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No. 1.

SCIENTIFIC NOVELTIES FOR THE NEW YEAR, 1878.

THE MATHEMATICIAN AND THE LITTLE CROOKED THING

THAT ASKS QUESTIONS.

A SERIES OF PUZZLES.

Dedicated to the **RISING GENERATION** *as well as the more advanced; Gentle Exercise for Intelligent Students during the Holidays, and worthy of the attention of all, from the Undergraduate to Professors and Presidents of the Learned Societies.*

WHAT IS THE SQUARE ROOT OF 1?

HINTS AND SUGGESTIONS.

PRICE, WITH CARD OF DIAGRAMS, SIXPENCE *(net)*.

SOLD BY
W. J. SMITH, 41, 42, & 43, NORTH STREET,
BRIGHTON.

EDWARD VERRALL,
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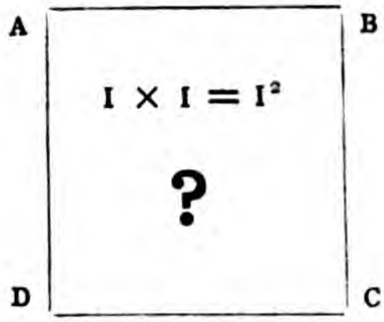
BRIGHTON.

THE MATHEMATICIAN AND THE LITTLE CROOKED THING THAT ASKS QUESTIONS.



HINTS AND SUGGESTIONS.

The contention is that One, the *square* area, might, could, would, should, or ought to have had, a *root;* but that, according to the conventional system, it has none. To multiply the side into the side, the BASIS of the system, means multiplication, but $1 \times 1 = 1$ involves no multiplication at all. These hints, and suggestions in aid, point to the manner in which a root may be found for the Unit of Area corresponding with the *Linear Unit.*

It is further contended that the whole system is one of mere convention, and not, as peradventure it ought to be, the application of the two Sciences, Arithmetic and Geometry, to each other; and that,—use the system as we may,—we are everywhere met with incompatibilities and incongruities, and with approximation instead of accuracy.

If these papers should lead to the discovery of important error in our conventional measurement of surface, such discovery will be the result of patient and persevering investigation, which has occupied the leisure of about forty years. Thousands of diagrams have been drawn, and it has been the Author's practice, after starting hundreds of new ideas, not to proceed with any one of them, without testing its applicability to the object, by reproducing the diagrams and re-calculating the results which have led to it.

These results have taken shape from time to time, and have been published; but although calculated to lead to a clearer understanding of these Puzzles, it is not pretended that what is now placed before the Reader may not be found, here and there, contradictory, in some measure, to those Essays on the difference between square and superficial measurement. There has been, moreover, a desultoriness and divergence in the style which was adopted in order to stimulate *competent* criticism, by working the tactics of Critics who seek to avoid the trouble of it by *pooh-poohing* or ridicule. This style may render the treatises unintelligible to those inexperienced in these investigations; or, to use the idea of one of the Critics —to *nine-tenths* of the public, who, he holds, "cannot understand" such subjects. It has been, hitherto, a forlorn attempt to raise the energies of the *other tenth*, whom the Critic pronounces to be "pledged to disbelief."

A Solution will (D.V) be published to every puzzle. A little attention will shew that they are not very hard "nuts to crack." THE PUZZLE in this instance, No. 1, is to explain in what manner convention can be reconciled with the adaptation of arithmetic to geometry. And as, compatibly with that object, the diagrams point out *several ways* by which a firm basis may be found for a starting point,—without violence to definition, and supported by Euclid's renowned forty-seventh,—this Puzzle involves another, viz., *which to choose*, in preference to the conventional $1 \times 1 = 1$, which is a myth.

It is hoped that the publication of these papers may excite some opposition, which the author will endeavour to meet, whether addressed to him publicly or privately. He will thankfully receive, and give every attention to any efforts in furtherance or in discouragement of his object:—that of substituting actual for merely proximate reckoning, probably dispensing with the extraction of the Square Root and interminable decimals.

The author has further to notice some difficulties which he has encountered, and which, if not already surmounted, he hopes to overcome as he proceeds. He thinks that a safer and surer process to accomplish his object, and one more likely to elicit attention, is to advance gradually by sap and mine, rather than by assault, backed by the heavy ordnance of long treatises which it takes trouble to read and consider,—trouble not readily bestowed by Philosophers on NOVELTIES and matters out of their own track.

The square may be a convenient measure of area for the Mechanic or the Surveyor. The Mechanic may cut his material conveniently, and use it up by means of a T square as a guide; and it may assist the Architect to plan and cover his land, or the Surveyor to measure it. Convention may establish a standard or standards to bind all: but Convention has its limits, and cannot force a Circular unit which measures an inch every way, into the sides of a square 1×1, whose diagonal is greater than that unit. This is no less a fact than that the Mathematician cannot measure the circumference,—nor permit others to do it; unless they try for it in an orthodox manner as impracticable as the $\sqrt{2}$.

Another difficulty that has been encountered is this. A decimal system of Notation does not fit quadrature or measurement in squares. Hence the interminable decimals which no Algebra can get over, nor Logarithms make accurate compensation for. This I presume to be their object, and that they are,—*mathematically,* —*unsuccessful expedients.*

These hints and suggestions will assist the enquirer to understand the drift of the diagrams, about which there is no mystery, nor desire to substitute "ingenuity and artifice" for plain matter-of-fact demonstration;

such a process as has been used,—assuming a square to be a superficial unit,—to shew how, $.25 \times .25 = .0625$ $\therefore \frac{625}{10000} \div 625 = \frac{1}{16}$ is the area of a sixteenth, although, if 1×1 (Fig. 1) $= 1$, certainly *one quarter* multiplied by *one quarter* might, could, would, should, or ought to have been, *one quarter*.

It only remains in the way of introduction to offer the following explanations :—

Fig. I

is the Conventional square of A B $= 1$.

Is not this, which is called A B^2, the square of A B + B C + C D + D A $= 1 + 1 + 1 + 1$ not of A B. This makes 1 the $\sqrt{4}$? Our Ancestors took no heed of the perimeter, *an important element in the reckoning;* for possibly its measure is the extreme length which can be used to determine the area of the square in arithmetical figures arising out of the linear unit.

Incongruously, the square of *half* in this diagram is one quarter, and E F, the *geometrical square root of half*, is not *half*, but an impracticable decimal.

Fig. II.

The line A B $= 1$,—length without breadth,—seems a very inconvenient figure to adopt in order to indicate area or surface corresponding to the line.

Fig. III.

The square A D C E has breadth equal to its length, and contains the extreme surface which A B $= 1$ will inclose. May not A D, therefore, be the SQUARE ROOT OF ONE? inasmuch as A D^2 is the square *on* the quarter, and $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4}$ or 1. Here is no violence to definition.

