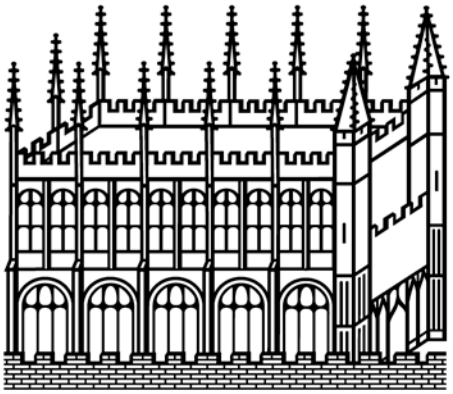
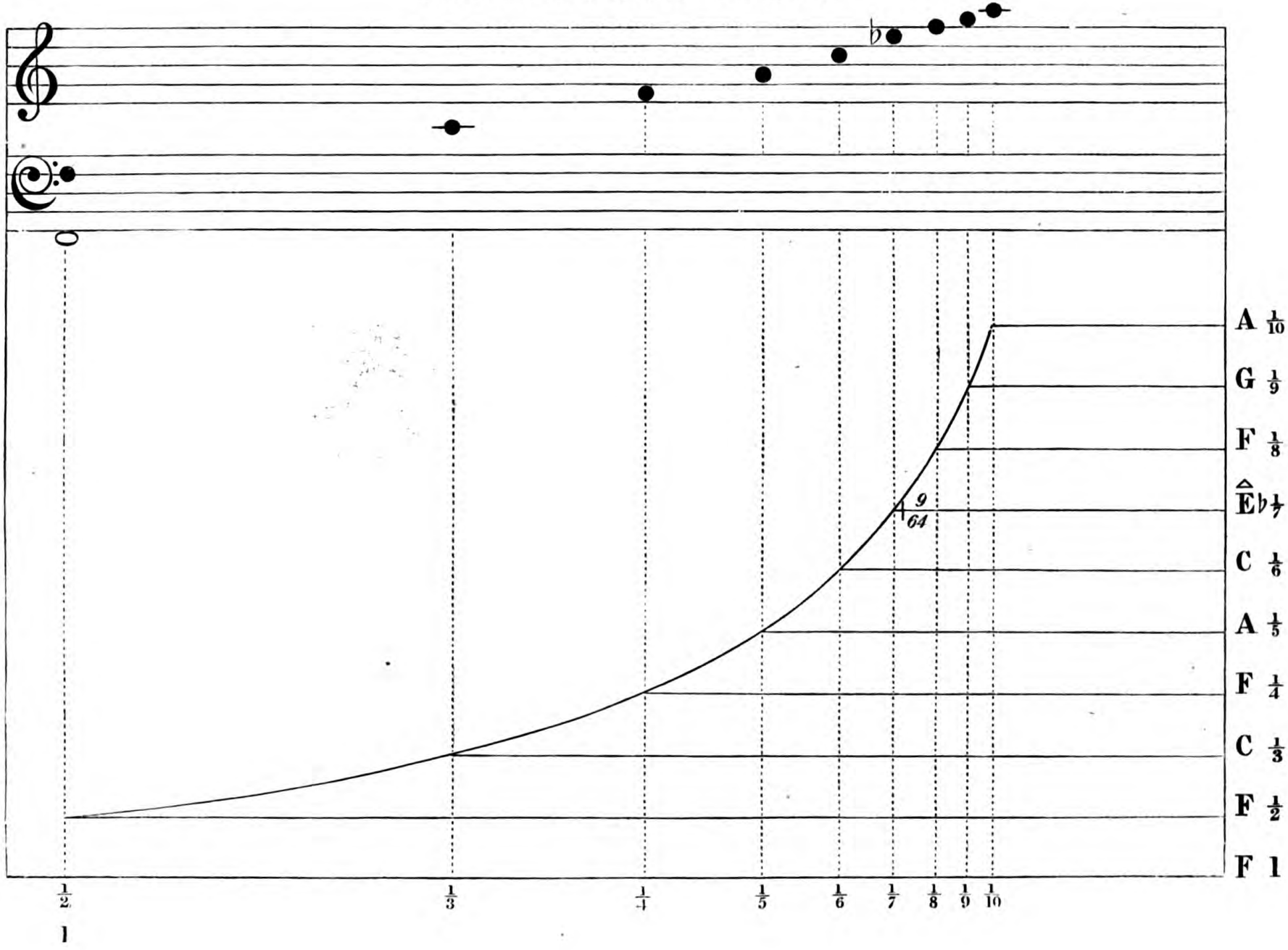


BROWN'S ELEMENTS OF MUSICAL SCIENCE

THE HARMONIC CURVE.



ELEMENTS

OF

MUSICAL SCIENCE.

BY

ROBERT BROWN.

"It must be considered, that no man who undertakes merely to methodize or illustrate a system, pretends to be its author."—JOHNSON'S LIVES OF THE POETS.

"There is not a more effectual way of learning the rules of any art, than by attending to the practice of those who have performed in it most successfully."—BEATTIE ON TRUTH.

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DEDICATION.

TO

SIR JOHN F. W. HERSCHEL, BARONET, K.H., F.R.S.,

&c. &c. &c.

DEAR SIR JOHN,

IT is with sincere gratitude that I acknowledge your kindness, in permitting me to prefix to the following pages the illustrious name of HERSCHEL.

As every true friend to Science, who ventures to appear as an author, must be extremely desirous to have his principles well considered with regard to their *truth*; so I am persuaded that nothing will conduce more to this, than appearing under the sanction of a name to which Science and Philosophy are so deeply indebted.

I have the honour to be, with much regard,

DEAR SIR JOHN,

Your obedient humble Servant,

ROBERT BROWN.

ROCKHAVEN, 11*th April*, 1860.

PREFACE.

THE design of the following work, is to encourage and promote the study of Harmony. The method which the Author has been led to adopt, appears to him to possess advantages, both theoretical and practical, sufficient to warrant a publication, which would otherwise be justly considered superfluous.

The great practical advantage aimed at, consists in simplifying the study by reducing all the combinations of notes to their simplest elements, which, in general, do not exceed five or six Chords, or compound sounds; and in the most elaborate music, eleven or twelve: and by representing those sounds to the eye, in a running commentary under the Bass. By this method, the student is gradually taught to analyze the works of the Great Composers for himself.

The example of writing the Fundamental Harmony on a separate line, was set by Mr. Logier, in his valuable work on the "Science of Music and Practical Composition;" and especially in his published extracts from the works of Classical Composers, entitled "Theoretical and Practical Study for the Pianoforte," which are invaluable to a student of Music. But in these works, the chords over the Bass are expressed by the figures of Thorough Bass, which denote merely the places occupied by the notes on the Staff. The Author's improvement on Mr. Logier's method (which he considers of great importance), consists in expressing the Harmony over the Fundamental Bass, by signs which represent the true Chords or Sounds.

The student who has learned this method, will find it a valuable key to the writings, as well as the compositions, of experienced Harmonists; and if he copy out their examples,

analyzing them in this way, he will be saved from much irksome labour and discouragement afterwards.

The fundamental Chords are few in number, and the Rules for treating them, few and simple. The number of Chords is much increased by Inversions: and new Rules being given for each Inversion, the Rules are so multiplied as to become a burden on the memory, until long practice has rendered them familiar. But by referring every chord to its fundamental or direct form, and exhibiting that form to the eye, the Rule for treating the direct Chord, is applied with ease to all its Inversions.

For the sake of readers whose sole object is Practical Harmony, the Theory is chiefly reserved to the Appendix; and no more of it introduced into the body of the work than is necessary to explain the derivation and proportions of the Chords. The simple calculations which are required for this purpose, may be passed over by those readers who have an invincible dislike to figures; and prefer setting out from first principles understood by the ear, such as Octaves, Fifths, Major Thirds, &c.

The Theory of Harmony, as it is here treated, is extremely simple; requiring no skill in mathematics, but quite intelligible to any one who is familiar with the simplest rules of arithmetic. As for the Logarithms that are employed for the purpose of shortening some of the calculations, the reader has no occasion to consult Logarithmic Tables; but if he will take on trust the numbers that are given as the Logarithms of the four primary Intervals, Octave, Fifth, Major Third, and Grave Seventh, he will soon find sufficient proof of their correctness as he proceeds to combine them, by Addition and Subtraction, in the composition of the other Intervals.

The Theory of Harmony is, in this work, established on the basis of known facts in Natural Philosophy. The various Chords, with their Resolutions, instead of being, for the most part, referred to the ear of the musician, are subjected to a strict analysis, and described in arithmetical proportion, the only true measure of such quantities. By means of this analysis, the author has been enabled to trace the derivation of the other combinations of sound from the two Harmonic Chords; to investigate the subject of Intervals; to show

the effect of Modulation by perfect Fifths, on the pitch of notes; to demonstrate the true nature of the Chromatic Scales; to explain the different methods of Temperament; and to throw light on some unsettled controversies. He has attempted to demonstrate, that the true foundation of Harmony is the Harmonic Scale, which is the spontaneous production of vibrating strings and tubes; that the Scale of Triads is deduced from the Harmonic Scale; and the Diatonic Scales, both Major and Minor, from the Scale of Triads. In following out this principle, the result at which he has arrived is this: that with one or two exceptions, which are specified in their proper places, all the approved Chords and Intervals of Music may be obtained from the notes of three Dominant Harmonies, taken on the notes of one Harmonic Triad or Common Chord.

The seven unequal degrees into which a correct ear divides the Octave, cannot be accounted for, except by considering that the notes which produce them belong to the Chords of a Fundamental Bass suggested by the ear. Hence it appears, that Chords are prior, in order of nature, to the notes of the Scale. It has been assumed, however, that the Diatonic Scale is the foundation of all Music; not only of Melody, but of Harmony also. As a consequence of this assumption, the Octave of the Subdominant or Fourth of the Scale, has been commonly held to be the Seventh that is added to the Triad, in the Dominant Harmony; and the prime number 7, not being found in the Diatonic Scale, has been excluded from the system; by which means, the common theory has been rendered not only defective, but erroneous. The true Harmonic Seventh 4 : 7 has thus been supplanted by the Diatonic Interval G F, 9 : 16; a false Interval, which is at once detected on applying the test of arithmetical proportion. In consequence of this error, the writers on Theory, when they treat of the Dominant Seventh and its derivatives, immediately lose their way among false Intervals, and are compelled to leave the musician to the direction of his ear.*

For instruction in the higher branches of the Musical Art, the author refers his reader to the writings of professional men, who are the proper masters of the subject. He would

---

* See Appendix M.

gladly have restricted his work to the mere elements: but in order to make it intelligible to readers who have not studied Harmony, he found it necessary to take a cursory view of the principal branches commonly treated in works on that subject; for which he is, of course, indebted to the practical writers. He has borrowed largely from Dr. Callcott's incomparable Grammar, which ought to be the pocket companion of every student of Music. He begs leave to express his obligations to Mr. Horsley; whose recent work on Harmony, with the separate exercises on Thorough Bass, should be in every one's hands: to Professor Donaldson, for his interesting and instructive lectures on the Theory of Music, in the University of Edinburgh; and to his worthy friend Mr. W. H. Callcott, for much kind and valuable assistance.

ROCKHAVEN, 22*nd June*, 1860.

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EXPLANATION OF SYMBOLS USED IN THIS WORK.

INTERVALS . . . . . + Major.
− Minor.
× Augmented.
÷ Diminished.

CHORDS*. . . . + Major Triad.
− Minor Triad.
5 ÷ Diminished Triad.
7 + Dominant Seventh.
7 ÷ Diminished Seventh.
6 × + Augmented Sixth.

DELTA . . . . . Δ over a letter, denotes the Grave or Dominant Seventh, distinguished from the Minor Seventh; as $\overset{\Delta}{F}$.

ONE CLEF.

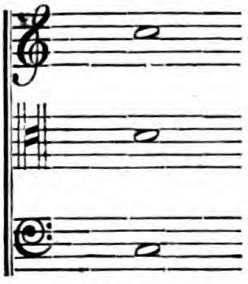
F Altissimo, 3 Octaves above the Bass.

F Soprano, 2 Octaves above the Bass.

F Alto, 1 Octave above the Bass.

F Basso.

TWO CLEFS.



C Treble.

C Tenor, an Octave below the Treble.

C Bass.

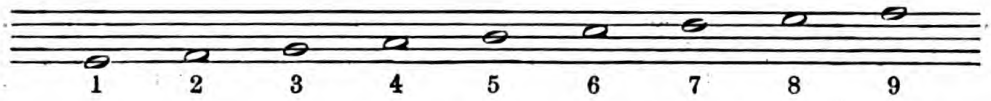
* See Chap. XII. at the beginning, for more Signatures.

ELEMENTS OF MUSICAL SCIENCE.

CHAPTER I.

NOTATION.

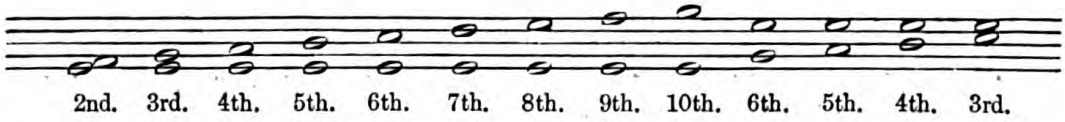
1. The Notes of Music are represented to the eye, upon a scale called the Staff, consisting of five horizontal lines; which, with the four spaces between them, contain nine Degrees:



These Degrees are not counted from any determinate point, but from one note to another. They are always counted upwards, when the contrary is not specified.

2. When more degrees are required, the Scale is extended by means of short temporary lines, above or below the Staff, called Added lines, or Leger lines; thus:—

3. The difference of pitch between two notes, is called an Interval. The Intervals take their names from the Degrees on the Staff, including both extremes. Thus, from 1 to 2 is called a Second; from 1 to 3 a Third; from 2 to 3 a Second; and so on.



4. These names are to be carefully distinguished from the fractional parts of a string; to which they bear no analogy.

5. The degrees of the Staff being too few to express all the necessary Intervals, the name of the same degree is given to several Intervals, essentially different from each other, and distinguished by the epithets Major, Minor, Augmented, Diminished, &c.

6. To render those distinctions visible, instead of leaving them to be inferred from the context, I use the common arithmetical signs; as follows:

\+ is read Major.
− ,, Minor.
× ,, Augmented.
÷ ,, Diminished.

7. The terms Major and Minor, belong properly to Thirds, Sixths, and Sevenths; which are the characteristic Intervals of the Major and Minor Modes. The term Augmented, is applied to Perfect, or Major, Intervals, increased by a Chromatic Semitone; and the term Diminished, to Perfect, or Minor, Intervals, decreased by the same.

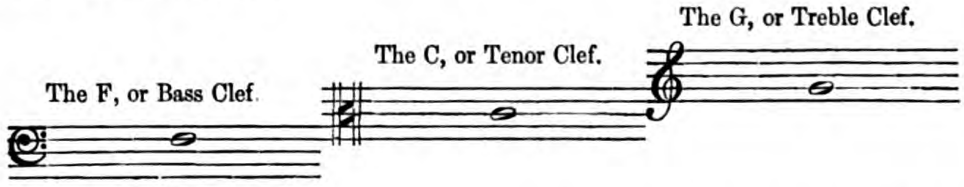
8. The Notes of Music take their names from the first seven letters, ascending in alphabetical order. When the series exceeds seven Degrees, the same letters are repeated:

A	B	C	D	E	F	G,	A	B	C	&c.
1	2	3	4	5	6	7	8	9	10.	

9. A figure called a Sharp, ♯, placed before a note, raises it a Chromatic Semitone; a Flat, ♭, lowers it as much; and a Natural, ♮, by removing the Sharp or Flat, restores the original note. This refers to the notes of the Scale.*

10. The Degrees of the Staff do not, of themselves, indicate the pitch of the Notes: but a figure or symbol, called a Clef (*Clavis*, a Key), placed at the beginning of the Staff, determines the pitch of a particular note, from which the rest are counted. Hence, no doubt, the names Key, and Key-note.

11. The Clefs are three in number; representing the notes F, C, and G: the Tenor Clef note C, being about the middle of the ordinary compass of human voices; F a fifth below, and G a fifth above it:



12. When the Tenor Clef is placed, as above, on the middle line of the Staff, it is called the Alto, or Viola Clef. The upper two lines of the Alto correspond to the lower two lines of the Treble; and the lower two lines of the Alto to the upper two lines of the Bass: the Tenor Clef note occupying the middle line, which corresponds to the Added line between the Treble and the Bass.

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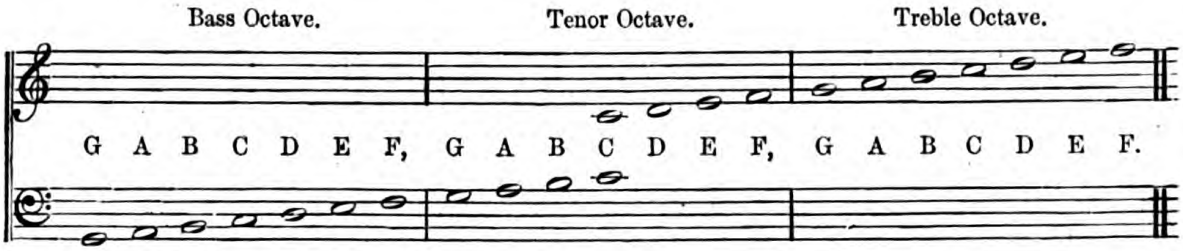
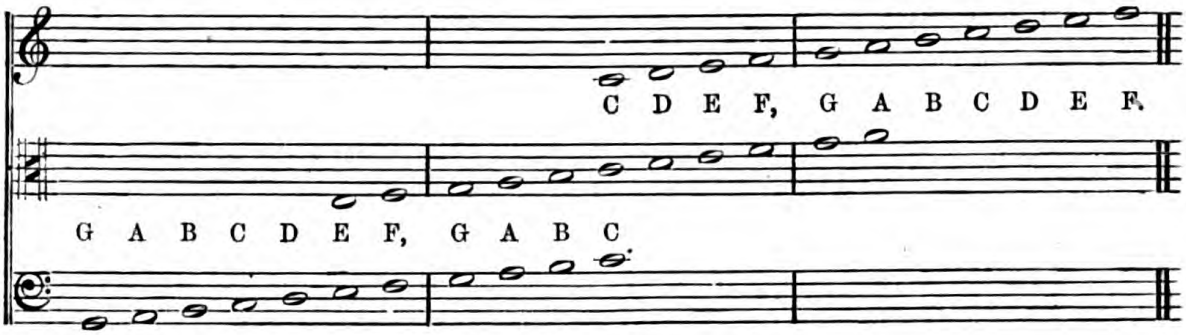
* The Grave Seventh, which is also expressed by a Flat, will be explained in its place. See Chapter X. 24.

13. The Treble and Bass Clefs are used in music for keyed instruments. The ascending Scale, in those Clefs, is as follows:



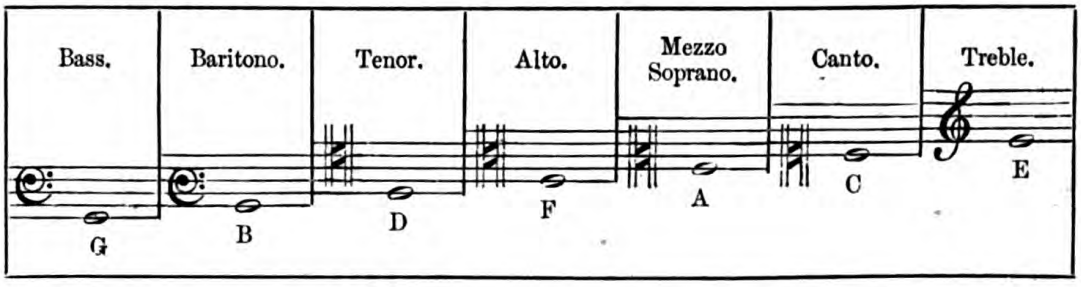
In this arrangement, each Degree of the Staff represents two different notes.

14. The same Scale is thus written in the three Clefs:



By this arrangement, each degree of the Staff represents three different notes.

15. This happy invention, however, was not reckoned complete, till each Degree of the Staff was made to denote, in succession, every note of the Scale!



16. Thus an artificial difficulty was introduced into the study of Music, which, so far as I know, has no parallel in any other science.

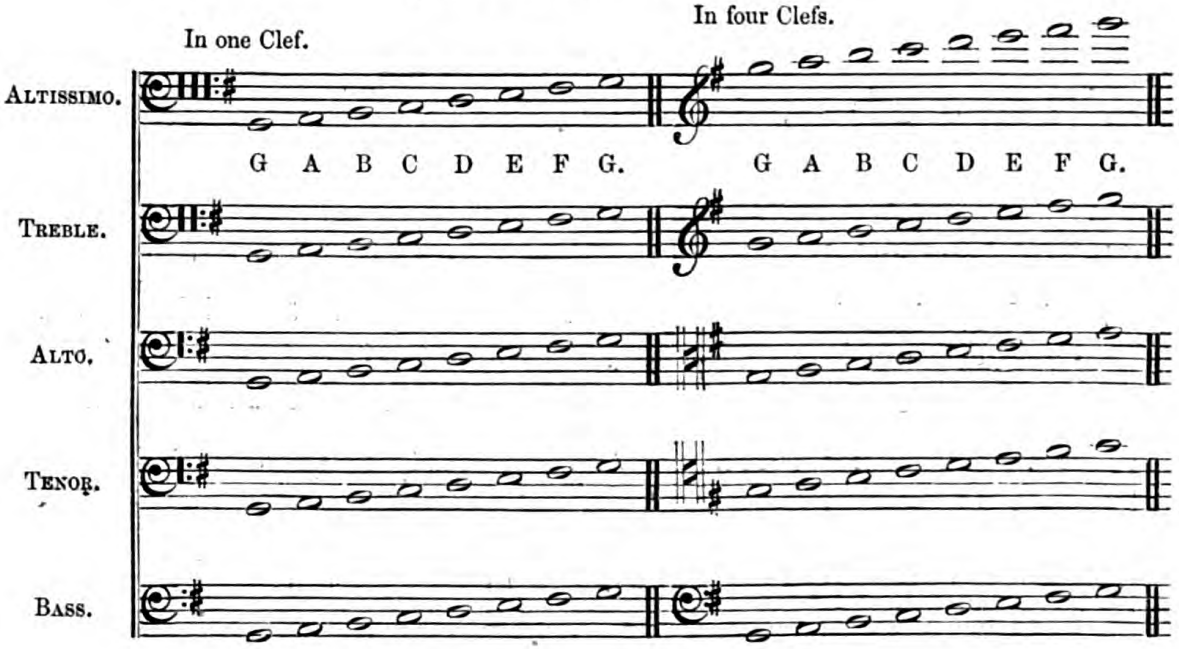
17. The effect of this difficulty has been, to discourage the study; and to confine the use of the Musical Score to professional men of a high class. This has long been complained of. So far back as the year 1672, an Essay was published by Thomas Salmon, M.A. of Trinity College, Oxford, containing a proposal for reducing all Music to one Clef; which, though

never adopted, has been quoted with approbation, by different writers.* The same proposal has been made, at different times, in France. †

18. As the Degrees on the Staff take their names from the letters of the alphabet, Mr. Salmon determines each Degree to one particular letter; naming the Degrees on every staff, as they are named in the Bass, from which all Intervals are counted. The only difference of pitch, therefore, between the notes of one Staff, and the corresponding notes of another, consists of Octaves; which he distinguishes from each other, by the initial letters of the words Treble, Mean, and Bass, instead of the Clefs.

19. As this method opens the way to the use of the Musical Score, I intend to avail myself of it when I come to that part of the subject; retaining, however, the F Clef, which is familiar to every one; and distinguishing the other parts from the Bass, by short Bars attached to the Clef, which direct the performer to take the notes one, two, or even sometimes three Octaves higher.

SCALE OF FOUR OCTAVES.



* See Malcolm's "Treatise on Music," 8vo, Edinburgh, 1721, page 378. Dr. Burney's "History of Music," vol. iii. page 473; where the reader will find a list of nine clefs, that were in use when Mr. Salmon's Essay appeared.

† "Our countryman Salmon's proposal for reducing all Music to one Clef, has frequently been revived in France, without the least allusion to him or his work; which are both so much out of the question, that the French writers have frequently disputed among themselves the right to the invention. And so late as January 1786, a proposal was published in the 'Journal de Paris,' for adopting a single Clef, as a new discovery."—Burney's *History*, vol. iv. page 627.

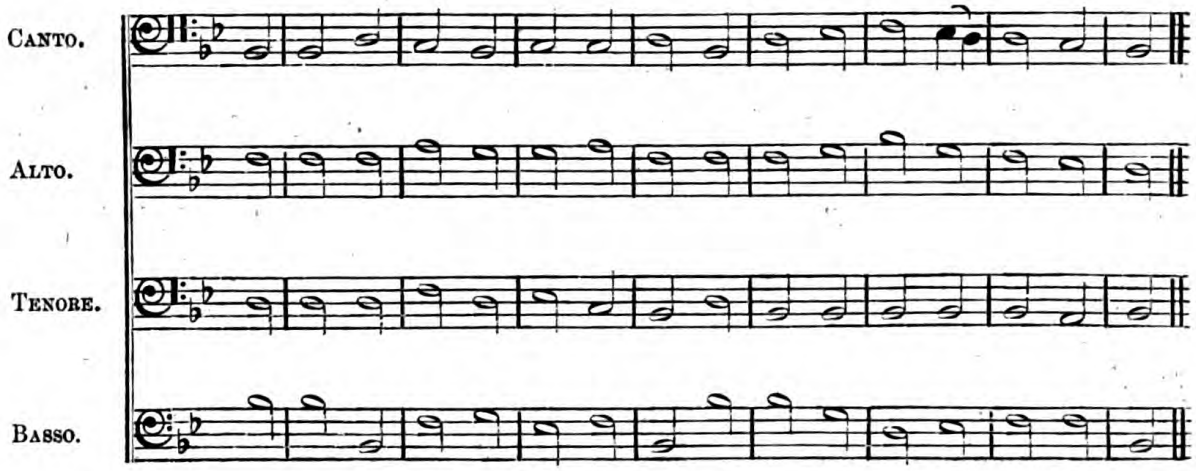
20. The Baritono and Mezzo Soprano Clefs, appear to have been long given up as useless. The Canto or Soprano, though still used on the Continent, has lately been superseded, in this country, by the Treble Clef: but the knowledge of it is necessary, for the sake of the standard Music printed before the change.

21. Vocal Music, of the higher class, in four parts, is still printed in the four remaining Clefs,—Treble, Alto, Tenor, and Bass. The plea urged for this practice, which I shall give in the words of Mr. Shield,* is *necessity*. "Experience," says he, "has taught me, that many musical amateurs have an aversion to a variety of Clefs: and from the appearance of the Viola Clef so early, they will perhaps conclude that I mean to perplex them with the seven which are used in old music. But I shall not employ more than four, throughout the whole of this work; and these are absolutely necessary, on account of the different compasses of voices and instruments."

22. The alleged necessity arises out of the different compass of voices and instruments: that is to say, the four Clefs are necessary, in order to confine the notes of each part, as much as possible, within its own Staff.

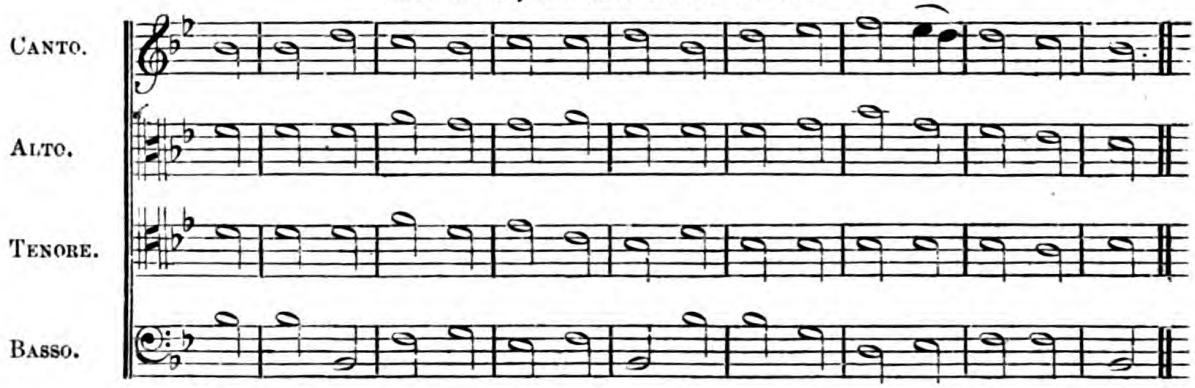
23. After having scored a number of Choruses, with all the vocal and instrumental parts, in one Clef, I am convinced that the alleged necessity is purely imaginary. The reader will judge of this for himself, when he has examined the Scores in the twentieth Chapter of this work; and compared them with the editions of Handel's Works, published by Dr. Arnold, and the Handel Society. In the mean time, the following Example is given as a specimen of the two methods of notation :—

LUTHER'S HYMN, SCORED IN ONE CLEF.



* "Introduction to Harmony," page 11, note.

THE SAME, IN THE FOUR CLEFS.

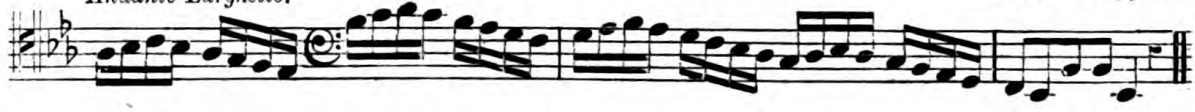


24. A passage in the Bass, that rises much above the Staff, is usually printed in the Tenor Clef:

"Thou didst blow."

Andante Larghetto.

"Israel in Egypt."

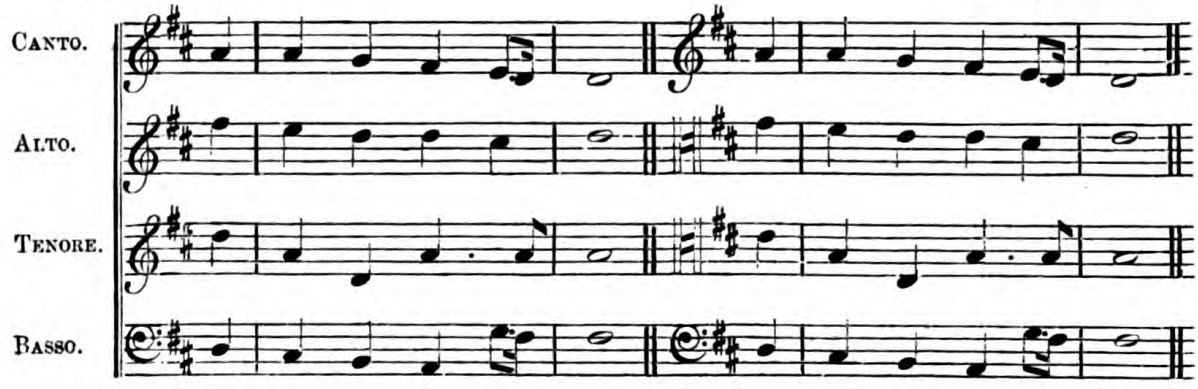


25. A shift of an Octave would be more natural, and much easier:

26. In music intended for the use of amateurs, the Alto and Tenor parts have, of late years, been frequently printed in the Treble Clef, an Octave higher than the true pitch. The error is, no doubt, corrected to the ear, when those parts are sung by men's voices: but the appearance of the music is disfigured to the eye, by the derangement of the parts. The pitch would be corrected by using the Tenor Clef on the third *space;* the notes being printed exactly as in the Treble Clef, thus:—

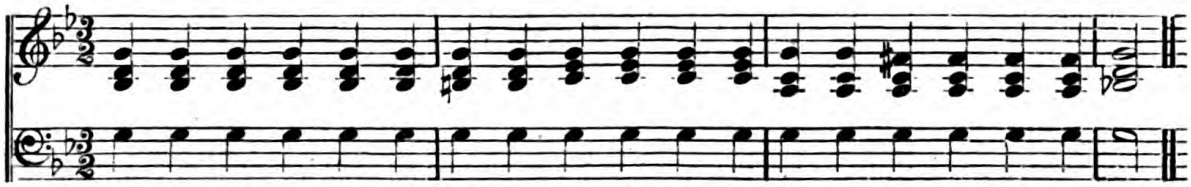
"The Kingdom of this World."

"Hallelujah Chorus."

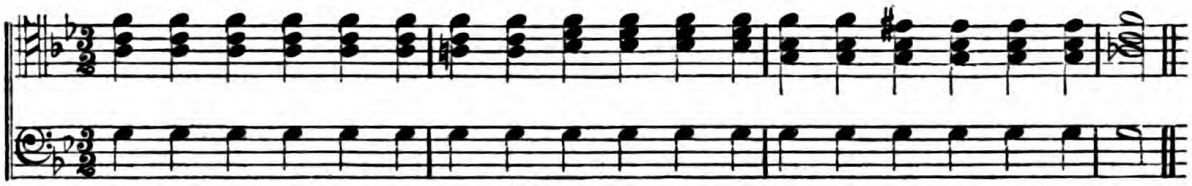


27. It is sometimes convenient to use the Tenor Clef in the same way, when an accompaniment is too low for the Treble Clef; as in the following Example:—

From Latrobe's "Selections." CALDARA.



The same in the Tenor Clef, on the third Space.

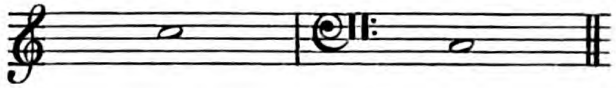


28. To transpose the other Clefs into the F Clef, observe the following rules:—

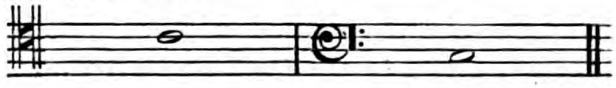
The Tenor and Alto take the first Octave.
The Canto and Treble, the second Octave.

Write the Alto, 1 Degree lower:—

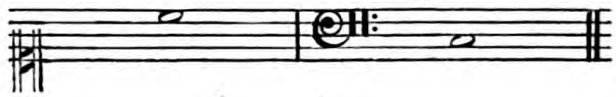
The Treble, 2 Degrees lower:—



The Tenor, 3 Degrees lower:—



and the Canto, 4 Degrees lower:—



CHAPTER II.

THE HARMONIC SCALE.

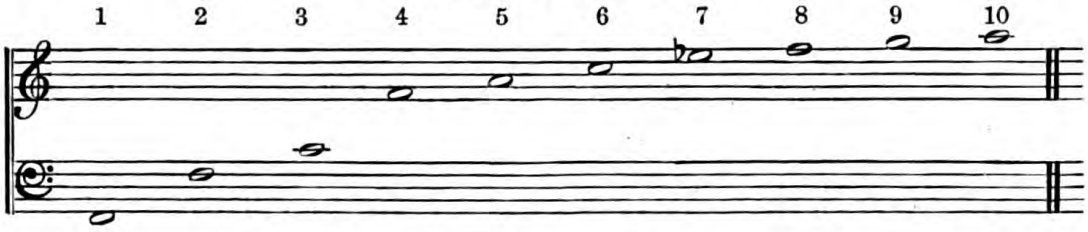
1. WHEN a musical string is vibrating, besides one principal sound, there are heard, at the same time, certain subordinate sounds, ascending in pitch, and decreasing in power, called Harmonics. The cause of these, as ascertained by experiments, is a spontaneous division of the string into aliquot parts; which, without interrupting the principal sound, or interfering with each other, produce subordinate vibrations, and subordinate sounds. *

2. The fractional parts into which the string divides itself, are one-half, one-third, one-fourth, and so on. The divisors are the prime numbers 2, 3, 5, 7, and their compounds. The rate of vibration being directly as the divisors, and inversely as the length of the aliquot parts of the string, † the divisors may be called Harmonic Numbers. ‡

Parts of the String, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$.

Harmonic Numbers, 2, 3, 4, 5, 6, 7, 8, 9, 10.

HARMONIC SCALE.



3. An Interval is the difference of pitch between two musical sounds; or the proportion of two Harmonic Numbers to each other. It is denoted by the numbers corresponding to the two sounds, placed either in the form of a fraction, as $\frac{2}{3}$, or in the proportional form 2 : 3 : the former indicating the proportion of the fractional part to the whole string; and the latter, the proportional rates of vibration in the two sounds. §

4. Although all the Intervals within the compass of the Harmonic Scale, are used in compositions for a full band, yet, in treating of Harmony, it is convenient and customary to neglect the Octaves contained in Intervals greater than the Ninth; and to consider those Intervals as replicates of the smaller Intervals that remain. Thus 1 : 3, the Twelfth, neglecting the Octave 1 : 2, becomes 2 : 3, the Fifth; 1 : 5, the Seventeenth, omitting the

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* Appendix C. † Appendix B. ‡ Appendix D. § Appendix A.

Double Octave 1 : 4, becomes 4 : 5, the Major Third; and 1 : 7, the Twenty-first, omitting the Double Octave 1 : 4, becomes 4 : 7, the Grave Seventh.

5. The Degrees of the Harmonic Scale consist of Intervals decreasing regularly, in a geometrical ratio, from the Octave to the Minor Tone:—

1 : 2 . . . . Octave.
2 : 3 . . . . Fifth.
3 : 4 . . . . Fourth.
4 : 5 . . . . Major Third.
5 : 6 . . . . Minor Third.
6 : 7 . . . . Grave Third.
7 : 8 . . . . Tone Maximus.
8 : 9 . . . . Tone Major.
9 : 10 . . . . Tone Minor.

6. Besides the Degrees, the Harmonic Scale contains the following Intervals, without going higher than the Ninth:—

3 : 5 . . . . Major Sixth.
4 : 7 . . . . Grave Seventh.
5 : 7 . . . . Grave Fifth.
5 : 8 . . . . Minor Sixth.
4 : 9 . . . . Ninth.
5 : 9 . . . . Minor Seventh.
7 : 9 . . . . Acute Third.
7 : 10 . . . . Acute Fourth, or Tritone.

7. The Intervals already noticed, being heard together in the Harmonics of a single note, are all more or less consonant: especially if they be properly accompanied; that is to say, if the more consonant Intervals, which are lower in the Harmonic Scale, be sufficiently predominant in the Chord, as they are in the Scale itself.

8. All other Intervals are produced by combining the Harmonics of different Roots.*

9. The Intervals that are essentially necessary in studying the structure of Chords, are the Octave, Fifth, Major Third, and Grave Seventh.

---

* Appendix E.

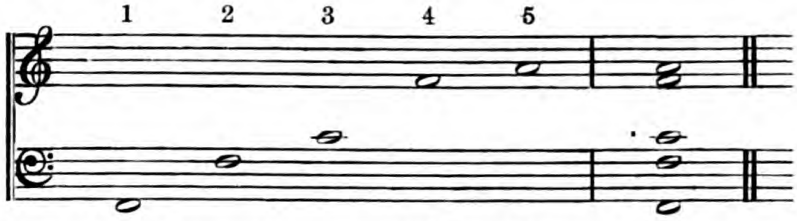
CHAPTER III.

HARMONIC CHORDS.

1. THE Harmonic Scale, although it is properly one Chord, directly contains the two Fundamental Concords, from which all other Chords are derived: namely, the Harmonic Triad, or Major Common Chord; and the Harmonic Tetrad, or Dominant Harmony.

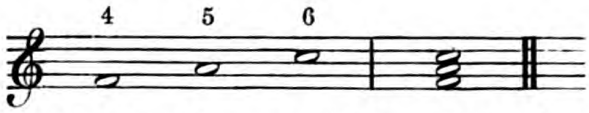
I. HARMONIC TRIAD.

2. The first five notes of the Harmonic Scale, when heard together, are called the Fundamental Concord, or Major Common Chord:—



This is reckoned the most pleasing form in which the Chord can be produced by five notes.

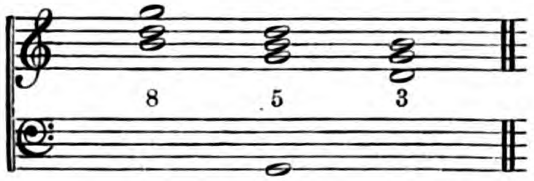
3. In the fourth, fifth, and sixth notes of the same Scale, the essential notes of this Chord are found together, in the form of a Triad:—



In this form, the Chord is called the Harmonic Triad, or Major Triad; and consists of a Fifth, harmonically divided * into Major Third and Minor Third: 4 : 5, and 5 : 6.

F A C . . . . 4 . 5 . 6.

4. The upper notes of a Triad are capable of various positions; which are usually reduced to three, according as the Octave,† Fifth, or Third, is uppermost:—



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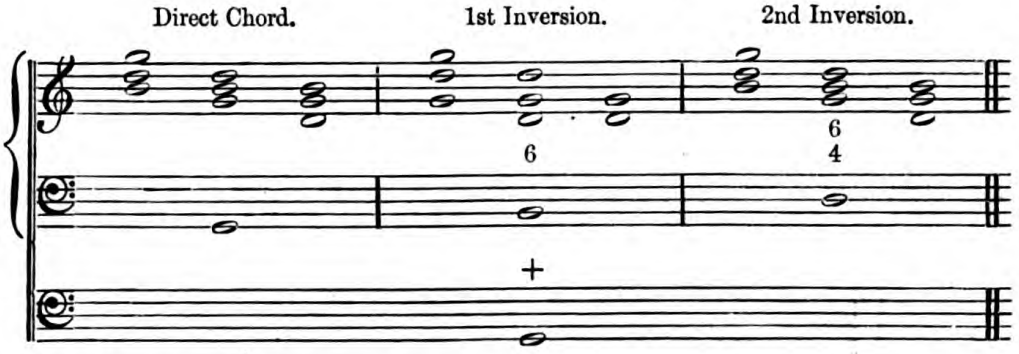
* See Appendix E, 3.

† In four parts, the Octave to the Root is generally added to the three notes of the Triad; but the name Triad is retained.

5. When the Root, or Fundamental note, is removed to a higher part, and another note of the Chord is taken as a Bass, the Chord is said to be inverted.

Positions and Inversions of the Major Triad.

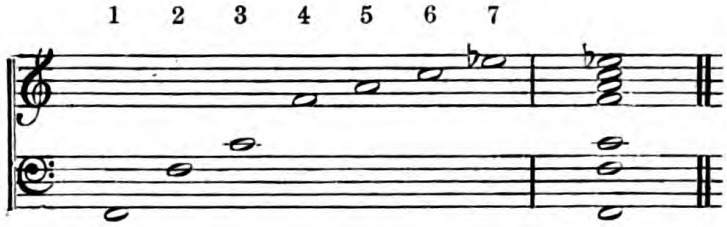


6. The first Inversion of a Triad is called a Chord of the Sixth; the Third being understood. The second Inversion is called a Chord of the Fourth and Sixth.

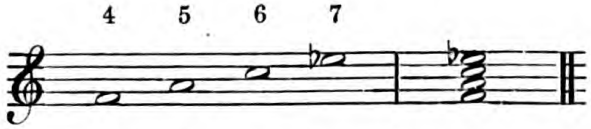
Direct Chord	G	B	D	. . . . 4 . 5 . 6
1st Inversion. . . .	B	D	G	. . . . 5 . 6 . 8
2nd Inversion. . . .	D	G	B	. . . . 3 . 4 . 5.

II. HARMONIC TETRAD.

7. The first seven notes of the Harmonic Scale, heard together, are called the Fundamental Seventh, or Dominant Harmony:—



8. In the fourth, fifth, sixth, and seventh notes of the same Scale, the essential notes of this Chord are found together in the form of a Tetrad; or, as it is commonly called, a Chord of the Seventh:—



In this form it is called the Fundamental Seventh; or more commonly the Dominant

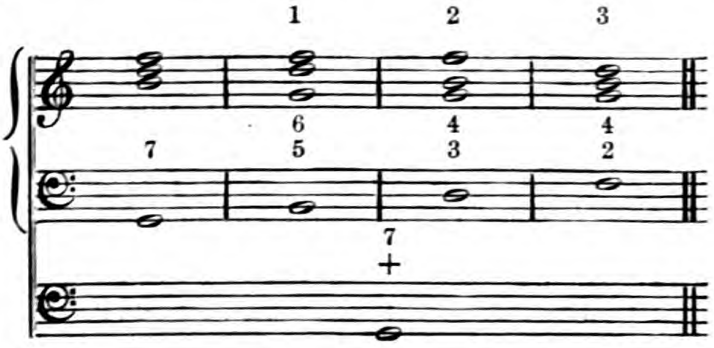
Seventh, from its being peculiar to the Dominant, or Fifth of the Key. In this work it is distinguished by a small Delta, Δ, over the letter that names the note:—

$$F\ A\ C\ \overset{\Delta}{E}\flat \ldots . 4 . 5 . 6 . 7.$$

9. This Chord consists of a Grave Seventh, harmonically divided into Major Third, Minor Third, and Grave Third:—

$$\frac{4}{5} \quad \frac{5}{6} \quad \frac{6}{7}$$

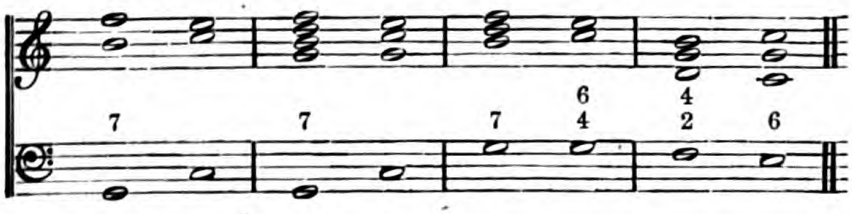
10. The Dominant Seventh, in common with other Tetrads, has three Inversions:—



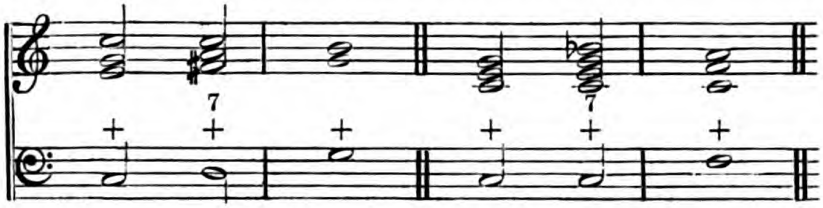
11. The first Inversion is called a Chord of the Fifth and Sixth; the Third being understood. The second is called a Chord of the Third and Fourth; the Sixth being understood. The Third is called a Chord of the Second, or Second and Fourth; the Sixth being understood.

Direct Chord	G	B	D	$\overset{\Delta}{F}$	. . . . 4 . 5 . 6 . 7
1st Inversion	B	D	F	G	. . . . 5 . 6 . 7 . 8
2nd Inversion	D	F	G	B	. . . . 6 . 7 . 8 . 10
3rd Inversion	F	G	B	D	. . . . 7 . 8 . 10 . 12.

12. The Dominant Harmony, when treated regularly, is followed by the Tonic Harmony; which is called its Resolution. The Major Third ascends a Semitone to the Tonic, or Key-note, and the Seventh descends one Degree to the Third of the Key. When these two changes are made, the Chord is said to be resolved:—

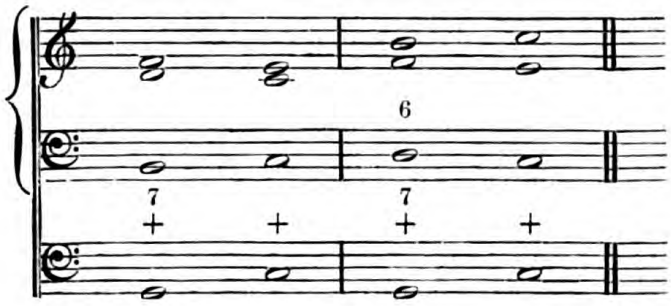


13. When any other note of the Scale takes the Dominant Harmony, it becomes a Dominant, and introduces a new Key. This is called Modulation.*



14. In composition, the Root of the Dominant Seventh is often omitted; and the Chord assumes the form of a Diminished Triad, consisting of Grave Fifth, harmonically divided into Minor Third and Grave Third; 5 : 6, and 6 : 7.

B D $\overset{\Delta}{F}$ . . . . 5 . 6 . 7.



15. The Dominant Seventh note, bears the same name or letter with the Subdominant or Fourth of the Scale: but differs from it in pitch, by an Interval of 63 : 64, or Komma Major :—

* Chord of the Subdominant . . . F A C F . . . . 4 . 5 . 6 . 8 × 8 = 32 . 40 . 48 . 64.

Chord of the Dominant . . . . . G B D $\overset{\Delta}{F}$ . . . . 4 . 5 . 6 . 7 × 9 = 36 . 45 . 54 . 63.

16. In the Resolution of the Dominant Seventh, the Major Third rises a Diatonic Semitone, 15 : 16, and the Seventh falls a Hemitone Minor, 21 : 20 :—

G B D $\overset{\Delta}{F}$ . . . . 4 . 5 . 6 . 7 × 3 = 12 . 15 . 18 . 21

C – E . . . . 4 . 5 . – . – × 4 = – . 16 . – . 20.

---

* See Chapter XV.

CHAPTER IV.

THE SCALE OF TRIADS.

1. Two Harmonic Triads, in which the Fifth of the one is the Bass of the other, may be called adjacent Triads. A series of adjacent Triads produces a Scale, consisting of alternate Major and Minor Thirds, which is of great use in the study of Harmony, and which I shall call the Scale of Triads.

SCALE OF TRIADS.

B♭ D F A C E G B D F♯ A, &c.

2. Any three adjacent Triads in this Scale, contain the notes of a Diatonic Scale. For example:—

3. To construct the Diatonic Scale of C: on the note F, set up the Major Triad F A C; on C, a similar Triad C E G; and on G another, G B D. Then—

4. Because F C G are perfect Fifths ascending, F is to C, and C to G, as 2 to 3. (Chap. II. 5.) Wherefore, if F be taken = 4, C will be = 6, and G = 9. Multiply the proportions of those Triads, 4, 5, 6, by 4, 6, 9, the value of F C G:—

Triad of F . . . . F A C . . . . 4 . 5 . 6 × 4 = 16 . 20 . 24.
Triad of C . . . . C E G . . . . 4 . 5 . 6 × 6 = 24 . 30 . 36.
Triad of G . . . . G B D . . . . 4 . 5 . 6 × 9 = 36 . 45 . 54.

Set down these notes in succession, with the proportions thus found:—

16	20	24	30	36	45	54.
F	A	C	E	G	B	D.

To reduce them within an Octave, multiply F A, and divide D, by 2. Then arrange them in alphabetical order, beginning with C, and adding its Octave:—

24	27	30	32	36	40	45	48.
C	D	E	F	G	A	B	C.

5. This is the Diatonic Scale of C, and of the Major Mode. Its Fundamental Basses * are the Roots of the three Triads from which it was constructed; viz.:—

F, the Subdominant, or Fourth of the Scale;
C, the Tonic, or Key-note; and
G, the Dominant, or Fifth of the Key.

---

* "The following excellent observation of Dr. Pepusch cannot be two often, or two strongly, impressed upon the mind of the student, viz.:—'All melodies have the perfect concords of the key they are in, for their Fundamental Basses.'"—Callcott's *Grammar*, Art. 328, note; 2nd Ed. 305.

6. From the construction it is evident, that the fountain of the Diatonic Scale is not the Tonic, or Key-note, as is commonly supposed, but the Subdominant.

7. The Degrees of the Diatonic Scale, are ascertained from the proportions of the notes as already given:—

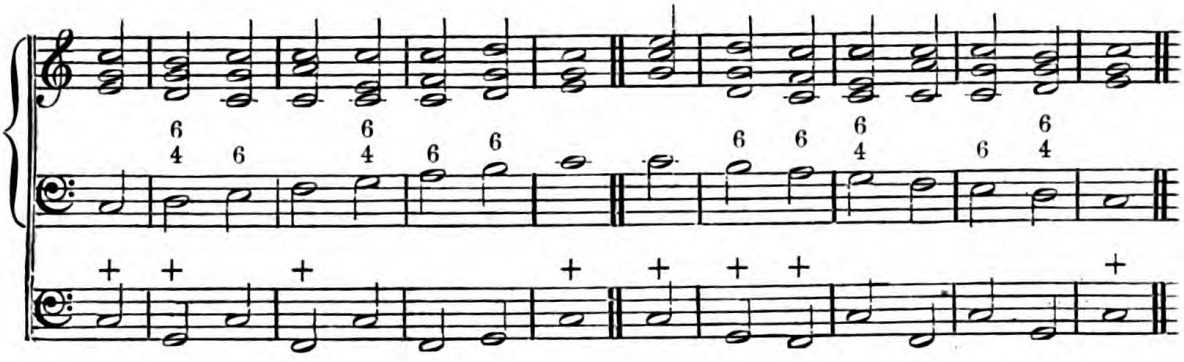
C	D	. . . .	24	:	27 ÷ 3	=	8	:	9 Major Tone.
D	E	. . . .	27	:	30 ÷ 3	=	9	:	10 Minor Tone.
E	F	. . . .	30	:	32 ÷ 2	=	15	:	16 Diatonic Semitone.
F	G	. . . .	32	:	36 ÷ 4	=	8	:	9 Major Tone.
G	A	. . . .	36	:	40 ÷ 4	=	9	:	10 Minor Tone.
A	B	. . . .	40	:	45 ÷ 5	=	8	:	9 Major Tone.
B	C	. . . .	45	:	48 ÷ 3	=	15	:	16 Diatonic Semitone.

8. The names by which the notes of the Scale are commonly distinguished, are the following:—

First	The Tonic, or Key-note.
Third	The Mediant.
Fifth	The Dominant.
Fourth	The Subdominant.
Sixth	The Submediant.
Second	The Supertonic.
Major Seventh . .	The Leading note.

Each of these notes has its peculiar effect upon the ear, when the Key is kept in mind. (See Callcott's *Grammar*, Part 2, Chap. 5.)

9. The Diatonic Scale, accompanied by Common Chords:—



THE MINOR TRIAD.

10. The Scale of Triads has already been considered as a series of Major Triads; each of which has one note in common with the Triad immediately above and below it. The same Scale is now to be considered as a series of alternate Major and Minor Triads: each having

two notes in common with the Triad above and below it. Thus the notes of the Diatonic Scale contain five Triads: three Major, and two Minor:—

	16	20	24	30	36	45	54
	F	A	C	E	G	B	D
Major Triad	F	A	C	–	–	–	–
Minor Triad .	–	A	C	E	–	–	–
Major Triad	–	–	C	E	G	–	–
Minor Triad .	–	–	–	E	G	B	–
Major Triad	–	–	–	–	G	B	D

11. From this Table it is evident that the Minor Triad or Common Chord, is a compound Chord, taking its Minor Third from the Triad below,* and its Major Third from the Triad above it; and so has two Harmonic Roots. Its proportions are 10, 12, 15:—

Triad of A . . . . A C E . . . . 20 . 24 . 30 ÷ 2 = 10 . 12 . 15
Triad of E . . . . E G B . . . . 30 . 36 . 45 ÷ 3 = 10 . 12 . 15.

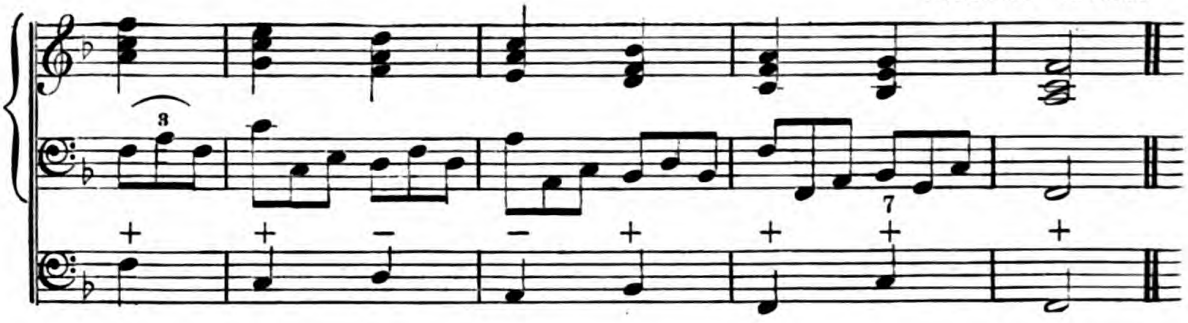
12. The Major and Minor Triads are compared by reducing their Basses to the same terms; thus:—

Major Triad, 4 . 5 . 6 × 5 = 20 . 25 . 30.
Minor Triad, 10 . 12 . 15 × 2 = 20 . 24 . 30.

The difference lies in the middle term, which is 25 in the Major, and 24 in the Minor Triad. This Interval, 24 : 25, is commonly called a Chromatic Semitone.

13. The descending Scale, accompanied by three Major and two Minor Triads:—

PURCELL'S "Ground."



* Lower in pitch.

CHAPTER V.

THE MINOR MODE.

1. IN modern Music there are two Modes or Scales, distinguished by the epithets Major and Minor. The Minor Mode imitates the Major; has its Tonic, Dominant, and Subdominant; and admits of a perfect close on its Tonic.

2. The characteristic distinction between the two Modes, lies in the Thirds of the Fundamental Chords, which are Major in the one, and Minor in the other. The Minor, however, admits of exceptions to this rule; especially in the Dominant Chord, in which the Third is generally Major: but the Third of the Tonic, or Key-note, is always Minor.

3. It has been shown in the preceding Chapter, that the Scale of Triads consists of alternate Major and Minor Triads, interwoven with each other: and that the Diatonic Scale of the Major Mode is derived from three adjacent Major Triads. It is now to be shown how the Scale of the Minor Mode is produced by three Minor Triads.

4. To construct the Scale of the Minor Mode, lay down three Minor Triads on a series of ascending Fifths, and multiply them by the proportions of their respective Basses, which are in an ascending ratio of 2 to 3. Then set down their notes in ascending order:—

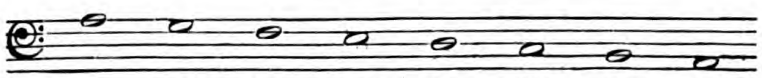
Triad of D . . . . D F A . . . . 10 . 12 . 15 × 8 = 80 . 96 . 120.
Triad of A . . . . A C E . . . . 10 . 12 . 15 × 12 = 120 . 144 . 180.
Triad of E . . . . E G B . . . . 10 . 12 . 15 × 18 = 180 . 216 . 270.

80	96	120	144	180	216	270.
D	F	A	C	E	G	B.

To reduce them within an Octave, multiply D F, and divide B, by 2. Then arrange them in alphabetical order, beginning with A, and adding its Octave:—

120	135	144	160	180	192	216	240.
A	B	C	D	E	F	G	A.

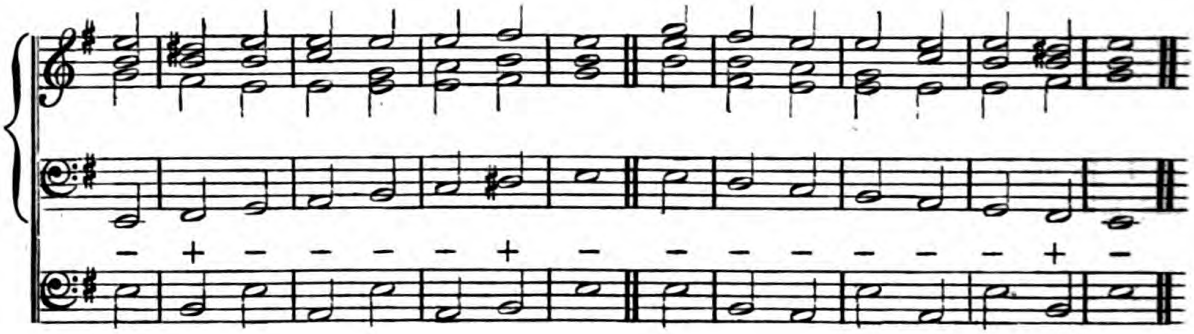
5. This series, reversed, gives the descending Scale of A Minor; of which D A E, although not the Harmonic Roots, are considered as the Fundamental Basses.



240 216 192 180 160 144 135 120

6. When the Chord of the Dominant leads to the Tonic, it is always taken with a Major Third; as in the following example:—

THE MINOR SCALE, ACCOMPANIED BY COMMON CHORDS.



7. In the ascending Scale, in order to avoid the Interval of the Augmented Second, C D♯, which is out of the Diatonic series, it is usual to take the Sixth Major, as well as the Seventh.

8. The Degrees of the Minor Scale, with the exception just now mentioned, are the same with those of the Major, but differently arranged.

9. Each Minor Scale is called the relative Minor to the Major Scale on its right hand, in the Scale of Triads:—

		+ B♭	D	+ F	A	+ C	E	+ G	B	D
		16	20	24	30	36	45	54	–	81
Scale of C Major				F	A	C	E	G	B	D
Relative Minor	A	D		F	A	C	E	G	B	–

10. The two Relative Scales are commonly supposed to consist of the same notes: but D at the end of C Major, which is Fifth to G, is a Comma (80:81) higher in pitch than D at the beginning of A Minor, which is Major Third to B♭: as may be seen by comparing the figures under the Scale of Triads.

11. Hence it is evident, that the Minor Triad D F A, to be in tune, must be taken on the Subdominant of A Minor, not on the Supertonic of C Major: wherefore it belongs to the Key of A Minor, not to C Major. This affords a good reason for its not being reckoned by the older writers, a Harmony of the Major Mode.*

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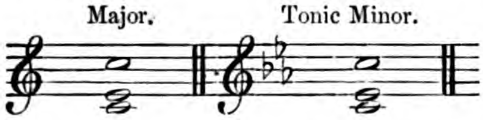
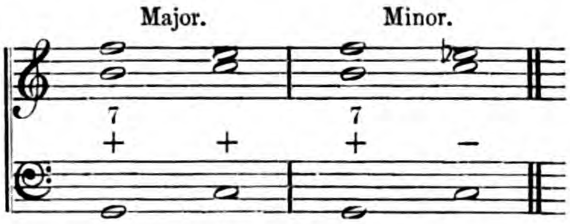
* "Dr. Pepusch, although he expressly allows the Harmonies of A and E in C Major, makes no mention of D."—Callcott's *Grammar*, Art. 332, note; 2nd Ed. 309.

12. The Tonic Minor of a Major Mode, is produced by flattening the Thirds of the Tonic, Subdominant, and Dominant:—



13. The Chord of the Dominant Seventh is the same in the Minor Mode as in the Major; but differs in its Resolution, by descending a Tone instead of a Semitone:—



14. In the Resolution of the Dominant Harmony to the Minor Tonic, the Major Third rises a Diatonic Semitone, and the Seventh falls a Tone Minimus, 35 : 32:—

$$\begin{array}{llll} E & G\sharp & B & \overset{\Delta}{D} \ldots\ 4 \,.\, 5 \,.\, 6 \,.\, 7 \times 15 = 60 \,.\, 75 \,.\, 90 \,.\, 105 \\ - & A & - & C \ldots\ 10 \,.\, 12 \,.\, - \,.\, - \times 8 = - \,.\, 80 \,.\, - \,.\, 96. \end{array}$$

$$\begin{array}{ll} G\sharp & A \ldots\ 75 : 80 \div 5 = 15 : 16 \\ \overset{\Delta}{D} & C \ldots\ 105 : 96 \div 3 = 35 : 32. \end{array}$$

CHAPTER VI.

CHORDS OF ADDITION.

1. OF the three Triads from which the Diatonic Scale is derived, the Tonic, which holds the middle place, takes no addition: the Dominant, on the right hand, besides having its own Grave Seventh, frequently borrows a note from the adjacent Triad *above*, called the Added Ninth; and the Subdominant, on the left, a note from the adjacent Triad *below*, called the Added Sixth. The Chords thus augmented, are called Chords of Addition.

I. ADDED NINTH.

2. The Chord of the Added Ninth, is the Dominant Harmony, with the Fifth of the adjacent Triad above:—

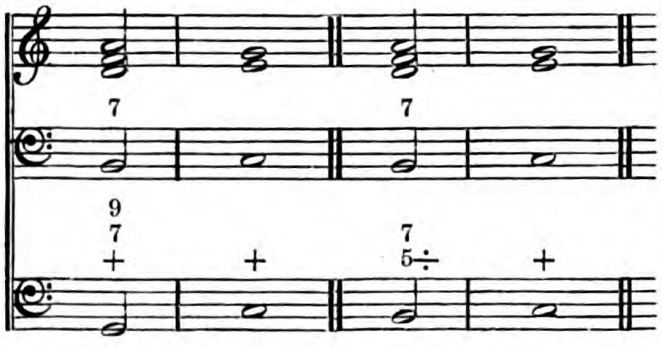
G B D $\overset{\Delta}{F}$ A . . . . 4 . 5 . 6 . 7 . 9.

Dominant Harmony . . . . G B D $\overset{\Delta}{F}$ . . . . 4 . 5 . 6 . 7 × 2 = 8 . 10 . 12 . 14.
Major Triad of D . . . . . D F♯ A . . . . . 4 . 5 . 6 . — × 3 = 12 . 15 . 18.
Added Ninth . . . . G B D $\overset{\Delta}{F}$ A . . . . 8 . 10 . 12 . 14 . 18 ÷ 2 = 4 . 5 . 6 . 7 . 9.

3. The Added Ninth is resolved by descending a Second, to the Fifth of the Tonic: and in composition, the Root is almost always omitted; leaving a Diminished Triad with Minor Seventh:—

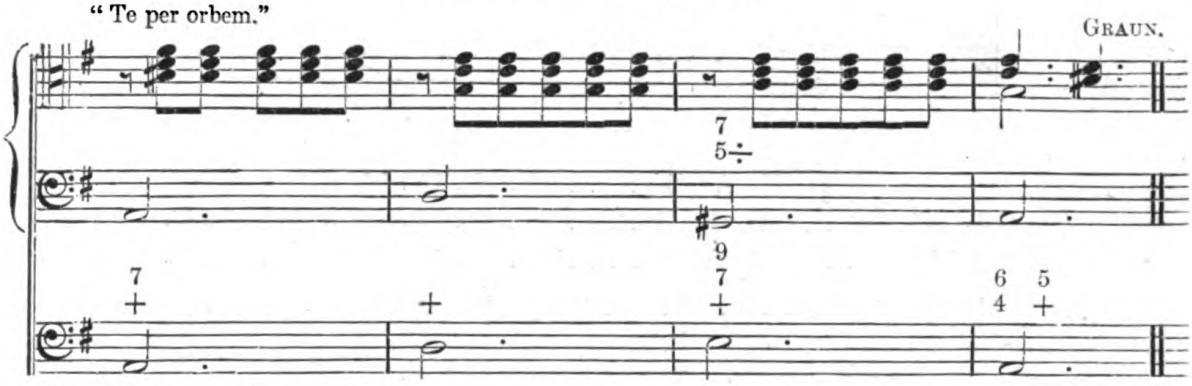
B D $\overset{\Delta}{F}$ A . . . . 5 . 6 . 7 . 9.

Sometimes, where the Resolution is regular, and leads the eye at once to the Fundamental Bass, the Major Third is given in the Bass, with the signature of the Diminished Triad and Seventh; instead of the Root, with the Dominant Seventh and Added Ninth. In the following Example it is given both ways:—



4. In the Resolution of the Added Ninth, the Major Third rises a Diatonic Semitone, 15 : 16; the Seventh falls a Hemitone Minor, 21 : 20; and the Ninth falls a Major Tone, 9 : 8 :—

G B D F A . . . . 4 . 5 . 6 . 7 . 9 × 3 = 12 . 15 . 18 . 21 . 27.
C – E G . . . . 4 . 5 . 6 . – . – × 4 = – . 16 . – . 20 . 24.
A G . . . . 27 : 24 ÷ 3 = 9 : 8.



II. ADDED SIXTH.

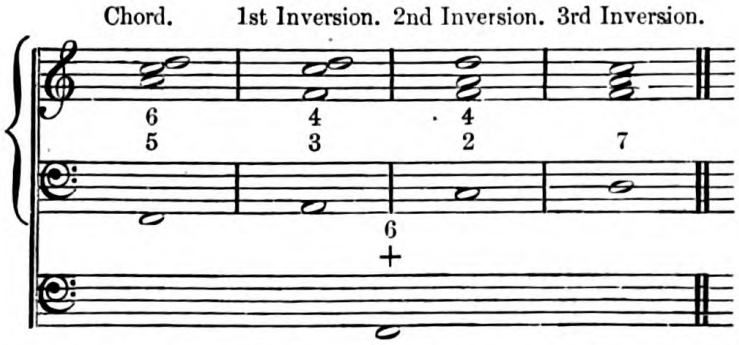
5. There are three Chords that bear this name; one belonging to the Major Mode, and two to the Minor :—

1st. Major Triad, with Major Sixth.
2nd. Minor Triad, with Minor Sixth.
3rd. Minor Triad, with Major Sixth.

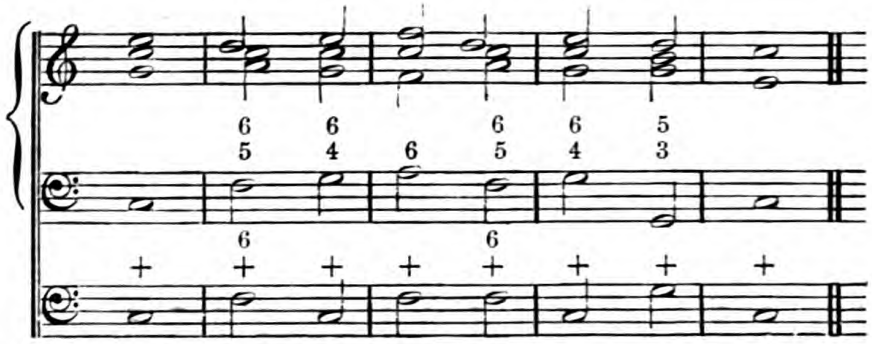
6. The Chord of the Added Sixth, is the Triad of the Subdominant, with the Third of the adjacent Triad below, or rather its Octave, added :—

F A C D . . . . 12 . 15 . 18 . 20.
Subdominant . . . . F A C . . . . 4 . 5 . 6 × 3 = 12 . 15 . 18.
Triad below . . . . B♭ D F . . . . 4 . 5 . 6 × 4 = 16 . 20 . 24.

1st.—MAJOR TRIAD WITH MAJOR SIXTH.

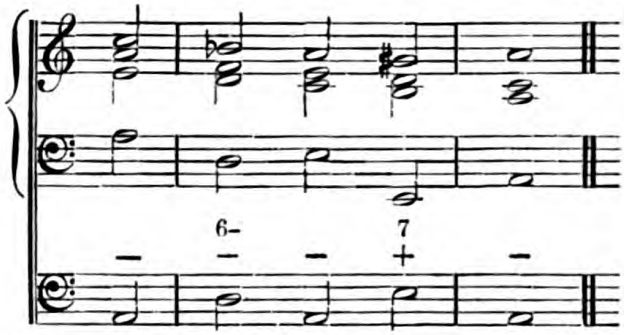


7. This Chord may be followed either by the Tonic or the Dominant Harmony. In this Example it is the Tonic :—



8. 2nd.—MINOR TRIAD, WITH MINOR SIXTH.

D F A B♭ . . . . 10 . 12 . 15 . 16.
Subdominant. . . . D F A . . . . 10 . 12 . 15 × 3 = 30 . 36 . **45.**
Triad below . . . . G B♭ D . . . . 10 . 12 . 15 × 4 = 40 . 48 . **60.**
D F A B♭ . . . . 30 . 36 . 45 . 48 ÷ 3 = 10 . 12 . 15 . **16.**

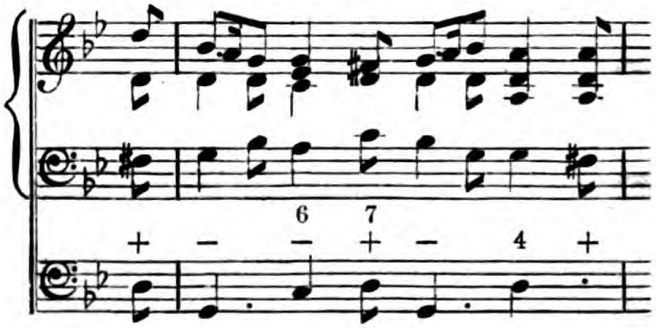


9. This Minor Sixth, though simpler in its proportions, and more analogical in its derivation, is much seldomer used than the Major Sixth, which follows.

10. 3rd.—MINOR TRIAD, WITH MAJOR SIXTH.

D F A B . . . . 30 . 36 . 45 . 50.
Subdominant. . D . . . . . D F A . . . . . 10 . 12 . 15 × 3 = 30 . 36 . 45.
Triad below . . G . . . . . G B♭ D . . . . . 10 . 12 . 15.
Tonic Major to G . . . . . G B D . . . . . 4 . 5 . 6 × 10 = 40 . 50 . 60.

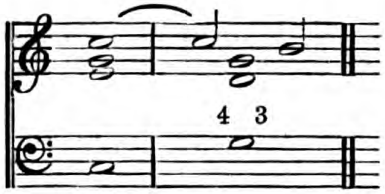
"How beautiful are the feet." HANDEL.



CHAPTER VII.

CHORDS OF SUSPENSION.

1. ANY note of a Common Chord, that descends one Degree to the next Chord, on the accented part of the Measure (XIX. 2), may, before descending, divide the time of the new Chord with the succeeding note:—as in this Example, where the note C is said to be prepared in the first Chord, suspended and resolved in the second:—

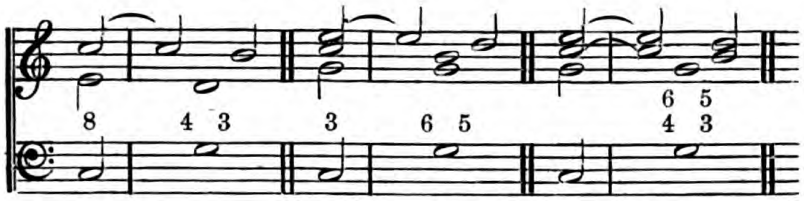


2. The Dominant Seventh, Added Sixth, and Added Ninth, may prepare Suspensions in the same way.

3. The concords into which the Suspensions are resolved, being the Third, Fifth, and Eighth, the suspended notes are properly the Fourth, Sixth, and Ninth; together with the Major Seventh, which is commonly resolved by ascending to the Eighth.

4. The notes that may be suspended, are indicated by the progression of the Fundamental Bass. Thus:—

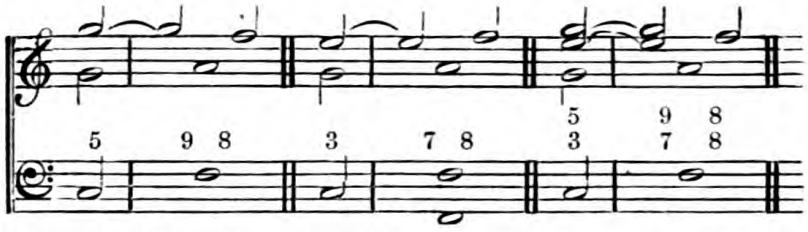
5. The ascent of a Fifth, prepares the Fourth and Sixth:—



The suspended Fourth is Octave to the Root, and the suspended Sixth is Octave to the Third, of the adjacent Triad below:—

Triad of G	G	B	D	4 . 5 . 6 × 3 = 12 . 15 . 18.
Triad below	C	E	G	4 . 5 . 6 × 4 = 16 . 20 . 24.
Suspended Fourth	G	C	D	12 . 16 . 18 ÷ 2 = 6 . 8 . 9.
Suspended Sixth	G	B	E	12 . 15 . 20.
Suspended Fourth and Sixth .	G	C	E	12 . 16 . 20 ÷ 4 = 3 . 4 . 5.

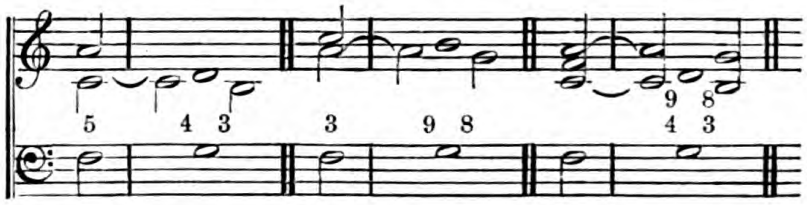
6. The ascent of a Fourth, prepares the Ninth and Major Seventh:—



The suspended Ninth is the Fifth, and the suspended Seventh is the Third, of the adjacent Triad above:—

Triad of F . . . . . — F A C . . . . . 4 . 5 . 6 × 2 = 8 . 10 . 12.
Triad above . . . . . — C E G . . . . . 4 . 5 . 6 × 3 = 12 . 15 . 18.
Suspended Ninth . . F A C G . . . . 8 . 10 . 12 . 18 ÷ 2 = 4 . 5 . 6 . 9.
Suspended Seventh . F A C E . . . . . 8 . 10 . 12 . 15.

7. The ascent of a Second, prepares the Fourth and Ninth:—



In this case, the Chord that prepares the Suspensions, is F A C; and the Chord into which the suspended notes are resolved, is G B D. These two Triads are unconnected, having the Triad C E G between them:—

$$\overset{+}{F} \quad A \quad \overset{+}{C} \quad E \quad \overset{+}{G} \quad B \quad D.$$

8. The suspended Fourth C, however, being Fifth in the Triad F A C, is also the Root of the intervening Triad C E G, and therefore belongs to a Triad that is adjacent to G B D; for which reason, the Fourth G C is a true Interval:—

Triad of F . . . . . F A C . . . . 4 . 5 . 6 × 8 = 32 . 40 . 48.
Triad of G . . . . . G B D . . . . 4 . 5 . 6 × 9 = 36 . 45 . 54.
Suspended Fourth . G C D . . . . 36 . 48 . 54 ÷ 6 = 6 . 8 . 9.

9. On the contrary, the suspended Ninth A, not being contained in the adjacent Triad C E G, belongs only to the unconnected Triad F A C; for which reason, the Ninth G A, as prepared in the Triad F A C, is a false Interval, 9 : 20; equal to Ninth 4 : 9, minus Comma 80 : 81:—

Suspended Ninth . . . . G B A . . . . 36 . 45 . 80.
G A . . . . . 36 : 80 ÷ 4 = 9 : 20.

It is therefore evident, that this Ninth, to be in tune, must be raised a Comma, as if it were prepared in the adjacent Triad above, D F♯ A :—

Suspended Ninth . . . . G B A . . . . 36 . 45 . 81 ÷ 9 = 4 . 5 . 9.

10. When the Fourth and Ninth are suspended together, the Ninth should be taken in tune, as above :—

Fourth and Ninth . . . . G C D A . . . . 36 . 48 . 54 . 81 ÷ 3 = 12 . 16 . 18 . 27 ;

but the two suspended notes, being taken from unconnected Triads, still contain between them the false Interval 16 : 27, equal to Major Sixth 3 : 5, plus Comma 80 : 81.

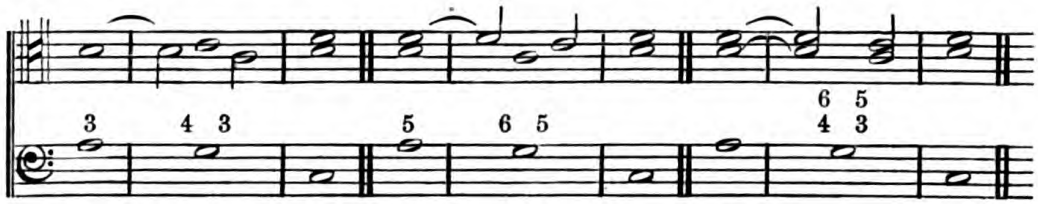
If this Chord be taken exactly as it is prepared (Sect. 7, 8) :—

G C D A . . . . 36 . 48 . 54 . 80 ÷ 2 = 18 . 24 . 27 . 40.

it contains two false Intervals ; viz. :—

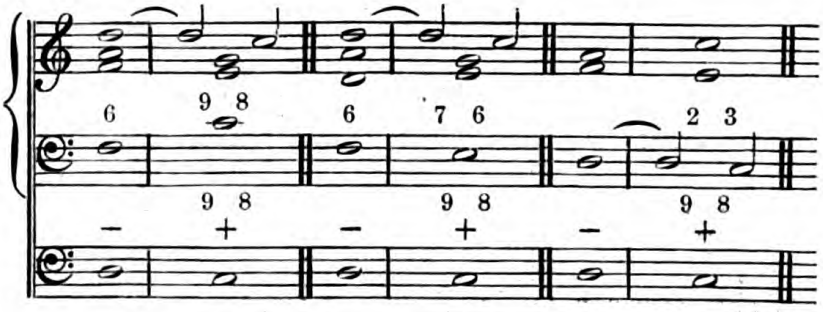
G A . . . . 9 : 20 = Ninth, minus Comma.
D A . . . . 27 : 40 = Fifth, minus Comma.

11. The descent of a Second, prepares the Fourth and Sixth :—



These Chords, as prepared, are exactly in tune.

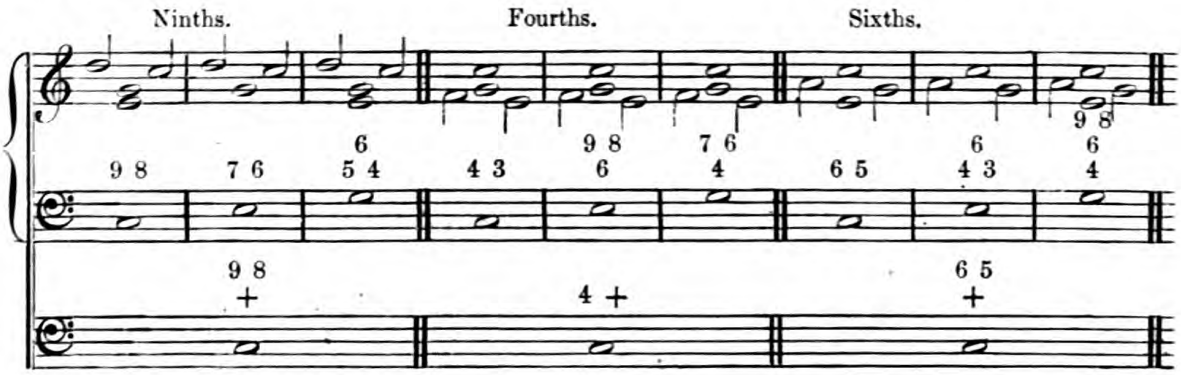
12. The descent of a Second prepares the Ninth ; but as it is against rule to prepare a Ninth by an Eighth, one of the Chords must be inverted, unless the suspended note be in the Bass :—*



* "The Major Ninth is prepared in a Third, in a Fifth, and sometimes in a Sixth ; but never in an Eighth."—Dr. Pepusch, page 33. "Corelli prepared the Ninth with a Third, or with a Fifth : but whenever any of his pupils prepared it by an Eighth, he used to cry out '*Cattivo*,' which is the Italian word for bad."—Shield, page 29.

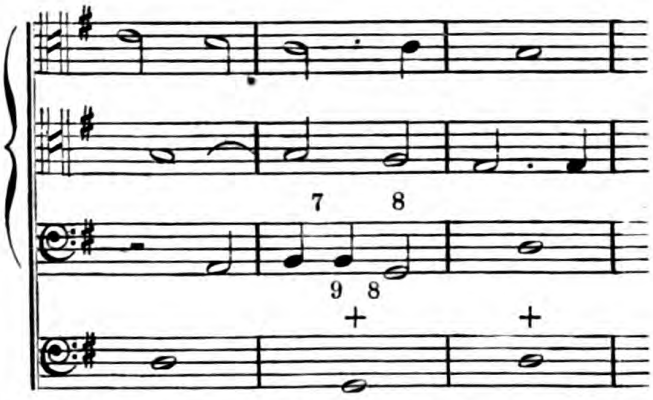
13. As the Chords of Suspension are resolved into the Common Chords, so their Inversions are resolved into the Inversions of the Common Chords: the first Inversion resolving into the Chord of the Sixth; and the Second into the Chord of the Fourth and Sixth.

14. The names and signatures of the Chords of Suspension, being taken from the Intervals which the suspended notes make with the actual Bass, change with every Inversion of the Chord. Thus, by a rise of a Third in the Bass, the Ninth is reduced to a Seventh, the Seventh to a Fifth, the Sixth to a Fourth, and the Fourth to a Second, or its Octave a Ninth:—



15. The Resolution must not be heard at the same time with the Suspension, in any part of the Chord;* except in the Bass.

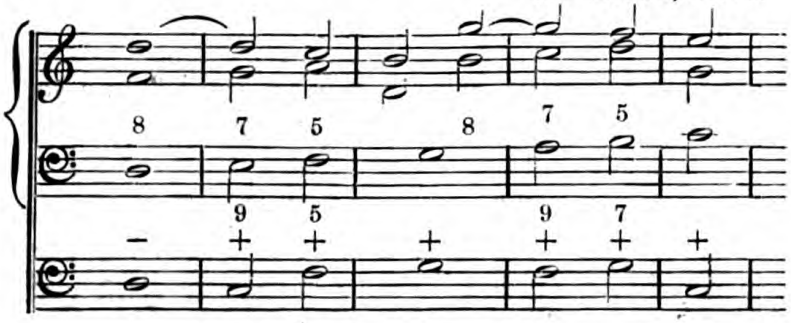
16. Suspended notes are, for the most part, resolved on the same Bass. Sometimes however, they are resolved, with fine effect, on a different note of the same Chord; as in "Non nobis":—



17. Sometimes also, although rarely, they are resolved on a different Fundamental Bass:—

* See the Example, Glee "Discord," Chap. XX. 33; where this rule is transgressed by a great composer, on purpose to produce a discordant effect.

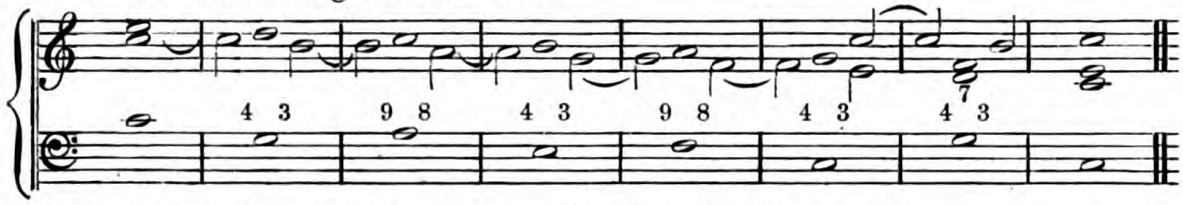
DR. PEPUSCH, Ex. 79.



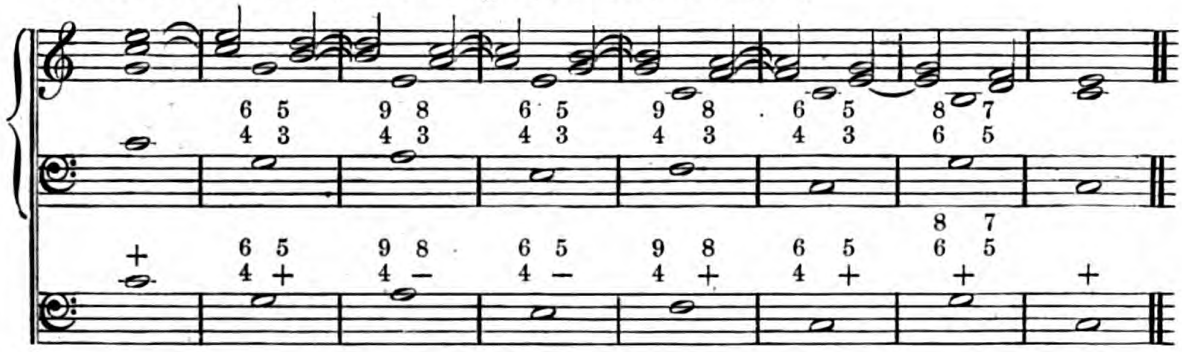
18. And sometimes they pass through more than one succeeding Chord, before they are resolved.

19. The Fourth and the Ninth are the most effective Suspensions.

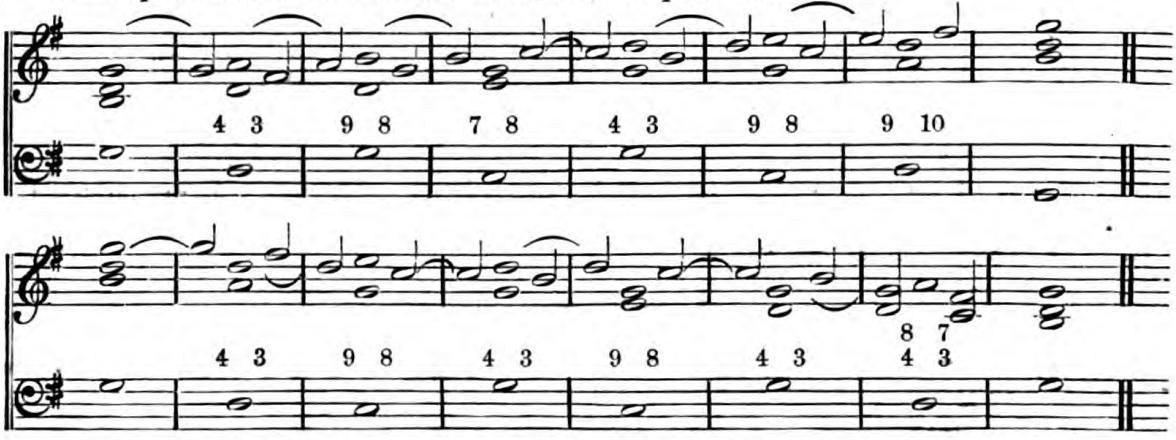
The Fourth alternating with the Ninth:—



The Fourth and Ninth, alternating with Fourth and Sixth:—



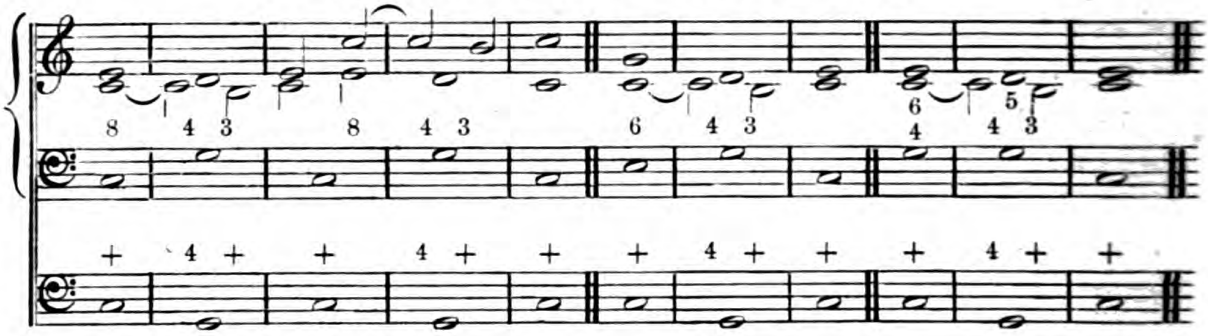
Accompaniment to the Diatonic Scale, with Suspensions:—



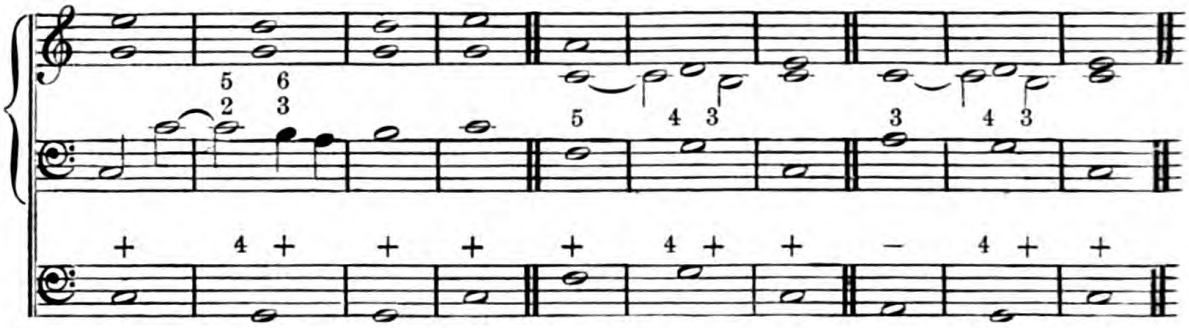
20. The Ninth is sometimes resolved by ascending; as in the seventh Measure of the last Example.

21. The following Examples of Suspensions are selected from a copious collection in Dr. Pepusch's "Treatise on Harmony:"—*

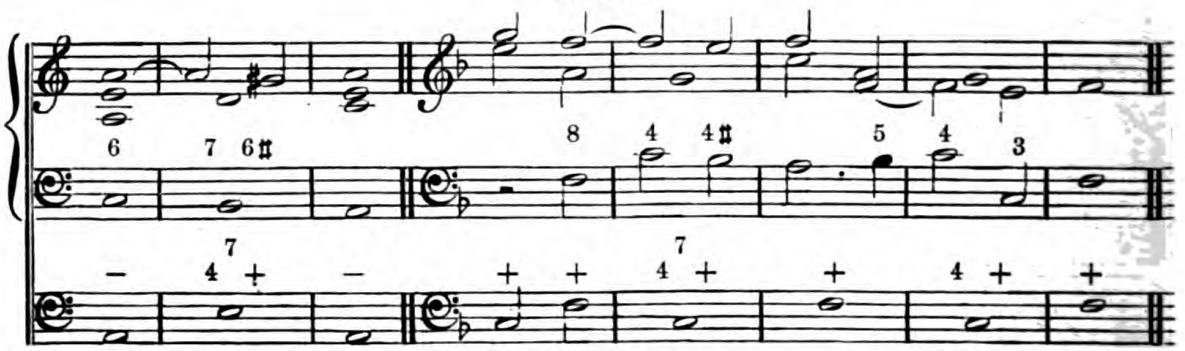
FOURTHS, VARIOUSLY PREPARED AND RESOLVED.



FOURTHS.

