BROWN'S
MUSICAL
ARITHMETIC.

AN

INTRODUCTION

TO

MUSICAL ARITHMETIC;

WITH ITS APPLICATION

TO

TEMPERAMENT.

BY

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PREFACE.

Arithmetical proportion has been so sparingly introduced in works on Music, that this little treatise may lay claim to some measure of novelty. The first principles have long been before the public, in Dr. Holder's interesting "Treatise on the Natural Grounds and Principles of Harmony," published in 1694; but, so far as appears, they have never yet been applied to the illustration of Musical Chords, much less to the tuning of Instruments by Temperament. It is now attempted to accommodate them to both of these purposes: with what success must be left to the judgment of the intelligent reader.

Musical Intervals take their names from the number of degrees which they occupy on the staff. This has been found sufficient for the purposes of writing and reading music, and treating of it in a practical way; but when accurate measurement is required for theoretical purposes, they are expressed by double numbers, denoting the comparative rate of vibration in the two sounds of the Interval. Thus the Octave, in which the upper note vibrates twice while the lower vibrates once, is represented by the figures 1, 2.

As these terms denote merely the proportion of one note to another, it is evident that, in that form, Intervals cannot be added together as real quantities. But if they be expressed by single, instead of double numbers, they may be added and subtracted like other sums. This is done to our hand, in the method of tuning keyed Instruments by what is called the Equal Temperament, which divides the Octave into twelve equal degrees, or Semitones. Thus, the Semitone is represented by 1, the Tone (2 Semitones) by 2, the Minor Third (3 Semitones) by 3, and so on; adding 1 for every Semitone, till 12 completes the Octave. Now these degrees, although they are equal Semitones, are not equal quantities; for, if measured by the part of the string that produces them, they decrease regularly as they ascend in the scale: nevertheless they are rightly called equal Semitones, because each of them bears the same proportion to the one that is immediately under it. So the numbers 1 to 12 represent the Scale of Semitones, as tuned on our pianofortes.

These numbers, so applied, are called Logarithms; 12 being called the Logarithm of the Octave, and so of the rest. By using decimal fractions, or high numbers, it is easy to adapt this method to all kinds of Musical Intervals. Thus, the two Semitones make up the Minor Tone; the two Tones, the Major Third; the two Thirds the Fifth; the Fifth and Fourth the Octave:—

		Log.
Chromatic Semitone . . .	24 : 25 . . .	·708.
Diatonic Semitone . . .	15 : 16 . . .	1·116.
Minor Tone	9 : 10 . . .	1·824.
Major Tone	8 : 9 . . .	2·040.
Major Third	4 : 5 . . .	3·864.
Minor Third	5 : 6 . . .	3·156.
Fifth	2 : 3 . . .	7·020.
Fourth	3 : 4 . . .	4·980.
Octave	1 : 2 . . .	12·000.

This Logarithm of the Equal Semitone, or twelfth part of the Octave, is so easily explained, and so convenient for comparison, that I have adopted it in the present little work, in preference to taking the Minor Tone as unity, which I did in former treatises. In comparing the different methods of Temperament, I have used only two places of decimals, which give the Intervals true to the hundredth part of the equal Semitone.

Any one who desires greater accuracy, may obtain it to the thousandth part, by using decimals to three places, as given in Art. 24.

These Logarithms render the calculation of Intervals exceedingly simple; and afford an excellent means of testing the pretensions of any new methods of Temperament which may be proposed.

Rockhaven, 29th December, 1864.

INTRODUCTION

TO

MUSICAL ARITHMETIC.

CHAPTER I.

THE PITCH OF MUSICAL SOUNDS.

1. Musical sounds are produced by the regular vibration of elastic bodies, communicated to the air.

2. Slower vibrations produce graver sounds, or lower notes; and more rapid vibrations produce more acute sounds, or higher notes. Grave sounds below a certain pitch, and acute sounds above a certain pitch, are inaudible; but the vanishing point differs in different ears.

3. The rate of vibration is measured by an instrument invented on purpose, consisting of a kind of clock-work. But it is easier and more convenient to measure the length of the sounding body, whether string or tube; which, in similar circumstances, is always proportioned, inversely, to the number of vibrations.

4. The pitch of a musical sound, is its position in the Scale of acuteness and gravity. As no natural standard exists whereby the pitch of any particular note can be determined, a note is chosen which approximates to the middle of the range of human voices; it is called C, or *Do*, and its pitch is arbitrarily fixed at or near 256 vibrations in a second; which number, being the eighth power of 2, is easily remembered. This note, which occupies the added line between the Treble and Bass staves, is called the Tenor Clef note. By the Germans it is named once marked c. A range of four Octaves,

having this note in the middle, comprehends the most useful part of the Scale :—

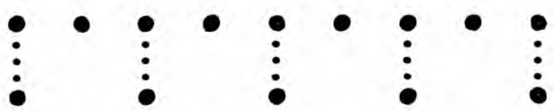
Thrice marked $\overline{\overline{\overline{c}}}$ . . . .
Twice marked $\overline{\overline{c}}$ . . . . . .
Once marked $\overline{c}$ . . . . . .
Small c . . . .
Great C . . . .

The Octave below Great C is called Double C, and marked C C.

5. The difference of pitch between two musical sounds is called an Interval. The measure of an Interval, is the proportion of the rate or velocity of the vibrations in the two sounds. Thus,

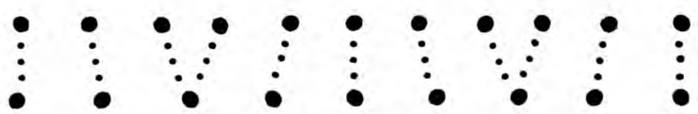
6. In the Interval called an Octave, each vibration of the lower note, is accompanied by two vibrations of the higher note; and the Interval is correctly expressed by 1 to 2, or 1 : 2, or $\frac{1}{2}$.

