element of the co-ordinate divided by the element of the time; the elementary variation, therefore, of this quotient, is equal to the preceding product. The consideration of the other two co-ordinates affords two similar equations; thus the determination of the motion of a body becomes an investigation of pure analysis, which is reduced to the integration of three differential equations. This integration is easy when the force is directed to a fixed point, but often the nature of forces renders it impossible; nevertheless the consideration of the differential equations leads to some interesting principles of mechanics, such as the following: The elementary variation of the square of the velocity of a body, submitted to the action of any accelerating forces whatever, is equal to double the sum of the products of each force, by the small space which the body advances in the direction of that force in a small instant of time: from which it may be concluded that the velocity acquired by a heavy body, descending along a line or curved surface, is the same as if it had fallen vertically from the same height.

Many philosophers, struck with the order which prevails in nature, and with the fecundity of the means by which phenomena are produced, have concluded that she always accomplishes her ends by ways the most simple.

In extending this idea to mechanics, they have sought what was the economy of nature in the employment of forces, and after many fruitless attempts, they at last discovered that of all the curves that a body might describe in going from one point to another, it always chose that in which the integral of the product of the mass of the body by its velocity, and by the element of the curve, is a *minimum*. Thus the velocity of a body moving in a curved surface and not solicited by any force being constant, it proceeds from one point to another by the shortest line. The preceding integral is called the *action of a body*, and the re-union of similar integrals, relative to the bodies of a system is called the *action of the system*. According to these philosophers the economy of nature consists in sparing this action, so that it may always be the least possible. It is this which constitutes *the principle of the least action*. But this principle is nothing but a curious result of the primordial laws of motion, laws, which as we have seen, are the most simple and natural that can be imagined, and which seem to be derived from the essence of matter itself. All the laws, mathematically possible, offer analogous results, it ought not therefore to be elevated to the dignity of a final cause, for so far from having given birth to the laws of motion, it has not even contributed to their discovery, without which we should still dispute upon what was to be understood by the least action of nature.

CHAP. III.

Of the Equilibrium of a System of Bodies.

The most simple case of equilibrium between several bodies, is that of two material points meeting each other with equal and opposite velocities. Their velocities will be destroyed, and the points reduced to a state of rest, by their mutual impenetrability, a property of matter by which two bodies cannot at the same moment occupy the same point of space. But if two bodies, of different masses, meet with opposite velocities, what then is the relation of their velocities to their masses, in case of equilibrium? To solve this problem, let us suppose a system of material points, contiguous to each other, arranged upon the same straight line, and impelled by a common velocity in the direction of this straight line; let us suppose also a second system of contiguous material points arranged upon the same straight line, having a common velocity, but in a direction contrary to the first system, so that the two systems, after mutually striking each other, may remain in equilibrio. It is clear that if the first system was only composed of a single material point, every point in the second system would destroy in the striking point a part of its velocity equal to the velocity of this system; in the case of equilibrium, therefore, the velocity of the striking point should be equal to the product of the velocity of the second system by the number of its point, and we may substitute for the first system a single point impelled by a velocity equal to this product. For the second system likewise may be substituted a material point impelled by a velocity equal to the product of the velocity of the first system by the number of its points. Therefore instead of the two systems we shall have two points that will sustain each other in equilibrio with contrary velocities, of which one will be the product of the velocity of the first system, by the number of its points, and the other will be the product of the velocity of the points of the second system, by their number; these products, therefore, in the case of equilibrium, should be equal to each other.

The mass of a body is the sum of its material points. The product of the mass by the velocity is called the *quantity of motion*, the same thing is understood of the *force of a body*. In the case of equilibrium between two bodies, or of two systems of material points that strike each other in contrary directions, the quantities of motion, or the forces opposed to each other should be equal, and consequently these velocities should be proportional to the masses.

Two material points cannot act on each other but in the direction of the straight line that joins them, the action which the first exerts upon the second communicates to it a quantity of motion; now the second may be conceived to be sollicited by this quantity, and by another equal and directly opposite; the action of the first body is therefore entirely employed in destroying this last quantity of motion; but to do this it must employ a quantity of motion equal and contrary to that which is to be destroyed. Hence it appears that in the mutual action of bodies, action and reaction are always equal and in contrary directions. We see likewise that this equality does not suppose any particular force to reside in matter, but results from the impossibility that a body should acquire motion from the action of another without depriving it of a portion, in the same manner as a vase can only be filled at the expence of another which communicates with it.

The equality of action and reaction manifests itself in all the operations of nature. Iron attracts the loadstone in the same manner as it is itself attracted, the same is observed in electric attractions and repulsions, in the developement of elastic forces, and even in the forces of animals, for whatever may be the nature of the motive power of man, and of animals, it is clear they receive by the reaction of matter a force equal and contrary to that which they communicate; and thus in this respect they are subject to the same laws as inanimate beings.

The reciprocity of velocity and mass in the case of equilibrium, serves to determine the ratio of the masses in different bodies. Those of homogeneous bodies are proportional to their volumes, which geometry teaches us to measure; but all bodies are not homogeneous, and the differences which exist either in their integrant particles, or in the number and magnitude of the intervals or pores which separate these particles, produce very great ones in the masses which are contained under the same volume. Geometry then becomes insufficient to determine the ratio of these masses, and we are obliged to have recourse to the science of mechanics.

If we conceive two globes of different

matter. If two globes be taken of different materials, whose diameters be supposed to vary till they become in equilibrio with each other when impelled by equal and contrary velocities, we may be assured that then, they each contain an equal number of material points, and consequently that their masses are equal. The ratio which the volumes of these substances bears to their equality of mass will thus be obtained, and afterwards we may determine by geometry the ratio of the masses of any two volumes whatever of the same substance. But this method would be excessively laborious, for the numberless comparisons which the wants of society require for the purposes of commerce; fortunately nature offers us in the weight of bodies a very simple method of comparing their masses.

It has been shewn in the preceding chapter that every material point on the same part of the Earth's surface, tends to descend with the same velocity by the action of gravity. The sum of these tendencies is what constitutes the weight of a body, and thus the weights are proportional to the masses. Hence it follows that if two bodies suspended by a thread passing over a pulley, are in equilibrio when an equal portion of the thread is on each side the pulley, the masses of these bodies are equal, since tending to move with the same velocity by the action of gravity, their action upon each other is the same as if they struck each other with equal and contrary velocities. Two bodies may also be placed in equilibrio by means of a balance, whose arms are perfectly equal, which will be a proof of the equality of their masses. The ratio between the masses of different bodies is likewise ascertained by means of an exact balance, a number of small equal weights, by observing how many of these weights are necessary to produce an equilibrium.

The density of a body depends on the number of its material points included in a given volume, it is therefore proportional to the ratios of the mass to the volume. If a body existed without pores its density would be the greatest possible, and by comparing its density with that of other bodies we should be able to determine the quantity of matter they contained. But being ignorant of such a substance, we can only ascertain the relative density of bodies; these densities are in proportion to the weight under the same volume, since the weights are proportional to the masses; taking therefore as unity the density of any substance whatever, for instance distilled water at the temperature of melting ice, the density of another body, will be the ratio of its weight to that of an equal volume of water; and this is called its *specific gravity*.

What we have said, seems to suppose that matter is homogeneous, and that bodies only differ from each other in their figure, and by the magnitude of their pores and integrant particles, but it is possible that there may be essential differences in the very nature of these particles; but this is indifferent to the science of mechanics, which only considers bodies with relation to their motions. We may therefore, without fear of error, admit the homogeneous nature of matter, provided that by equal masses, we understand masses, which being sollicited by equal and contrary velocities, produce an equilibrium.

In the theory of equilibrium and of the motion of bodies, we omit every consideration of the figure and of the number of the pores which they contain. But attention is paid to the difference of their respective densities, by supposing them formed of material points more or less dense, which in fluids are perfectly free, and which in hard bodies are united by inflexible straight lines without mass, and which in soft and elastic bodies are both flexible and extensive. It is evident that in these suppositions, bodies will offer the same appearances which they really present to us.

The conditions of equilibrium of a system of bodies may always be determined by the laws of the composition of forces, explained in the First Chapter of this Book; for we may conceive the force by which every material point in the system is animated, to be applied to that point in its direction, in which are united the directions of those forces which destroy it, or which by combining with it, form a resulting force, which in the case of equilibrium is destroyed by the fixed points of the system. Let us consider, for example, two material points attached to the extremity of an inflexible lever, and suppose the forces which solicit them to be in the plane of the lever. In conceiving these forces united at the point of concourse of their directions, the resulting force arising from their combination, should, to produce an equilibrium, pass through the fulcrum, which can alone destroy it, and it appears that from the law of the composition of forces, that the producing forces are in this case reciprocally to each other as the perpendiculars drawn from the point of support or fulcrum to the direction of the forcer.

If we imagine two ponderous bodies attached to the extremities of a rectilinear and inflexible lever; whose mass is infinitely small, relatively to that of the bodies, the directions parallel to the force of gravity may be conceived to meet at an infinite distance, in this case the forces which solicit the bodies, or what is the same thing, their weights, to produce an equilibrium, must be reciprocally as the perpendiculars drawn from the point of support to the directions of the forces, but these perpendiculars are in proportion to the arms of the lever. Thus the weights of bodies in equilibrio are reciprocally as the arms of the lever to which they are attached.

A very small weight may therefore be made to sustain a very large one in equilibrio, by means of a lever and other machines which are referrable to the same principle; and by this method we can lift an enormous weight by a very slight effort, but for this purpose, the power must describe a great space to elevate the weight a very small height, so that what is gained in force is lost in time, and this takes place in all machines. But time may often be disposed of at pleasure, when we can only employ a very limited force. In other cases where a great velocity is to be produced, it may be effected by means of a lever, by applying the force to the shortest arm; it is in this possibility of augmenting at pleasure either the mass or velocity of the moving power, that the principal advantage of machinery consists.

In examining attentively in a great number of cases the conditions of equilibrium of a system of bodies, and the relation of each force to the velocity of the body to which it is applied, when the equilibrium of the system begins to give way, the following principle has been recognized, which contains in the most general manner the conditions of equilibrium of a system of material points, solicited by any force whatever.

If the position of a system be changed

by an infinitely small quantity (in a manner consistent with the conditions of the connection of its different parts), every material point will advance in the direction of the force by which it is solicited, a quantity, equal to the part of this direction comprised between the first position of the point and the perpendicular drawn from the second position of the point upon this direction: that is, in the case of equilibrium, the sum of the products of every force, by the quantity which the point to which it is applied advances in its direction, is equal to nothing. In this consists the principle of virtual velocities, for which we are indebted to John Bernoilli; but in practice, it should be observed that the products must be taken negatively of those points which move in a direction contrary to their forces. It must be recollected also, that the force is the product of the mass of a material point by the velocity which it would receive if entirely free.

If we conceive the position of every

point of the system to be determined by three rectangular co-ordinates; the sum of the products of every force, by the quantity which the point which it solicits advances in its direction, when the position of the system is deranged by an infinitely small quantity, will be expressed by a linear function of the variation of all the co-ordinates of these points. These variations will have with each other certain relations, which will depend on the manner in which the different parts of the system are connected together. In reducing, therefore, by means of the conditions of this connection, the arbitrary variations to the least possible number, in the preceding sum, which should be zero in the case of equilibrium, we must, for the equilibrium to take place in every direction, make the coefficients of the remaining variations separately equal to zero, which will give as many variations as there are arbitrary variations. These equations combined with those which arise from the connection of the parts of the system will contain all the conditions of its equilibrium. Let us consider, for example, a system of ponderable points connected together in an invariable manner, and suppose it attached to a fixed point, round which it can turn freely in all directions; let us imagine three axes passing through this fixed point, and perpendicular to each other, and the position of every point of the system determined by three coordinates parallel to these axes, and the direction of its gravity decomposed according to the direction of these co-ordinates.

If the system be now moved an infinitely small quantity round the first axis, it is evident that the quantity which every point advances in the direction of the force parallel to this axis is equal to nothing. The quantity that it advances in the direction of the force parallel to the second axis, is equal to the product of the angular motion of rotation of the system, by the co-ordinate parallel to the third axis; and the quantity which it advances in the direction of the force parallel to the third axis, is equal (but with a contrary sign) to the angular motion of rotation of the system by the co-ordinate parallel to the second axis: thus, according to the preceding principle, the system should be in equilibrio about its first axis, when the sum of the products of the mass of every point by its force parallel to the second axis, and by the co-ordinate parallel to the third axis, is equal to the sum of the products of the mass of every point, by the force parallel to the third axis, and by the co-ordinate parallel to the second.

That the equilibrium may subsist [in every position of the system round the principal axis, it is necessary that the preceding equality should subsist, whatever may be the forces parallel to the second and third axes, since it must be independant of the direction of gravity relatively to these axes; each of these preceding sums should therefore be made equal to zero, and as the partial actions of gravity decomposed parallel to any axis whatever, are the same for every point of the system, the sum of the products of each of these points, by its co-ordinate parallel to the second or third axis, is zero. If the same equality subsists relatively to the co-ordinates parallel to the first axis, it is easy to see that it will subsist equally, with respect to any other axis passing through the fixed point; hence it follows that whatever be the situation of the system round this point, it cannot turn round any axis, but will remain in equilibrio, in consequence of the action of gravity. The point which possesses this remarkable property is called the *centre of gravity* of the system, it is such that whatever be the position of the system, the resulting force of all the efforts of gravity pass constantly through it.

To determine it, we refer its position and that of the points of the system to three fixed axes perpendicular to each other, and one of the co-ordinates of this centre, multiplied by the entire mass of the system, is equal to the sum of the products of the mass of each point by its corresponding co-ordinate. Thus the determination of this centre, of which gravity first suggested the idea, is independant of it. The consideration of this centre, extended to a system of bodies, whether ponderable or not, is of great use in the science of mechanics.

CHAP. IV.

Of the Equilibrium of Fluids.

It has been observed in the First Book, that elastic fluids, such as air, owe their origin to heat, and incompressible fluids, such as water, to the pressure of the atmosphere. But to determine the laws of their equilibrium, we have only to consider them as formed of an infinite number of particles perfectly movable among each other, and yielding to the smallest pression which is exerted on one side more than on the other.

It follows from this characteristic property of fluids, that the force that solicits every particle of the free surface of a fluid in equilibrio is perpendicular to this surface; gravity is consequently perpendicular to the surface of stagnant waters, which is for this reason horizontal.

In virtue of the mobility of its parts a ponderable fluid may exercise a pressure much greater than its weight. A small column of water, for example, terminated by a large horizontal surface presses as much the base on which it reposes as a cylinder of water of the same base and height. To understand the truth of this paradox we may imagine a fixed cylindric vase, whose bottom or base is moveable, suppose this vase filled with water, and its base kept in equilibrio by a force equal and contrary to the pression it experiences, it is clear that the equilibrium would still subsist, even if a part of the fluid was to consolidate itself and to unite with the sides of the vessel; for, in general, the equilibrium of a system of bodies, is not deranged, by the supposition of several of them uniting, or becoming attached to fixed points; we may thus form an infinity of vases of different figures, having all the same base and the same height as the cylindric vase, and in which the water will exercise the same pressure on the movable base.

The pression which a fluid exerts against any surface is perpendicular to each of its elements, otherwise the fluid particle contiguous to it would slide, by the decomposition of the pression which it experienced. If a fluid acts only by its weight, its entire pression is equal to the weight of a prism of the fluid, whose base is equal to the surface pressed, and whose height is the distance of the centre of gravity of this surface from the level surface of the fluid.

A body plunged in a fluid, loses a part of its weight, equal in weight to a volume of the displaced fluid ;—for before the immersion, the surrounding fluid formed an equilibrium to the weight of this volume of fluid, which without disturbing the equilibrium may be supposed to have formed itself into a solid mass, the resulting force of all the actions of the fluid upon this mass, should be in equilibrio with its weight, and pass through its centre of gravity; now it is clear that these actions are the same upon a body which occupies its place, the action of the fluid destroys therefore a part of the weight of the body equal to the weight of the volume of the displaced fluid.

Thus bodies weigh less in air than in vacuo; this difference, though but small, is not to be neglected in delicate experiments.

By means of a balance which carries at the extremity of one of its arms a body which can be plunged in a fluid, we can measure very exactly the diminution of weight which the body experiences in this immersion, and determine its specific gravity, or its relative density compared with that of the fluid. This is the ratio of the weight of a body in vacuo to the diminution of its weight when it is entirely immersed in the fluid. It is thus that the specific gravities of bodies are determined, compared with distilled water.

For a body lighter than a fluid to be in equilibrio at its surface, its weight must be equal to the volume of the displaced fluid ; it is moreover indispensable that the centres of gravity of this portion of the fluid and of the body, be in the same vertical, for the resulting force of the actions of gravity upon all the particles of the body pass through its centre of gravity, and the resulting force of all the actions of the fluid upon the body, pass through the centre of gravity of the displaced fluid ; these resulting forces, therefore, must be on the same line, since they destroy each other, the centres of gravity are therefore in the same vertical.

There exist two states of equilibrium entirely distinct. In one, if the equilibrium is ever so little deranged, the bodies which compose the system only oscillate about their primitive position, and the equilibrium is said to be firm or stable.

This stability is absolute if it takes place, whatever be the nature of the oscillations ; it is relative if it only takes place in oscillations of a certain kind. In the other state of equilibrium, if the system be deranged, all the bodies deviate more and more from their primitive state.

We may form a just idea of these two states by considering an ellipse placed vertically on an horizontal plane. If the ellipse is in equilibrio on its smaller axis, it is clear that when deranged a little from this situation, it tends to return, making oscillations which the friction and resistance of the air soon destroy; but if the ellipse be in equilibrio upon its greatest axis, upon being once deranged it will deviate more and more, and finish by reversing itself upon its lesser axis. The stability of an equilibrium, therefore, depends on the nature of the small oscillations which the system, upon being deranged, makes about its primitive position. Very often this investigation is attended with great difficulty; but in many cases, and particularly in that of floating bodies, it is sufficient to judge of the stability of an equilibrium to determine if the force which solicits the system when deranged from this state, tends to restore it to the same state again. In bodies floating on water, or any other fluid, it may be determined by the following rule:

If through the centre of gravity of the section of the surface of the water, of a floating body, we conceive a horizontal axis, such, that the sum of the products of every element of the section, multiplied by the square of its distance from this axis, be less than relatively to any other horizontal axis drawn through the same centre; the equilibrium will be stable in every direction, when this sum surpasses the product of the volume of the displaced fluid by the height of the centre of gravity of the body, above the centre of gravity of this volume.

This rule is principally useful in the construction of vessels which require sufficient stability to enable them to resist the efforts of storms which tend to submerge them. In a ship the axis drawn from the stern to the prow is that relatively to which the sum abovementioned is a minimum; it is easy, therefore, to ascertain and measure its stability by the preceding rule.

Two fluids contained in a vase, dispose themselves in such a manner that the heaviest occupies the lowest part of the vessel, and the surface which separates them is horizontal.

If two fluids communicate with each other by means of a bent tube the surface which separates them in a state of equilibrium is horizontal, and their heights above this surface are reciprocally as their specific densities. In supposing to the whole atmosphere the density of the air at the temperature of melting ice, and compressed by a column of mercury of two feet and a half, its height will be equal to 23690 feet; but because the density of the atmospheric strata diminish as they are elevated above the surface of the earth, the height of the atmosphere is much greater.

To trace the general laws of the equilibrium of a fluid mass solicited by any forces whatever, we shall observe that every point in the interior of this mass suffers a pression which in the atmosphere is measured by the height of the barometer, and which may be done in a similar manner by any other fluid. In considering every particle as an infinitely small parallelopiped the pression of the surrounding fluid will be perpendicular to that of the parallelopiped, which will tend to move itself perpendicularly to every one of its faces, by virtue of the difference of the pressions on two opposite faces. From these different pressions arise three forces perpendicular to each other, which must be combined with the other forces which solicit the fluid particle. And since this particle should be in equilibrio, in consequence of all these forces, the principle of virtual velocities will give the general equations of its equilibrium, whatever may be the position of the entire mass. The conditions of the integrability of these differential equations will determine the relations that should exist between the forces by which the fluid is solicited, to render the equilibrium possible: their integration will give the pression which every fluid particle is subject to, and this pression will determine its elasticity and density, if the fluid is elastic and compressible.

CHAP. V.

Of the Motion of a System of Bodies.

LET us consider first the action of two material points of different masses, and which, moving on the same straight line, meet and strike each other. We may conceive immediately before the shock their velocities to be decomposed in such a manner that they may have a common velocity, and two contrary velocities, such as that if they have these alone they would have remained in equilibrio. The velocity common to each of the points, is not altered by their mutual action, and therefore will subsist alone after the shock. To determine it we shall observe, that the quantity of motion of the two points in virtue of this common velocity *plus* the quantity of motion due to the velocities destroyed, represents the sum of the quantities of motion before the shock, provided we take the quantities of motion due to contrary velocities with contrary signs; but by the condition of equilibrium, the sum of the quantities of the motion due to the velocities destroyed equals nothing; therefore the quantity of motion due to the common velocity, is equal to that which primitively existed between the two points, and consequently this velocity is equal to the sum of the quantities of motion divided by the sum of the masses.

To find the velocity after the shock when the points are perfectly elastic, we must add or subtract from the common velocity which they would have if they were unelastic, the velocity which they will acquire or lose according to this hypothesis: perfect elasticity doubles these effects by completely restoring the springs which the shock had compressed, the velocity of each point after the shock may therefore be obtained by subtracting its velocity before the shock from double the common velocity; from which it is easy to infer that the sum of the products of each mass, by the square of the velocity is the same before as after the shock of the two points; and this is true universally in the shock of any number of bodies which are perfectly elastic, in whatever manner they act upon each other.

The shock of two material points is merely ideal; but it is easy to reduce to the same case the shock of any two bodies whatever by observing that if these bodies strike each other according to the straight line connecting their centres of gravity, and perpendicular to their surface of contact, they will act one upon the other as if their masses were united in these centres; therefore motion is communuicated to them in the same manner as between two material points whose masses are respectively these bodies.

Such are the laws of communication of motion, laws which experience confirms, and which are derived mathematically from the two fundamental laws of motion explained in Chap. II. of this Book.

Several philosophers have endeavoured to determine them from the consideration of final causes. Descartes, persuaded that the quantity of motion in the universe should always preserve itself the same, has deduced from this false hypothesis, false laws of the communication of motion, offering an example of the errors to which we are exposed, when we attempt to guess at the laws of nature, by attributing to her particular views.

When a body receives an impulsion in the direction passing through the centre of gravity, all its parts move with an equal velocity. If the direction passes on one side of this point, the different parts of the body acquire unequal velocities, and from this inequality of velocity, there results a motion of rotation of the body about its centre of gravity, at the same time that this centre is carried on with the velocity that it would have taken if the direction of impulsion had passed through this point. This case is that of the Earth and planets. To explain the double motion of rotation and translation of the Earth, it is sufficient to suppose that it received a primitive impulsion in a direction passing at a small distance from its centre of gravity; and this distance in the hypothesis of the homogenity of this planet, is nearly the $\frac{1}{160}$th part of its radius. It is infinitely improbable that the primitive impulsion of the planets should have been in the directions of their centres of gravity, all the celestial bodies therefore should turn upon themselves. For the same reason the Sun, which turns on itself, must have received an impulsion which, not passing through its centre of gravity, carries it along through space with the planetary system, except an impulsion in a contrary direction should have totally destroyed this motion, which is not at all probable.

The impulsion given to a homogeneous sphere, which does not pass through its centre, causes it constantly to turn round a diameter which is perpendicular to a plane passing through its centre, and the direction of the force impressed. New forces which solicit all its points, and whose resulting force passes through the centre, do not alter the parallelism of its axis of rotation.

It is thus that the axis of the Earth remains always very nearly parallel to itself in its revolution round the Sun, without any occasion to suppose with Copernicus an annual motion of the poles of the Earth, round those of the ecliptic. If the body be of any figure whatever, its axis of rotation may vary at every instant. The investigation of these variations is the most interesting problem relating to hard bodies, by its connection with the precession of the equinoxes, and the libration of the Moon. Its solution has led to this curious and useful result, namely, that in every body there exist three axes, perpendicular to each other, round which it may revolve uniformly when not solicited by external forces ; these axes have, for this reason, been called the principal axes of rotation.

A body, or a system of ponderable bodies, of any figure, oscillating about a fixed and horizontal axis, forms a compound pendulum. There exists no other in nature, and the simple pendulum of which we have spoken above, is an ideal geometrical conception for the purpose of simplifying the subject. To this it is easy to refer compound pendulums, in which all the points are firmly united together.

If we multiply the length of a simple pendulum, whose oscillations are of the same length as a compound pendulum, by the entire mass of this latter pendulum, and by the distance of its centre of gravity from its axis of oscillation, the product will be equal to the sum of the products of every particle of the compound pendulum by the square of its distance from the axis of oscillation. It is by this rule discovered by Huygens, that from experiments on compound pendulums the length of the simple pendulum has been determined which vibrates seconds.

Let us imagine a pendulum making very small oscillations, and at the moment when it deviates the most from the vertical, a small force to be impressed on it in a direction perpendicular to the plane of its motion; it will describe an ellipse round the vertical. To represent this motion we may suppose a fictitious pendulum which continues to vibrate as this would have done if it had not received this new impulsion, and at the same time that this latter pendulum vibrates on each side the imaginary one as if this pendulum was immovable and vertical, and thus the motion of the real pendulum is the result of two simple oscillations which exist together, and which it is easy to determine.

This method of considering the small oscillations of bodies, may be extended to any system whatever. If the system be supposed deranged from its state of equilibrium by very small impulsions, and afterwards to receive new impulsions, it will then oscillate, with respect to the successive situations which it would have taken by virtue of its first impulsions, in the same manner as it would oscillate relatively to its state of equilibrium, if these new impulsions had been the only ones impressed on this state. Thus the very small oscillations of a system of bodies, however complicated, may be considered as performed by simple oscillations, perfectly similar to those of the pendulum. For if we conceive the system to be a little deranged from its state of equilibrium, in such a manner that the force soliciting each body of the system may be directed towards the point which it occupied in this state, and at the same time proportional to its distance from this point; it is evident that this will be the case during the oscillation of the system, and that the velocity of each body will be proportional to its distance from a state of equilibrium. They will arrive, therefore, at the same instant at this position, and will oscillate in the same manner as the simple pendulum. But the state of derangement which we have supposed to the system is not *unique*. If we cause one of the bodies to deviate from its state of equilibrium, and then investigate the position of the remaining bodies, which may satisfy the preceding conditions, an equation is obtained of a degree equal to the number of bodies in the system, which will give as many simple oscillations as there are bodies. Let us conceive the first of these oscillations to exist in the system, and that at any instant, all the bodies of the system deviate from their position, proportionally to the relative quantities of the second simple oscillation. In consequence of the co-existence of these oscillations, the system will oscillate, relatively to the successive positions which it would have taken in consequence of the first simple oscillation, in the same manner as it would have oscillated about its state of equilibrium in consequence of the second oscillation. Its motion will therefore be composed of the two first simple oscillations. The third oscillation may be combined with these in the same manner, and by continuing thus to combine the rest, we may represent all the motions of the system in the most general manner.

And hence results an easy method of determining the absolute stability of its equilibrium. If in all the positions relative to each simple oscillation, the forces which solicit the bodies tend to bring them back to their state of equilibrium, it is then stable, it will be otherwise or will only have a relative stability if in some one of its positions, the forces tend to make the bodies deviate from it.

In the waves we have a sensible example of the co-existence of small oscillations. When a point in the surface of stagnant water is slightly deranged, we see circular waves form and extend around it; by agitating the surface in another point, new waves are formed which mingle with the first and they dispose themselves over the surface, already agitated by the first waves, as they would have done if this surface had been entirely tranquil; so that each remains distinctly perceptible when thus united. That which the eye distinguishes in the waves of water, the ear also perceives in sounds or undulations of the air, which are propagated simultaneously without alteration, producing very distinct impressions.

The principle of the co-existence of simple oscillations which we owe to Daniel Bernouilli, is one of those general results which is interesting from the facility which it gives to the imagination of representing phenomena and their successive changes. It may easily be deduced from the analytic theory of the small oscillations of a system. They depend on differential linear equations whose complete integrals are the sum of the particular integrals. Thus the simple oscillations dispose themselves one upon the other to form the motion of the system. It is interesting to follow in this manner the intellectual truths of analysis, in the phenomena of nature. The system of the world affords numberless instances of a similar correspondence, and it is this which constitutes the greatest charm of mathematical speculations.

It seems natural to reduce the laws of the motion of bodies to some general principle, in the same manner as the laws of their equilibrium have been reduced to the principle of virtual velocities. To effect this, let us consider the motion of a system of bodies acting one upon another without being solicited by accelerating forces. Their velocities change at every instant, but we may conceive each velocity at any instant as composed of that which takes place in the following instant, and of another velocity which should be destroyed at the beginning of this second instant. If the velocity destroyed be known, it will be easy to calculate the velocity in the second instant by the law of the decomposition of forces: now, it is clear that if these bodies had only been animated by the velocities destroyed, they would mutually have maintained themselves in a state of equilibrium. Thus the laws of equilibrium will give the ratios of the velocities destroyed, and it will be easy to conclude the velocities which remain and their directions. The infinitesmal calculus will then give the successive variations in the motion of the system, and its position at every instant. It is evident that if the bodies are animated by accelerating forces, the same decomposition of velocities may be employed, but then the equilibrium must subsist between the destroyed velocities and these forces.

This method of reducing the laws of motion to those of equilibrium, and for which we are principally indebted to D'Alembert, is very general and luminous. And we should be surprised that it had escaped the notice of former geometricians, who had occupied themselves on the principles of dynamics, if we did not know how often it happens that the most simple ideas offer themselves the last to the human mind.

It remained only to combine the principle just explained with that of virtual velocities, to give to the science of mechanics all the perfection of which it is susceptible.

This is what Lagrange has accomplished, and by this means has reduced the investigation of the motion of any system of bodies to the integration of differential equations; the object of mechanics is thus accomplished, and it belongs then to pure analysis to finish the solution of the problem. The most simple manner of forming these equations is this:

If we imagine three fixed axes perpendicular to one another, and if at any instant the velocity of every material point of a system of bodies be decomposed into three others parallel to these three, every partial velocity may be considered as uniform during this instant, and *we may conceive the point to be solicited at the end of this instant*, in a direction parallel to one of these axes, by three velocities, namely, by the velocity in this instant, by the little variation it receives in the following instant, and by this same variation applied in a contrary direction. The two first of these velocities subsist in the following instant, the third should be destroyed by the forces which sollicit the point, and by the action of the other points of the system: thus if we suppose the instantaneous variations of the partial velocities of every point of the system to be applied to this point in a contrary direction, the system should remain in equilibrio in consequence of all these variations of the forces which solicit it. By the principle of virtual velocities the equations of this equilibrium may be obtained, which, combined with those arising from the connection of the parts of the system, give the differential equations of the motion of each of its points. In like manner the laws of the motion of fluids may be reduced to those of their equilibrium. In this case the conditions relative to the connection of the parts of the system are reduced to this; that if the fluid be incompressible, the volume of any particle of it must remain constantly the same, and that it will depend on the pression according to some given law if the fluid be compressible and elastic. The equations which express these conditions, and the variations in the motion of the fluid, contain the partial differences of the co-ordinates of the particle, taken either with relation to the times or to the primitive co-ordinates. The integration of this species of equations is attended with great difficulties, and can only be effected in certain particular cases, relating to the motion of ponderable fluids in vases, to the theory of sounds, and to the oscillations of the atmosphere and ocean.

The consideration of the differential equations of the motion of bodies, has led to the discovery of several very general and useful principles in mechanics, and which are an extension of those which have been developed already relatively to the motion or point in the second chapter of the second Book. A material point moves uniformly in a straight line, if not induced to quit it by the action of some foreign cause. In a system of bodies which act on each other, without being subject to the action of foreign causes, the common centre of gravity moves uniformly in a straight line, and its motion is the same as if all the bodies were supposed united in this point, and the forces which solicit them immediately applied to it, so that the quantity and direction of the resulting force remains always the same.

We have seen that the radius vector of a body, solicited by a force directed towards a fixed point, describes areas proportional to the times. If we suppose a system of bodies acting one upon the other in any manner whatever, and solicited by a force directed towards a fixed point, if from this point a radius vector be drawn to each of them, and these radii projected on an invariable plane passing through the point, the sum of the products of the mass of each body, by the area traced by the projection of its radius vector, is proportional to the time. In this consists the principle of the *conservation of areas.*

The *vis viva* of a system of bodies is the sum of the products of the mass of each body by the square of its velocity. When a body moves on a line or surface without being subject to a foreign action, this *vis viva* is constantly the same, since the velocity is constant. If the bodies of a system experience no other actions than those arising from these mutual tractions and pressions, either directly or by the intermediate agency of rods or threads which are inextensible and unelastic, the *vis viva* of the system remains constant, even in the case where several of the bodies are constrained to move in curved lines or surfaces. This is the principle of the conservation of the *vis viva.*

In the motion of a point solicited by forces of any kind, the variation of the *vis viva* is equal to twice the sum of the products of the mass of a point by each of the accelerating forces, multiplied respectively by the elementary quantities which the point advances towards their origins. In the motion of any system, the double of the sum of all these products is the variation of the *vis viva* of the system.

When the *vis viva* attains its maximum or minimum, this variation is nothing, the system then, according to the principle of virtual velocity, will be in equilibrio, in consequence of the accelerating forces which solicit it; and thus in all the situations which a system takes successively, that where the *vis viva* is the greatest or least, is that in which it will remain in equilibrio. There is, besides, this remarkable circumstance, that if in this situation the *vis viva* is constantly a maximum, whatever be the velocities of the bodies when they arrive at it, the equilibrium will be stable; but it will not be so if the *vis viva* be constantly a minimum.

This is an evident consequence of what has been stated above, relative to the simple oscillation of a system of bodies.

Finally, we have seen in the second Chapter, that the sum of the integrals of the products of the mass of each body of the system, by its velocity, and by the element of the curve which it describes, is a minimum; this constitutes the principle of the least action, which principle differs from that of the uniform motion of the centre of gravity, of the conservation of areas, and of the *vis viva*, in this, that these principles are the real integrals of the differential equations of the motion of bodies, whereas that of the least action is only a singular combination of these equations.

Another important remark remains to be made upon the extent of these different principles—that of the uniform motion of the centre of gravity of a system of bodies, and of the conservation of areas, subsist even in the case, when by the mutual actions of the bodies there happens a sudden change in their motion, which renders these principles extremely useful in many circumstances; but the principle of the conservation of the *vis viva*, and that of the least action, require that the variations in the motion of the system take place by imperceptible degrees.

When the system suffers a sudden change, either by the mutual action of the bodies, or by meeting with obstacles, the *vis viva* receives at each of these changes a diminution equal to the sum of the products of each mass, multiplied by the sum of the squares of the variations which this change effects in its velocity, decomposed parallel to any three axes perpendicular with each other.

All these principles would subsist, regard being had to the relative motion of the bodies of a system, if it was carried on by a general motion common to the foci of the forces which we have supposed fixed. They will likewise take place in the relative motion of bodies upon the surface of the earth, for it is impossible to judge of the relative motion of a system of bodies by the appearance only of its relative motion.

END OF VOL. I.

W. Flint, Printer, Old Bailey.

THE

SYSTEM OF THE WORLD.

BY

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TRANSLATED FROM THE FRENCH

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IN TWO VOLUMES.

VOL. II.

LONDON:

PRINTED FOR RICHARD PHILLIPS, BRIDGE STREET, BLACKFRIARS.

1809.

W. Flint, Printer, Old Bailey.

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BOOK IV.

OF THE THEORY OF UNIVERSAL GRAVITATION.

Opinionum commenta delet dies, naturæ judicia confirmat.
CIC. DE NAT. DEOR.

HAVING, in the preceding Books, explained the laws of the celestial motions, and those of the action of forces producing motion, we have now to compare them together, to learn what forces animate the solar system, to arrive without the assistance of any hypothesis, but by strict geometrical reasoning, at the principle of universal gravitation from which they are derived. It is in the celestial regions that the laws of mechanics are observed with the greatest precision; on the earth so many causes tend to complicate their result, that it is very difficult to unravel them, and still more difficult to submit them to calculation. But the bodies of the solar system, separated by immense distances and subject to the action of a principal force, whose effect is easily calculated, are not disturbed in their respective motions by forces sufficiently considerable to prevent us from including under general formulæ, all the changes which a succession of ages has produced or may hereafter produce in the system. There is no question here of vague causes, which cannot be submitted to analysis, and which the imagination modifies at pleasure to accommodate them to the phenomena. The law of universal gravitation has this inestimable advantage, that it may be reduced to calculation, and by a comparison of its results with observation, it presents the most certain method of verifying its existence. We shall see that this great law of nature represents all the celestial phenomena even in their minutest details, that there is not one single inequality of their motions which is not derived from it, with the most admirable precision, and that it explains the cause of several singular motions, just perceived by astronomers, and which were either too complicated or too slow for them to recognize their law. Thus, so far from having to fear that new observations will disprove this theory, we may be assured before-hand, that they will only confirm it more and more; and we may be assured that its consequences are equally certain as if they actually had been observed. The most profound geometry was indispensable to establish these theories: I have collected them in my Treatise of Celestial Mechanics. I shall confine myself here to present the principal results of this work, indicating the steps that led to them, and explaining the reasons as far as can be done without the assistance of analysis.

CHAP. I.

Of the Principle of Universal Gravitation.