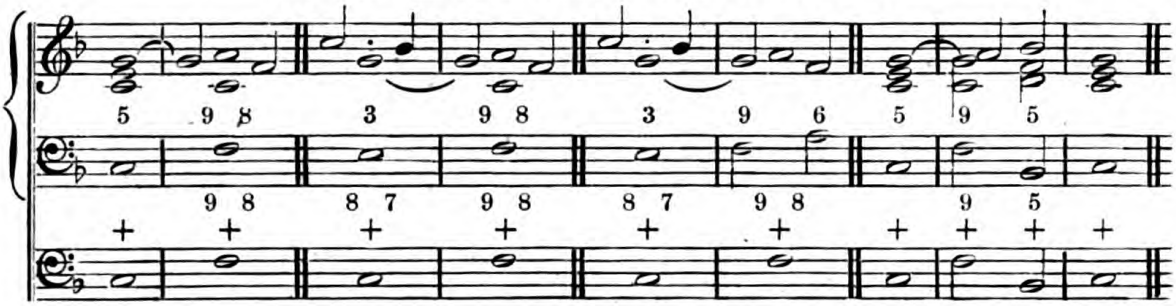


FOURTHS.

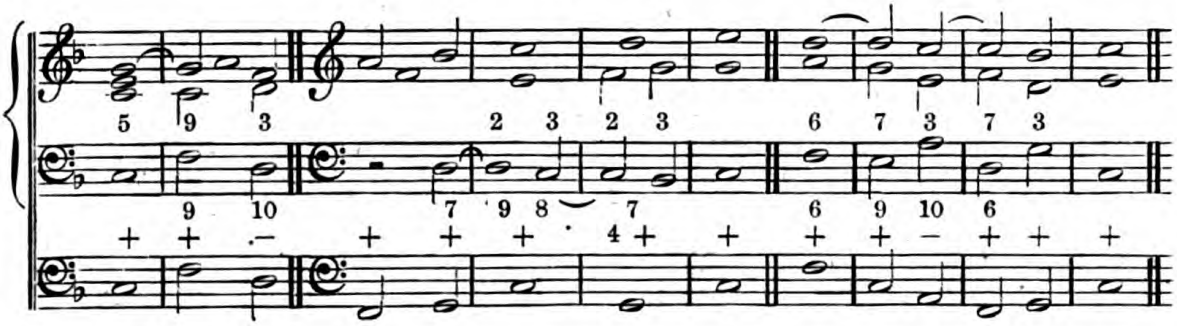


* In these Examples, the notes that are not accounted for in the signature of the Fundamental Harmony, are Passing notes; which will be explained in Chap. XVIII.

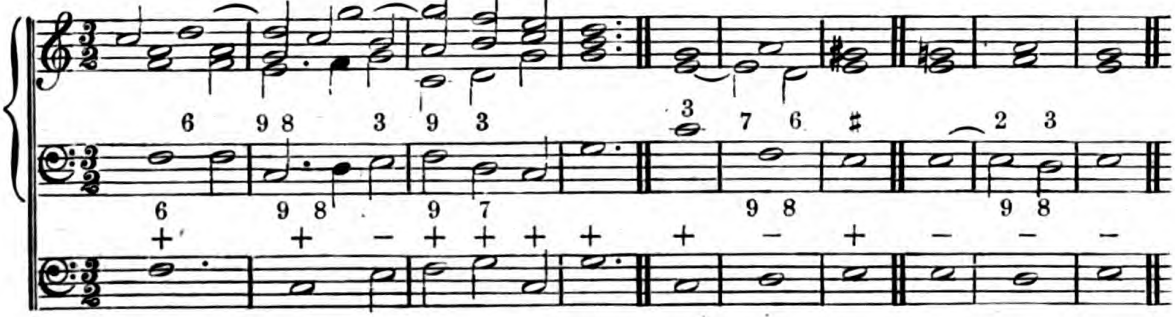
NINTHS VARIOUSLY PREPARED AND RESOLVED.



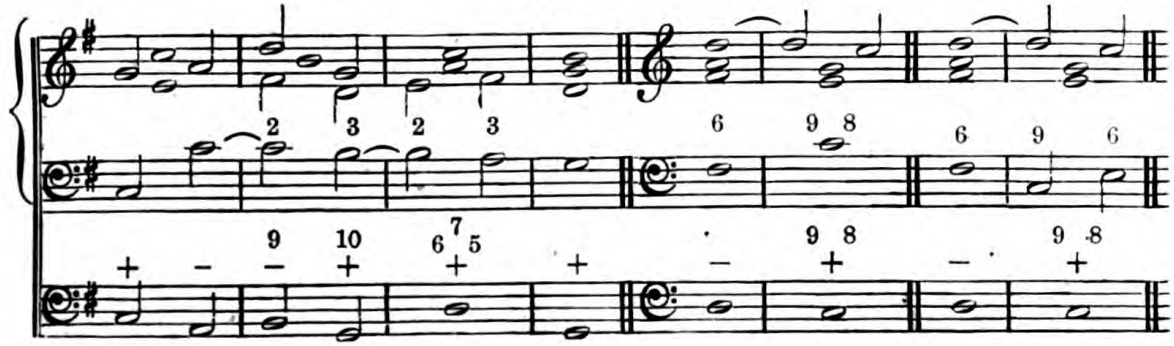
NINTHS.



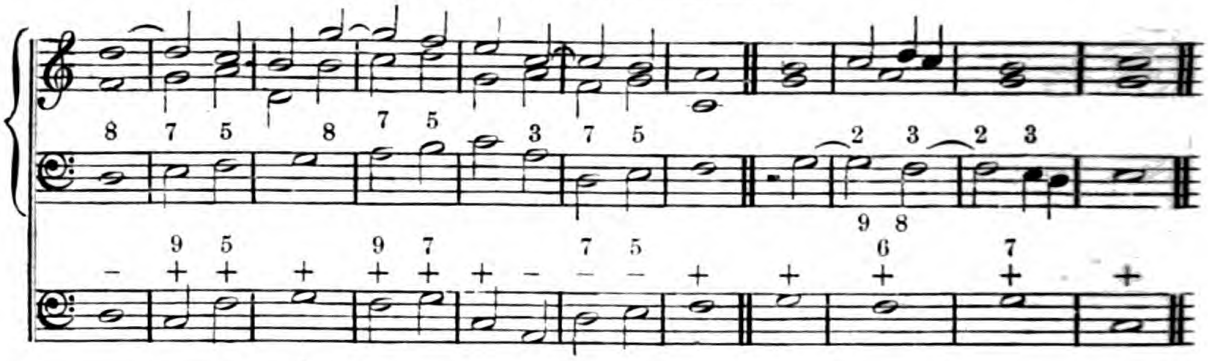
NINTHS.



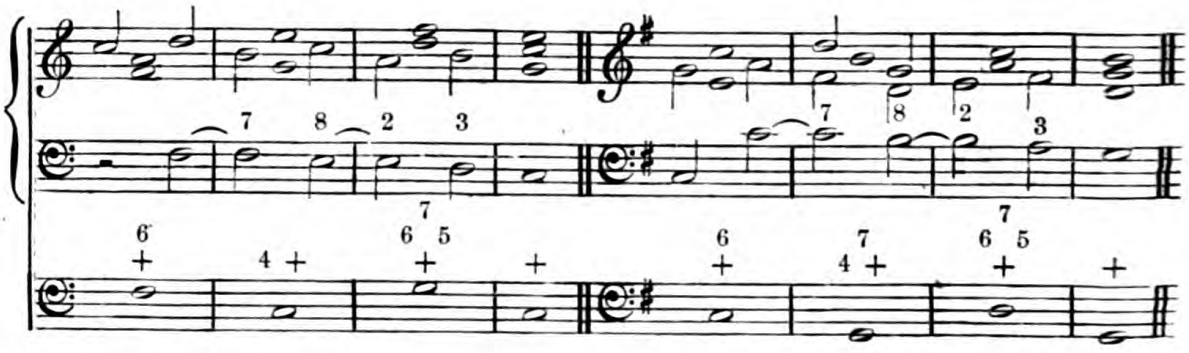
NINTHS.



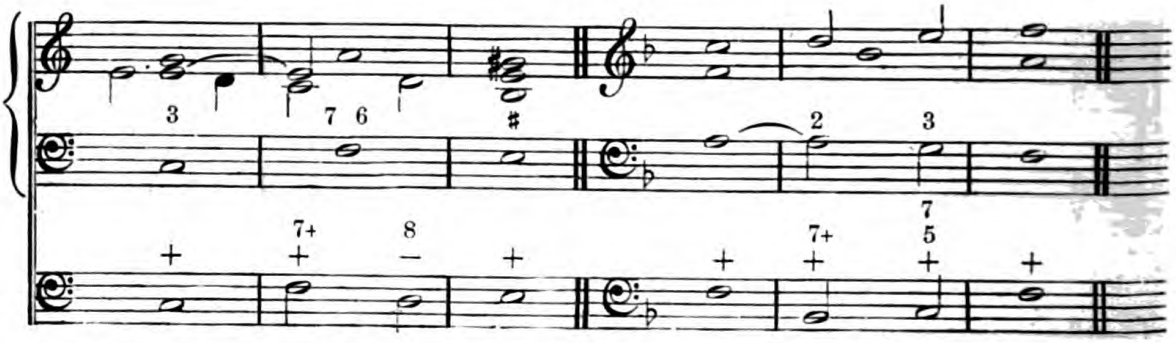
NINTHS AND MINOR SEVENTHS.



SIXTHS.



MAJOR SEVENTHS.

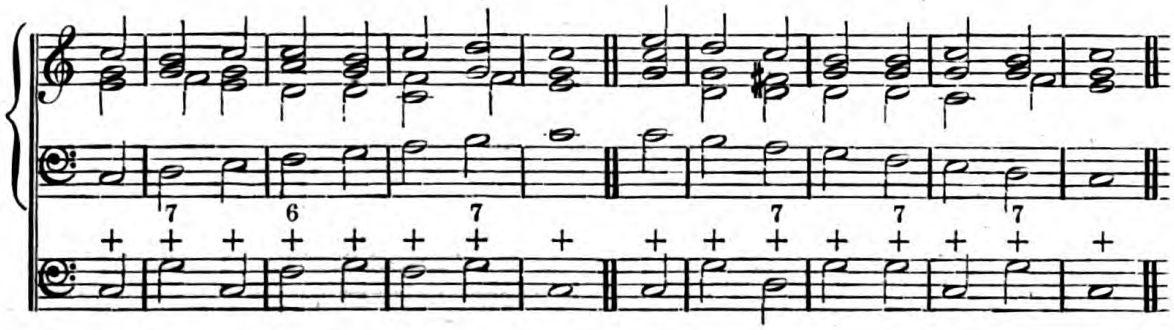


CHAPTER VIII.

THE RULE OF THE OCTAVE.

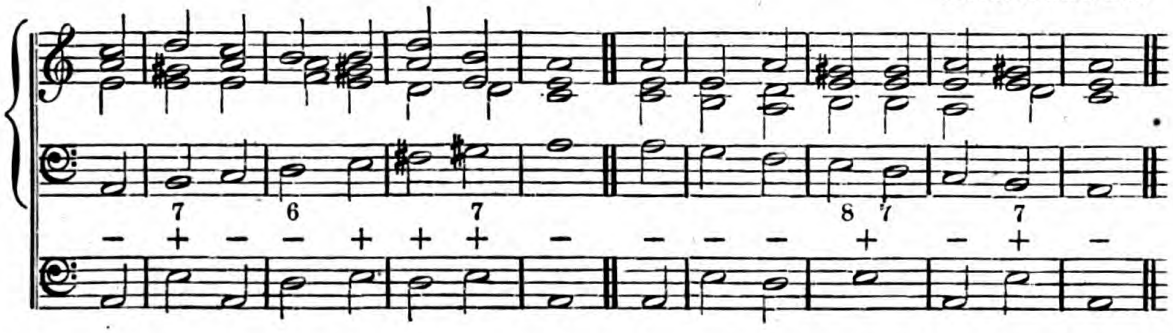
1. IN the common accompaniment called the Rule of the Octave, the Diatonic Scale is taken as a Bass, and accompanied by the Common Chords, Dominant Seventh, and Added Sixth :—

MAJOR MODE.



MINOR MODE.

ALBRECHTSBERGER.



2. In the descending Scale of the Major Mode, at the third note, the Harmony modulates into the Key of G, by taking the Dominant Harmony on D,* which resolves into G as its Tonic. This is done by sharpening the Third to D, and adding a Grave Seventh. At the fifth note, the Dominant Harmony is again taken on G; which, by resolving into C, restores the original Key.

---

* See Chap. III. 13.

3. By this Modulation, a change is made on the Fourth of the Scale;* which, instead of being the Subdominant note F, as in the ascending Scale, becomes, in descending, the Dominant Seventh to G: a graver note by an Interval of 63 : 64, or Komma Major:—

Subdominant Chord . . . . F A C $\overset{\vartriangle}{F}$ . . . 4 . 5 . 6 . 8 × 8 = 32 . 40 . 48 . 64.
Dominant Chord . . . . . G B D $\overset{\vartriangle}{F}$ . . . 4 . 5 . 6 . 7 × 9 = 36 . 45 . 54 . 63.

4. In consequence of this change, two degrees of the Scale are altered: F G becoming 7 : 8, Tone Maximus; and E F, 20 : 21, Hemitone Minor.

$\overset{\vartriangle}{F}$ G . . . 63 : 72 ÷ 9 = 7 : 8.
E $\overset{\vartriangle}{F}$ . . . 60 : 63 ÷ 3 = 20 : 21.

5. The complete Scale for the Major Mode should therefore include the grave Seventh on the Dominant: thus,

$\overset{+}{F}$	A	$\overset{+}{C}$	E	$\overset{+}{G}$	B	D	$\overset{\vartriangle}{F}$
16	20	24	30	36	45	54	63

---

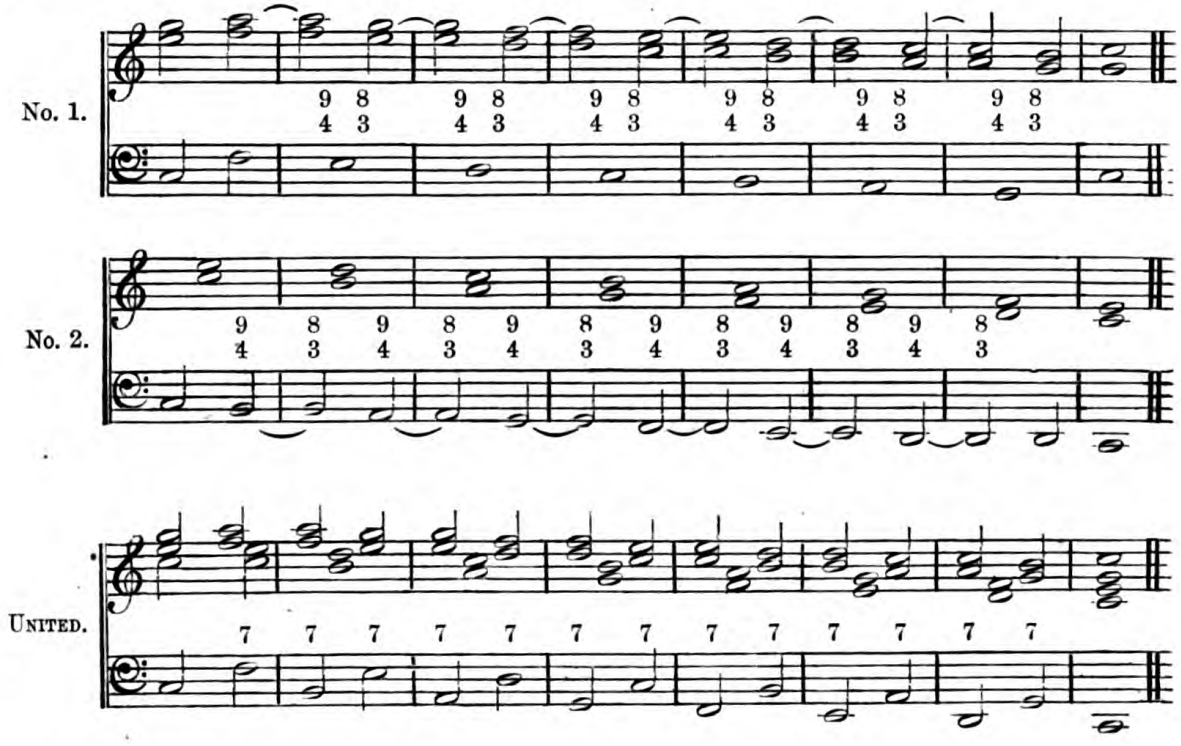
* This explains the grave Fourth, noticed by Mr. Holden as a characteristic of the descending Scale. His words are as follows:—"There is yet something farther to be observed in regard to the fourth note; which is not quite so grave a tone in ascending, as in the descending gamut. This the more expert musician will readily perceive: and he who cannot at present, for want of practice, enter into the examination of such minute differences, will yet arrive at the knowledge of them much sooner, by having them previously pointed out to him. It is for this reason that we distinguish the descending Fourth by the name of a grave Fourth, and mark it with a grave accent."—Holden's *Essay*, 4to, 1770, art. 23.

CHAPTER IX.

THE DIATONIC SEQUENCE OF SEVENTHS.

1. ANY similar succession of Chords, ascending or descending diatonically, is called a Sequence.*

2. Two descending Scales, the one a Fifth above the other—each having a Fourth and Ninth suspended and resolved on every note; † the one series being resolved on the accented part, and the other on the unaccented part of the Measure (XIX. 2); may be compounded together: the Basses of the two Scales being taken alternately:—



3. This is the celebrated Diatonic Sequence of Sevenths; of which M. Rameau remarks, "There is so great a connexion in this succession of Chords, and so natural a stream of Harmony, that custom often acquires it before knowledge."

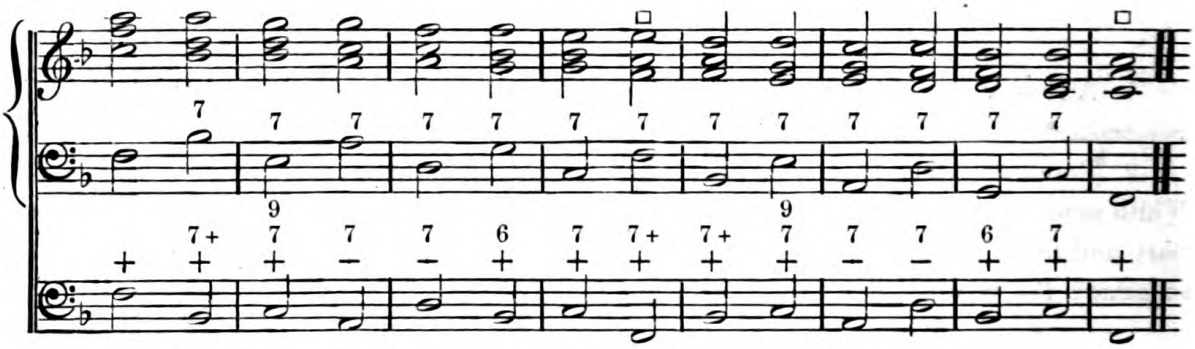
---

* Callcott's *Grammar*, Art 459; 2nd Ed. 425.

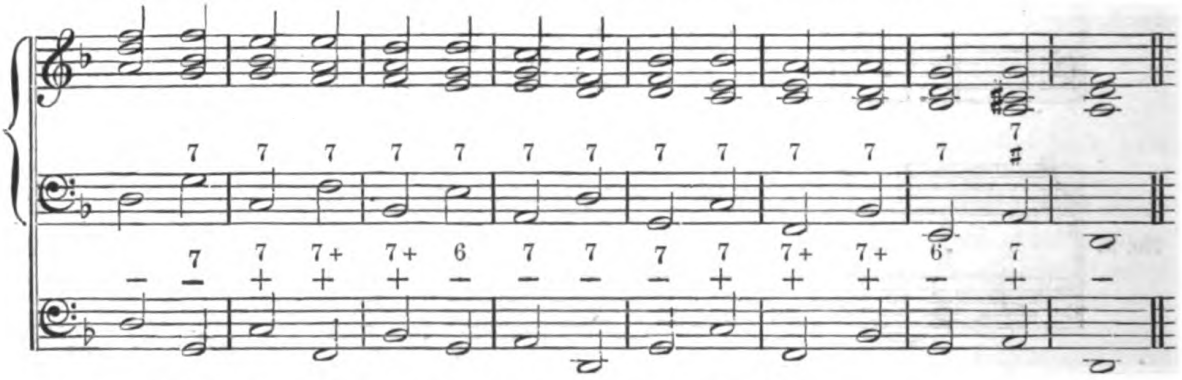
† In these two Examples, the Ninths are irregularly prepared by Eighths: but the irregularity disappears when they are combined.

4. In this progression, every Seventh is prepared by the Third of the preceding Chord, and resolved by the Third of the following.

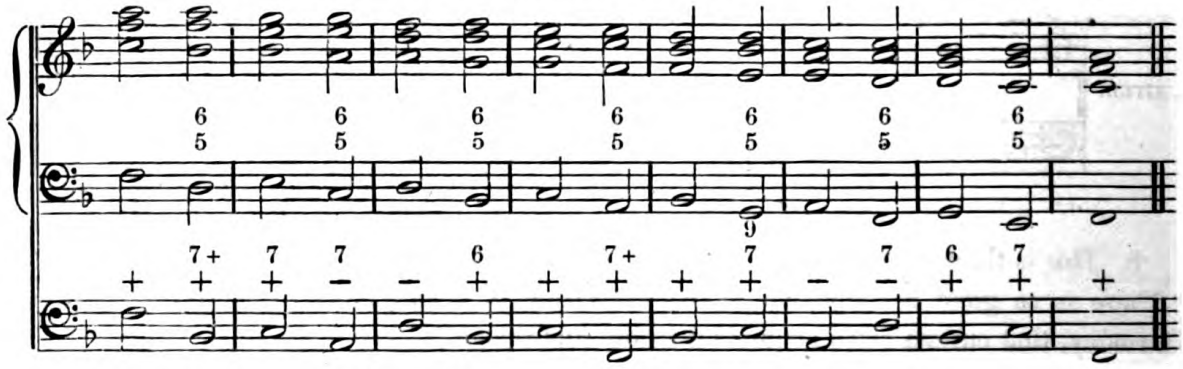
5. The Diatonic Sequence of Sevenths consists of two rhythmical Sections; each terminated, in the Fundamental Bass, by a regular Cadence of Subdominant, Dominant, and Tonic.



6. The Minor Mode consists of alternate Cadences, Major and Minor:—



7. Sevenths in the first Inversion, alternating with Triads:—



8. The Suspended notes in the Sequence of Sevenths, are commonly called Notes of Syncopation, to distinguish them from the common Suspensions; which are generally resolved on the same Fundamental Bass.

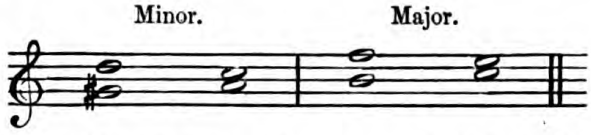
CHAPTER X.

COMPOUND DOMINANT HARMONIES.

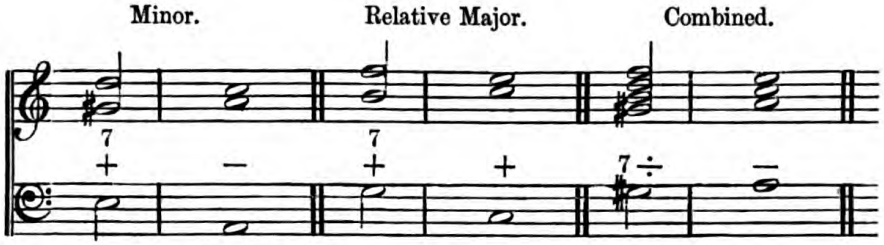
1. THE Chords of the Diminished Seventh and Augmented Sixth, have an effect upon the ear peculiar to themselves; and in the works of the great Composers, often produce effects of surpassing grandeur. Their true nature and proportions do not appear to have been hitherto known, or attended to. After much study, I have come to the conclusion that they are composed of different Dominant Harmonies united; and have named them accordingly.

I. THE DIMINISHED SEVENTH.

2. The characteristic notes of the Dominant Harmony, are the Major Third and Grave Seventh; resolving into the Tonic and its Third:—



3. In the Minor Mode, the characteristic notes of the Dominant Harmony, and their Resolution, combined with the same notes of the Relative Major, produce the Chord of the Diminished Seventh, and its Resolution:—



4. The proportions of the Diminished Seventh are:—

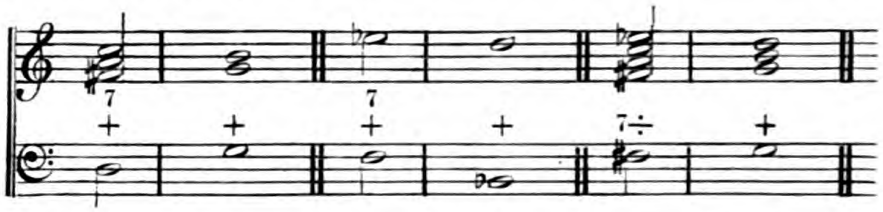
G♯ B $\overset{\Delta}{D}$ $\overset{\Delta}{F}$ . . . . 25 . 30 . 35 . 42.

E G♯ $\overset{\Delta}{D}$ . . . . 4 . 5 . 7 × 5 = 20 . 25 . 35.

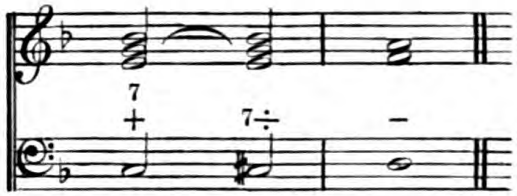
G B $\overset{\Delta}{F}$ . . . . 4 . 5 . 7 × 6 = 24 . 30 . 42.

5. This Chord was at first considered as peculiar to the Minor Mode: but the Major

Mode has adopted it, by combining its own Dominant Harmony with the Dominant Seventh note to the Minor Third above:—



6. The Chord of the Diminished Seventh, consists of a Diminished Triad with Minor Third *over* it. When it is prepared by the Dominant Seventh, it consists of a Diminished Triad with the Minor Third *under* it; which may be called the second form of the Chord:—



Minor Dominant . . . A C♯ E G . . . . 4 . 5 . 6 . 7 × 5 = 20 . 25 . 30 . 35.

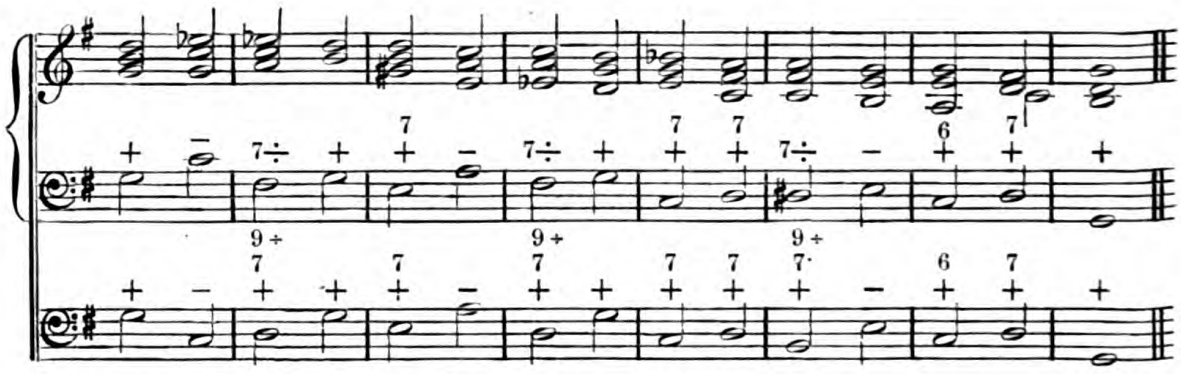
Relative Major . . . . C E G B♭. . . . 4 . 5 . 6 . 7 × 6 = 24 . 30 . 36 . 42.

First form . . . . C♯ E G B♭. . . . 25 . 30 . 35 . 42.

Second form . . . C♯ E G B♭. . . . 25 . 30 . 36 . 42.

In the first form, the Fifth of the Chord is taken from the Minor Dominant Chord; and in the second form, from the Relative Major.

7. In the following Example, the Diminished Seventh is resolved first into the Major, and then into the Minor Chord:—



8. In the Resolution of the Diminished Seventh, the lowest note, which is the Major Third of the Dominant Harmony, rises a Diatonic Semitone, 15 : 16, to the Tonic. The

Seventh descends a Minor Hemitone, 20 : 21, to the Fifth of the Tonic. The Fifth, which is Dominant Seventh to the Root, descends as follows:—

To the Major Third of the Tonic .	20 : 21	Hemitone Minor.
Second form	25 : 27	Diatonic Semitone + Comma.
To the Minor Third of the Tonic .	32 : 35	Tone Minimus.
Second form	8 : 9	Tone Major.

These varieties in the Resolution occasion no difficulty to the performer, whose ear is always directed by the Fundamental Bass.

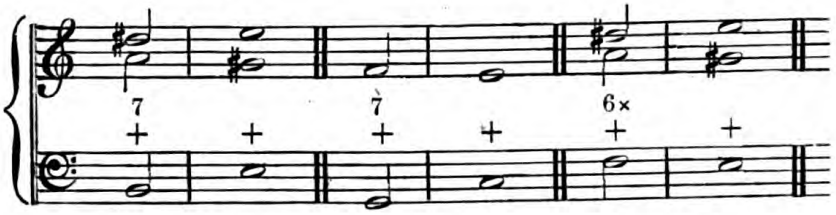
II. THE AUGMENTED SIXTHS.

9. The Chords of the Augmented Sixth are Dominant Harmonies, resolving regularly, but always into the *Major* Tonic. They are three in number, viz.:—

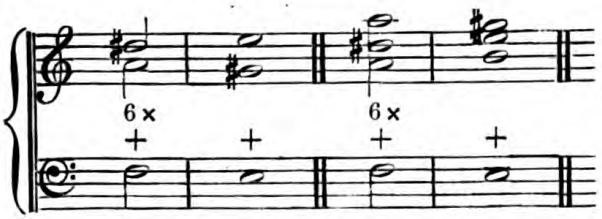
1st.—The Italian Sixth.
2nd.—The French Sixth.
3rd.—The German Sixth.

1st.—THE ITALIAN SIXTH.

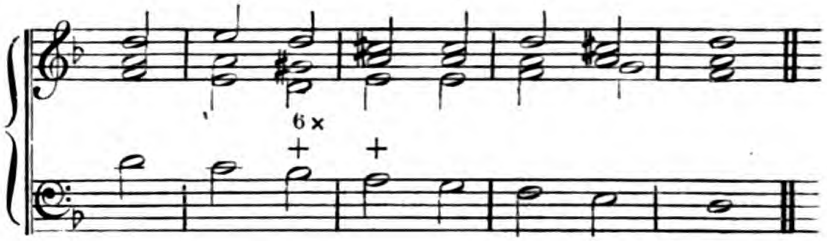
10. When the characteristic notes of the Dominant Harmony (Sect. 2) in the Minor Mode, are inverted, and the Dominant Seventh note of the Major Triad below is taken as a Bass, a Chord is formed, called the Italian Sixth. This Chord, which consists of Major Third and Augmented Sixth, resolves into the Major Tonic: the Sixth, which is Major Third of the Dominant Harmony, rising a Diatonic Semitone to the Tonic; and the two Dominant Sevenths descending a Minor Hemitone:—



11. In four parts, the Third is doubled; and the lower Third is resolved by ascending to the Fifth, while the upper Third descends:—

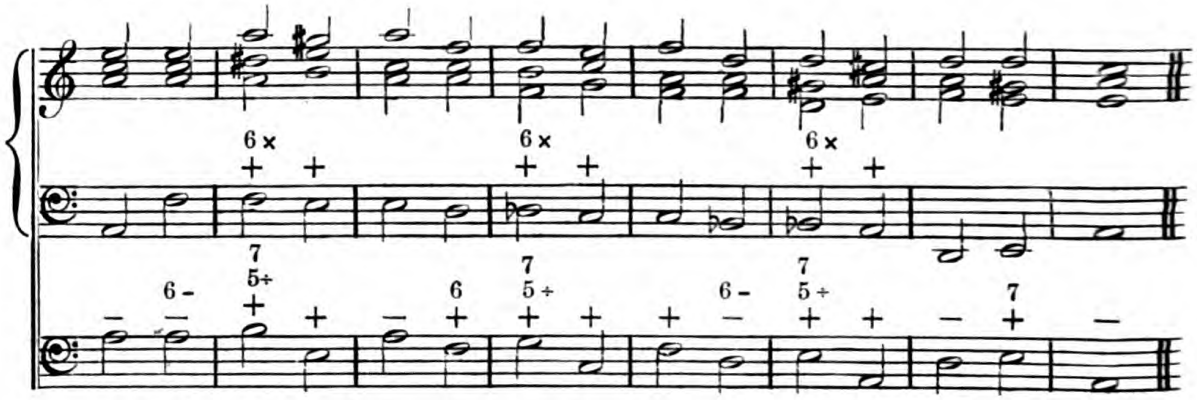


12. This Chord occurs naturally in the descending Scale of the Rule of the Octave, Minor Mode; when it modulates, as in the Major Mode, into the Key of the Dominant:—



If the Dominant Seventh were employed in this Modulation, the Sixth of the Scale, being the Fifth of that Chord, would be Major, instead of Minor. It seems not improbable that this may have led to the discovery of the Chord.

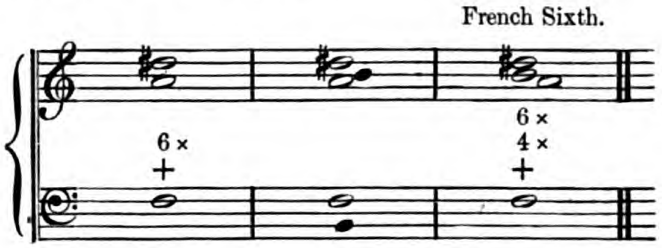
13. In the following Example, the Bass of the Italian Sixth is twice prepared by *the* Minor Added Sixth:—



14. The Inversions of this Chord are not used.*

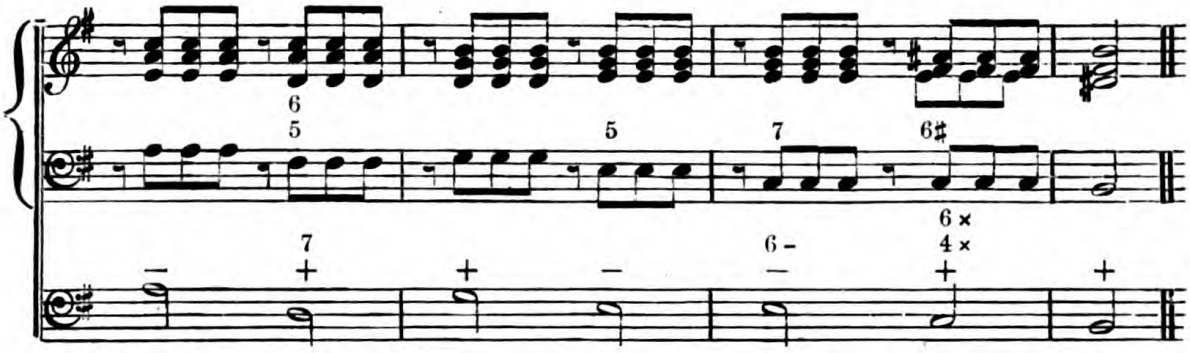
2nd.—THE FRENCH SIXTH.

15. When the Octave to the Fundamental Bass is introduced into the Chord of the Italian Sixth, it is called the French Sixth:—



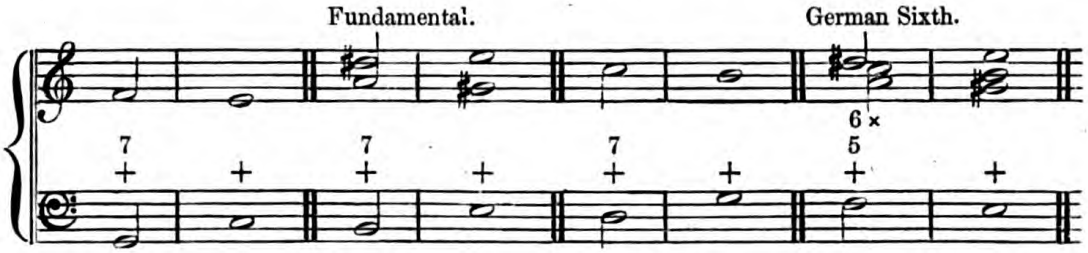
* See Callcott's *Grammar*, Art. 477, note; 2nd Ed. 441.

16. This Chord, on account of its harshness, is very seldom used. The following Example, however, occurs in a celebrated composition of Graun :—

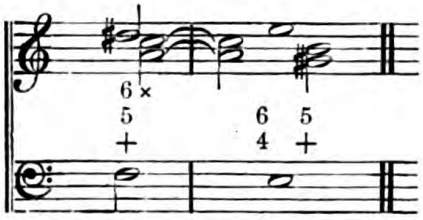


3rd.—THE GERMAN SIXTH.

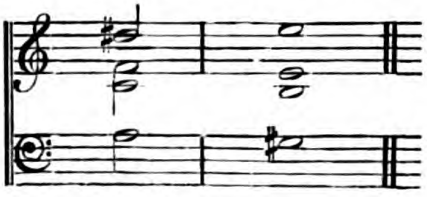
17. When the characteristic notes of the Dominant Harmony in the Minor Mode are inverted, and combined with the Dominant Seventh notes of the Major Triads below and above it, a Chord is produced which is called the German Sixth. The Bass of this Chord, which has three Roots, is the Dominant Seventh to the lowest Root :—



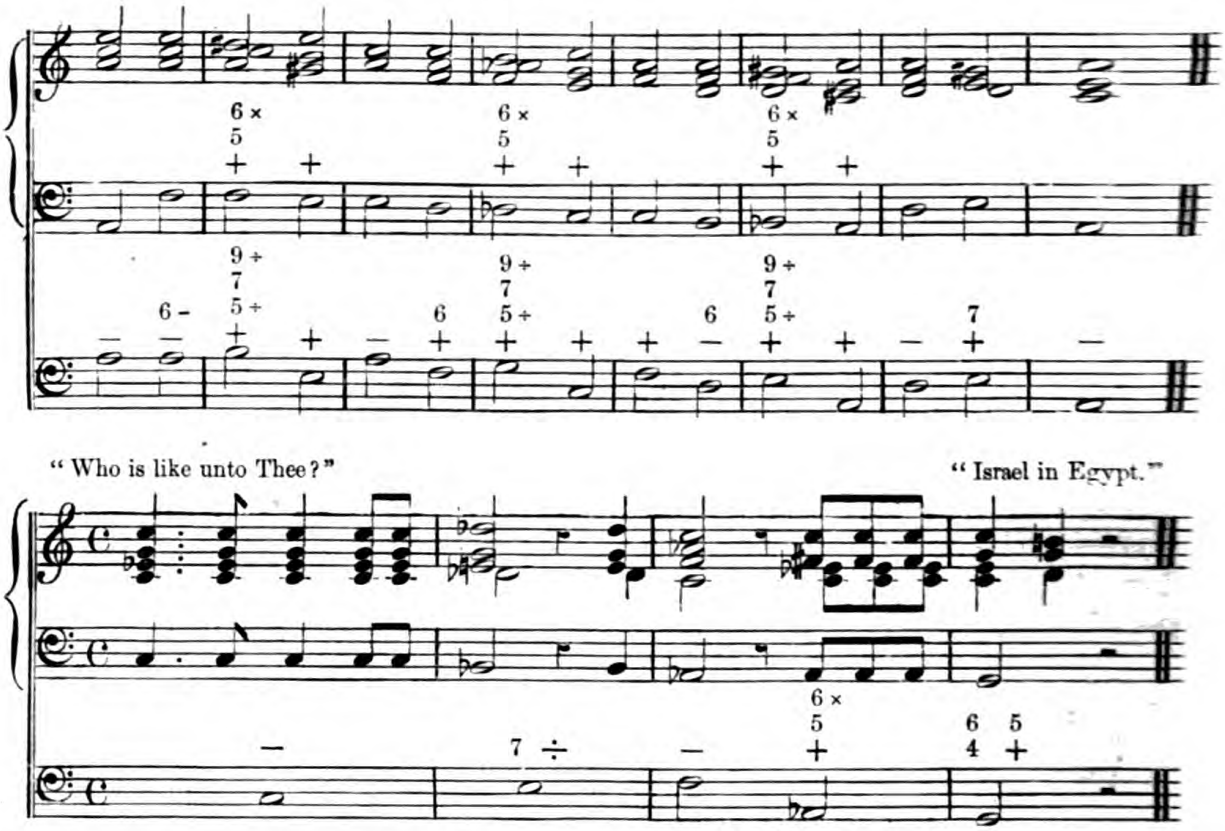
18. The consecutive Fifths (XIV. 10) between the Chord and its Resolution, are generally avoided by suspending the Fourth and Sixth :—



19. The first Inversion of this Chord is allowed, but no other; thus :—



20. Examples of the German Sixth:—



21. The proportions of the Augmented Sixth are:—

Italian Sixth . . . . $\overset{\triangle}{F}$ $\overset{\triangle}{A}$ — D♯ . . . . 28 . 35 . — . 50.
French Sixth . . . . $\overset{\triangle}{F}$ $\overset{\triangle}{A}$ B D♯ . . . . 28 . 35 . 40 . 50.
German Sixth . . . $\overset{\triangle}{F}$ $\overset{\triangle}{A}$ $\overset{\triangle}{C}$ D♯ . . . . 28 . 35 . 42 . 50.

and their derivation as follows:—

Seventh to G . . . . . G — $\overset{\triangle}{F}$ . . . . 4 . — . 7 × 4 = 16 . — . 28.
Harmony of B . . . . . B D♯ $\overset{\triangle}{A}$ . . . . 4 . 5 . 7.
Inverted . . . . . $\overset{\triangle}{A}$ B D♯ . . . . 7 . 8 . 10 × 5 = 35 . 40 . 50.
Seventh to D . . . . . D — $\overset{\triangle}{C}$ . . . . 4 . — . 7 × 6 = 24 . — . 42.

22. In the Resolution of these Chords, the Sixth, which is the Major Third of the Dominant Harmony, rises a Diatonic Semitone: the Bass, Third, and Fifth, which are all Dominant Sevenths, descend a Minor Hemitone, 20 : 21.

23. The Bass of the Augmented Sixth, corresponds to the Fifth of the Dominant Harmony lowered a Major Hemitone, 14 : 15. Wherefore, by lowering the Bass a Major Hemitone, the Inversions of the Dominant Harmony, that have the Fifth of the Root for their Bass, are changed into the Chords of the Augmented Sixth. Thus :—

Diminished Triad . . . . B D $\overset{\vartriangle}{F}$ . . . . . 1st Inversion, D $\overset{\vartriangle}{F}$ B.

Dominant Seventh . . . G B D $\overset{\vartriangle}{F}$ . . . . 2nd Inversion, D $\overset{\vartriangle}{F}$ G B.

Diminished Seventh . . B D $\overset{\vartriangle}{F}$ $\overset{\vartriangle}{A}\flat$. . . . 1st Inversion, D $\overset{\vartriangle}{F}$ $\overset{\vartriangle}{A}\flat$ B

Italian Sixth . . . . $\overset{\vartriangle}{D}\flat$ $\overset{\vartriangle}{F}$ B.

French Sixth . . . . $\overset{\vartriangle}{D}\flat$ $\overset{\vartriangle}{F}$ G B.

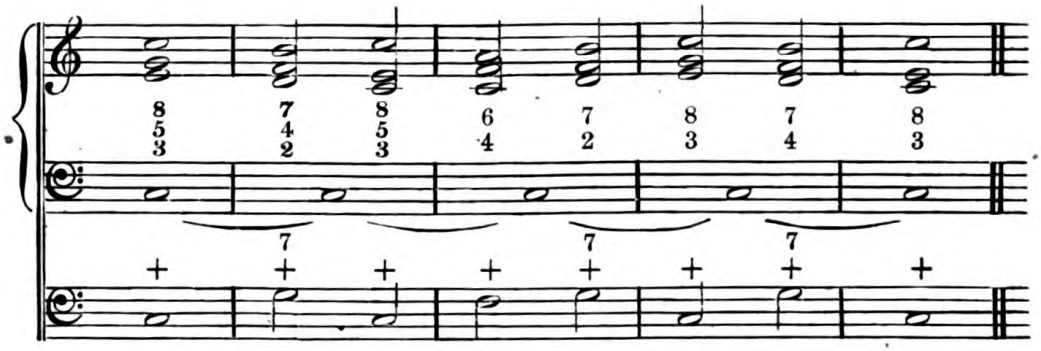
German Sixth . . . $\overset{\vartriangle}{D}\flat$ $\overset{\vartriangle}{F}$ $\overset{\vartriangle}{A}\flat$ B.

24. This change on the Bass is produced by a Flat; which, on the Grave or Dominant Seventh note, represents the Major Hemitone 14 : 15, not the Chromatic Semitone.

CHAPTER XI.

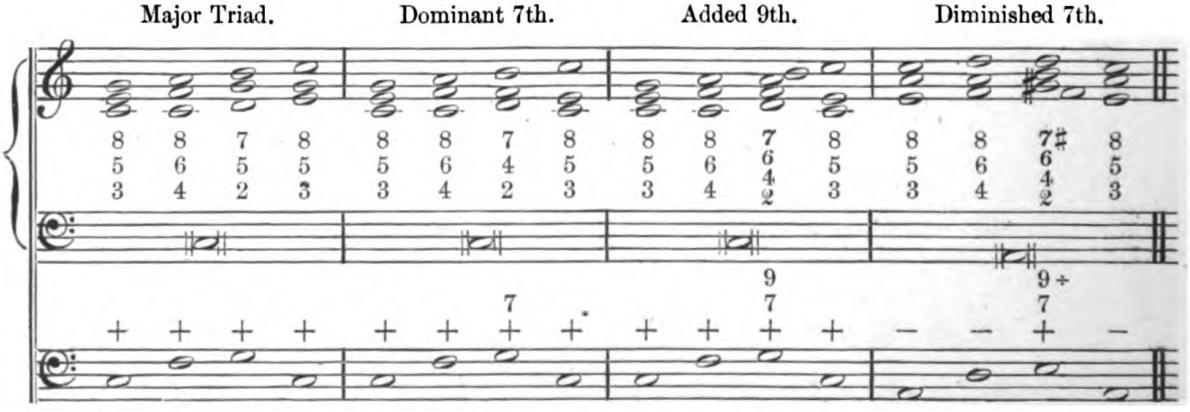
PEDAL HARMONIES.

1. "WHEN the Dominant Harmony is taken, unprepared, upon the Tonic Bass as a holding note, whether preceded by the Tonic, or by the Subdominant Harmony; the passage is termed a Tonic Pedal note, or Organ point:"—*



In this Example, the Dominant Harmony is taken three times on the Tonic Bass C; which note is common to the other two Chords, viz. the Tonic and the Subdominant.

2. The different Chords of the Dominant Harmony may be thus arranged on the Tonic Bass:—



3. The Intervals being counted from the Tonic Bass, the Third of the Dominant Harmony becomes a Major Seventh; the Fifth becomes a Ninth, the Seventh an Eleventh, and the

* Callcott's *Grammar*, Art. 472; 2nd Edit. 437.

Ninth a Thirteenth: for which reason, these Chords are often called Chords of the Ninth, Eleventh, and Thirteenth. They are figured, however, an Octave lower; 2, 4, 6.

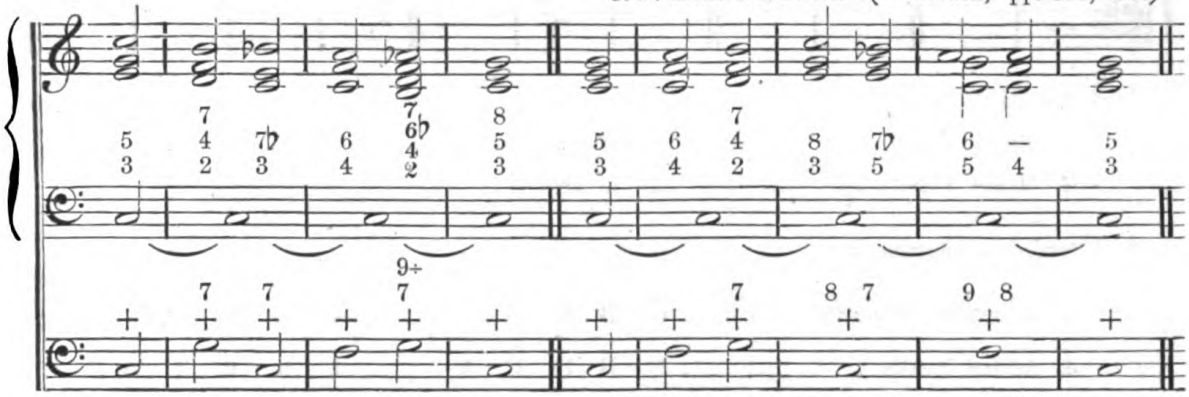
4. The proportions of the Pedal Harmonies are as under:—

Ninth	C	G	B	D	. . . . .	8 . 12 . 15 . 18.
Eleventh	C	B	D	$\overset{\vartriangle}{F}$	. . . . .	8 . 15 . 18 . 21.
Thirteenth Major . . .	C	B	D	$\overset{\vartriangle}{F}$	A	8 . 15 . 18 . 21 . 27.
Thirteenth Minor . . .	A	G♯	B	$\overset{\vartriangle}{D}$	$\overset{\vartriangle}{F}$	40 . 75 . 90 . 105 . 126.

5. The last of these is Mr. Corfe's "Terrific Chord." How remote it is from simple Harmony, appears by the high numbers, and by its derivation from the Harmonies of A, E, and G.

6. Examples of a Tonic Pedal Harmony:—

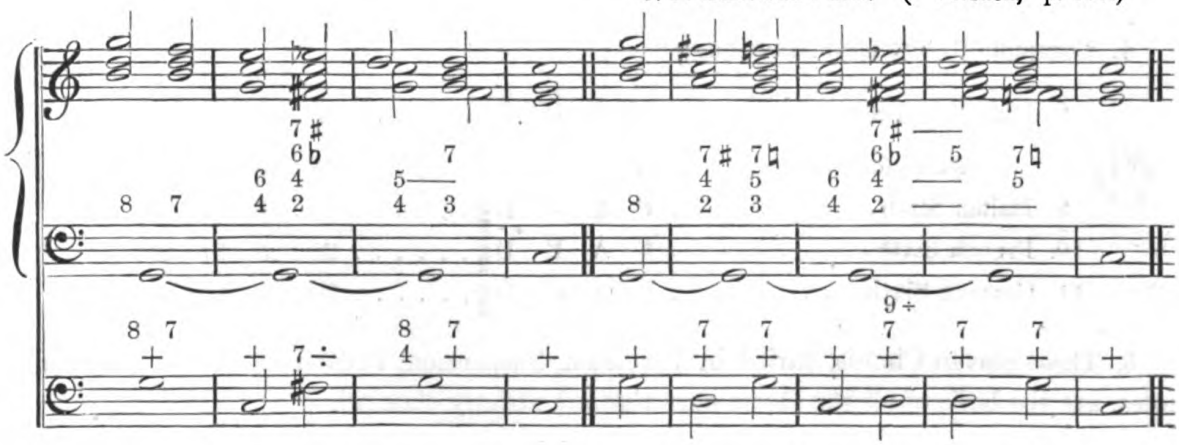
C. P. EMANUEL BACH. ("Versuch," pp. 329, 330.)



7. Sometimes the Pedal Harmonies are taken on the Dominant, instead of the Tonic; and sometimes the holding note occupies an upper part, instead of the Bass.

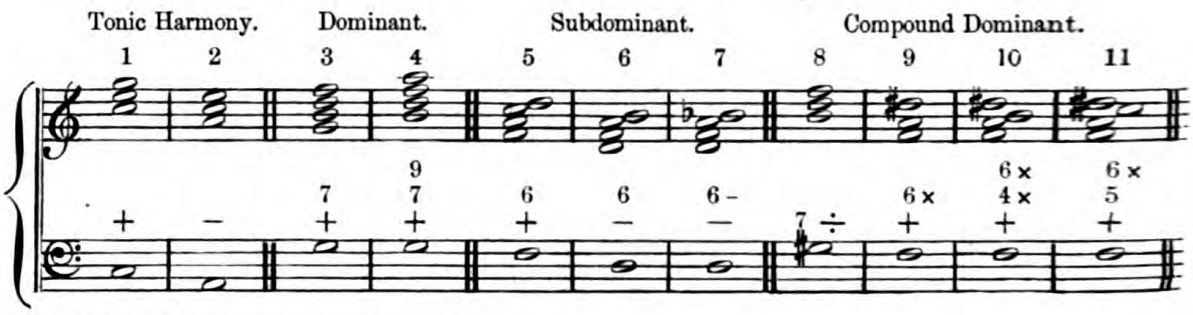
8. Examples of a Dominant Pedal Harmony:—

C. P. EMANUEL BACH. ("Versuch," p. 330.)



CHAPTER XII.

RECAPITULATION OF CHORDS.



1. Tonic Harmony :—

1. Major Triad . . . . . C E G . . . . . 4 . 5 . 6.
2. Minor Triad . . . . . A C E . . . . . 10 . 12 . 15.

2. Dominant Harmony :—

3. Dominant Seventh . . . . . G B D $\overset{\Delta}{F}$ . . . . . 4 . 5 . 6 . 7.
4. Added Ninth . . . . . G B D $\overset{\Delta}{F}$ A . . . . 4 . 5 . 6 . 7 . 9.

3. Subdominant Harmony :—

5. Added Sixth, Major Mode . . F A C D . . . . . 12 . 15 . 18 . 20.
6. Added Sixth, Minor Mode . . D F A B . . . . . 30 . 36 . 45 . 50.
7. Minor Added Sixth . . . . . D F A B♭ . . . . . 10 . 12 . 15 . 16.

4. Compound Dominant Harmonies :—

8. Diminished Seventh . . . . . G♯ B $\overset{\Delta}{D}$ $\overset{\Delta}{F}$ . . . . . 25 . 30 . 35 . 42.
Second form . . . . . G♯ B D $\overset{\Delta}{F}$ . . . . . 25 . 30 . 36 . 42.
9. Italian Sixth . . . . . F A — D♯ . . . . . 28 . 35 . 50.
10. French Sixth . . . . . F A B D♯ . . . . . 28 . 35 . 40 . 50.
11. German Sixth . . . . . F A C D♯ . . . . . 28 . 35 . 42 . 50.

5. These eleven Chords, varied by Inversion, Suspension, Pedal Basses, and omission of notes, are the basis of all the Harmonies that are strictly classical.

6. Let the notes of a Major Triad, as F A C, be taken as the Basses of three Dominant Harmonies: and out of the notes of those three Chords, may be produced all the Chords enumerated above, except the Added Sixth Major, of the Minor Mode:—

1. F A C $\overset{\vartriangle}{E}\flat$ . . . . 4 . 5 . 6 . 7 × 4 = 16 . 20 . 24 . 28.
2. A C♯ E $\overset{\vartriangle}{G}$ . . . . 4 . 5 . 6 . 7 × 5 = 20 . 25 . 30 . 35.
3. C E G $\overset{\vartriangle}{B}\flat$ . . . . 4 . 5 . 6 . 7 × 6 = 24 . 30 . 36 . 42.

1. Major Triad . . . . . F A C . . . . . 16 . 20 . 24.
1. Dominant Seventh . . . . F A C $\overset{\vartriangle}{E}\flat$ . . . . 16 . 20 . 24 . 28.
1. 3. Minor Triad . . . . . A C E . . . . . 20 . 24 . 30.
1. 3. Added Ninth . . . . . F A C $\overset{\vartriangle}{E}\flat$ G . . 16 . 20 . 24 . 28 . 36.
1. 3. Added Sixth . . . . . C E G A . . . . 24 . 30 . 36 . 40.
1. 3. Minor Sixth . . . . . A C E F . . . . 20 . 24 . 30 . 32.
2. 3. Diminished Seventh . . . C♯ E $\overset{\vartriangle}{G}$ $\overset{\vartriangle}{B}\flat$ . . . . 25 . 30 . 35 . 42.
1. 2. Italian Sixth . . . . . $\overset{\vartriangle}{E}\flat$ $\overset{\vartriangle}{G}$ — C♯ . . . . 28 . 35 . — . 50.
1. 2. French Sixth . . . . . $\overset{\vartriangle}{E}\flat$ $\overset{\vartriangle}{G}$ A C♯ . . . . 28 . 35 . 40 . 50.
1. 2. 3. German Sixth . . . . . $\overset{\vartriangle}{E}\flat$ $\overset{\vartriangle}{G}$ $\overset{\vartriangle}{B}\flat$ C♯ . . . . 28 . 35 . 42 . 50.

7. The Added Sixth Major, of the Minor Mode, requires an additional Triad, a Fifth above the last:—

A C♯ E . . . . 20 . 25 . 30
C E G . . . . 24 . 30 . 36.
G B D . . . . 36 . 45 . 54.

Added Sixth, Minor Mode . . . . E G B C♯ . . . . 30 . 36 . 45 . 50.

8. The Chords of Harmony, when referred to their Fundamental Basses, are few in number, and simple in their construction.

9. The Harmonic Triad, or Major Common Chord, and the Harmonic Tetrad, or Dominant Harmony, are Harmonics of a single string:—

Harmonic Triad . . . . F A C . . . . . 4 . 5 . 6.
Harmonic Tetrad . . . F A C $\overset{\vartriangle}{E}\flat$ . . . . 4 . 5 . 6 . 7.

10. The Minor Triad is compounded of the Harmonics of two strings, whose vibrations are to one another as 2 to 3:—

Triad of F . . . . . F A C . . . . 4 . 5 . 6 × 2 = 8 . 10 . 12.
Triad of C . . . . . C E G . . . . 4 . 5 . 6 × 3 = 12 . 15 . 18.
Minor Triad . . . . A C E . . . . . 10 . 12 . 15.

11. The Added Ninth consists of the Dominant Harmony, with the Fifth of the adjacent Triad above:—

Dominant Harmony . G B D $\overset{\triangle}{F}$ . . . . . 4 . 5 . 6 . 7 × 2 = 8 . 10 . 12 . 14.
Triad above . . . . . D F♯ A . . . . . 4 . 5 . 6 . — × 3 = 12 . 15 . 18.
Added Ninth. . . . . G B D $\overset{\triangle}{F}$ A . . 8 . 10 . 12 . 14 . 18 ÷ 2 = 4 . 5 . 6 . 7 . 9.

12. The Added Sixth consists of the Subdominant Triad, with the Octave of the Third in the adjacent Triad below:—

Subdominant Triad . . . F A C . . . . . 4 . 5 . 6 × 3 = 12 . 15 . 18.
Triad below . . . . . B♭ D F . . . . . 4 . 5 . 6 × 4 = 16 . 20 . 24.
Added Sixth . . . . . F A C D . . . . . 12 . 15 . 18 . 20.

Note.—The D which forms the Added Sixth, is lower by a Comma than the Supertonic of the Scale.

MINOR MODE.

Subdominant Triad . . . D F A . . . . . 10 . 12 . 15 . — × 3 = 30 . 36 . 45.
Triad below . . . . . G B♭ D . . . . . 10 . 12 . 15 . — × 4 = 40 . 48 . 60.
Added Minor Sixth . . . D F A B♭. . . . 30 . 36 . 45 . 48 ÷ 3 = 10 . 12 . 15 . 16.

13. The Major Added Sixth of the Minor Mode, consists of the Subdominant Triad, with the Octave of the Third in the adjacent Triad below, altered from Minor to Major:—

Subdominant Triad . . . D F A . . . . . 10 . 12 . 15 × 3 = 30 . 36 . 45.
Triad below . . . . . G B♭ D . . . . . 10 . 12 . 15.
Tonic Major . . . . . G B D . . . . . 4 . 5 . 6 × 10 = 40 . 50 . 60.
Added Major Sixth . . . D F A B . . . . 30 . 36 . 45 . 50.

Note.—B, the Added Major Sixth, is lower by a Comma than the Supertonic of the Scale, with which it is commonly confounded.

14. The Suspended Fourth is Octave to the Root, and the Suspended Sixth is Octave to the Third, of the adjacent Triad below:—

Major.—Triad of C	C	E	G	. . . .	4 . 5 . 6 × 3	= 12 . 15 . 18.
Triad below	F	A	C	. . . .	4 . 5 . 6 × 4	= 16 . 20 . 24.
Suspended Fourth . . .	C	F	G	. . . .	12 . 16 . 18 ÷ 2	= 6 . 8 . 9.
Suspended Sixth	C	E	A	. . . .	12 . 15 . 20	. . . . 12 . 15 . 20.
Minor.—Triad of A	A	C	E	. . . .	10 . 12 . 15 × 3	= 30 . 36 . 45.
Triad below	D	F	A	. . . .	10 . 12 . 15 × 4	= 40 . 48 . 60.
Suspended Fourth . . .	A	D	E	. . . .	30 . 40 . 45 ÷ 5	= 6 . 8 . 9.
Suspended Sixth	A	C	F	. . . .	30 . 36 . 48 ÷ 6	= 5 . 6 . 8.

15. The Suspended Ninth is Fifth, and the Suspended Seventh is Third, of the adjacent Triad above:—

Major.—Triad of C	C	E	G	. . . . .	4 . 5 . 6 . — × 2	= 8 . 10 . 12.
Triad above	G	B	D	. . . . .	4 . 5 . 6 . — × 3	= 12 . 15 . 18.
Suspended Ninth . . .	C	E	G	D	8 . 10 . 12 . 18 ÷ 2	= 4 . 5 . 6 . 9.
Suspended Seventh .	C	E	G	B	8 . 10 . 12 . 15	. . . . . 8 . 10 . 12 . 15.
Minor.—Triad of A	A	C	E	. . . . .	10 . 12 . 15 . — × 2	= 20 . 24 . 30.
Triad above	E	G	B	. . . . .	10 . 12 . 15 . — × 3	= 30 . 36 . 45.
Suspended Ninth . . .	A	C	E	B	20 . 24 . 30 . 45	. . . . . 20 . 24 . 30 . 45.
Suspended Seventh . .	A	C	E	G	20 . 24 . 30 . 36 ÷ 2	= 10 . 12 . 15 . 18.

16. The Diminished Seventh is a compound of the Dominant Harmonies of the Minor Mode, and its Relative Major:—

A Minor	E	G♯	B	D̊	. . . . 4 . 5 . 6 . 7 × 5	= 20 . 25 . 30 . 35.
C Major	G	B	D	F̊	. . . . 4 . 5 . 6 . 7 × 6	= 24 . 30 . 36 . 42.
Diminished Seventh . .	G♯	B	D̊	F̊	. . . . 25 . 30 . 35 . 42.	
Second form	G♯	B	D	F̊	. . . . 25 . 30 . 36 . 42.	

17. The Italian and French Sixths are compounded of two Dominant Harmonies, taken on the two notes of a Major Third:—

Dominant Seventh of G	G	—	F̊		. . . . 4 . — . 7 × 4	= 16 . — . 28.
Dominant Harmony of B . . .	B	D♯	Å		. . . . 4 . 5 . 7.	
Inverted	Å	B	D♯		. . . . 7 . 8 . 10 × 5	= 35 . 40 . 50.
Italian Sixth	F̊	Å	—	D♯	. . . . 28 . 35 . — . 50.	
French Sixth	F̊	Å	B	D♯	. . . . 28 . 35 . 40 . 50.	

18. The German Sixth is a compound of three Dominant Harmonies, taken on the notes of the Harmonic Triad :—

Dominant Seventh of G . . . . G – $\overset{\triangle}{F}$ . . . . 4 . – . 7 × 4 = 16 . – . 28.

Dominant Harmony of B . . . B D♯ $\overset{\triangle}{A}$ . . . . 4 . 5 . 7.

Inverted . . . . . $\overset{\triangle}{A}$ B D♯. . . . 7 . 8 . 10 × 5 = 35 . 40 . 50.

Dominant Seventh of D . . . . D – $\overset{\triangle}{C}$ . . . . 4 . – . 7 × 6 = 24 . – . 42.

German Sixth . . . . $\overset{\triangle}{F}$ $\overset{\triangle}{A}$ $\overset{\triangle}{C}$ D♯. . . . 28 . 35 . 42 . 50.

19. The Pedal Harmonies are Dominant Harmonies, with the Root of the Triad below, placed under them. See Chap. XI. 4.

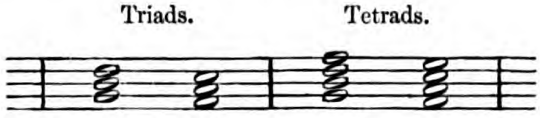
CHAPTER XIII.

THOROUGH BASS.

1. THOROUGH BASS, or Figured Bass, is a system of musical short-hand, in which the Bass of an accompaniment is expressed by notes, and the Chords by figures over them.

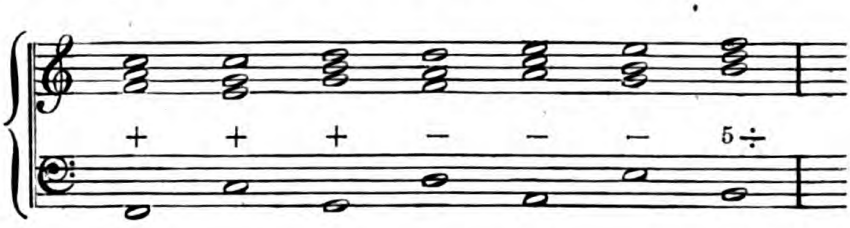
2. The figures of Thorough Bass point to the Intervals as they would stand on the Staff, if the notes were written. These Intervals are counted, not from the Fundamental Bass, but from the written Bass under the figures; and so partake of the ambiguity of the notation on which they are founded. Yet as the figures represent full Chords, they are often useful in ascertaining the Harmony; especially where any notes of a Chord are omitted in the written parts.

3. Most Chords are reducible to Triads or Tetrads; the former occupying three adjacent lines or spaces on the Staff, and the latter four:—



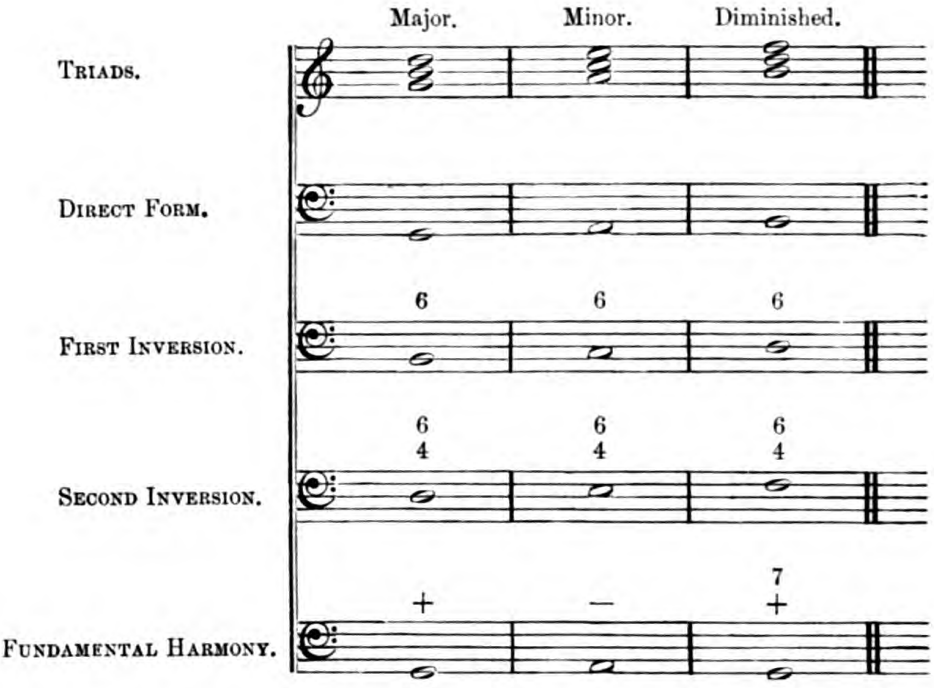
4. There are three Triads—Major, Minor, and Diminished. The last belongs to the Dominant Harmony; having, for its Bass, the Major Third of the Dominant. *

5. A Bass-note without a figure, is accompanied by a Triad; which is Major, Minor, or Diminished, according to its place in the Scale:—

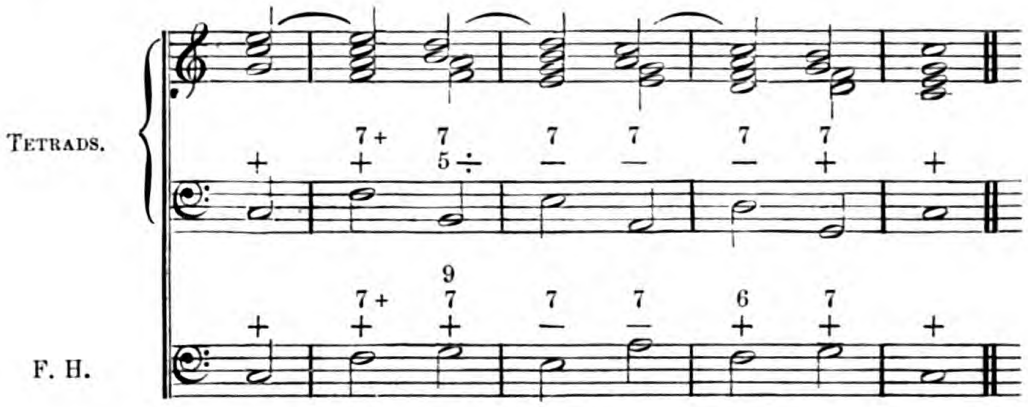


6. The first Inversion of a Triad, is called a Chord of the Sixth; the Third being understood. The second Inversion is called a Chord of the Fourth and Sixth. They are figured as follows:—

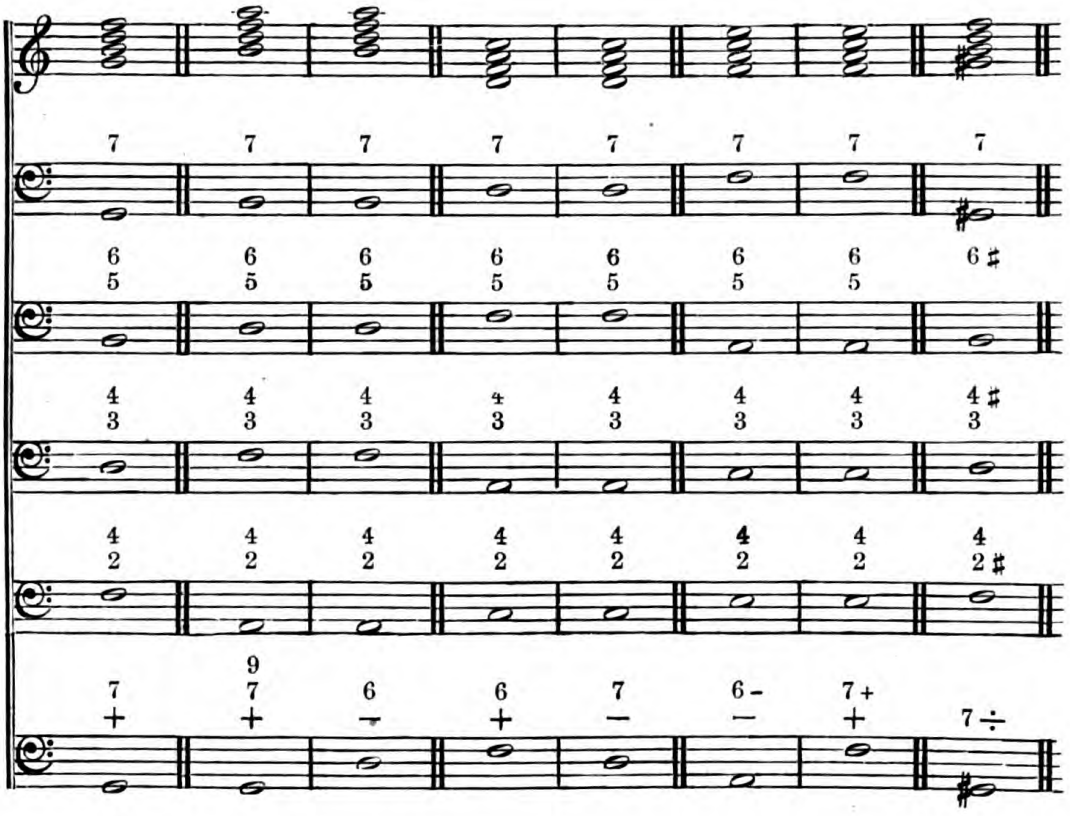
* Chap. III. 14.



7. The Tetrads, or Chords of the Seventh, consist of the same Triads, with Sevenths over them; which are Major, Minor, or Dominant Sevenths, according to their places in the Scale:—



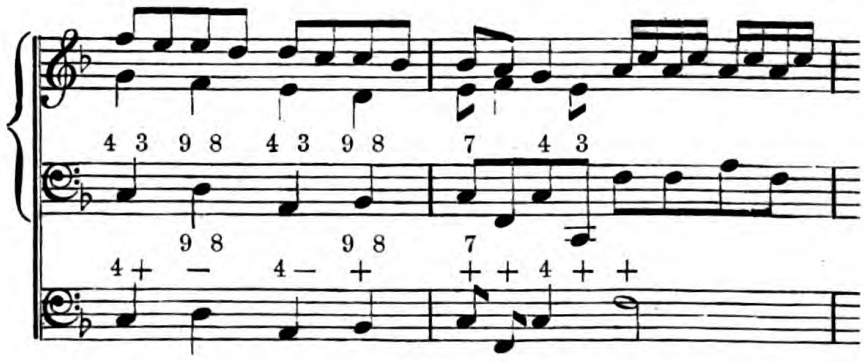
8. A Tetrad, in its direct form, is figured with a 7. The first Inversion is called a Chord of the Fifth and Sixth, the Third being understood. The second Inversion is called a Chord of the Third and Fourth, the Sixth being understood; and the third Inversion, a Chord of the Second, or Second and Fourth, the Sixth being understood. They are figured as follows:—



9. Various other Signatures will be found in the Examples under CHORDS OF SUSPENSION, COMPOUND DOMINANT HARMONIES, and PEDAL HARMONIES.

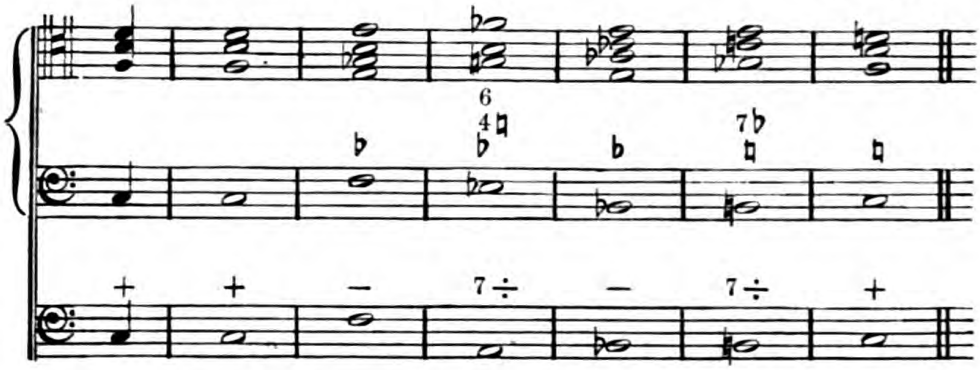
10. The figures 3, 5, 8, or any of them, denote the Common Chords; but are seldom used, except where a Common Chord follows a Different Chord upon the same Bass :—

CORELLI (Concerto 6).



11. When an accidental Sharp, Flat, or Natural, is required on a note of the Chord, it is written before or after the figure that expresses the note. A Sharp, Flat, or Natural, over a Bass-note, without a figure, affects the Third of the Chord :—

"He sent a thick darkness." "Israel in Egypt."



12. The following abbreviations are used in Figured Bass :—

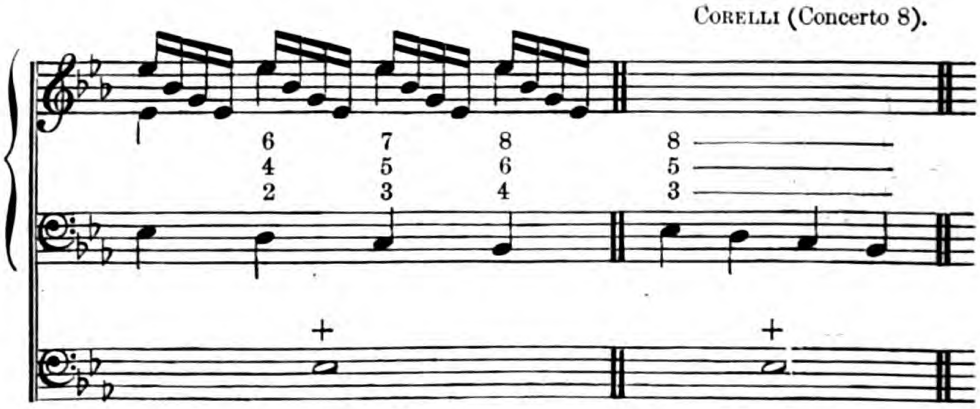
Chords	6	8	8	6	8	8	7	8	9	10
	4	5	5	4♯	5	6	5	5	5	8
	2	3	4	2	3	3	3	3	3	5
Abbreviations. . .	2	3	4	4♯	5	6	7	8	9	10

13. A dash through the figure is often used instead of a Sharp, as :—

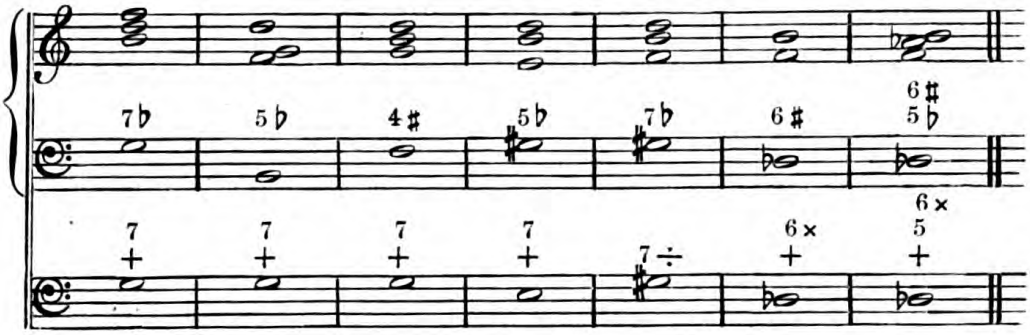
4 5 6 7

and a horizontal line after a figure, over a new Bass-note, intimates that the note represented by the figure, is to be struck, or continued, on the new Bass :—

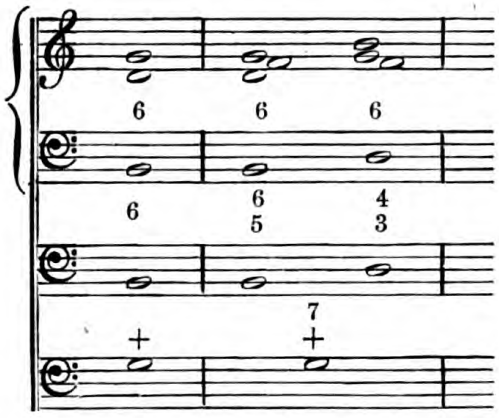
CORELLI (Concerto 8).



14. Irregular signatures, containing Sharps or Flats where the notes are natural, are evidently intended, by the Composers who use them, to give a hint of what the Chord is :—



15. The first and second inversions of the Dominant Harmony were formerly called Chords of the Sixth, and figured with a 6; the former being termed the Great Sixth, and the latter the Small Sixth. The performer was expected to know enough of Harmony to distinguish them from the first Inversion of the Triad, and from each other, although the three Chords were all figured alike :—



These ambiguous signatures occur in some of the Examples quoted in this work: particularly in the Scales of Mr. Bach, Chap. XX. sect. 36.

CHAPTER XIV.

PROGRESSION.

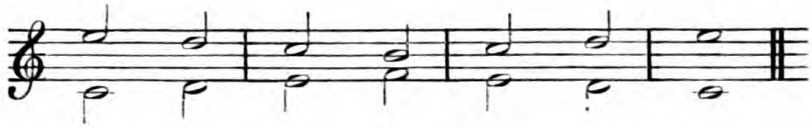
1. IN studying the progression of Chords, it is necessary to consider the motion of the different parts separately. Each part should move easily and naturally, forming, if possible, some kind of melody by itself.

2. The motions of the parts are distinguished into Direct, or Similar; Contrary; and Oblique.

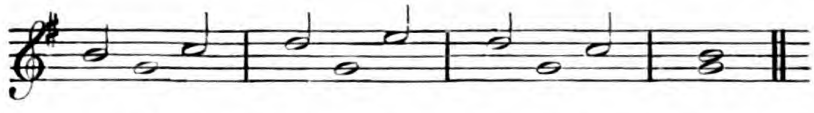
3. In the Direct motion, the parts move in the same direction, ascending or descending:—

This motion is, for the most part, restricted to Thirds and Sixths.

4. In the Contrary motion, one part rises, while the other falls:—



5. In the Oblique motion, one part continues at the same pitch, while the other rises or falls:—



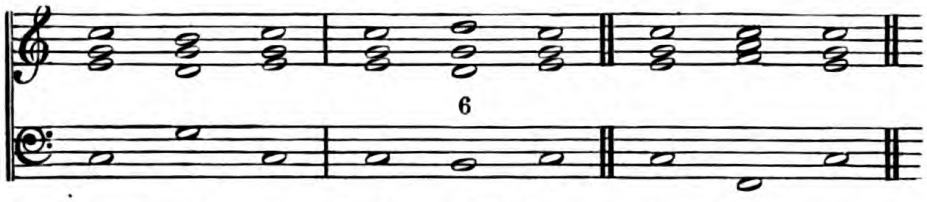
6. The Contrary and Oblique motions, where they can be used, are to be preferred to the Direct; in which the forbidden progressions are most apt to take place.

7. When four parts are required, one of the notes of the Triad is doubled, or its Octave is added. This is termed, *doubling the part*. The rule for doubling, follows the order of the Harmonic numbers, 1, 2, 3, 4, 5, 6, 7; the Root 1 being, in that series, doubled twice, (1, 2, 4); the Fifth 3, doubled once, (3, 6); and the Major Third 5, not doubled at all. It is therefore best to double the Root; next the Fifth: and not to double the Major Third,

except for the purpose of avoiding greater faults, or accommodating the melody. This rule does not affect the Minor Third.

RULE I.

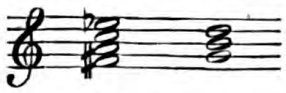
8. The general rule for correct accompaniment is, that the Chords should be taken closely; avoiding all unnecessary leaps, and changing the position with every change of the Chord:—



In these Examples, the Direct motion is avoided, between the extreme parts.

RULE II.

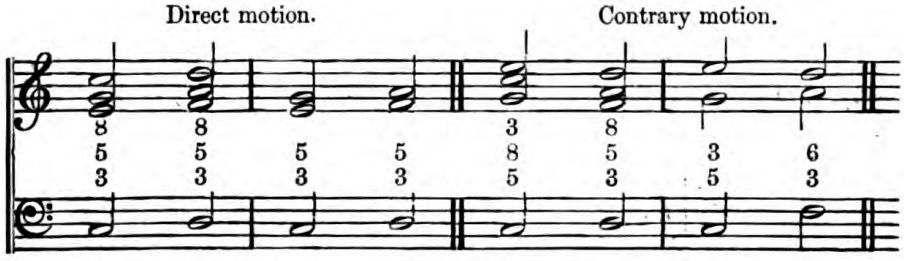
9. "All irregular motions of the parts are to be avoided. Every Major or Sharp Interval ought to ascend; and every Minor or Flat Interval ought to descend: that is to say, the part in which those Intervals are found, is to rise after the sharp, and fall after the flat:"—



"This Rule, however, is always subordinate to that of avoiding consecutive Octaves or Fifths; and is not to be regarded, when the Melody is to produce an effect contrary to the Rule. The internal parts of Harmony, however, are to be regulated by these observations." *

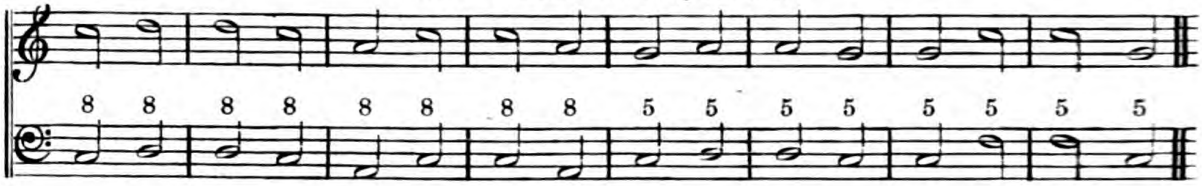
RULE III.

10. Consecutive Fifths and Octaves, between the same parts, are forbidden in the direct motion. This Rule does not affect the Diminished Fifth:—

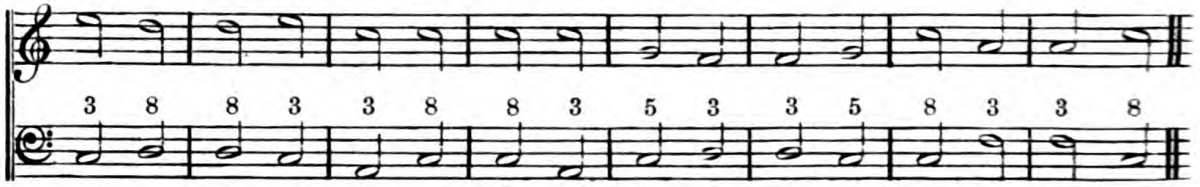


* Callcott's *Grammar*, Art. 324; 2nd Ed. 302.

From "HECK's *Thorough Bass.*"



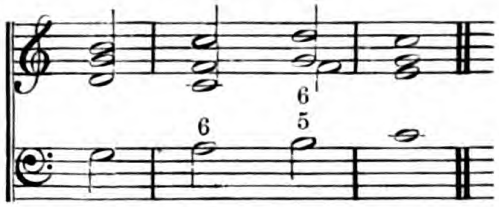
Corrected by Contrary and Oblique motion.



11. Consecutive Fifths and Octaves are most apt to occur, when the Fundamental Bass moves one Degree: which is called a Diatonic progression, and infers a transition to an unconnected Triad. Thus the Diatonic progression F G, passes over the intermediate Chord of C, which connects F and G together; F A C, C E G, G B D.

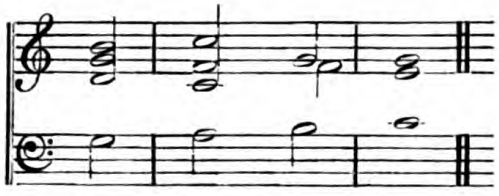
EXCEPTIONS.

12. (1.)—In the first Inversion of the Perfect Cadence, consecutive Fifths are admitted between the upper parts:—

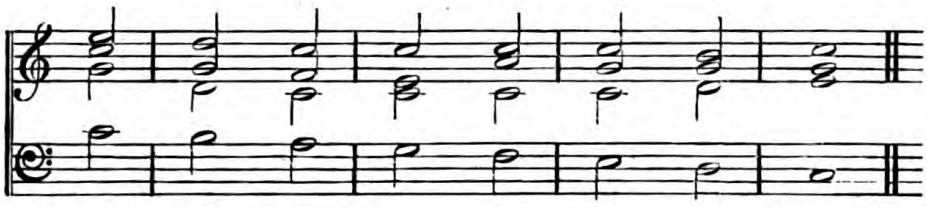


This Example is taken from the Rule of the Octave itself, as given by Rameau, Shield, and most of the approved writers.

13. Albrechtsberger avoids the consecutive Fifths, by omitting the Fifth of the Dominant Harmony, and the Octave of the Tonic; whereby the closing Cadence is rendered less effective:—



14. (2.)—In an accompaniment of Common Chords to the descending Scale, the most harmonious arrangement produces consecutive Fifths and Octaves in the upper parts, between the second and third Chords:—



15. From these exceptions it seems evident, that in certain cases consecutive Fifths and Octaves may be used, provided they are properly accompanied. What those cases and accompaniments are, may be hereafter discovered and defined. In the mean time we may rest satisfied with the judgment of one of the best Harmonists of our day; which I give in his own words:—

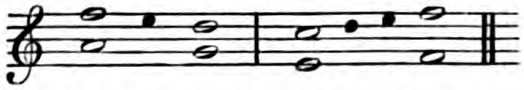
16. "Concerning the rules which forbid consecutive Fifths and Eighths, it may be observed, that they are founded on the practice of the most classical musicians; and that every breach of them has always been considered as a fault. Octaves, by their want of variety, weaken the harmony; and Fifths are disagreeable from their want of unity. It must, however, be remarked, that in scoring for instruments, and even for voices, Unisons and Octaves are frequently employed: but then, it must be remembered, that whatever may be the number of parts, they are, in reference to the Harmony, considered as one.

17. "Consecutive Fifths, which the ear condemns at once, should be avoided 'if possible;' and the 'possible' will always be discovered by him who, by study and practice, has possessed himself with the purest principles of his art. But it is right to state, that consecutive Fifths are not now treated with the unrelenting severity which was formerly exercised towards them. Schneider, in his 'Elements of Musical Harmony and Composition,' has discussed the subject in a dispassionate manner. He endeavours to show, that under certain circumstances, they are allowable: but he concludes his observations by the following excellent admonition:—'To the beginner, however, we can give no other counsel, than that of following, unconditionally, the law in question; until, by the study of acknowledged masterpieces, and by frequent opportunities of hearing good music, he shall have acquired a correct tact in these matters.'"—Horsley's *Introduction to Harmony*, pp. 120, 121.

RULE IV.

18. The progression from Thirds or Sixths, to Fifths or Octaves, between the same parts, is forbidden in the Direct motion, when the upper part moves more than one Degree:—

19. These progressions are supposed to contain what are called Hidden Fifths or Octaves; as A E, G D, and E E, F F, in the following Example, indicated by the black passing notes:—

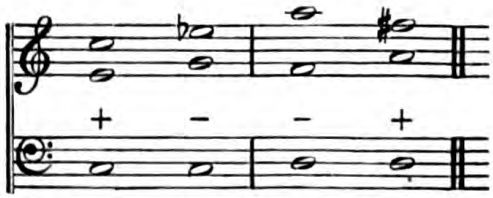


20. Mr. Horsley, after laying down particular rules for the progression of the Intervals in question, remarks:—"These Rules, in all their strictness, regard Harmony in two parts only: and they are relaxed as the number of parts is increased. But it is well observed by Albrechtsberger, that the extreme parts of a composition, (the Treble and Bass, for example), should always be regulated by them." *

RULE V.

21. When a Major Chord is to be changed to Minor, or a Minor Chord to Major, the alteration should take place on the same Degree, and in the same part:—

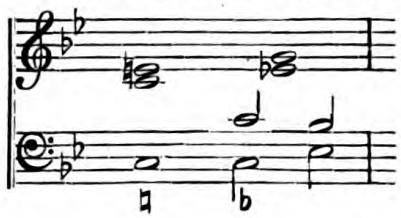
22. By the breach of this Rule, the Interval between the original and the altered note, instead of a Chromatic Semitone, becomes a Diminished or an Augmented Eighth; which is called a False Relation:—



* Horsley's *Introduction*, page 123.

23. Mr. Horsley shows, by copious Examples from the Fathers of English Church Music, that this Rule was not attended to in their time. In some of them, the note is altered on the same degree, but not in the same part :—

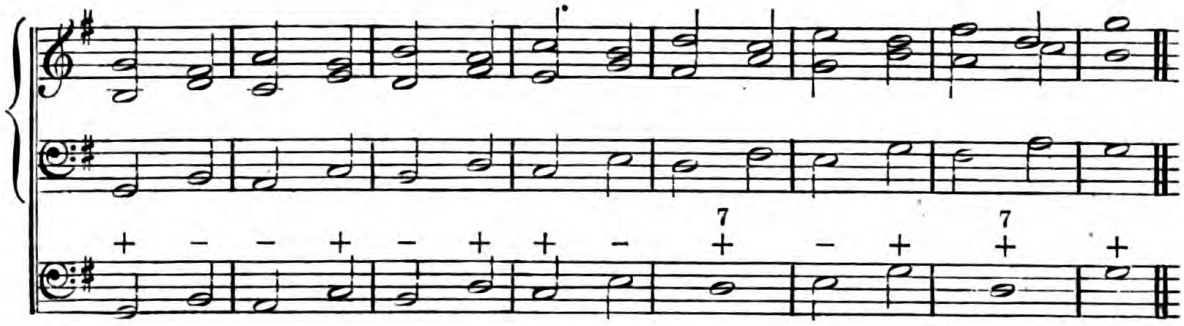
FARRANT. (Boyce, vol. i. p. 56.)



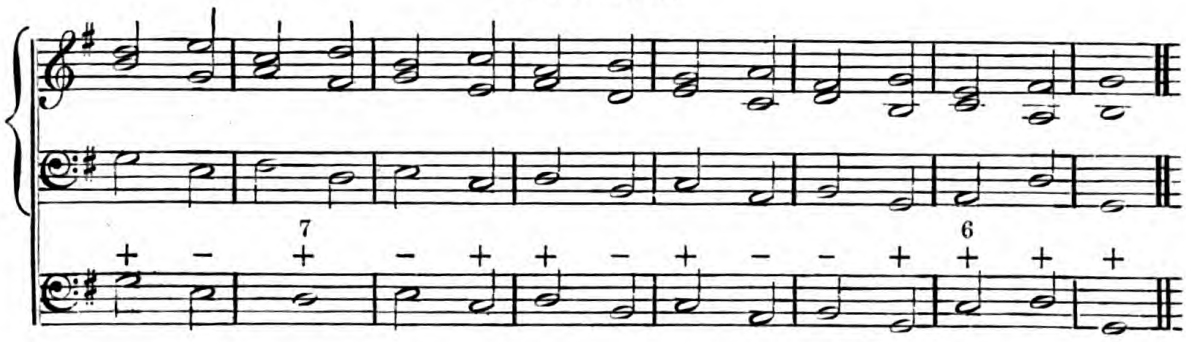
24. In the following Examples of Triads in Sequence, or regular progression, the contrary motion is observed throughout.