Figs. IV. & VI.

Breadth being as above provided, we are led to ask whether $\overset{\text{area}}{\frac{1}{4}} + \overset{\text{area}}{\frac{1}{4}} + \overset{\text{area}}{\frac{1}{4}} + \overset{\text{area}}{\frac{1}{4}} = \overset{\text{area}}{\frac{4}{4}}$ may not be the square of one, and $\frac{1}{2}$ A B of Fig. I, or A D Figs. IV. & VI., consequently the SQUARE ROOT OF ONE.

Fig. IV.

Justifies the last question by the application of Euclid's 47th Proposition to the same diagram. For if A D, Fig III., or A E, Fig IV., be $\sqrt{1/4}$, then $A\ E^2 + A\ F^2 = 1/4 + 1/4 = 1/2$ or $E\ F^2$. And as F H is the diagonal of $E\ F^2$ or E F G H $\therefore E\ F^2 + E\ H^2 = F\ H^2 =$ A B C D $= \frac{4}{4}$ or 1 area.

Fig. V.

Is not the oblong A B C D equal to the squares A B C D, Figs. IV. & VI. or $1/4 + 1/4 + 1/4 + 1/4 = \frac{4}{4} = 1$?

Fig. VII.

Analysis of the Conventional Square A B C D, Fig. 1, by means of Euclid's 47th.

		areas		
$A\ B^2$	Conventional	$=1$	Suggestive	4
$E\ F^2$	,,	$=1/2$	,,	2
$I\ K^2$	,,	$=1/4$	,,	1
$N\ O^2$	,,	$=1/8$	,,	1/2
$R\ S^2$	,,	$=\frac{1}{16}$	,,	1/4

Figs. VIII. & IX.

These diagrams, it is hoped, will be found to meet some of the difficulties arising out of the decimal system; to which the only clue which I will at present give, is

$$A\ B = .10 \times .10 = .100 \text{ or } \tfrac{100}{10} = 10.$$

$$.1 + .1 + .1 + .1 + .1 + .1 + .1 + .1 + .1 + .1 = 1.0$$

wherefore as $(E\ F^2 = .3 \times .3 =).9 + .1^2 = \frac{10}{10} = 1$, E F = ? may be the Square Root of One.

The above is merely introductory. The subject of the "square root of one" will be handled at much greater length in the "Solution of the Puzzle No. 1," preparing for the Press.

The term "Solutions" may be incorrectly applied. They will rather be short treatises on the several subjects, after the fashion of "Joyce's Scientific Dialogues." It may take many steps to reach the desired object. The second Puzzle will be to "*decide where Doctors disagree.*" It will (D.V.) contain diagrams connected with a puzzle issued in

1874, and criticisms on a published solution of it, *with replies.*

Several Papers are contemplated if this scheme should find encouragement.

No. 2.—To decide on differences of opinion regarding the puzzle published in 1874; with criticisms and replies.

No. 3.—What is the Square Root of SEVEN? A mathematician in remote antiquity suggested that 49 and 50 were both the square of seven. In my treatise on the difference between Square and Superficial measurement (1865), I pledged myself to find out what he meant. I have started another covey, and this will be a continuation of No. 1,—as will be

No. 4.—Analysis of a Square inch, foot, yard, and mile.

No. 5.—Why do learned Societies advise their Governments to send in search of a *North Pole?* This involves suggestions as to the Nature of Magnetism.

No. 6.—Is Newton's law of the squares of the distances a direct or inverse law? Curious optical experiment in illustration.

No. 7.—May not Newton's *ultimate* idea that "atoms are held together by some unknown but enormous force," rather than attraction and gravitation, be illustrated by the Electric Telegraph, the Telephone, and recent discoveries?

The Author hopes to raise interesting and profitable discussion by these Papers. Probably they will not exceed the present in price, viz., Introduction, with Diagram, 6d., and "Solution," 1s., each. They are intended to be short, and some subjects may be taken together; nor has the Author determined the order in which they will be published, if published at all.

10, Carlton Terrace,
Portslade-by-Sea, Shoreham,
January, 1878.

W. PETERS.

1879, and criticisms on a published solution of it, and replies.

Several Papers are contemplated if this scheme should find encouragement.

No. 3. [illegible] of opinion [illegible] already published [illegible] correspondence and replies.

No. 4.—What is the Square Root of minus [illegible] mathematician [illegible] suggested that [illegible] were both the [illegible] of [illegible]. In my treatise on the [illegible] directed [illegible] Square [illegible] (1891) [illegible] I [illegible] another [illegible] will [illegible]

No. 5.—Analysis [illegible]

No. 6.—[illegible] Natural [illegible] their [illegible] ments [illegible] the nature of [illegible]

No. 7.—[illegible] the distance [illegible]

No. 8.—[illegible] rather than [illegible]

[illegible]

[illegible] PHILLIPS.

[illegible]

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WHAT IS THE SQUARE ROOT OF 1?

Fig. II.

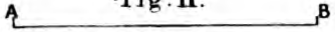


Fig. III.

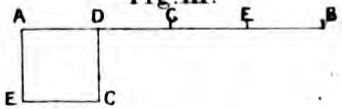


Fig. IV.

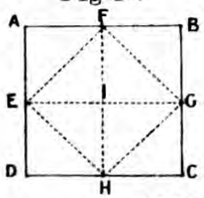
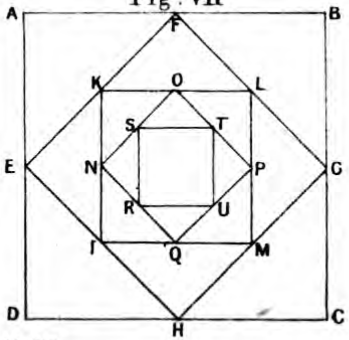
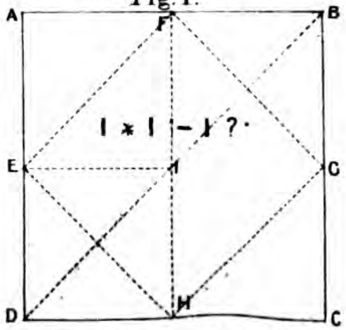


Fig. VII



SUGGESTIVE

Fig. I.



CONVENTIONAL

Fig. V.

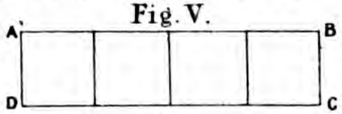


Fig. VI.

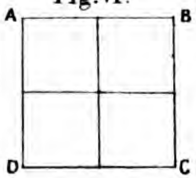


Fig. VIII.