Of all the phenomena of the solar system, the elliptic motion of the planets and of the comets seems the most proper to conduct us to the general law of the forces by which they are animated. Observation has shewn that the areas described by the radii vectores of the planets and comets about the Sun are proportional to the times. Now we have seen in Chap. II. of the preceding Book, that for this to take place, the force which deflects the path of these bodies from a right line must constantly be directed towards the origin of their radii vectores. The tendency of the planets and comets to the Sun is therefore a necessary consequence of the proportionality of these areas to the times in which they are described.

To determine the law of this tendency, let us suppose the planets moved in circular orbits, which supposition does not greatly differ from the truth. The squares of their real velocities will then be proportional to the squares of the radii of these orbits divided by the squares of the times of their revolutions. But by the law of Kepler the squares of these times are to each other as the cubes of their radii. The squares of the velocities are therefore reciprocally as these radii. It has been shewn above that the central forces of several bodies moving in circular orbits, are as the squares of the velocities, divided by the radii of the circumferences described; the tendency therefore of the planets to the Sun is, reciprocally, as the squares of the radii of their orbits supposed circular. This hypothesis, it is true, is not rigorously exact, but the constant relation of the squares of the times to the cubes of the greater axes of their orbits, being independant of their excentricities, it is natural to think it would subsist also in the case of the orbits being circular. Thus, the law of gravity towards the Sun varying reciprocally as the square of the distance is clearly indicated by this relation, analogy leads us to suppose that this law, which extends from one planet to another, subsists equally for the same planet at its different distances from the Sun, and its elliptic motion confirms this beyond a doubt. To comprehend this, let us follow this motion from the departure of the planet from its perihelion : its velocity is then at its maximum, and its tendency to recede from the Sun, surpassing its gravity towards it, its radius vector augments and forms an obtuse angle with the direction of its motion. The force of gravity towards the Sun decomposed according to this direction, continually diminishes the velocity, till it arrives at the aphelion ; at this point the radius vector becoming perpendicular to the curve its velocity is a minimum, and its tendency to recede from the Sun being less than its gravity towards it, the planet will approach it describing the second part of its ellipse. In this part the gravity towards the Sun increases its velocity in the same manner as it before decreased it, and the planet will arrive at its perihelion with its primitive velocity, and recommences a new revolution similar to the first. Now the curvature of the ellipse at the aphelion and perihelion being the same, the radii of curvature are the same, and consequently the centrifugal forces of these two points are as the squares of the velocities. The sectors described in the same time being equal, the aphelion and perihelion velocities are reciprocally as the corresponding distances of the planet from the Sun; the squares of these velocities are therefore reciprocally as the squares of these same distances; but at the perihelion and aphelion distances, the centrifugal forces in the osculatory circumferences are evidently equal to the gravity of the planet towards the Sun, which is therefore in the inverse proportion to the squares of these distances Thus the theorems of Huygens on the centrifugal force were sufficient to demonstrate the tendency of the planets towards the Sun: for it is highly probable that this law, which extends from one planet to another, and which is verified in the same planet at its aphelion and perihelion, extends also to every part of the planetary orbit, and at the same time to every distance from the Sun. But to establish it in an incontestable manner, it was requisite to determine the general expression of the force which, directed towards the focus of an ellipse, obliges a projectile to describe that curve. And it was Newton who demonstrated that this force was reciprocally as the square of the radius vector. It was essential also todemonstrate rigorously that the force of gravity,towards the Sun, only varies in one planet from that of another from their different distances from it.

This great geometrician shewed, that this followed necessarily from the law of the squares of the periodic times being reciprocally as the cubes of the distance. Supposing, therefore, all the planets in repose at the same distance from the Sun, and abandoned to their gravity towards its centre, they would descend from the same height in equal times; this result should likewise extend to the comets, notwithstanding the greater axes of their orbits are unknown, for we have seen in the second Book that the magnitude of the areas described by their radii vectores, supposes the law of the squares of their periodic times proportional to the cubes of their axes.

A general analysis, which embraces every possible result from a given law, shews us that not only an ellipse but any other conic section may be described by virtue of the force which retains the planets in their orbits; a comet may therefore move in an hyperbola, but then it would only be once visible, and would after its apparition recede from the limits of the solar system to approach other suns, which it would again abandon, thus visiting the different systems that are scattered in the immensity of the heavens. It is probable, considering the infinite variety of nature, that such bodies exist. Their apparition should be a very rare occurrence ; the comets we usually observe, are those which, having closed orbits, return at the end of intervals more or less considerable, into the regions of space that are in the vicinity of the Sun. The satellites tend also perpetually to the Sun. If the Moon was not subject to its action, instead of describing an orbit almost circular round the earth, it would soon finish by abandoning it; and if this satellite and those of Jupiter were not sollicited towards the Sun, according to the same law as the planets, sensible inequalities would result in their motions, which have not been recognized by observation. The planets, comets, and satellites are therefore subject to the same law of gravity towards the Sun. At the same time that the satellites move round their planet, the whole system of the planet and its satellites is carried by a common motion and retained by the same force, round the Sun. Thus the relative motion of the planet and its satellites, is nearly the same as if the planet was at rest, and not acted on by any external force.

We are thus conducted without the aid of hypothesis, by a necessary consequence of the laws of the celestial motions, to consider the Sun as the centre of a force, which, extending infinitely into space, diminishes as the square of the distance increases, and which attracts all bodies that are in the sphere of its activity. Every one of the laws of Kepler discovers a property of this attractive force. The law of the areas proportional to the times, shews us that it is constantly directed towards the centre of the Sun; the elliptic orbits of the planets shew that this force diminishes as the square of the distance increases; finally, the laws of the squares of the periodic times proportional to the cubes of the distance, demonstrate that the gravity of all the planets towards the Sun is the same at equal distances; we shall call this gravity *the solar attraction* when we speak of it as relative to the centre of the Sun towards which it is directed; for without knowing the cause, we may by one of those conceptions, common to geometricians, suppose an attractive power residing in the centre of the Sun.

The errors to which observations are liable, and the small alterations in the elliptic motion of the planets, leave a little uncertainty in the results which we have just deduced from the laws of motion; and it may be doubted if the solar gravity diminishes exactly in the inverse ratio of the square of the distance. But a very small variation in this law, would produce a very sensible difference in the motions of the perihelia of the planetary orbits. The perihelion of the terrestrial orbit, would have an annual motion of * 200″ if we only increased by one ten-thousandth part, the power of the distance to which the solar gravity is reciprocally proportional; this motion is only † 36″ 4, according to observation, and of this we shall hereafter see

* 1° 5″. † 11′ 6.

the cause. The law of the square of the distance, is then at least extremely near; and its extreme simplicity should induce us to adopt it, as long as observations do not compel us to abandon it. At the same time we must not measure the simplicity of the laws of nature by our facility of conception; but when those which appear to us the most simple, accord perfectly with observations of the phenomena, we are justified in supposing them rigorously exact.

The gravity of the satellites towards the centre of their planet, is the necessary consequence of the proportionality of the areas described by their radii vectores to the times, and the law of the diminution of this force, according to the square of the distance, is indicated by the ellipticity of their orbits. But this ellipticity is hardly to be perceived in the orbits of the satellites of Jupiter, Saturn, and Uranus, which renders the law of the diminution of the force difficult to ascertain by the motion of any one single satellite; but the constant ratio of the squares of the times of their revolutions, with the cubes of their distances, indicates it beyond a doubt, by demonstrating, that from one satellite to another, the gravity towards the planet is reciprocally as the square of the distance from its centre.

This proof is wanting for the earth, it having but one satellite, but it may be supplied by the following considerations.

The force of gravity extends to the summits of the highest mountains, and the small diminution which it there experiences, does not permit us to doubt, but that at still greater altitudes it would also be sensible. Is it not natural to extend this to the Moon, and to suppose that the force which retains it in its orbit, is its gravity towards the earth, in the same manner as the solar gravity retains the planets in their orbits round the Sun? For in fact these two forces seem to be of the same nature: they both of them penetrate the most intimate parts of matter, animating them with the same velocities; for we have seen that the solar gravity sollicits equally all bodies placed at equal distances from the Sun, and that the terrestrial gravity also causes all bodies to fall through the same height in equal times.

A heavy body forcibly projected horizontally from a great height, falls on the earth at a great distance, describing a curve which is sensibly parabolic, it will fall still farther if the force is greater; and supposing it about seven thousand metres in a second, it would not fall to the Earth, but would circulate round it like a satellite, setting aside the resistance of the air. To form a moon of this projectile, it must be taken to the height of that body, and there receive the same motion of projection.

But what compleats the demonstration of the identity of the moon's tendency towards the earth with gravity, is that, to obtain this tendency, it is sufficient to diminish the terrestrial gravity according to the general law of the variation of the attractive force of the celestial bodies. Let us enter into the details that are suitable to the importance of this subject.

The force which at every instant deflects the Moon from the tangent of her orbit, causes it to describe, in one second, a space equal to the versed sine of the arc which it describes in that time; since this sine is the quantity that the Moon, at the end of a second, deviates from the direction it had in the beginning. This quantity may be determined by the distance of the Earth, inferred from the lunar parallax, in parts of the terrestrial radius; but to obtain a result independent of the inequalities of the Moon, we must take for the mean parallax that part of it which is independent of these inequalities. This part is according to observation * 10541″, relatively to the radius drawn from the centre of gravity of the earth, to the parallel of which the square of the sine of the latitude is equal to $\frac{1}{3}$. We select this parallel, because the attraction of the Earth on the points corresponding to its surface is, at the distance of the Moon, very nearly equal to the mass of the Earth,

* 56′ 54″ 9.

divided by the square of the distance from its centre of gravity. The radius drawn from a point of this parallel to the centre of gravity of the Earth is 6369374 metres, from whence it may be computed that the force which sollicits the Moon towards the Earth, causes it to fall 0^{me}. 00101727 in one second of time. It will be shewn hereafter, that the action of the Sun diminishes the lunar gravity $\frac{1}{358}$th part. The preceding height must therefore be augmented $\frac{1}{358}$th part, to render it independent of the action of the Sun; it then becomes 0^{me}. 00102011. But in its relative motion round the Earth, the Moon is sollicited by a force equal to the sum of the masses of the Earth and Moon, divided by the square of their mutual distance; therefore to obtain the height which the Moon would fall through in one second by the action of the Earth alone, the preceding space must be diminished in the ratio of the mass of the Earth to the Sun, of the masses of the Earth and Moon. But by the phenomena of the tides, it appears that the mass of the Moon is equal $\frac{1}{58,7}$ of that of the earth, multiplying therefore this space by $\frac{58,7}{59,7}$, we have 0^{me}. 00100300 for the height which the Moon falls through in one second by the action of the Earth.

Let us now compare this height with that which results from observation made on the pendulum. Under the parallel above mentioned, the length of the pendulum vibrating seconds is (by Chapter XII. Book I.) equal to 3^{me}. 65706. But on this parallel, the attraction of the Earth is less than the force of gravity by $\frac{2}{3}$ of the centrifugal force due to the motion or notation of the Earth at the equator; and this force is $\frac{1}{288}$th part of that of gravity; the preceding space must therefore be augmented $\frac{1}{432}$d part, to have the space due to the action of terrestrial gravity alone, which on this parallel is equal to the mass divided by the square of the terrestrial radius, we shall therefore have 3^{me}. 66553 for this space. At the distance of the Moon it should be diminished in the ratio of the square of the radius of the terrestrial spheroid to the square of the distance of the

Moon: for this it is sufficient to multiply it by the square of the tangent of the lunar parallax, or † 10541″, this will give 0^{me}. 00100483 for the height which the Moon should fall through in one second by the attraction of the Earth. This quantity derived from experiments on the pendulum, differs very little from that which results from direct observation of the lunar parallax; to make them coincide, it is sufficient to diminish the parallax * 6″, and to reduce it to ‡ 10535″. This is the parallax resulting from the theory of gravity, differing only $\frac{1}{1600}$th part from that derived from direct observation, to which I think it preferable, considering the exactness of the elements from which it is computed. It would be sufficient to diminish a little the mass of the Moon, to obtain by this theory of gravity the same parallax that is given by observation: but all the phenomena of the tides concur in giving to this satellite a mass more considerable, and

* 2″. † 56′ 55″ 2. ‡ 56′ 53″ 3.

such very nearly as we have used in the above computation. But however that may be, the small difference between the two parallaxes is within the limits of the errors of observation, and of the elements employed in the calculation. It is then certain, that the force which retains the Moon in its orbit is the terrestrial gravity diminished in proportion to the square of the distance. Thus the law of the diminution of gravity, which, in planets accompanied by several satellites, is proved by a comparison of their periodic times with their distances, is demonstrated for the Moon, by comparing its motion with that of projectiles at the surface of the Earth.

The observations of the pendulum made on the summits of mountains had already indicated this diminution of the terrestrial gravity; but they were insufficient to discover the law, because of the small height of the most elevated mountains, compared with the radius of the Earth: it was requisite to find a body very remote from us, as the Moon, to render the law perceptible, and to convince us that the force of gravity on the Earth is only a particular case of a force which pervades the whole universe.

Every phenomenon throws new light and confirms the laws of nature. It is thus that the comparison of experiments on gravity with the lunar motion, shews us, that the origin of the distances of the Sun and of the planets in the calculation of their attractive forces, should be placed in their centres of gravity; for it is evident this takes place on the Earth, whose attractive force is of the same nature as that of the Sun and planets.

The Sun, and those planets which are accompanied by satellites, being thus endowed with an attractive force varying inversely as the square of the distance, a strong analogy leads us to attribute the same property to the other planets. The spherical figure common to all these bodies, indicates that their particles are united round their centers of gravity by a force which, at equal distances, equally sollicits them towards these points; but the following considerations leave no doubt on this subject.

We have seen that if the planets and the comets were placed at the same distance from the Sun, their gravity towards it would be in proportion to their masses: now it is a general law in nature, that action and reaction are equal and contrary, all these bodies therefore react on the Sun, and attract in proportion to their masses; they are therefore endowed with an attractive force proportional to their masses, and inversely as the square of the distance. By the same principle the satellites attract the planets and the Sun according to the same law. This attractive property then is common to all the celestial bodies: it does not disturb the elliptic motion round the Sun, when we consider only their mutual action; for the relative motion of the bodies of a system are not changed by giving them a common velocity: by impressing therefore, in a contrary direction to the Sun and to the planet, the motion of the first of these two bodies, and the action which it experiences on the part of the second, the Sun may be considered as immovable; but the planet will be sollicited towards it with a force reciprocally as the squares of the distance and proportional to the sum of the masses: its motion round the Sun will therefore be elliptic. And we see by the same reasoning that it would be so if the planet and Sun were carried through space, with a motion common to each of them. It is equally evident that the elliptic motion of a satellite is not disturbed by the motion of translation of its planet, nor would it be by the action of the Sun, if it was always exactly the same on the satellite and the planet. Nevertheless, the action of a planet on the Sun influences the length of its revolution, which is diminished as the mass of the planet is more considerable, so that the relation of the square of its periodic time to the cube of the major axis of its orbit, depends on its mass. But since this relation is nearly the same for all the planets, their masses must evidently be very small compared with that of the Sun, which is equally free for the satellites with respect to their principal planets. This we may readily suppose from the smallness of their volumes.

The attractive property of the heavenly bodies does not only belong to them in a mass, but belongs to each of their particles. If the Sun only acted on the centre of the Earth, without attracting particularly every one of its particles, there would arise in the ocean oscillations infinitely more considerable, and very different from those which we observe. The gravity of the Earth therefore to the Sun is the result of the gravity of all its particles, which consequently attract the Sun in proportion to their respective masses; besides each body on the earth, tends towards its centre proportionally to its mass, it reacts therefore, on it, and attracts it in the same ratio. If that was not the case, and if any part of the Earth, however small, attracted another part without being attracted by it, the centre of gravity would move in space in virtue of the force of gravity, which is impossible.

The celestial phenomena compared with the laws of motion, conduct us therefore, to this great principle of nature, namely, *that all the particles of matter attract each other in proportion to their masses, and inversely as the squares of their distance.*

Already we may perceive in this universal gravitation the cause of the perturbations to which the heavenly bodies are subject; for the planets and comets being subject to the action of each other, they must deviate a little from the laws of the elliptic motion, which they would otherwise exactly follow, if they only obeyed the action of the Sun. The satellites also deranged in their motions round their planets by their mutual action and that of the Sun, deviate a little from these laws.

We perceive, then, that the particles of the heavenly bodies, united by their attraction, should form a mass nearly spherical; and that the result of their action at the surface of the body, should produce all the phenomena of gravitation. We see, moreover, that the motion of rotation of the celestial bodies should slightly alter their spherical figure, and flatten them at the poles ; and then the resulting force of all their mutual actions not passing through their centres of gravity, should produce in their axes of rotation similar motions to those discovered by observation. Finally, we may perceive why the particles of the ocean, unequally acted on by the Sun and Moon, should have oscillations similar to the ebbing and flowing of the tides. But these different effects of the principle of gravitation, must be particularly developed to give it all the certainty of which physical truth is susceptible.

CHAP. II.

Of the Masses of the Planets, and of Gravity at their Surface.

It appears on the first view of the subject impossible to determine the respective masses of the Sun and planets, and to measure the height from which bodies fall in a given time, from the action of gravity at their surface. But the connection of truths with each other conducts us to results which appeared inaccessible, when the principle on which they depend was unknown. Thus the measure of the intensity of gravity at the surface of the planets is rendered practicable by the discovery of universal gravitation. Let us return to the theorems on centrifugal force given in the preceding book. The result derived from them is, that the gravity of a satellite towards its planet is to the gravity of the Earth towards the Sun, as the mean radius of the orbit of the satellite divided by the square of the time of its sidereal revolution, is to the mean distance of the Earth from the Sun, divided by the square of a sidereal year. To reduce these gravities to the same distance from the bodies which produce them, they must be multiplied respectively by the squares of the radii of the orbits which they describe. And as at equal distances the masses are proportional to their attractions, the mass of the Earth is to that of the Sun, as the cube of the mean radius of the orbit of the satellite, divided by the square of the time of its sidereal revolution, is to the cube of the mean distance of the Earth from the Sun, divided by the square of the sidereal year. Let this result be applied to Jupiter. For this purpose we shall observe that the mean radius of the orbit of the fourth satellite subtends at the mean distance of Jupiter from the Sun an angle of *1530′′ 86, seen at the mean distance of

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* 8′ 15″ 9.

the Earth from the Sun, this radius appears under an angle of * 7964″75; the radius of the circle contains † 636619″ 8: thus the mean radii of the orbits of the fourth satellite, and of the terrestrial orbit, are in the proportion of these two last numbers. The duration of the sidereal revolution of the fourth satellite is ‡ 16^{d} 6890, and the sidereal year is § 365^{d} 2564. Setting out from these data the mass of Jupiter is found to be $\frac{1}{1066.08}$, that of the Sun being represented by unity. To obtain greater exactness, it is necessary to augment by unity the denominator of this fraction, because the force which retains Jupiter in its relative orbit round the Sun, is the sum of the attractions of the Sun and of Jupiter. The mass of this planet then is $\frac{1}{1067.08}$. I have determined by the same method the masses of Saturn and of Uranus. That of the Earth may be calculated in the same manner, but the following

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* 43′ 0″ 5.

† 57° 17′ 44″ 8.

‡ 16^{d} 16^{h} 32′ 9″ 6.

§ 365^{d} 6^{h} 9′ 12″ 9.

method is yet more precise. If the mean distance of the Earth from the Sun be taken for unity, the arc described by the Earth in a second of time, will be the proportion of the circumference, to radius, divided by the number of seconds in the sidereal year, or by *36525638″4; dividing the square of this arc by the diameter, we shall get $\frac{1479565}{1020}$ for its versed sine, it is the quantity which the Earth falls towards the Sun during one second, in consequence of its relative motion round it. It has been seen, in the preceding chapter, that upon the terrestrial parallel, the sine of the latitude of which is $\frac{1}{3}$, the attraction of the Earth causes bodies to fall through the Earth $3^{me.}$ 66553 in one second. To reduce this attraction to the mean distance of the Earth from the Sun, it must be multiplied by the square of the sine of the solar parallax, and this product divided by the number of metres contained in this distance. Now the terrestrial radius, upon the parallel we are considering, is 6369374 me-

* 31558151″2.

tres; dividing this number, therefore, by the sine of the solar parallax, or by * 27″ 2, we shall get the mean radius of the terrestrial orbit expressed in metres. It follows from hence that the effect of the Earth's attraction at the mean distance of this planet from the Sun, is equal to the product of the fraction $\frac{366553}{6369374}$ by the cube of the sine of 27″ 2; it is consequently equal to $\frac{4.48855}{1020}$: taking this fraction from $\frac{1479565}{1020}$ we shall have $\frac{1479560.5}{1020}$ for the effect of the Sun's attraction at the same distance. The masses of the Sun and Earth are therefore in the proportion of the numbers 1479560.5 and 4.48855; from whence it follows that the mass of the Earth is $\frac{1}{329630}$. If the parallax of the Sun is a little different from what we have admitted, the value of the mass of the Earth should vary as the cube of this parallax compared to that of 27″ 2.

The following determinations of the masses of those planets which have no sa-

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* 8″ 5.

tellites, have been obtained by the secular changes which the action of these bodies produces in the elements of the solar system. I have determined the masses of Venus and Mars from the secular diminution of the obliquity of the ecliptic, supposed to be * 154″ 3, and from the acceleration of the Moon's mean motion, fixing it at †34.36 for the first century, setting out from 1700. The mass of Mercury has been determined by its volume, supposing the densities of this planet and of the Earth inversely as their mean distances from the Sun. An hypothesis really very precarious, but which corresponds with sufficient exactness to the respective densities of the Earth, Jupiter and Saturn. It will be necessary to rectify all these values when time shall have demonstrated more correctly the secular variations in the celestial motions and orbits.

The masses of those planets which are accompanied by satellites should be also rectified by very precise observations of

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* 50″ 3. † 11″ 1.

the greatest elongation of the satellite from their planets, without neglecting the consideration of the ellipticity of their orbits.

Masses of the Planets, that of the Sun being taken as unity.

Mercury	$\frac{1}{2025810}$
Venus	$\frac{1}{383137}$
The Earth	$\frac{1}{329630}$
Mars	$\frac{1}{1846082}$
Jupiter	$\frac{1}{1067,09}$
Saturn	$\frac{1}{3359,40}$
Uranus	$\frac{1}{19504}$

The densities of bodies are proportional to their masses divided by their volumes, and when they are nearly spherical their volumes are as the cubes of their radii. Their densities therefore are as their masses divided by the cubes of their radii; but to obtain greater accuracy, that radius of a planet must be taken which corresponds to that parallel, the square of the sine of whose latitude is $\frac{1}{3}$, and which is equal to one third of the sum of the radius of the pole, added to twice the radius of the equator. It is thus found that taking the mean density of the Sun for unity, those of the Earth, Jupiter, Saturn, and Uranus are 3.9393, 0.8601, 0.4951 and 1.1376.

We ought to observe that the error in the measures of the apparent diameter of the planets, and the irradiation, which we have not considered on account of the great difficulty of appreciating it, may influence these calculations very perceptibly. We shall again observe that the preceding value of the density of the Earth is independent of the solar parallax; for both its mass and volume, compared to the Sun, increase as the cube of this parallax.

The measures of the greatest elongations of satellites from their planets merit particularly the attention of observers, since on this depends the knowledge of the masses and densities of the planets. Newton has proposed a very simple method to divest the apparent diameter of the effect of irradiation. It consists in observing, during the night, the light of a lamp through an opening placed at a considerable disance, and small enough only to suffer a part of the light to be visible. The brilliance of the light and the opening is to be diminished till the lamp appears exactly the same size and brightness as the planet; the proportion of the opening to the distance of the observer, will give with great precision the diameter of the planet. The appearances of Saturn's ring may be thus represented, the dimensions of the interior and exterior ring measured, concerning which irradiation produces so much uncertainty. To obtain the intensity of gravitation at the surface of the Sun and planets, let us consider that if Jupiter, and the Earth were exactly spherical, and deprived of their rotatory motion, gravity at their equators would be proportionate to the masses of these bodies, divided by the squares of their diameters; now at the mean distance from the Sun to the Earth, Jupiter's equatorial diameter is *626″26, and that of the Earth's equator is † 54″5;

* 3′ 22. † 17″ 5.

representing then the weight of a body at the terrestrial equator by unity, the weight of this body transported to the equator of Jupiter would be 2,509, but this weight must be diminished by about a ninth from the effects of the centrifugal force due to the rotation of these planets. The same body would weigh 27,65 at the equator of the Sun, and falling bodies would describe one hundred metres in the first second of their descent.

CHAP. III.

Of the Perturbations of the Elliptic Motion of the Planets.

If the planets only obeyed the action of the Sun, they would revolve round it in elliptic orbits, but they act mutually upon each other and upon the Sun, and from these various attractions there result perturbations in their elliptic motions, which are to a certain degree perceived by observation, and which it is necessary to determine to have exact tables of the planetary motions. The rigorous solution of this problem, surpasses at present the powers of analysis, and we are obliged to have recourse to approximations. Fortunately the smallness of the masses compared to the Sun, and the smallness of their excentricity and inclination of their orbits, affords considerable facility to this object. It is still, however, sufficiently complicated, and the most delicate and intricate analysis is requisite to detect among the infinite number of inequalities to which the planets are subject, those which are sensible to observation and to assign their values.

The perturbations of the elliptic motion of the planets may be divided into two distinct classes. Those of the first class affect the elliptic motion of the planets, they increase with extreme slowness and are called *secular inequalities.* The other class depends on the configurations of the planets, both with respect to each other and to their nodes and perihelia, and being re-established every time these configurations become the same, they have been called *periodical inequalities* to distinguish them from secular inequalities, which are equally periodic but whose periods are much longer and independent of the mutual configurations of the planets.

The most simple manner of considering

these various perturbations, consists in imagining a planet moving according to the laws of the elliptic motion upon an ellipse whose elements vary by imperceptible gradations, and conceiving at the same time the true planet to oscillate round the imaginary planet in a small orbit, the nature of which must depend on its periodic inequalities. Thus its secular inequalities are represented by the imaginary planet, and its periodic inequalities by its motion round this same planet.

Let us first consider those secular inequalities which, by developing themselves, in the course of ages, should change at length both the form and position of the planetary orbits. The most important of these inequalities is that which may affect the mean motion of the planets. By comparing together the observations which have been made since the re-establishment of astronomy, the motion of Jupiter appears to be quicker and that of Saturn slower, than by a comparison of the same observations with those of the ancient astronomers:

from which it has been inferred that the first of these motions has accelerated, while the second has retarded from one century to another. And to take into account these variations, astronomers have introduced into their tables of planets, two secular equations increasing with the squares of the times, one additive to the motion of Jupiter, the other subtractive from that of Saturn. According to Halley the secular equation of Jupiter is *106″02 for the first century reckoned from 1700, the corresponding equation of Saturn is †156″94. It was natural to look for the cause of these equations in the mutual actions of these two planets, the most considerable of our system. Euler, who first directed his attention to this problem, found a secular equation, equal for both the planets, and additive to their mean motions, which is inconsistent with observation. Lagrange obtained a result which accorded more nearly with them. Other geometricians obtained other equations. Struck

* 34″ 3. † 50″ 8.

with this difference, I examined again this subject, applying the greatest possible care to the investigation, and I arrived at the true analytical expression for the secular inequality of the mean motions of the planets. In substituting the numerical values, relative to Jupiter and Saturn in this expression, I was surprised to find that it became equal to nothing. I suspected that this was not peculiar to these planets, and that if this expression was put in the most simple form of which it was susceptible, (by reducing to the least possible number the different quantities which it contains by means of the relations which subsist between them) all its terms would destroy each other. Calculation confirmed this suspicion, and taught me that, in general, the mean motions of the planets and their distances from the Sun are invariable; at least when we neglect the fourth powers of the excentricities and inclinations of the orbits, and the squares of the perturbating masses, which is more than sufficient for the actual purposes of astronomy. Lagrange has since confirmed this result, and shewn, by a beautiful method, that it is even true, when the powers and products of any order whatever of the excentricities and inclinations are taken into the calculation. Thus the variations of the mean motions of Jupiter and Saturn do not depend on their secular inequalities.

The permanency of the mean motion of the planets and of the greater axes of their orbits, is one of the most remarkable phenomena in the system of the world. All the other elements of the planetary ellipses are variable, all these ellipses approach to and depart insensibly from the circular form; their inclination to a fixed plane or to the ecliptic augments and diminishes, and their perihelia and nodes are continually changing their places. These variations which are performed with extreme slowness, arise from the mutual actions of the planets on each other, and require several centuries for their completion. They are nearly proportional to the times. They have already become apparent by observation; we have seen, in the first Book, that the perihelion of the Earth's orbit has a direct annual motion of *36″7, and that its inclination to the equator diminishes every century †154″3. It was Euler that first investigated the cause of this diminution, which all the planets contribute to produce by the respective situation of the planes of their orbits. The ancient observations are not exact enough, and the modern are too near each other to fix the exact quantity of these great changes, nevertheless they combine to prove their existence, and to shew that their progress is the same as is conformable to the law of gravitation. If we knew exactly the masses of the planets, future observations might be anticipated, and the true values assigned to the secular inequalities of the planets; but we only know the masses of those planets which are accompanied by satellites, the masses of the others can only

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* 11″ 5. † 49″ 9.

be determined when the progress of time shall have fully developed the quantity of these inequalities from whence these masses are to be computed. We may then in imagination look back to the successive changes which the planetary system has undergone, and foretell those which future ages will offer to astronomers, and the geometrician will at once comprehend in his formulæ both the past and future state of the world. The table of Chap. V. of the second Book, contains the secular variation which results from the preceding masses which we have assigned to the planets.

Many interesting questions here present themselves to our notice. Have the planetary ellipses always been, and will they always be nearly circular? Among the number of the planets have any of them ever been comets whose orbits have gradually approached to the circular form by the mutual attractions of the other planets? Will the obliquity of the ecliptic continually diminish till at length it coincides with the equator, and the days and nights become equal on the earth throughout the year? Analysis answers these questions, in a most satisfactory manner. I have succeeded in demonstrating that whatever be the masses of the planets, in as much as they all move in the same direction, in orbits of small excentricity, and little inclined to each other; their secular inequalities will be periodic, and contained within narrow limits, so that the planetary system will only oscillate about a mean state, from which it will deviate but by a very small quantity; the planetary ellipses therefore always have been, and always will be nearly circular, from whence it follows that no planet has ever been a comet, at least if we only calculate upon the mutual actions of the planetary system. The ecliptic will never coincide with the equator, and the whole extent of its variations will not exceed * three degrees.

The motions of the planetary orbits and of the stars will one day embarrass astronomers when they attempt to compare pre-

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* 2° 42.

cise observations separated by long intervals of time; already this difficulty begins to be manifest; it would be interesting therefore to find some plane that should remain invariable, that, is constantly parallel to itself during all these changes. There fortunately exists such a one, which possesses this remarkable property, to which the orbits of the planets may be referred, just as naturally as the motion of a system of bodies to its centre of gravity. This plane may easily be determined by the following rule.

If, at any instant of time whatever, and upon any plane passing through the centre of the Sun, we draw straight lines to the ascending nodes of the planetary orbits referred to this plane, and if we take on these lines, reckoning from the centre of the Sun, lines equal to the tangents of the inclinations of these orbits to this plane; and if at the extremities of these lines we suppose masses equal to the masses of the planets multiplied respectively into the square roots of the parameters of their orbits, and by the cosines of their inclinations; and lastly, if we determine the centre of gravity of this new system of masses, then the straight line drawn from the centre of the Sun to this point will be the tangent of the inclination of the invariable plane to the assumed plane; and continuing this line to the heavens, it will there mark its ascending node.

Whatever changes the succession of ages may produce in the planetary orbits, and whatever be the plane to which they are referred, the plane determined by this rule will always be the same. It is true its position depends on the masses of the planets; but those which have satellites have the greatest influence on its position, and the masses of the others will soon be sufficiently known to determine it with exactness. In adopting the preceding values of the planets, and the elements of their orbits, as given in Chap. V. Book II. we find that the longitude of the ascending nodes of the invariable plane was * 114°.3877 at the commencement of 1750, and at the same time its inclination to the ecliptic

* 102° 56′ 56″ 1.

was * 1° 719. In this computation we have neglected the comets, which nevertheless ought to enter into the determination of the invariable plane, since they make part of the solar system. It would be easy to include them in the preceding rule, if their masses and the elements of their orbits were known. But in our present ignorance of the nature of these objects, we suppose their masses too small to influence the planetary system, and this is the more probable, since the theory of the mutual attraction of the planets suffices to explain all the inequalities observed in their motions. But if the action of the comets should become sensible in length of time, it should principally affect the position of the plane, which we suppose invariable, and in this new point of view the consideration of this plane will still be useful, if the variations of this plane could be recognised, which would be attended with great difficulties.

The theory of the secular and periodic

* 1° 35′ 40″ 9.

inequalities of the motions of the planets, founded on the law of universal gravitation, has given to our astronomical tables a precision which proves the correctness and utility of this theory. By its means the solar tables which before deviated† two minutes from the observations, have acquired the same precision as the observations themselves. It is particularly in the motions of Jupiter and Saturn that these inequalities are most sensible, but they present themselves under a form so complicated, and the length of their periods is so considerable, that it would have required several ages to have determined their law by observations alone, which has in this instance been anticipated by theory.

After having established the invariability of the mean motions of the planets, I suspected that the alterations observed in the mean motions of Jupiter and Saturn proceeded from the action of comets. Lalande had remarked in the motion of Saturn, irregularities which did not appear

† 1′ 4″.

to depend on the action of Jupiter : he found its returns to the vernal exquinox more rapid than its returns to the autumnal equinox, although the positions of Jupiter and Saturn both to each other and to their aphelia, were nearly the same. Lambert likewise observed that the mean motion of Saturn which seemed to diminish from century to century by the comparison of antient with modern observations, appeared on the contrary to accelerate by the comparison of modern observations with each other, at the same time that Jupiter presented phenomena exactly contrary. All this seemed to indicate that causes independent of the action of Jupiter and Saturn on each other had altered their motions. But on mature reflections, the order of the variations observed in the mean motions of these planets, appeared to me to agree so well with the theory of their mutual attraction, that I did not hesitate to reject the hypothesis of a foreign cause.

It is a remarkable result of the mutual action of the planets on each other, that if we only consider the inequalities which have very long periods, the sum of the masses of every planet, divided respectively by the greater axis of their orbits, is always pretty nearly constant. From this it follows that the squares of the mean motions being reciprocally as the cubes of these axes, if the motion of Saturn is retarded by the action of Jupiter, that of Jupiter should be accelerated by the action of Saturn, which is conformable to observation. I perceived, moreover, that the law of these variations was the same as corresponded to the preceding theory. In supposing with Halley the retardation of Saturn to be * 256″94 for the first century, reckoned from 1700, the corresponding acceleration of Jupiter should be † 109″80, and Halley found ‡ 106″02 by observation. It was therefore very probable that the variations observed in the mean motions of Jupiter and Saturn, were the effects of their mutual action; and since it is certain that this

* 1′23″. † 35″ 5. ‡ 34″ 3.

action cannot produce any inequality either constantly increasing or periodic, but of a period independent of the configuration of these planets, and that it cannot effect in it any irregularities but what are relative to this configuration, it was natural to think that there existed in their theory a considerable inequality of this kind, of a very long period, and which was the cause of these variations.

The inequalities of this kind, although very small and almost insensible in differential equations, augment considerably in the integrations, and may acquire very great values in the expressions of the longitudes of the planets. I easily recognized the existence of similar inequalities in the differential equations of the motions of Jupiter and Saturn. These motions become very nearly commensurable; and five times the mean motion of Saturn is very nearly equal to twice that of Jupiter: from which I concluded that the terms which have for their argument five times the mean longitude of Saturn, minus twice that of Jupiter, might by integration become very sensible, although multiplied by the rules and products of three dimensions of the excentricities and inclinations of the orbits. I considered therefore that these terms were the probable cause of the variations observed in the mean motions of these planets. The probability of this cause, and the importance of the object, determined me to undertake the laborious calculation, necessary to determine this question. The result of this calculation fully confirmed my conjecture; and it appeared, that in the first place there exists in the theory of Saturn a great inequality of * 9027″7 at its maximum, and of which the period is 917¾ years; and, secondly, that the motion of Jupiter is subject to a similar inequality, whose period and law are the same, but its amount is only † 3856″5. It is to these two inequalities, formerly unknown, that we must attribute the apparent retardation of Saturn, and apparent acceleration of Jupiter. These phenomena attain-

* 48′ 44″ 9. † 20′ 49″ 5.

ed their maximum about the year 1560; since this epoch, their mean apparent motions have approximated to their true mean motions, and they were equal in 1790. This explains the reason why Halley, in comparing the antient and modern observations, found the mean motion of Saturn slower, and that of Jupiter more rapid than by the comparison of modern observations with each other, instead of which these last indicated to Lambert an acceleration in the motion of Saturn, and a retardation in that of Jupiter. And it is very remarkable that the quantities of these phenomena, deduced from observation alone by Halley and Lambert, are very nearly the same as result from the two great inequalities which I have just mentioned. If astronomy had been revived four centuries and a half later, the observations would have presented the contrary phenomena. The mean motions which the astronomy of any people have assigned to Jupiter and Saturn, may afford us information concerning the time of its foundation. Thus it appears that the Indian astronomers determined the mean motions of these planets, in that part of the period of the preceding inequalities, when the motion of Saturn was the slowest, and that of Jupiter the most rapid. Two of their principal astronomical epochs, the one 3102 A.C. the other 1491 A.C. answer nearly to this condition. The nearly commensurable relation that exists in the motions of Jupiter and Saturn, occasions other very perceptible inequalities, the most considerable of which affects the motion of Saturn; it would be entirely confounded in the equation of the centre, if twice the mean motion of Jupiter was exactly equal to five times that of Saturn. The difference observed in this century in the intervals of the returns of Saturn to the equinoxes both of spring and autumn, arises principally from this cause.