25. Progression by Seconds and Thirds :—

ASCENDING.



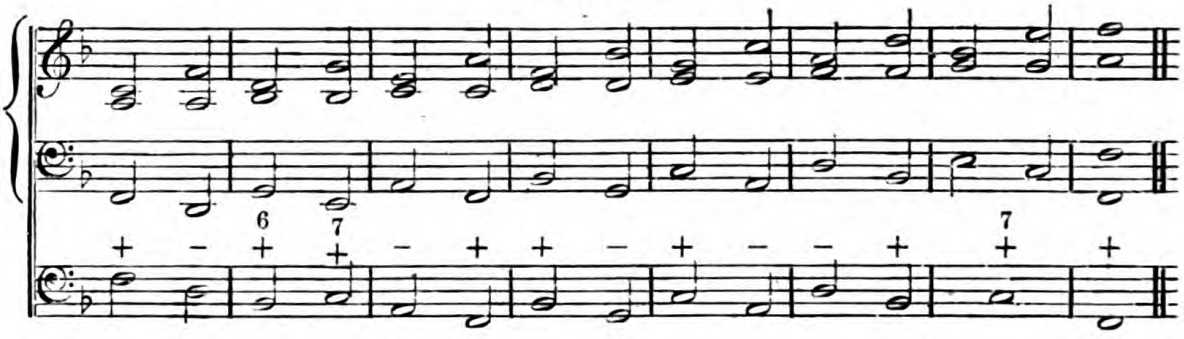
DESCENDING.



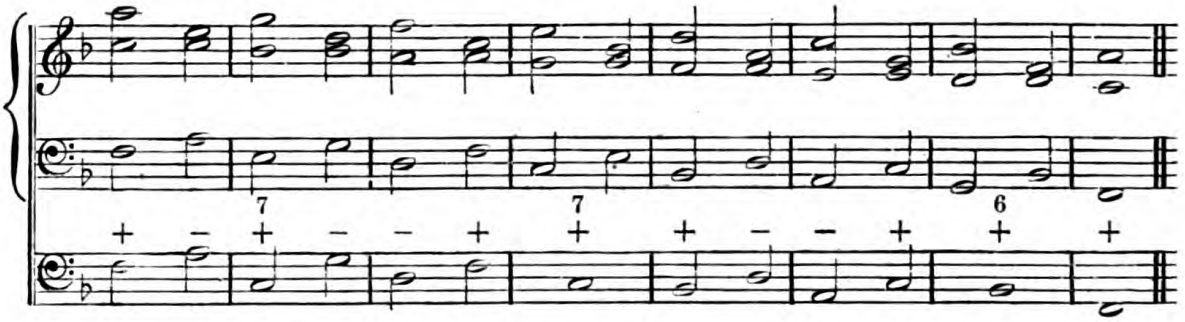
26. In the preceding Examples, where the Bass moves alternately by one degree of the Scale, (which is called a Diatonic progression), the Fifth of one of the Chords is omitted, and the Octave taken in its place, to avoid the forbidden progression of two consecutive Fifths.

27. Progression by Thirds and Fourths :—

ASCENDING.

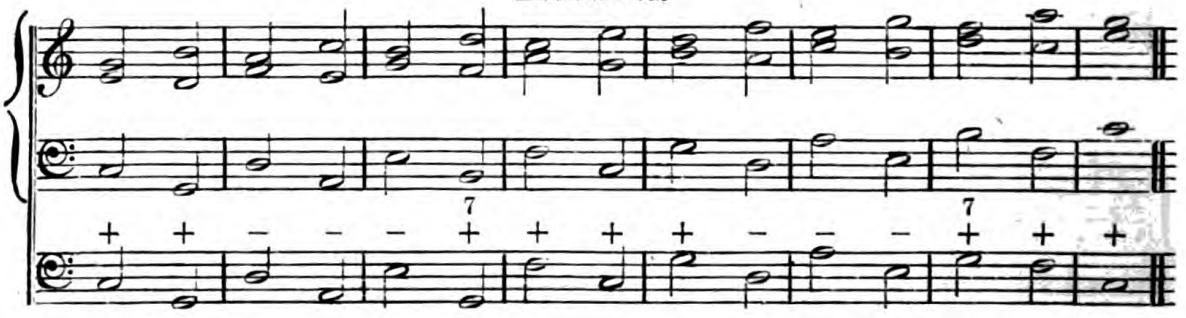


DESCENDING.

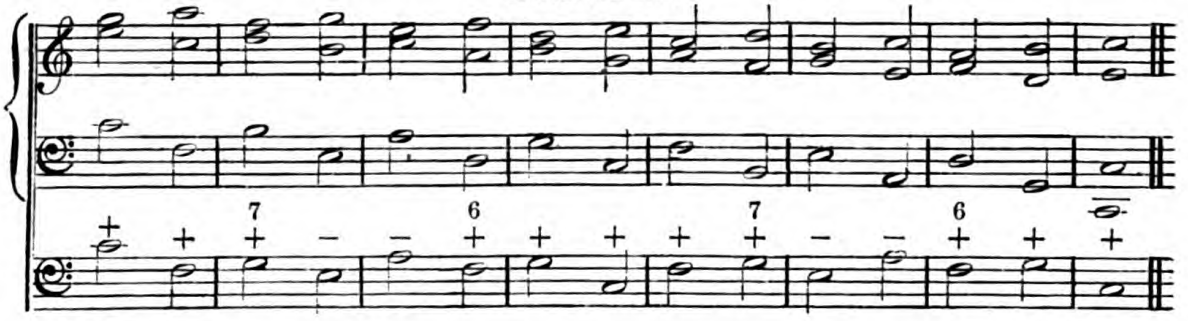


28. Progression by Fourths and Fifths :—

ASCENDING.

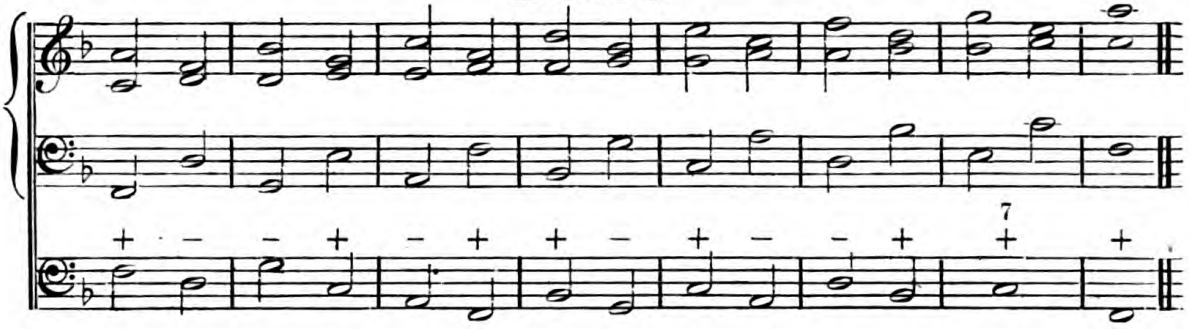


DESCENDING.

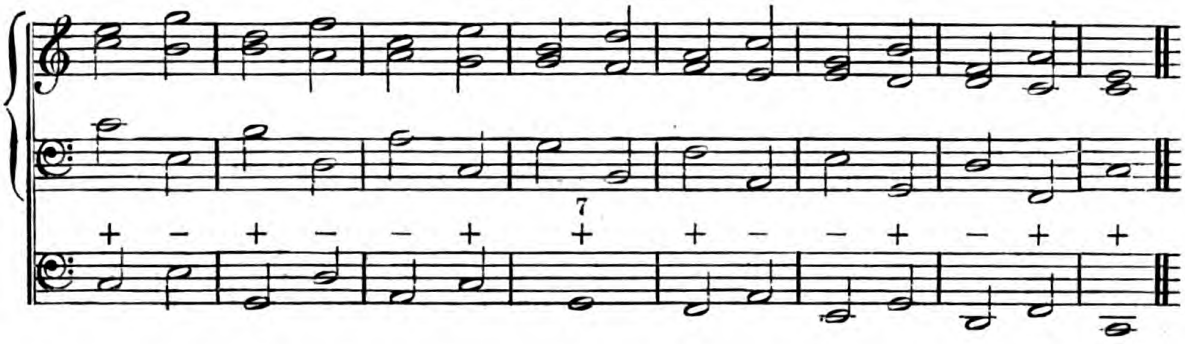


29. Progression by Fifths and Sixths :—

ASCENDING.



DESCENDING.



CHAPTER XV.

MODULATION.

1. THE Chords which have been briefly described, may be all exemplified in one Major Key, with its Relative Minor. But much pleasing variety is produced by a judicious use of what is called Modulation, or Change of Key: passing from one Key to another, according to certain rules; sometimes through several Keys; and returning at last to the original Key.

2. The order of Fifths, being of continual use in the study and practice of Music, and especially so in Modulation, should be always ready in the memory.

Order of ascending Fifths . . . . F C G D A E B.
Reversed in descending . . . . . B E A D G C F.

3. In modulating from one Major or Minor Key to another, the natural order is that of ascending or descending Fifths; every two adjacent Keys in that order, having more connexion with each other, than with more remote Keys. Thus,—

Key of F. Triads B♭ D F. F A C. C E G.
Key of C . . . . . F A C. C E G. G B D.
Key of G . . . . . C E G. G B D. D F♯ A.
Key of D . . . . . G B D. D F♯ A. A C♯ E.

In the Keys of F and C, the Triads F A C, C E G, are common to both. The Keys of F and G have in common the Triad C E G only: and the Keys of F and D have no Triad in common.

4. Now F being the Subdominant, and G the Dominant of the Key of C, it is evident that the Keys which are most closely connected with any given Key, are those of its Subdominant and Dominant; which, for that reason, are termed by Dr. Callcott, its attendant Keys.

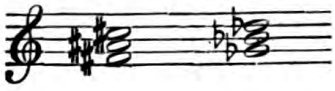
5. The scale of Keys may be extended upwards or downwards, at pleasure. But in order to avoid the confusion of Double Sharps and Flats, the number of Keys is commonly restricted to twenty-four, or twelve Major Keys with their Relative Minors; corresponding to the number of Keys on the finger-board of our instruments.

6. The order of Keys, ascending by Fifths from F, is :—

F C G D A E B F♯.

Instead of proceeding to the next Key, which is C♯ with seven Sharps, it is usual to substitute D♭ with five Flats; returning to the natural Key of C, through A♭, E♭, B♭, and F. By this substitution, which is called an Enharmonic Change, the names of the notes are changed, without altering the pitch.

Enharmonic Change from F♯ to G♭.



7. By means of this Enharmonic Change, the Bass, continuing to ascend by Fifths, instead of adding to the number of Sharps, subtracts from the Flats; and so leads back to the original Key.

8. Table of the twenty-four Keys :—

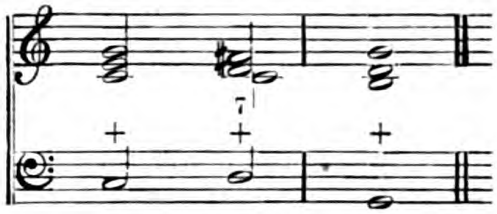
C Major . . . A Minor . . . Natural.
G Major . . . E Minor . . . 1 Sharp, F.
D Major . . . B Minor . . . 2 Sharps, F C.
A Major . . . F♯ Minor . . . 3 Sharps, F C G.
E Major . . . C♯ Minor . . . 4 Sharps, F C G D.
B Major . . . G♯ Minor . . . 5 Sharps, F C G D A.
F♯ Major . . . D♯ Minor . . . 6 Sharps, F C G D A E.
D♭ Major . . . B♭ Minor . . . 5 Flats, B E A D G.
A♭ Major . . . F Minor . . . 4 Flats, B E A D.
E♭ Major . . . C Minor . . . 3 Flats, B E A.
B♭ Major . . . G Minor . . . 2 Flats, B E.
F Major . . . D Minor . . . 1 Flat, B.
C Major . . . A Minor . . . Natural.

9. The Scale of Triads contains alternate Major and Minor Keys; each of which has six notes of the seven in common with the Keys immediately above and below :—

C Major . . . . F A C E G B D.
E Minor . . . . . A C E G B D F♯.
G Major . . . . . C E G B D F♯ A.
B Minor . . . . . E G B D F♯ A C♯.
D Major . . . . . G B D F♯ A C♯ E.

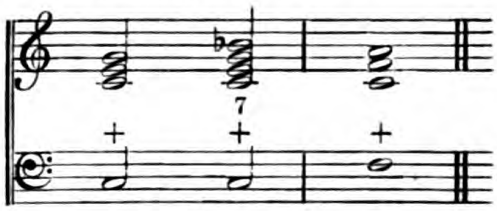
10. In modulating by ascending Fifths, the Supertonic becomes the Dominant of the new Key, and bears the Dominant Harmony.

To the key of the Dominant.



11. In modulating by descending Fifths, the Tonic becomes the Dominant of the new Key.

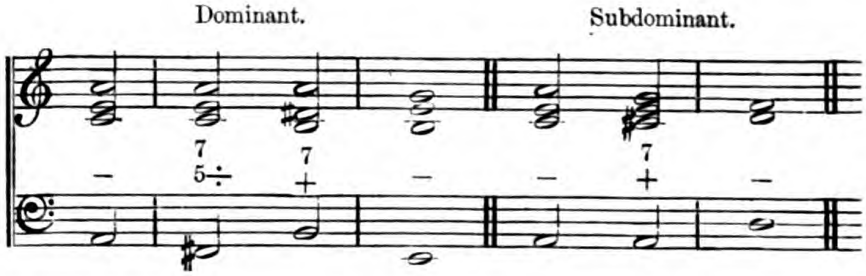
To the key of the Subdominant.



12. The other Modulations in common use, which I quote from Callcott's *Grammar*, are the following :—

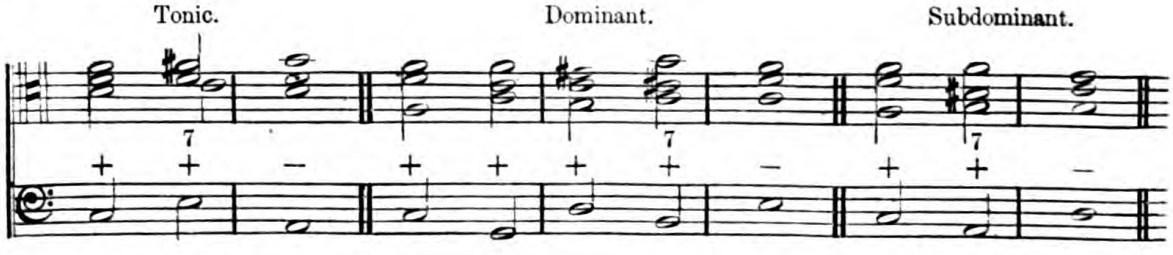
13. From the Minor Tonic, to the Keys of the Dominant and Subdominant :—

Dominant. Subdominant.

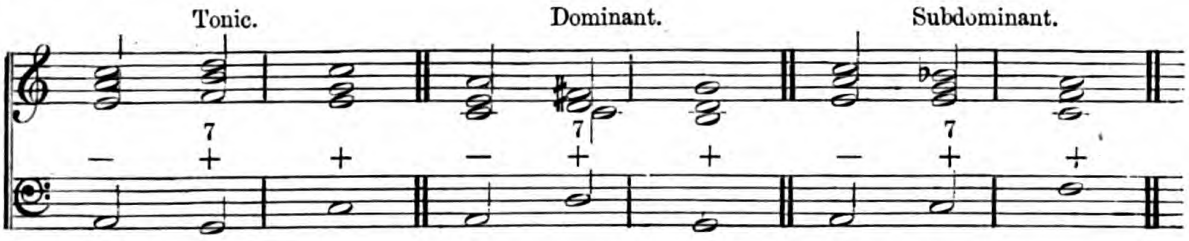


14. From the Major Mode to the Relative Minors of the Tonic, Dominant, and Subdominant :—

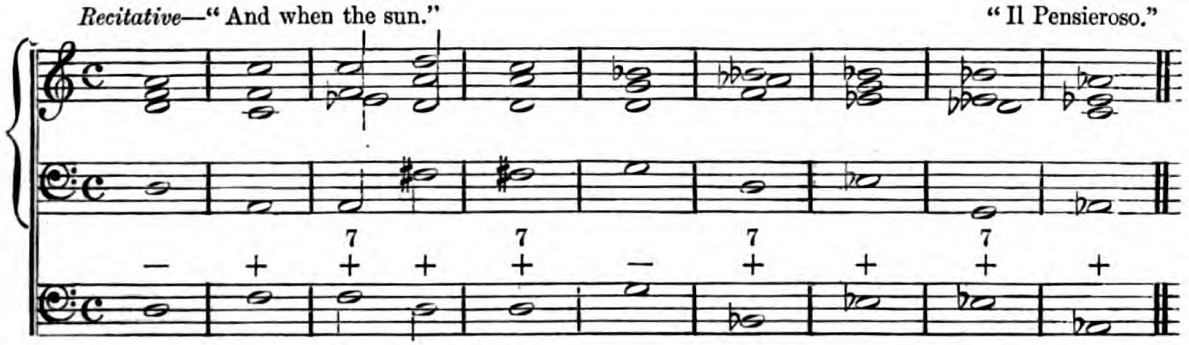
Tonic. Dominant. Subdominant.



15. From the Minor Mode, to the Relative Majors of the Tonic, Dominant, and Subdominant :—



16. Rapid Modulations are usual in Recitative, to avoid monotony; as in the introduction to Handel's song, "Hide me from day's garish eye."



17. Examples of Modulation into various Keys, from the masterly pen of C. P. Emanuel Bach, are given in Chapter XX. Sect. 37.

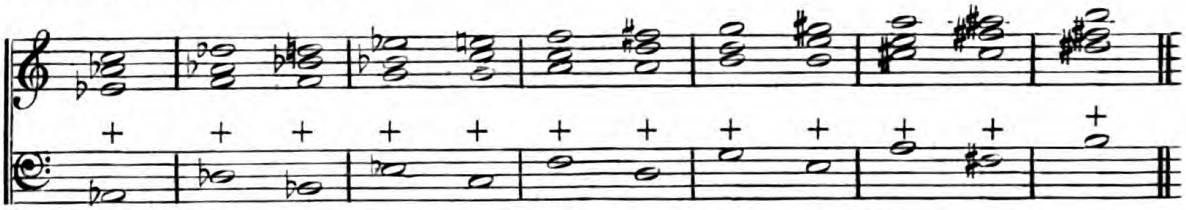
18. See also numerous Examples in Albrechtsberger's "Harmony and Composition," vol. II., pages 74 to 98.

CHAPTER XVI.

CHROMATIC MODULATION.

1. A SERIES of ascending or descending Semitones is called a Chromatic Scale. The most regular Scales of this kind are produced by a Fundamental Bass moving round the circle of Keys.

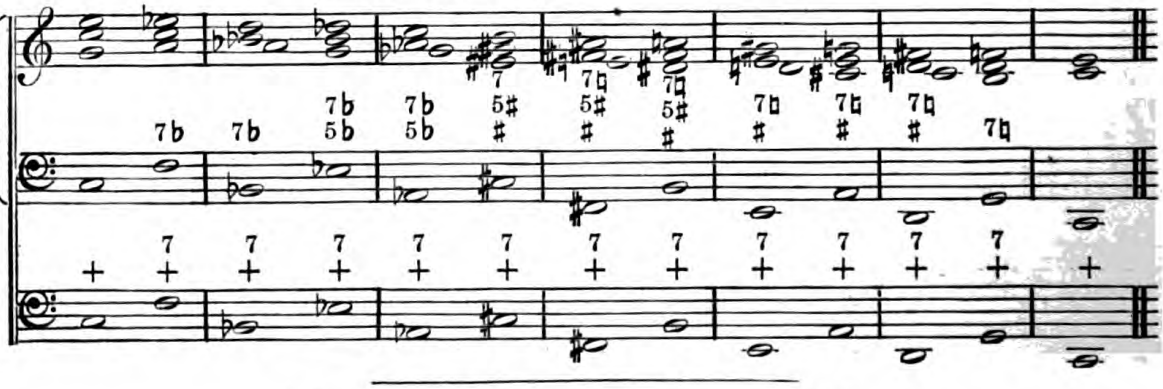
2. An ascending Chromatic Scale is formed by a series of Major Triads, whose Basses ascend by Fourths, and descend by Minor Thirds; the Key continually changing. In this Scale, the Chromatic notes are the Major Third and the Octave alternately: and the degrees are alternate Diatonic and Chromatic Semitones; producing, by their sums, a succession of *Minor* Tones.*



3. A descending Scale is produced by reversing the foregoing: the Bass descending by Fourths, and ascending by Minor Thirds.

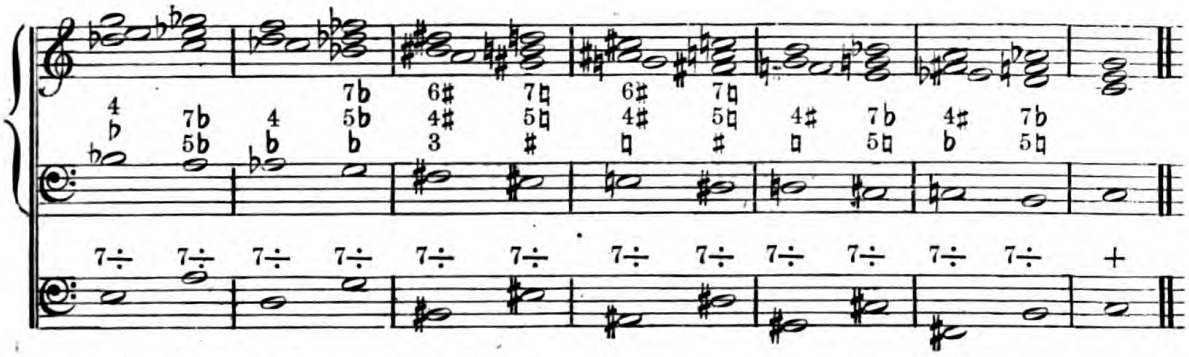
4. Another descending Chromatic Scale is formed by the Chromatic Sequence of Sevenths; a series of Dominant Harmonies, descending by Fifths, and ascending by Fourths; the Key changing continually. In this Scale, the Chromatic notes are the Major Third and the Dominant Seventh alternately: and the degrees are alternate Major and Minor Hemitones; producing, by their sums, a succession of *Major* Tones.†

CHROMATIC SEQUENCE OF SEVENTHS.



* See Appendix G, 1. † See Appendix G, 4.

5. The same scale is produced by a descending Sequence of Diminished Sevenths :—



6. An ascending Chromatic Scale is produced by reversing the last. How it can be reconciled with the Rules for the treatment of the Dominant Harmony, does not appear: but it is used by the best composers.

7. On pianofortes tuned in the usual way, the Chromatic Scale consists of twelve equal Semitones.

8. The rapid Chromatic divisions, with which some public performers astonish their hearers, are not subject to rule.*

---

* Emiliani told me, that in running a Chromatic division on the violin, he divided the Scale by the ear, not into regular semitones, but into small parts, perhaps 18 or 20 in the Octave.

CHAPTER XVII

ENHARMONIC MODULATION.

1. AN ascending series of three Major Thirds, falls short of the Octave by an Interval of 125 : 128, called the Enharmonic Diesis:—

64	80	100	125	128.
F	A	C♯	E♯	F.

This Interval is the difference between the Diatonic and Chromatic Semitones; or between a sharp note and the flat note above it; as G sharp and A flat.

2. When the name of a note is exchanged for that of another which is separated from it by the Diesis; as when E sharp is changed into F natural, or G sharp into A flat, or *vice versa*; the note is said to be Enharmonically changed. This change does not alter the pitch of the note on which the Chord depends; which would introduce a false progression into the Fundamental Bass. (See Chap. XX. 8.)

3. By means of Enharmonic changes, the Chord of the Diminished Seventh, which, on keyed instruments, consists of three Minor Thirds, affords a Modulation into all the twenty-four Keys.

4. Take for example the Diminished Seventh G♯ B D F; in which G♯, the Major Third of the Dominant Harmony, points, as Leading-note, to A as its Tonic. Its first Inversion is B D F G♯: but by substituting A ♭ for G♯, the Chord is changed into B D F A ♭, the Diminished Seventh of a new Key; resolving, from its Leading-note B, into the Tonic C.

5. The second Inversion of the Chord G♯ B D F, is D F G♯ B; which being Enharmonically changed into D F A ♭ C ♭, resolves, from its Leading-note D, into the Tonic E ♭.

6. The third Inversion is F G♯ B D; which, being Enharmonically changed into E♯ G♯ B D, resolves from its Leading note E♯ into F♯ as its Tonic.

7. Hence, from three Diminished Sevenths, with their Inversions, Enharmonically changed, the way is open to twelve Keys; which may be either Major or Minor:

Root F♯.	Chord	A♯ C♯ E G	resolves into	B.
	1st Inversion. .	C♯ E G B♭	,,	D.
	2nd Inversion. .	E G B♭ D♭	,,	F.
	3rd Inversion. .	G B♭ D♭ F♭	,,	A♭.
Root C♯.	Chord	E♯ G♯ B D	resolves into	F♯.
	1st Inversion. .	G♯ B D F	,,	A.
	2nd Inversion. .	B D F A♭	,,	C.
	3rd Inversion. .	D F A♭ C♭	,,	E♭.
Root G♯.	Chord	B♯ D♯ F♯ A	resolves into	C♯.
	1st Inversion. .	D♯ F♯ A C	,,	E.
	2nd Inversion. .	F♯ A C E♭	,,	G.
	3rd Inversion. .	A C E♭ G♭	,,	B♭.

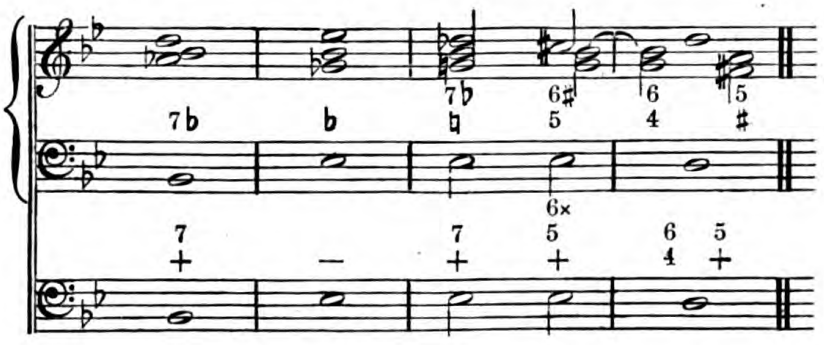
Twelve Keys . . . . A♭ E♭ B♭ F C G D A E B F♯ C♯.

8. Another species of Enharmonic Modulation, is that by which the Dominant Seventh is changed into the German Sixth. In this change, the Seventh rises Demisemitone Minor, an Interval of 49 : 50.

Dominant Seventh	G	B	D	F	. . . . 4 . 5 . 6 . 7 × 7 =	28 . 35 . 42 . 49.
German Sixth	G	B	D	E♯	. . . . .	28 . 35 . 42 . 50.

9. These two Chords are confounded on keyed instruments:* but the Dominant Seventh is resolved by descending, and the German Sixth by ascending.

10. A remarkable instance of this Modulation, is quoted by Dr. Callcott from Handel's "Solomon," in the Chorus "Draw the tear from hopeless love;" to express the words, "Full of death and wild despair:"†



* See Appendix H, 6, B.

† Callcott's *Grammar*, Art. 500; 2nd Ed. 463.

CHAPTER XVIII.

SUBSIDIARY NOTES.

1. THE Notes of Harmony may be divided into Essential and Subsidiary. The Essential notes are such as belong to the Chords of the Fundamental Harmony: to which may be added the notes of regular Suspension and Syncopation. In general, the Essential notes only, are expressed by the figures of Thorough Bass.

2. The Subsidiary notes may be classed as follows:

1st. Passing Notes, or Notes of Transition.
2nd. Appoggiaturas and After-notes.
3rd. Anticipations.

1st. PASSING NOTES.

3. Notes of Transition, being necessary to flowing melody, constitute a great part of most musical compositions. They occur most frequently between the notes of a Chord: as in the following Example, which consists of a single Chord with passing notes.

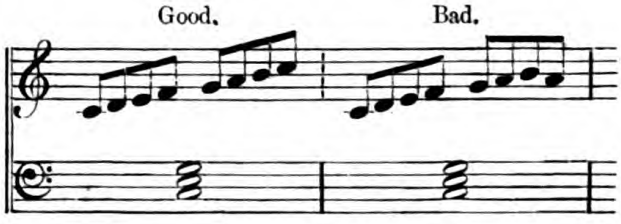
"Hailstone Chorus." "Israel in Egypt."

4. In this passage, the notes of the Harmony are C E G; which are chiefly on the accented parts of the Measure. The rest are Passing Notes.

5. The Rules for Notes of Transition, as laid down by Mr. Horsley, are the following:*—

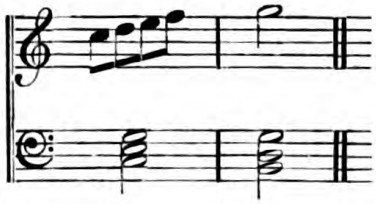
1st. That the passages which contain them, shall terminate on one of the component parts of the Harmony. Thus, a run on C must end on C, E, or G; a run on G must end on G, B, or D; and so on:—

Good. Bad.

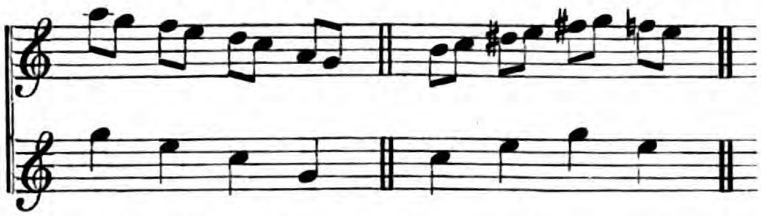


* Horsley, pp. 86, 87, 88.

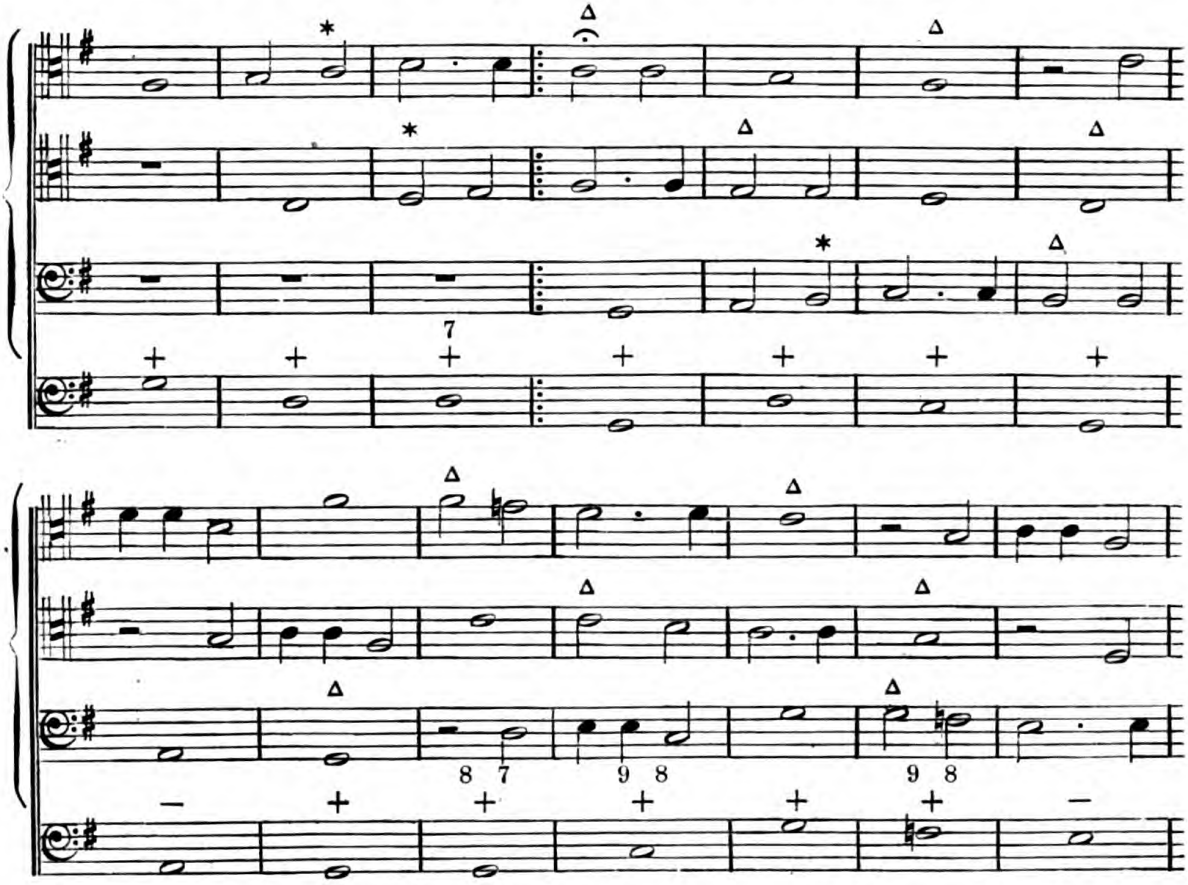
2nd. Every note of Transition must proceed to the note of the following Harmony, by a degree :—

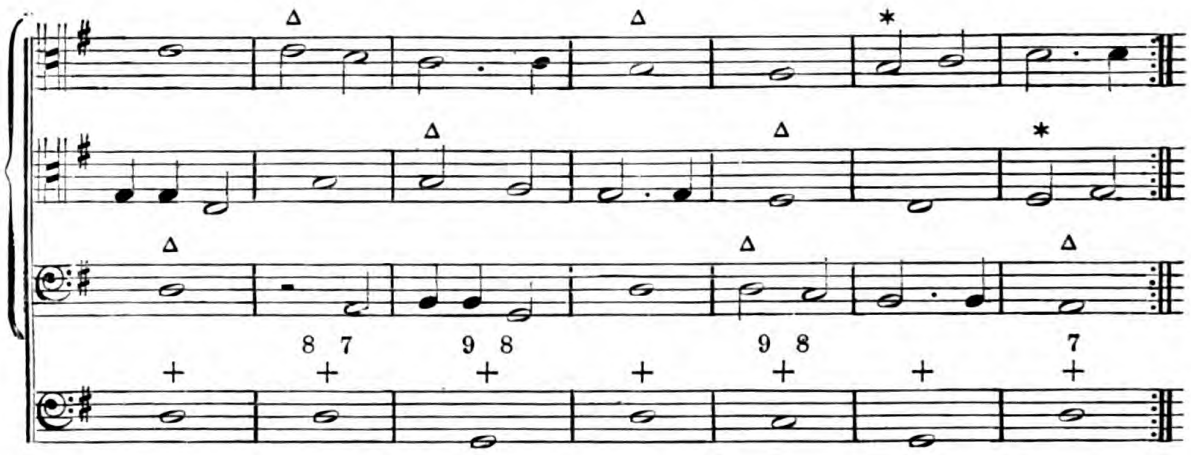


3rd. Every sound of the Triad may be preceded by the note above, or the note below; which notes may be altered by the Chromatic Semitone :—



6. Sometimes the notes of Transition occur between the notes of different Chords. An example of this is found in the celebrated Canon "Non nobis," by Bird : —

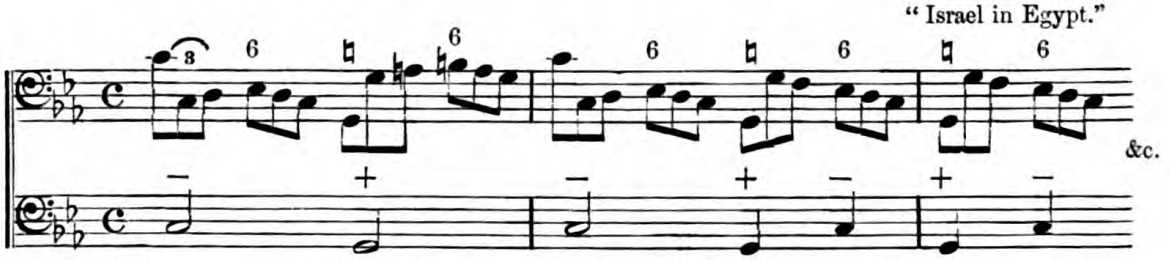




7. In this Example, the passing notes are marked with an asterisk. B in the second measure, and E in the third, are passing notes within the chord of D. But in the fifth measure, B is a passing note between the chords of D and C. In the last measure but one, A is a passing note between the Chords of C and G; and in the last measure, E is a passing note between the Chords of G and D.

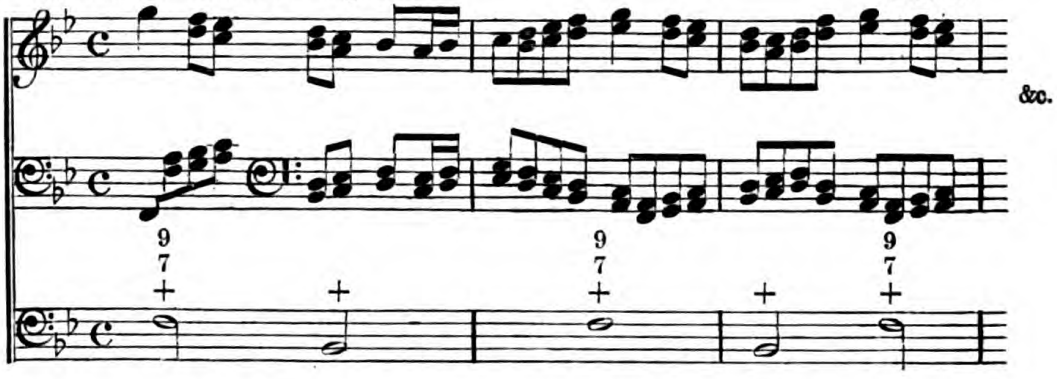
8. In the Chorus, "But the waters overwhelmed their enemies," a sublime effect is produced by alternate Minor and Major Chords, with passing notes in every chord.

"Israel in Egypt."



9. The Examples of Transient discords given by Mr. Horsley, will reward a careful study. I borrow one of them, as a specimen of double, triple, and even quadruple Transition:—

Mass No. 1. Novello's arrangement. HAYDN.



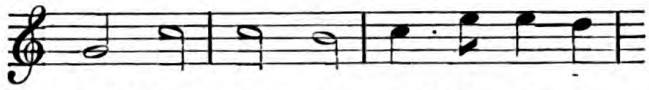
10. This passage, with the exception of eight quavers on the Tonic B♭, is written on the Added Ninth of the Dominant F. The notes of the Bass rise an Octave, where the Clef is repeated with the short bar attached to it  (Chap. I. 19).

2nd. APPOGGIATURAS.

11. "The Appoggiatura is a small note, placed before a large one of longer duration, from which it generally borrows half the value; and always occurs on the strong (or accented) part of the Measure:"—*

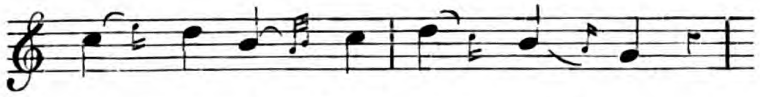
Written :—

Performed :—

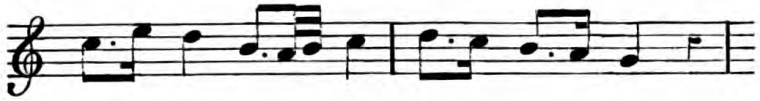


12. "When a small note follows a larger one, and depends upon that for its time, it is called an After-note, to distinguish it from the Appoggiatura. This grace always occurs on the weak (or unaccented) part of the Measure:"—†

Written :—



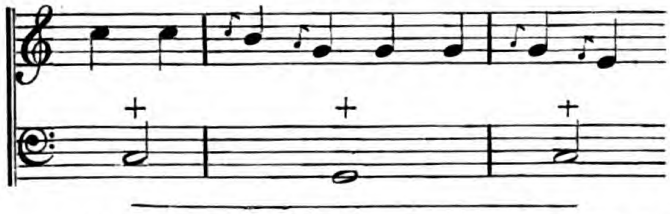
Performed :—



13. "In modern music, all the Discords of Transition may be reduced to Appoggiaturas and After-notes. Thus, the quavers in the following phrase, may be turned into crotchets preceded by Appoggiaturas:"—

Sonata 1. PLEYEL.

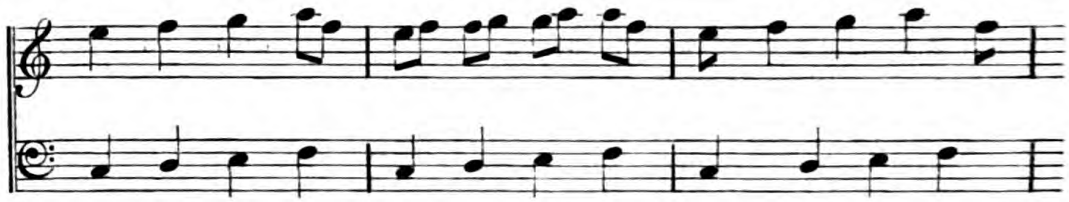
14. "The reduction of this phrase, shows the real notes of the Harmony; and explains the nature of irregular transition, in which Appoggiaturas are always employed."‡



* Callcott's *Grammar*, Art. 106; 2nd Ed. 103. † *Ib.* Art. 108; 2nd Ed. 105.
‡ *Ib.* Art. 393, 394; 2nd Ed. 361, 362.

3rd. ANTICIPATIONS.

15. "When a note is diminished by half its value, and the following degree employed to fill up its time upon the former Bass, such change is termed Anticipation. These anticipated notes are considered wholly as relating to Melody: and are not noticed by the figures of Thorough Bass:"—

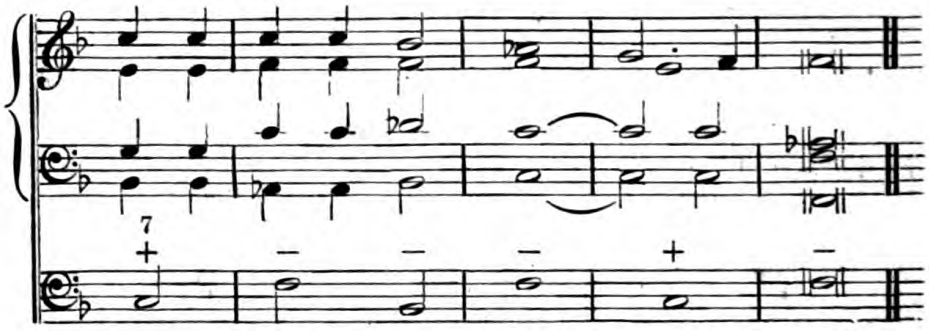


16. "In the foregoing Example, taken from Koch's 'Lexicon,' the first measure contains the simple notes; the second shows the Anticipation in quavers; and the third in syncopated notes."*

17. Examples of Double and Triple Anticipations, may be seen in Mr. Horsley's work, page 91.

18. At a final Cadence, it is usual, in performance, to anticipate the Tonic note, in the Dominant Harmony. This is sometimes done in the printed copies; as in the following Example from Handel's "Messiah," at the close:—

Chorus—"All we like sheep."



* Callcott's *Grammar*, Art. 412, 413; 2nd Ed. 378, 379.

CHAPTER XIX.

PROSODY.

1. As the Harmony of a passage often depends upon the Prosody or Rhythm,* it will be necessary to make some allusion to that department of Music: in which I shall do little more than define the terms; referring the reader to the Fourth part of Dr. Callcott's "Grammar," where the whole subject is admirably explained and illustrated.

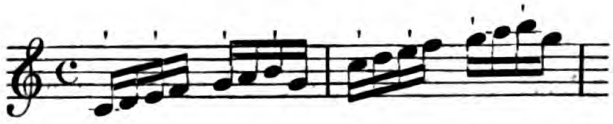
ACCENT.

2. Every piece of Music is divided by bars into equal parts called Measures. In Common time, each Measure is divided into four parts or times; the first and third of which are accented, the second and fourth unaccented. In Triple time, each Measure is divided into three times; the first accented, the second unaccented, and the third unaccented in respect of the first, but having a subordinate accent when compared with the second.

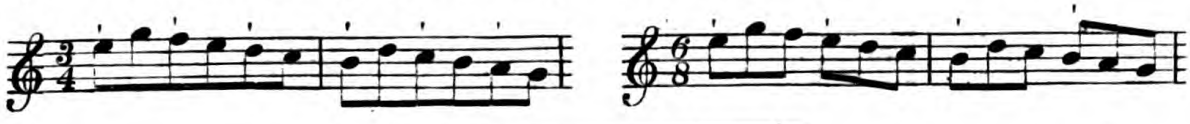
3. When the times are subdivided into shorter notes, the accents are indicated by the grouping of the notes.

4. In the following Example, the accents are on the first and third *pairs* of notes; the second and fourth pairs are unaccented:—

5. In the next Example, the accents are on the first and third notes of every group: the groups being regulated by the times of the measure:



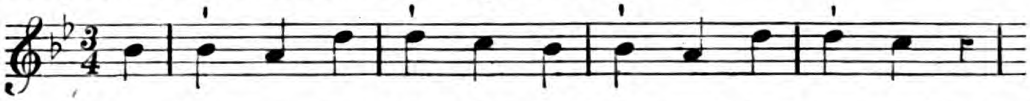
6. In Compound time, the difference between three-crotchet and six-quaver time, is known by the accent, and by the groups which regulate the accent.



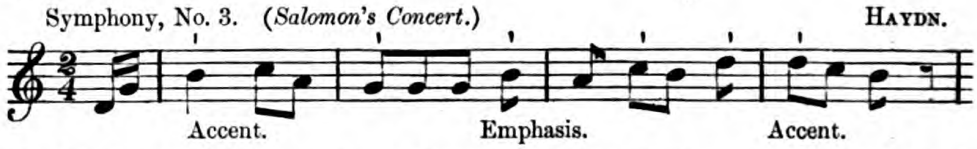
* Dr. Callcott shows, by an Example, that the same series of notes, accented in three different ways, bears three different Harmonies.—*Grammar*, Art. 507, 508; 2nd Ed. 468, 469.

7. These inferior accents, which belong to the times of the measure, do not interfere with the great and predominant accent that belongs to the note which follows the bar.

8. When the first note of a movement is unaccented, it is necessary to begin with part of a measure; that the first accented note may follow the bar:—



9. Sometimes a strong accent is laid upon the part of the measure that is usually unaccented. This kind of accent, which is also marked by the grouping of the notes, is termed by Dr. Callcott, *Emphasis.*



10. Of this kind is the accent on Syncopated or Driving notes, which begin on the unaccented, and end on the accented part of the Measure:—

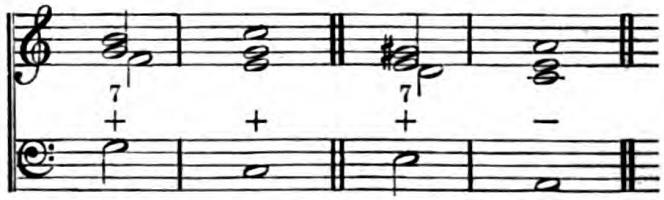
11. In this Example, the Emphasis is on the Syncopated minims, which begin on the second, and end on the third part of the measure.

CADENCES.*

12. A Cadence in Harmony, consists of two distinct Chords, the last of which is generally accented: and is used to terminate the Sections and Periods of musical rhythm.

13. When the Basses of both Chords, are the Roots of their respective Triads, the Cadence is termed Radical. Of these Cadences there are four in general use: the Perfect, Imperfect, False, and Mixed: to which may be added the Plagal, or Church Cadence, which is only a variation of the Imperfect; and the Authentic, which is only the ancient name of the Perfect.

14. The Perfect Cadence consists of the Dominant Harmony, followed by that of the Tonic:—



The first, or leading harmony, is always Major.

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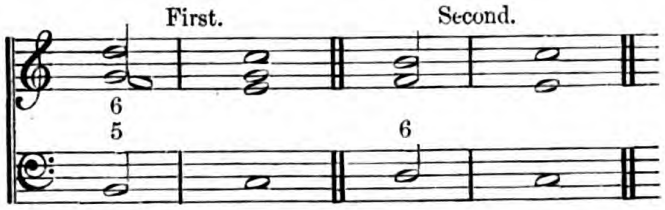
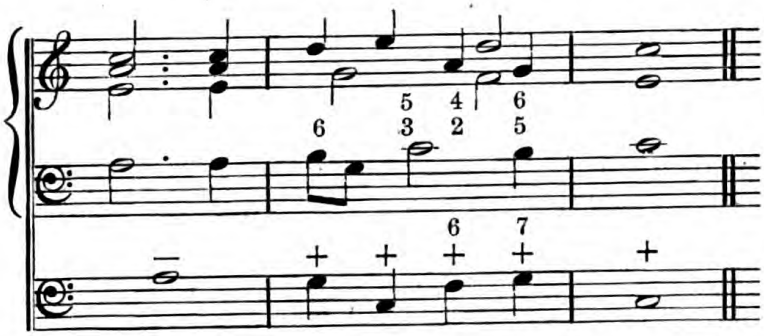
* See Callcott's *Grammar*, Part III. Chap. 4; from which the following brief abstract is taken.

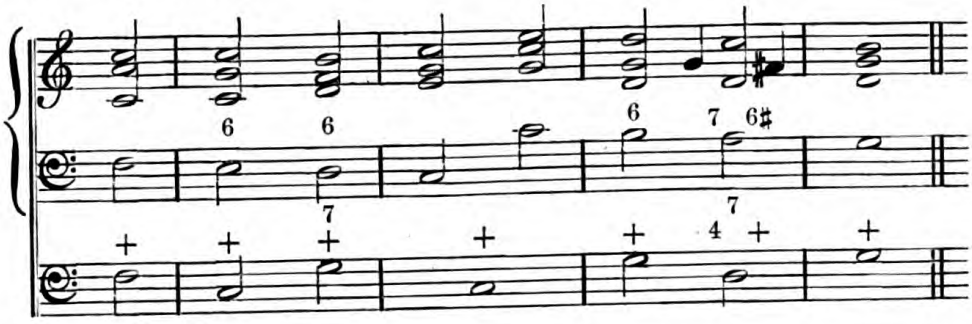
15. The Perfect Cadence has two Inversions :—



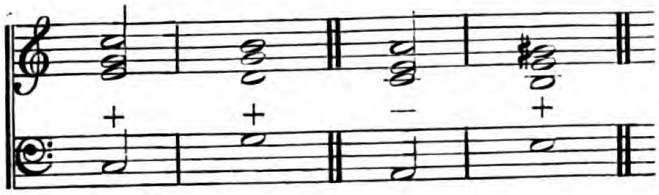
16. The first Inversion is called the Cadence of the Leading-note.



17. The second Inversion is called the Cadence of the Sharp Sixth. It is sometimes used as a final Cadence on the Tonic, as in "Non nobis :" but more generally on the Sixth of the descending Scale, when it commonly bears a suspended Seventh :—

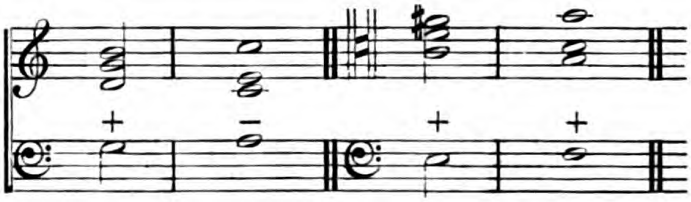


18. The Imperfect Cadence consists of the Tonic Harmony, followed by that of the Dominant without its Seventh ; and is the Perfect Cadence reversed :—

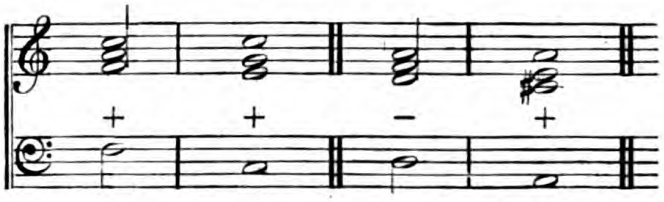


The second or final harmony, is always Major.

19. The False Cadence consists of the Dominant, followed by the Submediant in Diatonic gradation, taken in place of the Tonic.

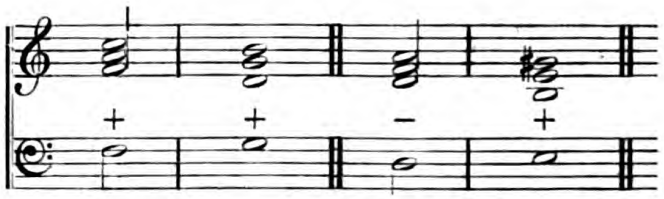


20. The Plagal Cadence consists of the Subdominant Harmony, followed by the Tonic :—

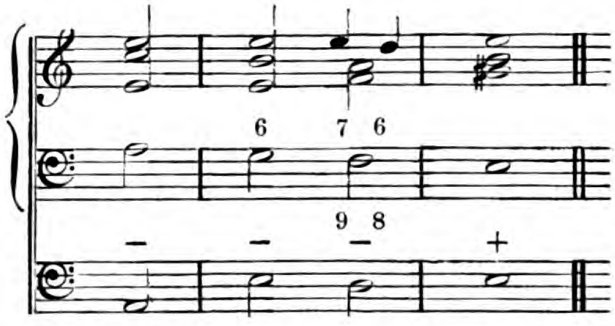


In this Cadence, the final harmony is always Major.

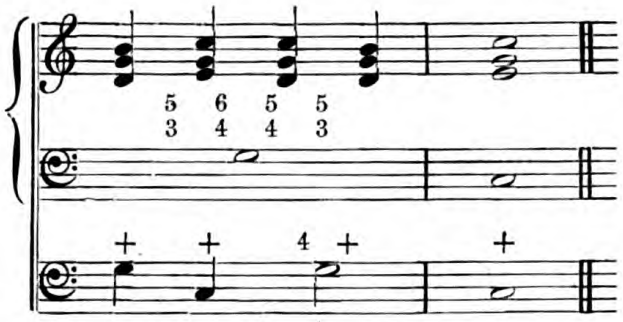
21. The Mixed Cadence consists of the Subdominant Harmony, followed by the Dominant :—



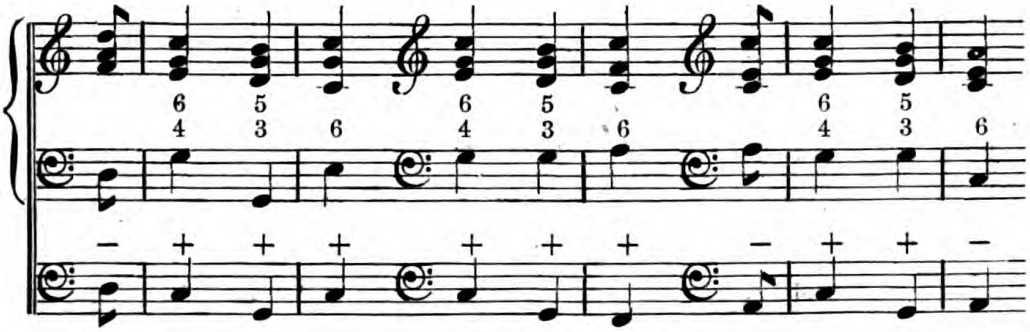
22. The first Inversion of the Mixed Cadence, is called the Cadence of the Major or Minor Sixth. It is chiefly used in the Minor mode; and is liable to the antecedent suspension of the Seventh :—



23. These Cadences may also become protracted, by using other Harmonies on the Dominant. Thus is formed what Dr. Pepusch calls the Grand Cadence :—

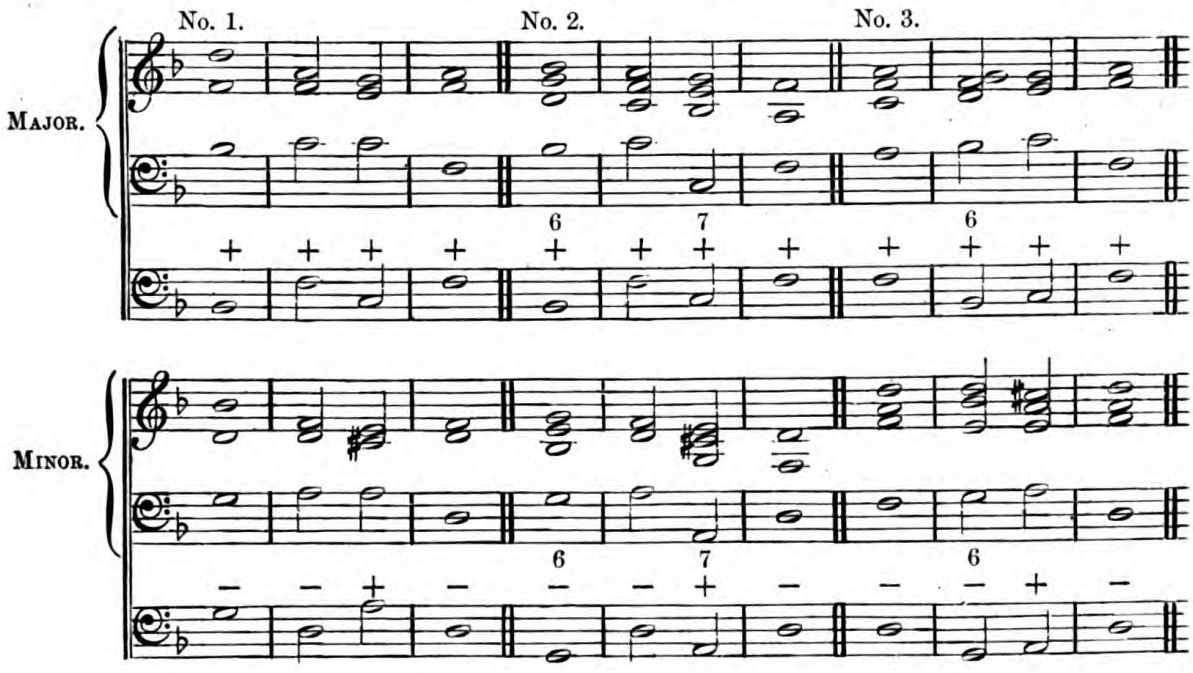


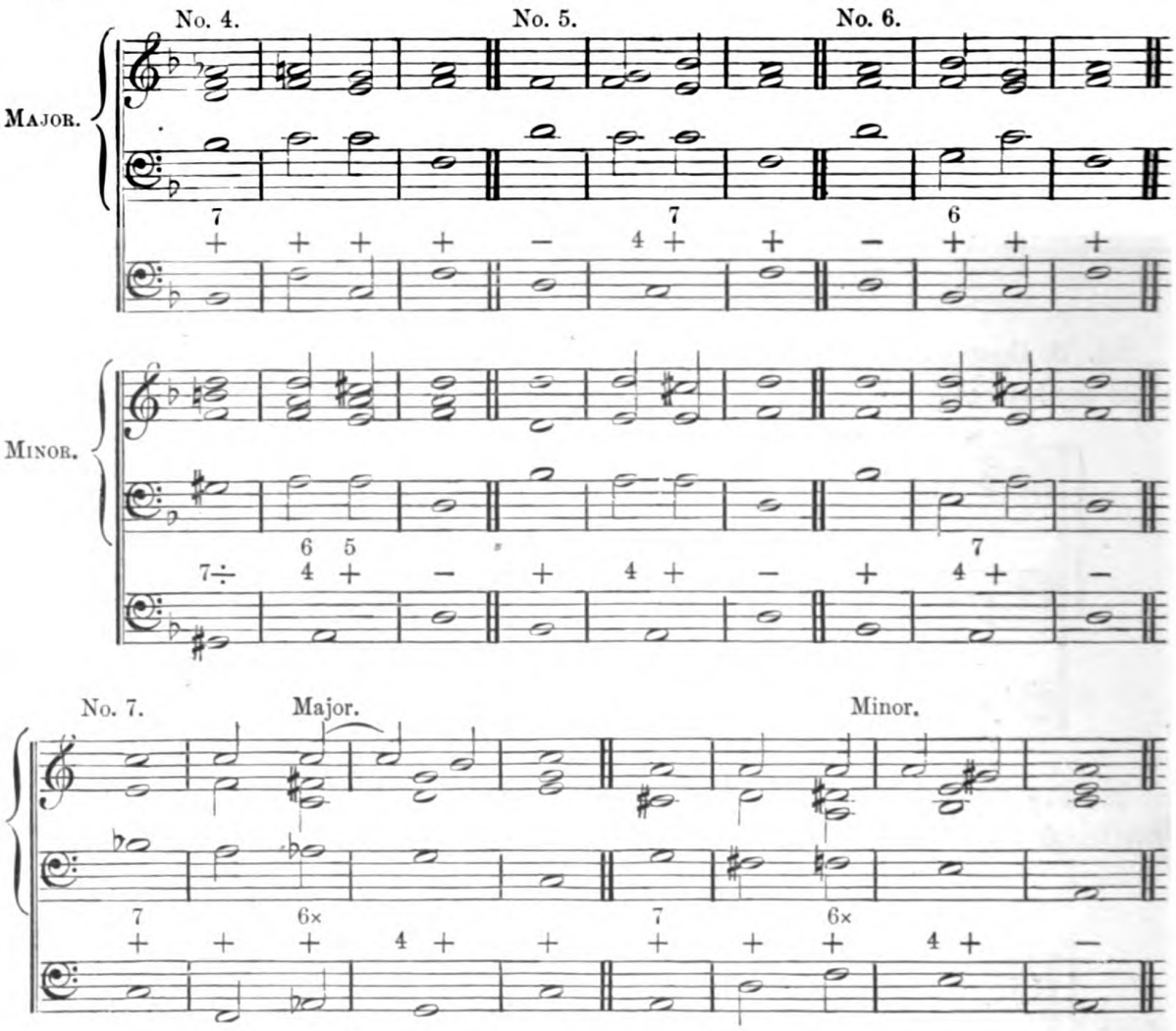
24. To these may be added those Deceptive Cadences, which, by varying the final Chord avoid the final close :—



25. For the following Table of Cadences, I am indebted to Professor Donaldson of Edinburgh :—

FORMULÆ OF PERFECT CADENCES.





METRICAL DIVISION.

26. A musical Composition is divided, by double bars, into Strains. A Strain consists of one or more Periods. The Periods are divided into Sections; the Sections into Phrases; and the Phrases into Feet.

27. A Foot, as defined by Dr. Callcott, is a small portion of Melody, with one principal accent, including the value of a simple Measure. It is analogous to the Foot in metrical poetry.

28. In Simple time, when the first note of the Foot is accented, the Foot and the Measure coincide:—

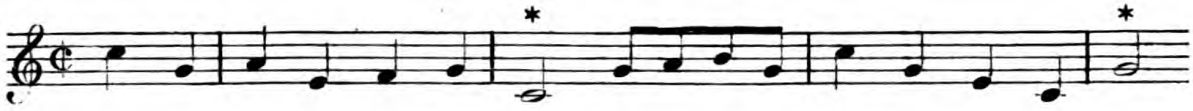
HAYDN'S Symphony, No. 3.

29. In Compound time, the Measures contain two Feet or three.

HAYDN, Op. 40. Sonata 3.

30. The Cæsure, is the last accent of a Phrase, Section, or Period. In all simple measures, it is distinguished by the place of the bar: as in this Example from Dr. Arne's "Judith."

"Conquest is not to bestow,
In the spear, or in the bow."



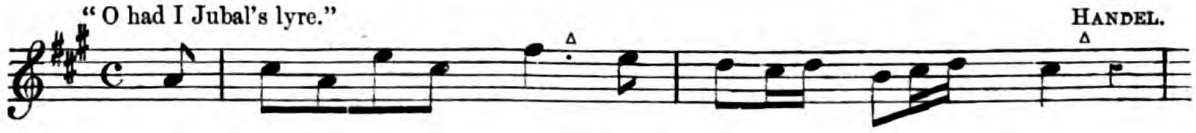
31. Dr. Callcott's remarks upon the Cæsure, are very valuable, on account of its great importance to the expression of the passage. See Callcott's *Grammar*, Part IV. Chap. 3.

32. A Phrase, is a short Melody, which contains no perfect or satisfactory musical idea. In Simple time, it generally contains two Feet, or the value of two Measures.

BEETHOVEN. Op. 2.

33. In the Compound time of the older writers, a Phrase sometimes consists of a single Measure.

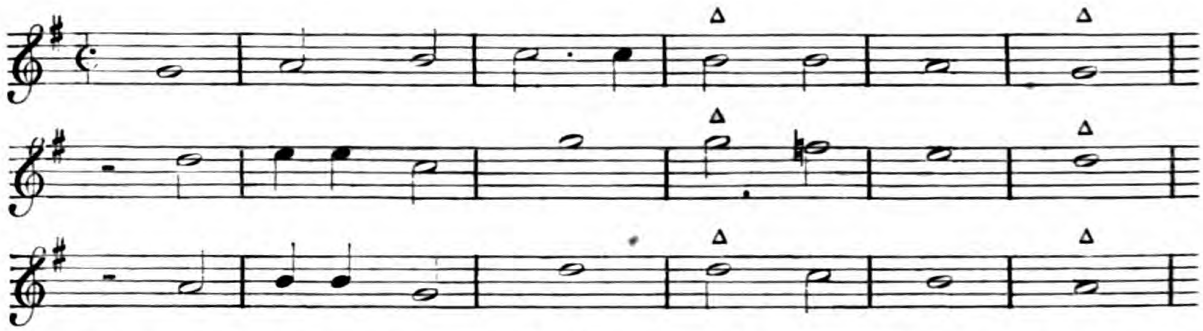
"O had I Jubal's lyre." HANDEL.



34. When three Measures are employed, instead of two, the Phrase is termed Extended, or Irregular.

35. In Figurate Counterpoint, anciently termed *Descant*, where Imitations, Fugues, and Canons are employed, the Phrases, as they occur, are interwoven in the different parts.

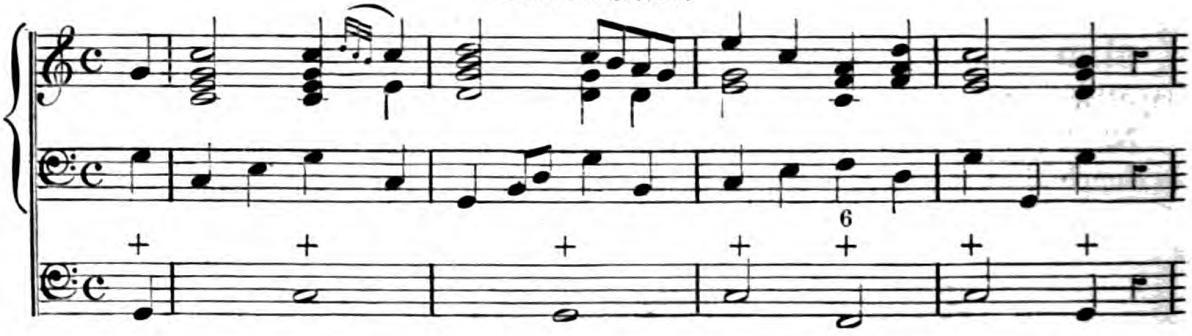
36. In those pieces of Music termed Canons, in which the same melody is continually heard in the different parts, the Phrases are, of course, interwoven throughout the whole composition. "Of this kind of Music," says Dr. Callcott, "the finest specimen now extant, is the celebrated 'Non nobis,' by Bird; the Phrases of which are as follows':"—



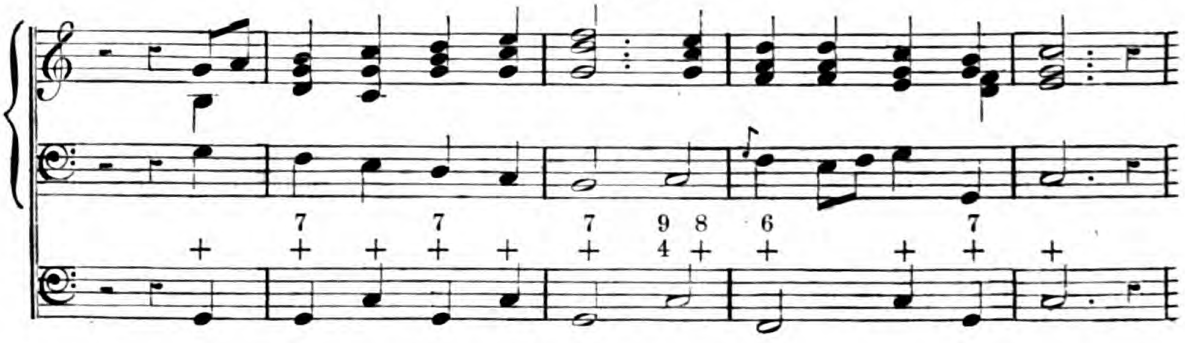
37. A Section consists of two regular Phrases, the last of which is terminated by a Cadence.

38. The Section takes the name of Tonic or Dominant, from its final harmony: as in the beginning of the Chorus "The heavens are telling," in Haydn's "Creation:"—

Dominant Section.



Tonic Section.



39. The extended Section may consist of five, six, seven, or more Feet. The contracted Section generally consists of three Feet; and differs from the extended Phrase, by terminating with a Cadence.

40. "In the ancient style of Music," says Dr. Callcott, "great effects are produced by interweaving Phrases, Sections, &c.; and also by intermixing subjects of different rhythms. These effects continually occur in the choruses of Handel."

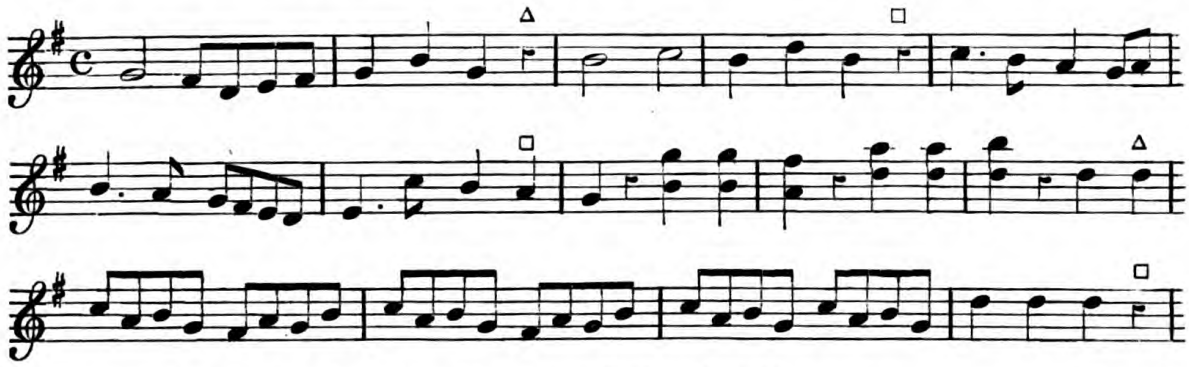
41. A Codetta is a short Phrase, or any other passage, which does not constitute part of a regular Section, but serves to connect one Section or Period with another.

42. A Period consists of one or more Sections, occasionally interspersed with independent Feet, Phrases, or Codettas.

43. The Period ends with a Radical Cadence (Sect. 13). The Deceptive Cadences, noticed Sect. 24, are exceptions to this Rule.

44. Those Periods which terminate with the Perfect Cadence, are, from their last harmony, termed Tonic Periods; and those which conclude with the Imperfect Cadence, Dominant Periods.

45. The following Example of a Dominant Period, consisting of three Sections, is taken from Kozeluch, Op. 23, Sonata 1. The second Section, being interwoven with the third, is contracted, and consists of three Measures only. The third Section is formed of two extended Phrases, with one Measure repeated: and concludes on the Dominant:—



46. As the Periods of modern Music are distinguished by the accuracy of their phraseology, being, for the most part, regular; so those of the old school are generally interwoven, and the Cæsure note of one period becomes the first note of the next.

47. The Coda, or final Period, is a concluding passage, which sometimes occurs after a protracted perfect Cadence. In some pieces it contains several Sections; in others merely a single Phrase.

48. To show what great effects are produced by this addition, after the last Perfect Cadence of the movement has been heard, Dr. Callcott adduces the "Hallelujah Chorus" of Handel's "Messiah;" the last seven Measures of which contain a Coda on the chords of Subdominant and Tonic, concluding with the Plagal Cadence. See Chap. XX. 44.

49. On this Coda, the Doctor remarks: "Such were the simple but sublime notes, which occurred to the genius of this truly great Composer: and the Chorus in which they occur, will ever remain a striking memorial of the immortal talents of Handel."*

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* See Callcott's *Grammar*, Part IV. and Part I. Chap. 3, Sect. 4; from which the foregoing brief abstract is taken.

50. I cannot better conclude this chapter, than by quoting Dr. Callcott's abridgment of Mr. Bach's ideas, upon the true method of playing keyed instruments.*

"To form a clear, pleasing, and expressive performer, three things are requisite:—

"1st. To play correctly; by covering every note with the finger, before it is struck, when possible; so that in the most difficult passages, the motion of the hands may be scarcely perceived.

2nd. "To make the instrument sing; by taking one finger off the key, at the instant the other strikes the following note; and by never playing the notes short or detached, except when expressly marked.

3rd. "To play with expression: by forcing the finger down upon the key, (already covered and lightly touched,) according to the Accent or Emphasis.

---

* Callcott's *Grammar*, page 48, note. C. P. Emanuel Bach, *Versuch*, Leipzig, 1787, pp. 13, 88. 95.

CHAPTER XX.

ANALYSIS OF THE SCORE.

1. WE are now come to the practical application of the principles and rules laid down in the course of this work.

2. The works of the great Composers, are the proper field of study for the Musician. The record of the Composer's ideas, is the Score. The Score is the original and entire draught of a Composition, or its transcript; wherein all the parts, whether vocal, or instrumental, or both, are ranged perpendicularly over the Bass; exhibiting to the eye, at one view, the whole design and construction of the Harmony.*

3. "The Harmony of a well-arranged Score," says Mr. Shield, "is the picture which charms the mind of a well-educated musician; who appreciates and feels all its beauties during his silent admiration. But the ear must have been previously formed to the true intonation, and the eye to the accurate perception of harmonious combinations."†

4. The reading of the Musical Score, however, has unfortunately been encumbered with artificial difficulties, which can scarcely be surmounted, except by men whose particular department of professional duty, requires the constant practice of it. In reading Chords on the Score, we begin, of course, with the Bass: then change to the Tenor Clef; then the Alto; then Soprano or Treble; then Bass again; and so on. In some of Handel's Double Choruses, there are thirteen changes of Clef, between the Bass and the top of the page. Besides the variety of Clefs, the more modern Scores contain parts written for particular instruments, in other Keys; which are still more perplexing.

5. The irksomeness of reading and analyzing a Score constructed on such principles, will be best understood by those who have made the attempt.‡ The only alternative left to the student, is to undertake the labour of transposing the whole Score into one Clef.

6. Another evil, inseparable from the use of different Clefs, is the multiplying of errors, in transcribing, and in printing; the difficulty of detecting which, is proportional to the difficulty of reading.

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* See Dr. Busby's *Dictionary of Music*, art. "Score." † Shield's *Thorough Bass*, p. 90.

‡ The late Dr. Mainzer told me, that while he was learning the principles of Music in his youth, his studies were well watered with his tears.

7. In studying the Score, the Fundamental Harmony, which is the groundwork of the whole, is first to be attended to; and then the texture and management of the parts. The latter I leave to those writers who have treated of the higher branches of the art.

8. The unity of a Musical Composition, is preserved by the progression of the Fundamental Bass; which always moves, as Rameau expresses it, by consonant intervals; and to which the notes of every chord must accommodate themselves.

9. In treating of the Fundamental Bass, it is convenient to consider it, in certain progressions, as moving by the smaller Interval of a Second, rather than by its Inversion the Seventh. With this explanation, the Intervals by which the Fundamental Bass moves, are nine in number; all derived either from one Triad, or from two adjacent Triads.

	4	5	6
From one Triad	F	A	C.

F	C	. . .	2 : 3	. . .	Fifth.	C	F	. . . 3 : 4 . . .	Fourth.
F	A	. . .	4 : 5	. . .	Major Third.	A	F	. . . 5 : 8 . . .	Minor Sixth.
A	C	. . .	5 : 6	. . .	Minor Third.	C	A	. . . 3 : 5 . . .	Major Sixth.

	8	10	12	15	18.
From two adjacent Triads	F	A	C	E	G.

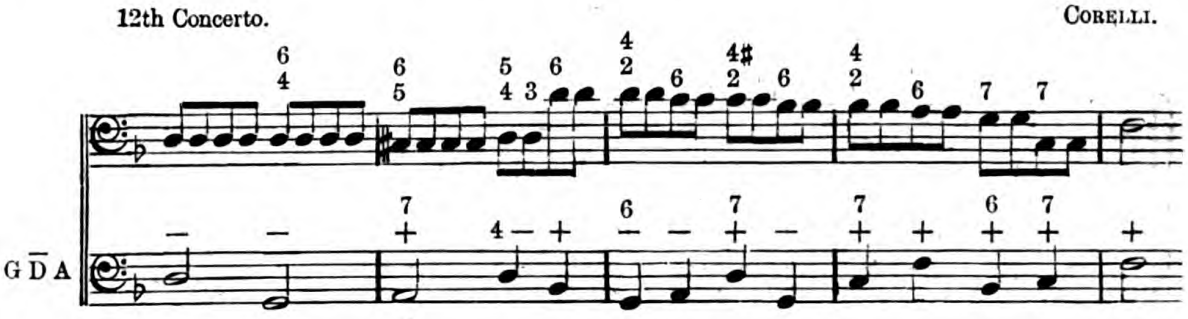
F	G	. . . 8 : 9 . . .	Major Tone.
G	A	. . . 9 : 10 . . .	Minor Tone.
E	F	. . . 15 : 16 . . .	Diatonic Semitone.

All these Intervals are exemplified in the progression of Triads, Chap. XIV. 24—29.

10. As the rules of Harmony were not derived from principles known before, but from the practice of men of genius who had devoted their talents to the composition of Music; so it will be found, that the works of the great Composers are the most conformable to the rules of the art; and therefore the fittest subjects for study and analysis.

11. In proceeding to analyze a piece of music, whether it be written out in Score, or as a condensed accompaniment, or merely as a figured Bass, a Staff should be left below it, to contain the Fundamental Harmony. Where the Bass is figured, the figures should always be copied, as they are useful in directing to the Harmony, especially where the Chords are not full.

12. Of these three methods, the Score is the most satisfactory; and, if it be written out in one Clef, the easiest. The figured Bass by itself, is the most difficult: but it may be used by those who are familiar with that method of notation; thus:—

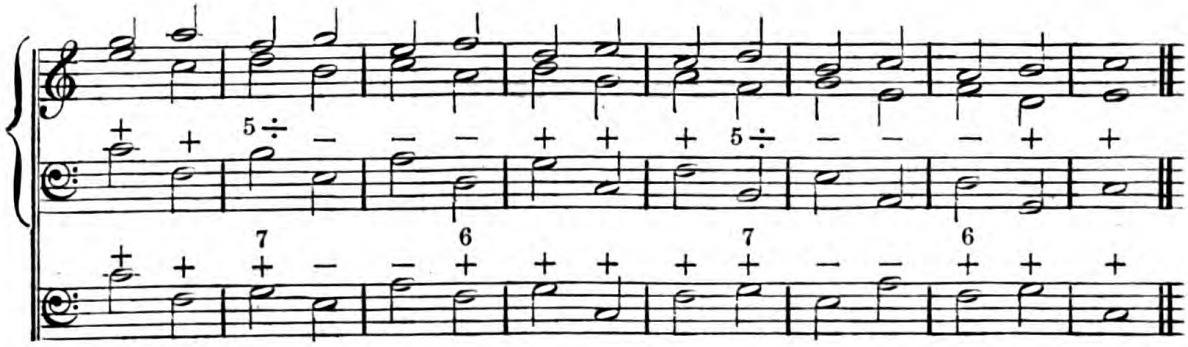


13. Most Compositions begin and end with the Tonic Harmony, or Common Chord of the Key. There may be twenty parts in the Score; but they are all confined to that Chord.

14. The Key having first been ascertained, the Fundamental Basses of the Key, which are the Subdominant, Tonic, and Dominant, may be written in letters on the margin of the Staff that was left for the Fundamental Harmony, as in the last Example. The Chords are then to be examined, and compared with the Fundamental Basses written on the margin. The notes of each Chord are to be gathered from the Score, without respect to their Positions and Inversions; and their Fundamental Basses inserted in the Staff reserved for that purpose.

15. If the notes of a Chord can be reduced to a Major Triad, the lowest note of the Triad is the Fundamental Bass. If they can be reduced to a Minor, or a Diminished Triad, they are ambiguous; for which reason, it will be useful to examine the various Triads separately.

16. Diatonic Sequence of Triads, Major Mode (F $\overset{+}{C}$ G):—



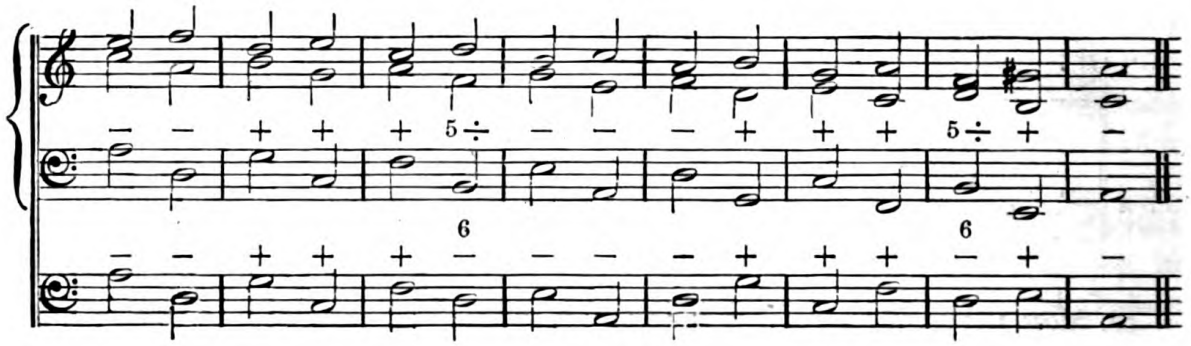
The Tonic, Dominant, and Subdominant, bear Major Triads;

The Mediant and Submediant, Minor Triads;

The Minor Triad on the Supertonic, is the Added Sixth of the Subdominant;

The Diminished Triad on the Leading-note, is the Dominant Harmony without the Root.

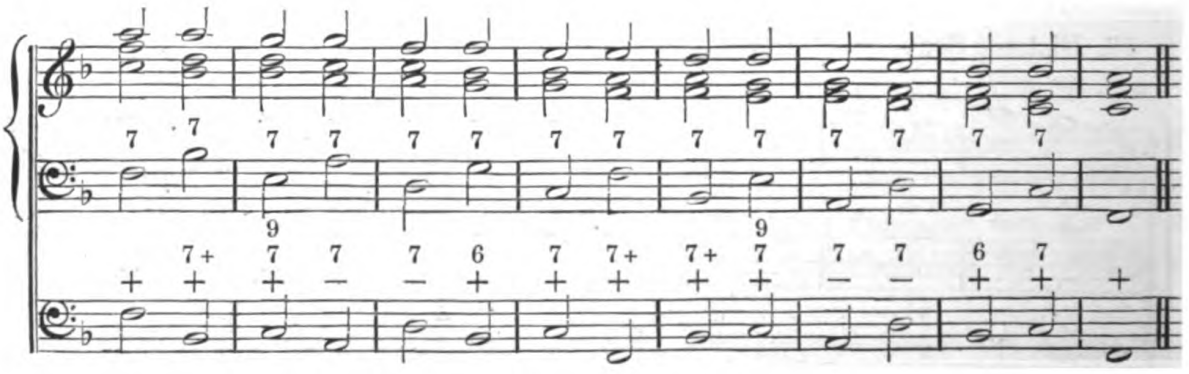
17. Diatonic Sequence of Triads, Minor Mode (D $\bar{A}$ E):—



The Tonic and Subdominant, bear Minor Triads;
The Dominant, sometimes a Minor, and sometimes a Major Triad;
The Mediant, Submediant, and Minor Seventh, Major Triads;
The Diminished Triad on the Supertonic is the Added Sixth of the Subdominant.

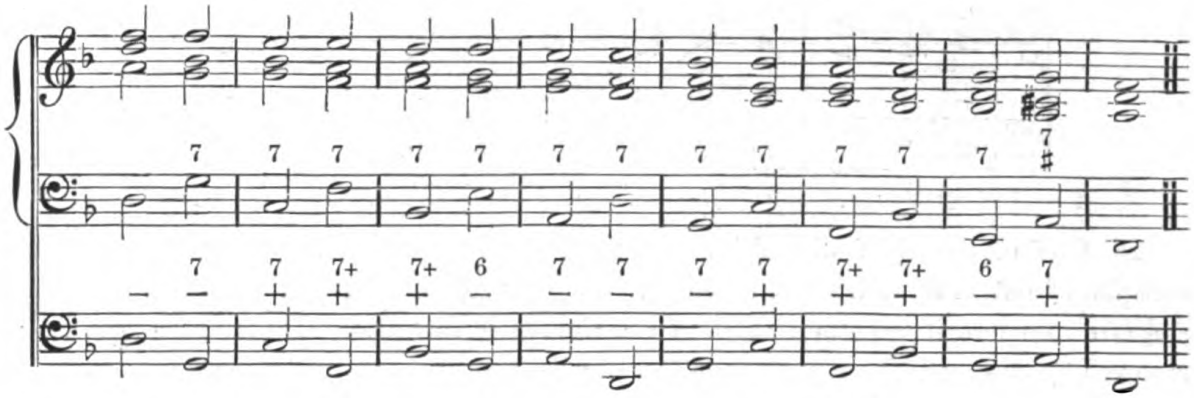
18. If the notes of a Chord are reducible to a Tetrad, or Chord of the Seventh, they are still more equivocal. The Tetrads that occur in the Diatonic series, are as follows:—

19. Diatonic Sequence of Sevenths, Major Mode (B ♭ $\dot{F}$ C):—



The Dominant bears the Harmonic or Dominant Seventh;
The Tonic and Subdominant carry Suspended Major Sevenths;
The Mediant and Submediant bear Suspended Minor Sevenths;
The Minor Seventh on the Supertonic, is the Added Sixth of the Subdominant;
The Diminished Triad with Minor Seventh on the Leading-note, is the Added Ninth on the Dominant.

20. Diatonic Sequence of Sevenths, Minor Mode (G $\bar{D}$ A):—



The Tonic, Subdominant, and Dominant, bear Minor Sevenths, except at the close, where the latter bears the Dominant Harmony;

The Mediant and Submediant carry Suspended Major Sevenths;

The Minor Seventh on the Supertonic, is the Added Sixth of the Subdominant;

The Minor Seventh on the Seventh of the Key, is the Dominant Harmony of the Relative Major.

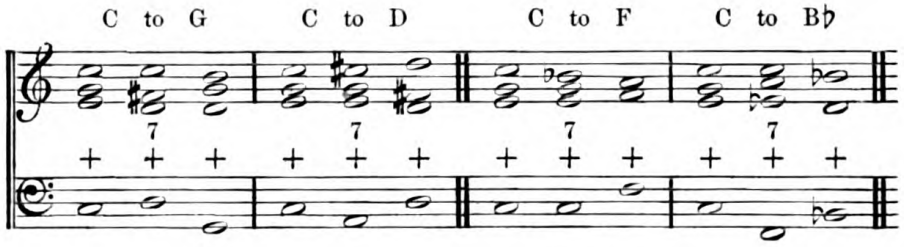
21. The foregoing Examples comprehend the various Chords that are found in the natural Scale; viz. the Major and Minor Common Chords, the Dominant Seventh, the Added Ninth, and the Added Sixths of the Major and Minor Modes.

22. The Compound Dominant Harmonies, are known by their peculiar Intervals—the Diminished Seventh, and the Augmented Sixth; which always require accidental Sharps, Flats, or Naturals.

23. Suspensions that are regularly prepared and resolved, are expressed in the Fundamental Harmony, by figures, as in Thorough Bass. As the Suspended notes are prepared in the preceding Chord, it is unnecessary to confuse the signature by using the Major and Minor signs; except in the case of the Major Seventh, which requires the Major sign, to distinguish it from the Dominant Harmony. The Major and Minor signs, however, are employed in the resolution of the Chord, in place of the figure 3. Where the Resolution is not regular, it may sometimes be better not to confuse the appearance of the Harmony, by noticing the suspended note in the signature.

24. The Subsidiary notes, described in Chap. XVIII. are unnoticed in the Fundamental Harmony; for which reason, the student should be able to recognise them, when they come in his way.

25. Modulations, or changes of Key, are of course accompanied by changes in the Fundamental Bass. They are introduced by the Dominant Harmony: in which the Third is sharpened when modulating upwards, and the Seventh flattened when modulating downwards:—

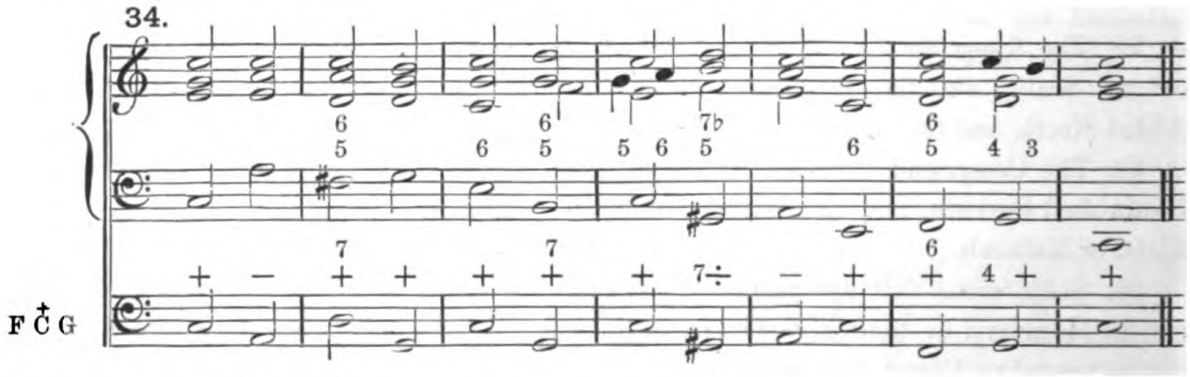


26. One of the chief difficulties in finding the Fundamental Bass, arises from the omission of one or more notes of a Chord; which is very common in the best compositions, and often renders it uncertain which of two Chords is intended. The ambiguity is to be removed by observing the connexion of the Harmony, or by consulting the ear.

27. After the Fundamental Bass has been written out, it may be played, an octave below, as an additional Bass, even on the same instrument, while the music is performed by another person. This is a pleasing exercise, as well as a good way to try the correctness of the work, and to detect any false notes.

I now proceed to the analysis of a few short portions of Harmony.

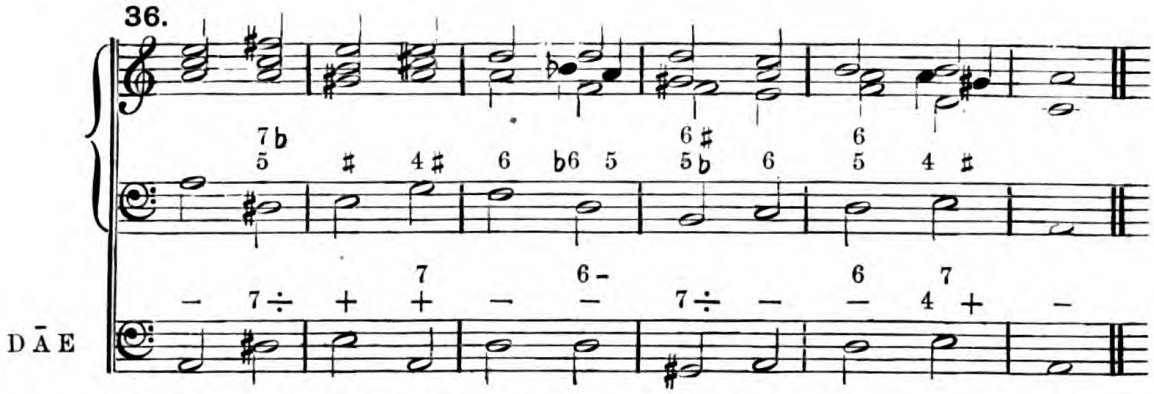
28. From the Scales of C. P. Emanuel Bach:—



This Example, which is in the Key of C Major, contains two transient Modulations; into G the Dominant, and A the Relative Minor. It begins with the Tonic Harmony of C, followed by its Relative Minor A. Then the Dominant Harmony, taken on D, leads to the Key of G Major. Then the Tonic Harmony of C is again struck; then G, with the Dominant Harmony, followed by C the Tonic. In this last Chord, the note A occurs; which may be considered as a passing-note to B in the next Chord, if the motion of the Fundamental Bass in Minims, is not to be interrupted; otherwise the Bass may be divided into two Crotchets, C and A, each having its proper Triad.

