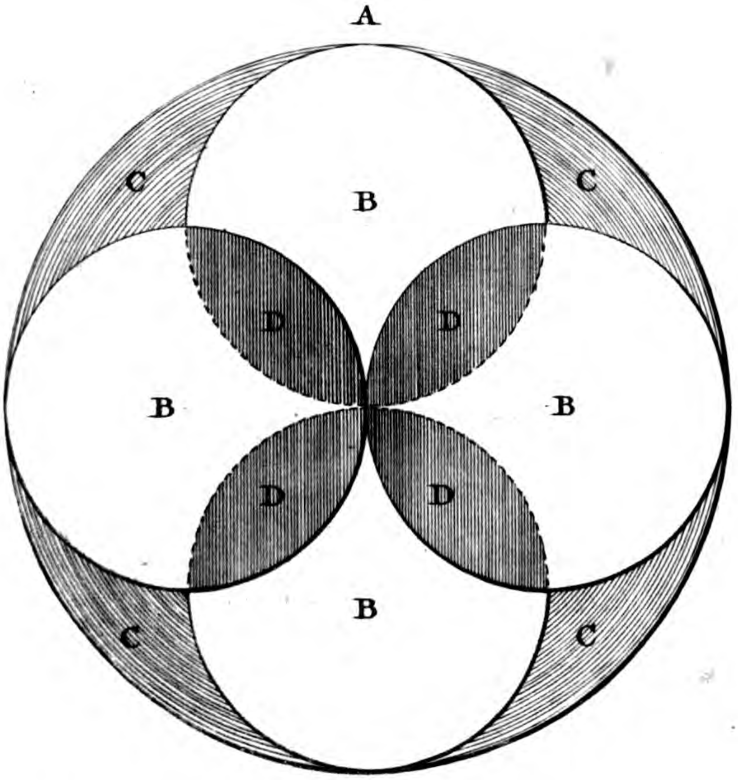
THE PROBLEM, OF NAPOLEON BUONAPARTE TO HIS STAFF.



Resolved and drawn by John Bennett.

London. John Bennett. 4, Three Tun Passage, Newgate Street, 1838.

THE ARCANUM.

COMPRISING

A CONCISE THEORY, OF

PRACTICABLE ELEMENTARY AND DEFINITIVE

GEOMETRY;

EXHIBITING THE

VARIOUS TRANSMUTATIONS OF

SUPERFICES AND SOLIDS;

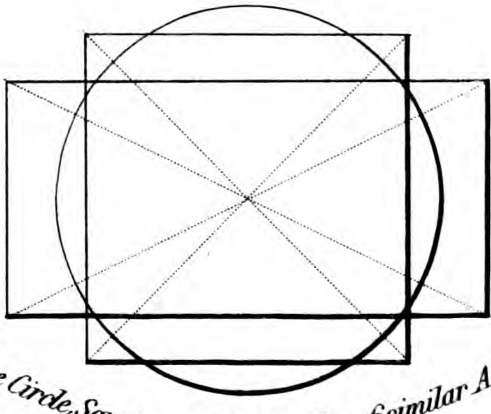
Obtaining also

THEIR ACTUAL CAPACITY BY THE

MATHEMATICAL SCALE:

INCLUDING SOLUTIONS TO THE YET

UNANSWERED PROBLEMS OF THE ANCIENTS.



The Circle, Square, and Rectangle of similar Areas.

BY JOHN BENNETT, ENGINEER, &c.

Author of original Geometrical Illustrations, Artificers Lexicon, Pocket Directors, &c.

LONDON,

John Bennett, 4, Three Tun Passage, Newgate Street. 1838.

PREFACE.

THE whole principal of Geometry consists in three chaste representations only; viz., the circle, the square, and the triangle; all the other geometrical figures emerge in, and emanate from them, as their primitive source: for instance, from the circle are engendered ellipses; from the square we derive all rectangles; and from the triangle, all three-sided figures, and the boundary lines of the whole individually, form innumerable varieties; therefore, upon this hypothesis, the straight line and circle may, with great truth, be said to comprehend the whole doctrine and mystery displayed by their manifold transmutations.

This work is therefore intended to initiate the student into this liberal art, which has hitherto been held in the highest esteem; and although it has invariably been shewn by previous authors, only to be possessed but with infinite difficulty and great application, however, by proper attention and inconsiderable perseverance to the axioms and principles herewith disclosed, a view will most readily be gained to this invaluable science, which will amply compensate for the time bestowed thereon; for, in whatever art or mystery any design may be enveloped, a knowledge of Geometry goes materially to relieve it, and greatly assists in the performance of the ultimate object which is borne in view.

The rapid strides to perfection which is manifested in the engineering department alone, proves at once, the absolute necessity for every person engaging therein, making himself thoroughly conversant with Geometry; not only by rendering more simple the complications usually attending every invention in its first introduction, but also to lead to others of still equal, or even greater utility; it is, therefore, proposed by this work, to conduct with easy, plain, and demonstrable facts, and by imperceptible degrees, by such means thereby lessening in the proceedure, all the difficulties arising out of abtruse portions, by simplicity, and the purest accuracy of arrangement.

It may be necessary to state, that throughout the whole of this work, no advantage has been taken of any author extant; therefore the dispositions the following axioms may be evinced to assume, are in this work purely original. I shall beg the liberty, in this humble address, to propose to my readers the following interrogatory, viz:---The all powerful influence which created *one*, might truly be declared omnipotent; but the previous hand, that increased the amount to countless millions, *made none.*

Now from such variable deductions as these, the science of Geometry flows; its simplicity expands with the progress of understanding its innovation, and may, with much propriety, be compared to the preceding question, which the intelligent reader may be pleased to show, should be he inclined thereto.

January 1*st*, 1839.

INTRODUCTION.

THE frontispiece to this work, commences with the sublimely beautiful problem of Napoleon Buonaparte, to his staff; the manner of obtaining this very valuable and desirable axiom, is as follows: during the publication of the work entitled "Geometrical Illustrations," and on the 9th of May, 1836, a paper was left for the Author thereof, at the Publisher's; the following is a literal copy: viz.

"Napoleon on his voyage from Egypt, amused himself and staff with circular geometry; what circular geometry might be, was only to be collected from the tradition, that the problem given by the future Emperor was, '*To divide the circumference of a circle into four equal parts, by means of circles only.*' The story however created the impression, that the idea which had passed through the mind of that eminent practical geometer, was, that in the properties of the circle, or still more probably in the sphere, might be discovered the elements of geometrical organization."

As a reference to the frontispiece, and by the diagram there exhibited, we shall show that the above question is completely answered, to its fullest extent and meaning, as far as superficies extend. First, the great circle A, is considered to represent the circle proposed by Napoleon; therefore, let two lines be drawn through the centre thereof, which being divided into four parts, as shown by B, B, B, B; four circles are then produced, which occupy a space equal to the great circle A: each small circle will be purely one fourth only; for the parts which are cut into at D, are equalized again in quantity of space, by the vacuities formed at C. From the nature of this problem, it will be presumed by all that the talent of Napoleon must be of the first, or superior order; and therefore, it is not so much to be wondered at, that at certain periods, and in the various transactions during his life, some actions were to be peculiarly marked, and attended with great success. This problem however, will also serve to elucidate innumerable incontrovertible evidences; and from such demonstrations as these, much more may yet be evinced, to furnish abundantly the means of facilitating scientific researches, which otherwise would be lost to society altogether; and, doubtless, after this clear and palpable exposition, much speculative inquiry may be made into the whole system, not unlikely to lead ultimately to the most surprising, and also very beneficial results.

As this diagram represents the superficial sections of the sphere only, the next number, or part, will introduce the divisions of the sphere, whereby a solid will be made to exhibit its fair proportions in the same manner.

DEFINITION OF THE DIAGRAMS,

PRATICALLY ILLUSTRATED.

FIGURE I.

The circle is a plane figure, comprehended by a single curved line, called its circumference, to which right lines, drawn from a point in the middle, called the centre, are equal to each other.

The area of the circle was found by multiplying the circumference by the fourth part of the diameter, or half the circumference by half the diameter; for every circle may be conceived to be a polygon of an infinite number of sides, and the semi diameter must be equal to the perpendicular of such a polygon, and the circumference of the circle equal to the periphery of the polygon ; therefore, half the circumference, multiplied by half the diameter, gives the area of the circle.

FIGURE II.

The square is a quadrilateral figure, both equilateral and equiangular. To find the area of the square, obtain the length of one side; multiply this by itself, and the product will be the area of the square.

FIGURE III.

The equilateral triangle is that which hath all its sides equal. The three preceding figures, viz., the circle, square, and triangle, are the primitive and leading figures throughout the whole of this science; and into which all others are connected to, or related with, throughout the whole of their various and manifold transactions. In them also, are generated and established, this extensive and very comprehensive system, from which arises the production of all geometrical appearances, let whatever be their external formation.

FIGURE IV.

The ellipsis is generated from the circle, in the same manner as the rectangle is from the square: it is also produced from the section of the cone, by a plane cutting both its sides, but not parallel to the base.

FIGURE V.

The rectangle, or parallelogram, is a quadrilateral right lined figure, whose opposite sides are parallel and equal to each other. It is generated by an equal motion of a right line, always parallel to itself. When it has all its four angles right, and only its opposite sides equal, it is called a rectangle, &c.

FIGURE VI.

The plane triangle, of which there are great varieties, is contained under three right lines, one side of which is right angled, as at A, in the figure.

FIGURE VII.

Represents the circle, half of which is dotted, leaving the upper half, or semi circular arch.

FIGURE VIII.

Demonstrates, in the same manner, the ellipsis; half of which describes the elliptical arch.

FIGURE IX.

Shows another section of the ellipsis, which produces the parabolical arch.

FIGURE X.

Is the quarter circle, or quadrant, being the fourth part of the area of the circle.

FIGURE XI.

Is the quarter ellipsis, containing also the fourth part of the area of the ellipsis.

FIGURE XII.

Is the half of the parabola, which contains the fourth part of the area of the ellipsis.

FIGURE XIII.

The square A, and rectangle B, occupying similar areas; the triangles which compose the bodies of both, are similar in figure, and area. Now by this method, any rectangle whose width proves to be double its length, may be squared; because the triangles are the same in figure, and space; consequently, equalized in quantity, whatever be their application.

FIGURE XIV.

Evinces the rectangle, formed immediately from the square, and holding the like space: for the two dotted triangles at the bottom make good the parts which would otherwise be left vacant.

FIGURE XV.

The rectangle A B C D, is produced from the square E F G C; or it may be the square E F G C, by this method discloses the rectangle A B C D: the dotted line H, will be found exactly in the centre of either; consequently, they may be reciprocally introduced; but in this axiom, as in Figure xiii. the rectangle is double the width of the square, or two similar squares joined.

FIGURE XVI.

Elucidates the last axiom in the plainest manner possible; for the rectangle by this means is made to produce its square, without any superfluous lines; notwithstanding which, the former diagram is of infinite use, and will if it did nothing else, serve to illustrate an important object; namely, that of divesting all geometrical questions of superfluous lines.

FIGURE XVII.

In this figure, the small square in the centre, will be found to contain half the area of its circumscribing square; and from the introduction of the dotted diagonal lines, the octagon is found and produced.

FIGURE XVIII.

The right angled triangle, is that which has one right angle, and is represented at A.

FIGURE XIX.

The equilateral triangle has all its sides equal.

FIGURE XX.

The scaleneous triangle has no two sides equal.

FIGURE XXI.

Is the rectangle, quadrangle, or parallelogram: it is generated by an equal motion of a right line, always parallel to itself; it has all its angles right, and opposite sides equal and parallel.

FIGURE XXII.

This figure shows, by the dotted lines, how many variations may be found from it; nevertheless, this diagram introduces but a trifling quantity compared to the numberless alterations which may be obtained from it.

FIGURE XXIII.

The dotted rectangle, as this figure, is made to produce another, whose area remains precisely the same; for it may be perceived, the abstractions which are made from the bottom, are replaced above; thereby altering the figure, making none whatever in the area.

FIGURE XXIV.

The same figure as the last, with this difference only; the deduction is made from the upper member, and added below; and therefore it may properly be said to be reversed.

FIGURE XXV.

The triangles which are taken from this rectangle, are replaced at the top and bottom; therefore, by such means, the figure assumes another shape; but the area remains the same as the dotted one.

FIGURE XXVI.

The same may be said of this; for what is removed from the one end, is added to the other.

FIGURE XXVII.

This figure is the same in nature as Figure xxv. the angles only are placed upon the opposite sides.

FIGURE XXVIII.

The dotted rectangle is made to form two distinct figures, which are equal in area to the whole. The triangle A, although bearing no similarity to the triangle B, their areas are equal notwithstanding.

FIGURE XXIX.

Is a representation of the great sphere, or any solid body, contained under the same formation, the divisions of which have never been geometrically obtained with correctness, as this diagram proposes to elucidate. Enclosed within the great circle are described four minor ones, each of which contain to the utmost exactness one-fourth part of the solidity of the larger body; the whole compose a mass of equal gravitation with the superior. The method for obtaining these sections are more particularly displayed, and may be seen on reference to the frontispiece, which is fully described in the Introduction.

The great sphere of the earth, being thus apportioned, exhibits plainly that which has been so much sought after, and an abundance of valuable information may be gained by the simplicity manifested in the working of this problem. In order, therefore, to enumerate some of the properties depending upon this leading question, the following may be stated as qualities, viz., Fire, Air, Earth, and Water; Heat, Cold, Wet, and Dry. The cardinal points; East, West, North, and South. The four quarters. as Europe, Asia, Africa, and America.

From this illustration, there would appear ample scope for much interesting matter, relating to the foregoing subject; but as this work is confined to the investigation of the problems geometrically, as they appear, all further speculation must be left to the student, who will profit much by studying the sublimity of this question.

FIGURE XXX.

The rectangle with two triangles abstracted from the dotted base line. They are again introduced upon the upper line, altering this figure in its appearance; nevertheless the area remains the same.

FIGURE XXXI.

The dotted triangle contains the area of the rectangle; as a proof of which, the dotted lines of the sides of the triangle cut the upper and lower lines of the rectangle in their centre.

FIGURE XXXII.

The dotted triangle in this figure contains the same area as the rectangle, and either of the four smaller triangles are equal in area to either of the four smaller rectangles. It will also be seen by this figure, that the dotted rhomboides, described within this rectangle, will contain only half the area of it.

FIGURE XXXIII.

The internal square in this figure is one-fourth of the area of the large square; the next square, half; and the triangles this figure produce, will serve to illustrate the various proportions they bear to each other.

FIGURE XXXIV.

The dotted square will be found, upon examination, to hold the area of the whole of the triangles, by which it is encompassed.

FIGURE XXXV.

The equilateral triangle within this square, being divided into four parts, which are equilateral triangles also, they are disposed at each of the sides of the square, altering materially the shape of the figure, but causing not the least deviation from its extension.

FIGURE XXXVI.

The abstractions from the angles and centre of this square, cause much alteration in this figure; but they make no more than the area of the square, the component parts which are removed can be again replaced, forming the original boundary lines.

FIGURE XXXVII.

The circle, to which the dotted line A is the diameter, thereby forming it into two parts, are equal in area to each other.

FIGURE XXXVIII.

The circle divided by the horizontal dotted line A B, and also the dotted vertical line C D, dividing it into four parts, the whole of which are of equal extent.

FIGURE XXXIX.

The circle divided into three equal parts, A, B, C, the whole of which are of similar areas.

FIGURE XL.

The circle divided into five equal parts, and such parts are equal to each other.

FIGURE XLI.

The circle divided into six equal parts, A, B, C, D, E, F, which are also equal; the triangle formed at G, H, I, has all its sides equal; it therefore demonstrates the equilateral triangle.

FIGURE XLII.

The circle in this diagram is divided into seven equal parts, A, B, C, D, E, F, G, the whole of which correspond in their areas.

FIGURE XLIII.

The triangle taken from this figure constitutes one-fourth part of it.

FIGURE XLIV.

The two triangles taken from this square amount to no more than one-fourth part of its area.

FIGURE XLV.

The triangle A B, being detached from this square, will lessen its area one-eighth part.

FIGURE XLVI.

The triangle A and B occupy one-fourth part of the area of this square only.

FIGURE XLVII.

The area of this rectangle is not increased by the alteration, for the additions placed at the sides and ends will make good the deductions taken from other parts of it.

FIGURE XLVIII.

The substance of this rectangle is embodied within the spaces A B, forming two dotted triangles.

FIGURE XLIX.

The abstraction from the base line of this figure is replaced above ; therefore, its surface is neither added to or diminished.

FIGURE L.

The rectangle formed into the dotted square. This can be done in every instance where the length of the rectangle happens to double the width; but in no other case without a different process.

FIGURE LI.

The abstractions from this rectangle, at A, B, C, are added again at E, I, F, G, H; therefore, its limits are neither added to or diminished.

FIGURE LII.

The same may be said of this figure, for the space unoccupied in the middle is divided, and the half is placed at each end.

FIGURE LIII.

The equilateral triangle formed upon a base line. Set the foot of the dividers at each end, extend them to the length of it, and dot them, as this diagram shows.

FIGURE LIV.

The same figure as the last, reversed.

FIGURE LV.

The unequal triangle. Neither of the sides being alike.

FIGURE LVI.

Two sides of this triangle are equal, as may be seen by the dotted chord line.

FIGURE LVII.

The sides of this triangle are equal, but the base line is not the same length. This triangle would form a rectangle of half the width and half the depth.

FIGURE LVIII.

This triangle has all its sides unequal, and in order to make it equal, take the altitude (that is, the depth from the apex to the base line) vertically, and to the centre of the base, which, multiplied by half the length of the base line, will give the area. Such area may be made use of in forming the right angle, equilateral triangle, or any other angle, which may be required.

FIGURE LIX.

The circumference of the circle. The circle given, introduce the dotted square ABCD, within the body of the circle, as described; from the vertical centre dotted line, draw the line ED, which will contain one-fourth part of the circumference of the circle.

FIGURE LX.

The sphere halved. Draw the dotted line BC, cutting the circle in its centre; also the vertical line AD: bisect AC in F, and BA in E; draw the dotted lines EH and FL; draw also the dotted line EGF, and from G draw the line GH, which will be the diameter of a sphere, containing half the solidity of the larger one, as shown at GIKM.

FIGURE LXI.

The square in this diagram is divided into four equal parts, the small square A containing the fourth part only.

FIGURE LXII.

This square is the same as the last, and is intended to elucidate the mathematical scale; therefore, the explanation herewith given will serve as a guide to all the questions throughout this work; notwithstanding the scale may be introduced in other parts, it will, nevertheless, be of the same capacity.

In this figure, the side is supposed to occupy twenty-four inches, or two feet; that is, a square of two feet, whose contents will be, viz. $2 \times 2 = 4$; consequently, the fourth part will contain but one only. There are twenty-four parts to each side; then $24 \times 24 = 576$: the fourth part of 576 is 144. The small square A contains twelve parts; then $12 \times 12 = 144$, the same quantity.

FIGURE LXIII.

The square, in this figure, is divided equally in the middle by one line; thereby forming two rectangles, or parallelograms.

FIGURE LXIV.

This square, as the last, is divided by one line diagonally, introducing two triangles; it is therefore evident that either of the triangles in this figure, or the rectangles in the last, must be equal in area to each other; consequently, the triangle A, in this figure, is equal in area to the rectangle A, in the last.

FIGURE LXV.

This square is halved by a compound line, but the divisions are equal, and the dotted square AB, taken from figure 66 below, will be found of equal area to A or B.

FIGURE LXVI.

The internal diagonal square A being half the area of the larger square, and equal in area to either of the compound figures in the last diagram, is also equal to the rectangle A, figure 63, and to the triangle A, figure 64; the small square B will consist of one-fourth of the larger square, and half of the smaller square, and therefore equal to the dotted rectangle CCCC.

FIGURE LXVII.

The small square A, in this figure, comprehends no more than half the area of the larger dotted square B, and will be equal in area to either of the indented halves shown at AB, figure 65.

FIGURE LXVIII.

The square halved by the scale. Divide the side AB into twenty-four equal parts, set off seventeen as shown; the same must be performed upon the base line BC, and the square described from those parts will be half the larger square, which the foregoing diagram will effectually prove.

FIGURE LXIX.

Three-fourths of this square is taken by the scale, viz. by setting off twenty-one parts from C to A, and from CD, the square ABCD containing three-fourths of the area of the larger square.

FIGURE LXX.

Bisect the sides of this square into equal parts, as described, dotted at ABCD; draw the dotted diagonal line FE, and BE; bisect AE in G, draw the dotted line GD; then will the line DH be the side of a square, containing three-fourths of the area of the square ABCD.

FIGURE LXXI.

The rectangle halved diagonally, introducing two right angled triangles.

FIGURE LXXII.

The rectangle in this diagram is divided by the vertical centre line, thereby making two figures, which are not squares, but nearly thereto; the rectangle, at figure 71, being the same dimensions as this figure, it is made evident that either of these squares must be equal in area to either of the triangles in the former.

FIGURE LXXIII.

The triangle A, in the middle of this rectangle, holds the same area as one of the squares in figure 72, and is equal in area to the two triangles shown occupying the other portion of this rectangle.

FIGURE LXXIV.

The rectangle ABCD, given; let the same be formed into a square, holding a similar area. In order to bring it nearer square, this figure is halved, as shown at G, and the half ACG, is added below at DEF, forming another rectangle of shorter dimension, and more convenient for the purpose of squaring, as will be shown in the next figure.

FIGURE LXXV.

The rectangle shown in the last figure, EFB, will be seen to form one side of this figure, and EGB, the other side reversed. Let BE be centered in H, and with the side HE form the other dotted side of an equilateral triangle, on the base line FE; then will the dotted line H upon FE be the side of a square holding the same area as the rectangle ABCD, in the former figure.

FIGURE LXXVI.

This rectangle can be squared by the line AB, or a square holding the area of the rectangle, may be obtained by making one side of the square, with the line AB, which is formed by centering the rectangle at A, and producing the diagonal line to the opposite angle at B.

FIGURE LXXVII.

This problem is to be found in other publications, and is only introduced in this merely as a proof of its simplicity. This rectangle is of the same dimensions as figure 76, and the method for squaring it is by extending the base line from B to C, the same length as the depth, from A to B; after which, centering the base line in D, and striking the dotted semicircle E, the line AB to be continued to E, as shown, then will the line BE be the side of a square, which will be equal in area to the rectangle. It will be perceived in figure 76, the rectangle is the same dimension as figure 77; and it will be ascertained also, that the line AB, in figure 76, is of the same length as EB, in figure 77. Therefore the same result is produced by these two different methods; and as figure 76 is much more simple in its performance and more easily attained, may be considered preferable.

FIGURE LXXVIII.

This rectangle is halved by the dotted diagonal line, making two right angled triangles, as A and B: the trapezium B formed in the middle, is equal in area to one of the triangles, or half of the rectangle.

FIGURE LXXIX.

This rectangle is halved by the scale, by setting out the base line into twenty-four parts, taking off seventeen for the base line of a rectangle holding half its area; for the width, let the side be also divided into twenty-four parts, and seventeen taken, as is shown in this diagram. The rectangle A will be found to contain half the area of the larger rectangle, and equal in area to the trapezium BB, in figure 78.

FIGURE LXXX.

Shows another method for halving the rectangle; let it be centered as shown at A, draw the dotted lines AC and AH, also AC and EB, draw the diagonal DC, and where these intersections are made at FF, such will be the extent of half this rectangle as shown at G. It will be necessary to observe, that B, figure 78, A, figure 79, and G, figure 80, have all equal areas, although their figures as to height and width are not similar.

FIGURE LXXXI.

The rectangle ACDB, is shown dotted in an altered figure above, but the area of the original is not exceeded. ACE of the rectangle is reversed, and shown dotted above at ACE: ABE and ECD of the rectangle, contain the area of HEGC, and BED is equal to HFG. Therefore, the contents of the upper dotted figure ACFG, is the same as the rectangle ABCD.

FIGURE LXXXII.

The rectangle, as shown at ABCD, is altered to the figure FED and GCH; but such alteration does not in the least alter the area, for both figures are of equal quantities.

FIGURE LXXXIII.

A semicircle given, for a circle to be produced, containing the same area. Let A be the semicircle, draw the horizontal dotted line AD, and also the dotted line CD, centre CD in E; and from the centre point C, draw the dotted circle E, which circle will contain the same area as the semicircle A.

FIGURE LXXXIV.

The quarter circle formed into a circle. Let A be the quarter circle, centre FH in I, and EF in G, draw the dotted line GC, and also CI, parallel to the sides as shown; then will CI be the semi-diameter of a circle, containing the area of the quarter circle A. By the scale divide the side EF into four parts, as shown; set off two, for the centre of the circle, as is described dotted in the diagram.

FIGURE LXXXV.

The triangle either halved, or doubled in area, by the circle. Let ABC be the triangle given, inclose the same in a dotted circle as described; centre GC in H, draw the line DH, and with the distance DH, continue the line DH to F; introduce the line of diameter EF, parallel to BC, and at the point I introduce the larger dotted circle, from which the triangle EDF is obtained, which will be twice the area of the triangle ABC.

FIGURE LXXXVI.

The segment of a circle, either halved or doubled, in area. Let A be the larger segment; ascertain its centre as shown at C, continue the dotted circle; introduce the dotted square within this circle, with the radius CD; introduce also the dotted circle A, and also the inner dotted square; draw the dotted lines as are shown at their intersections, when the segment B, will contain half the area of the segment A.

FIGURE LXXXVII.

The square doubled in area, by the circle. The square A given, centre it as shown at C, strike the circle from the points as described; centre DE, in F; continue CF, with the distance CF, to G; at the extremity of which, introduce the larger circle B, and in the interior describe the larger square, as is shown, which will be double the area of the smaller square A.

FIGURE LXXXVIII.

The rectangle doubled. The doubling of the circle A at B in this figure, is shown in fig. 87; it will, therefore, serve to describe this also.

FIGURE LXXXIX.

In this diagram, the rectangle A, the segment BB, the triangle C, and the square D, are doubled by the dotted lines below at EE, &c. In this figure, the whole of the figures are introduced, but the method in which this is elucidated is from the same principle as the previous figures, commencing at fig. 83.

FIGURE XC.

The small segment A, in this figure, is doubled in area by the segment B; C also doubles B, and D doubles C; consequently, the half of D, which is shown by the dotted line E, is equal in area to the whole extent of C.

FIGURE XCI.

The inner triangle A, constitutes one-fourth part of the area of the larger triangle from the introduction of the circle, and the dotted parts left at the sides, shown by B, are each one-fourth part of this triangle; consequently, the triangle A, in the centre, is equal in area to either of the three sides B.

FIGURE XCII.

The diagonal line A shown within this circle, being formed into a square, as described, will contain nearly one-third of the area of this circle: the larger square produced from the diagonal line B, will double the area of the square A.

FIGURE XCIII.

The quarter circle A, at the bottom corner of this diagram, is doubled in area by B; C doubles B, D doubles C, E doubles D, and F doubles E; consequently, there is four times the area in F more than what is occupied by D. Either of the four dotted divisions G, hold the same quantity of space as D, which the dotted segment H serves to elucidate; and the whole of these sections are obtained by the dotted lines III.

FIGURE XCIV.

The small ellipsis A is doubled in its area by B, and D doubles C; consequently, half of D shown by the dotted line E, will be equal in area to the whole of C.

FIGURE XCV.

The diagonal line A of this square, being divided into eight parts, as shown, will produce the equilateral triangle, equal in area to half the square, and also to the rectangle B: the triangular half of this square, and the equilateral triangle, must, under those circumstances, be the same.

FIGURE XCVI.

The square A by this scale is converted into a rhomboides, for the parts which are taken from the square at A are introduced in the formation of the rhomboides at B.

FIGURE XCVII.

The square A, shown within the circle B, occupies one-fourth part of the area of the circle, for the dotted line AC, being divided into five parts, three are taken for the square, as is described.

FIGURE XCVIII.

The square A, in this diagram, occupies one half of the area B, and is demonstrated by the scale C, wherein six parts are taken for it, as is demonstrated.

FIGURE XCIX.

The circle divided into three equal parts at A, B, and C ; let the line of diameter be divided into twenty-five parts, seven of which make the centre of triangular third part B in the middle, the sides A and C form the remainder; the dotted lines D show the surface of the middle part over that of the side, describing the part supposed lost at D, to be again admitted at E.

FIGURE C.

This circle is also divided vertically into three equal parts, seven of which are apportioned to the middle part, and nine to each of the sides; the middle part is shown dotted over A, and the part shown at D which is left of the circle, is introduced at EE; which serves to demonstrate the middle portion to be of equal area to either of the sides.

FIGURE CI.

Half the area of this circle is taken from the middle by the scale; viz. divide half the diameter into twenty-four equal parts, seventeen of which will be half the area of this circle, as is shown by the dotted inner circle A.

FIGURE CII.

The four projections at A from this circle, cause the remaining part to form another figure; the segments contain nearly a fourth part of the whole circle when together.

FIGURE CIII.

The centre rhomboides in the body of this rectangle constitute half its area.

FIGURE CIV.

In this rectangle, eight triangles are shown, which are all equal in area, although they are not the same in figure.

FIGURE CV.

The rectangular octagon containing only the area of the dotted rectangle, for it will be perceived, the part which is abstracted from the rectangle at A, is introduced again at B.

FIGURE CVI.

Within this rectangle the rhomboides are introduced, crossing each other from the angles; and, with the addition of the two dotted lines, this figure is made to contain twenty-four triangles.

FIGURE CVII.

