



Bodleian Libraries

UNIVERSITY OF OXFORD

This book is part of the collection held by the Bodleian Libraries and scanned by Google, Inc. for the Google Books Library Project.

For more information see:

<http://www.bodleian.ox.ac.uk/dbooks>

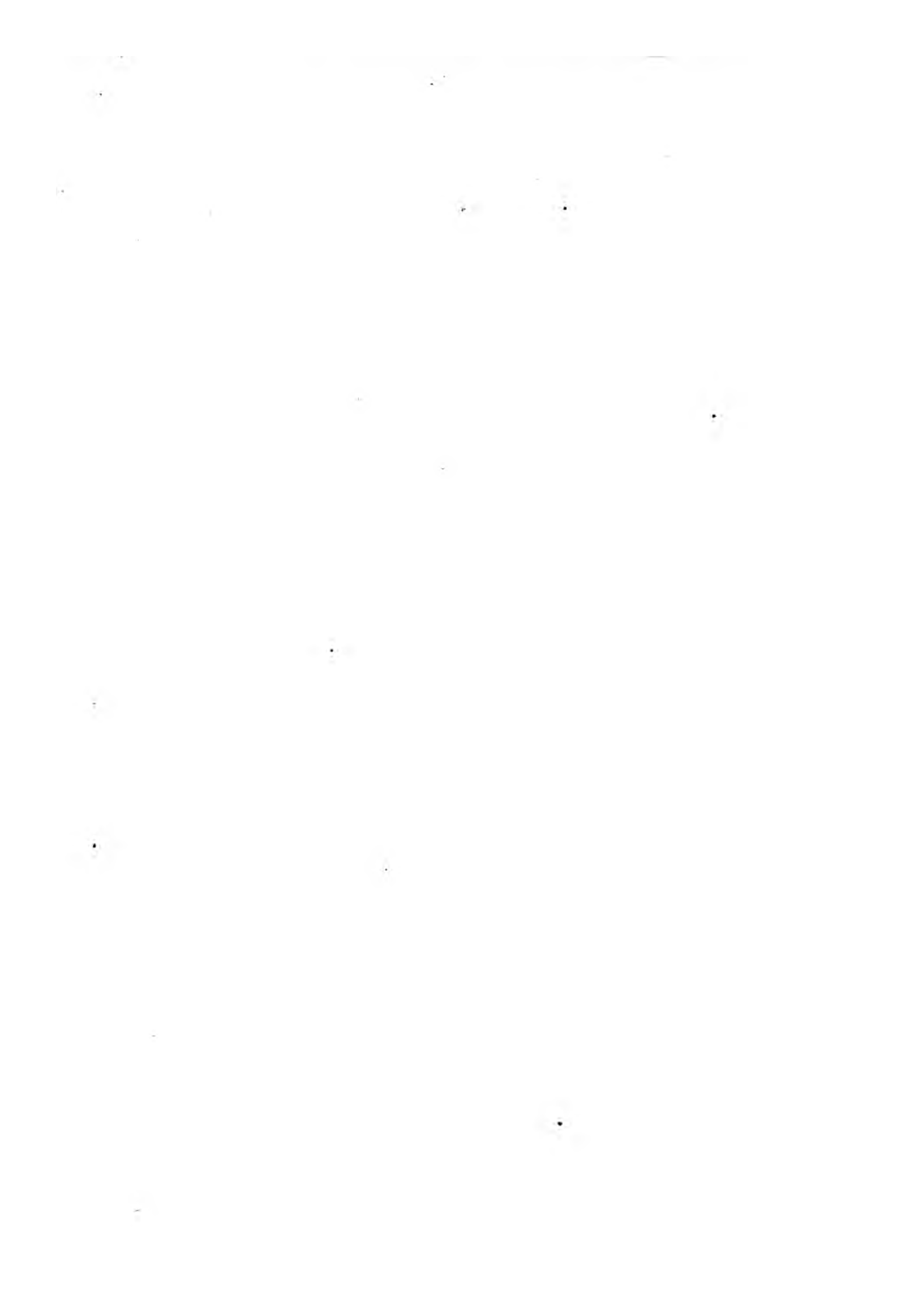


This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 2.0 UK: England & Wales (CC BY-NC-SA 2.0) licence.

TEMPLETON'S
WORK SHOP
COMPANION

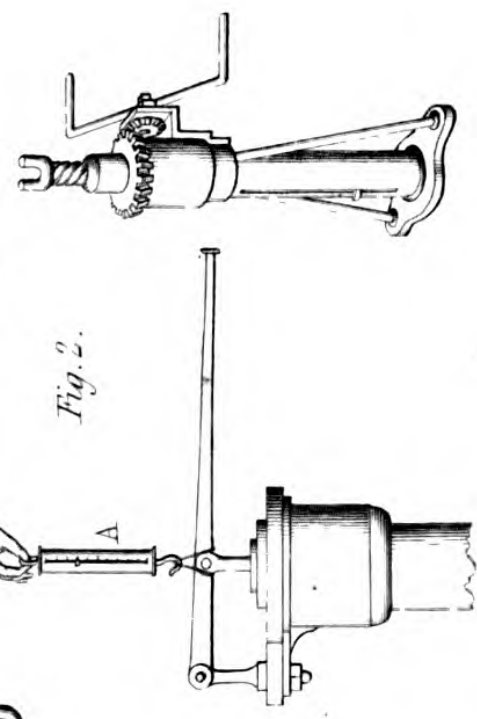
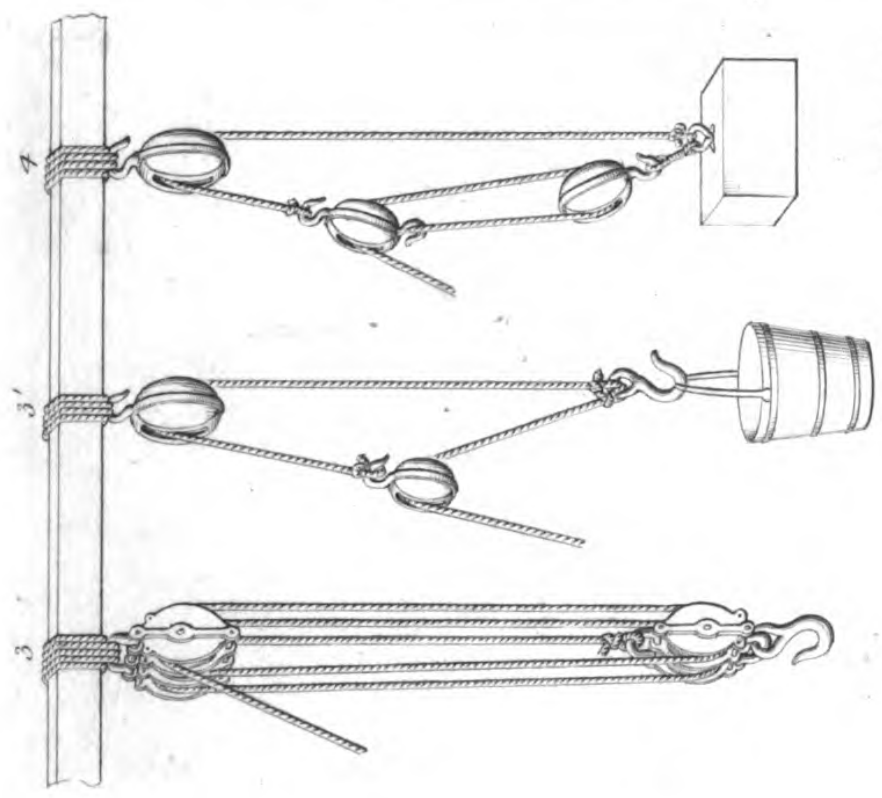
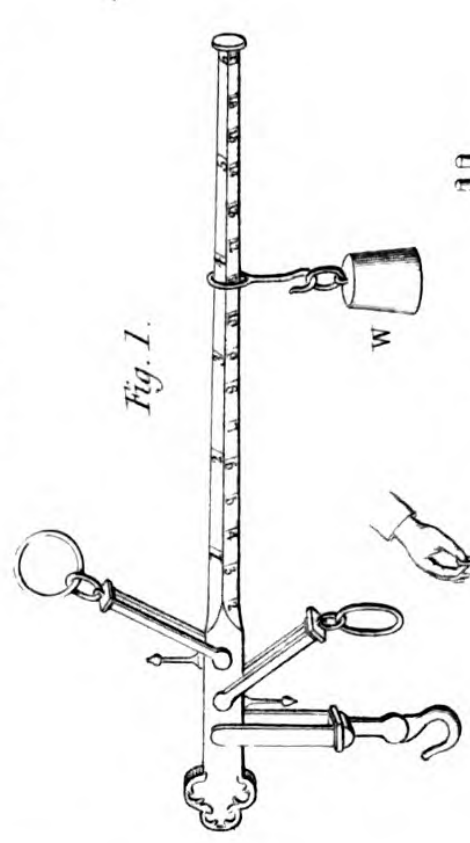
45. 1008.





APPLICATIONS OF MECHANIC POWERS.

Plate D.



J. W. Lowry & Co.

John Wade & Co. High Holborn 1845.

THE
OPERATIVE MECHANIC'S
WORKSHOP COMPANION,
AND
THE SCIENTIFIC GENTLEMAN'S
Practical Assistant:

COMPRISING
A GREAT VARIETY OF THE MOST USEFUL RULES IN
MECHANICAL SCIENCE,
DIVESTED OF MATHEMATICAL COMPLEXITY;
WITH NUMEROUS TABLES OF PRACTICAL DATA
AND CALCULATED RESULTS,
For Facilitating Mechanical and Commercial Transactions.

BY W. TEMPLETON,
AUTHOR OF THE ENGINEER'S POCKET-BOOK (PUBLISHED ANNUALLY),
&c., &c., &c.

London:
JOHN WEALE, HIGH HOLBORN.
1845.

PRINTED BY W. HUGHES,
KING'S HEAD COURT, GOUGH SQUARE.

TO THE PUBLIC.

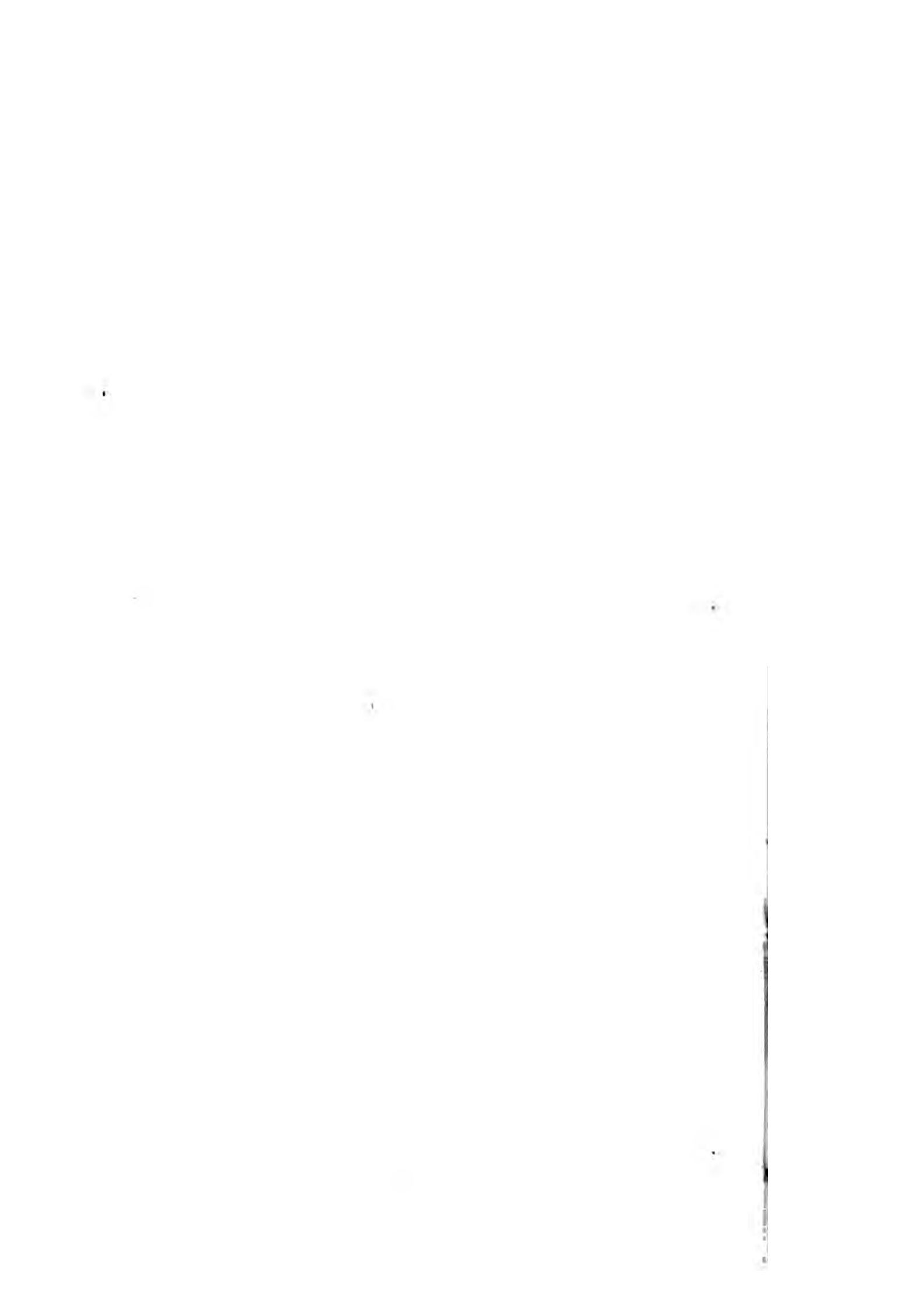
THE subjects of the following pages are chiefly compiled at the instigation of pressing solicitations by numerous individuals, no few of whom complain of the want of a convenient Text Book of Reference in which Mechanical and Commercial demands are judiciously combined. Others, with apparently equal reasons, for the sake of instruction, complain of the want of a compendium in which tuition and practical reference are discriminately arranged; and generally, that for purposes of estimation, no properly portable work is yet in existence: hence it is with a wish to diminish those wants in some degree that the present attempt has been made to supply the necessary information.

In regard to the selection, great care has been taken that the subjects be not only intimately interwoven, but also of such a nature as business transactions call most frequently into requisition. The elementary or educational portion will be found exceedingly plain, simple, and aptly adapted; also the practical rules much abbreviated through the application of decimal approximates, by which results are obtained sufficiently exact, and the means of calculation considerably facilitated.

That the production might be rendered still more worthy of public patronage, numerous diagrams, and several engravings, are annexed, which add much to its value in regard to practical utility; and it only remains for me to add a hope, that the Operative, the Man of Science, and an interested Public, may give due appreciation to the work, in proportion to its merits.

W. T.

London, 1st January, 1845.



CONTENTS.

	PAGE		PAGE
GEOMETRY.			
To erect a perpendicular on a right line	1	To make a triangle equal to a given quadrilateral	13
To erect a perpendicular at the end of a line	<i>ib.</i>	To form a square equal to a given circle	<i>ib.</i>
To bisect a given angle	2	To form an octagon from a given square	<i>ib.</i>
To describe a circle through three given points out of a right line	<i>ib.</i>	To form a square equal to two given squares	14
To find the centre of a given circle	3	To form a circle equal to two given circles	<i>ib.</i>
To find the length of any given arc of a circle	<i>ib.</i>	To draw a line equal to any portion of a circle's circumference	15
To draw a tangent to a circle	<i>ib.</i>	To draw a spiral with uniform spaces	<i>ib.</i>
To draw lines towards the centre of a circle, the centre being inaccessible,	<i>ib.</i>	To draw a volute for the Ionic column	16
To describe the segment of a circle	4	To draw a scroll for hand rails	<i>ib.</i>
To describe an ellipse or oval	5	To find the angles and lengths of materials for pyramidal frustums	17
To describe an elliptic arch	<i>ib.</i>	To describe the proper form of material by which to form a cone	18
To describe a parabola	6	Sector by which to obtain angles	<i>ib.</i>
To measure an intercepted line	7		
To obtain the distance of an inaccessible object	<i>ib.</i>	GEOMETRY APPLIED TO MECHANICS.	
To find the distance between two inaccessible objects	8	To delineate a vee-threaded screw	18
To design a beam of strongest section	<i>ib.</i>	To delineate a square-threaded screw	19
To find the proper position for the eccentric in a steam engine	9	To delineate a hollow spiral	20
To find the length of valve levers	<i>ib.</i>	To determine the proper forms for a pair of bevel wheels	<i>ib.</i>
To define the throw of an eccentric	10	Proportions for the construction of toothed wheels	21
To describe any regular polygon	<i>ib.</i>	To delineate wheels by orthographic projection	<i>ib.</i>
To construct a square upon a right line	<i>ib.</i>	Delineation of an undershot water-wheel	22
To form a square equal to a given triangle	11		
To form a square equal to a given rectangle	<i>ib.</i>	DECIMAL ARITHMETIC.	
To form a rectangle equal to a given square	<i>ib.</i>	Definitions	23
To bisect any given triangle	12	Reduction	<i>ib.</i>
To describe a circle in a given triangle	<i>ib.</i>	Applied examples	24
To form a rectangle in a given triangle	<i>ib.</i>	Definitions of Arithmetical Signs	27
To make a rectangle equal to a given triangle	13	British Standard Measures	28
		British Special Measures	29
		Decimal Approximations	30
		Decimal Equivalents	31

MENSURATION.	PAGE		PAGE
To measure the surface of a square, rectangle, rhomboid, &c.	32	Proportional breadths for six-sided nuts	59
Two sides of a triangle given, to find the third side	33	Table of the Weight of Sheet Iron, Copper, and Brass	60
Utility of triangles	34	Comparative weights of different bodies	61
To find the area of a triangle	<i>ib.</i>	Table of the Weights of Cast Iron Pipes	62
Table of Polygons	36	Weights of leaden pipes	<i>ib.</i>
Definitions of the circle	37	To find the weights of pipes of various metals	63
Rules in relation to the circle	38	Weight of a cubic inch of various metals	<i>ib.</i>
To find the diameter of a circle when any chord and versed sine are given	39	Table of the Weights of Cast Iron Balls	64
To find the length of any arc of a given circle	40	Table to facilitate the Measure of Timber	65
To find the area of the sector of a circle	<i>ib.</i>	Table of Cubic or Solid Measure	66
To find the area of a circular ring	41	To measure battens, deals, and planks	67
To find the area of an ellipse	<i>ib.</i>	Table of Scantling Timber	68
To find the area of a parabola	42		
To find the solidity or capacity of any cubical figure	<i>ib.</i>	INSTRUMENTAL ARITHMETIC.	
To find the convex surface and solidity of a cylinder	43	Explanation of the slide rule	69
To find the length of any cylindrical helix	44	Numeration	70
To find the convex surface and solidity of a cone	45	To multiply by the slide rule	<i>ib.</i>
To find the solidity or capacity of any frustum of a cone or pyramid	46	Proportion	71
To find the solid contents of a wedge	47	Rule of Three Inverse	<i>ib.</i>
To find the convex surface and solidity of a sphere or globe	48	Square and cube roots	72
To find the convex surface and solidity of the segment of a globe	49	Measure of squares, rectangles, &c.	73
To find the convex surface and solidity of a cylindrical ring	<i>ib.</i>	Measure of circles and polygons	<i>ib.</i>
To determine the proper length of iron for a ring of given diameter	50	Tables of Gauge-Points for the Slide Rule	74
To determine the length of angle iron to form a ring of given diameter	51	Mensuration of solidity and capacity	75
To measure the capacity of a locomotive tender tank	52	To compute the power of steam engines	76
Table of Specific Gravities and Properties of Metals	53	Of steam engine boilers	<i>ib.</i>
Table of Specific Gravities and Properties of Timber	54	COMMERCIAL TABLES.	
Table of Specific Gravities of Liquids, Gases, &c.	55	Tables by which to facilitate the Calculation of British Money	77
Weights of various measures of water	<i>ib.</i>	Table of Equivalent Prices	84
Table of the Weight of Square and Round Bar Iron	56		
Table of the Weight of Flat Bar Iron	57	STRENGTH OF MATERIALS.	
		Definitions	86
		Table of Tenacities, Resistance to Compression, &c. of Various Bodies	87
		Table of Comparative Strength of Ropes and Chains	88
		Table of Metallic Alloys	<i>ib.</i>
		Resistance of bodies to lateral pressure	89
		Table of Practical Data	<i>ib.</i>

CONTENTS.

vii

	PAGE		PAGE
To find the dimensions of a beam of timber to sustain a given weight	89	To determine the proper diameters of wheels to given peculiarities	114
To determine the absolute strength of a rectangular beam of timber	90	To find the proportional wheels for screw-cutting by a lathe	115
To determine the dimensions of a beam with a given degree of deflection	91	Table of Change Wheels for Screw Cutting	117
Cast iron beams of strongest section	92	Diameters of small wheels	118
Of wooden beams, trussed	93	Table of the Strength of Wheels of Cast Iron	<i>ib.</i>
Absolute strength of cast iron beams	<i>ib.</i>	Table of the Diameters of Wheels to contain a given number of Teeth	119
Table of Dimensions for Cast Iron Beams	94	FRICITION	123
To find the weight of a cast iron beam	95	PROPERTIES OF WATER AND AIR	125
Resistance to flexure by vertical pressure	<i>ib.</i>	<i>Effects produced by water in its natural state</i>	126
To determine the dimensions for a column of timber	96	The pressure of fluids	127
Table by which to determine the Dimensions of Cast Iron Columns	97	The hydraulic press	128
Resistance of bodies to twisting	98	The weights of bodies obtained by displacement of fluids	<i>ib.</i>
Relative strength of metals to resist torsion	<i>ib.</i>	The resistance of water to bodies passing through it	129
Table of Squares, Cubes, &c. of Numbers	99	Of water flowing through orifices	<i>ib.</i>
MECHANIC POWERS.		Discharging of water by rectangular apertures	130
Definitions, &c.	100	Flowing of water through pipes	132
Rules. First kind of lever	101	Table of the Diameters of Pipes for the Discharging of Water	133
— Second "	102	Laws of the gravity of water	<i>ib.</i>
— Third "	103	Rules relating to water-wheels	134
Lever on a safety valve, &c.	<i>ib.</i>	Turbines, their effects, &c.	135
Wheel and pinion, or crane	<i>ib.</i>	Rule to calculate the powers of turbines	136
Rules, &c.	104	Overshot water-wheel, notice of	137
The pulley, with applications	<i>ib.</i>	<i>Effects produced by steam</i>	<i>ib.</i>
Inclined plane	106	Table of the Elastic Force of Steam	138
Table of Inclinations and amount of Opposing Resistance	107	Of the latent heat in steam	140
Table of Inclined Planes	109	Steam as a motive power	141
The wedge	110	Temperature of steam	<i>ib.</i>
The screw	<i>ib.</i>	Expansive force of steam	142
The endless screw, or screw applied to a wheel	111	Table of Hyperbolic Logarithms	143
CONTINUOUS CIRCULAR MOTION.		Condensation of steam	<i>ib.</i>
Definitions, &c.	112	Boiling points of impure water	144
Proportional diameters of wheels to the number of revolutions	<i>ib.</i>	<i>Effects produced by air</i>	<i>ib.</i>
Of a train of wheels and pinions	113	Table of the Expansion of Air by Heat	145
Diameters or number of teeth in wheels in proportion to their velocities	<i>ib.</i>	Table relating to Pumps	146
		Oxygen of the atmosphere	147
		Resistance of the atmosphere	<i>ib.</i>
		Table of Atmospheric Force	148
		Effect of wind-mills	<i>ib.</i>

	PAGE		PAGE
STEAM ENGINE BOILERS	149	To determine the velocity for the	
To determine the amount of heating		piston of a steam engine . . .	155
surface in a boiler	<i>ib.</i>	Table of Approximate Velocities for	
Of waggon-shaped boilers	150	Pistons	156
Of cylindrical boilers	151	Table for Parallel Motions	157
Marine boilers	<i>ib.</i>		
Locomotive boilers	152	LOGARITHMS	158
Heating powers of combustibles	153	Table of Logarithms	162
Observations on the giving an order		Table of Circumferences and Areas	
for a steam engine	<i>ib.</i>	of Circles	163
Table of Dimensions for Steam En-		Table of Square and Cube Roots of	
gine Cylinders	154	Numbers	171
Unit of nominal power	<i>ib.</i>	Table of Per Centage and Discount	172
To estimate the power of an engine	155		

LIST OF PLATES.

A. Delineation of Screws	<i>to face page</i>	18
B. Illustration to the Drawing of Bevel Wheels		20
C. Orthographic Projection		22
D. Applications of Mechanic Powers		102
E. and F. Parallel Motions		156
G. Elevation and Section of an Overshot Water-Wheel, constructed by		
Messrs. Donkin & Co.		137
H. Details		<i>ib.</i>
K. Boilers of the 'Braganza' steam vessel, by Messrs. Bury, Curtis, and		
Kennedy		151
L. Locomotive Boiler		152

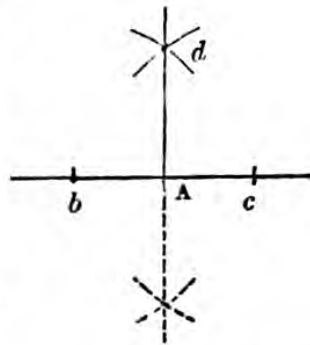
THE WORKSHOP COMPANION.

PRACTICAL GEOMETRY.

GEOMETRY is the science which investigates and demonstrates the properties of lines on surfaces and solids; hence, PRACTICAL GEOMETRY is the method of applying the rules of the science to practical purposes.

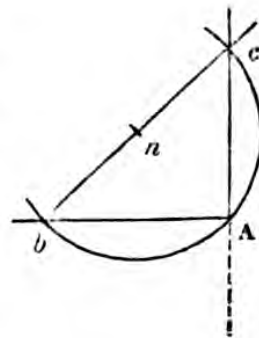
1. *From any given point, in a straight line, to erect a perpendicular; or, to make a line at right angles with a given line.*

On each side of the point A from which the line is to be made, take equal distances, as $A b$, $A c$; and from b and c as centres, with any distance greater than $b A$, or $c A$, describe arcs cutting each other at d ; then will the line $A d$ be the perpendicular required.



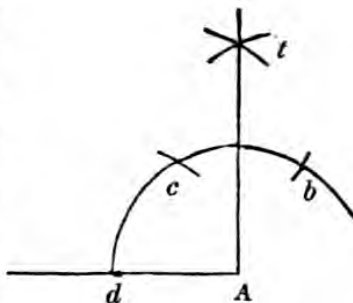
2. *When a perpendicular is to be made at or near the end of a given line.*

With any convenient radius, and with any distance from the given line $A b$, describe a portion of a circle, as $b A c$, cutting the given point in A; draw, through the centre of the circle n , the line $b n c$, and a line from the point A, cutting the intersections at c , is the perpendicular required.



3. *To do the same otherwise.*

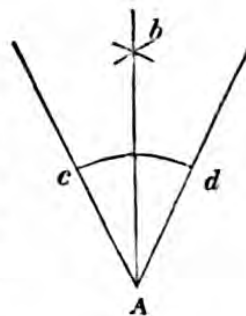
From the given point A , with any convenient radius, describe the arc $d c b$; from d , cut the arc in c , and from c , cut the arc in b ; also from c and b as centres, describe arcs cutting each other in t ; then will the line $A t$ be the perpendicular as required.



Note.—When the three sides of a triangle are in the proportion of 3, 4, and 5 equal parts respectively, two of the sides form a right angle; and observe that in each of these or the preceding problems, the perpendiculars may be continued below the given lines, if necessarily required.

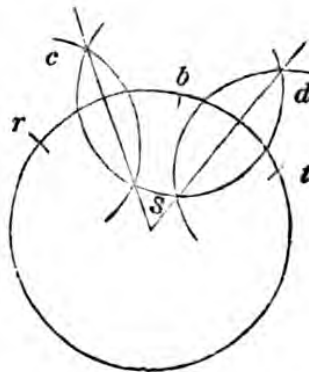
4. *To bisect any given angle.*

From the point A as a centre, with any radius less than the extent of the angle, describe an arc, as $c d$; and from c and d as centres, describe arcs cutting each other at b ; then will the line $A b$ bisect the angle as required.



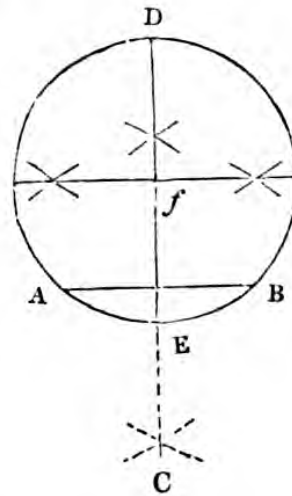
5. *To find the centre of a circle, or radius, that shall cut any three given points, not in a direct line.*

From the middle point b as a centre, with any radius, as $b c$, $b d$, describe a portion of a circle, as $c s d$; and from r and t as centres, with an equal radius, cut the portion of the circle in $c s$ and $d s$; draw lines through where the arcs cut each other, and the intersection of the lines at s is the centre of the circle as required.



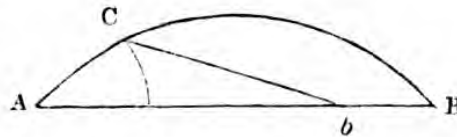
6. *To find the centre of a given circle.*

Bisect any chord in the circle, as $A B$, by a perpendicular $C D$; bisect also the diameter $E D$ in f , and the intersection of the lines at f is the centre of the circle required.



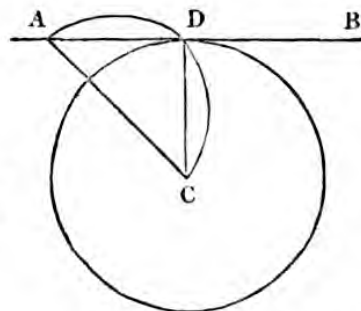
7. *To find the length of any given arc of a circle.*

With the radius $A C$, equal to $\frac{1}{4}$ th the length of the chord of the arc $A B$, and from A as a centre, cut the arc in c ; also from B as a centre, with equal radius, cut the chord in b ; draw the line $C b$, and twice the length of the line is the length of the arc nearly.



8. *Through any given point, to draw a tangent to a circle.*

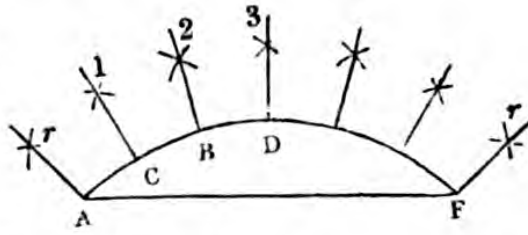
Let the given point be at A ; draw the line $A C$, on which describe the semicircle $A D C$; draw the line $A D B$, cutting the circumference in D , which is the tangent as required.



9. *To draw from, or to the circumference of a circle, lines tending towards the centre, when the centre is inaccessible.*

Divide the whole or any given portion of the circumference into the desired number of equal parts;

then with any radius less than the distance of two divisions, describe arcs cutting each other, as A 1, B 1, C 2, D 2, &c.; draw the lines C 1, B 2, D 3, &c., which lead to the centre as required.

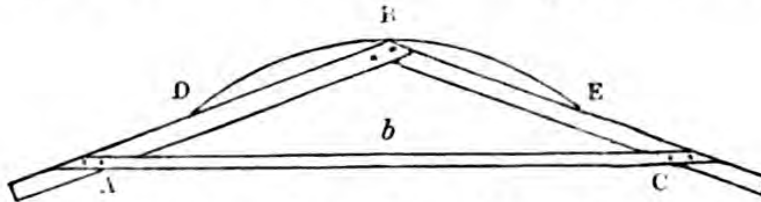


To draw the end lines,

As A r , F r , from C describe the arc r , and with the radius C 1, from A or F as centres, cut the former arcs at r , or r , and the lines A r , F r , will tend to the centre as required.

10. *To describe an arc, or segment of a circle, of large radii.*

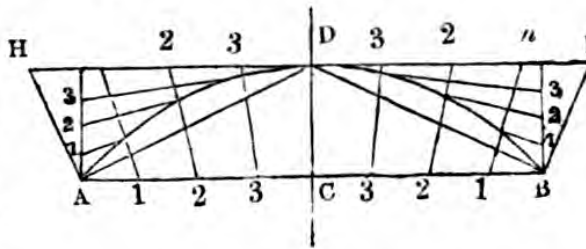
Of any suitable material, construct a triangle, as A B C; make A B, B C, each equal in length to the chord of the arc D E, and height, twice that of



the arc B b . At each end of the chord D E fix a pin, and at B, in the triangle, fix a tracer (as a pencil), move the triangle along the pins as guides, and the tracer will describe the arc required.

11. *Or otherwise,*

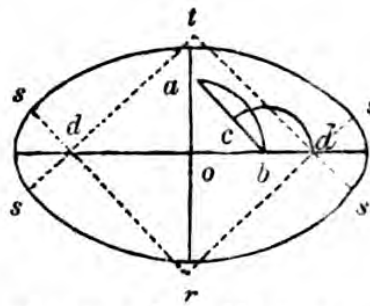
Draw the chord A C B, also draw the line H D I, parallel with the chord and equal to the height of the segment; bisect the chord in C, and erect the perpendi-



cular CD ; join AD , DB ; draw AH perpendicular to AD , and BI perpendicular to BD ; erect also the perpendiculars An , Bn , divide AB and HI into any number of equal parts; draw the lines 11 , 22 , 33 , &c., likewise divide the lines An , Bn , each into half the number of equal parts; draw lines to D from each division in the lines An , Bn , and through where they intersect the former lines, describe a curve, which will be the arc or segment required.

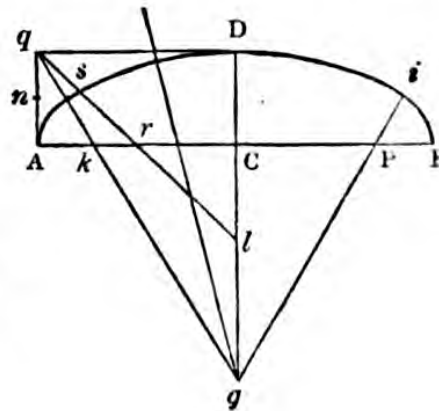
12. *To describe an ellipse, having the two diameters given.*

On the intersection of the two diameters as a centre, with a radius equal to the difference of the semi-diameters, describe the arc ab , and from b as a centre, with half the chord bca , describe the arc cd ; from o , as a centre, with the distance od , cut the diameters in dr , dt ; draw the lines r, s, s and t, s, s ; then from r and t describe the arcs s, s, s, s ; also from d and d , describe the smaller arcs s, s, s, s , which will complete the ellipse as required.



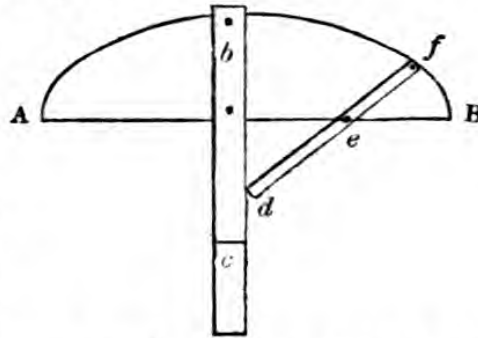
13. *To describe an elliptic arch, the width and rise of span being given.*

Bisect with a line at right angles the chord or span AB , erect the perpendicular Aq , and draw the line qD equal and parallel to AC ; bisect AC and Aq in r and n , make Cl equal to CD , and draw the line lrq ;



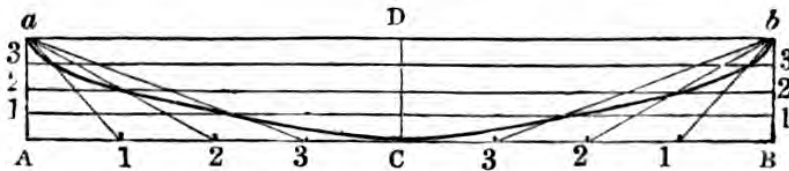
draw also the line nsD ; bisect sD with a line at right angles, and meeting the line CD in g ; draw the line gq , make CP equal to Ck , and draw the line gPi ; then from g as a centre with the radius gD , describe the arc sDi , and from k and P as centres, with the radius Ak , describe the arcs As and Bi , which completes the arch as required. *Or,*

14. Bisect the chord AB , and fix at right angles any straight guide, as bc ; prepare of any suitable material a rod or staff, equal to half the chord's length, as def ; from the end of the staff, equal to the height of the arch, fix a pin e , and at the extremity a tracer f ; move the staff, keeping its end to the guide and the fixed pin to the chord, and the tracer will describe one-half the arc required.



15. *To describe a parabola, the dimensions being given.*

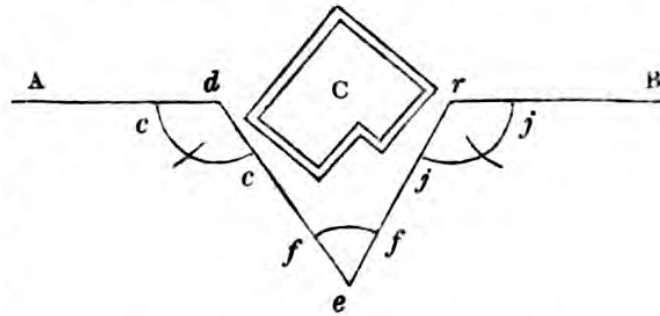
Let AB equal the length, and CD the breadth of the required parabola; divide CA , CB into any



number of equal parts, also divide the perpendiculars Aa and Bb into the same number of equal parts; then from a and b draw lines meeting each division on the line ACB , and a curve line drawn through each intersection will form the parabola required.

16. *To obtain by measurement the length of any direct line, though intercepted by some material object.*

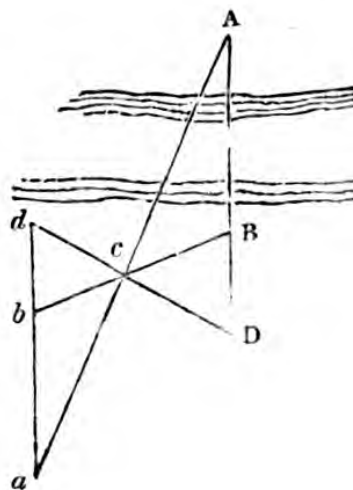
Suppose the distance between A and B is required, but the right line is intercepted by the object C. On the point *d*,



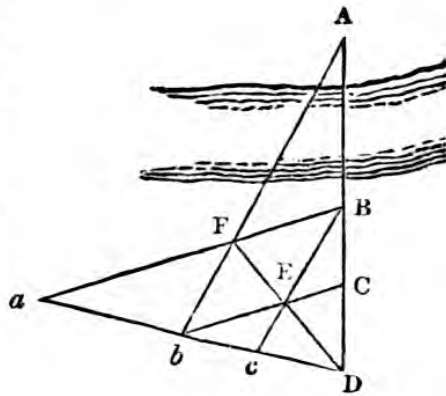
On the point *d*, with any convenient radius, describe the arc *cc*, make the arc twice the radius in length, through which draw the line *dce*, and on *e* describe another arc equal in length to once the radius, as *eff*; draw the line *efr* equal to *efd*; on *r* describe the arc *jjj*, in length twice the radius; continue the line through *rj*, which will be a right line, and *de*, or *er*, equal the distance between *dr*, by which the distance between A and B is obtained as required.

17. *To ascertain the distance geometrically, of any inaccessible object on an equal plane.*

Let it be required to find the distance between A and B, being inaccessible; produce the line in the direction of A B to any point, as D; draw the line D *d* at any angle to the line A B; bisect the line D *d*, through which draw the line B *b*, making *cb* equal to B *c*; draw the line *d b a*; also through *c*, in the direction *c A*, draw the line *a c A*, intersecting the line *d b a*; then *b a* equal B A, the distance required.

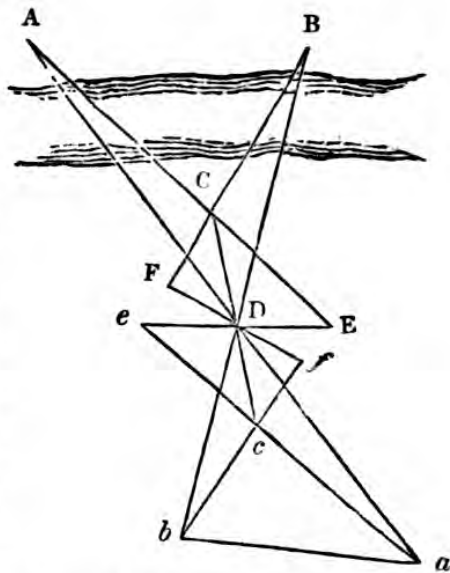


18. *Otherwise.* Prolong AB to any point D , making BC equal to CD ; draw the line Da at any angle with DA , and the line Cb similar to Bc ; draw also the line DEF , which intersects the line Da , then ab equal BA , or the distance required.



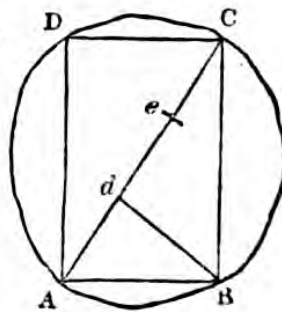
19. *To measure the distance between two objects, both being inaccessible.*

From any point C draw any line Cc , and bisect it in D ; take any point E in the prolongation of AC , and draw the line Ee , making De equal to DE ; in like manner take any point F in the prolongation of BC , and make Df equal to FD . Produce AD and ec till they meet in a , and also BD and fc till they meet in b ; then ab equal AB , or the distance between the objects as required.



20. *A round piece of timber being given, out of which to cut a beam of strongest section.*

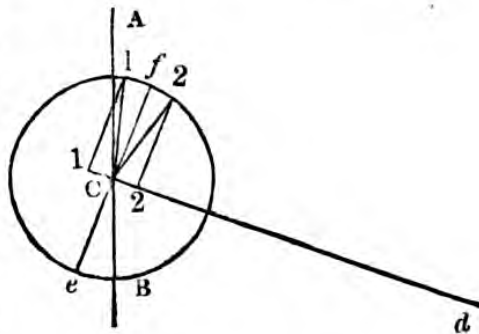
Divide into three equal parts any diameter in the circle, as Ad , eC ; from d or e , erect a perpendicular meeting the circumference of the circle, as dB ; draw AB and BC ,



also AD equal to BC , and DC equal to AB , and the rectangle will be a section of the beam as required.

21. *To find the proper position for an eccentric, in relation to the crank in a steam engine, the angle of eccentric rod, and travel of the valve, being given.*

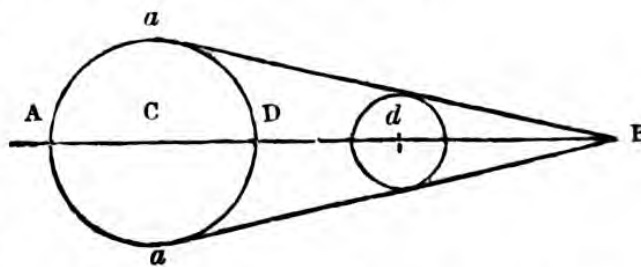
Draw the right line AB , as the situation of the crank at commencement of the stroke; draw also the line Cd , as the proper given angle of eccentric rod with the crank; then from C as centre, describe a circle



equal to the travel of the valve; draw the line ef at right angles to the line Cd , draw also the lines 11 , and 22 , parallel to the line ef ; and at a distance from ef on each side, equal to the lap and lead of the valve, draw the angular lines $C1$, $C2$, which are the angles of eccentric with the crank, for forward or backward motion, as may be required.

22. *The throw of an eccentric, and the travel of the valve in a steam engine, also the length of one lever for communicating motion to the valve, being given, to determine the proper length for the other.*

On any right line, as AB , describe a circle AD , equal to the throw of eccentric & travel of valve;

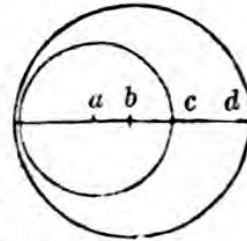


then from C as a centre, with a radius equal to the length of lever given,

cut the line AB , as at d , on which describe a circle, equal to the throw of eccentric or travel of valve, as may be required; draw the tangents Ba , Ba , cutting each other in the line AB , and dB is the length of the lever as required.

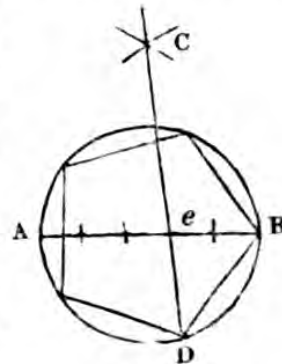
Note.—The throw of an eccentric is equal to the sum of twice the distance between the centres of formation and revolution, as ab , or to the degree of eccentricity it is made to describe, as cd . And

The travel of a valve is equal the sum of the widths of the two steam openings, and the valve's excess of length more than just sufficient to cover the openings.



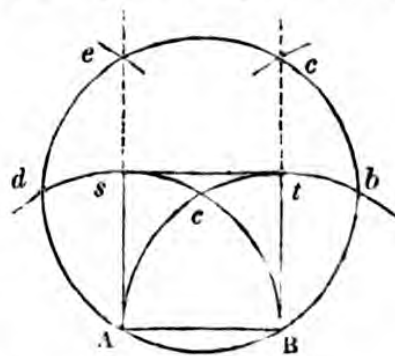
23. *To inscribe any regular polygon in a given circle.*

Divide any diameter, as AB , into so many equal parts as the polygon is required to have sides; from A and B as centres, with a radius equal to the diameter, describe arcs cutting each other in C , draw the line CD through the second point of division on the diameter e , and the line DB is one side of the polygon required.



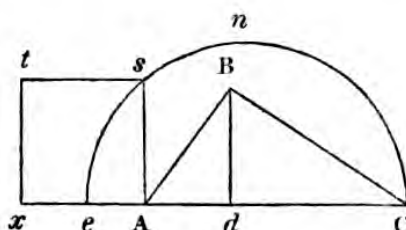
24. *To construct a square upon a given right line.*

From A and B as centres, with the radius AB , describe the arcs $Ac b$, $Bc d$, and from c with an equal radius describe the circle or portion of a circle ed , AB , bc ; from bd cut the circle at e and c ; draw the lines Ae , Bc , also the line st , which completes the square as required.



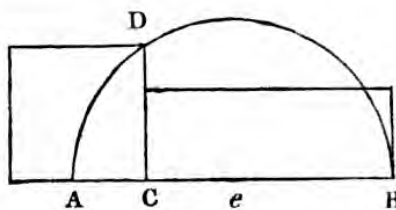
25. *To form a square equal in area to a given triangle.*

Let $A B C$ be the given triangle; let fall the perpendicular $B d$, and make $A e$ half the height $d B$; bisect $e C$, and describe the semicircle $e n C$; erect the perpendicular $A s$, or side of the square, then $A s t x$ is the square of equal area as required.



26. *To form a square equal in area to a given rectangle.*

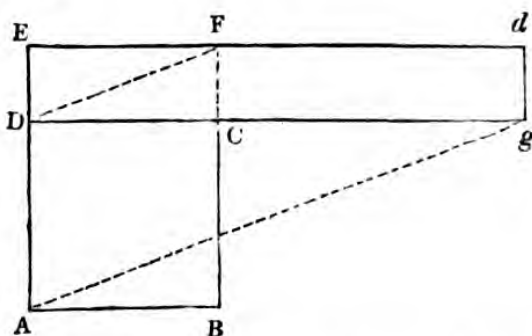
Let the line $A B$ equal the length and breadth of the given rectangle; bisect the line in e , and describe the semicircle $A D B$; then from



A with the breadth, or from B with the length of the rectangle, cut the line $A B$ at C , and erect the perpendicular $C D$, meeting the curve at D , and $C D$ equal a side of the square required.

27. *To find the length for a rectangle whose area shall be equal to that of a given square, the breadth of the rectangle being also given.*

Let $A B C D$ be the given square, and $D E$ the given breadth of rectangle; continue the line $B C$ to F , and draw the line $D F$; also, continue the line $D C$ to g , and draw

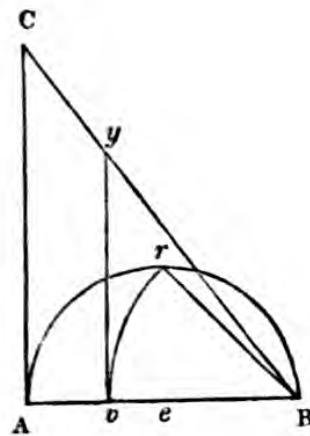


the line $A g$ parallel to $D F$; from the intersection of

the lines at g , draw the line gd parallel to DE , and Ed parallel to Dg ; then $EDdg$ is the rectangle as required.

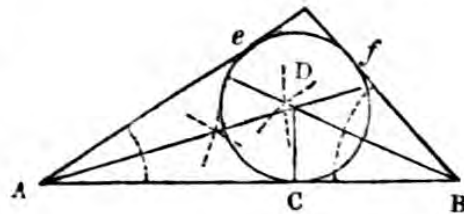
28. *To bisect any given triangle.*

Suppose ABC the given triangle; bisect one of its sides, as AB in e , from which describe the semicircle $A r B$; bisect the same in r , and from B , with the distance Br , cut the diameter AB in v ; draw the line vy parallel to AC , which will bisect the triangle as required.



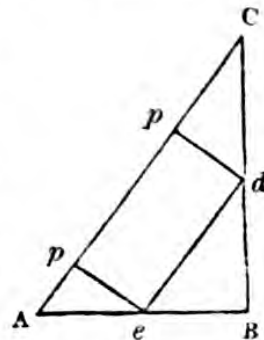
29. *To describe a circle of greatest diameter in a given triangle.*

Bisect the angles A and B , and draw the intersecting lines AD , BD , cutting each other in D ; then from D as centre, with the distance or radii DC , describe the circle Cef , as required.



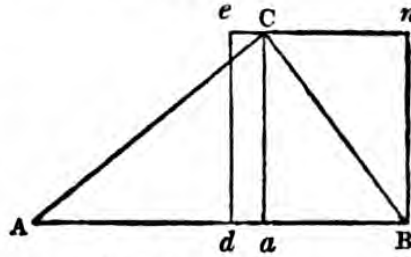
30. *To form a rectangle of greatest surface, in a given triangle.*

Let ABC be the given triangle; bisect any two of its sides, as AB , BC , in e and d ; draw the line ed ; also at right angles with the line ed , draw the lines ep , dp , and $eppd$ is the rectangle required.



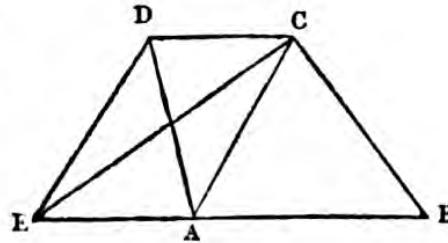
31. *To make a rectangle equal to a given triangle.*

Suppose $A B C$ the given triangle; bisect $A B$ in d , and erect the perpendicular $d e$; erect also the perpendicular $B n$, draw the line $e n$ parallel to $A B$, and equal to $a C$; then $d e B n$ is the rectangle as required.



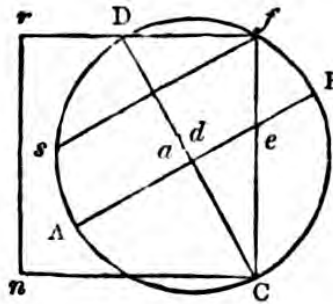
32. *To make a triangle equal to a given quadrilateral, as $A B C D$.*

Prolong the line $B A$, and draw the line $A C$; draw also the line $D E$ parallel to $A C$, and cutting the line $B A$ in E ; then draw the line $E C$, and $C E B$ is the triangle required.



33. *To form a square nearly equal in area to a given circle.*

Let $A C B D$ be the given circle; draw the diameters $A B$ and $C D$, bisect the radius $d B$ in e , and draw the line $C e f$; draw also at right angles the lines $C n$ and $f r$, making each equal to $C f$; join $n r$, and $n C f r$ is the square as required.

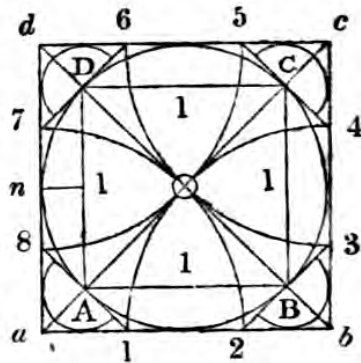


Note.—The line $s f$ is equal to one-fourth the circumference of the circle.

34. *To inscribe or describe a square within or without a given circle; also to form an octagon from a given square.*

Describe a circle, as $A B C D$, in diameter equal to

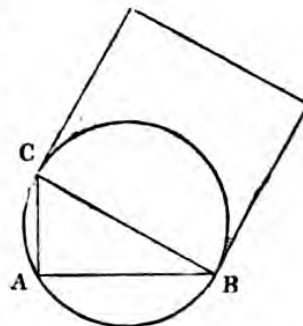
the extremity of the angles, or breadth of square required; draw at right angles the diameters $A C B D$, and within the circle draw the lines $A 1 B$, $B 1 C$, $C 1 D$, and $D 1 A$, meeting the diameters $A B C$ and D , which completes the inscribed square;—again, with the distance $1 n$ from $A B C$ and D , as centres, describe small arcs; draw lines from each as tangents, meeting the diameters at $A B C$ and D ; which will complete the square, equal in breadth to the circle's diameter.



Then from the extremity of the angles $a b c$ and d , as centres with the distance $a o$, $b o$, &c., cut the sides of the square in 1, 2, 3, &c.; draw the lines 23, 45, 67, 81; which completes the octagon as required.

35. *To form a square equal to two given squares, or, a circle equal to two given circles.*

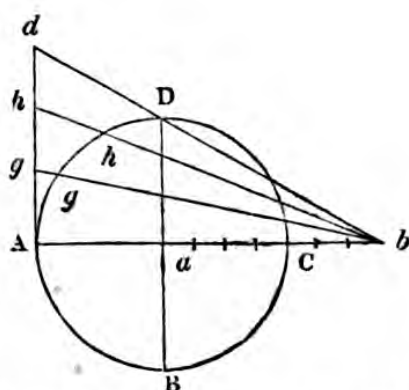
Let $A B$, $A C$, equal the sides of the given squares, or diameters of the given circles; make the angle at A a right angle, and draw the line $C B$, which is the side of a square equal to both the given squares; or bisect the line $C B$ as a diameter, on which describe the circle $C B$, and it is equal to the two given circles as required.



36. *To draw a right line equal to any given portion of the circumference of a circle.*

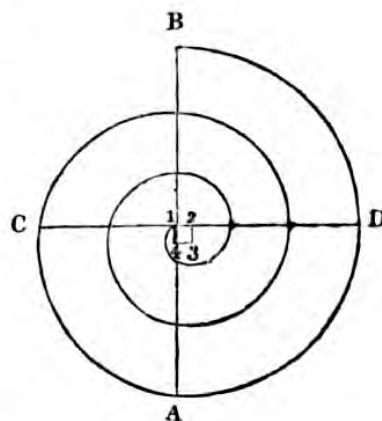
Let $A B C D$ be a given circle, the whole circum-

ference of which is required; draw at right angles the diameters $A C$, $B D$, divide the radius $a C$ into four equal parts, and make $C b$ equal to three of them; draw the tangent $A d$ parallel to $B D$, draw the line $b D d$, then will $A d$ equal one-fourth of the whole circumference; and if lines be drawn from b , through points in the circumference, meeting the line $A d$, as $g g$, $h h$, &c., the corresponding parts will be equal to each other.



37. *To draw a spiral with spaces of uniform distance.*

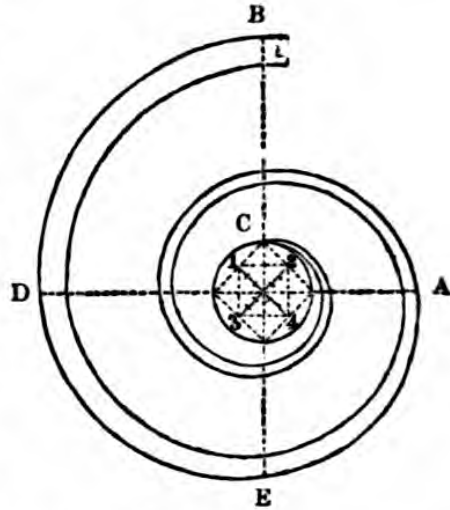
Bisect the height of the spiral, as $A B$, and divide either half into the number of spaces or revolutions required; then again subdivide any one space into four equal parts, one of which add to half the height of the spiral, and through which draw the line $C D$ at right angles to $A B$, thus forming the centre of the spiral, around which and equal



to one of the subdivisions form a square, its sides being parallel with the lines $A B$ and $C D$, the angles of which are the centres from whence to describe the various curves; as from 1, with the distance 1 B , describe the curve $B D$; from 2, with the distance 2 D , describe $D A$; from 3, with the distance 3 A , describe $A C$, &c., &c., and from the same centres the spiral may be continued to any extent required.

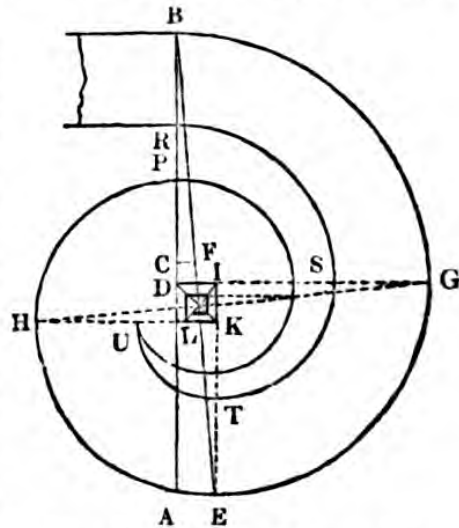
38. *To draw a volute for the capital of the Ionic column.*

Draw at right angles CA , CB , and on the centre C describe a circle equal to one-fourth of the height CB ; form a square in the circle, the diagonals of which correspond with or cut the diameters AD , EB ; bisect the square with lines crossing each other in the centre C , and parallel with the sides of the square; divide each into six equal parts, which are the centres from which the volute is to be described; thus, from 1, with the distance $1B$, describe the curve BD ; from 2, with $2D$, describe DE ; from 3, with $3E$, describe EA , &c., approaching the centre by degrees until the volute is completed as required.