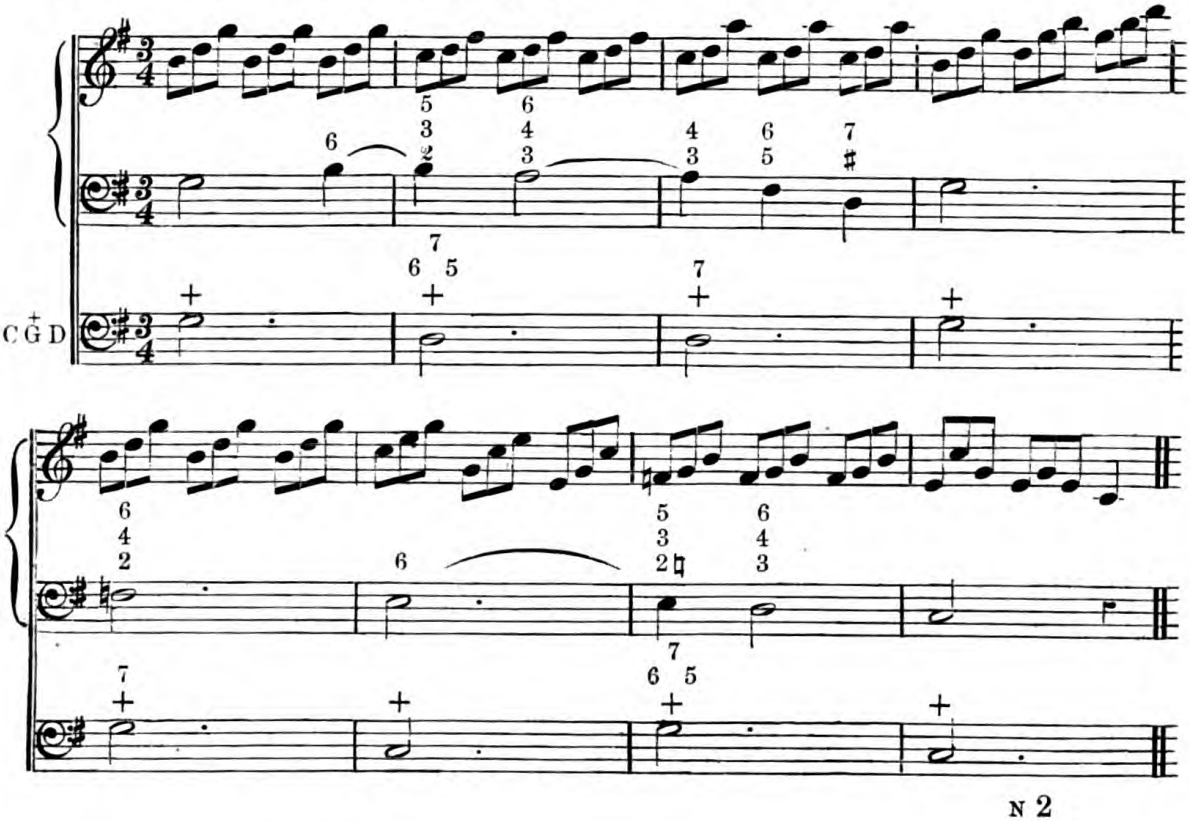
The next Chord is the Diminished Seventh on G♯, (the Fundamental Bass of which is E,) leading to the Key of A minor. Then the Tonic Harmony of C is again heard; followed by the Perfect Cadence, *viz.* F the Subdominant, with its Added Sixth; G the Dominant, with a Suspended Fourth; and C the final Chord.

29. From the Scales of C. P. Emanuel Bach:—



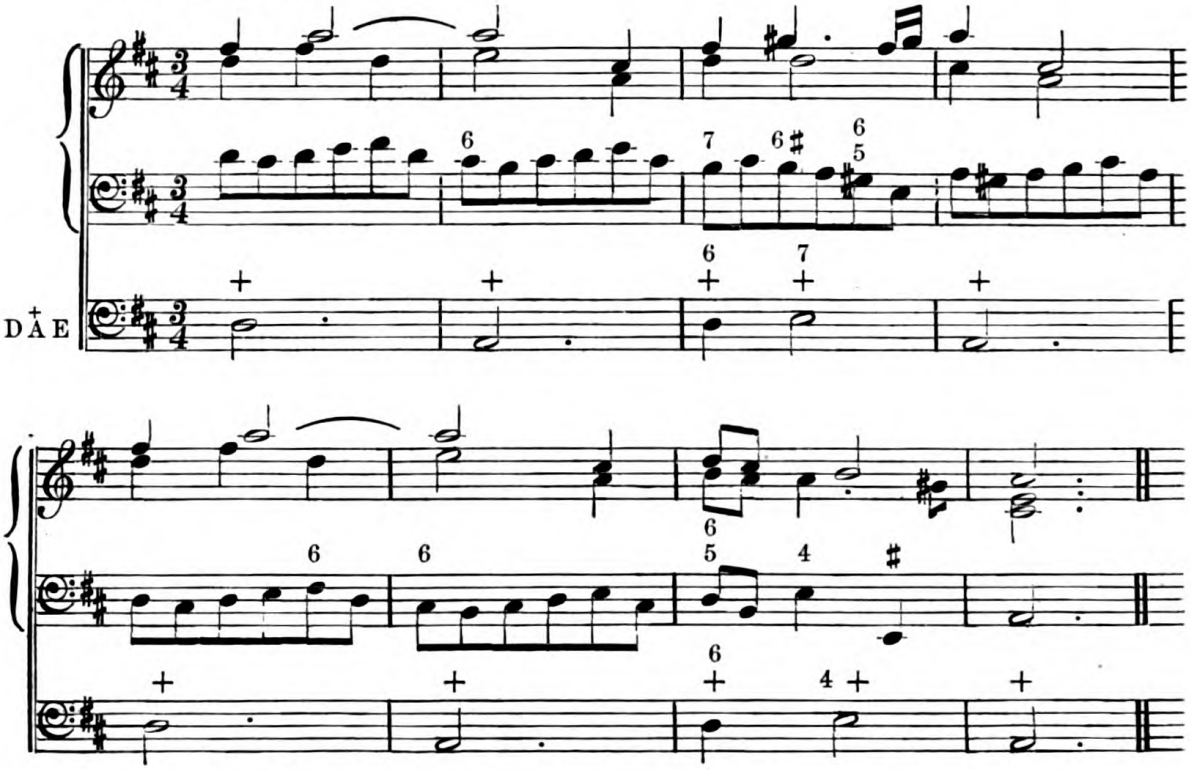
In this Example, which is in the Key of A Minor, after the Tonic Harmony, the Diminished Seventh is taken on D♯, leading into E Major. Next, the Dominant Harmony is taken on A, leading into D Minor, the Key of the Subdominant. Then the Added Sixth (which is here Minor), is taken on D as a Subdominant; followed by the Dominant Harmony on E, with Diminished Ninth, in the form of a Diminished Seventh on G♯; leading back to A Minor: after which there is a Perfect Cadence, of Subdominant Harmony, Dominant Harmony with Suspended Fourth, and Tonic Harmony.

30. From Shield's "Introduction," p. 53:—



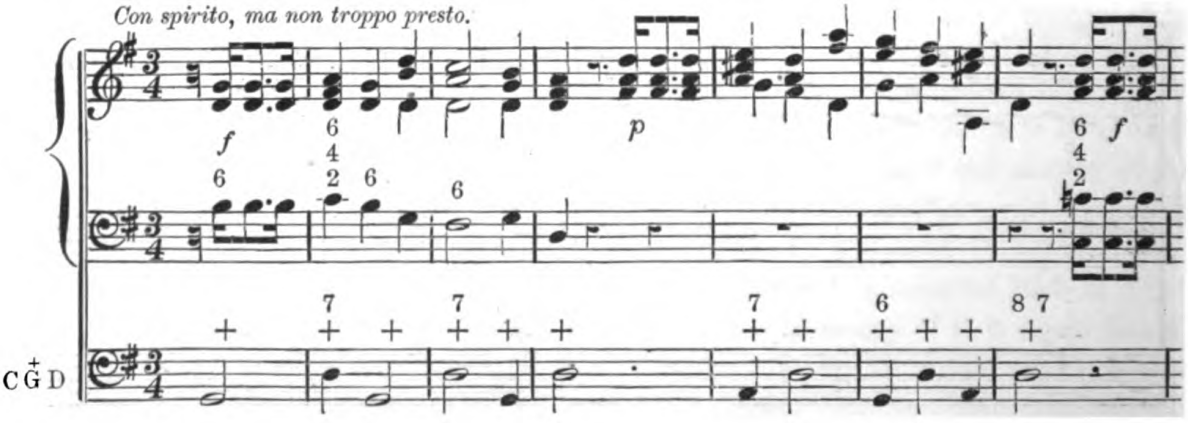
In this Example, which consists of the Tonic and Dominant Harmonies, beginning in the Key of G, and modulating into C, with Suspended Sixths in the second and seventh Measures, the notes of the Chords take the form of Arpeggios.

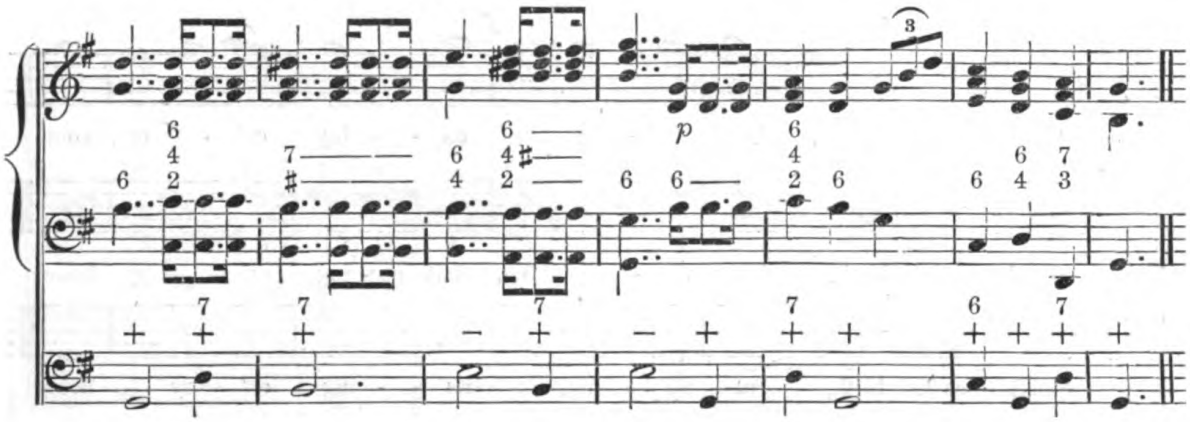
31. Corelli, Concerto 4 :—



In this Example, the notes of the Bass, that do not belong to the Chords written below, are passing notes; also the note C, in the Treble of the Subdominant Chord at the final Cadence.

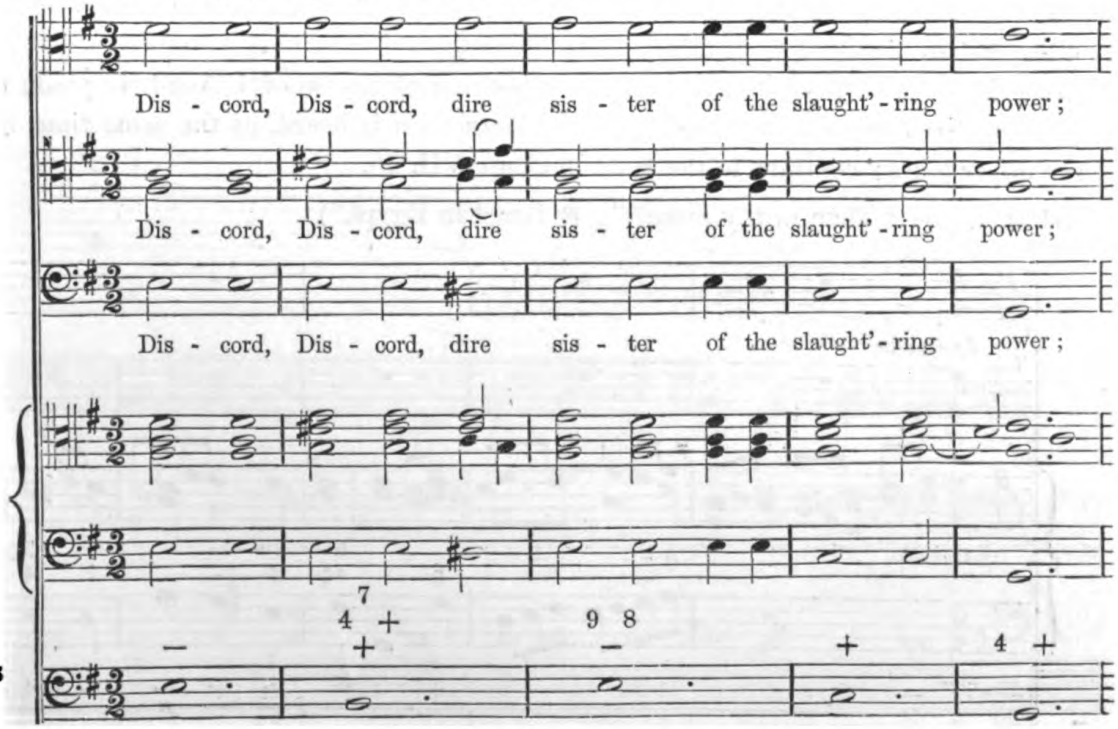
32. Shield's "Introduction," p. 55 :—

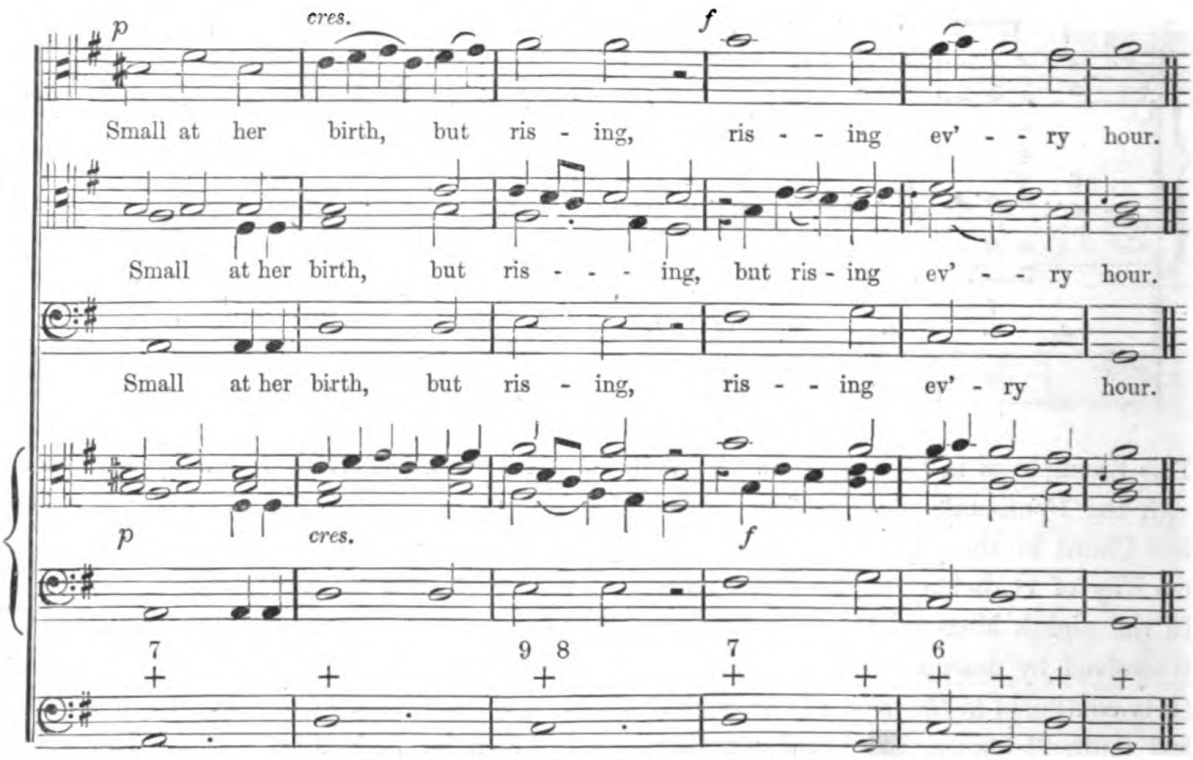




This Example is in G major. In the first four Measures, the Tonic Harmony alternates with the Dominant. In the fifth Measure, the Key changes to D, the Dominant. The first Chord in the sixth Measure, contains only the two notes G E; which, being now in the Key of D, belong to the Subdominant of that Key, with its Added Sixth, G B D E. In the eighth Measure, the Dominant Harmony being again taken on D, the Seventh, C, is resolved by descending to B: but the Major Third F♯, instead of ascending directly to G, is continued in the next Chord, and resolved in the one that follows it. In the ninth and tenth Measures, the Dominant Harmony is taken on B, leading into the Key of E, the Relative Minor: after which, the original Key is restored.

33. From a Glee by Samuel Webbe, sen.:—

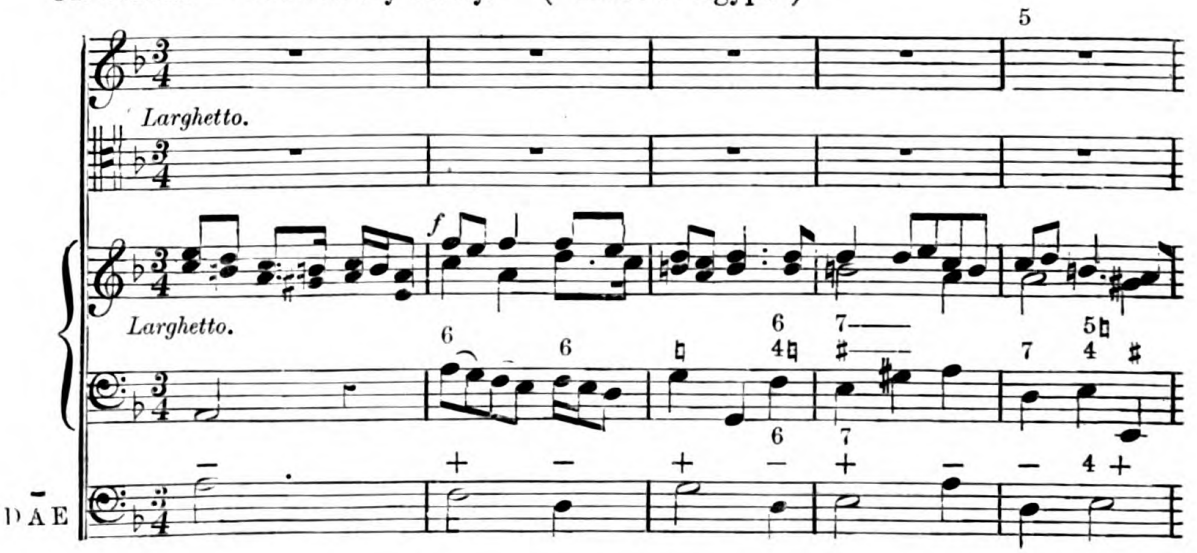


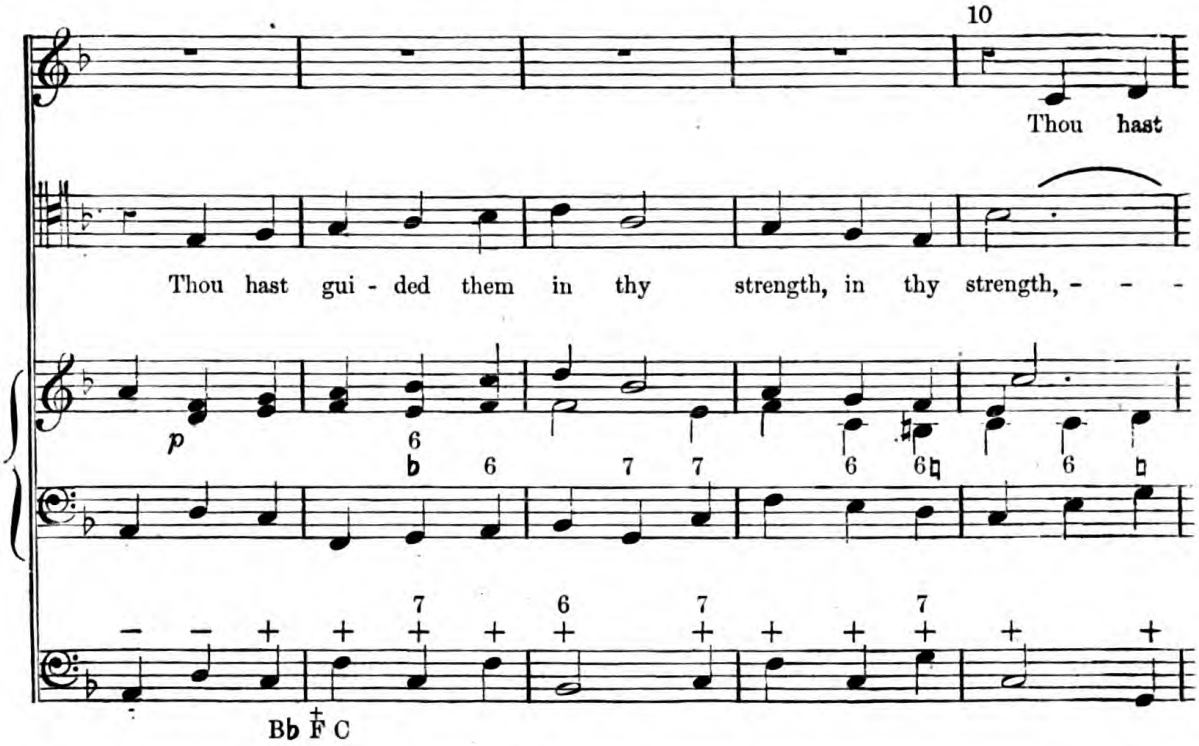


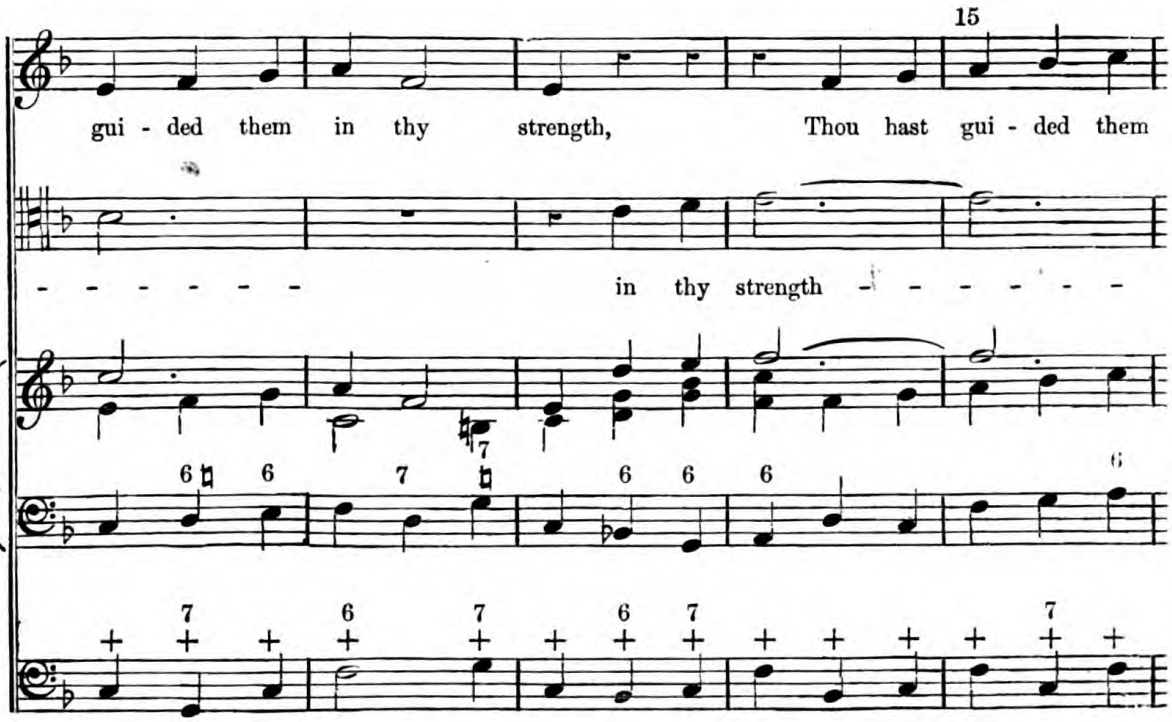
This passage begins in E Minor; followed, with fine effect, by the Plagal Cadence in G, the Relative Major. Then there is a transient Modulation into D Major; after which the strain concludes with two Cadences of Subdominant, Dominant, and Tonic, in G the Relative Major.

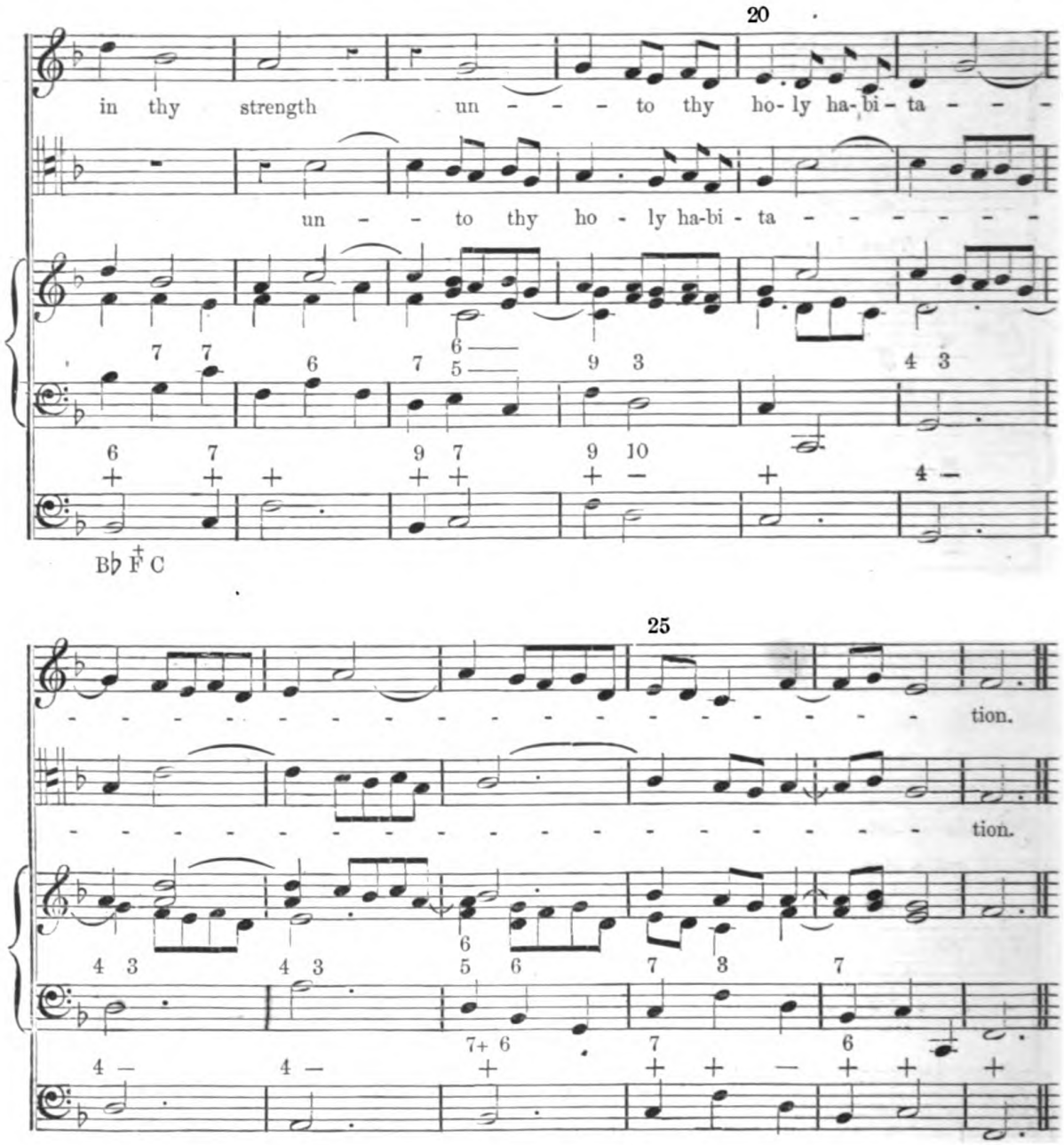
The discordant effect intended in the repetition of the word Discord, is produced by suspending the Fourth in the Bass, while its Resolution is heard, at the same time, in one of the upper parts; contrary to the Rule in Chap. VII. 15.

34. Duetto—"Thou in thy mercy." ("Israel in Egypt.")




10
Thou hast
Thou hast gui - ded them in thy strength, in thy strength, - - -
p
Bb F C


15
gui - ded them in thy strength, Thou hast gui - ded them
- - - - - - - in thy strength - - - - - - -

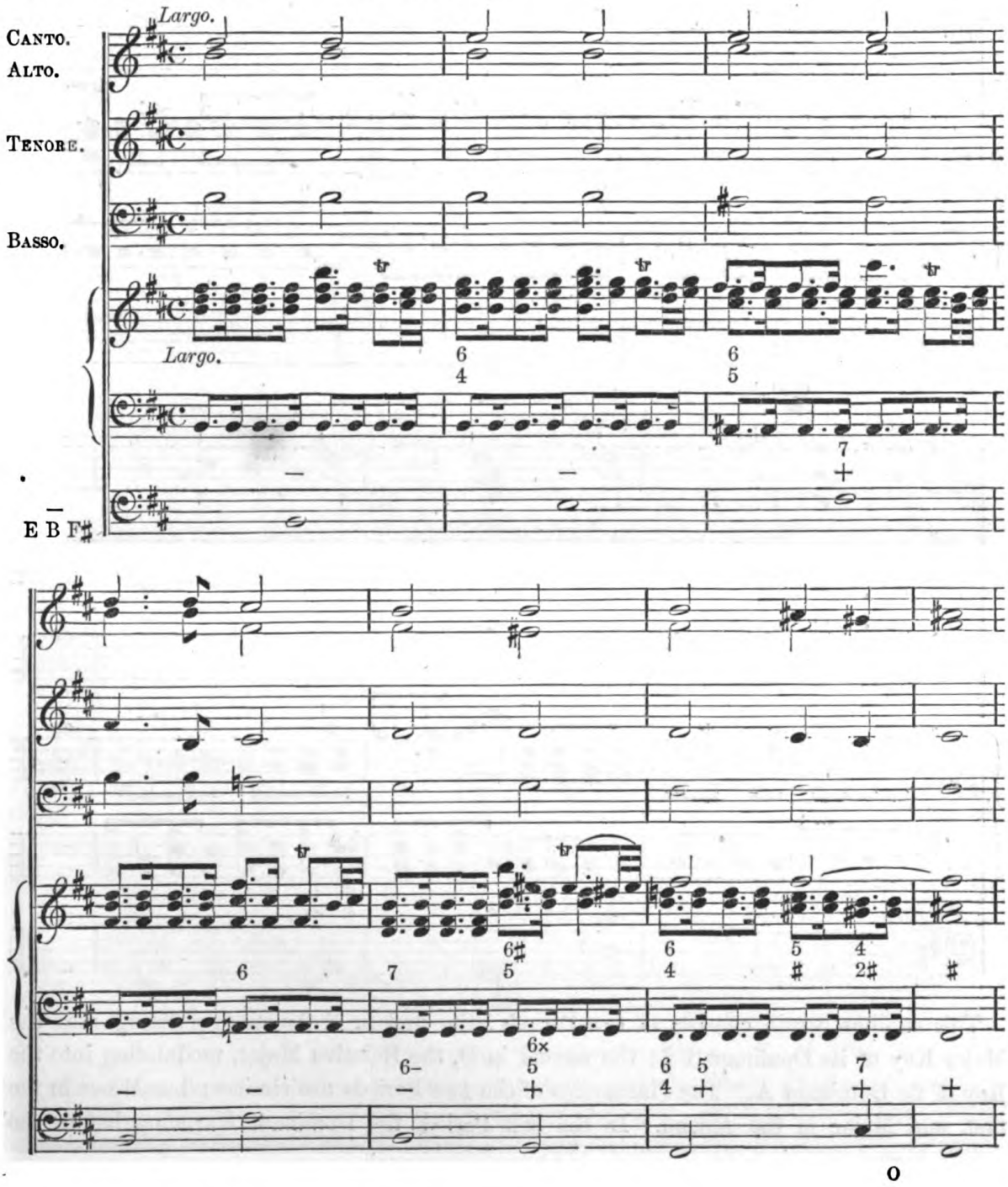


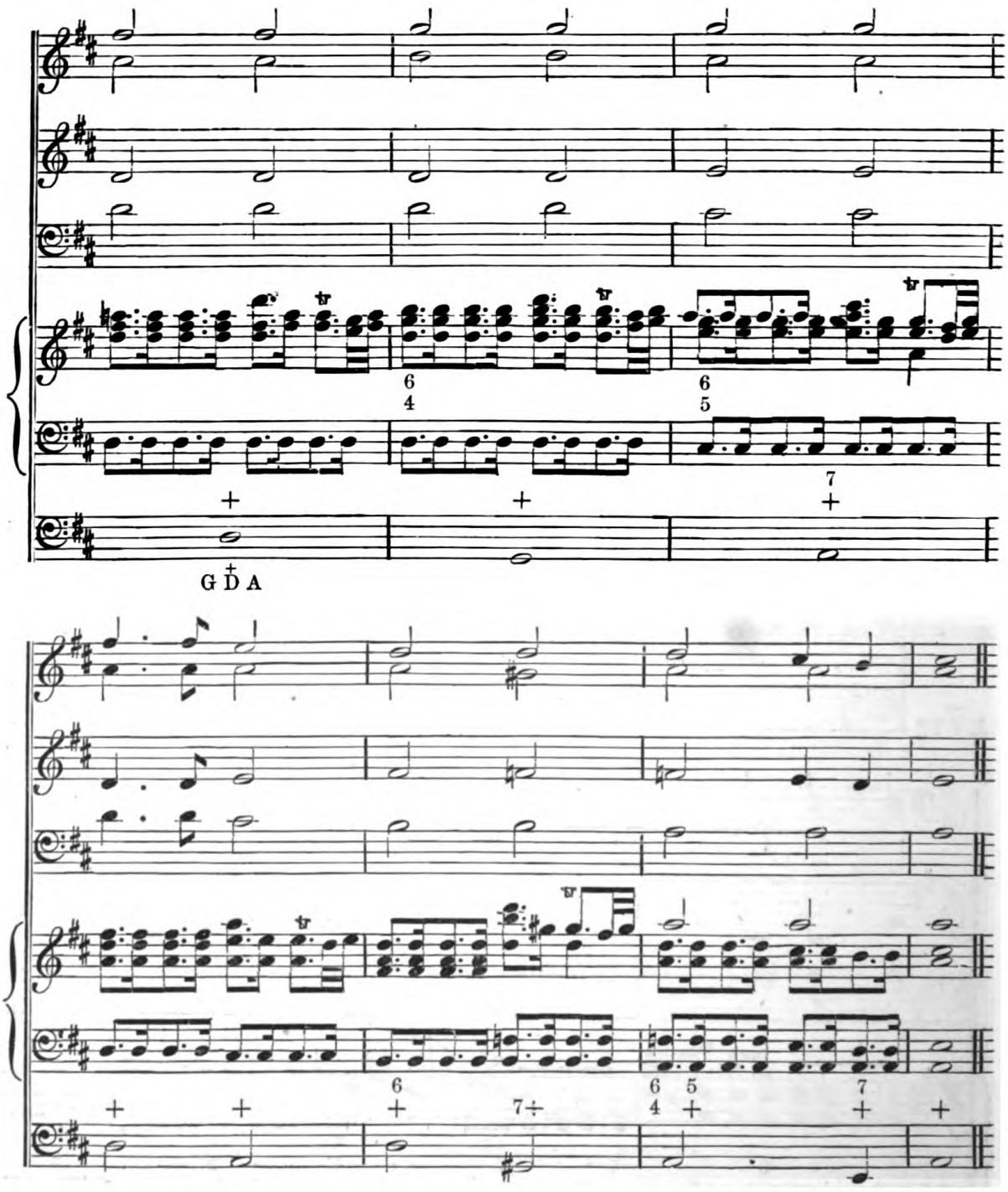
This beautiful passage begins with a Symphony in A Minor, connecting it with what went before. The first Chord is A Minor: next is F, the Relative Major of the Subdominant, followed by D Minor, the Subdominant itself. Then G, the Relative Major of the Dominant, followed by the Subdominant, Dominant, and Tonic Harmonies of the Key, which are repeated.

The vocal part is in F Major. In the ninth Measure, the Dominant Harmony on G,

leads into C Major: and at the thirteenth Measure, the Key returns to F; which Key, with occasional Chords from the Relative Minor, continues to the end of the passage. In the 24th and 26th Measures, the Added Sixth is introduced by a Suspended Major Seventh, which, in the latter, is not noticed in the signature of the Fundamental Harmony.

35. "Sanctus." From Graun's "Te Deum."



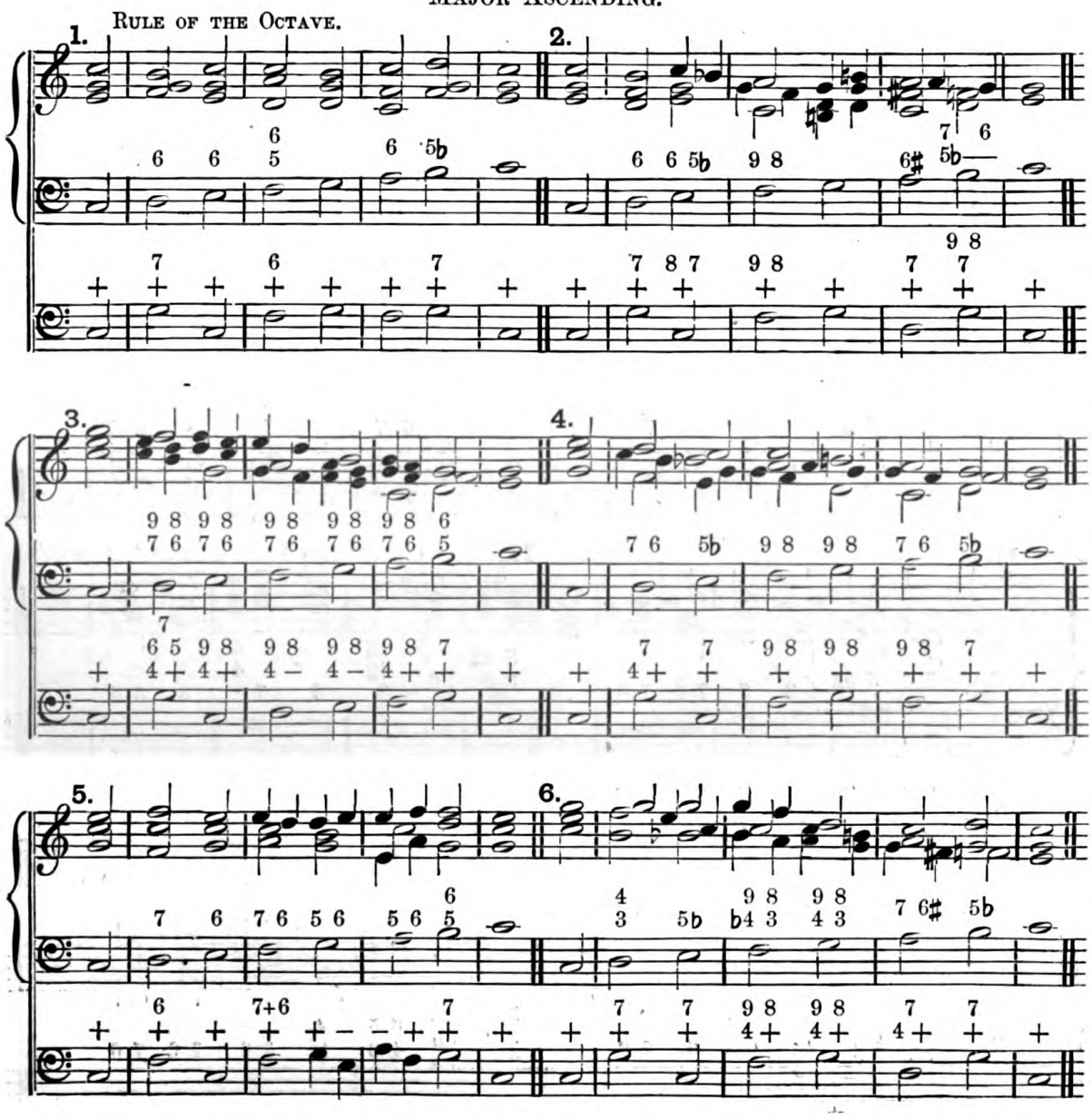


This sublime Strain consists of two Periods: the first in B Minor, modulating into the Major Key of its Dominant F♯; the second in D, the Relative Major, modulating into the Key of its Dominant A. The Harmonies of the two Periods are similar; but Minor in the first, and Major in the second. In the first Period, the Dominant Harmony before the

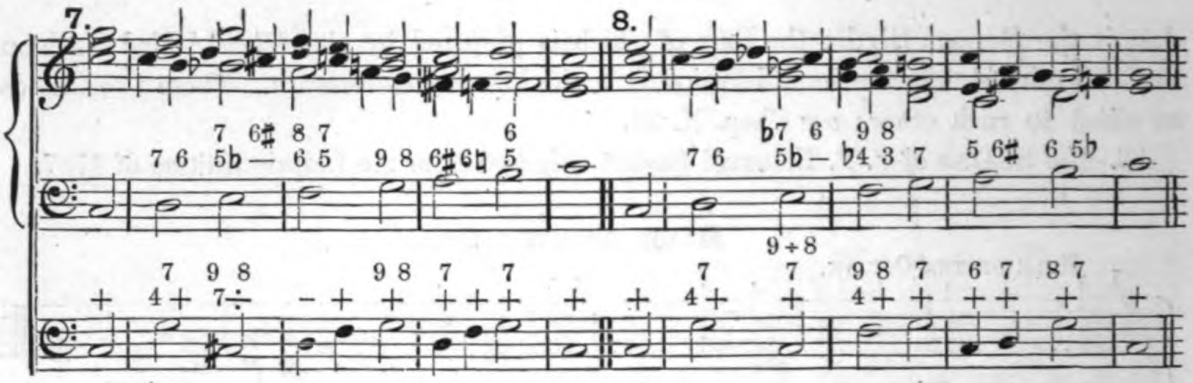
close, is the German Sixth; the Bass of which is prepared by the *Minor* Added Sixth on the Subdominant. In the second Period, it is the Diminished Seventh. These two Chords are allied to each other; see Chap. X. 23.

36. The SCALES of C. P. Emanuel Bach.* Figured from the Leipsic Edition of 1787.

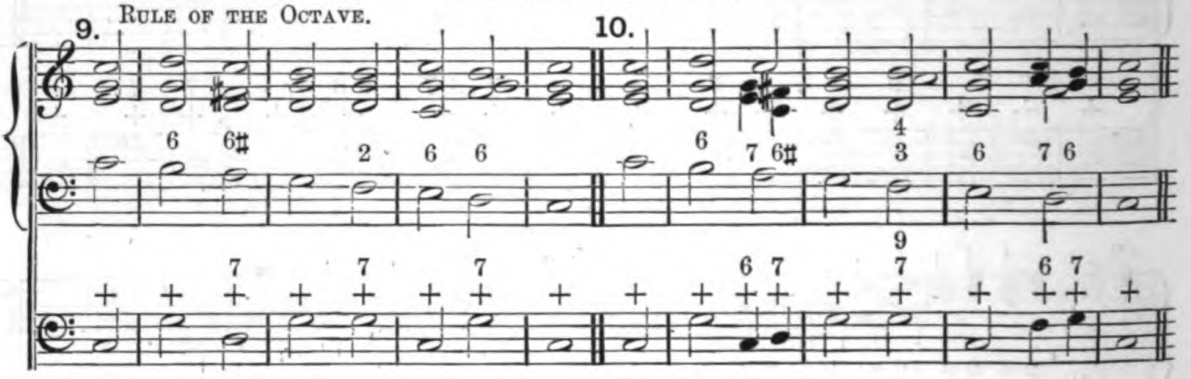
MAJOR ASCENDING.

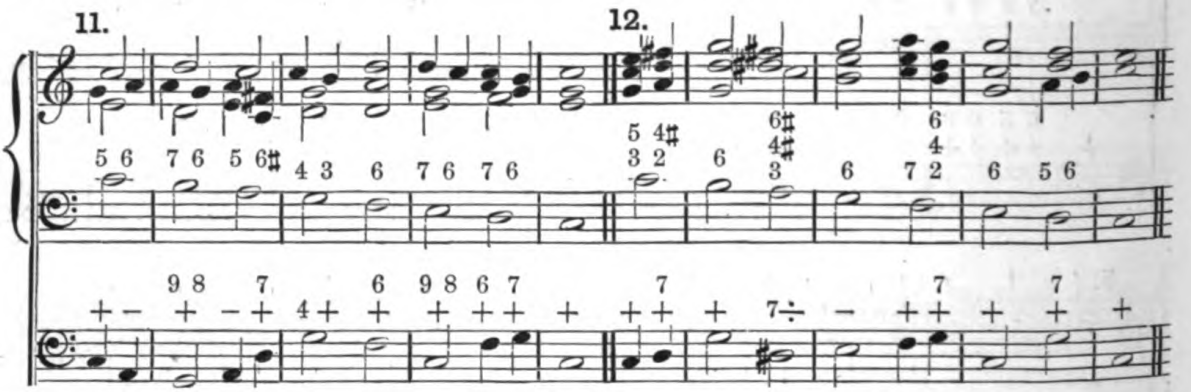


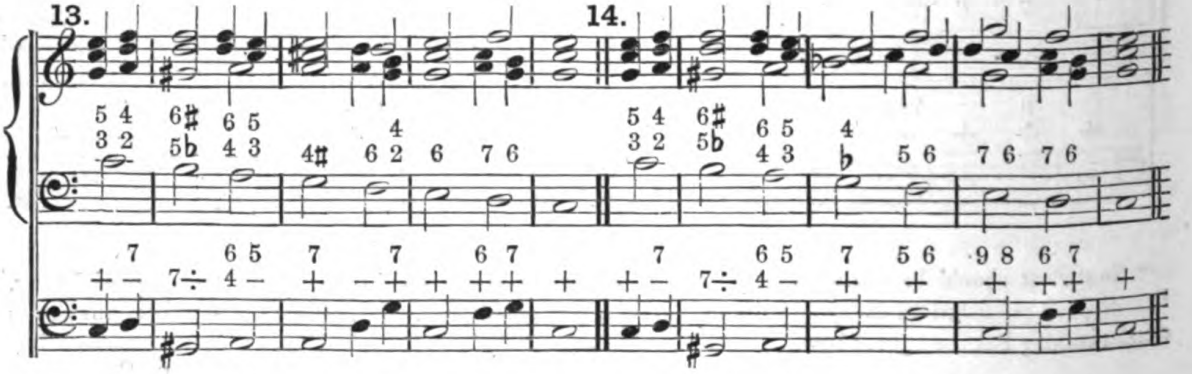
* No student should be without the Scales and Modulations of Emanuel Bach. They are arranged by Mr. Hamilton, and published by Messrs. Cocks and Co. at a very small price. Mr. Shield's fac-simile of the Scales contains nothing but the figured bass.



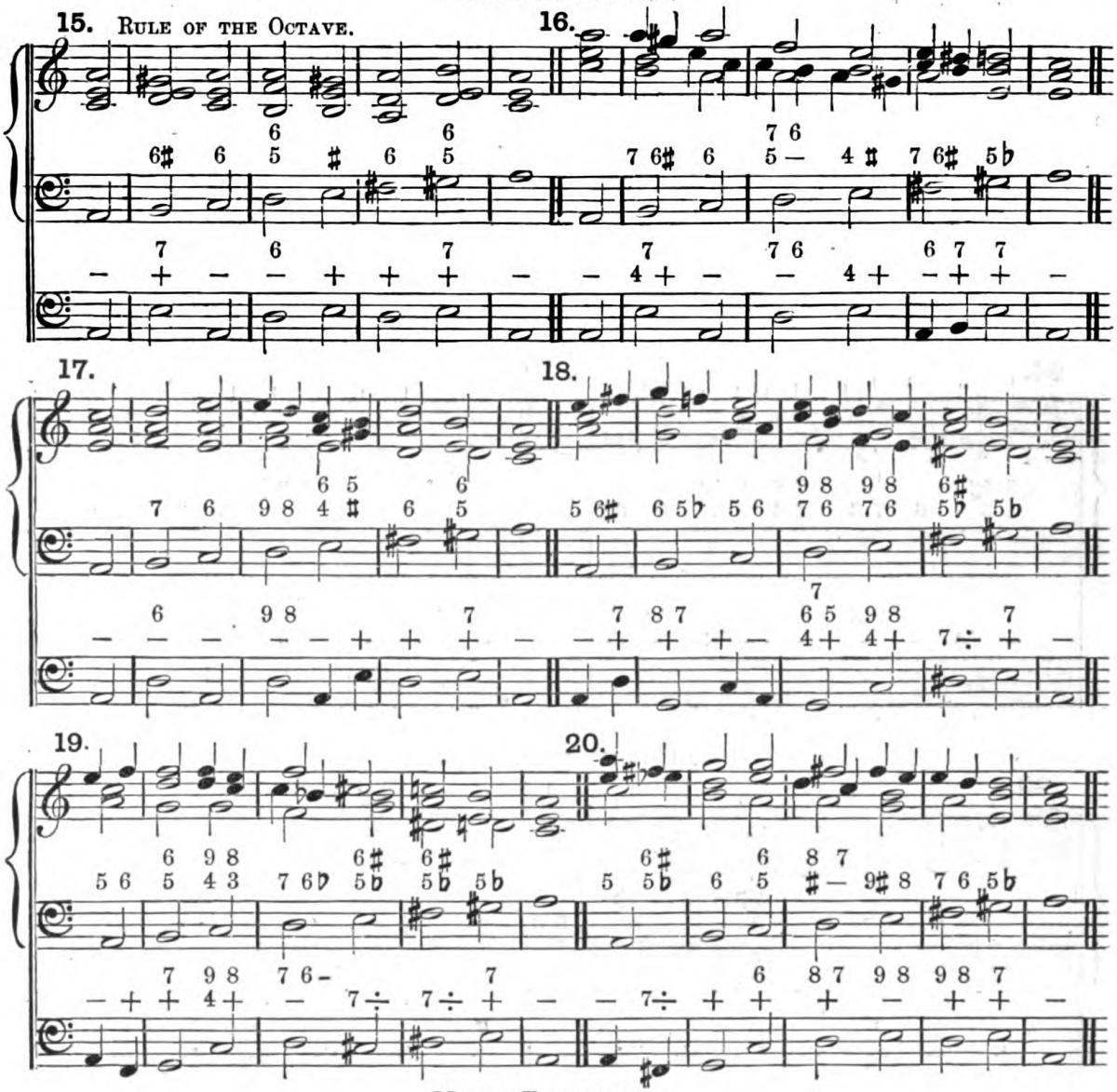
MAJOR DESCENDING.



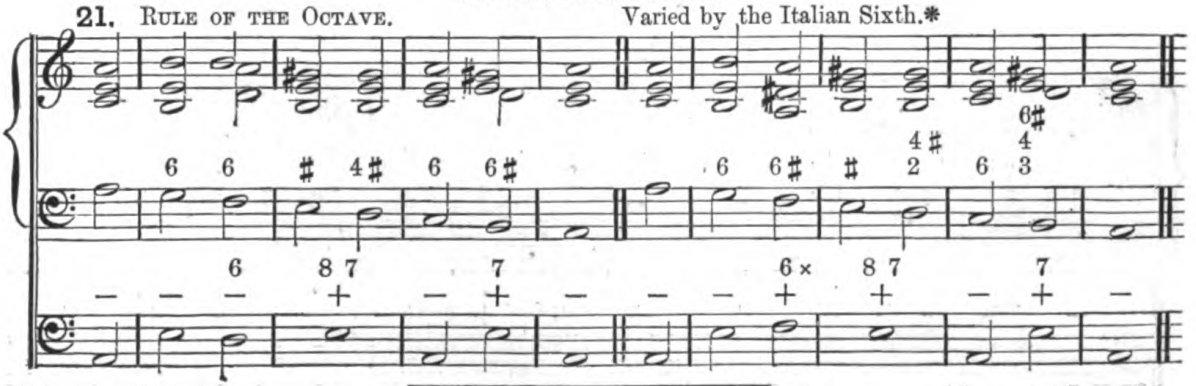




MINOR ASCENDING.

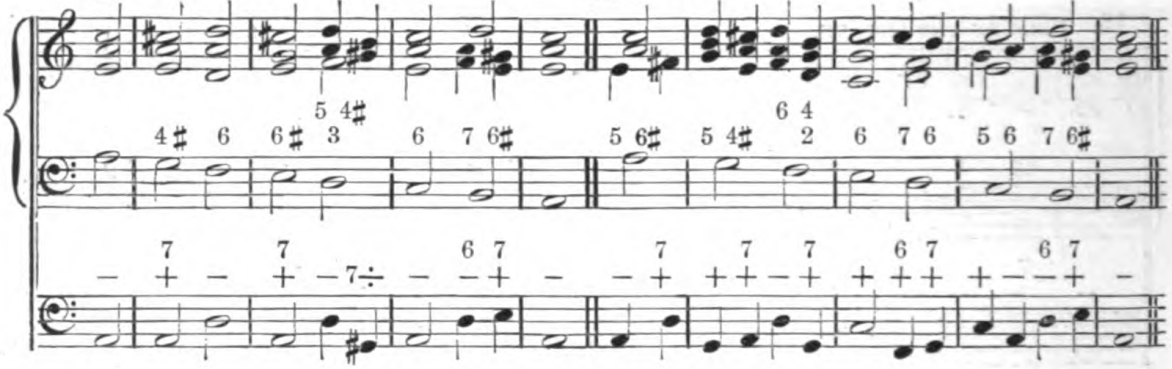


MINOR DESCENDING.



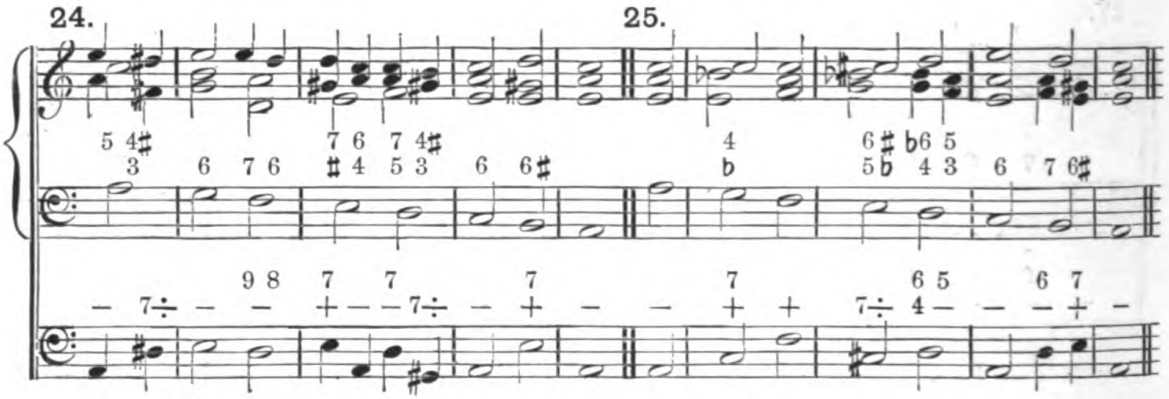
* Callcott's Grammar, Art. 487; 2nd Ed. 450.

22.



23.

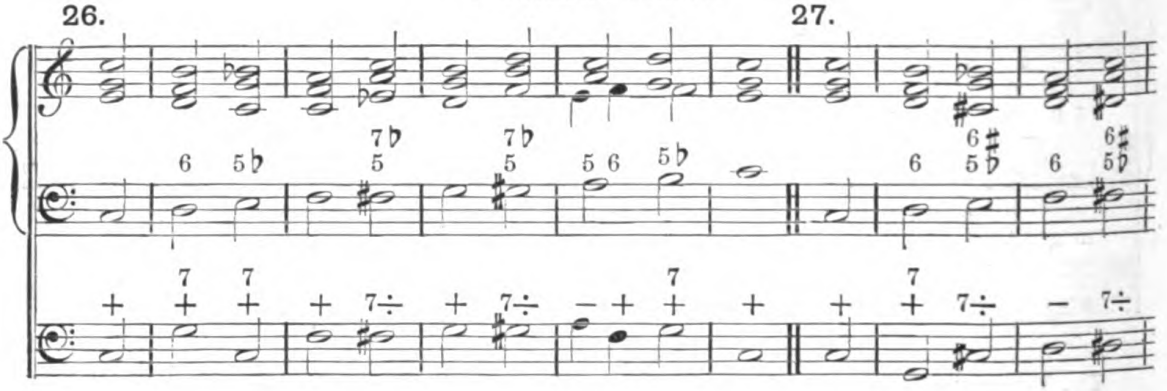
24.



25.

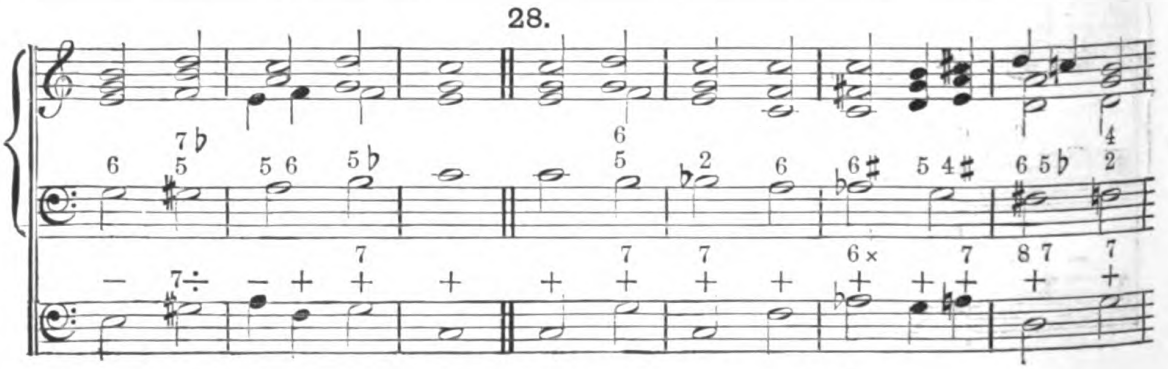
CHROMATIC MAJOR.

26.

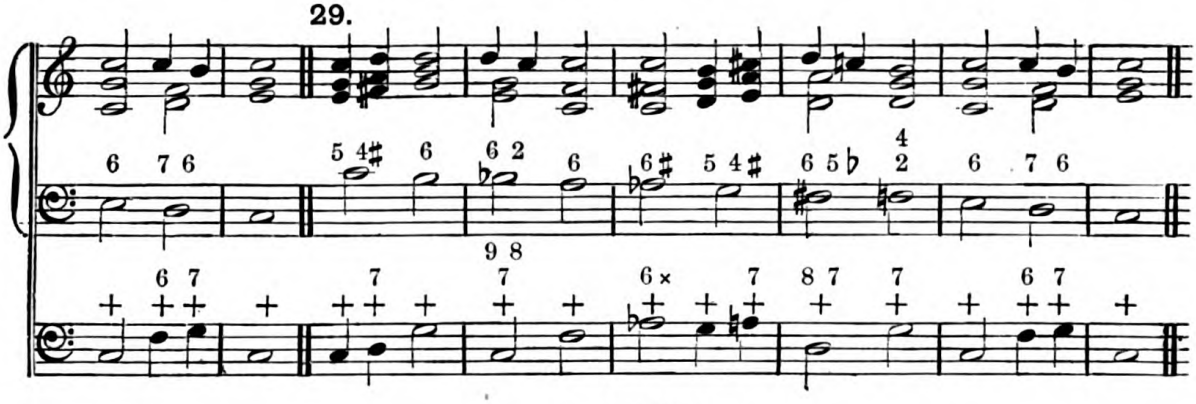


27.

28.

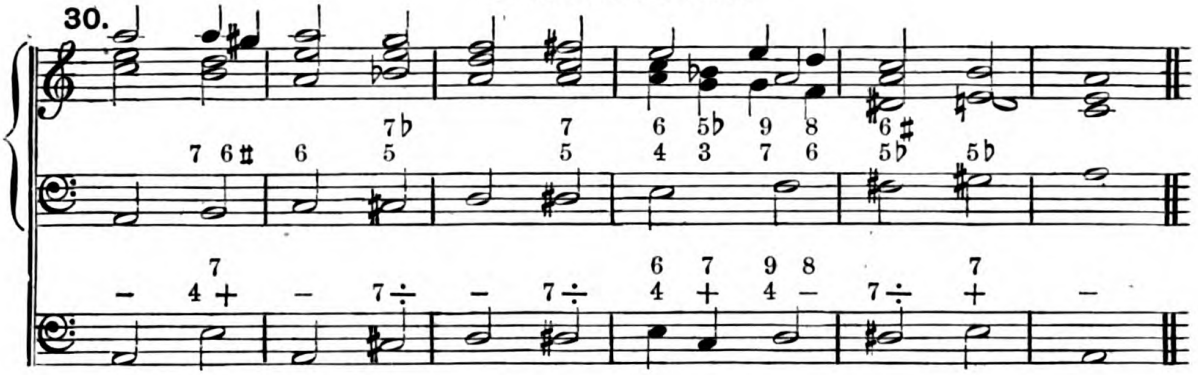


29.

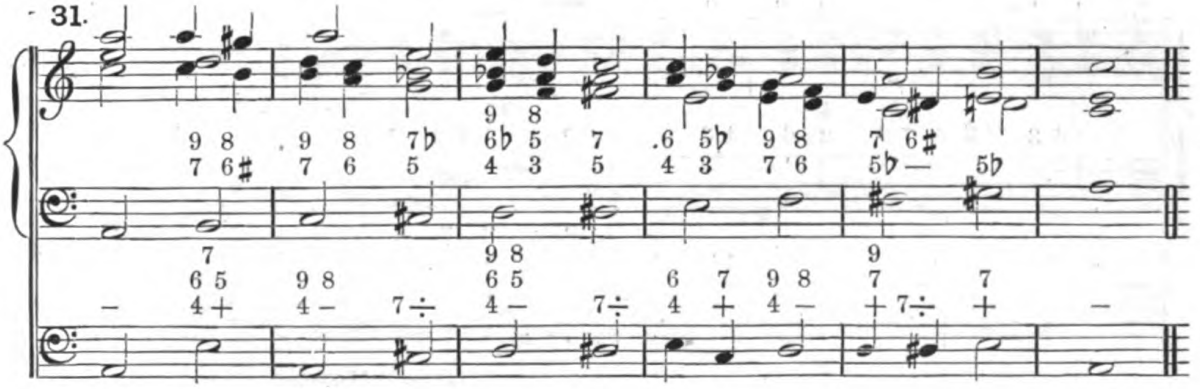


CHROMATIC MINOR.

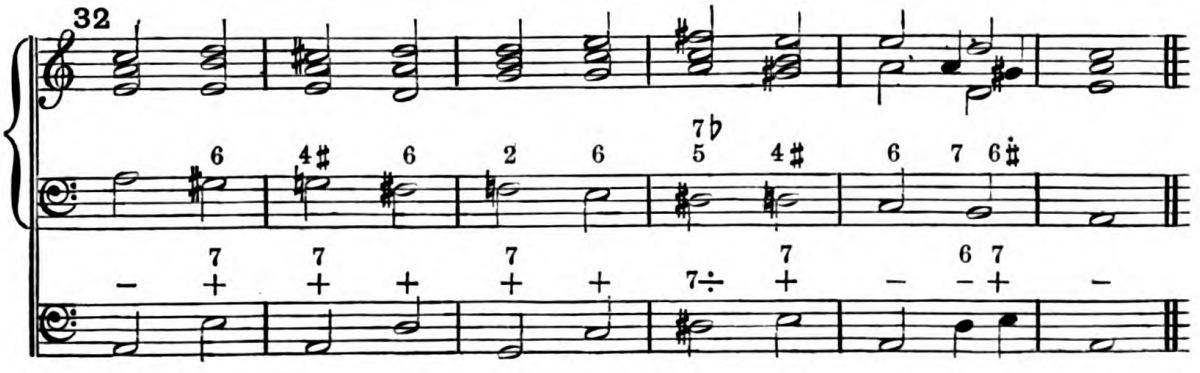
30.



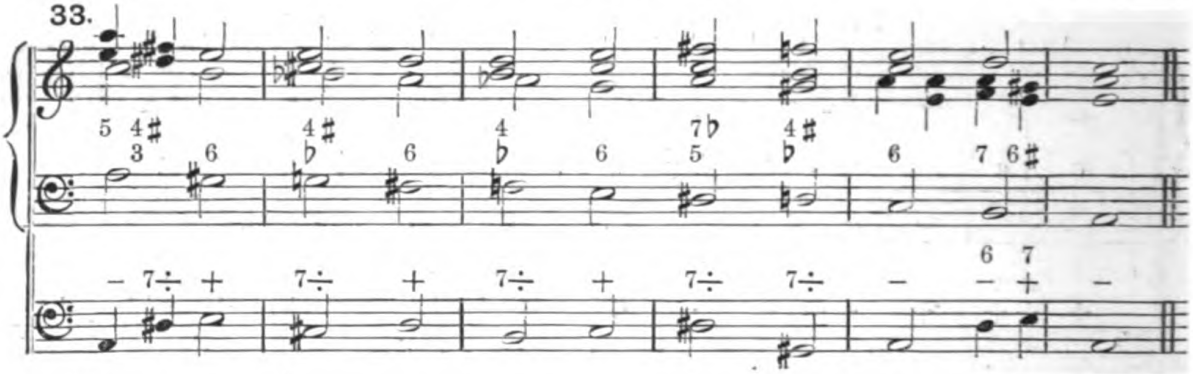
31.



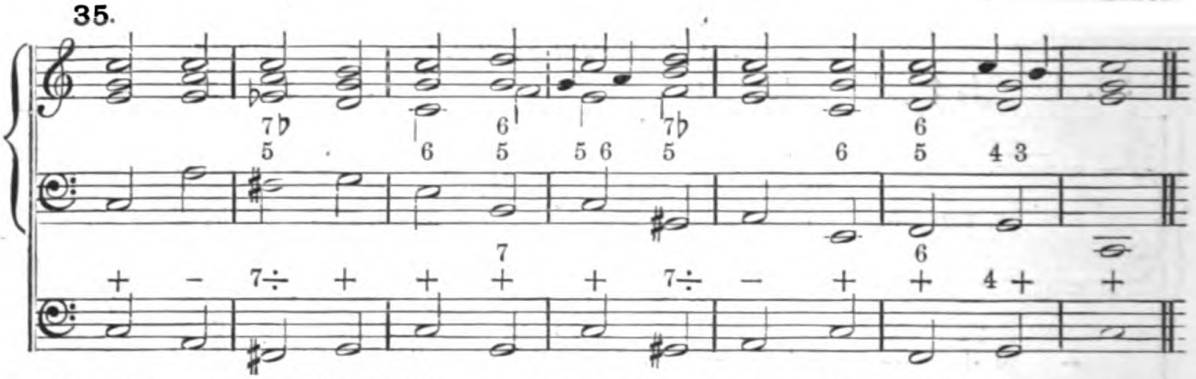
32.



33.

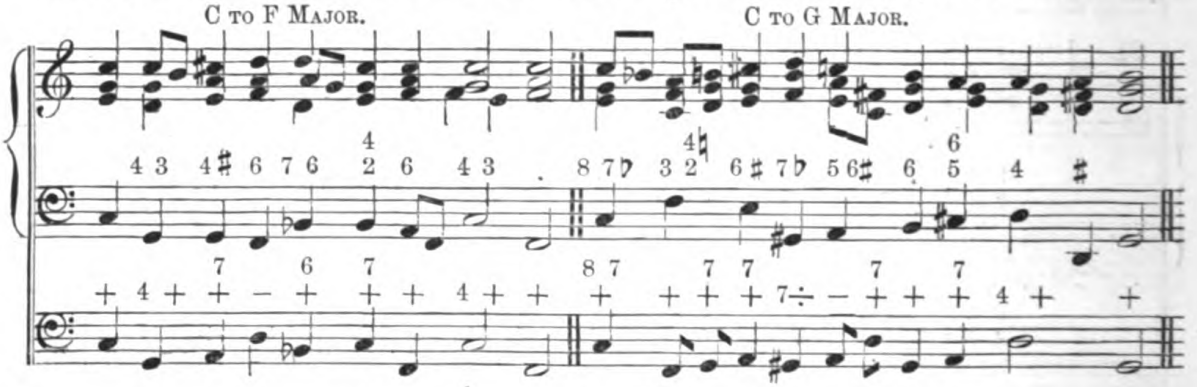


35.



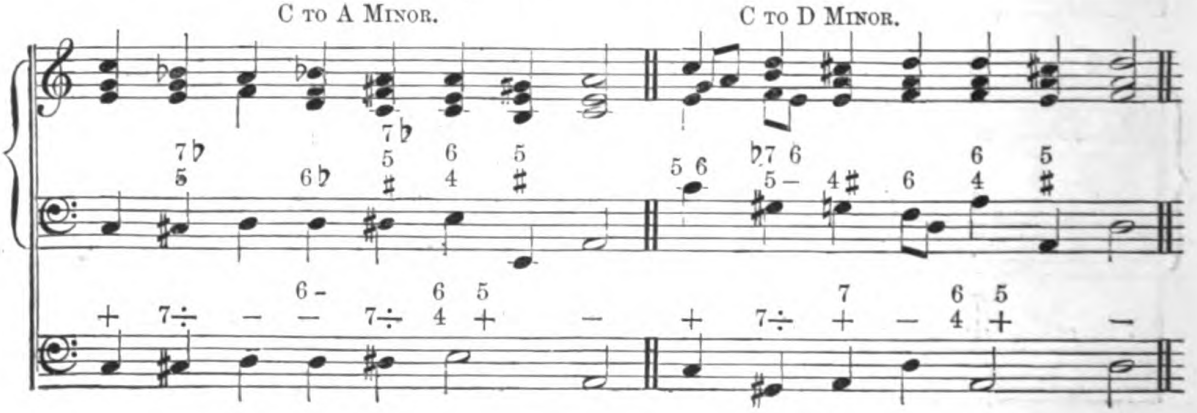
37. MODULATIONS of C. P. Emanuel Bach. Figured from the Leipsic Edition of 1787.

C TO F MAJOR.

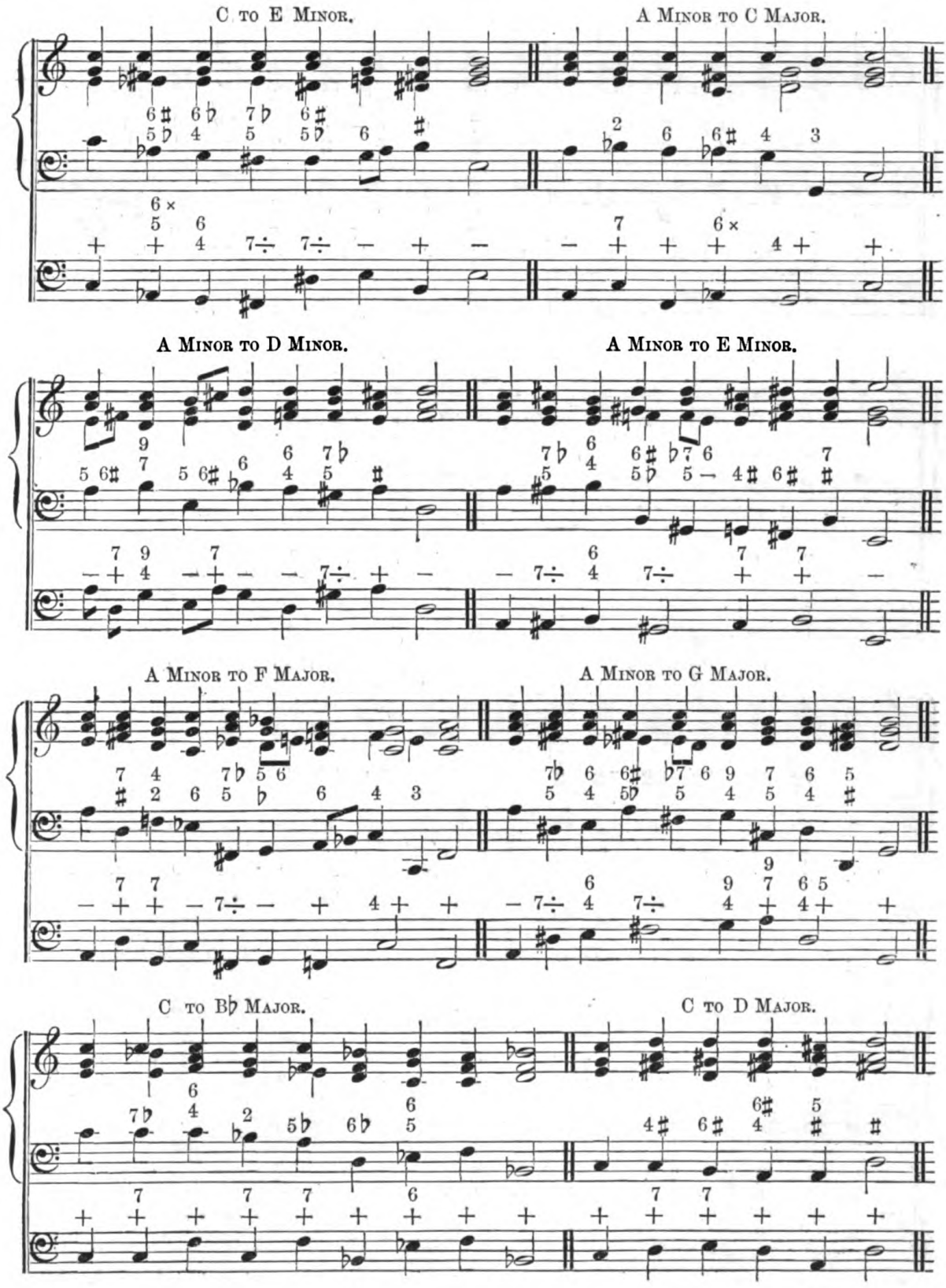


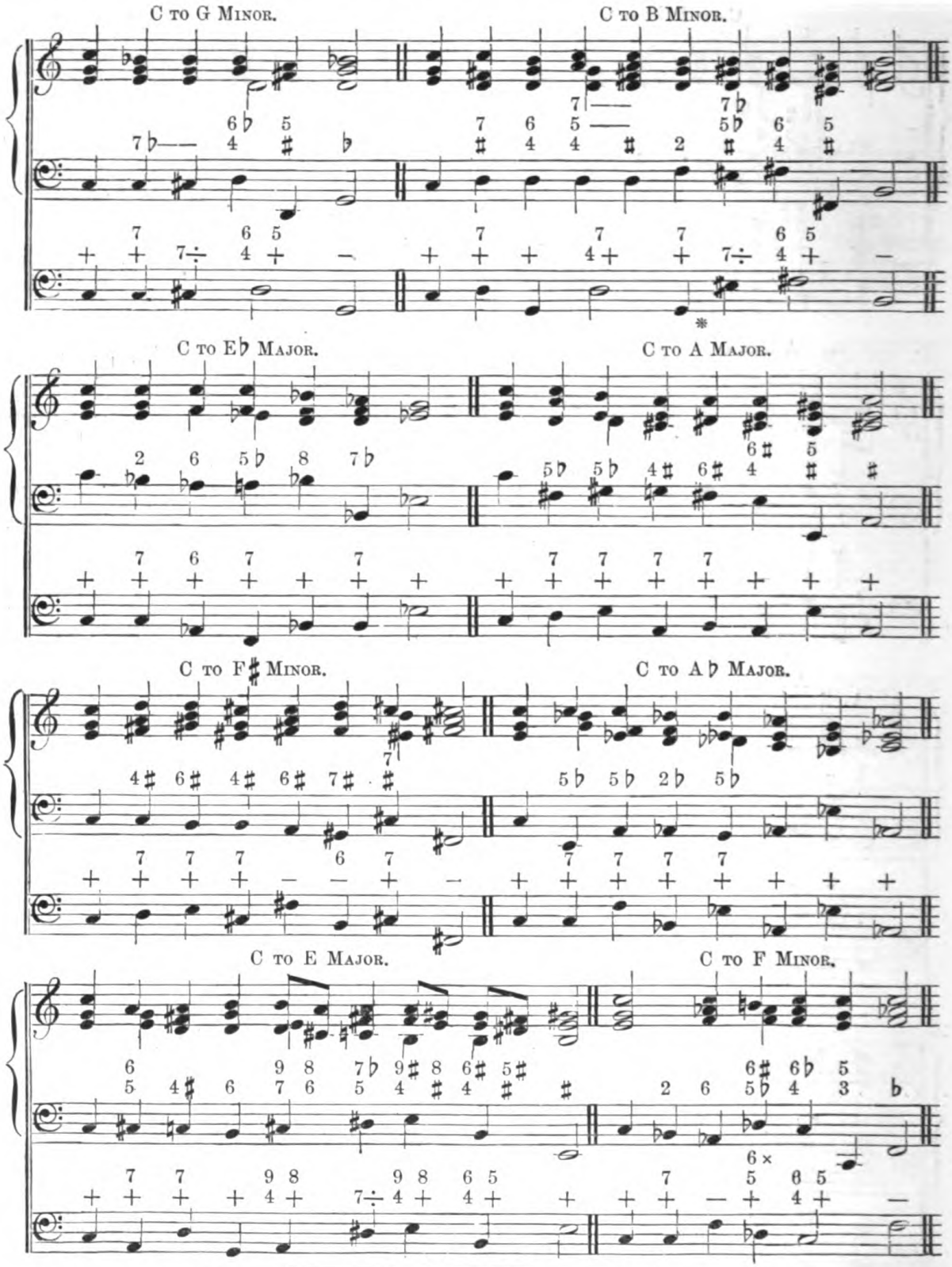
C TO G MAJOR.

C TO A MINOR.

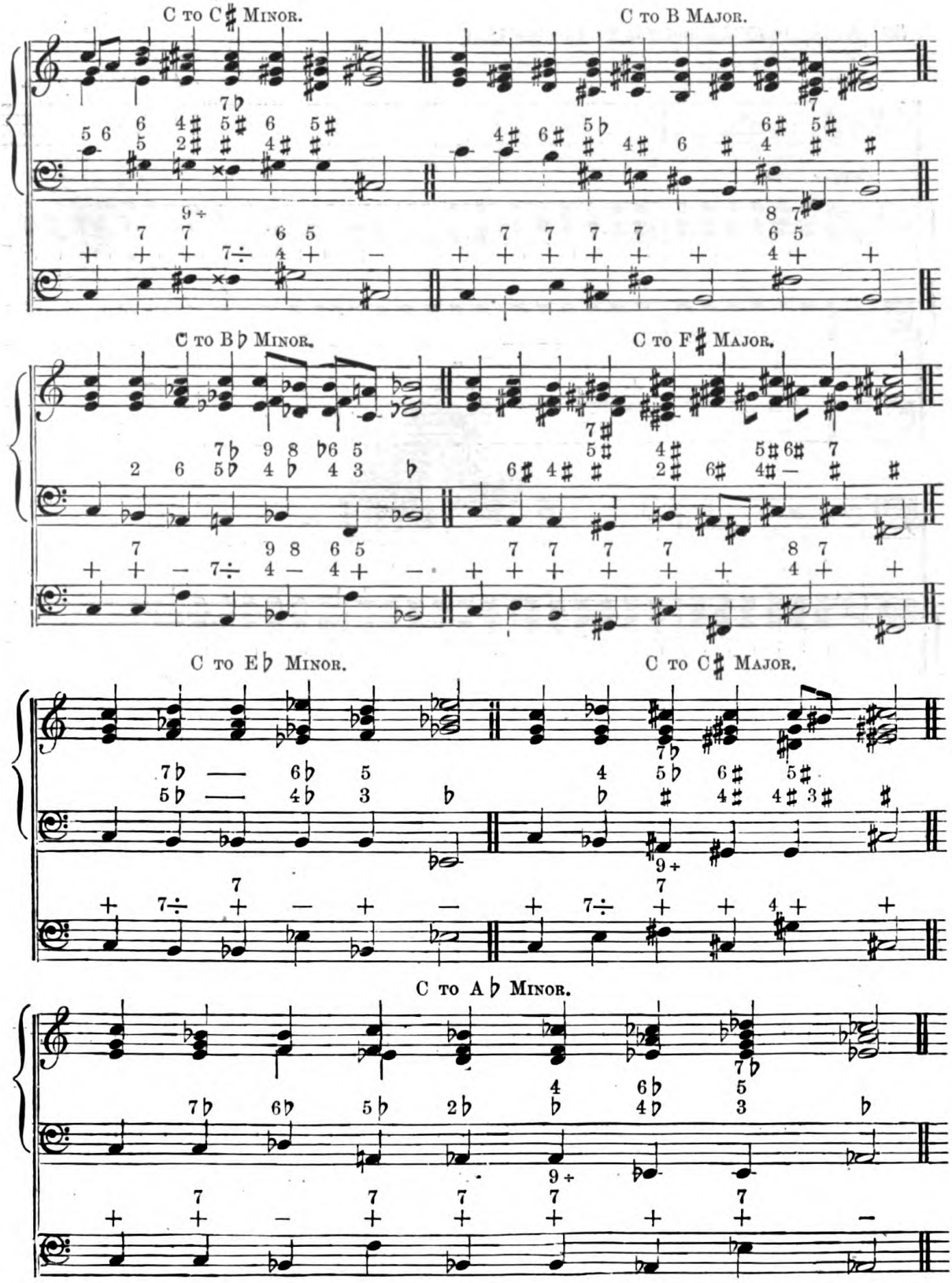


C TO D MINOR.

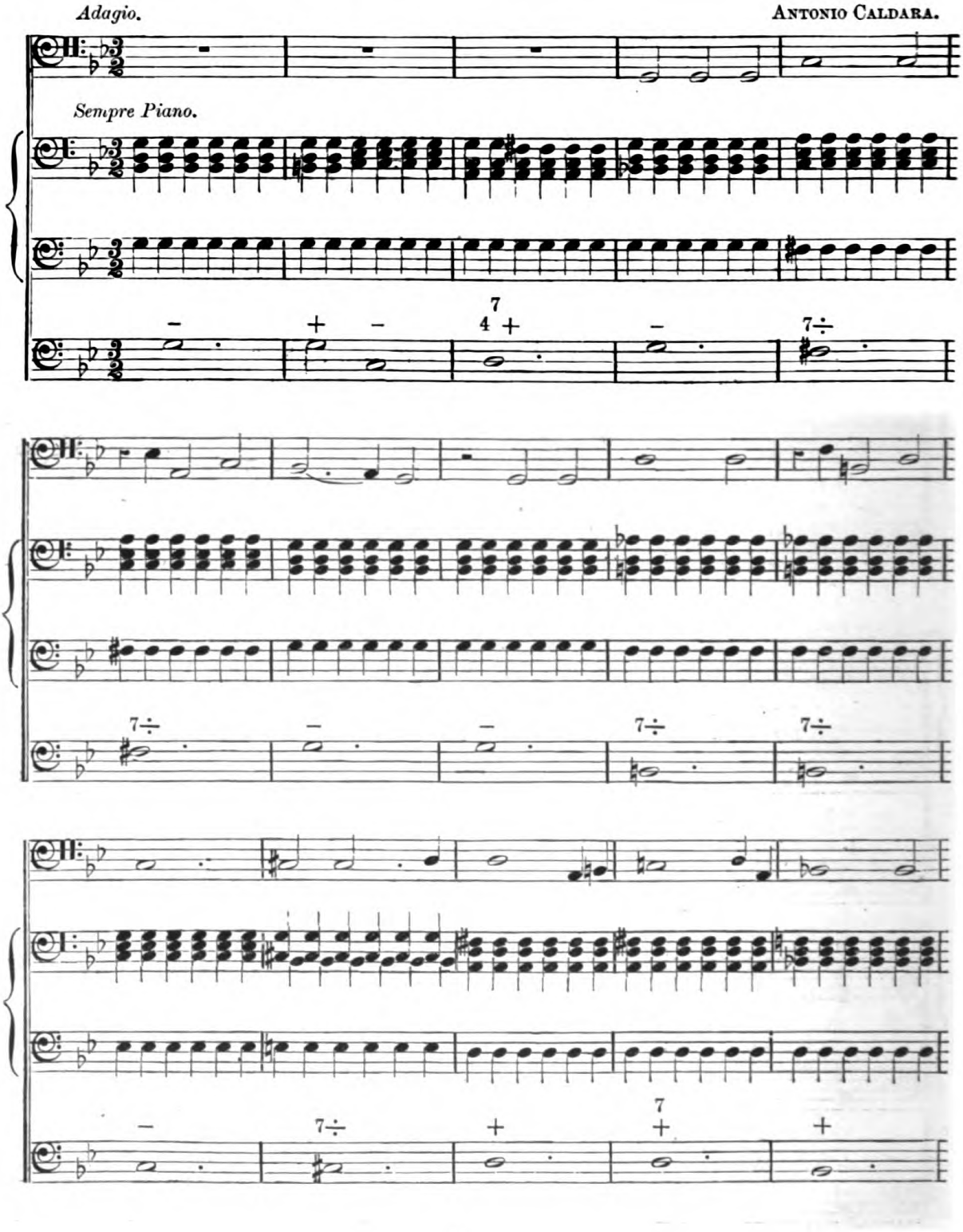

C to E Minor.
A Minor to C Major.
A Minor to D Minor.
A Minor to E Minor.
A Minor to F Major.
A Minor to G Major.
C to B♭ Major.
C to D Major.

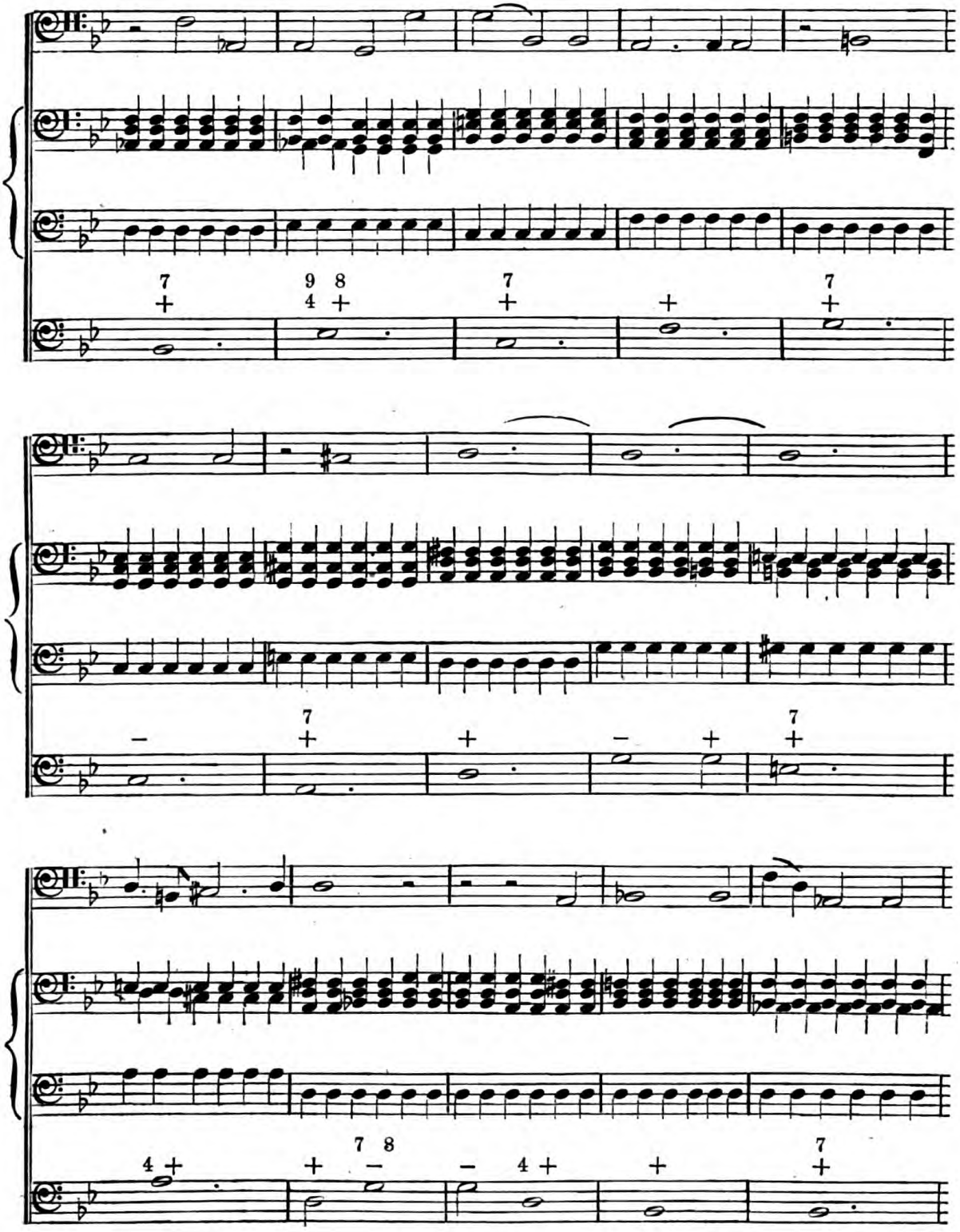


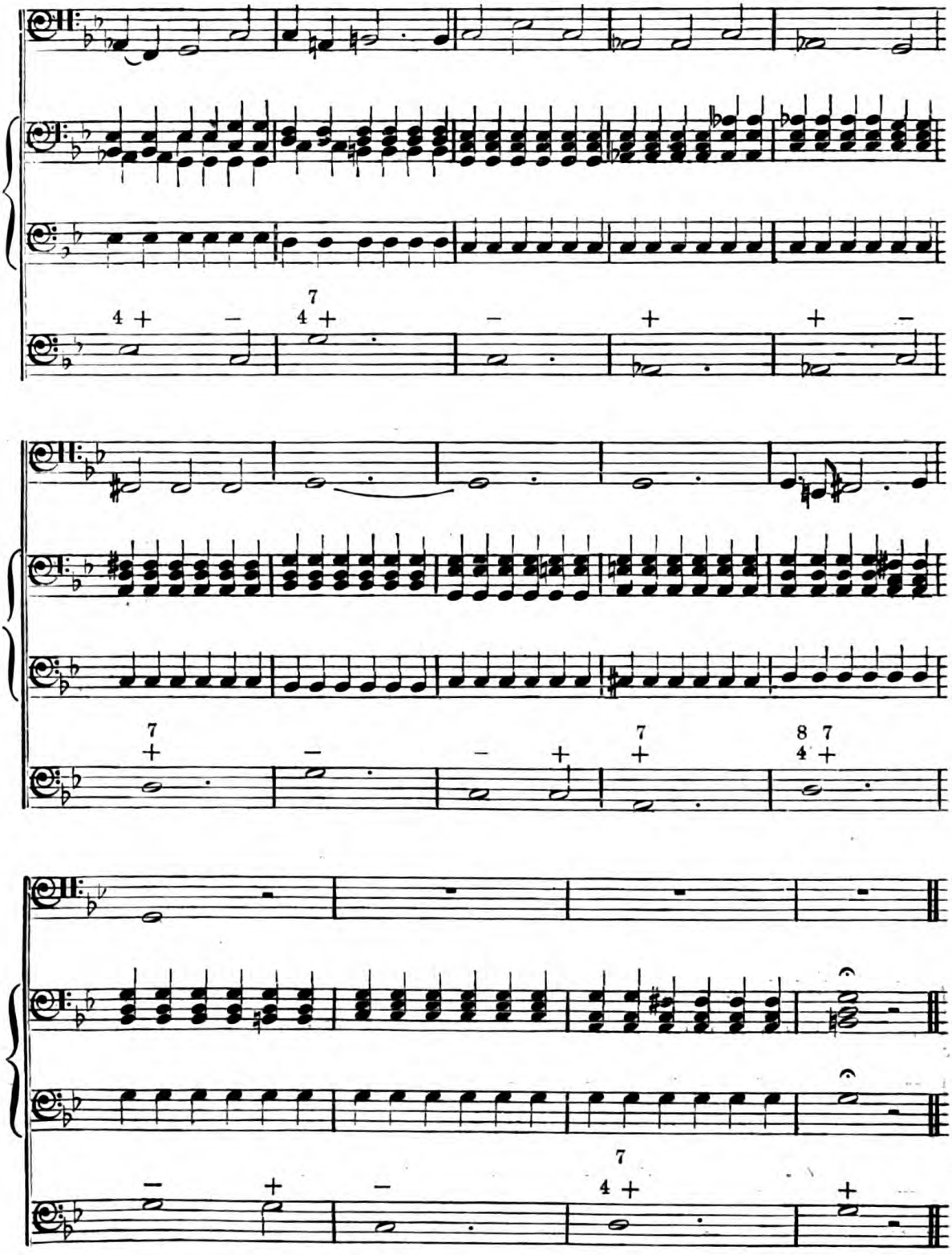
* The progression of the Fundamental Bass here (G, E♯), appears to be irregular. This is occasioned by an Enharmonic change of F into E♯ in the Bass (see Chap. XVII, 3), which may be called a licence.


C to C♯ Minor.
C to B Major.
C to B♭ Minor.
C to F♯ Major.
C to E♭ Minor.
C to C♯ Major.
C to A♭ Minor.