The octagonal ellipsis is produced within the rectangle, as this diagram describes.

FIGURE CVIII.

The octagonal ellipsis may be found to suit any dimensions, as to the length of the transverse and conjugate, by the method the dotted lines show in this diagram.

FIGURE CIX.

Is the equilateral triangle divided into two parts, A and B. Centre the base line C; draw the centre dotted vertical line AB, and divide the sides E and D in the same manner; draw the centre dotted line DE, and at the intersections introduce the lines FG; then will B contain the same area as A.

FIGURE CX.

The equilateral triangle divided into two equal parts, by a line parallel to one of its sides. Draw the vertical centre line C, divide the same into seven parts or divisions, as shown; the third part from the base line will equally divide this figure, as is described by the line AB.

FIGURE CXI.

The equilateral triangle doubled in area. The triangle A given; divide one of its sides into seventeen parts, as shown, to which add seven more, as described; draw another line parallel to the former; the additional part introduced at B will be of equal area to A, consequently the triangle will be doubled in extent.

FIGURE CXII.

The equilateral triangle divided into three parts. Centre each of the sides, and draw the lines from the points of the triangle to the centre of the sides; then will the divisions A, B, and C be equal to each other in capacity.

FIGURE CXIII.

Centre this triangle as the last, and from the angles draw the lines as shown, which will divide this also into three equal parts.

FIGURE CXIV.

The equilateral triangle divided into three parts, and each part to be an equilateral triangle also. This problem is described as one of infinite difficulty, from the following insertion in the "New Dictionary of Arts and Sciences," published in 1754:— "The trisection of the triangle is one of those great problems whose solution has been so much sought by mathematicians for these two thousand years; being, in this respect, on a footing with the quadrature of the circle and the duplicature of the cube." The three divisions A, B, and C are equal in area; but in order to obtain the equilateral triangle, whose area shall be similar to either of them, let the sides be divided into twelve equal parts, as the diagram represents; seven parts, as shown dotted at D, will give the equilateral triangle, containing only one-third of the area of the original, consequently the triangle formed by the dotted side DE, will also be equal in area to either A, B, or C. It is very evident that this method could only be obtained by the scale, which performs the operation with infinite ease and great truth; and it also serves to show the immense utility, as well as benefit, which may be derived by its adoption in all intricate questions.

FIGURE CXV.

The equilateral triangle divided into four parts of similar areas. The triangle A given, let fall the perpendicular AD, centre AB in C, and AF in E; draw the lines DC and DE; then will this triangle exhibit four equal divisions, two of which being similar to the other.

FIGURE CXVI.

The equilateral triangle divided into three parts. Let the perpendicular line AB be divided into seven equal parts; set off three from the bottom line B; draw the line FG parallel to B, with the vertical line above the same; then will the portions D, C, E be equal in area to each other.

FIGURE CXVII.

The equilateral triangle again divided into three equal parts. Centre the side AB in C, introduce the dotted line CD parallel to the base; draw the dotted diagonal lines DB, CE, at their intersection, as shown in F; draw the dotted line GH, and divide the distance between DC and GH; draw also the line as shown, with the vertical line F from the centre of the base line EB; then will the divisions I, K, L be equalised in area.

FIGURE CXVIII.

The triangle divided into two equal parts by a line parallel to its base. Let fall the perpendicular CD, as shown; let it be divided into seven parts; set off two, as shown by the line AB; when the upper and lower parts of this triangle will each contain the same quantity of surface as the other.

FIGURE CXIX.

The triangle divided into four parts, two parts of which are equilateral triangles. Divide the base line AB into four parts, as shown; draw the lines CD and DE parallel to the sides; draw also the horizontal line D parallel to the base AB; then will F, G, H, I be of similar areas; F and I will be equal triangles, G and H trapeziums.

FIGURE CXX.

The equal triangle again divided into four parts, each containing the same quantity of space. Draw the vertical line AB, in the centre of which, at C, draw the lines DC and EC; such lines will then introduce four parts to this triangle, whose surfaces will be exact and similar.

FIGURE CXXI.

In this diagram is introduced the square, which, for the others which follow, is divided by four lines, all parallel to each other and the base line; making five parallelograms.

FIGURE CXXII.

This square being divided diagonally by four lines, making five spaces, are all of them equal in area, and also equal in area to the parallelograms in the last figure. The finding of these divisions may be seen on the diagonal line AB, which is divided into twenty-two parts; seven are taken for each of the triangles, three to each of the next divisions, and two only for the centre one: therefore the centre one, or any one of the others, is equal in space to either of the five parallelograms shewn in the last problem.

FIGURE CXXIII.

The square No. 5, in the middle of this figure, is equal to either Nos. 1, 2, 3, or 4 of the other part of it, and either of these portions, and the whole of them, contain equal spaces with those shown in figures 121 and 122. The divisions set out upon the line AB serve to regulate this disposition, and they would be of no importance whatever as to the dimensions: the same method would regulate and divide a square of fifty feet, with the same facility and accuracy as this.

FIGURE CXXIV.

This square is shown for the sake of illustration, plainly divided into six divisions, each division being a rectangle or parallelogram.

FIGURE CXXV.

This square, as the last, is divided into six parts or divisions, assuming a different shape to those, notwithstanding which their areas are precisely the same.

FIGURE CXXVI.

In this square six other figures are assumed, whose areas are the same as the last, but very different in appearance. The triangle and rectangle within the body of this square are of equal space to the surrounding figures, Nos. 1, 2, 3, and 4.

FIGURE CXXVII.

The three centre figures occupying the body of this square amount to half of the area of the same; the other parts, when added, will make the remainder. It must be observed that either of these figures, A, B, or C, are equal in area to any of those in the foregoing diagrams, 124, 125, or 126.

FIGURE CXXVIII.

The same plan is pursued in regard to this diagram; these triangles are of precisely the same area as the last. The scale upon the diagonal line might lead the student to speculate a little upon these figures, whereby much useful information relative to squares and triangles may be gained.

FIGURE CXXIX.

The square is herewith shown divided into four equal parts, which are squares also.

FIGURE CXXX.

The circle is shown divided into four equal parts, as the preceding figure.

FIGURE CXXXI.

It is made to appear that the two preceding figures, although the one a square and the other a circle, are of equal areas, as this figure serves to demonstrate in the simplest manner possible, viz., the dividing of the fourth part of the circle into ten parts; the middle four are taken for the circle. By this figure the two former ones are perfectly equal in area, although very different in formation.

FIGURE CXXXII.

The areas of the triangles in these figures correspond in area with the squares in the three last.

FIGURE CXXXIII.

The rectangles produced by this figure are of precisely the same area as the last.

FIGURE CXXXIV.

The circle divided into four parts, each containing the same area. Divide AC, and strike the inner circle A, as shown, which will be one-fourth of the large circle. The other three divisions are found equally divided upon less and greater circumferences of the circles.

FIGURE CXXXV.

The circle given, let a square be produced, having the same area only. There are in this work several methods for squaring the circle and circling the square; but, as it happens, some methods are inconvenient, others impracticable. I have therefore ventured to introduce another, very simple, but not the less correct. The division of the semi into eight parts can very readily be done, as well as dividing the square into seven and adding a part for the semi, as is described. Several ways are introduced for the performance of this important question; time and circumstances must operate in making the most eligible use of them.

FIGURE CXXXVI.

The circle, with the dotted triangle B within it. Let the dotted line B be run on to A; centre AB; introduce a line, as shown, parallel to B, which will be the side of a triangle, containing the same area as the circle.

FIGURE CXXXVII.

The circle and rectangle, containing similar areas as the last figure; the rectangle being formed from the triangle, as the dotted diagonal line AB denominates.

FIGURE CXXXVIII.

The ellipsis and circle, containing similar areas. Divide the diameter of the circle into twenty-seven parts; set off five parts and a half for the centre of one circle, the same at the end of the line for the centre of the other; strike a centre circle, from which obtain a vertical line, and with the centres A A complete the upper and lower parts of the ellipsis, as is described dotted in this diagram.

FIGURE CXXXIX.

The equilateral triangle formed within the circle. Divide the circle into six parts; take three for the points of the triangle, as is described.

FIGURE CXL.

The equilateral triangle given, to be divided into four equilateral triangles, the whole of whose areas are equal to each other. Drop the dotted perpendicular; centre each of the sides, as shown; then draw the lines AB, BC, and AC; which centre triangle will be one-fourth of the area of the larger, making three others of equal qualities.

FIGURE CXLI.

The triangle divided into four equal parts, as A, B, C, D. Divide the vertical line E into seven parts, as shown; set off two, as described on the vertical line, when the line FG will divide the sides into parts, whose areas are equal to each other. The dotted triangle shown on one side is equal in area to either of the others.

FIGURES CXLII. AND CXLIII.

The triangle divided vertically into three equal parts, as A, B, C. Divide the base line of the triangle into twelve parts; take two from the centre, as shown, for the one part; the parts on either side will be of equal area to it: or, the side may be divided into six parts; one taken on each side of the apex will perform the same operation.

FIGURE CXLIV.

The right angled triangle divided into aliquot parts, viz., one-fourth, one-half, three-fourths. Divide the side into twenty-four equal parts; twelve of which constitute the fourth, seventeen the half, and twenty-one the three-fourths, as this diagram demonstrates.

FIGURE CXLV.

The equilateral triangle is in this diagram divided into parts, equal in quantity to their respective proportions ; viz., fourth, half, and three-fourths. The same method is shewn in other parts of this work, by the divisions of the sides into twenty-four parts.

FIGURE CXLVI.

In this figure, the equilateral triangle is produced upon the base line of the square AB, by centering CD in E: the dotted lines form the equal-sided, or equilateral triangle, whose area will amount to half the area of the square.

FIGURE CXLVII.

By the inversion of the four triangles, ABCD, the area, or covering surface, in this diagram, is doubled ; and from this problem any rectangle, whose length being twice its width, may be squared ; that is to say, a square formed in this manner holding a similar area to the rectangle. There are other diagrams in this work showing also this problem ; but this is introduced from its being extremely simple, and may be performed in some instances where the others might be attended with difficulty.

FIGURE CXLVIII.

The right angled triangle, rectangle, and square, holding similar areas. The triangle given as shown in this diagram ; let the hypothenuse be divided, as described, into eleven parts ; draw the lines, as shown, intersecting those parts ; and from the dotted diagonal line, the square is found ; and from the centre of the triangle, the rectangle is found.

FIGURE CXLIX.

The equilateral triangle, rectangle, and square, equalised in areas, the vertical line describing the quantity of parts requisite for such purpose.

FIGURE CL.

The divisions, as are described at AA, attached to this square, the octagonal figure can be produced, let the dimension of the square be what it may; but the octagon obtained in this manner will be less in area than the square, the quantity that is contained in the four angles.

FIGURE CLI.

The inner octagon B is generated by the crossed squares which are dotted in this diagram, and will be found to contain only half the area of A.

FIGURE CLII.

The equilateral triangle which is shown dotted at A in the internal part of this square, is found by the divisions shown upon the vertical centre line; and the triangle B, from the same divisional method, will be found to occupy the like area of the square.

FIGURE CLIII.

The area, or content of this figure, made square, may be obtained by the method shown in Fig. 154, following:—

FIGURE CLIV.

In this diagram is shown the area of the last figure, the divisional parts upon the base line of each describing the quantity requisite to determine the contents of these figures, in order that they may hold similar areas.

FIGURE CLV.

Herein will be shown another very effective method of squaring the rectangle and triangle, as the square ABCD, in this diagram, serves to elucidate.

FIGURE CLVI.

The square formed into three rectangles. To commence this operation, the square must be divided into three equal parts, or rectangles, as this figure demonstrates ; therefore, in the next figure, one of these rectangles will be shown squared.

FIGURE CLVII.

The third of the square A, B, and C, are three equal divisions of this square ; A only is made square (the others being precisely the same), which is clearly demonstrated by taking seven parts out of twelve ; consequently, the square A is equal in area to either of the rectangles in the last figure ; and the dotted lines in this form the same.

FIGURE CLVIII.

The unequal sided figure ABCD, given, let the same be made square. Draw the diagonal lines AD, and CB, as are shown dotted at EF; let their quantity be added together, and equally divide the same into four parts, as the square GHIK illustrates, which will be found to contain the same area as ABCD.

FIGURE CLIX.

The four dotted semis, or half circles, in this diagram, will be found to contain once and a half the area of the square.

FIGURE CLX.

The frustrum of the side of a pyramid formed into a square. Let the diagonal dotted line AA be centred; draw the vertical dotted line B, as shown in this diagram, which will be the centre of the square, holding the same area.

FIGURE CLXI.

The square AAAA being given, let the same be halved in its area, and to be also square. Draw the dotted diagonal lines as described by D; introduce the chord lines B and C; from their intersections produce the square EFGH, which will contain half the area or space of the square A.

FIGURE CLXII.

In this diagram, another method is shown for the production of half of the square. In this problem there will be found some beautiful properties, to which the attention of the student is particularly requested, from its extreme simplicity.

FIGURE CLXIII.

This unequal four-sided figure will be found to be divided into aliquot parts, as is described upon the face of the diagram, by the method of division shown thereon upon each of its sides.

FIGURE CLXIV.

The cube halved, or the halving of the cube. This great problem of the ancients, for the solution of which large sums of money were repeatedly offered by different kingdoms, has never been performed by any previous geometrician: it is herewith simply demonstrated, as this figure illustrates: viz., divide the side A into twenty-four equal parts, nineteen of those parts will form the side of a cube containing half the solidity. It is again proved in another manner, by the diagonal line B, which being divided into five equal parts, four of which will also be found to contain the half also. The beautiful simplicity which is attached to this diagram, alone proves it to be of infinite value; for where the dimension of the side of a cube resolves itself into fractional parts, a positive solution cannot, by arithmetic, be gained; but in this problem, it matters not what the dimensions amount to, for upon a knowledge thereof, the solution is in a very little time, with perfect ease, gained.

FIGURE CLXV.

This figure shows the sections or divisions of the cube A; B is three-fourths, C half, D one-fourth, and E one-eighth part; the whole of which is described by the divisions upon the base line in this diagram.

FIGURE CLXVI.

This figure shows a method of consolidating a rectangle ; for instance, if the length and width are of inconvenient proportions for use or practice, let this process be adopted, in order to obtain another dimension, which will be seen will contain precisely the same quantity.

FIGURE CLXVII

Shows the consolidation of another figure of a different dimension, the dotted lines containing the whole quantity of the rectangle.

FIGURE CLXVIII.

In this diagram it will be plainly shown that the depth of any rectangular figure may be augmented in size by this method ; but it will also be seen, that it must be attended by the loss in the length.

FIGURE CLXIX.

In this figure the rectangle is divided into two equal parts, by the centre line AB.

FIGURE CLXX.

This figure, as the last, showing the indented triangles A and B, which are also of equal areas.

FIGURE CLXXI.

The diagonal division of this rectangle is also equalised.

FIGURE CLXXII.

The ellipsis is herewith produced from the dotted rectangle, and the centres ABCD.

FIGURE CLXXIII.

The ellipsis divided into sixteen equal central divisions, which is also obtained from the rectangle.

FIGURE CLXXIV.

The outside and inside ellipses are created from the dotted rectangle shown in this diagram.

FIGURE CLXXV.

This figure is entirely fabricated from the crossed rectangles, as shown in Fig. 48.

FIGURE CLXXVI.

The solid rectangular cross illustrated in perspective.

THE DIVISIONS OF THE SPHERE.

This illustration of the divisions of the sphere, although much sought by mathematicians, has never been attempted by any: the method herewith laid down, from its simplicity, must be acknowledged by all to be a proof of its value. The ancient geometricians never contemplated the possibility of performing such an operation, not being able, in their day, to halve the cube. In this figure it will be seen, that whatever diameter the sphere may be comprised in, the object can be none; for all that is required for the performance of this most valuable operation, is merely the diameter AB, and the central vertical line. Let the two circles be inscribed, each of which contain, in solidity, one-fourth of the principal figure; therefore, the line AE being half the diameter, contains one-fourth, the line BC, one-half, and the line BD, three-fourths of the solidity. The method by which such diameters are obtained, are too obvious, upon the face of the diagram, to require any further explanation.

THE END.

W. M. KNIGHT, PRINTER, GREEN ARBOUR SQUARE, OLD BAILEY.

SUPERFICES.

THE CIRCLE, SQUARE, AND TRIANGLE; LEADING GEOMETRICAL EMBLEMS.

FIG. 1.

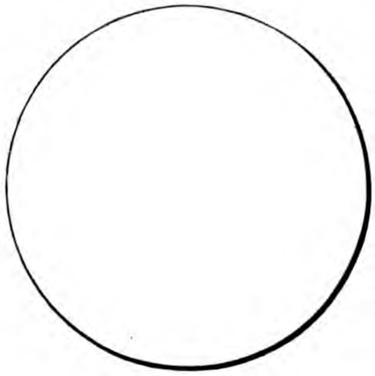


FIG. 2.

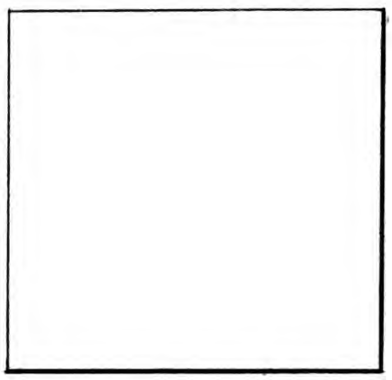


FIG. 3.

Drawn by John Bennett.

London, John Bennett, 1838.

London; John Bennett, 1838.

SUPERFICES.

London. John Bennett, 1838.

FIG. 4.

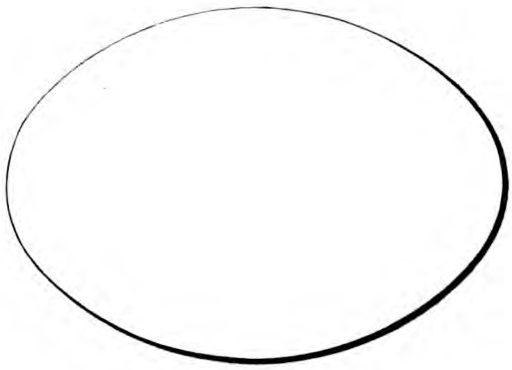


FIG. 5.

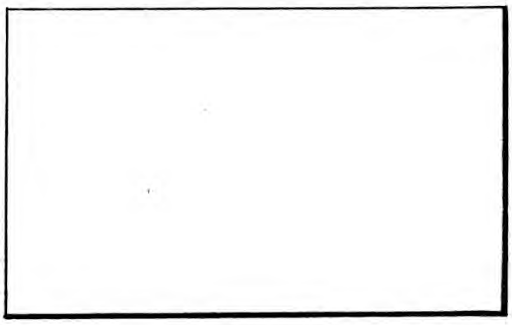
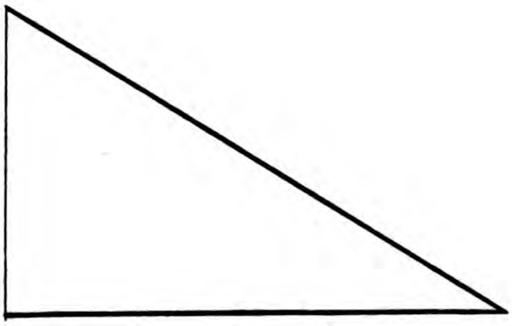


FIG. 6.



Drawn by John Bennett.

London; John Bennett, 1838.

FIG. 7.

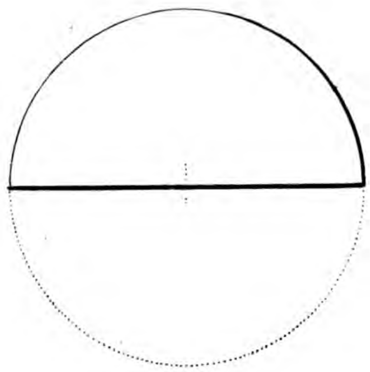


FIG. 8.

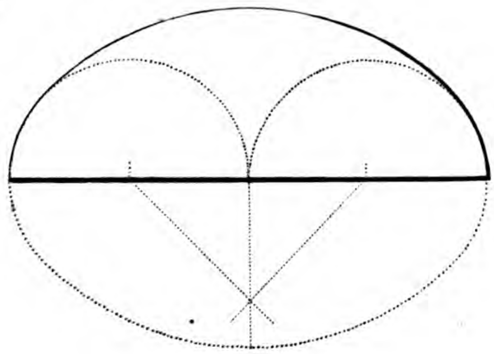
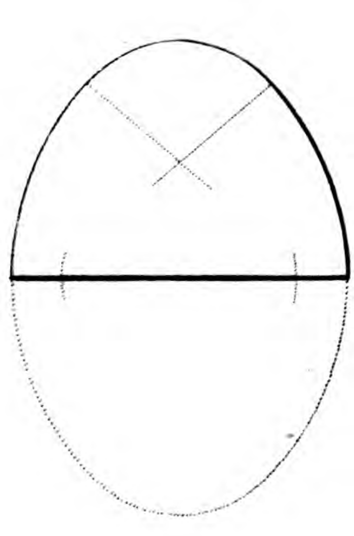


FIG. 9.



London, John Bennett, 1838.

FIG. 11.

FIG. 12

London; John Bennett, 1838.

London, John Bennett, 1838.

FIG. 10.

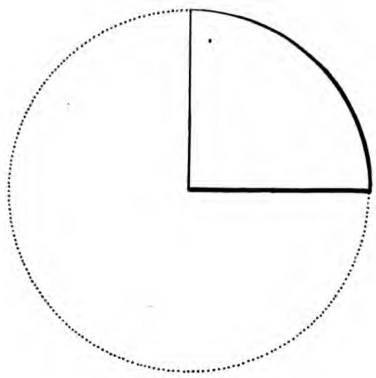
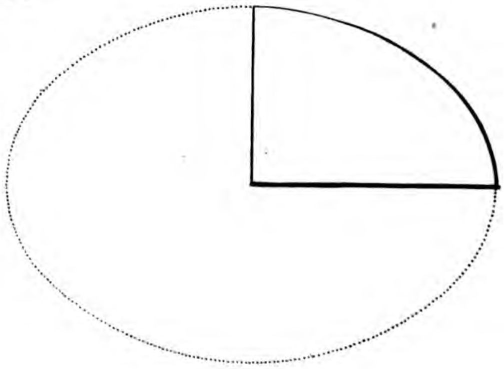


FIG. 11.



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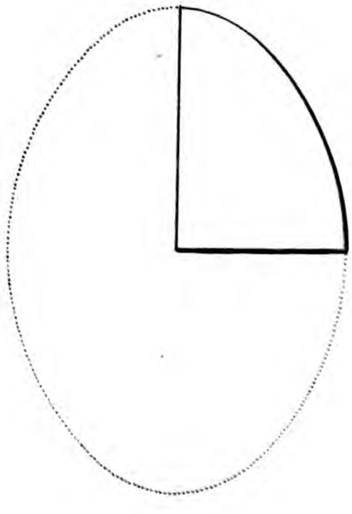
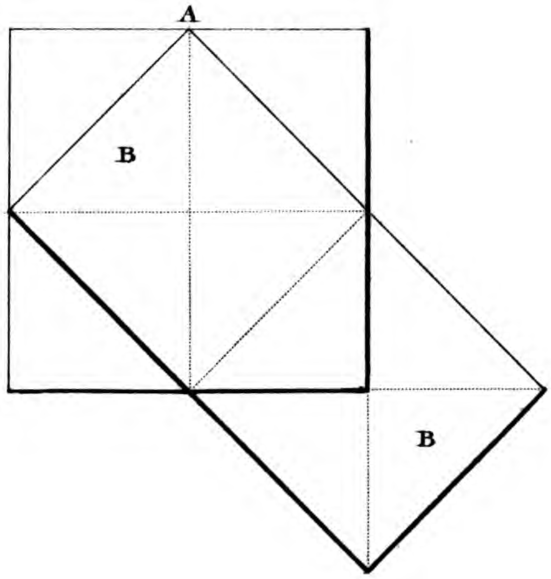


London; John Bennett, 1838.

FIG. 13.



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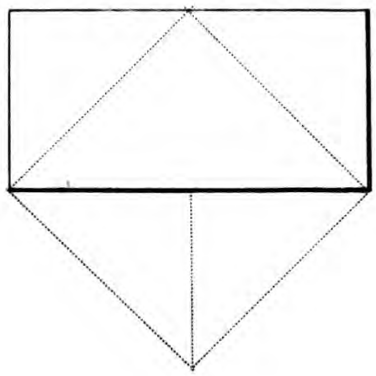
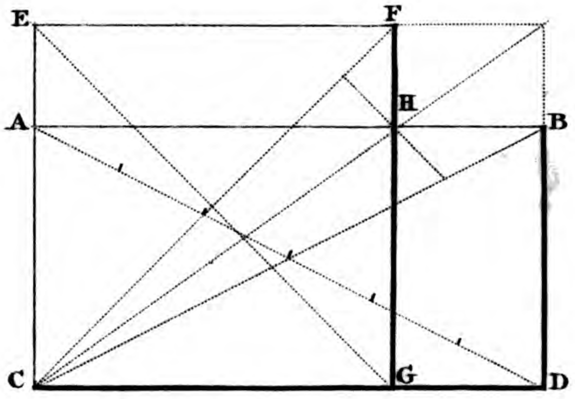
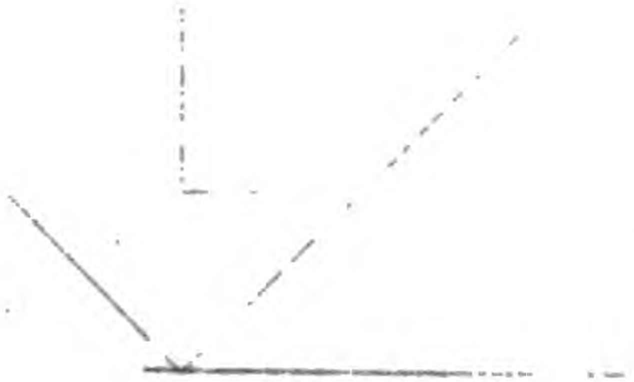


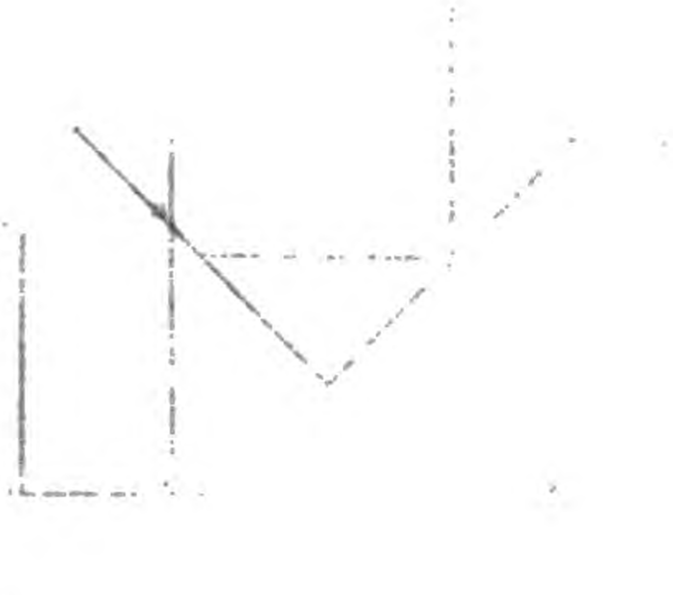
FIG. 15.



Invented and drawn by John Bennett.

London, John Bennett, 1838.





London; John Bennett, 1838.

London; John Bennett 1838

FIG. 16.

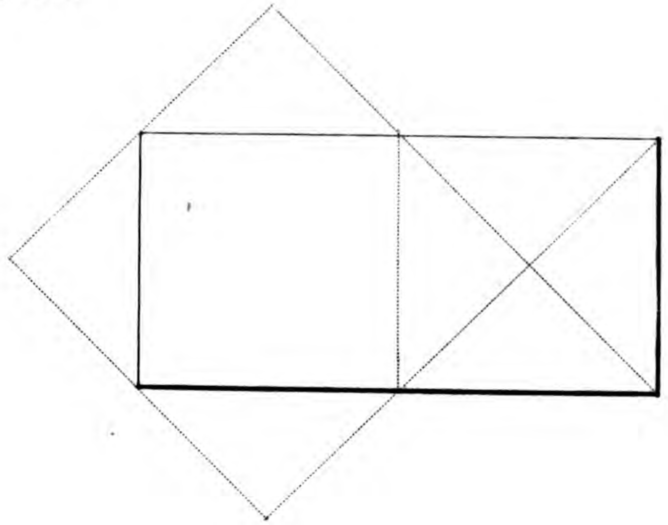
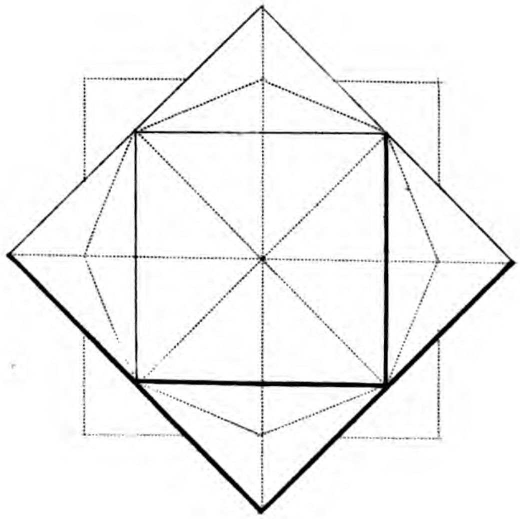


FIG. 17.