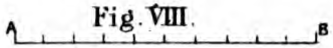
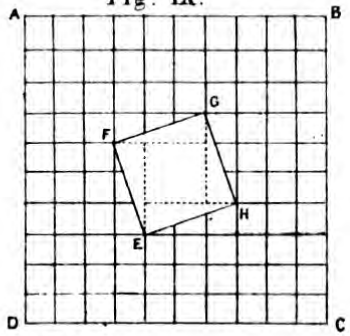


Fig. IX.



SUGGESTIVE

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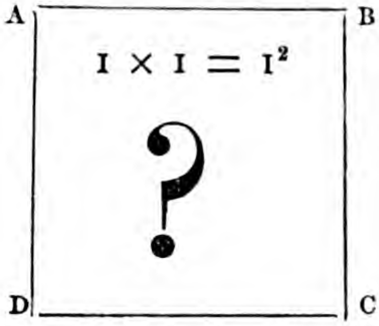

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WHAT IS THE SQUARE ROOT OF 1?

SOLUTION.

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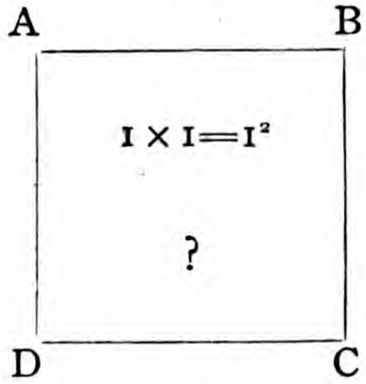

?

What is the square root of one **?**

MATHEMATICIAN.

One, of course. Every schoolboy will tell you that.

? 2. No doubt he will, because he is taught so! Can you or he show HOW IT IS?

MATH. *How it is!* Is not one by one, one? The side or root multiplied into the side gives the area. The area of the square is one; and as $1 \times 1 = 1$ and $A\,B = 1$ is the side of the square; and as A B C D the area is equal to one; therefore $A\,B = 1$ is THE SQUARE ROOT OF ONE; Q. E. D.

? 3. Carefully expressed, and thoroughly understood. BUT—Is this any adaptation of Arithmetic to Geometry?

MATH. No doubt it is. It has been universally accepted from generation to generation. I never thought of looking into such a matter as that.

? 4. You are easily satisfied.

MATH. How can there be any doubt of the fact?

? 5. Of the fact that it has been accepted from time immemorial there is demonstration enough to satisfy everybody. But I speak of the adaptation of your answer to Arithmetic and Geometry combined?

MATH. And there ought to be as little doubt of that.

? 6. "*La philosophie ordonne que nous doûtons de tout.*" *Il me semble que ceci n'est pas une démonstration.*

MATH. Our ancestors, from time immemorial, and their successors—sages of all ages—have in all ages received and never thought of raising a doubt, that *one* is the *square root* of *one.*

? 7. I cannot help that. We have inherited from our forefathers, it is true, a system—*or what you treat as a system*—of which this is the BASIS. But it is *not* a system. Incompatibilities and incongruities, you must see, crop up and spread from first to last. Mere antiquity—assume what authority it may—is not mathematical demonstration. And it is the province of THE LITTLE CROOKED THING THAT ASKS QUESTIONS to pursue his vocation as long as there is a crooked question to ask. Does it become a spirit of research to "subside" while there is error—peradventure a depth of error—to fathom? I trow not. And the more direct and straightforward the manner in which ANY little crooked thing may do it, the better: especially if he can stimulate answers *to the purpose* for the benefit of himself and others. He needs not to heed what crooked answers he may get.

MATH. Well! go on.

? 8. Will you THINK a little? Is not your answer No. **2** neither more nor less than what has been called "the ladies' reason"—one multiplied by one is the square of one, "BECAUSE IT IS?" There is no *shadow of an attempt* at proof—indeed, you mathematicians, when asked for proof, fall back on what you call CONVENTION. My object is to upset convention and to substitute proof.

MATH. You will find that a puzzler.

? 9. *After years of perseverance*, a more simple process than you contemplate; and I hope to make it intelligible to the most moderate capacity: no one ought to be afraid of it. When our ancestors determined, in ages gone by, that root and area, in the instance of THE UNIT only, should be represented by the same arithmetical figure, it *may have occurred to them* (as Peter Pindar attributed to George the Third) to "remember to forget" *that* "*C'est le premier pas qui côute.*" But, resolved on going a-head before they had established a firm footing, they preferred to make a false step at starting rather than stick where they were. It is admitted that they *went* A LITTLE *wrong*. They may have gone egregiously wrong.

MATH. That, certainly, is the merest assertion.

? 10. Whether mere assertion or not—though, by-the-bye, I have suggested not asserted it—it is a probability founded on fact. You have settled that $1 \times 1 = 1$. What follows as the next step?

MATH. Two, I suppose you mean.

? 11. Yes. And how did they get over that?

MATH. They took the Multiplication Table, multiplied the side or root into itself, and resolved that, as $1 \times 1 = 1$, so *four* should be the square of *two*, and two the root of it.

? 12. Surely there must be some incongruity about this! Besides—What had the Multiplication Table to do with Geometry? Did they get old Euclid's sanction? Did he concur with them that $AB \times AB = AB^2$? Surely this is neither calculation nor proof of anything.

MATH. I suppose they could not get on without a Multiplication Table, or something like it.

? 13. But $1 \times 1 = 1$ is not Multiplication, *nor anything like it.* Your's is not a sufficient answer. It is just $AB \times AB = AB^2$, which is not mathematically intelligible. One stands and has ever stood, wherever it is placed, in its integrity: it may be multiplied, by means of its position, into ten, a hundred, a thousand, and so on—but it can never be MULTIPLIED INTO ITSELF. De Morgan taxed Mr. James Smith, one of us circle squarers, with "cracking himself into lateral contiguity with himself." I have heard of "rolling two single gentlemen into one." One multiplied by one, is very like rolling a *very ttenuated* single gentleman into himself, in order to make a man of him. A *line* has neither breadth nor thickness.

MATH. Don't trifle with us. One by one equals one; and two by two are *four.*

? 14. Certainly once one is one: but $2 \times 2 = 4$ means *twice* two make four. Your Multiplication Table, like the witches with Macbeth, "palters with you in a double sense"—or rather *you* try to *trifle with me* in a double sense. The Multiplication Table honestly says that four is the square of *twice* two. Do take the pains carefully to inspect Fig. V. of the diagrams.

MATH. I see an oblong of *four* smaller squares—A B C D.

? 15. These may be quarters: but more of that anon. Are not those four geometrical squares *twice* two such squares?

MATH. They are; but—What is that to the point? Put your oblong into a square and $2 \times 2=4$. You cannot get over that. You have done it I see in your figure VI.