39. *To draw a scroll for the termination of a hand rail, &c.*

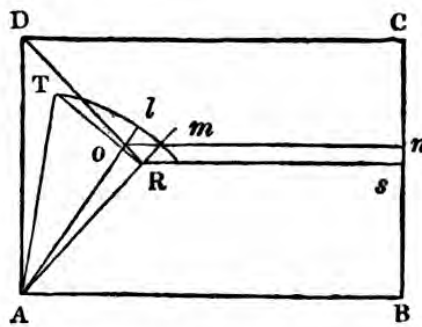
Let AB equal the given breadth, draw AE perpendicular to AB , which divide into eleven equal parts, and AE equal to one of them; join BE , bisect AB in C and BE in F , make CD equal to CF , and draw DG perpendicular to AB ; from F , with the radius FE , or FB , describe an arc cutting DG at G ; draw GH perpendicular to BE , cutting BE at O (or centre of the scroll); draw the diagonals



DOK, IOL, perpendicular to DOK; draw IK parallel to BA, KL parallel to ID, &c., meeting the diagonals; from D as a centre, with the distance DB, describe the arc BG; from I as a centre, with the distance IG, describe GE; from K, with the distance KE, describe KH, &c., proceeding in the same manner until the outside of the scroll is completed; make BR equal to the breadth of the rail; then from D, with the distance DR, describe the arc RS; from I, and distance IS, describe ST; and from T, with KT, describe TU, which completes the scroll as required.

40. *To find the various angles and proper dimensions of materials whereby to construct any figure whose form is the frustum of a proper or inverted pyramid, as hipped roofs, mill hoppers, &c., &c.*

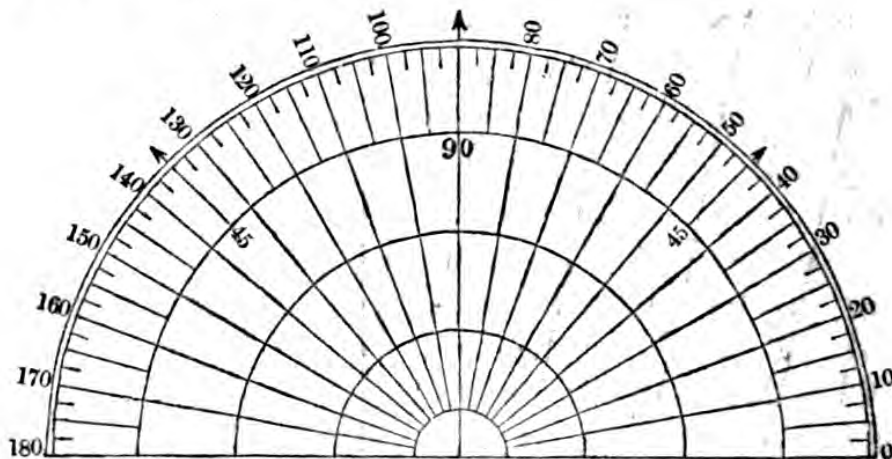
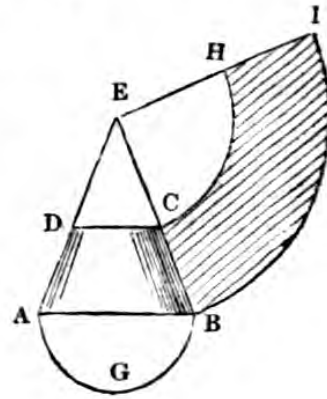
Let ABCD be the given dimensions of plan for a roof, the height RT also being given; draw the diagonal AR, meeting the top or ridge Rs on plan; from R, at right angles with AR and equal to the required



height, draw the line RT, then TA, equal the length of the struts or corners of the roof; from A, with the distance AT, describe an arc Tl, continue the diagonal AR until it cuts the arc Tl, through which, and parallel with the ridge Rs, draw the line mn, which determines the required breadth for each side of the roof; from A, meeting the line mn, draw the line Ao, or proper angle for the end of each board by which the roof might require to be covered; and the angle at T is what the boards require to be made in the direction of their thickness, when the corners or angles require to be mitred.

41. *To describe the proper form of flat plate, by which to construct any given frustum of a cone.*

Let $A B C D$ represent the required frustum, continue the lines $A D$ and $B C$ until they meet at E ; then from E as centre, with the radius $E C$, describe the arc $C H$; also from E with the radius $E B$, describe the arc $B I$; make $B I$ equal in length to twice $A G B$, draw the line $E I$, and $B C I H$ is the form of plate as required.



Sector from which angles may be obtained.

GEOMETRY APPLIED TO MECHANICS.

42. *To delineate a vee-threaded screw, the pitch and diameter of the screw being given. (See Plate A, fig. 1.)*

Upon the end of the line, or vertical centre of the screw $A B$, describe the semicircles $C D$, $c d$, the one being equal to the greatest diameter of the screw, and the other the lesser diameter, or diameter at the

SCREWS DELINEATED,

Plate A.

Fig. 2.

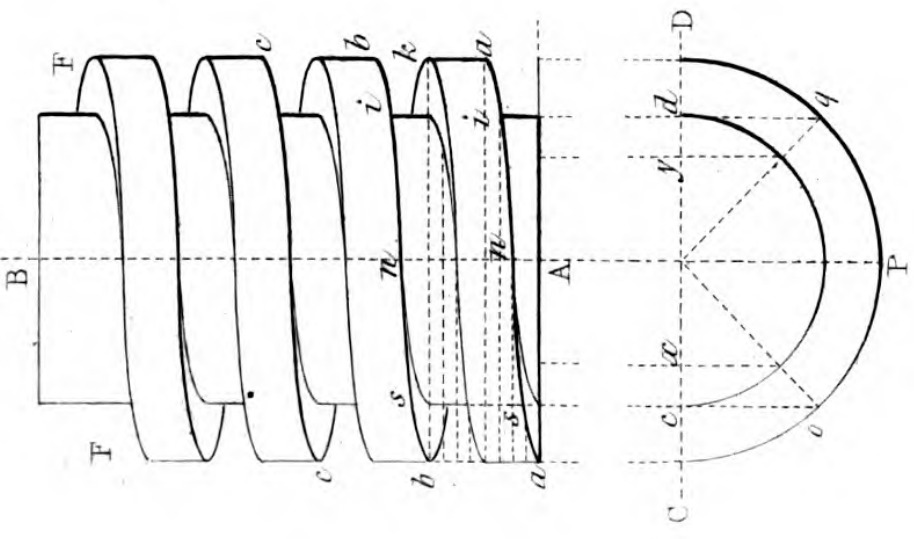


Fig. 3.

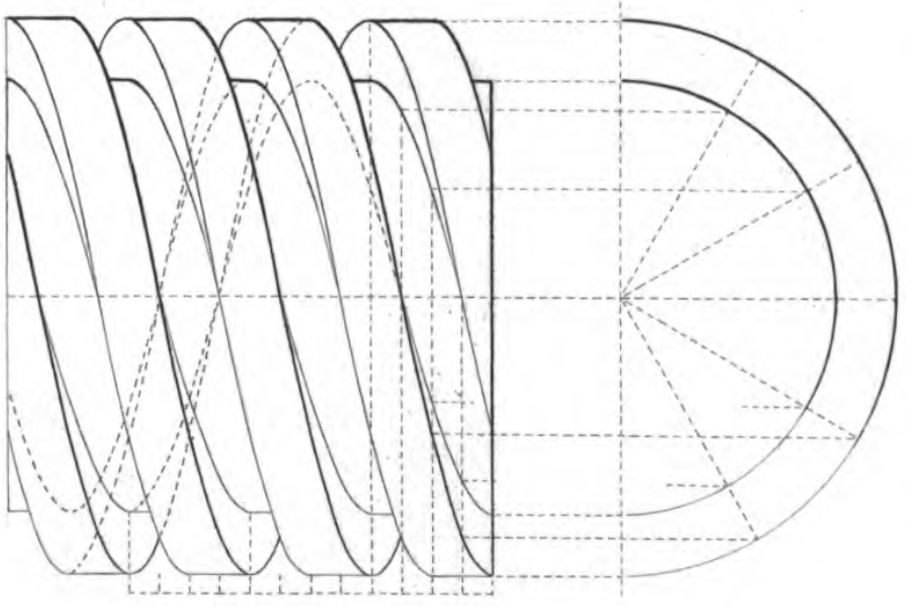
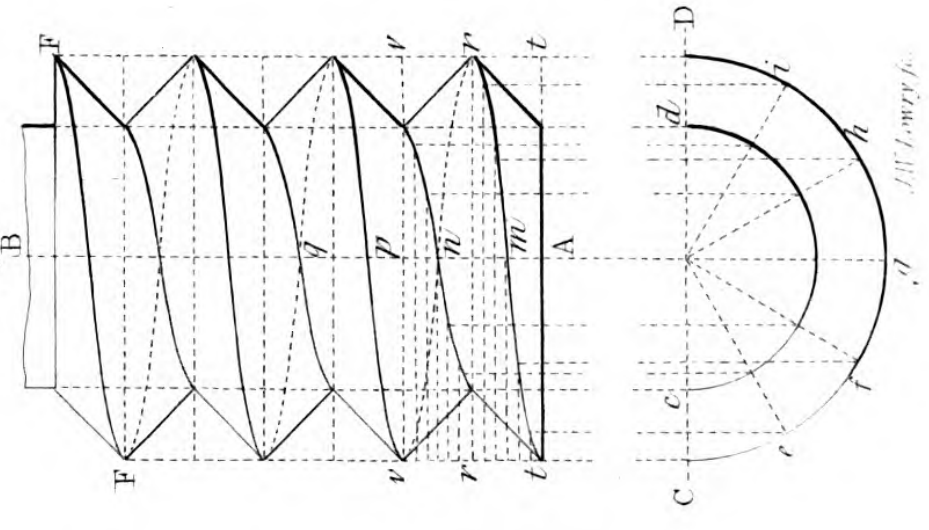


Fig. 1.



Published by John Bode & High Holborn, 1844.

ALL RIGHTS RESERVED.

bottom of the threads ; divide each semi-circumference into any number of equal parts, as C, e, f, g, h, i, D , from which draw lines parallel to the line AB ; divide the lines CF, DF , into equal divisions of half the required pitch or consecutive threads, as t, r, v , &c., draw the lines tt, rr, vv , &c., parallel with the diameter CD , and subdivide any two connected divisions into the same number of equal parts contained in both semicircles, from which draw lines meeting the vertical lines ; then by hand, or otherwise, and through the intersections, draw the waved lines, m, n, p, q , &c., and a thread of the screw is delineated as required.

Note.—The same process might be continued throughout the whole length of the screw, but it is much more convenient, when the proper curves are obtained, to form a suitable ruler : lay it in its proper situation upon each division, and draw the lines as required.

43. *To delineate a square-threaded screw, the pitch and diameter of screw being given. (See Plate A, fig. 2.)*

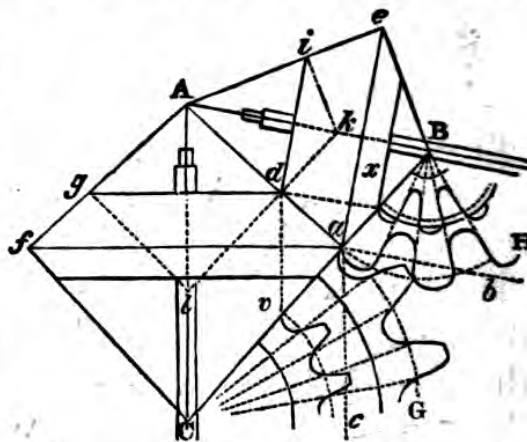
On the line AB representing the centre of the screw, describe the semicircles CD, cd , equal to the diameters at the tops and bottoms of the threads ; divide each semi-circumference into four equal parts, draw lines from each and parallel to the line AB ; draw also the lines CF, DF , which divide into the proper required pitch, as aa, bb, cc , &c. ; divide any two connected pitches or divisions, as a, b , into four equal parts, from which draw lines parallel to the diameter CD , meeting the vertical lines o, p, q , and forming intersections through which the waved lines s, n, i, s, n, i , or tops of the threads, must be traced by hand or otherwise ; draw also the lines x, y , forming intersections through which to trace the curve surface exhibited between and caused by the angular return

of the thread ; describe also the curves k and b , which terminates the returning thread, and completes the delineation as required.

Fig. 3, as to mode of construction, is exactly similar to that of fig. 2, but intended, by displacement of the cylinder, to delineate a continuous vein of the spiral in its proper form around the whole circumference ; hence, being deemed by the preceding figure already sufficiently described, further elucidation must be considered unnecessary.

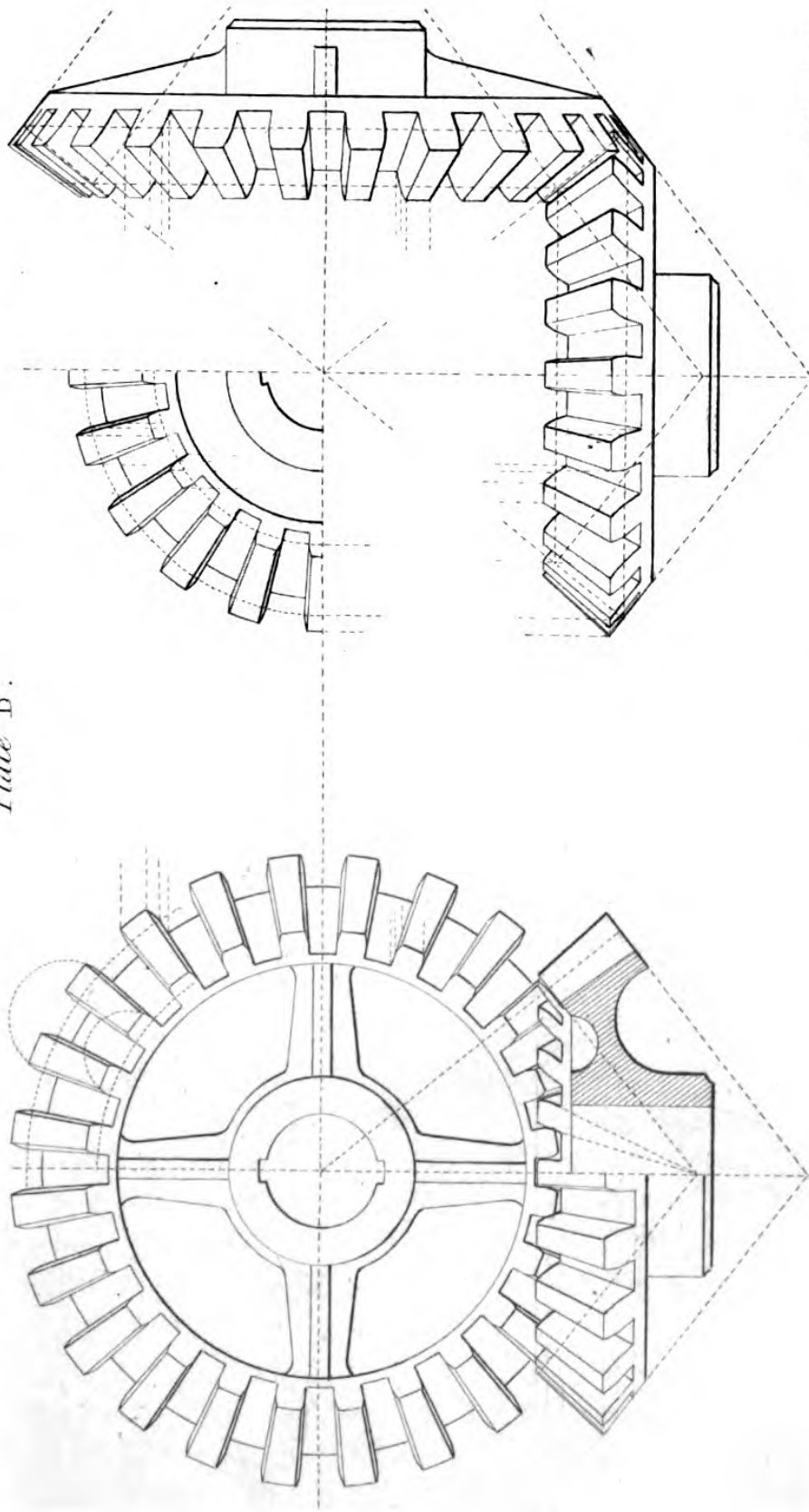
44. *To determine the proper forms for a pair of bevel wheels, the required angle of the shafts and diameters of the wheels being given.*

Draw at the given angles lines representing the shafts on which the wheels are to be fixed, as $A B$, $A C$; make the lines $a b$, $a c$, parallel with and at a distance from $A B$, $A C$, equal to the radius of each respective wheel at the greatest pitch circle ; draw the line $A a$ through the intersection at d ; then from a at right angles with $A B$, $A C$, draw the lines $a e$, $a f$, making each in length equal to the wheel's diameter ; draw the lines $A e$, $A f$, and from a , with the intended breadth of the wheels on the face, cut the line $A a$ in d ; draw the lines $d i$, $d g$, parallel to $a e$, $a f$, (hence the proper conical forms of the wheels and the pitch circles ;) draw at right angles with $A a$, and through the intersection of the lines $a e$, $a f$, the line $C B$, also the lines $B e$, $c f$, $d k i$, $d l g$; from B and C , with



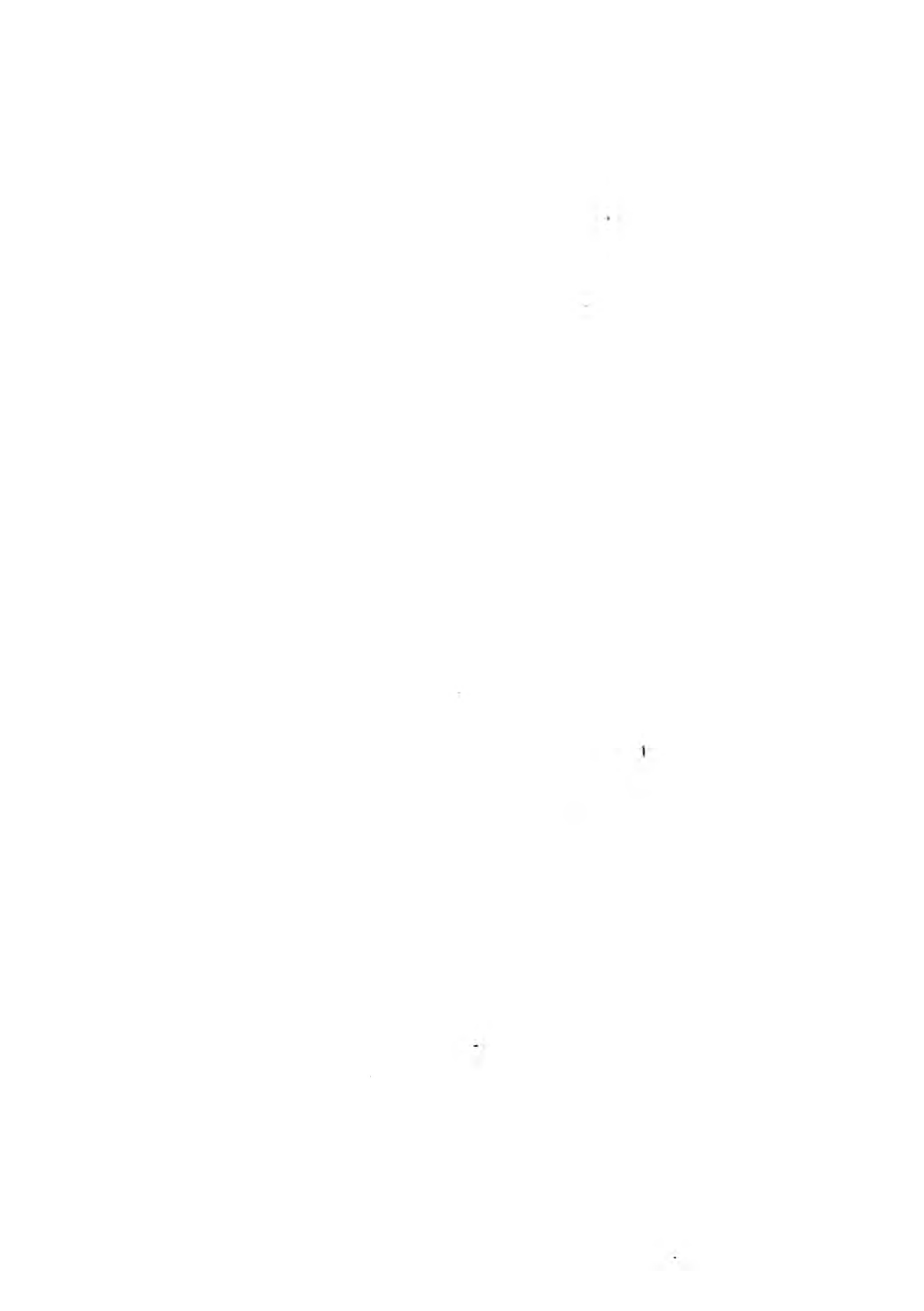
ILLUSTRATIONS TO THE DRAWING OF BEVEL WHEELS.

Plate B.



J.W. Lowry & Co.

Published by John Weale & High Holborn, 1844.



the radius $B a$, $C a$, describe portions of circles, as $a G$, $a H$, on which describe the greatest dimensions of, and proper form of the teeth; then from d , and parallel with $A B$, $A C$, draw the lines $a v$, $a x$, cutting the line $C B$ in v and x ; from B and C , with the distances $C v$, $B x$, describe the portions of circles, which determines the dimensions of the teeth on the interior pitch circle, and completes the proper forms of the wheels as required.

Proportions for the construction of toothed wheels.

Length of the teeth = $\frac{5}{7}$ of the pitch.

Thickness „ = $\frac{4}{9}$ do.

Breadth on face = $2\frac{1}{2}$ times the pitch.

Edge of the rim

Projecting rib inside do. } each $\frac{4}{9}$ of the pitch.

Thickness of flat arms

Breadth of arms at rim = 2 teeth and $\frac{1}{4}$ the pitch, increasing in breadth towards the centre of the wheel, in the proportion of $\frac{1}{2}$ an inch for every foot in length.

Thickness of the ribs or feathers on the arms = $\frac{1}{4}$ of the pitch.

Thickness of metal around the eye, or centre, = $\frac{7}{9}$ of the pitch.

Wheels and other circular bodies are very conveniently transferred from plan to that of a projected perspective by means of a peculiar appropriation of straight lines, commonly called orthographic projection; the principle of which will be readily understood by reference to the diagrams and illustrations given for the purpose in Plate C.

Fig. 1 is a circle divided into equal parts, and its form in projection is required, vn being supposed the line of intersection: parallel with the diameter of the

circle bf , draw at right angles, and through the centre, the line ce ; draw also the line or chord fg , cutting the line vn in g ; then, with the distance vg , describe the quadrant irs ; bisect the arc in r , from which and parallel with vn , draw the line rt ; draw also the lines ft , gt , which determines the breadth of the ellipsis or projected form of the circle, and which may be drawn as described (page 5, fig. 12): hence, all the other lines being so distinctly described by mere inspection of the diagrams, further explanation is unnecessary.

Fig. 2 is a projected representation of an undershot water wheel, of which A is the plan, vn the line of intersection, and B the diameter and breadth, laid at a proper angle or inclination, as determined by the principles of the diagrams: the breadth of the ellipsis is found and formed as there described, and the lines in the illustration, or fig. 2, whereby to obtain the proper angles for projection, render sufficiently explicit the proper mode by which the representation is effected.

ORTHOGRAPHIC PROJECTION.

Plate C.

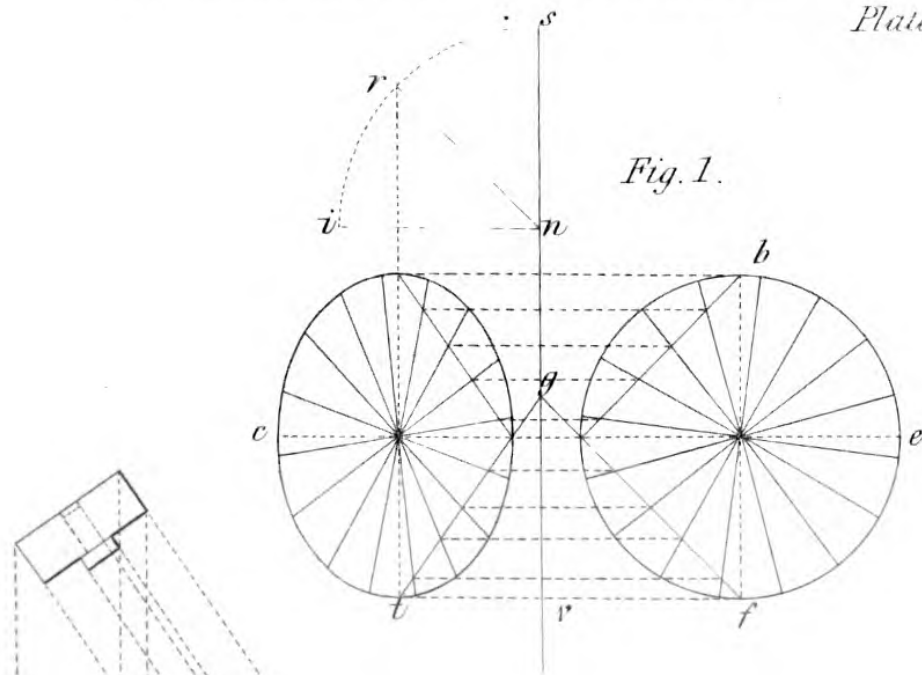


Fig. 1.

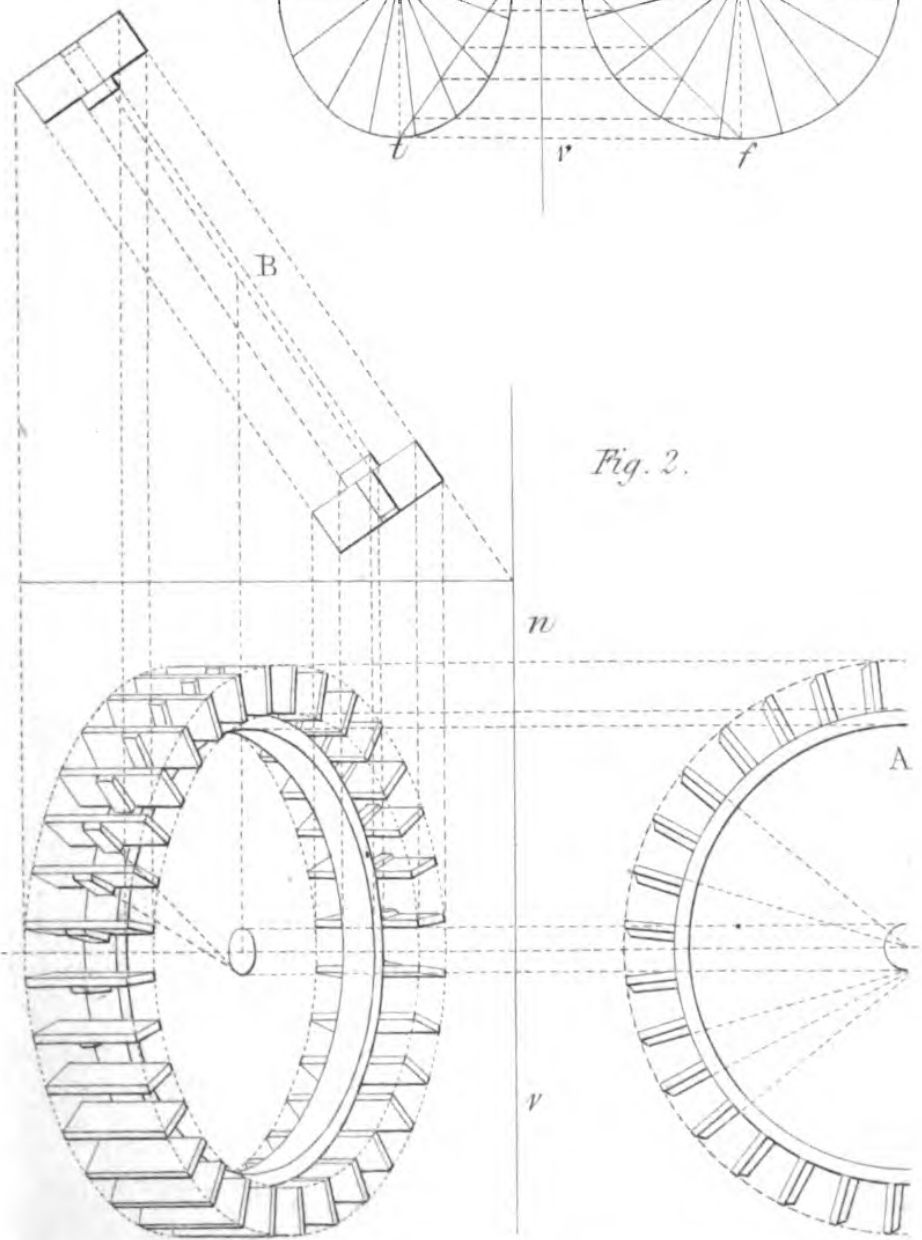


Fig. 2.

J.W. Lowry D.

DECIMAL ARITHMETIC.

DECIMAL ARITHMETIC is the most simple and explicit mode of performing practical calculations ; on account of its doing away with the necessity of fractional parts in the fractional form, thereby reducing long and tedious operations to a few figures arranged and worked in all respects according to the usual rules of common arithmetic.

Decimals simply signify tenths ; thus, the decimal of a foot is the tenth part of a foot, the decimal of that tenth is the hundredth of a foot, the decimal of that hundredth is the thousandth of a foot, and so might the divisions be carried on and lessened to infinity ; but in practice it is seldom necessary to take into account any degree of less measure than a one-hundredth part of the integer or whole number. And, as the entire system consists in supposing the whole number divided into tenths, hundredths, thousandths, &c., no peculiarity of notation is required, otherwise than placing a mark or dot, to distinguish between the whole and any part of the whole : thus 34·25 gallons signify 34 gallons 2 tenths and 5 hundredths of a gallon ; 11·04 yards signify 11 yards and 4 hundredths of a yard ; 16·008 shillings signify 16 shillings and 8 thousandth parts of a shilling : from which it must appear plain, that ciphers on the right hand of decimals are of no value whatever ; but placed on the left hand, they diminish the decimal value in a ten-fold proportion,—for ·6 signify 6 tenths ; ·06 signify 6 hundredths ; and ·006 signify 6 thousandths, of the integer or whole number.

REDUCTION.

Reduction means the construing or changing of

vulgar fractions to decimals of equal value; also finding the fractional value of any decimal given.

Rule 1. Add to the numerator of the fraction any number of ciphers at pleasure, divide the sum by the denominator, and the quotient is the decimal of equivalent value.

Rule 2. Multiply the given decimal by the various fractional denominations of the integer or whole number, cutting off from the right hand of each product for decimals a number of figures equal to the given number of decimals, and thus proceed until the lowest degree, or required value, is obtained.

Ex. 1. Required the decimal equivalent, or decimal of equal value, to $\frac{3}{12}$ of a foot.

$$\frac{3.00}{12} = .25, \text{ the decimal required.}$$

Ex. 2. Reduce the fraction $\frac{1}{8}$ of an inch to a decimal of equal value.

$$\frac{1.000}{8} = .125, \text{ the decimal required.}$$

Ex. 3. What is the decimal equivalent to $\frac{7}{8}$ of a gallon?

$$\frac{7.000}{8} = .875, \text{ the decimal equivalent.}$$

Ex. 4. Required the fractional value of the decimal .40625 of an inch.

$$\begin{array}{r}
 \phantom{\text{Multiply by } \frac{1}{8}} \cdot 40625 \\
 \text{Multiply by } \frac{1}{8} \quad 8 \\
 \hline
 3.25000 \\
 \times \frac{2}{18} = \frac{1}{9} \quad 2 \\
 \hline
 \phantom{\times \frac{2}{18} = \frac{1}{9}} \cdot 50000 \\
 \times \frac{2}{32} = \frac{1}{16} \quad 2 \\
 \hline
 1.00000 \quad \frac{3}{8} \text{ and } \frac{1}{32} \text{ of an inch, the value required.} \\
 \hline
 \hline
 \end{array}$$

Ex. 5. What is the fractional value of $\cdot 625$ of a cwt.?

$$\begin{array}{r}
 \cdot 625 \\
 \text{Multiply by 4 qrs.} \quad 4 \\
 \hline
 2\cdot 500 \\
 \times 28 \text{ lbs.} \quad 28 \\
 \hline
 14\cdot 000 = 2 \text{ quarters and } 14 \text{ lbs., the value} \\
 \hline \hline
 \text{required.}
 \end{array}$$

Ex. 6. Ascertain the fractional value of $\cdot 875$ of an imperial gallon.

$$\begin{array}{r}
 \cdot 875 \\
 \text{Multiply by 4 quarts} \quad 4 \\
 \hline
 3\cdot 500 \\
 \times 2 \text{ pints} \quad 2 \\
 \hline
 1\cdot 000 = 3 \text{ quarts and } 1 \text{ pint, the value} \\
 \hline \hline
 \text{required.}
 \end{array}$$

Ex. 7. What is the fractional value of $\cdot 525$ of a £. sterling?

$$\begin{array}{r}
 \cdot 525 \\
 \text{Multiply by 20 sh.} \quad 20 \\
 \hline
 10\cdot 500 \\
 \times 12 \text{ pence} \quad 12 \\
 \hline
 6\cdot 000 = 10 \text{ shillings and } 6 \text{ pence, the value} \\
 \hline \hline
 \text{required.}
 \end{array}$$

Independent of the mark or dot which distinguishes between integers and decimals, the fundamental rules, viz., Addition, Subtraction, Multiplication, and Division, are in all respects the same as in Simple Arithmetic; and an example in each, illustrative of placing the separating point, will no doubt render the whole system sufficiently intelligible, even to the dullest capacity.

Ex. 1. Add into one sum the following integers and decimals.

16.625; 11.4; 20.7831; 12.125; 8.04; and 7.002.

16.625

11.4

20.7831

12.125

8.04

7.002

75.9751 = the sum required.

Ex. 2. Subtract 119.80764 from 234.98276.

234.98276

119.80764

115.17512 = the remainder required.

Ex. 3. Multiply 62.10372 by 16.732.

62.10372

16.732

12420744

18631116

43472604

37262232

6210372

1039.11944304 = the product required.

Observe that the number of figures in the product from the right-hand accounted as decimals, are equal to the number of decimals in the multiplier and multiplicand taken together.

Ex. 4. Divide 39.375 by 9.25.

9.25)39.375(4.256 = the quotient required.

$$\begin{array}{r}
 3700 \\
 \hline
 2375 \\
 1850 \\
 \hline
 5250 \\
 4625 \\
 \hline
 6250 \\
 5550 \\
 \hline
 700 \\
 \hline
 \hline
 \end{array}$$

Observe that the number of decimals in the divisor and quotient together, must be equal to the number in the dividend.

Note.—The operation might be still continued, so as to reduce the quotient to a degree of greater exactitude, but in practice it is quite unnecessary, being even now reduced to a measure of greater nicety than is commonly required.

Definitions of Arithmetical Signs employed in the following calculations, which ought to be particularly attended to.

- = sign of equality, and signifies equal to, as 3 added 4 = 7.
- + „ addition „ plus or more, as 5 + 3 = 8.
- − „ subtraction „ minus or less, as 8 − 3 = 5.
- × „ multiplication „ multiplied by, as 8 × 3 = 24.
- ÷ „ division „ divided by, 24 ÷ 4 = 6 or $\frac{24}{4} = 6$.
- : :: : proportion „ that 2 is to 3 as 4 is to 6, &c.
- √ „ square root } „ evolution, or the extrⁿ. of roots;
- $\sqrt[3]{}$ „ cube root } „ thus, $\sqrt{64} = 8$ and $\sqrt[3]{64} = 4$.
- 4² „ to be squared } „ involution, or the raising of powers;
- 4³ „ to be cubed } „ thus, 4² = 16, and 4³ = 64.
- $\overline{3 + 5} \times 4 = 32$ „ { that 3 plus 5 multiplied by 4 = 32.
- $\sqrt{5^2 - 3^2} = 4$ 5 squared, minus 3 squared, the square root of the remainder = 4.
- $\frac{\sqrt[3]{20 \times 12}}{30} = 2$, 20 multiplied by 12, and divided by 30, the cube root of the quotient = 2.

BRITISH STANDARD MEASURES.

1. *Measures of length.*

12 inches	= 1 foot.
3 feet	= 1 yard.
5½ yards	= 1 pole or rod.
40 poles	= 1 furlong.
8 furlongs, 1760 yards, or 5280 feet	= 1 mile.

2. *Measures of surface, or square measure.*

144 square inches	= 1 square foot.
9 square feet	= 1 square yard.
30¼ square yards	= 1 square pole.
40 square poles	= 1 rood.
4 roods, or 4840 square yards	= 1 acre.

3. *Measures of solidity, or cubic measure.*

1728 cubic inches	= 1 cubic foot.
27 cubic feet	= 1 cubic yard.

4. *Measures of capacity.*

LIQUIDS.

8·665 cubic inches	= 1 gill.
4 gills	= 1 pint.
2 pints	= 1 quart.
4 quarts, or 277¼ cubic inches	= 1 gallon.

GRAIN, FRUITS, &c.

2 gallons	= 1 peck.
4 pecks, or 2218·192 cubic inches	= 1 bushel.
8 bushels	= 1 quarter.
5 quarters	= 1 load.

5. *Measures of weight.*

TROY.

24 grains	= 1 pennyweight.
20 pennyweights	= 1 ounce.
12 ounces	= 1 pound.

AVOIRDUPOIS.

27·34375 troy grains	= 1 dram.
16 drams	= 1 ounce.
16 ounces	= 1 pound.
14 pounds	= 1 stone.
2 stones	= 1 quarter.
4 quarters, or 112 lbs.	= 1 cwt.
20 cwt.	= 1 ton.

BRITISH SPECIAL MEASURES.

1. *Lineal measures for land.*

7·92 inches	= 1 link.
100 links or 22 yards	= 1 chain.
80 chains	= 1 mile.
69·121 miles	= 1 geog. degree.

2. *Square measures for land.*

62·7264 square inches	= 1 square link.
10,000 square links	= 1 square chain.
10 square chains	= 1 acre.

3. *Nautical measures.*

1 nautical mile	= 6082·66 feet.
3 miles	= 1 league.
20 leagues	= 1 degree.
360 degrees	= the earth's circumference.

Miscellaneous special measures.

6 lineal feet	= 1 fathom.
100 square feet	= 1 square of flooring.
272 sq. feet, at 14 in. in thickness	= 1 rod of brick-work.
600 square feet of inch-boards	= 1 load.
40 cubic feet of round timber }	= 1 ton or load.
50 cubic feet of hewn timber }	
40 cubic feet	= 1 ton of shipping.
120 deals	= 1 hundred.
120 nails	= 1 hundred.
1200 do.	= 1 thousand.
500 bricks	= 1 load.
32 bushels of lime	= 1 do.
36 do. sand	= 1 do.
19½ cwt.	= 1 fother of lead.
108 cubic feet	= 1 stack of wood.
42 gallons	= 1 tierce
63 do.	= 1 hogshead
84 do.	= 1 puncheon
126 do.	= 1 pipe
252 do.	= 1 tun
36 do.	= 1 barrel
54 do.	= 1 hogshead
72 do.	= 1 puncheon
108 do.	= 1 butt

old wine measure.
old ale measure.

DECIMAL APPROXIMATIONS FOR FACILITATING
CALCULATIONS IN MENSURATION.

Lineal feet multiplied by	·00019	= miles.
" yards	" ·000568	= " "
Square inches	·007	= square feet.
" yards	·0002067	= acres.
Circular inches	·00546	= square feet.
Cylindrical inches	·0004546	= cubic feet.
" feet	·02909	= cubic yards.
Cubic inches	·00058	= cubic feet.
" feet	·03704	= cubic yards.
" "	6·232	= imperial gallons.
" inches	·003607	= " "
Cylindrical feet	4·895	= " "
" inches	·002832	= " "
Cubic inches	·263	= lbs. av. ^s of cast iron.
" "	·281	= " wrought do.
" "	·283	= " steel.
" "	·3225	= " copper.
" "	·3037	= " brass.
" "	·26	= " zinc.
" "	·4103	= " lead.
" "	·2636	= " tin.
" "	·4908	= " mercury.
Cylindrical inches	·2065	= " cast iron.
" "	·2168	= " wrought iron.
" "	·2223	= " steel.
" "	·2533	= " copper.
" "	·2385	= " brass.
" "	·2042	= " zinc.
" "	·3223	= " lead.
" "	·207	= " tin.
" "	·3854	= " mercury.
Avoirdupois lbs.	·009	= cwts.
" "	·00045	= tons.

DECIMAL EQUIVALENTS TO FRACTIONAL PARTS OF LINEAL MEASURES.

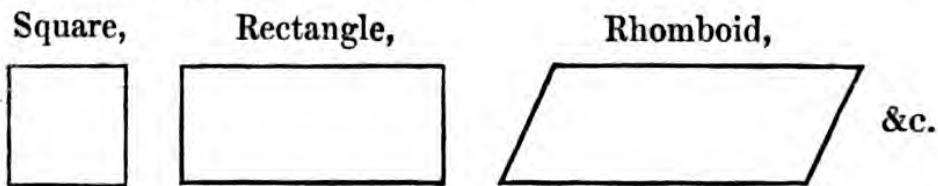
One inch, the integer or whole number.					
·96875	$\frac{7}{8}$ & $\frac{3}{32}$	·625	$\frac{5}{8}$	·28125	$\frac{3}{8}$ & $\frac{1}{32}$
·9375	$\frac{7}{8}$ & $\frac{1}{16}$	·59375	$\frac{5}{8}$ & $\frac{3}{32}$	·25	$\frac{1}{4}$
·90625	$\frac{7}{8}$ & $\frac{1}{32}$	·5625	$\frac{5}{8}$ & $\frac{1}{16}$	·21875	$\frac{1}{4}$ & $\frac{3}{32}$
·875	$\frac{7}{8}$	·53125	$\frac{5}{8}$ & $\frac{1}{32}$	·1875	$\frac{1}{4}$ & $\frac{1}{16}$
·84375	$\frac{3}{4}$ & $\frac{3}{32}$	·5	$\frac{5}{8}$	·15625	$\frac{1}{4}$ & $\frac{1}{32}$
·8125	$\frac{3}{4}$ & $\frac{1}{16}$	·46875	$\frac{1}{2}$ & $\frac{3}{32}$	·125	$\frac{1}{8}$
·78125	$\frac{3}{4}$ & $\frac{1}{32}$	·4375	$\frac{1}{2}$ & $\frac{1}{16}$	·09375	$\frac{3}{32}$
·75	$\frac{3}{4}$	·40625	$\frac{1}{2}$ & $\frac{1}{32}$	·0625	$\frac{1}{16}$
·71875	$\frac{5}{8}$ & $\frac{3}{32}$	·375	$\frac{3}{4}$	·03125	$\frac{1}{32}$
·6875	$\frac{5}{8}$ & $\frac{1}{16}$	·34375	$\frac{3}{4}$ & $\frac{1}{32}$		
·65625	$\frac{5}{8}$ & $\frac{1}{32}$	·3125	$\frac{3}{4}$ & $\frac{1}{16}$		
	are equal to		are equal to		are equal to
One foot, or 12 inches, the integer.					
·9166	11 inches.	·4166	5 in.	·0625	$\frac{1}{16}$ of in.
·6333	10 "	·3333	4 "	·0528	$\frac{1}{19}$ "
·75	9 "	·25	3 "	·04166	$\frac{1}{24}$ "
·6666	8 "	·1666	2 "	·03125	$\frac{1}{32}$ "
·5833	7 "	·0833	1 "	·02083	$\frac{1}{48}$ "
·5	6 "	·07291	$\frac{7}{8}$ "	·01041	$\frac{1}{96}$ "
	are equal to		are equal to		are equal to
One yard, or 36 inches, the integer.					
·9722	35 inches.	·6389	23 inches.	·3055	11 inches.
·9445	34 "	·6111	22 "	·2778	10 "
·9167	33 "	·5833	21 "	·25	9 "
·8889	32 "	·5556	20 "	·2222	8 "
·8611	31 "	·5278	19 "	·1944	7 "
·8333	30 "	·5	18 "	·1666	6 "
·8056	29 "	·4722	17 "	·1389	5 "
·7778	28 "	·4445	16 "	·1111	4 "
·75	27 "	·4166	15 "	·0833	3 "
·7222	26 "	·3889	14 "	·0555	2 "
·6944	25 "	·3611	13 "	·0277	1 "
·6667	24 "	·3333	12 "		
	are equal to		are equal to		are equal to

MENSURATION.

MENSURATION is that branch of Mathematics which is employed in ascertaining the extension, solidities, and capacities of bodies, capable of being measured.

1. MENSURATION OF SURFACE.

To measure or ascertain the quantity of surface in any right-lined figure whose opposite sides are parallel to each other, as a



Rule.—Multiply the length by the breadth; the product is the area or superficial contents.

Application of the rule to practical purposes.

1. The side of a square piece of board is $8\frac{3}{16}$ inches in length; required the area or superficies.

Decimal equivalent to the fraction $\frac{3}{16} = \cdot 1875$ (see page 31); and $8\cdot 1875 \times 8\cdot 1875 = 67\cdot 03515625$ square inches, the area.

2. The length of the fire grate under the boiler of a steam engine is 4 feet 7 inches, and its width 3 feet 6 inches; required the area of the fire grate.

7 in. = $\cdot 5833$ and 6 in. = $\cdot 5$ (see Table of Equivalents, p. 31): hence $4\cdot 5833 \times 3\cdot 5 = 16\cdot 04155$ square feet, the area.

3. Required the number of square yards in a floor whose length is $13\frac{1}{2}$, and breadth $9\frac{3}{4}$ feet.

$$13.5 \times 9.75 = 131.625 \div 9 = 14.625 \text{ square yards.}$$

Note 1.—The above rule is rendered equally applicable to figures whose sides are not parallel to each other, by taking the mean breadth as that by which the contents are to be estimated.

2. The square root of any given sum equals the side of a square of equal area.

3. Any square whose side is equal to the diagonal of another square, contains double the area of that square.

4. Any sum or area (of which to form a rectangle) divided by the breadth, the quotient equals the length; or divided by the length, the quotient equals the breadth of the rectangle required.

TRIANGLES.

Any two sides of a right-angled triangle being given, to find the third side.

Rule 1.—Add together the squares of the base and perpendicular, and the square root of the sum is the hypotenuse or longest side.

Rule 2.—Add together the hypotenuse and any one side, multiply the sum by their difference, and the square root of the product equals the other side.

Application to practical purposes.

1. Wanting to prop a building with raking shores, the top ends of which to be 25 feet from the ground, and the bottom ends, 16 feet from the base of the building; what must be their length, independent of any extra length allowed below the surface of the ground?

$$\sqrt{25^2 + 16^2} = \sqrt{881} = 29.6816 \text{ feet, or } .6816 \times 12 = 8 \text{ inches; consequently, 29 feet 8 inches nearly.}$$

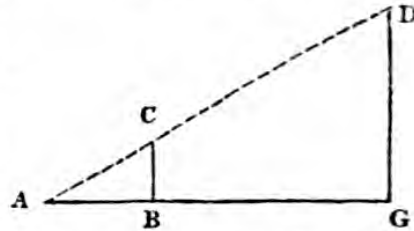
2. From the top of a wall 18 feet in height, a line was stretched across a canal for the purpose of ascer-

taining its breadth; the length of the line, when measured, was found to be 40 feet; required the breadth from the opposite embankment to the base of the wall.

$40 - 18 = 22$, and $\overline{40 + 18} \times 22 = \sqrt{1276} = 35.72$, or 35 feet 9 inches nearly, the width of the canal.

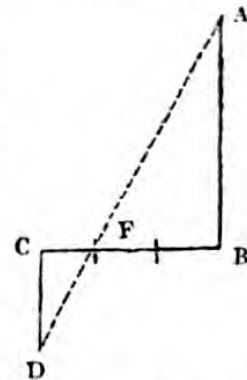
Triangles similar to each other are proportional to each other; hence their utility in ascertaining the heights and distances of inaccessible objects.

Thus, suppose the height of an inaccessible object D is required, I find by means of two staffs or otherwise, the height of the perpendicular BC and the length of the base line AB ; also the distance from A to the base of the object GD ;



then $AB : BC :: AG : GD$. And suppose $AB = 6$ feet,
 $BC = 2$ feet, and $AG = 150$
 $6 : 2 :: 150 : 50$ feet, the height of D from G .

Again, suppose the inaccessible distance A be required, make the line BA , BC , a right angle, and BC of three or four equal parts of any convenient distance, through one of which and in a line with the object A , determine the triangle CDF , then the proportion will be as

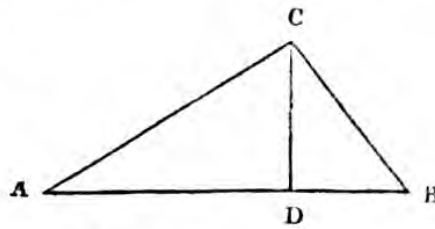


$CF : CD :: BF : BA$. Let $CF = 10$ yards, $CD = 53$, and
 $BF = 30$, $10 : 53 :: 30 : 159$ yards, the distance from B .

To find the area of a triangle when the base and perpendicular are given.

Rule.—Multiply the base by the perpendicular height, and half the product is the area.

1. The base of the triangle ADB is $11\frac{3}{2}$ inches in length, and the height DC, $3\frac{3}{8}$ inches; required the area.



$\frac{3}{2} = .09375$ and $\frac{3}{8} = .375$ (see page 31) :

hence $\frac{11 \cdot 09375 \times 3 \cdot 375}{2} = 18 \cdot 72075$ square inches, the area.

2. The base of a triangle is 53 feet 3 inches, and the perpendicular 7 feet 9 inches; required the area or superficies.

$$\frac{53 \cdot 25 \times 7 \cdot 75}{2} = 206 \cdot 34375 \text{ square feet, the area.}$$

When only the three sides of a triangle can be given, to find the area.

Rule.—From half the sum of the three sides, subtract each side severally; multiply the half sum and the three remainders together, and the square root of the product is equal the area required.

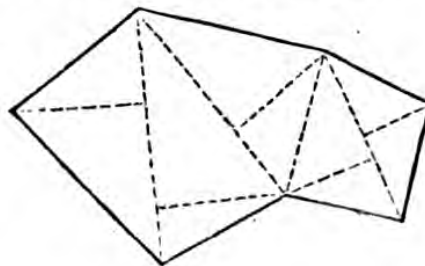
Required the area of a triangle, whose three sides are respectively 50, 40, and 30 feet.

$$\frac{50 + 40 + 30}{2} = 60, \text{ or half the sum of the three sides.}$$

$$\begin{aligned} 60 - 30 &= 30 \text{ first difference,} \\ 60 - 40 &= 20 \text{ second difference,} \\ 60 - 50 &= 10 \text{ third difference,} \end{aligned}$$

then $30 \times 20 \times 10 \times 60 = \sqrt{360000} = 600$, the area required.

Triangles are employed to great advantage in determining the area of any rectilinear figure, as the annexed, and by which the measurement is rendered comparatively simple.



POLYGONS.

Polygons being composed of triangles, may of course be similarly measured; hence in regular polygons, multiply the length of a side by the perpendicular height to the centre, and by the number of sides, and half the product is the area.

Table relative to the Construction and Estimation of Polygons.

Name.	No. of sides.	Angle at centre.	Angle at circum.	Perpen. side being 1.	Length of side, radius being 1.	Radius of circle, side being 1.	Radius of circle, per. being 1.	Area, side being 1.
Triangle .	3	120°	60°	0.2886	1.73	.579	2	0.4330
Square . .	4	90	90	0.5	1.412	.705	1.41	1
Pentagon	5	72	108	0.6882	1.174	.852	1.238	1.7204
Hexagon .	6	60	120	0.8660	1	1	1.156	2.5980
Heptagon	7	51 $\frac{2}{7}$	128 $\frac{2}{7}$	1.0382	.867	1.16	1.11	3.6339
Octagon .	8	45	135	1.2071	.765	1.307	1.08	4.8284
Nonagon .	9	40	140	1.3737	.681	1.47	1.062	6.1818
Decagon .	10	36	144	1.5388	.616	1.625	1.05	7.6942
Undecagon	11	32 $\frac{8}{11}$	147 $\frac{3}{11}$	1.7028	.561	1.777	1.04	9.3656
Dodecagon	12	30	150	1.8660	.516	1.94	1.037	11.1961

Application of the Table.

1. The radius of a circle being $6\frac{1}{2}$ feet, required the side of the greatest heptagon that may be inscribed therein.

$$\cdot 867 \times 6.5 = 5.6355, \text{ or } 5 \text{ feet } 7\frac{1}{2} \text{ inches nearly.}$$

2. Each side of a pentagon is required to be 9 feet; required the radius of circumscribing circle.

$$\cdot 852 \times 9 = 7.668, \text{ or } 7 \text{ feet } 8 \text{ inches.}$$

3. A perpendicular from the centre to either side of an octagon is required to be 12 feet; what must be the radius of circumscribing circle?

$$1.08 \times 12 = 12.96, \text{ or } 12 \text{ feet } 11\frac{1}{2} \text{ inches.}$$

4. Each side of a hexagon is $4\frac{1}{2}$ yards; required its superficial contents.

$$4\frac{1}{2}^2 \times 2.598 = 52.6095 \text{ square yards.}$$

THE CIRCLE AND ITS SECTIONS.

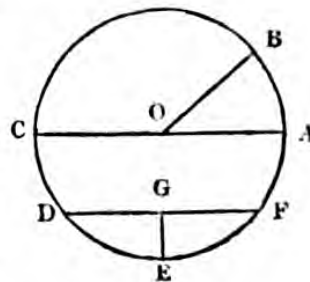
Observations and definitions.

1. The circle contains a greater area than any other plane figure bounded by the same perimeter or outline.

2. The areas of circles are to each other as the squares of their diameters; any circle twice the diameter of another, contains four times the area of the other.

3. The radius of a circle is a straight line drawn from the centre to the circumference, as O B.

4. The diameter of a circle is a straight line drawn through the centre, and terminated both ways at the circumference, as C O A.



5. A chord is a straight line joining any two points of the circumference, as D F.

6. The versed sine is a straight line joining the chord and circumference, as E G.

7. An arc is any part of the circumference, as C D E.

8. A semicircle is half the circumference cut off by a diameter, as C E A.

9. A segment is any portion of a circle cut off by a chord, as D E F.

10. A sector is a part of a circle cut off by two radii, as A O B.

General rules in relation to the circle.

1. Multiply the diameter by 3·1416, the product is the circumference.

2. Multiply the circumference by ·31831, the product is the diameter.

3. Multiply the square of the diameter by ·7854, the product is the area.

4. Multiply the square root of the area by 1·12837, the product is the diameter.

5. Multiply the diameter by ·8862, the product is the side of a square of equal area.

6. Multiply the side of a square by 1·128, the product is the diameter of a circle of equal area.

Application of the rules as to purposes of practice.

1. The diameter of a circle being $7\frac{3}{16}$ inches, required its circumference.

$$7\cdot1875 \times 3\cdot1416 = 22\cdot58025 \text{ inches, the circumference.}$$

Or, the diameter being $30\frac{1}{2}$ feet, required the circumference.

$$3\cdot1416 \times 30\cdot5 = 95\cdot8188 \text{ feet, the circumference.}$$

2. A straight line, or the circumference of a circle

being 274.89 inches, required the circle's diameter corresponding thereto.

$$274.89 \times .31831 = 87.5 \text{ inches diameter.}$$

Or, what is the diameter of a circle, when the circumference is 39 feet?

$.31831 \times 39 = 12.41409$ feet, and $.41409 \times 12 = 4.96908$ inches, or 12 feet 5 inches, very nearly the diameter.

3. The diameter of a circle is $3\frac{3}{4}$ inches; what is its area in square inches?

$$3.75^2 = 14.0625 \times .7854 = 11.044, \text{ \&c. inches area.}$$

Or, suppose the diameter of a circle 25 feet 6 inches, required the area.

$$25.5^2 = 650.25 \times .7854 = 510.706, \text{ \&c. feet, the area.}$$

4. What must the diameter of a circle be, to contain an area equal to 706.86 square inches?

$$\sqrt{706.86} = 26.586 \times 1.12837 = 29.998 \text{ or } 30 \text{ inches, the diameter required.}$$

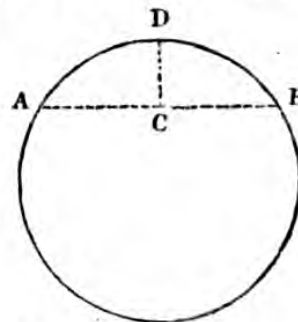
5. The diameter of a circle is $14\frac{1}{4}$ inches; what must I make each side of a square, to be equal in area to the given circle?

$$14.25 \times .8862 = 12.62835 \text{ inches, length of side required.}$$

Any chord and versed sine of a circle being given, to find the diameter.

Rule.—Divide the sum of the squares of the chord and versed sine by the versed sine, the quotient is the diameter of corresponding circle.

1. The chord of a circle AB equal $6\frac{1}{2}$ feet, and the versed sine CD equal 2 feet, required the circle's diameter.



$$\overline{6.5^2 + 2^2} = 46.25 \div 2 = 23.125 \text{ feet, the diameter.}$$

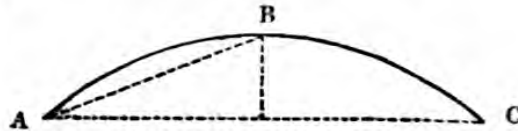
2. In a curve of a railway, I stretched a line 72 feet in length, and the distance from the line to the curve I found to be $1\frac{1}{4}$ ft.; required the radius of the curve.

$$\frac{72^2 + 1.25^2}{1.25 \times 2} = 5185.5625, \text{ and } \frac{5185.5625}{1.25 \times 2} = 2074.225 \text{ ft. the radius.}$$

To find the length of any given arc of a circle.

Rule.—From eight times the chord of half the arc, subtract the chord of the whole arc, and one-third of the remainder is equal the length of the arc.

Required the length of the arc ABC, the chord AB of half the arc being 4 feet 3 inches, and chord AC of the whole arc 8 feet 4 inches.



$$4.25 \times 8 = 34, \text{ and } 34 - 8.333 = \frac{25.667}{3} = 8.555 \text{ feet, the length of the arc.}$$

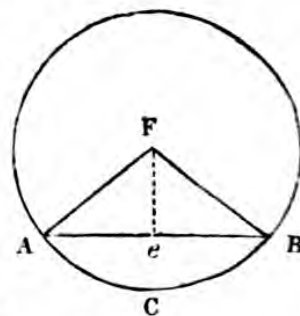
To find the area of the sector of a circle.

Rule.—Multiply the length of the arc by its radius, and half the product is the area.

The length of the arc ACB equal $9\frac{1}{2}$ feet, and the radii FA, FB, equal each 7 feet, required the area.

$$9.5 \times 7 = 65.5 \div 2 = 32.75 \text{ the area.}$$

Note.—The most simple means whereby to find the area of the segment of a circle, is, to first find the area of a sector whose arc is equal to that of the given segment; and if it be less than a semicircle, subtract the area of the triangle formed by the chord of the segment and radii of its extremities; but if more than a semicircle, add the area of the triangle to the area of the sector, and the remainder or sum is the area of the segment.