London; John Bennett, 1838.

FIG. 18.

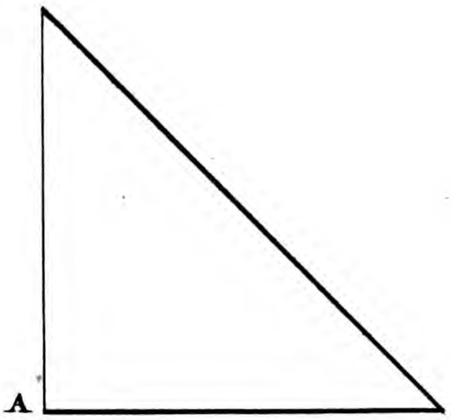


FIG. 19.

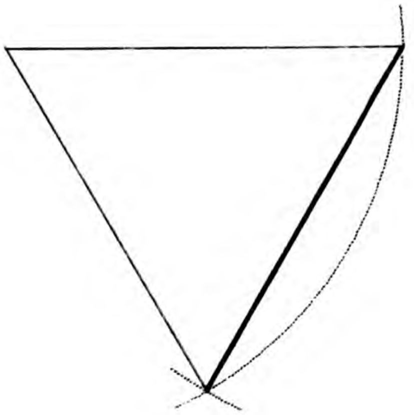


FIG. 20.

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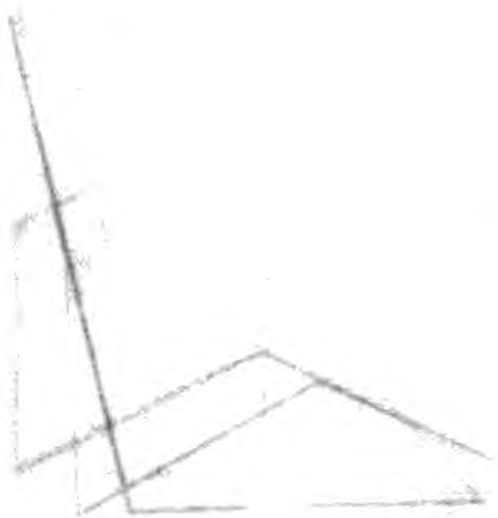
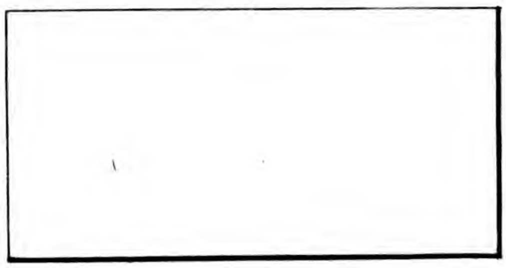


London, John Bennett. 1838.

FIG. 21.



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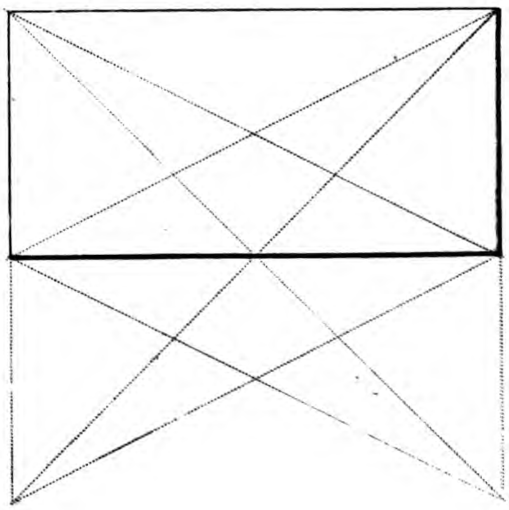

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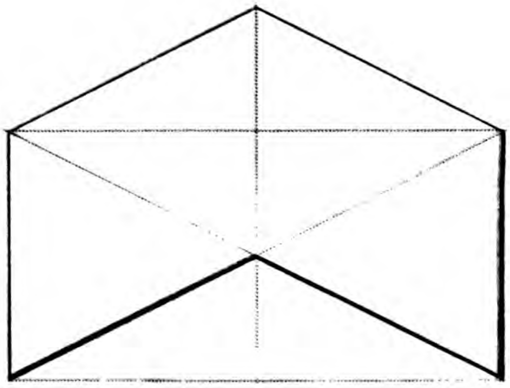


London, John Bennett 1838.

FIG. 24.

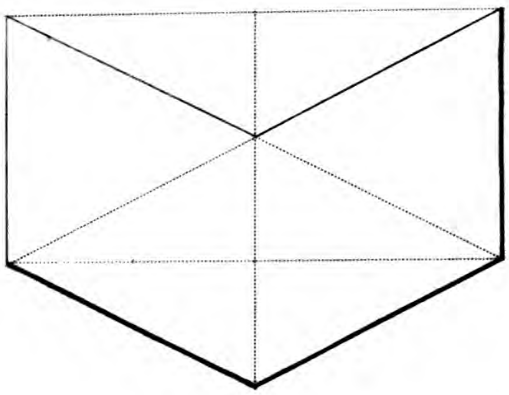
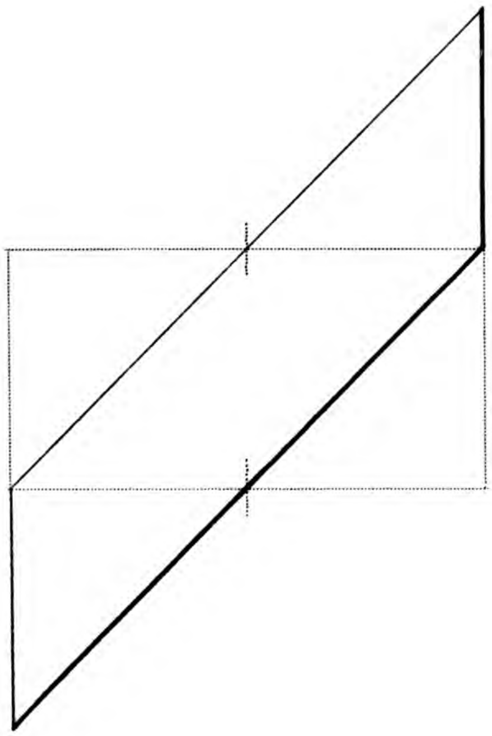


FIG. 25.



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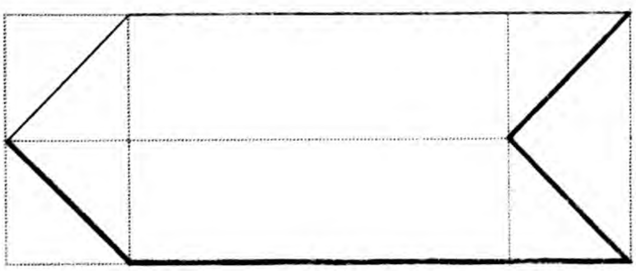

London; John Bennett, 1838.

FIG 28.

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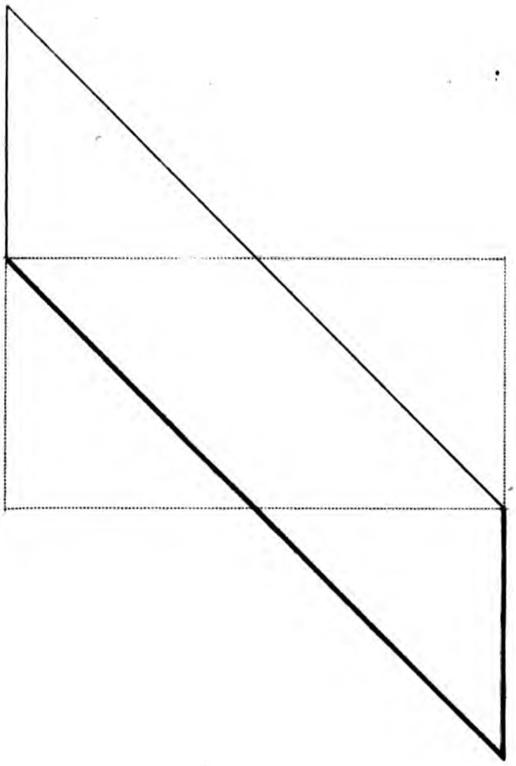
London; John Bennett, 1838.

FIG. 27.



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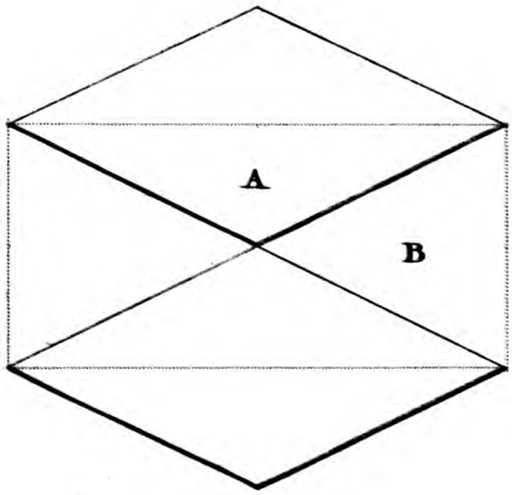


London; John Bennett, 1838.

THE GREAT SPHERE

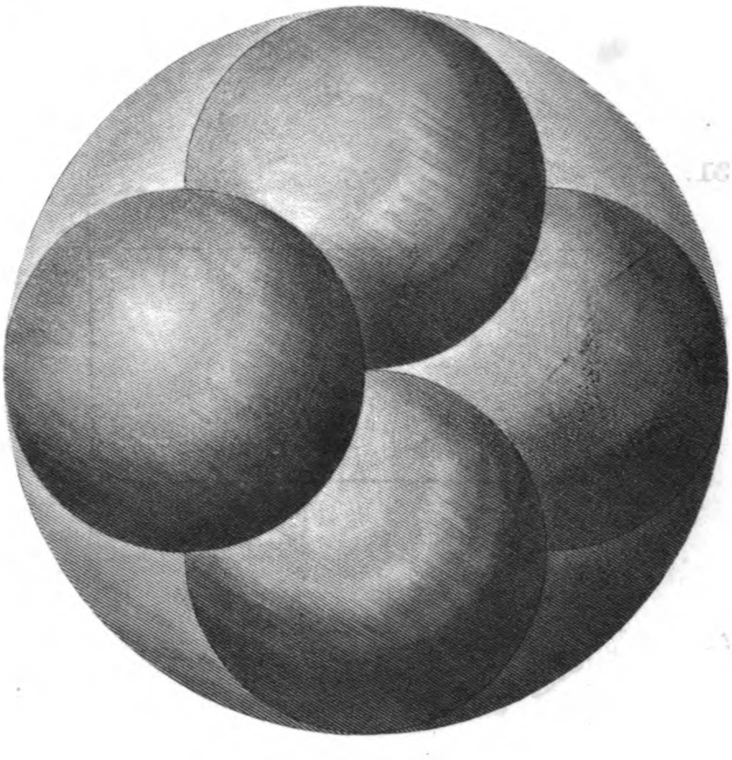
OF THE

EARTH,

DISPLAYING ITS

FOUR SECTIONS.

FIG. 29.



Resolved and drawn by John Bennett.

London. John Bennett, 1838.

FIG. 30.

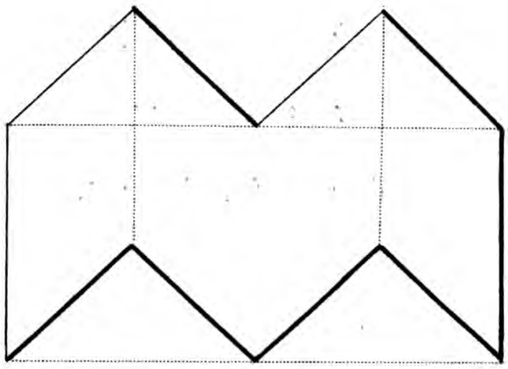


FIG. 31.

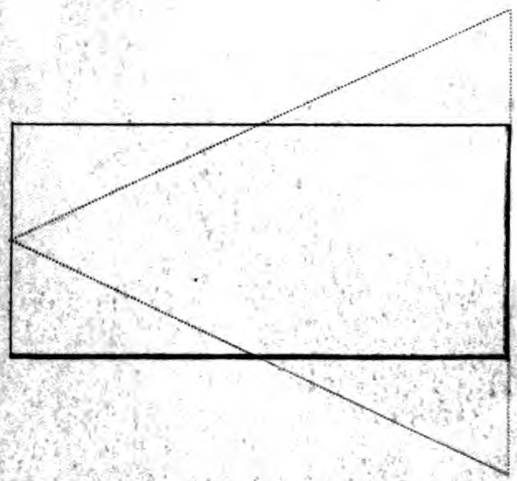
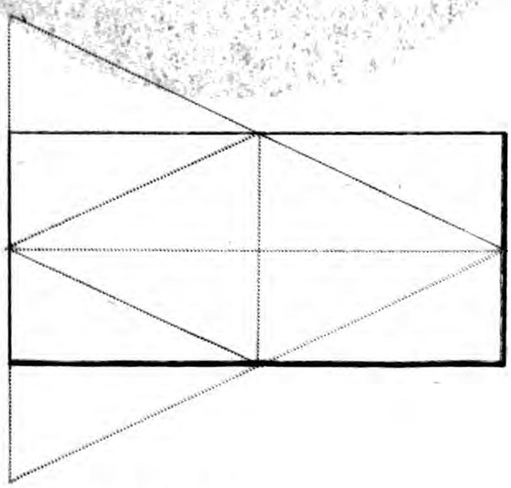


FIG. 32.



Invented and drawn by John Bennett.

London, John Bennett, 1838.

FIG. 33.

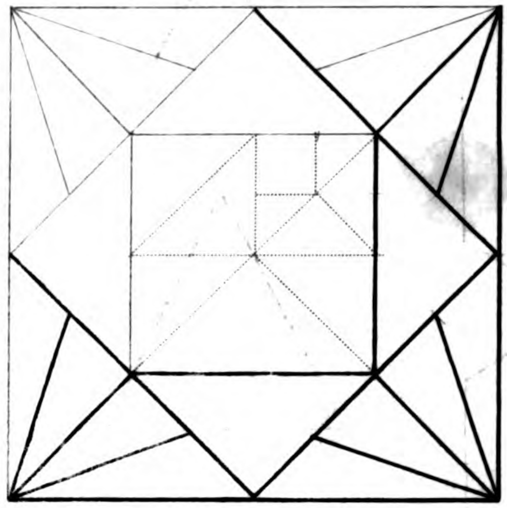
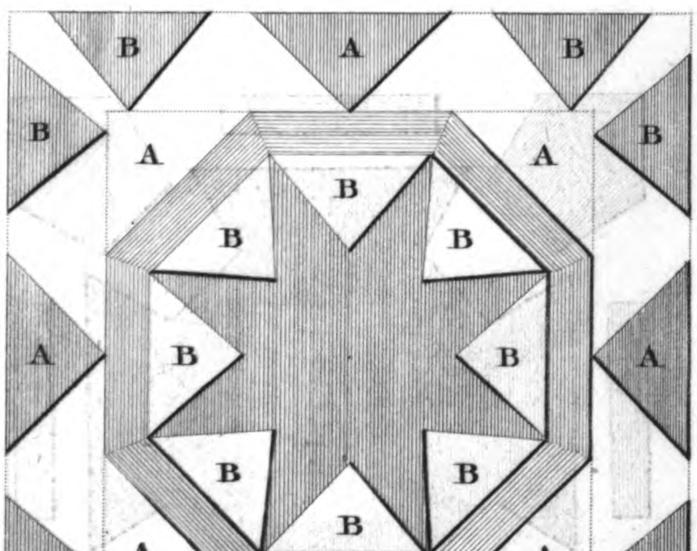


FIG. 34.



Invented and drawn by John Bennett.

London: John Bennett, 1838.

FIG. 35.

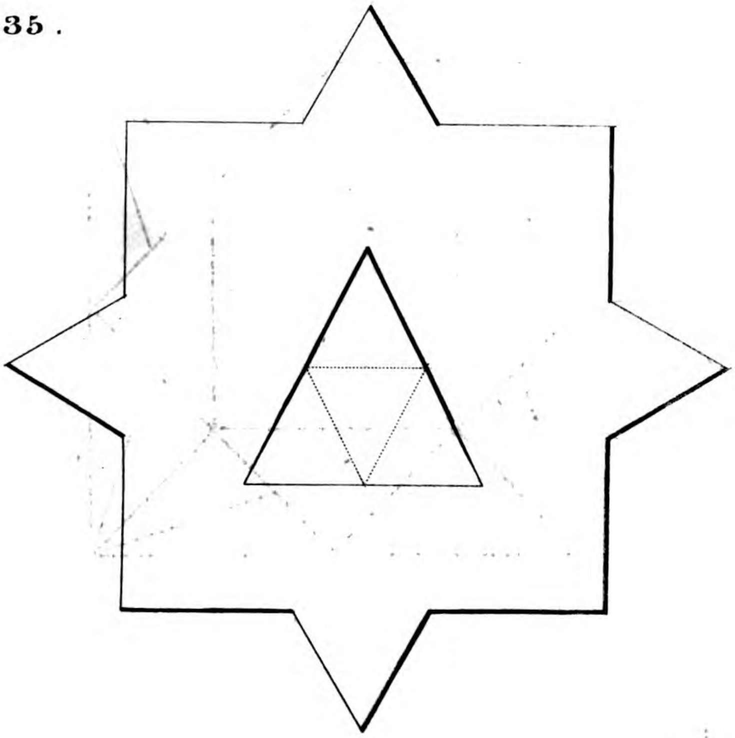
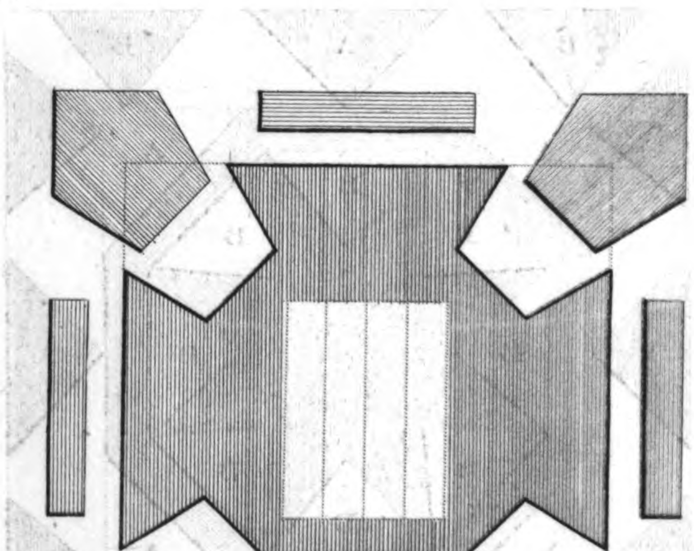


FIG. 36.



Invented and drawn by John Bennett.

London; John Bennett. 1838.

FIG. 37.

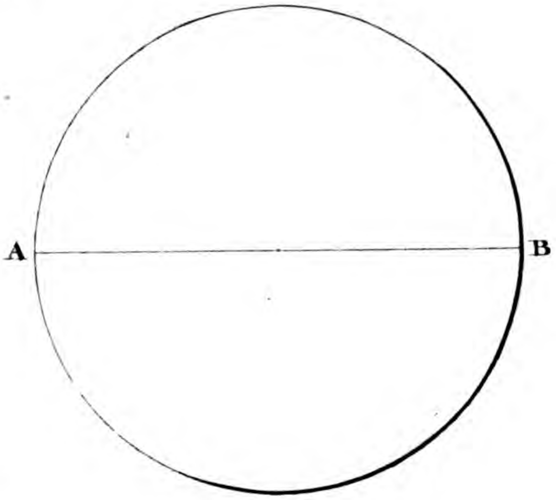


FIG. 38.

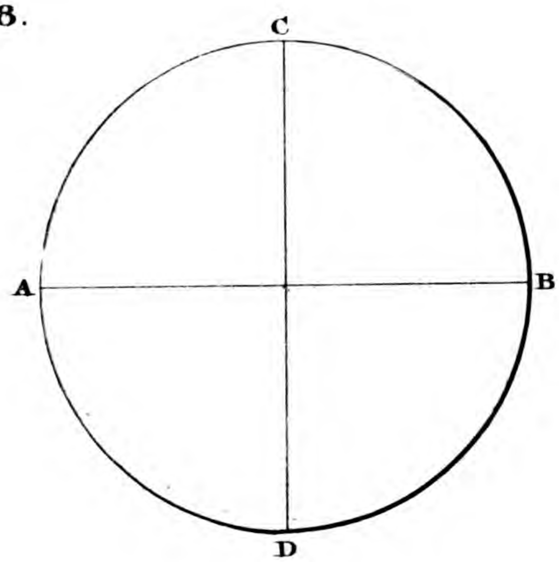
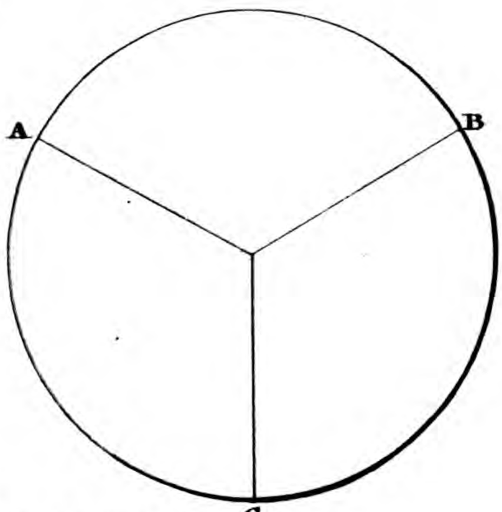


FIG. 39.



...nted and drawn by John Bennett.

London; John Bennett, 1838.

FIG. 40.

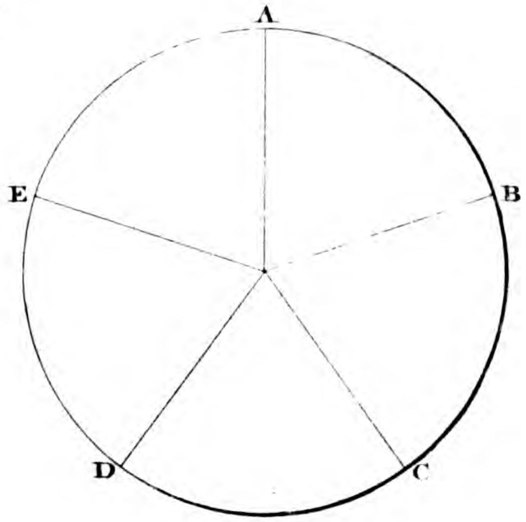


FIG. 41.

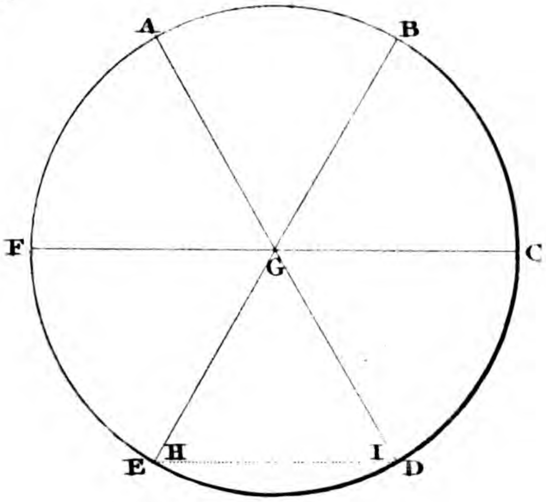
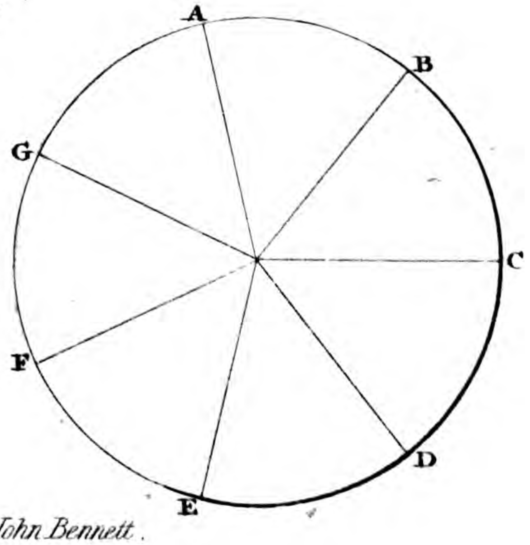


FIG. 42.



Invented and drawn by John Bennett.

London; John Bennett, 1838.

FIG. 43.

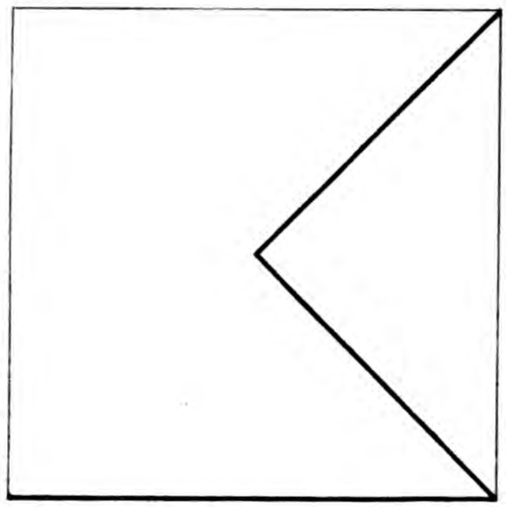
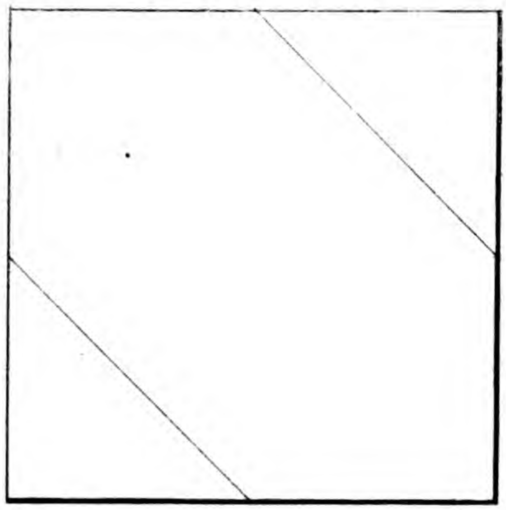


FIG. 44.



Invented and drawn by John Bennett.

London; John Bennett, 1838.

FIG. 45.

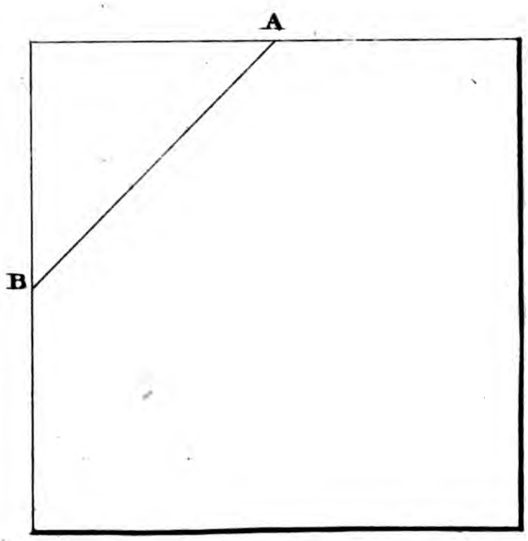
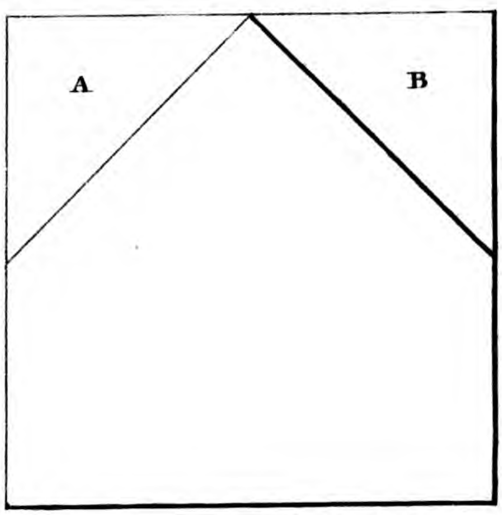


FIG. 46.



Invented and drawn by John Bennett.

London: John Bennett. 1838.

FIG. 47.

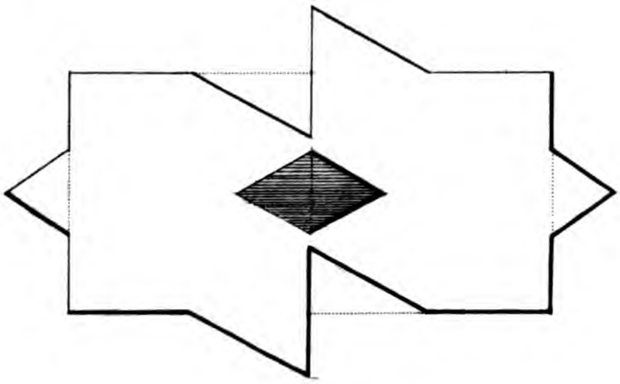


FIG. 48.