Every vibration of the lower note, coincides with every second vibration of the upper note; and the effect is almost the same as unison, or two notes of the same pitch sounding together:



7. In the Interval of the Fifth, every two vibrations of the lower note are accompanied by three of the upper note; and the Interval is expressed by 2 : 3, or $\frac{2}{3}$:

8. In the Interval of the Major Third, 4 : 5, the fourth vibrations of the lower note coincide with the fifth vibrations of the upper note :



9. The first ten numbers, taken as a Scale, produce nine Intervals, decreasing, in a geometrical ratio, from the Octave, 1 : 2, to the Minor Tone, 9 : 10 :—

1 : 2	. . . Octave.
2 : 3	. . . Fifth.
3 : 4	. . . Fourth.
4 : 5	. . . Major Third.
5 : 6	. . . Minor Third.
6 : 7	. . . Grave Third.
7 : 8	. . . Acute Second.
8 : 9	. . . Major Tone.
9 : 10	. . . Minor Tone.

10. Here the regular series of numbers and Intervals ends; for Harmony rejects all prime numbers above 7, and all their compounds. Wherefore, in order to continue the series of Harmonic numbers, it is necessary to combine the prime numbers 2, 3, 5, 7.

CHAPTER II.

HARMONIC NUMBERS.

11. In the following Table, the Thirds, Fifths, and Sevenths, are the Harmonic Intervals $\frac{4}{5}$, $\frac{2}{3}$, and $\frac{4}{7}$. All the even numbers are of course Octaves to lower numbers in the Table.

TABLE OF HARMONIC NUMBERS.

1	B♭	The Root.	10	D	Third to 8.
2	B♭	Octave to 1.	12	F	Fifth to 8.
3	F	Fifth to 2.	14	Â♭	Seventh to 8.
4	B♭	Octave to 2.	15	A	Third to 12.
5	D	Third to 4.	16	B♭	Octave to 8.
6	F	Fifth to 4.	18	C	Fifth to 12.
7	Â♭	Seventh to 4.	20	D	Third to 16.
8	B♭	Octave to 4.	21	Ê♭	Seventh to 12.
9	C	Fifth to 6.	24	F	Fifth to 16.

25	F♯	Third to 20.
27	G	Fifth to 18.
28	$\hat{A}\flat$	Seventh to 16.
30	A	Third to 24.
32	B♭	Octave to 16.
35	$\hat{C}$	Seventh to 20.
36	$\overset{\perp}{C}$	Fifth to 24.
40	D	Third to 32.
42	$\hat{E}\flat$	Seventh to 24.
45	E	Third to 36.
48	F	Fifth to 32.
49	$\hat{G}\flat$	Seventh to 28.
50	F♯	Third to 40.
54	G	Fifth to 36.
56	$\hat{A}\flat$	Seventh to 32.
60	A	Third to 48.
63	$\hat{B}\flat$	Seventh to 36.
64	$\overset{\perp}{B}\flat$	Octave to 32.
70	$\hat{C}$	Seventh to 40.
72	C	Fifth to 48.
75	C♯	Third to 60.
80	$\bar{D}$	Third to 64.
81	$\overset{\perp}{D}$	Fifth to 54.
84	$\hat{E}\flat$	Seventh to 48.
90	E	Third to 72.
96	F	Fifth to 64.
100	F♯	Third to 80.
105	$\hat{G}$	Seventh to 60.
108	G	Fifth to 72.
112	$\hat{A}\flat$	Seventh to 64.
120	A	Third to 96.
125	A♯	Third to 100.
126	$\hat{B}\flat$	Seventh to 72.
128	B♭	Octave to 64.
135	B	Third to 108.
140	$\hat{C}$	Seventh to 80.
144	C	Fifth to 96.
150	C♯	Third to 120.
160	$\bar{D}$	Third to 128.
162	$\overset{\perp}{D}$	Fifth to 108.
168	$\hat{E}\flat$	Seventh to 96.
175	$\hat{E}$	Seventh to 100.
180	E	Third to 144.
189	$\hat{F}$	Seventh to 108.
192	F	Fifth to 128.
200	F♯	Third to 160.
210	$\hat{G}$	Seventh to 120.
216	G	Fifth to 144.
224	$\hat{A}\flat$	Seventh to 128.
225	G♯	Third to 180.

12. In constructing such a Table, number 1 may, of course, represent any convenient note. I have chosen B♭, because it brings out the first three sharps in their order F♯ 25, C♯ 75, and G♯ 225. The various notes arise from the root by Octaves, Fifths, Major Thirds, and Grave Sevenths. Any note may be easily traced to the root thus: G♯ 225 is Third to 180; 180 is Third to 144; 144 is Fifth to 96; 96 is Fifth to 64; 64 is Octave to 32; 32 to 16, 16 to 8, 8 to 4, 4 to 2, and 2 to 1.

13. To distinguish different notes that bear the same name, I use the following method. The Root, with its Octaves, and the Fifths, all being basses of Major Triads, are called Major notes, and marked

with the Major or plus sign, +, as $\overset{+}{B}\flat$, $\overset{+}{F}$, $\overset{+}{C}$, &c. The Thirds, which bear Minor Triads, are called Minor notes, and marked with the Minor sign, –, as $\bar{D}$, $\bar{A}$, $\bar{E}$, &c. The Sevenths, being all grave or Dominant Sevenths, are marked with a circumflex accent, ∧, or a small delta, △, as $\hat{A}\flat$, $\hat{E}\flat$, $\hat{B}\flat$, &c. See in the preceding Table two Ds, 80, 81; two B flats, 63, 64; and two Cs, 35, 36.

CHAPTER III.

INTERVALS.

14. The following Table exhibits the various Intervals, in the order in which they arise out of the prime numbers and their compounds. But as the number 2, in composition, merely changes the Octave, it is here omitted, on purpose to avoid too many figures.

TABLE OF INTERVALS.