Thus, suppose the area of the segment $A C B e$ is required, and that the length of the arc $A C B$ equal $9\frac{3}{4}$ feet, $F A$ and $F B$ each equal 7 feet, and the chord $A B$ equal 8 feet 4 inches, also the perpendicular $e F$ equal $3\frac{3}{4}$ feet.

$$\frac{9.75 \times 7}{2} = 34.125 \text{ feet, the area of the sector.}$$

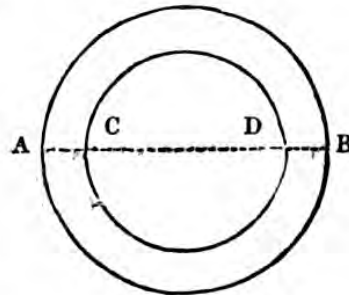
$$\frac{8.333 \times 3.75}{2} = 15.624 \text{ feet, area of the triangle.}$$

And $34.125 - 15.624 = 18.501$ feet, the area of the segment.

To find the area of the space contained between two concentric circles.

Rule.—Multiply the sum of the inside and outside diameters by their difference, and by $.7854$, the product is the area.

1. Suppose the external circle $A B$ equal 32 inches, and internal circle $C D$ equal 28 inches, required the area of the space contained between them.



$32 + 28 = 60$, and $32 - 28 = 4$, hence $60 \times 4 \times .7854 = 188.496$ in. the area.

2. The exterior diameter of the fly-wheel of a steam engine is 20 feet, and the interior diameter $18\frac{1}{2}$ feet; required the area of the surface or rim of the wheel.

$20 + 18.5 = 38.5$, and $20 - 18.5 = 1.5$, hence $38.5 \times 1.5 \times .7854 = 45.35$, &c. feet, the area.

To find the area of an ellipsis, or oval.

Rule.—Multiply the longest diameter by the shortest, and the product by $.7854$, the result is the area.

An oval is 25 inches by 16.5, what are its superficial contents?

$25 \times 16.5 = 412.5 \times .7854 = 323.9775$ inches, the area.

Note.—Multiply half the sum of the two diameters by 3.1416, and the product is the circumference of the oval or ellipsis.

To find the area of a parabola, or its segment.

Rule.—Multiply the base by the perpendicular height, and two-thirds of the product is the area.

What is the area of a parabola whose base is 20 feet and height 12?

$$20 \times 12 = \frac{240 \times 2}{3} = 160 \text{ feet, the area.}$$

Note.—Although the whole of the preceding practical applications or examples are given in measures of feet or inches, these being considered as the most generally familiar, yet the rules are equally applicable to any other unit of measurement whatever, as yards, chains, acres, &c., &c., &c.

2. MENSURATION OF THE SUPERFICIES, SOLIDITIES, AND CAPACITIES, OF BODIES.

To find the solidity or capacity of any figure in the cubical form.

Rule.—Multiply the length of any one side by its breadth and by the depth or distance to its opposite side, the product is the solidity or capacity, in equal terms of measurement.

Application of the rule to practical purposes.

1. Required the number of cubic inches in a piece of timber $23\frac{1}{2}$ inches long, $7\frac{3}{4}$ inches broad, and $3\frac{5}{8}$ inches in thickness.

$$23.5 \times 7.75 \times 3.625 = 660.203 \text{ cubic inches.}$$

2. A rectangular cistern is in length $8\frac{1}{2}$ feet, in breadth $5\frac{1}{4}$ feet, and in depth 4 feet; required its

capacity in cubic feet, also its capacity in British Imperial gallons.

$8.5 \times 5.25 \times 4 = 178.5$ cubic feet, and 178.5×6.232 (see Table of Decimal Approximations, p. 30) = 1112.412 gallons.

3. A rectangular cistern capable of containing 520 imperial gallons is to be $7\frac{1}{4}$ feet in length, and $4\frac{1}{2}$ feet in width; it is required to ascertain the necessary depth.

$7.25 \times 4.5 \times 6.232 = 203.318$, and $\frac{520.000}{203.318} = 2.557$ feet, or 2 feet $6\frac{3}{4}$ inches nearly.

4. A rectangular piece of cast iron 20 inches long and 6 inches broad, is to be formed of sufficient dimensions to weigh 150 lbs.; what will be the depth required?

$20 \times 6 \times .263$ (see Table of Decimal Approximations, Cast Iron, p. 30) = 31.96, and $\frac{150}{31.96} = 4.69$ in., or 4 and $\frac{11}{16}$ in., the thickness required.

To find the convex surface, and solidity or capacity, of a cylinder.

Rule 1.—Multiply the circumference of the cylinder by its length or height, the product is the convex surface.

Rule 2.—Multiply the area of the diameter by the length or height, and the product is the cylinder's solidity or capacity, as may be required.

Application of the rules.

1. The circumference of a cylinder is $37\frac{1}{2}$ inches, and its length $54\frac{3}{4}$ inches; required the convex surface in square feet.

$54.75 \times 37.5 \times .007$ (see Table of Approximations) = 14.371 square feet.

2. A cylindrical piece of timber is 9 inches dia-

meter, and 3 feet 4 inches in length; required its solidity in cubic inches, and also in cubic feet.

3 feet 4 inches = 40 inches, and $9^2 \times .7854 \times 40 = 2544.696$ cubic inches; then $2544.696 \times .00058 = 1.4759$ cubic feet.

3. Suppose a well to be 4 feet 9 inches diameter, and $16\frac{1}{2}$ feet from the bottom to the surface of the water; how many imperial gallons are therein contained?

$$4.75^2 \times 16.5 \times 4.895 = 1822.162 \text{ gallons.}$$

4. Again, suppose the well's diameter the same, and its entire depth 35 feet; required the quantity in cubic yards of material excavated in its formation.

$$4.75^2 \times 35 \times .02909 = 22.973 \text{ cubic yards.}$$

5. I have a cylindrical cistern capable of holding 7068 gallons, and its depth is 10 feet; now I want to replace it with one of an equal depth, but capable of holding 12,500 gallons; what must be its diameter?

$$4.895 \times 10 = 48.95, \text{ and } \frac{12500}{48.95} = \sqrt{255.3} = 15.9687 \text{ feet, or } 15 \text{ feet } 11\frac{5}{8} \text{ inches.}$$

6. A cylindrical piece of lead is required $7\frac{1}{2}$ inches diameter, and 168 lbs. in weight; what must be its length in inches?

$$7.5^2 \times .3223 = 18, \text{ and } \frac{168}{18} = 9.3 \text{ inches.}$$

To find the length of a cylindrical helix, or spiral, wound round a cylinder.

Rule.—Multiply the circumference of the base by the number of revolutions of the spiral, and to the square of the product add the square of the height; the square root of the sum is the length of the spiral.

Application of the rule.

1. Required the length of the thread or screw twisting round a cylinder 22 inches in circumference $3\frac{1}{2}$ times, and extending along the axis 16 inches.

$22 \times 3.5 = 77^2 = 5929$, and $16^2 = 256$, then $\sqrt{5929 + 256} = 78.64$ inches.

2. The well of a winding staircase is 5 feet diameter, and height to the top landing 25 feet, the hand-rail is to make $2\frac{1}{2}$ revolutions; required its length.

5 feet diameter = 15.7 feet circumference.

$15.7 \times 2.5 = 39.25^2 = 1540.5625$, and $25^2 = 625$, then

$\sqrt{1540 + 625} = 46.5$ feet, the length required.

To find the convex surface, solidity, or capacity of a cone or pyramid.

Rule 1.—Multiply the circumference of the base by the slant height, and half the product is the slant surface.

Rule 2.—Multiply the area of the base by the perpendicular height, and one-third of the product is the solidity or capacity, as may be required.

Application of the rules.

1. Required the area in square inches of the slant surface of a cone whose slant height equal $18\frac{3}{4}$ inches, and diameter at the base $6\frac{1}{4}$ inches.

$6.25 \times 3.1416 = 19.635$ circumference of the base; and

$$\frac{19.635 \times 18.75}{2} = 184.078125 \text{ square inches.}$$

2. Required the quantity of lead, in square feet, sufficient to cover the slant surface of a hexagonal pyramid whose slant height is .42 feet, and the breadth of each side at the base 4 feet 9 inches.

$$\frac{4.75 \times .42 \times 6 \text{ sides}}{2} = 598.5 \text{ square feet.}$$

3. What is the solidity of a cone in cubic inches, the diameter at the base being 15 inches, and perpendicular height $32\frac{1}{2}$ inches?

$$\frac{15^2 \times .7854 \times 32.5}{3} = 1914.4125 \text{ cubic inches.}$$

4. In a square solid pyramid of stone 67 feet in height, and $16\frac{1}{2}$ feet at the base, how many cubic feet?

$$\frac{16.5 \times 16.5 \times 67}{3} = 6080.25 \text{ cubic feet.}$$

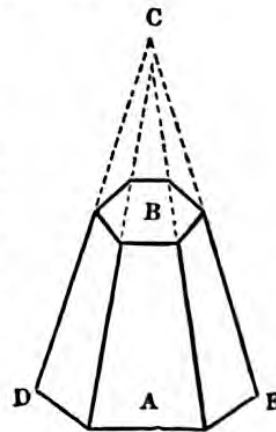
To find the solidity or capacity of any frustum of a cone or pyramid.

Rule.—If the base be a circle, add into one sum the two diameters, or, if a regular polygon, the breadth of one side at the top and at the base; then from the square of the sum subtract the product of these diameters or breadths; multiply the remainder by .7854, if a circle, or by the tabular area (see Table of Polygons, p. 36), and by one-third of the height, and the product is the content in equal terms of unity.

Note.—Where the whole height of the cone or pyramid can be obtained, of which the given frustum forms a part, the most simple method is first to find the whole contents, then the contents extending beyond the frustum, and subtracting the less from the greater, leaves the contents of the frustum required.

Application of the rules.

1. The perpendicular height AB of the frustum of a hexagonal pyramid CDE , is $7\frac{1}{2}$ feet, and the breadth of each side at top and base equal $3\frac{3}{4}$ and $2\frac{1}{2}$ feet; required the solid contents of the frustum in cubic feet.

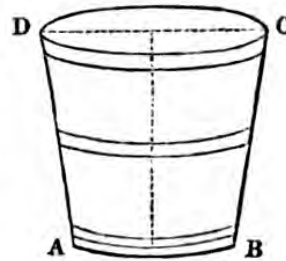


$3.75 + 2.5 = 6.25$, and $6.25 \times 6.25 = 39.0625$, then $3.75 \times 2.5 = 9.375$, and $39.0625 - 9.375 = 29.6875 \times 2.598$ (tabular area, p. 36) $= 77.138 \times 2.5$ or $\frac{1}{3}$ of the height $= 192.845$ cubic feet.

2. Required the solidity of the frustum of a cone, the top diameter of which is 7 inches, the base diameter $9\frac{1}{2}$, and the perpendicular height 12.

$\overline{7 + 9.5^2} = 272.25$, and $7 \times 9.5 = 66.5$, then $272.25 - 66.5 = 205.75 \times .7854 = 161.576 \times 4$ or $\frac{1}{3}$ of the height $= 646.3$ cubic inches.

3. A vessel in the form of an inverted cone, as ABCD, is 5 feet in diameter at the top, 4 feet at the bottom, and 6 feet in depth; required its capacity in imperial gallons.



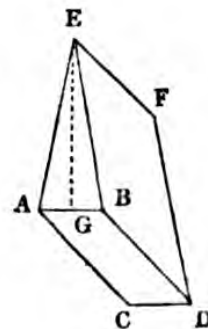
$\overline{5 + 4} = 9^2 = 81$, and $5 \times 4 = 20$, hence $81 - 20 = 61 \times .7854$, and by 2 or $\frac{1}{3}$ of the depth $= 95.8188$ cubic feet, and $\times 6.232 = 597.1427$ gallons.

To find the solid contents of a wedge.

Rule.—To twice the length of the base add the length of the edge; multiply the sum by the breadth of the base, and by the perpendicular height from the base, and one-sixth of the product is the solid contents.

Application of the rule.

Required the solidity of a wedge in cubic inches, the base ABCD being 9 inches by $3\frac{1}{2}$, the edge EF, 7 inches, and the perpendicular height GE, 15.



$$\frac{\overline{18 + 7} \times 3.5 \times 15}{6} = 218.75 \text{ cubic inches.}$$

To find the convex surface, the solidity, or the capacity of a sphere or globe.

Rule 1.—Multiply the square of the diameter by 3·1416, the product is the convex surface.

Rule 2.—Multiply the cube of the diameter by ·5236, the product is the solid contents.

Rule 3.—Multiply the cube of the diameter in feet by 3·263, or in inches by ·001888, the product is the capacity in imperial gallons.

Application of the rules.

1. Required the convex surface, the solidity, and the weight in cast iron of a sphere or ball $10\frac{1}{2}$ inches in diameter.

$$10\cdot5^2 \times 3\cdot1416 = 346\cdot3614 \text{ square inches.}$$

$$10\cdot5^3 \times \cdot5236 = 606\cdot132, \text{ \&c. cubic inches; and}$$

$$606\cdot132 \times \cdot263 \text{ (see Table of Approximations, p. 30)} = 159\cdot4 \text{ lbs.}$$

2. A hollow or concave copper ball is required 8 inches diameter, and in weight just sufficient to sink to its centre in common water; what is the proper thickness of copper of which it must be made?

$$\left. \begin{array}{l} \text{Weight of a cubic inch of water} = \cdot03617 \text{ lbs.} \\ \text{copper} = \cdot3225 \text{ ,,} \end{array} \right\} \text{ see p. 63.}$$

$$\frac{8^3 \times \cdot5236 \times \cdot03617}{2} = 4\cdot84828 \text{ cub. in. of water to be displaced.}$$

$$\text{And } \frac{4\cdot84828}{\cdot3225} = 15\cdot0334 \text{ cubic inches of copper in the ball.}$$

$$\text{Then } 8^2 \times 3\cdot1416 = 201\cdot0624, \text{ and } \frac{15\cdot0334}{201\cdot0624} = \cdot0747 \text{ inches, the}$$

thickness of copper required.

$$\cdot0747 \times 16 = \frac{1}{18} \text{ of an inch full, or 3 lbs. copper to a square foot.}$$

3. What diameter must I make a leaden ball, so as to weigh 72 lbs.?

$$\cdot5236 \times \cdot4103 = \cdot21483308, \text{ and } \frac{72}{\cdot21483308} = \sqrt[3]{340} = 6\cdot97$$

inches diameter.

To ascertain the amount of convex surface, also the solid contents, of the segment of a globe.

Rule 1.—Multiply the circumference of the globe or sphere by the height of the segment, and the product is the convex surface.

Rule 2.—To three times the square of the segment's radius, add the square of its height, multiply the sum by the height, and by $\cdot 5236$, the product is the solid contents.

Application of the rules.

1. Required the number of square feet in the convex surface of a sphere, the height of which is $9\frac{1}{2}$ feet, and the circumference of the sphere of which it is a part, equal $70\frac{1}{2}$ feet.

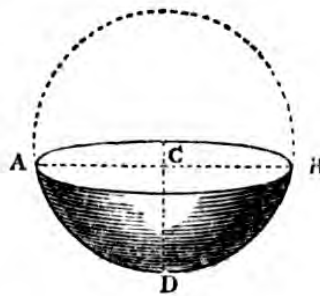
$$70\cdot 5 \times 9\cdot 5 = 669\cdot 75 \text{ square feet.}$$

2. The radius A C or B C of the spherical segment A D B equal 48 inches, and the height D C equal 12 inches ; required its solidity in cubic inches.

$$48^2 \times 3 = 6912, \text{ and } 12^2 = 144; \text{ then}$$

$$6912 + 144 \times 12 \times \cdot 5236 = 44334\cdot 75$$

cubic inches.



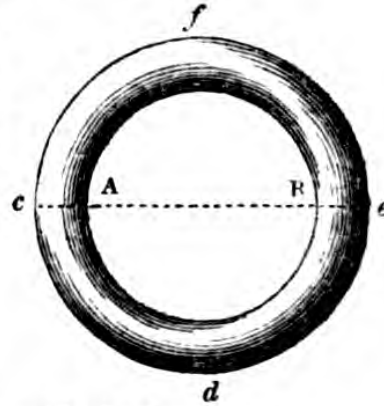
To find the convex surface and solidity of a cylindrical ring.

Rule 1. To the sectional diameter of the ring add the inner diameter of the circle, multiply the sum by the sectional diameter, and by $9\cdot 8696$, the product is the convex surface.

Rule 2. To the sectional diameter of the ring add the inner diameter of the circle, multiply the sum by the square of the sectional diameter, and by $2\cdot 4674$, the product is the solid contents.

Application of the rules.

The inner diameter AB of the cylindric ring $cdef$ equal 18 feet, and the sectional diameter cA or Be equal 9 inches; required the convex surface and solidity of the ring.



$$18 \text{ feet} \times 12 = 216 \text{ inches, and}$$

$$\frac{216 + 9 \times 9 \times 9 \cdot 8696}{\text{square inches.}} = 19985 \cdot 94$$

$$\frac{216 + 9 \times 9^2 \times 2 \cdot 4674}{\text{cubic inches.}} = 44968 \cdot 365$$

In the formation of a hoop or ring of wrought iron, it is found in practice that in bending the iron, the side or edge which forms the interior diameter of the hoop is upset or shortened, while at the same time the exterior diameter is drawn or lengthened; therefore, the proper diameter by which to determine the length of the iron in an unbent state, is the distance from centre to centre of the iron of which the hoop is composed; *hence the rule to determine the length of the iron.* If it is the interior diameter of the hoop that is given, add the thickness of the iron; but if the exterior diameter, subtract from the given diameter the thickness of the iron, multiply the sum or remainder by 3.1416, and the product is the length of the iron, in equal terms of unity.

Supposing the interior diameter of a hoop to be 32 inches, and the thickness of the iron $1\frac{1}{4}$, what must be the proper length of the iron, independent of any allowance for shutting?

$$32 + 1 \cdot 25 = 33 \cdot 25 \times 3 \cdot 1416 = 104 \cdot 458 \text{ inches.}$$

But the same is obtained simply by inspection in the Table of Circumferences.

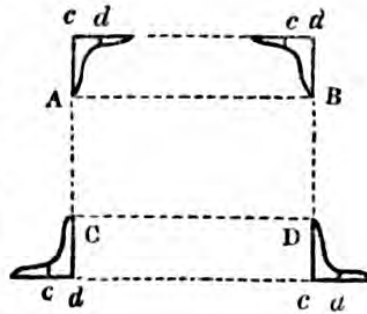
Thus, $33 \cdot 25 = 2 \text{ feet } 9\frac{1}{4} \text{ in.}$, opposite to which is $8 \text{ feet } 8\frac{1}{2} \text{ inches.}$

Again, let it be required to form a hoop of iron $\frac{7}{8}$ inch in thickness, and $16\frac{1}{2}$ inches outside diameter.

$$16.5 - .875 = 15.625, \text{ or } 1 \text{ foot } 3\frac{5}{8} \text{ inches ;}$$

opposite to which, in the Table of Circumferences, is 4 feet 1 inch, independent of any allowance for shutting.

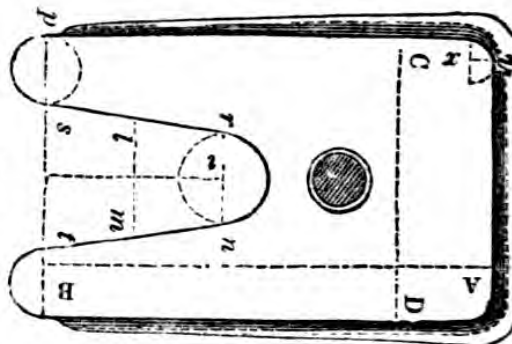
The length for angle iron, of which to form a ring of a given diameter, varies according to the strength of the iron at the root; and the rule is, for a ring with the flange outside, *add* to its required interior diameter, twice the extreme strength of the iron at the root; or, for a ring with the flange inside, *subtract* twice the extreme strength, and the sum or remainder is the diameter by which to determine the length of the angle iron. Thus, suppose two angle iron rings similar to the following be required, the exterior diameter A B, and interior diameter C D, each to be 1 foot $10\frac{1}{2}$ inches, and the extreme strength of the iron at the root $c d$, $c d$, &c. $\frac{7}{8}$ of an inch;



twice $\frac{7}{8} = 1\frac{3}{4}$, and $1 \text{ ft. } 10\frac{1}{2} \text{ in.} + 1\frac{3}{4} = 2 \text{ ft. } \frac{1}{4} \text{ in.}$, opposite to which, in the Table of Circumferences, is $6 \text{ ft. } 4\frac{1}{4} \text{ in.}$, the length of the iron for C D; and $1 \text{ ft. } 10\frac{1}{2} \text{ in.} - 1\frac{3}{4} = 1 \text{ ft. } 8\frac{3}{4} \text{ in.}$, opposite to which is $5 \text{ ft. } 5\frac{1}{4} \text{ in.}$, the length of the iron for A B.

But observe as before, that the necessary allowance for shutting must be added to the length of the iron, in addition to the length as expressed by the Table.

Required the capacity in imperial gallons of a Locomotive Engine Tender tank, 2 feet 8 inches in depth, and its superficial dimensions the following, with reference to the annexed plan.



Length, or dist. between A and B	= 10 ft. $2\frac{3}{4}$ in. or, 122.75 in.
Breadth	" C " D = 6 " $7\frac{1}{2}$ " 79.5 "
Length	" i " g = 3 " $10\frac{3}{4}$ " 46.75 "
Mean breadth of coke space or	} <i>lm</i> " = 3 " $1\frac{1}{4}$ " 37.25 "
Diameter of circle	
" " "	<i>ps</i> " = 1 " $6\frac{1}{2}$ " 18.5 "
Radius of back corners	<i>vx</i> " = 4 " 4 "

Then, $122.75 \times 79.5 = 9758.525$ square inches, as a rectangle.
 And $18.5^2 \times .7854 = 268.8$ " " area of circle formed by the two ends.

Total 10027.325 " " from which deduct the area of the coke space, and the difference of area between the semicircle formed by the two back corners, and that of a rectangle of equal length and breadth;

Then $46.75 \times 37.25 = 1731.4375$ area of *r, n, s, t*, in sq. ins.
 $32.25^2 \times .7854 = 408.4$ area of half the circle *rn*.

2

Radius of back corners = 4 inches;
 consequently $8^2 \times .7854 = 25.13$ the semicircle's area; and
 $8 \times 4 = 32 - 25.13 = 6.87$ inches taken off by rounding the corners.

Hence, $1731.4375 + 408.4 + 6.87 = 2146.707$, and
 $10027.325 - 2146.707 = 7880.618$ square inches, or
 whole area in plan,
 7880.618×32 the depth = 252179.776 cubic inches,
 and $252179.776 \times .003607$ (see Table of Approximations, page 30) = 909.61245 gallons.

PROPERTIES OF BODIES.

WOODS.							
Names.	Specific gravity, water, 1000.	Average wt. of a cubic foot in lbs.	Cubic feet in a ton.	Ultimate cohe- sive strength of an inch square prism in lbs.	Comparative		
					Stiffness.	Strength.	Resilience.
English oak	934	58	38 $\frac{1}{2}$	11880	100	100	100
Riga do.	872	54	41 $\frac{1}{2}$	12888	93	108	125
Dantzic do.	756	47	48	12780	117	107	99
American do.	672	42	53	10253	114	86	64
Beech	852	48	45	12225	77	103	138
Alder	800	46	48 $\frac{1}{2}$	9540	63	80	101
Plane	640	40	55	10935	78	92	108
Sycamore	604	38	59	9630	59	81	111
Chestnut	610	38	59	10656	67	89	118
Ash	845	52	43	14130	89	119	160
Elm	673	42	53	9720	78	82	86
Mahog. Spanish . .	800	50	45	7560	73	67	61
„ Honduras	637	40	55	11475	93	96	99
Walnut	671	42	53	8800	49	74	111
Teak	750	46	48 $\frac{1}{2}$	12915	126	109	94
Poona	640	40	55	12350	99	104	82
African oak	944	59	38	17200	101	144	138
Poplar	383	34	66	5928	44	50	57
Cedar	561	33	68	7420	28	62	106
Riga fir	753	47	48	9540	98	80	64
Memel do.	546	34	66	9540	114	80	56
Scotch do.	528	33	68	7110	55	60	65
Christ. Wh ^t . deal .	590	37	60	12346	104	104	104
Am ⁿ . white spruce	551	34	66	10296	72	86	102
Yellow pine	461	28	80	11853	95	99	103
Pitch pine	660	41	54 $\frac{1}{2}$	9796	73	82	92
Larch	530	31	72	12240	79	103	134
Cork	240	15	149	—	—	—	—

LIQUIDS.			GASES.	
Names.	Specific grav., water, 1000.	Weight of an imperial gall. in lbs.	Atmospheric air being the standard of comparison, or 1000.	
			Names.	Specific gravity.
Acid, sulphuric ..	1850	18·5	Hydriodic acid gas	4340
„ nitric	1271	12·7	Chlorine acid „	2500
„ muriatic....	1200	12·0	Carbonic acid „	1527
„ fluoric.....	1060	10·6	Nitrous oxide „	1527
„ citric	1034	10·3	Cyanogen „	1805
„ acetic	1062	10·6	Oxygen „	1111
Water from the Baltic	1015	10·2	Carbonic oxide „	972
Water from the Dead Sea.....	1240	12·4	Carbureted hydrogen gas	972
Water from the Mediterranean..	1029	10·3	Prussic acid „	937
Water, distilled ..	1000	10·0	Ammoniacal „	590
Oils, expressed...			Steam of water „	623
„ linseed	940	9·4	Hydrogen „	69
„ sweet almond	932	9·3	Weight of water at the com- mon temperature :	
„ whale.....	923	9·2	1 cubic inch =	·03617 lb.
„ hempseed...	926	9·3	1 „ foot =	62·5 „
„ olive.....	915	9·2	1 „ „ =	6·25 imp. galls.
Oils, essential....			1·8 „ feet =	1 cwt.
„ cinnamon... 1043		10·4	1 cylindrical inch =	·02842 lb.
„ lavender.... 894		8·9	1 „ foot =	49·1 „
„ turpentine .. 870		8·7	1 „ „ =	5 imp. galls.
„ amber..... 868		8·7	2·282 feet =	1 cwt.
Alcohol	825	8·2	11·2 imp. gallons =	1 cwt.
Ether, nitric	908	9·1	224 „ „ =	1 ton.
Proof spirit.....	922	9·2		
Vinegar	1009	10·1		

WEIGHT OF A LINEAL FOOT OF SQUARE AND
ROUND BAR IRON IN POUNDS.

Square iron.				Round iron.			
Inches square.	lbs.	Inches square.	lbs.	Inches diam.	lbs.	Inches diam.	lbs.
$\frac{1}{4}$	·208	2	13·33	$\frac{1}{4}$	·163	2	10·47
$\frac{5}{16}$	·325	$2\frac{1}{8}$	15·05	$\frac{5}{16}$	·255	$2\frac{1}{8}$	11·82
$\frac{3}{8}$	·468	$2\frac{1}{4}$	16·87	$\frac{3}{8}$	·368	$2\frac{1}{4}$	13·25
$\frac{7}{16}$	·638	$2\frac{3}{8}$	18·80	$\frac{7}{16}$	·501	$2\frac{3}{8}$	14·76
$\frac{1}{2}$	·833	$2\frac{1}{2}$	20·81	$\frac{1}{2}$	·654	$2\frac{1}{2}$	16·36
$\frac{9}{16}$	1·05	$2\frac{5}{8}$	22·96	$\frac{9}{16}$	·828	$2\frac{5}{8}$	18·03
$\frac{5}{8}$	1·30	$2\frac{3}{4}$	25·20	$\frac{5}{8}$	1·02	$2\frac{3}{4}$	19·79
$\frac{11}{16}$	1·57	$2\frac{7}{8}$	27·55	$\frac{11}{16}$	1·23	$2\frac{7}{8}$	21·63
$\frac{3}{4}$	1·87	3	30·00	$\frac{3}{4}$	1·47	3	23·56
$\frac{13}{16}$	2·20	$3\frac{1}{8}$	32·55	$\frac{13}{16}$	1·72	$3\frac{1}{8}$	25·56
$\frac{7}{8}$	2·55	$3\frac{1}{4}$	35·20	$\frac{7}{8}$	2·00	$3\frac{1}{4}$	27·65
$\frac{15}{16}$	2·92	$3\frac{3}{8}$	37·96	$\frac{15}{16}$	2·30	$3\frac{3}{8}$	29·82
1	3·33	$3\frac{1}{2}$	40·80	1	2·61	$3\frac{1}{2}$	32·07
$1\frac{1}{16}$	3·76	$3\frac{5}{8}$	43·81	$1\frac{1}{16}$	2·95	$3\frac{5}{8}$	34·40
$1\frac{1}{8}$	4·21	$3\frac{3}{4}$	46·87	$1\frac{1}{8}$	3·31	$3\frac{3}{4}$	36·81
$1\frac{3}{16}$	4·70	$3\frac{7}{8}$	50·05	$1\frac{3}{16}$	3·69	$3\frac{7}{8}$	39·31
$1\frac{1}{4}$	5·20	4	53·33	$1\frac{1}{4}$	4·09	4	41·88
$1\frac{5}{16}$	5·74	$4\frac{1}{4}$	60·20	$1\frac{5}{16}$	4·51	$4\frac{1}{4}$	47·28
$1\frac{3}{8}$	6·30	$4\frac{1}{2}$	67·50	$1\frac{3}{8}$	4·95	$4\frac{1}{2}$	53·01
$1\frac{7}{16}$	6·88	$4\frac{3}{4}$	75·20	$1\frac{7}{16}$	5·40	$4\frac{3}{4}$	59·06
$1\frac{1}{2}$	7·50	5	83·33	$1\frac{1}{2}$	5·89	5	65·45
$1\frac{9}{16}$	8·15	$5\frac{1}{4}$	92·43	$1\frac{9}{16}$	6·40	$5\frac{1}{4}$	72·61
$1\frac{5}{8}$	8·80	$5\frac{1}{2}$	101·03	$1\frac{5}{8}$	6·91	$5\frac{1}{2}$	79·36
$1\frac{11}{16}$	9·50	$5\frac{3}{4}$	110·40	$1\frac{11}{16}$	7·46	$5\frac{3}{4}$	86·73
$1\frac{3}{4}$	10·20	6	120·21	$1\frac{3}{4}$	8·01	6	94·60
$1\frac{13}{16}$	10·69	$6\frac{1}{2}$	130·20	$1\frac{13}{16}$	8·60	$6\frac{1}{2}$	110·60
$1\frac{7}{8}$	11·71	7	151·81	$1\frac{7}{8}$	9·20	7	128·28
$1\frac{15}{16}$	12·52	8	213·29	$1\frac{15}{16}$	9·33	8	167·51

Note.—The elastic power or direct tension of bar iron medium quality per square inch of cross section, equal 10 tons; and a bar is extended ·000096, or nearly one ten-thousandth part of its length for every ton of direct strain per square inch of its sectional area; note, also, that either of the metals,—iron, tin, or zinc,—at a red heat, possesses the property of decomposing water when in an aëriform state.

WEIGHT OF A LINEAL FOOT OF FLAT BAR IRON IN POUNDS.

Breadth in inches.	Thickness in parts of an inch.						
	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$
$\frac{1}{8}$.417	.52	—	—	—	—	—
$\frac{3}{16}$.52	.65	.78	.91	—	—	—
$\frac{1}{4}$.625	.785	.93	1.09	1.25	1.40	—
$\frac{5}{16}$.725	.91	1.09	1.27	1.46	1.64	1.82
1	.834	1.04	1.25	1.45	1.67	1.87	2.08
$1\frac{1}{8}$.937	1.17	1.40	1.64	1.87	2.10	2.34
$1\frac{1}{4}$	1.04	1.30	1.56	1.82	2.08	2.34	2.60
$1\frac{3}{8}$	1.14	1.43	1.71	2.00	2.29	2.57	2.86
$1\frac{1}{2}$	1.25	1.56	1.87	2.18	2.50	2.81	3.12
$1\frac{5}{8}$	1.35	1.69	2.03	2.36	2.70	3.04	3.38
$1\frac{3}{4}$	1.45	1.82	2.18	2.55	2.91	3.28	3.64
$1\frac{7}{8}$	1.56	1.95	2.34	2.73	3.12	3.51	3.90
2	1.66	2.08	2.50	2.91	3.33	3.75	4.16
$2\frac{1}{8}$	1.77	2.21	2.65	3.09	3.54	3.98	4.42
$2\frac{1}{4}$	1.87	2.34	2.81	3.28	3.75	4.21	4.68
$2\frac{3}{8}$	1.97	2.47	2.96	3.46	3.95	4.45	4.94
$2\frac{1}{2}$	2.08	2.60	3.12	3.64	4.16	4.68	5.20
$2\frac{5}{8}$	2.18	2.73	3.28	3.82	4.37	4.92	5.46
$2\frac{3}{4}$	2.29	2.86	3.43	4.01	4.58	5.15	5.72
$2\frac{7}{8}$	2.39	2.99	3.59	4.19	4.79	5.39	5.98
3	2.50	3.12	3.75	4.37	5.00	5.62	6.25
$3\frac{1}{4}$	2.70	3.38	4.06	4.73	5.41	6.09	6.77
$3\frac{1}{2}$	2.91	3.64	4.37	5.10	5.83	6.56	7.29
$3\frac{3}{4}$	3.12	3.90	4.68	5.46	6.25	7.03	7.81
4	3.33	4.16	5.00	5.83	6.66	7.50	8.33
$4\frac{1}{4}$	3.54	4.42	5.31	6.19	7.08	7.96	8.85
$4\frac{1}{2}$	3.75	4.68	5.62	6.56	7.50	8.43	9.37
$4\frac{3}{4}$	3.95	4.94	5.93	6.92	7.91	8.90	9.89
5	4.17	5.20	6.25	7.29	8.33	9.37	10.41
$5\frac{1}{4}$	4.37	5.46	6.56	7.65	8.75	9.84	10.93
$5\frac{1}{2}$	4.58	5.72	6.87	8.02	9.16	10.31	11.45
$5\frac{3}{4}$	4.79	5.98	7.18	8.38	9.58	10.78	11.97
6	5	6.26	7.50	8.75	10.00	11.25	12.50

WEIGHT OF FLAT BAR IRON.—(Continued.)

Breadth in inches.	Thickness in parts of an inch.						
	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	1	$1\frac{1}{8}$
1	2.29	—	—	—	—	—	—
$1\frac{1}{16}$	2.57	2.81	3.04	—	—	—	—
$1\frac{1}{4}$	2.86	3.12	3.38	3.64	3.90	—	—
$1\frac{3}{8}$	3.15	3.43	3.72	4.01	4.29	4.58	—
$1\frac{1}{2}$	3.43	3.75	4.06	4.37	4.68	5.00	5.62
$1\frac{5}{8}$	3.72	4.06	4.40	4.73	5.07	5.41	6.08
$1\frac{3}{4}$	4.01	4.37	4.73	5.10	5.46	5.83	6.56
$1\frac{7}{8}$	4.29	4.68	5.07	5.46	5.85	6.25	7.02
2	4.58	5.00	5.41	5.83	6.25	6.66	7.50
$2\frac{1}{8}$	4.86	5.31	5.75	6.19	6.64	7.08	7.96
$2\frac{1}{4}$	5.15	5.62	6.09	6.56	7.03	7.50	8.43
$2\frac{3}{8}$	5.44	5.93	6.43	6.92	7.42	7.91	8.90
$2\frac{1}{2}$	5.72	6.25	6.77	7.29	7.81	8.33	9.36
$2\frac{5}{8}$	6.01	6.56	7.10	7.65	8.20	8.75	9.84
$2\frac{3}{4}$	6.30	6.87	7.44	8.02	8.59	9.16	10.30
$2\frac{7}{8}$	6.58	7.18	7.78	8.38	8.98	9.58	10.78
3	6.87	7.50	8.12	8.75	9.37	10.00	11.25
$3\frac{1}{4}$	7.44	8.12	8.80	9.47	10.15	10.83	12.18
$3\frac{1}{2}$	8.02	8.75	9.47	10.20	10.93	11.66	13.12
$3\frac{3}{4}$	8.59	9.37	10.15	10.93	11.71	12.50	14.06
4	9.16	10.00	10.83	11.66	12.50	13.33	15.00
$4\frac{1}{4}$	9.73	10.62	11.51	12.39	13.28	14.16	15.92
$4\frac{1}{2}$	10.31	11.25	12.18	13.12	14.06	15.00	16.86
$4\frac{3}{4}$	10.88	11.87	12.86	13.85	14.84	15.83	17.80
5	11.45	12.50	13.54	14.58	15.62	16.66	18.75
$5\frac{1}{4}$	12.03	13.12	14.21	15.31	16.40	17.50	19.68
$5\frac{1}{2}$	12.60	13.75	14.89	16.04	17.18	18.33	20.62
$5\frac{3}{4}$	13.17	14.37	15.57	16.77	17.96	19.16	21.56
6	13.75	15.00	16.25	17.50	18.75	20.00	22.50
$6\frac{1}{2}$	14.88	16.24	17.60	18.95	20.30	21.66	24.36
7	16.04	17.50	18.94	20.41	21.86	23.33	26.24
$7\frac{1}{2}$	17.18	18.74	20.30	21.86	23.42	25.00	28.12
8	18.32	20.00	21.76	23.32	25.00	26.66	30.00

WEIGHT OF FLAT BAR IRON.—(Continued.)

Breadth in inches.	Thickness in inches.					
	1¼	1⅜	1½	1¾	2	2½
1½	6.24	6.86	—	—	—	—
1¾	7.28	8.02	8.74	—	—	—
2	8.32	9.16	10.00	11.66	—	—
2¼	9.36	10.30	11.24	13.12	15.00	—
2½	10.40	11.44	12.50	14.58	16.66	—
2¾	11.44	12.60	13.74	16.04	18.32	22.88
3	12.50	13.74	15.00	17.50	20.00	25.00
3¼	13.54	14.88	16.24	18.94	21.66	27.08
3½	14.58	16.04	17.50	20.40	23.32	29.16
3¾	15.62	17.18	18.74	21.86	25.00	31.24
4	16.66	18.32	20.00	23.32	26.66	33.32
4¼	17.70	19.46	21.24	25.78	28.32	35.40
4½	18.74	20.62	22.50	26.24	30.00	37.48
4¾	19.78	21.76	23.74	27.70	31.67	39.56
5	20.82	22.90	25.00	29.16	33.32	41.64
5¼	21.86	24.06	26.24	30.62	35.00	43.72
5½	22.90	25.20	27.50	32.08	36.66	45.80
5¾	23.94	26.34	28.74	33.54	38.32	47.88
6	25.00	27.50	30.00	35.00	40.00	50.00
6½	27.08	29.76	32.48	37.88	43.32	54.16
7	29.16	32.08	35.00	40.80	46.64	58.32
7½	31.24	34.36	37.48	43.72	50.00	62.48
8	33.32	36.64	40.00	46.64	53.32	66.64
8½	35.40	38.92	42.48	51.56	56.64	70.80
9	37.48	41.24	45.00	52.48	60.00	74.96

*Proportional breadths for hexagonal or six-sided nuts
for wrought iron bolts.*

Dia. of bolts.	Breadth of nuts.	Dia. of bolts.	Breadth of nuts.
⅜	¾ inch.	1⅛	1½ inch.
½	⅞ "	1¼	2⅜ "
⅝	1⅛ "	1⅜	2⅝ "
¾	1⅝ "	1½	2⅞ "
7⁄8	1½ "	1⅝	2¾ "
1	1¾ "	1¾	3 "

Note.—The thickness of the nut is equal the bolt's diameter.

WEIGHT OF A SUPERFICIAL FOOT OF PLATE OR SHEET IRON,
COPPER, AND BRASS, IN POUNDS.

Thickness in parts of an inch.		Thickness by the wire gauge.				Thickness by the wire gauge.			
Iron.	No.	Iron.	Copper.	Brass.	No.	Iron.	Copper.	Brass.	No.
$\frac{1}{32}$	1	12.5	14.5	13.75	16	2.5	2.9	2.75	16
$\frac{1}{16}$	2	12	13.9	13.2	17	2.18	2.52	2.4	17
$\frac{3}{16}$	3	11	12.75	12.1	18	1.86	2.15	2.04	18
$\frac{1}{4}$	4	10	11.6	11	19	1.7	1.97	1.87	19
$\frac{5}{16}$	5	8.74	10.1	9.61	20	1.54	1.78	1.69	20
$\frac{3}{8}$	6	8.12	9.4	8.93	21	1.4	1.62	1.54	21
$\frac{7}{16}$	7	7.5	8.7	8.25	22	1.25	1.45	1.37	22
$\frac{1}{2}$	8	6.86	7.9	7.54	23	1.12	1.3	1.23	23
$\frac{9}{16}$	9	6.24	7.2	6.86	24	1	1.16	1.1	24
$\frac{5}{8}$	10	5.62	6.5	6.18	25	.9	1.04	.99	25
$\frac{11}{16}$	11	5	5.8	5.5	26	.8	.92	.88	26
$\frac{3}{4}$	12	4.38	5.08	4.81	27	.72	.83	.79	27
$\frac{7}{8}$	13	3.75	4.34	4.12	28	.64	.74	.7	28
$\frac{1}{2}$	14	3.12	3.6	3.43	29	.56	.64	.61	29
$\frac{1}{2}$	15	2.82	3.27	3.1	30	.5	.58	.55	30

Note.—No. 1 wire gauge equal $\frac{5}{16}$ ths of an inch.

" 4	"	$\frac{1}{4}$	"
" 7	"	$\frac{3}{16}$	"
" 11	"	$\frac{1}{2}$	"
" 16	"	$\frac{3}{8}$	"
" 22	"	$\frac{1}{2}$	"

The great variety of thicknesses into which copper is manufactured, cause in trade the weight to be named whereby to determine the thickness required, the unit

COMPARATIVE WEIGHTS OF BODIES. 61

being that of a common sheet, so designated, viz., 4 feet by 2 feet, in lbs., thus :

A 70 lb. plate is $\frac{3}{16}$ ths of an inch in thickness.

46½	”	$\frac{1}{8}$	”
23	”	$\frac{1}{16}$	”
11½	”	$\frac{1}{32}$	”
6	”	$\frac{1}{64}$	&c., &c.

The thickness of lead is also in common determined or understood by the weight, the unit being that of a square or superficial foot ; thus,

4 lb. lead is $\frac{1}{16}$ th of an inch in thickness.

6	”	$\frac{1}{10}$	”
7½	”	$\frac{1}{8}$	”
11	”	$\frac{3}{16}$	”
15	”	$\frac{1}{4}$	”

COMPARATIVE WEIGHTS OF DIFFERENT BODIES.

Bar iron being 1.	Cast iron being 1.	Dry deal being 1.
Cast iron = .95	Bar iron = 1.07	Cast iron = 11.0
Steel = 1.02	Steel = 1.08	Cast tin = 11.2
Copper = 1.16	Brass = 1.16	Brass = 12.7
Brass = 1.09	Copper = 1.21	Copper = 13.3
Lead = 1.48	Lead = 1.56	Lead = 17.1

1. Suppose I have an article of plate iron, the weight of which is 728 lbs., but want the same of copper, and of similar dimensions, what will be its weight ?

$$728 \times 1.16 = 844.48 \text{ lbs.}$$

2. A model of dry pine weighing 32½ lbs., and in which the iron for its construction forms no material portion of the weight, what may I anticipate its weight to be in cast iron ?

$$32.5 \times 11 = 357.5 \text{ lbs.}$$

Note.—It frequently occurs in the formation or construction

CAST IRON PIPES.

of models, that neither the quality nor condition of the timber can be properly estimated, and in such cases it may be a near enough approximation to reckon 10 lbs. of cast iron to each lb. of model.

WEIGHTS OF 9-FOOT LENGTHS OF CAST IRON PIPES,
OF VARIOUS DIAMETERS.

Diameter of bore.	Thickness of metal.	Diameter of flange.	Thickness of flange.	Diameter of circle through holes.	Diameter and number of holes.		Weight in cwts. qrs. lbs.		
in.	in.	in.	in.	in.	in.				
2	$\frac{3}{8}$	$6\frac{1}{2}$	$\frac{9}{16}$	$4\frac{3}{4}$	$\frac{5}{8}$	4	0	3	0
3	$\frac{3}{8}$	$7\frac{1}{2}$	$\frac{5}{8}$	6	$\frac{5}{8}$	4	1	0	3
4	$\frac{1}{2}$	$9\frac{1}{2}$	$\frac{3}{4}$	$7\frac{3}{4}$	$\frac{3}{4}$	4	1	3	5
5	$\frac{1}{2}$	$10\frac{1}{2}$	$\frac{7}{8}$	$8\frac{3}{4}$	$\frac{3}{4}$	4	2	1	12
6	$\frac{5}{8}$	12	$\frac{7}{8}$	10	$\frac{7}{8}$	4	3	2	1
7	$\frac{5}{8}$	14	1	$11\frac{3}{4}$	$\frac{7}{8}$	6	4	3	17
8	$\frac{3}{4}$	15	1	$12\frac{3}{4}$	1	6	5	2	9
9	$\frac{3}{4}$	$16\frac{1}{2}$	$1\frac{1}{16}$	$14\frac{1}{4}$	1	6	6	1	12
10	$\frac{3}{4}$	$17\frac{1}{2}$	$1\frac{1}{8}$	$15\frac{1}{2}$	1	6	7	0	0
11	$\frac{7}{8}$	19	$1\frac{3}{16}$	$16\frac{3}{4}$	1	6	8	3	24
12	$\frac{7}{8}$	20	$1\frac{1}{4}$	$17\frac{3}{4}$	$1\frac{1}{8}$	6	9	3	5
13	$\frac{7}{8}$	21	$1\frac{1}{4}$	$18\frac{3}{4}$	$1\frac{1}{8}$	6	10	2	0
14	$\frac{7}{8}$	22	$1\frac{1}{4}$	$19\frac{3}{4}$	$1\frac{1}{8}$	8	11	0	26
15	$\frac{7}{8}$	23	$1\frac{1}{4}$	$20\frac{3}{4}$	$1\frac{1}{8}$	8	12	0	25
16	$\frac{7}{8}$	$24\frac{1}{2}$	$1\frac{5}{16}$	22	$1\frac{1}{4}$	8	12	3	8
17	$\frac{7}{8}$	$25\frac{1}{2}$	$1\frac{5}{16}$	23	$1\frac{1}{4}$	8	13	2	17
18	1	$26\frac{1}{2}$	$1\frac{3}{8}$	24	$1\frac{1}{4}$	8	16	1	15
19	1	28	$1\frac{3}{8}$	25	$1\frac{3}{8}$	8	17	2	13
20	1	29	$1\frac{3}{8}$	26	$1\frac{3}{8}$	8	18	0	26

Weights of leaden pipes.

$\frac{3}{4}$	inch bore weighs	10 lbs. per yard.		
1	"	"	12	"
$1\frac{1}{4}$	"	"	16	"
$1\frac{1}{2}$	"	"	18	"
$1\frac{3}{4}$	"	"	21	"
2	"	"	24	"

TO ASCERTAIN THE WEIGHTS OF PIPES OF VARIOUS METALS, AND ANY DIAMETER REQUIRED.

Thickness in parts of an inch.	Wrought iron.	Copper.	Lead.
$\frac{1}{32}$.326	11½ lbs. plate .38	2 lbs. lead .483
$\frac{1}{16}$.653	23½ " .76	4 " .967
$\frac{3}{32}$.976	35 " 1.14	5½ " 1.45
$\frac{1}{8}$	1.3	46½ " 1.52	8 " 1.933
$\frac{5}{32}$	1.627	58 " 1.9	9¼ " 2.417
$\frac{3}{16}$	1.95	70 " 2.28	11 " 2.9
$\frac{7}{32}$	2.277	80½ " 2.66	13 " 3.383
$\frac{1}{4}$	2.6	93 " 3.04	15 " 3.867

Rule.—To the interior diameter of the pipe, in inches, add the thickness of the metal; multiply the sum by the decimal numbers opposite the required thickness and under the metal's name; also by the length of the pipe in feet, and the product is the weight of the pipe in lbs.

1. Required the weight of a copper pipe whose interior diameter is $7\frac{1}{2}$ inches, its length $6\frac{1}{4}$ feet, and the metal $\frac{1}{8}$ of an inch in thickness.

$$7.5 + .125 = 7.625 \times 1.52 \times 6.25 = 72.4 \text{ lbs.}$$

2. What is the weight of a leaden pipe $18\frac{1}{2}$ feet in length, 3 inches interior diameter, and the metal $\frac{1}{4}$ of an inch in thickness?

$$3 + .25 = 3.25 \times 3.867 \times 18.5 = 232.5 \text{ lbs.}$$

Note.—Weight of a cubic inch of

Lead	equal	.4103	lbs.
Copper, sheet	"	.3225	"
Brass, do.	"	.3037	"
Iron, do.	"	.279	"
Iron, cast	"	.263	"
Tin, do.	"	.2636	"
Zinc, do.	"	.26	"
Water	"	.03617	"

WEIGHT OF CAST IRON BALLS.

Dia- meter in inches.	Weight in lbs.	Dia- meter in inches.	Weight in lbs.	Dia- meter in inches.	Weight in lbs.
2	1.10	6	29.72	10	137.71
2 $\frac{1}{4}$	1.57	6 $\frac{1}{4}$	33.62	10 $\frac{1}{4}$	148.28
2 $\frac{1}{2}$	2.15	6 $\frac{1}{2}$	37.80	10 $\frac{1}{2}$	159.40
2 $\frac{3}{4}$	2.86	6 $\frac{3}{4}$	42.35	10 $\frac{3}{4}$	171.05
3	3.72	7	47.21	11	183.29
3 $\frac{1}{4}$	4.71	7 $\frac{1}{4}$	52.47	11 $\frac{1}{4}$	196.10
3 $\frac{1}{2}$	5.80	7 $\frac{1}{2}$	58.06	11 $\frac{1}{2}$	209.43
3 $\frac{3}{4}$	7.26	7 $\frac{3}{4}$	64.09	11 $\frac{3}{4}$	223.40
4	8.81	8	70.49	12	237.94
4 $\frac{1}{4}$	10.57	8 $\frac{1}{4}$	77.32	12 $\frac{1}{4}$	253.13
4 $\frac{1}{2}$	12.55	8 $\frac{1}{2}$	84.56	12 $\frac{1}{2}$	268.97
4 $\frac{3}{4}$	14.76	8 $\frac{3}{4}$	92.24	12 $\frac{3}{4}$	285.37
5	17.12	9	100.39	13	302.41
5 $\frac{1}{4}$	19.93	9 $\frac{1}{4}$	108.98	13 $\frac{1}{4}$	320.80
5 $\frac{1}{2}$	22.91	9 $\frac{1}{2}$	118.06	13 $\frac{1}{2}$	338.81
5 $\frac{3}{4}$	26.18	9 $\frac{3}{4}$	127.63	13 $\frac{3}{4}$	357.93

1. What will be the weight of a hollow ball or shell of cast iron, the external diameter being $9\frac{1}{2}$ and internal diameter $8\frac{3}{4}$ inches?

Opposite $9\frac{1}{2}$ are 118.06, and
Opposite $8\frac{3}{4}$ are 92.24, subtract

—25.82 lbs., weight required.

2. Requiring to remove a cast iron ball 37.8 lbs. in weight, and in diameter $6\frac{1}{2}$ inches, and replace it by one of lead of an equal weight, what must be the diameter of the leaden ball?

Weight of lead to that of cast iron = 1.56 (see Table, page 61).

Then $\frac{6.5^3}{1.56} = \sqrt[3]{176} = 5.6$ inches, the diameter.

TABLES BY WHICH TO FACILITATE THE MENSURATION OF TIMBER.

1. *Flat or Board Measure.*

Breadth in inches.	Area of a lineal foot.	Breadth in inches.	Area of a lineal foot.	Breadth in inches.	Area of a lineal foot.
$\frac{1}{4}$	·0208	4	·3334	8	·6667
$\frac{1}{2}$	·0417	$4\frac{1}{4}$	·3542	$8\frac{1}{4}$	·6875
$\frac{3}{4}$	·0625	$4\frac{1}{2}$	·375	$8\frac{1}{2}$	·7084
1	·0834	$4\frac{3}{4}$	·3958	$8\frac{3}{4}$	·7292
$1\frac{1}{4}$	·1042	5	·4167	9	·75
$1\frac{1}{2}$	·125	$5\frac{1}{4}$	·4375	$9\frac{1}{4}$	·7708
$1\frac{3}{4}$	·1459	$5\frac{1}{2}$	·4583	$9\frac{1}{2}$	·7917
2	·1667	$5\frac{3}{4}$	·4792	$9\frac{3}{4}$	·8125
$2\frac{1}{4}$	·1875	6	·5	10	·8334
$2\frac{1}{2}$	·2084	$6\frac{1}{4}$	·5208	$10\frac{1}{4}$	·8542
$2\frac{3}{4}$	·2292	$6\frac{1}{2}$	·5416	$10\frac{1}{2}$	·875
3	·25	$6\frac{3}{4}$	·5625	$10\frac{3}{4}$	·8959
$3\frac{1}{4}$	·2708	7	·5833	11	·9167
$3\frac{1}{2}$	·2916	$7\frac{1}{4}$	·6042	$11\frac{1}{4}$	·9375
$3\frac{3}{4}$	·3125	$7\frac{1}{2}$	·625	$11\frac{1}{2}$	·9583
		$7\frac{3}{4}$	·6458	$11\frac{3}{4}$	·9792

Application and use of the Table.

1. Required the number of square feet in a board or plank $16\frac{1}{2}$ feet in length, and $9\frac{3}{4}$ inches in breadth.

Opposite $9\frac{3}{4}$ is $\cdot8125 \times 16\cdot5 = 13\cdot4$ square feet.

2. A board 1 foot $2\frac{3}{4}$ inches in breadth, and 21 feet in length; what is its superficial content in square feet?

Opposite $2\frac{3}{4}$ is $\cdot2292$, to which add the 1 foot; then $1\cdot2292 \times 21 = 25\cdot8$ square feet.

3. In a board $15\frac{1}{2}$ inches at one end, 9 inches at the other, and $14\frac{1}{2}$ feet in length; how many square feet?

$$\frac{15\cdot5 + 9}{2} = 12\frac{1}{4}, \text{ or } 1\cdot0208; \text{ and } 1\cdot0208 \times 14\cdot5 = 14\cdot8 \text{ sq. ft.}$$

2. *Cubic or Solid Measure.*

Mean $\frac{1}{4}$ girth in inches.	Cubic feet in each lineal foot.	Mean $\frac{1}{4}$ girth in inches.	Cubic feet in each lineal foot.	Mean $\frac{1}{4}$ girth in inches.	Cubic feet in each lineal foot.
6	·25	14	1·361	22	3·362
6 $\frac{1}{4}$	·272	14 $\frac{1}{4}$	1·41	22 $\frac{1}{4}$	3·438
6 $\frac{1}{2}$	·294	14 $\frac{1}{2}$	1·46	22 $\frac{1}{2}$	3·516
6 $\frac{3}{4}$	·317	14 $\frac{3}{4}$	1·511	22 $\frac{3}{4}$	3·598
7	·340	15	1·562	23	3·673
7 $\frac{1}{4}$	·364	15 $\frac{1}{4}$	1·615	23 $\frac{1}{4}$	3·754
7 $\frac{1}{2}$	·39	15 $\frac{1}{2}$	1·668	23 $\frac{1}{2}$	3·835
7 $\frac{3}{4}$	·417	15 $\frac{3}{4}$	1·722	23 $\frac{3}{4}$	3·917
8	·444	16	1·777	24	4
8 $\frac{1}{4}$	·472	16 $\frac{1}{4}$	1·833	24 $\frac{1}{4}$	4·084
8 $\frac{1}{2}$	·501	16 $\frac{1}{2}$	1·89	24 $\frac{1}{2}$	4·168
8 $\frac{3}{4}$	·531	16 $\frac{3}{4}$	1·948	24 $\frac{3}{4}$	4·254
9	·562	17	2·006	25	4·34
9 $\frac{1}{4}$	·594	17 $\frac{1}{4}$	2·066	25 $\frac{1}{4}$	4·428
9 $\frac{1}{2}$	·626	17 $\frac{1}{2}$	2·126	25 $\frac{1}{2}$	4·516
9 $\frac{3}{4}$	·659	17 $\frac{3}{4}$	2·187	25 $\frac{3}{4}$	4·605
10	·694	18	2·25	26	4·694
10 $\frac{1}{4}$	·73	18 $\frac{1}{4}$	2·313	26 $\frac{1}{4}$	4·785
10 $\frac{1}{2}$	·766	18 $\frac{1}{2}$	2·376	26 $\frac{1}{2}$	4·876
10 $\frac{3}{4}$	·803	18 $\frac{3}{4}$	2·442	26 $\frac{3}{4}$	4·969
11	·84	19	2·506	27	5·062
11 $\frac{1}{4}$	·878	19 $\frac{1}{4}$	2·574	27 $\frac{1}{4}$	5·158
11 $\frac{1}{2}$	·918	19 $\frac{1}{2}$	2·64	27 $\frac{1}{2}$	5·252
11 $\frac{3}{4}$	·959	19 $\frac{3}{4}$	2·709	27 $\frac{3}{4}$	5·348
12	1·	20	2·777	28	5·444
12 $\frac{1}{4}$	1·042	20 $\frac{1}{4}$	2·898	28 $\frac{1}{4}$	5·542
12 $\frac{1}{2}$	1·085	20 $\frac{1}{2}$	2·917	28 $\frac{1}{2}$	5·64
12 $\frac{3}{4}$	1·129	20 $\frac{3}{4}$	2·99	28 $\frac{3}{4}$	5·74
13	1·174	21	3·062	29	5·84
13 $\frac{1}{4}$	1·219	21 $\frac{1}{4}$	3·136	29 $\frac{1}{4}$	5·941
13 $\frac{1}{2}$	1·265	21 $\frac{1}{2}$	3·209	29 $\frac{1}{2}$	6·044
13 $\frac{3}{4}$	1·313	21 $\frac{3}{4}$	3·285	29 $\frac{3}{4}$	6·146

In the cubic estimation of timber, custom has established the rule of $\frac{1}{4}$ the mean girth being the side of the square considered as the cross sectional dimensions; hence, multiply the number of cubic feet per lineal foot as in the Table of Cubic Measure opposite the $\frac{1}{4}$ girth, and the product is the solidity of the given dimensions in cubic feet.

Suppose the mean $\frac{1}{4}$ girth of a tree $21\frac{1}{4}$ inches, and its length 16 feet, what are its contents in cubic feet?

$$3.136 \times 16 = 50.176 \text{ cubic feet.}$$

Battens, Deals, and Planks, as imported into this country, are each similar in their various lengths, but differing in their widths and thicknesses, and hence their principle distinction: thus, a batten is 7 inches by $2\frac{1}{2}$,—a deal 9 by 3,—and a plank 11 by 3,—these being what are termed the standard dimensions, by which they are bought and sold, the length of each being taken at 12 feet; therefore, in estimating for the proper value of any quantity, nothing more is required than their lineal dimensions, by which to ascertain the number of times 12 feet there are in the given whole.