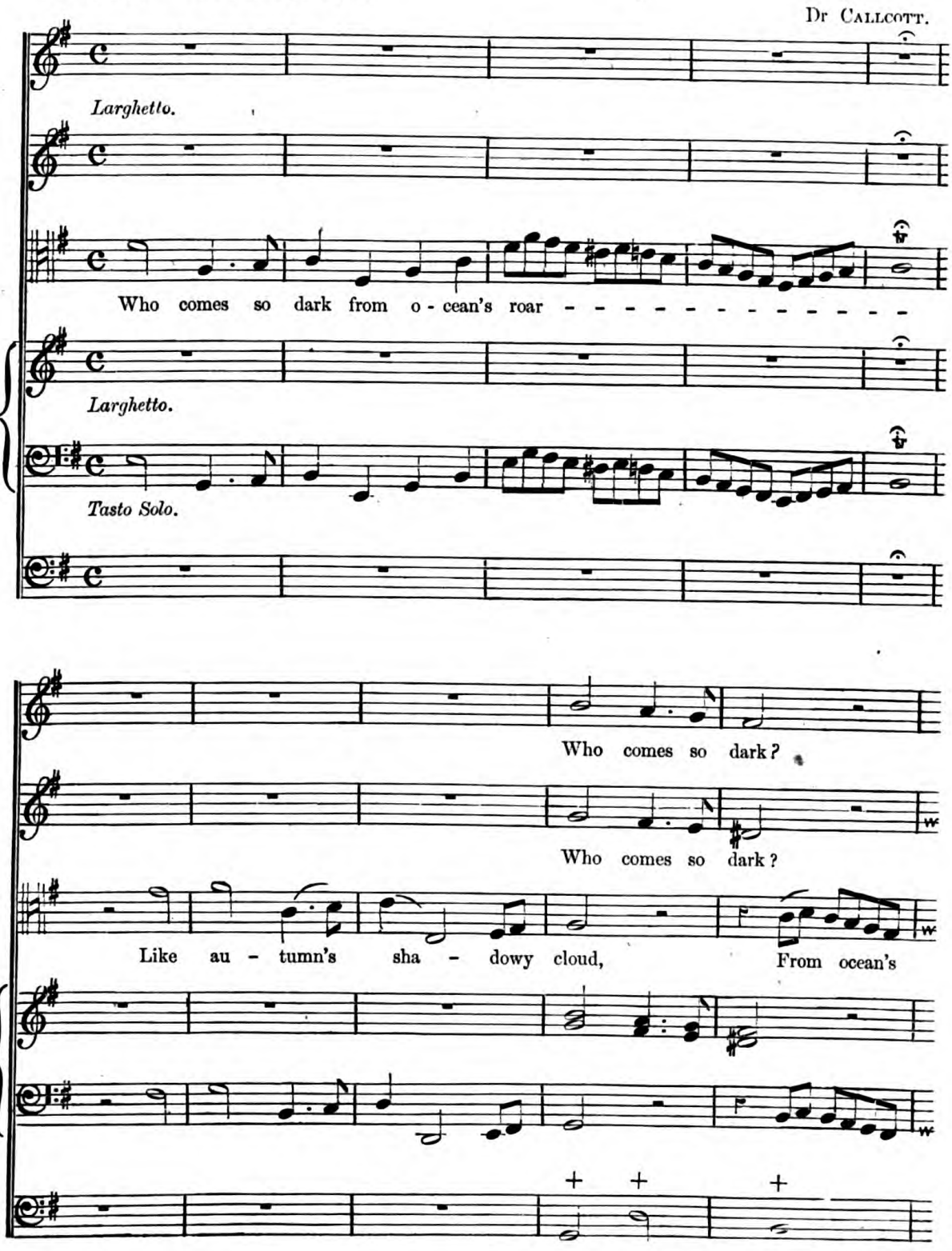
38. ARIA, "*Et Incarnatus.*" In One Clef.

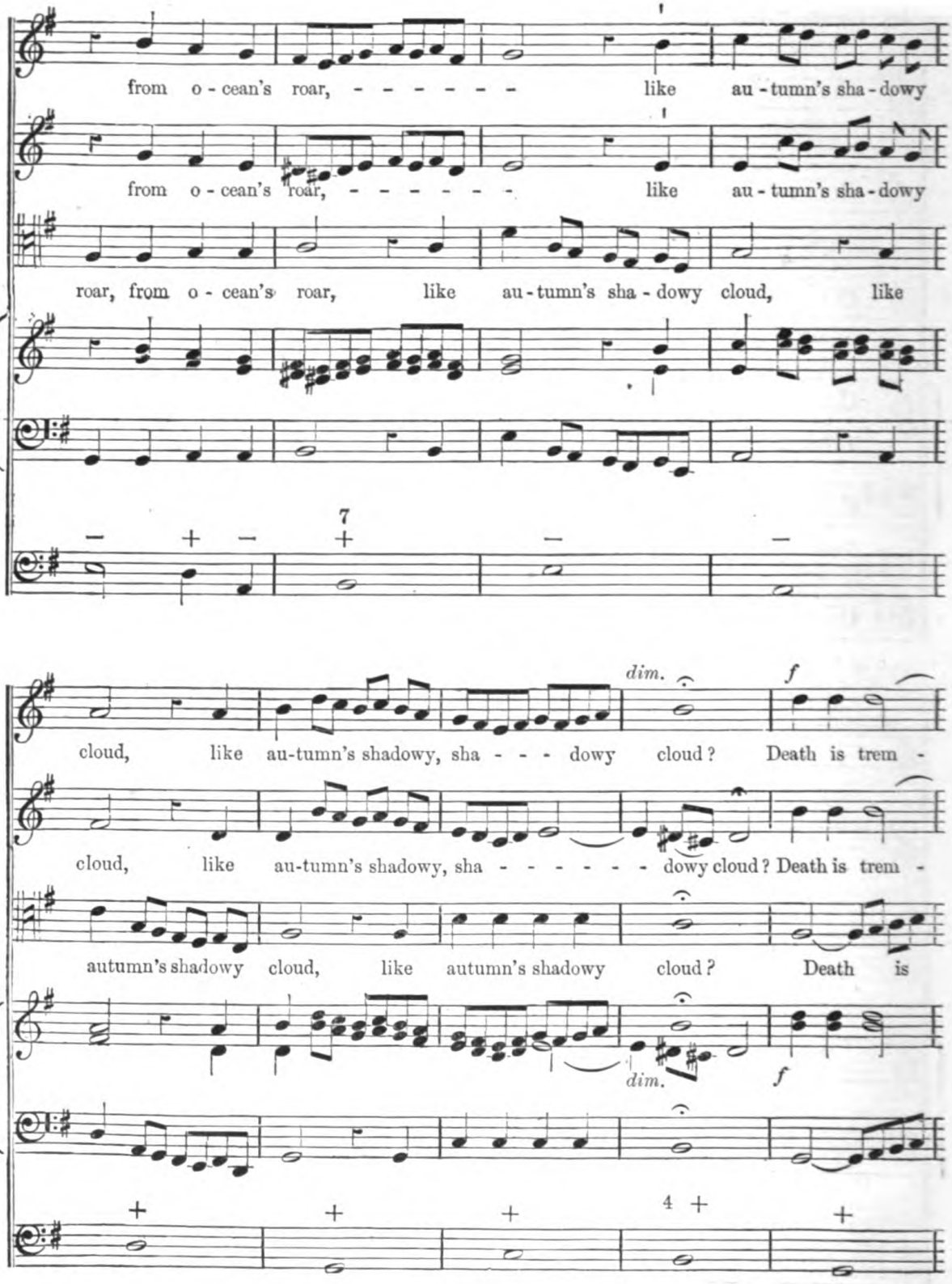


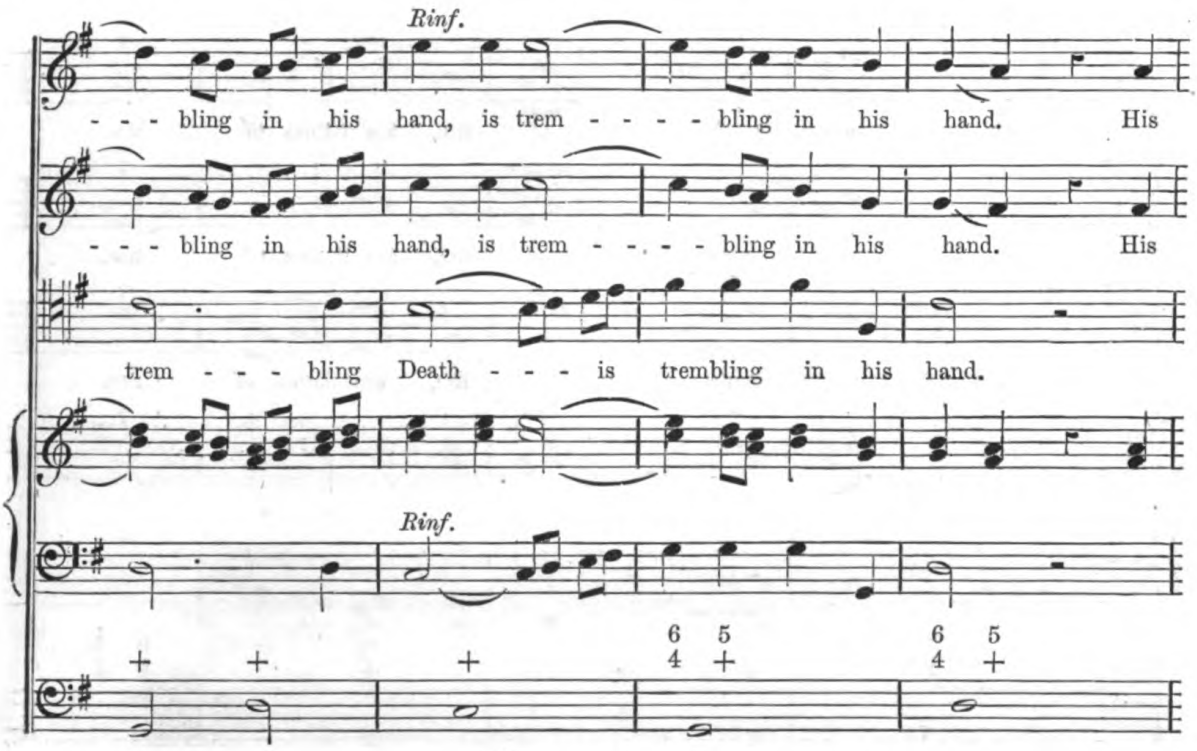

7
+
9 8
4 +
7
+
+
7
+
–
7
+
+
–
+
7
+
4 +
+
7 8
–
–
4 +
+
7
+

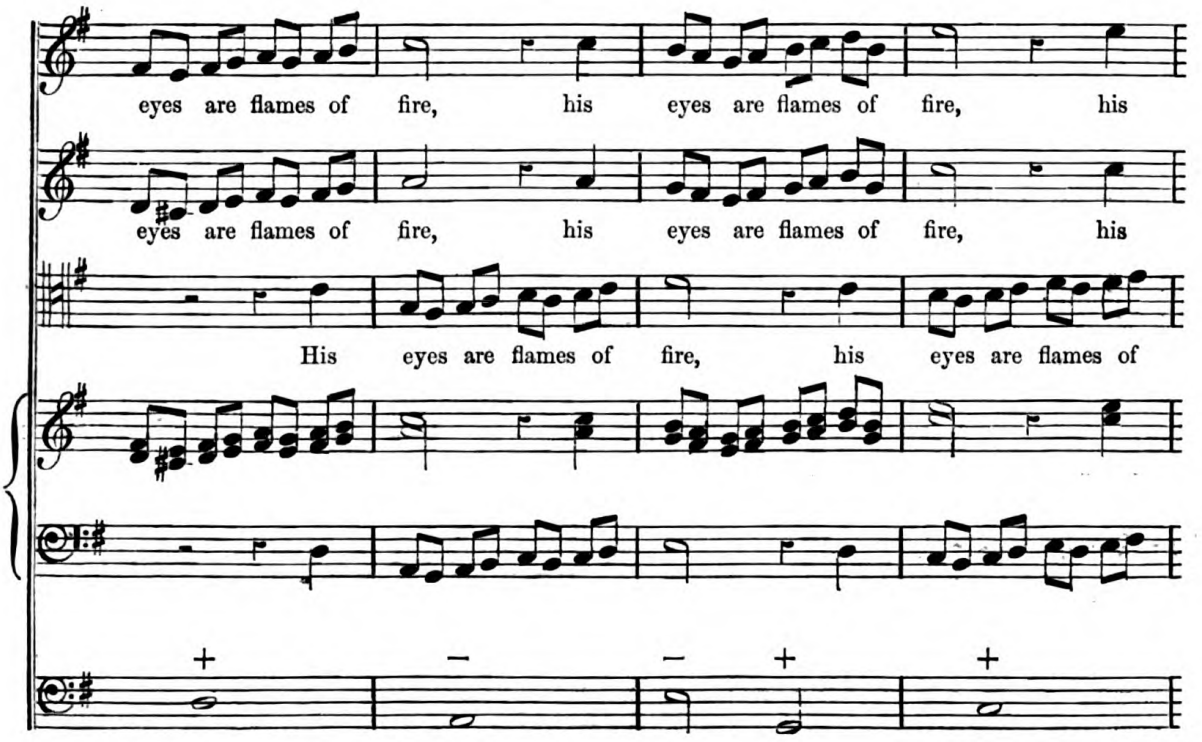


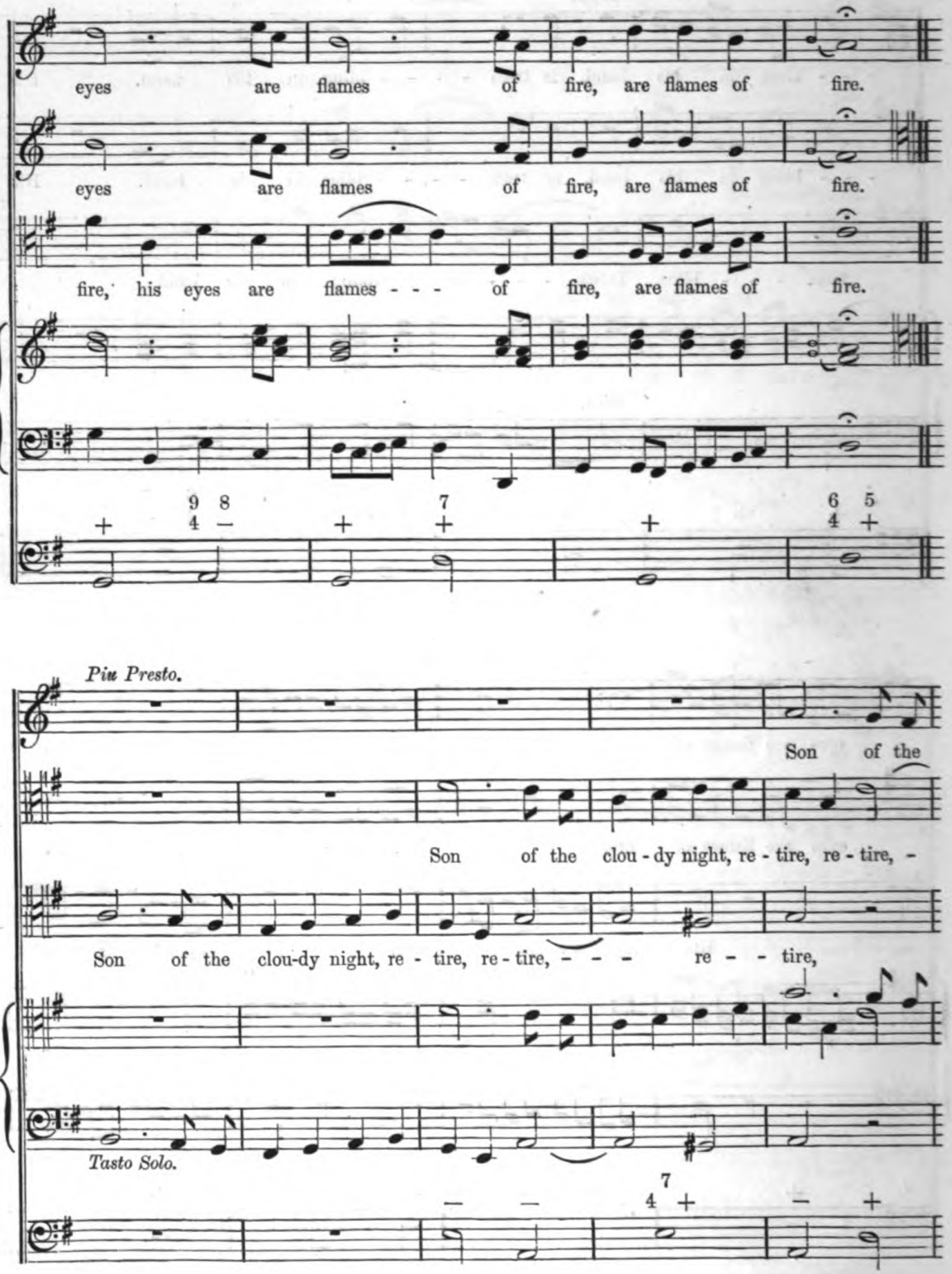
39. GLEE, "*Who comes so dark.*"

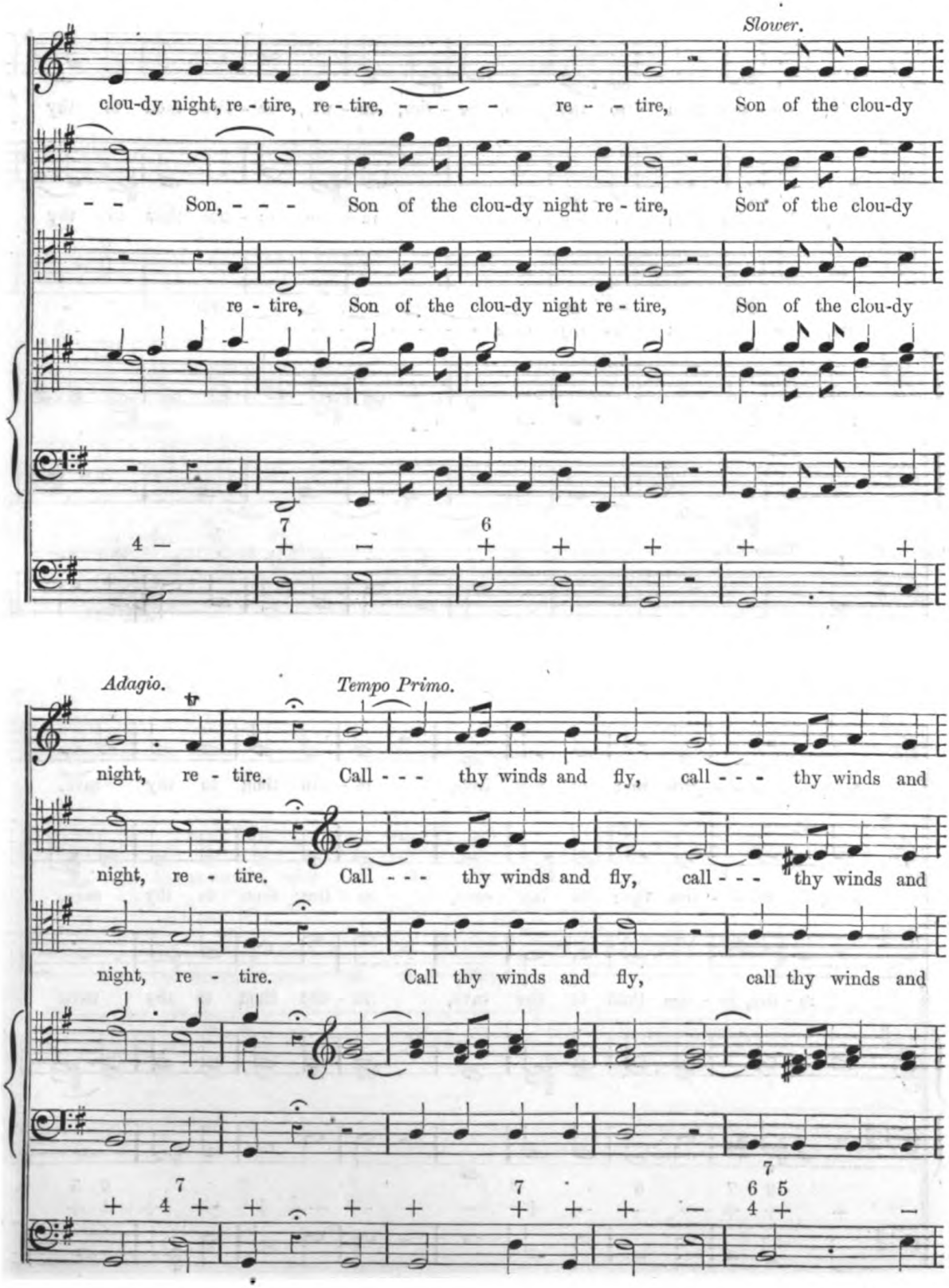


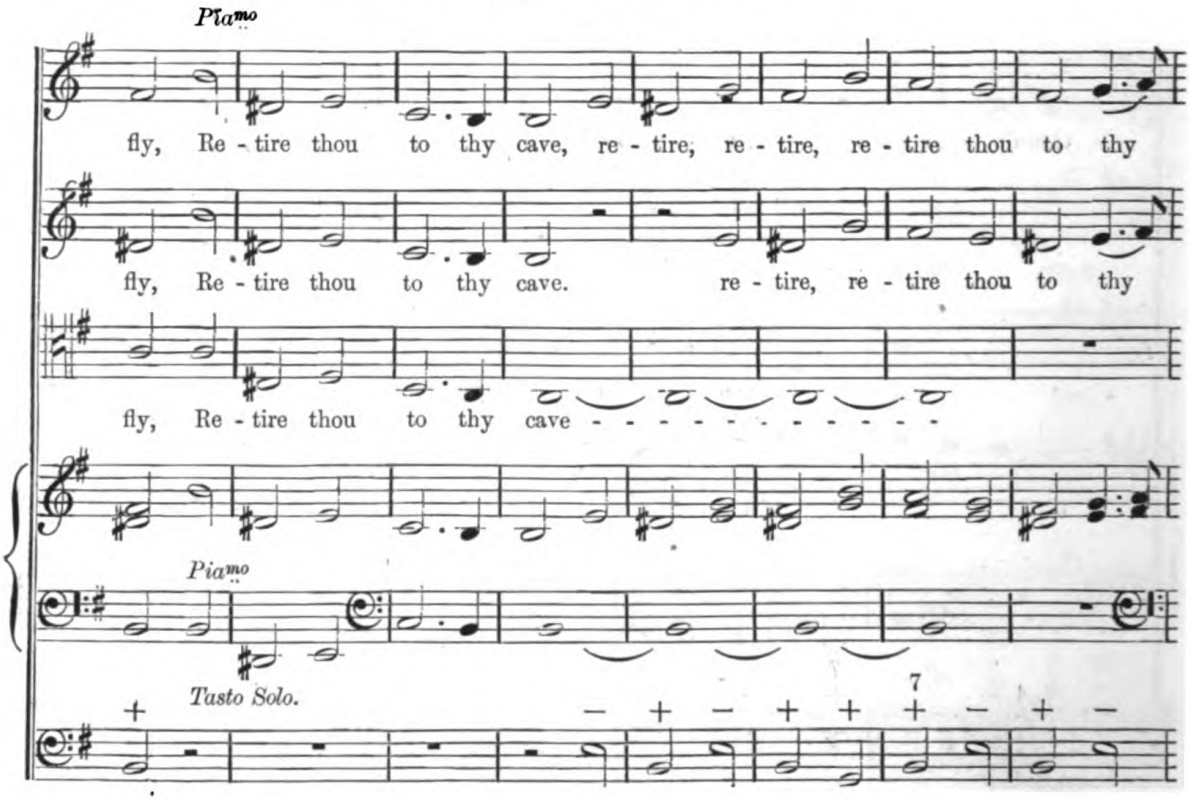

from o - cean's roar, - - - - - - - like au - tumn's sha - dowy
from o - cean's roar, - - - - - - - like au - tumn's sha - dowy
roar, from o - cean's roar, like au - tumn's sha - dowy cloud, like
7
dim.
f
cloud, like au-tumn's shadowy, sha - - - dowy cloud? Death is trem -
cloud, like au-tumn's shadowy, sha - - - - - - - dowy cloud? Death is trem -
autumn's shadowy cloud, like autumn's shadowy cloud? Death is
dim.
f
4

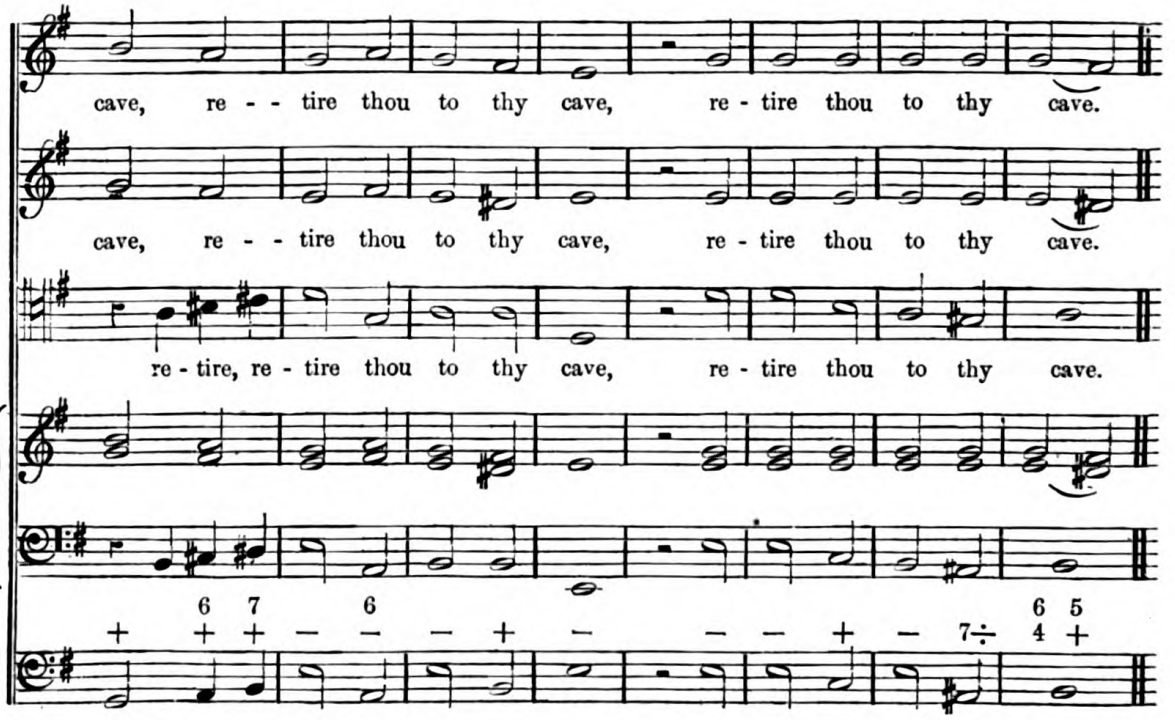

Rinf.
- - - bling in his hand, is trem - - - - bling in his hand. His
- - - bling in his hand, is trem - - - - bling in his hand. His
trem - - - bling Death - - - is trembling in his hand.
Rinf.
6 5
4 +
6 5
4 +

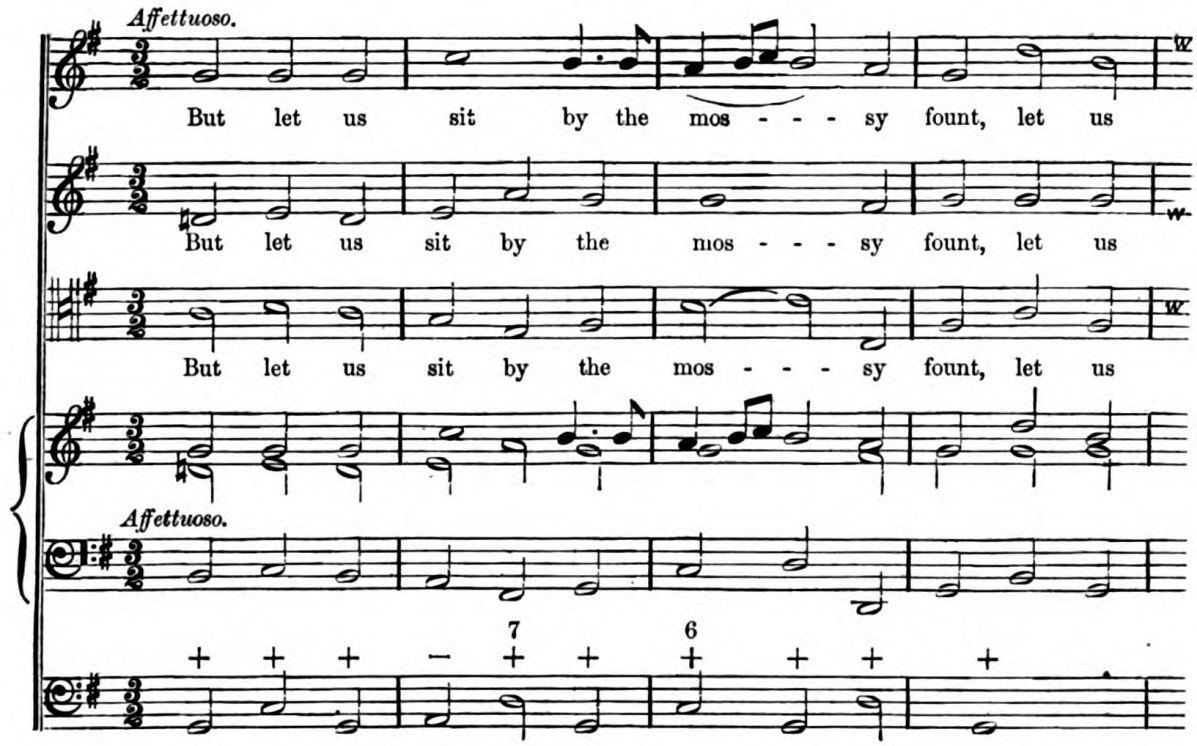

eyes are flames of fire, his eyes are flames of fire, his
eyes are flames of fire, his eyes are flames of fire, his
His eyes are flames of fire, his eyes are flames of

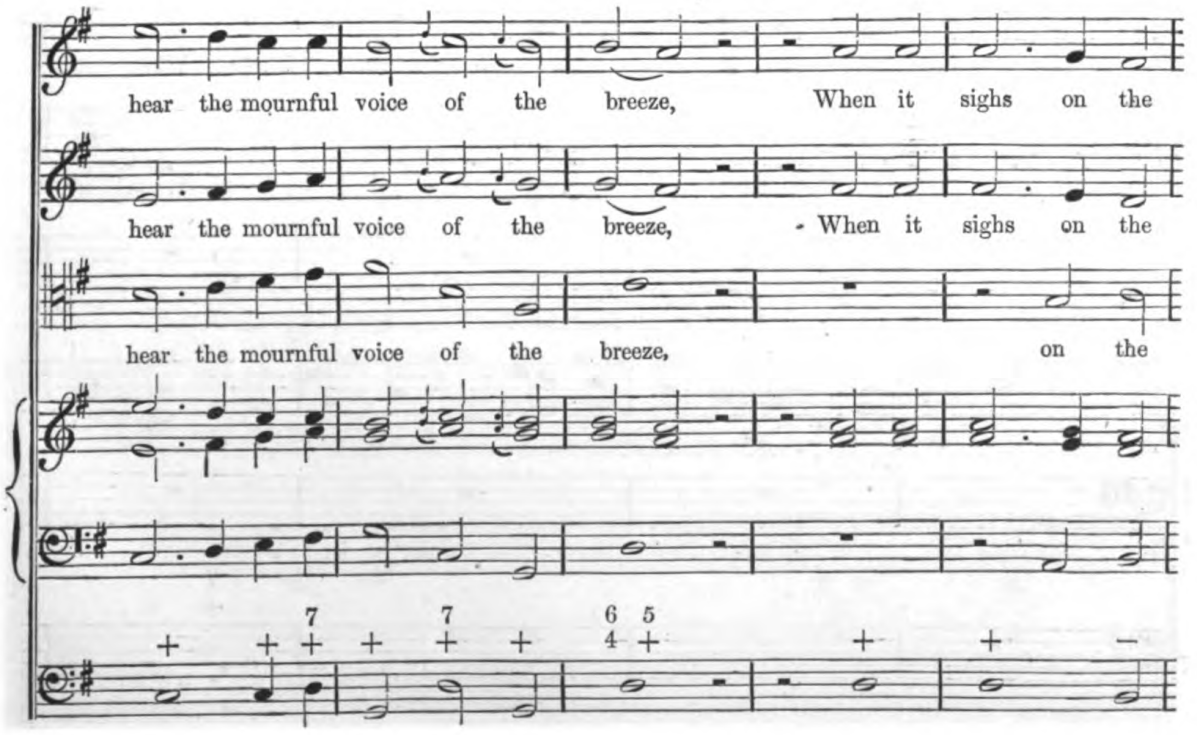

eyes are flames of fire, are flames of fire.
eyes are flames of fire, are flames of fire.
fire, his eyes are flames - - - of fire, are flames of fire.
9 8
4
7
6 5
4
Piu Presto.
Son of the
Son of the clou - dy night, re - tire, re - tire, -
Son of the clou-dy night, re - tire, re - tire, - - - re - - tire,
Tasto Solo.
7
4

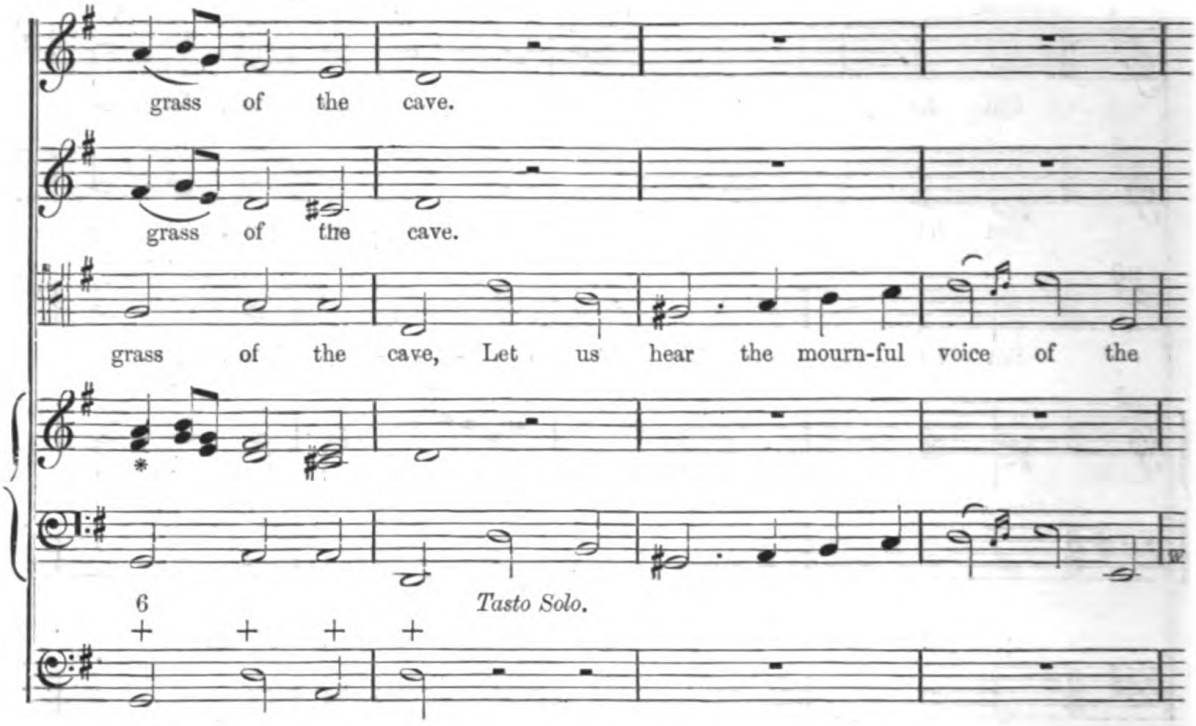

Slower.
clou-dy night, re - tire, re - tire, - - - - - re - - tire, Son of the clou-dy
- - Son, - - - Son of the clou-dy night re - tire, Son of the clou-dy
re - tire, Son of the clou-dy night re - tire, Son of the clou-dy
Adagio.
Tempo Primo.
night, re - tire. Call - - - thy winds and fly, call - - - thy winds and
night, re - tire. Call - - - thy winds and fly, call - - - thy winds and
night, re - tire. Call thy winds and fly, call thy winds and

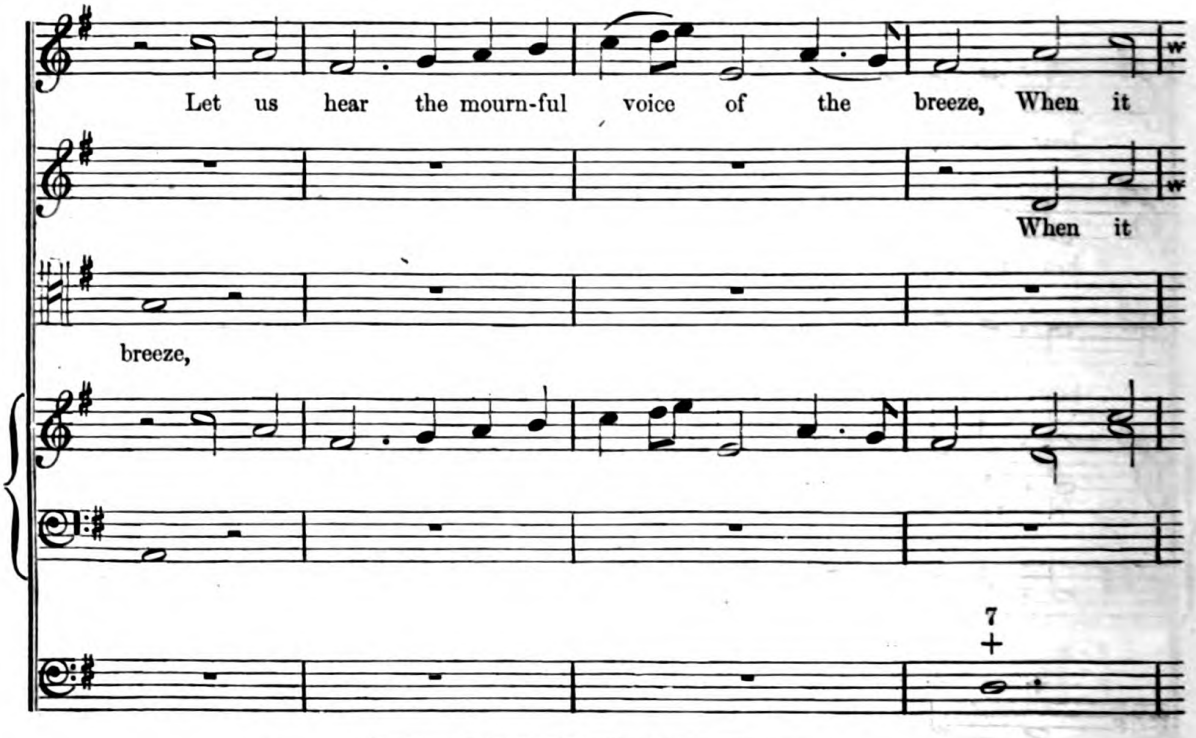

Piano
fly, Re - tire thou to thy cave, re - tire, re - tire, re - tire thou to thy
fly, Re - tire thou to thy cave. re - tire, re - tire thou to thy
fly, Re - tire thou to thy cave - - - - - - - - - - - -
Piano
Tasto Solo.
+ − + − + + 7 + − + −


cave, re - - tire thou to thy cave, re - tire thou to thy cave.
cave, re - - tire thou to thy cave, re - tire thou to thy cave.
re - tire, re - tire thou to thy cave, re - tire thou to thy cave.
6 7 6 6 5
+ + + − − − + − − − + − 7÷ 4 +

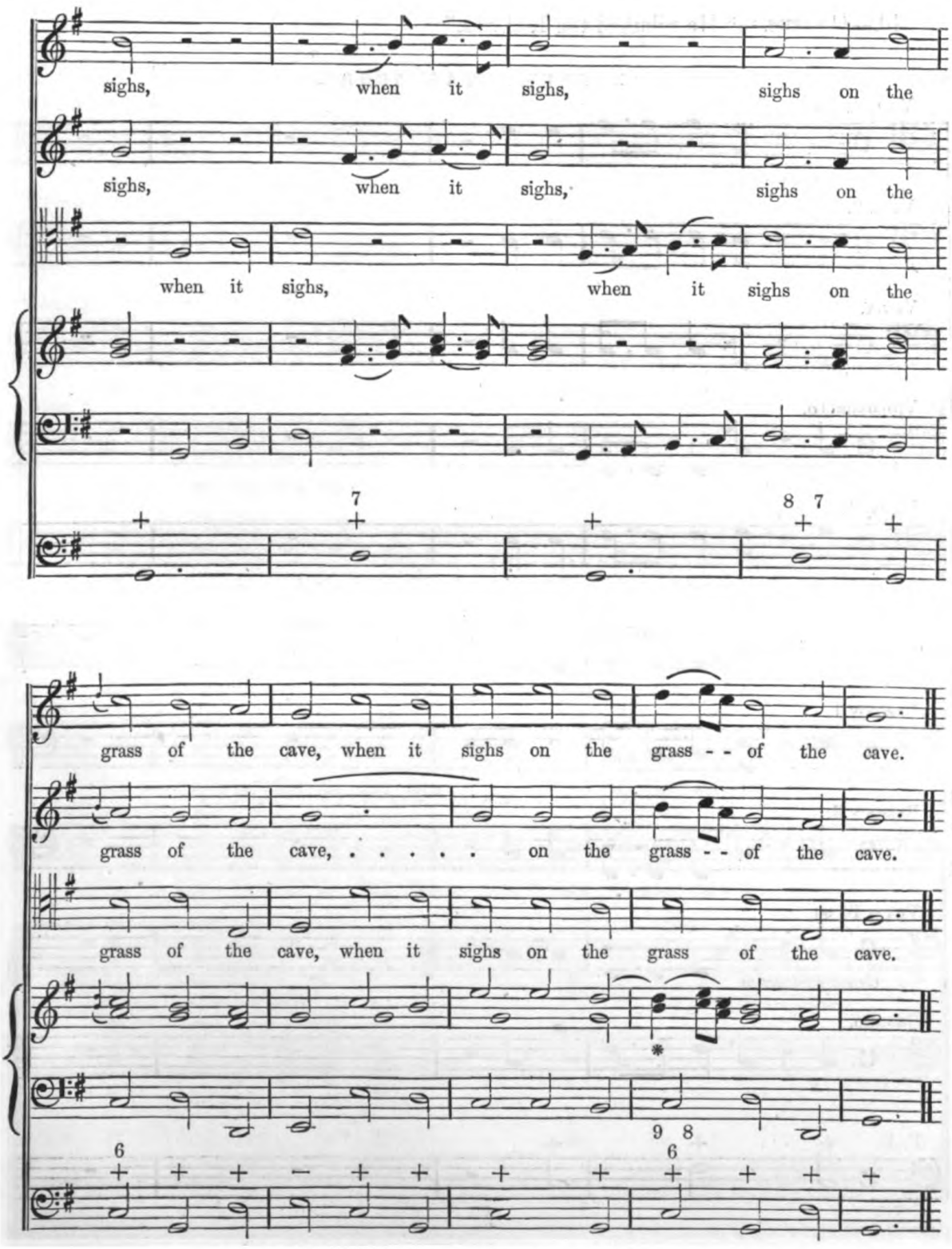

Affettuoso.
But let us sit by the mos - - - sy fount, let us
But let us sit by the mos - - - sy fount, let us
But let us sit by the mos - - - sy fount, let us
Affettuoso.
7
6


hear the mournful voice of the breeze, When it sighs on the
hear the mournful voice of the breeze, When it sighs on the
hear the mournful voice of the breeze, on the
7
7
6 5
4





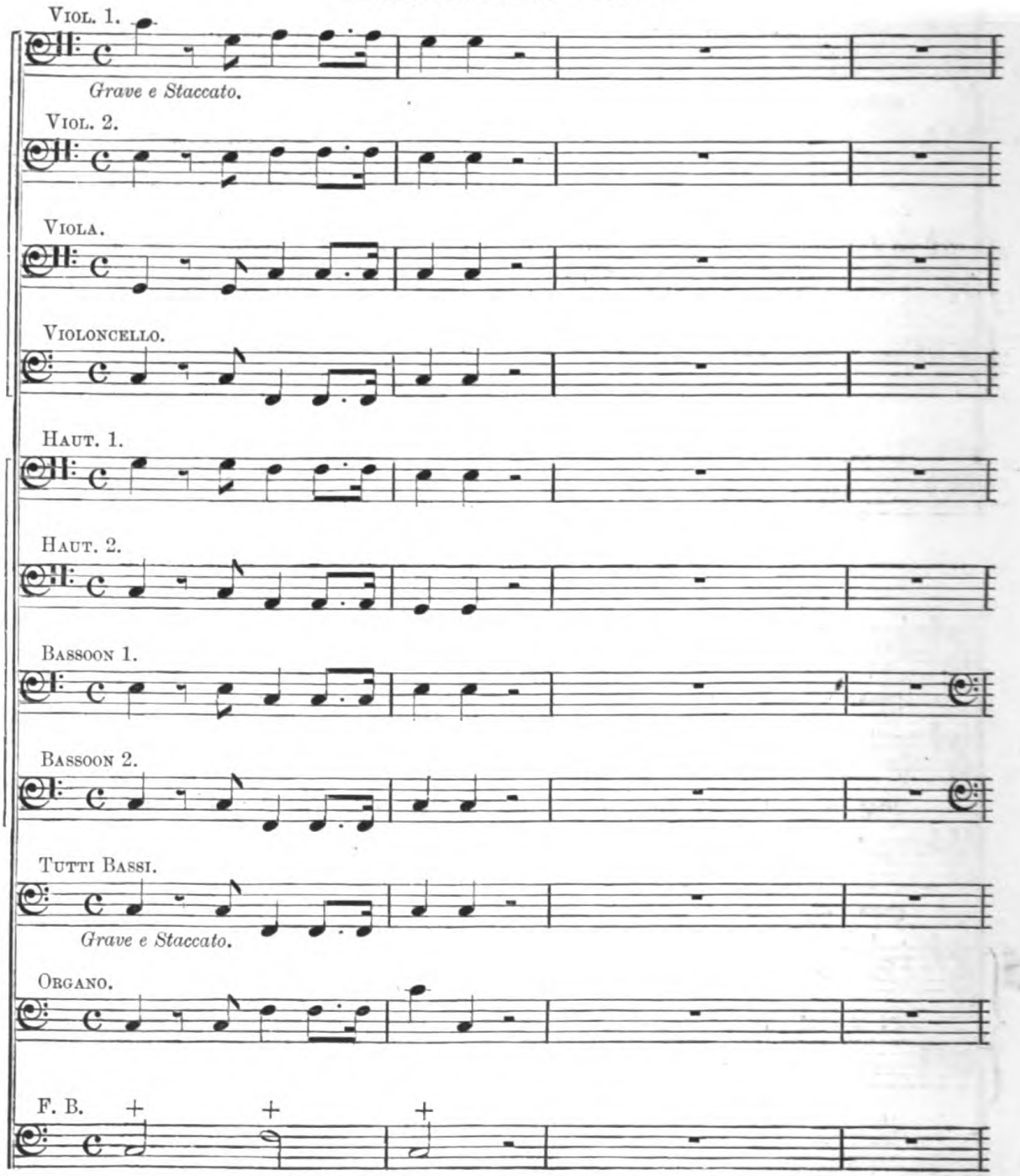
* These two notes, F♯ A, are introduced into the Chord, without preparation, by what is called a licence.



* In this Chord, the note B is introduced without preparation, by a licence.

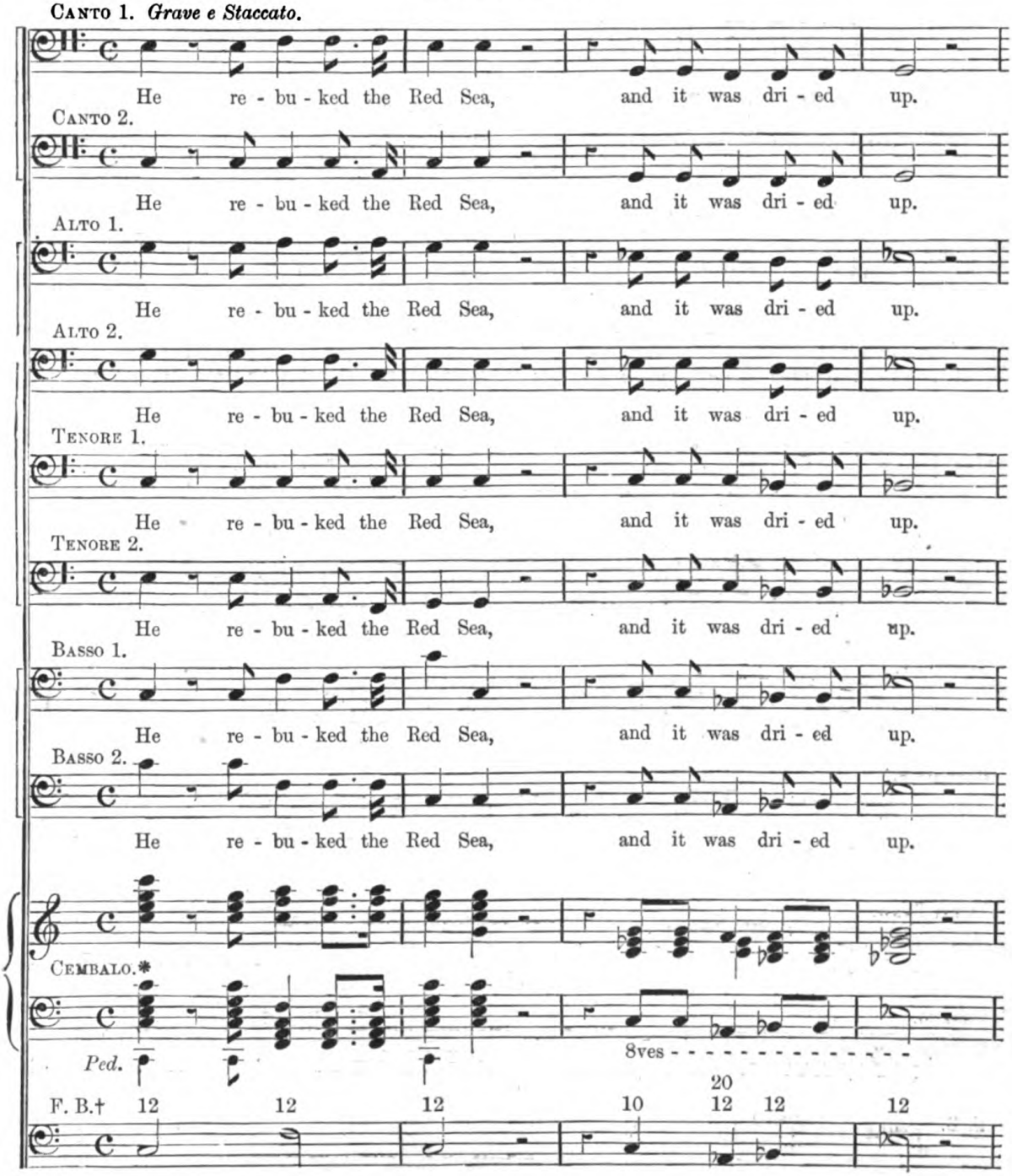
40.—CHORUS.—" He rebuked the Red Sea."

INSTRUMENTAL SCORE.



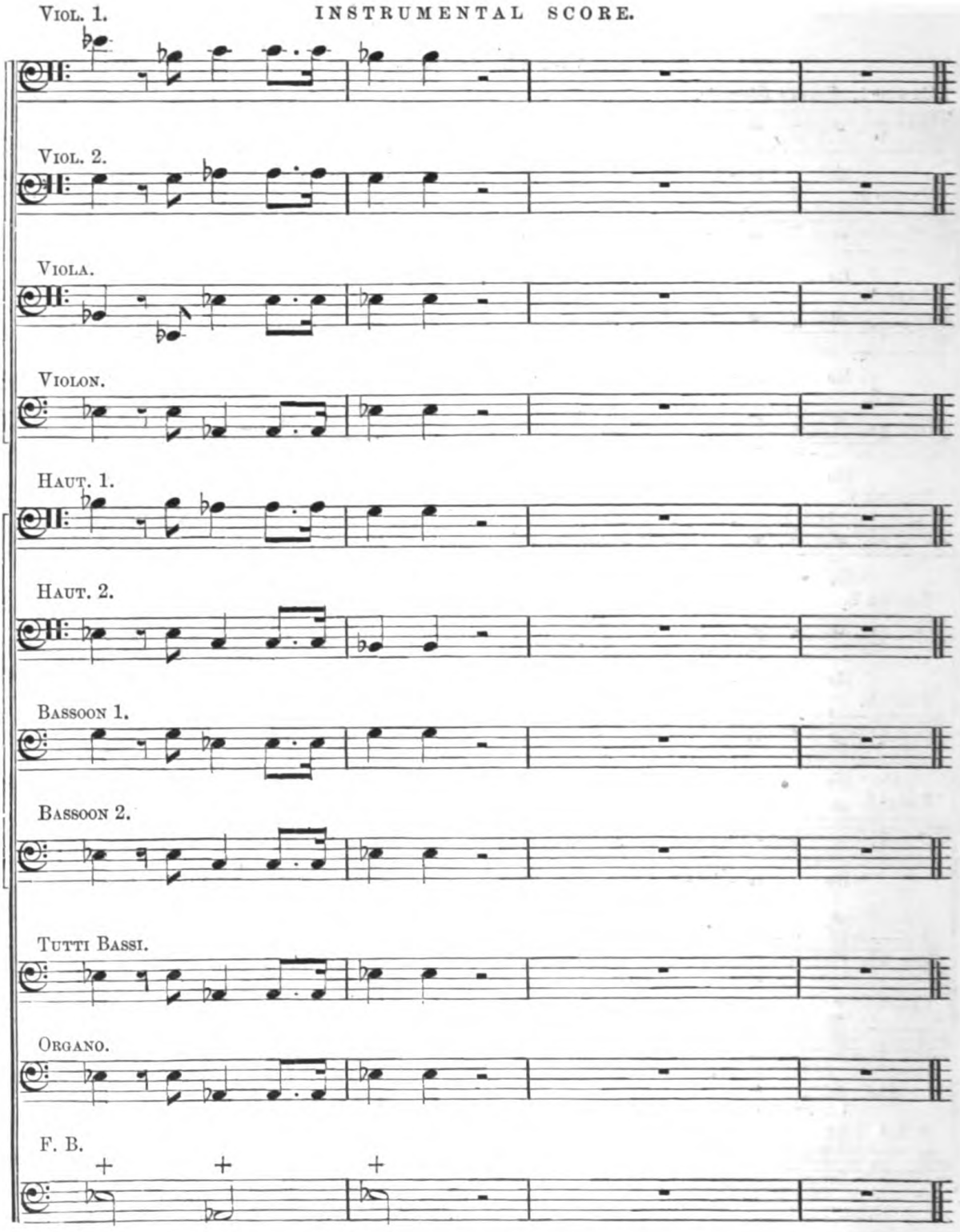
"Israel in Egypt."—HANDEL.

VOCAL SCORE.

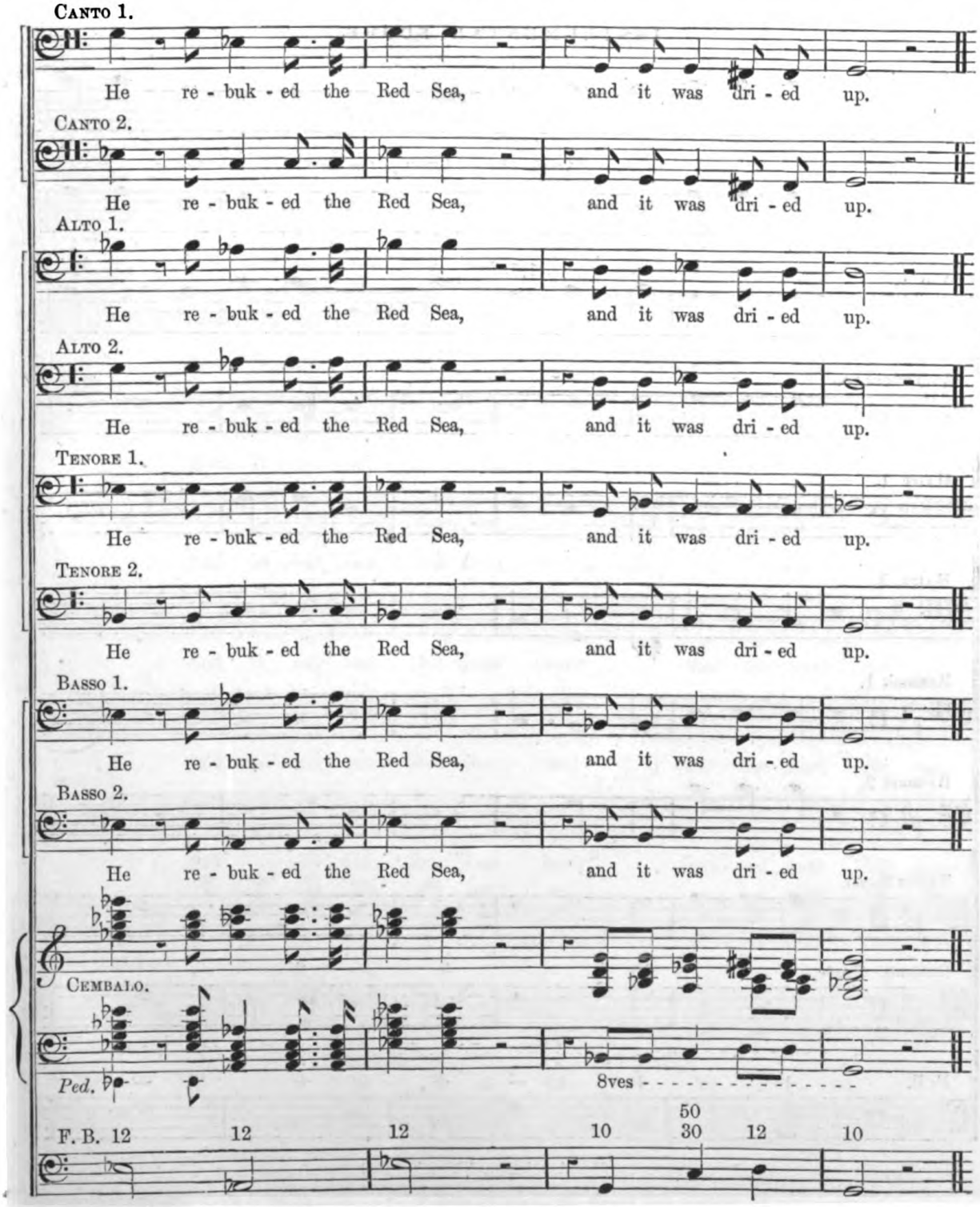


* Novello's arrangement. † See Appendix K, where this notation is explained.

INSTRUMENTAL SCORE.

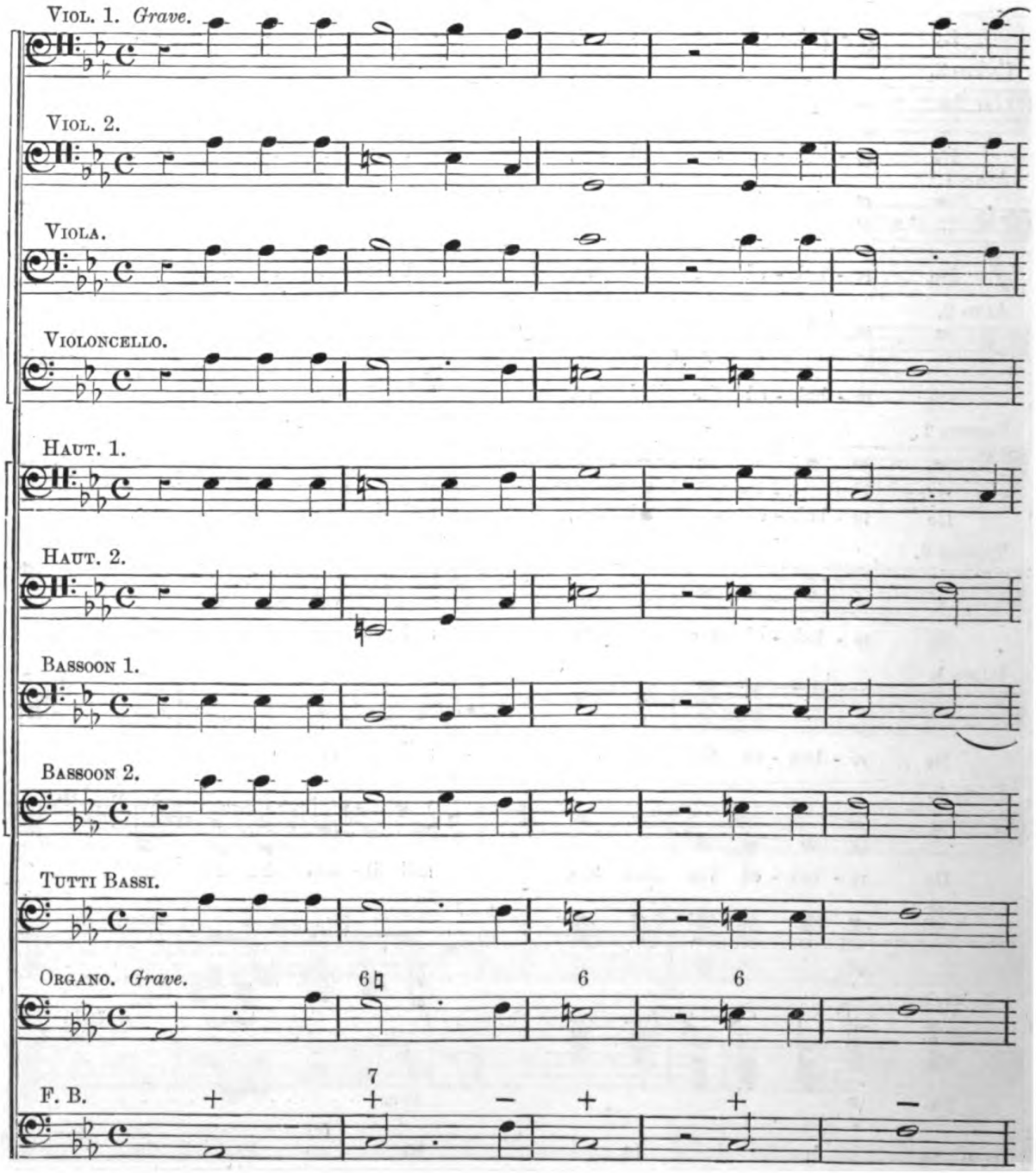


VOCAL SCORE.


CANTO 1.
He re - buk - ed the Red Sea, and it was dri - ed up.
CANTO 2.
He re - buk - ed the Red Sea, and it was dri - ed up.
ALTO 1.
He re - buk - ed the Red Sea, and it was dri - ed up.
ALTO 2.
He re - buk - ed the Red Sea, and it was dri - ed up.
TENORE 1.
He re - buk - ed the Red Sea, and it was dri - ed up.
TENORE 2.
He re - buk - ed the Red Sea, and it was dri - ed up.
BASSO 1.
He re - buk - ed the Red Sea, and it was dri - ed up.
BASSO 2.
He re - buk - ed the Red Sea, and it was dri - ed up.
CEMBALO.
Ped.
8ves
F. B. 12 12 12 10 50 30 12 10

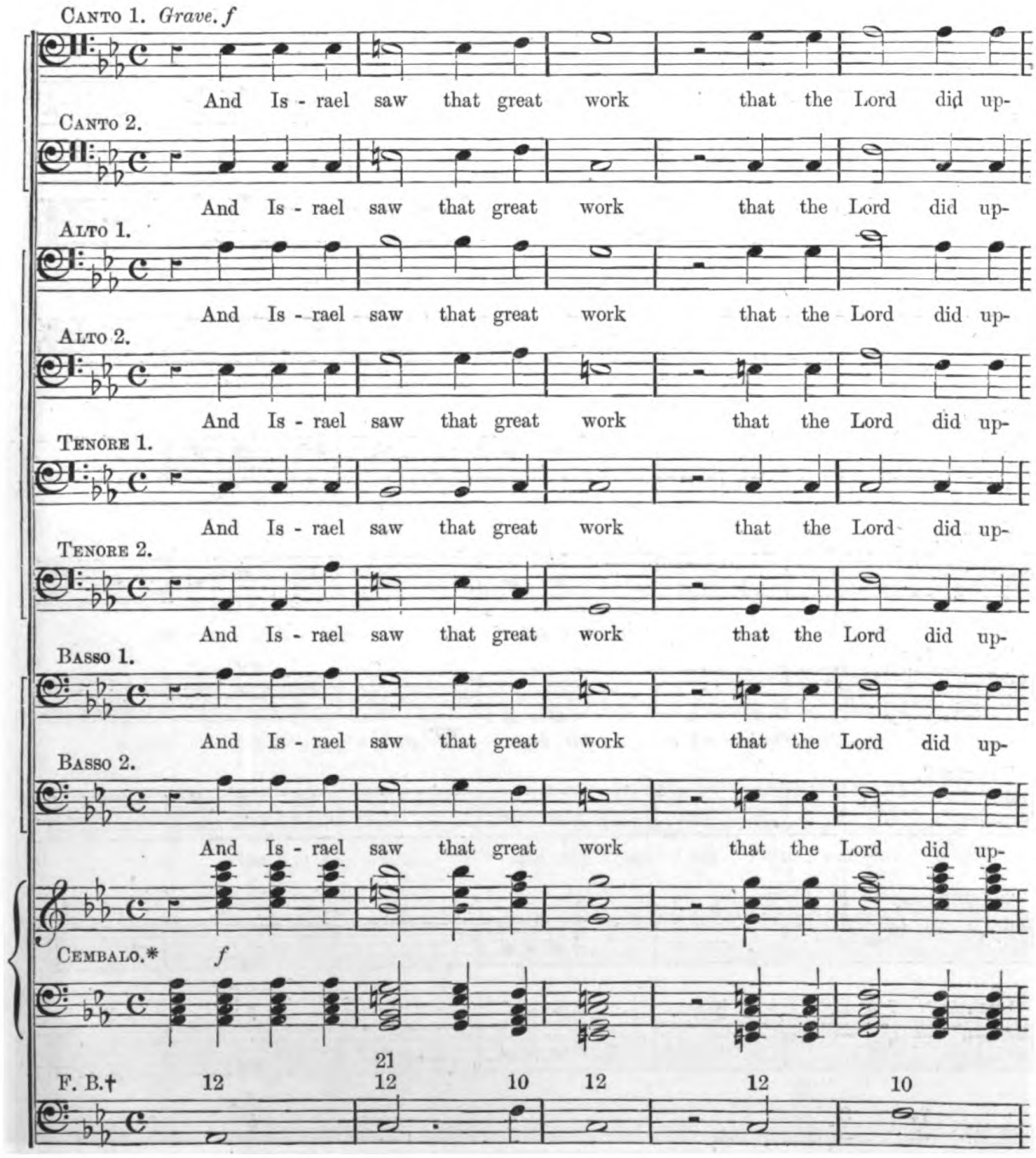
41. CHORUS.—" And Israel saw that great work."

INSTRUMENTAL SCORE.



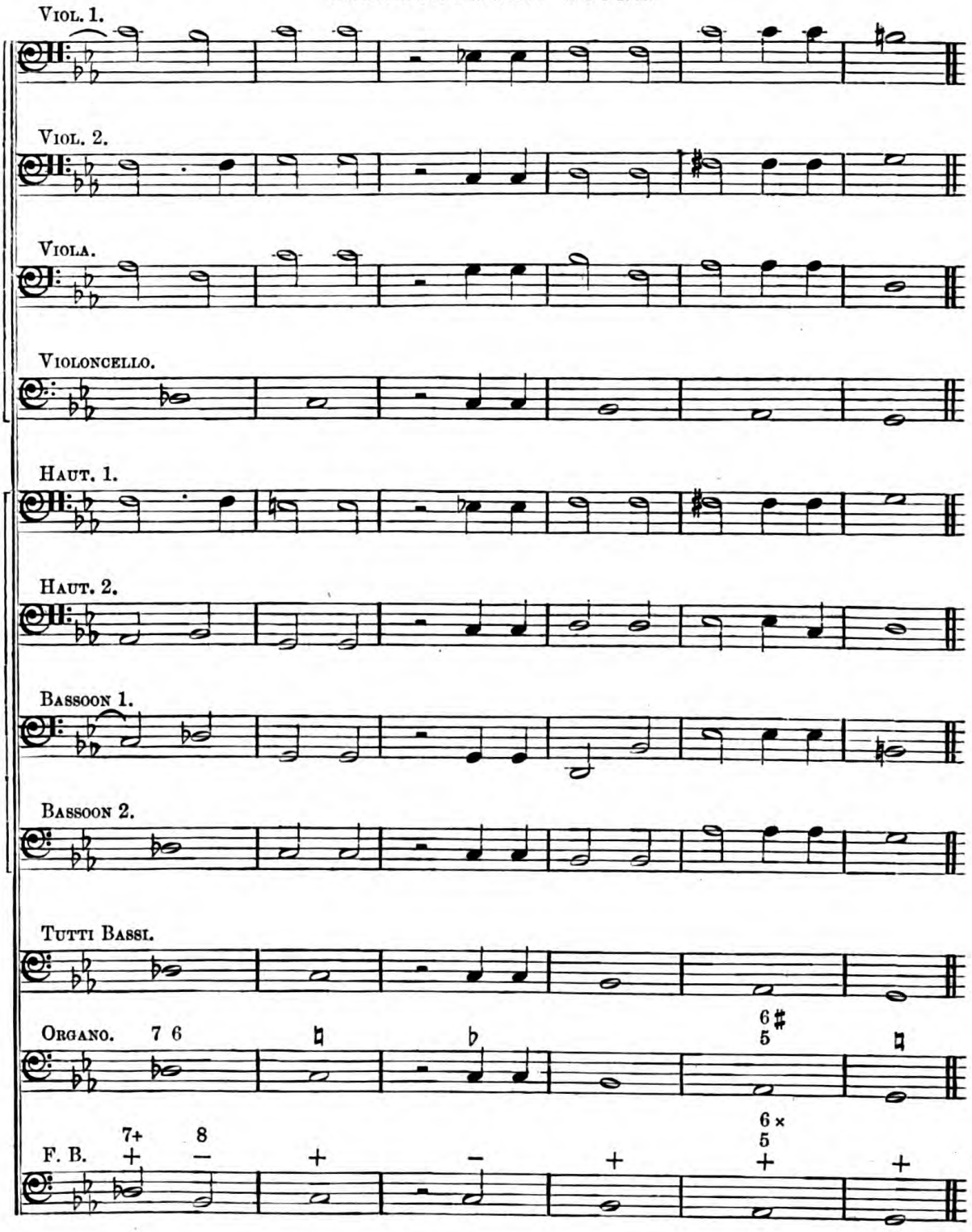
"Israel in Egypt."—HANDEL.

VOCAL SCORE.

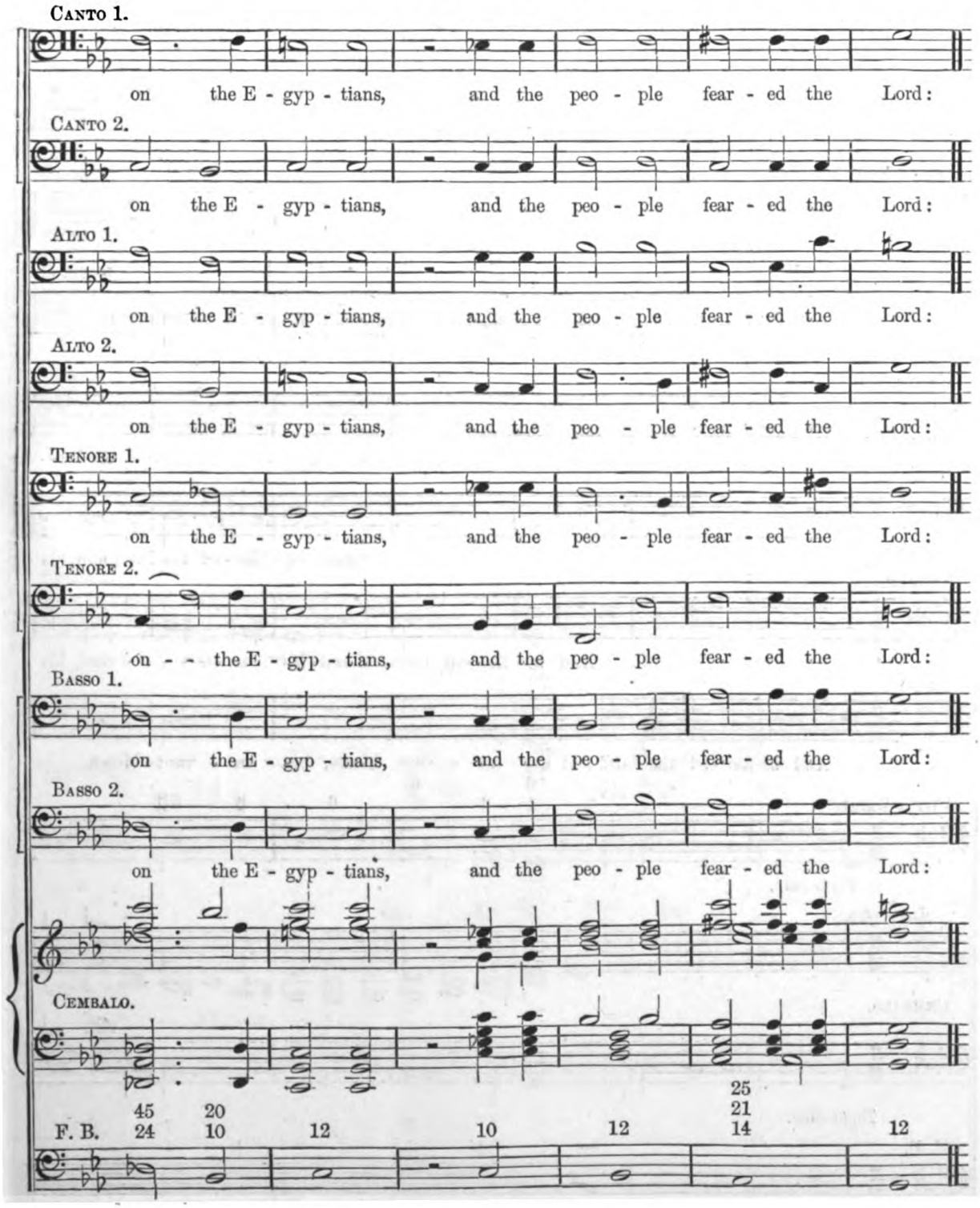


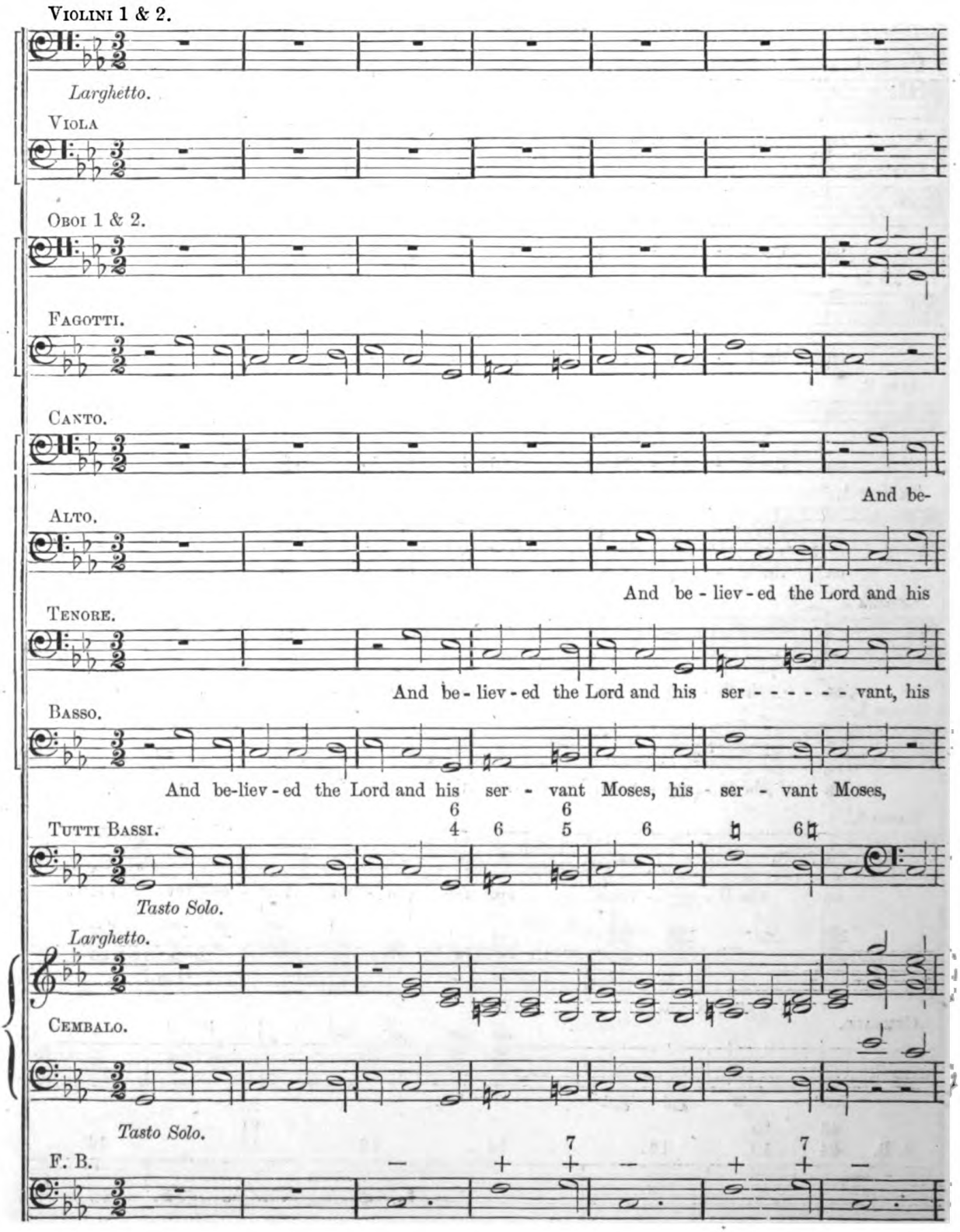
* Novello's arrangement. † See Appendix K, where this notation is explained.

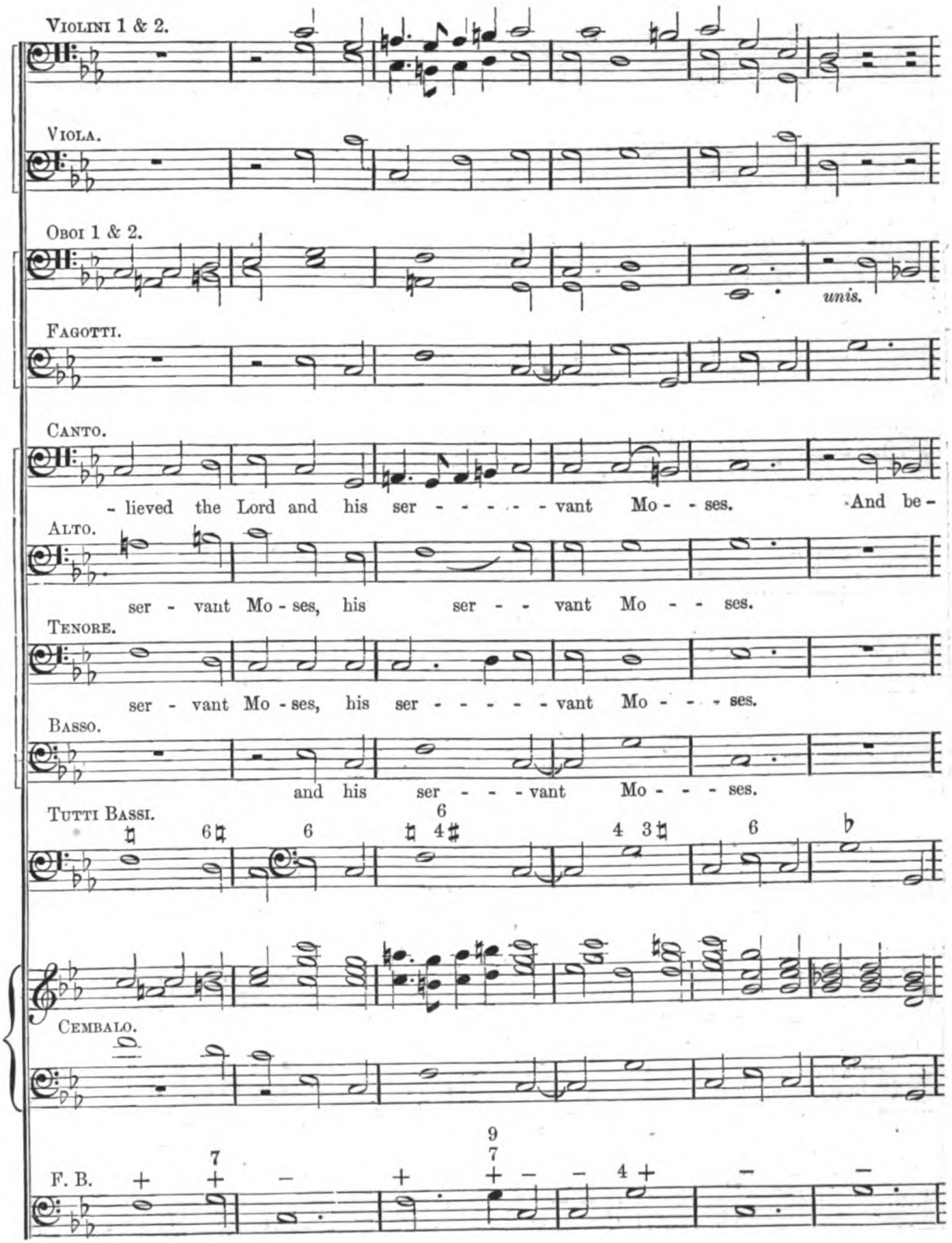
INSTRUMENTAL SCORE.

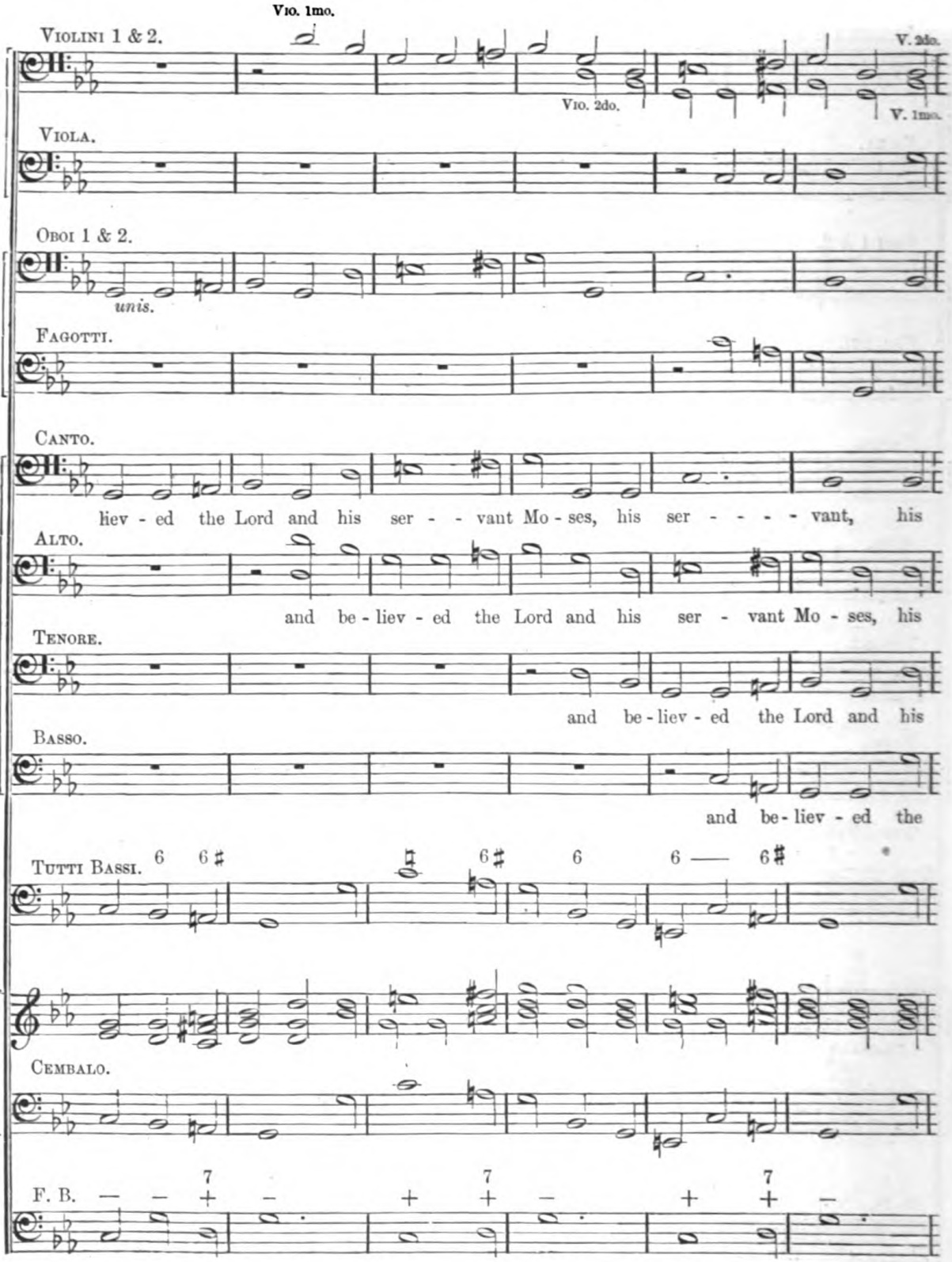


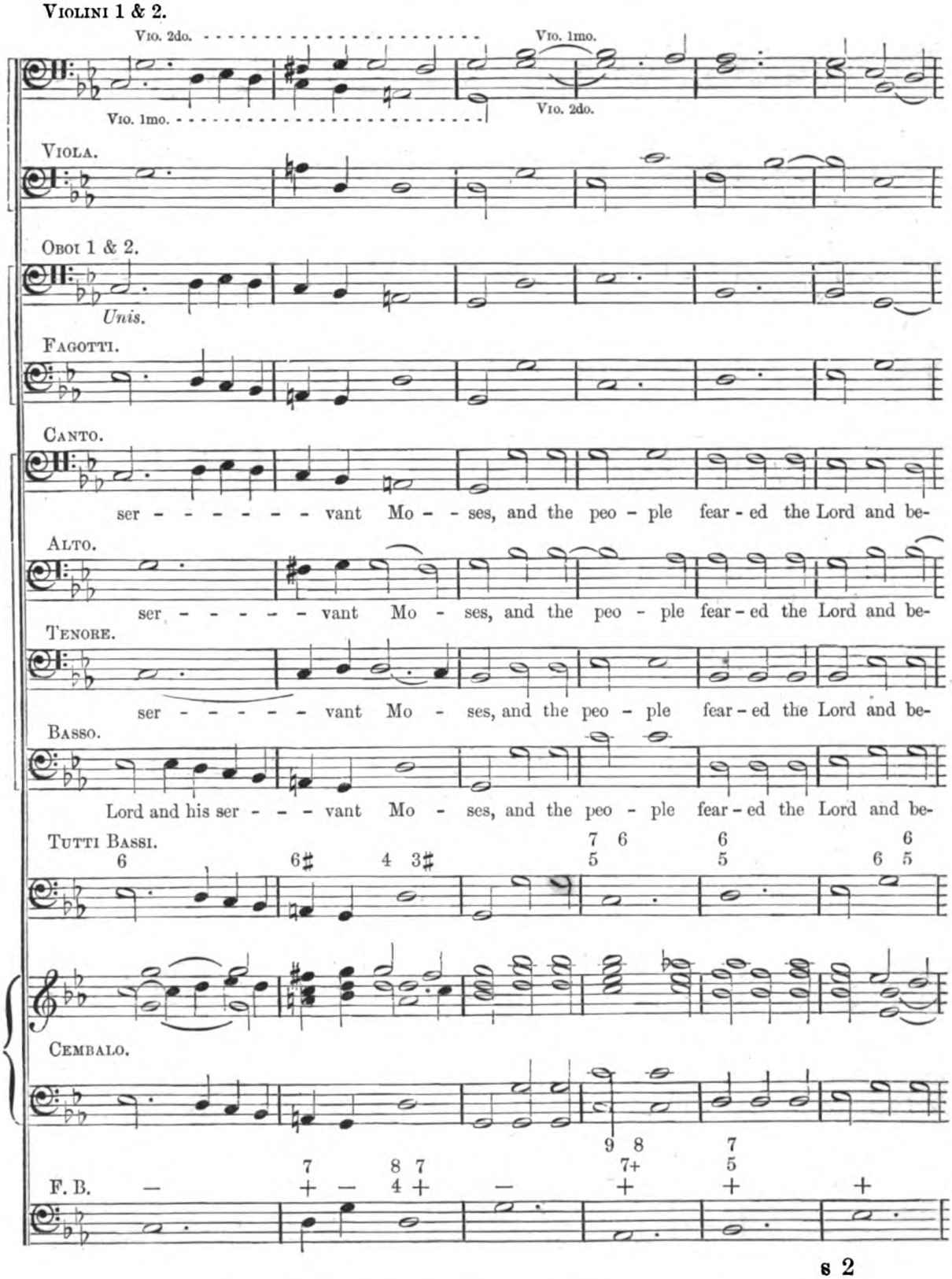
VOCAL SCORE.

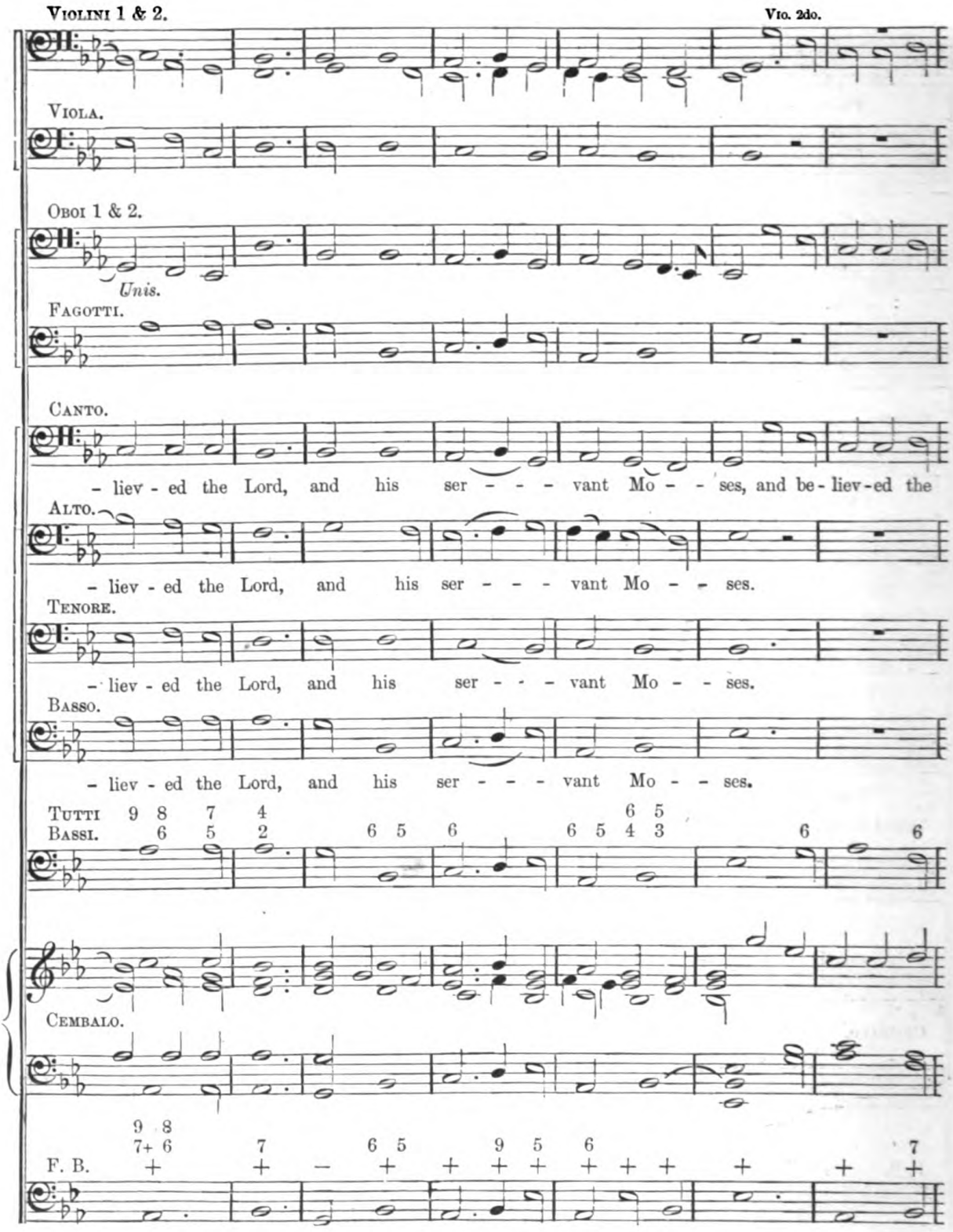

CANTO 1.
on the E - gyp - tians, and the peo - ple fear - ed the Lord:
CANTO 2.
on the E - gyp - tians, and the peo - ple fear - ed the Lord:
ALTO 1.
on the E - gyp - tians, and the peo - ple fear - ed the Lord:
ALTO 2.
on the E - gyp - tians, and the peo - ple fear - ed the Lord:
TENORE 1.
on the E - gyp - tians, and the peo - ple fear - ed the Lord:
TENORE 2.
on - - the E - gyp - tians, and the peo - ple fear - ed the Lord:
BASSO 1.
on the E - gyp - tians, and the peo - ple fear - ed the Lord:
BASSO 2.
on the E - gyp - tians, and the peo - ple fear - ed the Lord:
CEMBALO.
F. B.
45 20 25
24 10 12 10 12 21 14 12

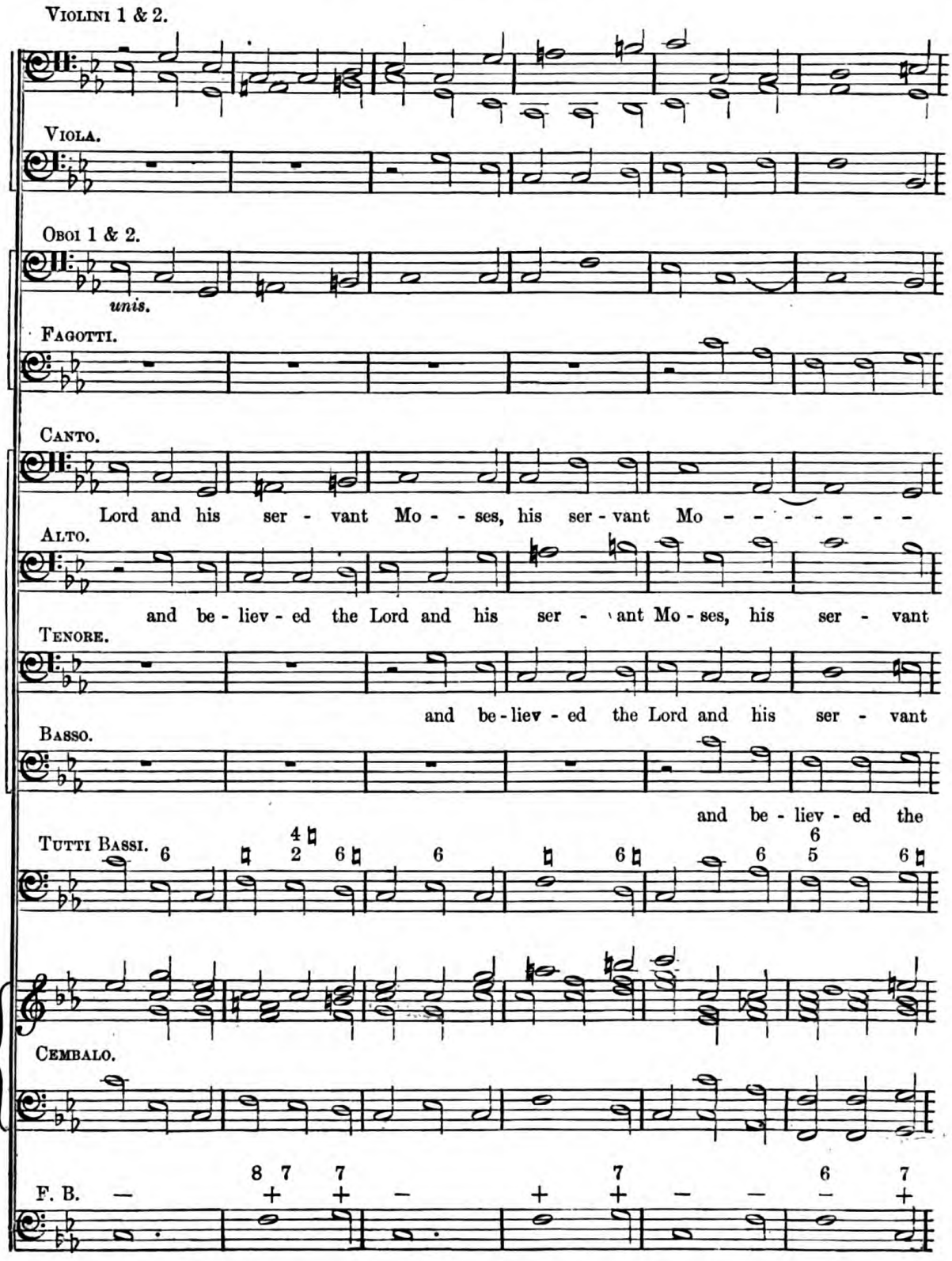

Violini 1 & 2.
Larghetto.
Viola
Oboi 1 & 2.
Fagotti.
Canto.
And be-
Alto.
And be - liev - ed the Lord and his
Tenore.
And be - liev - ed the Lord and his ser - - - - - - vant, his
Basso.
And be-liev - ed the Lord and his ser - vant Moses, his ser - vant Moses,
Tutti Bassi.
Tasto Solo.
Larghetto.
Cembalo.
Tasto Solo.
F. B.