Primes.	Intervals.
2 . . .	1 : 2, Octave.
3 . . .	2 : 3, Fifth.
	3 : 4, Fourth.
5 . . .	3 : 5, Major Sixth.
	4 : 5, Major Third.
	5 : 6, Minor Third.
	5 : 8, Minor Sixth.
7 . . .	4 : 7, Grave Seventh.
	5 : 7, Grave Fifth.
	6 : 7, Grave Third.
	7 : 8, Acute Second.
	7 : 10, Acute Fourth, or Tritone.
	7 : 12, Acute Sixth.
3, 3 . . .	4 : 9, Ninth.
	5 : 9, Minor Seventh.
	7 : 9, Acute Third.
	8 : 9, Major Tone.
	9 : 10, Minor Tone.
	9 : 16, Minor Seventh, less Comma.

Primes.		Intervals.
3, 5 . . .	8 :	15, Major Seventh.
	14 :	15, Hemitone Major.
	15 :	16, Diatonic Semitone.
3, 7 . . .	10 :	21, Grave Ninth.
	16 :	21, Grave Fourth.
	20 :	21, Hemitone Minor.
5, 5 . . .	14 :	25, Augmented Sixth.
	16 :	25, Augmented Fifth.
	18 :	25, Augmented Fourth.
	21 :	25, Augmented Second.
	24 :	25, Chromatic Semitone.
	25 :	36, Diminished Fifth.
	25 :	42, Diminished Seventh.
5, 7 . . .	32 :	35, Grave Second.
	35 :	36, Quarter Tone.
3, 3, 7 . . .	63 :	64, Komma Major.
3, 3, 3, 3 . . .	80 :	81, Comma.
5, 5, 5 . . .	125 :	128, Enharmonic Diesis.

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15. The simple arithmetical rules for the treatment of Intervals, are as follows :—

RULE I.

16. To reduce an Interval to its lowest terms: divide repeatedly by the Harmonic primes, 2, 3, 5, 7, so long as both terms are divisible by any of them ; thus :—

Reduce 150 : 252 to its lowest terms :

150 : 252 ÷ 2 = 75 : 126.
75 : 126 ÷ 3 = 25 : 42.—*Answer.*

RULE II.

17. To invert an Interval : invert the fraction that expresses it, and then halve the upper term, or double the lower ; thus :—

The Fourth, $\frac{3}{4}$, inverted, becomes $\frac{2}{3}$, the Fifth :

$$\frac{3}{4} \quad \frac{4}{3} \quad \frac{2}{3}$$

The Major Third $\frac{4}{5}$, inverted, becomes $\frac{5}{8}$, the Minor Sixth :

$$\frac{4}{5} \quad \frac{5}{4} \quad \frac{5}{8}$$

RULE III.

18. To add Intervals together: multiply the upper terms into each other, and likewise the lower terms: then reduce the Interval found, to its lowest terms; thus:—

Add together the Fifth $\frac{2}{3}$, and the Fourth $\frac{3}{4}$:

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}, \text{ Octave.}$$

Add together the Major, Minor, and Grave Thirds:

$$\frac{4}{5} \times \frac{5}{6} + \frac{6}{7} = \frac{120}{210} = \frac{4}{7}, \text{ Grave Seventh.}$$

19. The operation may frequently be abridged, by leaving out the same number, when it occurs in the numerator of one fraction, and the denominator of another; thus:—

$\frac{3}{4} \times \frac{4}{5}$, neglecting the two fours, is $= \frac{3}{5}$.

$\frac{4}{5} \times \frac{5}{6} \times \frac{6}{7}$, neglecting the fives and sixes, is $= \frac{4}{7}$.

or, by dividing, crosswise, any numerators and denominators that have a common measure; thus:—

$$\frac{24}{25} \times \frac{80}{81} = \frac{8}{5} \times \frac{16}{27} = \frac{128}{135}.$$

In this Example, 24 and 81 are divided by 3; 25 and 80 by 5.

RULE IV.

20. To subtract one Interval from another: multiply the terms of the two fractions crosswise: or invert the fraction that denotes the lesser Interval, and proceed as in Addition. Observe, that the greater fraction denotes the lesser Interval. Thus:—

From a Major Third $\frac{4}{5}$, subtract a Minor Third $\frac{5}{6}$:

$$\frac{4}{5} \times \frac{5}{6} = \frac{24}{25}: \text{ or } \frac{4}{5} \times \frac{6}{5} = \frac{24}{25}.$$

21. An Interval is said to be divided harmonically, when the terms into which it is divided are in arithmetical progression; as 2, 3, 4, increasing by 1; or 12, 15, 18, increasing by 3, and so on. Thus the following Intervals are harmonically divided:—

Octave	1 : 2 into 2 : 3 and 3 : 4 . .	Fifth and Fourth.
Fifth	2 : 3 into 4 : 5 and 5 : 6 . .	Major and Minor Third.
Major Third . .	4 : 5 into 8 : 9 and 9 : 10 . .	Major and Minor Tone.
Grave Seventh. .	4 : 7 into 4 : 5	Major Third.
	5 : 6	Minor Third, and
	6 : 7	Grave Third.

Hence the Fifth is called the mean proportional in the Octave; the Major Third, mean proportional in the Fifth, and so on. Intervals which cannot be divided harmonically, as the Minor Third 5 : 6, have no mean proportional.

22. The calculations of Intervals are generally short and simple. Where longer calculations are required, it is a saving of labour to use Logarithms, in which Addition and Subtraction do the work of Multiplication and Division. For example, the Octave is divisible into

12 Chromatic Semitones of . . .	24 : 25.
7 Enharmonic Dieses of. . . .	125 : 128.
3 Commas of	80 : 81.

To add these Intervals together by common Arithmetic, it is necessary to multiply $\frac{24}{25}$ by itself to the 12th power, $\frac{125}{128}$ to the 7th power, and $\frac{80}{81}$ to the third power; and then to multiply the products into each other. By Logarithms, the process is very short and simple:—

Chromatic Semitone	Log. ·708 × 12 =	8·496
Enharmonic Diesis	Log. ·408 × 7 =	2·856
Comma	Log. ·216 × 3 =	·648
	Logarithm of Octave	12·000

23. In a former work, as a convenient common measure for Intervals, I chose the Minor Tone, to be represented by 1, or unity. In this little treatise, because the Octave is divided, on our pianofortes,

into twelve equal Semitones, I have made the Octave to be represented by 12: and of course 1 represents the equal Semitone. By this means, every Interval on the pianoforte, is easily compared with the true Interval obtained by calculation.