? 16. I dispute not that it is the square of two, *as you understand it.* It *is* a square of *four;* but I deny that it is, in any sense, the mathematical square of *two:* any more than that one=the square of one—be it inch, foot, yard or mile—is proved by the Multiplication Table or by the formula $AB \times AB = AB^2$. Did you ever hear of a Multiplication Table beginning once one is one; once two is two; once three is three, and so forth?

MATH. No: because it is not needed.

? 17. Nay, rather, because there is no multiplication at all. And what became of THREE and FOUR in manufacturing your system?

MATH. You know as well as I. They multiplied each into itself, and so *three* by *three* became *nine,* and *four* by *four, sixteen.*

? 18. Mathematical leaps worthy of a kangaroo! One to four, four to nine, nine to sixteen: *hop, step, jump,* and like gravity, "*vires acquirit eundo.*" The sinews of the kangaroo, however, are the pure creation of the Multiplication Table. But still it is guiltless of multiplying one by, or into, one. And are not what you call the squares of

3 and of 4 the square of *three times three* and the square of *four times four?* The pure geometrican gives no heed to roots and multiples, in the sense in which you apply the terms. You are very positive about the square root of one—What is the square root of *two?*

MATH. The Tables will tell you—1.4142136·.

? 19. A decimal without an end, perhaps?

MATH. Very likely. But it is close enough for all practical purposes: quite a sufficient approximation. WHAT MORE DO YOU WANT?

? 20. A VERY CROOKED question to extort from a mathematician. I have been curious enough to *test the approximation*, and have found that 1.4142136·×1.4142136·= 2.00000010642496·. Very close, indeed; OUT just, or *nearly just*, ten million six hundred and forty-two thousand four hundred and ninety-six *two hundred billionths* of one. I admit it to be a *very close* approximation.

MATH. Nearly as close as figures will give it.

? 21. Just so. But why not *exact?*

MATH. Figures are very perverse and obstinate.

? 22. But I trust not indomitable. Surely, we may do what we like with our own? We make them.

MATH. Why need we interfere with them? They serve us well, and here they could do little more.

? 23. Very likely; and I am no more inclined to assist them over the ground, than I am to test the hundreds of decimals at the end of the noted tail 3.14159, *in the hope of arriving at the tip of it.* Calculation, in the instance of the square of 2, having *overshot* the mark, is very unlikely to recoil at nothing and hit it. Convention has raised up many hundreds of figures, and kicked them right and left; and all to extract the square root of two! but *that* all to very little purpose.

MATH. To sufficient *practical* purpose.

? 24. Is not—or ought not to be—the two with its long tail of decimals *the same two* as the two the square root of four, not on the diagram, (not the little four, Fig VI.), inasmuch as $AB^2+AD^2=DB^2$ of Fig. I. and $2DB^2=4$?

MATH. (*Hesitates*).

? 25. Two, you say, is the square root of *four*, and *by extraction* 1.4142136· is the length of the diagonal D B, whose square is 2.00000010642496·. Mind! we are now using the Conventional Diagram Fig. I. This is important. You will hardly question that $DB^2=AB^2+AD^2$ as you call it, or that $BC+BC=1+1=2$ is the root of the square on the diagonal of DB^2, which therefore is equal to *four?*

MATH. And so it is. $B=1$ and $2BC=2$ and $2\times 2=4$.

? 26. I see this: but I cannot let you off so easily. There is a little—a *very little—incongruity* to overcome. Have we not seen that the square of the extracted length of the diagonal D B, or DB^2, is some two hundred billionths more than 2. Is, therefore, *twice* DB^2 or $4BC^2$ viz. 4.00000021284992·, or 4 the square of 2? I am pointing out incongruities which I hope to explain and reconcile hereafter.

MATH. I have admitted that the root is an approximation only, and we cannot expect the square to be accurate.

? 27. You admit, then, that you are, and that our ancestors from time immemorial, and their successors—sages of all ages—have in all ages BEEN CONTENTED WITH

APPROXIMATIONS IN ADAPTING TWO EXACT SCIENCES TO EACH OTHER?

MATH. That cannot be denied. I must fall back on antiquity, and reluctantly admit it.

? 28. And what excuse do you make?

MATH. Figures resist us. THEY WON'T WORK. Has not De Morgan told us that *orthodox* circle squarers—for there *are* such mathematicians, whose perseverance is beyond all praise, have found their "progress arrested by the insufficiency of their Arithmetic and the limitations of their Geometry?" Have you not read his article on "Quadrature," in the "National Encyclopædia?"

? 29. I have: and do not doubt *that* conclusion. And so will all be baffled who attempt the square root of two till they can prove that 4.00000021284992·=4.

MATH. Still it is very close. and has worked its way throughout all ages.

? 30. I do not dispute your approximations, and accept your admissions of error. Your's may be even as a hair's breadth to the orbit of Saturn; but error is error; and *accuracy* is the NECESSITY of the Mathematician. The merchant or the banker striking a balance will not tolerate the error of a shilling. It may involve hundreds or thousands of pounds. It has been related to me that the Bank of England was so particular as to send to Southampton and have several cases of an Indian remittance of specie broken open and recounted, rather than pass an error of a sovereign. Can a cracked pitcher hold water? Was it not the *last feather* that broke the back of the camel? And is it not with SYSTEMS as with camels?—To proceed. Let us now draw the line A B, Fig. II., and use it apart from its appurtenances in Fig. I. What do you understand by A B, Fig. II.?

MATH. It represents, agreeably to Euclid's first Postulate, the distance between the points A and B.

? 31. And you agree that it has neither breadth or thickness?

MATH. Of course I do. We are bound by definition.

? 32. Is it quite consistent with definition and logical accuracy to describe a geometrical area, for purposes of arithmetical calculation, and for comparison of surface with surface, and of lines with surface, by the same figure or symbol which you have already appropriated to a line which has no breadth?

MATH. I see no objection to using the same figure for one *linear* and one *areal*. It has been done all along, and is, and must be, and ever will be done.

? 33. And if so, it will be for "the ladies' reason" still—It is because it is, and it shall be because it "must and always shall be." Surely, the mathematician will never give in whatever the ladies may do! Is there any other way of connecting the line A B with surface?

MATH. Is it not a straight line? Does it not "lie evenly between its extremities?" It may be divided, it may be produced or extended, but it, nay, even "*two* straight lines cannot inclose a space."

? 34. Agreed—two cannot: they run parallel and never meet, or they cross each other. But though two cannot, one can: AB=1 can.

MATH. But AB is a straight line, and you have agreed that "a straight line cannot inclose a space."