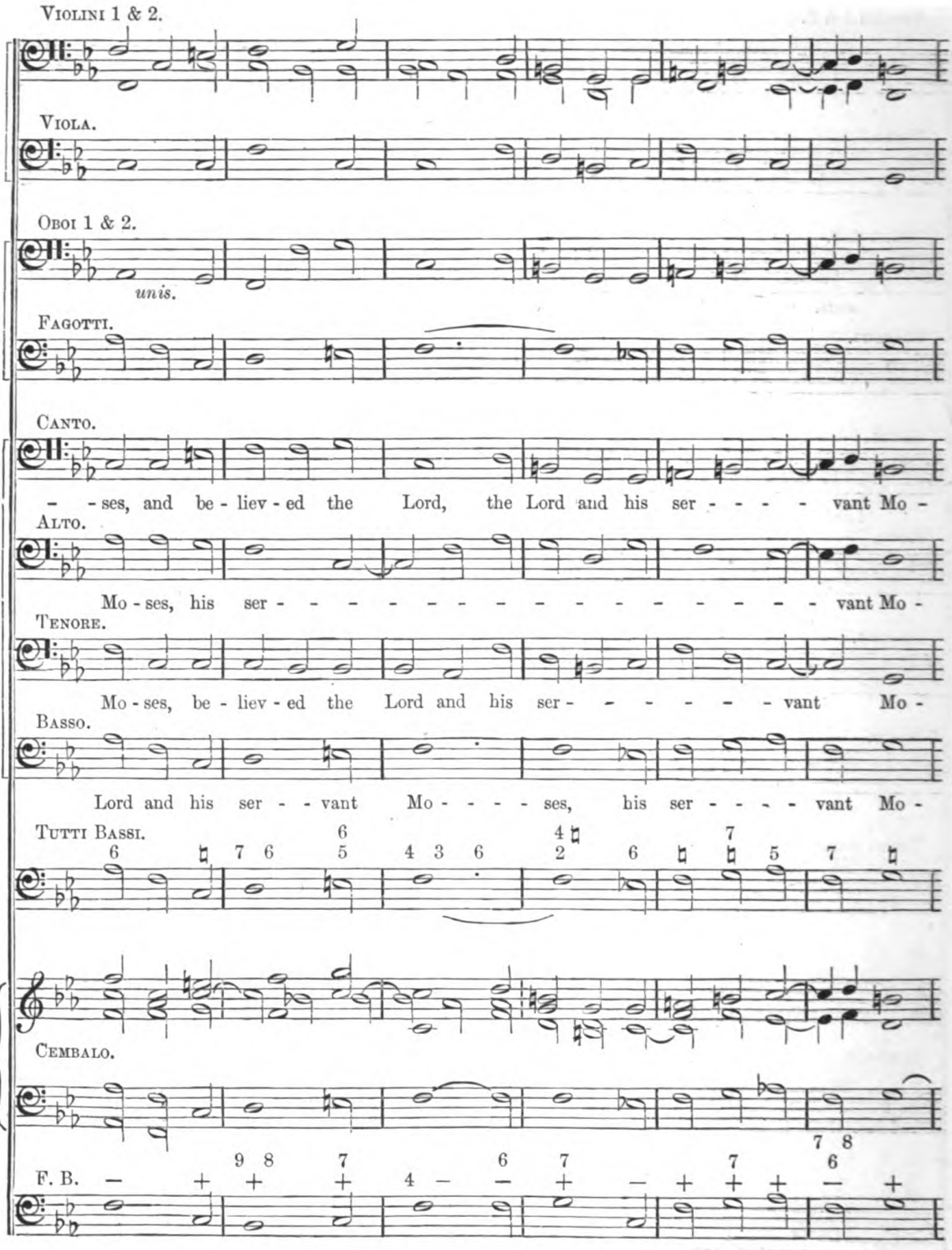

VIOLINI 1 & 2.
VIOLA.
OBOI 1 & 2.
unis.
FAGOTTI.
CANTO.
- lieved the Lord and his ser - - - - - vant Mo - - ses. And be -
ALTO.
ser - vant Mo - ses, his ser - - vant Mo - - ses.
TENORE.
ser - vant Mo - ses, his ser - - - - - vant Mo - - - ses.
BASSO.
and his ser - - - vant Mo - - - ses.
TUTTI BASSI.
CEMBALO.
F. B.

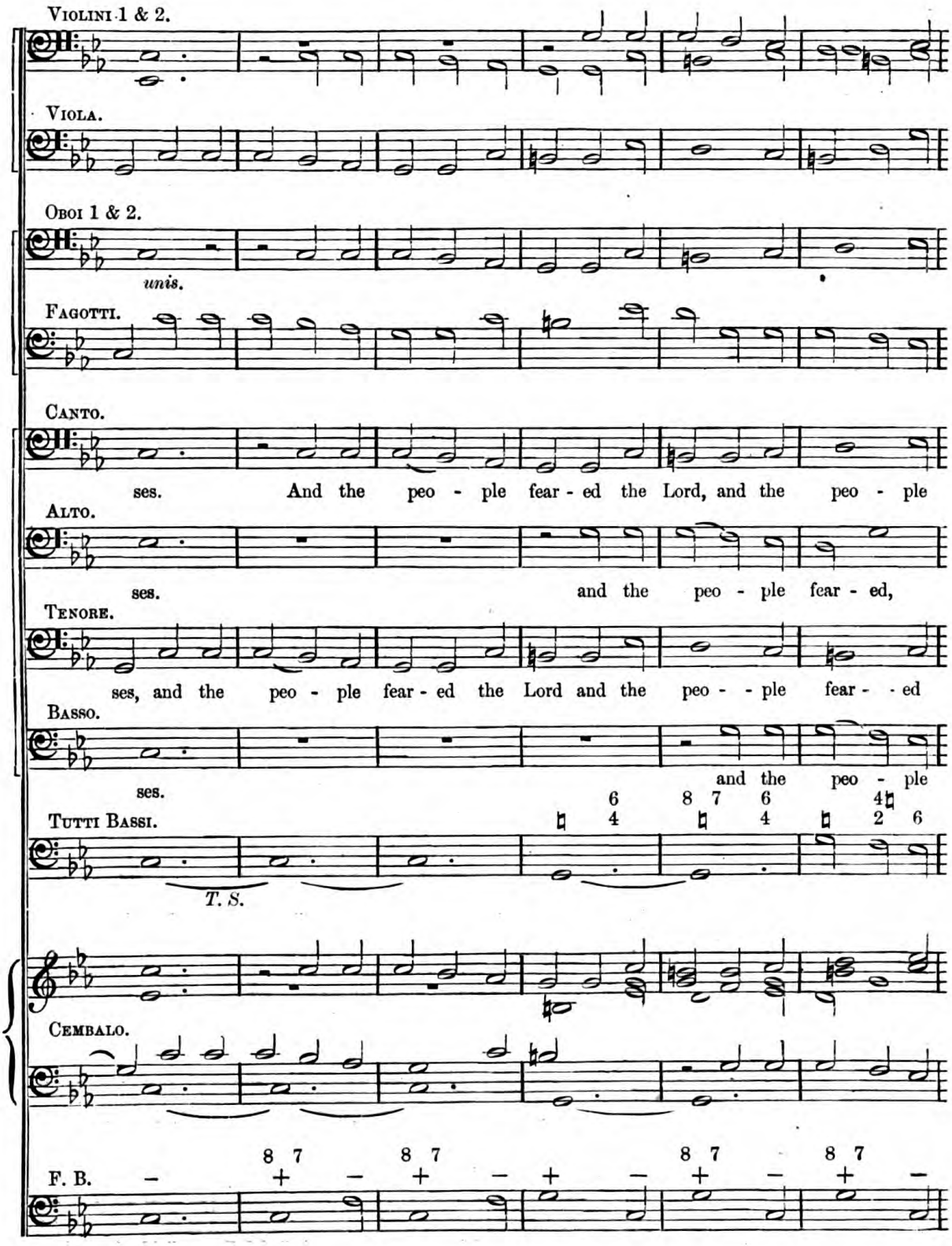

Vio. 1mo.
Violini 1 & 2.
Vio. 2do.
V. 2do.
V. 1mo.
Viola.
Oboi 1 & 2.
unis.
Fagotti.
Canto.
liev - ed the Lord and his ser - - vant Mo - ses, his ser - - - - vant, his
Alto.
and be - liev - ed the Lord and his ser - vant Mo - ses, his
Tenore.
and be - liev - ed the Lord and his
Basso.
and be - liev - ed the
Tutti Bassi.
6 6♯ 6♯ 6 6 — 6♯
Cembalo.
F. B.
7 7 7

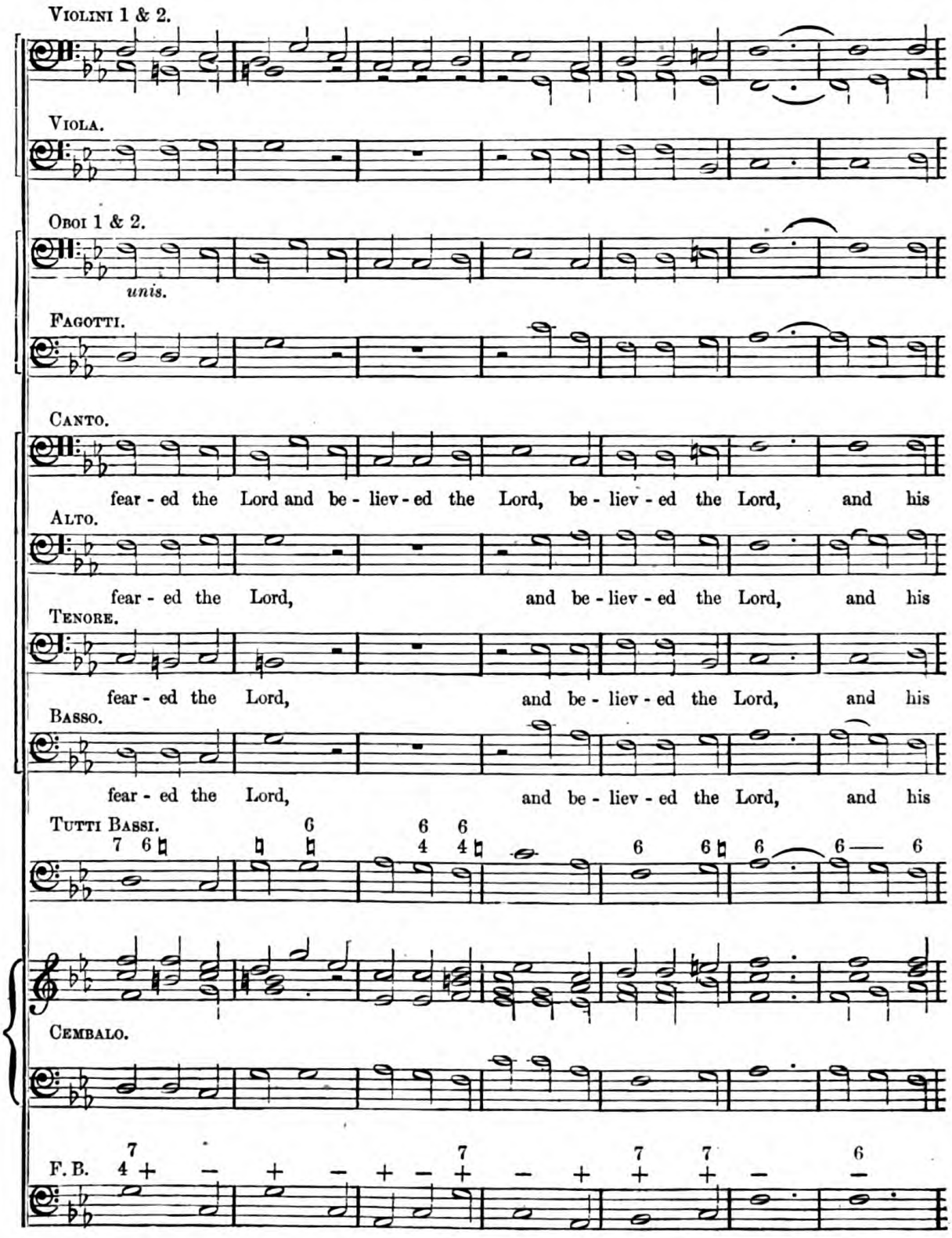

VIOLINI 1 & 2.
VIO. 2do.
VIO. 1mo.
VIO. 1mo.
VIO. 2do.
VIOLA.
OBOI 1 & 2.
Unis.
FAGOTTI.
CANTO.
ser - - - - - - - vant Mo - - ses, and the peo - ple fear - ed the Lord and be-
ALTO.
ser - - - - - - vant Mo - ses, and the peo - ple fear - ed the Lord and be-
TENORE.
ser - - - - - - vant Mo - ses, and the peo - ple fear - ed the Lord and be-
BASSO.
Lord and his ser - - - - vant Mo - ses, and the peo - ple fear - ed the Lord and be-
TUTTI BASSI.
CEMBALO.
F. B.
s 2

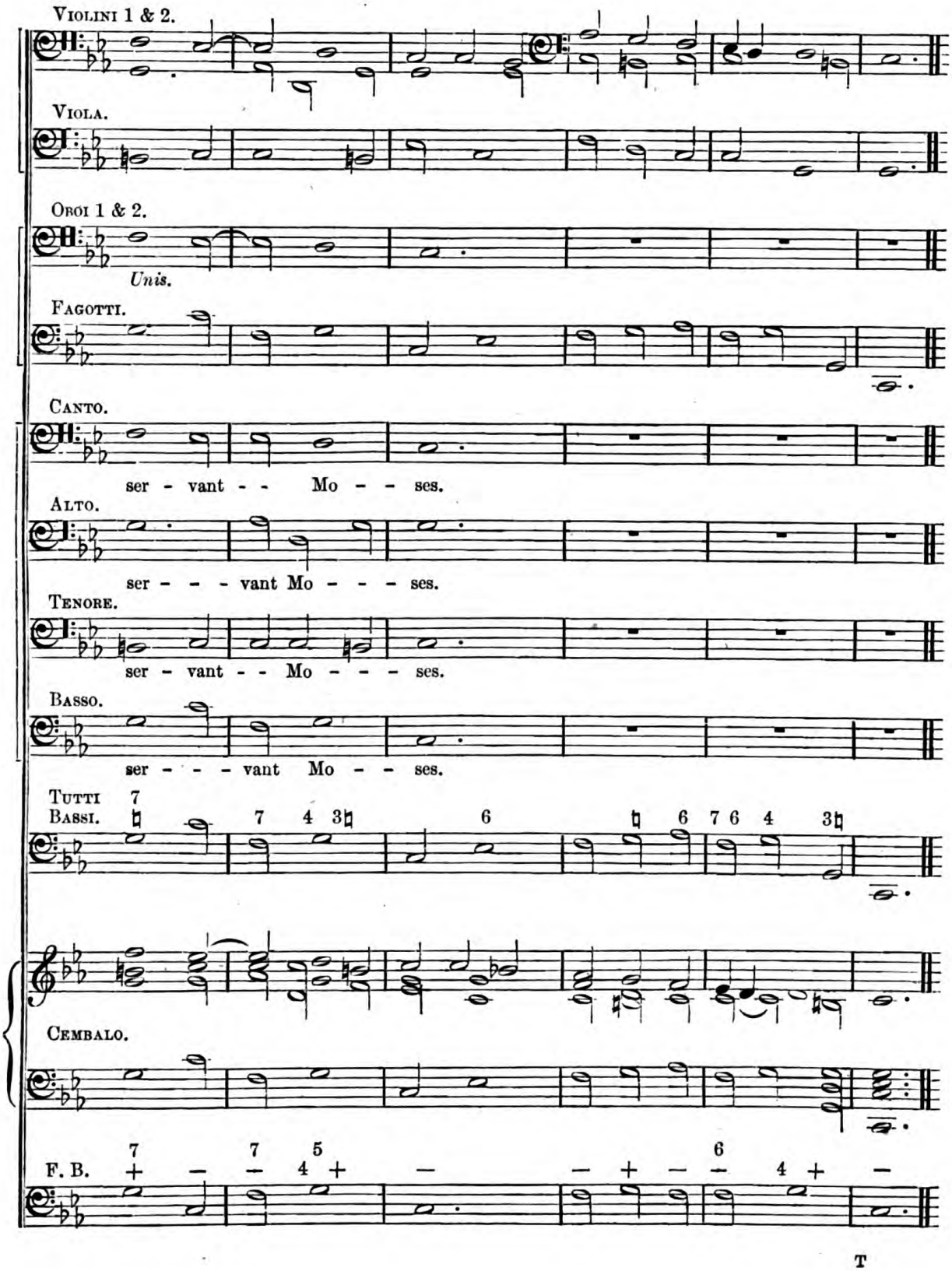

VIOLINI 1 & 2.
VIO. 2do.
VIOLA.
OBOI 1 & 2.
Unis.
FAGOTTI.
CANTO.
- liev - ed the Lord, and his ser - - - vant Mo - - ses, and be - liev - ed the
ALTO.
- liev - ed the Lord, and his ser - - - vant Mo - - ses.
TENORE.
- liev - ed the Lord, and his ser - - - vant Mo - - ses.
BASSO.
- liev - ed the Lord, and his ser - - - vant Mo - - ses.
TUTTI 9 8 7 4 6 5
BASSI. 6 5 2 6 5 6 6 5 4 3 6 6
CEMBALO.
9 8
F. B. 7+ 6 7 6 5 9 5 6 7


VIOLINI 1 & 2.
VIOLA.
OBOI 1 & 2.
unis.
FAGOTTI.
CANTO.
Lord and his ser - vant Mo - - ses, his ser - vant Mo - - - - - - - -
ALTO.
and be - liev - ed the Lord and his ser - vant Mo - ses, his ser - vant
TENORE.
and be - liev - ed the Lord and his ser - vant
BASSO.
and be - liev - ed the
TUTTI BASSI.
CEMBALO.
F. B.


VIOLINI 1 & 2.
VIOLA.
OBOI 1 & 2.
unis.
FAGOTTI.
CANTO.
- - ses, and be - liev - ed the Lord, the Lord and his ser - - - - vant Mo -
ALTO.
Mo - ses, his ser - - - - - - - - - - - - - - - - vant Mo -
TENORE.
Mo - ses, be - liev - ed the Lord and his ser - - - - - - vant Mo -
BASSO.
Lord and his ser - - vant Mo - - - - ses, his ser - - - - vant Mo -
TUTTI BASSI.
6 7 6 6 5 4 3 6 4 2 6 7 5 7
CEMBALO.
F. B. 9 8 7 6 7 7 6

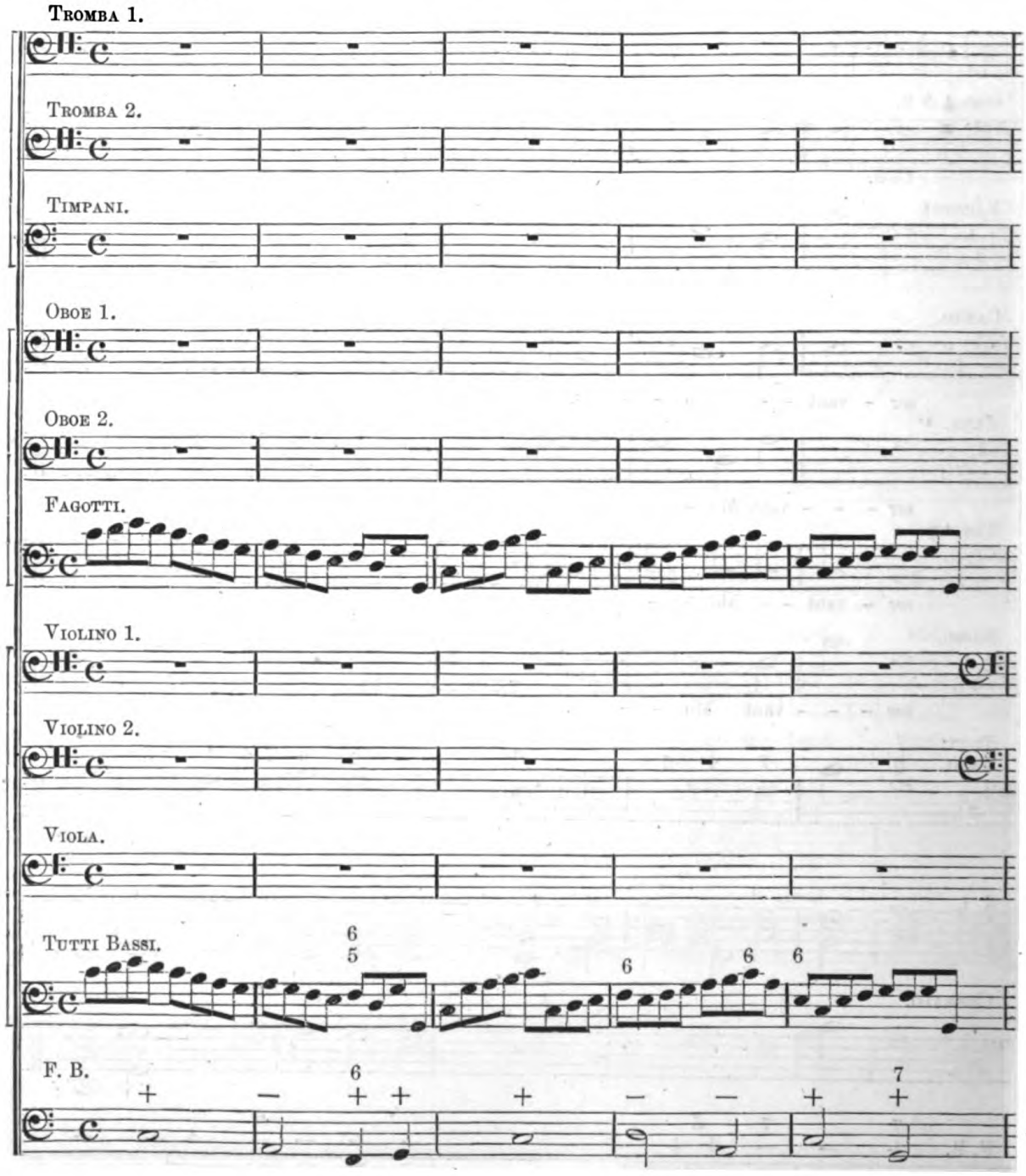

VIOLINI 1 & 2.
VIOLA.
OBOI 1 & 2.
unis.
FAGOTTI.
CANTO.
ses. And the peo - ple fear - ed the Lord, and the peo - ple
ALTO.
ses. and the peo - ple fear - ed,
TENORE.
ses, and the peo - ple fear - ed the Lord and the peo - - ple fear - - ed
BASSO.
ses. and the peo - ple
TUTTI BASSI.
T. S.
CEMBALO.
F. B.


Violini 1 & 2.
Viola.
Oboi 1 & 2.
unis.
Fagotti.
Canto.
fear - ed the Lord and be - liev - ed the Lord, be - liev - ed the Lord, and his
Alto.
fear - ed the Lord, and be - liev - ed the Lord, and his
Tenore.
fear - ed the Lord, and be - liev - ed the Lord, and his
Basso.
fear - ed the Lord, and be - liev - ed the Lord, and his
Tutti Bassi.
Cembalo.
F. B.


VIOLINI 1 & 2.
VIOLA.
OBOI 1 & 2.
Unis.
FAGOTTI.
CANTO.
ser - vant - - Mo - - ses.
ALTO.
ser - - - - vant Mo - - - - ses.
TENORE.
ser - vant - - Mo - - - - ses.
BASSO.
ser - - - - vant Mo - - ses.
TUTTI
BASSI.
7
7 4 3♮
6
♮ 6
7 6 4 3♮
CEMBALO.
F. B.
7 7 5 6
+ - - 4 + - - + - - 4 + -

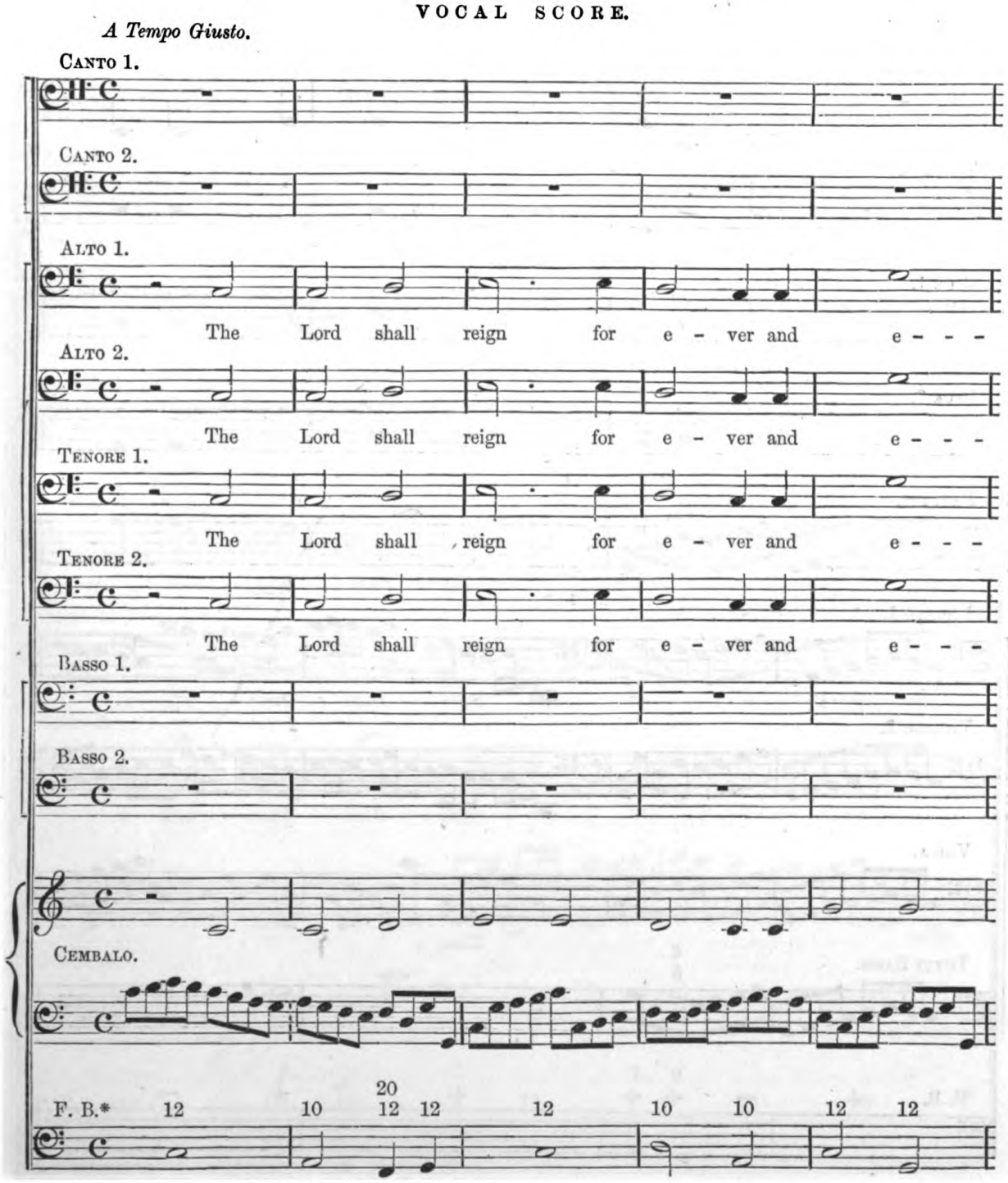
42. CHORUS, " The Lord shall reign."

INSTRUMENTAL SCORE.



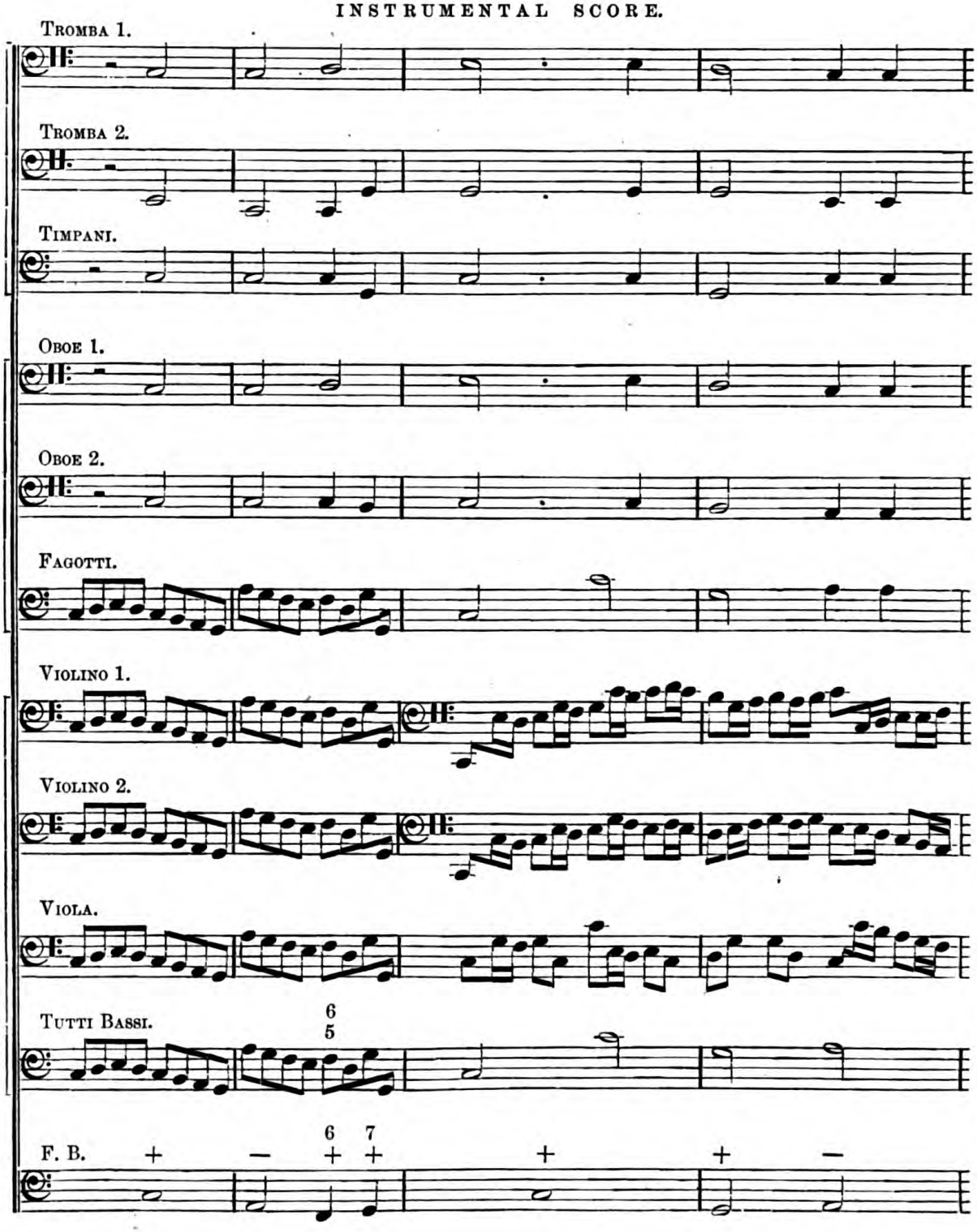
"Israel in Egypt."—HANDEL.

VOCAL SCORE.

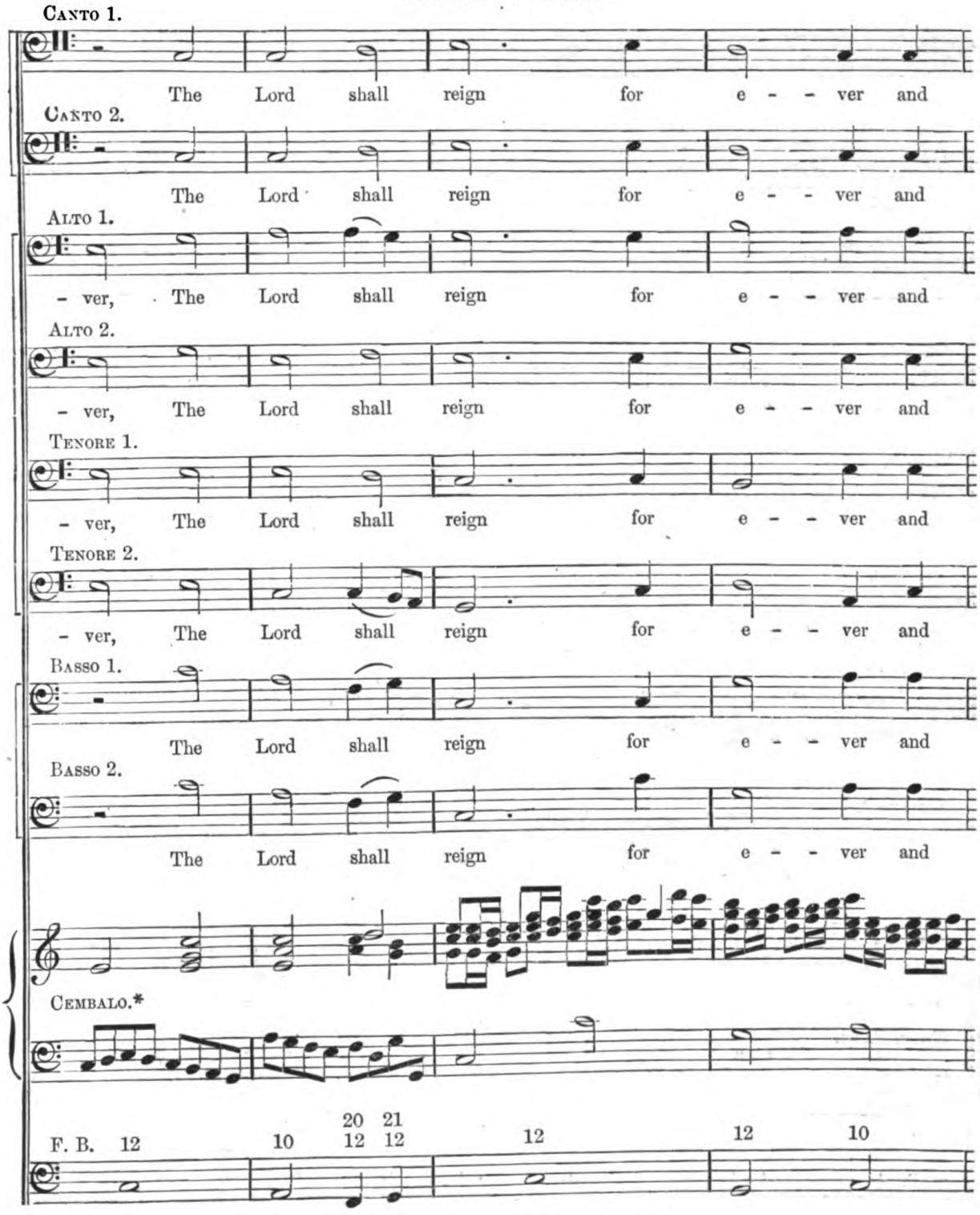


* See Appendix K, where this notation is explained.

INSTRUMENTAL SCORE.

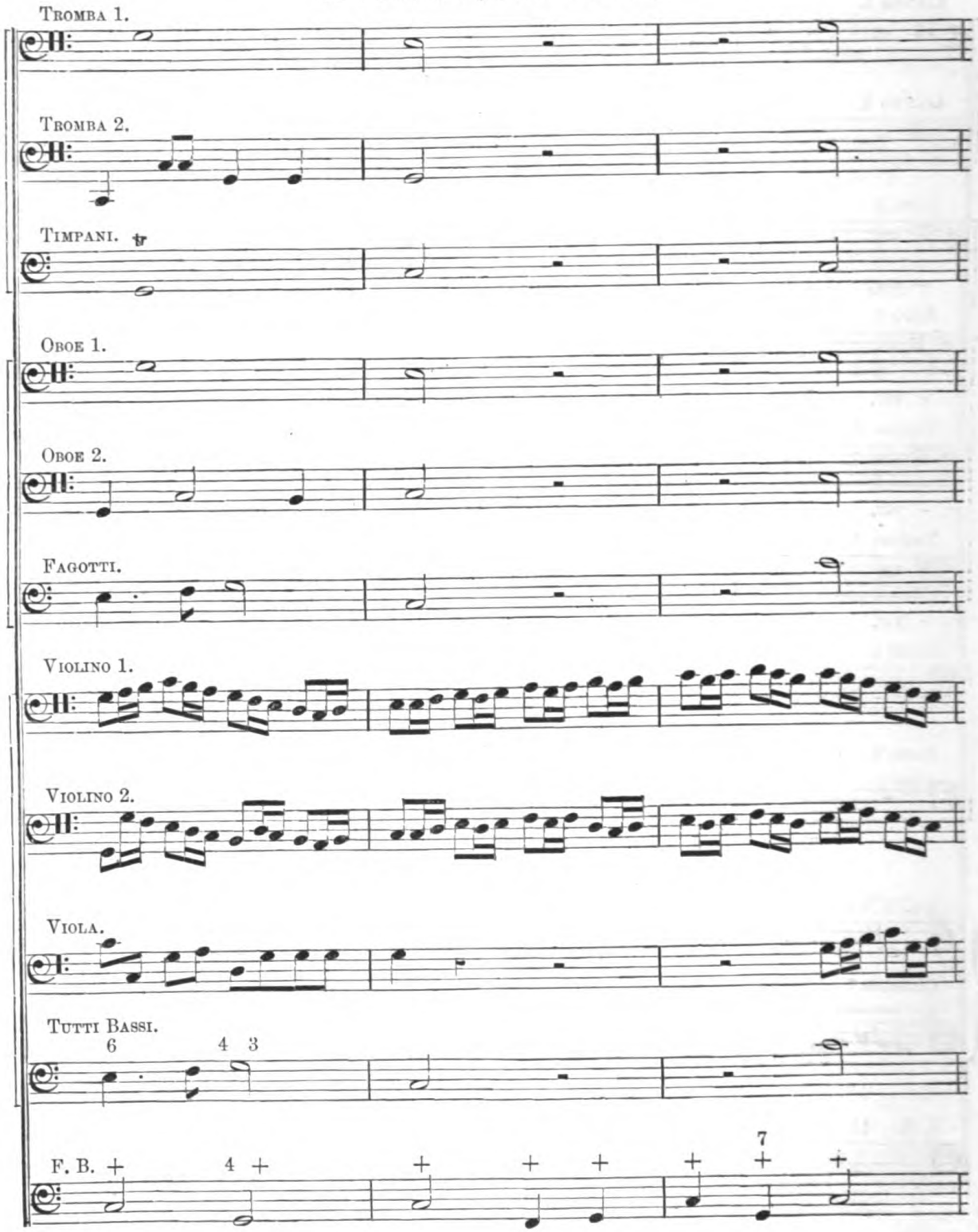


VOCAL SCORE.

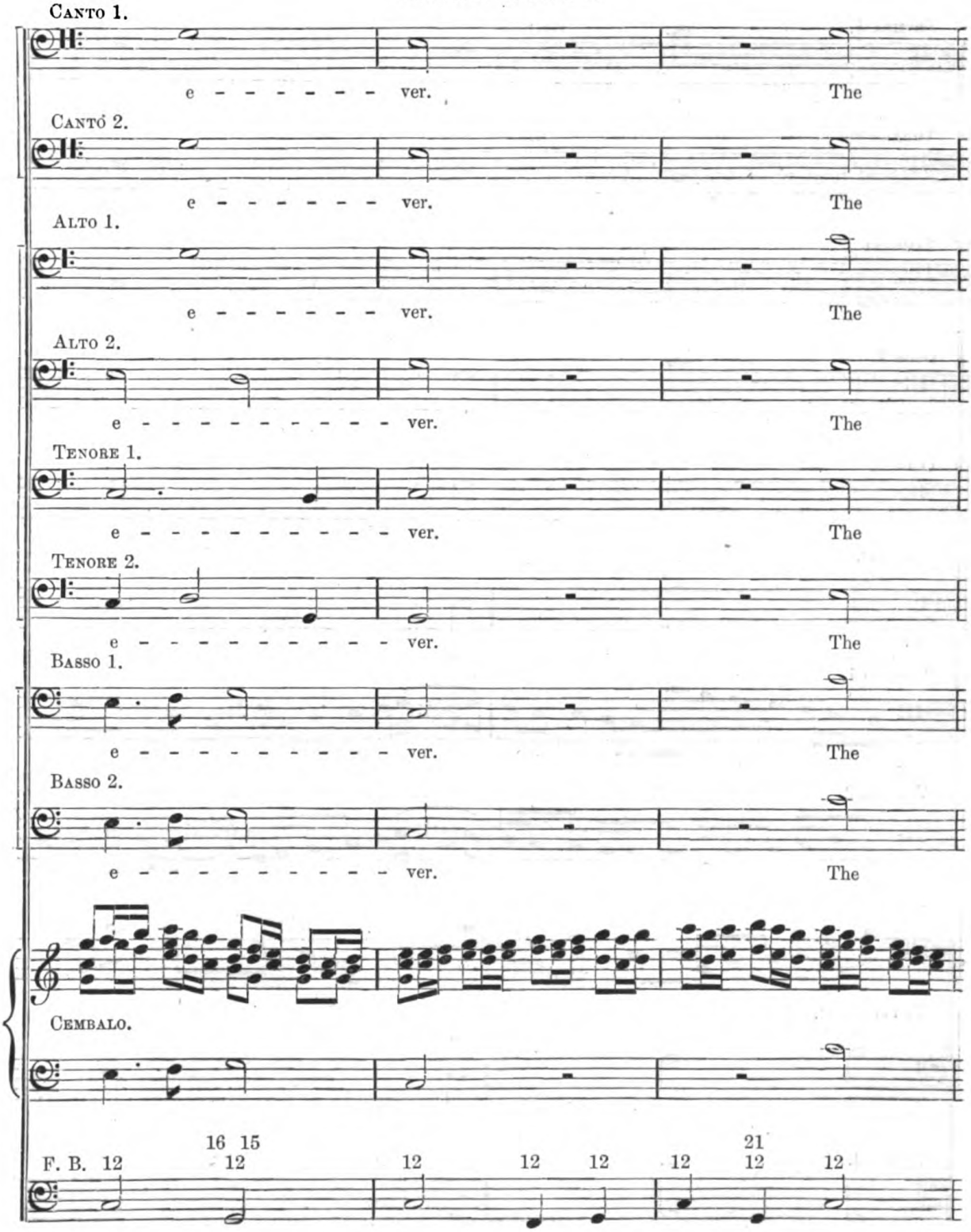


* Bishop's arrangement.

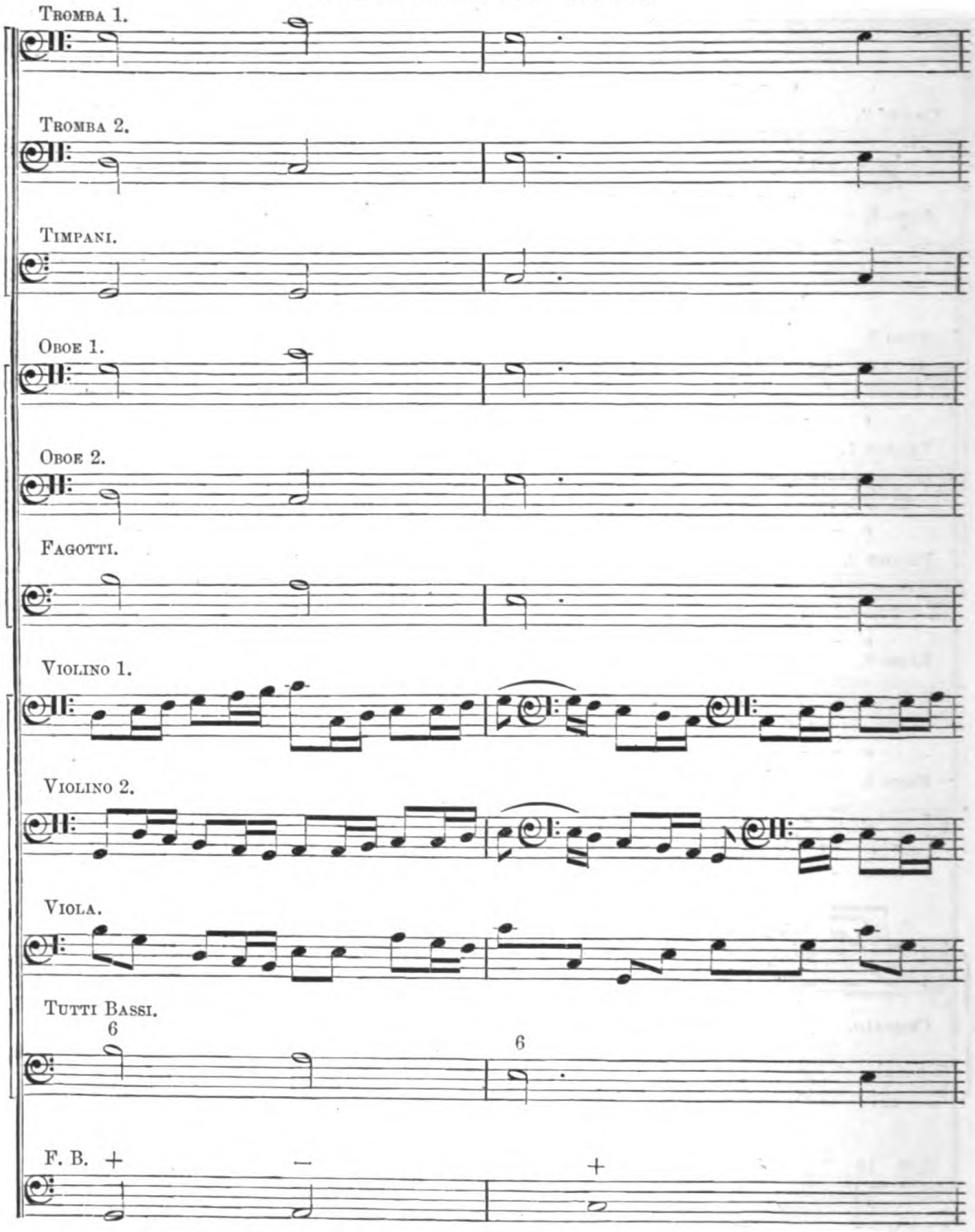
INSTRUMENTAL SCORE.



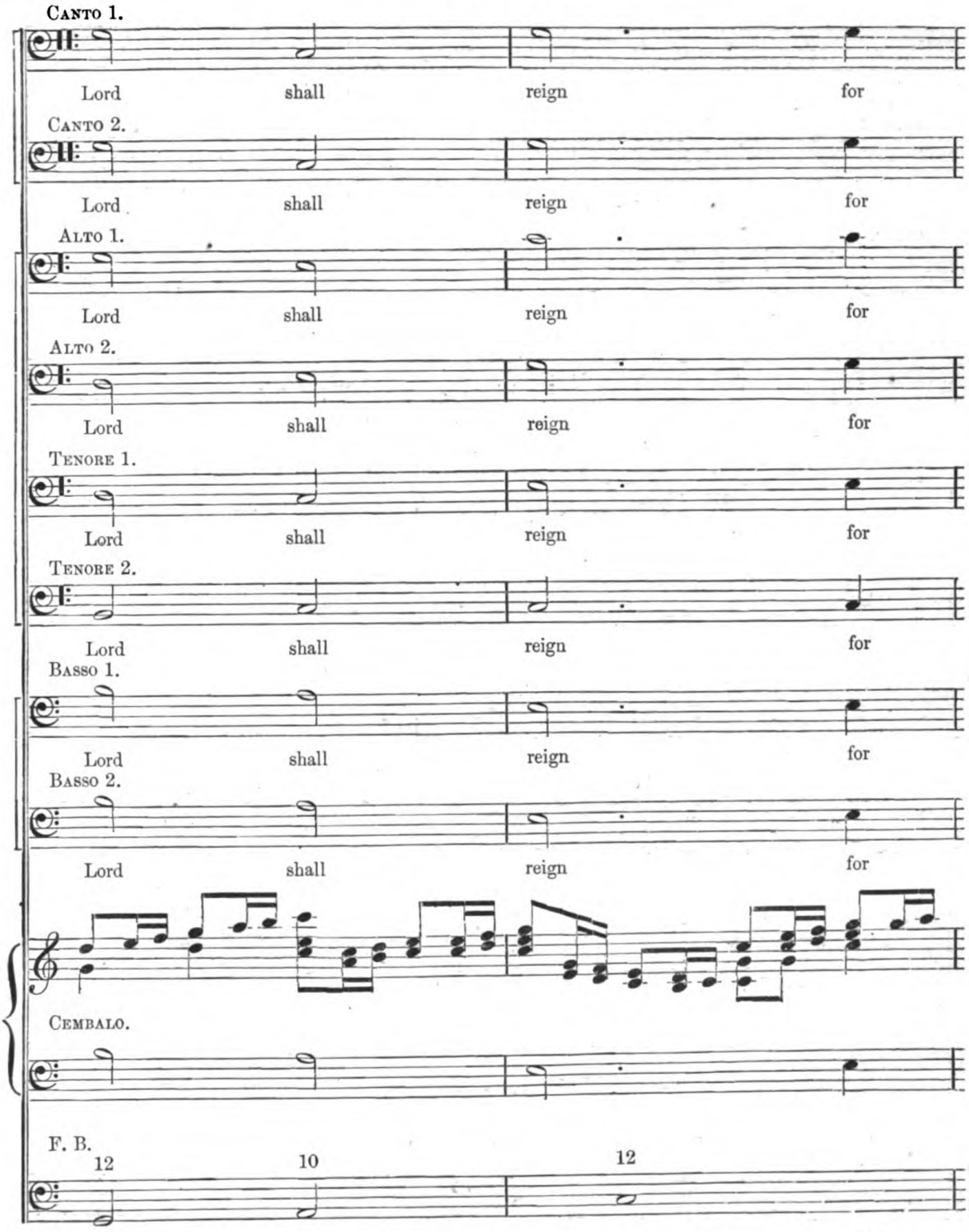
VOCAL SCORE.


CANTO 1.
e - - - - - - - ver.
The
CANTO 2.
e - - - - - - - ver.
The
ALTO 1.
e - - - - - - - ver.
The
ALTO 2.
e - - - - - - - - - ver.
The
TENORE 1.
e - - - - - - - - - ver.
The
TENORE 2.
e - - - - - - - - - ver.
The
BASSO 1.
e - - - - - - - - - ver.
The
BASSO 2.
e - - - - - - - - - ver.
The
CEMBALO.
F. B. 12
16 15
12
12
12
12
12
21
12
12

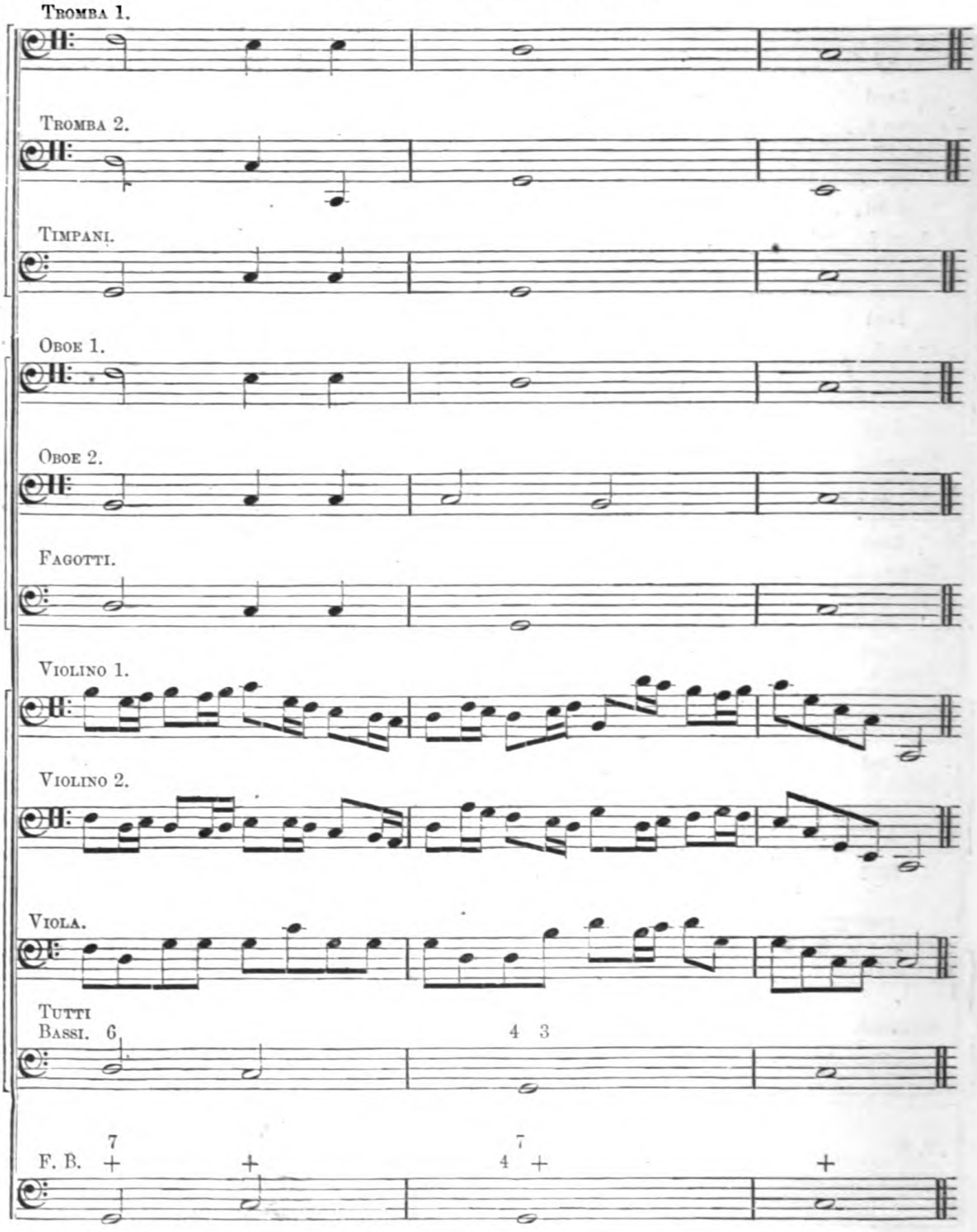
INSTRUMENTAL SCORE.



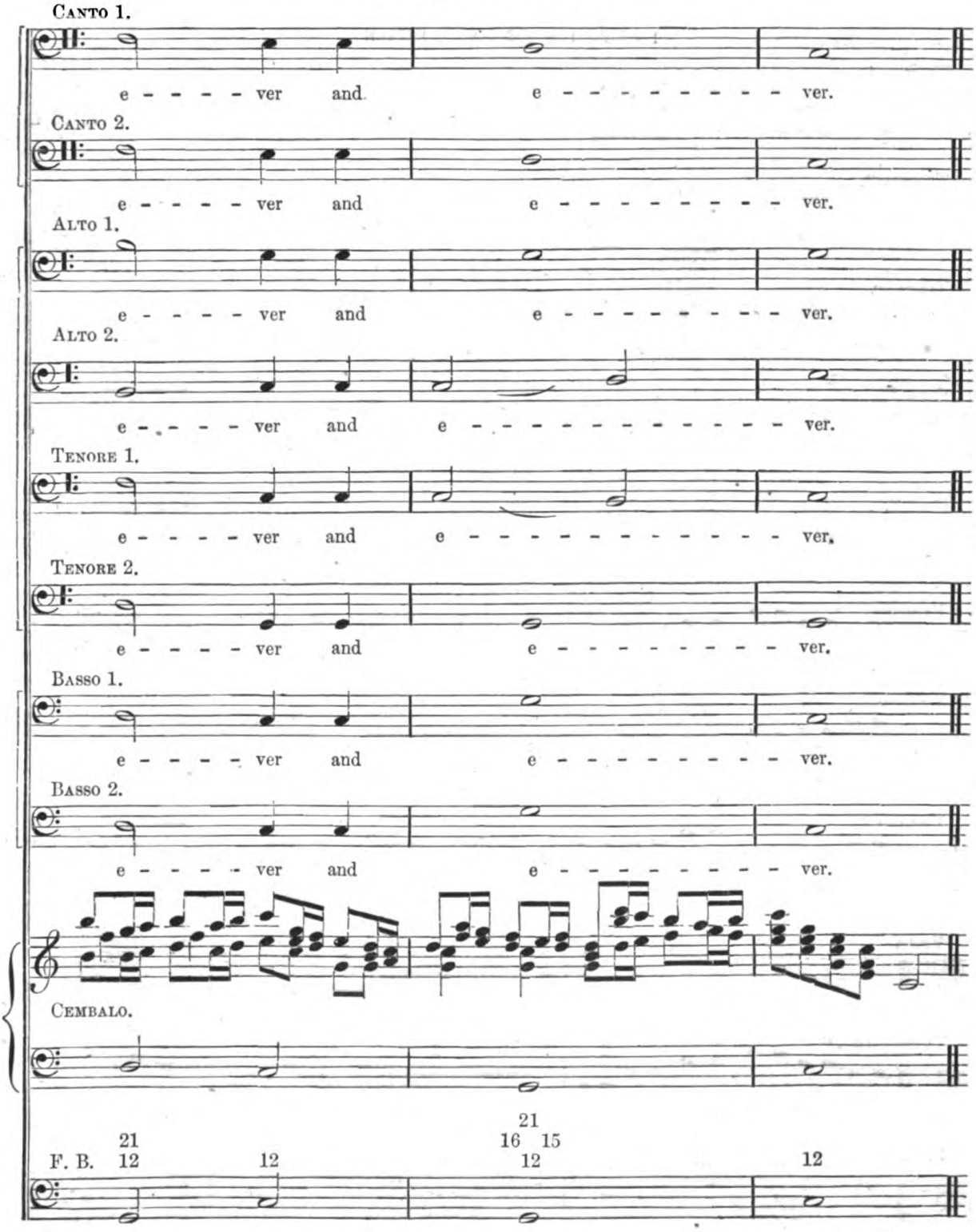
VOCAL SCORE.


CANTO 1.
Lord shall reign for
CANTO 2.
Lord shall reign for
ALTO 1.
Lord shall reign for
ALTO 2.
Lord shall reign for
TENORE 1.
Lord shall reign for
TENORE 2.
Lord shall reign for
BASSO 1.
Lord shall reign for
BASSO 2.
Lord shall reign for
CEMBALO.
F. B.
12
10
12

INSTRUMENTAL SCORE.

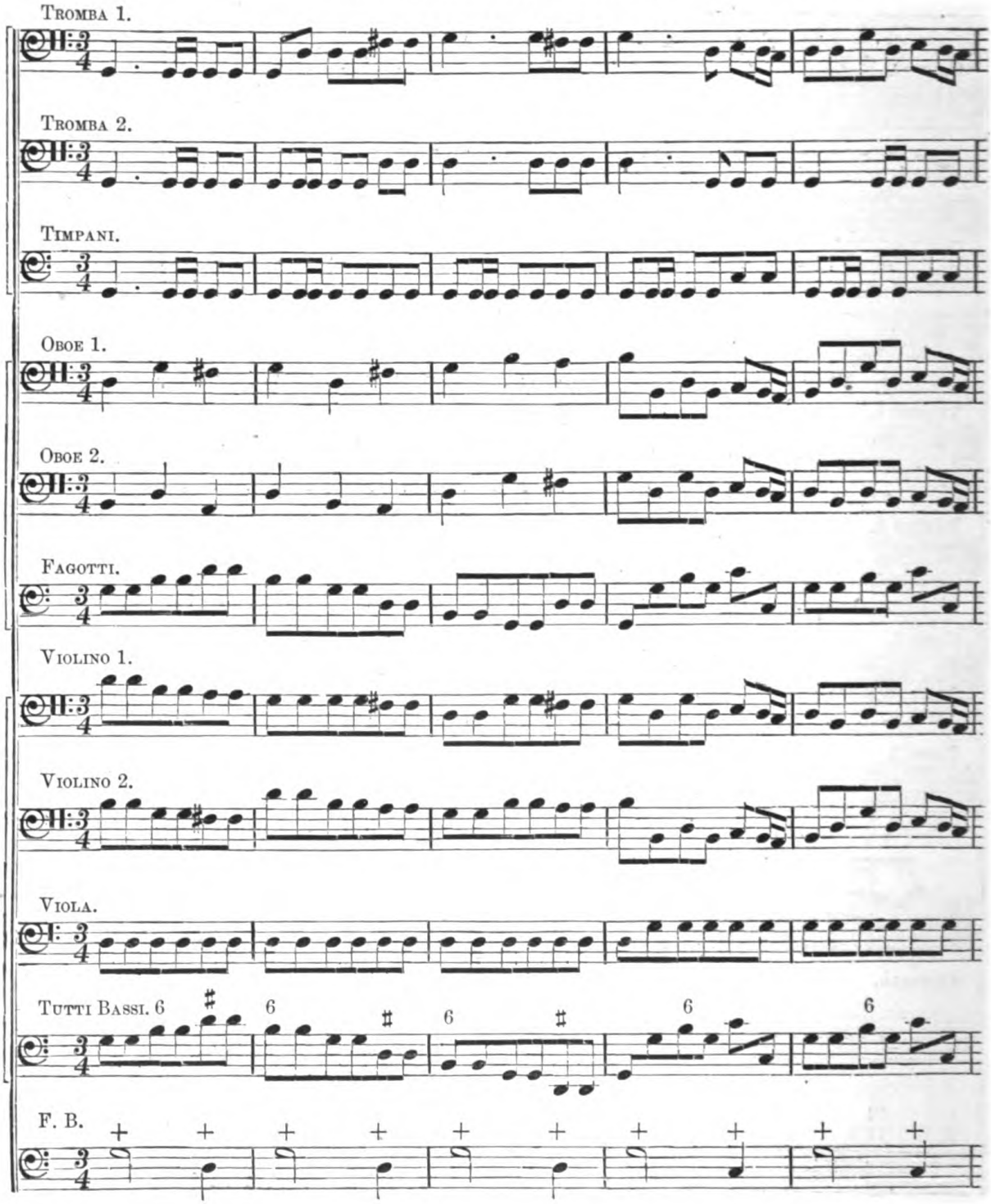


VOCAL SCORE.


CANTO 1.
e - - - - ver and e - - - - - - - - - ver.
CANTO 2.
e - - - - ver and e - - - - - - - - - ver.
ALTO 1.
e - - - - ver and e - - - - - - - - - ver.
ALTO 2.
e - - - - ver and e - - - - - - - - - - - - ver.
TENORE 1.
e - - - - ver and e - - - - - - - - - - - - ver.
TENORE 2.
e - - - - ver and e - - - - - - - - - ver.
BASSO 1.
e - - - - ver and e - - - - - - - - - ver.
BASSO 2.
e - - - - ver and e - - - - - - - - - ver.
CEMBALO.
F. B.
21 12
12
21 16 15 12
12

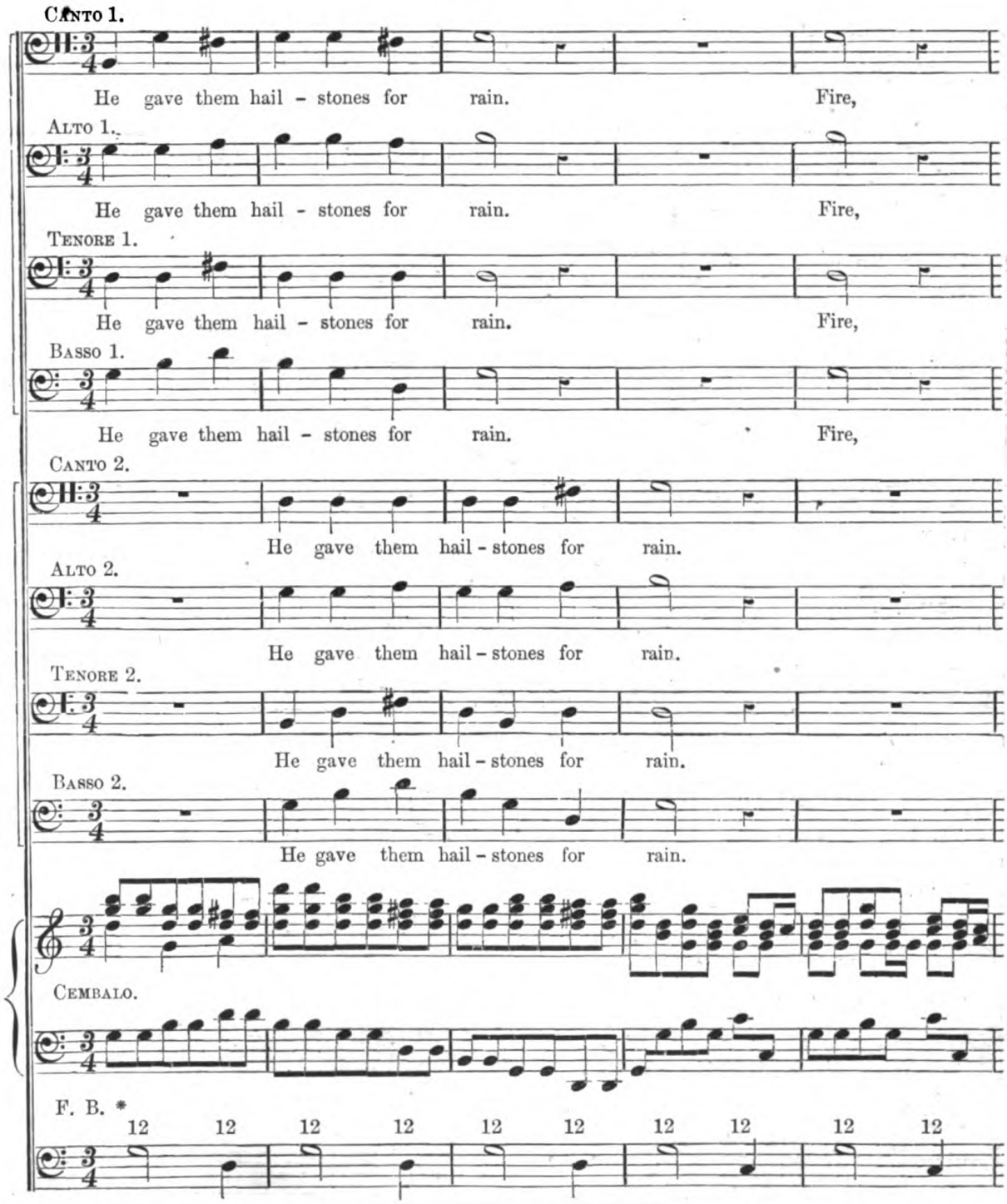
43. From the "Hailstone Chorus."

INSTRUMENTAL SCORE.



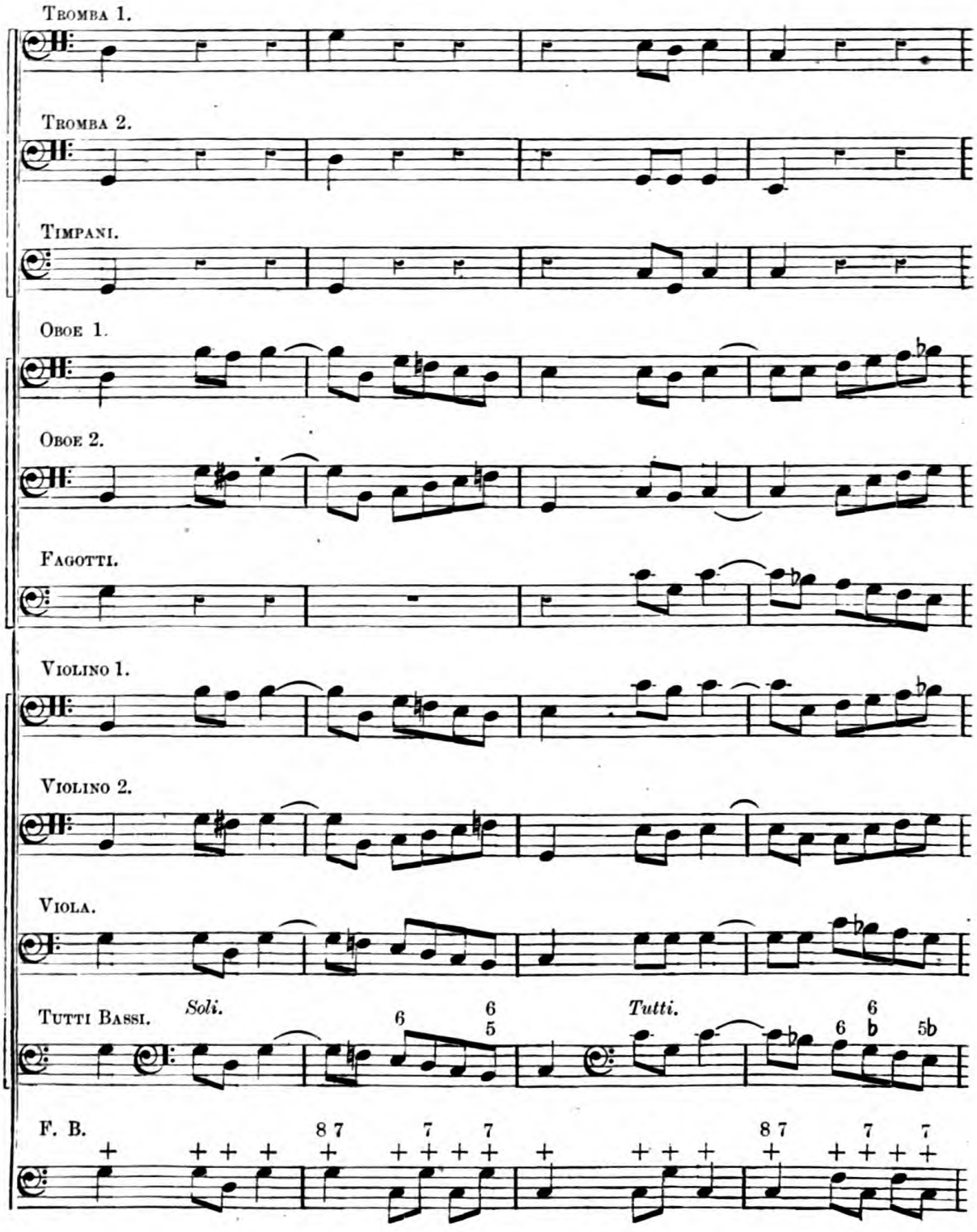
"Israel in Egypt."—HANDEL.

VOCAL SCORE.

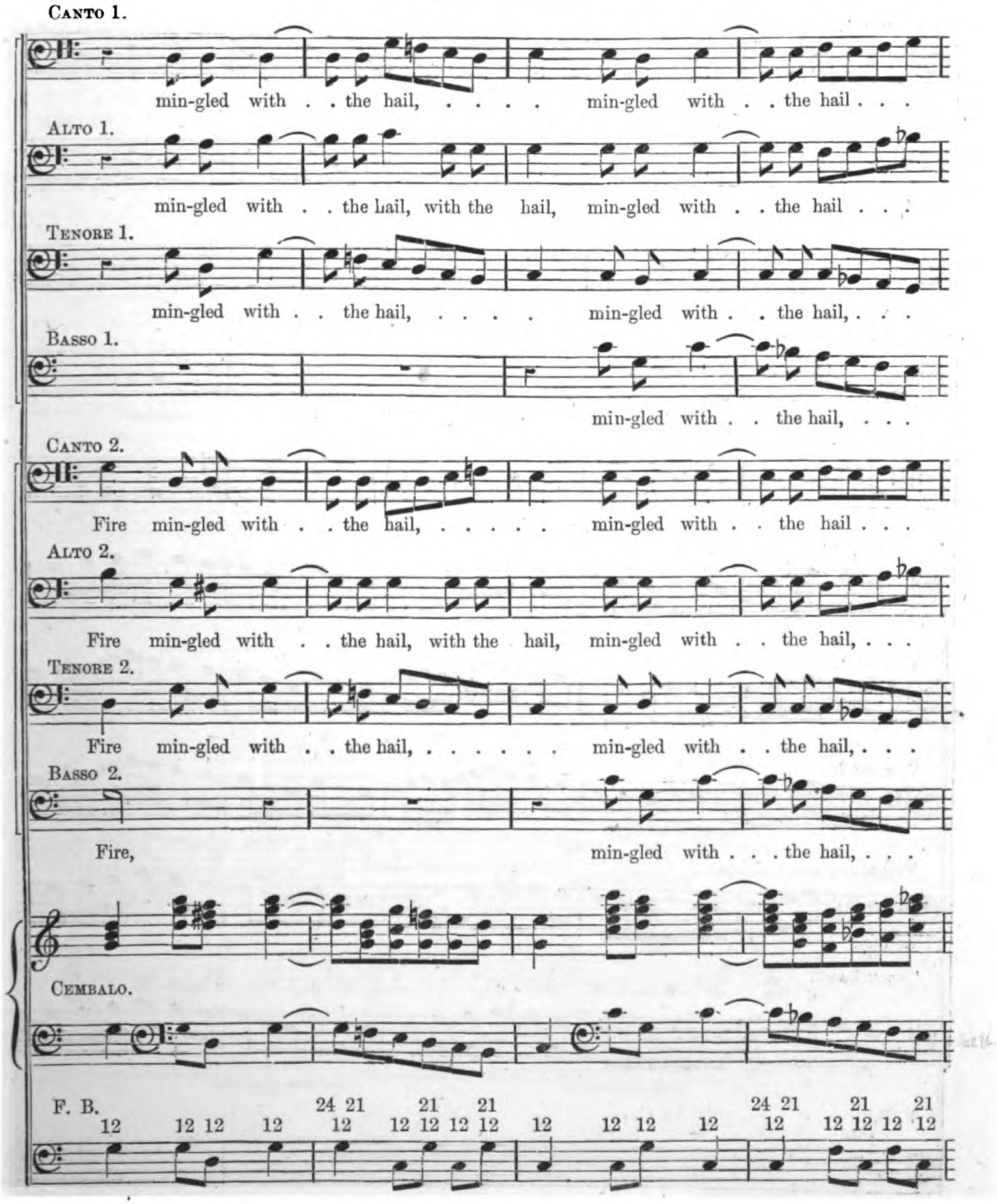


* See Appendix K, where this notation is explained.

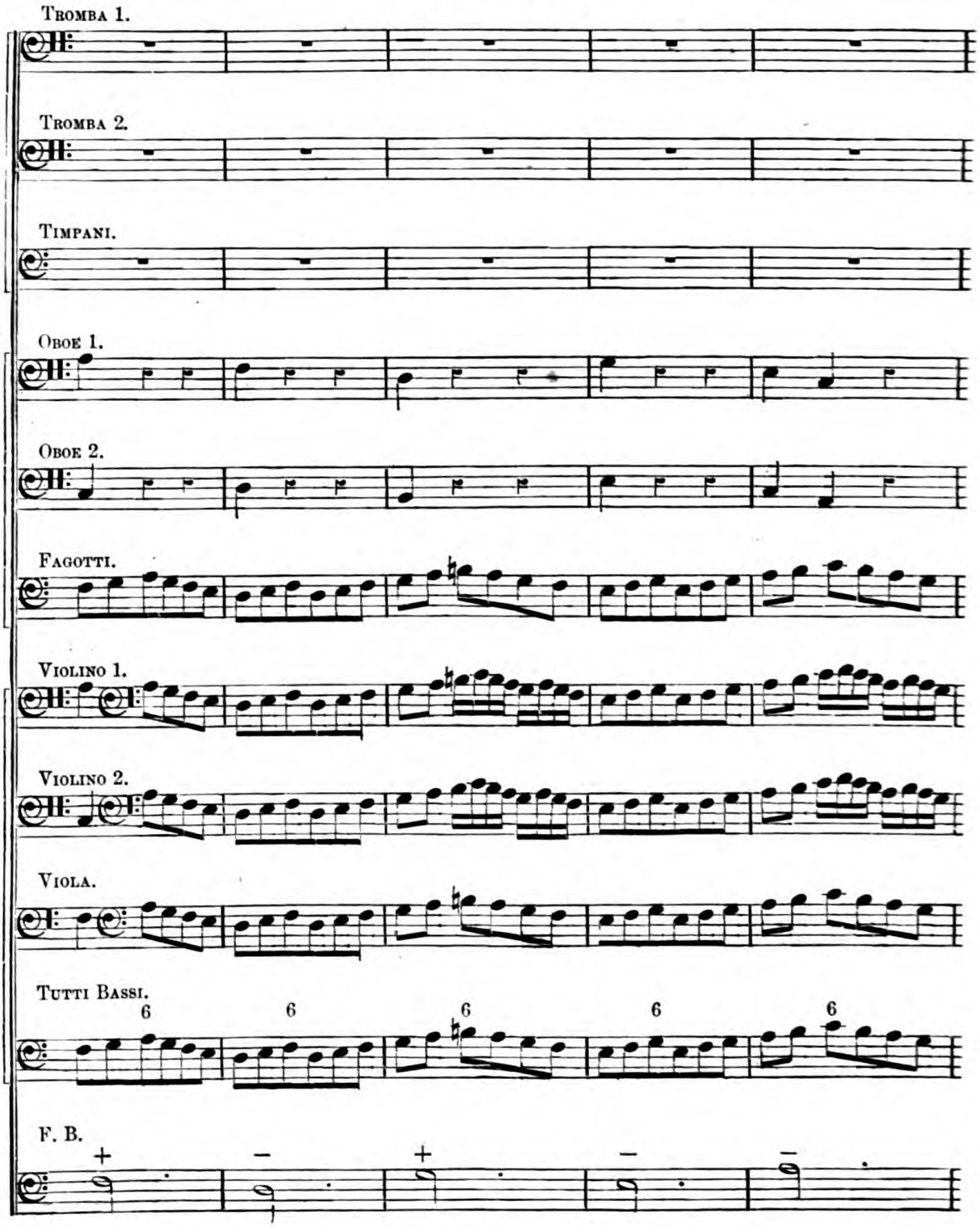
INSTRUMENTAL SCORE.



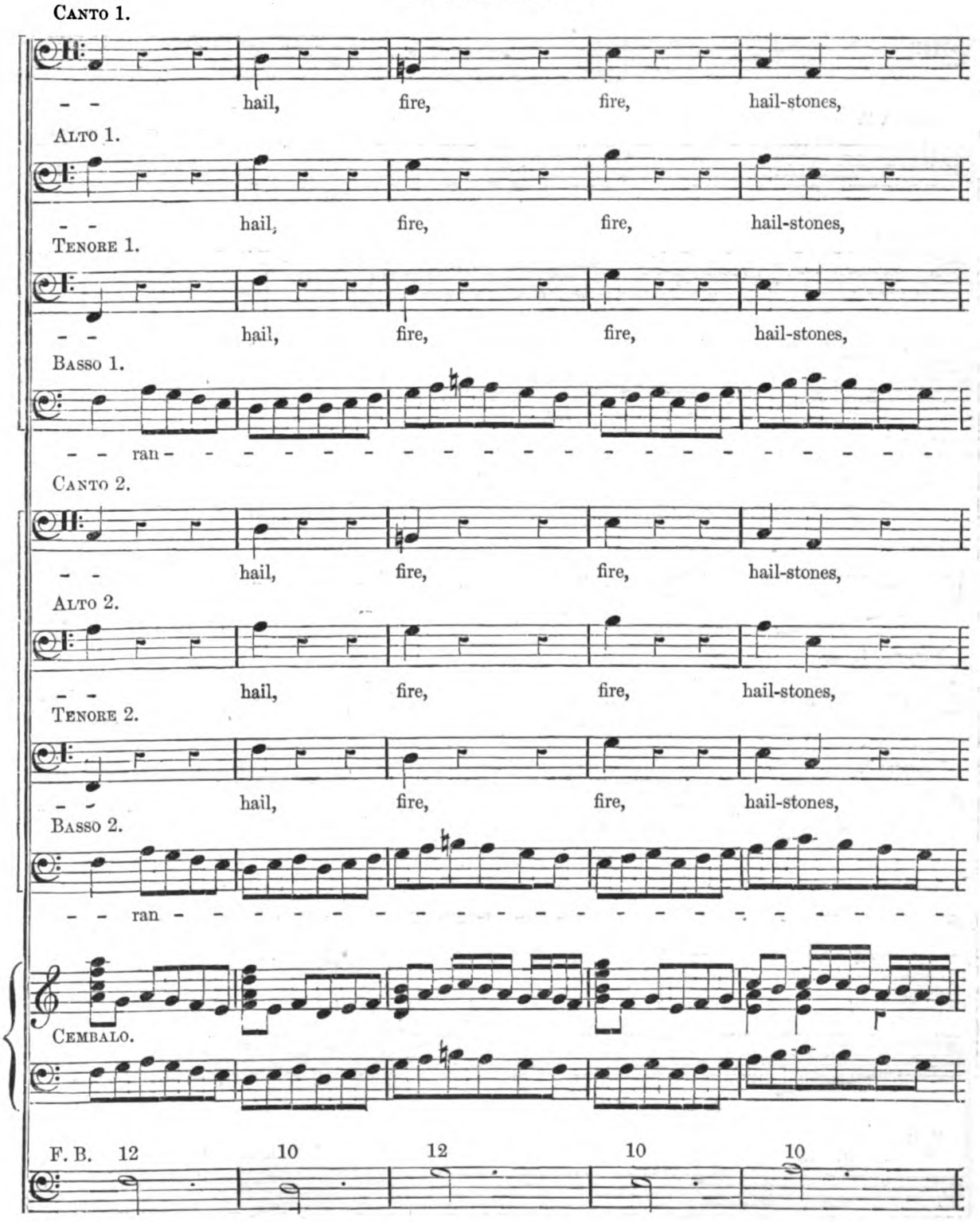
VOCAL SCORE.


CANTO 1.
min-gled with . . the hail, ... min-gled with . . the hail . . .
ALTO 1.
min-gled with . . the hail, with the hail, min-gled with . . the hail . . .
TENORE 1.
min-gled with . . the hail, ... min-gled with . . the hail, . . .
BASSO 1.
min-gled with . . the hail, . . .
CANTO 2.
Fire min-gled with . . the hail, ... min-gled with . . the hail . . .
ALTO 2.
Fire min-gled with . . the hail, with the hail, min-gled with . . the hail, . . .
TENORE 2.
Fire min-gled with . . the hail, ... min-gled with . . the hail, . . .
BASSO 2.
Fire, min-gled with . . . the hail, . . .
CEMBALO.
F. B.
12 12 12 12 24 21 12 12 12 12 21 21 12 12 12 12 12 24 21 12 12 12 12 21 21 12

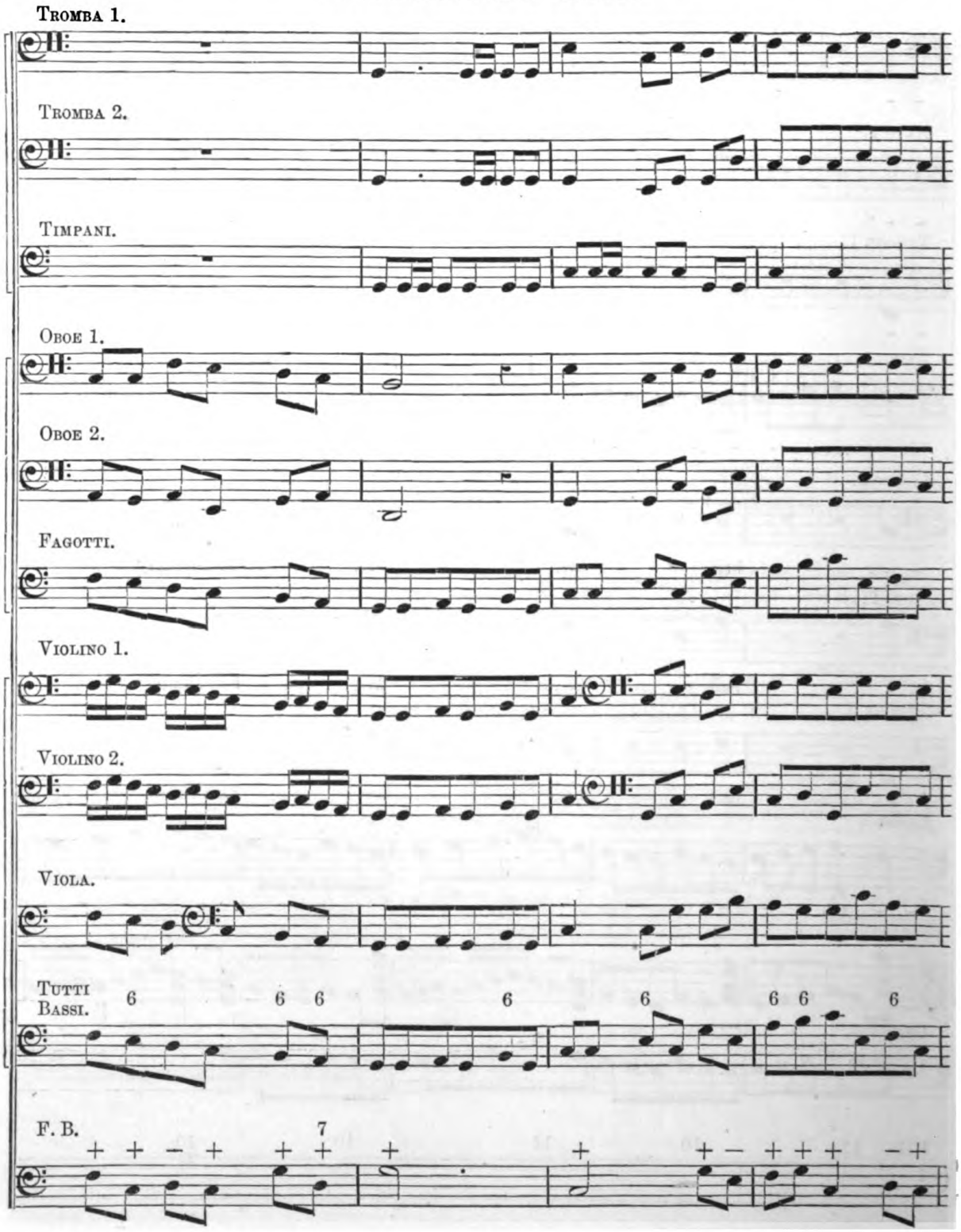
INSTRUMENTAL SCORE.



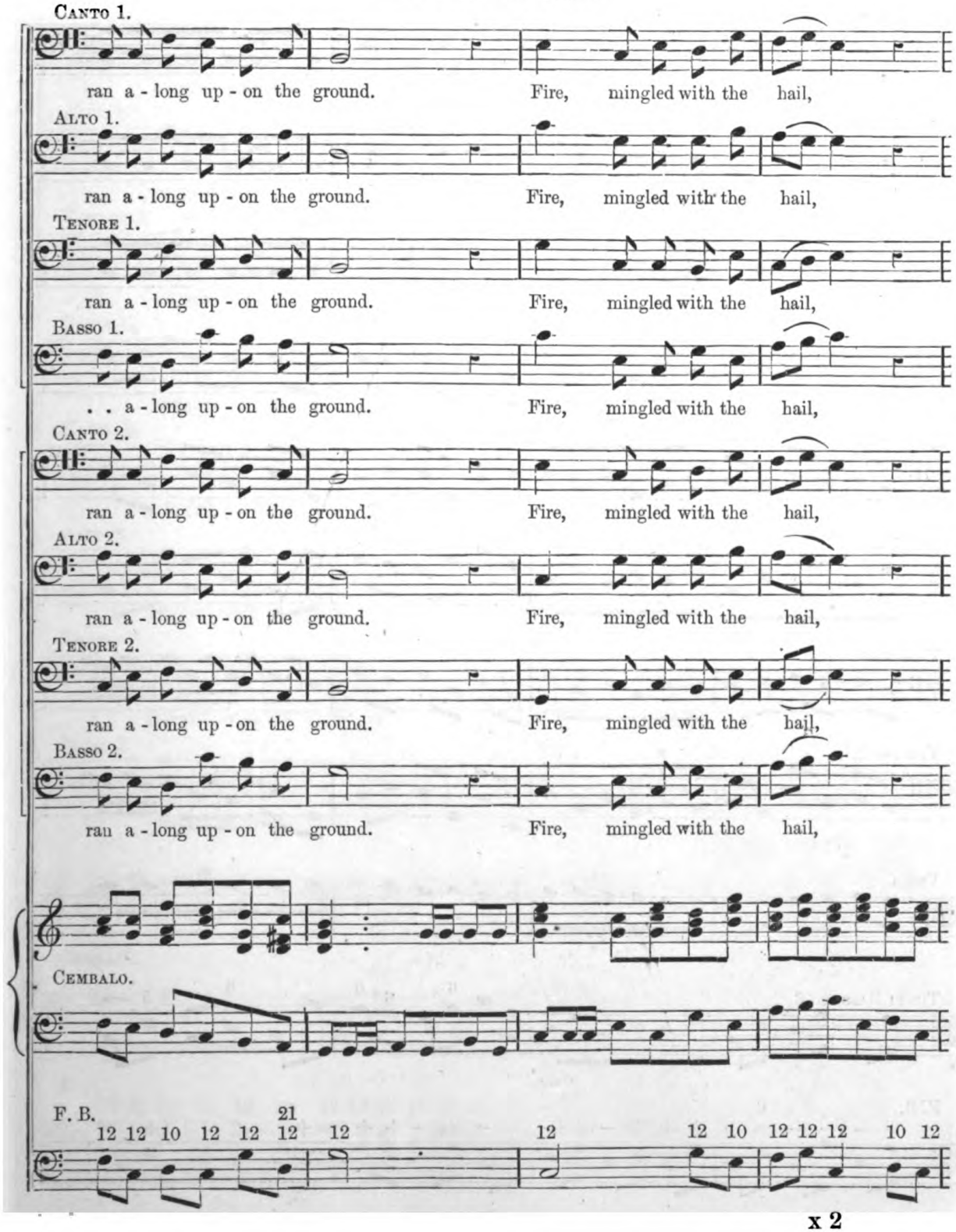
VOCAL SCORE.


CANTO 1.
- - hail, fire, fire, hail-stones,
ALTO 1.
- - hail, fire, fire, hail-stones,
TENORE 1.
- - hail, fire, fire, hail-stones,
BASSO 1.
- - ran -
CANTO 2.
- - hail, fire, fire, hail-stones,
ALTO 2.
- - hail, fire, fire, hail-stones,
TENORE 2.
- - hail, fire, fire, hail-stones,
BASSO 2.
- - ran -
CEMBALO.
F. B. 12 10 12 10 10

INSTRUMENTAL SCORE.