In general, when I had recognised these various inequalities, and examined more carefully than had been done before, those which had been submitted to calculation, I found that all the observed phenomena of the motions of these two planets adapted themselves naturally to the theory; they before had seemed to form an exception to the law of universal gravitation; they are now become one of the most striking examples of its truth. Such has been the fate of this brilliant discovery, that every difficulty that has arisen has only furnished a new subject of triumph for it, which is the most indubitable characteristic of the true system of nature.

I cannot in this place refrain from making a comparison of the real effects of this relation between the mean motion of Jupiter and Saturn, with those which astrology had attributed to it. In consequence of this relation, if the conjunction of the two planets arrives in the first point of Aries, it will in twenty years afterwards take place in Sagittarius, and in twenty years afterwards in Leo, it will continue to take place in these three signs for nearly two hundred years. In the same manner in the next two hundred years, it will go through the signs Taurus, Capricornus, and Virgo. In the next two hundred years it will proceed through the signs Gemini, Aquarius, and Libra ; and finally, in the last two hundred years it will describe the remaining signs Cancer, Pisces, and Scorpio ; after which it will again begin with the sign Aries as before. From hence arises a great year, each season of which is equal to two centuries. They attributed different temperatures to the different seasons of this year, as likewise to the signs which belonged to them. The assemblage of these three signs was called a *trigon.* The first trigon was that of Fire, the second of Earth, the third of Air, and the fourth of Water.—We may easily imagine that astrology made great use of these trigons, which even Kepler himself describes with great exactness, in several of his works : but it is very remarkable that sound astronomy in dissipating the imaginary influence that was supposed to attend this relation in the motion of the two planets, should have recognised in the harmony of this relation, the source of the greatest perturbations of the planetary system.

The planet Uranus, though lately discovered, offers already incontestable indications of the perturbations which it experiences from the action of Jupiter and Saturn. The laws of elliptic motion do not exactly satisfy its observed positions, and to represent them its perturbations must be considered. Their theory, by a very remarkable coincidence, places it in the years 1769, 1756, and 1690, in the same points of the heavens, where Monnier, Mayor, and Flamstead, had determined the position of three stars, which cannot be found at present: this leaves no doubt of the identity of these stars with the new planet.

CHAP. IV.

Of the Perturbations in the Elliptic Motion of Comets.

THE action of the planets produces, in the motion of comets, inequalities which are principally sensible in the intervals of their returns to the perihelion. Halley having remarked that the elements of the orbits of the comets observed in 1531, 1607, and 1682, were nearly the same, concluded that they belonged to the same comet which in the space of 151 years had made two revolutions. It is true that the period of the first revolution is thirteen months longer than the second. But this great astronomer thought, and with reason, that the attraction of the planets, particularly of Jupiter and Saturn, might have occasioned this difference, and after a vague estimation of this action for the course of the following period, he judged that it should retard the return of the comet, and he fixed it for the end of 1758, or the commencement of 1759. This prediction was too important in itself, and too intimately connected with the theory of universal gravitation, not to excite the curiosity of all those who were interested in the progress of the sciences; for about this time geometricians were very much engaged in extending the application of this theory. During the whole year of 1757, astronomers looked for this comet; and Clairaut, who had been one of the first to solve the problem of the three bodies, applied his solution to the determination of the inequalities which the comet had sustained by the action of Jupiter and Saturn. The 14th November, 1758, he announced in the academy of sciences, that the interval of the return of the comet to its perihelion, would be 618 days longer in the present actual period than in the former one, and that consequently the comet would pass its perihelion about the middle of April

1759. He observed, at the same time, that the small quantities neglected in this approximate calculation, might advance or retard this term a month. That moreover, a body which passes into regions so remote, and which escapes our sight during such long intervals, may be subject to the action of forces entirely unknown, as the attraction of other comets, or even of some planet, whose distance is too great to be ever visible to us. This philosopher had the satisfaction of seeing his prediction accomplished; the comet passed its perihelion the 12th March, 1759, within the limits of the errors of which he thought his results susceptible. After a new revision of his calculations, Clairaut fixed this passage at the 4th of April, and he would have brought it to the 25th March, if he had employed the mass of Saturn, such as is given in chap. II.; that is, within thirteen days of the actual observation. This difference will appear very small, if we consider the great number of quantities neglected, and the influence which the planet

Uranus might produce, whose existence was at that time unknown.

Let us remark, for the honour of the human understanding, that this comet, which in this century only excited the curiosity of astronomers and mathematicians, had been regarded in a very different manner, four revolutions before, when it appeared in 1456. Its long tail spread consternation over all Europe, already terrified by the rapid success of the Turkish arms, which had just destroyed the great empire. Pope Callixtus, on this occasion, ordered a prayer, in which both the comet and the Turks were included in one anathema.

In those times of ignorance, mankind were far from thinking, that the only mode of questioning nature is by calculation and observation : according as phenomena succeeded with regularity or without apparent order, they were supposed to depend either on final causes or on chance; and whenever any happened which seemed out of the natural order, they were considered as so many signs of the anger of heaven.

But these imaginary causes have successively given way to the progress of knowledge, and will totally disappear in the presence of sound philosophy, which sees nothing in them, but expressions of the ignorance of the truth.

To the terrors which the apparition of comets then inspired, succeeded the fear, that of the great number which traverse the planetary system in all directions, one of them might overturn the earth.

They pass so rapidly by us, that the effects of their attraction are not to be apprehended. It is only by striking the earth that they can produce any disastrous effect. But this circumstance, though possible, is so little probable in the course of a century, and it would require such an extraordinary combination of circumstances for two bodies, so small in comparison with the immense space they move in, to strike each other, that no reasonable apprehension can be entertained of such an event.

Nevertheless, the small probability of this circumstance may, by accumulating during a long succession of ages, become very great. It is easy to represent the effect of such a shock upon the earth: the axis and motion of rotation changed, the waters abandoning their antient position, to precipitate themselves towards the new equator; the greater part of men and animals drowned in a universal deluge, or destroyed by the violence of the shock given to the terrestrial globe; whole species destroyed; all the monuments of human industry reversed: such are the disasters which a shock of a comet would produce.

We see then why the ocean has abandoned the highest mountains, on which it has left incontestible marks of its former abode: we see why the animals and plants of the south may have existed in the climates of the north, where their relics and impressions are still to be found: lastly, it explains the short period of the existence of the moral world, whose earliest monuments do not go much farther back than three thousand years. The human race reduced to a small number of individuals, in the most deplorable state, occupied only with the immediate care for their subsistence, must necessarily have lost the remembrance of all sciences and of every art; and when the progress of civilization has again created new wants, every thing was to be done again, as if mankind had been just placed upon the earth. But whatever may be the cause assigned by philosophers to these phenomena, we may be perfectly at ease with respect to such a catastrophe during the short period of human life.

But man is so disposed to yield to the dictates of fear, that the greatest consternation was excited at Paris, and communicated to the provinces in 1773, by a memoir of Lalande, in which he determined, of those comets which had been observed, the orbits that most nearly approached the earth; so true it is, that error, superstition, vain terrors, and all the evils of ignorance are ever ready to start up, when the light of science is unfortunately extinguished.

The action of comets upon the solar

system has been hitherto insensible, which seems to indicate that their masses are inconsiderable. It is possible, however, that the minute errors of our best tables depend upon it. An exact theory of the perturbation of the planets, compared with very precise observations, is the only means of ascertaining this point, so important to the system of the world.

CHAP. V.

Of the Perturbations of the Motion of the Moon.

The Moon is attracted at the same time by the Sun and by the Earth, but its motion round the Earth is only disturbed by the difference of the action of the Sun upon these two bodies: if the Sun was at an infinite distance, it would act equally upon them, and in the direction of parallel lines; their relative motion, therefore, would not be affected by an action which was common to both; but its distance, though very great compared with that of the Moon, cannot be considered as infinite: the Moon is alternately nearer and farther from the Sun than the Earth, and the straight line joining the centres of the Sun and Moon, forms angles more or less acute with the radius vector of the Earth. Thus the Sun acts unequally and in different directions on the Earth and Moon; and from this diversity of action, inequalities must necessarily arise in the lunar motion, depending on the respective positions of the Moon and Sun. To determine these, we must at the same time consider the mutual actions and motions of these three bodies, the Sun, the Earth, and the Moon. This constitutes the famous problem of the three bodies, the exact solution of which surpasses the powers of analysis; but from the proximity of the Moon, compared with its distance from the Sun, and from the comparative smallness of its mass, an approximation may be obtained extremely near the truth. Nevertheless, the most delicate analysis is necessary to investigate all the terms, whose influence becomes sensible; of this the first steps that were made in this analysis afford sufficient proof.

Euler, Clairaut, and Dalembert, who resolved this problem nearly about the same time, agreed in finding by the theory of gravitation, the motion of the lunar perigee only half as great as it appears to be from observation. From which Clairaut concluded that the law of attraction was not quite so simple as had been imagined; and he supposed it to consist of two parts, one varying inversely as the squares of the distances, and sensible only at the great distance of the planets from the Sun, and that the other, increasing in a greater ratio as the distance diminished, became sensible at the distance of the Moon from the Earth. This conclusion was vehemently opposed by Buffon: he maintained that since the primordial laws of nature should be the most simple possible, they could only depend on one *modulus*, and their expression, therefore, must consist of one single term. This consideration should no doubt lead us not to complicate the law of attraction, except in case of extreme necessity; at the same time our ignorance respecting the nature of this force, does not permit us to pronounce with certainty as to the simplicity of its expression. However this may be, the metaphysician was in the right this time in his contest with the mathematician, who retracted his error on making this important discovery, that by carrying on the approximation farther than had been done at first, the law of attraction, reciprocally as the squares of the distances, gave the motion of the lunar perigee, exactly conformable to observation, which has since been confirmed by all those who have occupied themselves on this subject. It is impossible without the assistance of analysis, to explain the connection of all the inequalities of the Moon's motion with the combined action of the Sun and Earth upon this satellite. We shall observe, that the theory of universal gravitation has not only explained the motion of the node and of the perigee of the lunar orbit, together with the three great inequalities known by the names of *variation, evection,* and *annual equation,* all which astronomers had already recognized; but it has likewise developed a great number of others less considerable, which it would have been almost impossible to have found and ascertained by observation alone. In proportion as this theory has been brought to perfection, have the lunar tables acquired additional precision. This satellite, once so refractory, deviates now but little from the tables: but to give them that degree of precision, which is yet wanting, will require investigations at least as extensive as those which have been already made; for in every case the first steps which lead to a discovery, and the last which bring it to perfection, are the most difficult. It is possible, nevertheless, without analysis to explain the cause of the annual variation of the Moon, and of its secular equation. I shall the more willingly stop to explain the causes of these equations, because it will be seen that from them are derived the greatest inequalities of the Moon, which the course of ages may develope to observers, but which at the present period have been almost insensible.

In its conjunctions with the Sun, the Moon is nearer to it than the Earth, and experiences from it a more considerable action; the difference of the attractions of the Sun upon these two bodies, tends to diminish the gravity of the Moon towards the Earth. In a similar manner in the oppositions of the Moon to the Sun, this satellite being farther from the Sun than the Earth, is more weakly attracted: thus the difference of the actions of the Sun tends also in this case to diminish the gravity of the Moon to the Earth. In each case the diminution is very nearly the same, and equal to twice the product of the mass of the Sun, by the quotient of the radius of the lunar orbit, divided by the cube of the distance of the Sun to the Earth. In the quadratures, the action of the Sun upon the Moon, decomposed in the direction of the lunar orbit, tends to augment the gravity of the Moon to the Earth: but this augmentation is only half the value of the diminution which it experienced in the sysygies. Thus from all the actions of the Sun upon the Moon in the course of a synodical revolution, there results a mean force in the direction of the lunar radius vector, which diminishes the gravity of this satellite, and it is equal to half of the product of the mass of the Sun, by the quotient of the radius of the lunar orbit, divided by the cube of the distance of the Sun from the Earth.

To find the ratio which this product bears to the gravity of the Moon, we may observe, that this force of gravity which retains it in its orbit is nearly equal to the sum of the masses of the Earth and Moon, divided by the square of their mutual distance; and the force which retains the Earth in its orbit is very nearly equal to the mass of the Sun divided by its distance from the Earth. According to the theory of central forces, explained in the second Book, these two forces are as the radii of the orbits of the Sun and of the Moon, divided respectively by the squares of the times of their revolutions. Hence it follows that the preceding product is to the gravity of the Moon, as the square of the time of the sidereal revolution of the Moon is to the square of the time of the sidereal revolution of the Earth. This product therefore is very nearly $\frac{1}{179}$ of the lunar gravity, which by the mean action of the Moon is thus diminished by its $\frac{1}{358}$ part.

In consequence of this diminution, the Moon is sustained at a greater distance from the Earth, than if it was abandoned entirely to the action of its own force of gravity. The sector described by its radius vector is not altered, since the force which produces it is in the direction of this radius, but its real velocity and angular motion are diminished, and it is easy to see, that by placing the Moon at a greater distance, so that its centrifugal force might equal its gravity, diminished by the action of the Sun, and that its radius vector might describe the same sector that it would have described without this action; this radius would be augmented by its 358th part, and its angular motion diminished by a 179th part.

These quantities vary reciprocally as the cubes of the distances of the Sun to the Earth; when the Sun is perigee, its action being most powerful, dilates the lunar orbit, but this orbit contracts again, as the Sun approaches its apogee; thus the Moon describes in space, a series of epicicloids whose centres are on the terrestrial orbit, and which dilate and contract as the Earth approaches or recedes from the Sun. From hence an inequality arises in the lunar motion, very similar to the equation of the centre of the Sun, with this difference that it retards this motion, when that of the Sun augments, and that it accelerates it when the motion of the Sun diminishes. These two equations are thus always affected with contrary signs. The angular motion of the Sun is, as we have shewn in the first Book, reciprocally as the square of its distance at the perigee; this distance being $\frac{1}{60}$th less than the mean distance, its angular velocity is augmented $\frac{1}{30}$th; the diminution of $\frac{1}{179}$th produced by the action of the Sun in the lunar motion, is then greater by a twentieth; the increase of this diminution is therefore the 3580th part of this motion. Hence it follows that the equation of the centre of the Sun is to the annual equation of the Moon, as a thirtieth of the solar motion is to the 3580th of the lunar motion, which gives * 2398″ for the annual equation. It is about a seventh part less according to observation, this difference depends on the quantities that have been neglected in this first calculation.

The secular equation of the Moon is produced by a similar cause with the annual equation. Halley first remarked this equation, which Dunthorn and Mayer have confirmed by a profound discussion of the observations. These two learned astronomers have proved that the mean motion of the Moon cannot be reconciled with modern observations, and with the eclipses observed by the Chaldeans and Arabians. They have attempted to represent them by adding to the mean longitudes of this satellite a quantity proportional to the square of the number of centuries elapsed before or after the year

* 12′ 56″ 9.

1700. According to Dunthorn this quantity is * 30″9, for the first century: Meyer made it † 21″6, in his first tables which he increased to ‡ 27″8, in his last. And since that time Lalande, after a new investigation of the subject was led nearly to the same result as Dunthorn. The Arabian observations which have been chiefly made use of, are two eclipses of the Sun and one of the Moon, observed by Ibn Junis, near Cairo, towards the end of the tenth century, and extracted some time ago from a manuscript of this astronomer's existing in the library at Leyden. Doubts have risen concerning the reality of these eclipses; but the translation which M. Caussin has lately made of the part of this valuable manuscript which contains the observations has dissipated these doubts; it has moreover made us acquainted with twenty-five other eclipses observed by the Arabians, and which confirm the acceleration of the mean motion of the Moon.

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* 10″. † 7″. ‡ 9″.

Besides our modern observations compared with the Chaldean, are sufficient to establish the existence of the secular equation of the Moon. Delambre and Bouvard have determined, by means of a great number of observations both of the past and present century, the actual secular motion of this satellite, with a precision that leaves a very slight uncertainty : they found it only * 80″ less than that of Mayer, when the antient observations give a secular motion less by 6 or † 700 seconds. The lunar motion is therefore accelerated since the time of the Chaldeans, and the Arabian observations being made in the interval that separates them and confirming this supposition, it is impossible any longer to question the truth of it.

Now, what is the cause of this phenomenon? Does the theory of universal gravitation, which has so well explained the numerous inequalities of the Moon, account likewise for its secular variations? These questions are the more interesting to resolve, because if we succeed, we shall

* 26″ † 200″.

obtain the law of these secular variations of the Moon, for it is evident that the hypothesis of an acceleration proportional to the time, as admitted by astronomers, is only approximative and cannot extend to an unlimited period.

This object has greatly occupied the attention of geometricians, but their researches for a long time fruitless, having discovered nothing either in the action of the Sun or planets on the Moon, nor in the figures not exactly spherical of this satellite and the Earth, that could change the mean motion of the Moon, some rejected the secular equation altogether, others to explain it, had recourse to different hypothesis, such as the actions of comets, the resistance of an ether, and the successive transmission of gravity. Yet the correspondence of the other celestial phenomena with the theory of gravitation is so perfect, that we could not observe without great regret, that the secular variation of the Moon appeared to refuse to submit to it, and continued the only exception to a general and simple law whose discovery, by the grandeur and variety of the objects which it embraces, does so much honour to the human understanding. This reflection having determined me to reconsider this question after several attempts I was at last so fortunate as to discover its cause. *The secular equation of the Moon arises from the action of the Sun upon this satellite combined with the variation of the excentricity of the terrestrial orbit.* To form a just idea of this cause, we must recollect that the elements of the orbit of the Earth are subject to alteration from the action of the planets; its greater axis remains always the same, but its excentricity, its inclination to a fixed plane, and the position of its nodes and of its perihelion are incessantly changing. It must also be considered, that the action of the Sun upon the Moon diminishes by $\frac{1}{179}$, its angular velocity, and that this numerical co-efficient varies reciprocally as the cube of the distance of the Earth from the Sun. Now in expanding the inverse third power of the distance, into a series arranged according to the sines and cosines of the mean motions of the Moon, and of their multiples, taking for unity the semi-major axis of the terrestrial orbit; it is found that this series contains a term equal to three times the half of the square of the excentricity of this orbit; the expression of the diminution of the angular velocity of the Moon, contains therefore a term equal to the 179th part of this velocity multiplied by three times half the square of this excentricity, or what is equivalent, equal to the product of this square, by the angular velocity of the Moon, divided by 119.33. If the excentricity of the terrestrial orbit was constant, this term would be confounded with the mean angular velocity of the Moon; but its variation though very small, has nevertheless in progress of time a sensible influence on the motion of the Moon. It is evident that this motion will be accelerated when the excentricity diminishes, which has been the case ever since the most ancient observations to the present time, this acceleration will be changed into a retardation, when the excentricity arrived at its *minimum* will cease to decrease and begin to augment.

In the interval from 1700 to 1800, the square of the excentricity of the terrestrial orbit diminishes 0.0000015325, half the greater axis being taken as unity, the corresponding increase in the angular velocity, of the moon is therefore 0.0000000128425 of this velocity: this increase taking place successively and proportional to the time, its effect on the Moon's motion is only half what it would be if in the whole course of the century it was the same as in the end. To determine therefore this effect or the secular equation of the Moon at the end of a century, reckoning from 1700, we must multiply the secular motion of the Moon by the half of the very small increase in its angular velocity, but in a century the motion of the Moon is *5347405454, which gives †34″ 337 for its secular equation.

* 481266° 29′ 27″ † 11″ 1.

As long as the diminution of the square of the excentricity of the terrestrial orbit may be supposed proportional to the time, the secular equation of the Moon will increase sensibly as the square of the times; it would be sufficient therefore to multiply 34″337 by the square of the number of centuries contained between 1700 and the time for which the calculation is made. But I have found that in going back to the observations of the Chaldeans, the term proportional to the cube of the times, in the expression in series, of the secular equation of the Moon, becomes sensible, this term is equal to ‡0″13574 for the first century; it should be multiplied by the cube of the number of centuries reckoned from 1700, the product being taken as negative for the centuries anterior to this epoch. The mean action of the Sun upon the Moon depends also on the inclination of the lunar orbit to the ecliptic, and we

‡ 0″ 122166.

might suppose that the position of the ecliptic being variable, there should result inequalities in the motion of this satellite similar to those produced by the diminution of the excentricity of the terrestrial orbit; but the lunar orbit is constantly brought back by the action of the Sun to the same inclination to that of the Earth, so that the greatest and least declinations of the Moon are, in consequence of the secular variation in the obliquity of the ecliptic, subject to the same changes as the declinations of the Sun.

This constancy in the inclination of the lunar orbit is confirmed by all observations both ancient and modern.

The excentricity of this orbit experiences equally only an insensible alteration from the change of the excentricity of the terrestrial orbit.

It is not thus with the variation of the motion of the nodes and perigee, to which attention must be paid indispensibly, in investigations, the object of which are to perfect the lunar tables. In submitting these variations to analysis, I have found that the influence of the terms depending on the square of the perturbating force, and which, as we have seen, double the mean motion of the perigee, is yet greater on the variation of this motion. The result of this intricate analysis has given me a secular equation to be subtracted from the mean motion of the perigee, and equal to thirty-three tenths of the secular equation of the lunar motion ; so that the mean motion of the perigee is retarded, when that of the Moon accelerates. I have found likewise in the motion of the nodes of the lunar orbit upon the true ecliptic, a secular equation to be added to their mean longitude, and equal to seven tenths of the secular equation of the mean motion. Thus the motion of the nodes is retarded like that of the perigee when that of the Moon augments, and the secular equations of these three motions, are constantly in the proportion of the numbers 7, 33, and 10.

Future ages will develope these great inequalities which will produce one day variations at least equal to a fortieth of the circumference, in the secular motion of the Moon, and to a twelfth of the circumference in that of its perigee. These inequalities do not always continue increasing, they are periodical like those of the excentricity of the terrestrial orbit on which they depend, and do not re-establish themselves till after millions of years.

They must at length alter the periods which have been devised for the purpose of comprehending the entire numbers of revolutions of the Moon relatively to it nodes, to its perigee, and to the Sun, periods which differ sensibly in various parts of the immense period of the secular equation.

The luni-solar period of six hundred years, has been rigorously exact at a certain period which it would be easy to find by analysis if the masses of the planets were well determined; but this determination, so desirable for the perfection of astrononomical theory, is yet wanting. Fortunately Jupiter, whose mass we know exactly, is the planet which has the greatest influence on the secular equation on the Moon.

Already ancient observations, notwithstanding their imperfection, confirm these inequalities, and we may trace their progress either in these ancient observations or in the astronomical tables which have succeeded them to the present time. We have seen that the ancient eclipses, had made known the acceleration of the Moon's motion, before the theory of gravity had developed the cause.

In comparing modern observations and the eclipses observed by the Arabians, Greeks, and Chaldeans, with this theory, we find an agreement between them that appears surprising, when we consider the imperfection of ancient observations, the vague manner in which they have been transmitted to us, and the uncertainty which still exists concerning the excentricity of the earth's orbit, from our doubts respecting the masses of Venus and Mars. The developement of the secular equations of the moon is one of the most proper data to determine these masses.

It was particularly interesting to verify the theory of gravity, relatively to the secular equations of the motion of the Moon's nodes and perigee, the knowledge of which we owe to it. Astronomers not having attended to these equations, in the comparison of ancient and modern observations, should have found these motions too rapid; while at the same time they assigned too small a mean motion to the Moon when they did not consider its secular equation. It is this which Bouvard has confirmed by the comparison of a great number of modern observations. Above five hundred observations by de la Hire, Flamsteed, Bradley, and Maskelyne, disposed in the most favourable manner, and carefully discussed, have informed him that the secular motion of the perigee in the lunar tables inserted in the third edition of de Lalande's astronomy, must be diminished by about * fifteen minutes and three quarters. This motion thus erected no longer represents the ancient eclipses, which from hence demonstrates the existence of the secular equation of the perigee of the Moon.

To discover if the magnitude of this equation is the same as is given by the law of universal gravitation, Bouvard has first compared with the tables above-mentioned, twenty-one eclipses observed by the Greeks and Chaldeans, and this comparison has given him very nearly the secular equation of the perigee equal to thirty-three tenths of that of the mean motion: thereby two eclipses observed by the Arabians have conducted him to the same result, which he has again discovered by sixty eclipses, observed since the revival of astronomy in Europe till the commencement of the present century. This remarkable agreement between the results drawn from observations made at such very different epochs does not leave any doubt concerning the existence and magnitude of

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*8′ 30″.

the secular equationofthe lunar perigee,and confirms in an incontestible manner the relation of thirty-three to ten, which the theory of gravity establishes between this equation, and that of the Moon's mean motion. Bouvard has also confirmed by the comparison of the same eclipses, the secular equation of the nodes; and he has found that their motion in a century, given in the tables already cited ought to be diminished by *537''

The mean motions and the epochs of the tables of the Almageste and of the Arabians, indicate evidently these three secular equations of the lunar motion. The tables of Ptolemy are the result of immense calculations made by this astronomer and by Hipparchus; the labour of Hipparchus has not descended to us: we only know from the evidence of Ptolemy, that he had taken the greatest care to choose eclipses the most advantageous to the determination of the elements of which they were in

* 2' 50'' 2.

search. Ptolemy after two centuries and a half of new observations found nothing to change in the mean motion of the Moon as determined by Hipparchus; he corrected the motion of the nodes and perigee but a very small quantity; there is therefore reason to believe that the elements of the lunar tables of Ptolemy have been determined by a great number of eclipses, of which he only preserved those that appeared to him to coincide most with the mean results which had been obtained by Hipparchus and himself. Eclipses only make known correctly the mean sinodical motion of the moon, and its distances from its nodes and its perigee: we can only then depend upon these elements in the tables of the Almageste; now in going back to the first epoch of these tables, by means of motions determined only by modern observations, we do not find the mean distances of the Moon from its nodes, its perigee, and from the Sun, that are given in these tables at this epoch. The quantities which must be added to these distances, are very nearly these which result from the secular equations; the elements of these tables confirm therefore at the same time, the existence of these equations and the values which I have assigned to them.

The motions of the Moon relative to its nodes, to its perigee, and to the Sun, being slower in the tables of the Almageste than in our days, indicate also in these three motions an acceleration equally indicated whether by the corrections that Albategnius eight centuries after Ptolemy, made to the elements of these tables, by comparing them with a great number of eclipses observed in his time; or by the epochs of the tables which Ibn Junis constructed about the year one thousand, from the assemblage of the Chaldean, Greek and Arabian observations.

It is remarkable that the diminution of the excentricity of the terrestrial orbit should be much more sensible, in the lunar motion than in itself. This diminution which, since the most ancient eclipse we are acquainted with, has not altered the equation of the suns centre * 15′, has produced a † 100″ of variation in the Moon's longitude, and nearly ‡ 9° variation in its mean anomaly; we could hardly suspect it from the observations of Hipparchus and Ptolemy. Those of the Arabians indicated it with much probability; but the ancient eclipses, compared with the theory of gravitation, leave no doubt on this subject.

Here we see an example of the manner in which phenomena as they are developed, lead us to the knowledge of their true causes. When only the acceleration of the mean motion of the Moon was known, it might be attributed to the resistance of ether, or to the successive transmission of gravity. But analysis proves that these two causes cannot produce any sensible alteration in the mean motion of the nodes and of the lunar perigee, and that alone would suffice to exclude them, even when the true cause of the variations observed in these motions was unknown.

* 8′ 6″ † 90° ‡ 8° 6′.

The agreement of theory with observation proves that if the mean motions of the Moon are altered by causes foreign to the principle of universal gravitation, their influence is very small, and hitherto insensible.

Some partizans of final causes have imagined that the Moon was given to the Earth to afford it light in the absence of the Sun. But in this case nature would not have attained the end proposed, since we are often deprived at the same time of the light of each of them. To have accomplished this end it would have been sufficient to have placed the Moon at first in opposition to the Sun and in the plane of the ecliptic, at a distance from the Earth equal to one hundredth part of the distance of the Earth from the Sun, and to have given to the Earth and to the Moon, velocities parallel and proportional to their distances from the Sun. In this case the Moon being constantly in opposition to the Sun, would have described round it an ellipse similar to that of the Earth, these two bodies would thus have succeeded each other above the horizon, and as at this distance the Moon would not be eclipsed, its light would always replace that of the Sun.

Other philosophers, struck with the singular opinion of the Arcadians who thought themselves more ancient than the Moon, have imagined that this satellite may formerly have been a comet which passing near the Earth may have been forced by its attraction to accompany it. But by reascending by analysis back into the most distant ages, we find that the Moon has always moved in an orbit nearly circular, in the same manner as the planets round the Sun. Neither, therefore, has the Moon nor any other satellite ever been a comet.

CHAP. VI.

Of the Perturbations of the Satellites of Jupiter.

The first inequalities which observation discovered in the motion of these bodies, are also the first which are derived from the theory of their mutual attractions. We have seen in the second Book, that there exists

1. An equation in the motion of the first satellite equal to * 5258″, multiplied by the sine of double the excess of the mean longitude of the first satellite above that of the second.

2. An equation in the motion of the second satellite equal to † 11923″, multiplied by the sine of the excess of the first satellite above that of the third.

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* 28′ 23″ 5. † 1° 4′ 23″.

3. An equation in the motion of the third satellite equal to * 827″, multiplied by the sine of the excess of the longitude of the second satellite above that of the third.

Not only the theory of gravity gives these inequalities, as Lagrange and Bailly were the first to remark, but it shews us also, what observation seemed to indicate, that the inequality of the second satellite is the result of two inequalities, of which one being caused by the action of the first satellite, varies as the sine of the excess of the longitude of the first satellite above that of the second; and the other, produced by the action of the third, varies as the sine of double the excess of the longitude of the second satellite above that of the third. Thus the second satellite experiences a perturbation from the action of the first, similar to that which itself causes in the third; and it experiences from the third a similar perturbation to that which itself causes in the first.

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* 1′ 27″ 9.

These two inequalities are combined into one in consequence of the relation which exists between the mean motions and the mean longitudes of the three first satellites, for the mean motion of the first satellite *plus* twice that of the third, is equal to three times that of the second; and the mean longitude of the first satellite *minus* three times that of the second *plus* twice that of the third is constantly equal to a semi-circumference: but will these relations always exist, or are they only approximative, and will not the two inequalities of the second satellite, at present combined, be separated in the course of time? It is to theory that we must apply for a solution to this question.

The approximation which the tables gave to the preceding relations, made me suppose that they were rigorously exact; for it was against all probability that chance should have originally placed the three first satellites at the precise distances and positions suitable to the above relation: it was therefore extremely probable that it arose from some particular cause; I looked therefore for this cause in the mutual action of the satellites. A scrupulous investigation of this action, has shewn me that it has caused these relations to be rigorously exact; from whence I concluded, that in determining again by the examination of a great many distant observations, the mean longitudes of the three first satellites, it would be found that they would approximate still more to these relations, to which the tables should be made exactly to agree. I had the satisfaction of seeing this consequence of the theory confirmed, with remarkable precision, by the researches which Delambre has lately made concerning the satellites of Jupiter. It is not necessary that these relations should have taken place exactly at their origin, it was enough that they did not greatly differ, then the mutual actions of the satellites upon each other were sufficient to subject them to this law, and to maintain it unaltered; but the little difference between this and the primitive relation, has given rise to a small inequality of an arbitrary extent, and unequally distributed among the three satellites, and which I have distinguished under the name of *libration*. The two constant arbitrary quantities of this inequality, replace whatever arbitrary quantity is made to disappear by the two preceding relations, in the mean motions and in the epochs of the mean longitudes of the three first satellites; for the number of arbitrary quantities included in the theory of a system of bodies is necessarily sextuple the number of bodies: as observation does not indicate this inequality, it must evidently be very small, and even insensible.

The preceding relations would still subsist, even if the mean motions of the satellites were subject to secular variations analogous to that in the motion of the Moon. They would subsist also in the case of these motions being altered by the resistance of a medium, or by other causes, provided their effects were so small as not to be perceived in less than a century. In all these cases the secular equations so arrange themselves, by the reciprocal action of the satellites, that the secular equation of the first *plus* twice that of the third, will be constantly equal to three times that of the second. Thus the three first satellites of Jupiter form a system of bodies connected together by the preceding relations and inequalities, which their mutual action will maintain for ever, except some external cause should abruptly derange their respective positions.

The theory of gravitation has also enabled me to ascertain the cause of the singular variations observed in the excentricity of the orbit of the third satellite, which I mentioned in the second Book. These variations depend on two equations of the centre, very distinct from each other, to which its motion is subject, of which one relates to the perijoves proper to this satellite, and the other to the perijove of the fourth. The excentricities of the orbits of the four satellites, and their perijoves, are connected with each other by the mutual action of these bodies, in consequence of which the excentricity of the fourth satellite extends itself over the three others, but more feebly as they are more remote. It is very sensible in the orbit of the third, and combining itself with the excentricity peculiar to this orbit, it produces in the motion of the third satellite a compound equation of the centre, whose greatest value incessantly varies, and depends on a perijove, the motion of which is not uniform. The longitude of the perijove of the fourth satellite was * 159° 43, at the commencement of 1700, and its annual and sidereal motion is † 7852″; the longitude of the perijove proper to the third satellite was ‡ 194° 11 at the same period, and its annual and sidereal motion is ‖ 29776″. These perijoves coincided in 1684, and the two equations of the centre formed a single one, equal to their sum, the greatest value of which amounted to § 2661″. In 1775, these perijoves having arrived at contrary positions, the two equations of their centres

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* 133° 29′ 13″ 2.   † 42′ 14″ 4.   ‡ 174° 41′ 56″ 4.
‖ 2° 40′ 47″ 4.   § 14′ 22″ 1.

formed one equal to the difference, whose value was only * 759″. This explains the reason why Wargentin found, by comparing the observations, that the excentricity of this satellite was the greatest towards the beginning of the century, and least about the year 1760. He had at first endeavoured to explain these variations, by means of two equations of the centre, but not being aware that one of them depended on the perijove of the fourth satellite, and having also assigned to them incorrect values, he was forced to abandon them, and to recur to the hypothesis of a variable excentricity, whose variations he determined by experiment.

The mutual action of the satellites of Jupiter, produces at every instant a variation in the positions of their orbits. This is what the theory, compared with the observations, gives upon this subject. The equator of Jupiter is inclined † 34444″ to the plane of the orbit of that planet, the

* 4′ 5″ 9. † 3° 51′ 59″ 8.

longitude of its ascending node was * 347° 8519; at the commencement of 1700, its annual and sidereal motion is about † 6″. The orbit of the first satellite is only inclined ‡ 22″ to the plane of the equator of Jupiter; its nodes on this plane, coincide with the nodes of the same plane with the orbit of Jupiter, the orbit of the satellite being between these two planes.

The orbit of the second satellite moves in a fixed plane, inclined ‖ 221″ to the equator of Jupiter, and which passes through the line of the nodes of this equator, between this last plane, and that of the orbit of Jupiter. The orbit of Jupiter is inclined § 5182″ to this fixed plane, and its nodes with this plane have a retrograde motion, whose annual and sidereal value is equal ¶ 13° 3488, and whose period is thirty Julian years. The longitude of the ascending node was ** 179° 5185, in 1700.

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* 313° 4′. † 1″ 9. ‡ 7″ 1. ‖ 1′ 11″ 6.
§ 27′ 58″ 9. ¶ 12° 0′ 50″ 1. ** 161° 33′ 59′ 9.

The orbit of the third satellite moves on a fixed plane, inclined * 1030″ to the equator of Jupiter, and which passes through the line of the node of this equator, between this last plane, and that of the orbit of Jupiter; the orbit of the satellite is inclined † 2244″ to this fixed plane, and its nodes with this plane have a retrograde motion, whose annual and sidereal value is ‡ 2° 9149, and period 137 years; the longitude of its ascending node was § 136° 9630 in 1700. Astronomers who had recognised the motion of this node by observations, supposed the orbits of the second and third satellites in motion on the equator itself of Jupiter, but they were obliged by the observations to diminish a little the inclination of this equator, with the orbit of Jupiter, when they considered the motion of the third satellite.

Lastly, the orbit of the fourth satellite moves in a fixed plane, inclined ¶ 4630″ to

* 5′ 33″ 7. † 12′ 7″. ‡ 2° 37′ 24″ 2.
§ 123° 16′. ¶ 25′.

the equator of Jupiter, and which passes through the line of nodes of this equator, between this last plane and that of the orbit of Jupiter. The orbit of the satellite is inclined * 2772″ to this fixed plane, and its nodes with it have a retrograde motion, whose annual and sidereal value is † 7519″, and period 532 years, the longitude of the ascending node is ‡ 153°5185 in 1700.

The inclination of the orbit of the fourth satellite, with the orbit of Jupiter, continually varies in consequence of this motion. Having arrived at its mininum, about the end of the last century, it remained nearly stationary for a great number of years, and the nodes of the orbit of the satellite, with that of Jupiter, have had a direct annual motion of about § 8′. This circumstance was recognised by observations, and astronomers availed themselves of it, in their tables of this satellite; but for several years back,

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* 14′ 58″. † 40′ 36″. ‡ 138° 10′.
§ 4′ 19″ 2.

observation has indicated a very sensible augmentation in the inclination of its orbit with that of Jupiter, which without the assistance of the theory, would have rendered the construction of the tables very difficult.