24. In the following Table the various Intervals are divided into four classes, and the Logarithms are added:

1. INTERVALS OF THE DIATONIC SCALE.

		Log.
Ninth	4 : 9	14 · 040.
Octave	1 : 2	12 · 000.
Seventh Major	8 : 15	10 · 884.
Seventh Minor	5 : 9	10 · 176.
Sixth Major	3 : 5	8 · 844.
Sixth Minor	5 : 8	8 · 136.
Fifth	2 : 3	7 · 020.
Fourth	3 : 4	4 · 980.
Third Major	4 : 5	3 · 864.
Third Minor	5 : 6	3 · 156.
Tone Major	8 : 9	2 · 040.
Tone Minor	9 : 10	1 · 824.
Semitone Diatonic	15 : 16	1 · 116.

2. INTERVALS OF THE DOMINANT HARMONY.

		Log.
Grave Seventh	4 : 7	9 · 689.
Grave Fifth	5 : 7	5 · 825.
Grave Third	6 : 7	2 · 669.
Acute Second	7 : 8	2 · 311.
Acute Third	7 : 9	4 · 351.
Acute Fourth	7 : 10	6 · 175.
Acute Sixth	7 : 12	9 · 331.

3. AUGMENTED AND DIMINISHED INTERVALS.

		Log.
Augmented Sixth	14 : 25	10 · 039.
Augmented Fifth	16 : 25	7 · 728.
Augmented Fourth	18 : 25	5 · 688.
Augmented Second	21 : 25	3 · 019.
Diminished Seventh	25 : 42	8 · 981.
Diminished Fifth	25 : 36	6 · 312.

4. Semitones, and Smaller Intervals.

		Log.	
Semitone Diatonic	15 : 16	1·116.	
Semitone Chromatic	24 : 25	·708.	
Hemitone Major	14 : 15	1·195.	
Hemitone Minor	20 : 21	·845.	
Quarter Tone	35 : 36	·487	Neglected in Temperament.
Komma Major	63 : 64	·271	Neglected in Temperament.
Comma	80 : 81	·216	Neglected in Temperament.
Enharmonic Diesis	125 : 128	·408	Neglected in Temperament.
Diaschisma		·240.	

CHAPTER IV.

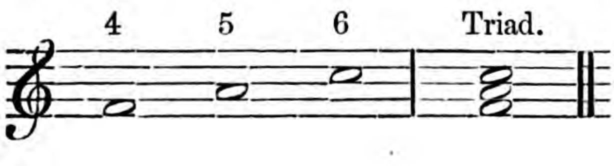
PRIMITIVE CHORDS.

25. The two Chords, from which all others are derived, are the Harmonic Triad, or Major Common Chord; and the Harmonic Tetrad, or Dominant Harmony.

1. Harmonic Triad.

26. The first five numbers produce the fundamental Concord, or Major Common Chord :—

27. In the fourth, fifth, and sixth numbers, the essential notes of this Chord are found together, in the form of a Triad :—

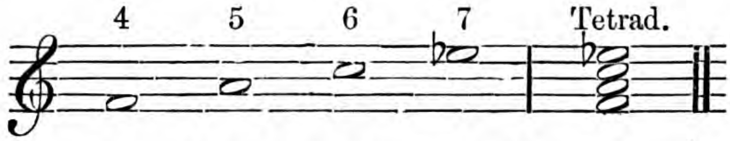


In this form, it is called the Harmonic Triad, or Major Triad. It consists of a Root, with its Major Third and Fifth. Its proportions, as shown above, are 4, 5, 6.

2. Harmonic Tetrad.

28. The first seven numbers produce the fundamental Seventh, or Dominant Harmony :—

29. In the fourth, fifth, sixth, and seventh numbers, the essential notes of this Chord are found together in the form of a Tetrad, or Chord of the Seventh :—



In this form, it is called the Harmonic Seventh, or fundamental Seventh; or more commonly the Dominant Seventh, from its being peculiar to the Dominant, or Fifth of the Key. It consists of a Root, with Major Third, Fifth, and Grave Seventh. Its proportions, as above, are 4, 5, 6, 7.

CHAPTER V.

THE SCALE.

30. Two similar Triads, in which the uppermost note of the one, coincides with the lowest note of the other, may be called Adjacent Triads. A series of adjacent Harmonic Triads, I shall call the Scale of Triads. The following is a part of it :—

$\overset{+}{B}\flat \quad \overset{-}{D} \quad \overset{+}{F} \quad \overset{-}{A} \quad \overset{+}{C} \quad \overset{-}{E} \quad \overset{+}{G} \quad \overset{-}{B} \quad \overset{+}{D} \quad \overset{-}{F}\sharp \quad \overset{+}{A}$

31. Any three adjacent Triads contain the notes of a Diatonic Scale. For example, take F, C, and G.

Triad of F . . .	F A C . . .	4 . 5 . 6 × 4 = 16 . 20 . 24.
Triad of C . . .	C E G . . .	4 . 5 . 6 × 6 = 24 . 30 . 36.
Triad of G . . .	G B D . . .	4 . 5 . 6 × 9 = 36 . 45 . 54.