Suppose I wish to purchase the following:

7 of	6 feet	$6 \times 7 = 42$ feet
5 "	14 "	$14 \times 5 = 70$ "
11 "	19 "	$19 \times 11 = 209$ "
and 6 "	21 "	$21 \times 6 = 126$ "

$$12)447(37.25 \text{ standard deals.}$$

TABLE SHOWING THE NUMBER OF LINEAL FEET OF SCANTLING OF VARIOUS DIMENSIONS, WHICH ARE EQUAL TO A CUBIC FOOT.

		In.	Ft. In.				In.	Ft. In.				In.	Ft. In.									
2 inches by	require in length	2	36	0	4 inches by	4	9	0	6 inches by	9½	2	6	7 inches by	7	2	11						
		2½	28	9		4½	8	0		10	2	5		7½	2	9	10½	2	3			
		3	24	0		5	7	2		10½	2	2		8	2	6	11	2	2	11½	2	1
		3½	20	7		5½	6	6		11	2	2		8½	2	5	12	2	0	9	2	3
		4	18	0		6	6	0		11½	1	11		9	2	3	9½	2	2	10	1	10
		4½	16	0		6½	5	6		10	2	10		10	2	1	10½	2	1	11½	1	9
		5	14	5		7	7	1		10½	3	5		11	1	10	11	1	10	12	1	8
		5½	13	1		7½	7½	1		11	3	3		11½	1	9	12	1	8	8	2	3
		6	12	0		8	8	0		11½	3	2		12	1	9	8½	2	1	9½	1	10
		6½	11	1		8½	8½	1		12	3	0		9	1	7	10	1	8	10½	1	8
		7	10	3		9	9	3		5	9	9		11	1	7	11	1	7	11½	1	7
		7½	9	7		9½	9½	7		10	10	10		12	1	6	12	1	6	9	1	9
8	9	0	10	10	8	10½	2	10	9	1	8	9	1	8	10	1	7					
8½	8	6	10½	10½	8	11	2	8	10½	1	6	11	1	5	11	1	5					
9	8	0	11	11	8	11½	2	6	11½	1	4	12	1	4	12	1	4					
9½	7	7	11½	11½	9	12	2	4	12	1	4	9	1	9	10	1	5					
10	7	3	12	12	9	5	9	9	8	2	3	8½	2	1	9½	1	8					
10½	6	10	5	5	9	5½	5	3	9	1	10	9	1	8	10	1	7					
11	6	6	6	6	10	6	4	10	10	1	8	10	1	6	11	1	6					
11½	6	4	6½	6½	10½	7	10	10	11	1	7	11	1	7	11½	1	5					
12	6	0	7	7	11	7½	10	10	12	1	6	12	1	6	12	1	4					
3 inches by	require in length	3	16	0	5 inches by	7	4	1	8 inches by	9	1	10	9 inches by	9	1	9						
		3½	13	8		7½	3	10		10	1	9		10	1	8	9½	1	8			
		4	12	0		8	3	7		10½	1	8		11	1	7	10	1	7			
		4½	10	8		8½	3	5		11	1	7		11½	1	7	10½	1	6			
		5	9	7		9	3	2		11½	1	7		12	1	6	12	1	6			
		5½	9	0		9½	3	0		12	1	6		9	1	9	9	1	9			
		6	8	0		10	2	10		10	1	8		9½	1	8	10	1	8			
		6½	7	4		10½	2	9		11	1	7		10	1	7	10½	1	6			
		7	6	10		11	2	8		11½	1	6		11	1	5	11	1	5			
		7½	6	4		11½	2	6		12	1	4		11½	1	4	11½	1	4			
		8	6	0		12	2	4		6	1	4		12	1	4	12	1	4			
		8½	5	8		6	4	0		6	1	4		9	1	9	9	1	9			
9	5	4	6½	3	8	7	1	8	9½	1	8	10	1	8								
9½	5	0	7	3	5	7½	1	8	10	1	8	10½	1	7								
10	4	10	7½	3	2	8	1	7	11	1	7	11	1	6								
10½	4	6	8	2	10	8	1	6	11½	1	5	11½	1	5								
11	4	4	8½	2	10	9	1	4	12	1	4	12	1	4								
11½	4	2	9	2	8	10	1	4	10	1	5	10	1	4								
12	4	0	9	2	8	11	1	4	11	1	4	11	1	4								
			9	2	8	11½	1	3	11½	1	3	11½	1	3								

INSTRUMENTAL ARITHMETIC,

OR UTILITY OF THE SLIDE RULE.

THE slide rule is an instrument by which the greater portion of operations in arithmetic and mensuration may be advantageously performed, provided the lines of division and gauge points be made properly correct, and their several values familiarly understood.

The lines of division are distinguished by the letters A B C D; A B and C being each divided alike, and containing what is termed a double radius, or double series of logarithmic numbers, each series being supposed to be divided into 1000 equal parts, and distributed along the radius in the following manner :

From 1 to 2	contains	301	of those parts,	being the log. of	2.
”	3	”	477	”	3.
”	4	”	602	”	4.
”	5	”	699	”	5.
”	6	”	778	”	6.
”	7	”	845	”	7.
”	8	”	903	”	8.
”	9	”	954	”	9.
			1000	being the whole number.	

The line D on the improved rules consists of only a single radius ; and although of larger radius, the logarithmic series is the same, and disposed of along the line in a similar proportion, forming exactly a line of square roots to the numbers on the lines B C.

NUMERATION.

Numeration teaches us to estimate or properly value the numbers and divisions on the rule in an arithmetical form.

Their values are all entirely governed by the value set upon the first figure, and being decimally reckoned, advance tenfold from the commencement to the termination of each radius: thus, suppose 1 at the joint be one, the 1 in the middle of the rule is ten, and 1 at the end one hundred: again, suppose 1 at the joint ten, 1 in the middle is 100, and 1 or 10 at the end is 1000, &c., the intermediate divisions on which complete the whole system of its notation.

TO MULTIPLY NUMBERS BY THE RULE.

Set 1 on B opposite to the multiplier on A; and against the number to be multiplied on B is the product on A.

Multiply 6 by 4.

Set 1 on B to 4 on A; and against 6 on B is 24 on A. The slide thus set, against 7 on B is 28 on A.

8	”	32	”
9	”	36	”
10	”	40	”
12	”	48	”
15	”	60	”
25	”	100, &c., &c.	

TO DIVIDE NUMBERS UPON THE RULE.

Set the divisor on B to 1 on A; and against the number to be divided on B is the quotient on A.

Divide 63 by 3.

Set 3 on B to 1 on A; and against 63 on B is 21 on A.

PROPORTION, OR RULE OF THREE DIRECT.

Rule.—Set the first term on B to the second on A ; and against the third upon B is the fourth upon A.

1. If 4 yards of cloth cost 38 shillings, what will 30 yards cost at the same rate ?

Set 4 on B to 38 on A ; and against 30 on B is 285 shillings on A.

2. Suppose I pay 31*s.* 6*d.* for 3 cwt. of iron, at what rate is that per ton ? 1 *ton* = 20 *cwt.*

Set 3 upon B to 31·5 upon A ; and against 20 upon B is 210 upon A.

RULE OF THREE INVERSE.

Rule.—Invert the slide, and the operation is the same as direct proportion.

1. I know that six men are capable of performing a certain given portion of work in eight days, but I want the same performed in three ; how many men must there be employed ?

Set 6 upon C to 8 upon A ; and against 3 upon C is 16 upon A.

2. The lever of a safety valve is 20 inches in length, and 5 inches between the fixed end and centre of the valve ; what weight must there be placed on the end of the lever to equipoise a force or pressure of 40 lbs. tending to raise the valve ?

Set 5 upon C to 40 upon A ; and against 20 on C is 10 on A.

3. If $8\frac{3}{4}$ yards of cloth, $1\frac{1}{2}$ yards in width, be a sufficient quantity, how much will be required of that which is only $\frac{7}{8}$ ths in width, to effect the same purpose ?

Set 1·5 on C to 8·75 on A ; and against ·875 upon C is 15 yards upon A.

SQUARE AND CUBE ROOTS OF NUMBERS.

On the engineer's rule, when the lines C and D are equal at both ends, C is a table of squares, and D a table of roots, as

Squares	1	4	9	16	25	36	49	64	81	on C.
Roots	1	2	3	4	5	6	7	8	9	on D.

To find the geometrical mean proportion between two numbers.

Set one of the numbers upon C to the same number upon D; and against the other number upon C is the mean number or side of an equal square upon D.

Required the mean proportion between 20 and 45.
Set 20 upon C to 20 upon D; and against 45 upon C is 30 on D.

To cube any number, set the number upon C to 1 or 10 upon D; and against the same number upon D is the cube number upon C.

Required the cube of 4.

Set 4 upon C to 1 or 10 upon D; and against 4 upon D is 64 upon C.

To extract the cube root of any number, invert the slide, and set the number upon B to 1 or 10 upon D; and where two numbers of equal value coincide on the lines B D, is the root of the given number.

Required the cube root of 64.

Set 64 upon B to 1 or 10 upon D; and against 4 upon B is 4 upon D, or root of the given number.

On the common rule, when 1 in the middle of the line C is set opposite to 10 on D, then C is a table of squares, and D a table of roots.

To cube any number by this rule, set the number upon C to 10 upon D; and against the same number upon D is the cube upon C.

MENSURATION OF SURFACE.

1. *Squares, Rectangles, &c.*

Rule.—When the length is given in feet and the breadth in inches, set the breadth on B to 12 on A; and against the length on A is the content in square feet on B.

If the dimensions are all inches, set the breadth on B to 144 upon A; and against the length upon A is the number of square feet on B.

Required the content of a board 15 inches broad and 14 feet long.

Set 15 upon B to 12 upon A; and against 14 upon A is 17.5 square feet on B.

2. *Circles, Polygons, &c.*

Rule.—Set .7854 upon C to 1 or 10 upon D; then will the lines C and D be a table of areas and diameters.

Areas	3.14	7.06	12.56	19.63	28.27	38.48	50.26	63.61	upon C.
Diam.	2	3	4	5	6	7	8	9	upon D.

In the common Rule, set .7854 on C to 10 on D; then C is a line or table of areas, and D of diameters, as before.

Set 7 upon B to 22 upon A; then B and A form or become a table of diameters and circumferences of circles.

Cir.	3.14	6.28	9.42	12.56	15.7	18.85	22	25.13	28.27	upon A.
Dia.	1	2	3	4	5	6	7	8	9	upon B.

Polygons from 3 to 12 sides.—Set the gauge-point upon C to 1 or 10 upon D; and against the length of one side upon D is the area upon C.

Sides		3	5	6	7	8	9	10	11	12
Gauge-points	.433	1.7	2.6	3.63	4.82	6.18	7.69	9.37	11.17	

Required the area of an equilateral triangle, each side 12 inches in length.

Set .433 upon C to 1 upon D; and against 12 upon D are 62.5 square inches upon C.

TABLE OF GAUGE-POINTS FOR THE ENGINEER'S RULE.							
Names.	F, F, F.	F, I, I.	I, I, I.	F, I.	I, I.	F.	I.
Cubic inches	578	83	1728	106	1273	105	121
Cubic feet	1	144	1	1833	22	121	33
Imp. gallons	163	231	277	294	353	306	529
Water in lbs.	16	23	276	293	352	305	528
Gold	814	1175	141	149	178	155	269
Silver	15	216	261	276	334	286	5
Mercury	118	169	203	216	258	225	389
Brass	193	177	333	354	424	369	637
Copper	18	26	319	331	397	345	596
Lead	141	203	243	258	31	27	465
Wro ^t . iron	207	297	357	338	453	394	682
Cast iron	222	32	384	407	489	424	733
Tin	219	315	378	401	481	419	728
Steel	202	292	352	372	448	385	671
Coal	127	183	22	33	28	242	42
Marble	591	85	102	116	13	113	195
Freestone	632	915	11	1162	14	141	21

FOR THE COMMON SLIDE RULE.							
Names.	F, F, F.	F, I, I.	I, I, I.	F, I.	I, I.	F.	I.
Cubic inches	36	518	624	660	799	625	113
Cubic feet	625	9	108	114	138	119	206
Water in lbs.	10	144	174	184	22	191	329
Gold	507	735	88	96	118	939	180
Silver	938	136	157	173	208	173	354
Mercury	738	122	127	132	162	141	242
Brass	12	174	207	221	265	23	397
Copper	112	163	196	207	247	214	371
Lead	880	126	152	162	194	169	289
Wro ^t . iron	129	186	222	235	283	247	423
Cast iron	139	2	241	254	304	265	458
Tin	137	135	235	25	300	261	454
Steel	136	183	22	233	278	239	418
Coal	795	114	138	146	176	151	262
Marble	370	53	637	725	81	72	121
Freestone	394	57	69	728	873	755	132

MENSURATION OF SOLIDITY AND CAPACITY.

General rule.—Set the length upon B to the gauge-point upon A ; and against the side of the square, or diameter on D, are the cubic contents, or weight in lbs. on C.

1. Required the cubic contents of a tree 30 feet in length, and 10 inches quarter girt.

Set 20 upon B to 144 (the gauge-point) upon A ; and against 10 upon D is 20·75 feet upon C.

2. In a cylinder 9 inches in length and 7 inches diameter, how many cubic inches ?

Set 9 upon B to 1273 (the gauge-point) upon A ; and against 7 on D is 346 inches on C.

3. What is the weight of a bar of cast iron 3 inches square, and 6 feet long ?

Set 6 upon B to 32 (the gauge-point) upon A ; and against 3 upon D is 168 lbs. upon C.

By the common rule.

4. Required the weight of a cylinder of wrought iron 10 inches long, and $5\frac{1}{2}$ diameter.

Set 10 upon B to 283 (G. Pt.) upon A ; and against $5\frac{1}{2}$ upon D is 66·65 lbs. on C.

5. What is the weight of a dry rope 25 yards long, and 4 inches circumference ?

Set 25 upon B to 47 (G. Pt.) upon A ; and against 4 on D is 53·16 lbs. on C.

6. What is the weight of a short-linked chain 30 yards in length, and $\frac{6}{16}$ ths of an inch in diameter ?

Set 30 upon B to 52 (G. Pt.) upon A ; and against 6 on D is 129·5 lbs. on C.

LAND SURVEYING.

If the dimensions taken are in chains, the gauge-point is 1 or 10 ; if in perches, 160 ; and if in yards, 4840. *Rule.*—Set the length upon B to the gauge-point on A ; and against the breadth upon A is the content in acres upon B.

1. Required the number of acres or contents of a field 20 chains 50 links in length, and 4 chains 40 links in breadth.

Set 20·5 on B to 1 on A; and against 4·4 on A is 9 acres on B.

2. In a piece of ground 440 yards long, and 44 broad, how many acres?

Set 440 upon B to 4840 on A; and against 44 on A is 4 acres on B.

POWER OF STEAM ENGINES.

Condensing Engines.—*Rule.* Set 3·5 on C to 10 on D; then D is a line of diameters for cylinders, and C the corresponding number of horses' power; thus,
 H.Pr. $3\frac{1}{2}$ 4 5 6 8 10 12 16 20 25 30 40 50 on C.
 C.D. 10in. $10\frac{3}{4}$ $12\frac{1}{4}$ $13\frac{1}{4}$ $15\frac{1}{2}$ 17 $18\frac{3}{4}$ $21\frac{1}{2}$ 24 $26\frac{3}{4}$ $29\frac{1}{2}$ $33\frac{3}{4}$ $37\frac{3}{4}$ on D.

The same is effected on the common rule by setting 5 on C to 12 on D.

Non-condensing Engines.—*Rule.* Set the pressure of steam in lbs. per square inch on B to 4 upon A; and against the cylinder's diameter on D is the number of horses' power upon C.

Required the power of an engine, when the cylinder is 20 inches diameter and steam 30 lbs. per square inch.

Set 30 on B to 4 on A; and against 20 on D is 30 horses' power on C.

The same is effected on the common rule by setting the force of the steam on B to 250 on A.

OF ENGINE BOILERS.

How many superficial feet are contained in a boiler 23 feet in length and $5\frac{1}{2}$ in width?

Set 1 upon B to 23 upon A; and against 5·5 upon B is 126·5 square feet upon A.

If 5 square feet of boiler surface be sufficient for each horse-power, how many horses' power of engine is the boiler equal to?

Set 5 upon B to 126·5 upon A; and against 1 upon B is 25·5 upon A.

TABLE BY WHICH TO FACILITATE THE CALCULATION OF BRITISH MONEY.

No.	L. S. D.			L. S. D.			L. S. D.			L. S. D.			L. S. D.			L. S. D.			L. S. D.		
	at $\frac{1}{4}d.$			at $\frac{1}{2}d.$			at $\frac{3}{4}d.$			at $1d.$			at $1\frac{1}{4}d.$			at $1\frac{1}{2}d.$			at $1\frac{3}{4}d.$		
20	0	0	0 $\frac{1}{2}$	0	0	1	0	0	1 $\frac{1}{2}$	0	0	2	0	0	2 $\frac{1}{2}$	0	0	3	0	0	3 $\frac{1}{2}$
30	0	0	0 $\frac{3}{4}$	0	0	1 $\frac{1}{2}$	0	0	2 $\frac{1}{4}$	0	0	3	0	0	3 $\frac{3}{4}$	0	0	4 $\frac{1}{2}$	0	0	5 $\frac{1}{4}$
40	0	0	1	0	0	2	0	0	3	0	0	4	0	0	5	0	0	5	0	0	7
50	0	0	1 $\frac{1}{4}$	0	0	2 $\frac{1}{2}$	0	0	3 $\frac{3}{4}$	0	0	5	0	0	6 $\frac{1}{4}$	0	0	7 $\frac{1}{2}$	0	0	8 $\frac{3}{4}$
60	0	0	1 $\frac{1}{2}$	0	0	3	0	0	4 $\frac{1}{2}$	0	0	6	0	0	7 $\frac{1}{2}$	0	0	9	0	0	10 $\frac{1}{2}$
70	0	0	1 $\frac{3}{4}$	0	0	3 $\frac{1}{2}$	0	0	5 $\frac{1}{4}$	0	0	7	0	0	8 $\frac{3}{4}$	0	0	10 $\frac{1}{2}$	0	1	0 $\frac{1}{2}$
80	0	0	2	0	0	4	0	0	6	0	0	8	0	0	10	0	1	0	0	1	2
90	0	0	2 $\frac{1}{4}$	0	0	4 $\frac{1}{2}$	0	0	6 $\frac{3}{4}$	0	0	9	0	0	11 $\frac{1}{4}$	0	1	1 $\frac{1}{2}$	0	1	3 $\frac{3}{4}$
100	0	0	2 $\frac{1}{2}$	0	0	5	0	0	7 $\frac{1}{2}$	0	0	10	0	1	0 $\frac{1}{2}$	0	1	3	0	1	5 $\frac{1}{2}$
110	0	0	2 $\frac{3}{4}$	0	0	5 $\frac{1}{2}$	0	0	8 $\frac{1}{4}$	0	0	11	0	1	1 $\frac{3}{4}$	0	1	4 $\frac{1}{2}$	0	1	7 $\frac{1}{4}$
120	0	0	3	0	0	6	0	0	9	0	1	0	0	1	3	0	1	6	0	1	9
130	0	0	3 $\frac{1}{4}$	0	0	6 $\frac{1}{2}$	0	0	9 $\frac{3}{4}$	0	1	1	0	1	4 $\frac{1}{4}$	0	1	7 $\frac{1}{2}$	0	1	10 $\frac{3}{4}$
140	0	0	3 $\frac{1}{2}$	0	0	7	0	0	10 $\frac{1}{2}$	0	1	2	0	1	5 $\frac{1}{2}$	0	1	9	0	2	0 $\frac{1}{2}$
150	0	0	3 $\frac{3}{4}$	0	0	7 $\frac{1}{2}$	0	0	11 $\frac{1}{4}$	0	1	3	0	1	6 $\frac{3}{4}$	0	1	10 $\frac{1}{2}$	0	2	2 $\frac{1}{4}$
160	0	0	4	0	0	8	0	1	0	0	1	4	0	1	8	0	2	0	0	2	4
170	0	0	4 $\frac{1}{4}$	0	0	8 $\frac{1}{2}$	0	1	0 $\frac{3}{4}$	0	1	5	0	1	9 $\frac{1}{4}$	0	2	1 $\frac{1}{2}$	0	2	5 $\frac{3}{4}$
180	0	0	4 $\frac{1}{2}$	0	0	9	0	1	1 $\frac{1}{2}$	0	1	6	0	1	10 $\frac{1}{2}$	0	2	3	0	2	7 $\frac{1}{2}$
190	0	0	4 $\frac{3}{4}$	0	0	9 $\frac{1}{2}$	0	1	2 $\frac{1}{4}$	0	1	7	0	1	11 $\frac{3}{4}$	0	2	4 $\frac{1}{2}$	0	2	9 $\frac{1}{4}$
200	0	0	5	0	0	10	0	1	3	0	1	8	0	2	1	0	2	6	0	2	11
300	0	0	7 $\frac{1}{2}$	0	1	3	0	1	10 $\frac{1}{2}$	0	2	6	0	3	1 $\frac{1}{2}$	0	4	9 $\frac{1}{2}$	0	4	4 $\frac{1}{2}$
400	0	0	10	0	1	8	0	2	6	0	3	4	0	4	2	0	5	0	0	5	10
500	0	1	0 $\frac{1}{2}$	0	2	1	0	3	1 $\frac{1}{2}$	0	4	2	0	5	2 $\frac{1}{2}$	0	6	3	0	7	3 $\frac{1}{2}$
600	0	1	3	0	2	6	0	3	9	0	5	0	0	6	3	0	7	6	0	8	9
700	0	1	5 $\frac{1}{2}$	0	2	11	0	4	4 $\frac{1}{2}$	0	5	10	0	7	3 $\frac{1}{2}$	0	8	10	0	10	2 $\frac{1}{2}$
800	0	1	8	0	3	4	0	5	0	0	6	8	0	8	4	0	10	0	0	11	8
900	0	1	10 $\frac{1}{2}$	0	3	9	0	5	7 $\frac{1}{2}$	0	7	6	0	9	4 $\frac{1}{2}$	0	11	3	0	13	1 $\frac{1}{2}$
1000	0	2	1	0	4	2	0	6	3	0	8	4	0	10	5	0	12	6	0	14	7
2000	0	4	2	0	8	4	0	12	6	0	16	8	1	0	10	1	5	0	1	9	2
3000	0	6	3	0	12	6	0	18	9	1	5	0	1	11	3	1	17	6	2	3	9
4000	0	8	4	0	16	8	1	5	0	1	13	4	2	1	8	2	10	0	2	18	4
5000	0	10	5	1	0	10	1	11	3	2	1	8	2	12	1	3	2	6	3	12	4
6000	0	12	6	1	5	0	1	17	6	2	10	0	3	2	6	3	15	0	4	7	6
7000	0	14	7	1	9	2	2	3	9	2	18	4	3	12	11	4	7	6	5	2	1
8000	0	16	8	1	13	4	2	10	0	3	6	8	4	3	4	5	0	0	5	16	8
9000	0	18	9	1	17	6	2	16	3	3	15	0	4	13	9	5	12	6	6	11	3
1000	1	0	10	2	1	8	3	2	6	4	3	4	5	4	2	6	5	0	7	5	10
2000	2	1	8	4	3	4	6	5	0	8	6	8	10	8	4	12	10	0	14	11	8
3000	3	2	6	6	5	0	9	7	6	12	10	0	15	12	6	18	15	0	21	17	6
4000	4	3	4	8	6	8	12	10	0	16	13	4	20	6	8	25	0	0	29	3	4
5000	5	4	2	10	8	4	15	12	6	20	16	8	26	0	10	31	5	0	36	9	2
6000	6	5	0	12	10	0	18	15	0	25	0	0	31	5	0	37	10	0	43	15	0
7000	7	5	10	14	11	8	21	17	6	29	3	4	36	9	2	43	15	0	51	0	10
8000	8	6	8	16	13	4	25	0	0	33	6	8	41	13	4	50	0	0	58	6	8
9000	9	7	6	18	15	0	28	2	6	37	10	0	46	17	6	56	5	0	65	13	6

MONEY TABLE—CONTINUED.

No.	L. S. D.			L. S. D.			L. S. D.			L. S. D.			L. S. D.			L. S. D.					
	at 2d.			at 2¼d.			at 2½d.			at 2¾d.			at 3d.			at 3¼d.			at 3½d.		
2	0	0	4	0	0	4½	0	0	5	0	0	5½	0	0	6	0	0	6½	0	0	7
3	0	0	6	0	0	6¾	0	0	7½	0	0	8¾	0	0	9	0	0	9¾	0	0	10½
4	0	0	8	0	0	9	0	0	10	0	0	11	0	1	0	0	1	1	0	1	2
5	0	0	10	0	0	11¼	0	0	11½	0	1	1¾	0	1	3	0	1	4¼	0	1	5½
6	0	1	0	0	1	1½	0	1	3	0	1	4½	0	1	6	0	1	7½	0	1	9
7	0	1	2	0	1	5¾	0	1	5½	0	1	7¼	0	1	9	0	1	10¾	0	2	0½
8	0	1	4	0	1	6	0	1	8	0	1	10	0	2	0	0	2	2	0	2	4
9	0	1	6	0	1	8¼	0	1	10½	0	2	0¾	0	2	3	0	2	5¼	0	2	7½
10	0	1	8	0	1	10½	0	2	1	0	2	3½	0	2	6	0	2	8½	0	2	11
11	0	1	10	0	2	1¾	0	2	3½	0	2	6¼	0	2	9	0	2	11¾	0	3	2½
12	0	2	0	0	2	3	0	2	6	0	2	9	0	3	0	0	3	3	0	3	6
13	0	2	2	0	2	5¼	0	2	8½	0	2	11¾	0	3	3	0	3	6¼	0	3	9½
14	0	2	4	0	2	7½	0	2	11	0	3	2½	0	3	6	0	3	9½	0	4	1
15	0	2	6	0	2	9¾	0	3	1½	0	3	5¼	0	3	9	0	4	0¾	0	4	4½
16	0	2	8	0	3	0	0	3	4	0	3	8	0	4	0	0	4	4	0	4	8
17	0	2	10	0	3	2¾	0	3	6½	0	3	10¾	0	4	3	0	4	7¼	0	4	11½
18	0	3	0	0	3	4½	0	3	9	0	4	1½	0	4	6	0	4	10½	0	5	3
19	0	3	2	0	3	6¾	0	3	11½	0	4	4¼	0	4	9	0	5	1¾	0	5	6½
20	0	3	4	0	3	9	0	4	2	0	4	7	0	5	0	0	5	5	0	5	10
30	0	5	0	0	5	7½	0	6	3	0	6	10½	0	7	6	0	8	1½	0	8	9
40	0	6	8	0	7	6	0	8	4	0	9	2	0	10	0	0	10	10	0	11	8
50	0	8	4	0	9	4½	0	10	5	0	11	5½	0	12	6	0	13	6½	0	14	7
60	0	10	0	0	11	3	0	12	6	0	13	9	0	15	0	0	16	3	0	17	6
70	0	11	8	0	13	1½	0	14	7	0	16	0½	0	17	6	0	18	11½	1	0	5
80	0	13	4	0	15	0	0	16	8	0	18	4	1	0	0	1	1	8	1	3	4
90	0	15	0	0	16	10½	0	18	9	1	7	7½	1	2	6	1	4	4½	1	6	3
100	0	16	8	0	18	9	1	0	10	1	2	11	1	5	0	1	7	1	1	9	2
200	1	13	4	1	17	6	2	1	8	2	5	10	2	10	0	2	14	2	2	18	4
300	2	10	0	2	16	3	3	2	6	3	8	9	3	15	0	4	1	3	4	7	6
400	3	6	8	3	15	0	4	3	4	4	11	8	5	0	0	5	8	4	5	16	8
500	4	3	4	4	13	9	5	4	2	5	14	7	6	5	0	6	15	5	7	5	10
600	5	0	0	5	12	6	6	5	0	6	17	6	7	10	0	8	2	6	8	15	0
700	5	16	8	6	11	3	7	5	10	8	0	5	8	15	0	9	9	7	10	4	2
800	6	13	4	7	10	0	8	6	8	9	3	4	10	0	0	10	16	8	11	13	4
900	7	10	0	8	3	9	9	7	6	10	6	3	11	5	0	12	3	9	13	2	6
1000	8	6	8	9	7	6	10	8	4	11	9	2	12	10	0	13	10	10	14	11	8
2000	16	13	4	18	15	0	20	16	8	22	18	4	25	0	0	27	1	8	29	3	4
3000	25	0	0	28	2	6	31	5	0	34	7	6	37	10	0	40	12	6	43	15	0
4000	33	6	8	37	10	0	41	13	4	45	16	8	50	0	0	54	3	4	58	6	8
5000	41	13	4	46	17	6	52	1	8	57	5	10	62	10	0	67	14	2	72	18	4
6000	50	0	0	56	5	0	62	10	0	68	15	0	75	0	0	81	5	0	87	10	0
7000	58	6	8	65	12	6	72	18	4	80	4	2	87	10	0	94	15	10	102	1	8
8000	66	13	4	75	0	0	83	6	8	91	13	4	100	0	0	108	6	8	116	13	4
9000	75	0	0	84	7	6	93	15	0	103	2	6	112	10	0	121	17	6	131	5	0

MONEY TABLE—CONTINUED.

No.	L. S. D.	L. S. D.	L. S. D.	L. S. D.	L. S. D.	L. S. D.	L. S. D.
	at 3 $\frac{3}{4}$ d.	at 4d.	at 4 $\frac{1}{4}$ d.	at 4 $\frac{1}{2}$ d.	at 4 $\frac{3}{4}$ d.	at 5d.	at 5 $\frac{1}{4}$ d.
2	0 0 7 $\frac{1}{2}$	0 0 8	0 0 8 $\frac{1}{2}$	0 0 9	0 0 9 $\frac{1}{2}$	0 0 10	0 0 10 $\frac{1}{2}$
3	0 0 11 $\frac{1}{4}$	0 1 0	0 1 0 $\frac{3}{4}$	0 1 1 $\frac{1}{2}$	0 1 2 $\frac{1}{4}$	0 1 3	0 1 3 $\frac{1}{2}$
4	0 1 3	0 1 4	0 1 5	0 1 6	0 1 7	0 1 8	0 1 9
5	0 1 6 $\frac{3}{4}$	0 1 8	0 1 9 $\frac{1}{4}$	0 1 10 $\frac{1}{2}$	0 1 11 $\frac{3}{4}$	0 2 1	0 2 2 $\frac{1}{4}$
6	0 1 10 $\frac{1}{2}$	0 2 0	0 2 1 $\frac{1}{2}$	0 2 3	0 2 4 $\frac{1}{2}$	0 2 6	0 2 7 $\frac{1}{2}$
7	0 2 2 $\frac{1}{4}$	0 2 4	0 2 5 $\frac{3}{4}$	0 2 7 $\frac{1}{2}$	0 2 9 $\frac{1}{4}$	0 2 11	0 3 0 $\frac{3}{4}$
8	0 2 6	0 2 8	0 2 10	0 3 0	0 3 2	0 3 4	0 3 6
9	0 2 9 $\frac{3}{4}$	0 3 0	0 3 2 $\frac{1}{4}$	0 3 4 $\frac{1}{2}$	0 3 6 $\frac{3}{4}$	0 3 9	0 3 11 $\frac{1}{4}$
10	0 3 1 $\frac{1}{2}$	0 3 4	0 3 6 $\frac{1}{2}$	0 3 9	0 3 11 $\frac{1}{2}$	0 4 2	0 4 4 $\frac{1}{2}$
11	0 3 5 $\frac{1}{4}$	0 3 8	0 3 10 $\frac{3}{4}$	0 4 1 $\frac{1}{2}$	0 4 4 $\frac{1}{4}$	0 4 7	0 4 9 $\frac{3}{4}$
12	0 3 9	0 4 0	0 4 3	0 4 6	0 4 9	0 5 0	0 5 3
13	0 4 0 $\frac{3}{4}$	0 4 4	0 4 7 $\frac{1}{4}$	0 4 10 $\frac{1}{2}$	0 5 1 $\frac{3}{4}$	0 5 5	0 5 8 $\frac{1}{4}$
14	0 4 4 $\frac{1}{2}$	0 4 8	0 4 11 $\frac{1}{2}$	0 5 3	0 5 6 $\frac{1}{2}$	0 5 10	0 6 1 $\frac{1}{2}$
15	0 4 8 $\frac{1}{4}$	0 5 0	0 5 3 $\frac{3}{4}$	0 5 7 $\frac{1}{2}$	0 5 11 $\frac{1}{4}$	0 6 3	0 6 6 $\frac{3}{4}$
16	0 5 0	0 5 4	0 5 8	0 6 0	0 6 4	0 6 8	0 7 0
17	0 5 3 $\frac{3}{4}$	0 5 8	0 6 0 $\frac{1}{4}$	0 6 4 $\frac{1}{2}$	0 6 8 $\frac{3}{4}$	0 7 1	0 7 5 $\frac{1}{4}$
18	0 5 7 $\frac{1}{2}$	0 6 0	0 6 4 $\frac{1}{2}$	0 6 9	0 7 1 $\frac{1}{2}$	0 7 6	0 7 10 $\frac{1}{2}$
19	0 5 11 $\frac{1}{4}$	0 6 4	0 6 8 $\frac{3}{4}$	0 7 1 $\frac{1}{2}$	0 7 6 $\frac{1}{4}$	0 7 11	0 8 3 $\frac{3}{4}$
20	0 6 3	0 6 8	0 7 1	0 7 6	0 7 11	0 8 4	0 8 9
30	0 9 4 $\frac{1}{2}$	0 10 0	0 10 7 $\frac{1}{2}$	0 11 3	0 11 10 $\frac{1}{2}$	0 12 6	0 13 1 $\frac{1}{2}$
40	0 12 6	0 13 4	0 14 2	0 15 0	0 15 10	0 16 8	0 17 6
50	0 15 7 $\frac{1}{2}$	0 16 8	0 17 8 $\frac{1}{2}$	0 18 9	0 19 9 $\frac{1}{2}$	1 0 10	1 1 10 $\frac{1}{2}$
60	0 18 9	1 0 0	1 1 3	1 2 6	1 3 9	1 5 0	1 6 3
70	1 1 10 $\frac{1}{2}$	1 3 4	1 4 9 $\frac{1}{2}$	1 6 3	1 7 8 $\frac{1}{2}$	1 9 2	1 10 7 $\frac{1}{2}$
80	1 5 0	1 6 8	1 8 4	1 10 0	1 11 8	1 13 4	1 15 0
90	1 8 1 $\frac{1}{2}$	1 10 0	1 11 10 $\frac{1}{2}$	1 13 9	1 15 7 $\frac{1}{2}$	1 17 6	1 19 4 $\frac{1}{2}$
100	1 11 3	1 13 4	1 15 5	1 17 6	1 19 7	2 1 8	2 3 9
200	3 2 6	3 6 8	3 10 10	3 15 0	3 19 2	4 3 4	4 7 6
300	4 13 9	5 0 0	5 6 3	5 12 6	5 18 9	6 5 0	6 11 3
400	6 5 0	6 13 4	7 1 8	7 10 0	7 18 4	8 6 8	8 15 0
500	7 16 3	8 6 8	8 17 1	9 7 6	9 17 11	10 8 4	10 18 9
600	9 7 6	10 0 0	10 12 6	11 5 0	11 17 6	12 10 0	13 2 6
700	10 18 9	11 13 4	12 7 11	13 2 6	13 17 1	14 11 8	15 6 3
800	12 10 0	13 6 8	14 3 4	15 0 0	15 16 8	16 13 4	17 10 0
900	14 1 3	15 0 0	15 18 9	16 17 6	17 16 3	18 15 0	19 13 9
000	15 12 6	16 13 4	17 14 2	18 15 0	19 15 10	20 16 8	21 17 6
000	31 5 0	33 6 8	35 8 4	37 10 0	39 11 8	41 13 4	43 15 0
000	46 17 6	50 0 0	53 2 6	56 5 0	59 7 6	62 10 0	65 12 6
000	62 10 0	66 13 4	70 16 8	75 0 0	79 3 4	83 6 8	87 10 0
000	78 2 6	83 6 8	88 10 10	93 15 0	98 19 8	104 3 4	109 7 6
000	93 15 0	100 0 0	106 5 0	112 10 0	118 15 0	125 0 0	131 5 0
000	109 7 6	116 13 4	123 19 2	131 5 0	138 10 10	145 16 8	153 2 6
000	125 0 0	133 6 8	141 13 4	150 0 0	158 6 8	166 13 4	175 0 0
000	140 12 6	150 0 0	159 7 6	168 15 0	178 2 6	187 10 0	196 17 6

MONEY TABLE—CONTINUED.

No.	L. S. D.	L. S. D.	L. S. D.	L. S. D.	L. S. D.	L. S. D.	L. S. D.
	at 5½d.	at 5¾d.	at 6d.	at 6¼d.	at 6½d.	at 6¾d.	at 7d.
2	0 0 11	0 0 11½	0 1 0	0 1 0½	0 1 1	0 1 1½	0 1 2
3	0 1 4½	0 1 5¼	0 1 6	0 1 6¾	0 1 7½	0 1 8¼	0 1 9
4	0 1 10	0 1 11	0 2 0	0 2 1	0 2 2	0 2 3	0 2 4
5	0 2 3½	0 2 4¾	0 2 6	0 2 7¼	0 2 8½	0 2 9¾	0 2 11
6	0 2 9	0 2 10½	0 3 0	0 3 1½	0 3 3	0 3 4½	0 3 6
7	0 3 3½	0 3 4¾	0 3 6	0 3 7¼	0 3 9½	0 3 11¼	0 4 1
8	0 3 8	0 3 10	0 4 0	0 4 2	0 4 4	0 4 6	0 4 8
9	0 4 1½	0 4 3¾	0 4 6	0 4 8¼	0 4 10½	0 5 0¾	0 5 3
10	0 4 7	0 4 9½	0 5 0	0 5 2½	0 5 5	0 5 7½	0 5 10
11	0 5 0½	0 5 3¼	0 5 6	0 5 8¾	0 5 11½	0 6 2¼	0 6 5
12	0 5 6	0 5 9	0 6 0	0 6 3	0 6 6	0 6 9	0 7 0
13	0 5 11½	0 6 2¾	0 6 6	0 6 9¼	0 7 0½	0 7 3¾	0 7 7
14	0 6 5	0 6 8½	0 7 0	0 7 3½	0 7 7	0 7 10½	0 8 2
15	0 6 10½	0 7 2¼	0 7 6	0 7 9¾	0 8 1½	0 8 5¼	0 8 9
16	0 7 4	0 7 8	0 8 0	0 8 4	0 8 8	0 9 0	0 9 4
17	0 7 9½	0 8 1¾	0 8 6	0 8 10¼	0 9 2½	0 9 6¾	0 9 11
18	0 8 3	0 8 7½	0 9 0	0 9 4½	0 9 9	0 10 1½	0 10 6
19	0 8 8½	0 9 1¼	0 9 6	0 9 10¾	0 10 3½	0 10 8¼	0 11 1
20	0 9 2	0 9 7	0 10 0	0 10 5	0 10 10	0 11 3	0 11 8
30	0 13 9	0 14 4½	0 15 0	0 15 7½	0 16 3	0 16 10½	0 17 6
40	0 18 4	0 19 2	1 0 0	1 0 10	1 1 8	1 2 6	1 3 4
50	1 2 11	1 3 11½	1 5 0	1 6 0½	1 7 1	1 8 1½	1 9 2
60	1 7 6	1 8 9	1 10 0	1 11 3	1 12 6	1 13 9	1 15 0
70	1 12 1	1 13 6½	1 15 0	1 16 5½	1 17 11	1 19 4½	2 0 10
80	1 16 8	1 18 4	2 0 0	2 1 8	2 3 4	2 5 0	2 6 8
90	2 1 3	2 3 1½	2 5 0	2 6 10½	2 8 9	2 10 7½	2 12 6
100	2 5 10	2 7 11	2 10 0	2 12 1	2 14 2	2 16 3	2 18 4
200	4 11 8	4 15 10	5 0 0	5 4 2	5 8 4	5 12 6	5 16 8
300	6 17 6	7 3 9	7 10 0	7 16 3	8 2 6	8 8 9	8 15 0
400	9 3 4	9 11 8	10 0 0	10 8 4	10 16 8	11 5 0	11 13 4
500	11 9 2	11 19 7	12 10 0	13 0 5	13 10 10	14 1 3	14 11 8
600	13 15 0	14 7 6	15 0 0	15 12 6	16 5 0	16 17 6	17 10 0
700	16 0 10	16 15 5	17 10 0	18 4 7	18 19 2	19 13 9	20 8 4
800	18 6 8	19 3 4	20 0 0	20 16 8	21 13 4	22 19 0	23 6 8
900	20 12 6	21 11 3	22 10 0	23 18 9	24 7 6	25 6 3	26 5 0
1000	22 18 4	23 19 2	25 0 0	26 0 10	27 1 8	28 2 6	29 3 4
2000	45 16 8	47 18 4	50 0 0	52 1 8	54 3 4	56 5 0	58 6 8
3000	68 15 0	71 17 6	75 0 0	78 2 6	81 5 0	84 7 6	87 10 0
4000	91 13 4	95 16 8	100 0 0	104 3 4	108 6 8	112 10 0	116 13 4
5000	114 11 8	119 15 10	125 0 0	130 4 2	135 8 4	140 12 6	145 16 8
6000	137 10 0	143 15 0	150 0 0	156 5 0	162 10 0	168 15 0	175 0 0
7000	160 8 4	167 14 2	175 0 0	182 5 10	189 11 8	196 17 6	204 3 4
8000	183 6 8	191 13 4	200 0 0	208 6 8	216 13 4	225 0 0	233 6 8
9000	206 5 0	215 12 6	225 0 0	234 7 6	243 15 0	253 2 6	262 10 0

MONEY TABLE—CONTINUED.

No.	L. S. D.	L. S. D.	L. S. D.	L. S. D.	L. S. D.	L. S. D.	L. S. D.
	at 7¼d.	at 7½d.	at 7¾d.	at 8d.	at 8¼d.	at 8½d.	at 8¾d.
2	0 1 2½	0 1 3	0 1 3½	0 1 4	0 1 4½	0 1 5	0 1 5½
3	0 1 9¾	0 1 10½	0 1 11¼	0 2 0	0 2 0¾	0 2 1½	0 2 2¼
4	0 2 5	0 2 6	0 2 7	0 2 8	0 2 9	0 2 10	0 2 11
5	0 3 0¼	0 3 1½	0 3 2¾	0 3 4	0 3 5¼	0 3 6½	0 3 7¾
6	0 3 7½	0 3 9	0 3 10½	0 4 0	0 4 1½	0 4 3	0 4 4½
7	0 4 2¾	0 4 4½	0 4 6¼	0 4 8	0 4 9¾	0 4 11½	0 5 1¼
8	0 4 10	0 5 0	0 5 2	0 5 4	0 5 6	0 5 8	0 5 10
9	0 5 5¼	0 5 7½	0 5 9¾	0 6 0	0 6 2¼	0 6 4½	0 6 6¾
10	0 6 0½	0 6 3	0 6 5½	0 6 8	0 6 10½	0 7 1	0 7 3½
11	0 6 7¾	0 6 10½	0 7 1¼	0 7 4	0 7 6¾	0 7 9½	0 8 0¼
12	0 7 3	0 7 6	0 7 9	0 8 0	0 8 3	0 8 6	0 8 9
13	0 7 10¼	0 8 1½	0 8 4¾	0 8 8	0 8 11¼	0 9 2½	0 9 5¾
14	0 8 5½	0 8 9	0 9 0½	0 9 4	0 9 7½	0 9 11	0 10 2½
15	0 9 0¾	0 9 4½	0 9 8¼	0 10 0	0 10 3¾	0 10 7½	0 10 11¼
16	0 9 8	0 10 0	0 10 4	0 10 8	0 11 0	0 11 4	0 11 8
17	0 10 3¼	0 10 7½	0 10 11¾	0 11 4	0 11 8¼	0 12 0½	0 12 4¾
18	0 10 10½	0 11 3	0 11 7½	0 12 0	0 12 4½	0 12 9	0 13 1½
19	0 11 5¾	0 11 10½	0 12 3¼	0 12 8	0 13 0¾	0 13 5½	0 13 10¼
20	0 12 1	0 12 6	0 12 11	0 13 4	0 13 9	0 14 2	0 14 7
30	0 18 1½	0 18 9	0 19 4½	1 0 0	1 0 7½	1 1 3	1 1 10½
40	1 4 2	1 5 0	1 5 10	1 6 8	1 7 6	1 8 4	1 9 2
50	1 10 2½	1 11 3	1 12 3½	1 13 4	1 14 4½	1 15 5	1 16 5½
60	1 16 3	1 17 6	1 18 9	2 0 0	2 1 3	2 2 6	2 3 9
70	2 2 3½	2 3 9	2 5 2½	2 6 8	2 8 1½	2 9 7	2 11 10½
80	2 8 4	2 10 0	2 11 8	2 13 4	2 15 0	2 16 8	2 18 4
90	2 14 4½	2 16 3	2 18 1½	3 0 0	3 1 10½	3 3 9	3 5 7½
100	3 0 5	3 2 6	3 4 7	3 6 8	3 8 9	3 10 10	3 12 11
200	6 0 10	6 5 0	6 9 2	6 13 4	6 17 6	7 1 8	7 5 10
300	9 1 3	9 7 6	9 13 9	10 0 0	10 6 3	10 12 6	10 18 9
400	12 1 8	12 10 0	12 18 4	13 6 8	13 15 0	14 3 4	14 11 8
500	15 2 1	15 12 6	16 2 11	16 13 4	17 3 9	17 14 2	18 4 7
600	18 2 6	18 15 0	19 7 6	20 0 0	20 12 6	21 5 0	21 17 6
700	21 2 11	21 17 6	22 12 1	23 6 8	24 1 3	24 3 10	25 10 5
800	24 3 4	25 0 0	25 16 8	26 13 4	27 10 0	28 6 8	29 3 4
900	27 3 9	28 2 6	29 1 3	30 0 0	30 18 9	31 17 6	32 16 3
000	30 4 2	31 5 0	32 5 10	33 6 8	34 7 6	35 8 4	36 9 2
000	60 8 4	62 10 0	64 11 8	66 13 4	68 15 0	70 16 8	72 18 4
000	90 12 6	93 15 0	96 17 6	100 0 0	103 2 6	106 5 0	109 7 6
000	120 16 8	125 0 0	129 3 4	133 6 8	137 10 0	141 13 4	145 16 8
000	151 0 10	156 5 0	161 9 2	166 13 4	171 17 6	177 1 8	182 5 10
000	181 5 0	187 10 0	193 15 0	200 0 0	206 5 0	212 10 0	218 15 0
000	211 9 2	218 15 0	236 0 10	233 6 8	240 12 6	247 18 4	255 4 2
000	241 13 4	250 0 0	258 6 8	266 13 4	275 0 0	283 6 8	291 13 4
000	271 17 6	281 5 0	290 12 6	300 0 0	309 7 6	318 15 0	328 2 6

MONEY TABLE—CONTINUED.

No.	L. S. D.	L. S. D.	L. S. D.	L. S. D.	L. S. D.	L. S. D.	L. S. D.
	at 9d.	at 9½d.	at 9¾d.	at 9¾d.	at 10d.	at 10¼d.	at 10½d.
2	0 1 6	0 1 6½	0 1 7	0 1 7½	0 1 8	0 1 8½	0 1 9
3	0 2 3	0 2 3¾	0 2 4½	0 2 5¼	0 2 6	0 2 6¾	0 2 7½
4	0 3 0	0 3 1	0 3 2	0 3 3	0 3 4	0 3 5	0 3 6
5	0 3 9	0 3 10¼	0 3 11½	0 4 0¾	0 4 2	0 4 3¼	0 4 4½
6	0 4 6	0 4 7¾	0 4 9	0 4 10½	0 5 0	0 5 1½	0 5 3
7	0 5 3	0 5 4¾	0 5 6½	0 5 8¼	0 5 10	0 5 11¾	0 6 1½
8	0 6 0	0 6 2	0 6 4	0 6 6	0 6 8	0 6 10	0 7 0
9	0 6 9	0 6 11¼	0 7 1½	0 7 3¾	0 7 6	0 7 8¼	0 7 10½
10	0 7 6	0 7 8½	0 7 11	0 8 1½	0 8 4	0 8 6½	0 8 9
11	0 8 3	0 8 5¾	0 8 8½	0 8 11¼	0 9 2	0 9 4¾	0 9 7½
12	0 9 0	0 9 3	0 9 6	0 9 9	0 10 0	0 10 3	0 10 6
13	0 9 9	0 10 0¾	0 10 3½	0 10 6¾	0 10 10	0 11 1¼	0 11 4½
14	0 10 6	0 10 9½	0 11 1	0 11 4½	0 11 8	0 11 11½	0 12 3
15	0 11 3	0 11 6¾	0 11 10½	0 12 2¼	0 12 6	0 12 9¾	0 13 1½
16	0 12 0	0 12 4	0 12 8	0 13 0	0 13 4	0 13 8	0 14 0
17	0 12 9	0 13 1¼	0 13 5½	0 13 9¾	0 14 2	0 14 6¼	0 14 10½
18	0 13 6	0 13 10½	0 14 3	0 14 7½	0 15 0	0 15 4¾	0 15 9
19	0 14 3	0 14 7¾	0 15 0½	0 15 5¼	0 15 10	0 16 2¾	0 16 7½
20	0 15 0	0 15 5	0 15 10	0 16 3	0 16 8	0 17 1	0 17 6
30	1 2 6	1 3 1½	1 3 9	1 4 4½	1 5 0	1 5 7½	1 6 3
40	1 10 0	1 10 10	1 11 8	1 12 6	1 13 4	1 14 2	1 15 0
50	1 17 6	1 18 6½	1 19 7	2 7 0½	2 1 8	2 2 8½	2 3 9
60	2 5 0	2 6 3	2 7 6	2 8 9	2 10 0	2 11 3	2 12 6
70	2 12 6	2 13 11½	2 15 5	2 16 10½	2 18 4	2 19 9½	3 1 3
80	3 0 0	3 1 8	3 3 4	3 5 0	3 6 8	3 8 4	3 10 0
90	3 7 6	3 9 4½	3 11 3	3 13 1½	3 15 0	3 16 10½	3 18 9
100	3 15 0	3 17 1	3 19 2	4 1 3	4 3 4	4 5 5	4 7 6
200	7 10 0	7 14 2	7 18 4	8 2 6	8 6 8	8 10 10	8 15 0
300	11 5 0	11 11 3	11 17 6	12 3 9	12 10 0	12 16 3	13 2 6
400	15 0 0	15 8 4	15 16 8	16 5 0	16 13 4	17 1 8	17 10 0
500	18 15 0	19 5 5	19 15 10	20 6 3	20 16 8	21 7 1	21 17 6
600	22 10 0	23 2 6	23 15 0	24 7 6	25 0 0	25 12 6	26 5 0
700	26 5 0	26 19 7	27 14 2	28 8 9	29 3 4	29 17 11	30 12 6
800	30 0 0	30 16 8	31 13 4	32 10 0	33 6 8	34 3 4	35 0 0
900	33 15 0	34 13 9	35 12 6	36 11 3	37 10 0	38 8 9	39 7 6
1000	37 10 0	38 10 10	39 11 8	40 12 6	41 13 4	42 14 2	43 15 0
2000	75 0 0	77 1 8	79 3 4	81 5 0	83 6 8	85 8 4	87 10 0
3000	112 10 0	115 12 6	118 15 0	121 17 6	125 0 0	128 2 6	131 5 0
4000	150 0 0	154 3 4	158 6 8	162 10 0	166 13 4	170 16 8	175 0 0
5000	187 10 0	192 14 2	197 18 4	203 2 6	208 6 8	213 10 10	218 15 0
6000	225 0 0	231 5 0	237 10 0	243 15 0	250 0 0	256 5 0	262 10 0
7000	262 10 0	269 15 10	277 1 8	284 7 6	291 13 4	298 19 2	306 5 0
8000	300 0 0	308 6 8	316 13 4	325 0 0	333 6 8	341 13 4	350 0 0
9000	337 10 0	346 17 6	356 5 0	365 12 6	375 0 0	384 7 6	393 15 0

MONEY TABLE—CONTINUED.

No.	L. S. D.	L. S. D.	L. S. D.	L. S. D.	L. S. D.	L. S. D.
	at 10 $\frac{3}{4}$ d.	at 11d.	at 11 $\frac{1}{4}$ d.	at 11 $\frac{1}{2}$ d.	at 11 $\frac{3}{4}$ d.	at 1s.
2	0 1 9 $\frac{1}{4}$	0 1 10	0 1 10 $\frac{1}{2}$	0 1 11	0 1 11 $\frac{1}{2}$	0 2 0
3	0 2 8 $\frac{1}{4}$	0 2 9	0 2 9 $\frac{3}{4}$	0 2 10 $\frac{1}{2}$	0 2 11 $\frac{1}{4}$	0 3 0
4	0 3 7	0 3 8	0 3 9	0 3 10	0 3 11	0 4 0
5	0 4 5 $\frac{3}{4}$	0 4 7	0 4 8 $\frac{1}{4}$	0 4 9 $\frac{1}{2}$	0 4 10 $\frac{3}{4}$	0 5 0
6	0 5 4 $\frac{1}{2}$	0 5 6	0 5 7 $\frac{1}{2}$	0 5 9	0 5 10 $\frac{1}{2}$	0 6 0
7	0 6 3 $\frac{1}{4}$	0 6 5	0 6 6 $\frac{3}{4}$	0 6 8 $\frac{1}{2}$	0 6 10 $\frac{1}{4}$	0 7 0
8	0 7 2	0 7 4	0 7 6	0 7 8	0 7 10	0 8 0
9	0 8 0 $\frac{3}{4}$	0 8 3	0 8 5 $\frac{1}{4}$	0 8 7 $\frac{1}{2}$	0 8 9 $\frac{3}{4}$	0 9 0
10	0 8 11 $\frac{1}{2}$	0 9 2	0 9 4 $\frac{1}{4}$	0 9 7	0 9 9 $\frac{1}{2}$	0 10 0
11	0 9 10 $\frac{1}{4}$	0 10 1	0 10 3 $\frac{3}{4}$	0 10 6 $\frac{1}{2}$	0 10 9 $\frac{1}{4}$	0 11 0
12	0 10 9	0 11 0	0 11 3	0 11 6	0 11 9	0 12 0
13	0 11 7 $\frac{3}{4}$	0 11 11	0 12 2 $\frac{1}{4}$	0 12 5 $\frac{1}{2}$	0 12 8 $\frac{3}{4}$	0 13 0
14	0 12 6 $\frac{1}{2}$	0 12 10	0 13 1 $\frac{1}{2}$	0 13 5	0 13 8 $\frac{1}{2}$	0 14 0
15	0 13 5 $\frac{1}{4}$	0 13 9	0 14 0 $\frac{3}{4}$	0 14 4 $\frac{1}{2}$	0 14 8 $\frac{1}{4}$	0 15 0
16	0 14 4	0 14 8	0 15 0	0 15 4	0 15 8	0 16 0
17	0 15 2 $\frac{3}{4}$	0 15 7	0 15 11 $\frac{1}{4}$	0 16 3 $\frac{1}{2}$	0 16 7 $\frac{3}{4}$	0 17 0
18	0 16 1 $\frac{1}{2}$	0 16 6	0 16 10 $\frac{1}{2}$	0 17 3	0 17 7 $\frac{1}{2}$	0 18 0
19	0 17 0 $\frac{1}{4}$	0 17 5	0 17 9 $\frac{3}{4}$	0 18 2 $\frac{1}{2}$	0 18 7 $\frac{1}{4}$	0 19 0
20	0 17 11	0 18 4	0 18 9	0 19 2	0 19 7	1 0 0
30	1 6 10 $\frac{1}{2}$	1 7 6	1 8 1 $\frac{1}{2}$	1 8 9	1 9 4 $\frac{1}{2}$	1 10 0
40	1 15 10	1 16 8	1 17 6	1 18 4	1 19 2	2 0 0
50	2 4 9 $\frac{1}{2}$	2 5 10	2 6 10 $\frac{1}{2}$	2 7 11	2 8 11 $\frac{1}{2}$	2 10 0
60	2 13 9	2 15 0	2 16 3	2 17 6	2 18 9	3 0 0
70	3 2 8 $\frac{1}{2}$	3 4 2	3 5 7 $\frac{1}{2}$	3 7 1	3 8 6 $\frac{1}{2}$	3 10 0
80	3 11 8	3 13 4	3 15 0	3 16 8	3 18 4	4 0 0
90	4 0 7 $\frac{1}{2}$	4 2 6	4 4 4 $\frac{1}{2}$	4 6 3	4 8 1 $\frac{1}{2}$	4 10 0
100	4 9 7	4 11 8	4 13 9	4 15 10	4 17 11	5 0 0
200	8 19 2	9 3 4	9 7 6	9 11 8	9 15 10	10 0 0
300	13 8 9	13 15 0	14 1 3	14 7 6	14 13 9	15 0 0
400	17 8 4	18 6 8	18 15 0	19 3 4	19 11 8	20 0 0
500	22 7 11	22 18 4	23 8 9	23 19 2	24 9 7	25 0 0
600	26 17 6	27 10 0	28 2 6	28 15 0	29 7 6	30 0 0
700	31 7 1	32 1 8	32 6 3	33 10 10	34 5 5	35 0 0
800	35 16 8	36 13 4	37 0 0	38 6 8	39 3 4	40 0 0
900	40 6 3	41 5 0	41 13 0	43 2 6	44 1 3	45 0 0
1000	44 15 10	45 16 8	46 17 6	47 18 4	48 19 2	50 0 0
2000	89 11 8	91 13 4	93 15 0	95 16 8	97 18 4	100 0 0
3000	134 7 6	137 10 0	140 12 6	143 15 0	146 17 6	150 0 0
4000	179 3 4	183 6 8	187 10 0	191 13 4	195 16 8	200 0 0
5000	223 19 2	229 3 4	234 7 6	239 11 8	244 15 10	250 0 0
6000	268 15 0	275 0 0	281 5 0	287 10 0	293 15 0	300 0 0
7000	313 10 10	320 16 8	328 2 6	335 8 4	342 14 2	350 0 0
8000	358 6 8	366 13 4	375 0 0	383 6 8	391 13 4	400 0 0
9000	403 2 6	412 10 0	421 17 6	431 5 0	440 12 6	450 0 0

EQUIVALENT PRICES TO COMMON WEIGHTS AND NUMBERS.