It is satisfactory to the geometrician to see these singular phenomena arise from analysis, which are perceptible to observation, but being at the same time the result of several simple inequalities, are too complicated for astronomers to ascertain their laws.

The different planes which we have just described, and on which the orbits of the satellites move, are not rigorously fixed; the plane of the equator of Jupiter, carrying them on in his motion, so that their nodes, with the orbit of this planet, being constantly the same with those of its equator; their inclinations, with the plane of this orbit, are always proportional to that of the equator. But all these motions are insensible, from the time of the discovery of the satellites, to the present day.

The orbit of each satellite participates a little in the motion of the adjoining orbit; for every thing is connected, in a system of bodies mutually subject to the action of each other. The satellites form round Jupiter a system similar to that of the planets round the Sun; and as their revolutions are very rapid, they present us in the short space of time since their discovery, all the great changes which a series of ages will produce in the planetary system. Thus the agreement of the theory of gravitation, with the variations observed in the motions of the satellites, leaves us no reason to doubt the variations which it indicates in the orbits of the planets, and which the most ancient observations, compared with our own, would scarcely render sensible.

This theory has banished all empiricism from the tables of the satellites of Jupiter. Those which Delambre has lately published, borrowing only from observation such data as necessarily depend on it, have the advantage of extending to all ages by rectifying these data as they become better known. We may conceive, that to establish the theory which served for a basis to these tables, it was necessary to know both the masses of the satellites, and the compression of Jupiter.

Five data, derived from observation, are necessary to determine these five unknown quantities. Those which I employ are the two principal inequalities of the first and second satellites; the period of the variations of the inclination of the orbit of the second satellite; the equation of the centre of the third satellite, which depends on the perijove of the fourth. Finally, the motion of this perijove. Taking for unity the mass of Jupiter, the masses of its satellites, as deduced from the preceding data, are as follows:

I. Satellite . .	0.0000172011
II. Satellite . .	0.0000237103
III. Satellite . .	0.0000872128
IV. Satellite . .	0.0000544681

These values may be corrected, when in the progress of time we become better acquainted with the secular variations of the orbits of the satellites.

The ratio of the two axes of Jupiter, resulting from these data, is equal 0.93041. This quantity has been measured several times, and the mean result is $\frac{13}{14}$, or 0.929, which does not differ a sensible quantity from the preceding result. But considering the great influence which the compression of the figure of Jupiter, has on the motion of the nodes, and of the perijoves of the satellites, we perceive that the ratio of the two axes of Jupiter, is given with greater precision by the observations of the eclipses, than by the most exact measures taken with a micrometer. The agreement of these measures, with the result of the theory, proves in a most satisfactory manner, that the action of gravity towards Jupiter, is composed of the gravities towards each of its particles; since in reasoning from this principle, we find the compression such as it really appears to be.

The eclipses of the first satellite of Jupiter, gave rise to the discovery of the successive motion of light, which the phenomenon of aberration has ascertained with still greater precision. It appeared to me that the theory of the motion of this satellite being now better known, and the observation of its eclipses become more numerous, their discussion should give the quantity of aberration more exactly than by direct observation. Delambre undertook this investigation at my request, and found the entire quantity of aberration * 62″5, which is exactly that which Dr. Bradley derived from his observations. It is very curious to observe such a perfect agreement, in results which have been obtained by such very different methods.

It follows from this agreement, that the velocity of light is uniform through the whole space comprehended by the terrestrial orbit. In fact, the velocity of light given by the aberration, is that which sub-

* 20″ 2.

sists at the circumference of the terrestrial orbit, and which, being combined with the motion of the Earth, produces this phenomenon. The velocity of light, as given by the eclipses of the satellites of Jupiter, is determined by the time which light employs to traverse the terrestrial orbit; these two velocities being the same, the velocity is uniform through the whole length of the diameter of the terrestrial orbit. It results also from these eclipses, that the velocity of light is uniform through the whole diameter of the orbit of Jupiter; for, from the excentricity of this orbit, the effect of the variations in the radii vectores, is very sensible in the eclipses of the satellites; and these exactly correspond to the hypothesis of a uniform velocity, in the motion of light.

If light is an emanation from luminous bodies, the uniformity of its velocity requires that it should be projected from each of them with the same force, and that its motion should not be sensibly retarded by their attraction. If we suppose light to consist in the vibrations of an elastic fluid, we must then, to explain the uniformity of their velocity, suppose the density of the fluid throughout the whole extent of the planetary system, proportionate to its elasticity. But the simplicity with which the aberration of the stars, and the phenomena of the refraction of light, in passing from one medium to another, are explained by considering light as an emanation from a luminous body, renders this hypothesis extremely probable.

CHAP. VII.

Of the Figure of the Earth and Planets, and of the Law of Gravity at their Surface.

It has been shewn in the First Book, what we have learnt from observations on the figure of the Earth, and of the planets: let us compare these results with those of universal gravitation.

The force of gravity towards the planets, is composed of the attractions of all their particles. If their mass was fluid, and without motion, their strata would be spherical, those nearer the centre being more dense. The force of gravity at their exterior surface, and at any distance whatever, without the sphere, would be exactly the same, as if the whole mass of the planet was compressed into the centre of gravity. It is in consequence of this remarkable property, that the Sun, the planets, comets, and satellites, act upon each other, very nearly as if they were so many material points. At very great distances. the attraction of the particles of a body of any figure, which are the most remote, and those which are nearest the particle attracted, compensate each other in nearly the same manner as if they were united in the centre of gravity; and if the ratio of the dimensions of the body be considered as a very small quantity of the first order, this result will be exact to a quantity of the second order. But in a sphere, it is rigorously true, and in a spheroid differing but little from a sphere, it is of the same order as the product of its excentricity, by the square of the ratio of its radius, to the distance of the point attracted. This property of the sphere, of attracting as if its mass was concentrated in its centre, contributes greatly to the simplicity of the motions of the heavenly bodies. It does not belong exclusively to the law of nature, it equally appertains to the law of the attraction varying proportionably to the simple distance, and cannot belong to any other law but those formed by the addition of these two. And of all the laws which render the force of gravity nothing at an infinite distance, that of nature is the only one in which the sphere possesses this property.

According to this law, a body placed within a spherical stratum of uniform thickness, is equally attracted by all its parts, so as to remain at rest in the midst of the various attractions which act upon it. The same circumstance takes place in an elliptic stratum, when the exterior and interior surfaces are similar and similarly situated. Supposing therefore the planets to be spheres of homogeneous matter, the force of gravity in their interior, must diminish as the distance from the centre; for the exterior part, relatively to the attracted particle, contributes nothing to its gravity, which entirely consists of the attraction of the internal sphere, whose radius is equal to the distance of this point from the centre. But this attraction is equal to the mass of the sphere, divided by the square of the radius, and the mass, is as the cube of this same radius. The force of gravity on the attracted particle, is therefore proportional to the radius. But if, (as is probably the case) the strata are more dense as they approach the centre, the force of gravity will diminish in a less ratio, than in the case of homogenity. The rotary motion of the planets causes them to differ a little from the spherical figure. The centrifugal force arising from this motion, causing the particles situated at the equator, to recede from the centre, and produce a flattening of the poles.

Let us consider first the effects of this circumstance in the most simple case, of the Earth's being an homogeneous fluid, and the whole force of gravity residing in its centre, and varying reciprocally as the square of the distance from this point. It will then be easy to prove that the terrestrial spheroid is an ellipsoid of revolution; for if we conceive two columns of fluids, communicating with each other at the centre, terminating, the one at the pole, the other at any point in the surface, these two columns ought to be in equilibrio. The centrifugal force does not alter the weight of the column directed to the pole, but diminishes the weight of the other column. This force is nothing at the centre of the Earth, and at the surface is proportional to the radius of the terrestrial parallel, or very nearly, as the cosine of the latitude ; but the whole of this force is not entirely employed in diminishing the force of gravity; for these two forces making an angle with each other, equal to the latitude, the centrifugal force, decomposed according to the direction of gravity, is weakened in the ratio of the cosine of this angle to radius. Thus, at the surface of the Earth, the centrifugal force diminishes the force of gravity, by the product of the centrifugal force at the equator, by the square of the cosine of the latitude; therefore the mean value of this diminution in the length of a fluid column, is the half of this product, and since the centrifugal force is $\frac{1}{289}$ of the force of gravity at the equator, this value is $\frac{1}{578}$ of the force of gravity, multiplied by the square of the cosine of the latitude. And since it is necessary, for the maintenance of the equilibrium, that the column by its length shall compensate the diminution of its weight, it should surpass the polar column by $\frac{1}{578}$ of its length, multiplied by the square of the above cosine. Thus the augmentation of the radii, from the pole to the equator, is proportional to the squares of these cosines, from whence it is easy to conclude, that the Earth is an ellipsoid of revolution, the equatorial and polar axis of which were in the proportion of 578 to 577.

It is evident that the equilibrium of the fluid mass would still subsist, supposing that a part should consolidate itself in the interior, provided the force of gravity remains the same.

To determine the law of gravity at the surface of the Earth, we should observe that the force of gravity to any point on this surface, is less than that at the pole, from its being situated farther from the centre. This diminution is nearly equal to double the augmentation of the terrestrial radius; it is equal therefore to the product of the $\frac{1}{289}$ part of the force of gravity by the square of the cosine of the latitude. The centrifugal force diminishes likewise the force of gravity by the same quantity; thus by the union of these two causes, the diminution of gravity from the pole to the equator, is $= 0,00694$, multiplied by the square of the cosine of the latitude, the force of gravity at the equator being taken as unity.

It has been shewn in the First Book, that the measures of meridional degrees, give the Earth an ellipticity greater than $\frac{1}{378}$, and that the measures of the pendulum indicate a diminution in the force of gravity, from the poles to the equator, less than 0.00694, and equal to 0.00567. The measures of the degrees and of the pendulum concur, therefore, to prove that the force of gravity is not directed to a single point, but is composed of the attractions of all the particles of the Earth.

This being the case, the law of gravity depends on the figure of the terrestrial spheroid, which depends itself on the law of gravity. It is this mutual dependance of the two unknown quantities on each other, that renders the investigation of the figure of the Earth very difficult. But fortunately the elliptic figure, the most simple of all the re-entering figures next to the sphere, satisfies the condition of the equilibrium of a fluid mass, subject to a motion of rotation, and of which all the particles attract each other reciprocally, as the squares of the distance. Newton, upon this hypothesis, and supposing the Earth a homogeneous fluid, found the ratio of the equatorial to the polar axis, to be 230 to 229.

It is easy to determine the law of variation of the force of gravity on the Earth, upon this hypothesis. For this purpose, let us consider two different points situated on the same radius, drawn from the centre to the surface of an homogeneous fluid, in equilibrio. All the similar elliptic strata, which cover any one amongst them, contribute nothing to its gravity. The resulting force of all the attractions which act on it, is derived entirely from the attraction of the interior spheroid, similar to the entire spheroid, and whose surface passes through the point in question. The similar and similarly situated particles of these two spheroids, attract the interior point, and the corresponding point of the exterior surface, proportionally to their masses, divided by the squares of their distances. These masses are in the two spheroids, as the cubes of their similar dimensions, and the squares of their distances, are as the squares of these dimensions. The attractions on similar particles, are proportional therefore to these dimensions; from which it follows, that the entire attractions of the two spheroids, are in the same ratio, and their directions parallel.

The centrifugal forces of the two points, now under consideration, are likewise proportional to the same dimensions. Therefore the force of gravity in each of them being the result of these two forces, will likewise be proportional to their distances from the centre of the fluid mass.

Now, if we conceive two fluid columns directed as before, to the centre of the spheroid, one from the pole, and the other from any point on the surface, it is evident, that if the ellipticity of the spheroid is very small, that is, if it differs but little from a sphere, that the force of gravity, decomposed according to the directions of these columns, will be nearly the same as the total gravity. Dividing, therefore, the length of these columns into an equal number of parts, infinitely small and proportional to their lengths, the weights of the corresponding parts will be to each other as the products of the lengths of the columns, by the force of gravity at the points of the surface where they terminate. The whole weight of these columns will therefore be to each other, in this ratio; and as these weights must be equal, to be in equilibrio, the force of gravity at their surface must consequently be reciprocally, as the length of these columns. Thus the length of the radius of the equator, surpassing the radius at the pole a 230th part, the force of gravity at the pole should likewise exceed that at the equator a 230th part.

This supposes the elliptic figure sufficient for the equilibrium of a homogeneous fluid mass. Maclaurin has demonstrated this in a beautiful manner, from which it results, that the equilibrium is rigorously possible; and that, if the ellipsoid differs little from a sphere, the ellipticity will be equal $\frac{5}{4}$ of the quantity, which expresses the proportion of the centrifugal force, to that of gravity, under the equator.

Two different figures of equilibrium may correspond to the same motion of rotation. But the equilibrium cannot exist with every motion of rotation. The shortest period of rotation of an homogeneous fluid in equilibrio, of the same density as the Earth, is * 0.1009 of a day, and this limit varies reciprocally, as the square root of the density. When the motion of rotation increases in rapidity, the fluid mass becoming more flattened at the poles, its period of rotation becomes less, and ultimately falls within the appropriate limits of a state of equilibrium. After a great many oscillations, the fluid, in consequence of the friction and resistances which it experiences, fixes itself at last in that state which is *unique*, and determined by the primitive motion of rotation. The axis drawn through the centre of gravity, of the fluid mass, and relative to which the moment of the forces was a maximum, at the origin, becomes the axis of rotation.

The preceding results afford us an easy method of verifying the hypothesis of the homogenity of the Earth. The irregularity of the measured degrees, may be

* $2^h\ 25'\ 17''$.

supposed to leave too much uncertainty, as to the ellipticity, to enable us to decide, if it is really such as the above hypothesis requires. But the regular increase of the force of gravity, from the equator to the pole, as determined by experiments on the pendulum, is sufficient to throw great light upon the subject.

In taking as unity the force of gravity at the equator, its increase at the pole, according to the hypothesis of homogenity, should be equal $\frac{1}{230} = 0.00435$. But by observations on the pendulum, this increase is 0.00567 : the Earth therefore is not homogeneous. And indeed it is natural to suppose, that the density of the strata increase as they approach the centre. It is even necessary, for the stability of the equilibrium of the waters of the ocean, that their density should be less than the mean density of the Earth ; otherwise, when agitated by the winds and other causes, they would overflow their limits, and inundate the adjoining continents.

The homogenity of the Earth being thus excluded by observation, we must, to determine its figure, suppose the sea covering a nucleus, composed of different strata, diminishing in density from the centre to the surface. Clairaut has demonstrated, in his beautiful work, that the equilibrium is still possible, in the supposition of an elliptic figure at the surface, and of the strata of the interior nucleus. In the most probable hypothesis, relative to the law of the densities and ellipticities of these strata, the ellipticity of the Earth is less than in the case of homogenity, and greater than if the force of gravity was directed to a single central point. The increase of the force is greater than in the first case, and less than in the second. But there exists between the increase of the force of gravity, taken as unity at the equator, and the ellipticity of the Earth, this remarkable analogy, that in all the hypotheses relative to the structure of the internal nucleus, which the sea incloses, the ellipticity of the Earth is just so much less than that which would take place in the case of homogenity, as the augmentation of the force of gravity exceeds that which should exist, according to the same supposition, and reciprocally, so that the fractions expressing the ellipticity, and the augmentation of the force of gravity always together, make a constant quantity equal $\frac{5}{2}$, of the proportion of the centrifugal force, to the force of gravity at the equator, which, on the Earth is $\frac{1}{115.2}$.

In attributing an elliptic figure to the strata of the terrestrial spheroid, the increase of its radii, the increase of the force of gravity, and the diminution of the degrees, from the pole to the equator, will vary as the squares of the cosine of the latitude, and these are connected with the ellipticity of the Earth, in such a manner, that the total increase of the radii is equal to the ellipticity. The total diminution of the degree, is equal to the ellipticity, multiplied by three times the degree at the equator; and the total increase of the force of gravity, is equal to the force of gravity at the equator, multiplied by the excess of $\frac{1}{115.2}$, above the ellipticity.

Thus the ellipticity of the Earth may be determined, either by direct measurement of degrees, or by observations on the length of the pendulum.

The observations of the pendulum give 0.00567, for the increase of the force of gravity, which taken from $\frac{1}{115,2}$, gives $\frac{1}{332}$, for the ellipticity of the Earth. If this hypothesis of the ellipse be conformable to nature, it should agree with the measures of degrees; but it implies errors that are altogether improbable: and this circumstance, joined to the difficulty of reconciling all these measures to the same elliptic figure, proves that the figure of the earth is much more complicated than had been believed. This will not appear surprising, if we consider the different depths of the sea, the elevation of the continents, and islands above its level, the heights of mountains, and the unequal density of the water, and different substances which are at the surface of this planet.

To embrace, in the most general manner

possible, the theory of the figure of the Earth and planets, it is necessary to determine the attraction of spheroids, differing little from spheres, and formed of strata, variable both in figure and density, according to any law whatever.

It will remain then to determine the figure which will agree with the equilibrium of a fluid, expanded over its surface, for we must imagine the planets covered with a fluid similar to the Earth, or their form would be entirely arbitrary. Dalembert has given, for this purpose, an ingenious method, which extends to a great number of cases, but which is deficient in that simplicity so desirable in such complicated investigations, and which constitutes their principal merit.

A remarkable equation of partial differences relative to the attraction of spheroids, led me, without the aid of integrations, and by differential methods only, to general expressions, for the radii of the spheroids; for the attractions upon any points whatever, either within the surfaces, or without them; for the condition of equilibrium of the fluids that surround them; for the law of gravity, and for the variation of the degrees at the surface.

All these quantities are connected with each other, by analogies extremely simple, from which results an easy method of verifying all the hypotheses that may be formed to represent either the variation of the force of gravity, or that of the values of different degrees of the meridian.

Thus Bouguer, with a view of reconciling the degrees measured at the equator, in France and in Lapland, supposed the Earth to be a spheroid of revolution, in which the increase of the degrees, from the equator to the pole; was proportional to the fourth power of the sine of the latitude. It is found that this hypothesis does not satisfy the increase of the force of gravity from the equator to Pello.—An increase, which according to observation, is equal to forty-five ten millionths of the whole gravity, and which would be only twenty-seven ten millionths in this hypothesis.

The above mentioned expressions give a direct and general solution of the problem which consists in determining the figure of a fluid mass in equilibrio, supposing it subjected to a motion of rotation, and composed of an infinity of fluids, of different densities, whose particles attract each other directly as their masses, and inversely as the squares of their distances.

Legendre had already solved this problem by a very ingenious analysis, which supposes the mass homogeneous. In this general supposition, the fluid necessarily takes the form of an ellipsoid of revolution, of which all the strata are elliptic, whose densities diminish at the same time that their ellipticities increase, from the centre to the surface.

The limits of compression of the whole ellipsoid, are $\frac{5}{4}$ and $\frac{1}{2}$ of the ratio of the centrifugal force, to the force of gravity at the equator. The first limit is relative to the hypothesis of homogenity, and the second, to the supposition of the strata, infinitely near the centre, being infinitely dense, and consequently the whole mass of the spheroid acting as if concentrated in that point. In the latter case, the force of gravity being directed to a single point, and varying inversely as the square of the distance, the figure of the Earth would be such as has been above determined; but in the general hypothesis, the line which determines the direction of the force of gravity from the centre to the surface of the spheroid, is a curve, every element of which is perpendicular to the stratum through which it passes.

It is remarkable, that the variations observed in the length of the pendulum, follow pretty correctly the law of the squares of the cosines of the latitudes, at the same time that the variations in the measured degrees, differ very sensibly from this law. The general theory of the attractions of spheroids, affords a simple explanation of this phenomenon; it shews us that the terms, which in the value of the terrestrial radius, differ from this law, become more sensible in the expression of the force of gravity, and still more sensible in the expression of degrees, where they may acquire a value sufficiently great to produce the phenomenon under consideration.

This theory likewise shews us, that the limits of the total increase of the force of gravity, taken at the equator as unity, are the products of 2 and $\frac{5}{4}$, by the ratio of the centrifugal force, to the force of gravity, the first limit referring to the case of an infinite density at the centre, the second to the case of homogenity. The increase, as derived from observation, being between these limits, indicates that the strata are more dense, as they approach the centre, which is conformable to the laws of hydrostatics. Thus the theory seems to accord with observation, as far as could be expected, considering our ignorance of the internal construction of the Earth.

The result of this agreement is, that in the calculation of the variations of the force of gravity, and of parallax, we may consider the terrestrial meridians as of an elliptic form, the compression of which is the excess of the fraction $\frac{1}{115.2}$, above the total increase of the force of gravity from the equator to the poles.

The radius drawn from the centre of gravity of the terrestrial spheroid, to its surface at the parallel, the square of the sine of whose latitude is $\frac{1}{3}$, determines the sphere, whose mass is equal to that of the Earth, and whose density is equal to its mean density; this radius is 6369374 metres, and the force of gravity on this parallel, is the same as at the surface of this sphere.

But what is the proportion of the mean density of the Earth, to that of a known substance at its surface? The effect of the attractions of mountains, on the oscillations of pendulums, and on the direction of the plumb-line, may conduct us to the solution of this interesting problem.

It is true that the highest mountains are always very small, in proportion to the

Earth ; but we may approach very near to the centre of their action, and this joined to the precision of modern observations, ought to render their effects perceptible.

The mountains of Peru, the highest in the world, seemed the most proper for this object. Bouguer did not neglect so important an observation in the journey which he undertook, for the measure of the meridional degrees at the equator.

But these great bodies being volcanic and hollow in their interior, the effect of their attraction was found to be much less than might be expected from their size. However it was perceptible ; the diminution of the force of gravity at the summit of Pichincha, would have been 0.00149, without the attraction of the mountain, and it was observed to be 0.00118. The effect of the deviation of the plumb-line, from the action of another mountain, surpassed * 20″. Dr. Maskelyne has since

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* 6″ 4.

measured, with great care, a similar effect, produced by the action of a mountain in Scotland: the result was, that the mean density of the Earth, is double that of the mountain, and four or five times greater than that of the common water. This curious observation deserves to be repeated several times on different mountains, whose interior construction is well known.

Let us apply the preceding theory to Jupiter.

The centrifugal force due to the motion of rotation of this planet, is nearly $\frac{1}{9}$ of the force of gravity at its equator; at least, if the distance of the fourth satellite from its centre, as given in the second Book, be adopted.

If Jupiter was homogeneous, the diameter of its equator might be obtained, by adding five-fourths of the preceding fraction to its shorter axis; taken as unity, these two axes would, therefore, be in the proportion of 41 to 36. According to observation, their proportion is that of 14 to 13. Jupiter, therefore, is not homogeneous. Supposing it to consist of strata, if the densities diminish from the centre to the surface, its ellipticity should be included between $\frac{5}{36}$ and $\frac{1}{18}$, the observed ellipticity being within these limits, proves the heterogenity of its strata, and by analogy that of the strata of the terrestrial spheroid, already rendered very probable in itself, and from the observations of the pendulum.

CHAP. VIII.

On the Figure of Saturn's Ring.

THE ring of Saturn, as has been shewn in the first Book, is formed of two concentric rings of very small thickness. By what mechanism do these rings sustain themselves round the planet? It is not probable that this should take place from the simple adhesion of their particles. Since, were this the case, the parts nearest to Saturn, sollicited by the constantly renewed action of gravity, would be at length detached from the rings, which would, by an insensible diminution, finally disappear, like all those works of nature which have not had sufficient force to resist the action of external causes. These rings support themselves then without effort, and only by the lines of equilibrium. But for this it is requisite to suppose them possessed of a rotary motion round an axis perpendicular to their plane, and passing through the centre of Saturn, so that their gravitation towards the planet, may be balanced by the centrifugal force due to this motion.

Let us imagine a homogeneous fluid spread round Saturn in the form of a ring, and let us see what ought to be its figure, for it to remain in equilibrium, in consequence of the mutual attraction of its particles, of their gravitation towards Saturn, and their centrifugal force. If, through the centre of the planet, a plane is imagined to pass, perpendicular to that of the ring, the section of the ring by this plane, is what I shall call the *generating curve*. Analysis proves that if the magnitude of the ring is small in proportion to its distance from the centre of Saturn, the equilibrium of the fluid is possible when the generating curve is an ellipse of which the greater axis is directed towards the centre of the planet. The duration of the rotation of the ring, is nearly the same as that of the revolution of a satellite, moved circularly at the distance of the centre of the generating ellipse. And this duration is about * four hours and a third, for the interior ring. Herschel has confirmed by observation this result, to which I had been conducted by the theory of gravitation.

The equilibrium of the fluid would also exist, supposing the generating ellipse variable in size and position, to the extent of the circumference of the ring; provided these variations are sensible only at a much greater distance than the axis of the generating section. Thus the ring may be supposed of an unequal breadth in its different parts, it may even be supposed of double curvature. These inequalities are indicated by the appearances and disappearances of Saturn's ring, in which the two arms of the ring have presented different phenomena. They are even neces-

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* $10^h\ 32'$.

sary to maintain the ring in equilibrium round the planet, since if it was perfectly similar in all its parts, its equilibrium would be deranged by the slightest force, such as the attraction of a satellite, and the ring would finally precipitate itself upon the planet.

The rings by which Saturn is surrounded, are consequently irregular solids, of unequal breadth in the different points of its circumference, so that their centres of gravity do not coincide with the centres of their figure. These centres of gravity may be considered as so many satellites, moving round the centre of Saturn, at distances dependant on the inequalities of the rings, and with angular velocities equal to the velocities of rotation of their respective rings.

It may be imagined, that these rings, sollicited by their mutual action, by that of the Sun, and of the satellites of Saturn, ought to oscillate round the centre of this planet, and that their nodes, formed with the plane of the orbit of this planet, should have a retrograde motion. It might be believed, that yielding to different forces, they should cease to be in the same plane; but Saturn having a rapid rotatory motion, and the plane of its equator being the same with that of its ring, and of its six first satellites, its action retains the system of these different bodies in the same plane. The action of the Sun, and of the seventh satellite, only changes the position of the plane of Saturn's equator, which in this motion carries with it the ring, and the orbits of the six first satellites, by a similar mechanism to that which retains the orbits of the satellites of Jupiter, and principally the orbit of the first, nearly in the plane of the equator of this planet.

Thus the constant position of Saturn's rings, and the orbits of the six first satellites in the same plane, indicate a considerable compression in this planet, and consequently a rapid motion of rotation, which has been confirmed by observation; and as all the satellites of Uranus move nearly in the same plane, we may conclude that this planet revolves upon itself, round an axis perpendicular to this plane.

CHAP. IX.

On the Atmosphere of the Celestial Bodies.

The thin, transparent, compressible, and elastic fluid which surrounds a body, and rests upon it, is called its atmosphere. We conceive a similar atmosphere surrounding every celestial body; the probability of its existence in all of them, is indicated by observation relative to the Sun and Jupiter. In proportion as the atmospherical fluid is elevated above the surface of a body, it becomes thinner, in consequence of its elasticity, which dilates it as it is less compressed. But if the particles of its surface were perfectly elastic, it would extend itself without ceasing, and finally would dissipate itself into space.

It is then requisite that the elasticity of the atmospherical fluid should diminish in a greater proportion than the weight which compresses it; and that there may exist a state of rarity, in which it may be without elasticity. It should be in this state at the surface of the atmosphere.

All the atmospheric strata should take after a time the rotatory motion, common to the body which they surround. For the friction of these strata against each other, and against the surface of the body, should accelerate the slowest motions, and retard the most rapid, till a perfect equality is established among them. In these changes, and generally in all those which the atmosphere undergoes, the sum of the products of the particles of the body, and of its atmosphere, multiplied respectively by the area which their radii vectores projected on the plane of the equator, describe round their common centre of gravity, are always equal in equal time.

Supposing then, that by any cause whatever, the atmosphere should contract itself, or that a part should condense itself on the surface of the body, the rotatory motion of the body, and of its atmosphere, would be accelerated, because the radii vectores of the area, described by the particles of the primitive atmosphere becoming smaller, the sum of the product of all the particles, by the corresponding area, could not remain the same, unless the velocity of rotation augments.

At its surface the atmosphere is only retained by its weight, and the form of this surface is such, that the force which results from the centrifugal and attractive forces of the body, is perpendicular to it. The atmosphere is flattened towards the poles, and distended at its equator, but this ellipticity has limits, and in the case where it is the greatest, the proportion of the axis of the pole and the equator, is as two to three.

The atmosphere can only extend itself at the equator, to that point where the centrifugal force exactly balances the force of gravity, for it is evident that beyond this limit, the fluid would dissipate itself. Relative to the Sun, this point is distant from its centre by the length of the radius of the orbit of a planet, the period of whose revolution is equal to that of the Sun's rotation.

The Sun's atmosphere then does not extend so far as Mercury, and consequently does not produce the zodiacal light, which appears to extend even to the terrestrial orbit. Besides, this atmosphere, the axis of whose poles should be at least two-thirds of that of the equator, is very far from having the lenticular form which observation assigns to the zodiacal light.

The point where the centrifugal force balances gravity, is so much nearer to the body, in proportion as its rotatory motion is more rapid. Supposing that the atmosphere extends itself as far as this limit, and that afterwards it contracts and condenses itself from the effect of cold at the surface of the body, the rotatory motion would become more and more rapid, and the farthest limit of the atmosphere would approach continually to its centre: it will then abandon successively in the plane of its equator, fluid zones, which will continue to circulate round the body, because their centrifugal force is equal to their gravity. But this equality not existing relative to those particles of the atmosphere, distant from the equator, they will continue to adhere to it. It is probable that the rings of Saturn are similar zones, abandoned by its atmosphere.

If other bodies circulate round that which has been considered, or if it circulates itself round another body, the limit of its atmosphere is that point where its centrifugal force, *plus* the attraction of the extraneous bodies, balances exactly its gravity. Thus the limit of the Moon's atmosphere, is the point where the centrifugal force due to its rotatory motion, plus the attractive force of the Earth, is in equilibrium with the attraction of this satellite. The mass of the Moon being $\frac{1}{587}$ of that of the Earth, this point is distant from the centre of the Moon, about the ninth part of the distance from the Moon to the Earth. If, at this distance, the primitive atmosphere of the Moon had not been deprived of its elasticity, it would have been carried towards the Earth, which might have retained it. This is perhaps the cause why this atmosphere is so little perceptible.

CHAP. X.

Of the Tides.

If the investigation of the laws of the equilibrium of the fluids which cover the planets, presents great difficulties, that of the motion of these fluids agitated by the attractions of the heavenly bodies, offer still greater.

Thus Newton, who occupied himself the first with this important problem, was satisfied with determining the figure in which the ocean would remain *in equilibrio,* under the action of the Sun and Moon. He supposed that the sea, at every instant, took this figure; and this hypothesis, which extremely facilitates the calculations, gave him results, in many respects conformable with the observations. In fact, this great geometrician had recourse to the action of the rotation of the Earth, to explain the retardation of the tides, beyond the passage of the Sun and Moon over the meridian; but his reasoning is unsatisfactory, and, moreover, appears contrary to the result of a rigorous analysis. The Academy of Sciences proposed this subject for a prize, in 1740; the successful pieces, contained the developement of the Newtonian theory, founded on the same hypothesis, of the ocean in equilibrium under the action of the attracting bodies. It is evident, nevertheless, that the rapidity of the Earth's motion prevents the waters that cover it, from taking at every instant, the figure suitable to the equilibrium of the forces, but the investigation of this motion, combined with that of the action of the Sun and Moon, was too difficult to be effected by the state of analysis at that time, and of the knowledge then possessed of the motions of fluids. But assisted by the discoveries which have since been made on both these subjects, I have again undertaken this problem, the most intricate in celestial mechanics. The only hypotheses which I shall permit myself are, that the ocean inundates the whole Earth, and that it meets with but slight obstructions in its motion; the rest of my theory is rigorously exact, and founded on the principles of the motion of fluids. By thus conforming to nature, I have the satisfaction to see my results agree with the observations, particularly with respect to the small difference which subsists between the two tides of one day, which difference, according to the theory of Newton, should be very great. I obtained this remarkable result, namely, that to make this difference disappear, it is only necessary to suppose the ocean to have every where the same depth. Daniel Bernoulli, in his Essay on the Tides, which divided the prize of the academy, in 1740, endeavoured to explain this phenomenon, by supposing that the motion of the Earth was too rapid to permit the tides to accommodate themselves to the theory. But it can be shewn by analysis, that this rapidity could not prevent the tides from being very unequal, if the depth of the ocean was not constant. We may see by this example, and by that of Newton, how much we should distrust the most plausible hypotheses, when not supported by rigorous calculation.

The preceding results, though very extensive, are still restricted by the supposition of a fluid, regularly spread over the Earth, and subject to very slight resistances in its motions. The irregularity in the depth of the ocean, the position and declivity of the shores, their situation relative to the neighbouring coasts, the friction of the waters against the bottom of the ocean, and the resistances they meet with: all these causes, which it is impossible to reduce to calculation, modify the oscillations of this great fluid mass. All that can be done is to analyse the general phenomena of the tides, which should result from the attractive forces of the Sun and Moon, and to derive from observation such data as are indispensably necessary to complete the theory of the tides for each particular port. These data are so many arbitrary quantities depending on the extent of the sea, its depth, and the local circumstances of the port. Under this point of view, we shall consider the oscillations of the ocean, and their correspondence with observations.

Let us first consider the action of the Sun alone upon the ocean, and suppose its motion uniform in the plane of the equator. It is evident, that if the Sun acted on the centre of gravity of the Earth, and of every particle of the ocean, by exerting equal and parallel forces, the whole system of the terrestrial spheroid would obey these forces by a common motion, and the equilibrium of the waters would not be at all altered. This equilibrium, then, is only deranged by the difference of these forces, and by the inequality of their directions. A particle of the ocean, placed directly under the Sun, is more attracted than the centre of the Earth. It tends, therefore, to separate itself from it, but it is retained by its gravity, which this tendency diminishes. Twelve hours afterwards, this particle is opposite to the Sun, which attracts it less forcibly than it does the centre of the Earth; the surface of the terrestrial globe therefore tends to separate itself from it, but the gravity of the particles retains it. This force is therefore diminished also in this case by the solar attraction. But since the distance of the Sun is very great, compared with the radius of the Earth, it is easy to see that the diminution of gravity in each case is very nearly the same. A simple decomposition of the action of the Sun upon the particles of the ocean, is sufficient to shew, that in any position of this body, relatively to these particles, its action in disturbing their equilibrium, becomes the same after twelve hours. And it may be established as a general principle in mechanics, that the state of a system of bodies, in which the primitive conditions of motion have disappeared by the resistances it meets with, is periodic, like the forces which solicit it. The state of the ocean should therefore be the same at each interval of half a-day, so that the tide should ebb and flow in this interval.

The law according to which the water rises and falls, may be thus determined. Let us conceive a vertical circle, whose circumference represents half a day, and whose diameter is equal to the whole tide, or the difference between the height of high and low water, and let the arcs of this circumference, reckoning from the lowest point, express the time elapsed since low water, the versed sines of these arcs will express the heights of the water, corresponding to these times. Thus, the ocean in rising, covers in equal times, equal arcs of this circumference. This law is exactly observed in the middle of the ocean, which is free on every side, but in our harbours, local circumstances produce some deviations. The sea employs rather a longer time to fall than to rise, which difference at Brest amounts to about * $10\frac{1}{2}$ minutes.

The greater the extent of the surface of the water, the more perceptible are the phenomena of the tides. In a fluid mass, the impressions which a fluid particle receives, are communicated to the whole. It is thus that the action of the Sun, which is insensible on an insulated particle, produces on the ocean such remarkable effects. Let us imagine, at the bottom of the sea, a curved canal, terminated at one of its extremities by a vertical tube, rising above the surface of the water, and which, if prolonged, would pass through the centre of the Sun.

The water will rise in this tube by the direct action of the Sun, which diminishes the gravity of its particles, and particularly by the pressure of the particles enclosed in the canal, which all make an effort to unite themselves beneath the Sun. The elevation of the water in the tube

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* 14′ 27″.

above the natural level of the sea, is the integral of all these infinitely small efforts. If the length of this canal is increased, this integral also becomes greater, because it extends over a larger space, and because there will be a greater difference in the quantity and direction of the forces, by which the extreme particles are sollicited.

By this example we see the influence which the extent of the sea has upon the phenomena of the tides, and the reason why they are insensible in the very small seas, as the Euxine and the Caspian. The magnitude of the tides depends also much on local circumstances. The oscillations of the ocean, when confined in a narrow channel, may become extremely great, and these may be augmented by the reflection of the waters from the opposite shore. It is thus, that the tides, very small in the South Sea islands, are very considerable in our harbours.