These notes, reduced within an Octave, and arranged in alphabetical order, produce the Scale of C:—

24	27	30	32	36	40	45	48
C	D	E	F	G	A	B	C

32. The Degrees of the Diatonic Scale are,

C D	24 : 27 ÷ 3 =	8 : 9	Major Tone.
D E	27 : 30 ÷ 3 =	9 : 10	Minor Tone.
E F	30 : 32 ÷ 2 =	15 : 16	Diatonic Semitone.
F G	32 : 36 ÷ 4 =	8 : 9	Major Tone.
G A	36 : 40 ÷ 4 =	9 : 10	Minor Tone.
A B	40 : 45 ÷ 5 =	8 : 9	Major Tone.
B C	45 : 48 ÷ 3 =	15 : 16	Diatonic Semitone.

33. The same Scale may be produced from the Octave, by Harmonic division; thus:—

Required to divide the Octave into degrees.

To divide the Octave C C, 1 : 2, harmonically, multiply its terms by 2, and the mean proportional will appear:

1 : 2 × 2 = 2 : 4; mean proportional 3.

C G C . . . . 2 . 3 . 4 × 12 = 24 . 36 . 48.

In the same way, the Fifth C G, 2 : 3, is divided into

C E G . . . . 4 . 5 . 6 × 6 = 24 . 30 . 36.

and the Major Third C E, 4 : 5, into

C D E . . . . 8 . 9 . 10 × 3 = 24 . 27 . 30.

To obtain a perfect Fourth to the Scale, invert the division of the Octave, placing the Fourth under the Fifth:

C F C . . . . 3 . 4 . 6 × 8 = 24 . 32 . 48.

then divide the Fifth F C harmonically:

F A C . . . . 4 . 5 . 6 × 8 = 32 . 40 . 48.

The only note wanting to complete the Scale is B. But as A C, 5 : 6, is a Minor Third, which has no mean proportional, the Fifth G D, 2 : 3, is harmonically divided into

G B D . . . . 4 . 5 . 6 × 9 = 36 . 45 . 54.

We have now the complete Scale,

24	27	30	32	36	40	45	48
C	D	E	F	G	A	B	C

and the Octave has been divided into seven degrees.

34. The Scale of Triads may be resolved into a series of alternate Major and Minor Triads, interwoven with each other; each Triad in the series, having two notes in common with the Triads above and below it:—

	16	20	24	30	36	45	54
	F	A	C	E	G	B	D
Major Triad	—	—	—	—	G	B	D
Minor Triad	—	—	—	E	G	B	—
Major Triad	—	—	C	E	G	—	—
Minor Triad	—	A	C	E	—	—	—
Major Triad	F	A	C	—	—	—	—

35. From this Table it is evident, that the Minor Triad is a compound Chord, taking its Minor Third from the Triad below, and its Major Third from the Triad above it; and so has two Harmonic Roots.

36. The proportions of the Minor Triad are 10, 12, 15:—

Triad of A . . . A C E . . . 20 . 24 . 30 ÷ 2 = 10 . 12 . 15.
Triad of E . . . E G B . . . 30 . 36 . 45 ÷ 3 = 10 . 12 . 15.

37. The descending Scale of the Minor Mode may be constructed in the same way as the Major, taking Minor instead of Major Triads. Its proportions are

240	216	192	180	160	144	135	120
A	G	F	E	D	C	B	A

CHAPTER VI.

COMPOUND CHORDS.

38. The Compound Chords are of two kinds; the one composed of Triads, the other of Dominant Harmonies.

39. Every compound Chord has, for its basis, a Triad, either Major or Minor, which may be called the fundamental Triad: but sometimes one, or even two notes of the Triad, are omitted.

1. Chords compounded of Triads.

40. These are commonly distinguished into Chords of Suspension, and Chords of Addition; but here we take them together.

41. The fundamental Triad borrows notes from the adjacent Triads, above and below it. The Fourth and the Sixth are taken from the Triad below; the Seventh, and the Ninth, or its Octave the Second, from the Triad above.

Major Mode.

Fundamental Triad.	C	E	G			4 . 5 . 6 × 3	= 12 . 15 . 18.
Triad below	F	A	C			4 . 5 . 6 × 4	= 16 . 20 . 24.

Suspended Fourth	C	F	G		12 . 16 . 18.
Suspended Fourth and Sixth	C	F	A		12 . 16 . 20.
Suspended Sixth	C	E	A		12 . 15 . 20.
Added Sixth	C	E	G	A	12 . 15 . 18 . 20.

Minor Mode.

Fundamental Triad	E	G	B	10 . 12 . 15 × 3	= 30 . 36 . 45.
Triad below	A	C	E	10 . 12 . 15 × 4	= 40 . 48 . 60.
Tonic Major	A	C♯	E	4 . 5 . 6 × 10	= 40 . 50 . 60.

Suspended Fourth	E	A	B		30 . 40 . 45.
Suspended Fourth and Sixth	E	A	C		30 . 40 . 48.
Suspended Sixth	E	G	C		30 . 36 . 48.
Added Sixth Minor	E	G	B	C	30 . 36 . 45 . 48.
Added Sixth Major	E	G	B	C♯	30 . 36 . 45 . 50.

Major Mode.

Fundamental Triad.	C E G . .	4 . 5 . 6	× 2 = 8 . 10 . 12.
Fundamental Tetrad	C E G B̂♭	4 . 5 . 6 . 7	× 2 = 8 . 10 . 12 . 14.
Triad above	G B D . .	4 . 5 . 6	× 3 = 12 . 15 . 18.

Suspended Seventh	C E G B . .	. 8 . 10 . 12 . 15.
Suspended Ninth	C E G D . .	. 8 . 10 . 12 . 18.
Suspended Seventh and Ninth.	C E G B D .	. 8 . 10 . 12 . 15 . 18.
Added Ninth	C E G B̂♭ D .	. 8 . 10 . 12 . 14 . 18.

Minor Mode.