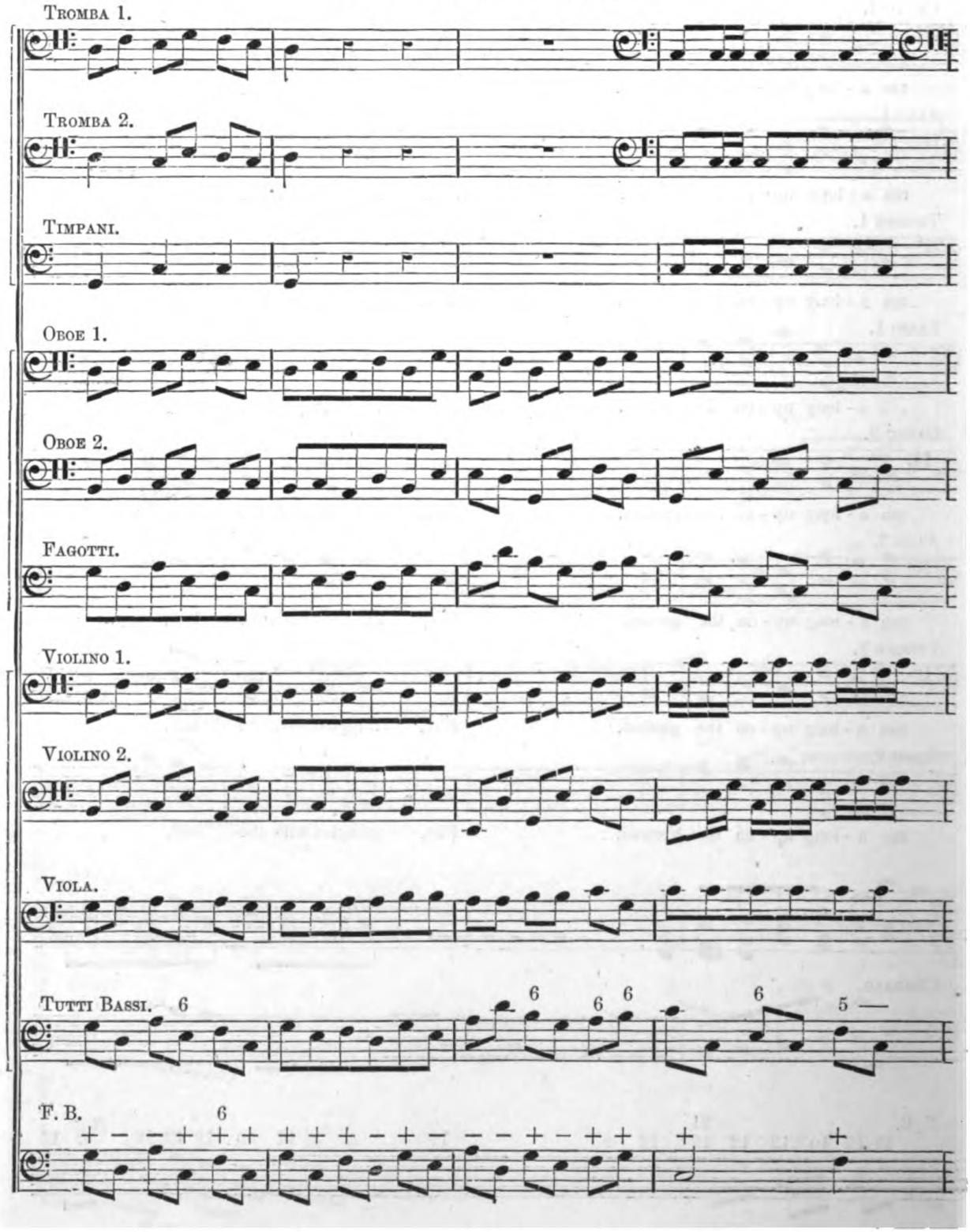


VOCAL SCORE.

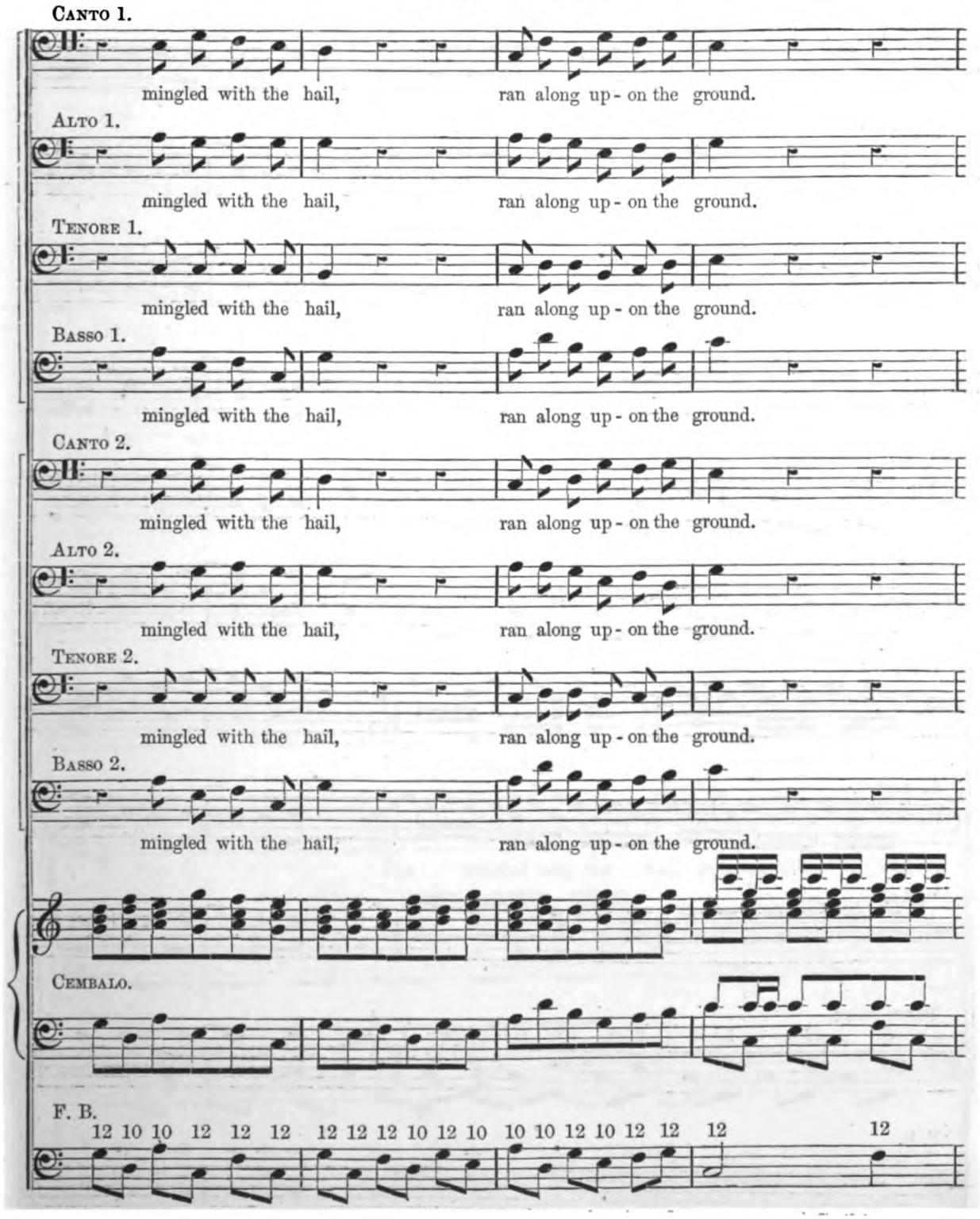


x 2

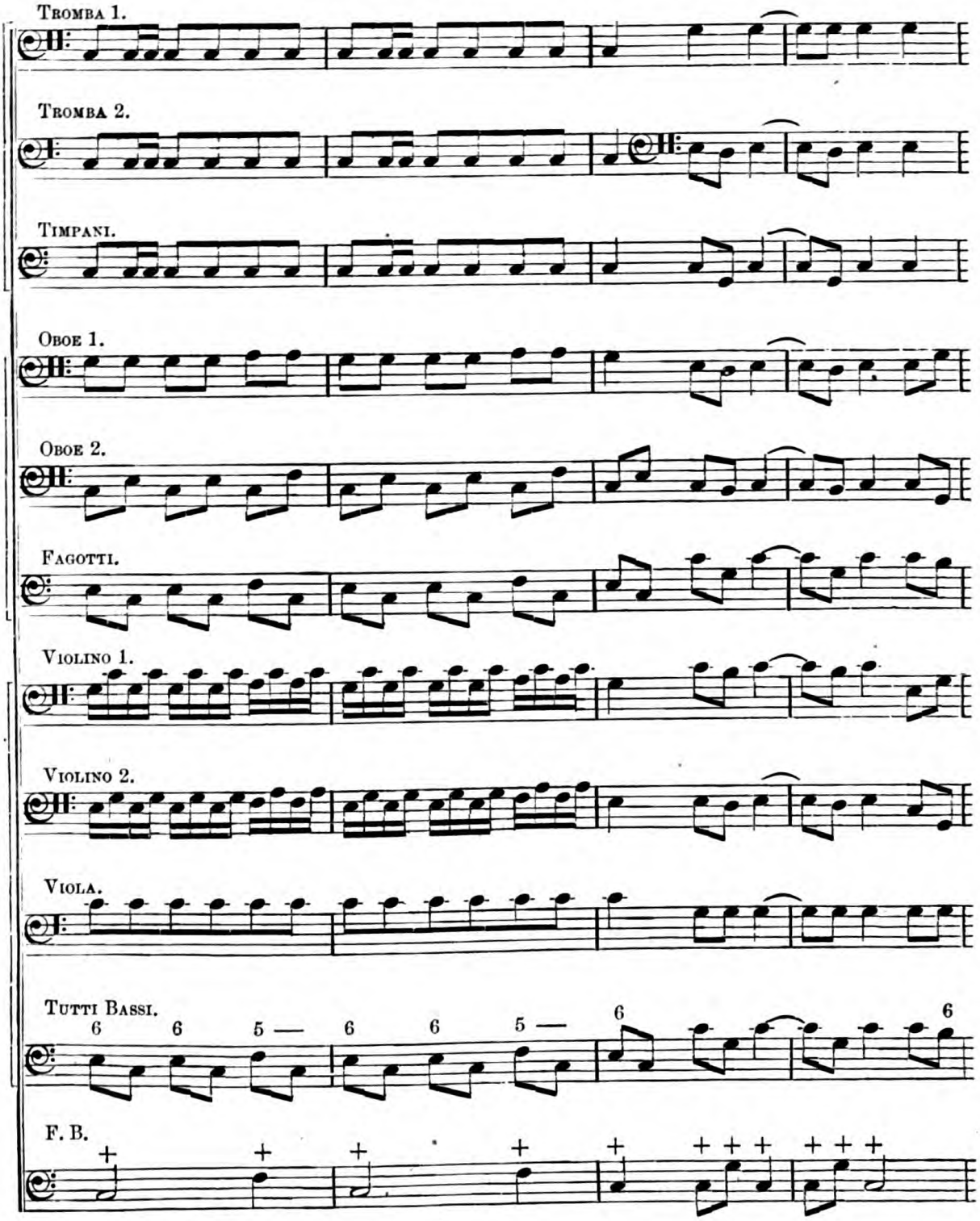
INSTRUMENTAL SCORE.



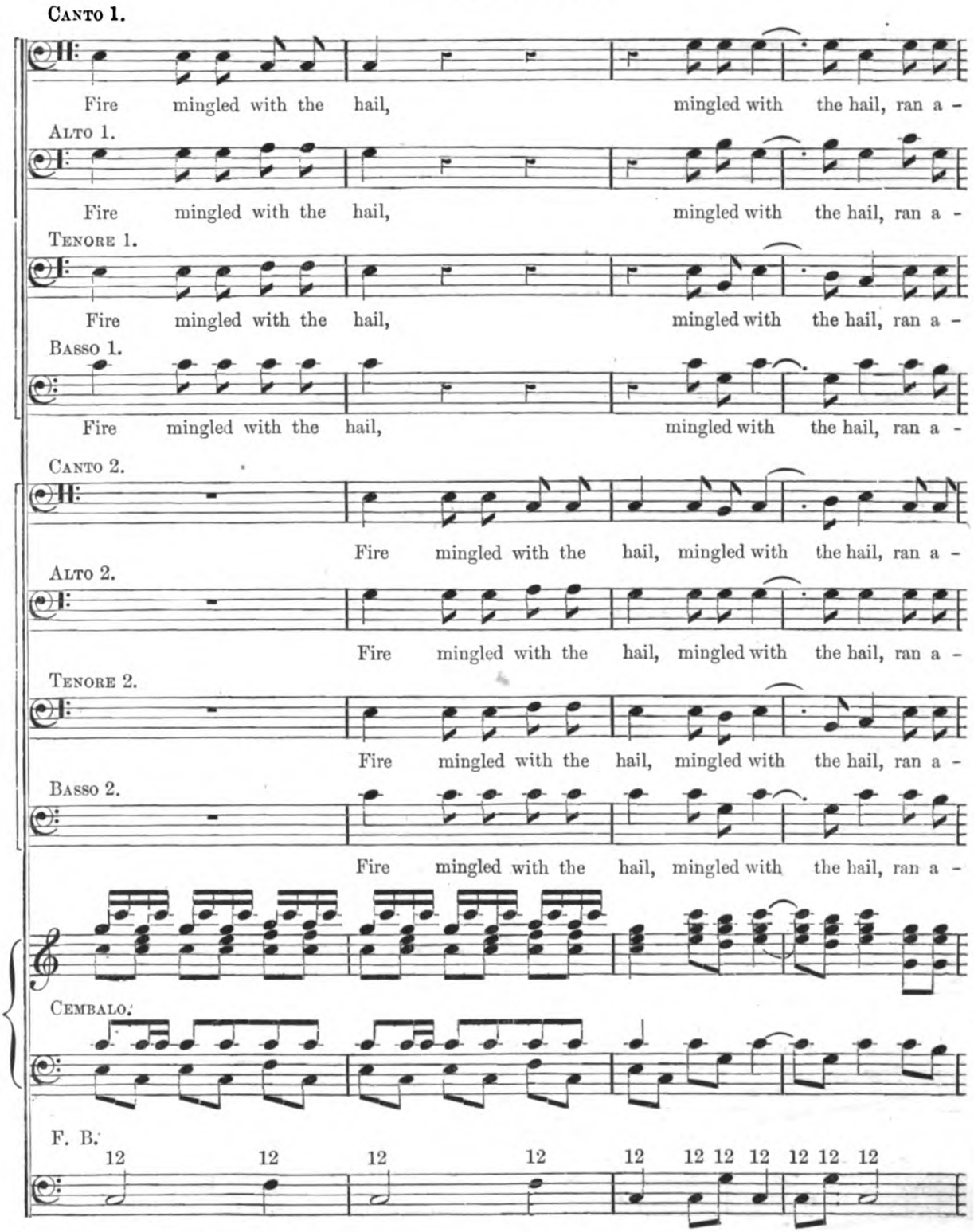
VOCAL SCORE.


CANTO 1.
mingled with the hail, ran along up - on the ground.
ALTO 1.
mingled with the hail, ran along up - on the ground.
TENORE 1.
mingled with the hail, ran along up - on the ground.
BASSO 1.
mingled with the hail, ran along up - on the ground.
CANTO 2.
mingled with the hail, ran along up - on the ground.
ALTO 2.
mingled with the hail, ran along up - on the ground.
TENORE 2.
mingled with the hail, ran along up - on the ground.
BASSO 2.
mingled with the hail, ran along up - on the ground.
CEMBALO.
F. B.
12 10 10 12 12 12 12 12 12 10 12 10 10 10 12 10 12 12 12 12

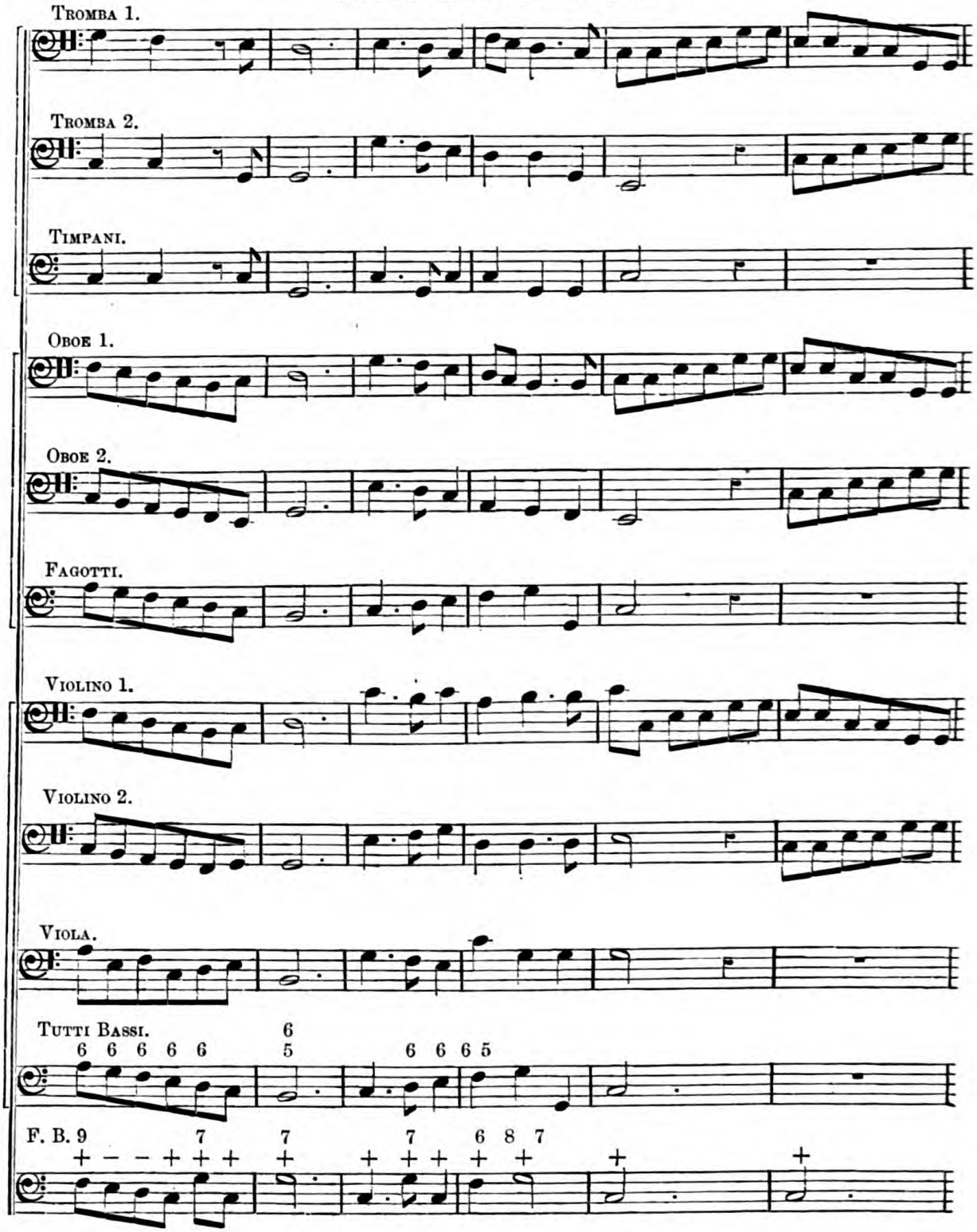
INSTRUMENTAL SCORE.



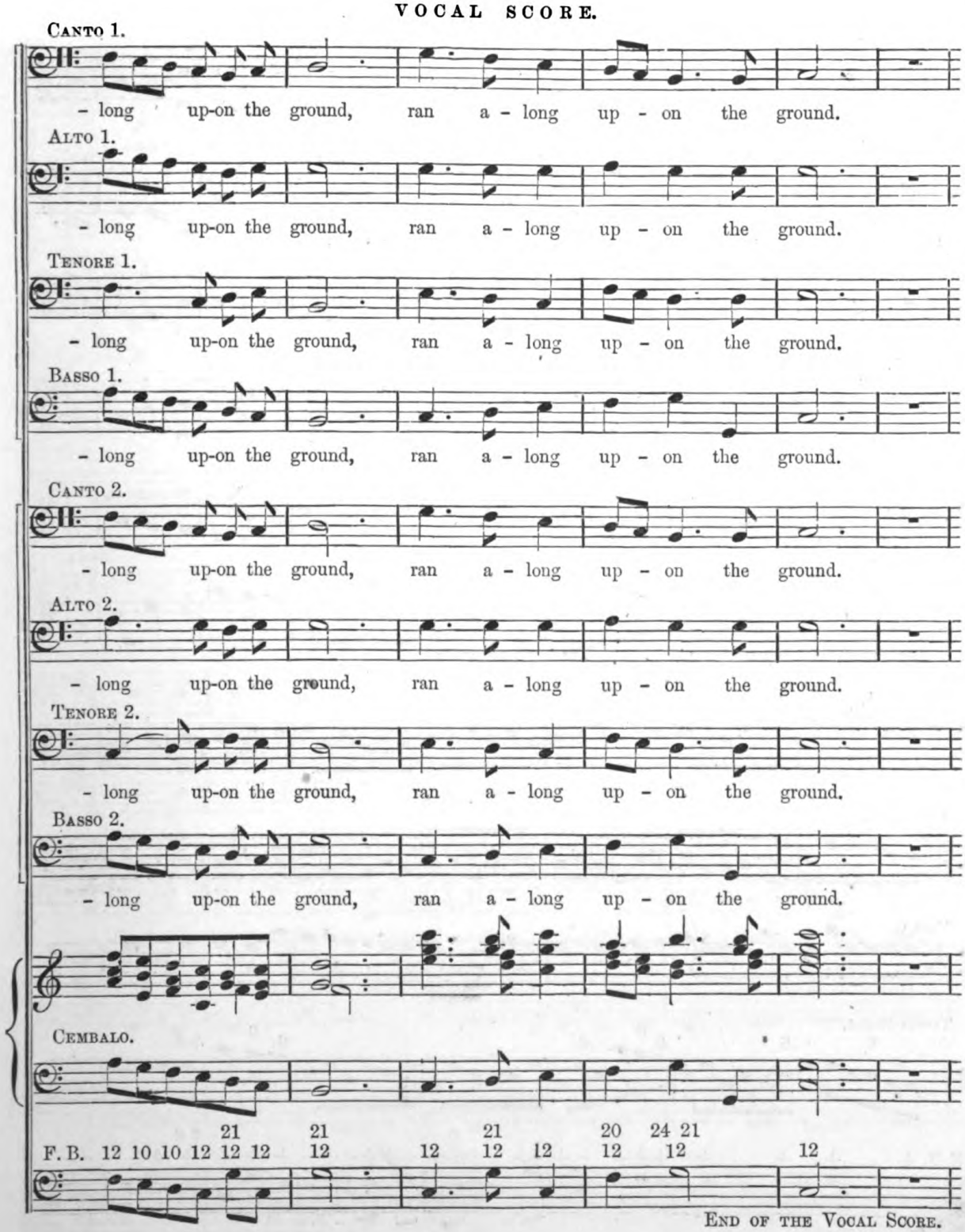
VOCAL SCORE.


CANTO 1.
Fire mingled with the hail, mingled with the hail, ran a -
ALTO 1.
Fire mingled with the hail, mingled with the hail, ran a -
TENORE 1.
Fire mingled with the hail, mingled with the hail, ran a -
BASSO 1.
Fire mingled with the hail, mingled with the hail, ran a -
CANTO 2.
Fire mingled with the hail, mingled with the hail, ran a -
ALTO 2.
Fire mingled with the hail, mingled with the hail, ran a -
TENORE 2.
Fire mingled with the hail, mingled with the hail, ran a -
BASSO 2.
Fire mingled with the hail, mingled with the hail, ran a -
CEMBALO.
F. B.
12 12 12 12 12 12 12 12 12 12 12

INSTRUMENTAL SCORE.



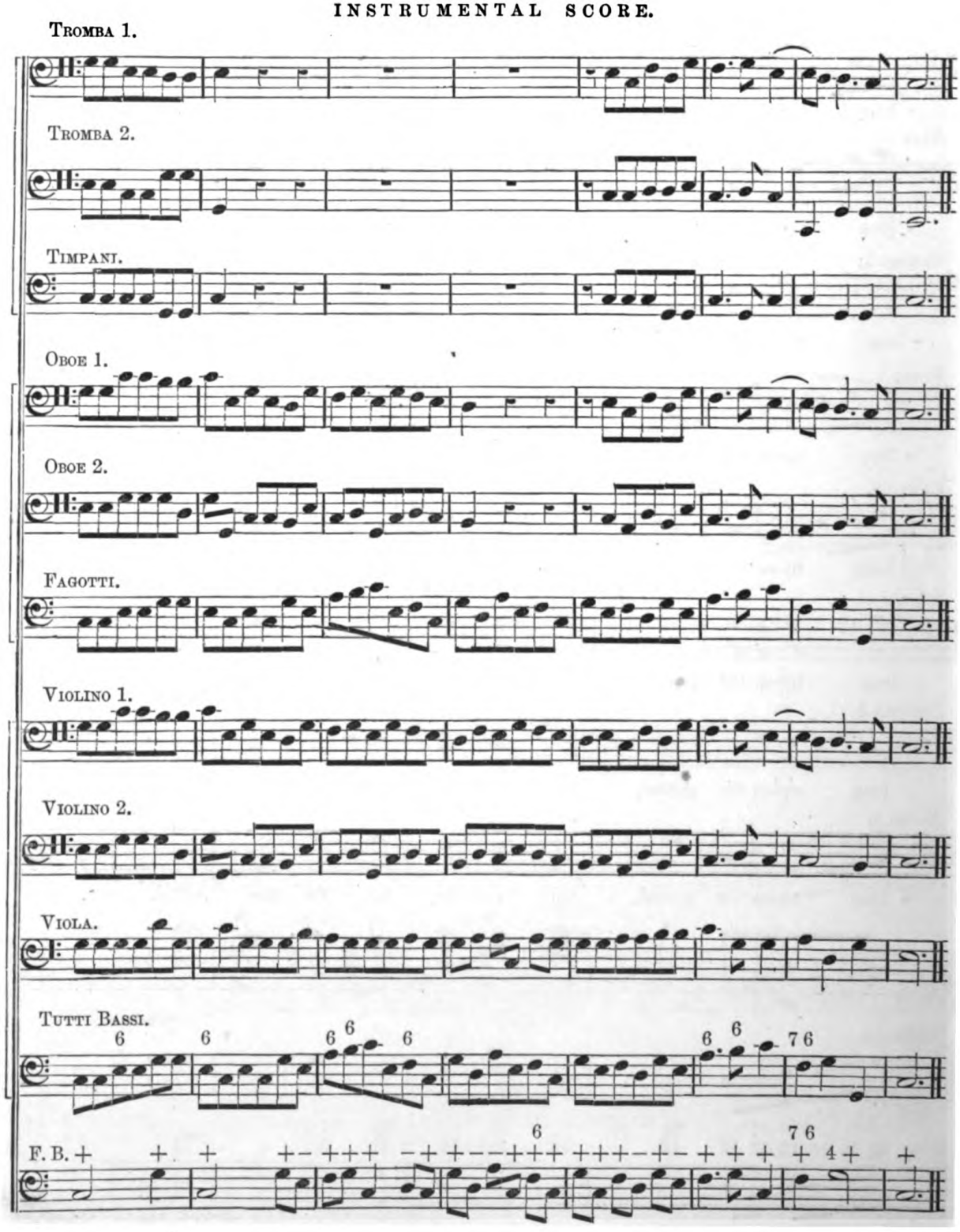
VOCAL SCORE.



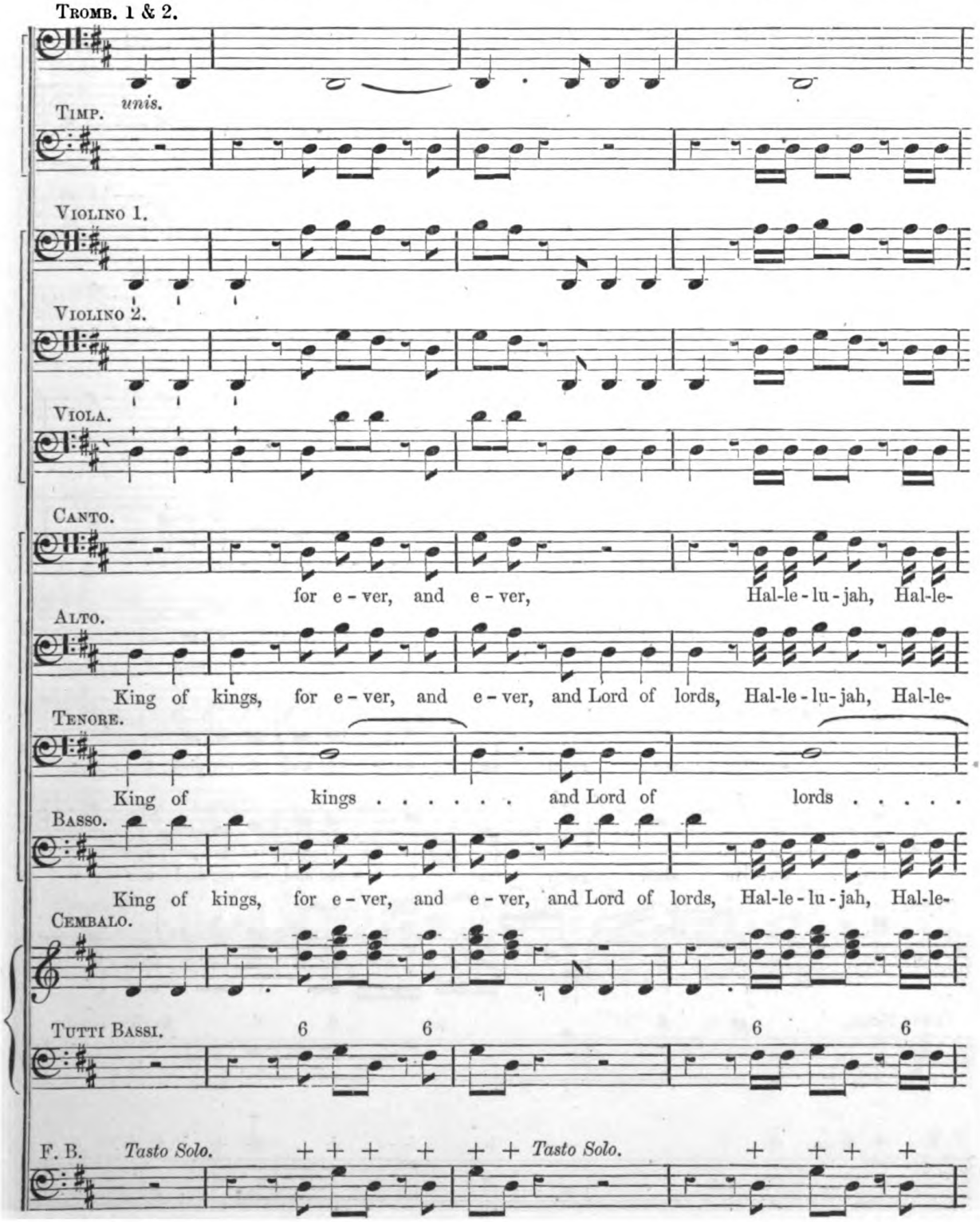
END OF THE VOCAL SCORE.

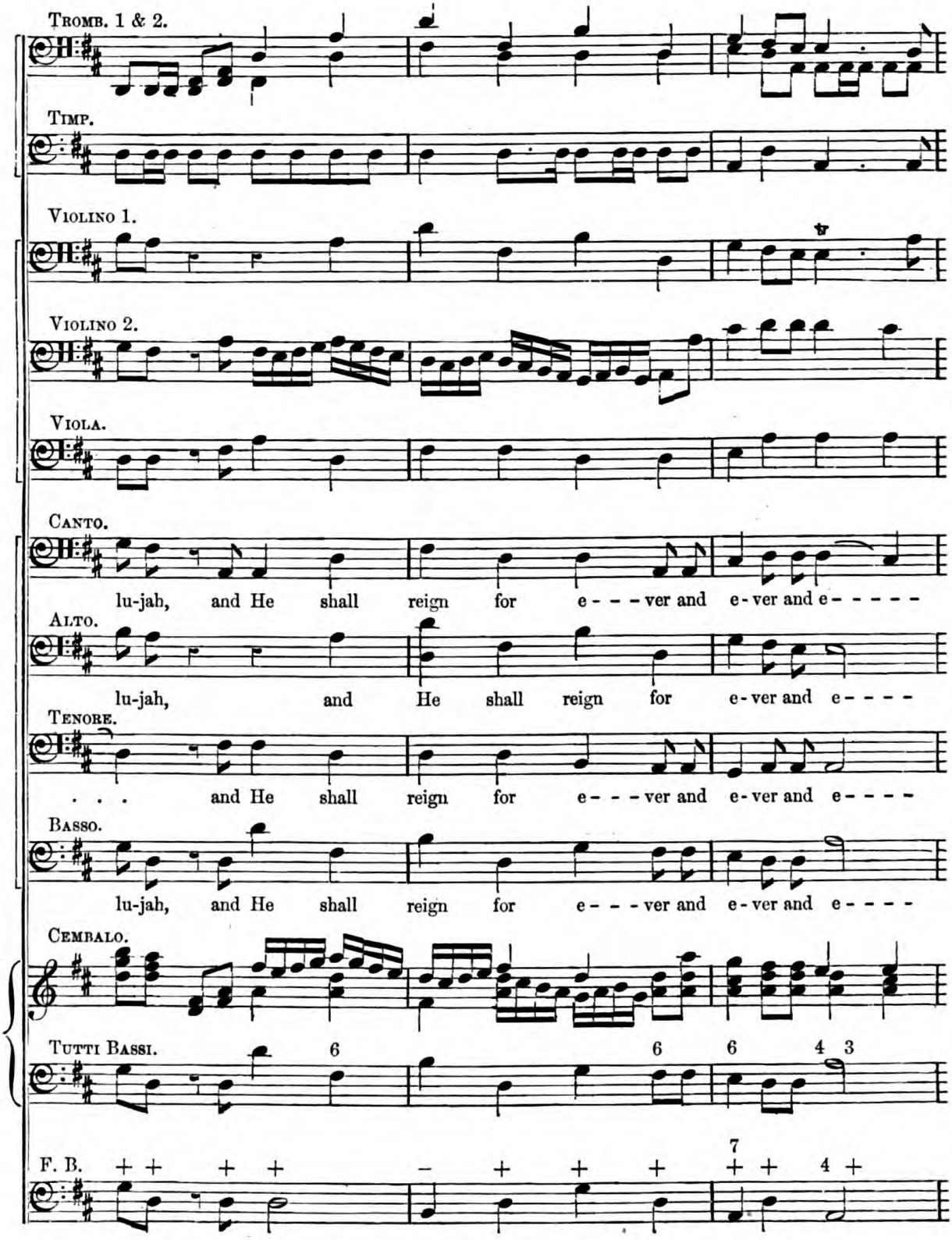
Volti.

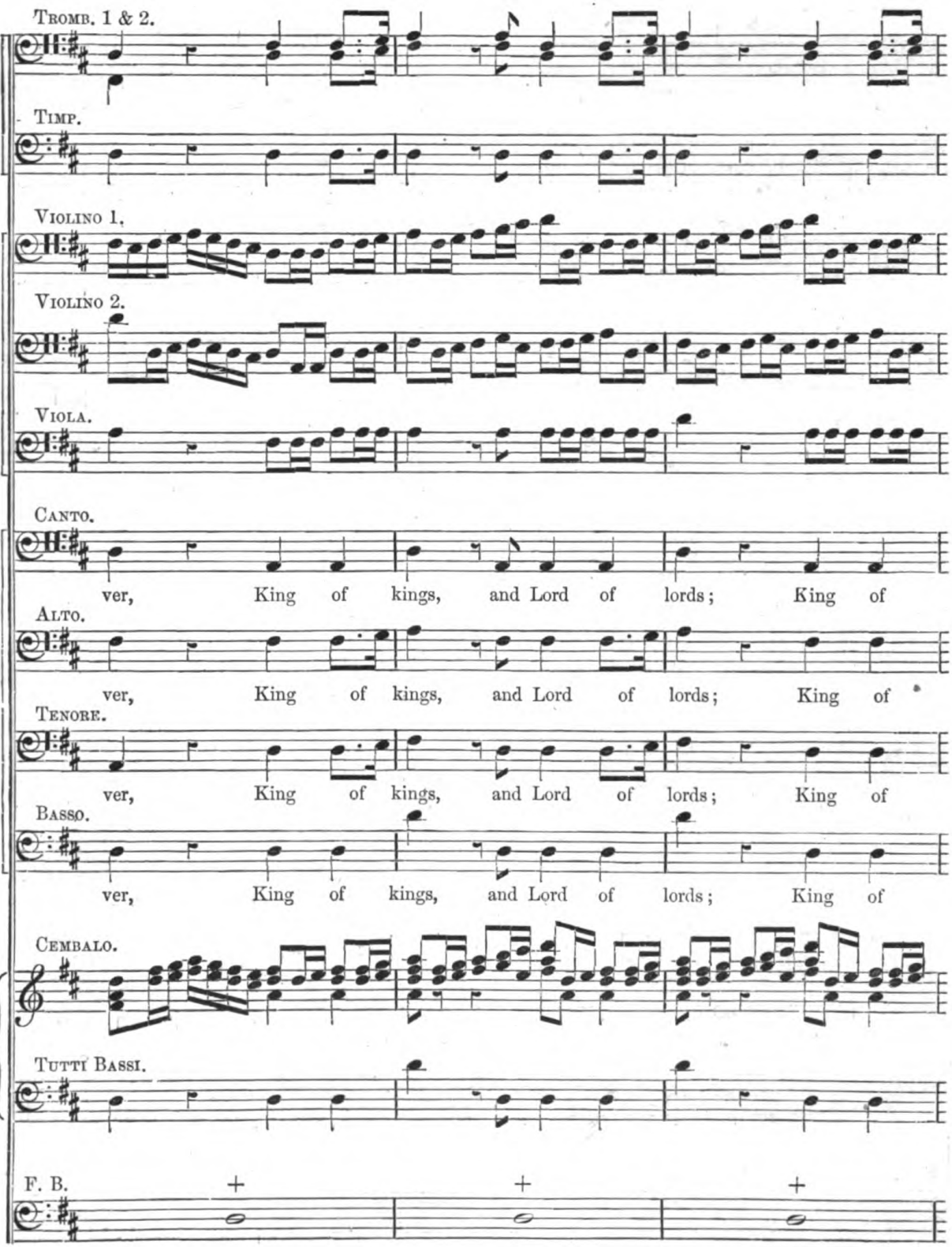
INSTRUMENTAL SCORE.

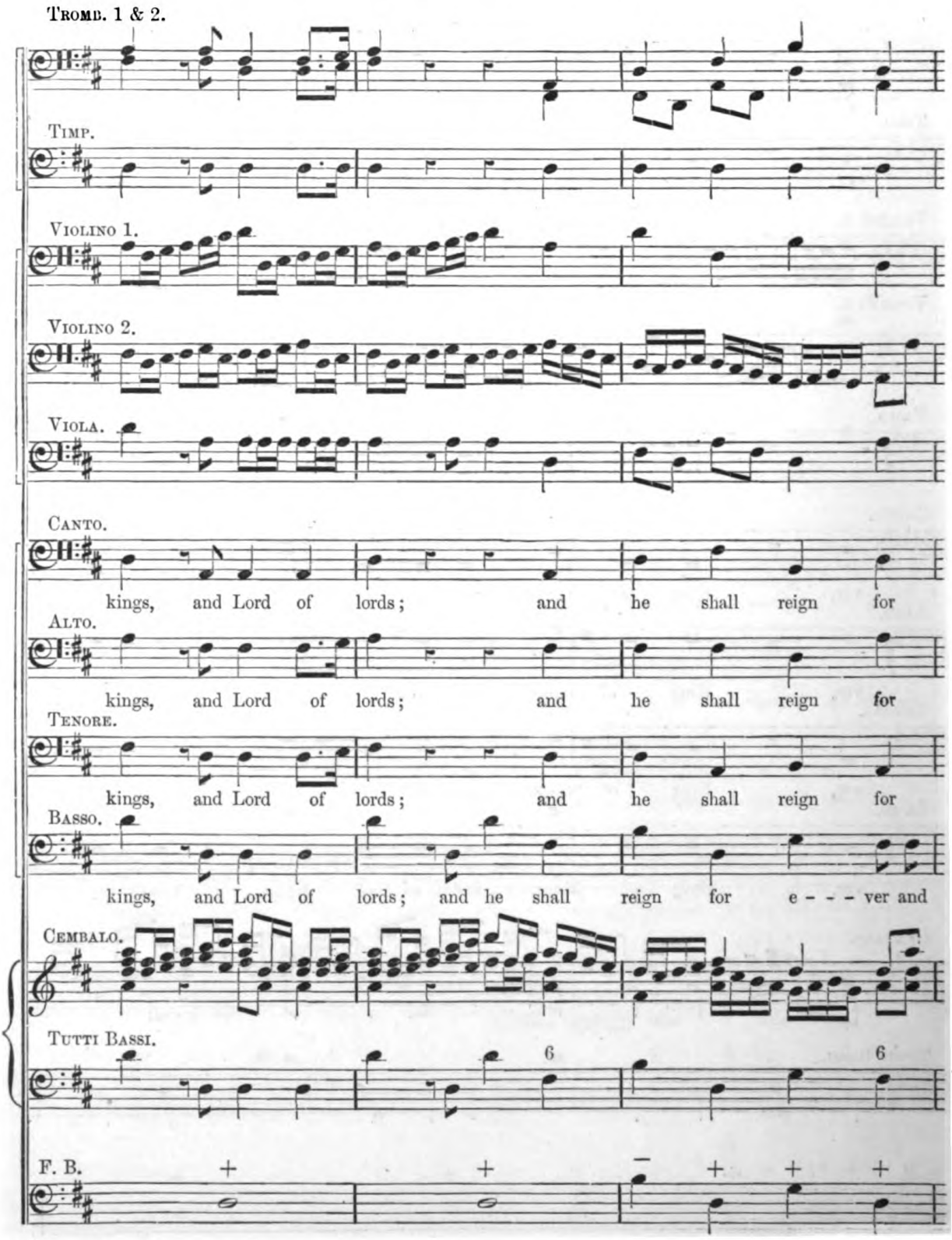


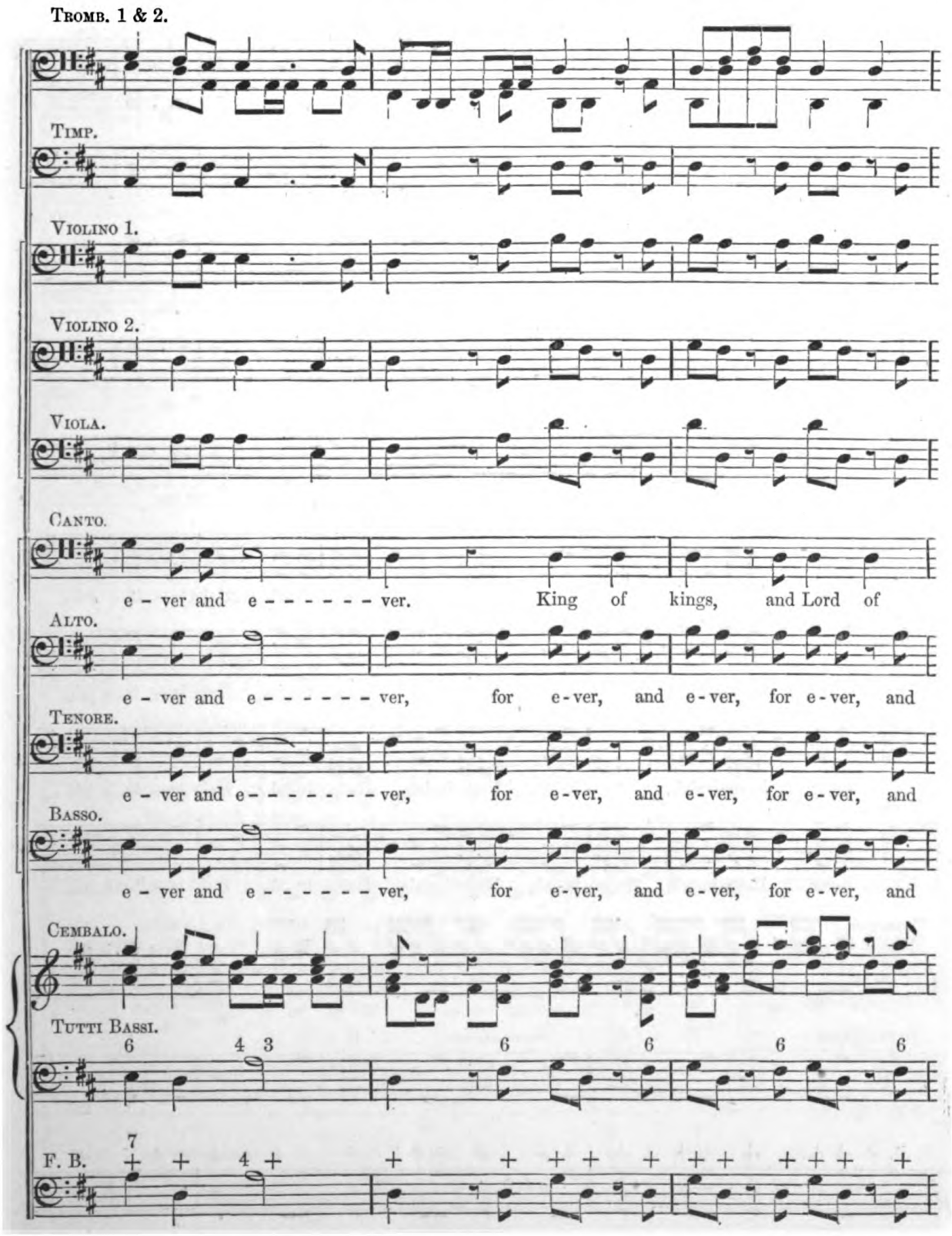
44. From the "Hallelujah Chorus." "Messiah."—HANDEL.

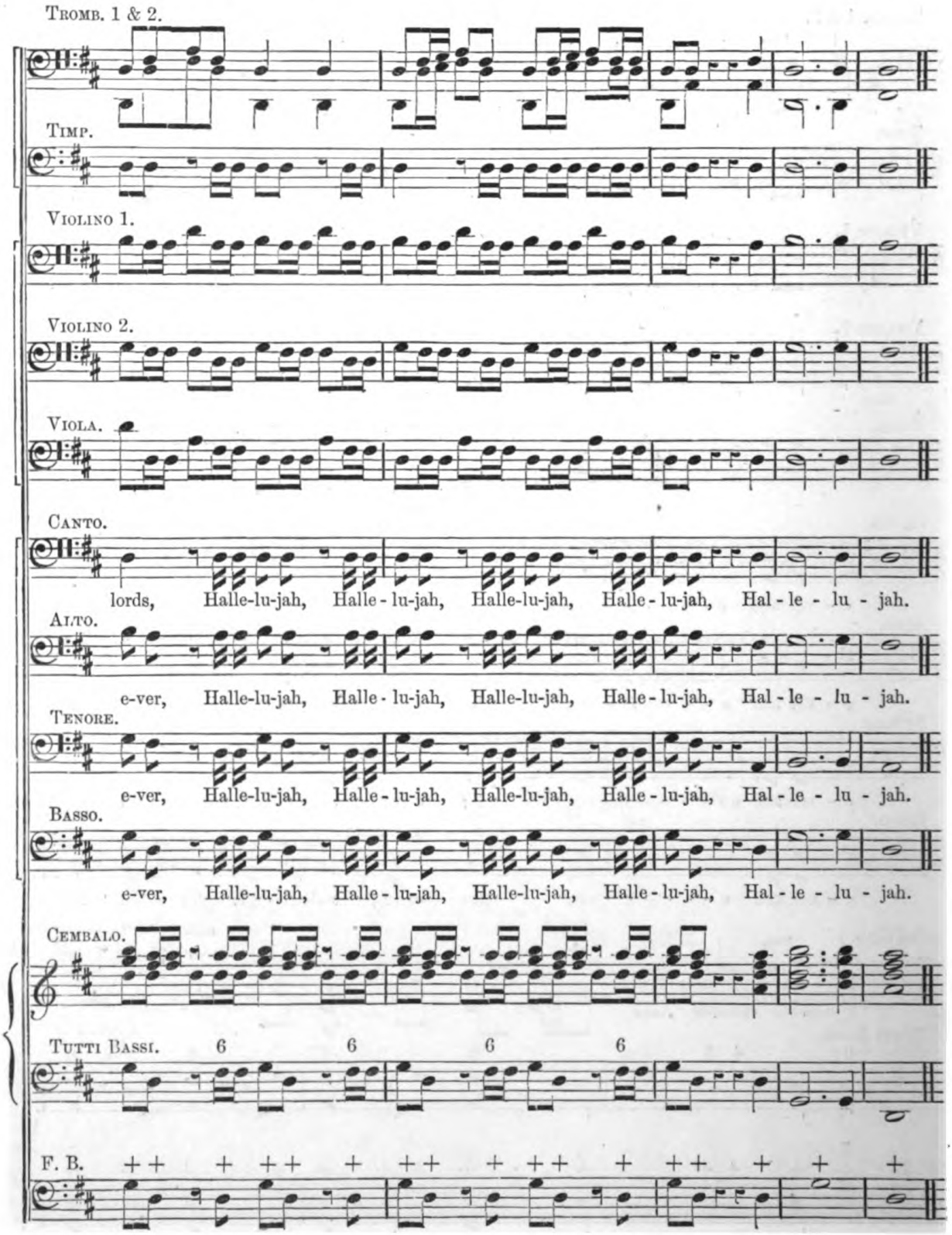



TROMB. 1 & 2.
TIMP.
VIOLINO 1.
VIOLINO 2.
VIOLA.
CANTO.
lu-jah, and He shall reign for e - - - ver and e - ver and e - - - - -
ALTO.
lu-jah, and He shall reign for e - ver and e - - - -
TENORE.
. . . and He shall reign for e - - - ver and e - ver and e - - - -
BASSO.
lu-jah, and He shall reign for e - - - ver and e - ver and e - - - -
CEMBALO.
TUTTI BASSI.
6 6 6 4 3
F. B.
7
+ + + + - + + + + + 4 +

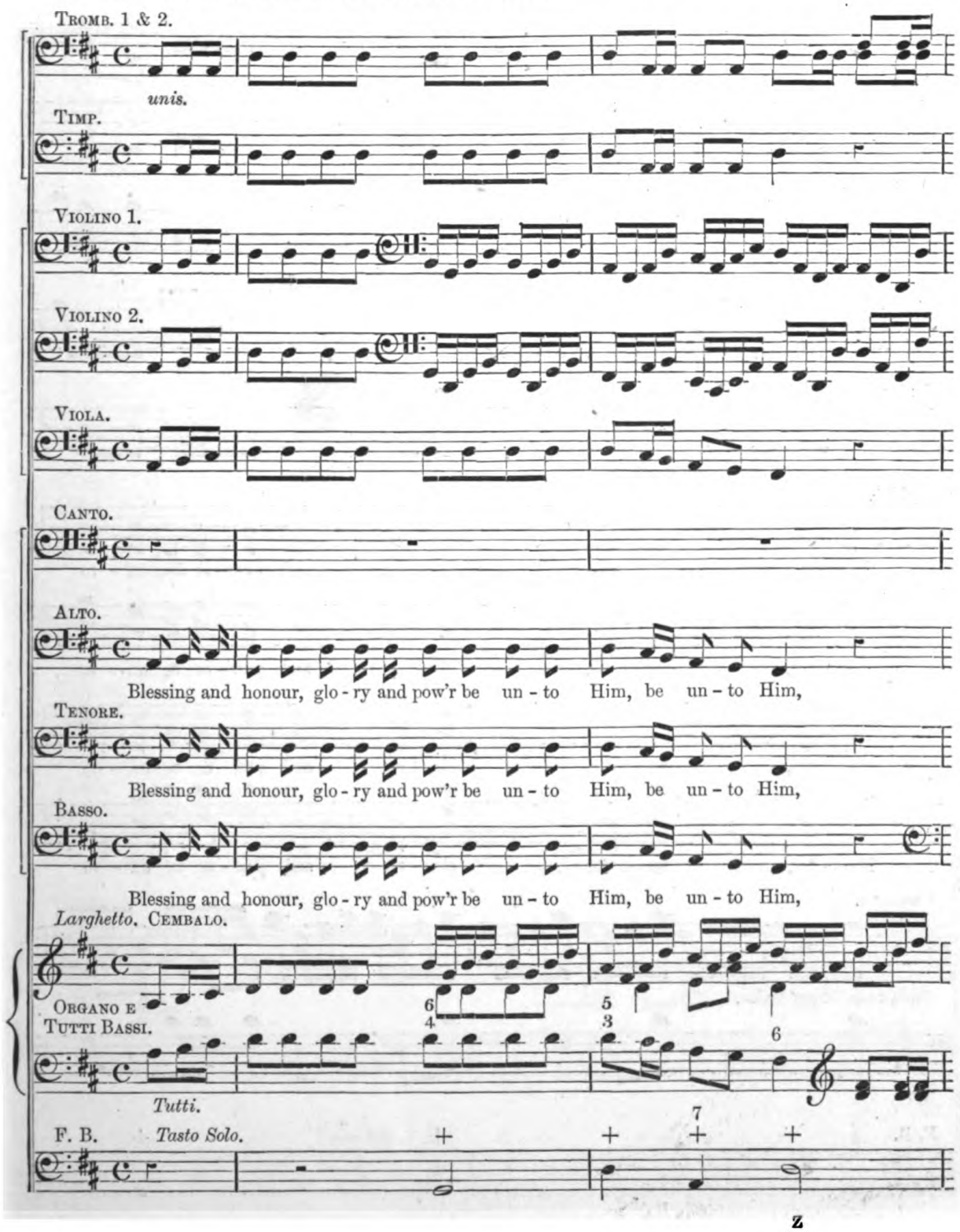

TROMB. 1 & 2.
TIMP.
VIOLINO 1.
VIOLINO 2.
VIOLA.
CANTO.
ver, King of kings, and Lord of lords; King of
ALTO.
ver, King of kings, and Lord of lords; King of
TENORE.
ver, King of kings, and Lord of lords; King of
BASSO.
ver, King of kings, and Lord of lords; King of
CEMBALO.
TUTTI BASSI.
F. B.

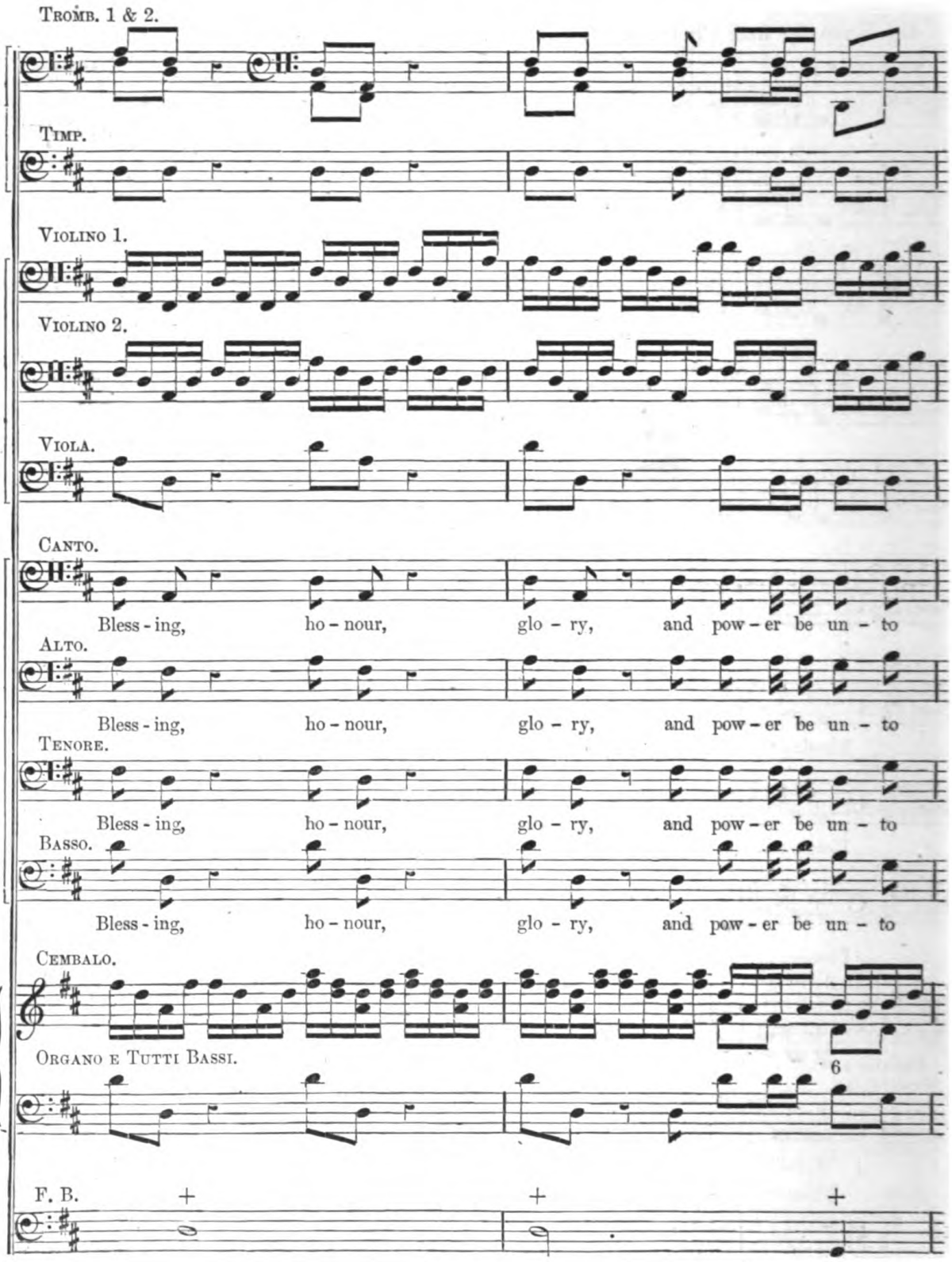

TROMB. 1 & 2.
TIMP.
VIOLINO 1.
VIOLINO 2.
VIOLA.
CANTO.
kings, and Lord of lords; and he shall reign for
ALTO.
kings, and Lord of lords; and he shall reign for
TENORE.
kings, and Lord of lords; and he shall reign for
BASSO.
kings, and Lord of lords; and he shall reign for e - - - ver and
CEMBALO.
TUTTI BASSI.
6
6
F. B.

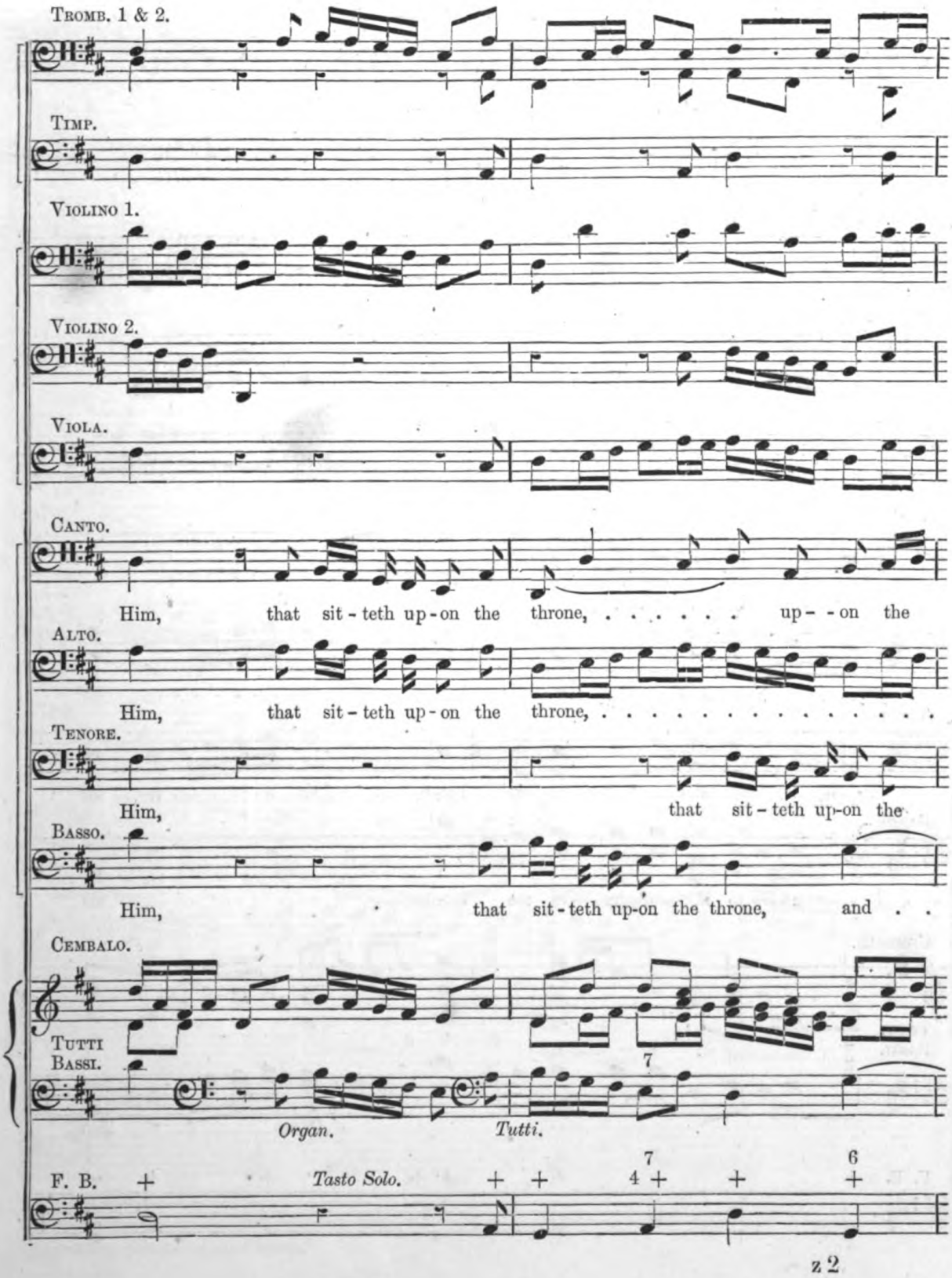

TROMB. 1 & 2.
TIMP.
VIOLINO 1.
VIOLINO 2.
VIOLA.
CANTO.
e - ver and e - - - - - - ver. King of kings, and Lord of
ALTO.
e - ver and e - - - - - - ver, for e - ver, and e - ver, for e - ver, and
TENORE.
e - ver and e - - - - - - - ver, for e - ver, and e - ver, for e - ver, and
BASSO.
e - ver and e - - - - - - ver, for e - ver, and e - ver, for e - ver, and
CEMBALO.
TUTTI BASSI.
6 4 3 6 6 6 6
F. B.
7
+ + 4 + + + + + + + + + + +

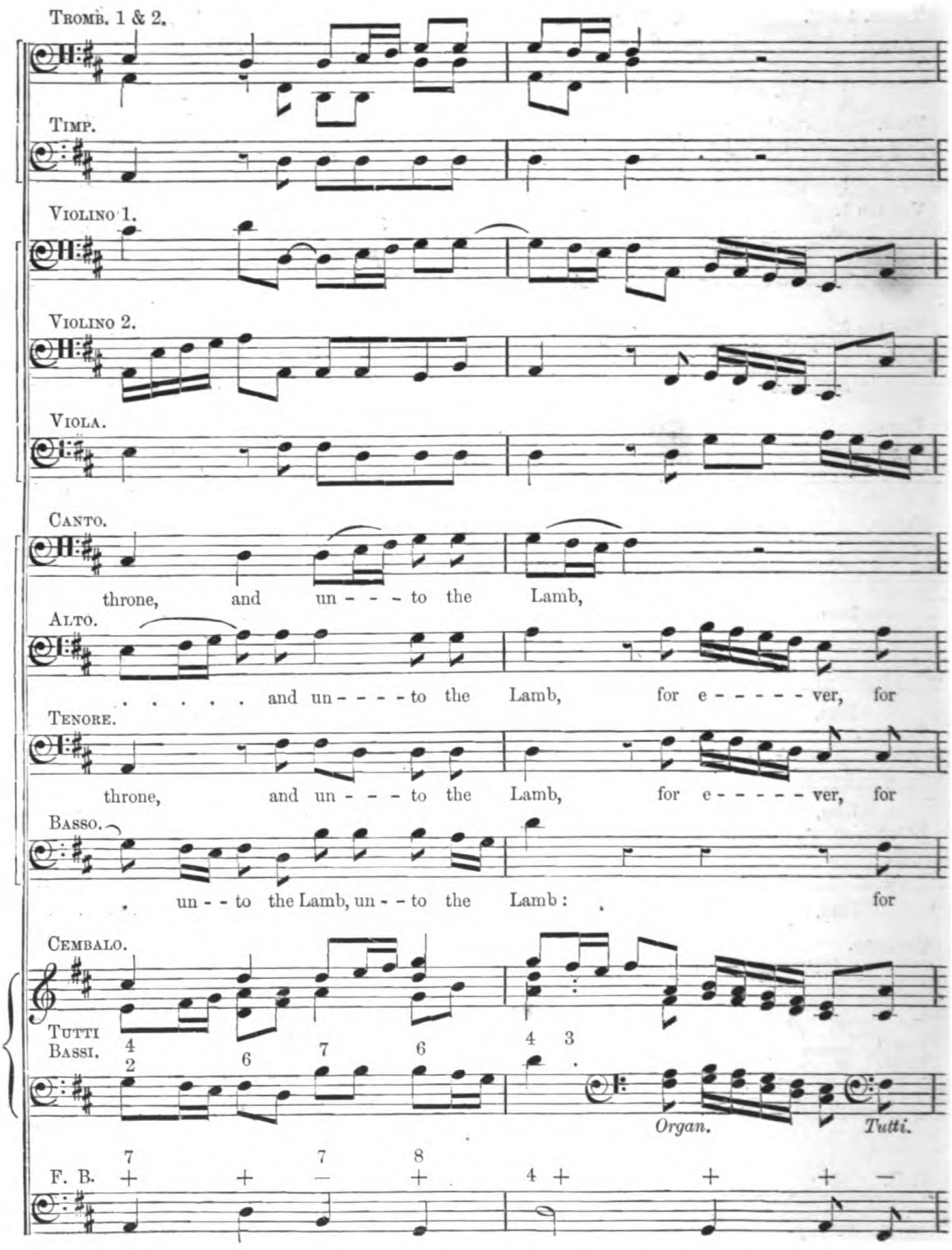

TROMB. 1 & 2.
TIMP.
VIOLINO 1.
VIOLINO 2.
VIOLA.
CANTO.
lords, Halle-lu-jah, Halle - lu-jah, Halle-lu-jah, Halle - lu-jah, Hal - le - lu - jah.
ALTO.
e-ver, Halle-lu-jah, Halle - lu-jah, Halle-lu-jah, Halle - lu-jah, Hal - le - lu - jah.
TENORE.
e-ver, Halle-lu-jah, Halle - lu-jah, Halle-lu-jah, Halle - lu-jah, Hal - le - lu - jah.
BASSO.
e-ver, Halle-lu-jah, Halle - lu-jah, Halle-lu-jah, Halle - lu-jah, Hal - le - lu - jah.
CEMBALO.
TUTTI BASSI. 6 6 6 6
F. B. +

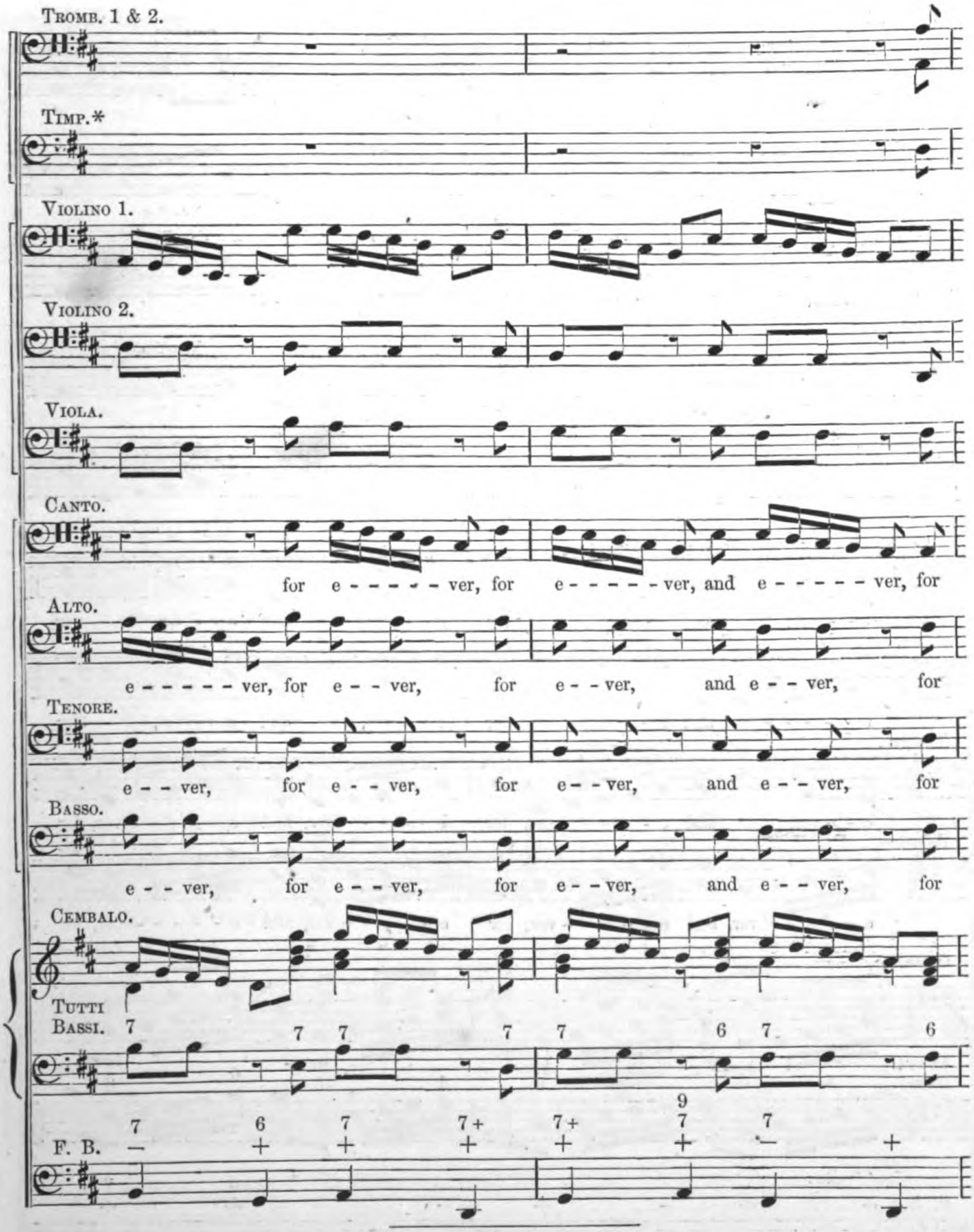
45. From the final Chorus of HANDEL's "Messiah."



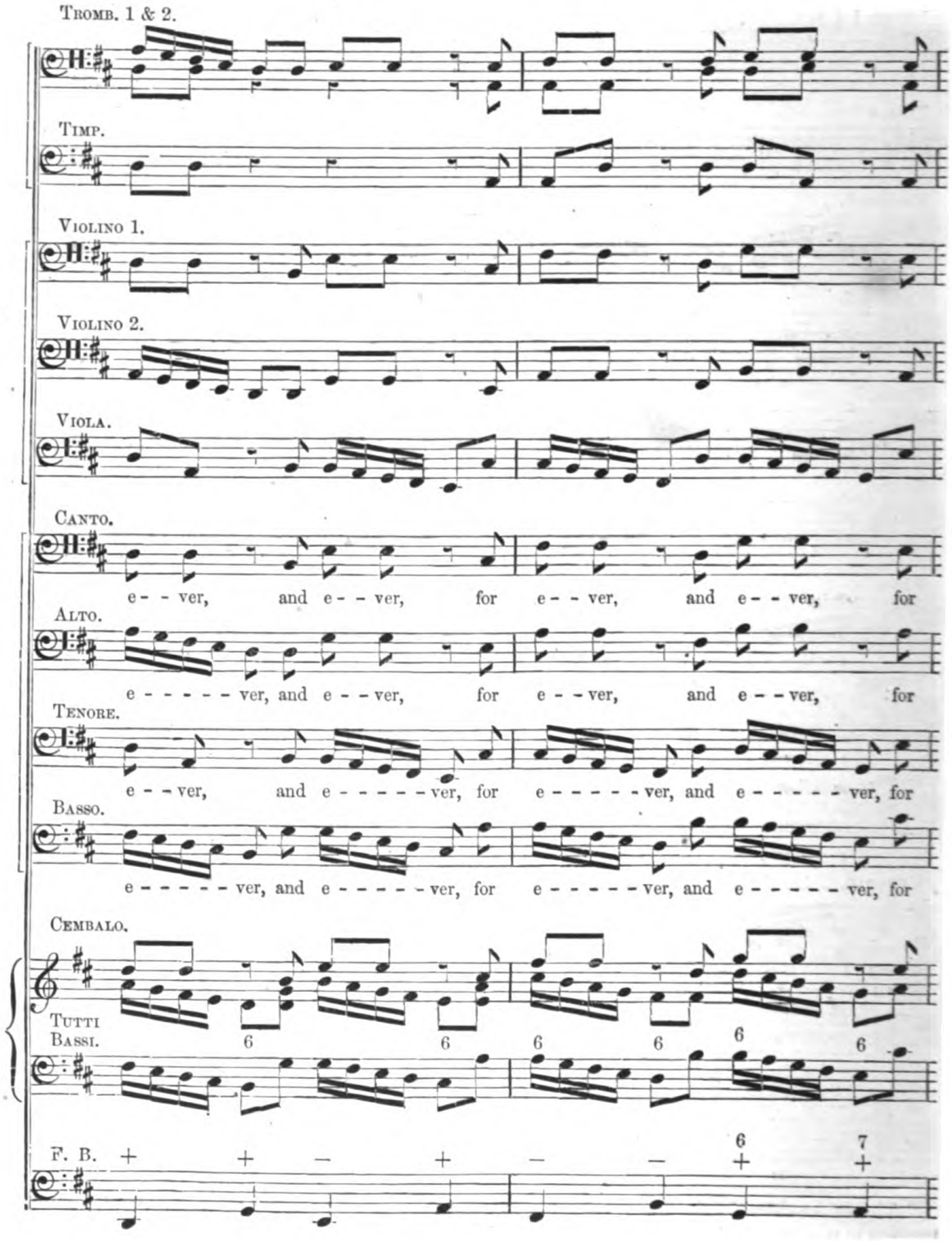

TROMB. 1 & 2.
TIMP.
VIOLINO 1.
VIOLINO 2.
VIOLA.
CANTO.
Bless - ing, ho - nour, glo - ry, and pow - er be un - to
ALTO.
Bless - ing, ho - nour, glo - ry, and pow - er be un - to
TENORE.
Bless - ing, ho - nour, glo - ry, and pow - er be un - to
BASSO.
Bless - ing, ho - nour, glo - ry, and pow - er be un - to
CEMBALO.
ORGANO E TUTTI BASSI.
6
F. B.

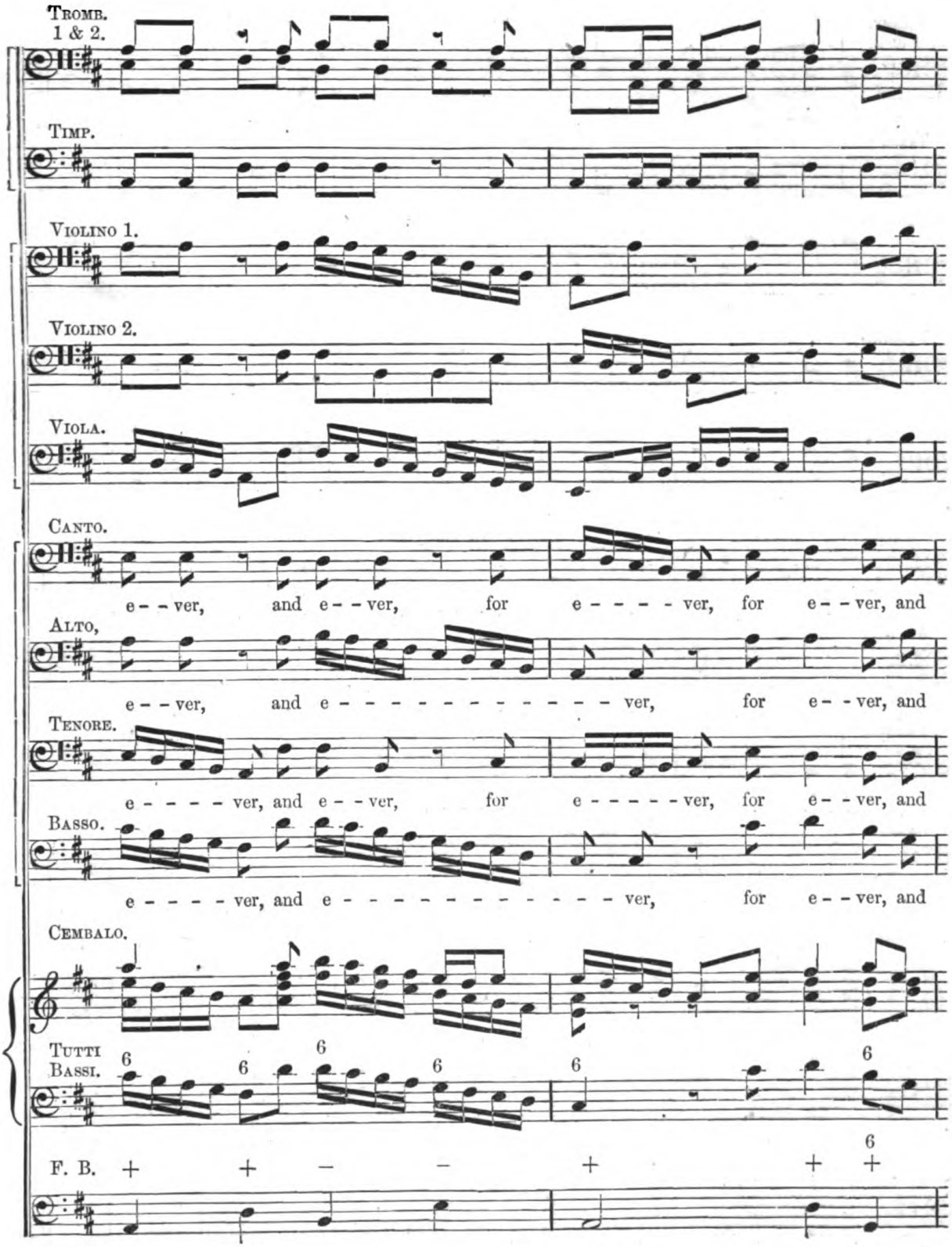

TROMB. 1 & 2.
TIMP.
VIOLINO 1.
VIOLINO 2.
VIOLA.
CANTO.
Him, that sit - teth up - on the throne, ... up - - on the
ALTO.
Him, that sit - teth up - on the throne, ...
TENORE.
Him, that sit - teth up - on the
BASSO.
Him, that sit - teth up - on the throne, and . .
CEMBALO.
TUTTI
BASSI.
Organ.
Tutti.
F. B.
Tasto Solo.
z 2

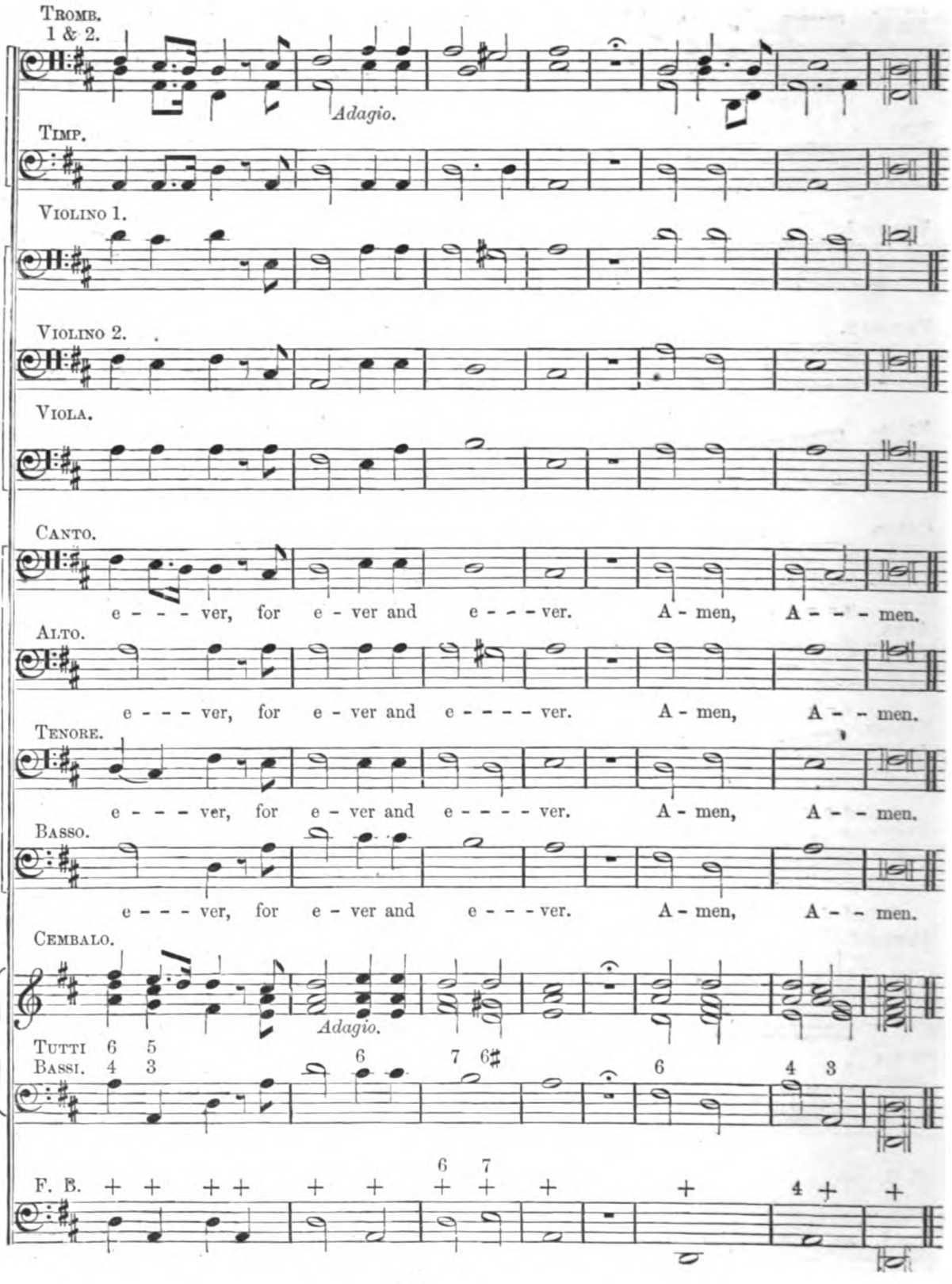

TROMB. 1 & 2.
TIMP.
VIOLINO 1.
VIOLINO 2.
VIOLA.
CANTO.
throne, and un - - - to the Lamb,
ALTO.
. . . . . and un - - - - to the Lamb, for e - - - - - ver, for
TENORE.
throne, and un - - - to the Lamb, for e - - - - - ver, for
BASSO.
un - - to the Lamb, un - - to the Lamb: for
CEMBALO.
TUTTI
BASSI.
4 6 7 6 4 3
2
Organ.
Tutti.
F. B.
7 7 8 4
+ + — + + + + —



* The notes written for the drums, being confined to the Tonic and Dominant of the Key, are not to be regarded in reckoning the Chords.


TROMB. 1 & 2.
TIMP.
VIOLINO 1.
VIOLINO 2.
VIOLA.
CANTO.
e - - ver, and e - - ver, for e - - ver, and e - - ver, for
ALTO.
e - - - - - ver, and e - - ver, for e - - ver, and e - - ver, for
TENORE.
e - - ver, and e - - - - - ver, for e - - - - - ver, and e - - - - - ver, for
BASSO.
e - - - - - ver, and e - - - - - ver, for e - - - - - ver, and e - - - - - ver, for
CEMBALO.
TUTTI
BASSI.
6 6 6 6 6 6
F. B. + + - + - - 6 + 7 +

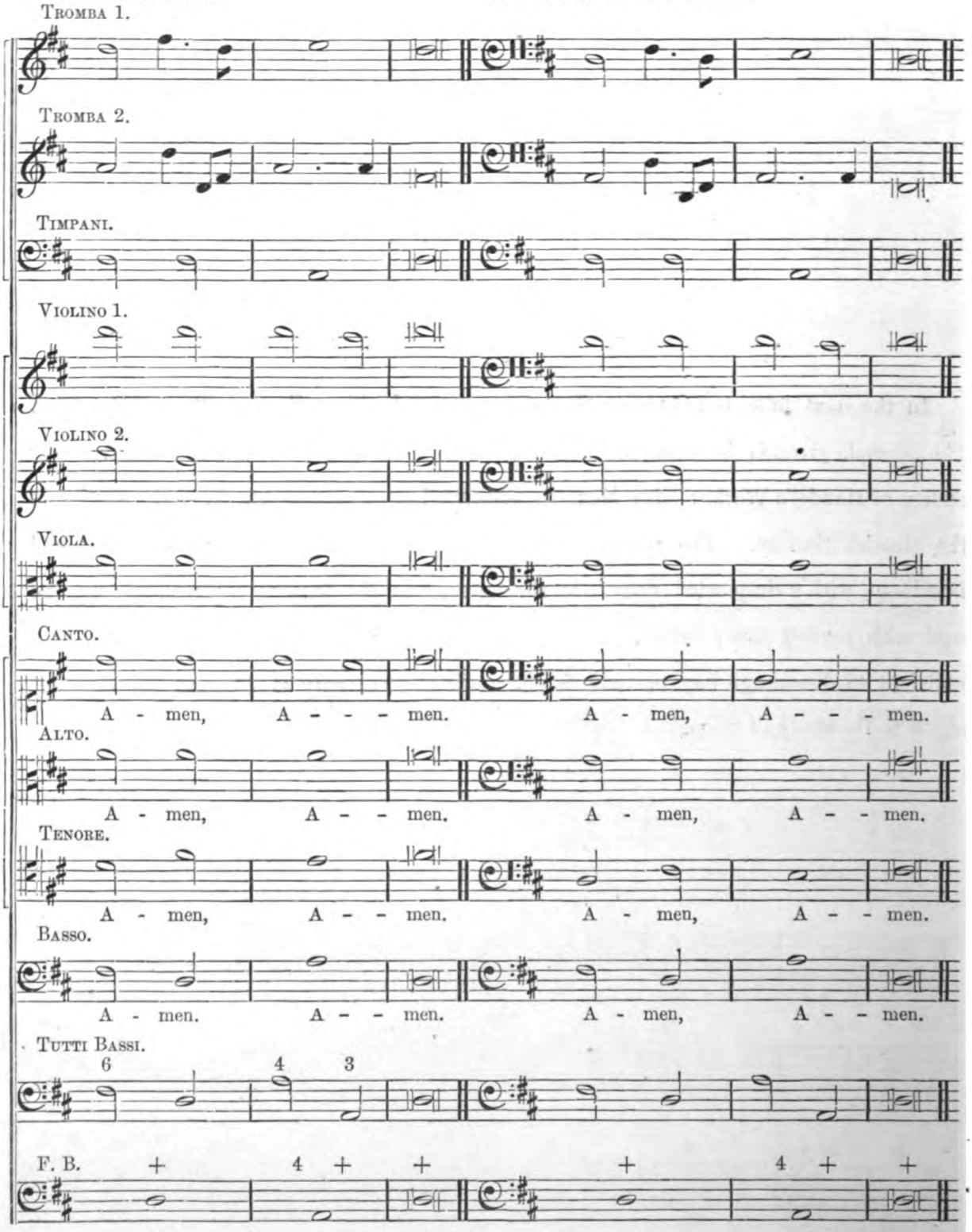

TROMB. 1 & 2.
TIMP.
VIOLINO 1.
VIOLINO 2.
VIOLA.
CANTO.
e - - ver, and e - - ver, for e - - - - - ver, for e - - ver, and
ALTO,
e - - ver, and e - - - - - - - - - - - - - - ver, for e - - ver, and
TENORE.
e - - - - - ver, and e - - ver, for e - - - - - ver, for e - - ver, and
BASSO.
e - - - - - ver, and e - - - - - - - - - - - - - - ver, for e - - ver, and
CEMBALO.
TUTTI BASSI.
6 6 6 6 6 6
F. B. + + - - + + 6 +


TROMB. 1 & 2.
Adagio.
TIMP.
VIOLINO 1.
VIOLINO 2.
VIOLA.
CANTO.
e - - - ver, for e - ver and e - - - ver. A - men, A - - - men.
ALTO.
e - - - ver, for e - ver and e - - - - ver. A - men, A - - men.
TENORE.
e - - - ver, for e - ver and e - - - - ver. A - men, A - - men.
BASSO.
e - - - ver, for e - ver and e - - - ver. A - men, A - - men.
CEMBALO.
Adagio.
TUTTI 6 5
BASSI. 4 3
6 7 6♯ 6 4 3
F. B. + + + + + + 6 7 + + + 4 + +

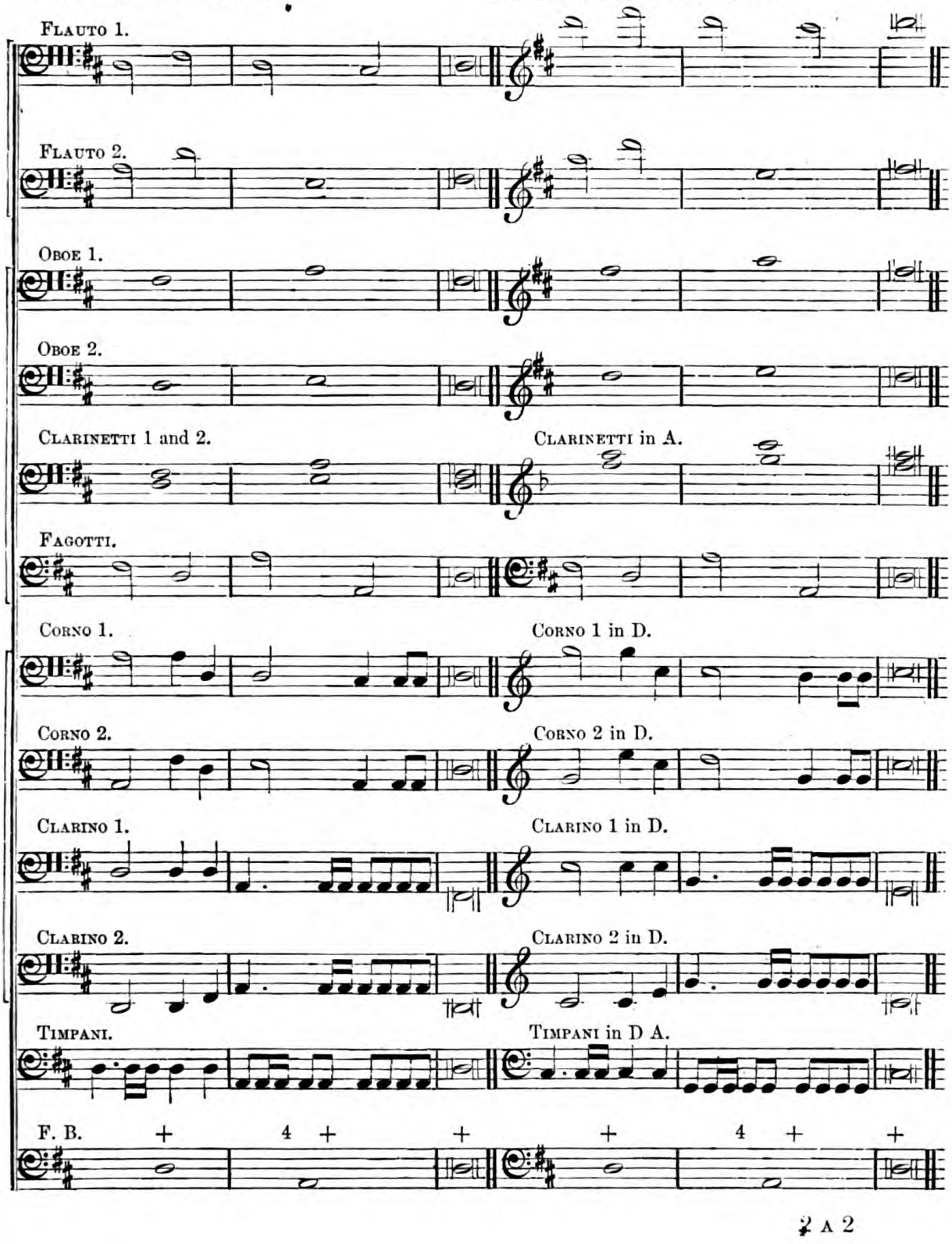
46.

In the next folio, the Common Score is contrasted with a Score written in One Clef. The example given is the Final Cadence of the Amen Chorus in the Messiah, from Arnold's edition of Handel's Works; with Mozart's additional accompaniments, from the edition of the Handel Society. The passage consists of two Major Triads, on the Tonic and Dominant, with a Suspended Fourth on the latter. In the one Score, the chords may be read with perfect ease; while, in the other, the confusion of Clefs and Signatures is sufficient to discourage any one who is not under the necessity of devoting his days and nights to the study of them.

HANDEL'S SCORE. SCORED IN THE F CLEF.



SCORED IN THE F CLEF. MOZART'S ACCOMPANIMENTS.



APPENDIX A.

ON THE PITCH OF SOUNDS.

1. THE pitch of a Musical Sound, is the place which it occupies in the scale of acuteness and gravity. There being no natural standard to determine the pitch of any particular sound, Pitch is properly a relative term: but practice has produced a standard, known by the name of Concert-pitch; in which the place of the Tenor Clef-note, C, is given to the sound of a two-foot organ-pipe. This note is understood to vibrate 256 times in a second. Wherefore the vibrations of the lowest string in the violoncello, which is two Octaves lower, are 64, or the third power of 4, in a second.*

2. Two similar strings, stretched by equal forces or weights, will sound a note of the same pitch.

3. If the two strings differ in length, the longer will sound a graver or lower note, and the shorter, a more acute or higher note; and that in an inverse proportion to the length of the strings.

4. The difference of pitch between two notes, is called an Interval.

5. The relative length of two similar strings, stretched by equal forces, is a true measure of the Interval between their sounds: which Interval is expressed by the proportion which the shorter string bears to the longer; as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, &c.