If the ocean covered a spheroid of revolution, and experienced no resistance to its motion, the instant of high water would be that of the passage of the Sun over the superior or inferior meridian; but it is not thus in nature; local circumstances produce great variations in the times of high water, even in harbours that are very near each other. To have a just idea of these variations, we may suppose a large canal communicating with the sea, and extending into the land; it is evident that the undulations which take place at its entrance, will be propagated successively through its whole length, so that the figure of its surface will be formed by the undulations of large waves in motion, which will be incessantly renewed, and will describe the whole length of the interval of half-a day. These waves will produce at every point of the canal, a flux and reflux, which will follow the preceding laws, but the hours of the flowing will be retarded, in proportion as the points are further from the entrance of the canal. What we have here said of a canal, may be applied to rivers whose surfaces rise and fall by similar waves, notwithstanding the contrary motion of their waves. These waves are observed in all rivers near their *embouchure*. They are propagated to great distances in great rivers; and at the strait of Pauxis, in the river of the Amazons, it is sensible at two hundred leagues from the sea. Let us next consider the action of the Moon, which we will suppose to move uniformly in the plane of the equator. It is evident that it must excite in the ocean, an ebb and flow similar to that resulting from the action of the Sun, and whose period is half a lunar day. Now it has been shewn in the preceding Book, that the total motion of a system agitated by very small forces, is equal to the sum of the partial motions, which every force would have impressed separately; the two partial tides, therefore, produced by the action of the Sun and Moon, combine without deranging each other, and from their combination results the tides, which we observe in our ports.

From hence arise the most remarkable phenomena of the tides. The instant of the lunar tide is not the same with that of the solar tide, since their periods are different. If these two tides coincide, the following lunar tide will retard upon the solar tide, by the excess of half a lunar day, above half a solar day, that is * 1752″ 5. These retardations accumulating from day to day, the full lunar tide will finish by coinciding with the low solar tide, and reciprocally. When the lunar and solar tides coincide, the combined tide is the greatest, which happens about the sysigies. The combined tide is the least, when the full tide, relative to one of the bodies, coincides with the low tide of the other, which is the reason of the tides being least at the quadratures. If the solar tide exceeded the lunar tide, it is clear that the hours of the greatest and least combined tides, would coincide with the times at which the solar tide would happen, if it alone existed. But if the lunar tide exceeds the solar tide, then the least combined tide coincides with the low solar

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* 9′ 27″ 8.

tide, and consequently its time is a quarter of a day's interval from the hour of the greatest combined tide. This offers an easy method of deciding, if the lunar tide is greater or less than the solar tide. All the observations concur in making the hour of the least tides differ by a quarter of a day from that of the greatest tide, which prove that the lunar tide exceeds the solar tide.

We have seen, in the first Book, that the mean value of the greatest tide in every month, is nearly 5^{me}.888, and that the mean least tide is 2^{me}.789. It is easy to conclude, after the requisite reductions, that the mean lunar tide, that which corresponds to the constant part of the parallax of the Moon, is three times less than the mean solar tide; or, in other words, that the action of the Moon, to elevate the waters of the ocean, is three times as great as that of the Sun.

The extent of the variation of the total tides, taken at the maximum, or minimum, is exactly the same by the theory of gravity, as by observation. Their increase, in departing from the minimum, is the double of their diminution, in departing from the maximum, as the observations indicate. Since the lunar tide exceeds the solar tide, the combined tide should be regulated chiefly by the lunar tide, and in a given time there should be as many tides as passages of the Moon over the superior or inferior meridian, which is conformable with what we observe. But the instant of the combined tide should oscillate round the instant of the lunar tide, according to some law depending on the phases of the Moon, and of the ratio of its action, to that of the Sun. The first of these instants precedes the second, from the greatest to the least tides, and follows it from the least to the greatest, so that the mean time of the combined tide being the same as the lunar tide, the mean retardation of the tides in one day is * 3505″.

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* 18′ 55″ 6.

According both to theory and observation, the height of the tides, and their retardation, vary according to the phases of the Moon. The least retardation coincides with the greatest height, and the greatest retardation with the least height, and the theory, by a remarkable coincidence, gives these retardations * 2705″ and † 5207″, the same as results from observation.

This agreement proves the justness of the theory, and the exactness of the supposed ratio between the actions of the Sun and the Moon. In changing this ratio a small quantity, a great discordance would arise in the heights and retardations, which therefore are capable of giving us this ratio with great precision.

We may here make an important remark, on which depends the explanation of several phenomena relating to the tides. If the spheroid which the sea covers, was a solid of revolution, the partial tides

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* 14′ 36″ 4.   † 28′ 7″.

would take place at the instant of the passages of their respective bodies over the meridian. Thus, when the sysygies happened at noon, the two tides, lunar and solar, would coincide with this instant, which would be that of the greatest combined tide. This greatest tide would take place at the same day as the sysigy; if the two partial tides followed nearly by the same interval, the passage of their respective bodies over the meridian. But the daily motion of the Moon in its orbit being considerable, the rapidity of this motion may influence very sensibly the interval between the passage of the Moon and the lunar tide.

We may form a just idea of this phenomenon, by imagining as above, a vast canal communicating with the ocean, and advancing very far into the continent, under the meridian of its *embouchure*. If we suppose that at this *embouchure*, the full tide takes place at the instant of the passage of the heavenly body over the meridian, and that it employs * twenty-one hours to arrive at its extremity, it is evident that at this last point, the solar tide will happen one hour after the passage of the Sun over the meridian. But two lunar days forming † 2.070 solar days, the lunar tide will only be ‡ 30′ later than the passage of the Moon; thus there will be § 70′ difference in the intervals of the solar and lunar tides, after their respective passages over the meridian.

From hence it follows that the maximum and minimum of the tides does not take place on the very days of the sysigy and quadrature, but one or two days after, when the interval of the lunar tide after the passage of the Moon, added to the interval of the passage of the Moon after that of the Sun, is equal to the interval of the solar tide, after the passage of the Sun. Thus, in the preceding example, the maximum and minimum, which, at the *embouchure* of the canal,

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* 50^h 24′. † 2^d 40′ 48.″ ‡ 16′ 12.″ § 37′ 48″.

take place the day of the sysygy and quadrature, will not arrive at its extremity till * twenty-one hours afterwards.

I have found, by comparing a great number of observations, by different methods, that at Brest the interval by which the greater tide follows the sysygy, is very nearly a day and a half. Hence it follows, that in this port the solar tide follows the passage of the Sun, † 18358″ and that the lunar tide follows the passage of the Moon ‡ 13101″. The hours of the tide at Brest are therefore the same as at the extremity of a canal, which should communicate with the ocean, if we suppose that at its *embouchure*, the partial tides take place at the instant of the passage of their respective bodies over the meridian, and that they employ a day and a half to arrive at its extremity, supposed 18358″ more to the eastward, than its *embouchure*. And in general, both observation and theory have shewn me, that each of the ports in

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* $2^d\ 2^h\ 24'$. † $4^h\ 24'\ 21''$. ‡ $3^h\ 8'\ 39''$.

France, may be considered relatively to the tides, as the extremity of a canal, at whose *embouchure* the partial tides take place at the very instant of the passage of the Sun and Moon, and are transmitted in a day and a half to its extremity, supposed situated to the eastward of its *embouchure,* by a quantity very different for the different ports.

It may be observed that the difference in the intervals, by which the partial tides follow the meridian passages of the bodies which produce them, do not essentially change any of the phenomena of the tides. For a system of bodies moving in the equator, it only retards one day and a half the phenomena, which would take place by calculation from an hypothesis, in which these intervals are nothing. Many philosophers have attributed the retardation of the phenomena of the tides, relatively to the phases of the Moon, to the time employed by its action, to transmit itself to the Earth. But this hypothesis is incompatible with the inconceivable activity of the attractive force, the proof of which will be shewn at the end of the Book ; it is not therefore to the time employed in this transmission, but to that necessary to communicate the original impressions through the ocean to our ports, that we are to attribute this delay.

The power of a celestial body to raise a particle of water placed between it and the centre of the earth, is equal to the difference of its action on the centre and on the particle; and this difference is double the quotient of the mass of the heavenly body, multiplied by the terrestrial radius, and divided by the cube of the distance of the centres of the celestial body and the Earth. This quotient relatively to the Sun, is by the Fifth Chapter, the hundred and seventy-ninth part of the force of gravity, which solicits the Moon towards the Earth, multiplied by the proportion of the terrestrial radius, to the distance of the Moon ; this force of gravity is very nearly equal to the sum of the masses of the Earth and Moon, divided by the square of the lunar distance; the power of the Sun to raise the waters of the sea is therefore eighty-nine times and a half less than the sum of the masses of the Earth and Moon, multiplied by the terrestrial radius and divided by the cube of the lunar distance.

But this force according to observation, is only a third of the force of the Moon, which is equal to double its mass, multiplied by the terrestrial radius, and divided by the cube of its distance; thus the mass of the Moon is to the sum of the masses of the Earth and Moon as 3 is to 179 ; from whence it follows that this mass is very nearly $\frac{1}{58.7}$ of that of the Earth. Its volume being only $\frac{1}{49.316}$ of that of the Earth, its density is 0.8401, the mean density of the Earth being taken as unity; and the weight which on the Earth is unity, transported to the surface of the Moon, would be reduced to 0.2291.

Nevertheless the irregularity in the depth of the sea, which, as has been shewn, produces a perceptible difference in the interval by which the lunar and solar tides follow the transits of their respective bodies over the meridian, may likewise influence the proportional altitudes of these two tides.

Let us imagine a port situated at the junction of two canals, communicating with the sea under the same meridian; let us also suppose that at their *embouchure,* the partial tide of each celestial body, happens at the very instant of its transit over the meridian. The tide in the port will be the result of the tides transmitted to it by each canal; if the tide employs one day to pass from the sea to the port by the first canal, and eight days and a half by the second, the difference of these intervals, being seven days and a half, the two solar tides of each canal will coincide in the port, and the compound solar tide will be equal to their sum. But as seven solar days and a half only produce seven lunar days and a quarter; the full lunar tide of the first canal should coincide with the last lunar tide of the second; thus the lunar tide in the port will be only the difference of the lunar tides, transmitted by the two canals. Supposing therefore that at the *embouchures* the tides may be proportional to the force of the celestial bodies, they will cease to be so in the port, where it may even happen that the lunar tide may be weaker than the solar.

It is important, therefore, when we wish to ascertain the proportional forces of the Sun and Moon from the phenomena of the tides, to be assured that the observed tides are in proportion to these forces. Analysis furnishes different means for this object; applying them to the observations made at Brest, I found that this proportion existed in a very approximate manner; thus the value which we have assigned to the mass of the Moon, should differ very little from the true value.

Hitherto we have supposed the Sun and Moon moved uniformly on the plane of the equator. Let us now vary their motions and their distances from the centre of the Earth. In developing the expressions of their actions upon the ocean, we may represent each term by the action of a body moved uniformly in a circle round the Earth, it will then be easy, by the principles already explained, to determine the flux and reflux of the ocean, corresponding to the different inequalities of the Sun and Moon. In submitting in this manner the phenomena of the tides to analysis, it is found that the tides produced by the Sun and Moon augment in an inverse ratio to the cubes of their distances. The tides ought therefore, *ceteris paribus,* to increase in the perigee and diminish in the apogee of the Moon. This phenomenon is extremely apparent at Brest; by examining the observations, I find that * 100″ of variation in the semi-diameter of the Moon, answers to half a metre of variation in the total tide, when the Moon is in the equator; and this result of observation is so conformable to that given by the theory, that we might from this alone have found

* 32″4.

the law of the variation of the Moon's action relative to its distance. The variations in the distance of the Sun from the Earth, are sensible in the heights of the tides, but in a much less degree than those of the Moon, because its action in elevating the waters is three times less, and its distance from the Earth varies in a less ratio. This result of the theory is also verified by observation. The action of the Moon being greater and its motion more rapid, when it is nearer the Earth, the combined tide in the sysygies perigee, ought to approximate to the lunar tide, which ought itself to approximate to the passage of the Moon; for we have seen that the partial tide approaches so much nearer to the body that causes it as its motion is more rapid, the tide's perigee on the day of the sysygy ought therefore to advance, and the tide's apogee to retard. It has been mentioned in the First Book that according to observation, every † minute of

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* 32″4.

increase or diminution in the lunar semi-diameter advances or retards the high tide by * 354″, which is very nearly what results from the theory. The parallax of the Moon influences also the interval of two consecutive tides of the morning and evening, about the sysygies, or in the vicinity of the maximum of the tides. According to the theory a minute of variation in the semi-diameter of the Moon, produces a variation in this interval of †258″, which is exactly confirmed by observation.

This phenomenon equally takes place at the quadratures, but the theory shews that it is three time less than at the sysygies, and this answers to the observations. To conceive this we must recollect that the daily retardation of the lunar tide, augments as the Moon's motion is the more rapid, as happens at the perigee; and that the retardation of the tides at the sysygies augments and approaches the

* 1′54″. † 1′23″.

daily retardation of the lunar tide, when the lunar force augments; these two causes, therefore combine in augmenting the interval of the tides at the sysigies' perigee. At the quadratures, when the lunar force increases, the daily retardation of the tide diminishes, by approaching the retardation of the lunar tide; thus the interval of the tides is increased by the rapidity of the motion of the Moon perigee, and diminished by the increase of the lunar force: these two causes, therefore, acting, then, in opposite directions, the increase in the retardation of the tide is only the effect of their difference, and for this reason is less than in the sysygies. Thus, having developed the theory of the tides, upon the supposition that the Sun and Moon move in the plane of the equator, we shall next consider the motion of these bodies, such as they really are in nature, and we shall see that new phenomena arise from their change of declination, which, compared with observation, tend still more to confirm the preceding theory.

This complicated case may be reduced to that of several bodies moved uniformly in the plane of the equator, but we must give to these bodies very different motions in their orbits. Some moving very slowly, produce a flux and reflux, whose period is half a day, others have a revolution equal to half the revolution of the earth, and they produce a flux and reflux, whose period is a day. Others have a revolution nearly equal to the *rotation of the Earth.* They produce a flux and reflux whose periods are of a month and of a year.

Let us examine these three species of tides.

The first contains not only the oscillations which we have considered above, and which depend on the motion of the Sun and Moon, and on the variations of their distances from the Earth, but likewise others depending on their declinations. In submitting these to analysis, we find, that the total tides of the equinoctial sysygies are greater than those of the solstitial sysygies, in the ratio of radius, to the square of the cosine of the declination of the Sun, or of the Moon, about the period of the solstices; we find, moreover, that the tides of the solstitial quadratures exceed those of the equinoctial quadratures, in a greater ratio than that of radius to the square of the cosine of the declination of the Moon, about the equinoctial quadratures. All the observations confirm these theoretical results, and leave no doubt of the diminution of the action of the bodies, as they deviate more or less from the equator.

The declinations of the Sun and Moon sensibly affect even the law of the diminution and augmentation of the tides, reckoning from the *maximum* or *minimum*. Both by theory and observation, their diminution is about one-third more rapid in the equinoctial sysygies, than in the solstitial. Their increase, both by theory and observation, is about twice as rapid in the equinoctial quadrature, as in those of the solstice.

The position of the nodes is likewise sensible in the height of the tides, by its influence on the declinations of the Moon.

The motion of the Moon, in right ascension, being more rapid in the solstices than at the equinoxes, must bring the lunar tide nearer to the Moon's meridian passage; the hour of the tides in the equinoctial sysygies should retard upon the hour of the solstitial sysygies; for the same reason, the hour of the tide, at the solstitial quadratures, should retard on that of the equinoctial quadratures; the theory gives this latter retardation about quadruple that of the first.

The declinations of the Sun and Moon influence likewise the daily retardation of the equinoctial and solstitial tides; it should be greater about the solstitial sysygies, than at the equinoctial, and still greater about the equinoctial quadrature, than at the solstitial; and in this second case, the difference in the retardation is four times greater than in the first: and observation confirms, with remarkable precision, all these theoretical results. The tides of the second class, whose period is a day, are proportional to the product of the sine, by the cosine of the declination of the body. They are nothing when it is in the equator, and they increase as it departs from it. By being combined with the tides of the first class, they render the two tides of the same day unequal. It is for this reason, that the morning tide at Brest is about $0^{me}.183$ greater than the evening tide, about the sysygies of the winter solstice, and less by the same quantity about the summer solstice, as has been observed in the First Book: for the same cause the morning tide is greater by $0^{me}.136$, about the equinoctial quadratures of autumn, and less by the same quantity at the equinoctial quadratures in the spring.

In general, the tides of the second class are not very considerable in our harbours, their magnitude is an arbitrary quantity, depending on local circumstances, which may augment them, and diminish at the same time the tides of the first class, so as to render the former insensible. For, let us conceive a large canal communicating by its two extremities with the ocean, the tide in a port situated on the border of this canal, will be the result of the undulations transmitted through its two *embouchures*. Now it may happen that the undulations of the first class arrive at such a time, that the maximum of one may coincide with the minimum of the other, and if these should be equal, it is clear that there would be neither flux nor reflux in this port, in consequence of these undulations. But there will be a flux produced by the undulations of the second class, which, having a period twice as long, will not correspond in such a manner, as for the maximum of those which enter by one embouchure, to coincide with the minimum of those which enter by the other.

In this case, there will be no tide when the Sun and Moon are in the plane of the equator; but it will become sensible when the Moon deviates from this plane, and then there will be one flux and one reflux in each lunar day, so that if the flux arrives at the setting of the Moon, the reflux will take place when it rises. This singular phenomenon has been observed at Batsha, a port in the kingdom of Tunquin, and in some other places. It is probable, that observations made in the different parts of the world, would afford all the intermediate varieties between the tides of Batsha, and those of our ports.

Let us consider, finally, the tides of the third class, whose periods are very long, and independant of the rotation of the Earth. If the length of this period was infinite, these tides would have no further effect than to change the permanent figure of the ocean, which would soon arrive at that state of equilibrium due to the forces which produce them. But it is evident that the length of these periods will produce nearly the same effect on the tides, as in the case of its being infinite. We may therefore consider the ocean as constantly in equilibrio, under the action of fictitious bodies, which produce tides of the third class, which may be determined in this hypothesis. These tides are very small, but are nevertheless sensible at Brest, and correspond to the result of calculations.

I have entered into a long detail on the tides, because it is the nearest and most perceptible result of the celestial attractions to us, and one most worthy the attention of philosophers. We see, by the exposition which I have made, the agreement of the theory of the tides, founded on the law of universal gravitation, with the phenomena of the heights and interval of the tides. If the Earth had no satellite, if its orbit was circular, and situated on the plane of the equator, we should only have had, to have enable us to recognize the action of the Sun upon the ocean, the hour of high water always the same, and the law of its formation. But the action of the Moon, combining with that of the Sun, produces in the tides varieties relative to its phases, which, by their agreement with observation, give a great probability to the truth of the theory of gravitation. All the inequalities of motion, produced by the declinations and distances of these two bodies, give rise to a number of phenomena, which, being recognized by observation, place this theory out of the shadow of doubt. It is thus that the varieties in the action of causes, establish their existence.

The action of the Sun and Moon on the Earth, a necessary consequence of the universal attraction, demonstrated by all the celestial phenomena, being directly confirmed by the phenomena of the tides, ought to leave no uncertainty on the subject. It is indeed brought now to such a degree of perfection, that not the least difference of opinion exists upon the subject, among men sufficiently learned in the science of geometry and mechanics, to comprehend its relation with the law of universal gravitation.

A long series of observations, more precise than have hitherto been made, will rectify the elements already known, and fix the value of those which are uncertain; and develope phenomena which before were obscured in the errors of observation. The tides are not less interesting to understand, than the inequalities of the heavenly bodies, and equally merit the attention of observers. We have hitherto neglected to follow them with sufficient precision, because of the irregularities they present. But I can assert, after a careful investigation, that these irregularities disappear by multiplying the observations; nor is it necessary that their number should be extremely great, particularly at Brest, whose situation is very favourable for this species of observation.

I have now only to speak of the method of determining the time of high water, on any day whatever. We should recollect, that each of our ports may be considered as the extremity of a canal, at whose *embouchure* the partial tides happen at the moment of the passage of the Sun and Moon over the meridian, and employ a day and a half to arrive at its extremity, supposed eastward of its embouchure, by a certain number of hours.—This number is what I call the fundamental hour of the port. It may easily be computed from the hour of the establishment of the port, by considering this as the hour of the full tide, when it coincides with the sysygy. The retardation of the tides, from one day to another, being then * 2705″, it will be † 3951″ for one day and a half, which quantity is to be added to the hour of the establishment, to have the fundamental hour. Now, if we augment the hours of the tides at the *embouchure* by ‡ 15 hours, *plus* the fundamental hours, we shall have the hours of the corresponding tides in our ports. Thus, the problem consists in finding the hours of the tides in a place whose longitude is known, on the supposition that the partial tides happen at the instant of the passage of the Sun and Moon over the meridian. For

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* 14′ 36″. † 21′ 20″. ‡ 36^{h}.

this purpose analysis affords very simple formulæ, which are easily reduced to tables, and very useful to be inserted in the ephemerides that are destined for navigators.

CHAP. XI.

On the Stability of the Equilibrium of the Ocean.

SEVERAL irregular causes, such as hurricanes and earthquakes, agitate the sea, elevate it to a great height, and sometimes oblige it to forsake its limits. Nevertheless, observation shews us that it has a tendency to return to its former state of equilibrium, and that the friction and resistances of all kinds that it experiences, would very soon bring it to this state, without the action of the Sun and Moon. This tendency constitutes the stable equilibrium, which we mentioned in the Third Book. We have there shewn that the stability of the equilibrium of a system of bodies may be absolute, or take place, whatever small derangement it may receive; or it may be relative, and depend on the nature of the primitive disturbance. To which class belongs the stability of the ocean? This is what observation cannot teach us with absolute certainty; for, although in the almost infinite variety of disturbances to which the ocean is liable, from the action of irregular causes, it may appear to return to its former state of equilibrium; yet we may nevertheless apprehend, that some extraordinary cause may communicate to it a shock, which though inconsiderable at its origin, may augment continually, and elevate it above the highest mountains: this would explain several phenomena in natural history. It is therefore interesting to investigate the conditions which are necessary for the absolute stability of the ocean, and to examine if these conditions exist in nature. In submitting this object to analysis, I have assured myself that this equilibrium is stable, if its density is less than the mean density of the Earth, which is extremely probable, for it is natural to think, that the strata are more dense as they approach the centre. We have besides seen, that this is proved by experiments on pendulums, by the measurement of degrees, and by the attractions of mountains. It appears then, that the equilibrium of the ocean is stable, and if, (as seems certain) the waters have formerly covered continents, which at present are elevated much above its level, we must not search for the cause in the want of stability in their equilibrium. I have likewise discovered, by the means of analysis, that this stability would cease to exist, if the mean density of the sea exceeded that of the earth, so that the stability of the equilibrium of the ocean, and the excess of the density of the terrestrial globe above that of the waters which cover it, are reciprocally connected one with the other.

CHAP. XII.

Of the Oscillations of the Atmosphere.

To arrive at the ocean, the action of the Sun and Moon must traverse the atmosphere, which must necessarily be subject to their influence, and experience similar oscillations to those of the ocean. From hence arise winds and variations in the barometer, the periods of which are the same as those of the flux and reflux of the ocean. But these winds are very inconsiderable, and almost insensible, in an atmosphere so much agitated by other causes. The extent in the oscillations of the barometer, is only one millimetre at the equator, where it is the greatest. Nevertheless, as local circumstances may considerably augment the oscillations of the ocean, they may equally increase the oscillations of the barometer, the observation of which merits the notice of philosophers.

We may here remark, that the action of the Sun and Moon, produce neither in the ocean nor in the atmosphere, any motion from east to west. That which is observed in the atmosphere, between the tropics, under the appellation of the trade-winds, proceeds therefore from some other cause —this seems to be the most probable :

The Sun, which we will suppose, for the sake of simplicity, in the plane of the equator, there rarifies by its heat the columns of air, and elevates them above their natural level, they should then re-descend by their weight, and be carried towards the poles in the superior part of the atmosphere, but at the same time, a current of cool air should arrive from the climates near the poles, to replace that which has been rarefied at the equator. Thus, two opposite currents of air are established, one in the inferior, the other in the superior part of the atmosphere. But the real velocity of the air, due to the rotation of the Earth, is so much the less as it is nearer the pole; it ought therefore, in advancing towards the equator, to turn slower than the corresponding parts of the Earth, and bodies placed at the terrestrial surface, should strike against it with the excess of their velocity, and experience by its re-action a resistance contrary to their motion of rotation: thus, to an observer who thinks himself immovable, the wind seems to blow in a direction opposite to the rotation of the Earth, that is, from west to east, which in fact is the direction of the trade winds.

If we consider all the causes which disturb the equilibrium of the atmosphere, its great mobility arising from its fluidity and elasticity; the influence of heat and cold on its elasticity; the great mass of vapour that it alternately absorbs and deposes; and lastly, the changes which the rotation of the Earth produces in the relative velocities of its particles, which for this reason are displaced in the direction of the meridians; we should not be surprized at the inconstancy and variety of its motions, which it would be very difficult to subject to any fixed and certain laws.

CHAP. XIII.

Of the Precession of the Equinoxes, and of the Nutation of the Axis of the Earth.

EVERY part of nature is linked together, and its general laws connect phenomena with each other, which, in appearance, have not the most remote analogy. Thus, the rotation of the terrestrial spheroid compresses the poles, and this compression, combined with the action of the Sun and Moon, produces the precession of the equinoxes, which, before the discovery of universal gravitation, did not appear to have any connection with the motion of the Earth.

Let us suppose this planet to be an homogeneous spheroid, protuberant at the equator, it may then be considered as composed of a sphere of a diameter equal to the axis of the poles, and of a meniscus surrounding the sphere, and whose greatest thickness corresponds with the equator of the spheroid. The particles of this meniscus may be considered as so many small moons adhering together, and which make their revolutions in a period equal to the revolution of the Earth on its axis.

The nodes of all their orbits should therefore have a retrograde motion, arising from the action of the Sun, in the same manner as the nodes of the lunar orbit; and from the connection of these bodies together, there should succeed a retrograde motion of the whole meniscus; but this meniscus divides its retrograde motion, with the sphere to which it is attached, which, for this reason, becomes slower; the intersection of the equator and the ecliptic, that is to say, the equinoctial points, should have a retrograde motion. Let us endeavour to investigate both the law and the cause of this phenomena.

And first we will consider the action of the Sun upon a ring, situated in the plane of the equator. If we conceive the mass of the Sun to be distributed uniformly over the circumference of its orbit, (supposed circular) it is evident that the action of this solid orbit will represent the mean action of the Sun. This action, upon every one of the points of the ring above the ecliptic, being decomposed into two, one in the plane of the ring, and the other perpendicular to it, it follows that the resulting force, arising from these last actions, on all the particles of the ring, is perpendicular to its plane, and situated on its diameter, which is perpendicular to the line of its nodes. The action of the solar orbit, on the part of the ring below the ecliptic, equally produces a resulting force, perpendicular to the plane of the ring, and situated in the inferior part of the same diameter. These two resulting forces combine to draw the ring towards the ecliptic, by giving it a motion round the line of nodes; its inclination, therefore, to the ecliptic, would be diminished by the mean action of the Sun, the nodes all the time continuing stationary; and this would be the case but for the motion of the ring, which we now suppose to turn round in the same time as the Earth. By this motion, the ring is enabled to preserve a constant inclination to the ecliptic, and to change the effect of the action of the Sun into a retrograde motion of the nodes. It gives to the nodes a variation, which otherwise would be in the inclination, and it gives to the inclination a permanency, which otherwise would rest with the nodes. To conceive the reason of this singular effect, let us suppose the situation of the ring varied an infinitely small quantity, in such a manner that the planes of its two positions intersect each other, in a line perpendicular to the line of nodes.

At the end of any instant whatever, we may decompose the motion of each of its points into two, one of which should subsist alone in the following instant, the other perpendicular to the plane of the ring, and which should be destroyed. It is evident that the resulting force of these second motions, relative to all the points of the upper part of the ring, will be perpendicular to its plane, and placed on the diameter which we just now considered, and this is equally true for the lower part of the ring. That this resulting force may be destroyed by the action of the solar orbit, and that the ring, by virtue of these forces, may remain in equilibrio on its centre, it is requisite that these forces should be contrary to each other, and their moments, relatively to this point, equal. The first of these conditions requires that the change of position, supposed to be given to the ring, be retrograde; the second condition determines the quantity of this change, and consequently the velocity of the retrograde motion of the nodes. And it is easily demonstrated, that this velocity is proportional to the mass of the Sun, divided by the cube of the distance from the Earth, and multiplied by the cosine of the obliquity of the ecliptic.

Since the planes of the ring, in its two consecutive positions, intersect each other in a diameter perpendicular to the line of its nodes, it follows that the inclination of these two planes to the ecliptic, is constant, and the inclination of the ring does not vary, by the mean action of the Sun.

That which has been explained relatively to a ring, may be demonstrated by analysis, to hold true in the case of a spheroid, differing but little from a sphere. The mean action of the Sun produces in the equinoxes a motion proportional to its mass, divided by the cube of its distance, and multiplied by the cosine of the inclination to the ecliptic. This motion is retrograde when the spheroid is flattened at the poles; its velocity depends on the compression of the spheroid, but the inclination of the equator to the ecliptic, always remains the same.

The action of the Moon produces likewise a similar retrogradation of the nodes of the terrestrial equator, in the plane of its orbit; but the position of this plane and its inclination to the equator incessantly varying, by the action of the Sun and the retrograde motion of the nodes produced by the action of the Moon, being proportional to the cosine of this inclination, this motion is variable.

Besides, in supposing it uniform, it would, according to the position of the lunar orbit, cause a variation both in the retrograde motion of the equinoxes, and in the inclination of the equator to the ecliptic. A calculation, by no means difficult, is sufficient to show, that the action of the Moon, combined with the motion of the plane of its orbit, produces 1. A mean motion in the equinoxes, equal to that which it would produce if it moved in the plane of the ecliptic. 2. An inequality subtractive, from this retrograde motion, and proportional to the sine of the longitude of the ascending node of the lunar orbit. 3. A diminution in the obliquity of the ecliptic, proportional to the cosine of this same angle. These two inequalities are represented at once by the motion of the extremity of the terrestrial axis (prolonged to the heavens) round a small ellipse, conformably to the laws explained in Chap. XI. of Book I.

The greater axis of this ellipse is to the lesser, as the cosine of the obliquity of the ecliptic is to the cosine of double this obliquity. We may comprehend from what has been said, the cause of the precession of the equinoxes, and of the nutation of the Earth's axis, but a rigorous calculation, and a comparison of its results with observation, is the true test of the truth of a theory. That of universal gravitation is indebted to d'Alembert, for the advantage of having been thus verified in the case of the two preceding phenomena. This great mathematician first determined by a beautiful analysis the motions of the axis of the Earth, by supposing the strata of the terrestrial spheroid to be of any density or figure whatever, and he not only found his results exactly conformable to observation, but obtained an accurate determination of the dimensions of the small ellipse described by the pole of the

Earth, as to which the observations of Bradley had left some little doubt.

The influence of a heavenly body, either upon the motion of the terrestrial axis of the Earth, or upon the ocean, is always proportional to the mass of that body, divided by the cube of the distance of that body from the Earth. The nutation of the Earth's axis being due to the action of the Moon alone, while the precession of the equinoxes arises from the combined action of the Sun and Moon, it follows that the observed values of these two phenomena, should give the ratio of their respective actions. If we suppose, with Bradley, the annual precession of the equinoxes to be *154″4, and the whole extent of the nutation †55″6, the action of the Moon is found to be double that of the Sun. But a very small difference in the extent of the nutation, produces a very considerable one in the ratio of the actions of these two bodies, to make it equal three

* 50″. † 17″9.

to one, as indicated by the observations of the tides, it is sufficient to suppose the extent of the nutation *62.2. Dr. Maskelyne, by re-examination of the observations of Bradley, finds this quantity †58″6, which differs but ‡3″6 from the result obtained by the phenomena of the flux and reflux of the ocean. So small a quantity being nearly insensible in the observations of the fixed stars, the ratio of the solar and lunar action is better determined by that of the tides; it seems to me, therefore, that we should fix the equation of the nutation at §31″1, that of the precession at 58″2, and the lunar equation of the tables of the Sun, ¶27″5. The phenomena of the precession and of the nutation throws a new light on the constitution of the terrestrial spheroid. They gave a limit to the compression of the earth supposed elliptic, hence it appears that this compression does not exceed $\frac{1}{305}$, which accords with the experiments that have

* 20″ 1. † 18″ 9. ‡ 1″ 1. § 10″. ¶ 8″ 9.

been made on pendulums. We have seen in Chap. VII. that there exists in the expression of the radius vector of the terrestrial spheriod, terms, which, but little sensible in themselves, and on the length of the pendulum, cause the degrees of the meridian to deviate considerably from the elliptic figure. These terms disappear entirely in the values of the precession and nutation, and for this reason, these phenomena agree with the experiments on pendulums. The existence of these terms, therefore reconciles the observations of the lunar parallax, those of the pendulums and degrees of the meridian, and the phenomena of precession and nutation.

Whatever figure and density we suppose in the strata of the Earth, whether or not it be a solid of revolution, provided it differs little from a sphere, we may always assign an elliptic solid of revolution, with which the precession and nutation will be the same. Thus in the hypothesis of Bouguer, of which we have spoken Chap. VII., and according to which the increase of the degrees varies as the fourth power of the sine of the latitude, these phenomena are exactlythe same as if the Earth were an ellipsoid whose ellipticity was $\frac{1}{183}$, but we have seen that observations do not permit us to suppose a greater ellipticity than $\frac{1}{305}$, so that these observations, and the experiments on pendulums, combine to disprove the hypothesis of Bouguer.

We have hitherto supposed the Earth entirely solid, but, this planet being covered in great part by the waters of the ocean, ought not their action to change the phenomena of the precession and nutation ? This question it is of importance to consider.

The ocean in consequence of its fluidity is obedient to the action of the Sun and of the Moon. It seems at first sight that their re-action should not affect the axis of the Earth. D'Alembert and every mathematician since, who has investigated these motions, have entirely neglected it, they have even commenced from that point to reconcile the observed quantity of the precession and nutation, with the measure of the terrestrial degrees. Nevertheless a more profound examination of this question has shewn us, that the fluidity of the sea is not a sufficient reason to neglect their effect in the precession of the equinoxes; for if on one hand, they obey the action of the Sun and Moon, on the other, the force of gravity tends to bring them back without ceasing, to a state of equilibrium, and permits them to make but small oscillations; it is therefore possible, that by their attraction and pression on the spheroid which they cover, they may communicate at least in part, the same motion to the axis of the Earth, which they would if they could possibly become solid. Besides we may by simple reasoning, be convinced that their action is of the same order as the action of the Sun and Moon, on the solid part of the Earth.

Let us imagine this planet homogeneous and of the same density as the ocean, and moreover that the waters take at every instant the figure that is requisite for the equilibrium of the forces that animate them. If in these hypotheses the Earth should suddenly become entirely fluid, it would preserve the same figure and all its parts would remain in equilibrio, and the axis of the Earth would have no tendency to move, and it is plain that the same should be the case, if a part of this mass should form, by becoming solid the spheroid which the ocean covers. The preceding hypotheses serve as a foundation to the theories of Newton, relative to the figures of the Earth, and of the tides.

It is remarkable, that among the infinite number of those which may be chosen on this subject, this great geometrician has selected two which give neither the precession nor the nutation; the re-action of the waters destroying the effect of the action of the Sun and Moon, upon the terrestrial nucleus, whatever may be its figure. It is true that these two hypotheses, particularly the last, are not conformable to nature, but we may see, *à priori,* that the effect of the re-action of the waters, although different from that which takes place in the hypothesis of Newton, is nevertheless of the same order.

The investigations which I have made on the oscillations of the ocean, have enabled me to determine this effect of the re-action of the waters in the true hypothesis of nature, and have led to this remarkable theorem.

Whatever may be the law of the depth of the ocean, and whatever the figure of the spheroid which it covers, the phenomena of the precession and nutation will be the same as if the ocean formed a solid mass with this spheroid.

If the Sun and Moon acted only on the Earth, the mean inclination of the equator to the ecliptic would be constant, but we have seen that the action of the planets continually changes the position of the terrestrial orbit: and produces a diminution of its obliquity to the equator, which is fully confirmed by observations ancient and modern, the same cause gives to the equinoxes a direct annual motion of 0″5707; thus the annual precession produced by the action of the Sun and Moon is diminished by this quantity in consequence of the action of the planets; without this action it would be * 155″20. These effects of the action of the planets are independent of the compression of the terrestrial spheroid, but the action of the Sun and Moon, upon the spheroid, modifies these effects and changes their laws.

If we refer to a fixed plane the position of the orbit of the Earth, and the motion of its axis of rotation, it will appear, that the action of the Sun in consequence of the variations of the ecliptic, will produce in this axis an oscillatory motion similar to the nutation, but with this difference, that the period of these variations being incomparably longer than that of

* 50″ 3.

the variations of the plane of the lunar orbit, the extent of the corresponding oscillation in the axis of the Earth is much greater than in the nutation. The action of the Moon produces in this same axis a similar oscillation, because the mean inclination of its orbit, to that of the Earth is constant. The displacement of the ecliptic, by being combined with the action of the Sun and Moon upon the Earth, produces upon its obliquity to the equator, a very different variation from that which would arise from this change of position only: the entire extent of this variation would be by this alteration of the ecliptic, about *12 degrees, and the action of the Sun and Moon, reduces it to about † 3 degrees.

The variation in the motion of the equinoxes, produced by these same causes, changes the duration of the tropical year in different centuries. The duration diminishes as this motion augments, which is the case

* 10° 48'. † 2° 42'.

at present, and the actual length of the year is shorter by about *12″, than in the time of Hipparchus. But this variation in the length of the year has its limits, which are restricted by the action of the Sun and Moon, upon the terrestrial spheroid. The extent of these limits would be about † 500″, by the alteration in the position of the ecliptic, but it is reduced to ‡ 120″ by this action.