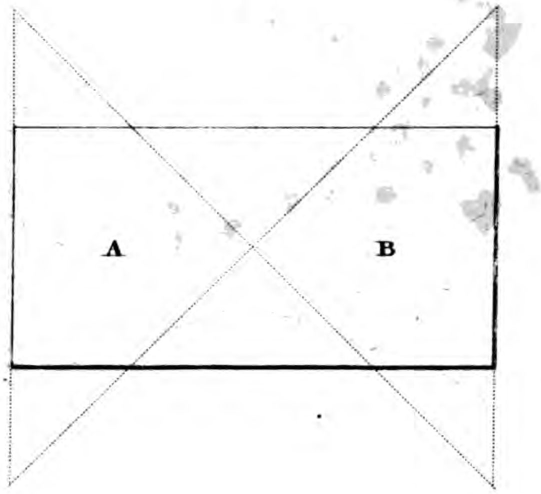
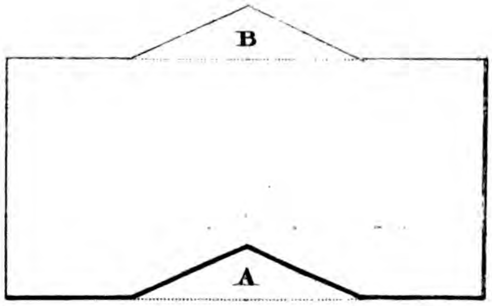


FIG. 49.



Invented and drawn by John Bennett.

London; John Bennett. 1838

FIG. 50.

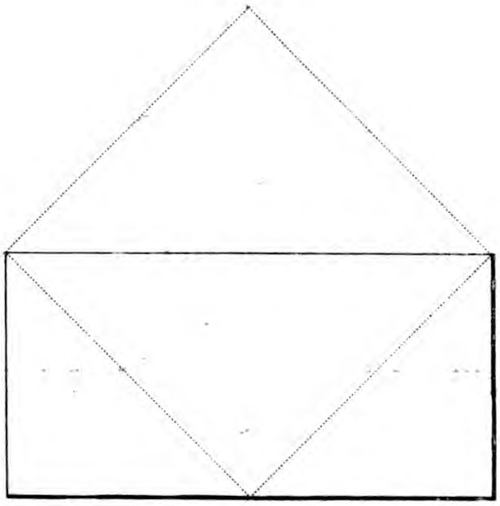


FIG. 51.

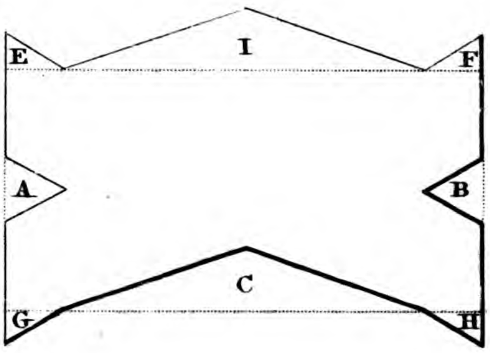
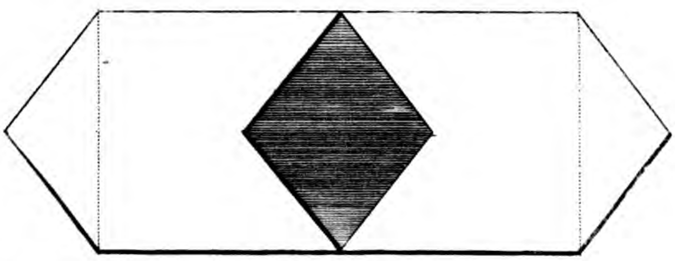


FIG. 52.



Invented and drawn by John Bennett.

London, John Bennett [illegible]

FIG. 53.

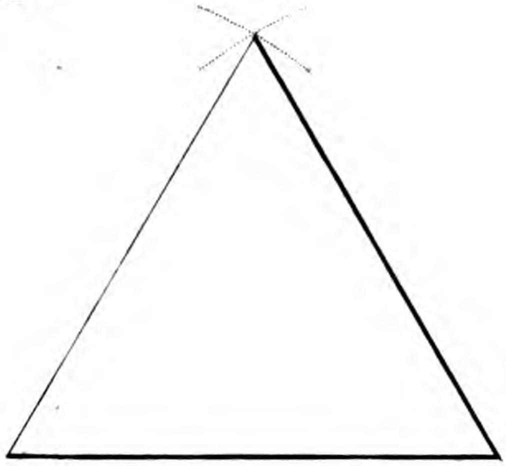


FIG. 54.

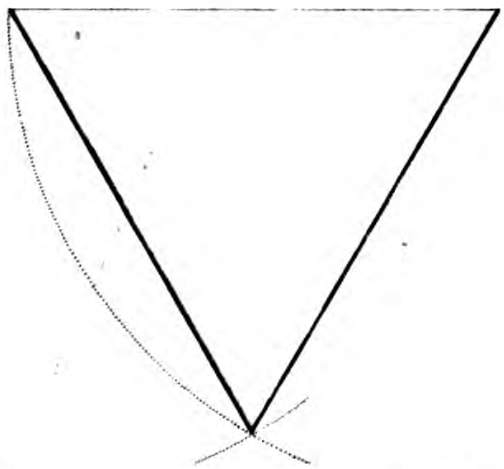
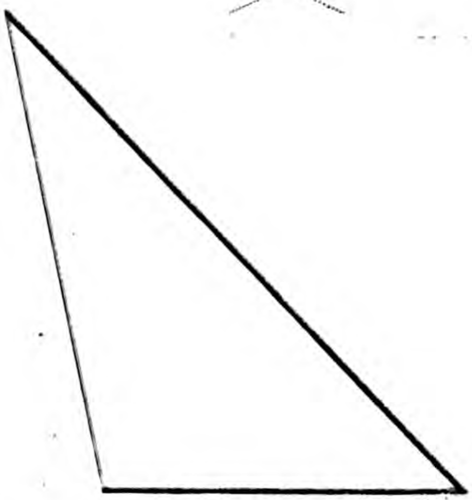


FIG. 55.



Drawn by John Bennett.

London, John Bennett 1836.

FIG. 56.

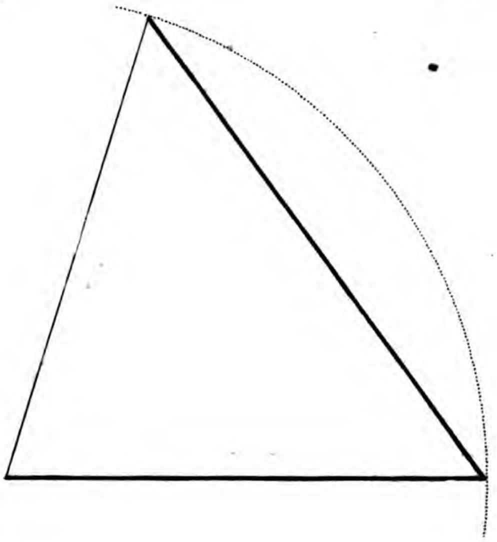


FIG. 57.

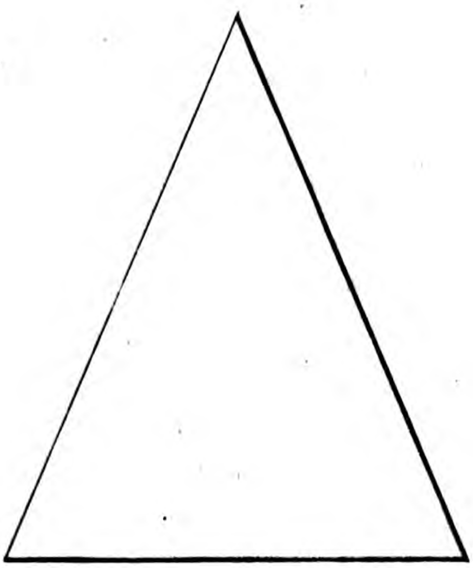
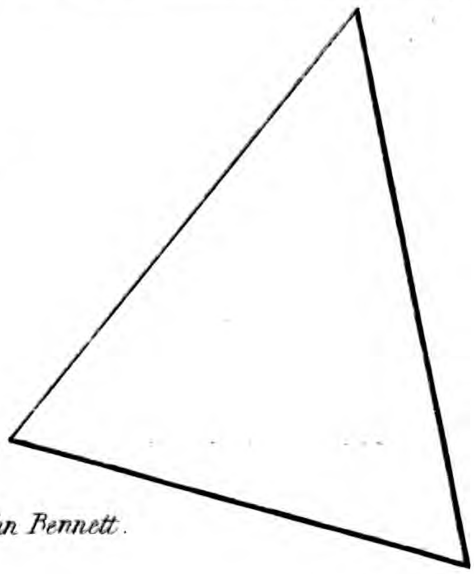


FIG. 58.



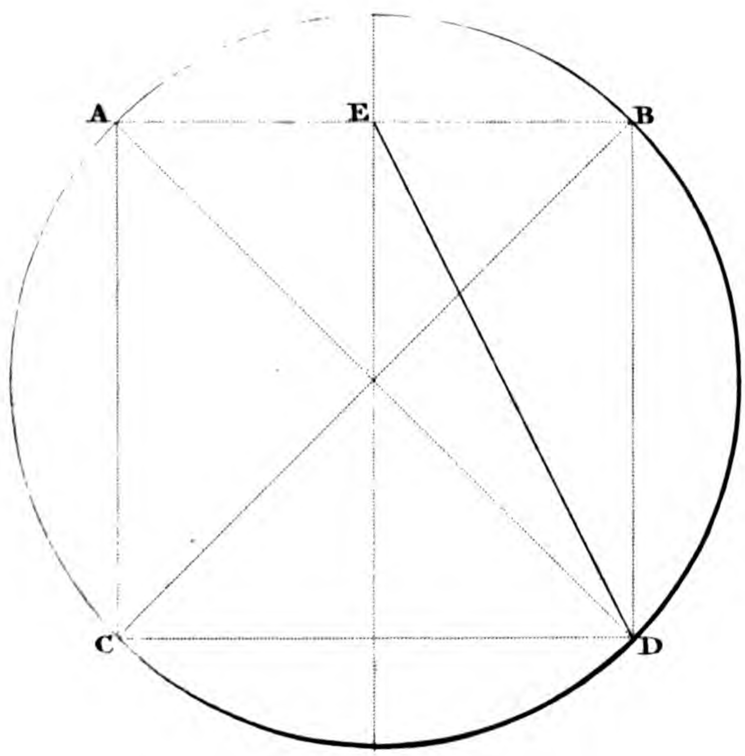
Drawn by John Bennett.

London: John Bennett, 1838.

CIRCUMFERENCE OF THE

CIRCLE.

FIG. 59.



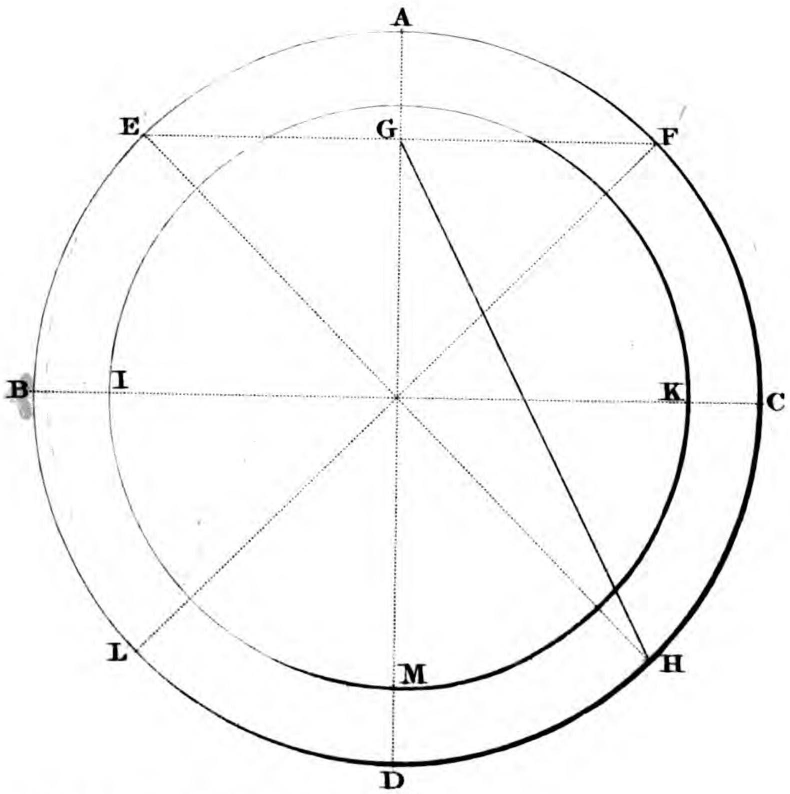
Resolved and drawn by John Bennett.

London, John Bennett, 1838.

THE SPHERE

HALVED.

FIG. 60.



Resolved and drawn by John Bennett.

London, John Bennett, 1838

FIG. 61.

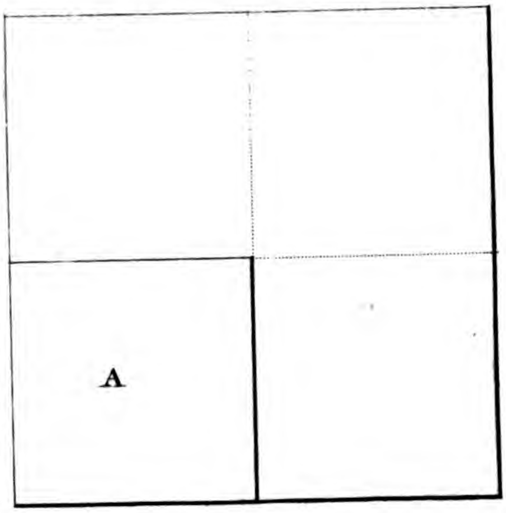
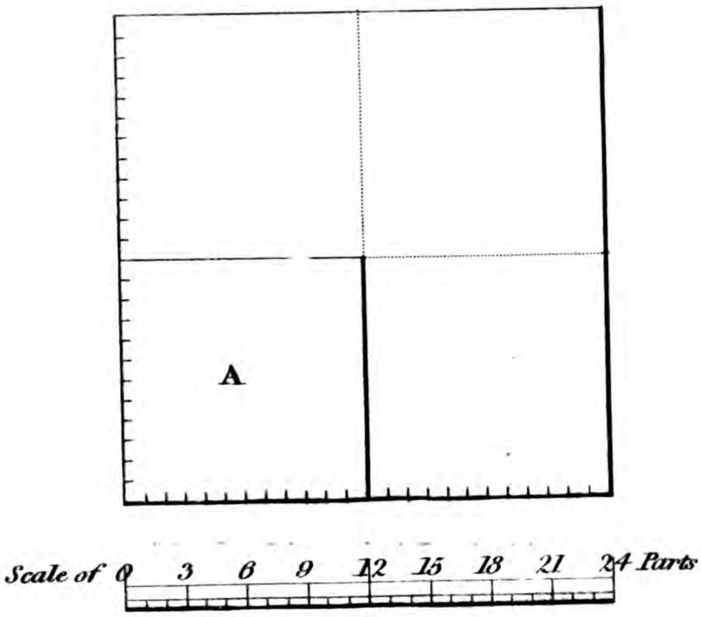


FIG. 62.



Drawn by John Bennett.

London, John Bennett, 1838.

FIG. 63.

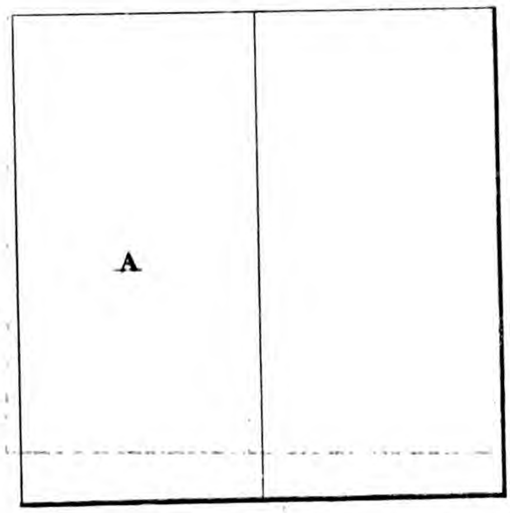
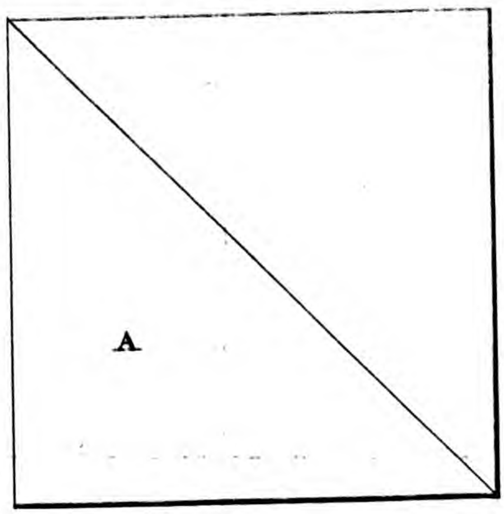


FIG. 64.



Drawn by John Bennett.

London, John Bennett, 1838.

FIG. 65.

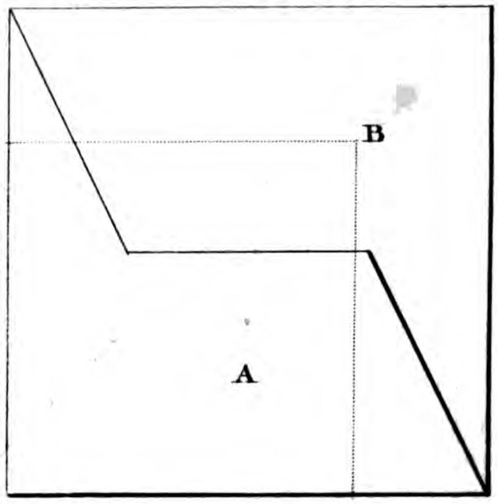
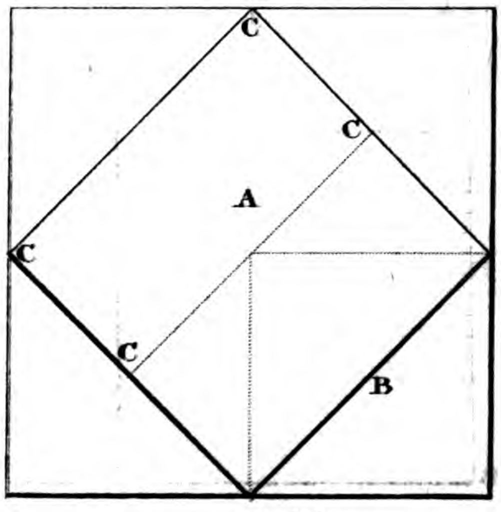


FIG. 66.



Invented and drawn by John Bennett.

London. John Bennett, 1838

FIG. 67.

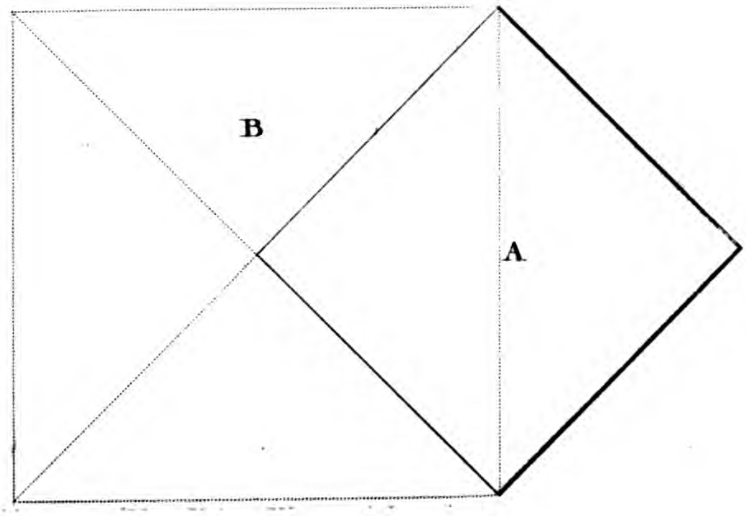
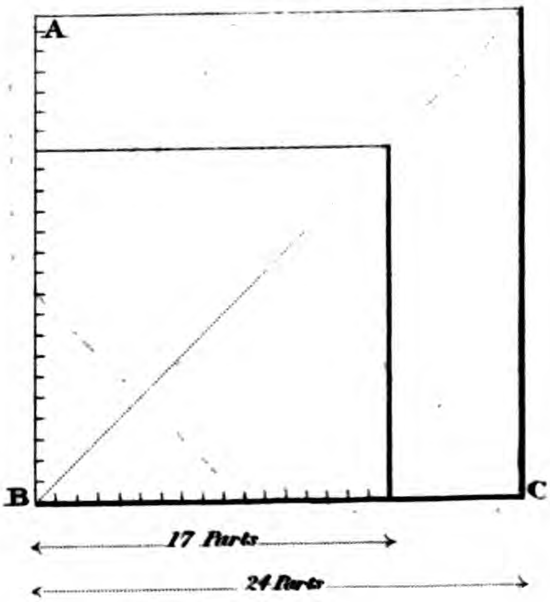


FIG. 68.



Invented and drawn by John Bennett.

London. John Bennett. 1838.

FIG. 69.

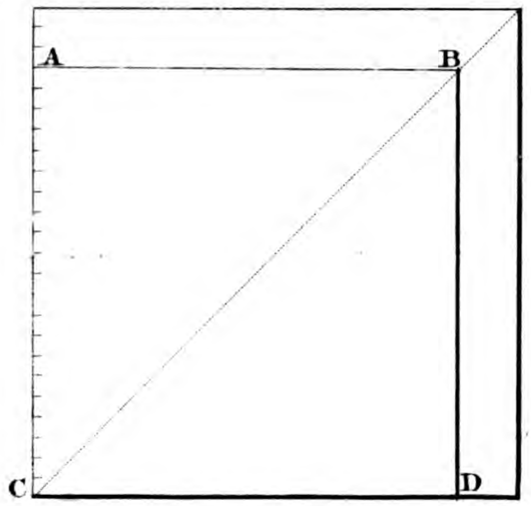
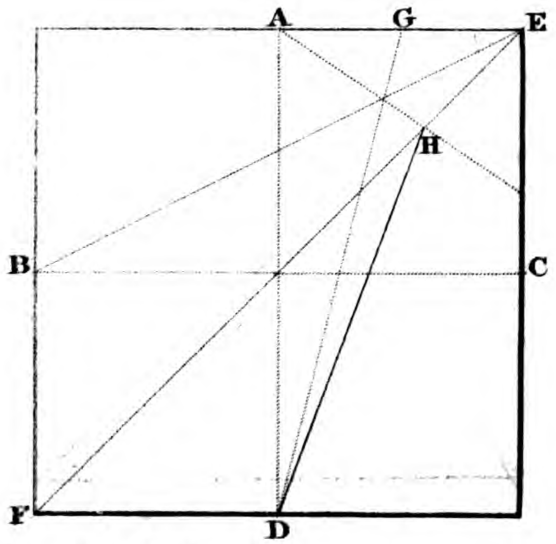


FIG. 70.



Invented and drawn by John Bennett.

London, John Bennett. 1838.

FIG. 71.

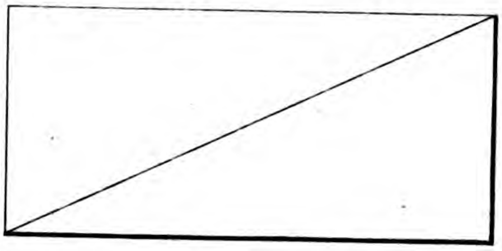


FIG. 72.

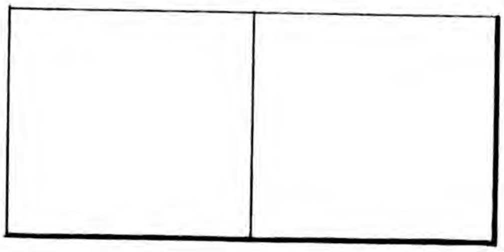
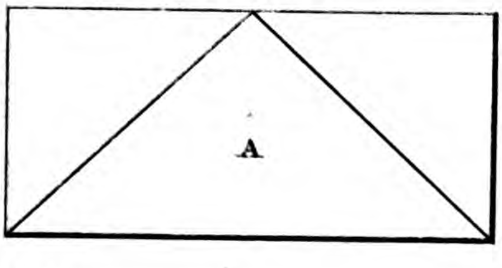


FIG. 73.



Drawn by John Bennett

London, John Bennett, 1858.

FIG. 74.

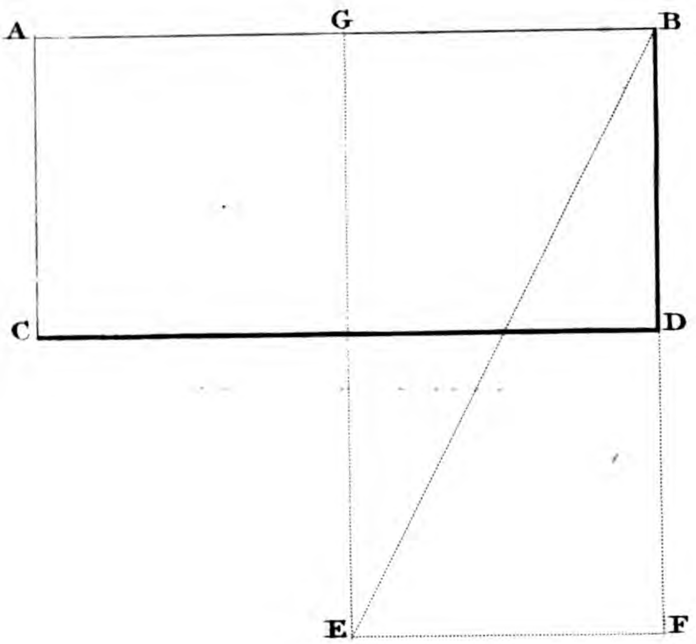
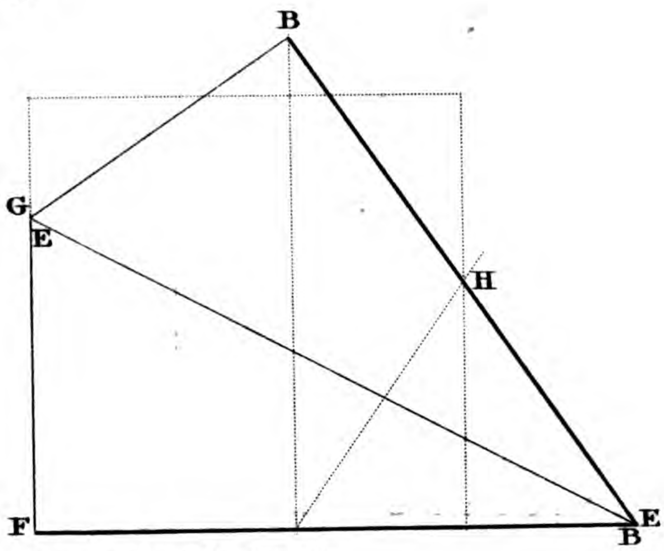


FIG. 75.



Invented and drawn by John Bennett.

London, John Bennett, 1838.

FIG. 76.

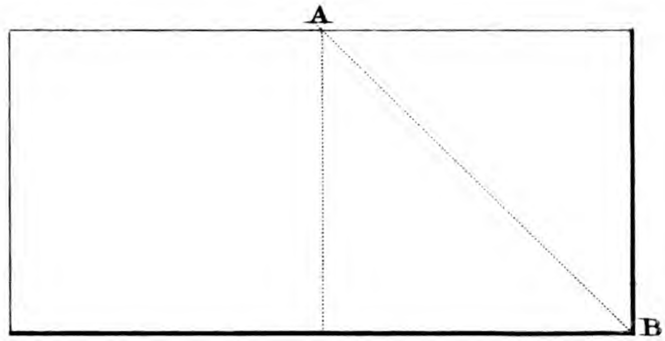
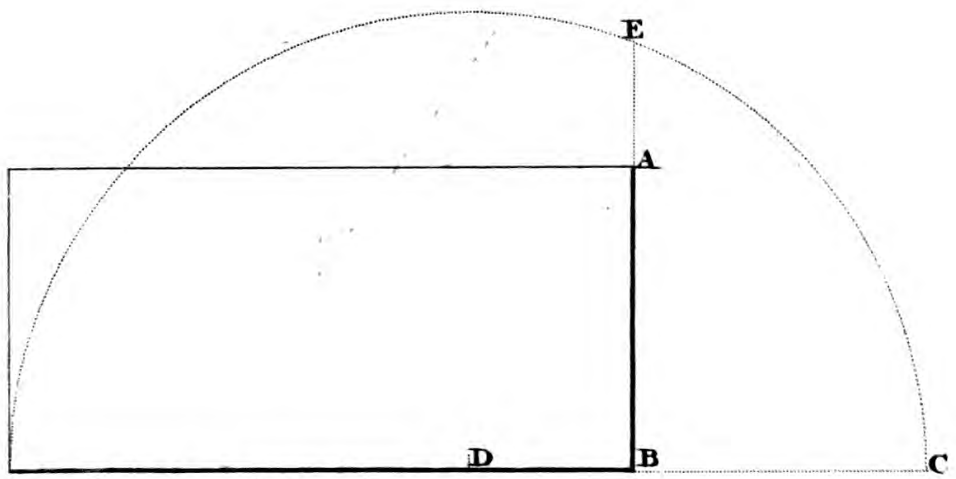


FIG. 77.



Drawn by John Bennett.

London, John Bennett, 1838.

FIG. 78.

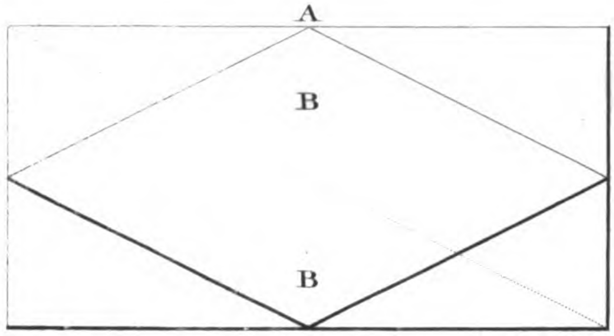


FIG. 79.