? 35. I do not dispute that, but you have admitted that

A B is the *distance* between the points A and B. This is when it "lies evenly between its extremities." Nevertheless, the distance may be measured on a plane, and represented by any other figures, and treated geometrically and arithmetically, *de facto*, as ONE. You have admitted that it can be *divided?*

MATH. I have.

? 36. Will you look at Fig. III.?

MATH. I see that you have bisected A B in C and the bisections A C and C B in D and E; and described a square or A D.

? 37. And A D, D C, C E and E A are in the same plane?

MATH. Just so.

? 38. And as you call A B C D Fig. I, AB^2, so A D C E is ?——.

MATH. AD^2, of course.

? 39. Is the line A B Fig. I, equal to the line A D C E A Fig. III.? Is not practically the point B removed to A?

MATH. I think I understand you.

? 40. In other words: Does the right line gain or lose anything, mathematical or *metaphysical*, by being converted into the four sides of the smaller square?

MATH. I reserve my right to dispute what I have had no occasion to think of before, and I do not, at present, see my way to answer. It has created four angles and there are four corners to turn.

? 41. There are. But Plato's, "youth of talents rare"—never deviated from the line. He might turn a corner of a square too sharply and off might go the wheel, or smash go the vehicle. But it is not so in Mathematics.

You mathematicians are *very accurate*. There is no reason why you should not be able to turn round a corner at right angles, without losing or gaining the *extimum-abstractionth* of a hair's breadth; or even of a geometrical point. How came you to say that A B can be divided and still be A B. Division implies separation. If you separate A D from D C, and C E from E B, then there would be spaces between the portions of the line A B. Yet you have admitted A D C E A to be equal to AB ?

MATH. Your observation savours of "Birnam Wood to Dunsinane," but on reflection I see that it is sensible and appropriate. We are all ready to admit that to bisect A B and to bisect the bisections, is to divide A B into four, with mathematical accuracy.

? 42. Then you would represent A D. arithmetically, as $\frac{1}{4}$ or .25 ?

MATH. Yes.

? 43. And $\frac{1}{4}$ or .25 means one quarter ?

MATH. Of course.

? 44. And as the square on AB=1 Fig. I. is 1 so the square on *one quarter*=.25 Fig. III. is *one quarter ?*

MATH. Oh, no! Our ancestors from the remotest of dates, "*avont changé tout cela*." They went a very different way to work. They invented decimal fractions.

? 45. I know they did. And apparently—*Il me semble*—for the very purpose of confounding the pretensions or objections of *Arithmetic*, and making figures fit convention in defiance of fact.

MATH. Explain yourself.

? 46. You say that, to multiply a quarter into itself produces a sixteenth. Is not this *just the opposite* to Multiplication?

MATH. It is; and so it has been from time immemorial.

? 47. Again I admit the antiquity of your system, but still contend that it is founded on error; or rather that it originated in error, and has no foundation. And investigation enables me not only to challenge your Geometry but your Arithmetic.

MATH. Challenge our Arithmetic!

? 48. You say that .0625 represents a sixteenth. "Multiplication" is merely a substitute for Addition. The Multiplication Table is a mere mechanical contrivance for convenience of reckoning, and for increasing not reducing. Your process is tortuous, and its issue *inverse*, not direct. Is this Mathematical?

MATH. Multiplication of fractions, you do not seem to understand, is a very different process to the multiplication of integers—just the opposite, as you say. You will see this explained in many treatises.

? 49. I have one of these at my side. This, far from discouraging me, establishes me in my opposition: it is "A Treatise on Arithmetic, in Theory and Practice, for the use of the Irish National Schools. Published by the direction of the Commissioners of National Education, Ireland. 1875." *Qu'est ce que c'est que* Theoretical Arithmetic?

MATH. Look at your Dictionary. "Theoretical" is "Pertinent to Theory or dependent on it; speculative, not practical."

? 50. Just as I supposed. But is not Arithmetic considered to be a science not only exact, but *purely prac-*

tical? The very title of this book savours of a modern consciousness that—as Sheridan might, not "*Malaprop*"*riately* have insinuated, there is "a little ingenuity and artifice" at work in your reckonings. But, to my quotations—line by line, that there may be no mistake—from page 97:—

"78. The smaller the divisor the larger the quo-
"tient—for, the smaller the parts of a given qnantity,
"the greater their number will be; but 0 is the least
"possible divisor, and therefore any quantity divided
"by 0 will give the largest possible quotient—which
"is *infinity*. Hence, though any quantity multiplied
"by 0 may be considered equal to 0, the multipli-
"cand being infinitely small; any number divided
"by 0 is equal to a number infinitely large.

"It appears strange, but yet it is true, that $\frac{5}{0}=\frac{1}{0}$
"for each is equal to the *greatest* possible number,
"and one therefore cannot be greater than another—
"the apparent contradiction arises from our being
"unable to form a true conception of an *infinite*
"quantity. It is necessary to bear in mind that 0,
"in such cases, indicates a quantity infinitely small
"rather than absolutely nothing."

From the time when I published, in 1846, a treatise challenging the soundness of the Newtonian Theory—being confirmed in my objections to it by the perusal of Whewell's Bridgewater Treatise—though I may have been considered to be extremely heterodox in matters of physical philosophy, albeit I believe my views to be sound—I do not think that I have ever perpetrated and put to paper anything so MATHEMATICALLY CURIOUS, and

SO CONTRADICTORY TO FACT, as this. Wherefore—whilst we have the "Order of things," the "Laws of Nature," and simple figures to direct us—I may be permitted to refer to and be guided by them, rather than modern treatises such as these, which I will venture to say—however cute—would have astonished Euclid or Cocker.

MATH. You are as perverse and obstinate as the very figures themselves.

? 51. As becomes the nature of the ? in search of the truth and *in defence of the figures.* When I said that "we make them," and "can do what we like with our own," I did not mean to despise them nor to insinuate that we are at liberty to *pervert* them from their proper functions and purposes, in order to support convention in exceeding its limitations. Figures, once constituted and their value established by convention, have an authority to regulate convention, quite independent of it—although always at command in maintaining what is right against what is wrong.

MATH. To show how we have perverted them, we ought to have your calculations. We should like to see your method of multiplying fractions.

? 52. And so you shall, in good time. But first let me ask, OF WHAT is .0625 the sixteenth?

MATH. Of AB^2, the square of A B, the Conventional square Fig. I.

? 53. That square, the square ON A B, is no more the square OF A B than it is the square of W X Y Z. Our ancestors in establishing their system—*with the square before them, it may reasonably be presumed*—overlooked the fact that it required the sides B C, C D, and D A to construct it; and thus the PERIMETER, a MOST IMPORTANT element of the problem before them, was ignored. After this observation, I will ask you to follow up my reckonings, both geometrical and arithmetical, beginning with AD^2 Fig. III., the sixteenth of what you call AB^2 Fig. I., but which I call the square A B C D or A B C D A=4.