Lastly, the day itself, such as we have defined it in the First Book, is subject by the displacement of the ecliptic, combined with the action of the Sun and Moon, to very small variations which are indicated by the theory, but are quite insensible to observation. According to this theory the rotation of the Earth is uniform, and the mean length of the day may be supposed constant, an important result for astronomy, as it is the measure of time, and of the revolutions of the heavenly bodies. If it should undergo any change,

* 3″ 8. † 27′. ‡ 38″ 8.

it would be recognized by the alteration in the number of these revolutions, which would be increased or diminished according to their length, but the action of the heavenly bodies does not cause any sensible alteration.

Nevertheless, it might be imagined, that the trade winds which blow constantly from east to west between the tropics, would diminish the velocity of the rotation of the Earth, by their action on the continents and mountains. It is impossible to submit this action to analysis, fortunately it may be demonstrated that this action on the rotation of the Earth is nothing, by means of the principle of the conservation of areas, which we have explained in the Third Book. According to this principle, the sum of all the particles of the Earth, the ocean and the atmosphere, multiplied respectively by the areas which their radii vectores describe round the centre of gravity of the Earth, projected on the plane of the equator, is constant in a given time.

The heat of the Sun can produce no effect, because it dilates bodies equally in every direction, and it is evident that if the rotation of the Earth should diminish, this sum would be less. Therefore the trade winds which are produced by the heat of the Sun, cannot alter the rotation of the Earth. To produce any sensible alteration in its period, some great change must take place in the parts of the terrestrial spheroid: thus a great mass taken from the poles to the equator, would make this rotation longer, it would become shorter if the denser materials were to approach the centre or axis of the Earth; but we see no cause that can displace such great masses to such great distances, as to produce any variation in the length of the day, which may be regarded as one of the most constant elements in the system of the world. It is the same with the points where the axis of rotation meets the surface. If the Earth turned round successively different diameters, making with each other considerable angles, the equator and the poles would change places on the Earth; and the ocean, flowing continually towards the new equator, would alternately overwhelm and then abandon the highest mountains; but all the investigations which I have made upon this change of position in the poles, convince me that it is insensible.

CHAP. XIV.

On the Libration of the Moon.

We have now only to explain the cause of the libration of the Moon, and of the nodes of its equator.

The Moon, in virtue of its motion of rotation, is a little flattened at its poles; but the attraction of the Earth, must have lengthened a little that axis which is turned towards it. If the Moon were homogeneous and fluid, it would (to be in equilibrio) assume the form of an ellipsoid, of which the lesser axis passed through the poles of rotation; the greater axis would be directed to the Earth, and in the plane of the lunar equator, and the mean axis would be situated in the same plane perpendicular to the other two. The excess of the greatest above the least axis would be quadruple the excess of the mean above the least, and nearly equal $\frac{1}{29,711}$, the least axis being taken as unity.

We may easily conceive that if the greater axis of the Moon deviates a little from the direction of the radius vector, which joins its centre with that of the Earth, the terrestrial attraction will tend to bring it down to this radius, in the same manner as gravity brings a pendulum towards the vertical. If the primitive motion of rotation of this satellite, had been sufficiently rapid to have overcome this tendency, the period of its rotation would not have been perfectly equal to that of its revolution, and the difference would have discovered to us successively every point in its surface. But at their origin the angular motions of rotation and revolution having differed but little; the force by which the greater axis of the Moon tended to deviate from the radius vector, was not sufficient to overcome the tendency of this same axis towards the radius, due to the terrestrial gravity, which by this means has rendered their motions rigorously equal and in the same manner as a pendulum, drawn aside from the vertical by a very small force, continually returns, making small vibrations on each side of it, so the greater axis of the lunar spheroid ought to oscillate on each side of the mean radius vector of its orbit. Hence would arise a motion of libration, of which the extent would depend on the primitive difference between the angular motions of rotation and revolution of the Moon. This difference must have been very small, since it has not been perceived by observation.

Thus we see that the theory of gravitation explains in a sufficiently satisfactory manner, the rigorous equality of these two mean motions of rotation and revolution in the Moon. It would be against all probability to suppose, that these two motions had been at their origin perfectly equal, but for the explanation of this phenomenon, it is enough that their primitive difference was but small, and then the attraction of the Earth would establish the equality which at present subsists.

The mean motion of the Moon being subject to great secular inequalities, which amount to several circumferences, it is evident that if its mean motion of rotation were perfectly uniform, this satellite would, by virtue of these inequalities, present successively to the Earth every point on its surface, and its apparent disk would change by imperceptible degrees, in proportion as these inequalities were developed; the same observers would see it always pretty nearly the same, and there would be no considerable difference but to observers separated by an interval of several ages. But the cause which has thus established an equality between the mean motions of revolution and rotation, should take away all hope from the inhabitants of the Earth, of seeing the opposite side of the lunar hemisphere. The terrestrial attraction, by continually drawing towards us the greater axis of the Moon, causes its motion of rotation to participate in the secular inequalities of its motion of revolution, and the same hemisphere to be constantly directed towards the Earth.

The same theory ought to be extended to all the satellites, in which an equality in their motion of rotation and of revolution round their planet has been observed.

The singular phenomenon of the coincidence of the nodes of the equator of the Moon, with those of its orbit, is another consequence of the terrestrial attraction. This was first demonstrated by Lagrange, who by a beautiful analysis was conducted to a complete explanation of all the observed phenomena of the lunar spheroid. The planes of the equator and of the orbit of the Moon, and the plane passing through its centre parallel to the ecliptic, have always very nearly the same intersection; the secular motions of the ecliptic, neither alter the coincidence of the nodes of these three planes, nor their mean inclination, which the attraction of the Earth constantly maintains the same.

We may observe here, that the preceding phenomena cannot subsist with the hypothesis in which the Moon, originally fluid and formed of strata of different densities, should have taken the figure suited to their equilibrium. They indicate between the axes of the Moon, a greater inequality than would take place in this hypothesis. The great inequalities which we observe at the surface of the Moon, have without doubt a sensible influence on these phenomena.

Whenever nature subjects the mean motions of the celestial bodies to determinate conditions, they are always accompanied by oscillations, whose extent is arbitrary. Thus the equality of the mean motions of revolution and rotation produce a real libration in this satellite. In like manner the coincidence of the mean nodes of the equator and lunar orbit, is accompanied by a libration of the nodes of this equator round those of the orbit, a libration so small as hitherto to have escaped observation. We have seen that the real libration of the greater lunar axis is insensible, and we have observed, (Chap. VI.) that the libration of the three satellites of Jupiter is also insensible. It is remarkable, that these librations, whose extent is arbitrary and might have been considerable, should nevertheless be so very small; we must attribute this to the same causes as originally established the conditions on which they depend.

But relatively to the arbitrary quantities, which relate to the initial motion of rotation of the celestial bodies, it is natural to think that without foreign attractions, all their parts, in consequence of the friction and resistance which is opposed to their reciprocal motion, would in process of time, acquire a permanent state of equilibrium, which cannot exist but with an uniform motion of rotation, round an invariable axis; so that observation should no longer indicate in this motion any other inequalities than those derived from these attractions. The most exact observations shew that this is the case with the Earth, the same result extends to the Moon, and probably to the other celestial bodies.

CHAP. XV.

Reflections on the Law of Universal Gravitation.

In considering the whole of the phenomena of the solar system, we may arrange them in the three following classes:

The first embraces the motions of the centre of gravity about the foci of the principal forces which animate them.

The second includes all that relates to the figure and oscillations of the fluids that surround them.

And the third comprehends the motions of these bodies round their centres of gravity. It is in this order that we have explained the different phenomena, and we have seen that they are necessary consequences of the principle of gravitation. This principle has made us acquainted with a great number of inequalities, which it would have been impossible to have unravelled by observation alone; it has furnished us the means of subjecting the heavenly motions to sure and precise rules. The astronomical tables, founded only on the theory of gravitation, borrow now from observation, only such arbitrary quantities as cannot otherwise be known, and we can only hope to add to their perfection by giving greater precision, both to our observations and our theory.

The motion of the Earth, which had obtained the assent of astronomers, from the simplicity with which it explained the celestial phenomena, has received from the principle of gravitation a new confirmation, which has carried it to the highest degree of evidence of which physical science is susceptible. We may increase the probability of a theory, either by diminishing the number of hypotheses on which it rests, or by augmenting the number of phenomena which it explains. The principle of gravity has procured these two advantages to the theory of the motion of Earth. As it is a necessary consequence of it, it adds no new supposition to this theory; but to explain the apparent motion of the stars, Copernicus admitted three distinct motions, one round the Sun, another round itself, and a third motion of its poles round those of the ecliptic. The principle of gravitation makes them all depend on one motion impressed on the Earth, in a direction not passing through the centre of gravity. In consequence of this motion, it revolves round the Sun, and on its own axis, it at the same time takes a flattened form, compressed at the poles, and the action of the Sun and Moon upon this figure, produces a slow motion on its poles, round the poles of the ecliptic. The discovery of this principle has then reduced to the least possible number, the suppositions on which Copernicus founded his theory. It has besides the advantage of connecting this theory with all the celestial phenomena. Without it, the ellipticity of the planetary orbits, the laws which the planets and comets follow in their revolution round the Sun, their secular and periodic inequalities, the numberless inequalities of the Moon, and of the satellites of Jupiter, the precession of the equinoxes, the nutation of the terrestrial axis, the motions of the lunar axis, and lastly, the ebbing and flowing of the sea, would only be insulated and unconnected phenomena. It is really a circumstance deserving our admiration, the manner in which all these phenomena, at first sight so unconnected, flow from one law which connects them with the motion of the Earth; so that, this motion once admitted, we are conducted by a series of geometrical reasoning to these phenomena. Each of them furnishes, therefore, a proof of its existence, and if we consider that there does not exist a single phenomena which cannot be referred to the law of gravity, and that this law determines with the greatest exactness the positions and motions of the heavenly bodies through the whole of their course, there will be no reason to fear that its truth will be questioned, in consequence of any phenomena hitherto unobserved ; and finally, when we see that Uranus with its satellites lately discovered, obey and confirm the same law, it is impossible to refuse assent to these proofs, and not to allow that nothing in natural philosophy is more completely demonstrated than the motion of the Earth, and the principle of universal gravitation, in proportion to the masses, and inversely as the squares of the distances.

Is this principle a primordial law of nature ? Or is it a general effect of an unknown cause ? Here we are stopped by our ignorance of the nature of the intimate properties of matter, and deprived of every hope of answering this question in a satisfactory manner. Instead of forming hypotheses on this subject, let us content ourselves with examining more particularly the manner in which this principle has been employed by philosophers.

They have admitted the following five suppositions:

1. That gravitation takes place between the most minute particles of bodies.

2. That it is proportional to their masses.

3. That it is inversely as the squares of the distances.

4. That it is transmitted instantaneously from one body to another.

5. And that it equally acts on bodies in a state of repose, and upon those which, moving in its direction, seem in part to withdraw themselves from its activity.

The first of these suppositions is, as we have seen, a necessary result of the equality which exists between action and reaction; every particle of the Earth attracting it, as the particle itself is attracted. This supposition is moreover confirmed by the measures of the degrees of the meridian, and by experiments on pendulums; for amidst all the irregularities of the measured degrees, we may perceive the traces of regular figure, which is conformable to the theory. The great influence the compression of Jupiter, has upon the nodes and perijoves of the orbits of its satellites, proves to us that the attraction of this planet is composed of the attractions of all its particles.

The proportionality of the attractive force to the masses, is demonstrated on the Earth by experiments on pendulums, the oscillations of which are of the same length of whatever substance they are composed. It is proved in the celestial regions, by the constant relation which exists between the squares of the periodic times of bodies, revolving about a common focus, to the cubes of the greater axis of their orbits.

We have seen in the First Chapter with what precision the almost absolute state of repose of the perihelia of the planetary orbits, indicate that the force of gravity varies according to the inverse square of the distance, and now that we know the cause of the motions of these perihelia, we may regard this law as rigorously exact. It is the same with all emanations which proceed from a centre, such as light; it seems as if all forces whose action is perceived at sensible distances follow this law. It has lately been observed, that the attractions and repulsions of electricity and magnetism decrease in proportion to the squares of the distances. A remarkable property of this law is, that if the dimensions of all the bodies of the universe, their mutual distances and velocities, were to be augmented or diminished proportionally, they would describe curves entirely similar to those described at present, and their appearances would be entirely the same. For the forces which animate them, being the result of attractions, proportional to the masses divided by the squares of the distances, they would augment and diminish proportionally as the dimensions of this imaginary universe. It may be remarked at the same time that this property can only belong to the law of nature. Thus the appearances of the motions of the universe, are independent of its absolute dimensions, as they are likewise of the absolute motion it may have in space, and we can only observe and recognize relative phenomena.

It is this law which gives to spheres the property of attracting each other mutually, as if their whole masses, were united at their respective centres. It terminates also the orbits and the figures of the celestial bodies, by lines and surfaces of the second order, at least if we neglect their perturbations and suppose them fluid.

We have no method of measuring the length of time in which gravity is propagated, because the action of the Sun having once attained the planets, it continues to act on them as if the attractive force was communicated instantaneously to the extremities of the system; we cannot therefore ascertain in how long a time it is transmitted to the Earth, no more than we could measure the velocity of light, were it not for the aberration and eclipses of Jupiter's satellites. But it is not the same with the small difference that may exist in the action of gravity upon bodies, according to the direction and quantity of their velocity. Analysis has shewn me, that there should result an acceleration in the mean motions of the planets round the Sun, and in the mean motions of the satellites about their planets.

I had imagined this method of explaining the secular equation of the Moon, when I believed with other geometricians that it was inexplicable on the principle of universal gravitation. I found that if it arose from this cause, we must suppose in the Moon, in order to release it entirely from its gravity towards the Earth, a velocity in the centre of this planet, at least six million times greater than that of light; the true cause of this equation being now known, we are certain that the activity of gravity is much greater than this. This force therefore acts with a velocity which we may consider as infinite, and we may conclude that the action of the Sun is transmitted in an indivisible instant to the extremities of the planetary system.

Do any other forces act on the heavenly bodies, besides their mutual attractions?

We are unacquainted with any, and we may affirm that their effect is totally insensible. We may be likewise equally certain that these bodies experience no sensible resistance from the fluids through which they pass, as light, the tails of comets, or the zodiacal light.

The attractive force disappears between bodies of inconsiderable magnitude, and reappears in their elements under a variety of different forms. The solidity of bodies, their crystallization, the refraction of light, the elevation and depression of fluids in capillary tubes, and generally all chemical combinations are the results of attractive forces, the knowledge of which forms the principal object of natural philosophy. Are these forces the same as that of the gravity observed in the celestial regions, and modified on the Earth by the figures of the integrant particles? To admit this hypothesis we must suppose much more space empty than full in all bodies, so that the density of their particles must be incomparably greater than the mean density of their whole volume. A spherical particle of one hundred thousandth of a foot in diameter, should have a density at least ten thousand millions of times greater than the mean density of the Earth, to exert at its surface an attraction equal to the terrestial gravity. But the attractive forces of bodies greatly surpass this gravity, since they inflect light, whose direction is not sensibly changed by the attraction of the Earth; the density of these particles therefore should be to that of bodies in a ratio, which the imagination would fear to admit, if their affinities depended on the law of universal gravitation. The ratio of the intervals, which separate the particles of bodies, to their respective dimensions, would be of the same order, as relatively to stars which form a nebula, which in this point of view may be considered as a great luminous body. There is no reason, however, which absolutely forbids us to consider all bodies in this manner. Many phenomena are favourable to the supposition, particularly the extreme facility with which light penetrates diaphonous bodies in all directions. The affinities would then depend on the form of the integrant particles, and we might then by the variety of these forms, explain all the variety of attractive forces, and reduce to one general law all the phenomena of astronomy and natural philosophy.

But the impossibility of ascertaining these figures, renders this investigation useless to the advancement of science. Some geometricians, to account for these affinities, have added to the laws of attraction, inversely as the squares of the distance, new terms which are insensible at small distances, but these terms would be the expressions of as many different forces, and besides being complicated with the figures of the particles, they would only complicate the explanation of the phenomena.

Amidst these uncertainties the wisest plan seems to be, to endeavour to determine by numberless experiments the laws of affinities, and to effect this, the most simple method appears to be, by comparing these forces with the repulsive force of heat, which may itself be compared with that of gravity. Some experiments already made with this view, afford us reason to hope that one day these laws will be perfectly known, and that then, by the application of analysis, the philosophy of terrestrial bodies may be brought to the same degree of perfection, which the discovery of universal gravitation has procured to astronomy.

THE

SYSTEM OF THE WORLD.

BOOK V.

Of the History of Astronomy.

THE order in which I have treated the principal results of the system of the world, is not that which the human intellect has followed in the investigation. Its progress has been embarrassed and uncertain. Frequently mankind have not arrived at the true cause of these phenomena, till all the hypotheses which imagination could suggest have been exhausted; and the truths that have been discovered, have almost always been combined with errors, which time and observation only have separated. I shall comprise in a small compass, an outline of these attempts and their success. We shall see that astronomy remained during a great many ages in its infancy, that it increased and flourished in the school of Alexandria, became then stationary till the time of the Arabs, who improved it by their observations, and lastly that it is within the three last centuries, it has rapidly risen to that state of perfection, in which we behold it at the present day.

CHAP. I.

Of the Astronomy of the Ancients; till the Foundation of the Alexandrine School.

The view of the firmament must at all times have fixed the attention of mankind, and more particularly in those happy climates, where the serenity of the air invited them to observe the stars. Agriculture required that the seasons should be distinguished and their returns known. It could not be long before it was discovered that the rising and setting of the stars, when they plunge themselves in the Sun's rays, or when they again disengage themselves from his light, might answer this purpose. Hence we find that among most nations, this species of observations may be traced back to such early times, till their origin is lost. But some rude remarks on the rising and setting of the stars, could not constitute a science. Astronomy did not commence till observations being registered and compared, and the celestial motions examined with greater care, some attempt was made to explain their motions and their laws.

The motion of the Sun in an orbit inclined to the equator ; the motion of the Moon, its phases and eclipses, the knowledge of the planets and their revolutions, and the sphericity of the Earth, were probably the objects of this ancient astronomy, but the few monuments that remain of it are insufficient to ascertain either its epoch or its extent. We can only judge of its great antiquity, by the astronomical periods which it has transmitted to us, by some just notions which the Egyptians and Chaldeans seem to have had of the system of the world, and by the exact relation of the ancient measures to the circumference of the Earth. Such has been the vicissitude of human affairs, that the arts by which alone the events of past ages can be transmitted in a durable manner, being of modern invention, the remembrance of the first inventors in the arts and sciences, has been entirely effaced. Great nations, whose names are hardly known in history, have disappeared from the soil which they inhabited; their annals, their language, and even their cities have been obliterated, and no remnant left of their science or their industry, but a confused tradition, and some scattered ruins, of doubtful and uncertain origin.

It appears that the practical astronomy of these early ages, was confined to the observations of eclipses, the rising and setting of the principal stars, with their occultations by the Moon and planets. The path of the Sun was followed by means of the stars which were eclipsed by the twilights, and perhaps by the variations in the meridian shadow of the gnomon. The motion of the planets was determined by the stars which they came nearest to, in their course. To distinguish these bodies, and recognize their various motions, the heaven was divided into constellations. And that zone from which the Sun, Moon and planets were never seen to deviate, was called the zodiac. It was divided into the twelve following constellations. Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus and Pisces. These were called *signs*, because they served to distinguish the seasons. Thus the entrance of the Sun into Aries, in the time of Hipparchus, marked the commencement of the spring, after which it described the other signs Taurus, Gemini, &c. but the retrograde motion of the equinoxes, changed the coincidence of the seasons; nevertheless, observers accustomed to mark the commencement of the spring by the entrance of the Sun into the sign of Aries, have continued to mark this in the same manner, and have distinguished the signs of the zodiac from the constellations, the first being ideal, and serving only to designate the course of the Sun in the ecliptic. Now that we endeavour to refer our ideas to the most simple expressions, we begin no longer to use the signs of the zodiac, but mark the positions of the heavenly bodies on the ecliptic, according to their distance from the equinoctial point.

Some of the names given to the constellations of the zodiac, appear to relate to the motion of the Sun. Cancer, for example, seems to indicate the retrogradation of this body from the solstice, and the balance denotes the equality of day and night. And other names seem to refer to the climate and agriculture of those nations to whom the zodiac owes its origin. The most ancient observations that have been transmitted to us with sufficient detail, are three eclipses of the Moon, observed at Babylon in the years 719 and 720 before the Christian æra. Ptolemy, who cites them in his Almagist, employs them in his determination of the motion of the Moon. It is certain, that neither he nor Hipparchus could obtain any that were more ancient, for the exactness of the comparison is in proportion to the interval which separates the extreme observations. This consideration should diminish our regret for the loss of nineteen hundred years of observations by the Chaldeans, and of which they boasted in the time of Alexander, and which Aristotle obtained by means of Calysthenes. But they could only have discovered the period of 6585 days, by a long series of observations. This period, called the σαρος, has the advantage of bringing back the Moon to nearly the same period, with respect to its node, its perigee, and to the Sun. Thus, the eclipses observed in one period, afford an easy method of calculating those which are to happen in the succeeding ones. The lunar-solar period of six hundred years, seems to have been known to the Chaldeans. These two periods suppose a knowledge nearly approximating to the true length of the year; it is also highly probable, that they had remarked the difference between the sidereal and tropical year, and that they were acquainted with the use of the gnomon and sun-dial. And finally, some of them were led from considering the spectacle of nature, to suppose that comets, like planets, are subject to fixed periods, which are regulated by external laws.

Astronomy is not less ancient in Egypt than in Chaldea. The Egyptians were acquainted, long before the christian æra, with the excess of the year, of one quarter of a day beyond 365 days: on this knowledge, they formed the sothic period of 1460 years, which, according to them, brought back the same seasons, months, and festivals of their years, whose length was 365 days. The exact direction of the sides of their pyramids with the four cardinal points, give us a very advantageous idea of their accuracy of observation. It is probable, that they had also methods of calculating eclipses. But that which reflects most honour to their astronomy, was the sagacious and important observation of the motion of Mercury and Venus about the Sun. The reputation of their priests attracted to them the greatest philosophers of Greece; and, according to all appearance, the school of Pythagoras is indebted to them for the sound notions they professed, relative to the system of the universe.

Among these people, astronomy was only cultivated in their temples, and by priests, who made no other use of their knowledge, than to consolidate the empire of superstition, of which they were the ministers. They carefully disguised it under emblems, which presented to credulous ignorance, heroes and gods, whose actions were only allegories of celestial phenomena, and of the operations of nature; allegories which the power of imitation, one of the chief springs of the moral world, has perpetuated to our own days, and been mingled with our religious institutions. The better to enslave the people, they profited by their natural desire of penetrating into futurity, and created astrology. Man being induced, by the illusions of his senses, to consider himself as the centre of the universe, it was easy to persuade him, that the stars influenced the events of his life, and could prognosticate to him his future destiny. This error, dear to his self-love, and necessary to his restless curiosity, seems to have been co-eval with astronomy. It has maintained itself through a very long period, and it is only since the end of the last century, that our knowledge of our true relations with nature, has caused them to disappear. In Persia and in India, the commencement of astronomy is lost in the darkness which envelopes the origin of these people. In no country do they go back so far as in China, by an incontestable series of historical monuments.

The prediction of eclipses, and the regulation of the calendar, were always regarded as important objects, for which a mathematical tribunal was established; but the scrupulous attachment of the Chinese for their ancient customs, which extended even to their astronomical rules, has contributed with them, to keep this science in a perpetual state of infancy.

The Indian tables indicate a much more refined astronomy, but every thing shews that it is not of an extremely remote antiquity. And here, with regret, I differ in opinion from a learned and illustrious astronomer, who, after having honoured his career by labours useful both to science and humanity, perished a victim to the most sanguinary tyranny, opposing the calmness and dignity of virtue, to the revilings of an infatuated people, who wantonly prolonged the last agonies of his existence.

The Indian tables have two principal epochs, which go back, one to the year 3102, the other to the year 1491 before the Christian æra. These epochs are connected with the mean motions of the Sun, Moon, and planets, in such a manner; that one is evidently fictitious; the celebrated astronomer, above alluded to, endeavours, in his Indian astronomy, to prove, that the first of these epochs is founded on observation. Notwithstanding, all the arguments are brought forward with that interest he so well knew how to bestow on subjects the most difficult, I am still of opinion, that this period was invented for the purpose of giving a common origin to all the motions of the heavenly bodies in the zodiac. In fact, computing, according to the Indian tables, from the year 1491, to 3102, we find a general conjunction of the Sun and all the planets, as these tables suppose, but their conjunction differs too much from the result of our best tables, to have ever taken place, which shews that the epoch to which they refer, was not established on observation. But, it must be owned, that some elements of the Indian astronomy, seem to indicate that they have been determined even before this first epoch. Thus the equation of the centre of the Sun, which they fix at 2°.4173, could not have been of that magnitude; but at the year 4300 before the Christian æra. But, independently of the errors to which the Indian observations are liable, it may be observed, that they only considered the inequalities of the Sun and Moon, relative to eclipses, in which the annual equation of the Moon is added to the equation of the centre of the Sun, and augments it about *22′, which is very nearly the difference between our determinations, and those of the Indians. Many elements, such as the equations of the centre of Jupiter and Mars, are so different in the Indian tables, from what they must have been at their first epoch, that we can conclude nothing in favour of their antiquity, from the other elements.

The whole of these tables, particularly the impossibility of the conjunction, at the epoch they suppose, prove, on the contrary, that they have been constructed, or at least rectified in modern times. Nevertheless, the ancient reputation of the Indians does not permit us to doubt, but that they have always cultivated astronomy, and the remarkable exactness of the mean motions which they have as-

* 11′ 52″.

signed to the Sun and Moon, necessarily required very ancient observations.

The Greeks did not begin to cultivate astronomy, till a long time after the Egyptians, of whom they were the disciples.

It is extremely difficult to ascertain the exact state of their astronomical knowledge, amidst the variety of fable which fills the early part of their history. It appears, however, that they divided the heavens into constellations, about thirteen or fourteen centuries before the Christian æra; for it is to this epoch that the sphere of Eudoxus should be referred. Their numberless schools for philosophy, produced not one single observer, before the foundation of the Alexandrine school. They treated astronomy as a science purely speculative, often indulging in the most frivolous conjectures.

It is singular, that at the sight of so many contending systems, which taught nothing, the simple reflection, that the only method of comprehending nature, is to interrogate her by experiment, never occurred to one of these philosophers, though so many were endowed with an admirable genius. But we must reflect, that the first observation only presenting insulated facts, little suited to attract the imagination, impatient to ascend to causes, they must have succeeded each with extreme slowness. It required a long succession of ages to accumulate a sufficient number, to discover, among the various phenomena, such relations which by extending themselves should unite with the interest of truth, that of such general speculations as the human understanding delights to indulge in.

Nevertheless, in the philosophic dreams of Greece, we trace some sound ideas, which their astronomers collected in their travels, and afterwards improved. Thales, born at Miletus, 640 years before our æra, went to Egypt for instruction: on his return to Greece, he founded the Ionian school, and there taught the sphericity of the Earth, the obliquity of the ecliptic, and the true causes of the eclipses of the Sun and Moon; he even went so far as to predict them, employing no doubt the periods which had been communicated to him by the priests of Egypt.

Thales had for his successors—Anaximander, Anaximenes, and Anaxagoras; to the first is attributed the invention of the gnomon and geographical charts, which the Egyptians appear to have been already acquainted with.

Anaxagoras was persecuted by the Athenians for having taught these truths of the Ionian school. They reproached him with having destroyed the influence of the gods on nature, by endeavouring to reduce phenomena to immutable laws. Proscribed with his children, he only owed his life to the protection of Pericles, his disciple and his friend, who succeeded in procuring a mitigation of his sentence, from death to banishment. Thus, *truth*, to establish itself on earth, has almost always had to combat established prejudices, and has more than once been fatal to those who have discovered it. From the Ionian school arose the chief of one more celebrated. Pythagoras, born at Samos, about

590 years before Christ, was at first the disciple of Thales. This philosopher advised him to travel into Egypt, where he consented to be initiated into the mysteries of the priests, that he might obtain a knowledge of all their doctrines. The Brachmans having then attracted his curiosity, he went to visit them, as far as the shores of the Ganges. On his return to his own country, the despotism under which it groaned, obliged him again to quit it, and he retired to Italy, where he founded his school. All the astronomical truths of the Ionian-school, were taught on a more extended scale in that of Pythagoras; but what principally distinguished it, was the knowledge of the two motions of the Earth on itself, and about the Sun. Pythagoras carefully concealed this from the vulgar, in imitation of the Egyptian priests, from whom, most probably, he derived his knowledge; but his system was more fully explained, and more openly avowed by his disciple Philalaus.

According to the Pythagoricians, not

only the planets, but the comets themselves, are in motion round the Sun. These are not fleeting meteors formed in the atmosphere, but the eternal works of nature. These opinions, so perfectly correct on the system of the universe, have been admitted and inculcated by Seneca, with the enthusiasm which a great idea, on the subject the most vast of human contemplation, naturally excited in the soul of a philosopher.

"Let us not wonder," says he, "that we are still ignorant of the law of the motion of comets, whose appearance is so rare, that we neither can tell the beginning nor the end of the revolution of these bodies, which descend to us from an immense distance. It is not fifteen hundred years, since the stars have been numbered in Greece, and names given to the constellations. The day will come, when, by the continued study of successive ages, things which are now hid, will appear with certainty, and posterity will wonder they have escaped our notice."

In the same school, they taught that the planets were inhabited, and that the stars were suns, disseminated in space, being themselves centres of planetary systems. These philosophic views should, from their grandeur and justness, have obtained the suffrages of antiquity; but having been taught with systematic opinions, such as the harmony of the heavenly spheres, and wanting, moreover, that proof which has since been obtained, by the agreement with observations, it is not surprising that their truth, when opposed to the illusions of the senses, should not have been admitted.

CHAP. II.

Of Astronomy, from the Foundation of the Alexandrine School, to the Time of the Arabs.

HITHERTO, the practical astronomy of different people, has only offered us some rude observations relative to the seasons and eclipses; objects of their necessities or their terrors. Their theoretical astronomy consisted in the knowledge of some periods, founded on very long intervals of time, and of some fortunate conjectures, relative to the constitution of the universe, but mixed with considerable error. We see, for the first time, in the school of Alexandria, a connected series of observations; angular distances were made with instruments suitable to the purpose, and they were calculated by trigometrical methods. Astronomy then took a new form, which the following ages have adopted and brought to perfection. The positions of the fixed stars were determined, the paths of the planets carefully traced, the inequalities of the Sun and Moon were better known, and, finally, it was the school of Alexandria that gave birth to the first system of astronomy, that had ever comprehended an entire plan of the celestial motions. This system was, it must be allowed, very inferior to that of the school of Pythagoras, but being founded on a comparison of observations, it afforded, by this very comparison, the means of its own destruction, and the true system of nature has been elevated on its ruins.

After the death of Alexander, his principal generals divided his empire among themselves, and Ptolemy Soter received Egypt for his share. His munificence and love of the sciences, attracted to Alexandria, the capital of his kingdom, a great number of the most learned men of Greece. Ptolemy Philadelphus, who inherited with the kingdom his father's love of the sciences, established them there under his own particular protection. A vast edifice, in which they were lodged, contained both an observatory, and that magnificent library which Demetrius Phalerus had collected with such trouble and expence. Here they were supplied with whatever books and instruments were necessary to their pursuits; and their emulation was excited by the presence of a prince, who often came amongst them to participate in their conversation and their labours.

Arystillus and Thimocares were the first observers of this rising school; they flourished about the year 300 before the Christian æra. Their observations of the principal stars of the zodiac, enabled Hipparchus to discover the precession of the equinoxes, and Ptolemy, from their observations of the planets, founded his theory of those bodies.

The next astronomer which the school of Alexandria produced, was Aristarchus, of Samos. The most delicate elements of astronomy, were the subjects of his investigation. He observed the summer solstice, the year 281 before the Christian æra. He determined the magnitude of the apparent diameter of the Sun, which he found equal to the 720th part of the whole circumference, which quantity is a mean between the two limits, which Archimedes assigned, a few years afterwards, to this diameter, by an ingenious method, according to which the solar diameter appeared to him greater than the 200th part of a right angle, and less than the 164th part. But that which reflects the greatest honour on the genius of Aristarchus, is the method by which he endeavoured to determine the distance of the Sun from the Earth. He observed the angle contained between the Sun and the Moon, at the moment he judged half of the lunar disk to be illuminated by the Sun, and having found it just 96°.7, he concluded that the Sun was eighteen or twenty times farther from us, than the Moon. Notwithstanding the inaccuracy of this result, it extended the boundaries of the universe much farther than had been done before. Aristarchus revived the opinion of the Pythagoricians, relative to the motion of the Earth. But as his writings have not been transmitted to us, we are ignorant to what extent he carried this theory in his explanation of the celestial phenomena. We only know that this judicious astronomer, having reflected that the motion of the Earth produced no change in the apparent position of the stars, placed them at a distance incomparably greater than the Sun. Thus it appears, that of all the ancient astronomers, Aristarchus had formed the most just notions of the magnitude of the universe.

The celebrity of his successor, Eratosthenes, is principally due to his measure of the Earth, and his observations on the obliquity of the ecliptic. Having, at the summer solstice, remarked a deep well, whose whole depth, was illuminated by the Sun, at Syene, in Upper Egypt, he compared this with the altitude of the Sun, observed at the same solstice at Alexandria. He found the celestial arc, contained between the zeniths of these two places, equal to the 50th part of the whole circumference, and as their distance was estimated at 500 stadia, he fixed at 250 thousand stadia, the length of the whole terrestrial circumference. The uncertainty that exists, as to the value of this stadium, does not permit us to appreciate the exactness of this measurement.

Aristotle, Cleomedes, Possidonius, and Ptolemy, have given four other evaluations of the circumference of the Earth, equivalent to 400, 300, 240, 180 thousand stadia. The simple relation of these measures to each other, leave room to conjecture, that these different quantities are translations of the same measure, in different stadia. The Alexandrian stadium was 400 great cubits, of the same length as the nilometer of Cairo, which, according to Freret, has not been altered for a great number of centuries, and may be traced back to the time of Sesostris; its magnitude is equal to 1.7119 feet, according to some measures lately made with great precision, which gives 684,76 feet, for the value of the stadium of Alexandria. As it is probable this stadium was that of Ptolemy, the circumference of the Earth, according to that astronomer, would be 123256800 feet, which differs but little from our actual measurement, which fixes it at 123178320 feet.*

If the measures of Possidonius, Cleomedes, and Aristotle, are identical with that of Ptolemy, the corresponding stadia are 513,570, 410,856, and 308,142 feet. Now, in comparing a great number of ancient itinerary distances with the actual known distances, we find in antiquity these different stadia so precisely, as to render the identity of these four measures of the Earth extremely probable. It is therefore very probable, that they all depend on some ancient and very exact measure, either executed with great care, or in which the errors were fortunately compensated, as has since happened in the measure of a

* Ten millions of metres, according to the new measurement.

degree, by Fernel, and even in that by Picard. It is true, we know, that Possidonius himself measured an arc of the terrestrial meridian; and his operation, as far as we can judge from the details that have been transmitted to us, was very inexact; but there is reason to think he only proposed to verify some ancient measures of the Earth, and that he found them to agree nearly with his own.

The observation of Eratosthenes, on the obliquity of the ecliptic, is very valuable, inasmuch as it confirms the diminution of it, determined *à priori*, by the theory of gravitation. He found the distance between the tropics less than 53.06, and greater than 52.96, which gives us a mean * 26.50, for the obliquity of the ecliptic. Hipparchus found no reason to alter this result by his observations.

But, of all the astronomers of antiquity, the science is most indebted to Hipparchus of Bythinia, for the great number and extent of his observations, by the important results he obtained, by comparing them

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* 23.51.

with those that had been formerly made by others; and for the excellent method which he pursued in his researches. He flourished at Alexandria about 140 years before our æra. Not content with what had already been done, he determined to recommence every thing, and not to admit any results but those founded on a new examination of former observations, or on new observations, more exact than those of his predecessors.

Nothing affords a stronger proof of the uncertainty of the Egyptian and Chaldean observations on the Sun and stars, than the necessity which compelled him to recur to the observations of the Alexandrine school, to establish his theories of the Sun, and of the precession of the equinoxes. He determined the length of the tropical year, by comparing one of his observations of the summer solstice, with one made by Aristarchus of Samos, forty-five years before; he found it 365,24667 days. This is in excess about four minutes and a half. But he remarks himself on the little reliance that can be placed on a determination from solstitial observations, and on the advantage of employing observations of the equinoxes. Hipparchus recognized that there elapsed 187 days from the vernal equinox, to that of the autumn, and 178 days only from this last equinox, to that of the spring. He observed, likewise, that these intervals were unequally divided by the solstices, so that 94 days and a half elapse from the vernal equinox to the summer solstice, and 92 days and a half from this solstice to the autumnal equinox.

To explain these differences, Hipparchus supposed the Sun to move uniformly in a circular orbit; but, instead of placing the Earth in the centre of it, he supposed it removed the 24th part of the radius, and fixed the apogee at the sixth degree of Gemini. From these data he formed the first solar tables to be found in the History of Astronomy. The equation of the centre, which they suppose was too great, it is very probable, that a comparison with eclipses, in which this equation is augmented by the annual equation of the Moon, confirmed Hipparchus in his error, and perhaps even led him into it. He was mistaken also in supposing circular the elliptic orbit of the Sun, and that the real velocity of this body was constantly uniform. The contrary is now demonstrated by direct measures of the Sun's apparent diameter; but such observations were impossible at the time of Hipparchus, whose solar tables, with all their imperfections, are a lasting monument of his genius, which Ptolemy, three centuries after, respected, but did not attempt to improve.