Per ton or 2240 lbs.	cwt. or 112 lbs.		qr. or 28 lbs.		stone or 14 lbs.		lb. or 1.	doz. or 12.	score or 20.	per 100.		per 120.		gross or 144.		per 1000.				
	L.	S.	D.	L.	S.	D.				L.	S.	D.	L.	S.	D.	L.	S.	D.	L.	S.
2 6 8	0	2	4	0	0	7	0	3	0	5	0	2	1	0	3	0	1	0	10	
3 10 0	0	3	6	0	0	10½	0	4½	0	7½	0	3	1½	0	4	6	1	11	3	
4 13 4	0	4	8	0	1	2	0	6	0	10	0	4	2	0	5	0	2	1	8	
5 16 8	0	5	10	0	1	5½	0	7½	1	0½	0	5	2½	0	6	3	0	2	12	1
7 0 0	0	7	0	0	1	9	0	9	1	3	0	6	3	0	7	6	0	3	2	6
8 3 4	0	8	2	0	2	0½	1	10½	1	5½	0	7	3½	0	8	9	0	3	12	11
9 6 8	0	9	4	0	2	4	1	1	1	8	0	8	4	0	10	0	4	3	4	
10 10 0	0	10	6	0	2	7½	1	1	1	10½	0	9	4½	0	11	3	0	4	13	9
11 13 4	0	11	8	0	2	11	1	3	2	1	0	10	5	0	12	6	0	5	4	2
12 16 8	0	12	10	0	3	2½	1	4½	2	3½	0	11	5½	0	13	9	0	5	14	7
14 0 0	0	14	0	0	3	6	1	6	2	6	0	12	6	0	15	0	6	5	0	
15 13 4	0	15	2	0	3	9½	1	7½	2	8½	0	13	6	0	16	3	6	15	2	
16 16 8	0	16	4	0	4	1	1	9	2	11	0	14	7	1	1	0	7	5	10	
17 10 0	0	17	6	0	4	4½	2	10½	3	1½	0	15	7½	1	2	6	7	16	3	
18 13 4	0	18	8	0	4	8	2	0	3	4	0	16	8	1	4	0	8	6	8	
19 16 8	0	19	10	0	4	11½	2	1½	3	6½	1	1	3	1	5	6	8	17	1	
21 0 0	1	1	0	0	5	3	2	3	3	9	1	2	6	1	7	0	9	7	6	
22 3 4	1	2	2	0	5	6½	2	4½	3	11½	1	3	9	1	8	6	9	17	11	
23 6 8	1	3	4	0	5	10	2	6	4	2	1	0	10	1	10	0	10	8	4	
24 10 0	1	4	6	0	6	1½	2	7½	4	6½	1	1	10½	1	11	6	10	18	9	
25 13 4	1	5	8	0	6	5	2	9	4	7	1	2	11	1	13	0	11	9	2	
26 16 8	1	6	10	0	6	8½	3	10½	4	9½	1	3	11½	1	14	6	11	19	7	
28 0 0	1	8	0	0	7	0	3	0	5	0	1	5	0	1	16	0	12	10	0	
29 3 4	1	9	2	0	7	3½	3	1½	5	2½	1	6	0½	1	17	6	13	0	5	
30 6 8	1	10	4	0	7	7	3	3	5	5	1	7	1	1	19	0	13	10	10	
31 10 0	1	11	6	0	7	10½	3	4½	5	7½	1	8	1½	2	0	6	14	1	3	
32 13 4	1	12	8	0	8	2	4	1	5	10	1	9	2	2	2	0	14	11	8	
33 16 8	1	13	4	0	8	5½	4	2	5	10	1	10	2½	2	3	6	15	2	1	
35 0 0	1	15	0	0	8	9	4	3	6	3	1	11	3	2	5	0	15	12	6	
36 3 4	1	16	2	0	9	0½	4	4	6	5	1	12	3½	2	6	6	16	2	11	
37 6 8	1	17	4	0	9	4	4	0	6	8	1	13	4	2	8	0	16	13	4	

38	10	0	1	18	6	0	9	7d	4	9d	4d	4d	4d	4d	4d	6	10d	1	14	4d	2	2	3	3	6	3	9	17	3	9
39	13	4	4	19	8	0	9	11	4	11d	4d	4d	4d	4d	4d	7	1	15	5	1	16	5	2	3	9	0	17	14	2	
40	16	8	2	0	10	0	10	2d	5	3d	4d	4d	4d	4d	4d	8	3	16	5	1	17	6	2	9	0	18	4	7		
42	0	0	2	2	0	0	10	6	5	3	4d	4d	4d	4d	4d	0	7	7	6	1	17	6	2	9	0	18	15	0		
43	3	4	2	3	2	0	10	9d	5	4d	4d	4d	4d	4d	4d	2	8	1	18	6d	2	6	3	0	0	19	5	5		
44	6	8	2	4	4	0	11	1	5	6d	4d	4d	4d	4d	4d	4	11	1	19	7	2	7	6	0	0	19	15	10		
45	10	0	2	5	6	0	11	4d	5	8d	4d	4d	4d	4d	4d	8	2	0	7d	2	8	9	9	0	0	20	6	3		
46	13	4	2	6	8	0	11	8	5	10	5d.	4d	4d	4d	4d	4	8	4	0	7d	2	10	0	0	0	20	16	8		
49	0	0	2	9	0	0	12	3	6	1d	5d	4d	4d	4d	4d	8	9	4	2	8	2	10	6	0	0	21	17	6		
51	6	8	2	11	4	0	12	10	6	5	5d	4d	4d	4d	4d	9	9	2	3	9	2	12	6	0	0	22	18	4		
53	13	4	2	13	8	0	13	5	6	8d	5d	4d	4d	4d	4d	10	7	7	5	11	2	14	6	0	0	23	19	2		
56	0	0	2	16	0	0	14	0	6	0	6d.	4d	4d	4d	4d	0	10	0	0	0	3	17	6	0	0	25	0	0		
58	6	8	2	18	4	0	14	7	6	3	6d.	4d	4d	4d	4d	5	10	5	12	1	3	2	6	0	0	26	0	10		
60	13	4	3	0	8	0	15	2	6	6	6d.	4d	4d	4d	4d	10	10	2	14	2	3	5	0	0	0	27	1	8		
63	0	0	3	3	0	0	15	9	6	9	6d.	4d	4d	4d	4d	11	3	3	16	3	3	7	6	0	0	28	2	6		
65	6	8	3	5	4	0	16	4	7	0	7d.	4d	4d	4d	4d	11	8	4	18	4	4	7	0	0	0	29	8	4		
67	13	4	3	7	8	0	16	11	7	3	7d.	4d	4d	4d	4d	12	1	1	20	5	3	10	6	0	0	30	4	2		
70	0	0	3	10	0	0	17	6	7	6	7d.	4d	4d	4d	4d	12	6	12	3	2	6	10	0	0	0	31	5	0		
72	6	8	3	12	4	0	18	1	7	9	7d.	4d	4d	4d	4d	12	11	3	4	7	6	13	6	0	0	32	5	10		
74	13	4	3	14	8	0	18	8	8	0	8d.	4d	4d	4d	4d	13	4	3	6	8	4	0	0	0	0	33	6	8		
77	0	0	3	17	0	0	19	3	8	3	8d.	4d	4d	4d	4d	13	9	9	8	9	4	2	6	0	0	34	7	6		
79	6	8	3	19	4	0	19	10	8	6	8d.	4d	4d	4d	4d	14	2	14	10	10	4	5	0	0	0	35	8	4		
81	13	4	4	1	8	0	1	0	8	9	8d.	4d	4d	4d	4d	14	7	14	11	11	4	7	6	0	0	36	9	2		
84	0	0	4	4	0	0	1	1	10	0	9d.	4d	4d	4d	4d	15	0	15	0	0	4	10	0	0	0	37	10	0		
86	6	8	4	6	4	0	1	1	1	3	9d.	4d	4d	4d	4d	15	5	16	1	0	10	6	0	0	0	38	10	10		
88	13	4	4	8	8	0	1	2	1	7	9d.	4d	4d	4d	4d	15	10	17	1	12	6	6	0	0	0	39	11	8		
91	0	0	4	11	0	0	1	2	2	9	9d.	4d	4d	4d	4d	16	3	18	2	4	15	0	0	0	0	40	12	6		
93	6	8	4	13	4	0	1	3	4	11	10d.	4d	4d	4d	4d	16	8	18	3	4	17	6	0	0	0	41	13	4		
95	13	4	4	15	8	0	1	3	11	11	10d.	4d	4d	4d	4d	17	1	19	4	4	19	0	0	0	0	42	14	2		
98	0	0	4	18	0	0	1	4	6	12	10d.	4d	4d	4d	4d	17	6	20	6	5	2	6	0	0	0	43	15	0		
100	6	8	5	0	4	0	1	5	1	13	10d.	4d	4d	4d	4d	17	11	21	7	6	3	6	0	0	0	44	15	10		
102	13	4	5	2	8	0	1	5	8	14	11d.	4d	4d	4d	4d	18	4	22	8	7	6	6	0	0	0	45	16	8		
105	0	0	5	5	0	0	1	6	3	15	11d.	4d	4d	4d	4d	18	9	23	9	8	6	12	0	0	0	46	17	6		
107	6	8	5	7	4	0	1	6	10	16	11d.	4d	4d	4d	4d	19	2	24	10	9	6	15	0	0	0	47	18	4		
109	13	4	5	9	8	0	1	7	5	17	11d.	4d	4d	4d	4d	19	7	25	11	10	6	18	0	0	0	48	19	2		
112	0	0	5	12	0	0	1	8	0	18	12d.	4d	4d	4d	4d	20	0	26	11	11	11	1	0	0	0	50	0	0		

Practical Utility of the preceding Tables.

1. What will be the cost of 1 ton 2 cwts. 3 qrs. 4 lbs. at $7\frac{3}{4}d.$ per lb.?

1 ton	=	2240 lbs.
2 cwts.	=	224 „
3 qrs.	=	84 „
4 lbs.	=	4 „
Total	=	<u>2552 „</u>

And 2000 at $7\frac{3}{4}d.$ (as per Table, p. 81)	=	£ 64	11	8
500	„	=	16	2 11
50	„	=	1	12 3½
2	„	=	0	1 3½
<u>2552</u>			<u>£ 82</u>	<u>8 2</u>

2. In the Table of Equivalent Prices, the fifth column from the left hand is the price per single article, or 1; hence the other columns on the right and left of that give the price at an equal rate, according to the various denominations by which the columns are headed.

Thus, suppose the price per lb. = $3\frac{1}{4}d.$, the price per ton = £30. 6s. 8d. Again, suppose the price per gross = £1. 10s., the price per dozen = 2s. 6d., &c.

STRENGTH OF MATERIALS.

MATERIALS of construction are liable to four different kinds of strain; viz., stretching, crushing, transverse action, and torsion or twisting: the first of which depends upon the body's tenacity alone; the second, on its resistance to compression; the third, on its tenacity and compression combined; and the fourth, on that property by which it opposes any acting force tending to change from a straight line, to that of a spiral direction, the fibres of which the body is composed.

In bodies, the power of tenacity and resistance to

compression in the direction of their length, is as the cross section of their area multiplied by the results of experiments on similar bodies, as exhibited in the following Table.

Table showing the Tenacities, Resistances to Compression, and other Properties of the common Materials of Construction.

Names of Bodies.	Absolute		Compared with Cast Iron		
	Tenacity in lbs. per sq. inch.	Resistance to compression in lbs. per sq. in.	Its strength is	Its extensibility is	Its stiffness is
Ash . . .	14130	—	0·23	2·6	0·089
Beech . . .	12225	8548	0·15	2·1	0·073
Brass . . .	17968	10304	0·435	0·9	0·49
Brick . . .	275	562	—	—	—
Cast iron . . .	13434	86397	1·000	1·0	1·000
Copper (wrought) . . .	33000	—	—	—	—
Elm . . .	9720	1033	0·21	2·9	0·073
Fir, or Pine, white . . .	12346	2028	0·23	2·4	0·1
„ „ red . . .	11800	5375	0·3	2·4	0·1
„ „ yellow . . .	11835	5445	0·25	2·9	0·087
Granite, (Aberdeen) . . .	—	10910	—	—	—
Gun-metal (copper 8, and tin 1) . . .	35838	—	0·65	1·25	0·535
Malleable iron . . .	56000	—	1·12	0·86	1·3
Larch . . .	12240	5568	0·136	2·3	0·058
Lead . . .	1824	—	0·096	2·5	0·0385
Mahogany, Honduras . . .	11475	8000	0·24	2·9	0·487
Marble . . .	551	6060	—	—	—
Oak . . .	11880	9504	0·25	2·8	0·093
Rope, (1 in. in circum.) . . .	200	—	—	—	—
Steel . . .	128000	—	—	—	—
Stone, Bath . . .	478	—	—	—	—
„ Craigleith . . .	772	5490	—	—	—
„ Dundee . . .	2661	6630	—	—	—
„ Portland . . .	857	3729	—	—	—
Tin (cast) . . .	4736	—	0·182	0·75	0·25
Zinc (sheet) . . .	9120	—	0·365	0·5	0·76

Comparative Strength and Weight of Ropes and Chains.

Circum. of rope in inches.	Weight per fathom in lbs.	Diameter of chain in inches.	Weight per fathom in lbs.	Proof strength in tons & cwt.		Circum. of rope in inches.	Weight per fathom in lbs.	Diameter of chain in inches.	Weight per fathom in lbs.	Proof strength in tons & cwt.	
3½	2¾	$\frac{5}{16}$	5½	1	5½	10	23	$\frac{7}{8}$	43	10	0
4¼	4¾	$\frac{3}{8}$	8	1	16¾	10¾	28	$\frac{1}{4}$ $\frac{5}{8}$	49	11	11
5	5¾	$\frac{7}{16}$	10½	2	10	11½	30½	1 in.	56	13	8
5¾	7	$\frac{1}{2}$	14	3	5½	12¼	36	$1\frac{1}{16}$	63	14	18
6½	9¾	$\frac{9}{16}$	18	4	3½	13	39	$1\frac{1}{8}$	71	16	14
7	11¼	$\frac{5}{8}$	22	5	2	13¾	45	$1\frac{3}{16}$	79	18	11
8	15	$1\frac{1}{16}$	27	6	4½	14½	48½	$1\frac{1}{4}$	87	20	8
8¾	19	$\frac{3}{4}$	32	7	7	15¼	56	$1\frac{5}{16}$	96	22	13
9½	21	$1\frac{3}{8}$	37	8	13½	16	60	$1\frac{3}{8}$	106	24	18

Note.—It must be understood and also borne in mind, that in estimating the amount of tensile strain to which a body is subjected, the weight of the body itself must also be taken into account; for according to its position so may it approximate to its whole weight, in tending to produce extension within itself; as in the almost constant application of ropes and chains to great depths, considerable heights, &c.

Alloys that are of greater Tenacity than the Sum of their Constituents, as determined by the Experiments of Muschenbroek.

Swedish copper 6 parts, Malacca tin 1;	tenacity per square inch	64,000 lbs.
Chili copper 6 parts, Malacca tin 1;	”	60,000 ”
Japan copper 5 parts, Banca tin 1;	”	57,000 ”
Anglesea copper 6 parts, Cornish tin 1;	”	41,000 ”
Common block-tin 4, lead 1, zinc 1;	”	13,000 ”
Malacca tin 4, regulus of antimony 1;	”	12,000 ”
Block tin 3, lead 1;	”	10,200 ”
Block tin 8, zinc 1;	”	10,000 ”
Lead 1, zinc 1;	”	4,500 ”

RESISTANCE TO LATERAL PRESSURE, OR TRANSVERSE ACTION.

The strength of a square or rectangular beam to resist lateral pressure acting in a perpendicular direction to its length, is as the breadth and square of the depth, and inversely as the length;—thus, a beam twice the breadth of another, all other circumstances being alike, equal twice the strength of the other; or twice the depth, equal four times the strength, and twice the length, equal only half the strength, &c., according to the rule.

Table of Data, containing the Results of Experiments on the Elasticity and Strength of various Species of Timber, by Mr. Barlow.

Species of Timber.	Value of E.	Value of S.	Species of Timber.	Value of E.	Value of S.
Teak . . .	174·7	2462	Elm	50·64	1013
Poona . . .	122·26	2221	Pitch pine . . .	88·68	1632
English oak .	105	1672	Red pine	133·	1341
Canadian do.	155·5	1766	New England fir	158·5	1102
Dantzic do.	86·2	1457	Riga fir	90	1100
Adriatic do.	70·5	1383	Mar Forest do.	63	1200
Ash	119	2026	Larch	76	900
Beech . . .	98	1556	Norway spruce .	105·47	1474

To find the dimensions of a beam capable of sustaining a given weight, with a given degree of deflection, when supported at both ends.

Rule.—Multiply the weight to be supported in lbs. by the cube of the length in feet; divide the product by 32 times the tabular value of E, multiplied into the given deflection in inches; and the

quotient is the breadth multiplied by the cube of the depth in inches.

Note 1.—When the beam is intended to be square, then the fourth root of the quotient is the breadth and depth required.

Note 2.—If the beam is to be cylindrical, multiply the quotient by 1.7, and the fourth root of the product is the diameter.

Ex. The distance between the supports of a beam of Riga fir is 16 feet, and the weight it must be capable of sustaining in the middle of its length is 8000 lbs., with a deflection of not more than $\frac{3}{4}$ of an inch; what must be the depth of the beam, supposing the breadth 8 inches?

$$\frac{16 \times 8000}{90 \times 32 \times .75} = 15175 \div 8 = \sqrt[3]{1897} = 12.35 \text{ in. the depth.}$$

To determine the absolute strength of a rectangular beam of timber when supported at both ends, and loaded in the middle of its length, as beams in general ought to be calculated to, so that they may be rendered capable of withstanding all accidental cases of emergency.

Rule.—Multiply the tabular value of S by four times the depth of the beam in inches, and by the area of the cross section in inches; divide the product by the distance between the supports in inches, and the quotient will be the absolute strength of the beam in lbs.

Note 1.—If the beam be not laid horizontally, the distance between the supports, for calculation, must be the horizontal distance.

Note 2.—One-fourth of the weight obtained by the rule, is the greatest weight that ought to be applied in practice as permanent load.

Note 3.—If the load is to be applied at any other point than the middle, then the strength will be, as the product of the two distances is to the square of half the length of the beam

between the supports ;—or, twice the distance from one end, multiplied by twice from the other, and divided by the whole length, equal the effective length of the beam.

Ex. In a building 18 feet in width, an engine boiler of $5\frac{1}{2}$ tons is to be fixed, the centre of which to be 7 feet from the wall ; and having two pieces of red pine 10 inches by 6, which I can lay across the two walls for the purpose of slinging it at each end,—may I with sufficient confidence apply them, so as to effect this object ?

$$\frac{2240 \times 5.5}{2} = 6160 \text{ lbs. to carry at each end.}$$

And 18 feet $- 7 = 11$, double each, or 14 and 22, then $\frac{14 \times 22}{18} = 17$ feet, or 204 inches, effective length of beam.

$$\text{Tabular value of S, red pine} = \frac{1341 \times 4 \times 10 \times 60}{204} = 15776 \text{ lbs.}$$

the absolute strength of each piece of timber at that point.

To determine the dimensions of a rectangular beam capable of supporting a required weight, with a given degree of deflection, when fixed at one end.

Rule.—Divide the weight to be supported, in lbs., by the tabular value of E, multiplied by the breadth and deflection, both in inches ; and the cube root of the quotient, multiplied by the length in feet, equal the depth required in inches.

Ex. A beam of ash is intended to bear a load of 700 lbs. at its extremity ; its length being 5 feet, its breadth 4 inches, and the deflection not to exceed $\frac{1}{2}$ of an inch.

Tabular value of E = $119 \times 4 \times .5 = 238$ the divisor ;
then $700 \div 238 = \sqrt[3]{2.94} \times 5 = 7.25$ inches, depth of the beam.

To find the absolute strength of a rectangular

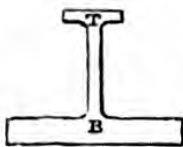
beam, when fixed at one end, and loaded at the other.

Rule.—Multiply the value of S by the depth of the beam, and by the area of its section, both in inches; divide the product by the leverage in inches, and the quotient equal the absolute strength of the beam in lbs.

Ex. A beam of Riga fir, 12 inches by $4\frac{1}{2}$, and projecting $6\frac{1}{2}$ feet from the wall; what is the greatest weight it will support at the extremity of its length?

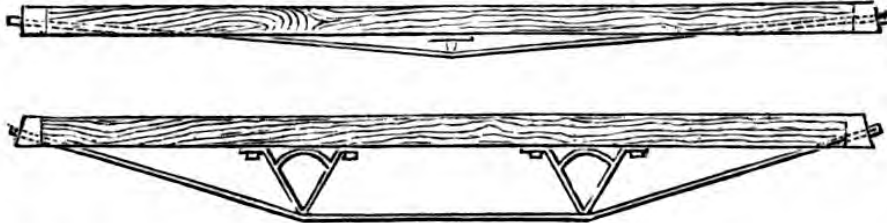
$$\begin{aligned} &\text{Tabular value of } S = 1100 \\ &12 \times 4.5 = 54 \text{ sectional area,} \\ \text{Then, } &\frac{1100 \times 12 \times 54}{78} = 9138.4 \text{ lbs.} \end{aligned}$$

When fracture of a beam is produced by vertical pressure, the fibres of the lower section of fracture are separated by extension, whilst at the same time those of the upper portion are destroyed by compression; hence exists a point in section where neither the one nor the other takes place, and which is distinguished as the point of neutral axis. Therefore, by the law of fracture thus established, and proper data of tenacity and compression given, as in the Table (p. 87), we are enabled to form metal beams of strongest section with the least possible material: thus, in cast iron the resistance to compression is nearly as $6\frac{1}{2}$ to 1 of tenacity; consequently a beam of cast iron to be of strongest section must be of the following form,



and a parabola in the direction of its length, the quantity of material in the bottom flange being about $6\frac{1}{2}$ times that of the upper: but such is not the case with beams of timber; for although the tenacity of timber be on an average twice that of its resistance to compression, its flexibility is so great, that any considerable length of

beam, where columns cannot be situated to its support, requires to be strengthened or trussed by iron rods, as in the following manner.



And these applications of principle not only tend to diminish deflection, but the required purpose is also more effectively attained, and that by lighter pieces of timber.

To ascertain the absolute strength of a cast iron beam of the preceding form, or that of strongest section.

Rule.—Multiply the sectional area of the bottom flange in inches by the depth of the beam in inches, and divide the product by the distance between the supports also in inches; and 514 times the quotient equal the absolute strength of the beam in cwts.

The strongest form in which any given quantity of matter can be disposed is that of a hollow cylinder; and it has been demonstrated that the maximum of strength is obtained in cast iron, when the thickness of the annulus or ring amounts to $\frac{1}{5}$ th of the cylinder's external diameter; the relative strength of a solid to that of a hollow cylinder being as the diameters of their sections.

A Table showing the Weight or Pressure a Beam of Cast Iron, 1 inch in breadth, will sustain without destroying its elastic force, when it is supported at each end, and loaded in the middle of its length, and also the deflection in the middle which that weight will produce. By Mr. Hodgkinson, Manchester.

Length.	6 feet.		7 feet.		8 feet.		9 feet.		10 feet.	
	Wt. in lbs.	Defl. in in.	Wt. in lbs.	Defl. in in.	Wt. in lbs.	Defl. in in.	Wt. in lbs.	Defl. in in.	Wt. in lbs.	Defl. in in.
3	1278	·24	1089	·33	954	·426	855	·54	765	·66
3½	1739	·205	1482	·28	1298	·365	1164	·46	1041	·57
4	2272	·18	1936	·245	1700	·32	1520	·405	1360	·5
4½	2875	·16	2450	·217	2146	·284	1924	·36	1721	·443
5	3560	·144	3050	·196	2650	·256	2375	·32	2125	·4
6	5112	·12	4356	·163	3816	·213	3420	·27	3060	·33
7	6958	·103	5929	·14	5194	·183	4655	·23	4165	·29
8	9088	·09	7744	·123	6784	·16	6080	·203	5440	·25
9	—	—	9801	·109	8586	·142	7695	·18	6885	·22
10	—	—	12100	·098	10600	·128	9500	·162	8500	·2
11	—	—	—	—	12826	·117	11495	·15	10285	·182
12	—	—	—	—	15264	·107	13680	·135	12240	·17
13	—	—	—	—	—	—	16100	·125	14400	·154
14	—	—	—	—	—	—	18600	·115	16700	·143
	12 feet.		14 feet.		16 feet.		18 feet.		20 feet.	
6	2548	·48	2184	·65	1912	·85	1699	1·08	1530	1·34
7	3471	·41	2975	·58	2603	·73	2314	·93	2082	1·14
8	4532	·36	3884	·49	3396	·64	3020	·81	2720	1·00
9	5733	·32	4914	·44	4302	·57	3825	·72	3438	·89
10	7083	·28	6071	·39	5312	·51	4722	·64	4250	·8
11	8570	·26	7346	·36	6428	·47	5714	·59	5142	·73
12	10192	·24	8736	·33	7648	·43	6796	·54	6120	·67
13	11971	·22	10260	·31	8978	·39	7980	·49	7182	·61
14	13883	·21	11900	·28	10412	·36	9255	·46	8330	·57
15	15937	·19	13660	·26	11952	·34	10624	·43	9562	·53
16	18128	·18	15536	·24	13584	·32	12080	·40	10880	·5
17	20500	·17	17500	·23	15353	·3	13647	·38	12282	·47
18	22932	·16	19656	·21	17208	·28	15700	·36	13752	·44

Note.—This Table shows the greatest weight that ever ought to be laid upon a beam for permanent load, and if there be any liability to jerks, &c., ample allowance must be made; also the weight of the beam itself must be included.

To find the weight of a cast iron beam of given dimensions.

Rule.—Multiply the sectional area in inches by the length in feet, and by 3·2, the product equal the weight in lbs.

Ex. Required the weight of a uniform rectangular beam of cast iron, 16 feet in length, 11 inches in breadth, and $1\frac{1}{2}$ inch in thickness.

$$11 \times 1\frac{1}{2} \times 16 \times 3\cdot2 = 844\cdot8 \text{ lbs.}$$

Resistance of bodies to flexure by vertical pressure.

When a piece of timber is employed as a column or support, its tendency to yielding by compression is different according to the proportion between its length and area of its cross section; and supposing the form that of a cylinder whose length is less than seven or eight times its diameter, it is impossible to bend it by any force applied longitudinally, as it will be destroyed by splitting before that bending can take place; but when the length exceeds this, the column will bend under a certain load, and be ultimately destroyed by a similar kind of action to that which has place in the transverse strain.

Columns of cast iron and of other bodies are also similarly circumstanced, this law having recently been fully developed by the experiments of Mr. Hodgkinson on columns of different diameters, and of different lengths.

When the length of a cast iron column with flat ends equals about thirty times its diameter, fracture will be produced wholly by bending of the material;—when of less length, fracture takes place partly by crushing and partly by bending: but, when the column is enlarged in the middle of its length from one and a half to twice its diameter at the ends, by being

cast hollow, the strength is greater by $\frac{1}{7}$ th than in a solid column containing the same quantity of material.

To determine the dimensions of a support or column to bear without sensible curvature a given pressure in the direction of its axis.

Rule.—Multiply the pressure to be supported in lbs. by the square of the column's length in feet, and divide the product by twenty times the tabular value of E; and the quotient will be equal to the breadth multiplied by the cube of the least thickness, both being expressed in inches.

Note 1.—When the pillar or support is a square, its side will be the fourth root of the quotient.

2. If the pillar or column be a cylinder, multiply the tabular value of E by 12, and the fourth root of the quotient equal the diameter.

Ex. 1. What should be the least dimensions of an oak support, to bear a weight of 2240 lbs. without sensible flexure, its breadth being 3 inches, and its length 5 feet?

Tabular value of E = 105,

$$\text{and } \frac{2240 \times 5^2}{20 \times 105 \times 3} = \sqrt[3]{8.888} = 2.05 \text{ inches.}$$

Ex. 2. Required the side of a square piece of Riga fir, 9 feet in length, to bear a permanent weight of 6000 lbs.

Tabular value of E = 96,

$$\text{and } \frac{6000 \times 9^2}{20 \times 96} = \sqrt[4]{253} = 4 \text{ inches nearly.}$$

Dimensions of Cylindrical Columns of Cast Iron to sustain a given load or pressure with safety.

Diam. in inches.	Length or height in feet.											
	4	6	8	10	12	14	16	18	20	22	24	
2	72	60	49	40	32	26	22	18	15	13	11	
2½	119	105	91	77	65	55	47	40	34	29	25	
3	178	163	145	128	111	97	84	73	64	56	49	
3½	247	232	214	191	172	156	135	119	106	94	83	
4	326	310	288	266	242	220	198	178	160	144	130	
4½	418	400	379	354	327	301	275	251	229	208	189	
5	522	501	479	452	427	394	365	337	310	285	262	
6	607	592	573	550	525	497	469	440	413	386	360	
7	1032	1013	989	959	924	887	848	808	765	725	686	
8	1333	1315	1289	1259	1224	1185	1142	1097	1052	1005	959	
9	1716	1697	1672	1640	1603	1561	1515	1467	1416	1364	1311	
10	2119	2100	2077	2045	2007	1964	1916	1865	1811	1755	1697	
11	2570	2550	2520	2490	2450	2410	2358	2305	2248	2189	2127	
12	3050	3040	3020	2970	2930	2900	2830	2780	2730	2670	2600	

Practical Utility of the preceding Table.

Ex. Wanting to support the front of a building with cast iron columns 18 feet in length, 8 inches in diameter, and the metal 1 inch in thickness; what weight may

I confidently expect each column capable of supporting without tendency to deflection?

Opposite 8 inches diameter, and under 18 feet = 1097

Also opposite 6 in. diameter and under 18 feet = 440

= 657 cwts.

Note.—The strength of cast iron as a column being 1·0000

" steel " = 2·518

" wrought iron " = 1·745

" (oak) Dantzic " = ·1088

" red deal " = ·0785

Elasticity of torsion, or resistance of bodies to twisting.

The angle of flexure by torsion is as the length and extensibility of the body directly, and inversely as the diameter; hence, the length of a bar or shaft being given, the power, and the leverage the power acts with, being known, and also the number of degrees of torsion that will not affect the action of the machine, to determine the diameter in cast iron with a given angle of flexure.

Rule.—Multiply the power in lbs. by the length of the shaft in feet, and by the leverage in feet; divide the product by fifty-five times the number of degrees in the angle of torsion; and the fourth root of the quotient equal the shaft's diameter in inches.

Ex. Required the diameters for a series of shafts 35 feet in length, and to transmit a power equal to 1245 lbs., acting at the circumference of a wheel $2\frac{1}{2}$ feet radius, so that the twist of the shafts on the application of the power may not exceed one degree.

$$\frac{1245 \times 35 \times 2.5}{55 \times 1} = \sqrt[4]{1981} = 6.67 \text{ inches in diameter.}$$

Relative strength of metals to resist torsion.

Cast iron . . . = 1·	Swedish bar iron . = 1·05
Copper . . . = ·48	English do. . = 1·12
Yellow brass . . = ·511	Sheer steel . . = 1·96
Gun-metal . . . = ·55	Cast do. . . = 2·1

Table of Squares, Cubes, and Fourth Power of Numbers.

Root.	Square.	Cube.	4th Power.	Root.	Square.	Cube.	4th Power.
1	1	1	1	7	49	343	2401
.1	1.21	1.331	1.4641	.1	50.41	357.911	2541.1681
.2	1.44	1.728	2.0736	.2	51.84	373.248	2687.3856
.3	1.69	2.197	2.8561	.3	53.29	389.017	2839.8241
.4	1.96	2.744	3.8416	.4	54.76	405.224	2998.6576
.5	2.25	3.375	5.0625	.5	56.25	421.875	3163.8625
.6	2.56	4.096	6.5536	.6	57.76	438.976	3346.2176
.7	2.89	4.913	8.3521	.7	59.29	456.533	3515.3041
.8	3.24	5.832	10.5876	.8	60.84	474.552	3701.4756
.9	3.61	6.859	13.0321	.9	62.41	493.039	3895.0081
2	4	8	16	8	64	512	4096
.1	4.41	9.261	19.4481	.1	65.61	531.441	4304.6721
.2	4.84	10.648	23.4256	.2	67.24	551.368	4521.2176
.3	5.29	12.167	27.8841	.3	68.89	571.787	4745.8221
.4	5.76	13.824	33.1776	.4	70.56	592.704	4979.5836
.5	6.25	15.625	39.0625	.5	72.25	614.125	5220.0625
.6	6.76	17.576	45.6976	.6	73.96	636.056	5470.0816
.7	7.29	19.683	53.1441	.7	75.69	658.503	5728.9761
.8	7.84	21.952	61.4656	.8	77.44	681.472	5996.9536
.9	8.41	24.389	70.7281	.9	79.21	704.969	6274.2141
3	9	27	81	9	81	729	6561
.1	9.61	29.791	92.3521	.1	82.81	753.571	6857.4961
.2	10.24	32.768	104.8576	.2	84.64	778.688	7173.9296
.3	10.89	35.937	118.5921	.3	86.49	804.357	7479.5201
.4	11.56	39.304	133.6336	.4	88.36	830.584	7807.4896
.5	12.25	42.875	150.0625	.5	90.25	857.375	8075.0525
.6	12.96	46.656	167.9616	.6	92.16	884.736	8493.6656
.7	13.69	50.653	187.4161	.7	94.09	912.673	8751.9281
.8	14.44	54.872	208.5136	.8	96.04	941.192	9231.6816
.9	15.21	59.319	231.3441	.9	98.01	970.299	9308.9601
4	16	64	256	10	100	1000	10000
.1	16.81	68.921	282.5761	.1	102.01	1030.301	10406.0401
.2	17.64	74.088	311.1696	.2	104.04	1061.208	10624.3216
.3	18.49	79.507	341.8801	.3	106.09	1092.727	11255.0881
.4	19.36	85.184	374.8096	.4	108.16	1124.864	11698.5856
.5	20.25	91.125	410.0625	.5	110.25	1157.625	12155.0625
.6	21.16	97.336	447.7456	.6	112.36	1191.016	12624.7696
.7	22.09	103.823	487.9681	.7	114.49	1225.043	13107.9601
.8	23.04	110.592	530.8416	.8	116.64	1259.712	13604.8896
.9	24.01	117.649	576.4801	.9	118.81	1295.029	14115.8161
5	25	125	625	11	121	1331	14641
.1	26.01	132.651	676.5201	.1	123.21	1367.631	15180.7041
.2	27.04	140.608	731.1616	.2	125.44	1404.928	15735.1936
.3	28.09	148.877	789.0481	.3	127.69	1442.897	16314.7361
.4	29.16	157.464	850.3056	.4	129.96	1481.544	16899.6016
.5	30.25	166.375	915.0625	.5	132.25	1520.875	17490.0625
.6	31.36	175.616	983.4496	.6	134.56	1560.896	18106.3936
.7	32.49	185.193	1055.6001	.7	136.89	1601.613	18749.9721
.8	33.64	195.112	1131.6496	.8	139.24	1643.032	19387.7776
.9	34.81	205.379	1211.7361	.9	141.61	1685.159	20153.3921
6	36	216	1296	12	144	1728	20736
.1	37.21	226.981	1384.5841	.1	146.41	1771.561	21345.8881
.2	38.44	238.328	1477.6336	.2	148.84	1815.848	22253.4556
.3	39.69	250.047	1575.2961	.3	151.29	1860.867	22898.6641
.4	40.96	262.144	1677.7216	.4	153.76	1906.624	23642.1376
.5	42.25	274.625	1785.0625	.5	156.25	1953.125	24414.0625
.6	43.56	287.496	1897.4736	.6	158.76	2000.376	25204.7376
.7	44.89	300.763	2015.1121	.7	161.29	2048.383	26011.6641
.8	46.24	314.432	2138.1376	.8	163.84	2097.152	26843.5456
.9	47.61	328.509	2266.7121	.9	166.41	2146.689	27692.3881

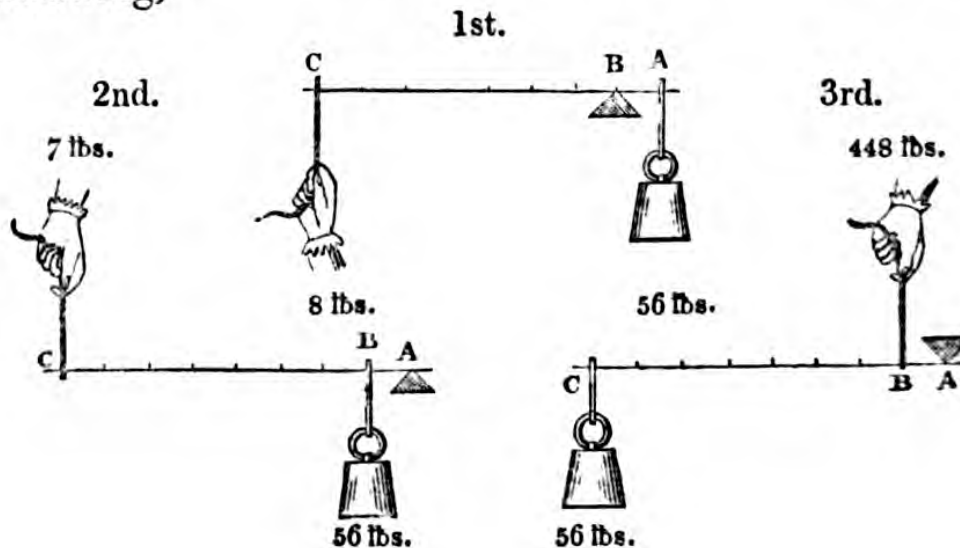
PRINCIPLES & PRACTICAL APPLICATIONS OF MECHANIC POWERS.

MECHANIC Powers, or the Elements of Machinery, are certain simple mechanical arrangements whereby weights may be raised or resistances overcome with the exertion of less power or strength than is necessary without them.

They are usually accounted six in number, viz., the *lever*, the *wheel and axle*, the *pulley*, the *inclined plane*, the *wedge*, and the *screw*; but properly two of these comprise the whole, namely, the *lever* and *inclined plane*,—the wheel and axle being only a lever of the first kind, and the pulley a lever of the second,—the wedge and the screw being also similarly allied to that of the inclined plane: however, although such seems to be the case in these respects, yet they each require, on account of their various modifications, a peculiar rule of estimation adapted expressly to the different circumstances in which they are individually required to act.

1. THE LEVER.

Lever, according to mode of application, as the following,



are distinguished as being of the first, second, or third kind; and although levers of equal lengths produce different effects, the general principles of estimation in all are the same; namely, the power is to the weight or resistance, as the distance of the one end to the fulcrum is to the distance of the other end to the same point.

In the *first kind*, the power is to the resistance, as the distance A B is to the distance B C.

In the *second*, the power is to the resistance, as the distance A B is to that of A C; and,

In the *third*, the resistance is to the power, as the distance A B is to that of A C.

Rule, first kind.—Divide the longer by the shorter end of the lever from the fulcrum, and the quotient is the effective force that the power applied is equal to.

Ex. 1. Let the handle of a pump equal 65 inches in length, and 10 inches from the shortest end to centre of motion; what is the amount of effective leverage thereby obtained?

$$65 - 10 = 55, \text{ and } \frac{55}{10} = 5\frac{1}{2} \text{ to } 1.$$

Ex. 2. Required the situation of the fulcrum on which to rest a lever of 15 feet, so that $2\frac{1}{2}$ cwt. placed at one end may equipoise 30 cwt. at the other, the weight of the lever not being taken into account.

$\frac{15 \times 2.5}{2.5 + 30} = 1.154$ feet from the end on which the 30 cwt. is to be placed.

The common steelyard, or Roman balance, as represented in fig. 1, Plate D, is a lever of the first kind, and so divided that one weight W, moved to or from the axis of motion, will equipoise and there indicate the weight of any article required to be known.

It is by the second kind of lever, that the greatest effect is obtained from any given amount of power; hence the propriety of the application of this principle to the working of force pumps, and shearing of iron, as by the lever of a punching-press, &c.

Rule, second kind.—Divide the whole length of lever, or distance from power to fulcrum, by the distance from fulcrum to weight, and the quotient is the proportion of effect that the power is to the weight or resistance to be overcome.

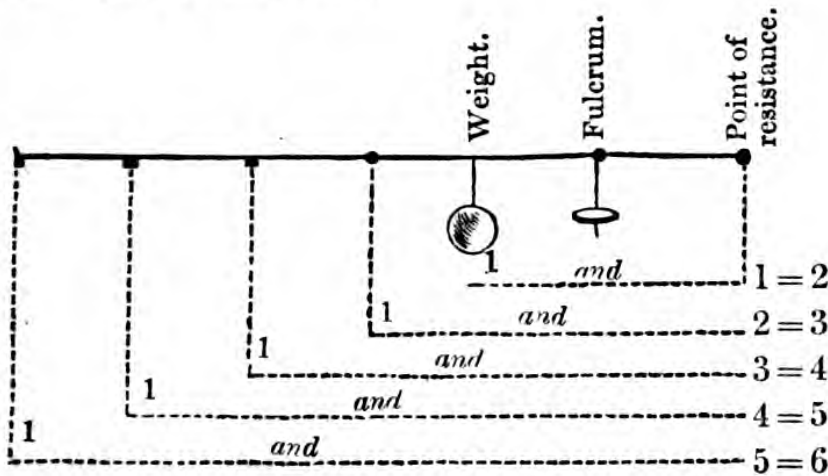
Ex. Required the amount of effect or force produced by a power of 50 lbs. on the ram of a Bramah's pump, the length of the lever being 3 feet, and distance from ram to fulcrum $4\frac{1}{2}$ inches.

3 feet = 36 inches, and $\frac{36}{4.5} = 8$, or the power and resistance are to each other as 8 to 1; hence $50 \times 8 = 400$ lbs. force upon the ram.

The lever on the safety valve of a steam boiler is of the *third kind*, the action of the steam being the power, and the weight or spring-balance attached the resistance; but in such application the action of the lever's weight must also be taken into account, and may be simply ascertained by such means as represented in fig. 2, Plate D, where A is a *Salter's balance* attached to the lever by a light line, immediately at the point of pressure on the valve, and which, raised by hand or otherwise, will indicate the lever's action at that point.

This is perhaps the most frequent application of the third kind of lever to mechanical advantage, and that in which great nicety is required in estimation of effect: hence observe, as in other levers, there are three distinct points that require to be particularly attended to; namely, the *weight*, *fulcrum*, and *re-*

stance, as shown in the annexed diagram to illustrate this particular case.



Thus, suppose the weight to be placed on any one of the divisions, it is still the same weight, or 1; but because of the principle of the lever, the resistance is increased equal to the number of times the weight is distant from the fulcrum; consequently the action of the lever tends to press down the valve equal the sum of the weight and resistance, or the number of times the weight is distant from the resistance.

2. THE WHEEL AND PINION, OR CRANE.

The mechanical advantage of the wheel and axle, or crane, is as the velocity of the weight to the velocity of the power; and being only a modification of the first kind of lever, it of course partakes of the same principles.

1. *To determine the amount of effective power produced from a given power by means of a crane with known peculiarities.*

Rule.—Multiply together the diameter of the circle described by the winch, or handle, and the number of revolutions of the pinion to 1 of the wheel; divide the product by the barrel's diameter in equal terms

of dimensions, and the quotient is the effective power to 1 of exertive force.

Ex. Let there be a crane the winch of which describes a circle of 30 inches in diameter; the pinion makes 8 revolutions for 1 of the wheel, and the barrel is 11 inches in diameter; required the effective power in principle, also the weight that 36 lbs. would raise, friction not being taken into account.

$$\frac{30 \times 8}{11} = 21.8 \text{ to 1 of exertive force; and } 21.8 \times 36 = 784.8 \text{ lbs.}$$

2. *Given any two parts of a crane to find the third that shall produce any required proportion of mechanical effect.*

Rule.—Multiply the two given parts together, and divide the product by the required proportion of effect; the quotient is the dimensions of the other parts in equal terms of unity.

Ex. Suppose that a crane is required, the ratio of power to effect being as 40 to 1, and that a wheel and pinion 11 to 1 is unavoidably compelled to be employed, also the throw of each handle to be 16 inches; what must be the barrel's diameter on which the rope or chain must coil?

$$16 \times 2 = 32 \text{ inches diameter described by the handle.}$$

$$\text{And } \frac{32 \times 11}{40} = 8.8 \text{ inches, the barrel's diameter.}$$

3. THE PULLEY.

The principle of the pulley, or more practically the block and tackle, is the distribution of weight on various points of support; the mechanical advantage derived depending entirely upon the flexibility and tension of the rope, and the number of pulleys or sheives in the lower or rising block: hence, by blocks and tackle of the usual kind, as shown in fig. 3, Pl. D,

the power is to the weight as the number of cords attached to the lower block; whence the following rules.

1. Divide the weight to be raised by the number of cords leading *to, from, or attached* to the lower block; and the quotient is the power required to produce an equilibrium, provided friction did not exist.

2. Divide the weight to be raised by the power to be applied; the quotient is the number of sheives in, or cords attached to the rising block.

Ex. Required the power necessary to raise a weight of 3000 lbs. by a four and five-sheived block and tackle, the four being the moveable or rising block.

Necessarily there are nine cords leading to and from the rising block.

$$\text{Consequently } \frac{3000}{9} = 333 \text{ lbs. the power required.}$$

Ex. 2. I require to raise a weight of 1 ton 18 cwt., or 4256 lbs., the amount of my power to effect this object being 500 lbs.; what kind of block and tackle must I of necessity employ?

$$\frac{4256}{500} = 8.51 \text{ cords; of necessity there must be 4 sheives}$$

or 9 cords in the rising block.

As the effective power of the crane may, by additional wheels and pinions, be increased to any required extent, so may the pulley and tackle be similarly augmented by purchase upon purchase; two of the most useful of such applications being represented in figs. 3 and 4, Plate D, the first of which is known by the term *runner and tackle*, and the second by that of *Spanish burton*.

4. THE INCLINED PLANE.

The *inclined plane* is properly the second elementary power, and may be defined the lifting of a load by regular instalments. In principle it consists of any right line not coinciding with, but laying in a sloping direction to, that of the horizon; the standard of comparison of which commonly consists in referring the rise to so many parts in a certain length or distance, as 1 in 100, 1 in 200, &c.,—the first number representing the perpendicular height, and the latter the horizontal length in attaining such height, both numbers being of the same denomination, unless otherwise expressed; but it may be necessary to remark, that the inclination of a plane, the sine of inclination, the height per mile, or the height for any length, the ratio, &c., are all synonymous terms.

The advantage gained by the inclined plane, when the power acts in a parallel direction to the plane, is as the length to the height or angle of inclination: hence the rule. Divide the weight by the ratio of inclination, and the quotient equal the power that will just support that weight upon the plane. Or, multiply the weight by the height of the plane, and divide by the length,—the quotient is the power.

Ex. Required the power or equivalent weight capable of supporting a load of 350 lbs. upon a plane of 1 in 12, or 3 feet in height and 36 feet in length.

$$\frac{350}{12} = 29.16 \text{ lbs.}, \text{ or } \frac{350 \times 3}{36} = 29.16 \text{ lbs. power, as before.}$$

Note.—The weight multiplied by the length of the base, and the product divided by the length of the incline, the quotient equal the pressure or downward weight upon the incline.

Table showing the Resistance opposed to the Motion of Carriages on different Inclinations of Ascending or Descending Planes, whatever part of the insistent weight they are drawn by.

Tens.	Hundreds.									
	100	200	300	400	500	600	700	800	900	
10	.01	.005	.00333	.0025	.002	.00167	.00143	.00125	.00111	
20	.00909	.00476	.00322	.00244	.00196	.00164	.00141	.00123	.0011	
30	.00833	.00454	.00312	.00238	.00192	.00161	.00139	.00122	.00109	
40	.00769	.00435	.00303	.00232	.00189	.00159	.00137	.0012	.00107	
50	.00714	.00417	.00294	.00227	.00185	.00156	.00135	.00119	.00106	
60	.00667	.004	.00286	.00222	.00182	.00154	.00133	.00118	.00105	
70	.00625	.00385	.00278	.00217	.00178	.00151	.00131	.00116	.00104	
80	.00588	.0037	.0027	.00213	.00175	.00149	.0013	.00115	.00103	
90	.00555	.00357	.00263	.00208	.00172	.00147	.00128	.00114	.00102	
	.00526	.00345	.00256	.00204	.00169	.00145	.00126	.00112	.00101	

Note.—Although this Table has been calculated particularly for carriages on railway inclines, it may with equal propriety be applied to any other incline, the amount of traction on a level being known.

Application of the preceding Table.

1. What weight will a tractive power of 150 lbs. draw up an incline of 1 in 340, the resistance on the level being estimated at $\frac{1}{240}$ th part of the insistent weight?

In a line with 40 in the left-hand column and under

200 is 00417

Also in the same line and under 300 is 00294

Added together = 00711

Then $\frac{150}{00711} = 21097$ lbs. weight drawn up the plane.

2. What weight would a force of 150 lbs. draw down the same plane, the friction on the level being the same as before?

Friction on the level = 00417

Gravity of the plane = 00294 subtract

= 00123

And $\frac{150}{00123} = 121915$ lbs. weight drawn down the plane.

Example of incline when velocity is taken into account.

A power of 230 lbs., at a velocity of 75 feet per minute, is to be employed for moving weights up an inclined plane 12 feet in height and 163 feet in length, the least velocity of the weight to be 8 feet per minute; required the greatest weight that the power is equal to.

$$\frac{230 \times 75 \times 163}{12 \times 8} = \frac{2811750}{96} = 29288 \text{ lbs., or } 13.25 \text{ tons.}$$

TABLE OF INCLINED PLANES,
Showing the ascent or descent per yard, and the corresponding ascent or descent per chain, per mile; and also the ratio.

Per yard.		Per chain.		Per mile.		Ratio.	
In parts of an in.	In decs. of an inch.	Inches.	Feet.	Inches.	Feet.	In decs. of an inch.	One in
$\frac{1}{64}$.0156	.344	2.29	9.625	64.17	.4375	82
$\frac{1}{48}$.0208	.458	3.06	11	73.33	.5	72
$\frac{1}{32}$.0312	.687	4.58	12.375	82.5	.5625	64
$\frac{1}{24}$.0417	.917	6.11	12.833	85.56	.5833	62
$\frac{1}{18}$.0625	1.375	9.17	13.2	88	.6	60
$\frac{1}{12}$.0833	1.833	12.22	13.75	91.67	.625	58
$\frac{1}{10}$.1	2.2	14.67	14.667	97.78	.6667	54
$\frac{1}{8}$.125	2.75	18.33	15.125	100.83	.6875	52
$\frac{1}{6}$.1667	3.667	24.44	15.4	102.67	.7	51
$\frac{1}{5}$.1875	4.125	27.50	16.5	110	.75	48
$\frac{1}{4}$.2	4.4	29.33	17.6	117.33	.8	45
$\frac{1}{3}$.25	5.5	36.67	17.875	119.17	.8125	44
$\frac{1}{2}$.3	6.6	44	18.333	122.22	.8333	43
$\frac{1}{16}$.3125	6.875	45.83	19.25	128.33	.875	41
$\frac{1}{12}$.3333	7.333	48.89	19.8	132	.9	40
$\frac{1}{10}$.375	8.25	55	20.167	134.44	.9167	39
$\frac{1}{8}$.4	8.8	58.67	20.625	137.5	.9375	38
$\frac{1}{6}$.4167	9.167	61.11	22	146.67	1 inch.	36

THE WEDGE.

The wedge is a double inclined plane, consequently its principles are the same: Hence when two bodies are forced asunder by means of the wedge in a direction parallel to its head,—Multiply the resisting power by half the thickness of the head or back of the wedge, and divide the product by the length of one of its inclined sides; the quotient is the force equal to the resistance.

Ex. The breadth of the back or head of a wedge being 3 inches, and its inclined sides each 10 inches, required the power necessary to act upon the wedge so as to separate two substances whose resisting force is equal to 150 lbs.

$$\frac{150 \times 1.5}{10} = 22.5 \text{ lbs.}$$

Note.—When only one of the bodies is moveable, the whole breadth of the wedge is taken for the multiplier.

THE SCREW.

The screw, in principle, is that of an inclined plane wound around a cylinder which generates a spiral of uniform inclination, each revolution producing a rise or traverse motion equal to the pitch of the screw, or distance between two consecutive threads,—the pitch being the height or angle of inclination, and the circumference the length of the plane when a lever is not applied; but the lever being a necessary qualification of the screw, the circle which it describes is taken, instead of the screw's circumference, as the length of the plane: hence the mechanical advantage is, as the circumference of the circle described

by the lever where the power acts, is to the pitch of the screw, so is the force to the resistance in principle.

Ex. 1. Required the effective power obtained by a screw of $\frac{7}{8}$ inch pitch, and moved by a force equal to 50 lbs. at the extremity of a lever 30 inches in length.

$$\frac{30 \times 2 \times 3.1416 \times 50}{.875} = 10760 \text{ lbs.}$$

Ex. 2. Required the power necessary to overcome a resistance equal to 7000 lbs. by a screw of $1\frac{1}{4}$ inch pitch, and moved by a lever 25 inches in length.

$$\frac{7000 \times 1.25}{25 \times 2 \times 3.1416} = 55.73 \text{ lbs. power.}$$

In the case of a screw acting on the periphery of a toothed wheel, the power is to the resistance, as the product of the circle's circumference described by the winch or lever, and radius of the wheel, to the product of the screw's pitch, and radius of the axle, or point whence the power is transmitted; but observe, that if the screw consist of more than one helix or thread, the apparent pitch must be increased so many times as there are threads in the screw. *Hence, to find what weight a given power will equipoise:*

Rule.—Multiply together the radius of the wheel, the length of the lever at which the power acts, the magnitude of the power, and the constant number 6.2832; divide the product by the radius of the axle into the pitch of the screw, and the quotient is the weight that the power is equal to.

Ex. What weight will be sustained in equilibrio by a power of 100 lbs. acting at the end of a lever 24 inches in length, the radius of the axle, or point whence the power is transmitted, being 8 inches, the

radius of the wheel 14 inches, the screw consisting of a double thread, and the apparent pitch equal $\frac{5}{8}$ of an inch?

$$\frac{14 \times 24 \times 100 \times 6.2832}{.625 \times 2 \times 8} = 21111.55 \text{ lbs., or } 9.4 \text{ tons, the power sustained.}$$

Note.—It is estimated that about one-third more power must be added to overcome the friction of the screw when loaded, than is necessary to constitute a balance between power and weight.

OF CONTINUOUS CIRCULAR MOTION.

IN mechanics, circular motion is transmitted by means of *wheels, drums, or pulleys*; and accordingly as the driving and driven are of equal or unequal diameters, so are equal or unequal velocities produced: hence the principle on which the following rules are founded.

1. WHEN TIME IS NOT TAKEN INTO ACCOUNT.

Rule.—Divide the greater diameter, or number of teeth, by the lesser diameter, or number of teeth, and the quotient is the number of revolutions the lesser will make for 1 of the greater.

Ex. How many revolutions will a pinion of 20 teeth make for 1 of a wheel with 125?

$$125 \div 20 = 6.25, \text{ or } 6\frac{1}{4} \text{ revolutions.}$$

Note.—Intermediate wheels of whatever diameters, so as to connect communication at any required distance apart, cause no variation of velocity more than otherwise would result were the first and last in immediate contact.

To find the number of revolutions of the last, to 1 of the first, in a train of wheels and pinions.

Rule.—Divide the product of all the teeth in the driving, by the product of all the teeth in the driven, and the quotient equal the ratio of velocity required.

Ex. 1. Required the ratio of velocity of the last, to 1 of the first, in the following train of wheels and pinions; viz., *pinions driving*,—the first of which contains 10 teeth, the second 15, and third 18;—*wheels driven*,—first 15 teeth, second 25, and third 32.

$$\frac{10 \times 15 \times 18}{15 \times 25 \times 32} = .225 \text{ of a revolution the wheel will make to 1 of the pinion.}$$

Ex. 2. A wheel of 42 teeth giving motion to one of 12, on which shaft is a pulley of 21 inches diameter, driving one of 6; required the number of revolutions of the last pulley to 1 of the first wheel.

$$\frac{42 \times 21}{12 \times 6} = 12.25, \text{ or } 12\frac{1}{4} \text{ revolutions.}$$

Note.—Where increase or decrease of velocity is required to be communicated by wheel-work, it has been demonstrated that the number of teeth on each pinion should not be less than 1 to 6 of its wheel, unless there be some other important reason for a higher ratio.

2. WHEN TIME MUST BE REGARDED.

Rule.—Multiply the diameter, or number of teeth in the driver, by its velocity in any given time, and divide the product by the required velocity of the driven; the quotient equal the number of teeth, or diameter of the driven, to produce the velocity required.

Ex. 1. If a wheel containing 84 teeth makes 20 revolutions per minute, how many must another contain to work in contact, and make 60 revolutions in the same time?

$$\frac{84 \times 20}{60} = 28 \text{ teeth.}$$

Ex. 2. From a shaft making 45 revolutions per minute, and with a pinion 9 inches diameter at the pitch line, I wish to transmit motion at 15 revolutions per minute; what at the pitch line must be the diameter of the wheel?

$$\frac{45 \times 9}{15} = 27 \text{ inches.}$$

Ex. 3. Required the diameter of a pulley to make 16 revolutions in the same time as one of 24 inches making 36.

$$\frac{24 \times 36}{16} = 54 \text{ inches.}$$

The distance between the centres and velocities of two wheels being given, to find their proper diameters.

Rule.—Divide the greatest velocity by the least; the quotient is the ratio of diameter the wheels must bear to each other. Hence, divide the distance between the centres by the ratio plus 1; the quotient equal the radius of the smaller wheel; and subtract the radius thus obtained from the distance between the centres; the remainder equal the radius of the other.

Ex. The distance of two shafts from centre to centre is 50 inches, and the velocity of the one 25 revolutions per minute, the other is to make 80 in the same time; the proper diameters of the wheels at the pitch lines are required.

$80 \div 25 = 3.2$, ratio of velocity, and $\frac{50}{3.2 + 1} = 11.9$, the radius

of the smaller wheel; then $50 - 11.9 = 38.1$, radius of larger; their diameters are $11.9 \times 2 = 23.8$, and $38.1 \times 2 = 76.2$ inches.

To obtain or diminish an accumulated velocity by means of wheels and pinions, or wheels, pinions, and pulleys, it is necessary that a proportional ratio of velocity should exist, and which is simply thus at-

tained:—Multiply the given and required velocities together, and the square root of the product is the mean or proportionate velocity.

Ex. Let the given velocity of a wheel containing 54 teeth equal 16 revolutions per minute, and the given diameter of an intermediate pulley equal 25 inches, to obtain a velocity of 81 revolutions in a machine; required the number of teeth in the intermediate wheel, and diameter of the last pulley.

$$\sqrt{81 \times 16} = 36 \text{ mean velocity.}$$

$$\frac{54 \times 16}{36} = 24 \text{ teeth, and } \frac{25 \times 36}{81} = 11.1 \text{ inches diameter of pulley.}$$

To determine the proportion of wheels for screw cutting by a Lathe.

In a lathe properly adapted, screws to any degree of pitch, or number of threads in a given length, may be cut by means of a leading screw of any given pitch, accompanied with change wheels and pinions; course pitches being effected generally by means of one wheel and one pinion with a *carrier*, or *intermediate wheel*, which cause no variation or change of motion to take place: hence the following

Rule.—Divide the number of threads in a given length of the screw which is to be cut, by the number of threads in the same length of the leading screw attached to the lathe; and the quotient is the ratio that the wheel on the end of the screw must bear to that on the end of the lathe spindle.