MATH. Stop! You have said that AD^2 Fig. III. may be the unit of surface, and A D Fig. III. THE SQUARE ROOT OF ONE! Is this consistent—and if so, How so?

? 54. Because it is the extreme square area which the line A B=1 will inclose. Do you admit that it is? Remember that I have said there are more ways than one to find the SQUARE ROOT OF ONE.

MATH. I do not see that there can be any doubt about the fact that A B can inclose no larger square than AD^2 Fig. III. It is so by construction.

? 55. Then I take my first stand in this redoubt. There is a *possibility* that I am laying a foundation for a combined geometrical and arithmetical *real* square, and have found one method of expressing its root.—But this Fig. III. has shown *breadth* in *connection with length;* and that the *breadth* which the area A B C D Fig. III.=1 will develop, is contained in the square *of which one quarter is the side*—and consequently, if the side be the root and the root the side, that AD^2 Fig. III. being an area of one quarter, and $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{4}{4}$ being *one* (not $\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$), AD^2, Fig. IV. may be the area of ONE, or of those four quarters, and consequently, not A D Fig. III., nor A B Fig. I., but *A D Fig. IV.* or *A E Fig. I.*, HALF not *a quarter*, may be THE SQUARE ROOT OF ONE in relation to the original line A B. Figure VII. will guide us as we proceed on this basis. We will begin with the small square in the middle, which is equal to the square AD^2 Fig. III., whose area shall be *one quarter*.

MATH. I admit that you have Euclid's authority for constructing the Figure VII., which you have no doubt done according to his forty-seventh. You seem to be aware that the square on the hypothenuse of two of the sides of a square is equal to the square of those two sides.

? 56. I have had this in mind throughout, in order to analyze the square A B C D A Fig. I., and having applied the principle to find the proportion of $RSTU = AD^2$ Fig. III. to the square A B C D Fig. VII., I commence with my reckoning that R S T U $=\frac{1}{4}$ area.

Fig. VII.		Fig. III.
R S T U=one quarter	area=	$AD^2=\frac{1}{4}$
N O P Q=half	„	$2AD^2=\frac{2}{4}$
I K L M=one	„	$4AD^2=\frac{4}{4}$
E F G H=two	„	$8AB^2=\frac{8}{4}$
A B C D=four	„	$16AB=\frac{16}{4}$

According to the foregoing reckonings, what you call the square of one may be the square of *sixteen*, for which I do not contend, leaving that as an "open question" for future consideration. I hold more strongly to its being a square of four—for which I do contend, believing that value to be in accordance with all the elements of our Problem and all the requirements of Arithmetic and Geometry. But this is not a complete and satisfactory process, for reasons at which I have hinted already.

MATH. Be more particular.

? 57. I have observed that a decimal system of notation

—or notation in a series of ten, whether integral or fractional—does not suit quadrature or measurement in squares; and, moreover, that the square is not a suitable measure for determining the superficial area.

MATH. Show this by some process of demonstration.

? 58. The unit A B may be used to *measure* a surface, not by the area which it encloses, but by the surface which it will measure *without* enclosing it. Nor is this incompatible with its measuring the area after it has been determined the other way.

MATH. Proceed with your demonstration how there may be superficial measurement, as you call it. If I draw A B Fig I. as a measure, keeping it all the while parallel to the side till it reaches the opposite side, and then reverse the process and measure the square in a direction perpendicular to the former, I shall find that the square contains a surface equal to the unit *every way*, and consequently it follows that, *as I call it*, the square is, and must be, *a unit square*, whether that unit be an inch, a foot, a yard, or a mile.

? 59. Your ideas of measurement and mine are much the same—but your conclusion unsound. Your surface is *not* the measure of the square every way. You have taken no account of the diagonal of the square, which is longer than your measure.

MATH. If there *could be* any *rational* objection to our system, I might say that this is feasible. How would *you* measure the square and find its relation to the linear unit?

? 60. I must be careful with this answer, otherwise I may expect that mathematicians, if there should be any who can have condescended to travel with me thus far, will pitch my little book into the waste-paper basket or the devouring element, as the due punishment of a libel on the tenth of mankind, which is said to be competent to "understand it," but are "pledged to disbelief." But as I hold myself perfectly at liberty to ask questions, I feel myself bound to answer as well as raise them. Cautiously, therefore, I proceed. I would place the rod, or measure, A B with its middle point in the middle of the square, and would move it from F to H in the direction of E. In this direction it would describe, from F to H, half of a certain well-known but perplexing geometrical figure; and at the same time from H to F, viâ G, the remainder of the said figure, making a curvilinear unit, containing a certain surface in relation to the square, which may be determined either according to the conventional method of reckoning in relation to one, or, according to my reckoning in Section 56, in relation to four. Now this process of measuring surface leaves four corners of the square intact, and therefore I hold that, apart from the value which we may give to the area, there *must be* a difference between actual superficial measurement and measurement in squares by the line A B=1. Q.E.D.

MATH. Still I believe that square and superficial have been terms identical in science, and that a difference between them has never been mooted.

? 61. Nor discussed. Hence the necessity, *perhaps*—"la philosophie ordonne que nous doûtons de tout"—for a ?. A great, and in your opinion an insuperable difficulty has baffled the ingenuity and mathematical acumen of sages in all ages. In the Asiatic Journal you will find that the Hindoos were hard at it in the remotest periods of heathen mythology. This difficulty may have been overcome by one of them, as I have endeavoured to show in my treatise on the difference between square and superficial measurement, published in 1865. The difficulty to which I allude has been to find the length described by the ends of the rod or measure, during the process which I have just indicated, by which we should arrive at the perimiter of that unit, and its actual measurement every way, neither more nor less. This subject has already been treated in the book just alluded to, and in the short pamphlet published in 1875, which will be enlarged in this series of papers, No. 2, if it should ever see the light.

MATH. You are opening my eyes, and I am strongly tempted to close this discussion and commit your paper to its proper doom. But, to tell you the truth, many of us have puzzled our brains until we have persuaded ourselves—ourselves not having succeeded—that the thing is impossible. Still, as orthodox Mathematicians have set their minds to the task, and I cannot but admit that you have raised objections to our system which may be worth considering, I feel disposed to forego my stern opposition, and try my hand at the knotty problem myself. With a little Algebra thrown in to assist in digesting your views, and the exercise of a little more patience than we are generally willing to bestow on novelties discovered by others, I may find that these impracticable decimals may be subdued, and a little perseverance may end in success.