Fundamental Triad.	A C E. .	. 10 . 12 . 15	× 2 = 20 . 24 . 30.
Fundamental Tetrad	A C♯ E Ĝ.	. 4 . 5 . 6 . 7	× 5 = 20 . 25 . 30 . 35.
Triad Above	E G B. .	. 10 . 12 . 15	× 3 = 30 . 36 . 45.

Suspended Seventh.	A C E G . .	. 20 . 24 . 30 . 36.
Suspended Ninth	A C E B . .	. 20 . 24 . 30 . 45.
Suspended Seventh and Ninth .	A C E G B .	. 20 . 24 . 30 . 36 . 45.
Added Ninth	A C♯ E Ĝ B .	. 20 . 25 . 30 . 35 . 45.

2. COMPOUND DOMINANT HARMONIES.

42. The Compound Dominant Harmonies are the Diminished Seventh, and the Augmented Sixths.

43. The Diminished Seventh is compounded of the Dominant Harmony of the Minor Mode, and that of its Relative Major:—

Fundamental Tetrad. .	E G♯ B D̂ .	. 4 . 5 . 6 . 7	× 5 = 20 . 25 . 30 . 35.
Relative Major. . . .	G B D F̂ .	. 4 . 5 . 6 . 7	× 6 = 24 . 30 . 36 . 42.
Diminished Seventh . . .	G♯ B D̂ F̂ . .		. 25 . 30 . 35 . 42.

44. The Augmented Sixth comprehends the three Chords, known by the names Italian, French, and German Sixths.

45. The French and Italian Sixths are compounded of the Dominant Harmony of the Minor Mode, with that of the nearest Major Harmony below:—

Fundamental Tetrad. .	B D♯ F♯ Â .	. 4 . 5 . 6 . 7	× 5 = 20 . 25 . 30 . 35.
Major Third below . .	G B D F̂ .	. 4 . 5 . 6 . 7	× 4 = 16 . 20 . 24 . 28.
French Sixth . . .	F̂ Â B D♯ . .		. 28 . 35 . 40 . 50.
Italian Sixth . . .	F̂ Â – D♯ . .		. 28 . 35 — 50.

46. The German Sixth is compounded of the Dominant Harmony of the Minor Mode with those of the two nearest Major Triads, above and below. In other words, it is composed of three Dominant Harmonies, taken on the three notes of a Major Triad.

Fundamental Tetrad	B D♯ F♯ $\hat{A}$.
Third above	D F♯ A $\hat{C}$.
Third below	G B D $\hat{F}$.

Or thus—

G B D $\hat{F}$	4 . 5 . 6 . 7 × 4	= 16 . 20 . 24 . 28.
B D♯ F♯ $\hat{A}$	4 . 5 . 6 . 7 × 5	= 20 . 25 . 30 . 35.
D F♯ A $\hat{C}$	4 . 5 . 6 . 7 × 6	= 24 . 30 . 36 . 42.
German Sixth . . $\hat{F}$ $\hat{A}$ $\hat{C}$ D♯ . .		28 . 35 . 42 . 50.

CHAPTER VII.

RECAPITULATION OF CHORDS.

47. The various Chords or Harmonies may be summed up as follows:—

TONIC HARMONY.

1. Major Triad	C E G	4 . 5 . 6.
2. Minor Triad	A C E	10 . 12 . 15.

DOMINANT HARMONY.

3. Dominant Seventh . . .	G B D $\hat{F}$ . . .	4 . 5 . 6 . 7.
4. Added Ninth	G B D $\hat{F}$ A . .	4 . 5 . 6 . 7 . 9.

SUBDOMINANT HARMONY.

5. Added Sixth, Major Mode . .	F	A	C	D	. .	12 . 15 . 18 . 20.
6. Added Sixth, Minor Mode. .	D	F	A	B	. .	30 . 36 . 45 . 50.
7. Minor Added Sixth	D	F	A	B♭	. .	10 . 12 . 15 . 16.

COMPOUND DOMINANT HARMONIES.

8. Diminished Seventh. . .	G♯	B	$\hat{\text{D}}$	$\hat{\text{F}}$	. . .	25 . 30 . 35 . 42.
9. Italian Sixth	$\hat{\text{F}}$	$\hat{\text{A}}$	—	D♯	. . .	28 . 35 — 50.
10. French Sixth	$\hat{\text{F}}$	$\hat{\text{A}}$	B	D♯	. . .	28 . 35 . 40 . 50.
11. German Sixth.	$\hat{\text{F}}$	$\hat{\text{A}}$	$\hat{\text{C}}$	D♯	. . .	28 . 35 . 42 . 50.

48. The following Table exhibits a comparative view of the principal Chords, with their proportions :—

Major Mode.

Major Triad	12 . 15 . 18.
Dominant Seventh	12 . 15 . 18 . 21.
Added Ninth	12 . 15 . 18 . 21 . 27.
Added Sixth	12 . 15 . 18 . 20.
Suspended Sixth	12 . 15 — 20.
Suspended Seventh	12 . 15 . 18 . 22½.
Suspended Seventh and Ninth . .	12 . 15 . 18 . 22½ . 27.
Suspended Ninth	12 . 15 . 18 . 27.
Suspended Fourth	12 . 16 . 18.
Suspended Fourth and Sixth . . .	12 . 16 . 20.

Minor Mode.

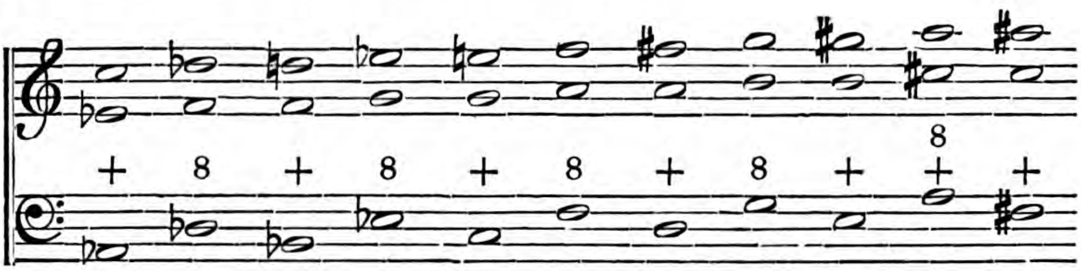
Minor Triad.	10 . 12 . 15.
Added Minor Sixth	10 . 12 . 15 . 16.
Suspended Seventh	10 . 12 . 15 . 18.
Suspended Sixth	10 . 12 . 16.