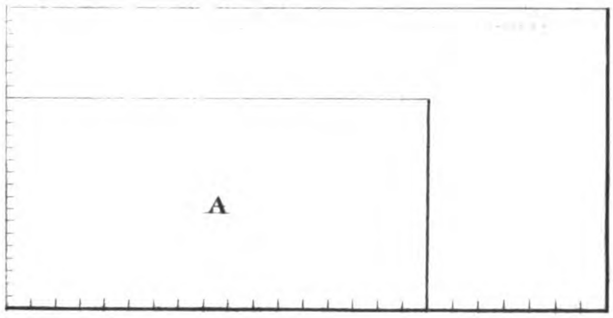
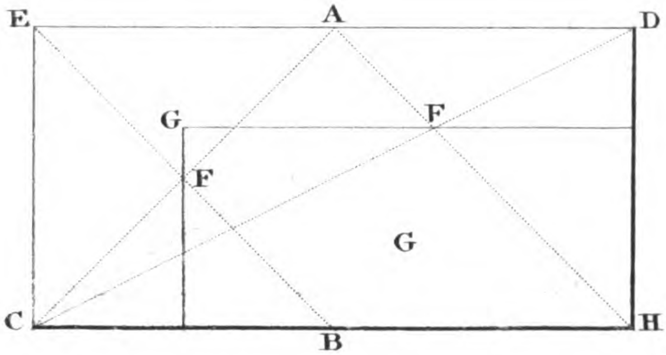


FIG. 80.



Invented and drawn by John Bennett

London, John Bennett, 1838

FIG. 81.

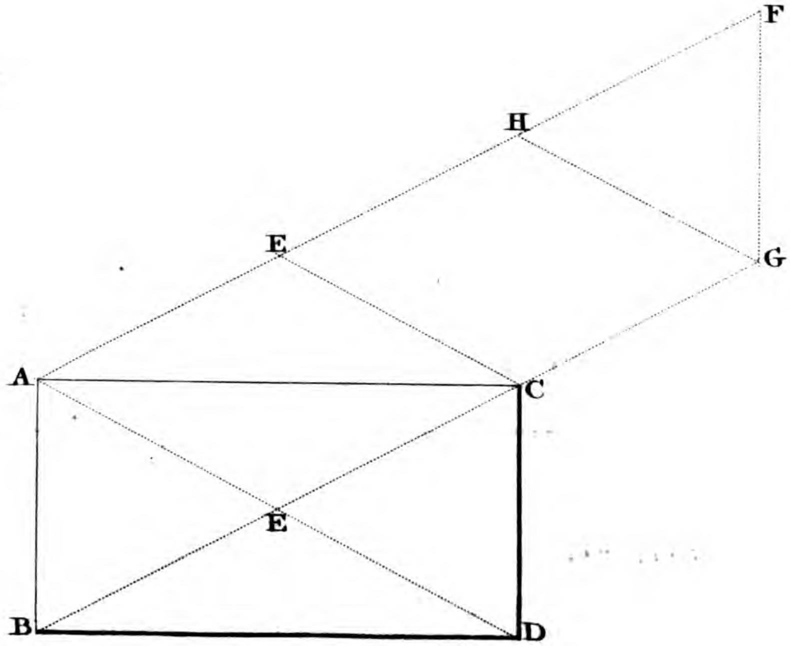
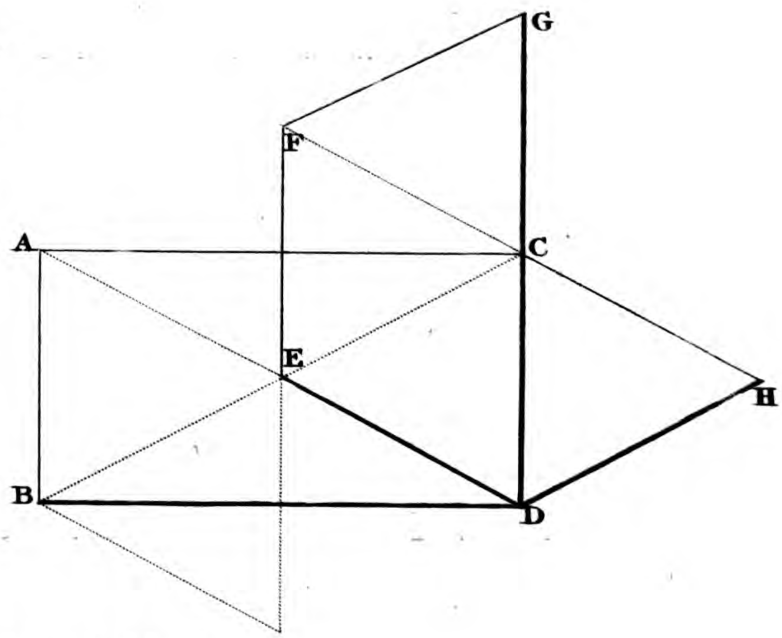


FIG. 82.



Invented and drawn by John Bennett.

London, John Bennett, 1838.

FIG. 83.

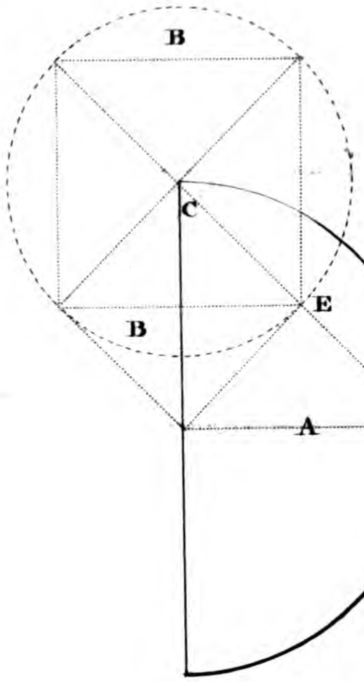
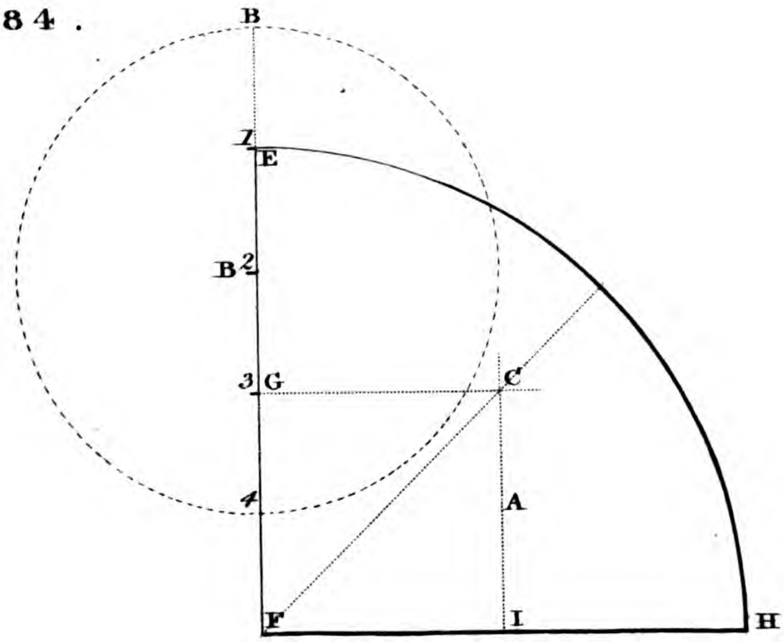


FIG. 84.



Invented & drawn by John Bennett.

London: John Bennett. 1838.

FIG. 85.

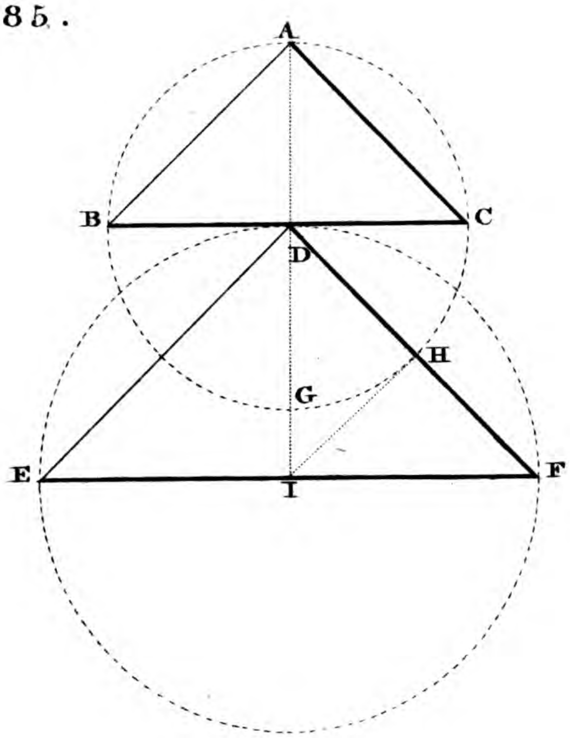
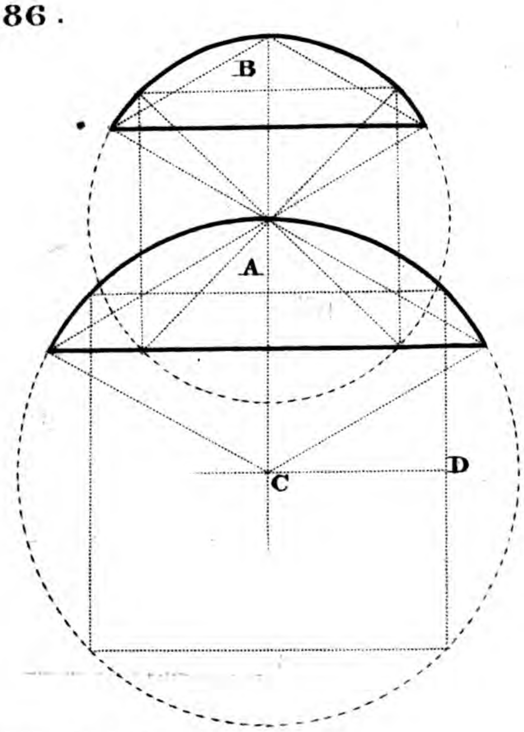


FIG. 86.



Invented & drawn by John Bennett.

London; John Bennett. 1838.

FIG. 87.

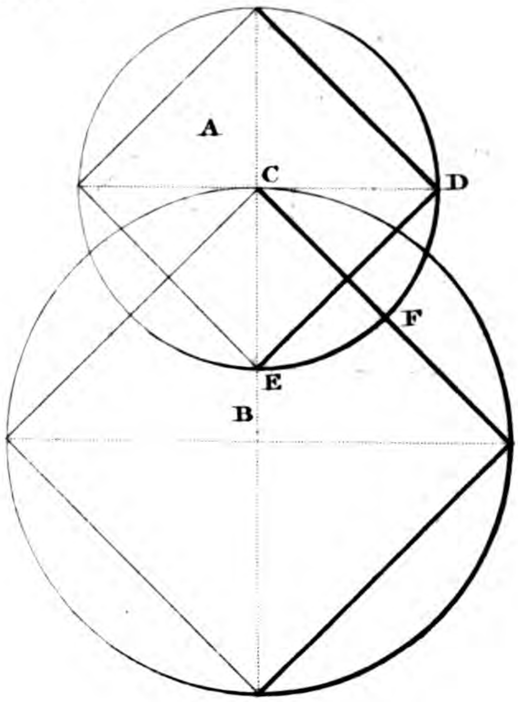
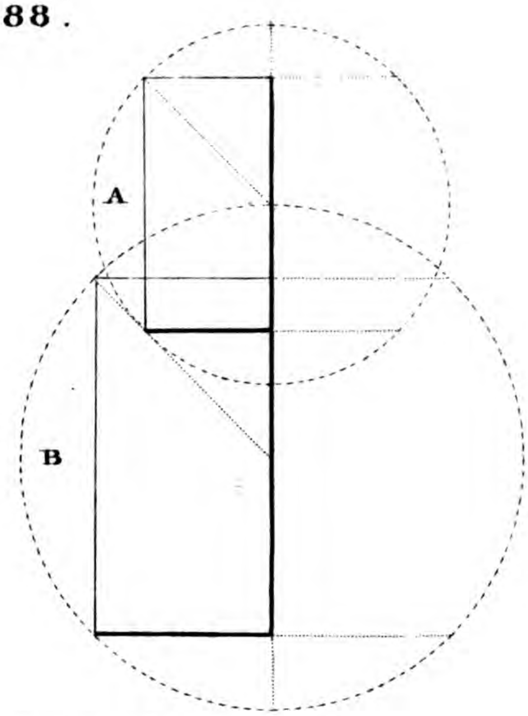


FIG. 88.



Invented & drawn by John Bennett.

London, John Bennett. 1838.

FIG. 89.

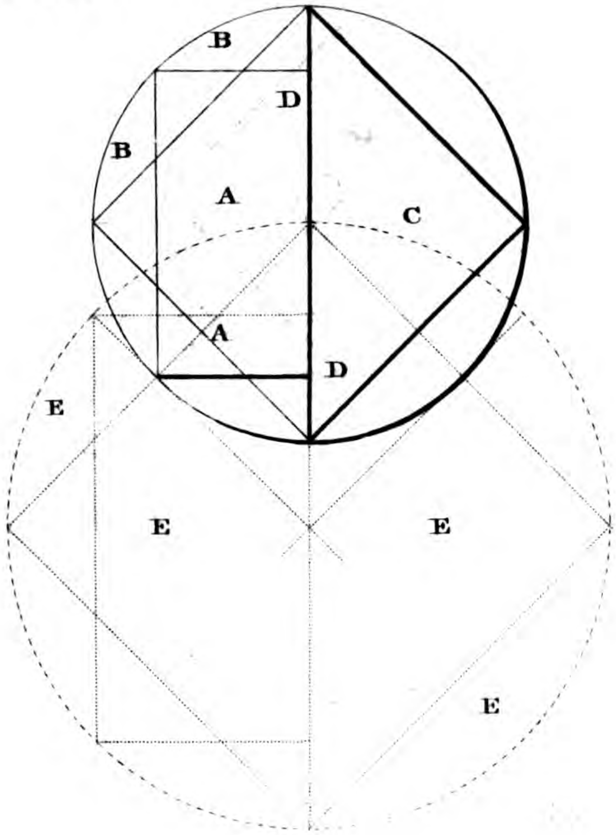
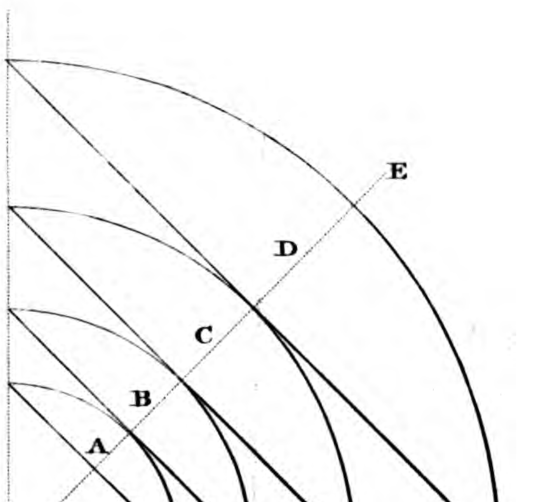


FIG. 90.



Invented & drawn by John Bennett.

London: John Bennett. 1838.

FIG. 91.

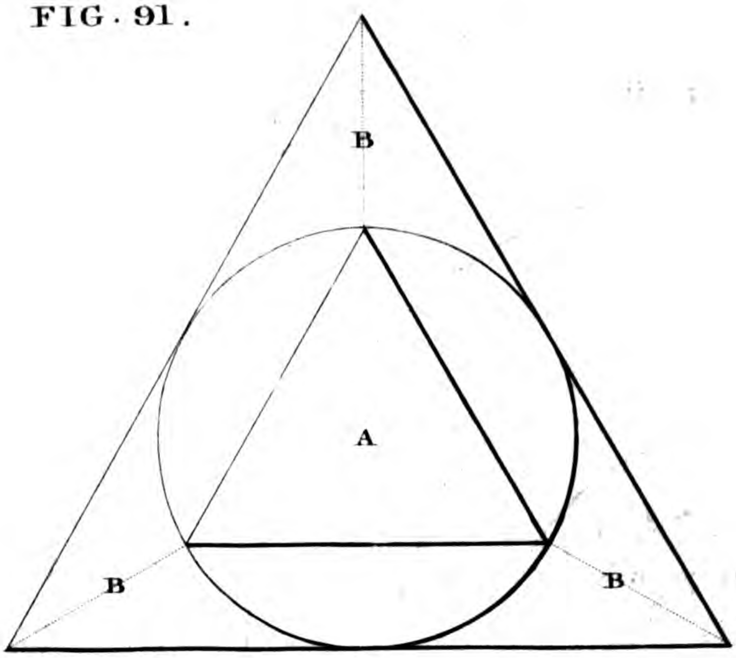
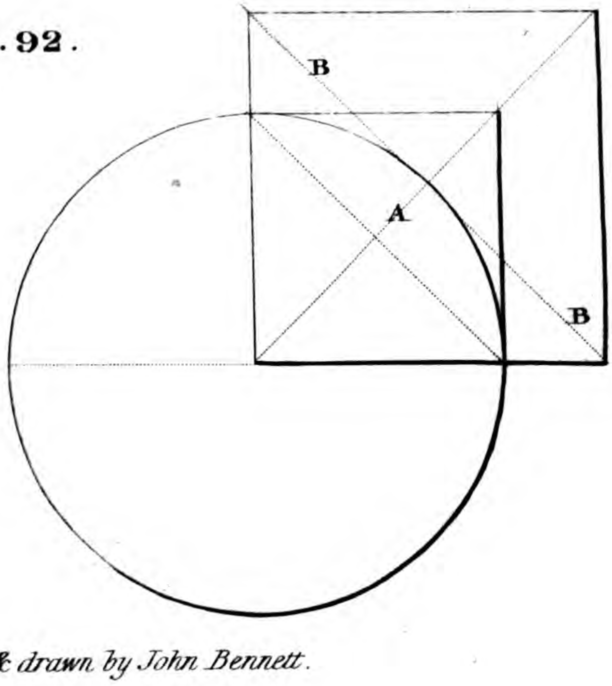


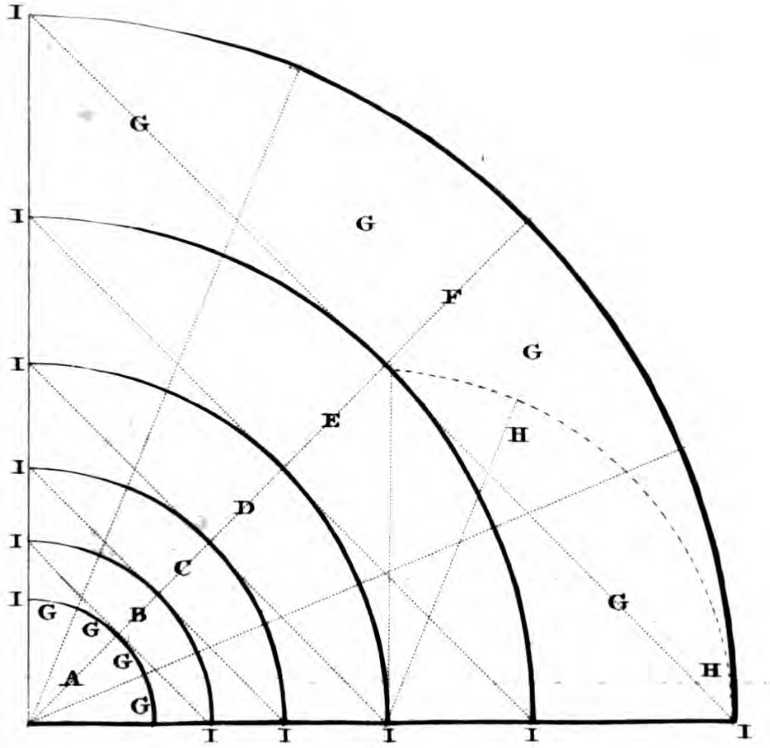
FIG. 92.



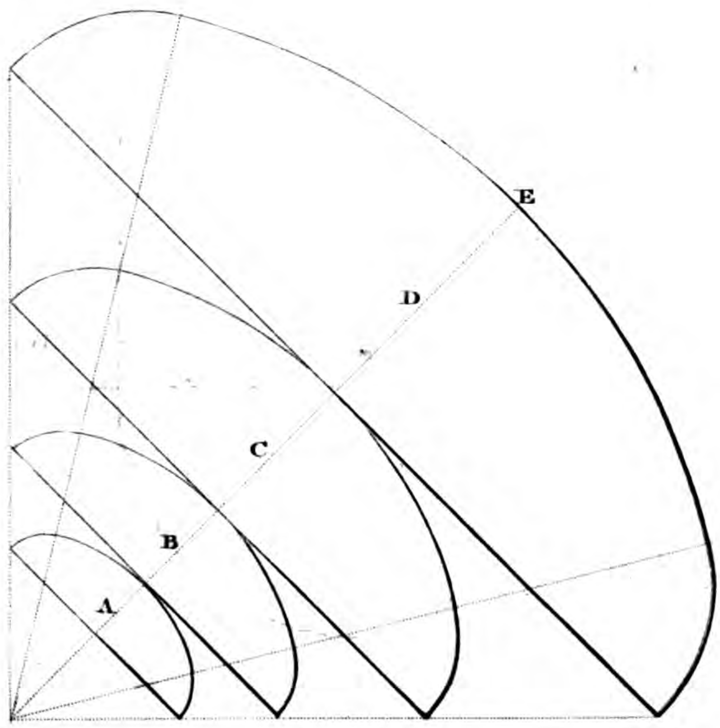
Invented & drawn by John Bennett.

London; John Bennett, 1838.

G. 93.



IG. 94.



Invented & drawn by John Bennett.

London; John Bennett, 1838.

FIG. 95.

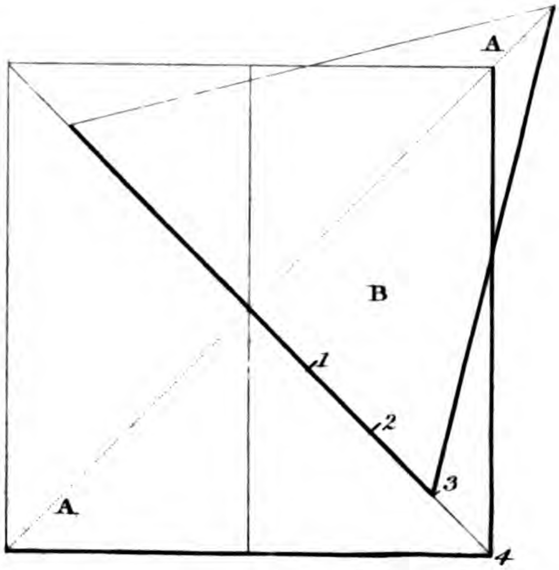
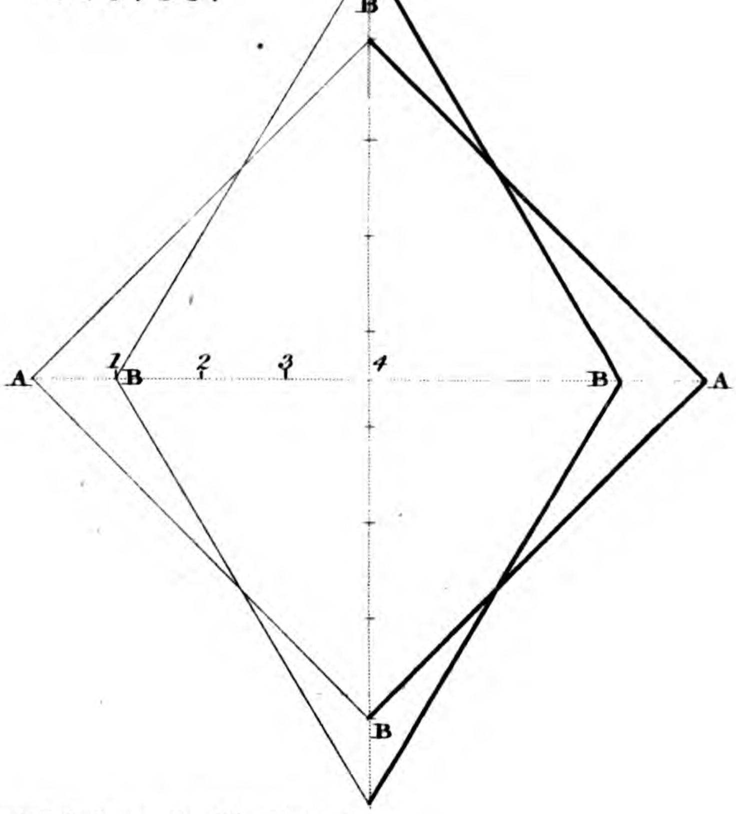


FIG. 96.



Invented & drawn by John Bennett.

London; John Bennett, 1838.

FIG. 97.

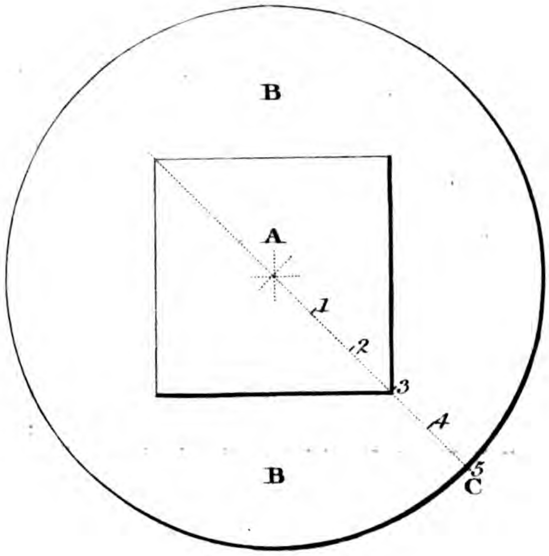
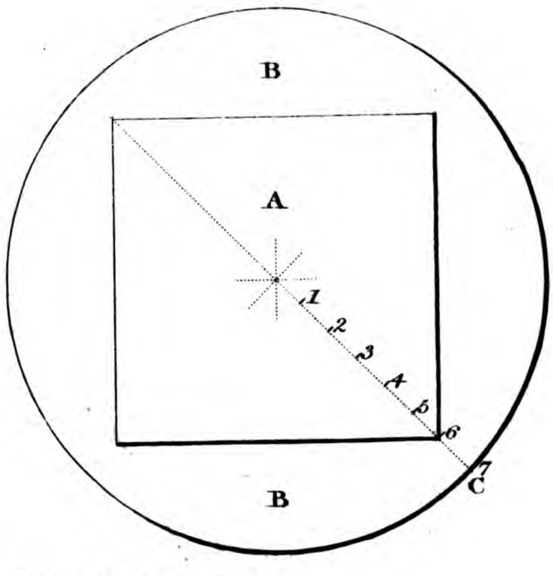


FIG. 98.



Invented & drawn by John Bennett.

London; John Bennett, 1838.

FIG. 99.

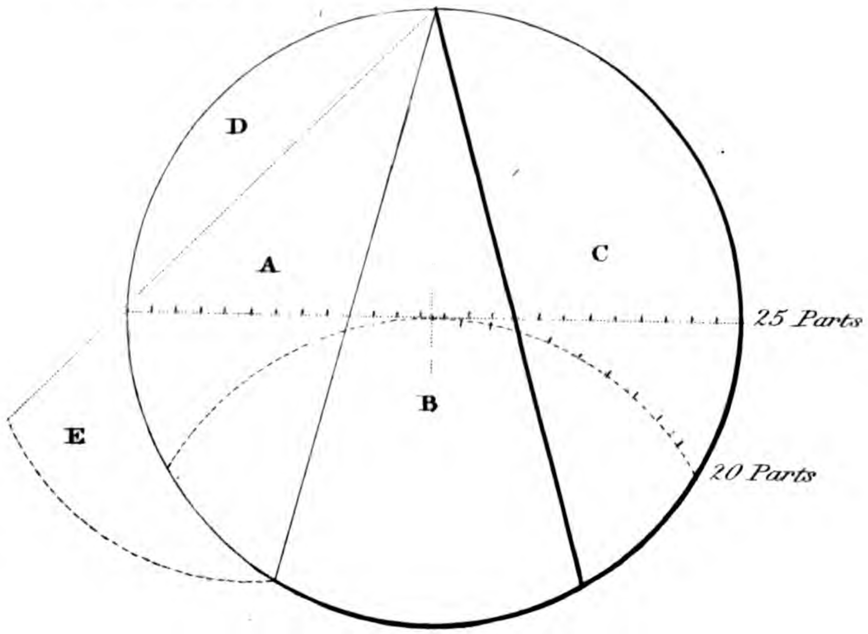
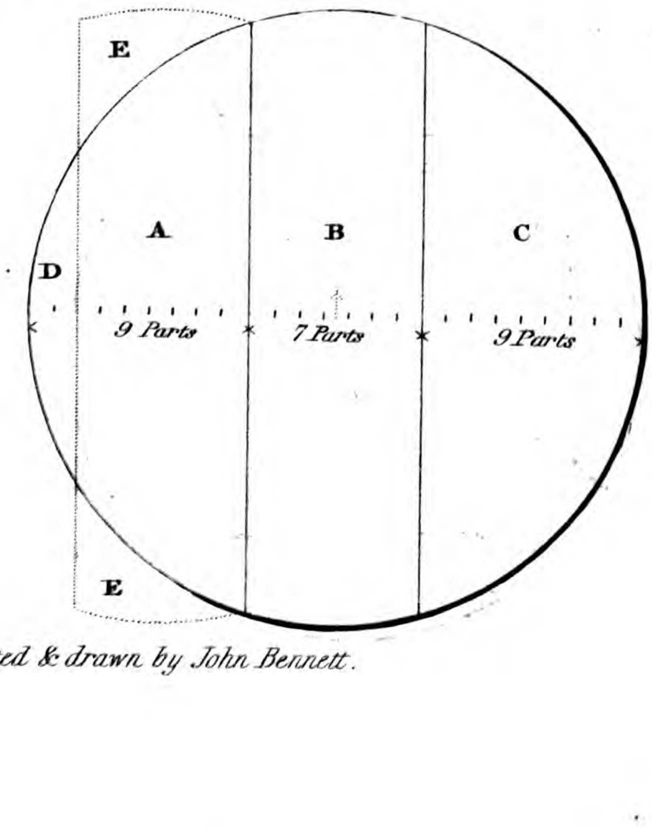


FIG. 100.