This great astronomer next considered the motions of the Moon; he measured the length of its revolution by comparing eclipses, and determined both the excentricity and inclination of its orbit, he ascertained the motion of its nodes and of its apogee, and from the determination of its parallax endeavoured to conclude that of the Sun, by the breadth of the cone of the terrestrial shadow, in an eclipse at the moment it was traversed by the Moon, which led him nearly to the same result as had been obtained by Aristarchus. He made a great number of observations on the planets, but too much the friend of truth to explain their motions by uncertain theories, he left the task of this investigation to his successors. A new star which appeared in his time induced him to undertake a catalogue of the fixed stars, to enable posterity to recognize any changes that might take place in the appearances of the heavens. He was sensible also of the importance of such a catalogue for the observations of the Moon and the planets. The method he employed was that of Arystillus and Timochares, and which we have already explained in the First Book. The reward of this long and laborious task was the important discovery of the precession of the equinoxes; in comparing his observations with those of these astronomers, he discovered that the stars had changed their situation with respect to the equator, but had preserved the same latitude with respect to the ecliptic, so that to explain these different changes, it is sufficient to give a direct motion to the celestial sphere round the poles of the ecliptic, which produces a retrograde motion of the equinoxes with respect to the stars. But he announced his discovery with some reserve, being doubtful of the accuracy of the observations of Arystillus and Timochares. Geography is indebted to Hipparchus for the method of determining places on the Earth, by their latitude and longitude, for which he first employed the eclipses of the Moon. Spherical trigonometry, also, owes its origin to Hipparchus, who applied it to the numberless calculations which these investigations required. His principal works have not been transmitted to us; they perished in the conflagration of the Alexandrine library, and we are only acquainted with them through the Almagest of Ptolemy.

The interval of near three centuries which separated these two astronomers, produced some observers, as Agrippa, Menelaus, and Theon. We may also notice in this interval the reformation of the calendar by Julius Cæsar, and the precise knowledge of the ebbing and flowing of the sea. Possidonius observed the law of this phenomenon, which appertains to astronomy by its evident relation to the motion of the Sun and Moon, and of which Pliny the naturalist has given a description remarkable for its exactness.

Ptolemy, born at Ptolemais in Egypt, flourished at Alexandria, about the year 130 of our æra. Hipparchus had conceived the project of reforming astronomy, and establishing the science on new foundations. Ptolemy continued this labour, too vast to be accomplished by a single individual, and has given a complete treatise on this science in his great work entitled the Almagest.

His most important discovery is the evection of the Moon. Astronomers previously had only considered the motion of this body relative to eclipses; by following it through its whole course, Ptolemy recognized, that the equation of the centre of the lunar orbit, was less in the sysigies than in the quadratures; he determined the law of this difference, and ascertained its value with great precision. To represent it, he made the Moon to move upon an epicycle carried by an excentric, according to a method attributed to Appollonius the geometrician, and which had before been employed by Hipparchus.

It was a general opinion of the ancients, that the uniform circular motion being the most simple and natural, was necessarily that of the heavenly bodies. This error maintained its ground till the time of Kepler, and for a long time impeded him in his researches. Ptolemy adopted it, and, placing the Earth on the centre of the celestial motions, he endeavoured to represent their inequalities in this false hypothesis. Eudoxus had previously imagined for this object, every planet attached to several concentric spheres, endowed with different motions; but this astronomer not having explained in what manner these spheres, by their action on the planets produce the variety of their motions. His hypothesis hardly deserves notice, in a treatise on astronomy. A much more ingenious hypothesis consists in moving along one circumference, of which the Earth occupies the centre, that of another circumference, on which moves that of a third, and so on, up to the last circumference, on which the body is supposed to move uniformly. If the radius of one of these circles surpasses the sum of the others, the apparent motion of the body round the Earth, will be composed of a mean uniform motion, and of several inequalities depending on the proportions of these several radii to each other, and the motions of their centres, and of that of the Star. By increasing their number, and giving them suitable dimensions, we may represent the inequalities of this apparent motion. Such is the most general manner of considering the hypothesis of cycles and excentrics, which Ptolemy adopted in his theories of the Sun, Moon, and planets. He supposed these bodies in motion round the Earth in this order of distances—the Moon, Mercury, Venus, the Sun, Mars, Jupiter, Saturn; astronomers were divided in their opinions as to the position of Mercury and Venus; Ptolemy followed the most ancient opinion, and placed them below the Sun; others placed them above, and finally, the Egyptians made them move round it. It is singular, that Ptolemy does not mention this hypothesis, which is equivalent to placing the Sun in the centre of the epicycles of these two planets, instead of making them revolve round an imaginary centre. But, being persuaded that his system could only be adapted to the three superior planets, he transferred it to the two inferior, and was misled by a false application of the principle of the uniformity of the laws of nature, which, if he had set out from the discovery of the Egyptians, on the motions of Mercury and Venus, would have led him to the true system of the world. But even, if epicycles could be made to represent the incqualities of the motions of the heavenly bodies, still it would be impossible to represent the variations in their distances. In the time of Ptolemy, these variations were almost insensible in the planets, whose apparent diameters could not then be measured. But his observations on the Moon should have taught him that his hypothesis was erroneous, according to which the diameter of the Moon perijee, in the quadratures, should be double of the diameter apogee in the sysigies. The motion in latitude of the planets, was another difficulty to be unexplained by this system; and every inequality which the improvements in the art of observing discovered, incumbered this system with a new epicycle, which, instead of being confirmed by the progress of the science, has only grown more and more complicated; and this should convince us, that it is not that of nature. But in considering it as a method of adapting the celestial motions to calculation, this first attempt of the human understanding towards an object so very complicated, does great honour to the sagacity of its author.

Ptolemy confirmed the motion of the equinoxes, discovered by Hipparchus, by comparing his observations with those of this great astronomer. He established the respective immobility of the Stars, their invariable latitude to the ecliptic, and their motion in longitude, which he found * 111″ in every year, as Hipparchus had suspected.

We now know, that this motion is very nearly † 154″ annually, which, considering the interval between the observations of Ptolemy and Hipparchus, implies an error of more than one degree in their observations. Notwithstanding the difficulty which attended the determination of the longitude of the Stars, when observers had no exact measure of time, we are surprised that so great an error should have been committed, particularly when we observe the agreement of the observations with each other, which Ptolemy cites as

* 35″.9. † 50″.

a proof of the accuracy of his result. He has been reproached with having altered them, but this reproach is not founded; his error, in the determination of the motion of the equinoxes, seems to have been derived from too great confidence in the result of Hipparchus, relative to the length of the tropical year and the motion of the Sun. In fact, Ptolemy determined the longitudes of the stars, by comparing them either with the Sun, or with the Moon, which was equivalent to a comparison with the Sun, since the synodical revolution of the Moon was well known by the means of eclipses. Now, Hipparchus having supposed the year too long, and consequently the motion of the Sun in longitude too slow, it is clear that this error diminished the longitudes of the Sun and Moon, employed by Ptolemy. The motion in longitude, which he attributed to the Stars, is too small by the arc described by the Sun in the time, equal to the error of Hipparchus in the length of the year.

In the time of Hipparchus, the tropical

year was 365.24234: this great astronomer supposed it 365,24667; the difference is 433″, and during this interval the Sun describes an arc of 47″; this, added to the annual precession of 111″, determined by Ptolemy, gives 158 for the precession, which he would have found, if he had computed from the length of the true tropical year, the error would then have been only 4″.

This remark has led to the examination of another question. It had been generally believed, that the catalogue of Ptolemy, was that of Hipparchus, reduced to his time by means of the annual precession of 111″. This opinion is founded on this circumstance, that the constant error in longitude of his Stars, disappear when reduced to the time of Hipparchus. But the explanation which we have given of the cause of this error, justifies Ptolemy from the reproach which has been imputed to him, of having taken the merit of Hipparchus to himself; and it seems right to believe him, when he asserts that he has observed all the Stars of his own catalogue, even to the Stars of the sixth magnitude. He adds, at the same time, that he found very nearly the same positions of the Stars, relatively to the ecliptic, as Hipparchus, so that the difference between these two catalogues must have been very small. Thus, the observations of Ptolemy on the stars, and the true value which he has assigned to the evection, are proofs of his exactness as an observer. It is true, that the three equinoxes which he has observed, are inaccurate; but it appears that, too much prepossessed in favour of the exactness of the solar tables of Hipparchus, he made his observations of the equinoxes, at that time very difficult, coincide with them, as the derangement of his armillary might have been sufficient to explain the errors.

The astronomical edifice, raised by Ptolemy, subsisted near fourteen centuries, and now that it is entirely destroyed, his Almagest, considered as a depositary of ancient observations, is one of the most precious monuments of antiquity. Ptolemy has not rendered less service to geography, in collecting all the known longitudes and latitudes of different places, and laying the foundation of the method of projections, for the construction of geographical charts. He composed a great treatise on optics, which has not been preserved, in which he explained the astronomical refractions: he likewise wrote treatises on the several sciences of chronology, music, gnomonics, and mechanics. So many labours, and on such a variety of subjects, manifest a very superior genius, and will ever obtain him a distinguished rank in the history of science. On the revival of astronomy, when his system gave way to that of nature, mankind avenged themselves on him for the despotism it had so long maintained; and they accused Ptolemy of having appropriated to himself the discoveries of his predecessors; but in his time, the works of Hipparchus, and of the astronomers of Alexandria, must have been sufficiently known, to have rendered excusable, his not distinguishing what belonged to them from his own discoveries. As to the long continuation of his errors, it must be attributed to the same causes which replunged Europe into darkness. The fame of Ptolemy has met with the same fate as that of Aristotle and Descartes. Their errors were no sooner recognized, than a blind admiration gave way to an unjust contempt, for even in science itself, the most useful revolutions are not always exempt from passion and prejudice.

CHAP. III.

Of the Astronomy of the Arabs, Chinese and Persians.

The progress of astronomy in the school of Alexandria terminated with the labours of Ptolemy. This school continued to exist for five centuries, but the successors of Ptolemy and Hipparchus contented themselves with commenting on their works without adding to their discoveries. With the exception of two eclipses, recorded by Theon, and some observations of Theon of Athens, the phenomena of the heavens continued unobserved during a period of more than six hundred years. Rome, once the seat of valour, glory, and learning, did nothing useful to science.

The consideration that was always attached by the republic to eloquence and military talents, seduced the imagination to those pursuits: and science, offering no advantage, was necessarily neglected in the midst of conquests undertaken by ambition, and of internal commotions in which liberty expired and yielded to the despotism of the emperors. The division of the empire, the necessary consequence of its vast extent, brought on its fall, and the light of science, extinguished by the barbarians, was only again revived among the Arabians.

This people, exalted by fanaticism, after having extended its religion and its arms over a great part of the Earth, had no sooner reposed in peace, than it devoted itself to letters and science.

It, however, was but a short time before that they destroyed their most beautiful ornament, by burning the famous library of Alexandria.

In vain the philosopher Philoponus exerted himself for its preservation. If these books, replied Omar, are conformable to the alcoran, they are useless; if they are contrary to it, they are detestable. Thus perished this immense treasure of erudition and genius. Repentance and regret soon followed this barbarous execution, for the Arabians were not long before they perceived their irreparable loss, and that they had deprived themselves of the most precious fruits of their conquests.

About the middle of the eighth century, the caliph Almansor gave great encouragement to astronomy, but among the Arabian princes who distinguished themselves for their love of the sciences, the most celebrated in history was Almamoun, of the family of the Abassides and son of the famous Aaron Rashid, so celebrated throughout Asia. Almamoun reigned in Bagdat in 814, having conquered the Greek emperor Michael III., he imposed on him, as one condition of peace, that he should have delivered to him the best books of Greece;—the Almagest was among the number, he caused it to be translated into the Arabian language, and thus diffused the astronomical knowledge which had formerly caused the celebrity of the Alexandrine school. Not content with encouraging learned men by his liberality, he was himself an observer, and determined the obliquity of the ecliptic; he likewise caused a degree of the meridian to be measured on the vast plain of Mesopotamia.

The encouragement given to astronomy by this prince and his successors, produced a great number of astronomers, among whom Albategnius deserves to be placed the first. We are indebted to him for an observation of the obliquity of the ecliptic which corrected for refraction and parallax gives * 26°2182 for this obliquity of the ecliptic. All the Arabian observations give nearly the same result, from whence we deduce the secular variation about † 159.″

Albategnius found the annual motion of the equinoxes equal to ‡ 168.″3, and the length of the tropical year equal to 365.24056. The first of these elements is

* 23° 35′ 46″. † 51″.5. ‡ 51″.2.

too great by 14″, the second is too small by more than a minute and a half, but these errors depend entirely on the observations of Ptolemy which Albategnius compared with his own; he would have come nearer the truth had he used only those of Hipparchus.

This great astronomer improved greatly the theory of the Sun. He reduced the excentricity of the solar ellipse to 0.017325, the radius of the orbit being taken as unity. At the commencement of 1750 it was 0.016814. Its diminution in an interval of 870 years was therefore 0.00511. The theory of gravity, adopting the most probable value of the masses of the planets gives .003967. This difference is within the limits of the errors to which the observations of Albategnius were liable.

These same observations conducted him likewise to the discovery of the proper motion of the Sun's apogee, he observed its place to be * 24°.76 in *Gemini,* which was more advanced since the time of Hipparchus than it should have been from

* 22° 17′.

the motion of the equinoxes only. According to our best tables the place of the apogee in 880 was * 26.°23, the observation of Albategnius was therefore only defective by † one degree and a half, which was a very precise determination for that age, considering the delicacy required to ascertain this element. These results are not only very valuable for their exactness, but particularly as they confirm the diminution of the excentricity of the solar orbit, demonstrated by the theory of gravitation and by the secular equation of the Moon; they likewise induce us to place great confidence in his determination of the obliquity of the ecliptic, which he relates to have been made with great care, by means of a radius of great length, and by taking all the precautions mentioned in the Almagest.

The most interesting part of the astronomy of the Arabs which have been preserved are these labours of Albategnius, in his work on the science of the stars, and two

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* 23°. † 36′.

eclipses of the Sun and one of the Moon observed by Ibn Junis, near Cairo, in 977, 978, and 979, which confirm the mean acceleration of the Moon. The Arabian astronomers chiefly occupied themselves in practical observations; they did not investigate the causes of the celestial phenomena, but retained without alteration the system of Ptolemy.

The Persians, after having for a long time submitted to the same sovereigns as the Arabians, and professing the same religion, about the middle of the eleventh century, shook off the yoke of the Caliphs. About this time their calendar received a new form, by the care of the astronomer Omar Cheyam ; it was founded on an ingenious intercalation, which consists in making in every thirty-three years six of them sextile.

Holagu Ilecoukan, one of their sovereigns, constructed a magnificent observatory, and entrusted the superintendance of it to Nassir Eddin. But no prince of this nation distinguished himself more for his zeal than Uleg Beigh, whom we should place in the first rank of observers. He formed himself a catalogue of stars at Sarmacand, the capital of his dominions, and likewise the best tables of the Sun and planets which existed before the time of Tycho Brahe. He fixed the precession of the equinoxes at 159″, and determined the obliquity of the ecliptic with instruments of very elaborate construction, he found it equal to 261475.

A century and a half previous to this, the Chinese astronomy affords us several observations of the Sun, made with great care with a very high gnomon, by Cocheon King, a celebrated astronomer; from these Lacaille concluded the length of the year the same as we now adopt, and the obliquity of the ecliptic 26.1519, 1278, the epoch of these observations; from this results a secular variation of 153″. It is on these observations, and those of Albategnius, that I have founded my determination at 134.3.

The Chinese astronomy likewise men-tions the occultation of several fixed stars by planets and a great many eclipses of the Sun and Moon. There no doubt exist in our libraries, manuscripts and other observations which would throw great light on the secular equations of the heavenly bodies, and on the masses of the planets, one of the principal things that remain unsettled in modern astronomy.

The investigation of these observations merit the attention of the learned in the oriental languages, for the great variations in the system of the world are not less interesting to be acquainted with, than the revolutions of empires.

CHAP. IV.

Of Astronomy in modern Europe.

IT is to the Arabians that modern Europe is indebted for the first rays of light that dissipated the darkness in which it was enveloped during twelve centuries. They have transmitted to us the treasure of knowledge which they received from the Greeks who were themselves disciples of the Egyptians; but by a deplorable fatality the arts and sciences have disappeared among all these nations, as soon as they had communicated them.

Despotism has for a long period extended its barbarism over those beautiful countries where science first had its origin, and those names which formerly rendered them celebrated, are now unknown in them.

Alphonso, king of Castille, was one of the first sovereigns who encouraged the revival of astronomy in Europe. This science can reckon but few such zealous protectors; but he was ill seconded by the astronomers whom he had assembled at a considerable expence, and the tables which they published did not answer to the great cost they had occasioned.

Endowed with a correct judgment, Alphonso was shocked at the confusion of the circles, in which the celestial bodies were supposed to move; he felt that the expedients employed by nature ought to be more simple. "If the Deity," said he, "had asked my advice, these things would have been better arranged." By these words, which are taxed with impiety, he meant to express that mankind were still far from knowing the true mechanism of the universe.

In the time of Alphonso, Europe was indebted to the encouragement of Frederic II. Emperor of Germany, for the first Latin translation of the Almagest of Ptolemy, which was made from the Arabic version.

We are now arrived at that celebrated epoch when astronomy, escaping from the narrow sphere which had hitherto confined it, raised itself by a rapid and continued progress to the height where we now behold it. Purbech, Regiomontanus, and Waltherus prepared the way to these prosperous days of the science, and Copernicus gave them birth by the fortunate explanation of the celestial phenomena, by means of the motion of the Earth on its axis, and round the Sun.

Shocked, like Alphonso, at the extreme complication of the system of Ptolemy, he tried to find among the ancient philosophers a more simple arrangement of the universe. He found that many of them had supposed Venus and Mercury to move round the Sun: that Nicetas, according to Cicero, made the Earth revolve on its axis, and by this means freed the celestial sphere from that inconceivable velocity which must be attributed to it to accomplish its diurnal revolution. He learnt from Aristotle and Plutarch that the Pythagoricians had made the Earth and planets move round the Sun, which they placed in the centre of the universe. These luminous ideas struck him; he applied them to the astronomical observations which time had multiplied, and had the satisfaction to see them yield, without difficulty, to the theory of the motion of the Earth. The diurnal revolution of the heavens was only an illusion due to the rotation of the Earth, and the precession of the equinoxes, is reduced to a slight motion of the terrestrial axis. The circles, imagined by Ptolemy, to explain the alternate direct and retrograde motions of the planets, disappeared. Copernicus only saw in these singular phenomena, the appearances produced by the motion of the Earth round the Sun, with that of the planets: and he concluded, from hence, the respective dimensions of their orbits, which, till then, were unknown. Finally, every thing in this system announced that beautiful simplicity in the expedients of nature, which delights so much when we are fortunate enough to discover it. Copernicus published it in his work, *On the Celestial Revolutions* ; not to shock received prejudices, he presented it under the form of an hypothesis. " Astronomers," said he, " in his dedication to Paul III., being permitted to imagine circles, to explain the motion of the stars, I thought myself equally entitled to examine if the supposition of the motion of the Earth, would render the theory of these appearances more exact and simple."

This great man did not witness the success of his work. He died suddenly by the rupture of a blood vessel, at the age of seventy-one years, a few days after receiving the first proof. He was born at Thorn, in Polish Prussia, the 19th of February, 1473. After learning the Greek and Latin languages, he went to continue his studies at Cracovia. Afterwards, induced by his taste for astronomy, and by the reputation which Regiomontanus had acquired, he undertook a journey to Italy, where this science was taught with success: being greatly desirous to render himself illustrious by the same career, he followed the lessons of Dominic Maria, at Bologna. When arrived at Rome, his talents obtained him the place of professor: he afterwards quitted this city, to establish himself at Fravenberg, where his uncle, then Bishop of Warmia, made him a canon. It was in this tranquil abode, that by thirty-six years of observation and meditation, he established his theory of the motion of the Earth. At his death, he was buried in the cathedral of Fravenberg, without any pomp or epitaph; but his memory will exist as long as the great truths which he has again introduced with such evidence, as to have at length dissipated the illusions of the senses, and surmounted the difficulties which ignorance of the laws of mechanics had opposed to them.

These truths had yet to vanquish obstacles of another kind, and which, arising from a respected source, would have stifled them if the rapid progress of all the mathematical sciences had not concurred to support them.

Religion was invoked to destroy an astronomical system, and one of its defenders, whose discoveries did honor to his age and country, was tormented by repeated prosecutions. Bethicus, the disciple of Copernicus, was the first who adopted his ideas; but they were not in great estimation till towards the beginning of the seventeenth century, and then they owed it principally to the labours and misfortunes of Galileo.

A fortunate accident had made known the most wonderful instrument ever discovered by human ingenuity, and which, by giving to astronomical observations a precision and extent hitherto unhoped for, displayed in the heavens new inequalities, and new worlds. Galileo hardly knew of the first trials of the telescope, before he bent his mind to bring it to perfection. Directing it towards the Stars, he discovered the four satellites of Jupiter, which shewed a new analogy between the Earth and planets; he afterwards observed the phases of Venus, and from that moment he no longer doubted of its motion round the Sun. The milky way displayed to him an infinite number of small stars, which the irradiation confounds to the naked eye, in a white and continued light; the luminous points which he perceived beyond the line which separated the light part of the Moon from the dark, made him acquainted with the existence and height of its mountains. At length he observed the appearances occasioned by Saturn's ring, the spots and rotation of the Sun. In publishing these discoveries, he shewed that they proved incontestibly, the motion of the Earth; but the idea of this motion was declared heretical by a congregation of cardinals; and Galileo, its most celebrated defender, was cited to the tribunal of the inquisition, and compelled to retract this theory, to escape a rigorous prison.

One of the strongest passions is the love of truth, in a man of genius. Full of the enthusiasm which a great discovery inspires, he burns with ardour to disseminate it, and the obstacles which ignorance and superstition, armed with power, oppose to it, only irritate and increase his energy. Galileo, convinced by his own observations of the motion of the Earth, had long meditated a new work, in which he proposed to develope the proofs of it. But to shelter himself from the persecution of which he had escaped being the victim, he proposed to present them, under the form of dialogues between three interlocutors, one of whom defended the system of Copernicus, combated by a Peripatetician. It is obvious, that the advantage would rest with the defender of this system; but, as Galileo did not decide between them, and gave as much weight as possible to the objections of the partisans of Ptolemy, he had a right to expect that tranquillity which his age and labours merited.

The success of these dialogues, and the triumphant manner with which all the difficulties against the motion of the Earth were resolved, roused the inquisition. Galileo, at the age of seventy, was again cited before this tribunal. The protection of the grand Duke of Tuscany, could not prevent his appearance. He was confined in a prison, where they required of him a second disavowal of his sentiments, threatening him with the punishment incurred by contumacy, if he continued to teach the system of Copernicus.

He was compelled to sign this formula of abjuration:

"*I Galileo, in the seventieth year of my*
"*age, brought personally to justice, being*
"*on my knees, and having before my eyes*
"*the holy evangelists, which I touch with*
"*my own hands, with a sincere heart and*
"*faith; I abjure, curse, and detest, the*
"*absurdity, error, and heresy, of the mo-*
"*tion of the Earth,*" &c.

What a spectacle! A venerable old man, rendered illustrious by a long life, consecrated to the study of nature, abjuring on his knees, against the testimony of his own conscience, the truth which he had so evidently proved. A decree of the inquisition, condemned him to a perpetual prison. He was released after a year, at the solicitations of the grand duke ; but, to prevent his withdrawing himself from the power of the inquisition, he was forbidden to leave the territory of Florence.

Born at Pisa, in 1564, he gave early indications of those talents which were afterwards developed. Mechanics owe to him many discoveries, of which the most important is the theory of falling bodies.

Galileo was occupied with the libration of the Moon, when he lost his sight ; he died three years afterwards, at Arcetre, in 1642, regretted by all Europe, which he left enlightened by his labours, and indignant at the judgment passed against so great a man, by an odious tribunal.

While this passed in Italy, Kepler, in Germany, developed the laws of the planetary motions. But, previous to the account of his discoveries, it is necessary to look back and to describe the progress of astronomy in the north of Europe, after the death of Copernicus.

The history of this science presents at this epoch, a great number of excellent observers. One of the most illustrious, was William IV. Landgrave of Hesse-Cassel. He had an observatory built at Cassel, which he furnished with instruments, constructed with care, and with which he observed a long time. He procured two celebrated astronomers, Rothman and Juste Byrye; and Tycho owed to his pressing solicitations, the advantages which Frederic, King of Denmark, obtained for him.

Tycho Brahe, who was one of the greatest observers that ever existed, was born at Knucksturp, in Norway. His taste for astronomy was manifested at the age of fourteen years, on the occasion of an eclipse of the Sun, which happened in 1560. At this age, when reflection is so rare, the justice of the calculation which announced this phenomenon, inspired him with an anxious desire to know its principles; and this desire was still further increased by the opposition of his preceptor and family. He travelled to Germany, where he formed connections of correspondence and friendship, with the most distinguished persons, who pursued astronomy either as a profession, or amusement, and particularly with the Landgrave of Hesse-Cassel, who received him in the most flattering manner.

On his return to his own country, he was fixed there by his sovereign, Frederic, who gave him the little island of Huena at the entrance of the Baltic. Tycho built a celebrated observatory there, which was called Uranibourg. There, during an abode of twenty-one years, he made a prodigious mass of observations, and many important discoveries. At the death of Frederic, envy, then unrestrained, compelled Tycho to leave his retreat. His return to Copenhagen did not appease the rage of his prosecutors; the minister, Walchendorp, (whose name, like that of all men who have abused the power entrusted to them, ought to be handed down to the execration of all ages,) forbad him to continue his observations. Fortunately, Tycho found a powerful protector in the Emperor Rodolph II. who settled on him a considerable pension, and lodged him commodiously at Prague. He died suddenly at this city, on the 24th of October, 1601, in the midst of his labours, and at an age when astronomy might have expected great services from him.

The invention of new instruments, and new improvements, added to the old ones a much greater precision in observation; a catalogue of stars very superior to those of Hipparchus, and Ulugh Beigh; the discovery of that inequality of the Moon, which is called variation; that of the inequalities of the motion of the nodes, and of the inclination of the lunar orbit; the interesting remark, that comets are beyond this orbit; a more perfect knowledge of astronomical refractions; finally, very numerous observations of the planets, which have served as the basis of the discoveries of Kepler. Such are the principal services which Tycho Brahe has rendered astronomy. Struck with the objections which the adversaries of Copernicus made to the motion of the Earth, and perhaps influenced by the vanity of wishing to give his name to an astronomical system, he mistook that of nature. According to him, the Earth is immovable in the centre of the universe; all the Stars move every day round the axis of the world; and the Sun, in its annual revolution, carries with it the planets. In this system, already known, the appearances are the same as in that of the motion of the Earth. We may, in general, consider any point we chuse; for example, the centre of the Moon as immovable, provided we assign the motion with which it is animated, in a contrary direction to all the stars.

But, is it not physically absurd to suppose the Earth immovable in space, while the Sun carries with it the planets in which it is included? How could the distance from the Earth to the Sun, which agrees so well with the duration of its revolution in the hypothesis of the motion of the Earth, leave any doubt of the truth of this hypothesis, in a mind constituted to feel the force of analogy. It must be confessed, that Tycho, though a great observer, was not fortunate in his research after causes; his unphilosophical mind had even imbibed the prejudices of astrology, which he tried to defend.

It would be, however, unjust to judge him with the same rigor as one who should refuse at present to believe the motion of the Earth, confirmed by the numerous discoveries made in astronomy since that period.

The difficulties which the illusions of the senses opposed to this theory, were not then completely removed. The apparent diameter of the fixed stars, greater than their annual parallax, gave to these stars in this theory, a real diameter, greater than that of the terrestrial orbit. The telescope, by reducing them to luminous points, made this improbable magnitude disappear. It could not be conceived how these bodies, detached from the Earth, could follow its motion. The laws of mechanics have explained these appearances; they have proved, what Tycho had again made doubtful, that a body, falling from a considerable height, and abandoned to the action of gravity alone, ought to fall very nearly in a vertical line, only deviating to the east, by a quantity difficult to estimate accurately by observation from its minuteness, so that at present there is as much difficulty in proving the motion of the Earth by a direct experiment, as formerly existed to prove that it should be insensible.

In his later years, Tycho Brahe had Kepler for a disciple and assistant. He was born in 1571, at Viel, in the duchy of Wirtemberg, and was one of those extraordinary men whom nature grants now and then to the sciences, to bring to light those great theories which have been prepared by the labour of many centuries.

The career of the sciences did not appear to him proper to satisfy the ambition he felt of rendering himself illustrious; but the ascendancy of his genius, and the exhortations of Maestlinus, led him to astronomy: and he entered into the pursuit with all the activity of a mind passionately desirous of glory.

The philosopher, endowed with a lively imagination, and impatient to know the causes of the phenomena which he sees, often obtains a glimpse of it, before observation can conduct him to it. Doubtless he might, with greater certainty, ascertain the cause from the phenomena; but the history of science proves to us, that this slow progress has not always been that of inventors.

What rocks has he to fear, who takes his imagination for his guide!

Prepossessed with the cause which it presents to him, instead of rejecting it when contradicted by facts, he alters them to make them agree with his hypotheses; he mutilates, if I may be allowed the expression, the work of nature, to make it resemble that of his imagination, without reflecting that time destroys with one hand these vain phantoms, and with the other confirms the results of calculation and experience.

The philosopher who is really useful to the cause of science, is he, who, uniting to a fertile imagination, a rigid severity in investigation and observation, is at once tormented by the desire of ascertaining the cause of the phenomena, and by the fear of deceiving himself in that which he assigns.

Kepler owed the first of these advantages to nature, and the second to Tycho Brahe. This great observer, whom he went to see at Prague, and who had discovered the genius of Kepler, in his earliest works, notwithstanding the mysterious analogies of numbers and figures with which it was filled, exhorted him to devote his time to observation, and procured him the title of imperial mathematician.

The death of Tycho, which happened a few years afterwards, put Kepler in possession of his valuable collection of observations, of which he made a most noble use, founding on them three of the most important discoveries that have been made in natural philosophy.

It was an opposition of Mars which determined Kepler to employ himself, in preference, on the motions of this planet. His choice was fortunate in this circumstance, that the orbit of Mars, being one of the most excentric of the planetary system, the inequalities of his motion were more perceptible, and therefore led to the discovery of their laws with greater facility and precision. Though the theory of the motion of the Earth had made the greater part of those circles with which Ptolemy had embarrassed astronomy, disappear, yet Copernicus had substituted many others to explain the real inequalities of the celestial bodies.

Kepler, deceived like him, by the opinion that their motions ought to be circular and uniform, tried a long time to represent those of Mars, in this hypothesis. Finally, after a great number of trials, which he has related in detail in his famous work called *Stella Martis*, he overcame the obstacle, which an error, supported by the suffrage of every period, had opposed to him; he discovered that the orbit of Mars is an ellipse, of which the Sun occupies one of the foci, and that the motion of the planet is such, that the radius vector, drawn from its centre to that of the Sun, describes equal areas in equal times. Kepler extended these results to all the planets, and published from this theory, in 1626, the Rudolphine tables, for ever memorable in astronomy, as being the first founded on the true laws of the planetary motions.

Without the speculations of the Greeks, on the curves formed from the section of a cone by a plane, these beautiful laws might have been still unknown. The ellipse being one of these curves, its oblong figure gave rise, in the mind of Kepler, to the idea of supposing the planet Mars, whose orbit he had discovered to be oval, to move on it, and soon, by means of the numerous properties which the ancient geometricians had found in the conic sections, he became convinced of the truth of this hypothesis. The history of the sciences offers us many examples of these applications of pure geometry, and of its advantages; for every thing is connected in the immense chain of truths, and often a single observation has been sufficient to shew the connection between a proposition apparently the most sterile, and the phenomena of nature which are only mathematical results of general laws.

The perception of this truth probably gave birth to the mysterious analogies of the Pythagoricians: they had seduced Kepler, and he owed to them one of his most beautiful discoveries. Persuaded that the mean distances of the planets from the Sun, ought to be regulated conformably to these analogies, he compared them a long time, both with the regular geometrical solids, and with the intervals of tones. At length, after seventeen years of meditations and calculation, conceiving the idea of comparing the powers of the numbers which expressed them, he found that the squares of the times of the planetary revolutions, are to each other as the cubes of the major axes of their orbits; a most important law, and which he had the advantage of observing in the system of satellites of Jupiter, and which extends to all the systems of satellites.

We might be astonished that Kepler should not have applied the general laws of elliptic motion to comets. But, misled by an ardent imagination, he lost the clue of the analogy, which should have conducted him to this great discovery. The comets, according to him, being only meteors, engendered in ether; he neglected to study their motions, and thus stopped in the middle of the career which was open to him, abandoning to his successors a part of the glory which he might yet have acquired. In his time, the world had just begun to get a glimpse of the proper method of proceeding in the search of truth, at which genius only arrived by instinct, frequently connecting errors with its discoveries. Instead of passing slowly y a succession of inductions, from insulated phenomena, to others more extended, and from these to the general laws of nature; it was more easy and more agreeable to subject all the phenomena to the relations of convenience and harmony, which the imagination could create and modify at pleasure.

Thus, Kepler explained the disposition of the solar system, by the laws of musical harmony. We behold him even in his latest works, amusing himself with these chimerical speculations, even so far as to regard them as the "*life and soul*" of astronomy. He has deduced from them the excentricity of the terrestrial orbit, the density of the Sun, its parallax, and other results; the inaccuracy of which, now discovered, is a proof of the errors to which we expose ourselves, in deviating from the rout traced by observation.

After having destroyed the epicycles which Copernicus had preserved; after having determined the curve which the planets describe round the Sun, and discovered the laws of their motion; Kepler too near to the principle from which these laws were derived, not to foresee it. Attempts to discover this principle, often exercised his active imagination; but the moment was not yet arrived, to make this last step, a more profound knowledge of mechanics, and a more perfect state of geometry.

However, amidst the fruitless trials of Kepler, and his numerous errors, the connection of facts conducted him to correct opinions on this subject, in the work in which he published his principal discoveries.

"Gravity," he says, "in his *Commentary on Mars*, is only a mutual and corporeal affection between similar bodies. "Heavy bodies do not tend to the centre "of the world, but to that of the round "body, of which they form a part; and "if the Earth were not spherical, heavy "bodies would not fall towards its centre, "but towards different points."

If the Moon and Earth were not retained at their respective distances, they would fall upon each other, the Moon passing through to $\frac{53}{54}$ of the distance, and the Earth passing through the remainder, supposing them equally dense. He believed also, that the attraction of the Moon was the cause of the tides, and he suspected, that the irregularities of the lunar motion were produced by the combined actions of the Sun and Earth on the Moon.

Astronomy likewise owes to Kepler, many useful discoveries. His work on optics, is full of new and interesting matter; he there explains the mechanism of vision, which was unknown before him. He assigned the true cause of the *lumiere cendrée* of the Moon; but he gave the honor of this discovery to his master, Maestlinus, entitled to notice from this discovery, and from having recalled Kepler to astronomy, and converted Galileo to the system of Copernicus.

Finally, Kepler, in his work entitled, *Stercometria Daliorum*, has presented some conceptions on affinity, which have influenced the revolution experienced by geometry towards the end of the last century.

With so many claims to admiration, this great man lived in misery, while judicial astrology, every where honored, was magnificently recompensed. The astronomers of his time, Descartes himself and Galileo, who might have received the greatest advantage from his discoveries, do not appear to have perceived their importance.

Fortunately the enjoyment which a man of genius receives from the truths which he discovers, and the prospect of a just and grateful posterity, console him for the ingratitude of his contemporaries.

Kepler had obtained pensions which were always ill paid : going to the diet of Ratisbon to solicit his arrears, he died in that city the 15th of November 1630. He had in his latter years the advantage of seeing the discovery of logarithms, and making use of them. This was due to Nepier, a Scottish baron ; it is an admirable contrivance, an improvement on the ingenious algorithm of the Indians, and which, by reducing to a few days the labour of many months, we may almost say doubles the life of astronomers, and spares them the errors and disgusts inseparable from long calculations ;—an invention so much the more gratifying to the human mind, as it is entirely due to its own powers : in the arts man makes use of the materials and forces of nature to increase his powers, but here all is his own work.

The labours of Huygens followed soon after those of Kepler and Galileo. Very few men have deserved so well of the sciences, by the importance and sublimity of their researches. The application of the pendulum to clocks is one of the most beautiful acquisitions which astronomy and geography have made, and to which fortunate invention, and to that of the telescope, the theory and practice of which Huygens considerably improved, they owe their rapid progress.

He discovered, by meaus of excellent

object-glasses which he succeeded in constructing, that the singular appearances of Saturn were produced by a very thin ring, with which the planet is surrounded: his assiduity in observing made him discover one of the satellites of Saturn.

He made numerous discoveries in geometry and mechanics; and if this extraordinary genius had conceived the idea of combining his theorems on centrifugal forces with his beautiful investigation on involutes, and with the laws of Kepler, he would have preceded Newton in his theory of curvilinear motion, and in that of universal gravitation. But it is not in such approximations that discovery consists.