CHAPTER VIII.

CHROMATIC SCALES.

49. A series of ascending or descending Semitones, is called a Chromatic Scale. The only regular Scales of this kind are produced by a Fundamental Bass moving round the circle of Keys.

50. An ascending Chromatic Scale is formed by a series of Major Triads, whose basses ascend by Fourths, and descend by Minor Thirds; the Key continually changing. In this Scale, the Chromatic notes are the Major Third and the Octave alternately: and the Degrees are alternate Diatonic and Chromatic Semitones; producing, by their sums, a succession of Minor Tones:—



This Scale ascends,

5 Diatonic Semitones	Log. 1 · 116	× 5	=	5 · 580
5 Chromatic Semitones . . .	· 708	× 5	=	3 · 540
Total				9 · 120

The Fundamental Bass,

Rises 5 Fourths	Log. 4 · 980	× 5	=	24 · 900
Falls 5 Minor Thirds	3 · 156	× 5	=	15 · 780
				9 · 120

To complete the Octave from A ♭,

Add F♯ G, Diatonic Semitone				1 · 116
G A♭, Diatonic Semitone				1 · 116
3 Commas lost	216	× 3	=	· 648
Octave				12 · 000

51. A descending Scale is produced by reversing the foregoing: the Bass descending by Fourths, and ascending by Minor Thirds.

52. Another descending Chromatic Scale is formed by the Chromatic Sequence of Sevenths; a series of Dominant Harmonies, descending by Fifths, and ascending by Fourths; the Key changing continually. In this Scale the Chromatic notes are the Major Third and the Dominant Seventh alternately; and the Degrees are alternate Major and Minor Hemitones; producing by their sums, a succession of Major Tones:—

This scale descends,

6 Major Hemitones	Log. 1 · 195	× 6	=	7 · 170
6 Minor Hemitones	· 845	× 6	=	5 · 070
	Total			12 · 240

The Fundamental Bass,

Falls 6 Fifths	Log. 7 · 020	× 6	=	42 · 120
Rises 6 Fourths	4 · 980	× 6	=	29 · 880
				12 · 240

To complete the Octave from D♭,

Add C♯ D♭, Enharmonic Diesis	· 408
	12 · 648
Subtract 3 Commas . . . . 216 × 3 =	· 648
Octave	12 · 000

53. The sum of two Degrees being, in the one Chromatic Scale, equal to a Minor Tone, and in the other to a Major Tone; the difference between which is a Comma: it follows, that at the end of an Octave, which contains 12 Chromatic Degrees, the last notes of the two Scales differ in pitch by six Commas; the one exceeding, and the other falling short of the original pitch, by three Commas.

CHAPTER IX.

TEMPERAMENT.

54. On keyed Instruments, the number of sounds within the Octave being usually limited to twelve, the same sound must represent several different notes. That this may occasion as little offence to the ear as possible, is the design of the methods of false tuning called Temperament.

55. Twelve notes tuned by ascending Fifths, or, what is equivalent, by rising Fifths and falling Fourths, take up the twelve keys of the finger-board.

E♭ B♭ F C G D A E B F♯ C♯ G♯.

If another note were added to the series, it would be D♯: but the key being already occupied by E♭, the problem is, to divide the Octave from E♭ to E♭ to the best advantage.

56. This is attempted by two different methods, commonly called the Equal Temperament, and the Vulgar, or Old Organ Temperament; which depend on the following propositions:

PROPOSITION 1.

57. The sum of 12 Fifths, minus 7 Octaves, is equal to the sum of 3 Commas, minus Diesis:—

Fifth . . . . Log.	7 · 020 × 12 =	84 · 240.	
Octave	12 · 000 × 7 =	84 · 000.	
			· 240.
Comma	216 × 3 =	· 648.	
Diesis		· 408.	
			· 240.

This Interval of 3 Commas, minus Diesis, is called Diaschisma. Its proportions are $\frac{524,288}{531,441}$, and its Logarithm, as above, · 240.

The Equal Temperament, which divides the Octave into twelve equal Semitones, depends on the first proposition.

PROPOSITION 2.

58. The sum of 4 Fifths, minus 2 Octaves, is equal to Major Third, plus Comma:—

Fifth . . . . . Log. 7·020 × 4 =	28·080.	
Octave . . . . 12·000 × 2 =	24·000.	
		4·080.
Major Third . . . . Log.	3·864.	
Comma	·216.	
		4·080.

PROPOSITION 3.

59. The sum of 3 Major Thirds, plus Diesis, is equal to an Octave:—

Major Third . . . . Log. 3·864 × 3 =	11·592.
Enharmonic Diesis . . . . Log.	·408.
Octave	12·000.

The Vulgar, or Organ Temperament, depends on the second and third propositions.

60. In the Equal Temperament, the sum of 12 Fifths is made to coincide with the sum of 7 Octaves, by tuning each of the Fifths flatter than the true Fifth, by $\frac{1}{12}$ Diaschisma. (Art. 57.)

61. The Vulgar Temperament proceeds by three operations; in each of which, the sum of 4 Fifths minus 2 Octaves, is made to coincide with a Major Third, by tuning the Fifths flatter than the true Fifth by $\frac{1}{4}$ Comma. By this means the Octave is divided into three Major Thirds, and the Enharmonic Diesis.

1st Operation	E♭	B♭	F	C	G.
2nd Operation	G	D	A	E	B.
3rd Operation	B	F♯	C♯	G♯	D♯.