Invented & drawn by John Bennett.

London; John Bennett. 1838.

FIG. 101.

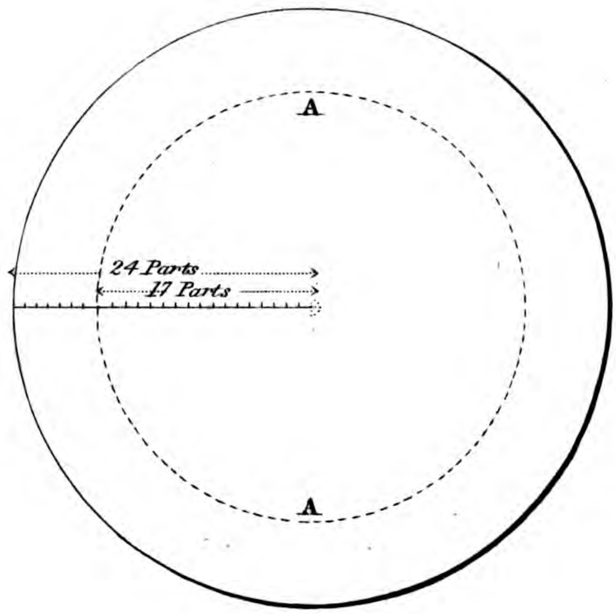
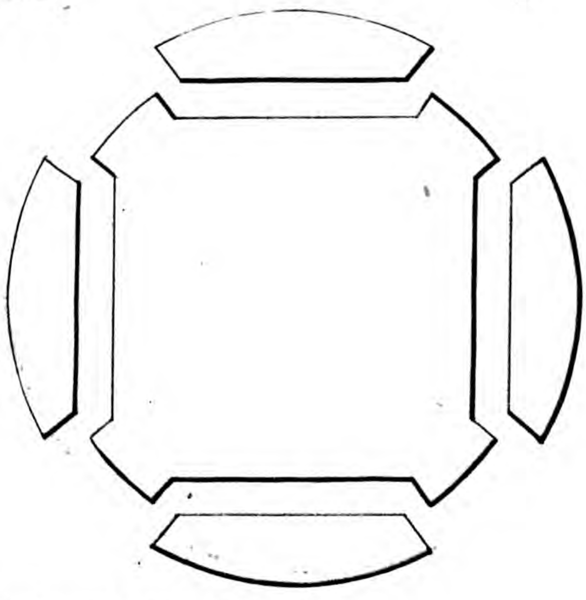


FIG. 102.



Invented & drawn by John Bennett.

London. John Bennett. 1838.

FIG. 103.

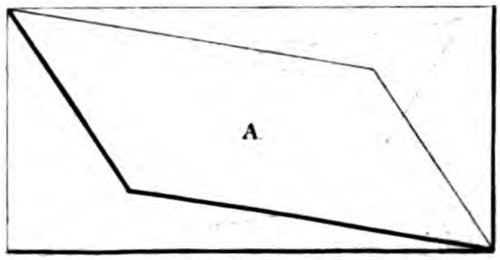


FIG. 104.

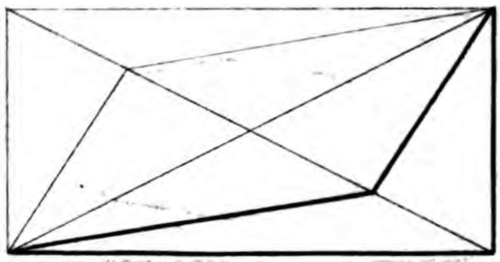
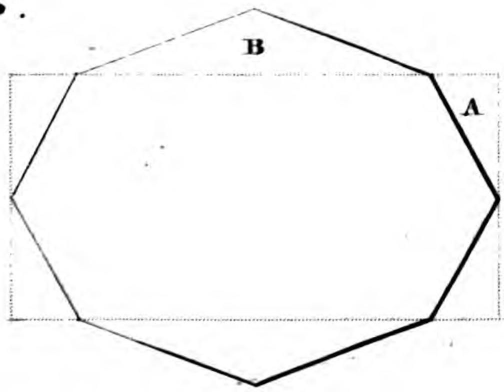


FIG. 105.



Invented & drawn by John Bennett.

London; John Bennett 1838.

FIG. 106.

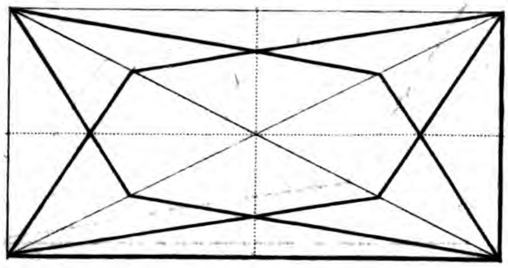


FIG. 107.

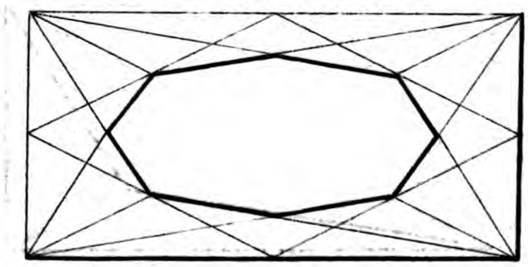
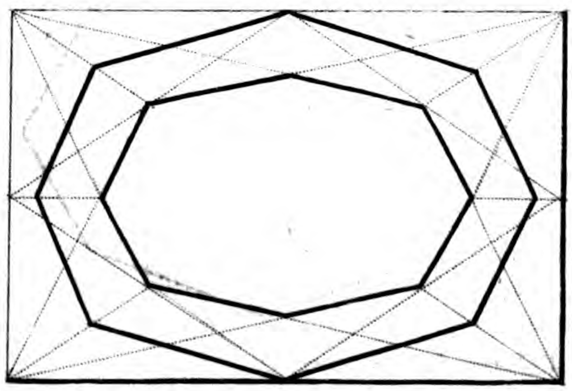


FIG. 108.



Invented & drawn by John Bennett.

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FIG. 109.

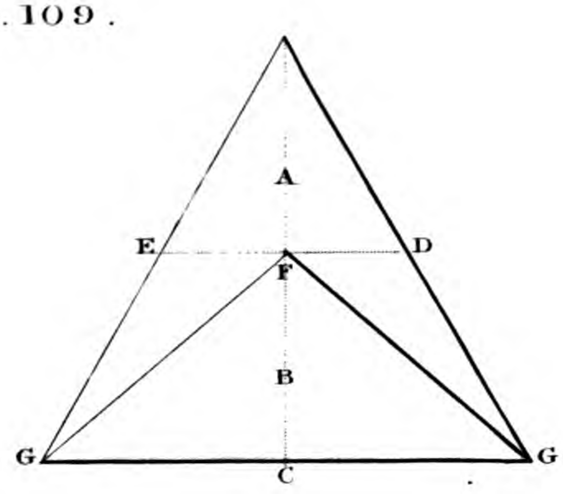


FIG. 110.

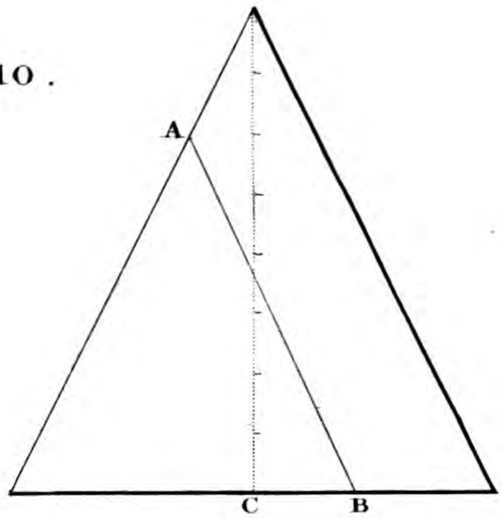
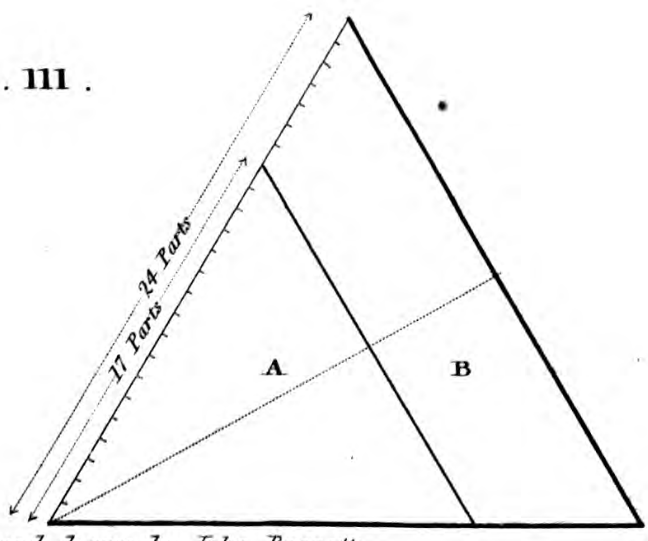


FIG. 111.



Invented and drawn by John Bennett.

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FIG. 112.

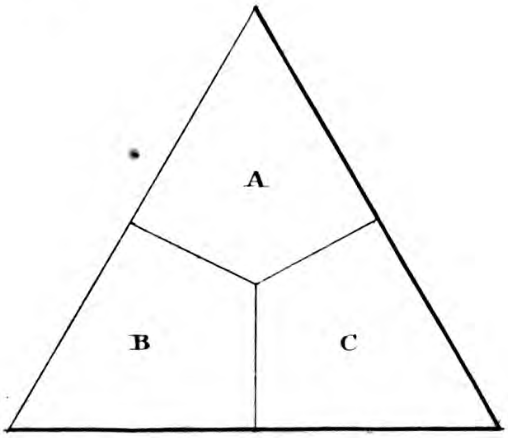


FIG. 113.

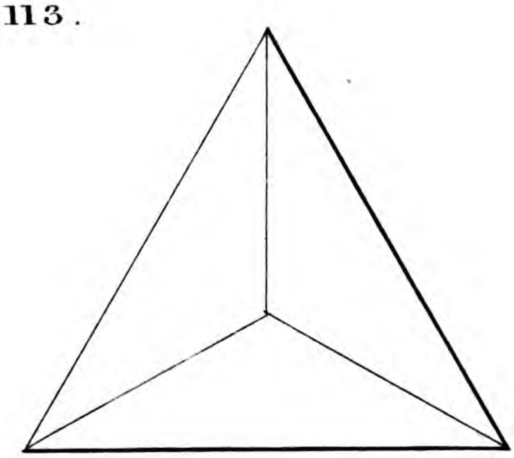
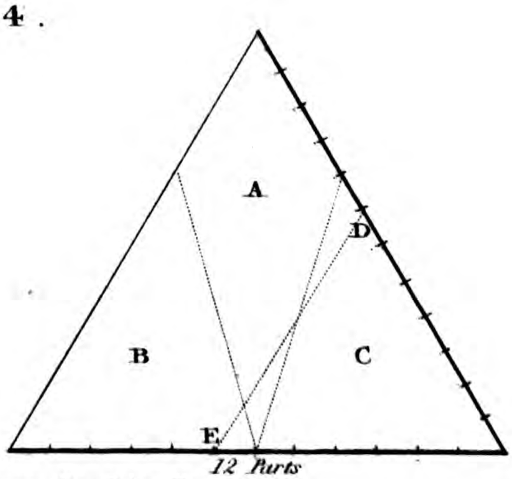


FIG. 114.



Invented and drawn by John Bennett.

London; John Bennett, 1838.

FIG. 115.

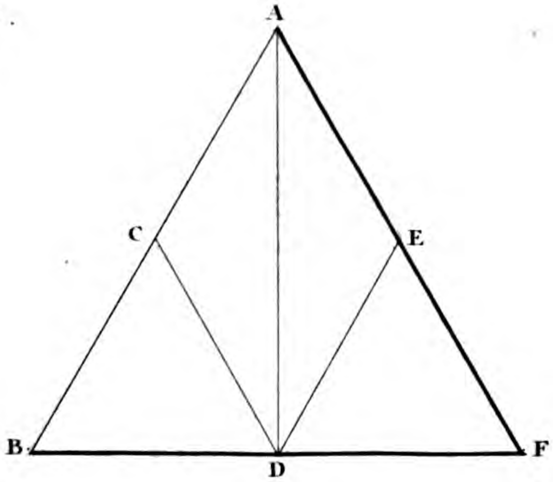


FIG. 116.

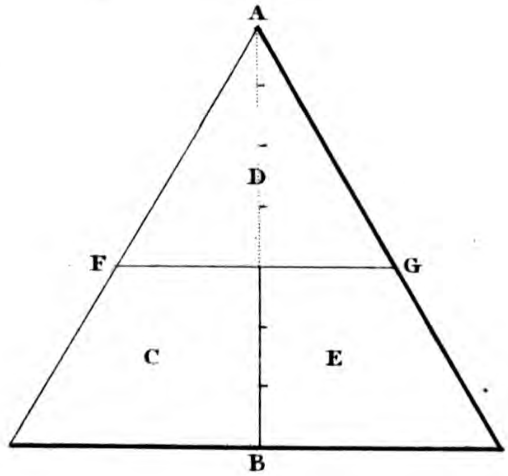
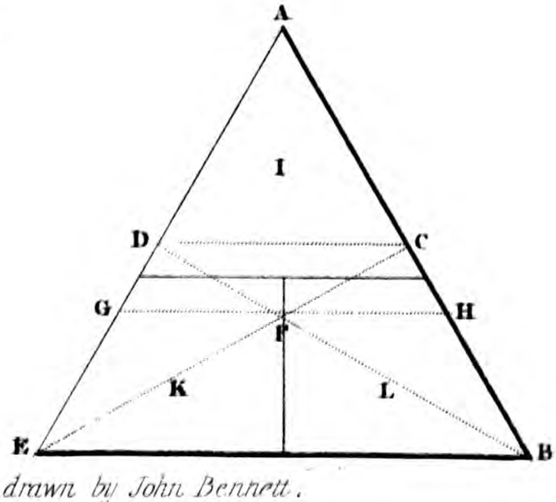


FIG. 117.



Invented and drawn by John Bennett.

London, John Bennett, 1838.

FIG. 118.

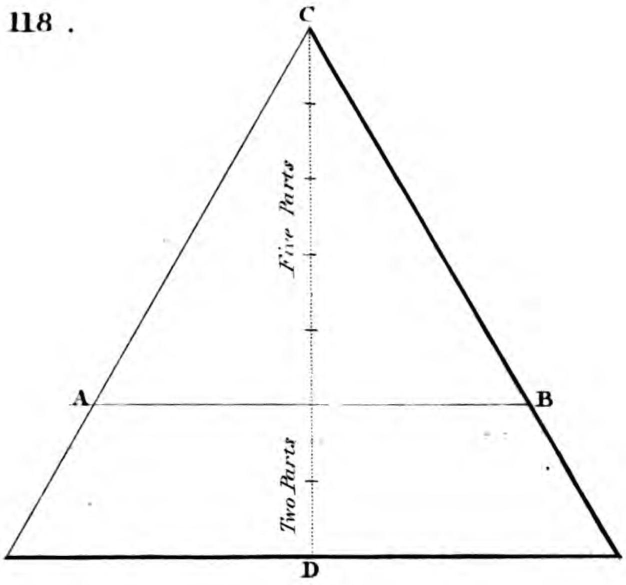


FIG. 119.

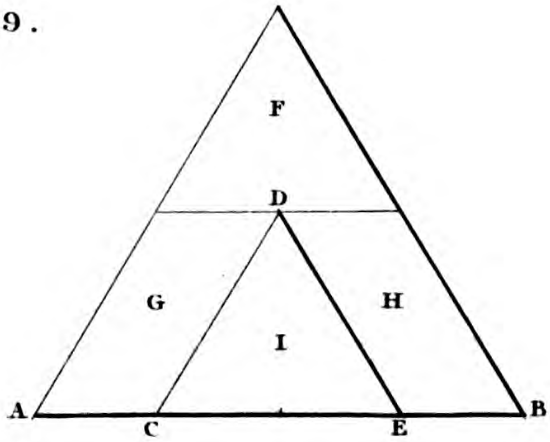
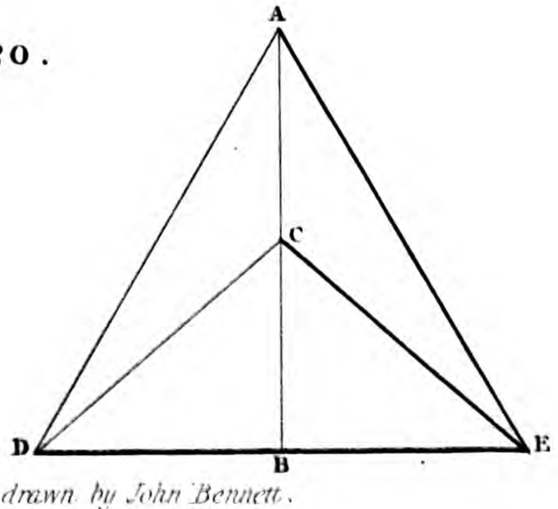


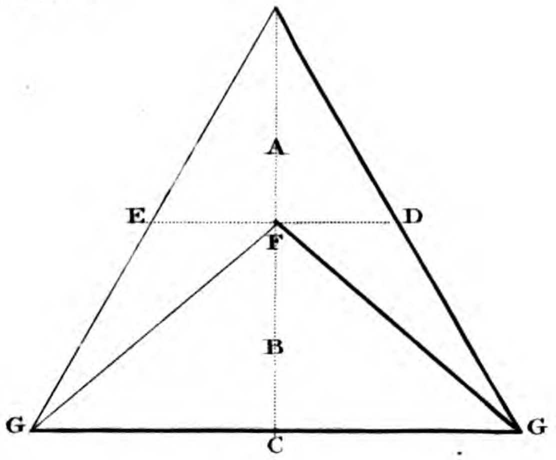
FIG. 120.



Invented and drawn by John Bennett.

London John Bennett. 1838.

FIG. 109.



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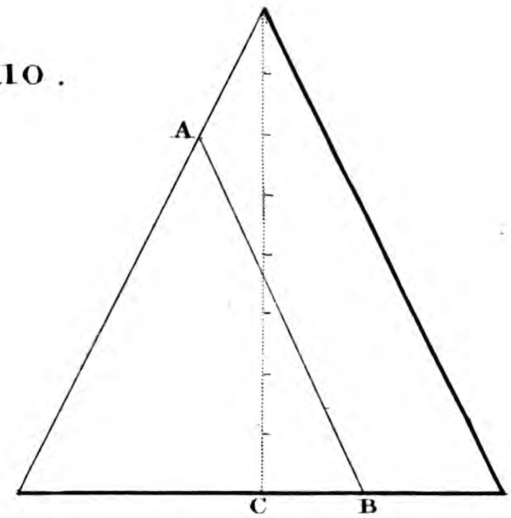

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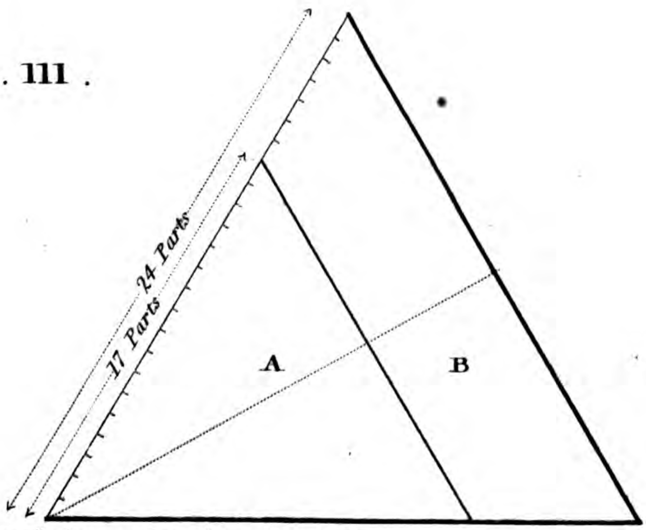

Invented and drawn by John Bennett.

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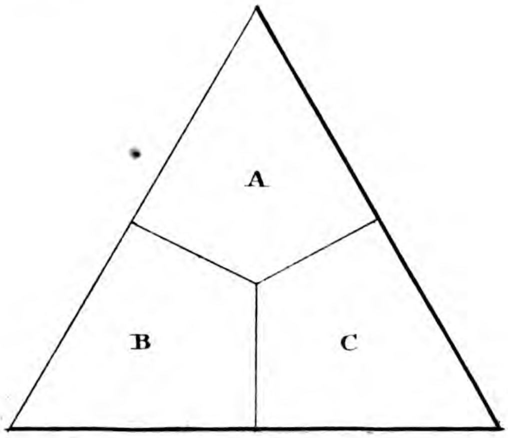
FIG. 112.



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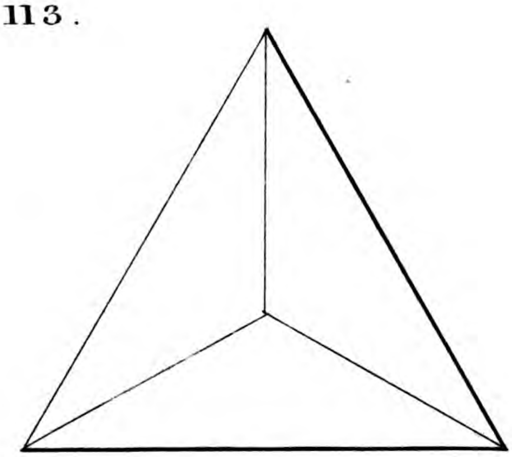

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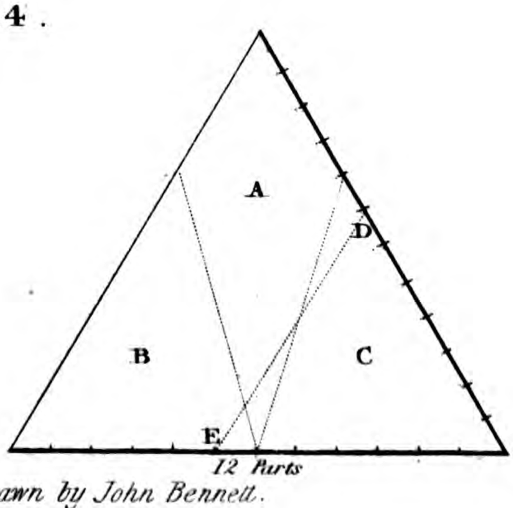
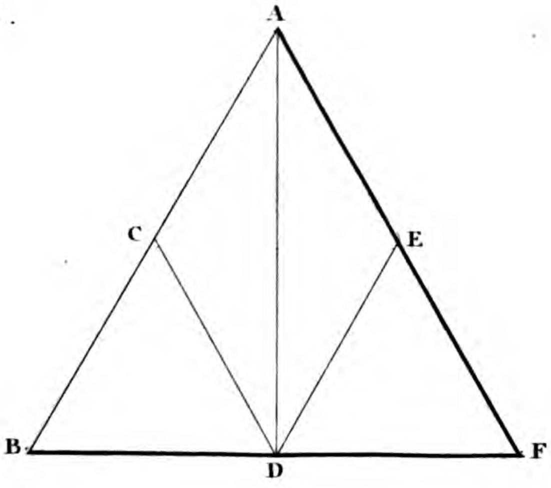


... and drawn by John Bennett.

London; John Bennett, 1838.

FIG. 115.



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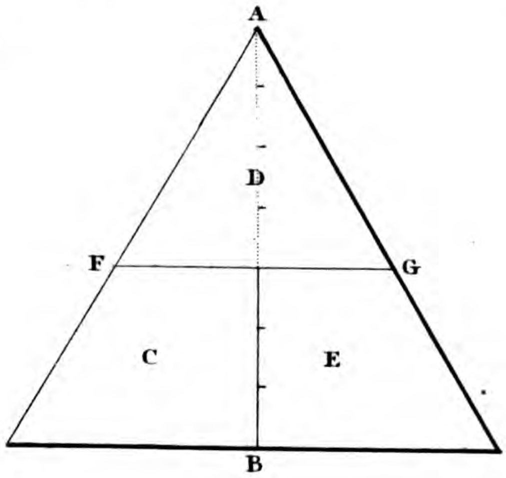
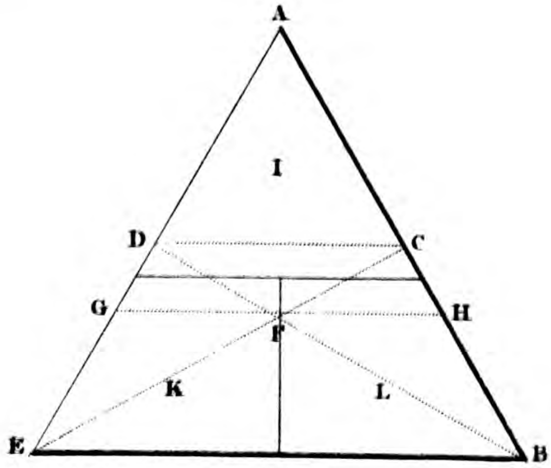


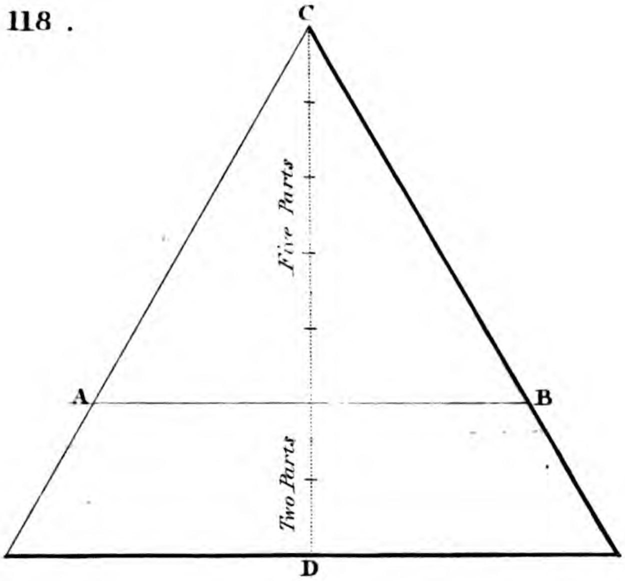
FIG. 117.



Invented and drawn by John Bennett.

London, John Bennett, 1838.

FIG. 118.



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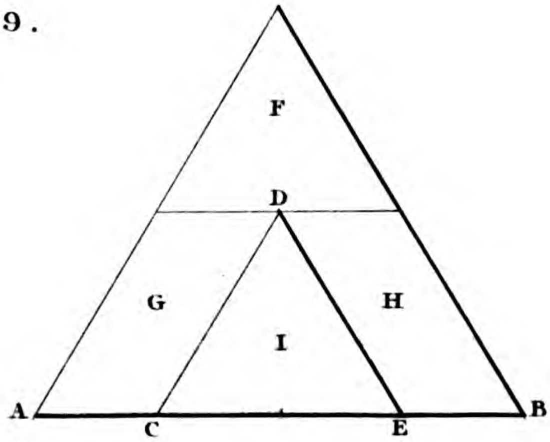

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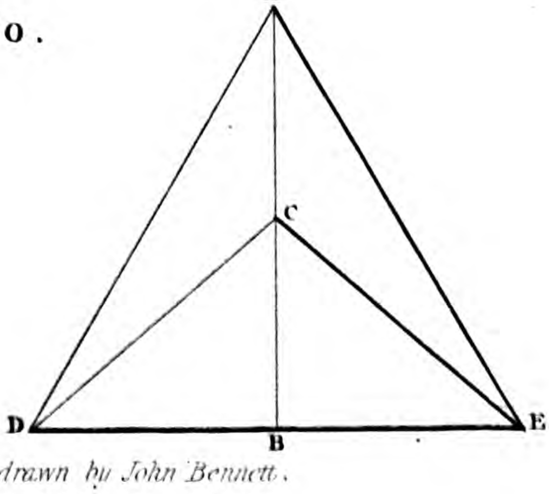


Invented and drawn by John Bennett.

London John Bennett. 1838.

FIG. 121.