? 62. You possibly are in the right way—but not so advanced as the late Professor De Morgan of the Athenæum, who, very likely, went farther into the matter than he would have been willing to admit in the face of the "Upper Tenth" in the regions of science, who are supposed by the critic in "Iron" to be "pledged to disbelief." Although leagued with this section of Philosophers, the late Professor had advanced so far that, whilst declaring the thing to be impossible, he was able to shew HOW *the thing* COULD *be done;* and *that* in terms convertible into a very simple formula.

MATH. If you discovered this, Why did you not bring your researches to bear, do it, and set the question at rest?

? 63. I did. I pursued his directions carefully to the letter. I reduced the rule which he had suggested to a formula, and found a diagram fitted to carry it out; told the Mathematician how De Morgan had provided that the reckoning should proceed; left him to choose his own length for the line, and even gave him the choice of a decimal, an integer, or a mixed number to start with; and assured him that I had never been foiled by an impracticable decimal as the result: showed him, in fact, how all could be done in De Morgan's way—and how De Morgan was proved to be right.

MATH. And with what result?

? 64. The Mathematician put his glass to the blind eye, and could not see it; or (perhaps a better illustration) either Algebra, which you expect to assist you, or *prejudice* put the cap on the telescope. *If* it be truly said of the *competent tenth* of mankind that they are "pledged to disbelief," they "kept the pledge" like the truest of teetotallers. But there must be a truce to these digressions. I will only add, for your encouragement, what the "Upper Tenth" probably are not aware of, that our ancestors and their successors, sages in all ages, had overlooked that the square has a perimeter, and that our conventional square is not the square of A B, but the square of A B C D or A B+B C+C D+D A.

MATH. I feel curious to learn how you would multiply fractions.

? 65. I would use the Multiplication Table only for its legitimate purpose — that of facilitating addition by shortening the process. The Table cannot be a medium of Mathematical Homœopathy for reducing fractions to infinitesimal proportions. Fractions are the means used for expressing proportions, and have only proportionate value as *subsidiary figures*. Take the instance of $\frac{1}{4}$. The denominator has its value, but it means not four but a *fourth* of four. The numerator indicates the number of fourths as $\frac{1}{4}$th, $\frac{2}{4}$ths, $\frac{3}{4}$ths, $\frac{4}{4}$ths, and as $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{4}{4}$
so $\frac{1}{4}\times\frac{1}{4}=\frac{4}{4}$
and four-fourths are one. I cannot multiply one-fourth into itself, inasmuch as, like the integer, $\frac{1}{4}$ cannot be multiplied into $\frac{1}{4}$. As regards the attempt to do it, by saying $\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$, this is more exquisitely mythical and unmathematical than multiplying one into itself. It is not rolling a quarter into a quarter; but "putting a gallon," not a quart, "into a pint pot." It was said, of Pitt I think, that he would, if he could, have "taxed the air he breathed." There would be strong objection raised to the Chancellor of the Exchequer minimizing the National Debt by decimating it and multiplying the

fractions into themselves. How did our ancestors arrive at a *sixteenth ?*

MATH. Very likely, much in the same way as you arrived at your A D^2 Fig. III., the basis of your reckoning in Section 56; and, having quadrisected the line A B, they found B C, C D, and D A to quadrisect; they did this: and then, joining the several points of quadrisection with their opposites, they found that they had subdivided the conventional square into *sixteen* smaller squares.

? 66. That is to say, They *changed the units* — A B became 4 and the area $4 \times 4 = 16$—and this suggested the necessity for an expedient to overcome the oversight of three-fourths of the perimeter, seeing that they had predetermined that the area of A B C D should be the unit of square measurement. How do you suppose that they proceeded?

MATH. They seem next to have fallen on the decimal scale, your Fig. VIII., and worked by tenths and no longer by quarters.

? 67. For some reason or other they *again* changed their units. A B became *ten* and not four. Ten linear tenths were equal to *one* linear: and ten-tenths multiplied by ten-tenths gave one hundred tenths or $\frac{100}{10}$, ro ten.

MATH. Not so; $\frac{10}{10} \times \frac{10}{10} = \frac{100}{100}$: one hundred *hundredths.*

? 68. This, like $1 \times 1 = 1$, is not multiplication. But $.1 + .1 + .1 + .1 + .1 + .1 + .1 + .1 + .1 + .1 = \frac{10}{10} = 1$ is the area of the uppermost oblong of the Figure IX.

Scale ten to the unit.

Linear ... $.1 + .1 + .1 + .1 + .1 + .1 + .1 + .1 + .1 + .1 = \frac{10}{10} = 1$

AREA.

(By your reckoning $.1 \times .1 = .1$.) First oblong.

$$.1+.1+.1+.1+.1+.1+.1+.1+.1+.1=\tfrac{10}{10}=1$$

Nine oblongs.

$$.9+.9+.9+.9+.9+.9+.9+9.+.9+.9=\tfrac{90}{10}=9$$

$$1+1+1+1+1+1+1+1+1+1= \quad 10$$

Having inspected this, Suppose you divide the scale of tenths into four parts. What expression would you give to the quarter of $AB=\tfrac{10}{10}=1$ linear?

MATH. On any scale a quarter would be represented decimally by .25.

? 69. Twenty-five what?

MATH. 25 hundredths.

? 70. Changing your units again! Mark! First AB=1; then AB=4; then AB=10; then AB=100. But to return to your calculations. Would not the result of multiplying the quarter or twenty-five hundredths into itself be *twenty-five* times *twenty-five* or six hundred and twenty-five *hundredths* $\tfrac{25}{100} \times \tfrac{25}{100}$ (my way) $\tfrac{625}{100}=6\tfrac{1}{4}$, or $.25 \times .25 = 6.25$ or $6\tfrac{1}{4}$, AS BY CONSTRUCTION?

MATH. No; assuredly not. You mistake the thing altogether. When you multiplied .25 by .25 there were two places of decimals in the Multiplier and two in the Multiplicand. You ought to have put a cipher to the left of the 625, and a decimal point to the left of that, to make up the four places, and then the fraction would have stood .0625.

? 71. Does not this mean that you multiply the 625 by ten and that the result is $\tfrac{625}{1000}$?

MATH. Certainly not. Having put a cipher to the left of the numerator you must add two to the right of the denominator, and the fraction stands thus: $\frac{625}{10000}$.

? 72. Or rather $\frac{0625}{10000}$?

MATH. Well! But it is the same thing. A cipher to the left is nothing.

? 73. Then why did you put it there?

MATH. To enable me to add the two ciphers to the denominator.

? 74. And this accounts for $AB = 100$, and $AB^2 = 100 \times 100 = 10000$?

MATH. But do you not see that $\frac{625}{10000} \div 625$ the greatest common measure, is equal to $\frac{1}{16}$? Does not this result shew you the beauty and accuracy of our system? After an elaborate and intricate process, we arrive at the fact that the area of a quarter *is* a sixteenth.