Ex. Let it be required to cut a screw with 5 threads in an inch, the leading screw being of $\frac{1}{2}$ inch pitch, or containing 2 threads in an inch; what must be the ratio of wheels applied?

$$5 \div 2 = 2.5, \text{ the ratio they must bear to each other.}$$

Then suppose a pinion of 40 teeth be fixed upon for the spindle,—

$$40 \times 2.5 = 100 \text{ teeth for the wheel on the end of the screw.}$$

But screws of a greater degree of fineness than about 8 threads in an inch are more conveniently cut by an additional wheel and pinion, because of the proper degree of velocity being more effectively attained; and these, on account of revolving upon a stud, are commonly designated the *stud-wheels*, or *stud-wheel* and *pinion*; but the mode of calculation and ratio of screw are the same as in the preceding rule;—hence, all that is further necessary is to fix upon any 3 wheels at pleasure, as those for the spindle and stud-wheels,—then multiply the number of teeth in the spindle-wheel by the ratio of the screw, and by the number of teeth in that wheel or pinion which is in contact with the wheel on the end of the screw; divide the product by the stud-wheel in contact with the spindle-wheel, and the quotient is the number of teeth required in the wheel on the end of the leading screw.

Ex. Suppose a screw is required to be cut containing 25 threads in an inch, the leading screw as before having 2 threads in an inch, and that a wheel of 60 teeth is fixed upon for the end of the spindle, 20 for the pinion in contact with the screw-wheel, and 100 for that in contact with the wheel on the end of the spindle;—required the number of teeth in the wheel for the end of the leading screw.

$$25 \div 2 = 12.5, \text{ and } \frac{60 \times 12.5 \times 20}{100} = 150 \text{ teeth.}$$

Or suppose the spindle and screw-wheels to be those fixed upon, also any one of the stud-wheels, to find the number of teeth in the other.

$$\frac{60 \times 12.5}{150 \times 100} = 20 \text{ teeth, or } \frac{60 \times 12.5 \times 20}{150} = 100 \text{ teeth.}$$

Table of Change Wheels for Screw Cutting, the leading screw being of $\frac{1}{2}$ inch pitch, or containing 2 threads in an inch.

Number of threads in inch of screw.	Numb. of teeth in		Number of threads in inch of screw.	Number of teeth in				Number of threads in inch of screw.	Number of teeth in			
	Lathe spindle-wheel.	Leading screw-wheel.		Lathe spindle-wheel.	Wheel in contact with spindle-wheel.	Pinion in contact with screw-wheel.	Leading screw-wheel.		Lathe spindle-wheel.	Wheel in contact with spindle-wheel.	Pinion in contact with screw-wheel.	Leading screw-wheel.
1	80	40	$8\frac{1}{4}$	40	55	20	60	19	50	95	20	100
$1\frac{1}{4}$	80	50	$8\frac{1}{2}$	90	85	20	90	$19\frac{1}{2}$	80	120	20	130
$1\frac{1}{2}$	80	60	$8\frac{3}{4}$	60	70	20	75	20	60	100	20	120
$1\frac{3}{4}$	80	70	$9\frac{1}{2}$	90	90	20	95	$20\frac{1}{4}$	40	90	20	90
2	80	90	$3\frac{3}{4}$	40	60	20	65	21	80	120	20	140
$2\frac{1}{4}$	80	90	10	60	75	20	80	22	60	110	20	120
$2\frac{1}{2}$	80	100	$10\frac{1}{2}$	50	70	20	75	$22\frac{1}{2}$	80	120	20	150
$2\frac{3}{4}$	80	110	11	60	55	20	120	$22\frac{3}{4}$	80	130	20	140
3	80	120	12	90	90	20	120	$23\frac{3}{4}$	40	95	20	100
$3\frac{1}{4}$	80	130	$12\frac{3}{4}$	60	85	20	90	24	65	120	20	130
$3\frac{1}{2}$	80	140	13	90	90	20	130	25	60	100	20	150
$3\frac{3}{4}$	80	150	$13\frac{1}{2}$	60	90	20	90	$25\frac{1}{2}$	30	85	20	90
4	40	80	$13\frac{3}{4}$	80	100	20	110	26	70	130	20	140
$4\frac{1}{4}$	40	85	14	90	90	20	140	27	40	90	20	120
$4\frac{1}{2}$	40	90	$14\frac{1}{4}$	60	90	20	95	$27\frac{1}{2}$	40	100	20	110
$4\frac{3}{4}$	40	95	15	90	90	20	150	28	75	140	20	150
5	40	100	16	60	80	20	120	$28\frac{1}{2}$	30	90	20	95
$5\frac{1}{2}$	40	110	$16\frac{1}{4}$	80	100	20	130	30	70	140	20	150
6	40	120	$16\frac{1}{2}$	80	110	20	120	32	30	80	20	120
$6\frac{1}{2}$	40	130	17	45	85	20	90	33	40	110	20	120
7	40	140	$17\frac{1}{2}$	80	100	20	140	34	30	85	20	120
$7\frac{1}{2}$	40	150	18	40	60	20	120	35	60	140	20	150
8	30	120	$18\frac{3}{4}$	80	100	20	150	36	30	90	20	120

Table by which to determine the Number of Teeth, or Pitch of Small Wheels, by what is commonly called the Manchester principle.

Diametral pitch.	Circular pitch.	Diametral pitch.	Circular pitch.
3	1·047	9	·349
4	·785	10	·314
5	·628	12	·262
6	·524	14	·224
7	·449	16	·196
8	·393	20	·157

Ex. 1. Required the number of teeth that a wheel of 16 inches diameter will contain of a 10 pitch.

$16 \times 10 = 160$ teeth, and the circular pitch = $\cdot 314$ inch.

Ex. 2. What must be the diameter of a wheel for a 9 pitch of 126 teeth?

$\frac{126}{9} = 14$ inches diameter, circular pitch $\cdot 349$ inch.

Note.—The pitch is reckoned on the diameter of the wheel instead of the circumference, and designated wheels of 8 pitch, 12 pitch, &c.

Strength of the Teeth of Cast Iron Wheels at a given velocity.

Pitch of teeth in inches.	Thickness of teeth in inches.	Breadth of teeth in inches.	Strength of teeth in horse-power, at			
			3 ft. per second.	4 ft. per second.	6 ft. per second.	8 ft. per second.
3·99	1·9	7·6	20·57	27·43	41·14	54·85
3·78	1·8	7·2	17·49	23·32	34·98	46·64
3·57	1·7	6·8	14·73	19·65	29·46	39·28
3·36	1·6	6·4	12·28	16·38	24·56	32·74
3·15	1·5	6	10·12	13·50	20·24	26·98
2·94	1·4	5·6	8·22	10·97	16·44	21·92
2·73	1·3	5·2	6·58	8·78	13·16	17·54
2·52	1·2	4·8	5·18	6·91	10·36	13·81
2·31	1·1	4·4	3·99	5·32	7·98	10·64
2·1	1·0	4	3·00	4·00	6·00	8·00
1·89	·9	3·6	2·18	2·91	4·36	5·81
1·68	·8	3·2	1·53	2·04	3·06	3·08
1·47	·7	2·8	1·027	1·37	2·04	2·72
1·26	·6	2·4	·64	·86	1·38	1·84
1·05	·5	2	·375	·50	·75	1·00

Table of the Diameters of Wheels at their pitch circle, to contain a required number of teeth at a given pitch.

Number of teeth	Pitch of the teeth in inches.												
	1 inch	1 1/8	1 1/4	1 3/8	1 1/2	1 5/8	1 3/4	2 inch	2 1/8	2 1/4	2 1/2	3 inch	
10	3 1/4	3 5/8	4	4 1/2	4 7/8	5 1/4	5 5/8	6 1/4	6 7/8	7 1/4	7 5/8	8 1/4	8 7/8
11	3 1/2	4	4 1/8	4 3/4	5 1/4	5 7/8	6 1/4	6 7/8	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4
12	3 3/4	4 1/4	4 1/2	4 5/8	5 1/4	5 7/8	6 1/4	6 7/8	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4
13	4	4 1/2	4 3/4	5 1/4	5 7/8	6 1/4	6 7/8	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4	9 5/8
14	4 1/8	4 1/2	4 5/8	5 1/4	5 7/8	6 1/4	6 7/8	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4	9 5/8
15	4 1/4	4 3/4	5 1/4	5 7/8	6 1/4	6 7/8	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4	9 5/8	10 1/4
16	4 1/2	5 1/4	5 7/8	6 1/4	6 7/8	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4	9 5/8	10 1/4	10 3/4
17	4 3/4	5 1/2	6 1/4	6 7/8	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4	9 5/8	10 1/4	10 3/4	11 1/4
18	5	5 1/2	6 1/4	6 7/8	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4	9 5/8	10 1/4	10 3/4	11 1/2
19	5 1/4	5 7/8	6 1/2	6 3/4	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4	9 5/8	10 1/4	10 3/4	11 1/2
20	5 1/2	6 1/4	6 1/2	6 3/4	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4	9 5/8	10 1/4	10 3/4	11 1/2
21	5 3/4	6 1/2	6 3/4	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4	9 5/8	10 1/4	10 3/4	11 1/2	11 3/4
22	6	6 1/4	6 1/2	6 3/4	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4	9 5/8	10 1/4	10 3/4	11 3/4
23	6 1/4	6 3/4	6 1/2	6 3/4	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4	9 5/8	10 1/4	10 3/4	11 3/4
24	6 1/2	6 3/4	6 1/2	6 3/4	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4	9 5/8	10 1/4	10 3/4	11 3/4
25	6 3/4	6 3/4	6 1/2	6 3/4	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4	9 5/8	10 1/4	10 3/4	11 3/4
26	6 3/4	6 3/4	6 1/2	6 3/4	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4	9 5/8	10 1/4	10 3/4	11 3/4
27	6 3/4	6 3/4	6 1/2	6 3/4	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4	9 5/8	10 1/4	10 3/4	11 3/4
28	6 3/4	6 3/4	6 1/2	6 3/4	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4	9 5/8	10 1/4	10 3/4	11 3/4
29	6 3/4	6 3/4	6 1/2	6 3/4	7 1/4	7 5/8	8 1/4	8 7/8	9 1/4	9 5/8	10 1/4	10 3/4	11 3/4

Diameter at the pitch circle in feet and inches.

DIAMETERS OF WHEELS.

Table of the Diameters of Wheels.—(Continued.)

Number of teeth.	Pitch of the teeth in inches.											
	1 inch	1 $\frac{1}{8}$	1 $\frac{1}{4}$	1 $\frac{3}{8}$	1 $\frac{1}{2}$	1 $\frac{5}{8}$	1 $\frac{3}{4}$	2 inch	2 $\frac{1}{8}$	2 $\frac{1}{4}$	2 $\frac{3}{8}$	3 inch
30	9 $\frac{1}{2}$	10 $\frac{3}{4}$	11 $\frac{1}{2}$	12	13	14	15	16	17	18	19	20
31	9 $\frac{7}{8}$	10 $\frac{7}{8}$	11 $\frac{1}{4}$	12 $\frac{1}{4}$	13 $\frac{1}{4}$	14 $\frac{1}{4}$	15 $\frac{1}{4}$	16 $\frac{1}{4}$	17 $\frac{1}{4}$	18 $\frac{1}{4}$	19 $\frac{1}{4}$	20 $\frac{1}{4}$
32	10 $\frac{1}{4}$	11 $\frac{1}{4}$	12	13	14	15	16	17	18	19	20	21
33	10 $\frac{1}{2}$	11 $\frac{1}{2}$	12 $\frac{1}{2}$	13 $\frac{1}{2}$	14 $\frac{1}{2}$	15 $\frac{1}{2}$	16 $\frac{1}{2}$	17 $\frac{1}{2}$	18 $\frac{1}{2}$	19 $\frac{1}{2}$	20 $\frac{1}{2}$	21 $\frac{1}{2}$
34	10 $\frac{3}{4}$	11 $\frac{3}{4}$	12 $\frac{3}{4}$	13 $\frac{3}{4}$	14 $\frac{3}{4}$	15 $\frac{3}{4}$	16 $\frac{3}{4}$	17 $\frac{3}{4}$	18 $\frac{3}{4}$	19 $\frac{3}{4}$	20 $\frac{3}{4}$	21 $\frac{3}{4}$
35	11	12	13	14	15	16	17	18	19	20	21	22
36	11 $\frac{1}{8}$	12 $\frac{1}{8}$	13 $\frac{1}{8}$	14 $\frac{1}{8}$	15 $\frac{1}{8}$	16 $\frac{1}{8}$	17 $\frac{1}{8}$	18 $\frac{1}{8}$	19 $\frac{1}{8}$	20 $\frac{1}{8}$	21 $\frac{1}{8}$	22 $\frac{1}{8}$
37	11 $\frac{1}{4}$	12 $\frac{1}{4}$	13 $\frac{1}{4}$	14 $\frac{1}{4}$	15 $\frac{1}{4}$	16 $\frac{1}{4}$	17 $\frac{1}{4}$	18 $\frac{1}{4}$	19 $\frac{1}{4}$	20 $\frac{1}{4}$	21 $\frac{1}{4}$	22 $\frac{1}{4}$
38	11 $\frac{1}{2}$	12 $\frac{1}{2}$	13 $\frac{1}{2}$	14 $\frac{1}{2}$	15 $\frac{1}{2}$	16 $\frac{1}{2}$	17 $\frac{1}{2}$	18 $\frac{1}{2}$	19 $\frac{1}{2}$	20 $\frac{1}{2}$	21 $\frac{1}{2}$	22 $\frac{1}{2}$
39	11 $\frac{3}{4}$	12 $\frac{3}{4}$	13 $\frac{3}{4}$	14 $\frac{3}{4}$	15 $\frac{3}{4}$	16 $\frac{3}{4}$	17 $\frac{3}{4}$	18 $\frac{3}{4}$	19 $\frac{3}{4}$	20 $\frac{3}{4}$	21 $\frac{3}{4}$	22 $\frac{3}{4}$
40	12	13	14	15	16	17	18	19	20	21	22	23
41	12 $\frac{1}{8}$	13 $\frac{1}{8}$	14 $\frac{1}{8}$	15 $\frac{1}{8}$	16 $\frac{1}{8}$	17 $\frac{1}{8}$	18 $\frac{1}{8}$	19 $\frac{1}{8}$	20 $\frac{1}{8}$	21 $\frac{1}{8}$	22 $\frac{1}{8}$	23 $\frac{1}{8}$
42	12 $\frac{1}{4}$	13 $\frac{1}{4}$	14 $\frac{1}{4}$	15 $\frac{1}{4}$	16 $\frac{1}{4}$	17 $\frac{1}{4}$	18 $\frac{1}{4}$	19 $\frac{1}{4}$	20 $\frac{1}{4}$	21 $\frac{1}{4}$	22 $\frac{1}{4}$	23 $\frac{1}{4}$
43	12 $\frac{1}{2}$	13 $\frac{1}{2}$	14 $\frac{1}{2}$	15 $\frac{1}{2}$	16 $\frac{1}{2}$	17 $\frac{1}{2}$	18 $\frac{1}{2}$	19 $\frac{1}{2}$	20 $\frac{1}{2}$	21 $\frac{1}{2}$	22 $\frac{1}{2}$	23 $\frac{1}{2}$
44	12 $\frac{3}{4}$	13 $\frac{3}{4}$	14 $\frac{3}{4}$	15 $\frac{3}{4}$	16 $\frac{3}{4}$	17 $\frac{3}{4}$	18 $\frac{3}{4}$	19 $\frac{3}{4}$	20 $\frac{3}{4}$	21 $\frac{3}{4}$	22 $\frac{3}{4}$	23 $\frac{3}{4}$
45	13	14	15	16	17	18	19	20	21	22	23	24
46	13 $\frac{1}{8}$	14 $\frac{1}{8}$	15 $\frac{1}{8}$	16 $\frac{1}{8}$	17 $\frac{1}{8}$	18 $\frac{1}{8}$	19 $\frac{1}{8}$	20 $\frac{1}{8}$	21 $\frac{1}{8}$	22 $\frac{1}{8}$	23 $\frac{1}{8}$	24 $\frac{1}{8}$
47	13 $\frac{1}{4}$	14 $\frac{1}{4}$	15 $\frac{1}{4}$	16 $\frac{1}{4}$	17 $\frac{1}{4}$	18 $\frac{1}{4}$	19 $\frac{1}{4}$	20 $\frac{1}{4}$	21 $\frac{1}{4}$	22 $\frac{1}{4}$	23 $\frac{1}{4}$	24 $\frac{1}{4}$
48	13 $\frac{1}{2}$	14 $\frac{1}{2}$	15 $\frac{1}{2}$	16 $\frac{1}{2}$	17 $\frac{1}{2}$	18 $\frac{1}{2}$	19 $\frac{1}{2}$	20 $\frac{1}{2}$	21 $\frac{1}{2}$	22 $\frac{1}{2}$	23 $\frac{1}{2}$	24 $\frac{1}{2}$
49	13 $\frac{3}{4}$	14 $\frac{3}{4}$	15 $\frac{3}{4}$	16 $\frac{3}{4}$	17 $\frac{3}{4}$	18 $\frac{3}{4}$	19 $\frac{3}{4}$	20 $\frac{3}{4}$	21 $\frac{3}{4}$	22 $\frac{3}{4}$	23 $\frac{3}{4}$	24 $\frac{3}{4}$
50	14	15	16	17	18	19	20	21	22	23	24	25

Diameter at the pitch circle in feet and inches.

0	1	2	3	4	5	6	7	8	9	10	11	0	1	2	3	4	5	6	7	8	9	10	11	0	1	2	3		
4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
8	9	10	11	0	1	2	3	4	5	6	7	8	9	10	11	0	1	2	3	4	5	6	7	8	9	10	11	0	
3	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	
4	5	6	7	7	8	9	10	11	0	1	2	3	4	5	6	7	8	9	10	11	0	1	2	3	4	5	6	7	
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
0	1	2	2	3	4	4	5	6	7	8	9	10	11	0	0	1	2	2	3	4	5	5	6	7	7	8	8	9	
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
10	11	11	0	1	2	3	4	4	5	6	6	7	8	8	9	10	10	11	0	0	1	2	2	3	4	4	4	5	
2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
8	9	9	10	11	0	0	1	2	2	3	4	4	5	6	6	7	7	8	9	9	10	11	0	1	1	2	2	3	
2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
6	7	7	8	8	9	10	10	11	0	1	1	2	2	3	4	4	5	5	6	6	7	7	8	8	9	9	10	11	
2	2	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
4	4	5	6	6	7	7	8	8	9	10	10	11	0	0	1	1	2	2	3	3	4	4	5	5	6	6	7	8	
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	
2	2	3	3	4	4	5	6	6	7	7	8	8	9	9	10	10	11	0	0	1	1	2	2	3	3	3	4	4	
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	
0	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10	11	0	0	1	1	1	1	
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	
1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	
8	8	9	9	9	10	10	11	11	0	0	1	1	1	2	2	2	2	2	3	3	3	3	4	4	4	5	5	5	6
1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	
0	6	6	7	7	8	8	8	9	9	10	10	10	11	11	0	0	0	1	1	1	1	2	2	2	2	2	2	2	
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
4	4	4	5	5	5	6	6	6	7	7	8	8	8	9	9	9	10	10	10	11	11	11	0	0	0	0	1	1	
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	

FRICTION.

FRICTION is an effect produced by bodies rubbing one upon another, which acts as a retarding influence in the motion of all mechanical contrivances, but might not unfrequently be considerably diminished by a due regard to its laws, and a proper attention to the selection of those materials on which a uniform smooth surface may be attained, and which, according to experiments, are least liable to tear or become hot, and cause a roughness to arise when in working contact.

Several doubts existed until lately respecting the laws of friction; but those doubts are now entirely removed through the experiments of Mr. G. Rennie, on his own account, and those of M. Morin, acting for and under the sanction of the French Government, from or by which the following laws have been fully established.

1. That when no unguent is interposed, the friction of any two surfaces (whether of quiescence or of motion) is directly proportional to the force with which they are pressed perpendicularly together; so that for any two given surfaces of contact, there is a constant ratio of the friction to the perpendicular pressure; that is, a double pressure will produce a double amount of friction, a triple pressure a triple amount, &c., of any other proportionate increase of the load.

2. That when no unguent is interposed, the amount of the friction is in every case wholly independent of the extent of the surfaces in contact; so that the force

with which two surfaces are pressed together being the same, their friction is the same, whatever may be the extent of their surfaces of contact.

3. That the friction of motion is wholly independent of the velocity of the motion ; that is, supposing two shafts or axles of equal bearings and of equal weights or pressures, but the one making 100 revolutions while the other makes but 50, the amount of friction in each is alike.

4. That when unguents are interposed, the amount of friction depends more upon the nature of the unguent than upon that of the surfaces of contact ; and hence, that the nature of the unguent to be applied must be governed by the pressure or insistent weight. Mr. Rennie found, that with the unguents tallow and anti-attribution, on axles under a pressure of from 1 to 5 cwt., the friction did not exceed $\frac{1}{39}$ th of the whole pressure ; but when softer unguents were applied, as oil, hog's-lard, &c., the ratio of the friction to the pressure considerably increased ; from which it is naturally inferred that the consistence of an unguent ought just to prevent the bodies coming into contact with each other.

5. That the friction of metals, without a stratum of unguent interposed, varies as their hardness, the hard metals producing less friction than the soft ones.

6. That without unguents, and within the limits of $32\frac{1}{2}$ lbs. pressure per square inch, the friction of hard metals upon hard metals may very generally be estimated at about $\frac{1}{8}$ th of the whole pressure.

7. That within the limits of their abrasion, the friction of metals is nearly alike ; but from 1.66 cwt. per square inch to 6 cwt. per square inch, the resistance increases in a very considerable ratio, being the greatest with steel on cast iron, and the least with brass on wrought iron.

PRACTICAL PROPERTIES OF WATER AND AIR;

OR OF LIQUIDS AND FLUIDS IN GENERAL.

WATER is the most abundant liquid we possess, and air is the most abundant fluid,—their properties contributing largely to the subserviency of man, not only as the support of his existence, and the comforts of his domestic enjoyments, but because of inherent energies which render them exceedingly advantageous, both as an efficient and disposable source of motive power.

By analysis it is ascertained, that water is composed of the gases *oxygen* and *hydrogen* in a state of chemical union; its distinguishing properties, like that of other liquids, being incompressibility,* gravity, capability of flowing, and constant tendency to press outwards in every direction; also that of being easily changed by the absorption of caloric to an aëriform state of any required density or degree of elastic force: hence the principle of the *hydraulic press*, the *water-wheel*, the *steam engine*, &c.

Atmospheric air, like water, is also a gaseous compound: of 100 parts of air, reckoning by weight, 75·55 parts are nitrogen, 23·32 oxygen, and 1·13 carbonic acid and watery vapour; and in such proportions

* Exact experiments have determined that water is not entirely incompressible, but the degree of compressibility is so small, being only about $46\frac{1}{2}$ millionth parts per atmosphere, that the terms incompressible, and non-elastic fluid, are not at all in practice inappropriate.

it is most conducive to animal existence. It also in this state possesses many important mechanical properties, a few of which are the following; viz., gravity, fluidity, compressibility, force or pressure, and elasticity; also expansibility by rarefaction or heat; and it is because of the oxygen it contains that combustion is supported. By change of temperature in the atmosphere, currents and winds are created; hence the means by which vessels are enabled to make distant voyages; also the cause which produces circular motion through the medium of the sails of a wind-mill; and it may likewise be observed, that without the pressure of the atmosphere the common pump would be without effect. Such preliminaries, being properly considered, tend much to pave the way to an efficient knowledge of those elements, *water* and *air*, either as regards the production of or retarding mechanical effects in their application as motive power.

1. *Effects produced by water in its natural state.*

Because of liquids possessing the properties of gravity and capability of flowing freely in every direction, sides of vessels, flood-gates, sluices, &c., sustain a pressure equal to the product of the area multiplied by half the depth of the fluid, and by its gravity, in equal terms of unity.

But when a sluice or opening through which a liquid may issue is under any given continued head, the pressure is equal the product of the area multiplied into the height from the centre of the opening to the surface of the fluid.

Ex. 1. Required the pressure of water on the sides of a rectangular cistern 18 feet in length, 13 in width, and 9 in depth.

The terms of measurement or unity are in feet : 1 cubic foot of water = 62·5 lbs. ; hence $\overline{18 \times 9 \times 2} + \overline{13 \times 9 \times 2} = 558 \times 4\cdot5 \times 62\cdot5 = 156937\cdot5$ lbs.

Weight of water on bottom = $18 \times 13 \times 9 \times 62\cdot5 = 131625$ lbs.

Ex. 2. Required the pressure on a sluice 3 feet square, and its centre 30 feet from the surface of the water.

$$3 \times 3 \times 30 \times 62\cdot5 = 16875 \text{ lbs. pressure.}$$

The weight of water or other fluid is as the quantity, but the pressure exerted is as the vertical height; hence, as fluids press equally in every direction, any vessel containing a fluid sustains a pressure equal to as many times the weight of the column of greatest height of that fluid, as the area of the vessel is to the sectional area of the column.

Ex. Let a cubical vessel whose sides are each 4 square feet have a tube inserted 1 inch in diameter and 6 feet in height, and let both vessel and tube be filled with water; required the whole weight of the water therein contained, and also the whole pressure exerted in tending to burst the vessel.

Cubic contents of the vessel = 8 feet, and each foot = 62·5 lbs. (see page 55).

Then, $62\cdot5 \times 8 = 500$. Area of pipe's section = ·7854 inches and height 72 in., also a cubic inch of water = ·03617 lbs.; hence $\cdot7854 \times 72 \times \cdot03617 = 2$ lbs. + 500 = 502 lbs. total weight of the water.

Again, the whole height of the column = 96 inches; then $\cdot7854 \times 96 \times \cdot03617 = 2\cdot33$ lbs. pressure of column on an equal area. 144 square inches = 1 square foot, and $\frac{144 \times 4 \times 6 \text{ sides}}{\cdot7854} = 4400\cdot4$ times the area of the pipe's

diameter in the whole surface; therefore $4400\cdot4 \times 2\cdot33 = 10253$ lbs., or total amount of pressure exerted.

Upon the preceding principles of water rests the utility of the hydraulic press. Thus, into a uniform cylinder of suitable strength is fitted a piston or ram moveable in a parallel direction, around which are properly fitted leather collars, to prevent any possibility of the water's escape; the water being injected by means of a force pump, and, by its non-compressible property, repelling the ram with a force equal to the number of times the end of the ram exceeds the area of the pump.

Ex. Required the repulsive force of a 6-inch ram, when a power of 50 lbs. is applied to the end of the lever, which is as 12 to 1 in effect, and the diameter of the pump or plunger $\frac{7}{8}$ ths of an inch.

$$\begin{aligned} \text{Area of ram} &= 28.2744 = 47; \\ \text{Area of pump} &= .6013 \end{aligned}$$

and $50 \times 12 \times 47 = 28200$ lbs., or 12 tons nearly.

When a body is partly or wholly immersed in water or other fluid, the vertical pressure of the fluid tends to raise the body with a force equal to the weight of the fluid displaced; hence the weight of any displaced quantity of a fluid by a buoyant body equals the weight of that body.

Ex. 1. Suppose a vessel with all its masts, stores, and general equipments is found to displace 35,000 cubic feet of sea water, what is the whole weight of the vessel?

Sea water average 64 lbs. per cubic foot.

$$\frac{35000 \times 64}{2240} = 1000 \text{ tons.}$$

Ex. 2. What must be the thickness of sheet-iron whereby to form a rectangular vessel 5 feet in length, 2 in width, and $1\frac{1}{4}$ in depth, that will just sink 3

inches in common water ; laps, seams, and rivets not being taken into account ?

5 ft. or 60 in.	× 15 = 900	× 2 = 1800	in. or sum of the sides.
" 60 "	× 15 =	1440	" bottom.
2 " 24 "	× 15 = 360	× 2 = 720	" ends.
Total			3960

And $3960 \times .281$ (see page 30) = 1112.7 the divisor.

Then $60 \text{ in.} \times 15 \times 3 \times .03617 = 156.254$ lbs. of water to displace.

Hence $\frac{156.254}{1112.7} = .14$ in., the thickness of iron required.

The resistance by which a moving body is opposed in passing through water, is as the square of the body's velocity ; hence, if a body be propelled at a certain velocity by a known power, to double that velocity will require four times the power ; to triple it, nine times the power, &c.

Water in flowing through an orifice or aperture under any given head, is governed by the same law of gravity as that of a solid body in vacuo descending through the same space, or falling from an equal height ; but as friction is created by the motion of two hard bodies in contact, so is friction also created by the action of the water in passing through the orifice or aperture : hence, an aperture twice the width of another will discharge more than a double quantity, because of the area advancing in a much greater ratio to that of the resistance. Thus, suppose an opening of 4 square feet be required : in a circular form its diameter would be about 2 feet 3 inches, and its circumference, or cause of friction, 7 feet ; in a square form, 2 feet by 2 feet, and the amount of its sides 8 feet ; but in a rectangular form, 4 feet in length and 1 in breadth, the cause of resistance is increased

to 10 feet; thus showing that the circular form is that which ought to be adopted in preference to any other for the conduction of water where practicability will admit.

When water issues out of a circular orifice in a thin plate at the bottom or side of a reservoir, the issuing stream tends to converge to a point at the distance of about half its diameter outside the orifice, and this contraction of the stream reduces the area of its section from 1 to $\cdot619$, or nearly $\frac{5}{8}$ ths. If a short parallel tube be attached, the vein of the stream is less contracted, and the area will equal $\cdot762$. But if the tube attached be the frustum of a cone whose greater end is the aperture, the length equal half the diameter of the aperture, and the area of the small end to the area of the larger as 1 to 1.6, there will be no contraction of the vein: hence the propriety of making the pipe in this form from a reservoir or other head of water through which the greatest quantity, according to its area, is required to pass.

To find the velocity of water issuing through a circular orifice at any given depth from the surface.

Rule.—Multiply the square root of the height or depth to the centre of the orifice by 8.1, and the product is the velocity of the issuing fluid in feet per second.

Ex. Required the velocity of water issuing through an orifice under a head of 11 feet from the surface.

$$\sqrt{11} = 3.3166 \times 8.1 = 26.864 \text{ feet, velocity per second.}$$

In the discharge of water by a rectangular aperture in the side of a reservoir, and extending to the surface, the velocity varies nearly as the square root of the height, and the quantity discharged per second equal

$\frac{2}{3}$ rds of the velocity due to the mean height, allowing for the contraction of the fluid according to the form of the opening, which renders the co-efficient in this case equal to 5.1 ; whence the following general rules.

1. *When the aperture extends to the surface of the fluid.* Multiply the area of the opening in feet by the square root of its depth also in feet, and that product by 5.1 ; then will $\frac{2}{3}$ rds of the last product equal the quantity discharged in cubic feet per second.

2. *When the aperture is under a given head.* Multiply the area of the aperture in feet by the square root of the depth also in feet, and by 5.1 ; the product is the quantity discharged in cubic feet per second.

Ex. 1. Required the quantity of water in cubic feet per second discharged through an opening in the side of a dam or weir, the width or length of the opening being $6\frac{1}{2}$ feet, and depth 9 inches, or .75 of a foot.

Square root of .75 = .866.

$$\text{Then } \frac{6.5 \times .75 \times .866 \times 5.1 \times 2}{3} = 14.3839 \text{ cubic feet.}$$

Ex. 2. What would be the quantity discharged through the above opening if under a head of water 4 feet in height ?

Square root of 4 = 2, and $2 \times 5.1 = 10.2$ feet velocity of the water per second.

And $6.5 \times .75 \times 2 \times 5.1 = 49.725$ cubic feet discharged in the same time.

PRACTICAL RULES BY WHICH TO DETERMINE THE NECESSARY HEAD, AND QUANTITY OF WATER DISCHARGED THROUGH CIRCULAR PIPES IN A GIVEN TIME.

Rule 1.—To the product of the pipe's length in feet, multiplied by the square of the quantity required in cubic feet per second, add the product of 50 times the pipe's diameter in feet, multiplied into the square of the required quantity per second; divide the sum by the product of 1542·133 into the fifth power of the diameter, and the quotient equal the *head* in feet to produce the velocity required.

Rule 2.—Multiply 1542·133 times the fifth power of the pipe's diameter in feet by the head of water in feet, and divide the product by the sum of the pipe's length and 50 times its diameter; the square root of the quotient equal the *quantity* discharged in cubic feet per second.

Ex. 1. Required the head of water necessary to produce a velocity of 2·988 feet, or 9·387 cubic feet per second, by a pipe of 2 feet in diameter and 180 feet in length.

$$\frac{9\cdot387^2 \times 180 + 50 \times 2 \times 9\cdot387^2}{1542\cdot133 \times 2^5} = \frac{15860\cdot8}{49348} = \cdot5 \text{ of a foot head.}$$

Ex. 2. What quantity of water per second will be discharged through a pipe of 2 feet diameter and 180 feet in length, when pressed by a head of water ·5 of a foot in height.

$$\frac{1542\cdot133 \times 2^5 \times \cdot5}{180 + 50 \times 2} = \sqrt{88\cdot1219} = 9\cdot387 \text{ feet per second.}$$

And the area of the end of the pipe in feet, multiplied into the velocity, equal the quantity in cubic feet.

Note.—The above rules apply strictly only to straight pipes; bends in a pipe diminish the velocity of a fluid equal to ·0038

times the sum of the sines of the several angles of inflection; hence, a bend in a pipe should not be sudden, and on no account should an angle be admitted.

Table of the Diameters of Pipes through which a required quantity of water may be discharged in a given time.

Cubic feet per minute.	Diameter in inches.	Cubic feet per minute.	Diameter in inches.	Cubic feet per minute.	Diameter in inches.
1	1	25	4 $\frac{3}{4}$	160	12 $\frac{1}{8}$
2	1 $\frac{3}{8}$	30	5 $\frac{1}{4}$	170	12 $\frac{1}{2}$
3	1 $\frac{5}{8}$	35	5 $\frac{5}{8}$	180	12 $\frac{3}{4}$
4	1 $\frac{7}{8}$	40	6	190	13 $\frac{1}{4}$
5	2 $\frac{1}{8}$	45	6 $\frac{1}{2}$	200	13 $\frac{5}{8}$
6	2 $\frac{3}{8}$	50	6 $\frac{3}{4}$	225	14 $\frac{3}{8}$
7	2 $\frac{5}{8}$	55	7 $\frac{1}{8}$	250	15 $\frac{1}{8}$
8	2 $\frac{3}{4}$	60	7 $\frac{1}{2}$	275	16
9	2 $\frac{7}{8}$	65	7 $\frac{3}{4}$	300	16 $\frac{5}{8}$
10	3	70	8	350	18
11	3 $\frac{1}{8}$	80	8 $\frac{5}{8}$	400	19 $\frac{1}{4}$
12	3 $\frac{1}{4}$	90	9 $\frac{1}{8}$	441	20 $\frac{1}{8}$
13	3 $\frac{1}{2}$	100	9 $\frac{5}{8}$	529	22
14	3 $\frac{5}{8}$	110	10	625	24
15	3 $\frac{3}{4}$	120	10 $\frac{1}{2}$	729	26
16	3 $\frac{7}{8}$	130	11	841	28
18	4	140	11 $\frac{3}{8}$	900	29
20	4 $\frac{1}{4}$	150	11 $\frac{3}{4}$	1000	30

The combined properties of gravity and fluidity which water possesses render it so available as a source of motive power; *gravity* being the property by which the power is produced, and *fluidity* that by which it is so commodiously qualified to the various modifications in which it is employed.

Water, it is ascertained, is subject to the same laws of gravity as those of solid bodies, and thereby accumulates velocity or effect in an equal ratio when falling through an equal space, or descending from an equal height: hence the velocity attained is as the

square root of the height of its fall ; and it is now quite satisfactorily decided, that because of the non-elastic property of water, its greatest effect is obtained when acting by gravity throughout its whole height, whether it be applied on a water-wheel, turbine, or other machine, through which circular motion is to be the immediate result.

In regard to *Water-Wheels* and other Machines through which motion is produced by the effort of water, much discrepancy of opinion has until lately existed, both as to form and velocity, besides other essential points requisite in gaining a maximum of effect with the least possible strain ; but these doubts are now in a great measure removed through experiments by the Franklin Institute in America, added to those in France by Morin, and the results of a patented machine by Whitelaw and Stirrat, Scotland, combined with pertinent observations and remarks by interested parties in this as well as in other countries : hence have been deduced the following demonstrative conclusions.

1. That to gain a maximum of effect by a horizontal water-wheel, the water must be laid upon the wheel on the stream side, and the diameter of the wheel so proportioned to the height of the fall, that the water may be laid on about $52\frac{3}{4}$ degrees distant from the summit of the wheel ; or the height of the fall being 1, the height or diameter of the wheel equal 1.108.

2. That the periphery of a water-wheel ought to move at a velocity equal to about twice the square root of the fall of the water in feet per second, and the number of buckets equal 2.1 times the wheel's diameter in feet ; also, that precautionary means be adopted for the escape of the air out of the buckets,

either by making the stream of water a few inches narrower than the wheel, or otherwise.

3. That because of water producing a less efficient power by impulse than gravity, turbines, or machines through which the motion is obtained by reaction, are greatly preferable to undershot, or low breast wheels.

4. That a head* of water is required sufficient to cause the velocity of its flowing to be as 3 to 2 of the wheel; $\frac{1}{9}$ th of the wheel's diameter being an approximate height, near enough for practical purposes.

5. That the effective power of a wheel constructed according to these restrictions is equal to the product of the number of cubic feet, and velocity in feet per minute, multiplied into $\cdot 001325$.

Example for general illustration.

Suppose a fall of water 25 feet in height, over which is delivered 112 cubic feet per minute, required the various peculiar requisites for a wheel to be in accordance with the preceding rules.

1st. $25 \times 1\cdot 08 = 27$ feet, the wheel's diameter.

2nd. $\sqrt{25} \times 2 = 10$ feet, velocity of the wheel in feet per second.

Also $27 \times 2\cdot 1 = 56\cdot 7$, say 57 buckets.

3rd. $27 \div 9 = 3$ feet, head of water required.

4th. $112 \times 10 \times 60 \times \cdot 001325 = 89$ horses' power.

The *turbine* of Fourneyron, in France, and the patented *Water-Mill* of Whitelaw and Stirrat, Scotland, have of late years attracted a considerable share of public attention, their simplicity of construction and asserted effects in like situations being equal to those of the best applied water-wheels. In their manner of construction they differ, but in principle they are the same; the action of each being created by a centrifugal and tangential force, caused by the weight or impulsion of a column

* By head is meant the distance from the surface of the water to that point at which it strikes upon the wheel.

of water whose height or altitude is equal to twice the height of the fall due to the water's velocity; and in order to produce a maximum of effect in either the one or the other by the pressure and centrifugal force of the effluent water, it is necessary that the emitting tubes, or helical channels of the machine, be so curved that the apertures shall be in a right line with the radius of the wheel.

In 1838 a number of experiments were made in France by Morin, with a view to the more general introduction of turbines, and a positive proof of their merited qualities in preference to wheels of impulse, which terminated considerably in favour of turbines, and from which the following deductions were made.

1. That turbines are equally adapted to great as to small falls of water.

2. That they are capable of transmitting an useful effect to from 70 to 78 per cent. of the absolute power.

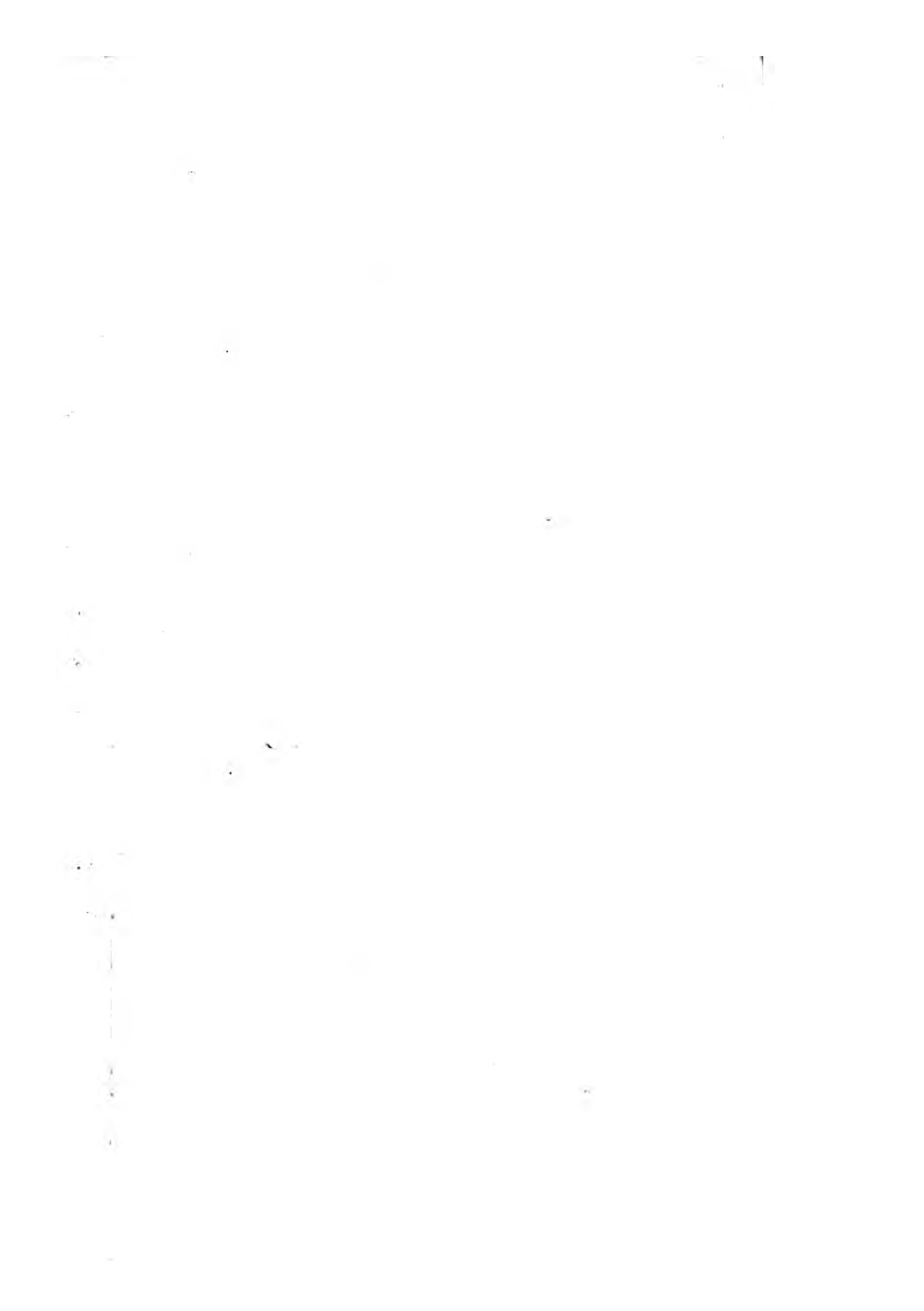
3. That their velocities may vary very considerably from the maximum effect, without differing very sensibly from it.

4. That they will work nearly as effectually when drowned to the depth of 5 or 6 feet as when free, and consequently, they will make use of the whole of the fall when placed below the level of extreme low water.

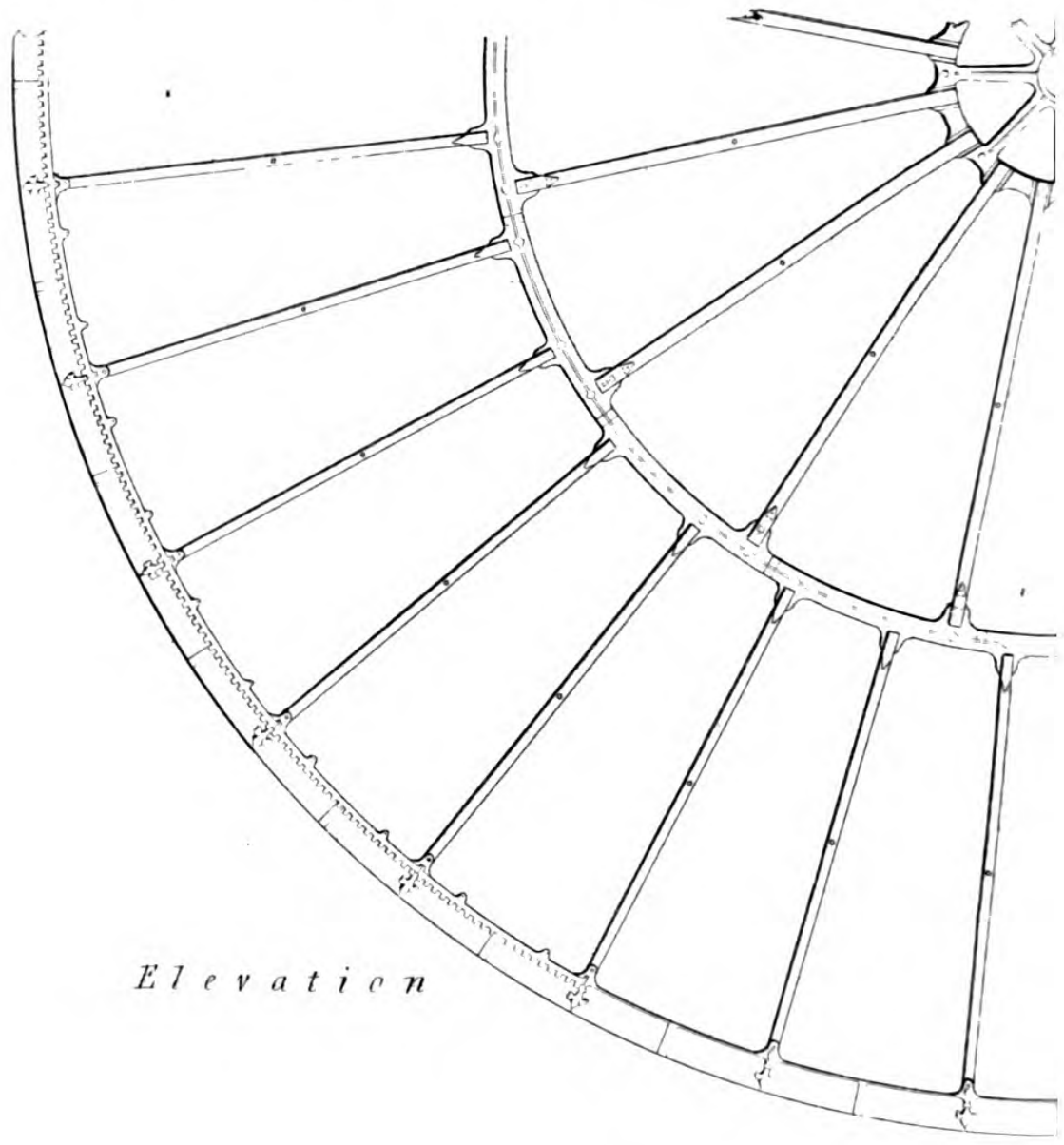
5. That they may receive variable quantities of water without altering the ratio of the power to the effect. Corresponding results have also been realized by Whitelaw and Stirrat's machine; hence what is said of the one is equally applicable to the other.

General Rule by Mr. Whitelaw, whereby to compute the power of their Turbine, or Water-Mill.

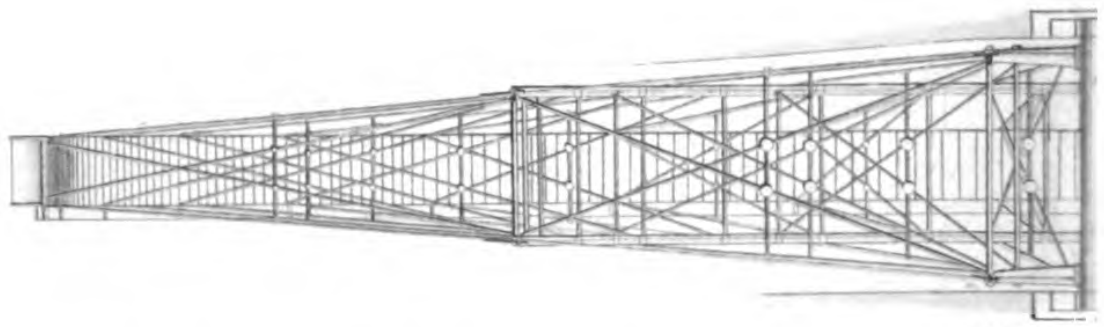
Rule.—Multiply the effective quantity of water flowing in cubic feet per minute by the height of the fall of the water in feet, and divide the product by 700;



AN OVRSHOT WATER WHEEL, CONSTRUCTED



Elevation

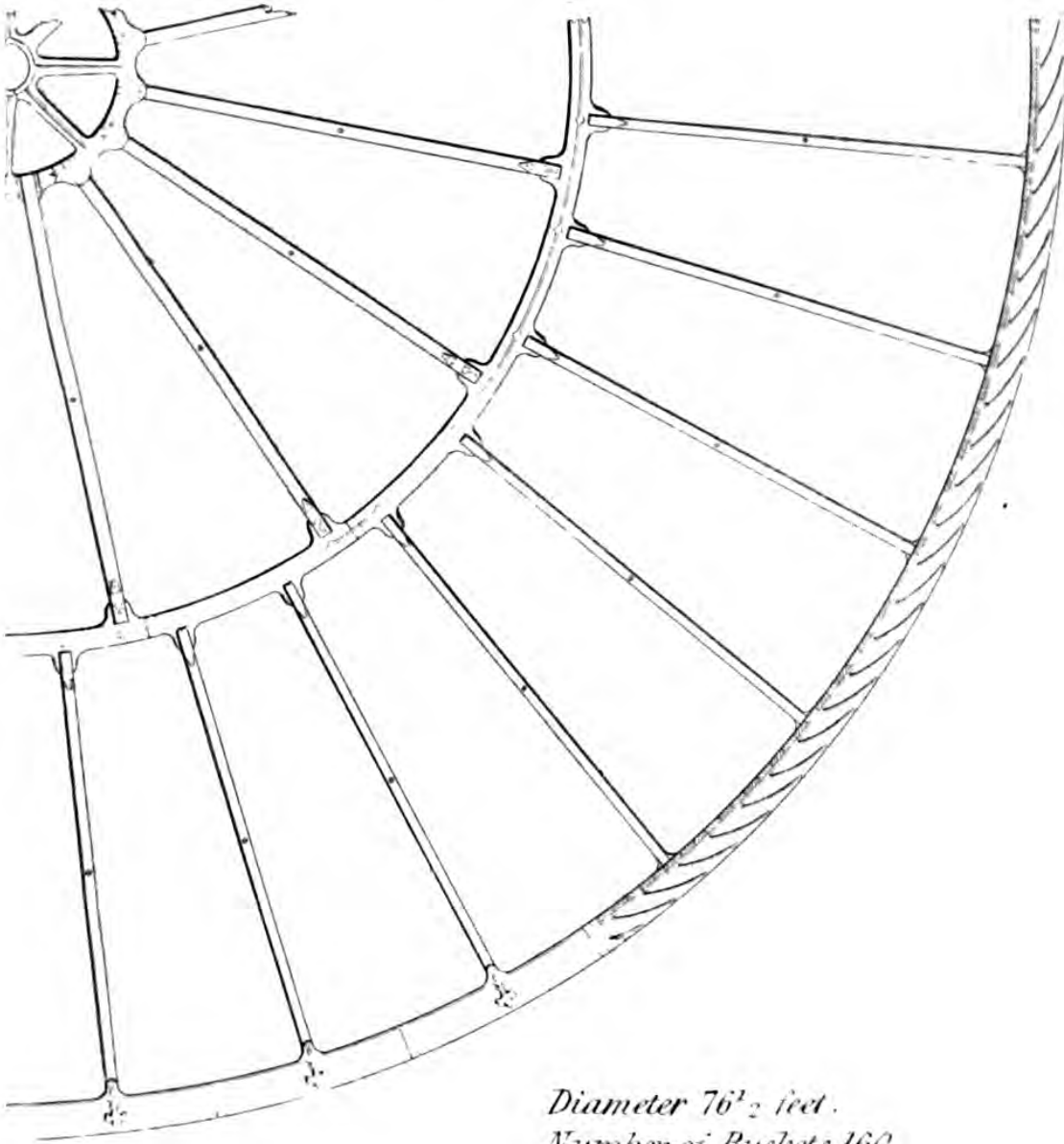


Sect

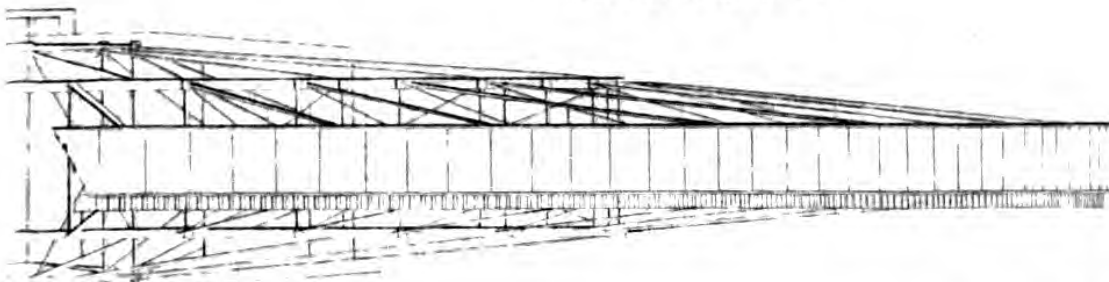
Published by John Weale 59 High Holborn 1844.

BY MESS^{RS} DONKIN & CO ENGINEERS, LONDON.

Plate G

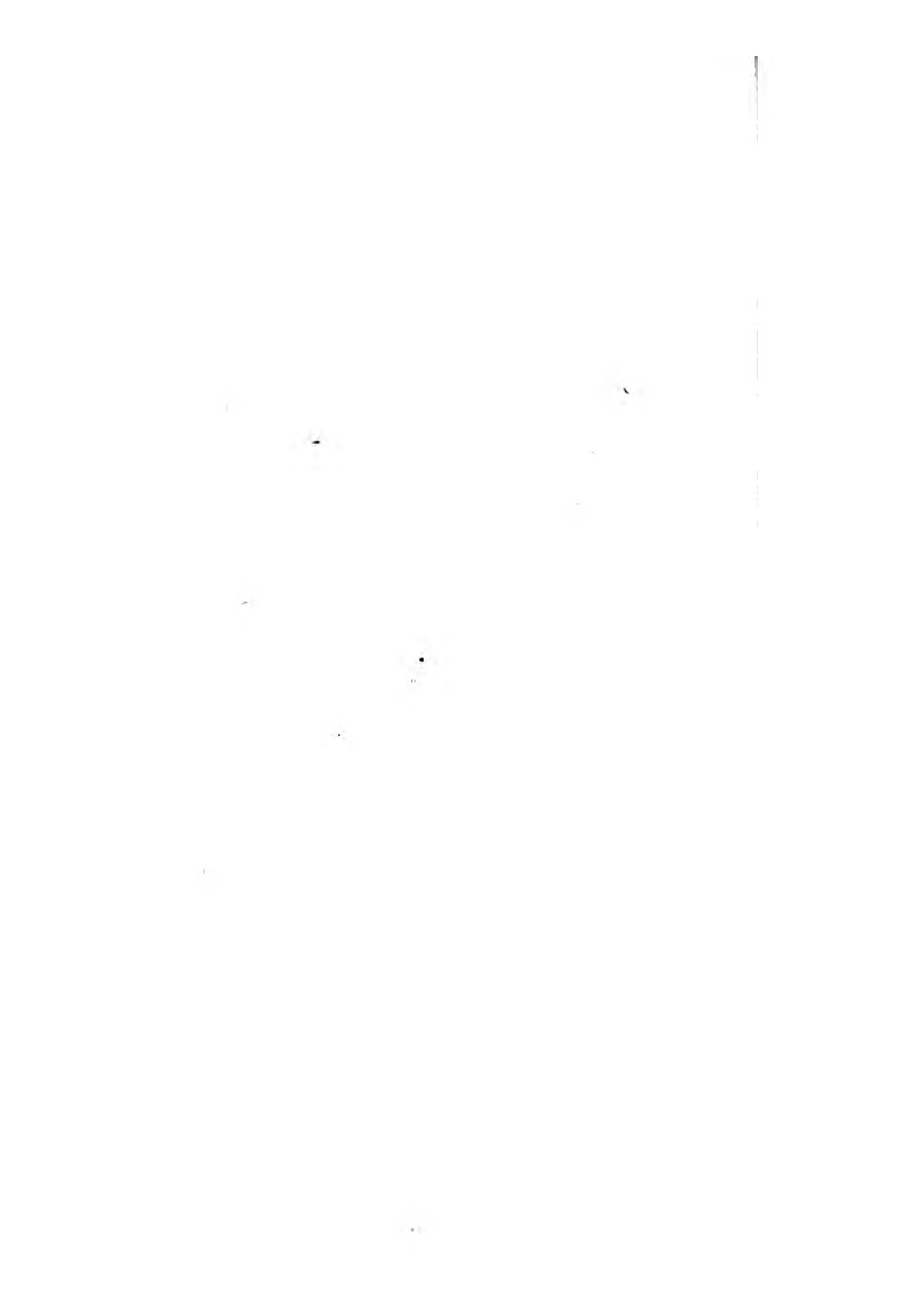


*Diameter 76 $\frac{1}{2}$ feet.
Number of Buckets 160.
Width of Wheel 2 feet.
Horses Power 30.*

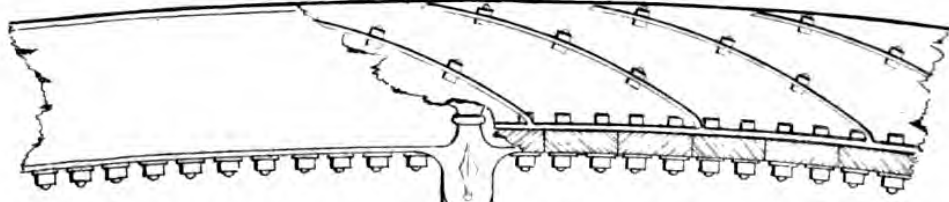


Don

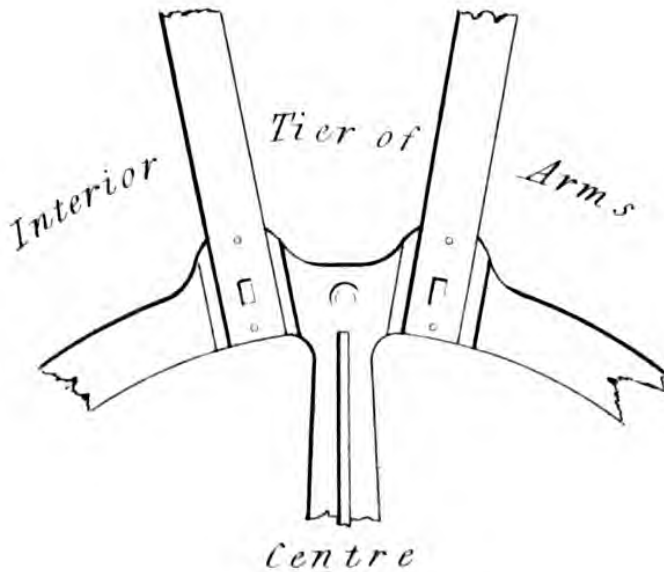
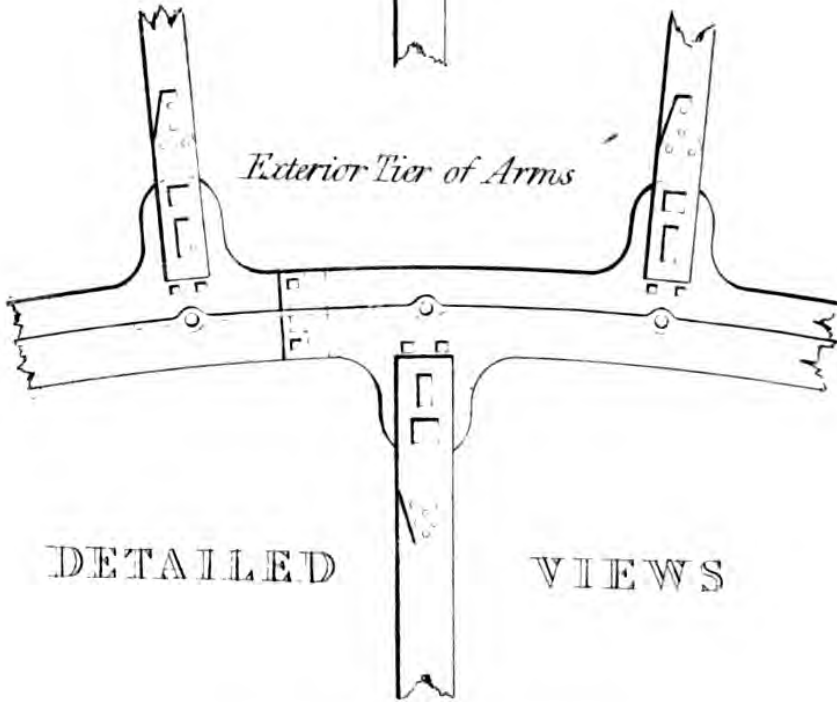
J.W. Lowry, Jr



Shroudings, Buckets, &c.