Towards the same time, Hevelius rendered himself useful to astronomy, by his immense labours. Few such indefatigable observers have existed; it is to be regretted that he would not adopt the application of telescopes to quadrants, an invention which gave a precision previously unknown to astronomy.

At this epoch astronomy received a new impulse from the establishment of learned societies.

Nature is so various in her productions and phenomena, of which it is so difficult to ascertain the causes, that it is requisite for a great number of men to unite their intellect and exertions to comprehend and develope her laws. This union is particularly requisite when the sciences in extending approximate, and require mutual support from each other.

It is then, that the natural philosopher has recourse to geometry, to arrive at the general causes of the phenomena which he observes, and the geometrician in his turn interrogates the philosopher, in order to render his own investigation useful, by applying them to experience: and to open in these applications a new road in analysis. But the principal advantage of learned societies is the philosophical feeling on every subject which is introduced into them, and from thence diffuses itself over the whole nation. The insulated philosopher may resign himself without fear to the spirit of system; he only hears contradiction at a distance; but in a learned society the shock of systematic opinions at length destroys them, and the desire of mutually convincing each other establishes between the members an agreement only to admit the results of observation and calculation. Thus experience has proved that since the origin of these establishments true philosophy has been generally extended.

By setting the example of submitting every opinion to the test of severe scrutiny, they have destroyed prejudices which had so long reigned among the sciences, and in which the highest intellects of the preceding ages had participated. Their useful influence on opinion accumulated in our own time, with an enthusiasm which at other periods would have perpetuated them. Finally, it is among them or by the encouragement they offer that those grand theories have been formed which are placed above the reach of the vulgar by their comprehensiveness; and which, extending themselves by their numerous occasions in which they are applicable, to nature and to the arts, are inexhaustible sources of delight and intelligence.

Of all the learned societies, the two most celebrated for the number and importance of their discoveries in the sciences, and particularly in astronomy, are the Academy of Sciences in Paris, and the Royal Society in London.

The first was created in 1666, by Louis XIV. who foresaw the lustre which the arts and sciences were to diffuse over his reign. This monarch, worthily seconded by Colbert, invited many learned strangers to fix themselves in his capital. Huygens availed himself of this flattering invitation; he published his admirable work *De horologio oscillatorio*, in the midst of the academy, of which he was one of the first members. He would have finished his days in this country, had it not been for the disastrous edict which, towards the end of the last century, deprived

France of so many valuable citizens. Huygens, departing from a country in which the religion of his ancestors was proscribed, retired to the Hague, where he was born the 14th of April, 1629, and died there the 15th of June, 1695.

Dominic Cassini was likewise induced to go to Paris, by the benefits of Louis XIV. During forty years of useful labours, he enriched astronomy with a crowd of discoveries: such are the theory of the satellites of Jupiter, the motions of which he determined from observations of their eclipses; the discovery of the four satellites of Saturn; those of the rotation of Jupiter, of the belts parallel to his equator, of the rotation of Mars, of the zodiacal light, a very approximate knowledge of the Sun's parallax, a very exact table of refractions, and, above all, the complete theory of the libration of the Moon.

The great number of astronomical academicians of extraordinary merit, and the limits of this historical abridgment, do not permit me to give an account of their labors; I shall content myself with observing that the application of the telescope to the quadrant, the invention of the micrometer and heliometer, the successive propagation of light, the magnitude of the Earth, its ellipticity, and the diminution of gravity at the equator, are all discoveries due to the Academy of Sciences.

Astronomy does not owe less to the Royal Society of London, the origin of which is a few years anterior to that of the Academy of Sciences. Among the astronomers which it has produced, I shall cite Flamstead, one of the greatest observers that have ever appeared. Halley, rendered illustrious by his travels, undertaken for the advantage of science, by his beautiful investigation concerning comets, which enabled him to discover the return of the comet in 1759; and by the ingenious idea of employing the transit of Venus over the Sun, in the determination of its parallax. I shall mention, lastly, Bradley, the model for observers, and who will be for ever celebrated for two of the most beautiful discoveries ever made in astronomy, the aberration of the fixed stars, and the nutation of the axis of the Earth.

When the application of the pendulum to clocks, and of telescopes to quadrants, had rendered the slightest changes in the position of the celestial bodies perceptible to observers, they endeavoured to determine the annual parallax of the fixed stars; for it was natural to suppose, that so great an extent as the diameter of the terrestrial orbit, would be sensible even at the distance of these stars. Observing them carefully, at every season of the year, there appeared slight variations; sometimes favorable, but more frequently contrary to the effects of parallax. To determine the law of these variations, an instrument of great radius, and divided with extreme precision, was requisite. The artist who executed it, deserves to partake of the glory of the astronomer who owed his discovery to him. Graham, a famous English watch-maker, constructed a great sector, with which Bradley discovered the aberration of the fixed stars, in the year 1727. To explain it, this great astronomer conceived the fortunate idea of combining the motion of the Earth with that of light, which Roemer had discovered at the end of the last century, by means of the eclipses of Jupiter's satellites. We should be surprised that none of the distinguished philosophers who then existed, and who knew the motion of light, should have paid any attention to the very simple effects which result from it, in the apparent position of the fixed stars. But, the human mind, so active in the formation of systems, has almost always waited till observation and experience have acquainted it with important truths, which its powers of reasoning alone might have discovered.

It is thus that the invention of telescopes has followed by more than three centuries that of lenses, and even then was only due to accident.

In 1745, Bradley discovered by observation, the nutation of the terrestrial axis. In all the apparent variations of the fixed stars, observed with extraordinary care, he perceived nothing which indicated a perceptible parallax. The measures of the degrees of the terrestrial meridian, and of the pendulum, multiplied in different parts of the globe, of which France gave the example, by measuring the whole arc of the meridian, which crosses it, and by sending the academician to the north and to the equator, to observe the magnitude of these degrees, and the intensity of the force of gravity. The arc of the meridian, comprised between Dunkirk and Barcelona, determined by very precise observations, and forming the base of the most natural and simple system of measures; the voyages undertaken to observe the two transits of Venus over the Sun's disk, in 1761 and 1769, and the exact knowledge of the dimensions of the solar system, which has been derived from these voyages; the invention of achromatic telescopes, of chronometers, of the sextant and repeating circle, the discovery of the planet Uranus, by Herschel, in 1781; that of its satellites, and of two new satellites of Saturn, due to the same observer, all the astronomical theories being brought to perfection, and all the celestial phenomena, without exception, being referred to the principle of universal gravitation. These, with the discoveries of Bradley, are the principal obligations which astronomy owes to our century, which, with the preceding, will always be considered as the most glorious epoch of the science.

CHAP. V.

Of the Discovery of universal Gravitation.

After having shewn by what successive efforts the human mind has attained the knowledge of the celestial motions, it only remains to consider the means by which it has arrived at the general principle, on which these laws depend. Descartes was the first who endeavoured to reduce the motions of the heavenly bodies to some mechanical principle. He imagined vortices of subtle matter, in the centre of which he placed these bodies. The vortex of the Sun forced the planet into motion; that of the planet, in the same manner, forced its satellite to revolve round it. The motion of comets traversing the heavens in all directions, destroyed these vortices, as they had before destroyed the solid christalline spheres of the ancient astronomers. Thus, Descartes was no happier in his mechanical, than Ptolemy in his astronomical theory. But their labours have not been useless to science. Ptolemy has transmitted to us, through fourteen centuries of ignorance, the few astronomical truths which the ancients had discovered. Descartes, born at a later period, and at a time when the universal curiosity was excited, which he himself had increased, by substituting in the place of ancient errors, others more seducing, and resting on the authority of his geometrical discoveries, was enabled to destroy the empire of Aristotle and Ptolemy, which might have stood the attack of a more careful philosopher; but by establishing as a principle, that we should begin by doubting of every thing, he himself warned us to examine his own system with severity, which could not long resist the new truths that were opposed to it. It was reserved for Newton to teach us the general principles of the heavenly motions. Nature not only endowed him with a profound genius, but placed his existence in a most fortunate period. Descartes had changed the face of the mathematical sciences, by the application of algebra to the theory of curves and variable functions. The geometry of infinites, of which this theory contained the germ, began to appear in various places. Wallis, Wren, and Huygens, had discovered the laws of motion; the discoveries of Galileo, on falling bodies, and of Huygens on evolutes and centrifugal force, led to the theory of motion in curves; Kepler had determined those described by the planets, and had formed a remote conception of universal gravitation; and finally, Hook had distinctly perceived that their motion was the result of a projectile force, combined with the attractive force of the Sun. The science of celestial mechanics wanted nothing more to bring it to light, but the genius of man, who, by generalizing these discoveries, should be capable of perceiving the law of gravitation: it is this which Newton accomplished in his immortal work on the mathematical principles of natural philosophy. This philosopher, so deservedly celebrated, was born at Woolstrop, in England, in the latter end of the year 1642, the year in which Galileo died. His first success in his early studies, announced his future reputation; a cursory perusal of elementary books, was sufficient for him to comprehend them; he next read the geometry of Descartes, the optics of Kepler, and the arithmetic of infinites, by Wallis, but soon aspiring to new inventions, he was, before the age of twenty-seven, in possession of his method of fluxions, and his theory of light. Anxious for repose, and averse to literary controversy, he delayed publishing his discoveries. His friend and preceptor, Dr. Barrow, exerted himself in his favor, and obtained for him the situation of professor of mathematics in the university of Cambridge; it was during this period, that, yielding to the request of Halley, and the solicitations of the Royal Society, he published his *Principia*. The university, of which he was a member, chose him for their representative, in the conventional parliament of 1788, and for that which was convened in 1701. He was knighted and appointed director of the mint, by Queen Anne: he was elected president of the Royal Society in 1703, which dignity he enjoyed till his death, in 1727. During the whole of his life he obtained the most distinguished consideration, and the nation to whose glory he had so much contributed, decreed him at his death public funeral honours.

In 1666, Newton retired into the country, and, for the first time, directed his thoughts to the system of the world. The descent of heavy bodies, which appears nearly the same at the summit of the highest mountains as at the surface of the Earth, suggested to him the idea, that gravity might extend to the Moon, and that being combined with some motion of projection, it might cause it to describe its elliptic orbit round the Earth. To verify this conjecture, it was necessary to know the law of the diminution of gravity. Newton considered, that if the Moon was retained in its orbit by the gravity of the Earth, the planets should also be retained in their orbits by their gravity towards the Sun, and demonstrated this by the law of the areas being proportional to the times. Now it results from the relation of the squares of the times to the cubes of the greater axis of their orbits, that their centrifugal force, and consequently their tendency to the Sun, diminishes inversely as the squares of the distances from this body. Newton, therefore, transferred to the Earth this law of the diminution of the force of gravity, and reasoning from the experiments of falling bodies, he determined the height which the Moon, abandoned to itself, would fall in a short interval of time. This height is the versed sine of the arc which it describes in the same interval; and this quantity the lunar parallax gives in parts of the radius of the Earth, so that, to compare the law of gravitation with observation, it was necessary to know the magnitude of this radius; but Newton having, at that time, an erroneous estimate of the terrestrial meridian, obtained a different result from what he expected, and suspecting that some unknown forces united themselves with the gravity of the Moon, he abandoned his original idea. Some years afterwards, a letter from Dr. Hook induced him to investigate the nature of the curve described by projectiles round the centre of the Earth. Picard had lately finished the measure of a degree in France, and Newton found, by this measure, that the Moon was retained in its orbit by the force of gravity alone, supposed to vary inversely as the square of the distance. By this law he found that bodies in their fall, describe ellipses, of which the centre of the Earth occupies one of their foci, and then, considering that the planetary orbits are likewise ellipses, having the Sun in one of their foci, he had the satisfaction to see, that the solution which he had undertaken from curiosity, could be applied to the greatest objects in nature. He arranged the several propositions relative to the elliptic motions of planets, and Dr. Halley having induced him to publish them, he composed his grand work, the Principiæ, which appeared in 1687. These details, which have been transmitted to us by his friend and cotemporary Dr. Pemberton, prove that this great philosopher had, so early as 1666, discovered the principal theorems on centrifugal force, which Huygens published six years afterwards, at the end of his work De Horologio Oscillatorio; for, indeed it is highly probable that the author of the method of fluxions, who seems then to have been in possession of it, should easily have discovered these theorems. Newton arrived at the law of the diminution of gravity, by the relation which subsists between the squares of the periodic times, and the cubes of the greater axes of their orbits, supposed circular. He demonstrated that this relation exists in elliptic orbits generally, and that it indicates an equal gravity of the planets towards the Sun, supposing them at an equal distance from its centre. The same equality of gravity towards the principal planet, exists likewise in all the systems of satellites, and Newton verified it on terrestrial bodies, by very accurate experiments.

This great geometrician, by considering this question generally, demonstrated that a projectile can move in any conic-section whatever, in consequence of a force directed towards its centre, and varying reciprocally as the square of the distances. He investigated the different properties of motion in this species of curves; he determined the conditions requisite for the section to be a circle, an ellipse, a parabola, or an hyperbola, which conditions depend entirely on the velocity and primitive position of the body.

Any velocity, position, and initial direction of a body being given, Newton assigned the conic section which the body should describe, and in which it ought consequently to move, which answers the reproach which John Bernouilli made him of not having demonstrated, that the conic sections are the only curves which a body, solicited by a force varying reciprocally as the squares of the distance, can describe. These investigations, applied to the motion of comets, informed him that these bodies move round the Sun, according to the same laws as the planets, with the difference only of their ellipses being very excentric; and he gave the means of determining by observation, the elements of these ellipses.

He learned from the comparison of the distance and duration of the revolutions of satellites, with those of the planets, the respective densities and masses of the Sun, and of planets accompanied by satellites, and the intensity of the force of gravity at their surface.

By considering that the satellites move round their planets very nearly, as if the planets were immovable, he discovered that all these bodies obey the same force of gravity towards the Sun.

The equality of action and reaction, did not permit him to doubt, but that the Sun gravitated towards the planets, and these towards their satellites; and even that the Earth is attracted by all the bodies that rest upon it. He extended this proposition afterwards by analogy, to all the celestial bodies, and established as a principle, *that all particles of matter attract each other directly as their mass, and inversely as the square of their distance.*

Arrived at this principle, Newton saw that the great phenomena of the system of the world might be deduced from it. By considering gravity at the surfaces of the celestial bodies, as the result of the attractions of all their particles, he ascertained these remarkable truths, that the attracting force of a body, or of a spherical stratum, on a point placed without it, is the same as if its mass was compressed into its centre; and that a point placed within a spherical stratum, or generally any stratum terminated by two elliptic surfaces, similar and similarly situated, is equally attracted on every side.

He proved that the motion of rotation of the Earth, ought to have flattened it in the direction of the poles, and he determined the law of the variation of the degrees and of gravity, supposing it homogeneous.

He saw that the action of the Sun and Moon on the terrestrial spheroid ought to produce a motion in its axis of rotation, to make the equinoxes retrograde, to elevate the waters of the ocean, and to produce in this great fluid mass the oscillations which are observed under the name of tides.

Lastly, he was convinced that the lunar irregularities were produced by the combined action of the Sun and Earth on this satellite. But with the exception of what concerns the elliptic motion of the planets and comets, the attraction of spherical bodies, and the intensity of gravity at the surface of the Sun, and of those planets that are accompanied by satellites, all these discoveries were only sketched by Newton. His theory of the form of the planets is limited by the supposition of their homogenity: his solution of the problem of the precession of the equinoxes, though very ingenious, is, notwithstanding the apparent agreement of his result with observation, in many respects defective; in the great number of the perturbations of the celestial motions, he has only considered those of the lunar motion, of which the most considerable, the evection, has escaped his investigation. He has perfectly established the existence of the principle which hè discovered, but the developement of its consequences and its advantages, has been the work of the successors of this great geometrician. The state of imperfection in which the infinitesmal calculus must have been in the hands of its inventor, has not permitted him to resolve completely the difficult problems which the theory of the system of the world presents; and he has been often obliged to give conjectures, at least uncertain till they have been verified by a rigorous calculation. Notwithstanding these inevitable defects, the importance and extent of his discoveries, the great number of original and profound conceptions, which have been the germ of the most brilliant theories of the geometricians of this century, and arranged with much elegance, insures to his *Principia* a pre-eminence over all other productions of human intellect.

The case is not the same with the sciences as with literature: this has limits which a man of genius may reach when he employs a language brought to perfection; he is read with the same interest in all ages; and time only adds to his reputation by the vain efforts of those who try to imitate him.

The sciences, on the contrary, without bounds like nature herself, increase infinitely by the labours of successive generations the most perfect work; by raising them to a height from which they can never again descend, gives birth to new discoveries which produce in their turn new works which efface the former from which they originated. Others will present in a point of view more general and more simple, the theories described in the *Principia*, and all the truths which it has displayed ; but it will remain as an eternal monument of the profundity of that genius which has revealed to us the greatest law of the universe.

This work and the equally original treatise by the same author on optics, have still the merit of being the best models which he proposed in the sciences, and in the delicate art of making experiments and submitting them to calculation. We there see the most beautiful applications of the method which consists in tracing the principal phenomena to their causes by a succession of inductions, and afterwards to redescend from these causes, to all the details of the phenomena.

General laws are impressed in all individual cases, but they are complicated with so many extraneous circumstances, that the greatest address is often necessary to develope them. The phenomena most proper for this object must be chosen, they must be multiplied that the attendant circumstances may be varied, and that whatever they have in common may be observed.

We thus ascend successively to relations more and more extended, and we arrive at length at general laws, which are verified either by proofs or by direct experiment, if that is possible, or by examining if they satisfy all the known phenomena.

This is the most certain method by which we can be guided in the search of truth. No philosopher has adhered more faithfully to this method than Newton; it conducted him to his discoveries in analysis, and it led him to the principle of universal gravitation, and to the properties of light. Other philosophers in England, cotemporaries of Newton, adopted it by his example, and it was the base of a great number of excellent works which then appeared.

The philosophers of antiquity following

a contrary path, and considering themselves as at the source of every thing, imagined general causes to explain them.

Their method, which was only productive of vain systems, had not greater success in the hands of Descartes. In the time of Newton, Leibnitz, Malebranche and other philosophers employed it with as little advantage.

At length the inutility of the hypotheses to which it led its followers, and the progress for which the sciences are indebted to the method of inductions has brought back all philosophers to this last method, which Chancellor Bacon has established with the whole force of reason and eloquence, and which Newton has yet more strongly recommended by his discoveries.

It is by means of synthesis that this great geometrician has explained his theory of the system of the world. It appears, however that he found the greater part of his theorems by analysis, the limits of which he has considerably extended, and to which he allows himself to have owed his general results on the quadratures of curves.

But his great predilection for synthesis, and his esteem for the geometry of the ancients, has induced him to represent his theorems, and even his method of fluxions under a synthetic form. And it is evident by the rules and examples which he has given of these transformations in many works, how much importance he attached to it. We may regret with the geometricians of his time, that he has not followed in the exposition of his discoveries, the path by which he arrived at them; and that he has suppressed the demonstration of many results, such as the equation of the solid of least resistance, preferring the pleasure of leaving it to be divined to that of enlightening his readers.

The knowledge of the method which has guided a man of genius is not less serviceable to the progress of the sciences, and even to his own glory, than his discoveries; and the principal advantage which has been derived from the famous dispute between Newton and Leibnitz, concerning the invention of the infinitesmal calculus, has been to make known the path of these two great men, in their first analytical labours.

The preference of Newton for the synthetical method, may be explained by the elegance with which he connected his theory of curvilinear motion, with the investigations of the ancients on the conic sections, and the beautiful discoveries which Huygens had published according to this method. Geometrical synthesis has besides the property of never losing sight of its object, and of enlightening the whole path which leads from the first axioms to their last consequences, while algebraic analysis soon makes us forget the principal object, to occupy ourselves with abstract combinations, and it is only at the end that it brings us back to it. But in thus quitting the object of investigation, after having assumed what is indispensably necessary to arrive at the required result, by directing all our attention to the operations of analysis and reserving all our forces to overcome the difficulties which present themselves, we are conducted by the universality of this method, by the inestimable advantage of thus transferring the train of reasoning in mechanical questions, to results often inaccessible to synthesis. The theory of the system of the world offers a great number of examples of this power of analysis to which this theory owes a degree of perfection which it would never have acquired if no other path had been followed than that traced by Newton. Such is the fecundity of analysis, that if we translate particular truths into this universal language, we shall find a number of new and unexpected truths arise merely from the form of expression. No language is so susceptible of the elegance which arises from the developement of a long train of expressions connected with each other, and all flowing from the same fundamental idea. Analysis unites to all these advantages that of always being able to conduct us to the most simple methods. Nothing more is requisite than to apply it in a convenient manner by a judicious choice of unknown quantities, and by giving to the results the form most easily reducible to geometrical construction, or to numerical calculation. The geometricians of this century, convinced of its superiority, have principally applied themselves to extend its domain, and enlarge it boundaries.

However, geometrical considerations ought not to be abandoned; they are of the greatest utility in the arts. Besides it is curious to imagine the different results of analysis represented in space; and reciprocally, to read all the affections of lines and surfaces, and all the variations in the motions of bodies, in the equations which express them. This approximation of geometry and analysis, diffuses a new light over the sciences; the intellectual operations of the latter, rendered perceptible by the images of the former, are more easy to comprehend, and more interesting to pursue ; and when observation realizes these images, and transforms these geometrical results into laws of nature, and when these, embracing the whole universe, display to our view its present and future state, the view of this sublime spectacle, presents to us one of the most noble pleasures reserved for mankind.

About fifty years have passed since the discovery of the theory of gravitation, without any remarkable addition to it. All this time has been requisite for this great truth to be generally understood, and to surmount the obstacles opposed to it by the system of vortices, and the authority of geometricians cotemporary with Newton, who combatted it perhaps from vanity, but who have nevertheless accelerated its progress by their labours on infinitesmal analysis.

At length, their successors have conceived the fortunate idea of applying this analysis to the celestial motions by reducing them to differential equations which they have rigorously integrated, or by converging approximation. They have thus explained by the law of gravitation all the known phenomena of the system of the world, and have given an unhoped for precision to astronomical tables. It has been necessary, for this object, to bring to perfection at once mechanics, optics, and analysis, which principally owe their rapid improvements to their being necessary to the purposes of physical astronomy. It might be rendered yet more correct and simple, but posterity will no doubt see with gratitude that the geometricians of this century have transmitted no astronomical phenomenon of which they have not determined the cause and the law.

Justice to France requires us to observe that if England had the advantage of giving birth to the discovery of universal gravitation, it is principally to the French geometricians, and to the encouragements of the Academy of Sciences, that the numerous developments of this discovery are due, and the revolution which it has produced in astronomy.

CHAP. VI.

Considerations on the System of the Universe, and on the Future Progress of Astronomy.

LET us now direct our attention to the arrangement of the solar system, and its relation with the stars. The immense globe of the Sun, the focus of these motions, revolves upon its axis in twenty-five days and a half. Its surface is covered with an ocean of luminous matter, whose active effervescence forms variable spots, often very numerous, and sometimes larger than the Earth. Above this ocean exists an immense atmosphere, in which the planets, with their satellites, move, in orbits nearly circular, and in planes little inclined to the ecliptic. Innumerable comets, after having approached the Sun, remove to distances, which evince that his empire extends beyond the known limits of the planetary system. This luminary not only acts by its attraction upon all these globes, and compels them to move around him, but imparts to them both light and heat; his benign influence gives birth to the animals and plants which cover the surface of the Earth, and analogy induces us to believe, that it produces similar effects on the planets; for, it is not natural to suppose that matter, of which we see the fecundity, develope itself in such various ways, should be sterile upon a planet so large as Jupiter, which, like the Earth, has its days, its nights, and his years, and on which observation discovers changes that indicate very active forces. Man, formed for the temperature which he enjoys upon the Earth, could not, according to all appearance, live upon the other planets; but ought there not to be a diversity of organization suited to the various temperatures of the globes of this universe? If the difference of elements and climates alone, causes such variety in the productions of the Earth, how infinitely diversified must be the productions of the planets and their satellites ? The most active imagination cannot form any just idea of them, but still their existence is extremely probable.

However arbitrary the system of the planets may be, there exists between them some very remarkable relations, which may throw light on their origin; considering them with attention, we are astonished to see all the planets move round the Sun from west to east, and nearly in the same plane, all the satellites moving round their respective planets in the same direction, and nearly in the same plane with the planets. Lastly, the Sun, the planets, and those satellites in which a motion of rotation have been observed, turn on their own axis, in the same direction, and nearly in the same plane as their motion of projection.

A phenomenon so extraordinary, is not the effect of chance, it indicates an universal cause, which has determined all these motions. To approximate somewhat to the probable explanation of this cause, we should observe that the planetary system, such as we now consider it, is composed of seven planets, and fourteen satellites. We have observed the rotation of the Sun, of five planets, of the Moon, of Saturn's ring, and of his farthest satellite; these motions with those of revolution, form together thirty direct movements, in the same direction. If we conceive the plane of any direct motion whatever, coinciding at first with that of the ecliptic, afterwards inclining itself towards this last plane, and passing over all the degrees of inclination, from zero to half the circumference; it is clear that the motion will be direct in all its inferior inclinations to a hundred degrees, and that it will be retrograde in its inclination beyond that; so that, by the change of inclination alone, the direct and retrograde motions of the solar system, can be represented. Beheld in this point of view, we may reckon twenty-nine motions, of which the planes are inclined to that of the Earth, at most $\frac{1}{4}$th of the circumference; but, supposing their inclinations had been the effect of chance, they would have extended to half the circumference, and the probability that one of them would have exceeded the quarter, would be $1 - \frac{1}{29}$, or $\frac{536870911}{536870912}$. It is then extremely probable, that the direction of the planetary motion is not the effect of chance, and this becomes still more probable, if we consider that the inclination of the greatest number of these motions to the ecliptic, is very small, and much less than a quarter of the circumference.

Another phenomenon of the solar system equally remarkable, is the small excentricity of the orbits of the planets and their satellites, while those of comets are much extended. The orbits of the system offer no intermediate shades between a great and small excentricity. We are here again compelled to acknowledge the effect of a regular cause; chance alone could not have given a form nearly circular, to the orbits of all the planets. This cause then must also have influenced the great excentricity of the orbits of comets, and what is very extraordinary, without having any influence on the direction of their motion; for, in observing the orbits of retrograde comets, as being inclined more than 100° to the ecliptic, we find that the mean inclination of the orbits of all the observed comets, approaches near to 100°, which would be the case if the bodies had been projected at random.

Thus, to investigate the cause of the primitive motions of the planets, we have given the five following phenomena: 1st, The motions of planets in the same direction, and nearly in the same plane. 2d, The motion of their satellites in the same direction, and nearly in the same plane with those of the planets. 3d, The motion of rotation of these different bodies, and of the Sun in the same direction as their motion of projection, and in planes but little different. 4th, The small excentricity of the orbits of the planets, and of their satellites. 5th, The great excentricity of the orbits of comets, although their inclinations may have been left to chance.

Buffon is the only one whom I have known, who, since the discovery of the true system of the world, has endeavoured to investigate the origin of the planets, and of their satellites. He supposes that a comet, in falling from the Sun, may have driven off a torrent of matter, which united itself at a distance, into various globes, greater or smaller, and more or less distant from this luminary. These globes are the planets and satellites, which, by their cooling, are become opaque and solid.

This hypothesis accounts for the first of the five preceding phenomena; for, it is clear that all bodies thus formed, must move nearly in the plane which passes through the centre of the Sun, and in the direction of the torrent of matter which produces them. The four other phenomena appears to me inexplicable by his theory. In fact, the absolute motion of the particles of a planet would then be in the same direction of the motion of its centre of gravity; but it does not follow that the rotation of the planet would be in the same direction. Thus, the Earth may turn from west to east, and yet the absolute direction of each of its particles may be from east to west. What I say of the rotatory motion of the planets, is equally applicable to the motion of their satellites in their orbits, of which the direction in the hypothesis he adopts, is not necessarily the same with the projectile motion of the planets.

The small excentricity of the motion of the planetary orbits, is not only very difficult to explain on this hypothesis, but the phenomenon contradicts it. We know by the theory of central forces, that if a body moving in an orbit round the Sun, touched the surface of this luminary, it would uniformly return to it at the completion of each revolution, from whence it follows, that if the planets had originally been detached from the Sun, they would have touched it at every revolution, and their orbits, far from being circular, would be very excentric. It is true, that a torrent of matter, sent off from the Sun, cannot correctly be compared to a globe which touches its surface. The impulse which the particles of this torrent receive from one another, and the reciprocal attraction exercised among them, may change the direction of their motion, and increase their perihelion distances; but their orbits would uniformly become very excentric, or at least it must be a very extraordinary chance that would give them excentricities so small as those of the planets. In a word, we do not see, in this hypothesis of Buffon, why the orbits of about eighty comets, already observed, are all very elliptical. This hypothesis, then, is far from accounting for the preceding phenomena. Let us see if it is possible to arrive at their true cause.

Whatever be its nature, since it has produced or directed the motion of the planets and their satellites, it must have embraced all these bodies, and considering the prodigious distance which separates them, they can only be a fluid of immense extent. To have given in the same direction, a motion nearly circular round the Sun, this fluid must have surrounded the luminary like an atmosphere. This view, therefore, of planetary motion, leads us to think, that in consequence of excessive heat, the atmosphere of the Sun originally extended beyond the orbits of all the planets, and that it has gradually contracted itself to its present limits, which may have taken place from causes similar to those which caused the famous star that suddenly appeared in 1572, in the constellation Cassiopæa, to shine with the most brilliant splendour during many months.

The great excentricity of the orbits of comets, leads to the same result; it evidently indicates the disappearance of a great number of orbits less excentric, which indicates an atmosphere round the Sun, extending beyond the perihelion of observable comets, and which, in destroying the motion of those which they have traversed in a duration of such extent, have re-united themselves to the Sun. Thus, we see that there can at present only exist such comets as were beyond this limit at that period. And as we can observe only those which in their perihelion approach near the Sun, their orbits must be very excentric: but, at the same time, it is evident that their inclinations must present the same inequalities as if the bodies had been sent off at random, since the solar atmosphere has no influence over their motions. Thus, the long period of the revolutions of comets, the great excentricity of their orbits, and the variety of their inclinations, are very naturally explained by means of this atmosphere.

But how has it determined the motions of revolution and rotation of the planets? If these bodies had penetrated this fluid, its resistance would have caused them to fall into the Sun. We may then conjecture, that they have been formed at the successive bounds of this atmosphere, by the condensation of zones, which it must have abandoned in the plane of its equator, and in becoming cold have condensed themselves towards the surface of this luminary, as we have seen in the preceding Book. One may likewise conjecture, that the satellites have been formed in a similar way by the atmosphere of the planets. The five phenomena, explained above, naturally result from this hypothesis, to which the rings of Saturn add an additional degree of probability.

Whatever may have been the origin of this arrangement of the planetary system, which I offer with that distrust which every thing ought to inspire that is not the result of observation or calculation; it is certain that its elements are so arranged, that it must possess the greatest stability, if foreign observations do not disturb it. Through this cause alone, that the motions of planets and satellites are nearly circular, and impelled in the same direction, and in planes differing but little from each other, it arises that this system can only oscillate to a certain extent, from which its deviation must be extremely limited; the mean motions of rotation and revolution of these different bodies are uniform, and their mean distances to the foci of the principal forces which animate them, are uniform. It seems that nature has disposed every thing in the heavens to insure the duration of the system by views similar to those which she appears to us so admirably to follow upon Earth, to preserve the individual and insure the perpetuity of the species.

Let us now look beyond the solar system. Innumerable suns, which may be the foci of as many planetary systems, are spread out in the immensity of space, and at such a distance from the Earth, that the entire diameter of it, seen from their centre, is insensible. Many stars experience both in their colour and splendour, periodical variations, very remarkable; there are some which have appeared all at once, and disappeared after having for some time spread a brilliant light. What prodigious change must have operated on the surface of these great bodies, to be thus sensible at the distance which separates them from us, and how much they must exceed those which we observe on the surface of the Sun? All these bodies which are become invisible, remain in the same place where they were observed, since there was no change during the time of their appearance, there exist then in space obscure bodies as considerable, and perhaps as numerous as the stars. A luminous star, of the same density as the Earth, and whose diameter should be two hundred and fifty times larger than that of the Sun, would not, in consequence of its attraction, allow any of its rays to arrive at us; it is therefore possible that the largest luminous bodies in the universe, may, through this cause, be invisible. A star, which, without being of this magnitude, should yet considerably surpass the Sun, would perceptibly weaken the velocity of its light, and thus augment the extent of its aberration. This difference in the aberration of stars and their situation, observed at the moment of their transient splendor, the determination of all the changeable stars, and the periodical variations of their light; in a word, the motions peculiar to all those great bodies, which, influenced by their mutual attraction, and probably by their primitive impulses, describe immense orbits, should, relatively to the stars, be the principal objects of future astronomy.

It appears that these stars, far from being disseminated at distances nearly equal in space, are united in various groups, each consisting of many millions of stars. Our Sun, and the most brilliant stars, probably make part of one of these groups, which, seen from the point where we are, seems to encircle the heavens, and forms the milky way. The great number of stars which are seen at once in the field of a large telescope, directed towards this way, proves its immense depth, which surpasses a thousand times the distance of Sirius from the Earth; as it recedes, it terminates, by presenting the appearance of a white and continued light, of small diameter, for then, the irradiation which exists even in the most powerful telescopes, covers and obscures the intervals between the stars. It is then probable, that those nebulæ, without distinct stars, are groups of stars seen from a distance, and which, if approached, would present appearances similar to the milky way.

The relative distances of the stars which form each group, are at least a hundred thousand times greater than the distance of the Sun from the Earth. Thus, we may judge of the prodigious extent of these groups, by the number of stars which are perceived in the milky way, if we afterwards reflect on the small extent and infinite number of nebulæ which are separated from one another by an interval incomparably greater than the relative distance of the stars of which they are formed; the imagination, lost in the immensity of the universe, will have difficulty to conceive its bounds.

From these considerations, founded on telescopic observations, it follows, that nebulæ, which appear so well defined, that their centres can be precisely determined, are, with regard to us, the celestial objects most fixed, and those to which it is best to refer the situation of all the stars. It follows then, that the motions of the bodies of our solar system are very complicated. The Moon describes an orbit nearly circular around the Earth; but, seen from the Sun, she describes a series of epicycloids, of which the centres are on the circumference of the terrestrial orbit. In like manner, the Earth describes a series of epicycloids, of which the centres on the arch which the Sun describes around the centre of gravity of our nebulæ; finally, the Sun himself describes a series of epicycloids, of which the centres are on the arch described by the centre of gravity of our nebulæ around that of the universe. Astronomy has already made one great step in making us acquainted with the motion of the Earth, and the series of epicycles which the Moon and the satellites describe upon the orbits of the planets. It remains to determine the orbit of the Sun, and the centre of gravity of its nebulæ; but, if ages are necessary to become acquainted with the motions of the planetary system, what a prodigious duration of time will it require to determine the motions of the Sun and stars? Observation begins to render them perceptible; an attempt has been made to explain them by a change of position in the Sun, indicated by its rotatory motion. Many observations are sufficiently well explained, by supposing the solar system carried towards the constellation Hercules. Other observations seem to prove, that these apparent motions of the stars are a combination of their real motion, with that of the Sun. Upon this subject, time will discover curious and important facts.

There still remains numerous discoveries to be made in our own system. The planet Uranus and its satellites, but lately known to us, leaves room to suspect the existence of other planets, hitherto unobserved. We cannot yet determine the rotatory motion, or the flattening of many of the planets, and the greatest part of their satellites. We know not, with sufficient precision, the density of all these bodies. The theory of their motions is a series of approximations, whose convergence depends, at the same time, on the perfection of our instruments, and the progress of analysis, and which must, by these means, daily acquire new degrees of correctness. By accurate and repeated measurement, the inequalities in the figure of the Earth, and the variation of weight on its surface, will be determined. The return of comets, already observed, new comets which will appear, the appearance of those, which, moving in hyperbolic orbits, can wander from system to system, the disturbance all those stars experience, and which, at the approach of a large planet, may entirely change their orbits, as is conjectured, happened by the action of Jupiter on the comet of 1770; the accidents, that the proximity, and even the shock of these bodies, may occasion in the planets, and in the satellites; in a word, the changes which the motions of the solar system experience, with respect to the stars; such are the principal objects which the system presents to astronomical researches, and future geometricians.

Contemplated as one grand whole, astronomy is the most beautiful monument of the human mind; the noblest record of its intelligence. Seduced by the illusions of the senses, and of self-love, man considered himself, for a long time, as the centre of the motion of the celestial bodies, and his pride was justly punished by the vain terrors they inspired. The labour of many ages has at length withdrawn the veil which covered the system. Man appears, upon a small planet, almost imperceptible in the vast extent of the solar system, itself only an insensible point in the immensity of space. The sublime results to which this discovery has led, may console him for the limited place assigned him in the universe. Let us carefully preserve, and even augment the number of these sublime discoveries, which form the delight of thinking beings.

They have rendered important services to navigation and astronomy; but their great benefit has been the having dissipated the alarms occasioned by extraordinary celestial phenomena, and destroyed the errors springing from the ignorance of our true relation with nature; errors so much the more fatal, as social order can only rest in the basis of these relations. *Truth, Justice*; these are its immutable laws. Far from us be the dangerous maxim, that it is sometimes useful to mislead, to deceive, and enslave mankind, to insure their happiness. Cruel experience has at all times proved that with impunity, these sacred laws can never be infringed.

FINIS.

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