? 75. You could have done without it: $1 \div 16 = .0625$. Rather I find—and the more closely I look into the subject the more clear it appears—that by a tortuous and inverse process, by changing your units and "cooking your accounts," you have arrived at a result which is contradicted by the adaptation of Arithmetic to Geometry, as explained in Section 56. That has advertence to the perimeter which you ignored, and to respect which is essential to the faithful adaptation of the sciences to each other.

MATH. You have given some explanation of what you mean by "tortuous and inverse;" but what *can* you mean by "cooking the account?"

? 76. In the progress of your elaborate and intricate process you arrived at $\frac{625}{100}$. In order to convert this into 625 something—*what* is not very clear—you put a cipher and decimal point to the left of the numerator .0625. To convert this $\frac{.0625}{100}$ into a vulgar fraction, you puffed out the .0, and simultaneously, with the same breath, you puffed in the light, and $\frac{625}{10000}=\frac{1}{16}$ was the result! The rationale of this process certainly is not obvious; and, if it can be explained in the treatises to which you have referred me, it must be done, I should think, through the medium of "*theoretical*" arithmetic. *Is it one* of those cases in which the Irish book, already quoted, tells me that "o indicates a quantity infinitely small, rather than absolutely nothing"? In such a case even the decimal of .o would not have been so easily puffed out. But—pardon me—I remember that you told me (Section 72) that o was good for nothing, and I asked why you put it there (Section 73).

MATH. Nor can I give any answer other than before: it was put there because it was necessary that it should be there.

? 77. I think that, now, I am safe to hold that my suggestion, in Section 45, that decimals were invented "for the very purpose of confounding the pretensions of arithmetic and making figures fit convention in defiance of fact," is proved to be *founded on fact.*

MATH. I should like now to see how you work out your Fig. IX.

? 78. In Section 68 I have already shewn how, by eschewing the Multiplication Table, I find, by a very simple process of demonstration, that the square contains a hundred tenths, not a hundred hundredths; and that the square E F G H=$\frac{10}{10}$ths or 1, on a scale of ten to the unit, the conventional ratio of numbers.

MATH. But I should like to see how you proceed after having divided the Scale Fig. VIII. into four parts.

? 79. Of course I reckon the sections of Fig. IX. as tenths, and instead of writing .25 and calling it $\frac{25}{100}$, I would write it down 2.5 or $2\frac{1}{2}$ *tenths*. It will now simplify the explanation to put the result into a tabular form, thus—

SCALE TEN TO THE UNIT.

All Tenths.

				Tenths.
$6\frac{1}{4}$	$6\frac{1}{4}$	$6\frac{1}{4}$	$6\frac{1}{4}$	$=25=2\frac{1}{2}$
$6\frac{1}{4}$	$6\frac{1}{4}$	$6\frac{1}{4}$	$6\frac{1}{4}$	$=25=2\frac{1}{2}$
$6\frac{1}{4}$	$6\frac{1}{4}$	$6\frac{1}{4}$	$6\frac{1}{4}$	$=25=2\frac{1}{2}$
$6\frac{1}{4}$	$6\frac{1}{4}$	$6\frac{1}{4}$	$6\frac{1}{4}$	$=25=2\frac{1}{2}$

Tenths $25+25+25+25=\frac{100}{10}=10$

The $6\frac{1}{4}$=your $\frac{1}{16}$ or .0625 or $\frac{625}{10000}$.

MATH. I see a mass of incongruity here, with reference to what has gone before.

? 80. Apparent, not actual. The following represents the same thing on a different scale:—

SCALE FOUR TO THE UNIT.

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$=\frac{4}{4}=1$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$=\frac{4}{4}=1$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$=\frac{4}{4}=1$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$=\frac{4}{4}=1$

$$1+1+1+1=\frac{16}{4}=4$$

The results entirely accord with the basis of reckoning suggested in Section 56, by which the actual square is to the conventional square as 1 to 4, as 25 to 100 on the first table, and as 4 to 16 on the second.

MATH. But you have given us another square, which you call $\frac{10}{10}$ or 1—viz., E F G H in the middle of Fig. IX.

? 81. I am not surprised at your objection. And my answer may still savour of "Birnam Wood and Dunsinane." But it is not inexplicable. $EF^2 = .3 \times .3 = .9$

$$+ \quad .1 \qquad .1$$

Square on Hypothenuse E F $\frac{10}{10}$ or 1.

I have designated E F in putting forward its claim to be THE SQUARE ROOT OF ONE as E F? It is, according to the tables, the interminable $\sqrt{10}$ or 3.1622777·. In my publication of 1865, I have gone, at great length, into this matter, and endeavoured to shew that the $\sqrt{10}$ is 3.2., and that as the enclosing square (contending for the claims of the perimeter) is 16 and $3.2 \times 4 = 12.8$, and the value of the one to the other is as 128 to 160 and $\frac{128}{160} \div 32 = \frac{4}{5}$, the one may be the *superficial* square in contradistinction to the conventional square.

MATH. To those who are "greedy of novelty" you have certainly suggested no small amount of innovation.

? 82. And I have run to the full length of my tether. I have suggested *three ways* of finding a SQUARE ROOT OF ONE, and I hope have kept within the requirements of Arithmetic and the limitations of Geometry.

MATH. Am I to understand that we are to multiply all our areas by four, and destroy the calculations of ages?

? 83. Very far beyond that. The latter is a very crooked question to ask, when you have the whole world at your back against a ? You have to settle that matter. But I refer you to Section 9; to the glaring fact that you have "gone *a little* wrong," and to the suggestion that closes it. Also to my observation at the commencement of Section 18. I have admitted that your reckonings are approximations, but they are *approximations to a multiple* to which I alluded. Pursuing the principles of Section 56, we may find that the false step at starting, independently of the original error of substituting one for four, by ignoring three-fourths of the perimeter, may lead to the discovery that, as contemplated in "Hints and Suggestions" regarding Fig. I., the result may possibly be, that "the measure of the perimeter may be the extreme length which can be used to determine the area of a square in arithmetical figures, arising out of the linear unit, and that the leaps of the Multiplication Table, gaining strength as they proceed in "Geometrical progression," "producing" a line *without a surface to inclose in it*, may be attended with "egregious error." This question will naturally arise for discussion in Paper No. 3, when we shall be treating of actual units rather than proportions. New complications will arise which, when unravelled, may reconcile the two exact sciences, and, by eschewing the Multiplication Table for all purposes but the legitimate one of facilitating the process of addition—may end in establishing a "Geometrical Progression" *in the same ratio as the "Arithmetical,"* the practical application of which would lead to very startling results.

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