6. The cause of this difference of pitch, is the different rate or velocity of the vibrations whereby the sound is produced; which are slower in the longer string, and quicker in the shorter, in exact proportion to their respective lengths. Wherefore,—

7. The relative rate of vibration, is also a true measure of the Interval between the sounds of two strings; which Interval is expressed by the proportion between the vibrations of the longer string, and those of the shorter; as 1:2, 2:3, 3:4, &c.

8. The length of the strings, and the rate of vibration, being proportioned to each other, though inversely: and the figures employed to express them, being the same: it makes no difference which of these measures be adopted. But as the former, being a process of division, soon leads to fractions, the latter is generally preferred, as more convenient. Thus the Diatonic Scale may be represented by numbers, denoting the relative vibration of the notes of the Scale:

24	27	30	32	36	40	45	48
C	D	E	F	G	A	B	C

* Since this Appendix was written, I have seen an interesting paper on the subject by Sir John Herschel; in which he justly censures the late raising of Concert-pitch in this country, and recommends 256 for the number of vibrations in the Tenor Clef note, as being the eighth Octave to a theoretical sound that would vibrate once in a second.

APPENDIX B.

ON THE VIBRATION OF MUSICAL STRINGS.

THE elements which affect the rate of vibration in musical strings, are three: viz. Length, Weight, and Tension.

1. In strings which are similar in all respects but Length, the vibrations are inversely as the Length. Thus,—

Let A and B be similar strings, differing in Length: the length of A being = 3, and that of B = 2: their respective rates of vibration will be, inversely, as 3 to 2; viz. A 2, B 3.

2. In strings which are similar in all respects but Tension, the vibrations are, directly, as the square root of the Tension.

Let C and D be similar strings; C being stretched by a weight = 4, and D by a weight = 16: their rates of vibration will be as 2, the square root of 4, to 4, the square root of 16: viz. C 2, D 4.

3. In strings which are similar in all respects but weight, the vibrations are, inversely, as the square root of the Weight.

Let E and F be similar strings, differing in Weight: the weight of E being = 16, and the weight of F = 9: their rates of vibration will be, inversely as 4, the square root of 16, to 3, the square root of 9: viz. E 3, F 4.

4. In strings of equal Length, the vibrations are as the square root of the Tension, divided by the square root of the Weight. Thus,—

Let G and H be strings of equal Length; the Weight of G being = 4, and the Tension = 16: the Weight of H = 9, and its Tension = 36: their rates of Vibration will be, as 4, the square root of 16, divided by 2, the square root of 4: to 6, the square root of 36, divided by 3, the square root of 9: viz.—

G . . . . $4 \div 2 = 2.$
H . . . . $6 \div 3 = 2.$

5. In strings of equal specific gravity, the weight of equal lengths of string is as the bulk or volume. The volume is as the areas of their transverse sections; the square roots of which are as their diameters. Wherefore,—

6. In strings of equal Specific Gravity, but of unequal Length and Thickness, the Vibrations are as the square root of the Tension, divided by the Diameter of the string multiplied into its Length: thus—

$$V = \frac{\sqrt{T}}{D \times L.}$$

7. As a general rule to find the comparative rate of Vibration: Divide the Weight of the string by the Length; extract the square root of the quotient, and multiply it by the Length: then divide the square root of the Tension by the product: thus—

$$V = \frac{\sqrt{T}}{L \times \sqrt{W \div L.}}$$

APPENDIX C.

HARMONICS.

1. FOR an exposition and demonstration of the theory of Harmonics, the reader is referred to Sir John Herschel's valuable and interesting Treatise on Sound, Part 2, Sect. 2, "On the Vibrations of Musical Strings or Chords," *Encyclopædia Metropolitana*, vol. iv: from which the following extracts are taken:—

"161. Some curious and important consequences follow from this. And, first, a cord, although vibrating freely, may yet have any number of points, equally distributed at aliquot parts of its whole length, which never leave the axis, and between which the vibrating portions are equal and similar, and lie alternately above and below the axis, and in reversed position as to right and left. Such points of rest are called nodes, or nodal points: the intermediate portions which vibrate, are termed bellies, or ventral segments.

"162. Secondly, if a string in the act of vibration be touched in any point, so as to reduce that point to rest, and retain it in the axis, then if, after the contact, it vibrate at all, it will divide itself into a certain number of ventral parts, similar and equal to each other, and separated by nodes: and each of these will vibrate as if the others had no existence, but instead of the nodes, were fixed points of attachment.

"163. Experience confirms this. If the string of a violin or violoncello, while maintained in the act of vibration by the action of the bow, be lightly touched with the finger, or a feather, exactly in the middle, or at one third of the length, it will not cease to vibrate: but its vibrations will be diminished in extent, and increased in frequency; and a note will become audible, fainter but much more acute than the original, or, as it is termed, the fundamental note of the string; and corresponding in the former case to a double, in the latter to a triple rapidity of vibration; the note heard in the former case being the Octave, in the latter the Twelfth, above the fundamental tone. If a small piece of light paper, cut into the form of an inverted V, be set astride on the string, it will be violently agitated, and probably thrown off, when placed in the middle of a ventral segment: while, at a node, it will ride quietly as if the string were (as it really is at those points) at perfect rest. The sounds thus produced, are termed Harmonics.

"164. But further, any number of the different modes of vibration of which a cord is thus susceptible, may be going on simultaneously, or be, as it were, superposed on each other.

This is a consequence of the principle in mechanics, of the superposition of small motions; which, when the excursions of the parts of a system from their places of rest, are infinitely small, admits of any or all the motions of which, from any causes, they are susceptible, to go on at once, without interfering with or disturbing each other.

"166. Experience again confirms this result of theory. It was long known to musicians that, besides the principal or fundamental note of a string, an experienced ear could detect in its sound, when set in vibration, especially when very lightly touched in certain points, other notes, related to the fundamental one by fixed laws of harmony, and which are called therefore, Harmonic Sounds. They are the very same which, by the production of distinct nodes, may be insulated, as it were, and cleared from the confusing effect of the co-existent sounds, as in Art. 163. They are, however, much more distinct in bells, and other sounding bodies, than in strings; in which only delicate ears can detect them." —Pages 781, 782.

2. The Harmonics particularly specified in the preceding extracts, are the Octave, and the Twelfth, or Fifth to the Octave; produced by the division of the string into two, and into three, ventral segments.

3. In following out the theory laid down, if the string be lightly touched at $\frac{1}{4}$ of its length, the Harmonic of the Double Octave will be heard, by the division of the string into 4 segments. In the same way, the points $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, will produce the Major Third, Fifth, and Grave Seventh, of the Double Octave.

4. Again, if the string be touched at $\frac{1}{8}$ of its length, the Harmonic of the third Octave will be heard; and at $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{12}$, the Harmonics will be the Second, Major Third, and Fifth, of the third Octave. But if the string be divided into segments by 11, or any other prime number above 7, no distinct harmonic sound will be obtained.

5. Hence it appears, that a vibrating string divides itself freely by the first seven numbers, but refuses to be divided by any prime number above 7. This primary division of the string comprehends all the Intervals usually styled Concords, and the Dominant Harmony, which will be noticed afterwards. (Appendix E, 9; Class 1.)

6. But according to the theory advanced in Art. 164, a subdivision by the same numbers may be superposed on the first division. Thus, $\frac{1}{3}$ of the string, divided by 3, is $= \frac{1}{9}$; $\frac{1}{5}$ divided by 3, is $= \frac{1}{15}$; $\frac{1}{5}$ divided by 5 is $= \frac{1}{25}$, and so on. This secondary division comprehends the Intervals derived from two adjacent Triads. (Appendix E, 9; Class 2.)

7. Still farther, the secondary division may be again subdivided by the same numbers: as $\frac{1}{9}$ by 3, $= \frac{1}{27}$; $\frac{1}{15}$ by 3, $= \frac{1}{45}$th. This is the foundation of the false Intervals derived from unconnected Triads; which are generally rejected in good Harmony. (Appendix E, 12.)

8. The numbers 2, 3, 5, are held, by all theorists, to be Harmonic primes. The only

question is with respect to 7, which represents the Grave or Dominant Seventh; and is rejected, with one consent, by almost all writers on Musical Theory.

9. Mr. Logier, so far as I know, is almost the only writer in the present century, who has adopted and advocated the Harmonic Seventh. That it was not unknown, however, during last century, is evident from the following passage in Kollmann's "Essay on Harmony," by which it appears that it was not only held in theory, but reduced to practice in organ-building. "Of these two Intervals $\frac{4}{7}$ and $\frac{7}{8}$, which are not in our modern system, but which are found in an organ at Berlin, (if I am not mistaken it is in Dreyfaltigkeits Kirche) see Kirnberger's 'Kunst des reines Satzes,' Part 1 and 2: also Jamard's 'Recherches sur la Théorie de la Musique,' Paris and Rouen."*

10. On this subject, I take the liberty of quoting some valuable remarks from the same Treatise on Sound, by Sir John Herschel, Art. 253.† "Mr. Logier has lately, in a Work of great practical utility, and very extensive circulation among musical Students, endeavoured to place the interpolation of the intermediate notes between those of the natural Scale, on *à priori* grounds, by assuming the flat seventh as the seventh *harmonic* of the fundamental note; that is to say, the note produced by subdividing into seven equal parts the length of a string whose fundamental tone is 1; or at least one of the Octaves of that note. There is something ingenious in this idea. In the first place it completes the series of the first 10 harmonics, or notes, whose vibrations are multiples by 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, of those of the fundamental tone. Again, it gives something like a plausible reason for the prominent importance of the Chord of the flat Seventh in music. This Chord, in fact, which, if we take 1 for a fundamental note, consists of the notes 1, 3, 5, 7♭, becomes, in this point of view, a *perfect concord*, consisting entirely of harmonics of 1, and its pulses will succeed each other on the ear, in a cycle comprising four vibrations of the fundamental tone, 1, five of the next, 3, six of the next, 5, and seven of the essential note, 7♭, as represented in fig. 35. The succession of pulses in the common chord is also represented in the same figure (*see* 'Encyclopædia,') and its regularity and pleasing variety, even to the eye, explains its agreeable effect on the ear. It is for musicians to say, whether they can make up their minds to regard the discord of the seventh in the light of a perfect concord, or no. There is certainly nothing at all *discordant*, in the vulgar sense of the word, *i. e.*, unpleasant, in its sound, and so far it may be regarded as at least '*discordia concors*;' but so far from possessing the essential character of a concord, that the ear is satisfied in hearing it, and expects and desires no more,—there is no discord which calls so urgently for resolution."‡

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* Kollmann's "Essay on Musical Harmony," folio, London, 1796, page 11.

† Encyclopædia Metropolitana, vol. iv. page 797.

‡ The word Discord is used, by writers on music, in very loose and improper senses. There is a grammar in music, as well as in language. The Dominant Seventh is no more proved to be a discord by requiring another chord to follow, than an active verb is, by requiring an accusative case. To call every Interval a discord, that

11. In this passage, Sir John points out the true origin and proportions of the Dominant Harmony. The objections which he afterwards alleges against the use of the Harmonic Seventh in Music, do not apply to anything advanced in this work.

12. The Octave may be harmonically divided these three ways :—

1st. Into two, by the Fifth :
C G C . . . . 2 . 3 . 4.

2nd. Into three, by the Fourth and Major Sixth :
C F A C . . . . . 3 . 4 . 5 . 6.

3rd. Into four, by the Dominant Harmony :
C E G B♭ C . . . . 4 . 5 . 6 . 7 . 8.

The Harmonic Sound between 6 and 8, which is distinctly audible in the Harmonics of a Bass string on the pianoforte, is the mean proportional between the Fifth and the Octave.

Fifth . . . . . 2 : 3 × 2 = 4 : 6.
Seventh . . . . . 4 : 7.
Octave . . . . 1 : 2 × 4 = 4 : 8.

13. In the descending Scale C B A, 48, 45, 40, it is required to find a flat Seventh between B and A. All the intermediate numbers, except 42, contain prime numbers above 7, and are excluded. 42, which contains the prime number 7, represents B flat, Harmonic Seventh to C 24 :—

48	45	42	40.
C	B	B♭	A.

It may also be remarked that the interval B♭ C is harmonically divided by B natural.

14. All theorists who reject the Harmonic Seventh, substitute for it the Subdominant of the Scale, which differs from it in pitch by Komma Major, 63 : 64.* This difference is quite sufficient to account for the Dominant Harmony being still called a discord :—

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occupies certain degrees on the musical staff, is no better. In reply to these objections, Sir John wrote to the author as follows : "I like your idea of the 'discord' *governing* its resolution, as a verb in grammar *governs* its accusative case. There is nothing discordant in the abstract assertion *I love;* but the mind is not content, and feels the sense to be incomplete, unless it be followed out by an accusative case : I love somebody, or something."

* The following passage occurs in Brewster's *Edinburgh Cyclopædia*, Art. "Comma" :—"No. 21, 63 : 64, is a Comma mentioned by M. Chladni, and is the error, or flattening, of the Minor Seventh, on the trumpet and horn."

That is to say, the Harmonic Seventh, which is produced by those instruments, is flatter than the Minor Seventh of the Dominant, by that Interval.

Triad of F . . . . F A C F . . . . 4 . 5 . 6 . 8 × 8 = 32 . 40 . 48 . 64.
Triad of G . . . . G B D — . . . . 4 . 5 . 6 . — × 9 = 36 . 45 . 54 . —

Dominant Harmony, with Subdominant . . . . . 36 . 45 . 54 . 64.
Dominant Harmony, with Harmonic Seventh . . . . . 36 . 45 . 54 . 63.

15. The two Chords may be compared by means of two similar strings of sufficient length, F and G, stretched by equal weights upon a graduated scale; the length of F being 126 degrees and of G 112, representing the Subdominant and the Dominant of a Diatonic Scale. One-half of F, will sound the Octave to the Subdominant; and four-sevenths of G, the Harmonic Seventh to the Dominant. If an instrument be tuned correctly to the pitch of string G, and the Chord G B D sounded upon it, with the half of string F, = 63 degrees, as a Seventh to it, and then altered to 64 degrees, corresponding to four-sevenths of string G, the pleasing effect will be sensible to the ear, of passing from an ill-tuned Chord to a perfect Concord.

16. The substitution of the Subdominant for the Harmonic Seventh, is attended with the following difficulties:

1st.—The Dominant Harmony, which, of all Chords, is next in importance to the fundamental Concord, or Major Common Chord, is derived from two unconnected Triads; and so placed beyond the limits within which the rest of the Chords are confined. (See Chap. XII. 6.)

Triad of F . . . . . F A C F . . . . 4 . 5 . 6 . 8 × 8 = 32 . 40 . 48 . 64.
Triad of G . . . . . G B D — . . . . 4 . 5 . 6 . — × 9 = 36 . 45 . 54 . —

Dominant Seventh . . G B D F . . . . . 36 . 45 . 54 . 64.

2nd.—In consequence of this, the Dominant Chord contains the following false Intervals:

G F . . . . 9 : 16 . . . . Minor Seventh, minus Comma.
B F . . . . 45 : 64 . . . . Diminished Fifth, minus Comma.
D F . . . . 27 : 32 . . . . Minor Third, minus Comma.

3rd.—The Compound Dominant Harmonies are much worse; being derived from Triads still more remote:

DIMINISHED SEVENTH.

Triad of F . . . . . F A C . . . . . 4 . 5 . 6 × 32 = 128 . — . —
Triad of E . . . . . E G♯ B . . . . . 4 . 5 . 6 × 15 = 60 . 75 . 90.
Triad of G . . . . . G B D . . . . . 4 . 5 . 6 × 18 = 72 . 90 . 108.

Diminished Seventh . . G♯ B D F . . . . . 75 . 90 . 108 . 128.

Augmented Sixth.

Triad of F	F A C	4 . 5 . 6 × 32 =	128 . 160 . 192.
Triad of B	B D♯ F♯.	4 . 5 . 6 × 45 =	180 . 225 . 270.
Italian Sixth	F A – D♯.	. . . .	128 . 160 . – . 225.
French Sixth	F A B D♯.	. . . .	128 . 160 . 180 . 225.
German Sixth	F A C D♯.	. . . .	128 . 160 . 192 . 225.

4th. When the Chord changes from the Minor Seventh to the Dominant Seventh, how can the performer alter the Seventh *by the ear*, till it is *in tune* with the new Chord, if the new Chord be a discord? See Appendix H, 6, A, with the question and answer.

5th. The regular progression of the Harmonic Intervals is awkwardly interrupted. This may be seen by the Frontispiece;* in which the place of the substituted Seventh ($\frac{9}{64}$, instead of $\frac{9}{63}$ or $\frac{1}{7}$) is shown by a mark on the line E♭. It is evident to the eye, that the curve, if it were drawn through that mark, would be distorted. In like manner, a correct ear will detect the false tuning of the Chord, if the Subdominant note, or one of its Octaves, be taken as a Dominant Seventh.

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* The Frontispiece represents the comparative lengths of nine similar strings, which, if stretched by equal forces or weights, would sound the Harmonics of another string, of which the fractions marked on the margin are aliquot parts.

APPENDIX D.

HARMONIC NUMBERS.

1. A MUSICAL Interval, or the difference of pitch between two sounds, is correctly expressed by two numbers, (which I shall call Harmonic Numbers,) denoting the relative velocity of the vibrations by which the two sounds are produced. Thus the Interval of the Octave, in which the vibrations of the two sounds are in the proportion of 1 to 2, is correctly expressed by the numbers 1 and 2.

2. The only Prime Numbers that are found in the Harmonics of a musical sound are 2, 3, 5, 7; and to these, with their Compounds, all Harmonic Numbers are confined.

3. The following Table contains all the Harmonic Numbers as far as 64.

1									
2	4								
3	6	9							
4	8	12	16						
5	10	15	20	25					
6	12	18	24	30	36				
7	14	21	28	35	42	49			
8	16	24	32	40	48	56	64		
9	18	27	36	45	54	63	72	81	
10	20	30	40	50	60	70	80	90	100.

4. The next Table contains all the Harmonic Numbers up to 64, with select numbers above it; referred to their respective notes, as counted from F, the fountain of the Scale of C (Chap. IV. 6). The Thirds, Fifths, and Sevenths, in the Table, are the Harmonic Intervals $\frac{4}{5}$, $\frac{2}{3}$, and $\frac{4}{7}$. All the even numbers are, of course, Octaves to lower numbers in the Table.

TABLE OF HARMONIC NUMBERS.

NUMBERS.			COMPONENTS.	LOGARITHMS.
1	F	The Root	.. ..	0·
2	F	Octave to 1	2	6·578,813
3	C	Fifth to 2	3	10·427,173
4	F	Octave to 2	2, 2	13·157,627
5	A	Third to 4	5	15·275,532
6	C	Fifth to 4	2, 3	17·005,986
7	E♭	Seventh to 4	7	18·469,064
8	F	Octave to 4	2, 2, 2	19·736,440
9	G	Fifth to 6	3, 3	20·854,345
10	A	Third to 8	2, 5	21·854,345

NUMBERS.			COMPONENTS.	LOGARITHMS.
12	C	Fifth to 8	2, 2, 3	23·584,800
14	$\overset{\Delta}{\text{E}}\flat$	Seventh to 8	2, 7	25·047,878
15	E	Third to 12	3, 5	25·702,705
16	F	Octave to 8	2^4	26·315,254
18	G	Fifth to 12	2, 3, 3	27·433,159
20	A	Third to 16	2, 2, 5	28·433,159
21	$\overset{\Delta}{\text{B}}\flat$	Seventh to 12	3, 7	28·896,237
24	C	Fifth to 16	2, 2, 2, 3	30·163,613
25	C♯	Third to 20	5, 5	30·551,064
27	D	Fifth to 18	3, 3, 3	31·281,518
28	$\overset{\Delta}{\text{E}}\flat$	Seventh to 16	2, 2, 7	31·626,691
30	E	Third to 24	2, 3, 5	32·281,518
32	F	Octave to 16	2^5	32·894,067
35	$\overset{\Delta}{\text{G}}$	Seventh to 20	5, 7	33·744,596
36	G	Fifth to 24	2, 2, 3, 3	34·011,972
40	A	Third to 32	2, 2, 2, 5	35·011,972
42	$\overset{\Delta}{\text{B}}\flat$	Seventh to 24	2, 3, 7	35·475,050
45	B	Third to 36	3, 3, 5	36·129,877
48	C	Fifth to 32	2^4, 3	36·742,427
49	$\overset{\Delta}{\text{D}}\flat$	Seventh to 28	7, 7	36·938,129
50	C♯	Octave to 25	2, 5, 5	37·129,877
54	D	Fifth to 36	2, 3, 3, 3	37·860,331
56	$\overset{\Delta}{\text{E}}\flat$	Seventh to 32	2, 2, 2, 7	38·205,505
60	E	Octave to 30	2, 2, 3, 5	38·860,331
63	$\overset{\Delta}{\text{F}}$	Seventh to 36	3, 3, 7	39·323,410
64	F	Octave to 32	2^6	39·472,881
72	G	Octave to 36	2, 2, 2, 3, 3 ..	40·590,785
75	G♯	Third to 60	3, 5, 5	40·978,236
80	$\overset{-}{\text{A}}$	Third to 64	2^4, 5	41·590,786
81	$\overset{+}{\text{A}}$	Fifth to 54	3, 3, 3, 3	41·708,691
90	B	Octave to 45	2, 3, 3, 5	42·708,691
96	C	Octave to 48	2^5, 3	43·321,240
100	C♯	Octave to 50	2, 2, 5, 5	43·708,691
108	D	Octave to 54	2, 2, 3, 3, 3 ..	44·439,145
112	$\overset{\Delta}{\text{E}}\flat$	Octave to 56	2^4, 7	44·784,318
120	E	Octave to 60	2^3, 3, 5	45·439,145
125	E♯	Third to 100	5, 5, 5	45·826,596
126	$\overset{\Delta}{\text{F}}$	Octave to 63	2, 3, 3, 7	45·902,223
128	F	Octave to 64	2^7	46·051,694
135	F♯	Third to 108	3, 3, 3, 5	46·557,050

NUMBERS.			COMPONENTS.	LOGARITHMS.
144	G	Octave to 72	2^4, 3, 3	47·169,599
150	G♯	Octave to 75	2, 3, 5, 5	47·557,050
160	$\bar{A}$	Octave to 80	2^5, 5	48·169,599
162	$\overset{+}{A}$	Octave to 81	2, 3, 3, 3, 3 ..	48·287,504
180	B	Octave to 90	2, 2, 3, 3, 5 ..	49·287,504
192	C	Octave to 96	2^6, 3	49·900,053
200	C♯	Octave to 100	2, 2, 2, 5, 5 ..	50·287,504
216	D	Octave to 108	2^3, 3, 3, 3	51·017,958
224	$\overset{\Delta}{E}$♭	Octave to 112	2^5, 7	51·363,132
225	D♯	Third to 180	3, 3, 5, 5	51·405,410
240	E	Octave to 120	2^4, 3, 5	52·017,958
250	E♯	Octave to 125	2, 5, 5, 5	52·405,409
256	F	Octave to 128	2^8	52·630,507

5. The Numbers in the preceding Table, are produced by multiplying the Components into each other. For example, No. 42, B♭:—

Components . . . . 2 × 3 × 7 = 42.

6. The Logarithm of any given Number, is found by adding together the Logarithms of the Components. For example, the Logarithm of 42, B♭:—

Components,	2 . . .	Log. 6 . 578,813
,,	3 . . .	Log. 10 . 427,173
,,	7 . . .	Log. 18 . 469,064
Log. of 42 . . .		35 . 475,050

7. The use of the Logarithms will be explained in Appendix E.

8. Abstract of the foregoing Table, omitting the Octaves:—

1. F. The Root.
3. C, Fifth to F.
5. $\bar{A}$, Third to F.
7. $\overset{\Delta}{E}$♭, Seventh to F.
9. G, Fifth to C.
15. E, Third to C.
21. $\overset{\Delta}{B}$♭, Seventh to C.
25. C♯, Third to $\bar{A}$.
27. D, Fifth to G.
35. $\overset{\Delta}{G}$, Seventh to $\bar{A}$.
45. B, Third to G.
49. $\overset{\Delta}{D}$♭, Seventh to $\overset{\Delta}{E}$♭.
63. $\overset{\Delta}{F}$, Seventh to G.
75. G♯, Third to E.
81. $\overset{+}{A}$, Fifth to D.
125. E♯, Third to C♯.
135. F♯, Third to D.
225. D♯, Third to B.

APPENDIX E.

INTERVALS.

1. AN Interval is the difference of pitch between two musical sounds; or the relation of two Harmonic Numbers to each other. It is correctly expressed by the numbers corresponding to the two sounds, placed in the form of a fraction; as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, &c., which indicate the fractional part of the string that sounds the upper note of the Interval; the lower note being that of the whole string. Or it may be expressed by the same numbers placed thus, 1:2, 2:3, 3:4, &c.; indicating the relation of the two numbers to each other; which is always in exact proportion to the comparative rate of vibration in the two sounds.

2. The following simple arithmetical Rules are necessary for the treatment of Intervals:*—

Rule 1.—To reduce an Interval to its lowest terms: divide repeatedly by the Harmonic Primes 2, 3, 5, 7, so long as both terms are divisible by any of them; thus:—

Reduce 150 : 252 to its lowest terms.

$$150 : 252 \div 2 = 75 : 126.$$

$$75 : 126 \div 3 = 25 : 42.—\textit{Answer.}$$

Rule 2.—To invert an Interval: invert the fraction that expresses it, and then halve the upper term, or double the lower; thus:—

The Fourth, $\frac{3}{4}$, inverted, becomes $\frac{2}{3}$, the Fifth:

$$\frac{3}{4} \quad \frac{4}{3} \quad \frac{2}{3}.$$

The Major Third, $\frac{4}{5}$, inverted, becomes $\frac{5}{8}$, the Minor Sixth:

$$\frac{4}{5} \quad \frac{5}{4} \quad \frac{5}{8}$$

Rule 3.—To add Intervals together: multiply the Numerators into each other, and likewise the Denominators: then reduce the Interval found, to its lowest terms; thus:—

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* See Dr. Holder.

Add together the Fifth, $\frac{2}{3}$, and the Fourth, $\frac{3}{4}$.

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2} \text{ Octave.}$$

Add together the Major, Minor, and Grave Thirds.

$$\frac{4}{5} \times \frac{5}{6} \times \frac{6}{7} = \frac{120}{210} = \frac{4}{7} \text{ Grave Seventh.}$$

The operation may be abridged, by leaving out the same numbers when they occur in the numerator of one fraction, and the denominator of another; thus:—

$\frac{3}{4} \times \frac{4}{5}$, neglecting the two fours, is $= \frac{3}{5}$.

$\frac{4}{5} \times \frac{5}{6} \times \frac{6}{7}$, neglecting the fives and sixes, is $= \frac{4}{7}$;

or, by dividing, crosswise, any numerators and denominators, that have a common measure: thus:—

$$\frac{24}{25} \times \frac{80}{81} = \frac{8}{5} \times \frac{16}{27} = \frac{128}{135}.$$

In this Example, 24 and 81 are divided by 3; 25 and 80 by 5.

Rule 4.—To subtract one Interval from another: multiply the terms of the two fractions crosswise: or invert the fraction that denotes the lesser Interval, and proceed as in Addition. Observe, that the greater fraction denotes the lesser Interval. Thus:—

From a Major Third, $\frac{4}{5}$, subtract a Minor Third, $\frac{5}{6}$.

$$\frac{4}{5} \times \frac{5}{6} = \frac{24}{25}; \qquad \text{or } \frac{4}{5} \times \frac{6}{5} = \frac{24}{25}.$$

3. An Interval is said to be divided harmonically, when the terms into which it is divided are in arithmetical progression; as 2, 3, 4, increasing by 1; or 12, 15, 18, increasing by 3; and so on. Thus the following Intervals are harmonically divided:—

Octave	1 : 2	into 2 : 3	and 3 : 4	. . . . Fifth and Fourth.
Fifth	2 : 3	into 4 : 5	and 5 : 6	. . . . Major and Minor Third.
Major Third	4 : 5	into 8 : 9	and 9 : 10	. . . . Major and Minor Tone.
Grave Seventh	4 : 7	into 4 : 5		. . . . . Major Third;
		5 : 6		. . . . . Minor Third; and
		6 : 7		. . . . . Grave Third.

Hence the Fifth is called the mean proportional in the Octave: the Major Third, mean proportional in the Fifth, and so on. Intervals which cannot be divided harmonically, as the Minor Third, 5 : 6, have no mean proportional.

4. The calculations of Intervals are generally short and simple: but where longer operations are required, it is a saving of labour to employ Logarithms; in which Addition and Subtraction do the work of Multiplication and Division. For example, the Octave is divisible into—

12 Chromatic Semitones of . . . . . 24 : 25;
7 Enharmonic Dieses of . . . . . 125 : 128; and
3 Commas of . . . . . 80 : 81.

To add these Intervals together by common arithmetic, it is necessary to multiply $\frac{24}{25}$ by itself to the 12th power; $\frac{125}{128}$ to the 7th power, and $\frac{80}{81}$ to the 3rd power; and then to multiply the three products into each other. By Logarithms, the process is short and simple:—

Chromatic Semitone	Log.	·387,451	× 12	= 4·649,412
Enharmonic Diesis	Log.	·225,098	× 7	= 1·575,686
Comma	Log.	·117,905	× 3	= ·353,715
		Logarithm of Octave		6·578,813

5. The Logarithm of an Interval is found by subtracting the Logarithm of the numerator from that of the denominator. Thus, to find the Logarithm of the Minor Tone $\frac{9}{10}$:—

Common Log. of 10 . . .	1·000,000,000
Common Log. of 9 . . .	·954,242,509
	·045,757,491

6. In the printed Tables of Logarithms, unity, or 1, which is the Logarithm of 10, represents the unusual Interval of a Twenty-fourth, equal to 3 Octaves, plus Major Third. If a different Interval be chosen to be represented by unity, the Logarithms of others may be found from it by the Rule of Three.

7. Some writers use the common Logarithms. Others make unity to represent a Comma; and others, as Euler, an Octave. The Octave is the best for the Harmonic Scale; but for the Intervals treated of in works on Harmony, I have adopted the Minor Tone, as the most convenient.

8. The primitive Intervals, from which all others are derived, are the Octave, Fifth, Major Third, and Grave Seventh; represented by the prime numbers 2, 3, 5, 7.

9. In classifying Intervals, I have followed the natural order suggested by Harmonics; viz:—

1st. Harmonics of the original note F.
2nd. Harmonics of C, the first Harmonic of F.
3rd. Harmonics of A, the second Harmonic of F.
4th. Harmonics of G, the first Harmonic of C.

TABLE OF HARMONICS.

HARMONICS OF FOUR STRINGS.							OCTAVES TO THE SAME.										
Harmonics of F. 1. ..	2	3	4	5	6	7	8	10	12	14	16	20	24	28	32	40	64
Harmonics of C. 3. ..	6	9	12	15	18	21	24	30	36	42	..	..	..	..	..	..	..
Harmonics of A. 5. ..	10	15	20	25	30	35	..	..	..	..	..	..	..	..	..	..	..
Harmonics of G. 9. ..	18	27	36	45	54	63	..	..	..	.	..	..	..	..	..	..	..

Class I.—Intervals derived from the Harmonics of one string, F.

1 : 2 . . . . . Octave.
2 : 3 . . . . . Fifth.
3 : 4 . . . . . Fourth.
3 : 5 . . . . . Major Sixth.
4 : 5 . . . . . Major Third.
5 : 6 . . . . . Minor Third.
4 : 7 . . . . . Grave Seventh.
5 : 7 . . . . . Grave Fifth.
6 : 7 . . . . . Grave Third.
5 : 8 . . . . . Minor Sixth.
7 : 8 . . . . . Tone Maximus.
7 : 10 . . . . . Acute Fourth, or Tritone.
7 : 12 . . . . . Acute Sixth.

Class II.—Intervals derived from the Harmonics of two strings, F, C, vibrating in the proportion of 1 to 3.

4 : 9 . . . . . Ninth.
5 : 9 . . . . . Minor Seventh.
7 : 9 . . . . . Acute Third.
8 : 9 . . . . . Tone Major.
9 : 10 . . . . . Tone Minor.
8 : 15 . . . . . Major Seventh.
14 : 15 . . . . . Hemitone Major.
9 : 16 . . . . . Minor Seventh, less Comma.
15 : 16 . . . . . Semitone Diatonic.
9 : 20 . . . . . Ninth, less Comma.
10 : 21 . . . . . Grave Ninth.
16 : 21 . . . . . Grave Fourth.
20 : 21 . . . . . Hemitone Minor.
15 : 32 . . . . . Minor Ninth.

Class III.—Intervals derived from the Harmonics of three strings, F C A, vibrating in the proportions 2, 3, 5 :—

14 : 25 . . . Augmented Sixth.
18 : 25 . . . Augmented Fourth.
21 : 25 . . . Augmented Second.
24 : 25 . . . Semitone Chromatic.
25 : 36 . . . Diminished Fifth.
25 : 42 . . . Diminished Seventh.
32 : 35 . . . Tone Minimus.

10. All the Intervals already enumerated, are contained within the limits of three Dominant Harmonies, taken on the three notes of the Harmonic Triad: thus,—

Chord of F . . . . F A C $\overset{\vartriangle}{\text{E}}\flat$ . . . . 4 . 5 . 6 . 7 × 4 = 16 . 20 . 24 . 28.

Chord of A . . . . A C$\sharp$ E $\overset{\vartriangle}{\text{G}}$ . . . . 4 . 5 . 6 . 7 × 5 = 20 . 25 . 30 . 35.

Chord of C . . . . C E G $\overset{\vartriangle}{\text{B}}\flat$ . . . . 4 . 5 . 6 . 7 × 6 = 24 . 30 . 36 . 42.

11. If the numbers contained in this small Table, be doubled or halved when it is necessary (which implies nothing more than taking the Octave above or below), they will produce all the Intervals referred to. For example:—

Diminished Seventh . . . . C$\sharp$ 25
$\overset{\vartriangle}{\text{B}}\flat$ 42

Augmented Sixth . . . . . $\overset{\vartriangle}{\text{E}}\flat$ 28 ÷ 2 = 14
C$\sharp$ 25

Minor Ninth . . . . . E 30 ÷ 2 = 15
F 16 × 2 = 32

12. The sounds of two strings vibrating in the proportion of 4 to 9 (or its Octave 8 to 9), being separated from each other by the sum of two Fifths, 4 : 9; their Harmonics, taken together, produce the following false or dissonant Intervals:—

Class IV.—False Intervals, derived from the Harmonics of four strings, F A C G, vibrating in the proportions 4, 5, 6, 9:—

16 : 27, Major Sixth . . . . . + Comma.
20 : 27, Fourth . . . . . + Comma.
25 : 27, Semitone Diatonic . . . . . + Comma.
27 : 28, Hemitone Minor . . . . . − Comma.
27 : 32, Minor Third . . . . . − Comma.
27 : 40, Fifth . . . . . − Comma.
32 : 45, Tritone . . . . . − Komma Major.
45 : 64, Grave Fifth . . . . . + Komma Major.

13. There are cases in which the False Sixth, 16 : 27, or its Octave, 8 : 27, cannot be avoided: such as, the Pedal Thirteenth, Chap. XI. 4; and the double Suspension of Fourth and Ninth, Chap. VII. 10.

14. The Semitones and smaller Intervals, are the differences between certain other Intervals, according to the following Table:—

TABLE OF SMALL INTERVALS.

2 : 3	Fifth .. .. Log.:	3 · 848				
5 : 7	Grave Fifth	3 · 193	· 655	Hemitone Major	$\overset{\vartriangle}{\mathrm{E}}\flat\ \mathrm{E}$	14 : 15
3 : 4	Fourth	2 · 730				
4 : 5	Major Third	2 · 118	· 612	Semitone Diatonic	E F	15 : 16
5 : 7	Grave Fifth	3 · 193				
3 : 4	Fourth	2 · 730	· 463	Hemitone Minor	$\mathrm{A}\ \overset{\vartriangle}{\mathrm{B}}\flat$	20 : 21
4 : 5	Major Third	2 · 118				
5 : 6	Minor Third	1 · 730	· 388	Semitone Chromatic ..	C C♯	24 : 25
5 : 6	Minor Third	1 · 730				
6 : 7	Grave Third	1 · 463	· 267	Quarter Tone	$\overset{\vartriangle}{\mathrm{G}}\ \mathrm{G}$	35 : 36
15 : 16	Semitone Diatonic	· 613				
24 : 25	Semitone Chromatic ..	· 388	· 225	Enharmonic Diesis	E♯ F	125 : 128
6 : 7	Grave Third	1 · 463				
7 : 8	Tone Maximus	1 · 267	· 196	Demisemitone Major ..	$\mathrm{C}\ \overset{\vartriangle}{\mathrm{D}}\flat$	48 : 49
7 : 10	Tritone	3 · 385				
5 : 7	Grave Fifth	3 · 193	· 192	Demisemitone Minor ..	$\overset{\vartriangle}{\mathrm{D}}\flat\ \mathrm{C}\sharp$	49 : 50
7 : 8	Tone Maximus	1 · 267				
8 : 9	Tone Major	1 · 118	· 149	Komma Major	$\overset{\vartriangle}{\mathrm{F}}\ \mathrm{F}$	63 : 64
8 : 9	Tone Major	1 · 118				
9 : 10	Tone Minor	1 · 000	· 118	Comma	$\bar{\mathrm{A}}\ \dot{\mathrm{A}}$	80 : 81
20 : 21	Hemitone Minor	· 463				
24 : 25	Semitone Chromatic ..	· 388	· 075	Semicomma Major	$\mathrm{E}\sharp\ \overset{\vartriangle}{\mathrm{F}}$	125 : 126
14 : 15	Hemitone Major	· 655				
15 : 16	Semitone Diatonic	· 612	· 043	Semicomma Minor	$\overset{\vartriangle}{\mathrm{E}}\flat\ \mathrm{D}\sharp$	224 : 225
48 : 49	Demisemitone Major ..	· 196				
49 : 50	Demisemitone Minor ..	· 192	· 004	Minima	$\mathrm{G}\sharp\ \overset{\vartriangle}{\mathrm{B}}\flat\flat$	$\frac{2400}{2401}$

The sum of Hemitone Major and Diatonic Semitone is equal to Tone Maximus.
The sum of the two Hemitones is equal to Tone Major.
The sum of the two Semitones is equal to Tone Minor.
The sum of Chromatic Semitone and Enharmonic Diesis is equal to Diatonic Semitone.
The sum of the two Demisemitones is equal to Chromatic Semitone.
The sum of Komma Major and Comma is equal to Quarter Tone.
The sum of Komma Major and Semicomma Major is equal to Enharmonic Diesis.
The sum of the two Semicommas is equal to Comma.

15. Intervals being derived from different prime numbers, it is evident that no one can be a measure of all the rest; and that as many measures are necessary, as there are prime numbers in the Intervals that are to be compared.

16. The Logarithms of the Intervals are the best approximation to a common measure. By means of the Logarithm adopted in this work, the Minor Tone becomes the common measure; and the fractional parts are given to three places of decimals, or thousandth parts of the measure.

17. Another means of comparing Intervals, is to reduce them to as many known quantities, as they contain prime numbers. Thus the Octave may be divided into 12 Chromatic Semitones, 3 Commas, 7 Major Kommas, and 7 Major Semicommas:—

D =	S. Chromatic Semitone, . .	Log. · 387,451	×	12	=	4 · 649,412
	C. Comma	Log. · 117,905	×	3	=	· 353,715
	K. Komma Major	Log. · 149,471	×	7	=	1 · 046,297
	X. Semicomma Major . . .	Log. · 075,627	×	7	=	· 529,389
	Log. of Octave					6 · 578,813

18. It is, however, more convenient for comparison, to give a separate column to the Enharmonic Diesis, D; which is equal to the sum of Komma Major and Semicomma Major. The following Table, which includes all the Intervals in the first three Classes, is constructed in that manner:—

INTERVALS.			LOGARITHMS.	KNOWN QUANTITIES.				
4 : 9	F G	Ninth	7·697	14 S	8 D	4 C	..	..
9 : 20	G A	Ninth, less Comma	7·579	14 S	8 D	3 C	..	..
15 : 32	E F	Ninth Minor	7·191	13 S	8 D	3 C	..	..
10 : 21	A $\overset{\vartriangle}{B}\flat$	Grave Ninth	7·042	13 S	7 D	3 C	..	X
1 : 2	F F	Octave	6·579	12 S	7 D	3 C	..	..
8 : 15	F E	Seventh Major	5·966	11 S	6 D	3 C	..	..
5 : 9	A G	Seventh Minor	5·579	10 S	6 D	3 C	..	..
14 : 25	$\overset{\vartriangle}{E}\flat$ C$\sharp$	Augmented Sixth	5·503	10 S	5 D	3 C	K	..
9 : 16	G F	Minor Seventh, less Comma	5·461	10 S	6 D	2 C	..	..
4 : 7	F $\overset{\vartriangle}{E}\flat$	Grave Seventh	5·311	10 S	5 D	2 C	..	X
7 : 12	$\overset{\vartriangle}{E}\flat$ C	Acute Sixth	5·116	9 S	5 D	3 C	K	..
25 : 42	C$\sharp$ $\overset{\vartriangle}{B}\flat$	Diminished Seventh	4·924	9 S	5 D	2 C	..	X
3 : 5	C A	Sixth Major	4·848	9 S	5 D	2 C	..	..
5 : 8	A F	Sixth Minor	4·461	8 S	5 D	2 C	..	..
2 : 3	F C	Fifth	3·848	7 S	4 D	2 C	..	..
25 : 36	C$\sharp$ G	Diminished Fifth	3·461	6 S	4 D	2 C	..	..
7 : 10	$\overset{\vartriangle}{E}\flat$ A	Tritone	3·385	6 S	3 D	2 C	K	..
5 : 7	A $\overset{\vartriangle}{E}\flat$	Grave Fifth	3·194	6 S	3 D	C	..	X
18 : 25	G C$\sharp$	Augmented Fourth	3·118	6 S	3 D	C	..	..
3 : 4	C F	Fourth	2·731	5 S	3 D	C	..	..
16 : 21	F $\overset{\vartriangle}{B}\flat$	Grave Fourth	2·581	5 S	2 D	C	..	X
7 : 9	$\overset{\vartriangle}{E}\flat$ G	Acute Third	2·385	4 S	2 D	2 C	K	..
4 : 5	F A	Third Major	2·118	4 S	2 D	C	..	..
5 : 6	A C	Third Minor	1·730	3 S	2 D	C	..	..
21 : 25	$\overset{\vartriangle}{B}\flat$ C$\sharp$	Augmented Second	1·655	3 S	D	C	K	..
6 : 7	C $\overset{\vartriangle}{E}\flat$	Grave Third	1·463	3 S	D	..	..	X
7 : 8	$\overset{\vartriangle}{E}\flat$ F	Tone Maximus	1·267	2 S	D	C	K	..
8 : 9	F G	Tone Major	1·118	2 S	D	C	..	..
9 : 10	G A	Tone Minor	1·000	2 S	D	..	..	..
32 : 35	F $\overset{\vartriangle}{G}$	Tone Minimus	·851	2 S	..	..	..	X
14 : 15	$\overset{\vartriangle}{E}\flat$ E	Hemitone Major	·655	S	..	C	K	..
15 : 16	E F	Semitone Diatonic	·612	S	D	..	..	..
20 : 21	A $\overset{\vartriangle}{B}\flat$	Hemitone Minor	·463	S	..	..	..	X
24 : 25	C C$\sharp$	Semitone Chromatic	·388	S	..	..	..	..
35 : 36	$\overset{\vartriangle}{G}$ G	Quarter Tone	·267	..	..	C	K	..
125 : 128	E$\sharp$ F	Enharmonic Diesis	·225	..	D	..	..	..
63 : 64	$\overset{\vartriangle}{F}$ F	Komma Major	·149	..	..	..	K	..
80 : 81	$\overset{-}{A}$ $\overset{+}{A}$	Comma	·118	..	..	C	..	..
125 : 126	E$\sharp$ $\overset{\vartriangle}{F}$	Semicomma Major	·075	..	..	..	..	X

19. In the next Table, the small Intervals are reduced to the Logarithm of the Comma, and to the following elements.—

K. Komma Major	63 : 64	. . . . .	Log. 1 · 267,7264
X. Semicomma Major	125 : 126	. . . . .	„ · 641,4294
Y. Semicomma Minor	224 : 225	. . . . .	„ · 358,5706
Z. Minima	2400 : 2401	. . . . .	„ · 033,5343

SMALL INTERVALS.			LOGARITHM OF COMMA.	KNOWN QUANTITIES.			
14 : 15	$\overset{\vartriangle}{E}\flat$ E	Hemitone Major	5 · 554	3 K	X	3 Y	Z
15 : 16	E F	Semitone Diatonic	5 · 195	3 K	X	2 Y	Z
20 : 21	A $\overset{\vartriangle}{B}\flat$	Hemitone Minor	3 · 927	2 K	X	2 Y	Z
24 : 25	C C♯	Semitone Chromatic	3 · 286	2 K	..	2 Y	Z
35 : 36	$\overset{\vartriangle}{G}$ G	Quarter Tone	2 · 268	K	X	Y	..
125 : 128	E♯ F	Enharmonic Diesis	1 · 909	K	X	..	..
48 : 49	C $\overset{\vartriangle}{D}\flat$	Demisemitone Major	1 · 660	K	..	Y	Z
49 : 50	$\overset{\vartriangle}{D}\flat$ C♯	Demisemitone Minor	1 · 626	K	..	Y	..
63 : 64	$\overset{\vartriangle}{F}$ F	Komma Major..	1 · 268	K	..	..	..
80 : 81	$\bar{A}$ $\dot{A}$	Comma	1 · 000	..	X	Y	..
125 : 126	E♯ $\overset{\vartriangle}{F}$	Semicomma Major	· 641	..	X	..	..
224 : 225	$\overset{\vartriangle}{E}\flat$ D♯	Semicomma Minor	· 359	..	..	Y	..
$\frac{2400}{2401}$	G♯ $\overset{\vartriangle}{B}\flat\flat$	Minima	· 034	..	..	..	Z

OF TRUE AND FALSE INTERVALS.

20. It is useful to attend to the distinction of true or perfect, and false or imperfect Intervals; the latter being either defective or redundant by a Comma.

21. Intervals may be divided into two classes, Direct and Inverted, according to their places in the Scale of Triads.

F A C E G B D

22. Thirds and Fifths are Direct Intervals, being read forwards . . . . as F C;
Fourths and Sixths are Inverted Intervals, being read backwards . as C F;
Sevenths and Ninths are Direct Intervals, being read forwards . . as F G.

23. Inverted Thirds and Fifths are false Intervals, Defective by Comma :—

F	A	C	E	G	B	D
8	10	12	15	18	—	24
16	20	24	30	36	45	54
32	40	48	60	—	—	—
64	80	—	—	—	—	—

D F . 27 : 32 . Minor Third . . . . minus Comma.
D A . 27 : 40 . Fifth . . . . . minus Comma.
B F . 45 : 64 . Diminished Fifth . minus Comma.

24. Direct Fourths and Sixths are False Intervals, redundant by Comma :—

A D . 20 : 27 . Fourth . . . . . plus Comma.
F B . 32 : 45 . Augmented Fourth plus Comma.
F D . 16 : 27 . Major Sixth . . . . plus Comma.

25. Inverted Sevenths and Ninths are False Intervals, defective by Comma :—

* G F . 18 : 32 ÷ 2 = 9 : 16 . . . . . Minor Seventh . minus Comma.
B A . 45 : 80 ÷ 5 = 9 : 16 . . . . . Minor Seventh . minus Comma.
D C . 27 : 48 ÷ 3 = 9 : 16 . . . . . Minor Seventh . minus Comma.
G A . 18 : 40 ÷ 2 = 9 : 20 . . . . . Ninth . . . . . minus Comma.
D E . 27 : 60 ÷ 3 = 9 : 20 . . . . . Ninth . . . . . minus Comma.

26. The reason of these False Intervals is, that they are derived from unconnected Triads.

---

* This is the Interval that is commonly mistaken for the Dominant Seventh.

APPENDIX F.

MODULATION.

1. THE Scale of Keys is the same with the Scale of Triads, Chap. XV. 3; wherefore the natural order of Modulation from one Key to another, is by Perfect Fifths, ascending or descending. On this account, it is necessary to consider the changes of pitch that take place in the Fundamental Bass, in moving along the Scale of Triads:—

$$\overset{+}{\text{E}}\flat \quad \text{G} \quad \overset{+}{\text{B}}\flat \quad \text{D} \quad \overset{+}{\text{F}} \quad \text{A} \quad \overset{+}{\text{C}} \quad \text{E} \quad \overset{+}{\text{G}} \quad \text{B} \quad \overset{+}{\text{D}} \quad \text{F}\sharp \quad \overset{+}{\text{A}} \quad \text{C}\sharp \quad \overset{+}{\text{E}} \quad \text{G}\sharp \quad \overset{+}{\text{B}}.$$

2. In the Scale of which this is a part, every three notes form a Triad, which is Major or Minor according to its place in the Scale: every five notes contain two adjacent Triads; and every seven notes, a Diatonic Scale.

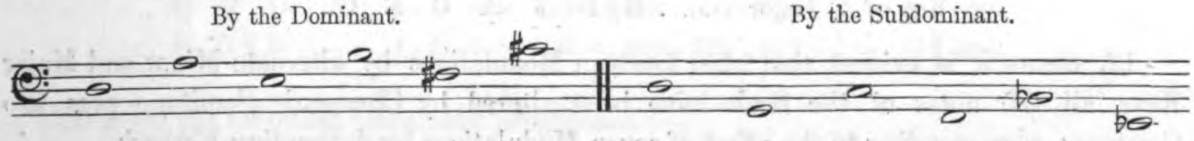
3. The notes that are distinguished by the Major sign +, bear Major Triads, and are higher in pitch by a Comma, than the intermediate notes, of the same name, that bear Minor Triads; as may be seen by comparing the two notes A in the series:—

$\overset{+}{\text{F}}$	A	$\overset{+}{\text{C}}$	$\overset{+}{\text{G}}$	$\overset{+}{\text{D}}$	$\overset{+}{\text{A}}$.
16	20	24	36	54	81.

The first A, which is Major Third to F, is = 20, and its double Octave = 80. The last A, which is Fifth to D, is = 81: the difference between them being 80 : 81, or a Comma.

4. Modulation by the Dominant ascends by Fifths and descends by Fourths. Modulation by the Subdominant descends by Fifths and ascends by Fourths:—

By the Dominant. By the Subdominant.



5. In modulating to the Key of the Dominant, the Triad of the Subdominant is dropped; the Tonic becomes Subdominant, the Dominant becomes Tonic, and a new Dominant Triad is taken on the Fifth of the old one:—

Key of C . . . . F A C . . C E G . . G B D
Key of G . . . . ——— . . C E G . . G B D . . D F♯ A.

6. In this Modulation, the notes F A remove from left to right, and rise a Comma; A, the Third of the Subdominant, being replaced by A the Fifth of the new Dominant; and F,

the Subdominant, by F ♯ the Third of the new Dominant; the last rising also a Chromatic Semitone.

7. On the contrary, in modulating to the Key of the Subdominant, the Triad of the Dominant is dropped: the Tonic becomes Dominant, the Subdominant becomes Tonic, and a new Subdominant is taken, a Fifth below the new Tonic:—

Key of C . . . . . F A C . . C E G . . G B D.
Key of F . . . . B♭ D F . . F A C . . C E G . . ———

8. Here the notes B D remove from right to left, and fall a Comma; D, the Fifth of the Dominant, being replaced by D, the Third of the new Subdominant; and B, the Third of the Dominant, by B♭, the new Subdominant; the last falling also a Chromatic Semitone.

9. Hence it is evident, that after seven Modulations by ascending Fifths, the seven notes of the Scale have risen a Chromatic Semitone, plus two Commas; and that after seven Modulations by descending Fifths, the seven notes of the Scale have fallen as much.

10. This remote Interval, $\frac{2048}{2187}$, which is called by the Greeks *Apotome*, is equal to the sum of seven Fifths, minus four Octaves:—*

Chromatic Semitone	Log.			· 387,451
Comma	Log.	· 117,905 × 2 =	· 235,810	
			———	· 623,261
Fifth	Log.	3 · 848,359 × 7 =	26 · 938,513	
Octave	Log.	6 · 578,813 × 4 =	26 · 315,252	
			———	· 623,261

11. In a descending series of alternate Major and Minor Keys, at every change of Key, one note removes from right to left, and falls a Comma; and at every change from a Minor Key to a Major, the note that removes, also falls a Chromatic Semitone:—

Key of C Major . . . . . F A C E G B D.
Key of A Minor . . . . . D F A C E G B.
Key of F Major . . . . B♭ D F A C E G.

12. Hence it is evident, that after fourteen Modulations by alternate Minor and Major Keys, all the notes of the Scale have been altered by Chromatic Semitone plus two Commas; corresponding to the effect of seven Modulations by descending Fifths.†

13. These changes in the pitch of the Fundamental Bass, do not apply to direct Modulation between the notes of the Diatonic Scales.

---

* $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{1} \times \frac{2}{1} \times \frac{2}{1} \times \frac{2}{1} = \frac{2048}{2187}$

† See Note, page 207.

APPENDIX G.

CHROMATIC MODULATION.

1. AN ascending Chromatic Scale is produced by a series of Major Triads whose Basses ascend by Fourths and descend by Minor Thirds, the Key continually changing. In this Scale, the Chromatic notes are the Major Third and the Octave alternately; and the degrees are alternate Diatonic and Chromatic Semitones, producing, by their sums, a succession of Minor Tones :—

$$\frac{15}{16} \times \frac{24}{25} = \frac{360}{400} = \frac{9}{10} \text{ Minor Tone.}$$

2. Taking C = 81 :

C		81.
F Fourth above C is	81 × 4 ÷ 3 =	108.
D Minor Third below F.	108 × 5 ÷ 6 =	90.
G Fourth above D	90 × 4 ÷ 3 =	120.
E Minor Third below G.	120 × 5 ÷ 6 =	100.

Multiply by those numbers the Major Triads of the notes :—

C	E	G	. . . .	4 . 5 . 6	× 81	=	324 . 405 . 486	. . . .	E	405.
F	A	C	. . . .	4 . 5 . 6	× 108	=	432 . 540 . 648	. . . .	F	432.
D	F♯	A	. . . .	4 . 5 . 6	× 90	=	360 . 450 . 540	. . . .	F♯	450.
G	B	D	. . . .	4 . 5 . 6	× 120	=	480 . 600 . 720	. . . .	G	480.
E	G♯	B	. . . .	4 . 5 . 6	× 100	=	400 . 500 . 600	. . . .	G♯	500.

This gives the following Chromatic progression :—

405	432	450	480	500
E	F	F♯	G	G♯
	$\frac{15}{16}$	$\frac{24}{25}$	$\frac{15}{16}$	$\frac{24}{25}$

which may be extended at pleasure.

3. A descending Chromatic Scale is produced by reversing the foregoing; the Bass descending by Fourths, and ascending by Minor Thirds.

4. Another descending Chromatic Scale is formed by the Chromatic Sequence of Sevenths; a series of Dominant Harmonies, descending by Fifths, and ascending by Fourths, the Key changing continually. In this Scale the Chromatic notes are the Major Third and the Dominant Seventh alternately: and the degrees are alternate Major and Minor Hemitones; producing, by their sums, a succession of Major Tones:

$$\frac{14}{15} \times \frac{20}{21} = \frac{280}{315} = \frac{8}{9} \text{ Major Tone.}$$

5. Take, for example, the following series of descending Fifths:—

81	54	36	24	16
A	D	G	C	F

To bring them within an Octave, multiply D G by 2, and C F by 4:—

81	108	72	96	64
A	D	G	C	F

Then multiply, by those numbers, the Dominant Chord of each of the notes:—

A	C♯	E	G̊	. . . .	4	.	5	.	6	.	7	× 81	= 324	.	405	.	486	.	567	. . . .	G	567.
D	F♯	A	C	. . . .	4	.	5	.	6	.	7	× 108	= 432	.	540	.	648	.	756	. . . .	F♯	540.
G	B	D	F	. . . .	4	.	5	.	6	.	7	× 72	= 288	.	360	.	432	.	504	. . . .	F	504.
C	E	G	B♭	. . . .	4	.	5	.	6	.	7	× 96	= 384	.	480	.	576	.	672	. . . .	E	480.
F	A	C	E♭	. . . .	4	.	5	.	6	.	7	× 64	= 256	.	320	.	384	.	448	. . . .	E♭	448.

This gives the following Chromatic progression:—

567	540	504	480	448
G̊	F♯	F̊	E	E̊♭
	20/21	14/15	20/21	14/15

which may be extended at pleasure.

6. The sum of two Degrees being, in the one Chromatic Scale, equal to a Minor Tone, and in the other, to a Major Tone; the difference between which is a Comma: it follows, that at the end of an Octave, which contains twelve Chromatic Degrees, the last notes of the two Scales differ in pitch by six Commas.

7. Dr. Callcott, speaking of the Chromatic Semitone, observes:—"This Semitone was termed by the Pythagoreans, *Apotome*, and the Diatonic Semitone was termed *Limma*. They contended, that the Apotome, or distance from B flat to B natural, was larger than the Limma, or distance from A to B flat. It is now, however, demonstrated by the experiments of Mersenne, &c. &c., that the theory of Zarlino and Salinas is true; namely, that the Interval from A to B flat, is the Major Semitone; and that from B flat to B natural, is the Minor Semitone: contrary to the nomenclature of Boethius and the Pythagoreans."*

* Callcott's *Grammar*, Art. 214; 2nd Ed. 203.

8. This apparent difference between the Ancients and the Moderns, may be reconciled by comparing the two Chromatic progressions explained in this Chapter:—

405	432	450	480	500	567	540	504	480	448
A	B♭	B♮	C	C♯	$\overset{\vartriangle}{\text{C}}$	B	$\overset{\vartriangle}{\text{B}}$♭	A	$\overset{\vartriangle}{\text{A}}$♭
	$\frac{15}{16}$	$\frac{24}{25}$	$\frac{15}{16}$	$\frac{24}{25}$		$\frac{20}{21}$	$\frac{14}{15}$	$\frac{20}{21}$	$\frac{14}{15}$

In the one, the Interval between B flat and B natural, is $\frac{24}{25}$, the Chromatic Semitone, which is less than the Diatonic Semitone $\frac{15}{16}$. In the other, the Interval between B and $\overset{\vartriangle}{\text{B}}$ flat, the same notes by name, is the Hemitone Major $\frac{14}{15}$, which is greater than the Diatonic Semitone.

NOTE referred to, page 204.

The gradual rise of pitch in a scale of Ascending Fifths, may be further illustrated thus:

The sum of 4 Fifths minus 2 Octaves, is equal to Major Third plus Comma. See Appendix L, Proposition 2:—

$$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{1} \times \frac{2}{1} = \frac{64}{81}$$

$$\frac{4}{5} \times \frac{80}{81} = \frac{320}{405} \div 5 \ . \ . = \frac{64}{81}$$

The interval from F flat to B sharp may be divided into 5 Major Thirds:—

F♭ A♭ A♭ C C E E G♯ G♯ B♯

But in ascending from F flat to B sharp by Fifths, each of the Major Thirds is augmented by Comma. Wherefore the interval from F flat to B sharp is equal to 5 Major Thirds plus 5 Commas:—

SCALE OF FIFTHS, ASCENDING FROM F FLAT TO B SHARP.

F♭ 16 . . . . . 16.	A♭ 16.	C 16.	E 16.	G♯ 16.
C♭ 16 × 3 ÷ 2 = 24.	E♭ 24.	G 24.	B 24.	D♯ 24.
G♭ 24 × 3 ÷ 2 = 36.	B♭ 36.	D 36.	F♯ 36.	A♯ 36.
D♭ 36 × 3 ÷ 2 = 54.	F 54.	A 54.	C♯ 54.	E♯ 54.
A♭ 54 × 3 ÷ 2 = 81.	C 81.	E 81.	G♯ 81.	B♯ 81.

APPENDIX H.

HARMONIC CHANGES.

1. As it is essential to the unity of a composition, that the Fundamental Bass should move by perfect Intervals; and as the notes of every chord must be regulated by the Fundamental Bass: it is frequently necessary to make a small change in the pitch of a note; which I shall call a Harmonic change; consisting generally of a Comma, or Komma Major.

2. The Fundamental Basses of the Diatonic Scale, are the Subdominant, Tonic, and Dominant: with the Sixth and Third, as Relative Minors to the Tonic and Dominant. When the Relative Minor of the Subdominant is introduced, it is borrowed from the *Key* of the Relative Minor; the Subdominant of which, being a Comma lower than the Supertonic of the Major Mode, makes a true Minor Triad with the Fourth and Sixth of the Scale:—

Triad of the Supertonic . . . . . D F A . . . . 81 . 96 . 120 ÷ 3 = 27 . 32 . 40.
Relative Minor of Subdominant. . . . D F A . . . . 80 . 96 . 120 ÷ 8 = 10 . 12 . 15.

3. When the Ninth is prepared by a rise of a Second in the Fundamental Bass, the Major Third, which prepares the Suspension, rises a Comma to produce a true Ninth:—

Preparation . . . . F A C – . . . . 4 . 5 . 6 . – × 16 = 64 . 80 . 96.
Suspension . . . . G B D A . . . . 4 . 5 . 6 . 9 × 9 = 36 . 45 . 54 . 81.

4. When the Dominant Seventh is prepared by the Subdominant, or Fourth of the Scale, a Harmonic change takes place by a descent of Komma Major, 63 : 64.

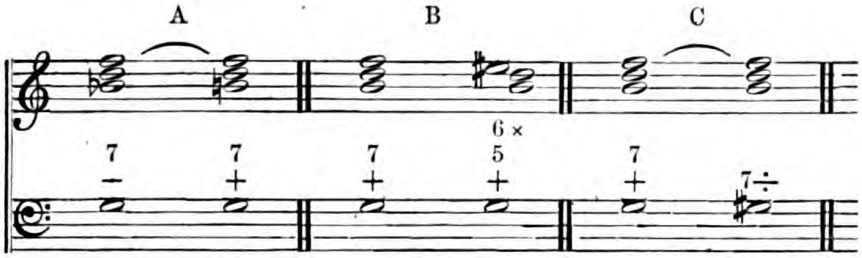
Subdominant Chord . . . . F A C F . . . . 4 . 5 . 6 . 8 × 8 = 32 . 40 . 48 . 64.
Dominant Seventh . . . . . G B D $\overset{\Delta}{F}$ . . . . 4 . 5 . 6 . 7 × 9 = 36 . 45 . 54 . 63.

5. When the Dominant Seventh is changed into the German Sixth (XVII. 8), the Seventh rises a Demisemitone Minor, 49 : 50:—

Dominant Seventh . . . . . F A C $\overset{\Delta}{E}\flat$ . . . . 4 . 5 . 6 . 7 × 7 = 28 . 35 . 42 . 49.
German Sixth . . . . . F A C D$\sharp$ . . . . . 28 . 35 . 42 . 50.

6. These Harmonic changes, though disregarded in keyed instruments, are well known

to the great masters of the violin. Some years ago, I put the following questions to Emiliani, and noted down his answers:—



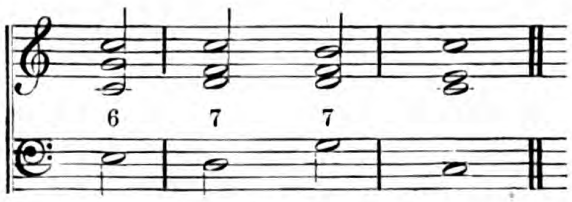
A.—When the Chord changes from the Minor Seventh to the Dominant Seventh, do you make any change on the Seventh? Do you flatten it a little?—"Yes, surely." *Q.* I make the difference between the two Sevenths to be as 36 to 35?—*A.* "But we alter the Seventh by the ear, till it is in tune with the new Chord."

B.—When the Dominant Seventh is changed to the German Sixth, do you make any difference between the F natural, and the E sharp?—*A.* "You cannot make the distinction on the pianoforte, but we do on the violin."*

C.—When the Dominant Seventh is changed to the Diminished Seventh by sharpening the Bass note, do you flatten the Fifth a little, or hold it on at the same pitch?—*A.* "I hold it on exactly at the same pitch."

The answer to the last question confirms the idea of two forms to the Diminished Seventh (X. 6), unless the second form should be considered as a Suspension; to which I see no particular objection.

7. When the Dominant Harmony is preceded, as in the following Example, by a Seventh on the Supertonic, the true Fundamental Bass of the latter Chord is, and has long been, a subject of controversy among theorists; some contending for D the Supertonic, and others for F the Subdominant.



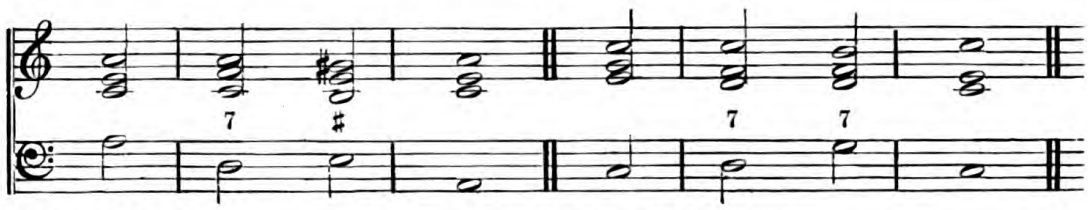
On this subject, Dr. Callcott writes as follows:—†

"Theorists are divided in their opinions concerning the Root of this Chord: but a great

* Emiliani related the following anecdote of Tartini, who is supposed to have had the finest ear that was ever known. On being asked who was the best living performer on the violin, he answered, "He that plays least out of tune."

† Callcott's *Grammar*, Art. 428; 2nd Ed. 394.

majority of authors are in favour of its derivation from the Second, or Supertonic of the Key. Rameau estimates the Root of this Harmony by its Resolution; calling it D when followed by G, and F when followed by C. Heck considers it as a compound of both the Harmonies of D and of F. Dr. Boyce, in his MSS., and with him the author of this work, thinks that the Root is decided by the Scale of the Key in which it is found: thus:—



D in A Minor; F in C Major."

8. That Dr. Callcott's opinion is correct, may be thus demonstrated. If D is to be held as the Fundamental Bass in C Major (for in the case of A Minor there is no dispute), it must be either the Supertonic of C Major, or the Subdominant of its Relative Minor, A. If the former, the Chord D F A C, 27, 32, 40, 48, is out of tune; the Bass being a Comma too sharp for the other notes of the Chord. If the latter, a note is admitted as a Fundamental Bass, which does not even belong to the Key. (See Sect. 2.)

9. Mr. Liston, who took the part of D, perceived this; but in avoiding the false Minor Triad, 27, 32, 40, which was inconsistent with his "perfect intonation," he got upon the other horn of the dilemma, by admitting a false progression in the Fundamental Bass. His words are these: "The Bass leaps downwards by Major (perfect) Fifth, too small by Comma; or upwards by Minor (perfect) Fourth, too great by Comma, to the true Fifth of the Key."

10. By taking F, the Subdominant of the Key, as the Fundamental Bass, and D as its Added Sixth (VI. 6), the progression of the Bass is perfect, and all the Chords are in tune.

APPENDIX K.

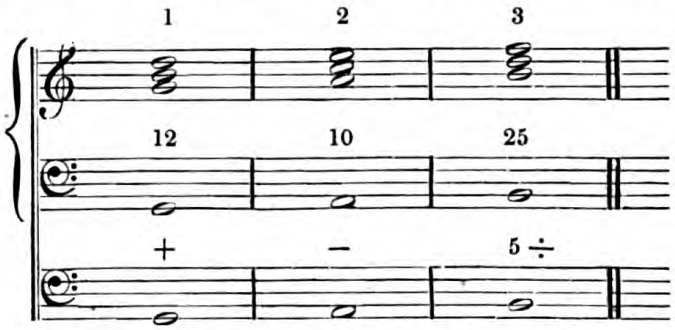
ARITHMETICAL NOTATION.

1. THE figures of Thorough Bass represent the names given to Intervals from the Degrees on the Staff. The Notation of the Fundamental Harmony in this work (although intended to represent Sounds, not Degrees), being founded partly on the Figured Bass, will, it is hoped, be intelligible to musical readers in general.

2. A system of Notation, however, more simple and philosophical, may be formed from the numbers that denote the proportions of the various Intervals and Chords. In constructing such a system, the first nine numbers, being already appropriated by the figures of Thorough Bass, may be avoided.

3. The three Triads may be represented by their lowest notes: thus,

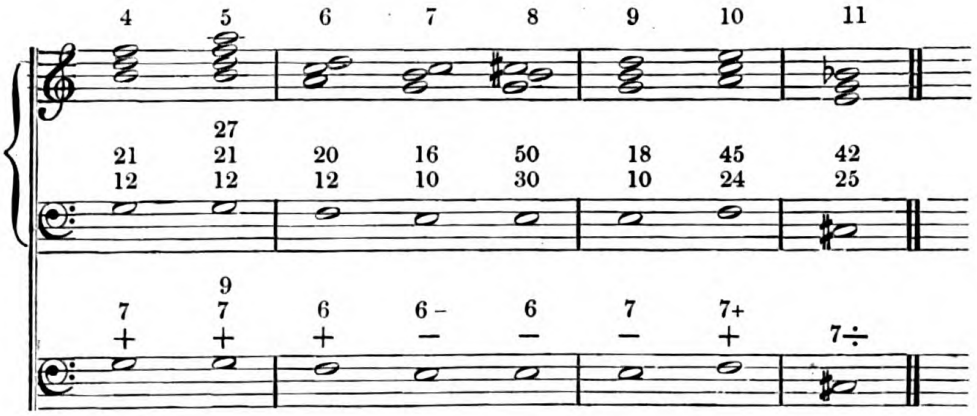
1. Major Triad . . . . . 4 . 5 . 6 × 3 = 12 . 15 . 18.
2. Minor Triad . . . . . 10 . 12 . 15 . . . 10 . 12 . 15.
3. Diminished Triad . . 5 . 6 . 7 × 5 = 25 . 30 . 35.



We may therefore express the Major Triad by 12, or one of its multiples; the Minor Triad by 10, or one of its multiples; and the Diminished Triad by 25.

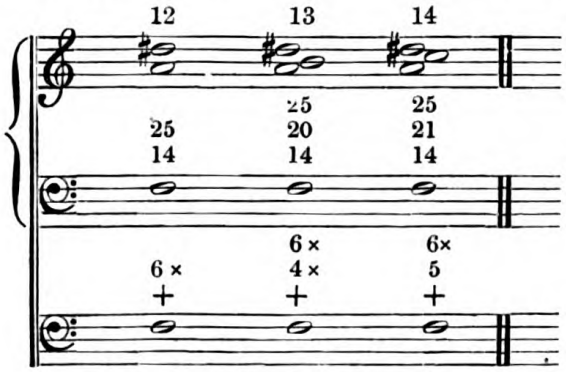
4. The various additions made to the Triads, may be expressed by figures above the numbers that denote the Triads: thus,

4. Dominant Seventh . . 4 . 5 . 6 . 7 × 3 = 12 . 15 . 18 . 21.
5. Added Ninth . . . . . 4 . 5 . 6 . 7 . 9 × 3 = 12 . 15 . 18 . 21 . 27.
6. Added Sixth, Major Mode . . . . . 12 . 15 . 18 . 20.
7. ————————Minor Mode, Minor Sixth . . . 10 . 12 . 15 . 16.
8. ————————Minor Mode, Major Sixth . . . 30 . 36 . 45 . 50.
9. Minor Seventh . . . . . 10 . 12 . 15 . 18.
10. Major Seventh . . . . . 24 . 30 . 36 . 45.
11. Diminished Seventh . . . . . 25 . 30 . 35 . 42.

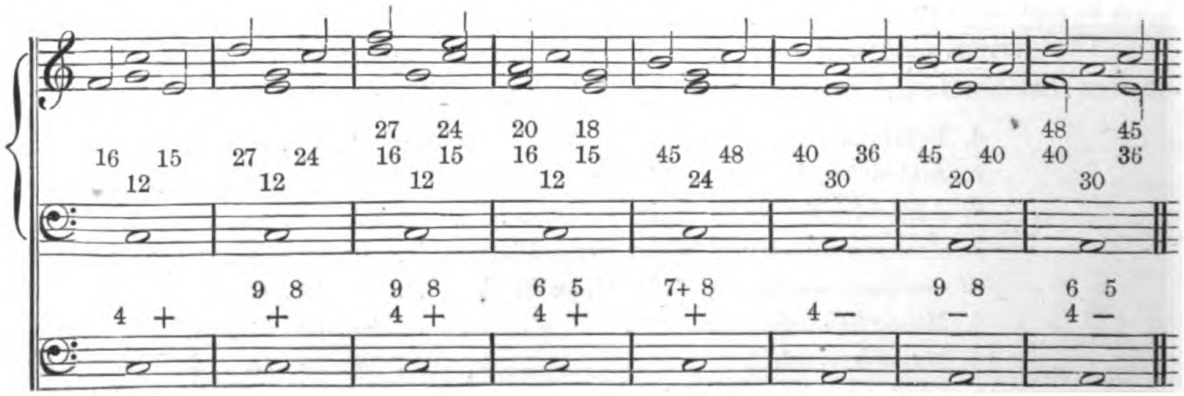


5. The Chords of the Augmented Sixth, omitting 35 the Major Third, which is common to them all, may be expressed by the remaining numbers, divided by 2 :—

12. Italian Sixth . . . 28 . 35 . — 50 ÷ 2 = 14 . — 25.
13. French Sixth . . . 28 . 35 . 40 . 50 ÷ 2 = 14 . 20 . 25.
14. German Sixth . . 28 . 35 . 42 . 50 ÷ 2 = 14 . 21 . 25.

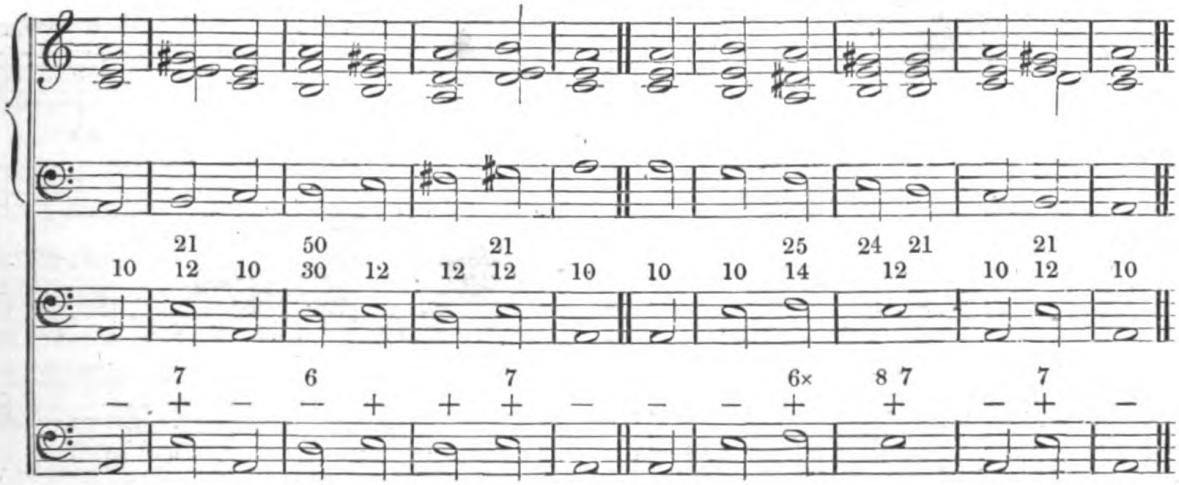


6. Suspensions resolved upon the same Bass, will be written thus :—

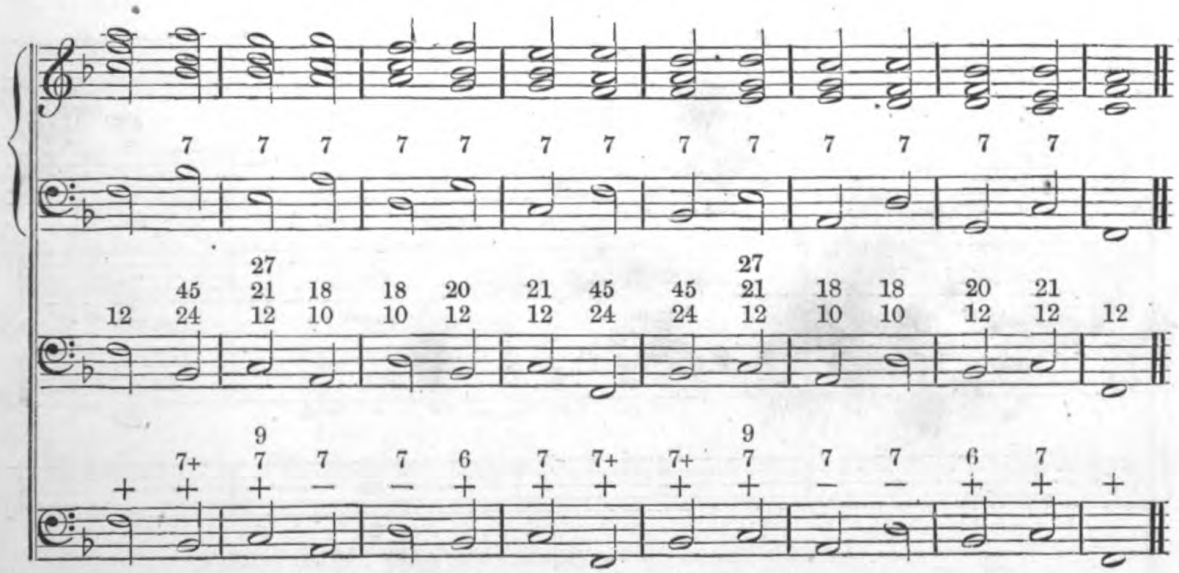


7. The following Examplès are given as specimens of this Notation :—

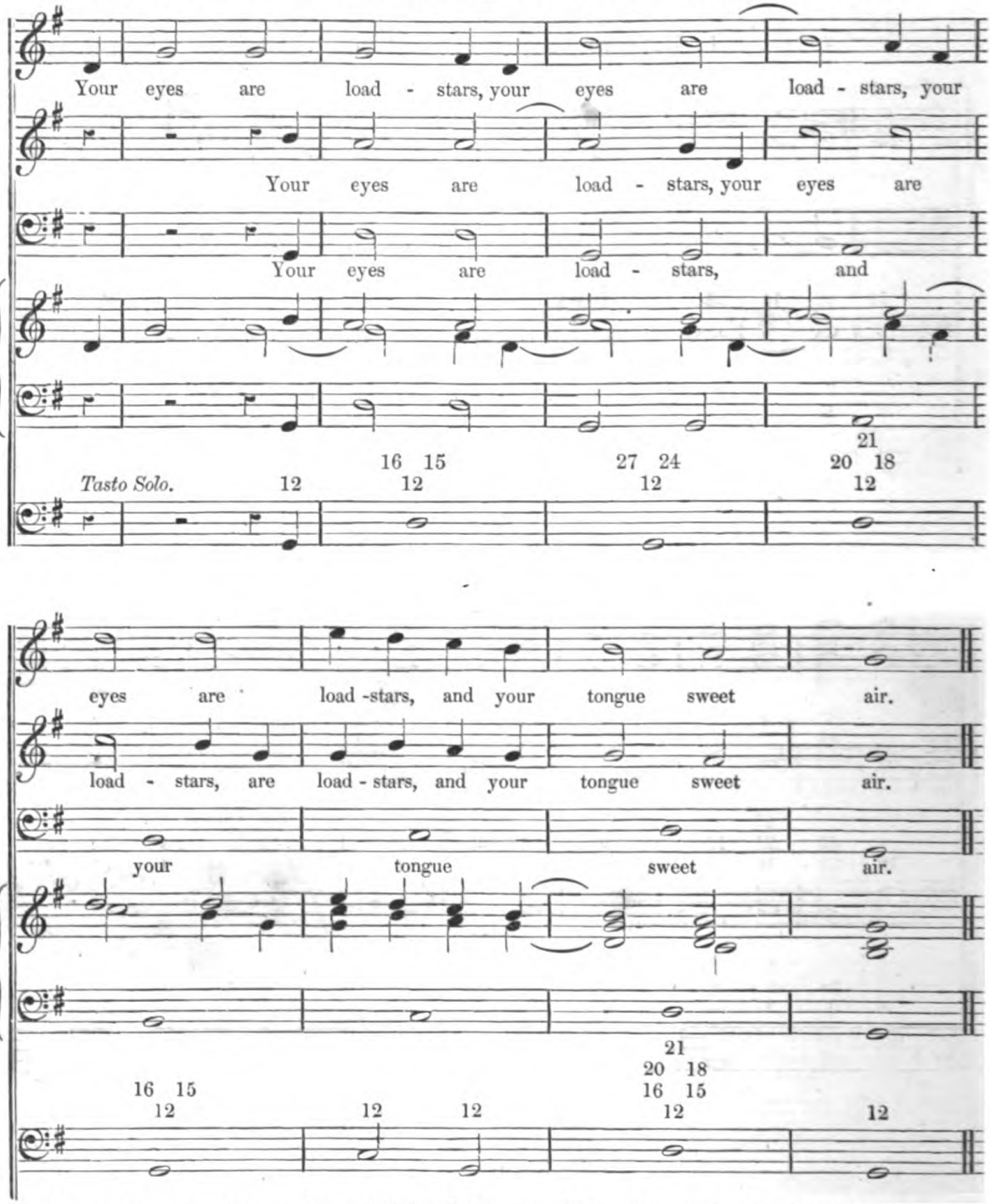
I. RULE OF THE OCTAVE, MINOR MODE.



II. DIATONIC SEQUENCE OF SEVENTHS.



III. CONCLUDING PERIOD OF MR. SHIELD'S GLEE, "Shakespeare's Loadstars."



8. More extensive Examples of this Notation will be found in the Double Choruses, Chap. XX. 40—43, under the Vocal Scores.

APPENDIX L.

TEMPERAMENT.

1. ON keyed instruments, the number of sounds within the Octave being usually limited to twelve, the number of keys; it follows that the same sound must represent several different notes. That this may occasion as little offence as possible to the ear, is the design of those methods of false tuning which are called Temperament.

2. The seven notes of the natural Scale, together with three sharp and two flat notes, take up the twelve keys; producing the following Scale:

C C♯ D E♭ E F F♯ G G♯ A B♭ B.

3. These twelve notes, arranged in ascending Fifths and descending Fourths, stand thus:—

4. If another note were added to the series, it would be D♯, on the thirteenth key of the board; but that key being already occupied by E♭, the problem is, to divide the Octave from E♭ to E♭, to the best advantage.

5. This is attempted by two different methods, commonly called the Vulgar and the Equal Temperaments; which depend on the following propositions:—

Proposition 1.

The Sum of 12 Fifths, minus 7 Octaves, is equal to the sum of three Commas, minus Diesis:—

Fifth	Log. 3 · 848,359 × 12	= 46 · 180,308	
Octave	Log. 6 · 578,813 × 7	= 46 · 051,691	
			· 128,617
Comma	Log. · 117,905 × 3	= · 353,715	
Diesis	Log. · 225,098	· 225,098	
			· 128,617

This Interval of 3 Commas minus Diesis, is called Diaschisma. Its proportions are $\frac{524,288}{531,441}$

Proposition 2.

The sum of 4 Fifths, minus 2 Octaves, is equal to Major Third plus Comma:—

Fifth	Log. 3 . 848,359 × 4 =	15 · 393,436
Octave	Log. 6 . 578,813 × 2 =	13 · 157,626
		2 · 235,810
Major Third	Log.	2 · 117,905
Comma	Log.	· 117,905
		2 · 235,810

Proposition 3.

The sum of 3 Major Thirds, plus Diesis, is equal to an Octave:—

Major Third . .	Log. 2 · 117,905 × 3 =	6 · 353,715
Diesis	Log.	· 225,098
Octave	Log.	6 · 578,813

6. The Equal Temperament, which divides the Octave into twelve equal Semitones, depends on the first proposition. In this Temperament, the sum of 12 Fifths is made to coincide with the sum of 7 Octaves, by tuning each of the Fifths flatter than the true Fifth by $\frac{1}{12}$ Diaschisma.

7. The Vulgar Temperament depends on the second and third propositions. This Temperament proceeds by three operations; in each of which the sum of 4 Fifths is made to coincide with a Major Third, by tuning each of the Fifths flatter than the true Fifth, by $\frac{1}{4}$ Comma. By this means, the Octave is divided into three Major Thirds and the Enharmonic Diesis.

1st Operation	E♭ B♭ F C G.
2nd Operation	G D A E B.
3rd Operation	B F♯ C♯ G♯ D♯.

But the key for D♯ being already occupied by E♭, the Diesis remains where the extremes of the tuning Scale meet, in the Interval between G♯ and E♭. This Interval, which is a diminished Sixth instead of a Fifth, is called by organ-builders, the Great Wolf.

8. By the Vulgar Temperament, all the Major Thirds in the Scale (Sect. 2), 8 in number, are perfect. Where they are wanting, their places are supplied by Wolf Intervals, or diminished Fourths; as B E♭ for B D♯; or G♯ C for A♭ C. The same observation applies to the Minor Sixths, which are inversions of the Major Thirds.

9. All the Minor Thirds in the Scale, 8 in number, are too flat by $\frac{1}{4}$ Comma, and their inversions, the Major Sixths, as much too sharp. Where they are wanting, their places are supplied by Wolf Intervals.

10. All the Fifths are too flat by $\frac{1}{4}$ Comma; and their inversions the Fourths as much too sharp: except where the extreme notes of the tuning Scale meet in the Great Wolf.

11. The places of the Wolf Intervals will be best understood, by attending to the Scale of Major Keys:—

$$\overset{3}{D}\flat \quad \overset{2}{A}\flat \quad \overset{1}{E}\flat \quad B\flat \quad F \quad C \quad G \quad D \quad A \quad \overset{1}{E} \quad \overset{2}{B} \quad \overset{3}{F}\sharp$$

12. In this Scale, the six Keys from B♭ with 2 flats, to A with 3 sharps, are in tune. The next two Keys, E♭ on the left, and E on the right, have each one Wolf Third; the next, A♭ and B have two; and the extreme Keys, D♭ and F♯, have three Wolf Thirds. The harsh Fifth, or Great Wolf, occurs in the three Keys on the left, E♭ A♭, and D♭; as may be seen by inspecting the following Table:—

TABLE OF WOLF INTERVALS IN THE MAJOR KEYS.

Key of F♯, 6 sharps.	Dominant	C♯	E♯	G♯	tuned	C♯ F	G♯ Wolf Third.
	Tonic	F♯	A♯	C♯	,,	F♯ B♭	C♯ Wolf Third.
	Subdominant . .	B	D♯	F♯	,,	B E♭	F♯ Wolf Third.
Key of B, 5 sharps .	Dominant	F♯	A♯	C♯	tuned	F♯ B♭	C♯ Wolf Third.
	Tonic	B	D♯	F♯	,,	B E♭	F♯ Wolf Third.
Key of E, 4 sharps .	Dominant	B	D♯	F♯	tuned	B E♭	F♯ Wolf Third.
Key of E♭, 3 flats . .	Subdominant . .	A♭	C	E♭	tuned	G♯ C	E♭ Wolf Third and Fifth.
Key of A♭, 4 flats . .	Tonic	A♭	C	E♭	tuned	G♯ C	E♭ Wolf Third and Fifth.
	Subdominant . .	D♭	F	A♭	,,	C♯ F	G♯ Wolf Third.
Key of D♭, 5 flats .	Dominant	A♭	C	E♭	tuned	G♯ C	E♭ Wolf Third and Fifth.
	Tonic	D♭	F	A♭	,,	C♯ F	G♯ Wolf Third.
	Subdominant . .	G♭	B♭	D♭	,,	F♯ B♭	C♯ Wolf Third.

13. In both methods of Temperament, the Subdominant of the Scale is used for the Dominant Seventh; which occasions a deviation from the true pitch, that rivals the Wolf Intervals themselves. It is no doubt for this reason, that so beautiful and important a concord has long been mis-named a discord.

14. The Logarithm of the Minor Tone being taken as unity, that of the Octave is 6·578,813. This number, divided by 12, gives the Logarithm of the equal Semitone = ·548,234; and enables us to compare it with the various Intervals represented by it:—

Hemitone Major	·655.
Diatonic Semitone	·612.
Hemitone Minor	·463.
Chromatic Semitone	·388.
Equal Temperament	·548.

15. On the comparative merits of the different systems of Temperament, Mr. Liston, the inventor of the Euharmonic Organ, observes as follows:—"The Vulgar Temperament is generally practised on the Organ; and is recommended as the best for that Instrument, by the ingenious Organ-builder Mr. Flight, in a little work which lies before us. On the Pianoforte, the most prevalent practice is, to divide the Scale of the Instrument, as nearly as possible, into twelve equal semitones. By this means, the Wolf Intervals disappear: but this is brought about by making the Thirds and Sixths, Major and Minor, so harsh, that they could not be tolerated, were the sounds held out as in the Organ. Custom, however, has reconciled the ears of the musical world to those Intervals: and the convenience of having every Key equally good, though none very good, recommends this method of tuning to general adoption."—*Brewster's Cyclopædia*, article "Music," Sect. 267.

16. The necessity of Temperament shows the imperfection of our Instruments; but proves nothing against the perfection of Musical Intervals themselves. The combination of sounds produced by one musical note with its Harmonics, is such, that whether we attend to the sounds, or to their arithmetical proportions, it is impossible to conceive anything more perfect: nay, any deviation evidently implies imperfection.

17. Whence comes it, that three strings, two of which are tuned to the first two Harmonics of the other, produce all the combinations of sound in Harmony? It is one of the wonderful works of HIM who is perfect in wisdom.

APPENDIX M.

EULER'S ADDITION TO THE SCALE OF THE OCTAVE.

It is just a century since the celebrated Euler, who took the theory of Music as he found it, wrote these remarkable words:—

"Such, then, is the real origin of the musical notes already in use; they are derived from the numbers 2, 3, and 5. Were we farther to introduce the number 7, that of" [the number of] "the tones of an Octave would be increased, *and the art of Music carried to a higher degree of perfection.** But here the mathematician gives up the musician to the direction of his ear."

Music, as an art, although not as a science, had already attained the degree of perfection which Euler speaks of. But instead of leaving the musician, as he does, to the direction of his ear, let us introduce the number 7 into the calculation of the Scale, and ascertain its effect in arithmetical proportion.

On F, C, G, the Fundamental Basses of the key of C, set up three Harmonic Tetrads, F A C $\overset{\triangle}{E}\flat$, C E G $\overset{\triangle}{B}\flat$, G B D $\overset{\triangle}{F}$ (Chap. III. 7). Then,

Because F C G are perfect Fifths ascending, F is to C, and C to G, as 2 to 3. (Chap. II. 5.) Wherefore, if F be taken = 4, C will be = 6, and G = 9.

Multiply the value of those Tetrads, 4, 5, 6, 7, by 4, 6, 9, the value of F C G:—

Tetrad of F . . . F A C $\overset{\triangle}{E}\flat$ . . . 4 . 5 . 6 . 7 × 4 = 16 . 20 . 24 . 28.

Tetrad of C . . . C E G $\overset{\triangle}{B}\flat$ . . . 4 . 5 . 6 . 7 × 6 = 24 . 30 . 36 . 42.

Tetrad of G . . . G B D $\overset{\triangle}{F}$ . . . 4 . 5 . 6 . 7 × 9 = 36 . 45 . 54 . 63.

Set down those notes, with the proportions thus found:—

F	A	C	$\overset{\triangle}{E}\flat$	E	G	$\overset{\triangle}{B}\flat$	B	D	$\overset{\triangle}{F}$
16	20	24	28	30	36	42	45	54	63

* Euler's 'Letters to a German Princess,' Letter VII.

To reduce them within an Octave, multiply F A by 4, and C E♭ E G B♭ B by 2:—

F	16 × 4 = 64	E	30 × 2 = 60
A	20 × 4 = 80	G	36 × 2 = 72
C	24 × 2 = 48	B♭	42 × 2 = 84
E♭	28 × 2 = 56	B	45 × 2 = 90

Then arrange them in alphabetical order, beginning with C, and adding its Octave:—

C	D	E♭	E	F	F	G	A	B♭	B	C
48	54	56	60	63	64	72	80	84	90	96

The notes of this Scale correspond exactly with the same notes in the Table of Harmonic Numbers, Appendix D, page 191.

To derive the three Dominant Harmonies from the notes of the Diatonic Scale, five Triads are necessary, on a Scale of ascending Fifths:

16	24	36	54	81
E♭	B♭	F	C	G.

Multiply the value of the Triads, 4, 5, 6, by the value of these notes:—

Triad of E♭ . . . . . E♭ G B♭ . . . . . 4 . 5 . 6 × 16 = 64 . 80 . 96.
Triad of B♭ . . . . . B♭ D F . . . . . 4 . 5 . 6 × 24 = 96 . 120 . 144.
Triad of F . . . . . F A C . . . . . 4 . 5 . 6 × 36 = 144 . 180 . 216.
Triad of C . . . . . C E G . . . . . 4 . 5 . 6 × 54 = 216 . 270 . 324.
Triad of G . . . . . G B D . . . . . 4 . 5 . 6 × 81 = 324 . 405 . 486.

This gives the following Scale of Triads:—

E♭	G	B♭	D	F	A	C	E	G	B	D
64	80	96	120	144	180	216	270	324	405	486.

To adapt the notes E♭, B♭, F, for the place of Sevenths, multiply by 4:—

E♭ B♭ F . . . . 64 . 96 . 144 × 4 = 256 . 384 . 576.

From the Scale thus formed, extract the following Tetrads:—

F A C E♭. . . . . 144 . 180 . 216 . 256 ÷ 4 = 36 . 45 . 54 . 64.
C E G B♭. . . . . 216 . 270 . 324 . 384 ÷ 6 = 36 . 45 54 . 64.
G B D F . . . . . 324 . 405 . 486 . 576 ÷ 9 = 36 . 45 . 54 . 64.

These are commonly given as the Dominant Harmonies of the keys B♭, F, and C; but being derived from unconnected Triads, they contain false Intervals (Ap. E. 20—26):—

G F, 9 : 16. B F, 45 : 64. D F, 27 : 32.

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