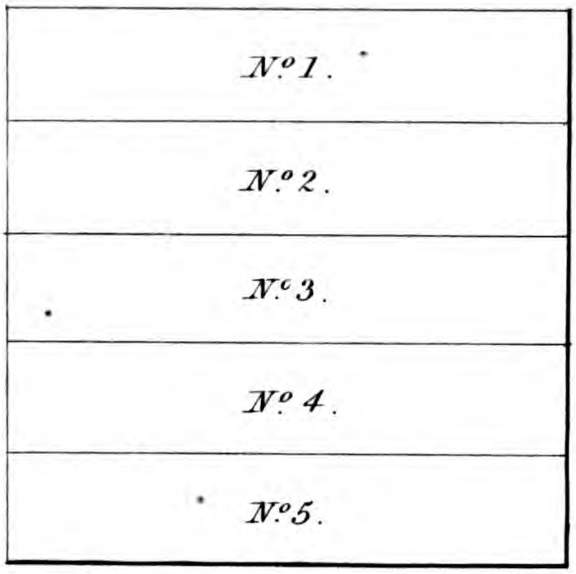


FIG. 122.

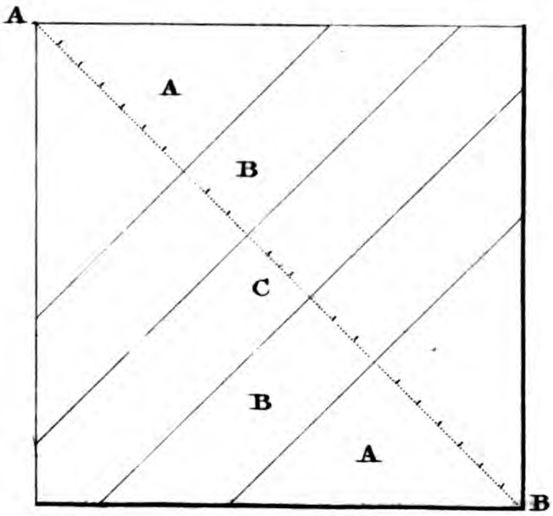
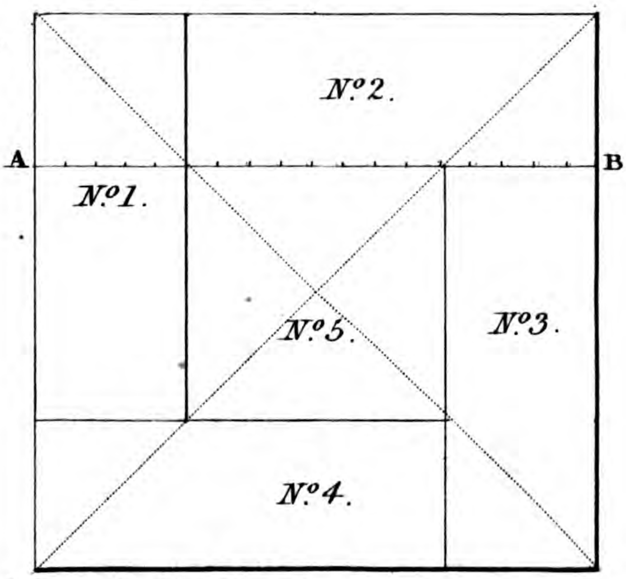


FIG. 123.



Invented and drawn by John Bennett.

London: John Bennett, 1838.

FIG. 124.

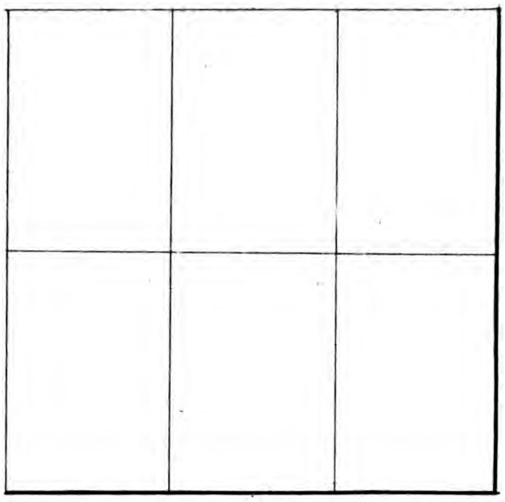


FIG 125.

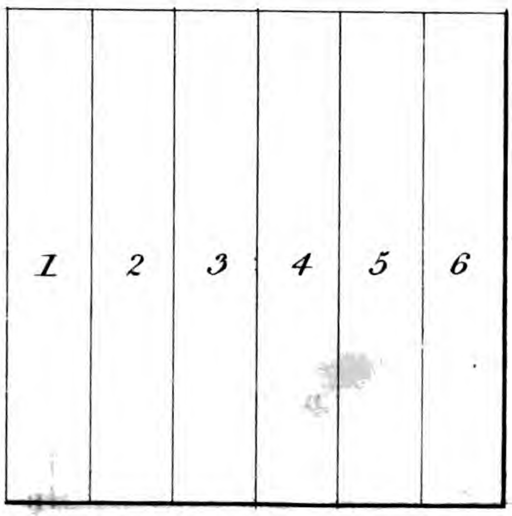
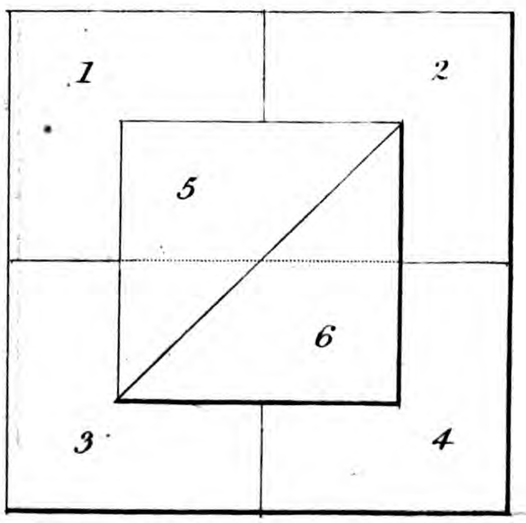


FIG. 126.



vented and drawn by John Bennett.

London, John Bennett, 1838.

FIG. 127.

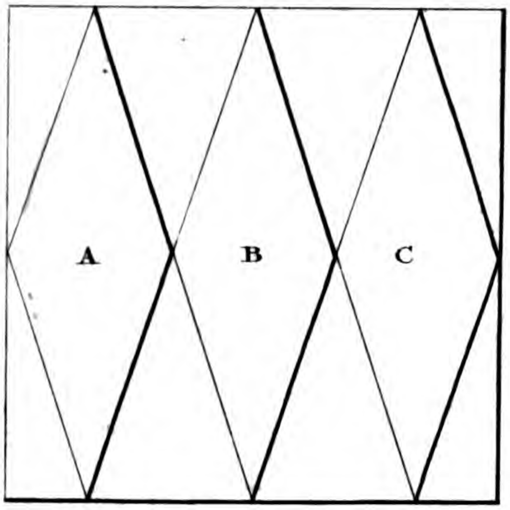


FIG. 128.

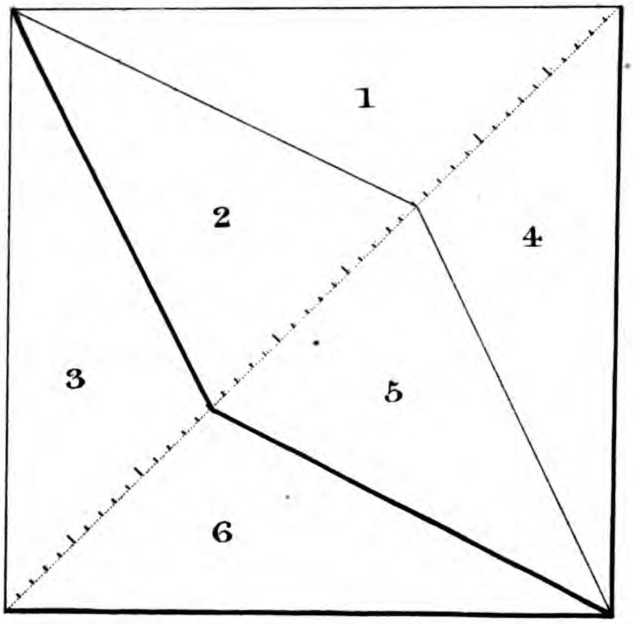
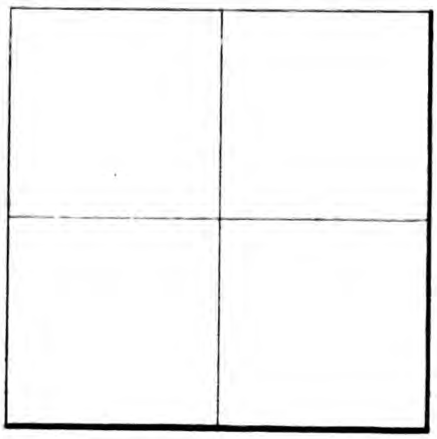


FIG. 129.



Invented and drawn by John Bennett.

London: John Bennett, 1838.

FIG. 130.

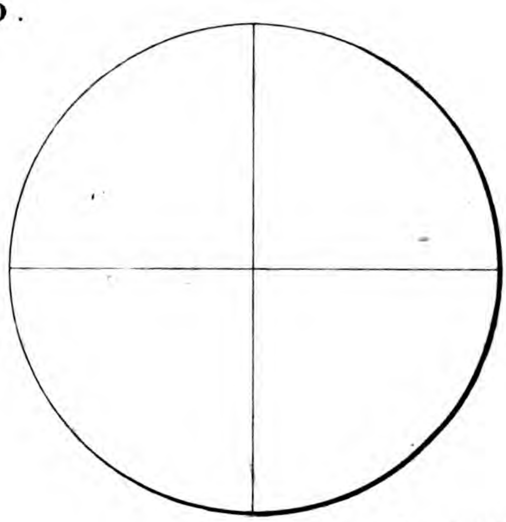


FIG. 131.

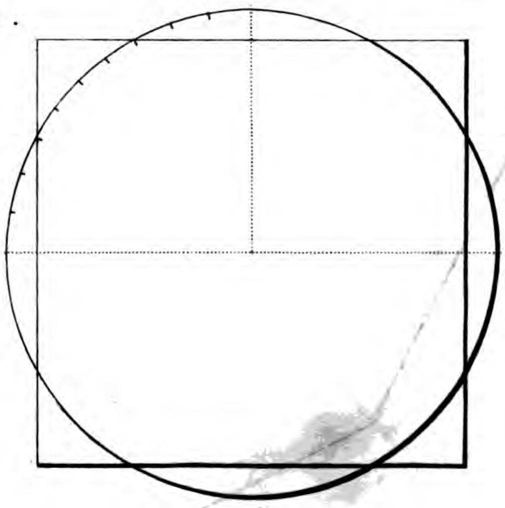
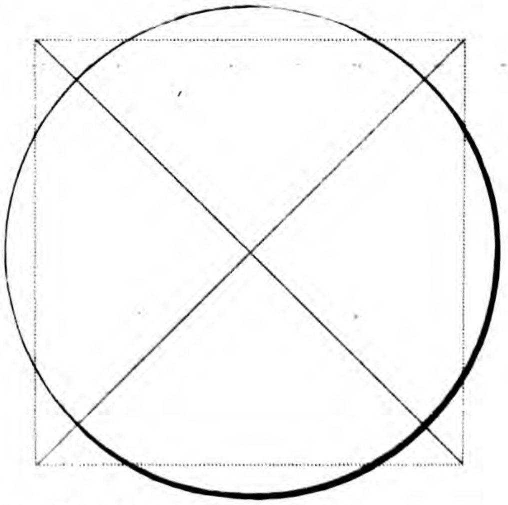


FIG. 132.



Invented and drawn by John Bennett.

London: John Bennett, 1838.

FIG. 133.

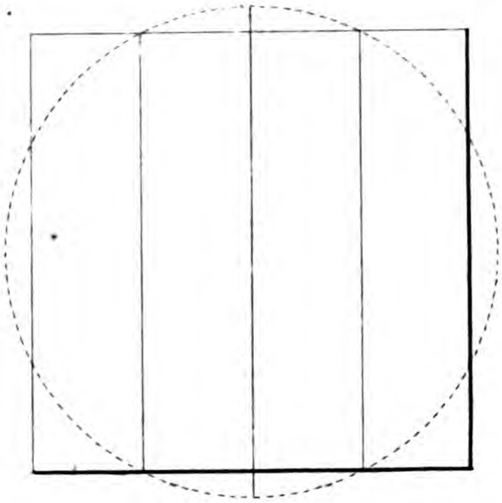


FIG. 134.

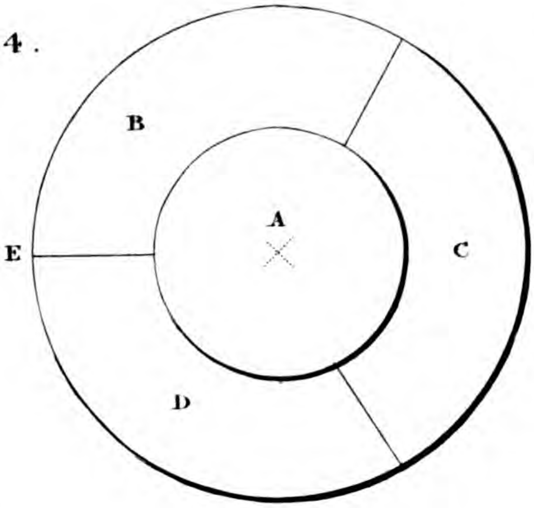
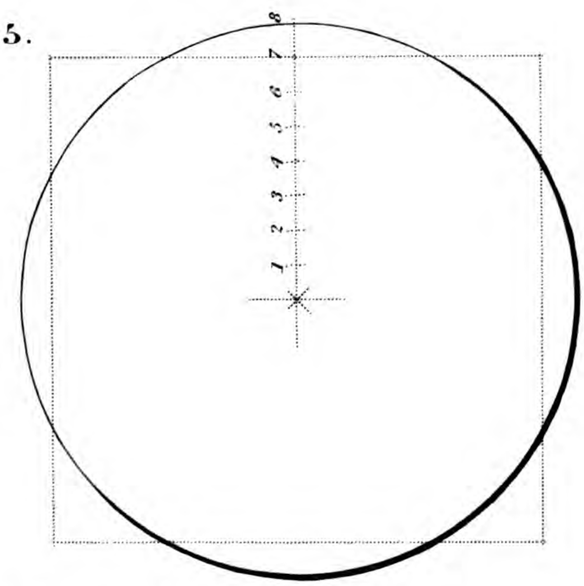


FIG. 135.



Invented and drawn by John Bennett.

London, John Bennett, 1838

FIG. 136.

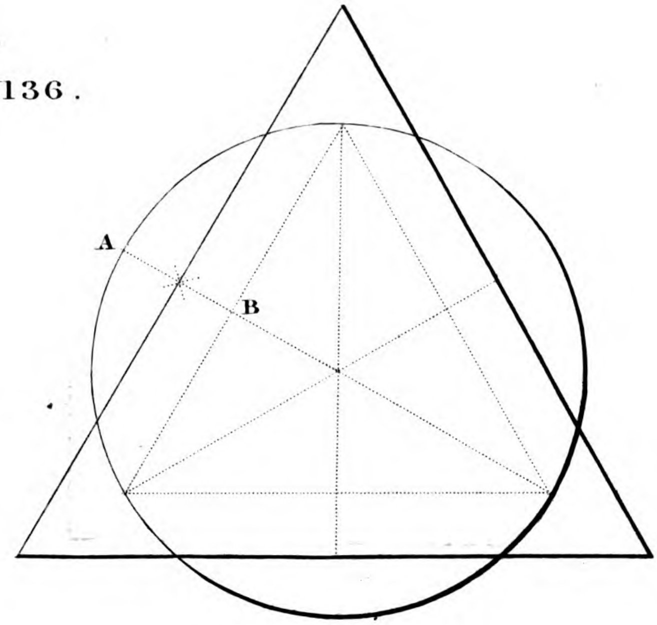


FIG. 137.

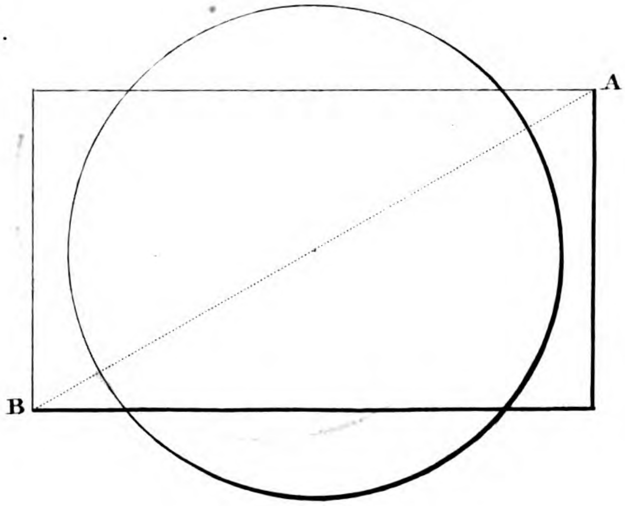


FIG. 138.

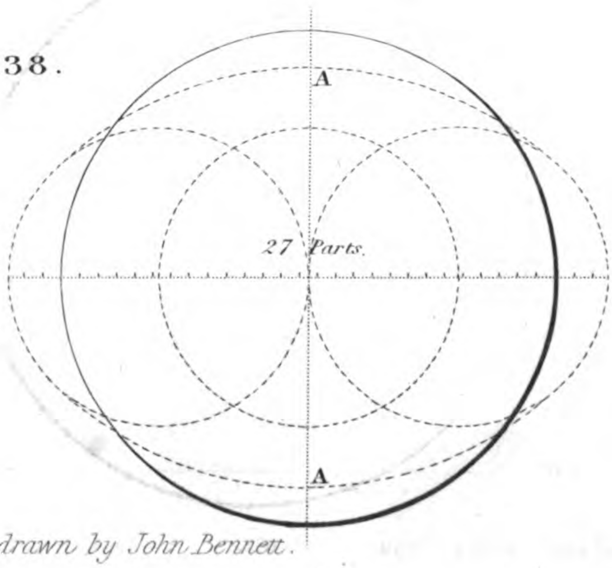


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FIG. 139.

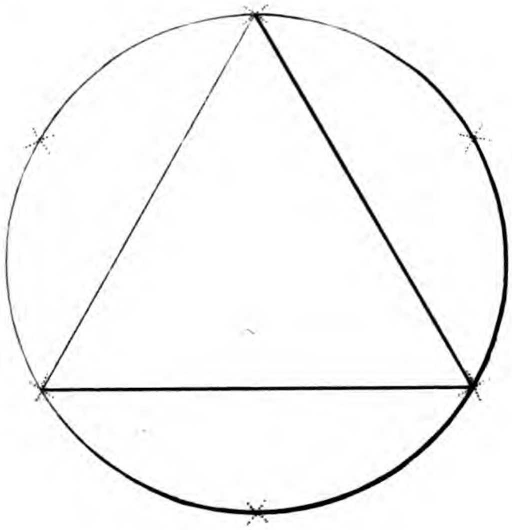


FIG. 140.

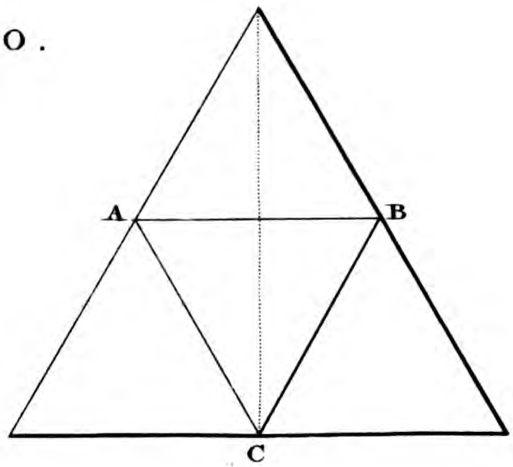
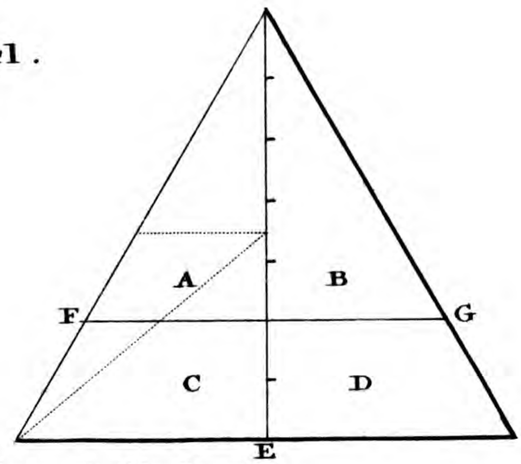


FIG. 141.



Invented and drawn by John Bennett.

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FIG. 142.

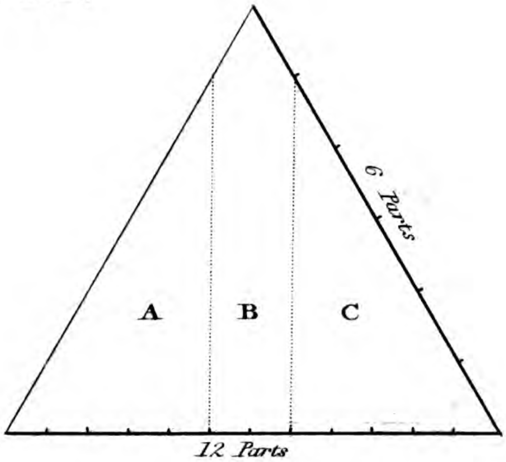

FIG. 143.

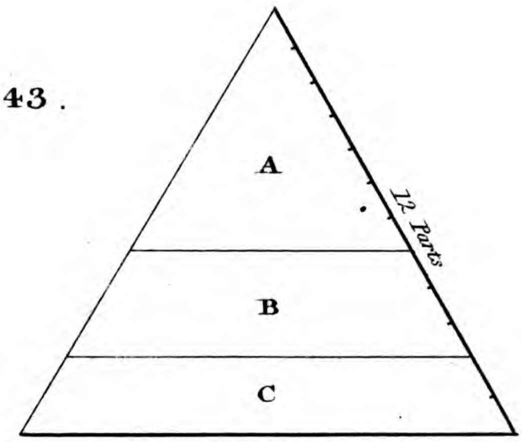

FIG. 144.

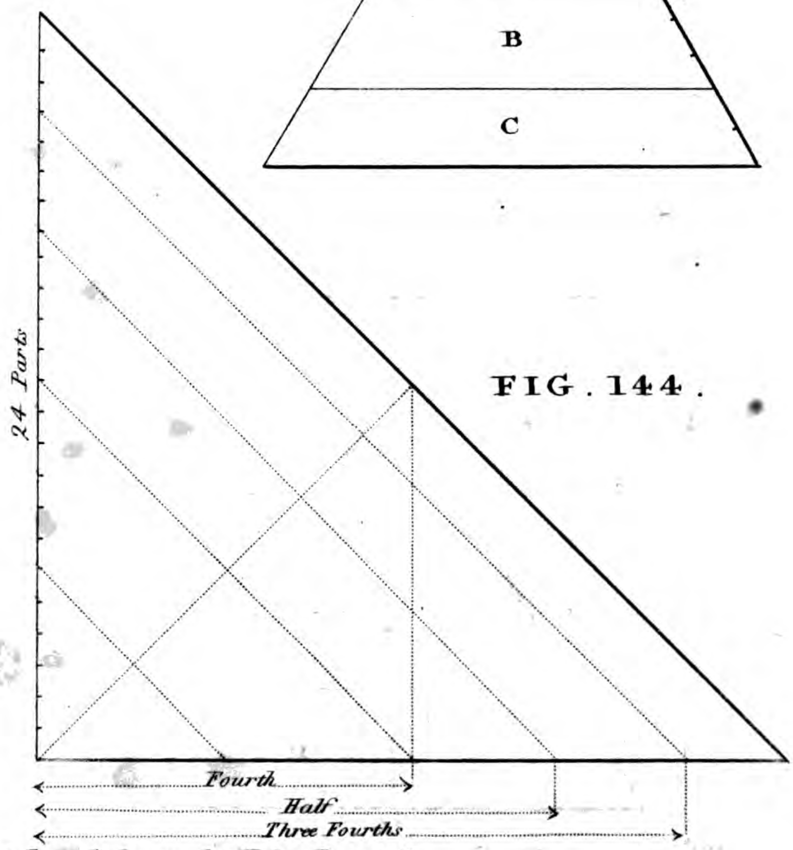

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FIG. 145.

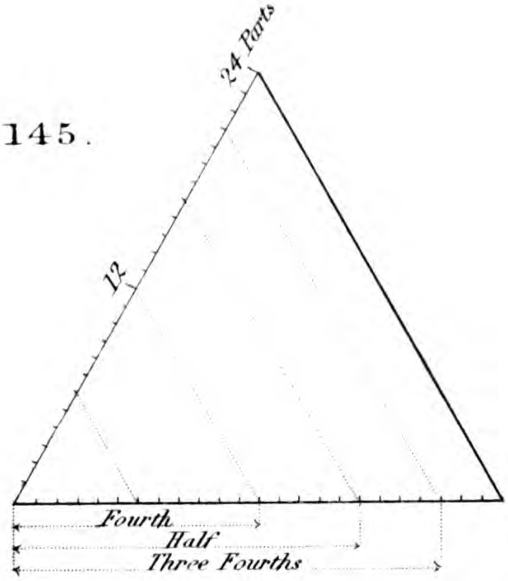

FIG. 146.

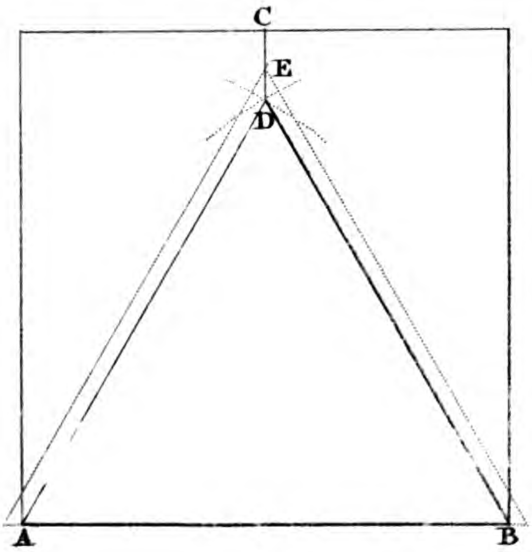

FIG. 147.

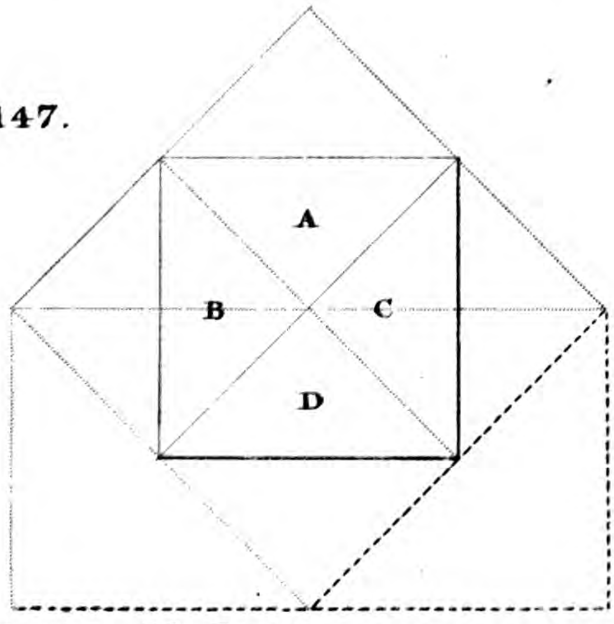

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FIG. 148.

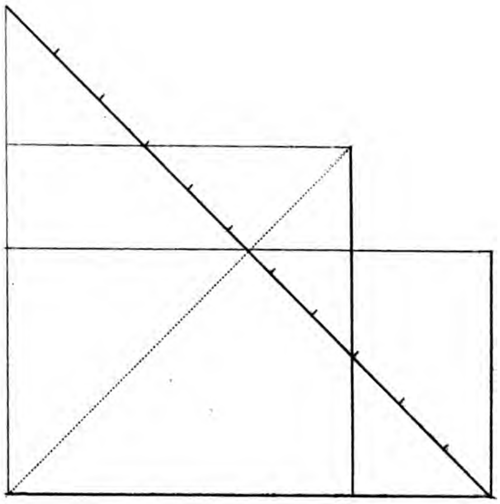

FIG. 149.

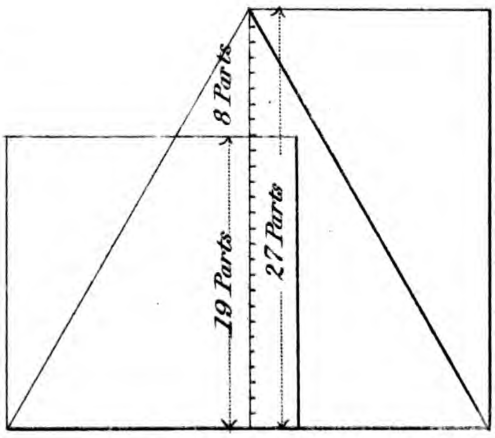

FIG. 150.

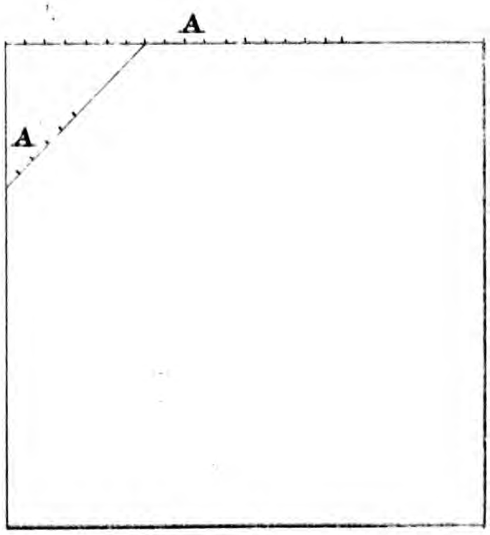

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FIG. 151.