Exterior Tier of Arms



the quotient equal the effect produced in horses' power.

Ex. Required the power produced by 1400 cubic feet per minute over a fall of 30 feet.

$$\frac{1400 \times 30}{700} = 60 \text{ horses' power.}$$

Plates G and H exhibit miniature views of a ponderous overshot water-wheel, chiefly of wrought iron, and in which novelty of construction, immense diameter, and great efficiency, have rendered it an object of universal interest throughout the district in which it is situated: its nominal power is that of 30 horses, but upwards of 40 of indicated resistance seems not materially to diminish its velocity; and no doubt, on account of the improved form of the buckets, the greatest amount of the water's effect is obtained.

2. *Effects produced by water in an aëriform state.*

When water in a vessel is subjected to the action of fire, it readily imbibes the heat or fluid principle of which the fire is the immediate cause, and sooner or later, according to the intensity of the heat, attains a temperature of 212° Fahrenheit. If at this point of temperature the water be not enclosed, but exposed to atmospheric pressure, ebullition will take place, and steam or vapour will ascend through the water, carrying with it the superabundant heat, or that which the water cannot under such circumstances of pressure absorb, to be retained and to indicate a higher temperature.

Water, in attaining the aëriform state, is thus uniformly confined to the same laws under every degree of pressure; but as the pressure is augmented, so is the indicated temperature proportionately elevated: hence the various densities of steam, and corresponding degrees of elastic force.

Table of the Elastic Force of Steam, and corresponding Temperature of the Water with which it is in contact.

Pressure per square inch, atmospheric pressure included.		Elastic Force in		Temperature in Degrees of			Volume of Steam compared with vol. of Water.
		Inches of Mercury.	Metres of Mercury.	Fahr.	Reaum.	Cent.	
lbs.	kilog.						
14.7	6.668	30.00	.762	212.0	80.0	100.0	1711
15	6.80	30.60	.778	212.8	80.4	100.4	1670
16	7.26	32.64	.829	216.3	81.9	102.4	1573
17	7.71	34.68	.880	219.6	83.3	104.2	1488
18	8.16	36.72	.932	222.7	84.7	105.9	1411
19	8.62	38.76	.984	225.6	86.0	107.6	1343
20	9.07	40.80	1.037	228.5	87.3	109.2	1281
21	9.52	42.84	1.089	231.2	88.5	110.7	1225
22	9.98	44.88	1.140	233.8	89.7	112.1	1174
23	10.43	46.92	1.192	236.3	90.8	113.5	1127
24	10.88	48.96	1.244	238.7	91.9	114.8	1084
25	11.34	51.00	1.296	241.0	93.0	116.1	1044
26	11.79	53.04	1.348	243.3	93.9	117.4	1007
27	12.25	55.08	1.400	245.5	94.9	118.6	973
28	12.70	57.12	1.452	247.6	95.8	119.8	941
29	13.15	59.16	1.503	249.6	96.7	120.9	911
30	13.61	61.21	1.555	251.6	97.6	122.0	883
31	14.06	63.24	1.607	253.6	98.5	123.1	857
32	14.51	65.28	1.659	255.5	99.3	124.2	833
33	14.97	67.32	1.711	257.3	100.1	125.2	810
34	15.42	69.36	1.763	259.1	100.9	126.2	788
35	15.87	71.40	1.814	260.9	101.7	127.2	767
36	16.33	73.44	1.866	262.6	102.5	128.1	748
37	16.78	75.48	1.918	264.3	103.2	129.1	729
38	17.23	77.52	1.970	265.9	104.0	129.9	712
39	17.69	79.56	2.022	267.5	104.7	130.8	695
40	18.14	81.60	2.074	269.1	105.4	131.7	679
41	18.59	83.64	2.126	270.6	106.0	132.6	664
42	19.05	85.68	2.178	272.1	106.7	133.4	649
43	19.50	87.72	2.229	273.6	107.4	134.2	635
44	19.96	89.76	2.281	275.0	108.0	135.0	622
45	20.41	91.80	2.333	276.4	108.6	135.8	610
46	20.86	93.84	2.385	277.8	109.2	136.6	598
47	21.32	95.88	2.437	279.2	109.9	137.3	586
48	21.77	97.92	2.489	280.5	110.4	138.1	575
49	22.22	99.96	2.541	281.9	111.1	138.8	564
50	22.68	102.00	2.592	283.2	111.6	139.6	554

The preceding Table is peculiarly adapted for estimating the power of steam engines on the condensing principle, because in such the effective force of the steam is the difference between the total force and the resisting vapour retained in the condenser. The following Table is more adapted for estimating the effects of non-condensing engines, as, in such, the atmospheric pressure is not generally taken into account, engines of this principle being supposed to work in a medium ; or, the atmospheric pressure on the boiler, to cause a greater density of steam, is equal to the resisting atmosphere which the effluent steam has to contend with on leaving the cylinder.

Table of the Elastic Force of Steam, the pressure of the Atmosphere not being included.

Elastic Force in			Tempera- ture in degrees of Fahr.	Volume of Steam of Water being 1.	Cubic in. of Water in a cubic foot of Steam.
Atmosphere.	lbs. & sq. in.	in. of Merc.			
1·19	2·5	5·15	220	1496	1·14
1·22	3	6·18	222	1453	1·18
1·29	4	8·24	225	1366	1·25
1·36	5	10·3	228	1282	1·33
1·70	10	20·6	240	1044	1·64
2·04	15	30·9	251	883	1·93
2·38	20	41·2	260	767	2·23
2·72	25	51·5	268	678	2·52
3·06	30	61·8	275	609	2·81
3·40	35	72·1	282	553	3·09
3·74	40	82·4	288	506	3·38
4·08	45	92·7	294	468	3·66
4·42	50	103·0	299	435	3·93
4·76	55	113·3	304	407	4·20
5·10	60	123·6	309	382	4·48

Steam, independent of the heat indicated by an immersed thermometer, also contains heat that cannot

be measured by any instrument at present known, and, in consequence of which, is termed latent or concealed heat; the only positive proof we have of its existence being that of incontestible results or effects produced on various bodies. Thus, if one part by weight of steam at 212° , be mixed with nine parts of water at 62° , the result is water at 178.6° ; therefore, each of the nine parts of water has received from the steam 116.6° of heat, and consequently the steam has diffused or given out $116.6 \times 9 = 1049.4 - 33.4 = 1016^{\circ}$ of heat which it must have contained. Again, it is ascertained by experiment, that if one gallon of water be transformed into steam at 212° , and that steam allowed to mix with water at 52° , the whole will be raised to the boiling point, or 212° . From these and other experiments, it is ascertained that the latent heat in steam varies from 940° to 1044° , the ratio of accumulation advancing from 212° , as the steam becomes more dense and of greater elastic force: hence the severity of a scald by steam to that by boiling water.

It is because of the latent heat in steam, or water in an aëriform state, that it becomes of such essential service in *heating, boiling, drying, &c.* In the heating of buildings, its *economy, efficiency, and simplicity of application*, are alike acknowledged;—the steam being simply conducted through all the departments by pipes, by extent of circulation condenses, the latent heat being thus given to the pipes, and diffused by radiation. In boiling, its efficiency is considerably increased if advantage be taken of sufficiently enclosing the fluid and reducing the pressure on its surface by means of an air-pump; thus, water in a vacuum boils at about a temperature of 98° , and in sugar-refining, where such means are employed, the syrup is boiled at 150° .

Steam is also of great utility as a productive source of motive power; and in this respect its properties are, *elastic force*, *expansive force*, and *reduction by condensation*. Elastic force signifies the whole urgency or power the steam is capable of exerting with undiminished effect. By expansive force is generally understood the amount of diminishing effect of the steam on the piston of a steam engine, reckoning from that point of the stroke where the steam of uniform elastic force is cut off; but it is more properly the force which steam is capable of exerting when expanded to a known number of times its original bulk. And

Condensation, here understood, is the abstraction or reduction of heat by another body, and consequently not properly a contained property of the steam, but an effect produced by combined agency, in which steam is the principal; because any colder body will extract the heat and produce condensation, but steam cannot be so beneficially replaced by any other fluid capable of maintaining equal results.

The rules formed by experimenters as corresponding with the results of their experiments on the elastic force of steam at given temperatures vary, but approximate so closely that the following rule, because of being simple, may in practice be taken in preference to any other.

Rule.—To the temperature of the steam in degrees of Fahrenheit, add 100, divide the sum by 177, and the 6th power of the quotient equal the force in inches of mercury.

Ex. Required the force of steam corresponding to a temperature of 312°.

$$\frac{312 + 100}{177} = 2.3277^6 = 159 \text{ inches of mercury.}$$

But the Table, page 138, is much better adapted to practical purposes, as the various results or effects are obtained simply by inspection.

To estimate the amount of advantage gained by using steam expansively in a steam engine.

When steam of a uniform elastic force is employed throughout the whole ascent or descent of the piston, the amount of effect produced is as the quantity of steam expended. But let the steam be shut off at any portion of the stroke, say, for instance, at one-half, it expands by degrees until the termination of the stroke, and then exerts half its original force; hence an accumulation of effect in proportion to the quantity of steam.

Rule.—Divide the length of the stroke by the distance or space into which the dense steam is admitted, and find the hyperbolic logarithm of the quotient, to which add 1, and the sum is the ratio of the gain.

Ex. Suppose an engine with a stroke of 6 feet, and the steam cut off when the piston has moved through 2; required the ratio of gain by uniform and expansive force.

$6 \div 2 = 3$; hyperbolic logarithm of 3 = 1.0986 + 1 = 2.0986
ratio of effect; that is, supposing the whole effect of the steam to be 3, the effect by the steam being cut off at $\frac{1}{3} = 2.0986$.

Again, let the greatest elastic force of steam in the cylinder of an engine equal 48 lbs. per square inch, and let it be cut off from entering the cylinder when the piston has moved $4\frac{1}{2}$ inches, the whole stroke being 18; required an equivalent force of the steam throughout the whole stroke.

$18 \div 4.5 = 4$, and $48 \div 4 = 12$.

Logarithm of 4 + 1 = 2.38629.

Then $2.38629 \times 12 = 28.635$ lbs. per square inch.

Table of Hyperbolic Logarithms.

No.	Logarithm.	No.	Logarithm.	No.	Logarithm.	No.	Logarithm.
1 $\frac{1}{4}$	·22314	3 $\frac{1}{2}$	1·25276	5 $\frac{3}{4}$	1·74919	8	2·07944
1 $\frac{1}{2}$	·40546	3 $\frac{3}{4}$	1·32175	6	1·79175	8 $\frac{1}{2}$	2·14006
1 $\frac{3}{4}$	·55961	4	1·38629	6 $\frac{1}{4}$	1·83258	9	2·19722
2	·69314	4 $\frac{1}{4}$	1·44691	6 $\frac{1}{2}$	1·87180	9 $\frac{1}{2}$	2·25129
2 $\frac{1}{4}$	·81093	4 $\frac{1}{2}$	1·50507	6 $\frac{3}{4}$	1·90954	10	2·30258
2 $\frac{1}{2}$	·91629	4 $\frac{3}{4}$	1·55814	7	1·94591	12	2·48490
2 $\frac{3}{4}$	1·01160	5	1·60943	7 $\frac{1}{4}$	1·98100	14	2·63905
3	1·09861	5 $\frac{1}{4}$	1·65822	7 $\frac{1}{2}$	2·01490	16	2·77258
3 $\frac{1}{4}$	1·17865	5 $\frac{1}{2}$	1·70474	7 $\frac{3}{4}$	2·04769	18	2·89037

In regard to the other case of expansion: When the temperature is constant the bulk is inversely as the pressure; thus, Suppose steam at 30 lbs. per square inch, required its bulk to that of original bulk, when expanded so as to retain a pressure equal to that of the atmosphere, or 15 lbs.

$$\frac{15 + 30}{15} = 3 \text{ times its original bulk.}$$

Condensation of steam for motive purposes, generally, is effected by cold water, the quantity of which may be estimated by the following rule. From 1000 plus the temperature of the steam, subtract the required temperature of the condensed water, divide the remainder by the temperature of the condensed water minus the temperature of the cold or condensing water, and the quotient equal the number of times that the quantity, for condensation, must exceed that by which the steam is formed.

Ex. Required the ratio or quantity of water for condensation to 1 of water for the formation of steam, the temperature of the steam being 220°, and the required temperature of condensed water 180°.

$$\frac{1000 + 220 - 180}{180 - 52} = 8 \text{ times the quantity.}$$

Water holding impurities in solution tends to retard its attaining the aëriform state, and so impairs the amount of its elastic force at an equal temperature, as exhibited in the following Tables. Thus, common water boils at 212° Fahrenheit.

Name of substance.	Proportionate quantity in 100 parts by weight of water.		Boiling points.
Salts in sea water.	3.03.		213.2° F.
Sulphate of soda	In common water.	31.5	213
Sulphate of iron		64	216
Alum		52	220
Sulphate of lime		45	220
Sulphate of magnesia		57.5	222
Muriate of soda		30	224
Nitrate of soda		60	246
Acetate of soda		60	256

Elastic Force of Steam in Inches of Mercury.

Common water } boiling point, 212° F. { elastic force 30 in.
Sea water . . . } at 212 ,, { ,, 23.05 ,,
Common water } boiling point, 216° F. { elastic force 32.5 in.
Sea water . . . } at 216 ,, { ,, 24.6 ,,
Common water } boiling point, 220° F. { elastic force 35.1 in.
Sea water . . . } at 220 ,, { ,, 26.5 ,,

Hence the propriety of procuring, for steam, water in its purest state.

3. Effects produced by air in its natural and also in a rarefied state.

The weight or pressure of the atmosphere is equal to the weight of a column of water 34 feet in height, or to a column of mercury 30 inches in height, or to 14.7 lbs. av. per square inch at a mean temperature. But air, like all other gases, is rendered lighter by

the application of heat, for then the particles of the mass are repelled from each other, or rarefied, and occupy a greater space. Rarefied air being specifically lightest, mounts above that of common density; hence change of temperature, and the principal cause of winds.

Table of the Expansion of Atmospheric Air by Heat.

Deg. of Fahr.	Bulk.	Deg. of Fahr.	Bulk.	Deg. of Fahr.	Bulk.
32°	1000	65°	1077	100°	1152
35	1007	70	1089	120	1194
40	1021	75	1099	140	1235
45	1032	80	1110	160	1275
50	1043	85	1121	180	1315
55	1055	90	1132	200	1364
60	1066	95	1142	212	1376

The pressure or gravity of the atmosphere being equal to a column of water 34 feet in height, is the means or principle on which rests the utility of the common pump, also of the syphon and all other such hydraulic applications. In a pump the internal pressure on the surface of the liquid is removed by the action of the bucket, and as by degrees the density becomes lessened, so the water rises by the external pressure to the above-named height; and at such height it will remain, unless by some derangement of construction taking place, the atmospheric fluid is allowed to enter and displace the liquid column. But observe, if the temperature of the water or other liquid be so elevated that steam or vapour arise through it, then, according to the vapour's accumulation of density, may the action of the pump be partially or wholly destroyed; and the only means of evasion in such cases is to place the working bucket beneath the surface of the liquid which is required to be raised.

Table showing the quantity of Water per lineal foot in Pumps or Vertical Pipes of different diameters.

Diameter of pump in inches.	Number of gallons per lineal ft.	Number of cubic feet per lineal ft.	Diameter of pump in inches.	Number of gallons per lineal ft.	Number of cubic feet per lineal ft.
2	·136	·0218	8	2·176	·3490
2 $\frac{1}{4}$	·172	·0276	8 $\frac{1}{4}$	2·314	·3712
2 $\frac{1}{2}$	·212	·0340	8 $\frac{1}{2}$	2·456	·3940
2 $\frac{3}{4}$	·257	·0412	8 $\frac{3}{4}$	2·603	·4175
3	·306	·0490	9	2·754	·4417
3 $\frac{1}{4}$	·359	·0576	9 $\frac{1}{4}$	2·909	·4666
3 $\frac{1}{2}$	·416	·0668	9 $\frac{1}{2}$	3·068	·4923
3 $\frac{3}{4}$	·478	·0766	9 $\frac{3}{4}$	3·232	·5184
4	·544	·0872	10	3·400	·5454
4 $\frac{1}{4}$	·614	·0985	10 $\frac{1}{4}$	3·572	·5730
4 $\frac{1}{2}$	·688	·1104	10 $\frac{1}{2}$	3·748	·6013
4 $\frac{3}{4}$	·767	·1230	10 $\frac{3}{4}$	3·929	·6302
5	·850	·1363	11	4·114	·6599
5 $\frac{1}{4}$	·937	·1503	11 $\frac{1}{4}$	4·303	·6902
5 $\frac{1}{2}$	1·028	·1649	11 $\frac{1}{2}$	4·496	·7212
5 $\frac{3}{4}$	1·124	·1803	11 $\frac{3}{4}$	4·694	·7529
6	1·224	·1963	12	4·896	·7853
6 $\frac{1}{4}$	1·328	·2130	12 $\frac{1}{4}$	5·312	·8521
6 $\frac{1}{2}$	1·436	·2304	13	5·746	·9217
6 $\frac{3}{4}$	1·549	·2489	13 $\frac{1}{2}$	6·196	·9939
7	1·666	·2672	14	6·664	1·0689
7 $\frac{1}{4}$	1·787	·2866	15	7·650	1·2271
7 $\frac{1}{2}$	1·912	·3067	16	8·704	1·3962
7 $\frac{3}{4}$	2·042	·3275	18	11·016	1·7670

Examples illustrative of the Utility of the Table.

1. Required the quantity of water lifted by each stroke of the bucket of a 9 $\frac{1}{2}$ -inch pump, the length of the stroke being 2 $\frac{1}{4}$ feet.

$$3·068 \times 2·25 = 6·903 \text{ gallons each stroke.}$$

2. What length of stroke with a 6-inch pump will be necessary to discharge 44 gallons of water per

minute, the number of strokes being 18 in the given time?

$$\frac{44}{1.224 \times 18} = 2 \text{ feet, the length of stroke.}$$

3. What must be the diameter capable of raising 25 cubic feet of water per minute, the length of the stroke being $2\frac{1}{2}$ feet, and making 16 effective strokes per minute?

$$\frac{25}{2.5 \times 16} = .625, \text{ or } 10\frac{3}{4} \text{ inches nearly.}$$

It is by the oxygen of the atmosphere that combustion is supported. The common combustibles of nature are chiefly compounds of carbon and hydrogen, which, during combustion, combine with the oxygen of the atmosphere, and are converted into carbonic acid and watery vapour, different species of fuel requiring different quantities of oxygen. The quantity required for the combustion of a pound of coal varies from 2 to 3 lbs., according to the quality of the coal. 60 cubic feet of atmospheric air is necessary to produce 1 lb. of oxygen.

The pressure or fluid properties of the atmosphere oppose bodies in passing through it, the opposing resistance increasing as the square of the velocity of the body, and the resistance per square foot in lbs. as its velocity in feet per second, multiplied into .002288. Thus, suppose a locomotive engine in a still atmosphere, at a velocity of 25 miles per hour, presents a resisting frontage of 20 feet; required the amount of opposing resistance at that velocity.

25 miles per hour equal 36.67 feet per second.

Then $36.67^2 \times .002288 \times 20 = 61.5$ lbs. constant opposing force.

Table of the Force and common Appellations given to Winds at different velocities.

Velocity of the wind in		Force in lbs. avoirdupois per square foot.	Common Appellations given to the Wind.
Miles per hour.	Feet per second.		
1	1.47	.005	Hardly perceptible.
2	2.93	.020	} Just perceptible.
3	4.40	.044	
4	5.87	.079	} Gentle pleasant wind.
5	7.33	.123	
10	14.67	.492	} Pleasant brisk gale.
15	22.00	1.107	
20	29.34	1.968	} Very brisk.
25	36.67	3.075	
30	44.01	4.429	} High winds.
35	51.34	6.027	
40	58.68	7.873	} Very high.
45	66.01	9.963	
50	73.35	12.300	A storm or tempest.
60	88.02	17.715	A great storm.
80	117.36	31.490	A hurricane.

In order to gain the greatest amount of the wind's impulsive effect to produce rotary or circular motion by the sails of a wind-mill, the total surface of the sails presented to the wind ought to be about $\frac{7}{8}$ ths of the circle's surface which is formed by their motion, and each sail angled to the plane of motion as follows, the whip or back being divided into six equal parts.

Distance from centre of motion	1	2	3	4	5	6	} Smeaton's rule.
Angle with plane of motion	.. 18°	19	18	16	12½	7	
By G. Forrester, Liverpool	.. 24°	21	18	14	9	3	

RULES, TABLES, &c., RELATIVE TO BOILERS AND THE STEAM ENGINE.

THE Boiler of a Steam Engine may be explained as that portion of the structure in which the vital principle of the engine is generated; consequently its construction is of the utmost importance, for upon the proper efficiency of the boiler depends in a great measure the efficiency of the engine.

Boilers not unfrequently, because of unavoidable peculiarities, are necessarily constructed of various forms; but for land or stationary engine boilers, if no thwarting circumstances intervene, either the waggon or cylindrical forms are commonly resorted to; the former for those of condensing engines, and the latter for those of the high-pressure principle.

In the construction of boilers, much attention ought to be paid in avoiding thin films of water where the action of the fire is great, because it is neither consistent with safety, nor can there be the proper quantities of steam generated according to the surface exposed, unless under some extraordinary degree of pressure. Also convex surfaces, exposed to the action of the steam, unless properly supported, ought strenuously to be avoided. Large water spaces, concave surfaces, or straight plates securely stayed, with ample steam room, are the chief requisites to be attended to.

1. *To determine the proper quantity of heating surface in a boiler for an engine with a cylinder of a given capacity, and steam at any density required.*

Rule.—Multiply 375 times the area of the cylinder in feet by the velocity of the piston in feet per minute, and divide the product by the volume of steam to 1

of water at the density required (see Table, page 138), and the quotient is the amount of effective heating surface in square feet.

Ex. Required the amount of effective heating surface in a boiler for an engine whose cylinder is $4\frac{1}{2}$ square feet in area, and the piston's velocity 224 feet per minute, the pressure of the steam to equal 5 lbs. per square inch above the pressure of the atmosphere.

$$\frac{375 \times 4.5 \times 224}{1282} = 295 \text{ square feet nearly; the fire grate be-}$$

ing in accordance with the following rule.

Multiply the number of square feet of heating surface by $\cdot 12$, the product equal the area of fire grate in square feet, thus :

$$295 \times \cdot 12 = 35.4 \text{ square feet of furnace bar.}$$

Note.—By effective heating surface is meant horizontal surfaces over fire, flame, or heated air; vertical or side surfaces requiring about $1\frac{3}{4}$ feet to equal in effect 1 of horizontal surface.

2. To determine the proper dimensions for a waggon-shaped boiler, when the amount of effective heating surface in square feet is obtained by the preceding rule.

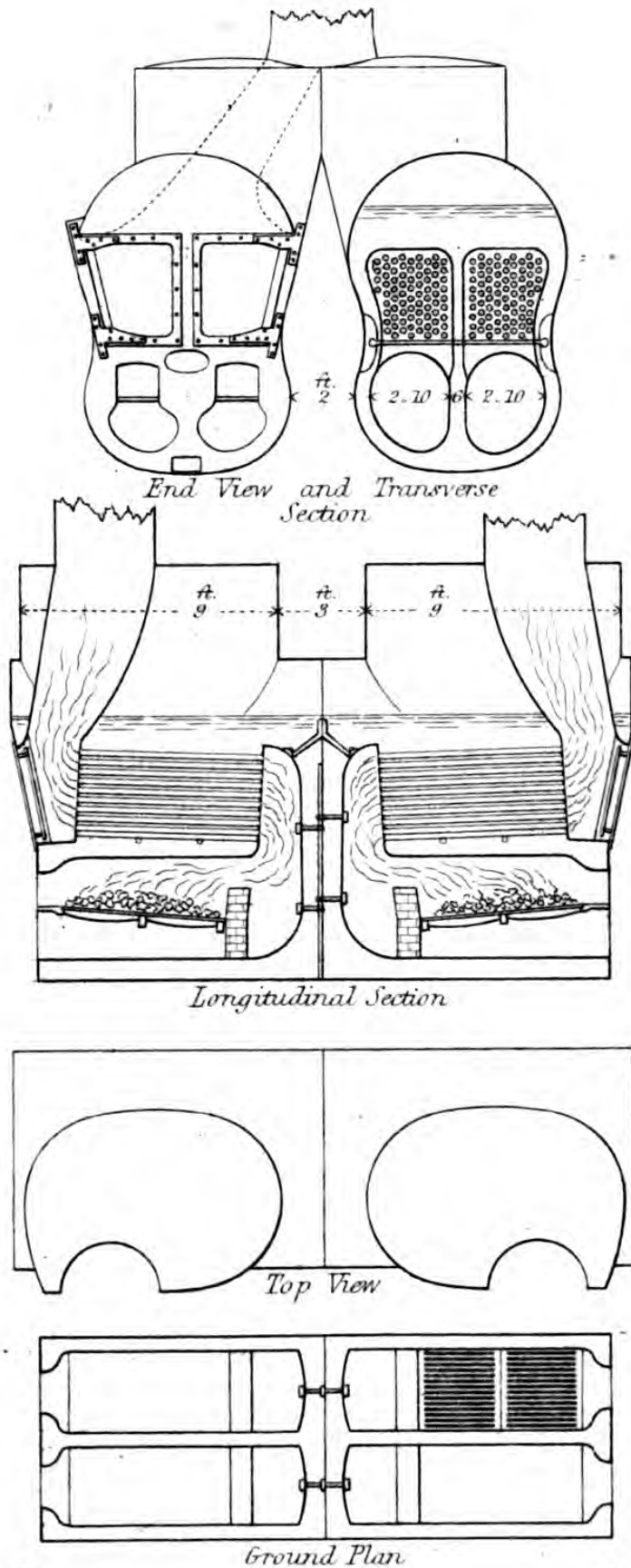
1. The bottom surface equal half the whole surface.
2. The length of the boiler equal twice the square root of bottom surface.
3. The width equal one-fourth the length; and
4. The height equal one-third the length.

Ex. Required the dimensions for a boiler of the waggon form that may present an effective heating surface of 295 square feet.

$$\begin{aligned} \text{Bottom surface} &= 295 \div 2, \text{ or } 147.5 \text{ square feet.} \\ \text{Length} \quad . \quad . &= \sqrt{147.5 \times 2}, \text{ or } 24.26 \text{ feet.} \\ \text{Width} \quad . \quad . &= 24.26 \div 4, \text{ or } 6.06 \text{ feet.} \\ \text{Height} \quad . \quad . &= 24.26 \div 3, \text{ or } 8.08 \text{ feet.} \end{aligned}$$

Note.—The amount of side or vertical surface equal twice





End View and Transverse Section

Longitudinal Section

Top View

Ground Plan

Plans of the Boilers on board the Peninsular Co's Steam Ship Braganza
By Messrs Bury, Curtis and Kennedy, Liverpool.

W. T., del.

G. Gladwin, sc

London: J. Weale, 1845

the length of the boiler, added to the width, and multiplied by 1.75 to obtain that of effective surface; hence,

$$\frac{147.5 \times 1.75}{24.26 \times 2 + 6.06} = 4.7 \text{ feet, depth of side flue.}$$

3. *To determine the dimensions for a cylindrical boiler.*

Rule.—Extract the square root of 1.34 times the effective heating surface in square feet, and twice the root equal the boiler's circumference in feet; also, the circumference equal the length.

Ex. Let a cylindrical boiler be required with an effective heating surface of 86 square feet; what must be its length and diameter in feet?

$$\sqrt{86 \times 1.34} = 10.74 \times 2 = 21.48 \text{ feet circumference, or 6 feet 10 inches diameter, and 21.48 feet in length.}$$

Note.—When an internal flue is to be inserted in a boiler, the external surface of the boiler may be diminished in length, equal to half the exposed surface of the flue. Observe, also, that the height of the contained water in boilers generally ought to be about $\frac{2}{3}$ rds the whole height of the boiler.

Marine Engine Boilers are now become so varied in their designs, that any attempt at enumerating—by ever so slight a description—those that may be considered worthy of record, would much exceed the limits of my present purpose; but having been favored by Messrs. Bury, Curtis, and Kennedy, with particulars of boilers lately constructed by them for the 'Braganza' steam vessel, and which are now giving the highest degree of satisfaction in every respect, I give the annexed designs and data, in preference to any others. (See Plate K.)

These boilers are constructed so as to work at 10 lbs. per square inch if required; they are four in number; the diameter of each cylinder is $62\frac{1}{8}$ inches, length of stroke $5\frac{1}{2}$ feet, or calculated velocity about 231 feet per minute; the nominal power of each

engine is 140 horses, and the heating surface per horse-power is 14·1 square feet.

Now $14\cdot1 \times 140 = 1974$ square feet to each engine.

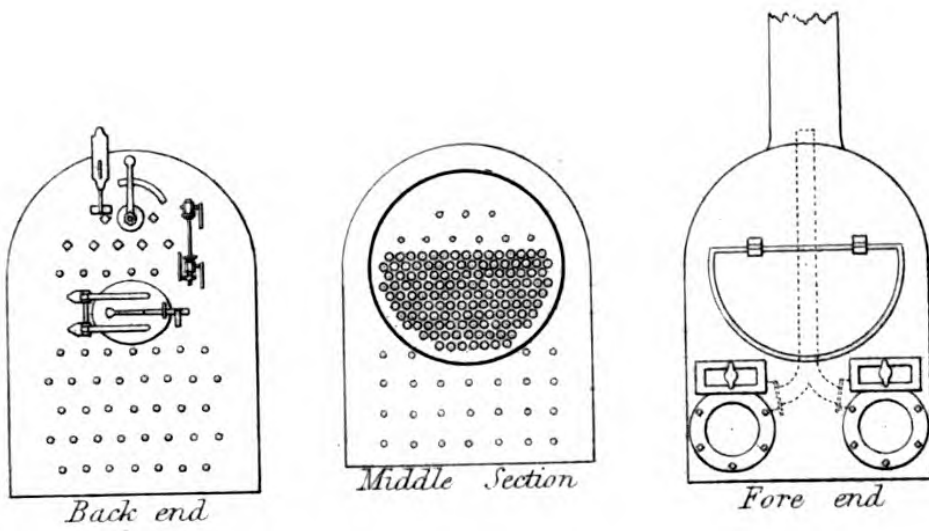
Then as per rule (page 149), 375 multiplied by the area of cylinder or 21 feet, and by the velocity per minute or 231 feet, and divided by 1044 (see Table, page 139) or volume of steam to 1 of water, at 10 lbs. per square inch, equal 1742·4 square feet; to which add about $\frac{1}{8}$ th of the quotient, or 217·8, for side or vertical surface, equal 1960·2 square feet to each engine,—thus nearly corresponding with *their* given quantity of heating surface.

Note.—In the four boilers there are 608 brass tubes, $6\frac{1}{2}$ feet in length, and 3 inches in diameter. The length of each fire-place is 5 feet 10 inches, and breadth 2 feet 10; there are ·56 of a square foot of fire-bar to each cubic foot of cylinder capacity, and ·16 of a square foot of tube aperture to each square foot of fire-bar, the openings of the tubes being reduced by the thickness of the tube hoops. The diameter of each chimney, of which there are two, is 3 feet 5 inches.

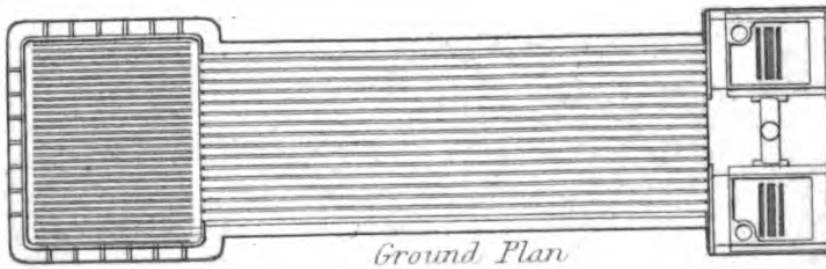
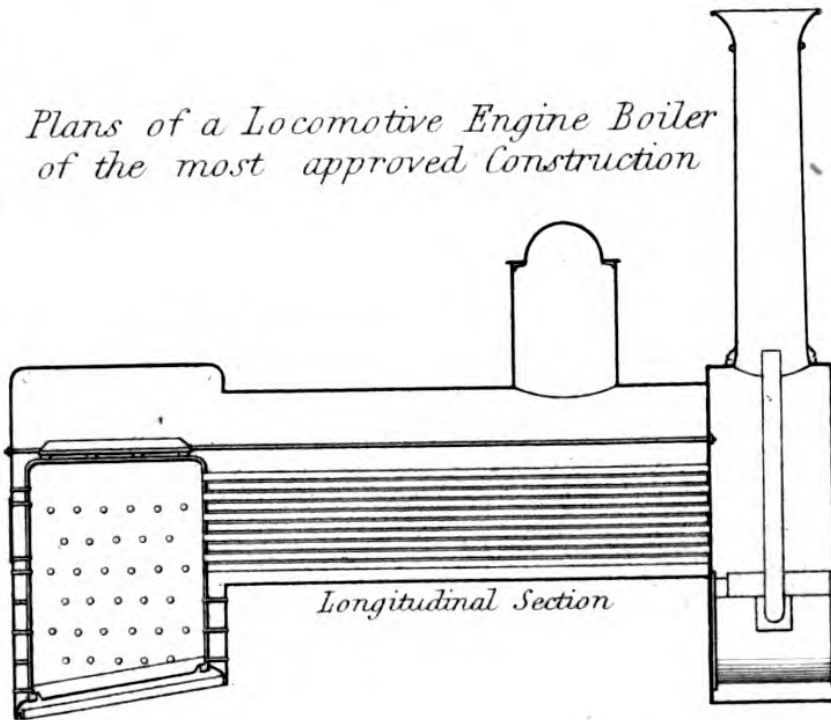
Locomotive Boilers, in the manner of their construction, are a class which, of necessity, demand the utmost degree of qualified attention, because of the great pressures they are required to sustain, and the unavoidably narrow limits in which such vast volumes of steam are required to be generated.

In boilers of this description, it is of the most essential consequence that the water spaces in the fire-box be of a sufficient width, say, on the back and sides, not less than $2\frac{3}{4}$ to 3 inches, and in the front, where the tendency of the fire is urged by the action of the blast-pipe, at least $3\frac{1}{2}$ inches.

The boiler which I have selected for illustration (see Plate L,) is of the most modern description; and from an intimate knowledge of its performances, I am enabled to assert, with full confidence, that it possesses efficient and economical properties to the production of steam.



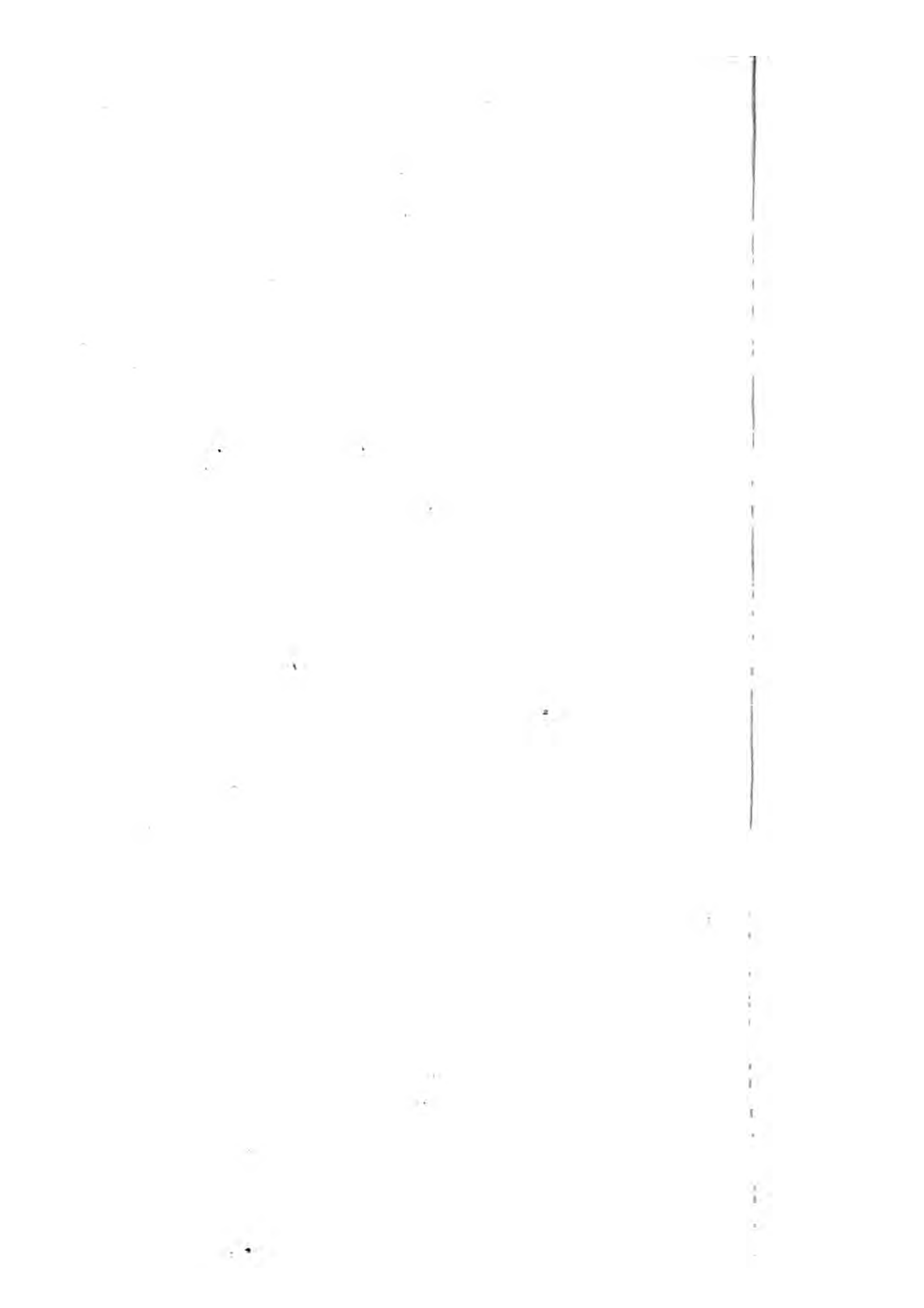
*Plans of a Locomotive Engine Boiler
of the most approved Construction*



W. T., del.

G. Gladwin, sc.

London: J. Weale, 1845



Specified particulars relative to the Boiler and Engine.

Diameter of cylinders	14 inches.
Length of stroke	18 "
Lap of the valve	1 "
Diameter of driving wheels	5½ feet.
Length of internal fire-box	2 feet 11½ inches.
Width of do.	3 " 5 "
Length of cylindrical part of boiler	8 " 8 "
Diameter of do.	3 " 4½ "
Length of tubes	8 " 11½ "
Number of tubes	133, of brass.
Interior diameter of do.	1¾ inches.
Diameter of blast-pipe	4 "

About 112 lbs. of coke consumed in this boiler evaporate 84 gallons of water; and from 20 to 25 lbs. of coke are consumed per mile.

Heating Powers of Combustible Substances.

Species of combustible.	lbs. of water heated from 32° to 212°.	lbs. of boiling water evaporated by 1 lb. of fuel.	lbs. of atmospheric air to each lb. of fuel.
Wood in its ordinary state ..	26	4.72	4.47
Wood charcoal	73	13.37	11.46
Coal	60	10.90	9.26
Coke	65	11.81	11.46
Turf	30	5.45	4.60
Turf charcoal	64	11.63	9.86

In regard to the giving of an order for a steam engine of a required number of horses' power, it has lately been argued and ultimately decided, that in a commercial point of view the order is not sufficiently completed by the dimensions of cylinder, boiler, &c., being of ample magnitude to produce the specified dynamical effect in horses' power; and not unless corresponding with the established custom by Boulton and Watt, or that of other manufacturers of equally well known respectability: hence, generally, the fol-

Following Tables may be better adapted to practice than even the most simple rule.

Table of Dimensions for Steam Engine Cylinders by celebrated makers.

Stationary Condensing Engines, by Boulton & Watt.			Marine Engines, by Maudsley, Napier, &c.			High-Pressure, or Non-Condensing Engines, by various makers.				
Nominal horses' power.	Diameter of cylinders in inches.	Lengths of strokes in feet.	Nominal horses' power.	Diameter of cylinders in inches.	Lengths of strokes in feet.	Nominal horses' power.	Diameters of cylinders, the force of the steam being, per square inch, 25 lbs. 30 lbs. 40 lbs. 50 lbs.			
6	14½	3	10	20	2	1	3¾	3½	3	2¾
8	16¼	3	15	24	2¼	2	5¼	4¾	4¼	3¾
10	18	3½	20	27	2½	3	6½	6	5	4½
12	19½	4	25	29½	2¾	4	7½	6¾	6	5¼
14	21	4½	30	32	3	5	8¼	7½	6½	5½
16	22½	4½	40	36	3½	6	9	8¼	7¼	6½
18	23½	5	50	40	4	7	9¾	9	7¾	6¾
20	24½	5	60	43	4	8	10½	9¾	8½	7½
22	26	5	70	46½	4½	9	11½	10¼	8¾	7¾
24	27	5½	80	47½	4½	10	11¾	11	9½	8½
25	27½	5½	90	50	4¾	11	12¼	11¾	9¾	8¾
26	28	5¾	100	53	5	12	13	12	10½	9¼
28	29	6	110	55½	5½	14	14	12¾	11¼	10
30	30	6	120	57	5½	16	15	13¾	12	10½
35	32½	6½	130	60¾	5¾	18	15¾	14½	12¾	11¼
40	34½	6½	150	65	6	20	16¾	15¼	13½	11¾
50	38½	7	200	74½	6	25	18½	17¼	15	13¼
60	42¼	7	250	84	6	30	20¼	19¾	16¼	14½

The *unit* of nominal power for steam engines, or the usual estimate of dynamical effect per minute of a horse, called by engineers a horse-power, is 33,000 lbs. at a velocity of 1 foot per minute; or, the effect of a load of 200 lbs. raised by a horse for 8 hours a day, at the rate of 2½ miles per hour, or 150 lbs. at the rate of 220 feet per minute.

To estimate by means of an indicator the amount of effective power produced by a steam engine.

Rule.—Multiply the area of the piston in square inches by the average force of the steam in lbs. and by the velocity of the piston in feet per minute ; divide the product by 33,000, and $\frac{7}{10}$ ths of the quotient equal the effective power.

Ex. Suppose an engine with a cylinder of $37\frac{1}{2}$ inches diameter, a stroke of 7 feet, and making 17 revolutions per minute, or 238 feet velocity, and the average indicated pressure of the steam 16.73 lbs. per square inch ; required the effective power.

$$\begin{aligned} \text{Area} &= 1104.4687 \text{ inches} \times 16.73 \text{ lbs.} \times 238 \text{ feet} \\ &\quad \underline{\hspace{10em} 33000} \\ &= \frac{133.26 \times 7}{10} 93.282 \text{ horses' power.} \end{aligned}$$

To determine the proper velocity for the piston of a steam engine.

Rule.—Multiply the logarithm of the *n*th part of the stroke at which the steam is cut off by 2.3, and to the product of which add .7. Multiply the sum by the distance in feet the piston has travelled when the steam is cut off, and 120 times the square root of the product equal the proper velocity for the piston in feet per minute.

Ex. Let the steam be cut off in an 8-foot stroke when the piston has travelled $\frac{1}{4}$ th of the length ; required its proper velocity.

$$\begin{array}{r} \text{Logarithm of } 4 = 0.60206 \\ \text{Multiplied by} \quad 2.3 \\ \hline 1.384738 \\ \text{To which add} \quad .7 \\ \hline 2.084738 \\ \hline 2 \end{array}$$

$$\sqrt{4.169476} = 2.04 \times 120 = 245 \text{ feet, velocity per minute.}$$

Table of Approximate Velocities for the Pistons of Steam Engines.

Condensing Engines.			Non-Condensing Engines.		
Length of stroke in feet.	Velocity in feet per minute.	Number of revolutions per minute.	Length of stroke in feet.	Velocity in feet per minute.	Number of revolutions per minute.
2	160	40	1½	186	62
2½	177½	35½	2	200	50
3	192	32	2½	212½	42½
3½	203	29	2¾	217¼	39½
4	214	26¾	3	222	37
4½	220½	24½	3½	231	33
5	230	23	4	236	29½
5½	236½	21½	4½	243	27
6	240	20	5	247½	24¾
7	245	17½	5½	253	23
8	256	16	6	264	22

Of the Parallel Motion in a Steam Engine.

When the power from the piston is communicated by means of a beam or lever moving upon an axis, the parallel motion becomes a very important portion of the machine, for then it forms the link of connexion, and by its properties renders the action of alternate circular motion, and reciprocating vertical motion, mutually agreeable, thereby properly ensuring to the piston rod a truly direct line to that of the cylinder; but to effect this, the greatest degree of exactitude of the various parts is required, otherwise extra friction is created, and the effective power of the engine proportionately diminished. (See Plates E and F.)

Fig. 1, Plate E, is a motion for a double cylinder engine.

„ 2, „ a motion for a pumping engine.

„ 3, „ the common condensing engine.

Fig. 1, Plate F, is for a marine engine.

„ 2, „ also for a marine engine.

„ 3, „ for a non-condensing or high-pressure engine.

PARALLEL MOTIONS.

Plate
E

Fig. 1.

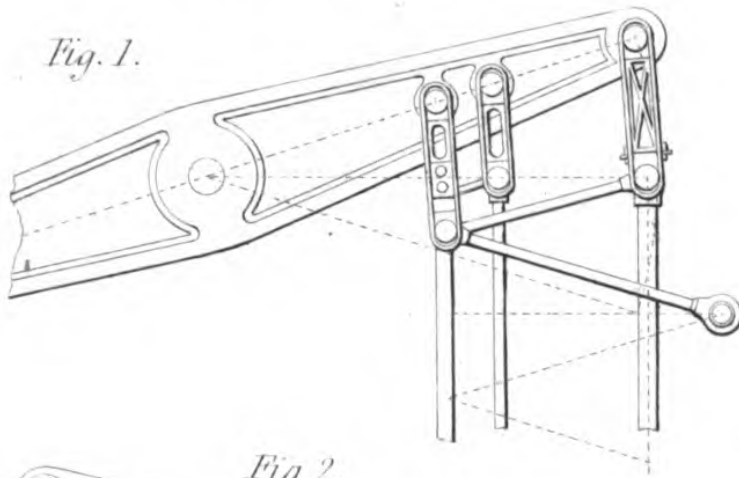


Fig. 2.

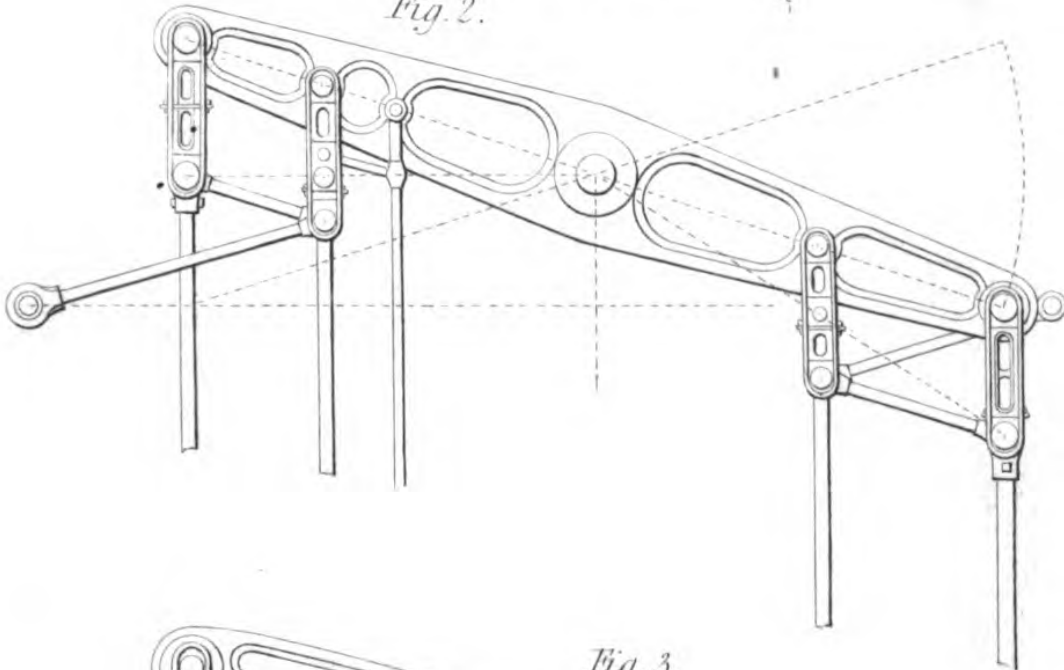
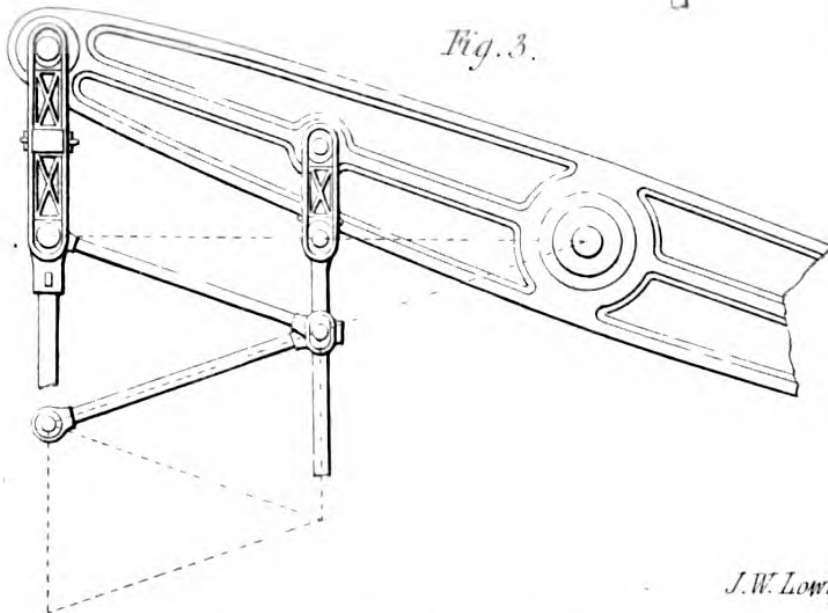


Fig. 3.



J. W. Lowry sc.

John Weale, 59, High Holborn 1844.

Table by which to determine the various Distances of the Moveable Points in a Parallel Motion.

Radius of beams in ft.	Parallel bars in ft.	Length of radius rods in ft. & in.	Radius of beams in ft.	Parallel bars in ft.	Length of radius rods in ft. & in.	Radius of beams in ft.	Parallel bars in ft.	Length of radius rods in ft. & in.	Radius of beams in ft.	Parallel bars in ft.	Length of radius rods in ft. & in.
4 feet	2	2 0	6½ feet	3	4 1⅛	8½ feet	3¾	6 0¼	10½ feet	5	6 0½
	2¼	1 4⅜		3¼	3 3		4	5 6¾			
	2½	0 10⅞		3½	2 6⅞		4¼	4 3			
	2¾	0 6⅞		3¾	2 0¼		4½	3 6⅝			
	3	0 4		4	1 6¾		4¾	2 11½			
4½ feet	2	3 1½	7 feet	4¼	1 2¼	9 feet	5	2 5⅝	11 feet	6¼	2 10¾
	2¼	2 3		4½	0 10½		5¼	2 0⅞		6½	2 5⅜
	2½	1 7¼		3	5 4		5½	1 7⅝		6¾	2 1
	2¾	1 1⅜		3¼	4 4		4	6 3		5¼	6 3½
	3	0 9		3½	3 6		4¼	5 3¾		5½	5 6
5 feet	3¼	0 5¾	7½ feet	3¾	3 2⅞	9½ feet	4½	4 6	11½ feet	5¾	4 9½
	2	4 6		4	2 3		4¾	3 9⅝		6	4 2
	2¼	3 4⅜		4¼	1 9		5	3 2½		6¼	3 7⅜
	2½	2 6		4½	1 4⅝		5¼	2 8⅞		6½	3 1⅜
	2¾	1 10⅞		4¾	1 0⅞		5½	2 2¾		6¾	2 8⅞
5½ feet	3	1 4	8 feet	5	0 9⅝	10 feet	5¾	1 10	12 feet	5½	6 6½
	3¼	0 11¼		3½	4 6⅞		6	1 6		5¾	5 9
	3½	0 7¾		3¾	3 9		4½	5 6⅝		6	5 0½
	2¼	4 8⅜		4	3 0¾		4¾	4 9		6¼	4 5
	2½	3 7¼		4¼	2 6		5	4 0⅝		6½	3 10⅞
6 feet	2¾	2 9	8½ feet	4½	2 0	9¾ feet	5¼	3 5⅝	11¾ feet	6¾	3 4
	3	1 6⅜		5	1 7		5½	2 10⅞		7	2 10⅝
	3¼	1 1⅜		5¼	0 11½		5¾	2 5¼		7¼	2 6
	3½	1 1¼		3½	5 9½		6	2 0½		5¾	6 8¼
	3¾	0 9⅞		4	4 9¾		5	5 0		6	6 0
6½ feet	3	3 0	9 feet	4	4 0	10½ feet	5¼	4 3½	12½ feet	6¼	5 3½
	3¼	2 3⅞		4¼	3 3¾		5½	3 8⅞		6½	4 7⅞
	3½	1 9½		4½	2 8⅝		5¾	3 1¾		6¾	4 1
	3¾	1 4¼		4¾	2 2¾		6	2 8		7	3 6⅞
	4	1 0		5	1 9⅝		6¼	2 3		7¼	3 1⅜
7 feet	4¼	0 8½	9½ feet	5¼	1 5¼	10¾ feet	6½	1 10½	13 feet	7½	2 8½
	2¾	3 10		3½	5 9½		4¾	5 9½		5¾	6 8¼
	3	3 0		4	4 9¾		5	5 0		6	6 0
	3¼	2 3⅞		4¼	3 3¾		5¼	4 3½		6¼	5 3½
	3½	1 9½		4½	2 8⅝		5½	3 8⅞		6½	4 7⅞

LOGARITHMS.

LOGARITHMS literally signify *ratios of numbers*; hence Logarithmic Tables may be various, but those in common use for the facilitating of arithmetical operations generally are of the following corresponding progressions, viz.,

Arithmetical, 0, 1, 2, 3, &c., or series of logarithms.

Geometrical, 1, 10, 100, 1000, &c., or ratio of numbers.

And thus it may be perceived, that if the log. of 10 be 1, the log. of any number less than 10 must consist wholly of decimals, because increasing by a decimal ratio. Again, if the log. of 100 be 2, the log. of any intermediate number between 10 and 100 must be 1, with so many decimals annexed; and in like manner, the log. of any intermediate number between 100 and 1000, must be 2, with decimals annexed proportionally, as before.

APPLICATION AND UTILITY OF COMMON LOGARITHMIC TABLES.

The whole numbers of the series of logarithms, as 1, 2, 3, &c., are called the indices, or characteristics of the logarithm, and which must be added to the logarithm obtained by the Table, in proportion to the number of figures contained in the given sum. Thus, suppose the logarithm be required for a sum of only two figures, the index is 1; if of three figures, the index is 2; and if of four figures, the index is 3, &c., being always a number less by unity than the number of figures the given sum contains.

Ex. The index of 8 is 0, because it is less than 10.
 The index of 80 is 1, because it is less than 100.
 The index of 800 is 2, because it is less than 1000.
 The index of 8000 is 3, because it is less than 10,000, &c.

The index of a decimal is always the number which denotes the significant figure from the decimal point, and is marked with the sign thus —, to distinguish it from a whole number.

Ex. The index of $\cdot 32549$ is -1 , because the first significant figure is the first decimal.

The index of $\cdot 032549$ is -2 , because the first significant figure is the second decimal.

The index of $\cdot 0032549$ is -3 , because the first significant figure is the third decimal, &c. of any other sum.

If the given sum for which the logarithm is required contains or consists of both integers and decimals, the index is determined by the integer part, without having any regard to the other.

1. To find the logarithm of any whole number under 100.

Look for the number under N in the first page of any Logarithmic Table; then immediately on the right of it is the logarithm required, with its proper index. Thus, the log. of 64 is 1.806180, and the log. of 72 is 1.857332.

2. To find the logarithm of any number between 100 and 1000, or any sum not exceeding 4 figures.

Find the first three figures in the left-hand column of the page under N, in which the number is situated, and the fourth figure, at the top or bottom of the page; then the logarithm directly under the fourth

figure, and in a line with the three figures in the column on the left, with its proper index, is the logarithm required. Thus, the log. of 450 is 2·653213, and the log. of 7464 is 3·872972. Or, the log. of 378·5 is 2·578066, and that of ·7854 is —1·895091.

3. *To find the number indicated by a given logarithm.*

Look for the decimal part of the given logarithm in the different columns, and if it cannot be found exactly, take the next less. Then under N in the left-hand column, and in a line with the logarithm found, are three figures of the number required, and on the top of the column in which the found logarithm stands is one figure more; place the decimal point as indicated by the logarithmic index, which determines the sum, properly valued, as required.

If the logarithm cannot be found exactly in the Tables, subtract from it the next less that can be found, and divide the remainder by the tabular difference; the quotient will be the rest of the figures of the given number, which, being annexed to the tabular number already found, is the proper number required.

Ex. Required the number answering to the logarithm 3·233568.

$$\begin{array}{r}
 \text{Given logarithm} = 3\cdot233568 \\
 \text{Next less is the log. of 1712} = 3\cdot233504 \\
 \hline
 \text{Remainder} \qquad \qquad \qquad 64 \\
 \text{Tab. Diff.} = 253, \text{ and } \frac{64}{253} = \cdot25 \\
 \text{Hence the number required} = 1712\cdot25.
 \end{array}$$

For practical purposes in mechanics, logarithms are seldom resorted to, unless for the raising of the powers of numbers or extraction of their roots: these

operations, when Tables are at hand, they very much facilitate; involution, or the raising of powers, being performed simply by multiplication, and evolution, or the extraction of roots, by division, as in simple arithmetic.

Ex. 1. Required the square or second power of 25·791.

$$\begin{array}{r} \text{Log. of } 25\cdot791 = 1\cdot411468 \\ \text{Multiplied by} \quad \quad \quad \underline{\quad\quad\quad} 2 \text{ the power required.} \end{array}$$

Logarithm $\underline{2\cdot822936}$ indicated number or square required = 665·175.

Ex. 2. What is the cube of 30·7146 ?

$$\begin{array}{r} \text{Logarithm} \quad = 1\cdot487345 \\ \text{Multiplied by} \quad \quad \quad \underline{\quad\quad\quad} 3 \text{ the power required.} \end{array}$$

Logarithm $\underline{4\cdot462035}$ indicated number or cube required = 28975·7.

Ex. 3. Required the square root of 365.

$$\begin{array}{r} \text{Log.} = \frac{2\cdot562293}{2} = 1\cdot281146 \text{ indicated number or root} \\ = 19\cdot105. \end{array}$$

Ex. 4. Find the cube root of 12345.

$$\begin{array}{r} \text{Log.} = \frac{4\cdot091491}{3} = 1\cdot363830 \text{ indicated number or root} \\ = 23\cdot1116. \end{array}$$

Table of Logarithms from 1 to 100.

N.	Log.	N.	Log.	N.	Log.	N.	Log.
1	0.000000	26	1.414973	51	1.707570	76	1.880814
2	0.301030	27	1.431364	52	1.716003	77	1.886491
3	0.477121	28	1.447158	53	1.724276	78	1.892095
4	0.602060	29	1.462398	54	1.732394	79	1.897627
5	0.698970	30	1.477121	55	1.740363	80	1.903090
6	0.778151	31	1.491362	56	1.748188	81	1.908485
7	0.845098	32	1.505150	57	1.755875	82	1.913814
8	0.903090	33	1.518514	58	1.763428	83	1.919078
9	0.954243	34	1.531479	59	1.770852	84	1.924279
10	1.000000	35	1.544068	60	1.778151	85	1.929419
11	1.041393	36	1.556303	61	1.785330	86	1.934498
12	1.079181	37	1.568202	62	1.792392	87	1.939519
13	1.113943	38	1.579784	63	1.799341	88	1.944483
14	1.146128	39	1.591065	64	1.806180	89	1.949390
15	1.176091	40	1.602060	65	1.812913	90	1.954243
16	1.204120	41	1.612784	66	1.819544	91	1.959041
17	1.230449	42	1.623249	67	1.826075	92	1.963788
18	1.255273	43	1.633468	68	1.832509	93	1.968483
19	1.278754	44	1.643453	69	1.838849	94	1.973128
20	1.301030	45	1.653213	70	1.845098	95	1.977724
21	1.322219	46	1.662758	71	1.851258	96	1.982271
22	1.342423	47	1.672098	72	1.857332	97	1.986772
23	1.361728	48	1.681241	73	1.863323	98	1.991226
24	1.380211	49	1.690196	74	1.869232	99	1.995635
25	1.397940	50	1.698970	75	1.875061	100	2.000000

Note.—The best Tables of Logarithms are those by Taylor, Gardiner, Hutton, Babbage, and Caillet. The smaller works are those by Lalande, Hassler, Renaud, Christison, and Wallace, and those published in the 'Library of Useful Knowledge.'

Tables of the Circumferences of Circles to the nearest fraction of practical measurement; also the Areas of Circles in inches and decimal parts, likewise of feet and decimal parts, as may be required.

Rules that may render the following Tables more generally useful.

1. Any of the areas in inches multiplied by $\cdot 04328$, or the areas in feet multiplied by $6\cdot 232$, the product is the number of imperial gallons at 1 foot in depth.
2. Any of the areas in feet multiplied by $\cdot 03704$, the product equal the number of cubic yards at 1 foot in depth.
3. The area of a circle in inches multiplied by the length or thickness in inches, and by $\cdot 263$, the product equal the weight in lbs. of cast iron.

Note.—The French cubic metre, or unit of solid measure, equal $35\cdot 31716$ English cubic feet. Also the litre, or unit for measures of capacity, equal $61\cdot 028$ English cubic inches, or about $\cdot 453$ of an imperial gallon.

Dia. in inch.	Circum. in inch.	Area in sq. inch.	Side of =sq.
$\frac{1}{16}$	$\cdot 196$	$\cdot 0030$	$\cdot 0554$
$\frac{1}{8}$	$\cdot 392$	$\cdot 0122$	$\cdot 1107$
$\frac{3}{16}$	$\cdot 589$	$\cdot 0276$	$\cdot 1661$
$\frac{1}{4}$	$\cdot 785$	$\cdot 0490$	$\cdot 2115$
$\frac{5}{16}$	$\cdot 981$	$\cdot 0767$	$\cdot 2669$
$\frac{3}{8}$	$1\cdot 178$	$\cdot 1104$	$\cdot 3223$
$\frac{7}{16}$	$1\cdot 374$	$\cdot 1503$	$\cdot 3771$
$\frac{1}{2}$	$1\cdot 570$	$\cdot 1963$	$\cdot 4331$
$\frac{9}{16}$	$1\cdot 767$	$\cdot 2485$	$\cdot 4995$
$\frac{5}{8}$	$1\cdot 963$	$\cdot 3068$	$\cdot 5438$
$\frac{11}{16}$	$2\cdot 159$	$\cdot 3712$	$\cdot 6093$
$\frac{3}{4}$	$2\cdot 356$	$\cdot 4417$	$\cdot 6646$
$\frac{13}{16}$	$2\cdot 552$	$\cdot 5185$	$\cdot 7200$
$\frac{7}{8}$	$2\cdot 748$	$\cdot 6013$	$\cdot 7754$
$\frac{15}{16}$	$2\cdot 945$	$\cdot 6903$	$\cdot 8308$
1 in.	$3\frac{1}{8}$	$\cdot 7854$	$\frac{7}{8}$
$1\frac{1}{16}$	$3\frac{1}{2}$	$\cdot 9940$	$\frac{7}{8}$ & $\frac{3}{32}$
$1\frac{1}{4}$	$3\frac{7}{8}$	$1\cdot 227$	1 in.
$1\frac{3}{8}$	$4\frac{1}{4}$	$1\cdot 484$	$1\frac{3}{16}$
$1\frac{1}{2}$	$4\frac{3}{4}$	$1\cdot 767$	$1\frac{5}{16}$
$1\frac{5}{8}$	$5\frac{1}{8}$	$2\cdot 074$	$1\frac{7}{16}$
$1\frac{3}{4}$	$5\frac{1}{2}$	$2\cdot 405$	$1\frac{9}{16}$
$1\frac{7}{8}$	$5\frac{7}{8}$	$2\cdot 761$	$1\frac{11}{16}$
2 in.	$6\frac{1}{4}$	$3\cdot 141$	$1\frac{3}{4}$
$2\frac{1}{8}$	$6\frac{5}{8}$	$3\cdot 546$	$1\frac{7}{8}$
$2\frac{1}{4}$	7	$3\cdot 976$	2 in.
$2\frac{3}{8}$	$7\frac{3}{8}$	$4\cdot 430$	$2\frac{1}{8}$
$2\frac{1}{2}$	$7\frac{1}{2}$	$4\cdot 908$	$2\frac{3}{16}$
$2\frac{5}{8}$	$8\frac{1}{4}$	$5\cdot 412$	$2\frac{5}{16}$
$2\frac{3}{4}$	$8\frac{3}{4}$	$5\cdot 939$	$2\frac{7}{16}$
$2\frac{7}{8}$	9	$6\cdot 491$	$2\frac{9}{16}$
3 in.	$9\frac{3}{8}$	$7\cdot 068$	$2\frac{5}{8}$
$3\frac{1}{8}$	$9\frac{7}{8}$	$7\cdot 669$	$2\frac{3}{4}$
$3\frac{1}{4}$	$10\frac{1}{4}$	$8\cdot 295$	$2\frac{7}{8}$
$3\frac{3}{8}$	$10\frac{3}{8}$	$8\cdot 946$	3 in.
$3\frac{1}{2}$	11	$9\cdot 621$	$3\frac{1}{8}$
$3\frac{5}{8}$	$11\frac{3}{8}$	$10\cdot 320$	$3\frac{1}{4}$
$3\frac{3}{4}$	$11\frac{3}{4}$	$11\cdot 044$	$3\frac{3}{8}$
$3\frac{7}{8}$	$12\frac{1}{8}$	$11\cdot 793$	$3\frac{7}{16}$

CIRCUMFERENCES AND

Dia. in inch.	Circum. in ft. in.	Area in sq. inch.	Area in sq. feet.	Dia. in inch.	Circum. in ft. in.	Area in sq. inch.	Area in sq. feet.
4 in.	1 0 $\frac{1}{2}$	12.566	.0879	9 in.	2 4 $\frac{1}{4}$	63.617	.4453
4 $\frac{1}{8}$	1 0 $\frac{7}{8}$	13.364	.0935	9 $\frac{1}{8}$	2 4 $\frac{5}{8}$	65.396	.4577
4 $\frac{1}{4}$	1 1 $\frac{3}{8}$	14.186	.0993	9 $\frac{1}{4}$	2 5	67.200	.4704
4 $\frac{3}{8}$	1 1 $\frac{3}{4}$	15.033	.1052	9 $\frac{3}{8}$	2 5 $\frac{3}{8}$	69.029	.4832
4 $\frac{1}{2}$	1 2 $\frac{1}{8}$	15.904	.1113	9 $\frac{1}{2}$	2 5 $\frac{3}{4}$	70.882	.4961
4 $\frac{5}{8}$	1 2 $\frac{1}{2}$	16.800	.1176	9 $\frac{5}{8}$	2 6 $\frac{1}{4}$	72.759	.5093
4 $\frac{3}{4}$	1 2 $\frac{7}{8}$	17.720	.1240	9 $\frac{3}{4}$	2 6 $\frac{3}{8}$	74.662	.5226
4 $\frac{7}{8}$	1 3 $\frac{1}{4}$	18.665	.1306	9 $\frac{7}{8}$	2 7	76.588	.5361
5 in.	1 3 $\frac{5}{8}$	19.635	.1374	10 in.	2 7 $\frac{3}{8}$	78.540	.5497
5 $\frac{1}{8}$	1 4 $\frac{1}{8}$	20.629	.1444	10 $\frac{1}{8}$	2 7 $\frac{3}{4}$	80.515	.5636
5 $\frac{1}{4}$	1 4 $\frac{1}{2}$	21.647	.1515	10 $\frac{1}{4}$	2 8 $\frac{1}{8}$	82.516	.5776
5 $\frac{3}{8}$	1 4 $\frac{7}{8}$	22.690	.1588	10 $\frac{3}{8}$	2 8 $\frac{1}{2}$	84.540	.5917
5 $\frac{1}{2}$	1 5 $\frac{1}{4}$	23.758	.1663	10 $\frac{1}{2}$	2 8 $\frac{7}{8}$	86.590	.6061
5 $\frac{5}{8}$	1 5 $\frac{5}{8}$	24.850	.1739	10 $\frac{5}{8}$	2 9 $\frac{3}{8}$	88.664	.6206
5 $\frac{3}{4}$	1 6	25.967	.1817	10 $\frac{3}{4}$	2 9 $\frac{3}{4}$	90.762	.6353
5 $\frac{7}{8}$	1 6 $\frac{3}{8}$	27.108	.1897	10 $\frac{7}{8}$	2 10 $\frac{1}{8}$	92.855	.6499
6 in.	1 6 $\frac{3}{4}$	28.274	.1979	11 in.	2 10 $\frac{1}{2}$	95.033	.6652
6 $\frac{1}{8}$	1 7 $\frac{1}{4}$	29.464	.2062	11 $\frac{1}{8}$	2 10 $\frac{7}{8}$	97.205	.6804
6 $\frac{1}{4}$	1 7 $\frac{5}{8}$	30.679	.2147	11 $\frac{1}{4}$	2 11 $\frac{1}{4}$	99.402	.6958
6 $\frac{3}{8}$	1 8	31.919	.2234	11 $\frac{3}{8}$	2 11 $\frac{3}{4}$	101.623	.7143
6 $\frac{1}{2}$	1 8 $\frac{3}{8}$	33.183	.2322	11 $\frac{1}{2}$	3 0 $\frac{1}{8}$	103.869	.7270
6 $\frac{5}{8}$	1 8 $\frac{3}{4}$	34.471	.2412	11 $\frac{5}{8}$	3 0 $\frac{1}{2}$	106.139	.7429
6 $\frac{3}{4}$	1 9 $\frac{1}{8}$	35.784	.2504	11 $\frac{3}{4}$	3 0 $\frac{7}{8}$	108.434	.7590
6 $\frac{7}{8}$	1 9 $\frac{1}{2}$	37.122	.2598	11 $\frac{7}{8}$	3 1 $\frac{1}{4}$	110.753	.7752
7 in.	1 10	38.484	.2693	12 in.	3 1 $\frac{5}{8}$	113.097	.7916
7 $\frac{1}{8}$	1 10 $\frac{3}{8}$	39.871	.2791	12 $\frac{1}{8}$	3 2	115.466	.8082
7 $\frac{1}{4}$	1 10 $\frac{3}{4}$	41.282	.2889	12 $\frac{1}{4}$	3 2 $\frac{1}{2}$	117.859	.8250
7 $\frac{3}{8}$	1 11 $\frac{1}{8}$	42.718	.2990	12 $\frac{3}{8}$	3 2 $\frac{7}{8}$	120.276	.8419
7 $\frac{1}{2}$	1 11 $\frac{1}{2}$	44.178	.3092	12 $\frac{1}{2}$	3 3 $\frac{1}{4}$	122.718	.8590
7 $\frac{5}{8}$	1 11 $\frac{7}{8}$	45.663	.3196	12 $\frac{5}{8}$	3 3 $\frac{5}{8}$	125.185	.8762
7 $\frac{3}{4}$	2 0 $\frac{3}{8}$	47.173	.3299	12 $\frac{3}{4}$	3 4	127.676	.8937
7 $\frac{7}{8}$	2 0 $\frac{3}{4}$	48.707	.3409	12 $\frac{7}{8}$	3 4 $\frac{3}{8}$	130.192	.9113
8 in.	2 1 $\frac{1}{8}$	50.265	.3518	13 in.	3 4 $\frac{3}{4}$	132.732	.9291
8 $\frac{1}{8}$	2 1 $\frac{1}{2}$	51.848	.3629	13 $\frac{1}{8}$	3 5 $\frac{1}{4}$	135.297	.9470
8 $\frac{1}{4}$	2 1 $\frac{7}{8}$	53.456	.3741	13 $\frac{1}{4}$	3 5 $\frac{5}{8}$	137.886	.9642
8 $\frac{3}{8}$	2 2 $\frac{1}{4}$	55.088	.3856	13 $\frac{3}{8}$	3 6	140.500	.9835
8 $\frac{1}{2}$	2 2 $\frac{5}{8}$	56.745	.3972	13 $\frac{1}{2}$	3 6 $\frac{3}{8}$	143.139	1.0019
8 $\frac{5}{8}$	2 3	58.426	.4089	13 $\frac{5}{8}$	3 6 $\frac{3}{4}$	145.802	1.0206
8 $\frac{3}{4}$	2 3 $\frac{3}{8}$	60.132	.4209	13 $\frac{3}{4}$	3 7 $\frac{1}{8}$	148.489	1.0294
8 $\frac{7}{8}$	2 3 $\frac{7}{8}$	61.862	.4330	13 $\frac{7}{8}$	3 7 $\frac{1}{2}$	151.201	1.0584

AREAS OF CIRCLES.

Dia. in inch.	Circum. in ft. in.	Area in sq. inch.	Area in sq. feet.	Dia. in inch.	Circum. in ft. in.	Area in sq. inch.	Area in sq. feet.
14	3 7 ⁷ / ₈	153.938	1.0775	19	4 11 ⁵ / ₈	283.529	1.9847
14 ¹ / ₈	3 8 ³ / ₈	156.699	1.0968	19 ¹ / ₈	5 0	287.272	1.9941
14 ¹ / ₄	3 8 ³ / ₄	159.485	1.1193	19 ¹ / ₄	5 0 ¹ / ₂	291.039	2.0371
14 ³ / ₈	3 9 ¹ / ₈	162.295	1.1360	19 ³ / ₈	5 0 ⁷ / ₈	294.831	2.0637
14 ¹ / ₂	3 9 ¹ / ₂	165.130	1.1569	19 ¹ / ₂	5 1 ¹ / ₄	298.648	2.0904
14 ⁵ / ₈	3 9 ⁷ / ₈	167.989	1.1749	19 ⁵ / ₈	5 1 ⁵ / ₈	302.489	2.1172
14 ³ / ₄	3 10 ¹ / ₄	170.873	1.1961	19 ³ / ₄	5 2	306.355	2.1443
14 ⁷ / ₈	3 10 ³ / ₄	173.782	1.2164	19 ⁷ / ₈	5 2 ³ / ₈	310.245	2.1716
15	3 11 ¹ / ₈	176.715	1.2370	20	5 2 ⁷ / ₈	314.160	2.1990
15 ¹ / ₈	3 11 ¹ / ₂	179.672	1.2577	20 ¹ / ₈	5 3 ¹ / ₄	318.099	2.2265
15 ¹ / ₄	3 11 ⁷ / ₈	182.654	1.2785	20 ¹ / ₄	5 3 ⁵ / ₈	322.063	2.2543
15 ³ / ₈	4 0 ¹ / ₄	185.661	1.2996	20 ³ / ₈	5 4	326.051	2.2822
15 ¹ / ₂	4 0 ⁵ / ₈	188.692	1.3208	20 ¹ / ₂	5 4 ³ / ₈	330.064	2.3103
15 ⁵ / ₈	4 1	191.748	1.3422	20 ⁵ / ₈	5 4 ⁷ / ₈	334.101	2.3386
15 ³ / ₄	4 1 ¹ / ₂	194.828	1.3637	20 ³ / ₄	5 5 ¹ / ₈	338.163	2.3670
15 ⁷ / ₈	4 1 ⁷ / ₈	197.933	1.3855	20 ⁷ / ₈	5 5 ¹ / ₂	342.250	2.3956
16	4 2 ¹ / ₄	201.062	1.4074	21	5 5 ⁷ / ₈	346.361	2.4244
16 ¹ / ₈	4 2 ⁵ / ₈	204.216	1.4295	21 ¹ / ₈	5 6 ³ / ₈	350.497	2.4533
16 ¹ / ₄	4 3	207.394	1.4517	21 ¹ / ₄	5 6 ⁷ / ₈	354.657	2.4824
16 ³ / ₈	4 3 ³ / ₈	210.597	1.4741	21 ³ / ₈	5 7 ¹ / ₈	358.841	2.5117
16 ¹ / ₂	4 3 ⁷ / ₈	213.825	1.4967	21 ¹ / ₂	5 7 ¹ / ₂	363.051	2.5412
16 ⁵ / ₈	4 4 ¹ / ₄	217.077	1.5195	21 ⁵ / ₈	5 7 ⁷ / ₈	367.284	2.5708
16 ³ / ₄	4 4 ⁵ / ₈	220.353	1.5424	21 ³ / ₄	5 8 ¹ / ₄	371.543	2.6007
16 ⁷ / ₈	4 5	223.654	1.5655	21 ⁷ / ₈	5 8 ⁵ / ₈	375.826	2.6306
17	4 5 ³ / ₈	226.980	1.5888	22	5 9 ¹ / ₈	380.133	2.6608
17 ¹ / ₈	4 5 ⁷ / ₈	230.330	1.6123	22 ¹ / ₈	5 9 ¹ / ₂	384.465	2.6691
17 ¹ / ₄	4 6 ¹ / ₈	233.705	1.6359	22 ¹ / ₄	5 9 ⁷ / ₈	388.822	2.7016
17 ³ / ₈	4 6 ¹ / ₂	237.104	1.6597	22 ³ / ₈	5 10 ¹ / ₄	393.203	2.7224
17 ¹ / ₂	4 6 ⁷ / ₈	240.528	1.6836	22 ¹ / ₂	5 10 ⁵ / ₈	397.608	2.7632
17 ⁵ / ₈	4 7 ¹ / ₈	243.977	1.7078	22 ⁵ / ₈	5 11	402.038	2.7980
17 ³ / ₄	4 7 ⁵ / ₈	247.450	1.7321	22 ³ / ₄	5 11 ¹ / ₂	406.493	2.8054
17 ⁷ / ₈	4 8 ¹ / ₈	250.947	1.7566	22 ⁷ / ₈	5 11 ⁷ / ₈	410.972	2.8658
18	4 8 ¹ / ₂	254.469	1.7812	23	6 0 ¹ / ₄	415.476	2.8903
18 ¹ / ₈	4 8 ⁷ / ₈	258.016	1.8061	23 ¹ / ₈	6 0 ⁵ / ₈	420.004	2.9100
18 ¹ / ₄	4 9 ¹ / ₄	261.587	1.8311	23 ¹ / ₄	6 1	424.557	2.9518
18 ³ / ₈	4 9 ³ / ₄	265.182	1.8562	23 ³ / ₈	6 1 ³ / ₈	429.135	2.9937
18 ¹ / ₂	4 10 ¹ / ₈	268.803	1.8816	23 ¹ / ₂	6 1 ⁷ / ₈	433.737	3.0129
18 ⁵ / ₈	4 10 ¹ / ₂	272.447	1.9071	23 ⁵ / ₈	6 2 ¹ / ₄	438.363	3.0261
18 ³ / ₄	4 10 ⁷ / ₈	276.117	1.9328	23 ³ / ₄	6 2 ⁵ / ₈	443.014	3.0722
18 ⁷ / ₈	4 11 ¹ / ₄	279.811	1.9586	23 ⁷ / ₈	6 3	447.690	3.1081

Dia. in ft. in.	Circum. in ft. in.	Area in sq. inch.	Area in sq. feet.	Dia. in ft. in.	Circum. in ft. in.	Area in sq. inch.	Area in sq. feet.
2 0	6 3 $\frac{3}{8}$	452.390	3.1418	2 10	8 10 $\frac{3}{4}$	907.922	6.3051
2 0 $\frac{1}{4}$	6 4 $\frac{1}{8}$	461.864	3.2075	2 10 $\frac{1}{4}$	8 11 $\frac{1}{2}$	921.323	6.3981
2 0 $\frac{1}{2}$	6 4 $\frac{7}{8}$	471.436	3.2731	2 10 $\frac{1}{2}$	9 0 $\frac{3}{8}$	934.822	6.4911
2 0 $\frac{3}{4}$	6 5 $\frac{3}{4}$	481.106	3.3410	2 10 $\frac{3}{4}$	9 1 $\frac{1}{8}$	948.419	6.5863
2 1	6 6 $\frac{1}{2}$	490.875	3.4081	2 11	9 1 $\frac{7}{8}$	962.115	6.6815
2 1 $\frac{1}{4}$	6 7 $\frac{1}{4}$	500.741	3.4775	2 11 $\frac{1}{4}$	9 2 $\frac{3}{4}$	975.908	6.7772
2 1 $\frac{1}{2}$	6 8 $\frac{1}{8}$	510.706	3.5468	2 11 $\frac{1}{2}$	9 3 $\frac{1}{2}$	989.800	6.8738
2 1 $\frac{3}{4}$	6 8 $\frac{7}{8}$	520.769	3.6101	2 11 $\frac{3}{4}$	9 4 $\frac{1}{4}$	1003.79	6.9701
2 2	6 9 $\frac{5}{8}$	530.930	3.6870	3 0	9 5	1017.87	7.0688
2 2 $\frac{1}{4}$	6 10 $\frac{1}{2}$	541.189	3.7583	3 0 $\frac{1}{4}$	9 5 $\frac{7}{8}$	1032.06	7.1671
2 2 $\frac{1}{2}$	6 11 $\frac{1}{4}$	551.547	3.8302	3 0 $\frac{1}{2}$	9 6 $\frac{5}{8}$	1046.35	7.2664
2 2 $\frac{3}{4}$	7 0	562.002	3.9042	3 0 $\frac{3}{4}$	9 7 $\frac{1}{2}$	1060.73	7.3662
2 3	7 0 $\frac{3}{4}$	572.556	3.9761	3 1	9 8 $\frac{1}{4}$	1075.21	7.4661
2 3 $\frac{1}{4}$	7 1 $\frac{1}{8}$	583.208	4.0500	3 1 $\frac{1}{4}$	9 9	1089.79	7.5671
2 3 $\frac{1}{2}$	7 2 $\frac{3}{8}$	593.958	4.1241	3 1 $\frac{1}{2}$	9 9 $\frac{7}{8}$	1104.46	7.6691
2 3 $\frac{3}{4}$	7 3 $\frac{1}{8}$	604.807	4.2000	3 1 $\frac{3}{4}$	9 10 $\frac{1}{2}$	1119.24	7.7791
2 4	7 3 $\frac{7}{8}$	615.753	4.2760	3 2	9 11 $\frac{3}{8}$	1134.12	7.8681
2 4 $\frac{1}{4}$	7 4 $\frac{3}{4}$	626.798	4.3521	3 2 $\frac{1}{4}$	10 0 $\frac{1}{8}$	1149.09	7.9791
2 4 $\frac{1}{2}$	7 5 $\frac{1}{2}$	637.941	4.4302	3 2 $\frac{1}{2}$	10 0 $\frac{7}{8}$	1164.16	8.0846
2 4 $\frac{3}{4}$	7 6 $\frac{1}{4}$	649.182	4.5083	3 2 $\frac{3}{4}$	10 1 $\frac{3}{4}$	1179.32	8.1891
2 5	7 7	660.521	4.5861	3 3	10 2 $\frac{1}{2}$	1194.59	8.2951
2 5 $\frac{1}{4}$	7 7 $\frac{7}{8}$	671.958	4.6665	3 3 $\frac{1}{4}$	10 3 $\frac{1}{4}$	1209.95	8.4026
2 5 $\frac{1}{2}$	7 8 $\frac{5}{8}$	683.494	4.7467	3 3 $\frac{1}{2}$	10 4	1225.42	8.5091
2 5 $\frac{3}{4}$	7 9 $\frac{1}{2}$	695.128	4.8274	3 3 $\frac{3}{4}$	10 4 $\frac{7}{8}$	1240.98	8.6171
2 6	7 10 $\frac{1}{4}$	706.860	4.9081	3 4	10 5 $\frac{5}{8}$	1256.64	8.7269
2 6 $\frac{1}{4}$	7 11	718.690	4.9901	3 4 $\frac{1}{4}$	10 6 $\frac{3}{8}$	1272.39	8.8361
2 6 $\frac{1}{2}$	7 11 $\frac{3}{4}$	730.618	5.0731	3 4 $\frac{1}{2}$	10 7 $\frac{1}{4}$	1288.25	8.9462
2 6 $\frac{3}{4}$	8 0 $\frac{5}{8}$	742.644	5.1573	3 4 $\frac{3}{4}$	10 8	1304.20	9.0561
2 7	8 1 $\frac{3}{8}$	754.769	5.2278	3 5	10 8 $\frac{3}{4}$	1320.25	9.1686
2 7 $\frac{1}{4}$	8 2 $\frac{1}{8}$	766.992	5.3264	3 5 $\frac{1}{4}$	10 9 $\frac{1}{2}$	1336.40	9.2112
2 7 $\frac{1}{2}$	8 2 $\frac{7}{8}$	779.313	5.4112	3 5 $\frac{1}{2}$	10 10 $\frac{3}{8}$	1352.65	9.3936
2 7 $\frac{3}{4}$	8 3 $\frac{3}{4}$	791.732	5.4982	3 5 $\frac{3}{4}$	10 11 $\frac{1}{8}$	1369.00	9.5061
2 8	8 4 $\frac{1}{2}$	804.249	5.5850	3 6	10 11 $\frac{7}{8}$	1385.44	9.6212
2 8 $\frac{1}{4}$	8 5 $\frac{3}{8}$	816.865	5.6729	3 6 $\frac{1}{4}$	11 0 $\frac{3}{4}$	1401.98	9.7364
2 8 $\frac{1}{2}$	8 6 $\frac{1}{8}$	829.578	5.7601	3 6 $\frac{1}{2}$	11 1 $\frac{1}{2}$	1418.62	9.8511
2 8 $\frac{3}{4}$	8 6 $\frac{7}{8}$	842.390	5.8491	3 6 $\frac{3}{4}$	11 2 $\frac{1}{4}$	1435.36	9.9671
2 9	8 7 $\frac{5}{8}$	855.300	5.9398	3 7	11 3	1452.20	10.0841
2 9 $\frac{1}{4}$	8 8 $\frac{1}{2}$	868.308	6.0291	3 7 $\frac{1}{4}$	11 3 $\frac{7}{8}$	1469.14	10.2021
2 9 $\frac{1}{2}$	8 9 $\frac{1}{4}$	881.415	6.1201	3 7 $\frac{1}{2}$	11 4 $\frac{5}{8}$	1486.17	10.3201
2 9 $\frac{3}{4}$	8 10	894.619	6.2129	3 7 $\frac{3}{4}$	11 5 $\frac{3}{8}$	1503.30	10.4391

Dia. in ft. in.		Circum. in ft. in.		Area in sq. inch.	Area in sq. feet.	Dia. in ft. in.		Circum. in ft. in.		Area in sq. inch.	Area in sq. feet.
3	8	11	6 $\frac{1}{4}$	1520.53	10.559	4	6	14	1 $\frac{5}{8}$	2290.22	15.904
3	8 $\frac{1}{4}$	11	7	1537.86	10.679	4	6 $\frac{1}{4}$	14	2 $\frac{3}{8}$	2311.48	16.051
3	8 $\frac{1}{2}$	11	7 $\frac{3}{4}$	1555.28	10.800	4	6 $\frac{1}{2}$	14	3 $\frac{1}{4}$	2332.83	16.200
3	8 $\frac{3}{4}$	11	8 $\frac{1}{2}$	1572.81	10.922	4	6 $\frac{3}{4}$	14	4	2354.28	16.349
3	9	11	9 $\frac{3}{8}$	1590.43	11.044	4	7	14	4 $\frac{3}{4}$	2375.83	16.498
3	9 $\frac{1}{4}$	11	10 $\frac{1}{8}$	1608.15	11.167	4	7 $\frac{1}{4}$	14	5 $\frac{1}{2}$	2397.48	16.649
3	9 $\frac{1}{2}$	11	10 $\frac{3}{8}$	1625.97	11.291	4	7 $\frac{1}{2}$	14	6 $\frac{3}{8}$	2419.22	16.800
3	9 $\frac{3}{4}$	11	11 $\frac{1}{4}$	1643.89	11.415	4	7 $\frac{3}{4}$	14	7 $\frac{1}{8}$	2441.07	16.951
3	10	12	0 $\frac{1}{2}$	1661.90	11.534	4	8	14	7 $\frac{7}{8}$	2463.01	17.104
3	10 $\frac{1}{4}$	12	1 $\frac{1}{4}$	1680.02	11.666	4	8 $\frac{1}{4}$	14	8 $\frac{5}{8}$	2485.05	17.257
3	10 $\frac{1}{2}$	12	2	1698.23	11.793	4	8 $\frac{1}{2}$	14	9 $\frac{1}{2}$	2507.19	17.411
3	10 $\frac{3}{4}$	12	2 $\frac{7}{8}$	1716.54	11.920	4	8 $\frac{3}{4}$	14	10 $\frac{1}{4}$	2529.42	17.565
3	11	12	3 $\frac{5}{8}$	1734.94	12.048	4	9	14	11	2551.76	17.720
3	11 $\frac{1}{4}$	12	4 $\frac{3}{8}$	1753.45	12.176	4	9 $\frac{1}{4}$	14	11 $\frac{7}{8}$	2574.19	17.876
3	11 $\frac{1}{2}$	12	5 $\frac{1}{4}$	1772.05	12.305	4	9 $\frac{1}{2}$	15	0 $\frac{5}{8}$	2596.72	18.033
3	11 $\frac{3}{4}$	12	6	1790.76	12.435	4	9 $\frac{3}{4}$	15	1 $\frac{5}{8}$	2619.35	18.189
4	0	12	6 $\frac{3}{4}$	1809.56	12.566	4	10	15	2 $\frac{1}{4}$	2642.08	18.347
4	0 $\frac{1}{4}$	12	7 $\frac{1}{2}$	1828.46	12.697	4	10 $\frac{1}{4}$	15	2 $\frac{7}{8}$	2664.91	18.506
4	0 $\frac{1}{2}$	12	8 $\frac{3}{8}$	1847.45	12.829	4	10 $\frac{1}{2}$	15	3 $\frac{3}{4}$	2687.83	18.665
4	0 $\frac{3}{4}$	12	9 $\frac{1}{8}$	1866.55	12.962	4	10 $\frac{3}{4}$	15	4 $\frac{1}{2}$	2710.85	18.825
4	1	12	9 $\frac{7}{8}$	1885.74	13.095	4	11	15	5 $\frac{1}{4}$	2733.97	18.985
4	1 $\frac{1}{4}$	12	10 $\frac{3}{4}$	1905.03	13.229	4	11 $\frac{1}{4}$	15	6 $\frac{1}{8}$	2757.19	19.147
4	1 $\frac{1}{2}$	12	11 $\frac{1}{2}$	1924.42	13.364	4	11 $\frac{1}{2}$	15	6 $\frac{7}{8}$	2780.51	19.309
4	1 $\frac{3}{4}$	13	0 $\frac{1}{4}$	1943.91	13.499	4	11 $\frac{3}{4}$	15	7 $\frac{3}{4}$	2803.92	19.471
4	2	13	1	1963.50	13.635	5	0	15	8 $\frac{1}{2}$	2827.44	19.635
4	2 $\frac{1}{4}$	13	1 $\frac{7}{8}$	1983.18	13.772	5	0 $\frac{1}{4}$	15	9 $\frac{1}{4}$	2851.05	19.798
4	2 $\frac{1}{2}$	13	2 $\frac{5}{8}$	2002.96	13.909	5	0 $\frac{1}{2}$	15	10	2874.76	19.963
4	2 $\frac{3}{4}$	13	3 $\frac{3}{8}$	2022.84	14.047	5	0 $\frac{3}{4}$	15	10 $\frac{3}{4}$	2898.56	20.128
4	3	13	4 $\frac{1}{4}$	2042.82	14.186	5	1	15	11 $\frac{5}{8}$	2922.47	20.294
4	3 $\frac{1}{4}$	13	5	2062.90	14.325	5	1 $\frac{1}{4}$	16	0 $\frac{5}{8}$	2946.47	20.461
4	3 $\frac{1}{2}$	13	5 $\frac{3}{4}$	2083.07	14.465	5	1 $\frac{1}{2}$	16	1 $\frac{1}{4}$	2970.57	20.629
4	3 $\frac{3}{4}$	13	6 $\frac{1}{2}$	2103.35	14.606	5	1 $\frac{3}{4}$	16	1 $\frac{7}{8}$	2994.77	20.797
4	4	13	7 $\frac{3}{8}$	2123.72	14.748	5	2	16	2 $\frac{3}{4}$	3019.07	20.965
4	4 $\frac{1}{4}$	13	8 $\frac{1}{8}$	2144.19	14.890	5	2 $\frac{1}{4}$	16	3 $\frac{1}{2}$	3043.47	21.135
4	4 $\frac{1}{2}$	13	8 $\frac{3}{8}$	2164.75	15.033	5	2 $\frac{1}{2}$	16	4 $\frac{1}{4}$	3067.96	21.305
4	4 $\frac{3}{4}$	13	9 $\frac{1}{4}$	2185.42	15.176	5	2 $\frac{3}{4}$	16	5 $\frac{1}{8}$	3092.56	21.476
4	5	13	10 $\frac{1}{2}$	2206.18	15.320	5	3	16	5 $\frac{7}{8}$	3117.25	21.647
4	5 $\frac{1}{4}$	13	11 $\frac{1}{4}$	2227.05	15.465	5	3 $\frac{1}{4}$	16	6 $\frac{1}{4}$	3142.04	21.819
4	5 $\frac{1}{2}$	14	0	2248.01	15.611	5	3 $\frac{1}{2}$	16	7 $\frac{1}{2}$	3166.92	21.992
4	5 $\frac{3}{4}$	14	0 $\frac{7}{8}$	2269.06	15.757	5	3 $\frac{3}{4}$	16	8 $\frac{1}{4}$	3191.91	22.166

CIRCUMFERENCES AND

Dia. in ft. in.		Circum. in ft. in.		Area in sq. inch.	Area in sq. feet.	Dia. in ft. in.		Circum. in ft. in.		Area in sq. inch.	Area in sq. feet.
5	4	16	9	3216.99	22.333	6	2	19	4½	4300.85	29.867
5	4¼	16	9¾	3242.17	22.515	6	2¼	19	5¼	4329.95	30.069
5	4½	16	10⅝	3267.46	22.621	6	2½	19	6	4359.16	30.271
5	4¾	16	11⅜	3292.83	22.866	6	2¾	19	6¾	4388.47	30.475
5	5	17	0⅝	3318.31	23.043	6	3	19	7⅝	4417.87	30.679
5	5¼	17	0⅞	3343.88	23.221	6	3¼	19	8⅜	4447.37	30.884
5	5½	17	1¼	3369.56	23.330	6	3½	19	9⅛	4476.97	31.090
5	5¾	17	2½	3395.33	23.578	6	3¾	19	9⅞	4506.67	31.296
5	6	17	3⅝	3421.20	23.758	6	4	19	10¾	4536.47	31.503
5	6¼	17	4⅛	3447.16	23.938	6	4¼	19	11½	4566.36	31.710
5	6½	17	4⅞	3473.23	24.119	6	4½	20	0¼	4596.35	31.919
5	6¾	17	5⅝	3499.39	24.301	6	4¾	20	1⅛	4626.44	32.114
5	7	17	6½	3525.66	24.483	6	5	20	1⅞	4656.63	32.337
5	7¼	17	7¼	3552.01	24.666	6	5¼	20	2⅝	4686.92	32.548
5	7½	17	8	3578.47	24.850	6	5½	20	3½	4717.30	32.759
5	7¾	17	8¾	3605.03	25.034	6	5¾	20	4¼	4747.79	32.970
5	8	17	9⅝	3631.68	25.220	6	6	20	5	4778.37	33.183
5	8¼	17	10⅜	3658.44	25.405	6	6¼	20	5¾	4809.05	33.396
5	8½	17	11⅛	3685.29	25.592	6	6½	20	6½	4839.83	33.619
5	8¾	17	11⅞	3712.24	25.779	6	6¾	20	7⅜	4870.70	33.824
5	9	18	0¾	3739.28	25.964	6	7	20	8⅛	4901.68	34.039
5	9¼	18	1½	3766.43	26.155	6	7¼	20	8⅞	4932.75	34.255
5	9½	18	2¼	3793.67	26.344	6	7½	20	9¾	4963.92	34.471
5	9¾	18	3⅛	3821.02	26.534	6	7¾	20	10½	4995.19	34.688
5	10	18	3⅞	3848.46	26.725	6	8	20	11¼	5026.26	34.906
5	10¼	18	4⅝	3875.99	26.916	6	8¼	21	0⅛	5058.02	35.125
5	10½	18	5½	3903.63	27.108	6	8½	21	0⅞	5089.58	35.344
5	10¾	18	6¼	3931.36	27.301	6	8¾	21	1⅝	5121.24	35.564
5	11	18	7	3959.20	27.494	6	9	21	2⅜	5153.00	35.784
5	11¼	18	7¾	3987.13	27.688	6	9¼	21	3¼	5184.86	36.006
5	11½	18	8⅝	4015.16	27.883	6	9½	21	4	5216.82	36.227
5	11¾	18	9⅜	4043.28	28.078	6	9¾	21	4¾	5248.87	36.450
6	0	18	10⅛	4071.51	28.274	6	10	21	5½	5281.02	36.674
6	0¼	18	10⅞	4099.83	28.471	6	10¼	21	6⅜	5313.27	36.897
6	0½	18	11¼	4128.25	28.668	6	10½	21	7⅛	5345.62	37.122
6	0¾	19	0½	4156.77	28.866	6	10¾	21	7⅞	5378.07	37.347
6	1	19	1¼	4185.39	29.065	6	11	21	8¾	5410.62	37.573
6	1¼	19	2⅛	4214.11	29.264	6	11¼	21	9½	5443.26	37.700
6	1½	19	2⅞	4242.92	29.466	6	11½	21	10¼	5476.00	38.027
6	1¾	19	3⅝	4271.83	29.665	6	11¾	21	11	5508.84	38.256

Dia. in ft. & in.		Circum. in ft. and in.		Area in feet.	Dia. in ft. & in.		Circum. in ft. and in.		Area in feet.
7	0	21	11 $\frac{7}{8}$	38.4846	10	0	31	5	78.5400
	1	22	3	39.4060		1	31	8 $\frac{1}{8}$	79.8540
	2	22	6 $\frac{1}{8}$	40.3388		2	31	11 $\frac{1}{4}$	81.1795
	3	22	9 $\frac{1}{4}$	41.2825		3	32	2 $\frac{3}{8}$	82.5160
	4	23	0 $\frac{3}{8}$	42.2367		4	32	5 $\frac{1}{2}$	83.8627
	5	23	2 $\frac{1}{8}$	43.2022		5	32	8 $\frac{5}{8}$	85.2211
	6	23	6 $\frac{3}{4}$	44.1787		6	32	11 $\frac{3}{4}$	86.5903
	7	23	11	45.1656		7	33	2 $\frac{7}{8}$	87.9697
	8	24	1 $\frac{1}{8}$	46.1638		8	33	6 $\frac{1}{8}$	89.3608
	9	24	4 $\frac{1}{8}$	47.1730		9	33	9 $\frac{1}{4}$	90.7627
	10	24	7 $\frac{1}{4}$	48.1926		10	34	0 $\frac{3}{8}$	92.1749
11	24	10 $\frac{3}{8}$	49.2236	11	34	3 $\frac{1}{2}$	93.5986		
8	0	25	1 $\frac{1}{2}$	50.2656	11	0	34	6 $\frac{5}{8}$	95.0334
	1	25	4 $\frac{5}{8}$	51.3178		1	34	9 $\frac{3}{4}$	96.4783
	2	25	7 $\frac{7}{8}$	52.3816		2	35	0 $\frac{7}{8}$	97.9347
	3	25	11	53.4562		3	35	4 $\frac{1}{8}$	99.4021
	4	26	2 $\frac{1}{8}$	54.5412		4	35	7 $\frac{1}{4}$	100.8797
	5	26	5 $\frac{1}{4}$	55.6377		5	35	10 $\frac{5}{8}$	102.3689
	6	26	8 $\frac{3}{8}$	56.7451		6	36	1 $\frac{1}{2}$	103.8691
	7	26	11 $\frac{1}{2}$	57.8628		7	36	4 $\frac{1}{2}$	105.3794
	8	27	2 $\frac{3}{4}$	58.9920		8	36	7 $\frac{3}{4}$	106.9013
	9	27	5 $\frac{3}{4}$	60.1321		9	36	10 $\frac{7}{8}$	108.4342
	10	27	9	61.2826		10	37	2 $\frac{3}{4}$	109.9772
11	28	0 $\frac{1}{8}$	62.4445	11	37	5 $\frac{1}{4}$	111.5319		
9	0	28	3 $\frac{1}{4}$	63.6174	12	0	37	8 $\frac{3}{8}$	113.0976
	1	28	6 $\frac{3}{8}$	64.8006		1	37	11 $\frac{1}{2}$	114.6732
	2	28	9 $\frac{1}{2}$	65.9951		2	38	2 $\frac{5}{8}$	116.2607
	3	29	0 $\frac{5}{8}$	67.2007		3	38	5 $\frac{3}{4}$	117.8590
	4	29	3 $\frac{3}{4}$	68.4166		4	38	8 $\frac{7}{8}$	119.4674
	5	29	7	69.6440		5	39	0	121.0876
	6	29	10 $\frac{1}{8}$	70.8823		6	39	3 $\frac{1}{4}$	122.7187
	7	30	1 $\frac{1}{4}$	72.1309		7	39	6 $\frac{3}{8}$	124.3598
	8	30	4 $\frac{3}{8}$	73.3910		8	39	9 $\frac{1}{2}$	126.0127
	9	30	7 $\frac{1}{2}$	74.6620		9	40	0 $\frac{5}{8}$	127.6765
	10	30	11 $\frac{5}{8}$	75.9433		10	40	3 $\frac{3}{4}$	129.3504
11	31	1 $\frac{3}{4}$	77.2362	11	40	6 $\frac{7}{8}$	131.0360		

170 CIRCUMFERENCES AND AREAS OF CIRCLES.

Dia. in ft. & in.		Circum. in ft. and in.		Area in feet.	Dia. in ft. & in.		Circum. in ft. and in.		Area in feet.
13	0	40	10	132.7326	16	0	50	3 $\frac{1}{8}$	201.0624
	1	41	1 $\frac{1}{8}$	134.4391		1	50	6 $\frac{1}{4}$	203.1615
	2	41	4 $\frac{3}{8}$	136.1574		2	50	9 $\frac{3}{8}$	205.2726
	3	41	7 $\frac{1}{2}$	137.8867		3	51	0 $\frac{1}{2}$	207.3946
	4	41	10 $\frac{5}{8}$	139.6260		4	51	3 $\frac{3}{4}$	209.5264
	5	42	1 $\frac{5}{8}$	141.3771		5	51	6 $\frac{1}{2}$	211.6703
	6	42	4 $\frac{7}{8}$	143.1391		6	51	10	213.8251
	7	42	8	144.9111		7	52	1 $\frac{1}{8}$	215.9896
	8	42	11 $\frac{1}{8}$	146.6949		8	52	4 $\frac{1}{4}$	218.1662
	9	43	2 $\frac{1}{4}$	148.4896		9	52	7 $\frac{3}{8}$	220.3537
	10	43	5 $\frac{1}{2}$	150.2943		10	52	10 $\frac{1}{2}$	222.5510
11	43	8 $\frac{5}{8}$	152.1109	11	53	1 $\frac{5}{8}$	224.7603		
14	0	43	11 $\frac{3}{4}$	153.9384	17	0	53	4 $\frac{7}{8}$	226.9806
	1	44	2 $\frac{7}{8}$	155.7758		1	53	8	229.2105
	2	44	6	157.6250		2	53	11 $\frac{1}{8}$	231.4625
	3	44	9 $\frac{1}{8}$	159.4852		3	54	2 $\frac{1}{8}$	233.7055
	4	45	0 $\frac{1}{4}$	161.3553		4	54	5 $\frac{3}{8}$	235.9682
	5	45	3 $\frac{1}{2}$	163.2373		5	54	8 $\frac{1}{2}$	238.2430
	6	45	6 $\frac{5}{8}$	165.1303		6	54	11 $\frac{5}{8}$	240.5287
	7	45	9 $\frac{3}{4}$	167.0331		7	55	2 $\frac{7}{8}$	242.8241
	8	46	0 $\frac{7}{8}$	168.9479		8	55	6	245.1316
	9	46	4	170.8735		9	55	9 $\frac{1}{8}$	247.4500
	10	46	7 $\frac{1}{8}$	172.8091		10	56	0 $\frac{1}{4}$	249.7781
11	46	11 $\frac{1}{4}$	174.7565	11	56	3 $\frac{1}{2}$	252.1184		
15	0	47	1 $\frac{1}{2}$	176.7150	18	0	56	6 $\frac{1}{2}$	254.4696
	1	47	4 $\frac{5}{8}$	178.6832		1	56	9 $\frac{5}{8}$	256.8303
	2	47	7 $\frac{3}{4}$	180.6634		2	57	0 $\frac{7}{8}$	259.2033
	3	47	10 $\frac{7}{8}$	182.6545		3	57	4	261.5872
	4	48	2 $\frac{1}{2}$	184.6555		4	57	7 $\frac{1}{8}$	263.9807
	5	48	5 $\frac{1}{8}$	186.6684		5	57	10 $\frac{1}{4}$	266.3864
	6	48	8 $\frac{1}{4}$	188.6923		6	58	1 $\frac{3}{8}$	268.8031
	7	48	11 $\frac{3}{8}$	190.7260		7	58	4 $\frac{1}{2}$	271.2293
	8	49	2 $\frac{5}{8}$	192.7716		8	58	7 $\frac{5}{8}$	273.6678
	9	49	5 $\frac{3}{4}$	194.8282		9	58	10 $\frac{3}{4}$	276.1171
	10	49	8 $\frac{7}{8}$	196.8946		10	59	2	278.5761
11	50	0	198.9730	11	59	5 $\frac{1}{8}$	281.0472		

SQUARE AND CUBE ROOTS OF NUMBERS. 171

No.	S. R.	C. R.	No.	S. R.	C. R.	No.	S. R.	C. R.	No.	S. R.	C. R.
1	1·0000	1·0000	55	7·4161	3·8029	109	10·4403	4·7768	163	12·7671	5·4625
2	1·4142	1·2599	56	7·4833	3·8258	110	10·4880	4·7914	164	12·8062	5·4737
3	1·7320	1·4422	57	7·5498	3·8485	111	10·5356	4·8058	165	12·8452	5·4848
4	2·0000	1·5874	58	7·6157	3·8708	112	10·5830	4·8202	166	12·8840	5·4958
5	2·2360	1·7099	59	7·6811	3·8929	113	10·6301	4·8345	167	12·9228	5·5068
6	2·4494	1·8171	60	7·7459	3·9148	114	10·6770	4·8488	168	12·9614	5·5178
7	2·6457	1·9129	61	7·8102	3·9364	115	10·7238	4·8629	169	13·0000	5·5287
8	2·8284	2·0000	62	7·8740	3·9578	116	10·7703	4·8769	170	13·0384	5·5396
9	3·0000	2·0800	63	7·9372	3·9790	117	10·8166	4·8909	171	13·0766	5·5404
10	3·1622	2·1544	64	8·0000	4·0000	118	10·8627	4·9048	172	13·1148	5·5612
11	3·3166	2·2239	65	8·0622	4·0207	119	10·9087	4·9186	173	13·1529	5·5720
12	3·4641	2·2894	66	8·1240	4·0412	120	10·9544	4·9324	174	13·1909	5·5827
13	3·6055	2·3513	67	8·1853	4·0615	121	11·0000	4·9460	175	13·2287	5·5934
14	3·7416	2·4101	68	8·2462	4·0816	122	11·0453	4·9596	176	13·2664	5·6040
15	3·8729	2·4662	69	8·3066	4·1015	123	11·0905	4·9731	177	13·3041	5·6416
16	4·0000	2·5198	70	8·3666	4·1212	124	11·1355	4·9866	178	13·3416	5·6252
17	4·1231	2·5712	71	8·4261	4·1408	125	11·1803	5·0000	179	13·3790	5·6357
18	4·2426	2·6207	72	8·4852	4·1601	126	11·2249	5·0132	180	13·4164	5·6462
19	4·3588	2·6684	73	8·5440	4·1793	127	11·2694	5·0265	181	13·4536	5·6566
20	4·4721	2·7144	74	8·6023	4·1983	128	11·3137	5·0396	182	13·4907	5·6670
21	4·5825	2·7589	75	8·6602	4·2171	129	11·3578	5·0527	183	13·5277	5·6774
22	4·6904	2·8020	76	8·7177	4·2358	130	11·4017	5·0657	184	13·5646	5·6877
23	4·7958	2·8438	77	8·7749	4·2543	131	11·4455	5·0787	185	13·6014	5·6980
24	4·8989	2·8844	78	8·8317	4·2726	132	11·4891	5·0916	186	13·6381	5·7082
25	5·0000	2·9240	79	8·8881	4·2908	133	11·5325	5·1044	187	13·6747	5·7184
26	5·0990	2·9624	80	8·9442	4·3088	134	11·5758	5·1172	188	13·7113	5·7286
27	5·1961	3·0000	81	9·0000	4·3267	135	11·6189	5·1299	189	13·7477	5·7387
28	5·2915	3·0365	82	9·0553	4·3444	136	11·6619	5·1425	190	13·7840	5·7488
29	5·3851	3·0723	83	9·1104	4·3620	137	11·7046	5·1551	191	13·8202	5·7589
30	5·4772	3·1072	84	9·1651	4·3795	138	11·7473	5·1676	192	13·8564	5·7689
31	5·5677	3·1413	85	9·2195	4·3968	139	11·7898	5·1801	193	13·8924	5·7789
32	5·6568	3·1748	86	9·2736	4·4140	140	11·8321	5·1924	194	13·9283	5·7889
33	5·7445	3·2075	87	9·3273	4·4310	141	11·8743	5·2048	195	13·9642	5·7988
34	5·8309	3·2396	88	9·3808	4·4479	142	11·9163	5·2171	196	14·0000	5·8087
35	5·9160	3·2710	89	9·4339	4·4647	143	11·9582	5·2293	197	14·0356	5·8186
36	6·0000	3·3019	90	9·4868	4·4814	144	12·0000	5·2414	198	14·0712	5·8284
37	6·0827	3·3322	91	9·5393	4·4979	145	12·0415	5·2535	199	14·1067	5·8382
38	6·1644	3·3619	92	9·5916	4·5143	146	12·0830	5·2656	200	14·1421	5·8480
39	6·2449	3·3912	93	9·6436	4·5306	147	12·1243	5·2776	201	14·1774	5·8577
40	6·3245	3·4199	94	9·6953	4·5468	148	12·1655	5·2895	202	14·2126	5·8674
41	6·4031	3·4482	95	9·7467	4·5629	149	12·2065	5·3014	203	14·2478	5·8771
42	6·4807	3·4760	96	9·7979	4·5788	150	12·2474	5·3132	204	14·2828	5·8867
43	6·5574	3·5033	97	9·8488	4·5947	151	12·2882	5·3250	205	14·3178	5·8963
44	6·6332	3·5303	98	9·8994	4·6104	152	12·3288	5·3368	206	14·3527	5·9059
45	6·7082	3·5568	99	9·9498	4·6260	153	12·3693	5·3484	207	14·3874	5·9154
46	6·7823	3·5830	100	10·0000	4·6415	154	12·4096	5·3601	208	14·4222	5·9249
47	6·8556	3·6088	101	10·0498	4·6570	155	12·4498	5·3716	209	14·4568	5·9344
48	6·9282	3·6342	102	10·0995	4·6723	156	12·4899	5·3832	210	14·4913	5·9439
49	7·0000	3·6593	103	10·1488	4·6875	157	12·5299	5·3946	211	14·5258	5·9533
50	7·0710	3·6840	104	10·1980	4·7026	158	12·5698	5·4061	212	14·5602	5·9627
51	7·1414	3·7084	105	10·2469	4·7176	159	12·6095	5·4175	213	14·5945	5·9720
52	7·2111	3·7325	106	10·2956	4·7326	160	12·6491	5·4288	214	14·6287	5·9814
53	7·2801	3·7562	107	10·3440	4·7474	161	12·6885	5·4401	215	14·6628	5·9907
54	7·3484	3·7797	108	10·3923	4·7622	162	12·7279	5·4513	216	14·6969	6·0000

DISCOUNT TABLE.

To find the square or cube root of a number consisting of integers and decimals.—*Rule.* Multiply the difference between the root of the integer part of the given number, and the root of the next higher number, by the decimal part of the given number, and add the product to the root of the given integer number; the sum is the root required.

Ex. Required the square root of 20·321.

Square root of 21 = 4·5825

Do. 20 = 4·4721

Diff. = $\cdot 1104 \times \cdot 321 + 4\cdot 4721 = 4\cdot 507$, &c., the root required.

PER CENTAGE AND DISCOUNT TABLE.

Rate per cent.	Rate per £. in shillings.	Rate per s. in pence.	Rate per cent.	Rate per £. in shillings.	Rate per s. in pence.
$\frac{1}{8}$	·025	·015	$5\frac{1}{2}$	1·1	·66
$\frac{1}{4}$	·05	·03	6	1·2	·72
$\frac{1}{2}$	·1	·06	$6\frac{1}{4}$	1·25	·75
$\frac{3}{4}$	·15	·09	$7\frac{1}{2}$	1·5	·9
1	·2	·12	$8\frac{3}{4}$	1·75	1·05
2	·4	·24	10	2	1·2
$2\frac{1}{2}$	·5	·3	$12\frac{1}{2}$	2·5	1·5
3	·6	·36	15	3	1·8
$3\frac{1}{2}$	·7	·42	20	4	2·4
4	·8	·48	25	5	3
$4\frac{1}{2}$	·9	·54	30	6	3·6
5	1	·6	40	8	4·8
To cover a discount of	Mult. the net price by	Divide the product by	To cover a discount of	Mult. the net price by	Divide the product by
5 p. ct.	20	19	$27\frac{1}{2}$ p. ct.	40	29
$7\frac{1}{2}$	40	37	30	10	7
10	10	9	$32\frac{1}{2}$	40	27
$12\frac{1}{2}$	8	7	35	20	13
15	20	17	$37\frac{1}{2}$	8	5
$17\frac{1}{2}$	40	33	40	5	3
20	5	4	$42\frac{1}{2}$	40	23
$22\frac{1}{2}$	40	31	45	20	11
25	4	3	$47\frac{1}{2}$	40	21

For 50 per cent., double the sum.

Ex. 1. Required the per centage on £127 at $3\frac{1}{2}$ per cent.

$127 \times \cdot 7 = 88\cdot 9$ shillings, or £ 4. 8s. 10 $\frac{3}{4}$ d.

Ex. 2. What is the per centage on 36 shillings at $2\frac{1}{2}$ per cent.?

$36 \times \cdot 3 = 10\cdot 8$ pence, or 10 pence and 3 farthings nearly.