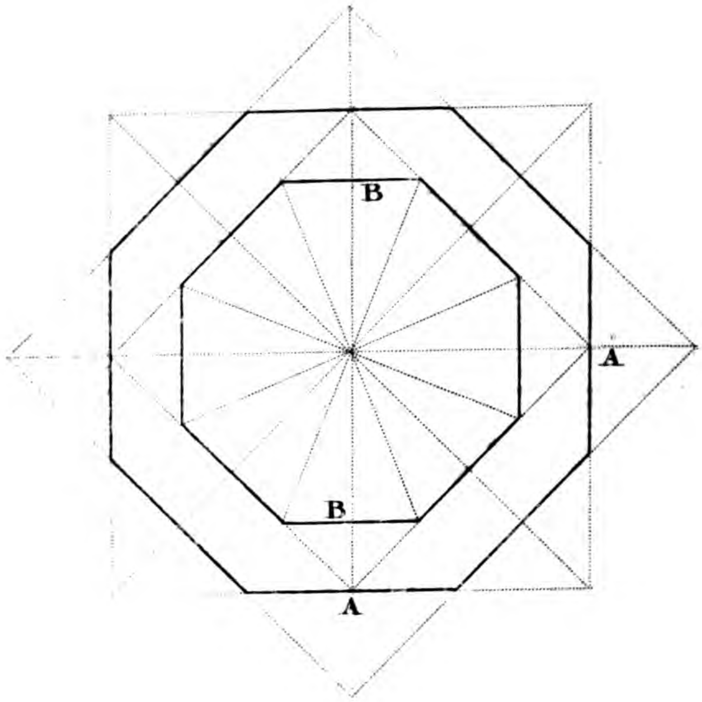
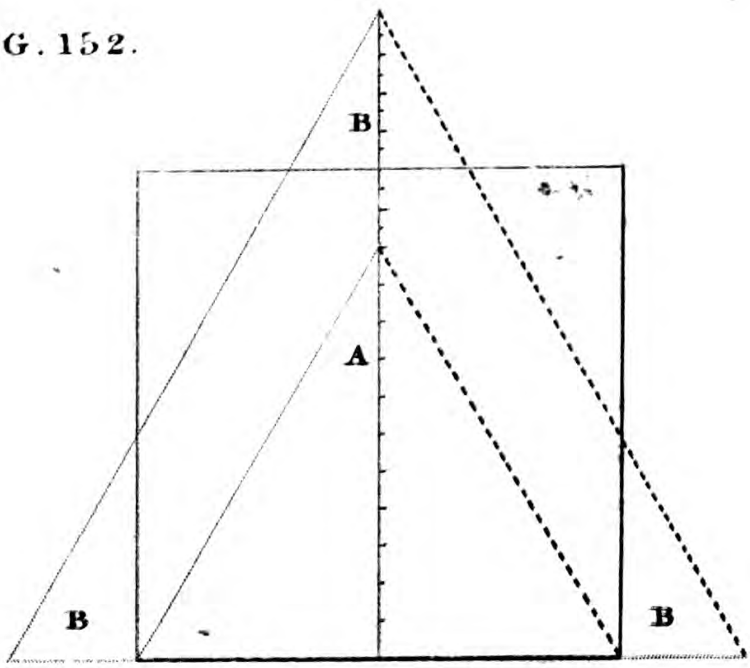


FIG. 152.



Invented and drawn by John Bennett.

London, John Bennett, 1838.

FIG. 153.

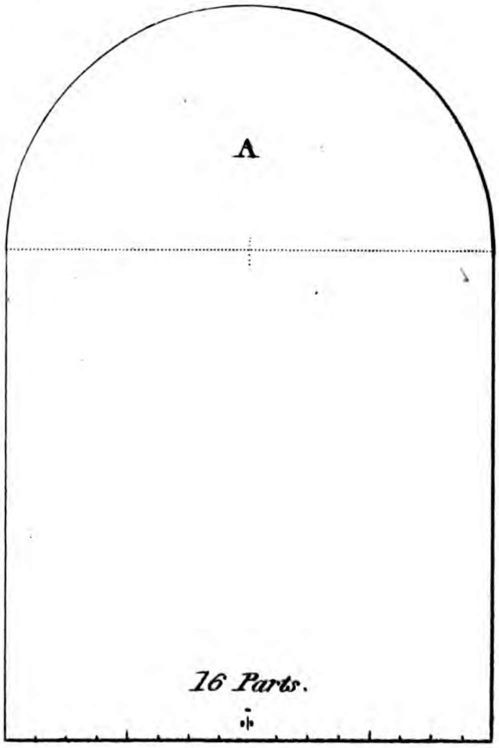
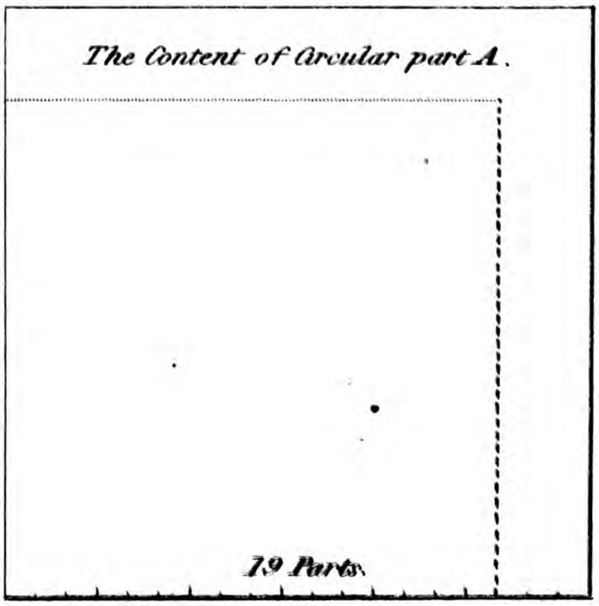


FIG. 154.



Invented and drawn by John Bennett.

London; John Bennett, 1838.

FIG. 155.

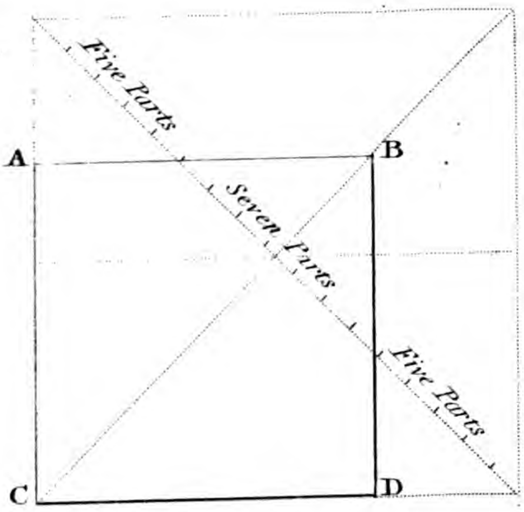


FIG. 156.

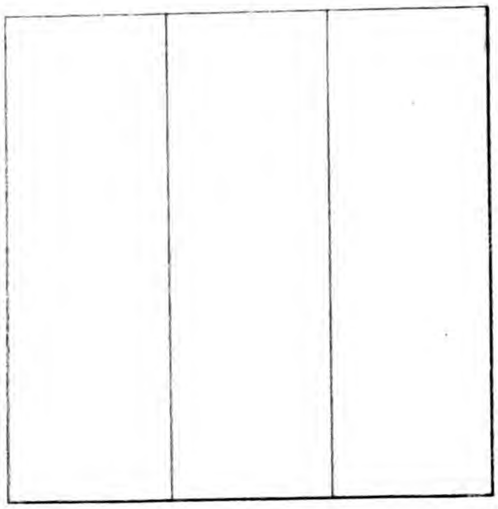
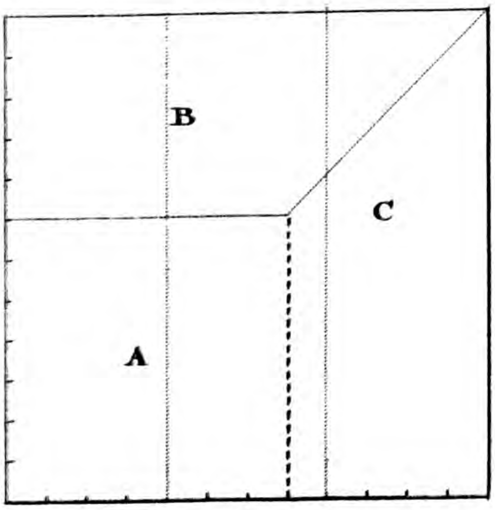


FIG. 157.



Invented and drawn by John Bennett.

London, John Bennett. 1838.

FIG. 158.

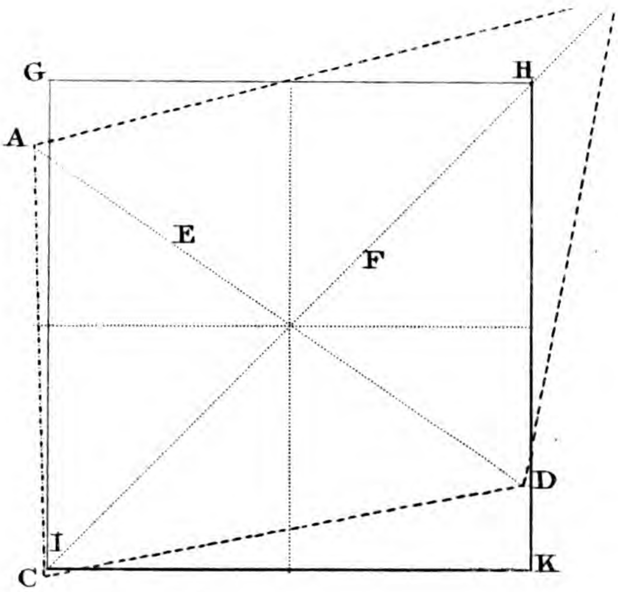


FIG. 159.

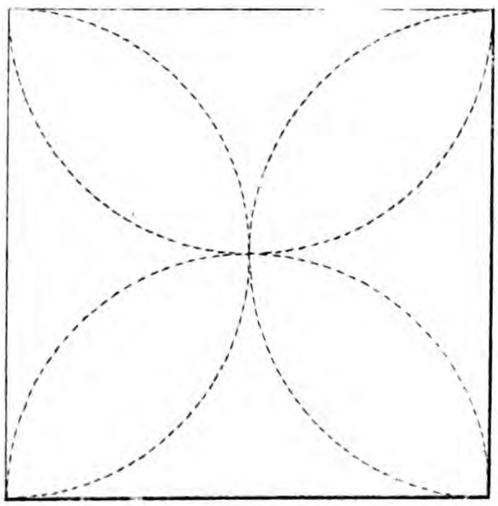
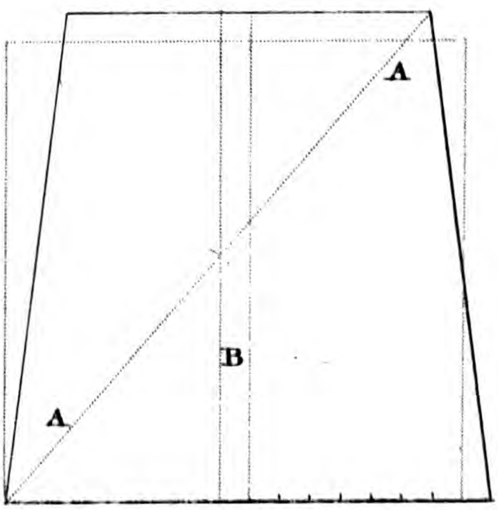


FIG. 160.



Invented and drawn by John Bennett.

London; John Bennett. 1838

FIG. 161.

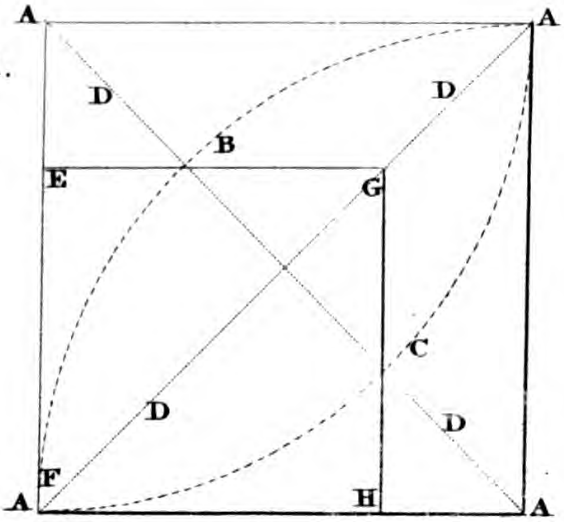


FIG. 162.

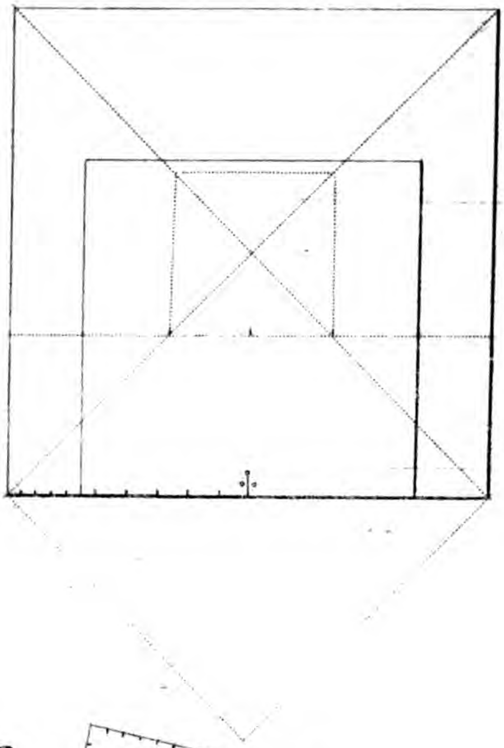
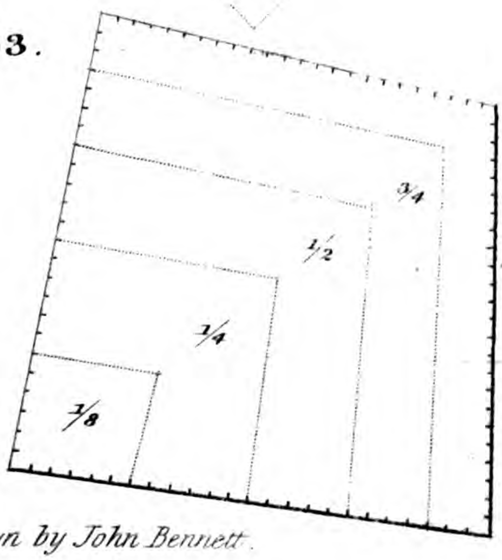


FIG. 163.



Invented and drawn by John Bennett.

London; John Bennett, 1838.

FIG. 164.

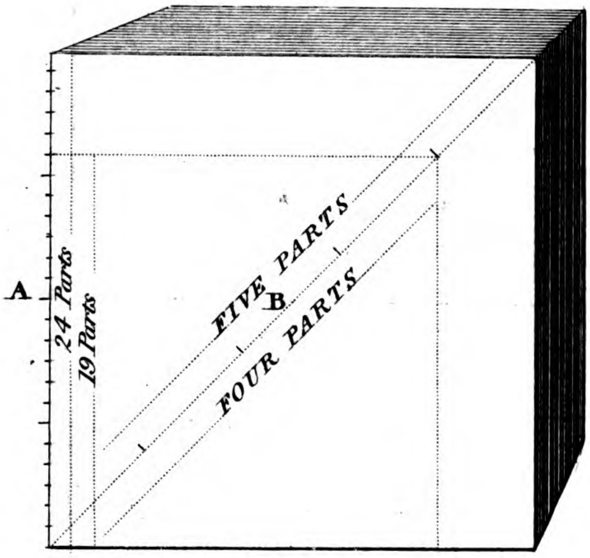
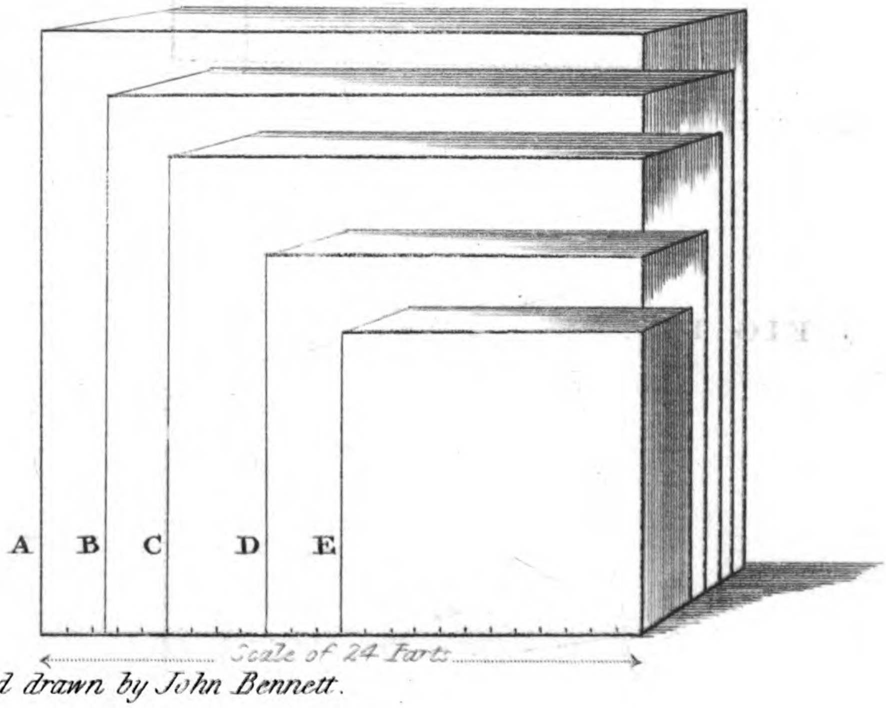


FIG. 165.



Invented and drawn by John Bennett.

London, John Bennett, 1838.

FIG. 166.

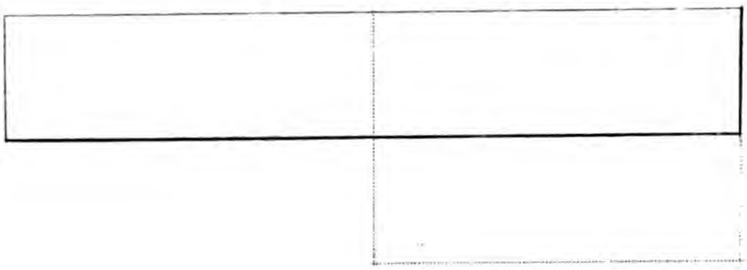


FIG. 167.

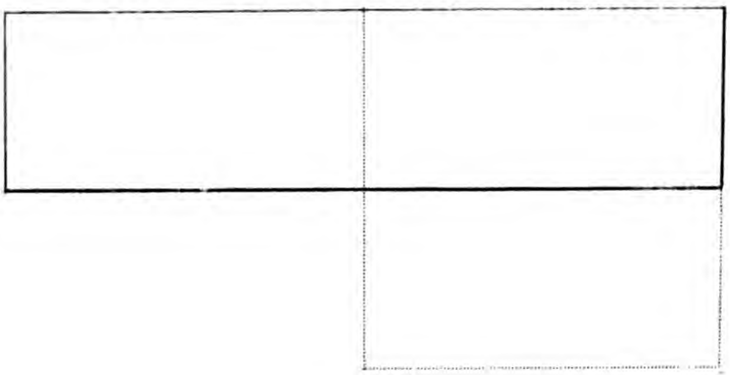
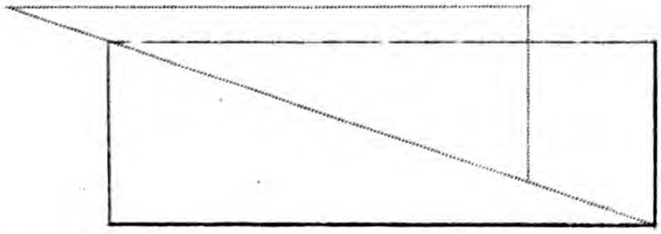


FIG. 168.



Invented and drawn by John Bennett.

London. John Bennett, 1838.

FIG. 169.

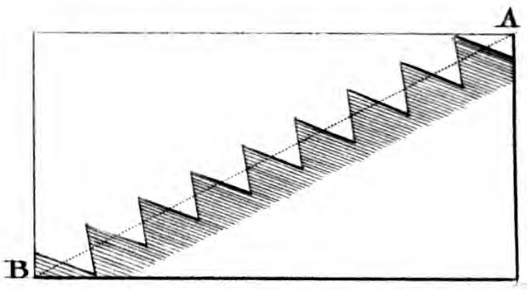


FIG. 170.

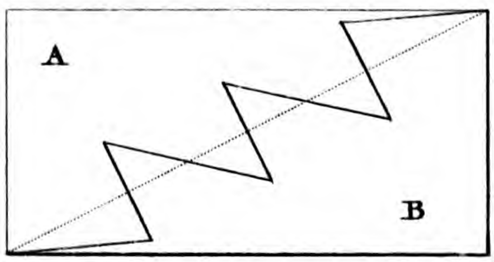
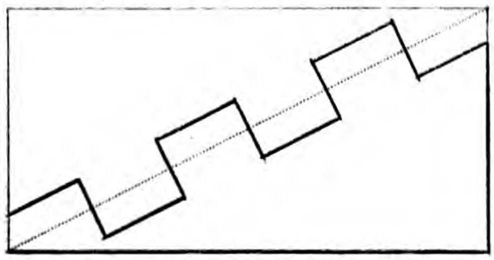


FIG. 171.



Invented and drawn by John Bennett.

London; John Bennett, 1838.

FIG. 172.

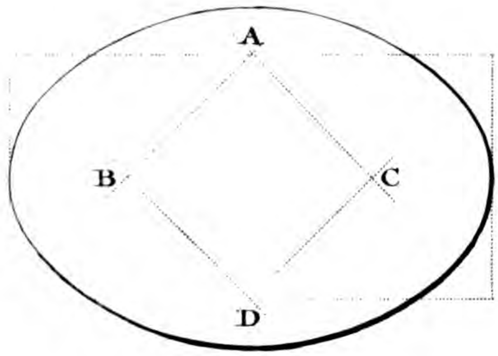


FIG. 173.

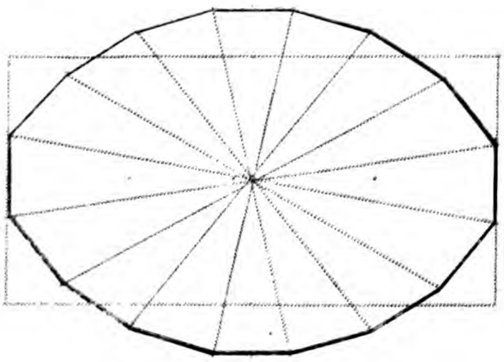
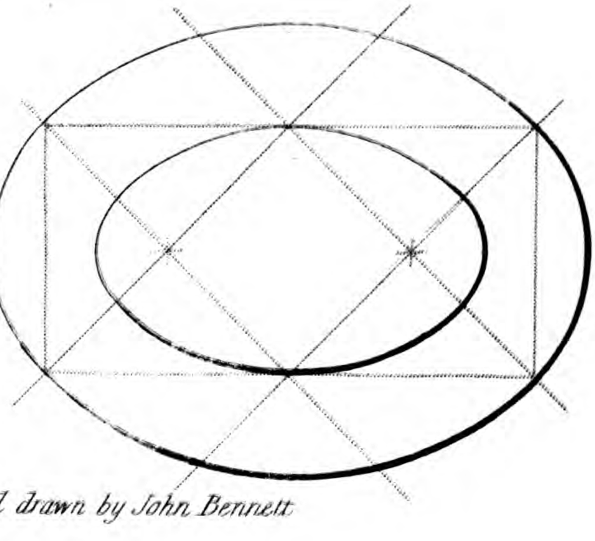


FIG. 174.



Invented and drawn by John Bennett

London, John Bennett, 1838.

FIG. 175.

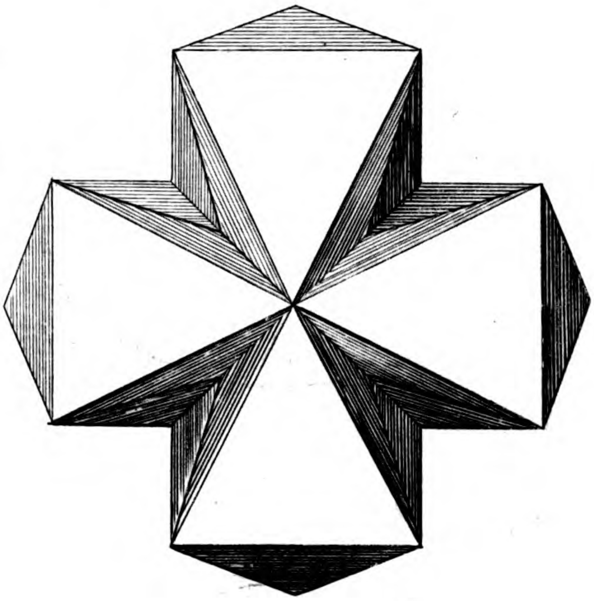
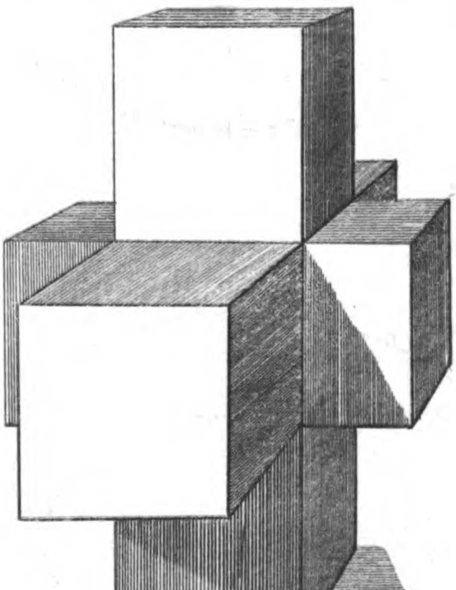


FIG. 176.



Invented and drawn by John Bennett.

London: John Bennett. 1838.

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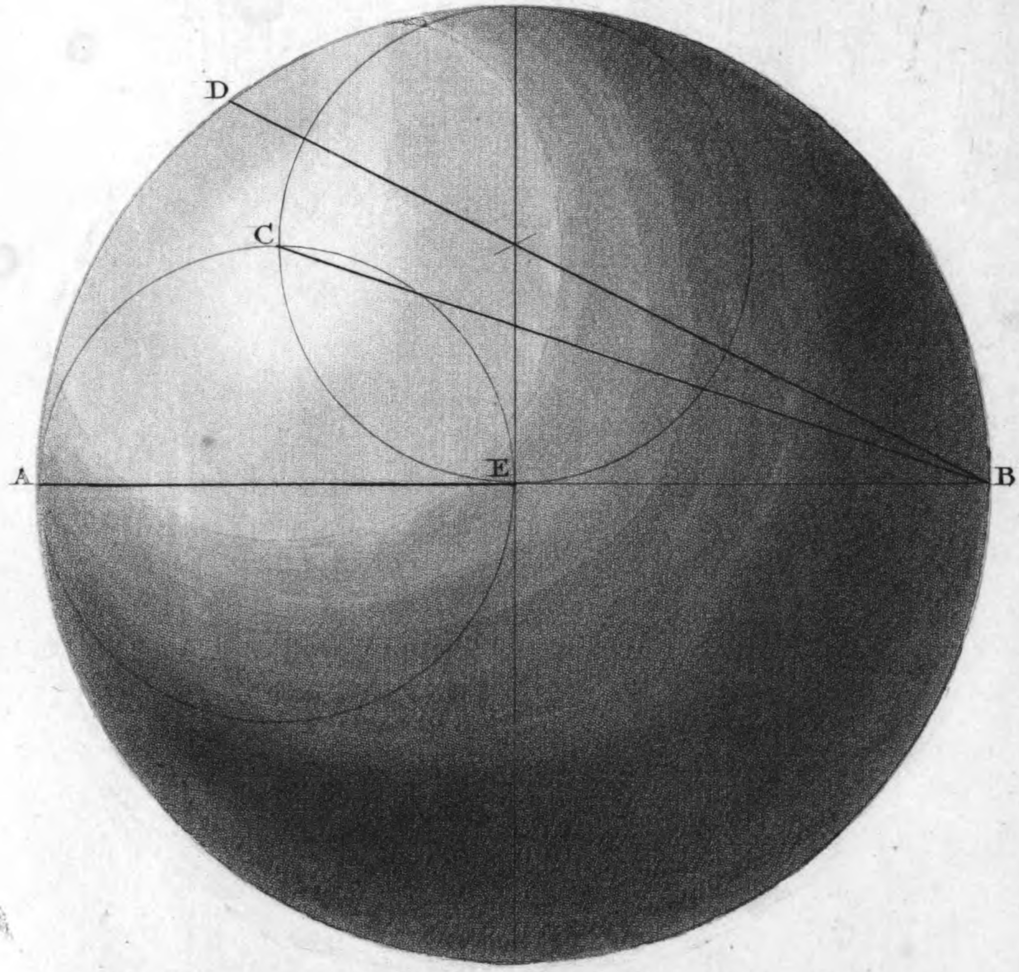

Invented and drawn by John Bennett.

London; John Bennett, 1839.