But the key for D♯ being already occupied by E♭, the Diesis remains where the extremes of the tuning Scale meet, between G♯ and E♭.

This Interval, which is a Diminished Sixth instead of a Fifth, is called by organ builders the Great Wolf.

62. By the Vulgar Temperament, all the Major Thirds in the tuning Scale are perfect. Where Major Thirds are wanting, their places are supplied by Wolf Intervals, or Diminished Fourths; as B E♭ for B D♯; or G♯ C for A♭ C. The same observation applies to the Minor Sixths, which are inversions of the Major Thirds.

63. All the Minor Thirds in the Scale, 8 in number, are too flat by $\frac{1}{4}$ Comma; and their inversions, the Major Sixths, as much too sharp. Where they are wanting, their places are supplied by Wolf Intervals.

64. All the Fifths are too flat by $\frac{1}{4}$ Comma, and their inversions, the Fourths, as much too sharp: except where the extreme notes of the tuning Scale meet in the Great Wolf.

65. In both Temperaments, the Subdominant of the Scale is used for the Dominant Seventh; which occasions a deviation from the true pitch, that rivals the Wolf Intervals themselves. It is no doubt for this reason, that so beautiful and important a concord has long been misnamed a discord.

66. Violins are tuned by perfect Fifths; Keyed Instruments by tempered Fifths.

67. To exhibit to the eye the method of tuning, the following Intervals are necessary:

Octave	Art. 24.	1. . .	Log. 12 · 000.
Fifth	Art. 24.	1. . .	7 · 020.
Diesis	Art. 24.	4. . .	· 408.
Comma	Art. 24.	4. . .	· 216.
Diaschisma. . . .	Art. 57.	. .	· 240.

68. The Fifth of the Equal Temperament, (Art. 60.) is equal to perfect Fifth, minus $\frac{1}{12}$ Diaschisma; or

$$7 \cdot 020 - \cdot 020 = 7 \cdot 000.$$

69. The Fifth of the Vulgar Temperament (Art. 61) is equal to perfect Fifth, minus $\frac{1}{4}$ Comma; or

$$7 \cdot 020 - \cdot 054 = 6 \cdot 966.$$

70. The tuning of the Equal Temperament proceeds by tempered Fifths of 7·000; rejecting the Octaves as they occur, that is, subtracting 12, when the sum exceeds that number:

Tuning Scale.			Tuned Scale.	
E♭	. . . .	0.	E♭	0.
B♭	. . . .	7.	E	1.
F	7 + 7 — 12 =	2.	F	2.
C	2 + 7 . . . =	9.	F♯	3.
G	9 + 7 — 12 =	4.	G	4.
D	4 + 7 . . . =	11.	G♯	5.
A	11 + 7 — 12 =	6.	A	6.
E	6 + 7 — 12 =	1.	B♭	7.
B	1 + 7 . . . =	8.	B	8.
F♯	8 + 7 — 12 =	3.	C	9.
C♯	3 + 7 . . . =	10.	C♯	10.
G♯	10 + 7 — 12 =	5.	D	11.
D♯	5 + 7 . . . =	12.	D♯ or E♭	12.

71. The tuning of the Vulgar Temperament proceeds by tempered Fifths of 6·966; rejecting the Octaves as before.

Tuning Scale.

	E♭	. . . .	0·
	B♭	. . . .	6·966.
	F	6·966 + 6·966 — 12 =	1·932.
	C	1·932 + 6·966 . . . =	8·898.
	G	8·898 + 6·966 — 12 =	3·864.
	D	3·864 + 6·966 . . . =	10·830.
	A	10·830 + 6·966 — 12 =	5·796.
	E	5·796 + 6·966 — 12 =	·762.
	B	·762 + 6·966 . . . =	7·728.
	F♯	7·728 + 6·966 — 12 =	2·694.
	C♯	2·694 + 6·966 . . . =	9·660.
	G♯	9·660 + 6·966 — 12 =	4·626.
Diesis {	D♯	4·626 + 6·966 . . . =	11·592.
	E♭	11·592 + ·408 . . . =	12·000.

VULGAR TEMPERAMENT.

TUNED SCALE.

		Differences.
E♭	0·	
E	·762	·762.
F	1·932	1·170.
F♯	2·694	·762.
G	3·864	1·170.
G♯	4·626	·762.
A	5·796	1·170.
B♭	6·966	1·170.
B	7·728	·762.
C	8·898	1·170.
C♯	9·660	·762.
D	10·830	1·170.
E♭	12·000	1·170.

The differences between the notes of this Scale are alternate Chromatic and Diatonic Semitones.

72. We shall now compare with the true Intervals, those of the Equal Temperament, and of the Vulgar Temperament exclusive of the Wolves.

EQUAL TEMPERAMENT.

1. INTERVALS OF THE DIATONIC SCALE.

		True.	Equal.	Error.
Ninth	4 : 9	14·04	14·00	4 ♭.
Octave	1 : 2	12·00	12·00	0.
Seventh Major	8 : 15	10·88	11·00	12 ♯.
Seventh Minor	5 : 9	10·18	10·00	18 ♭.
Sixth Major	3 : 5	8·84	9·00	16 ♯.
Sixth Minor	5 : 8	8·14	8·00	14 ♭.
Fifth	2 : 3	7·02	7·00	2 ♭.
Fourth	3 : 4	4·98	5·00	2 ♯.
Third Major	4 : 5	3·86	4·00	14 ♯.
Third Minor	5 : 6	3·16	3·00	16 ♭.
Tone Major	8 : 9	2·04	2·00	4 ♭.
Tone Minor	9 : 10	1·82	2·00	18 ♯.

EQUAL TEMPERAMENT.

2. Intervals of the Dominant Harmony.

		True.	Equal.	Error.
Grave Seventh	4 : 7	9 · 69	10 · 00	31 ♯.
Grave Fifth	5 : 7	5 · 83	6 · 00	17 ♯.
Grave Third	6 : 7	2 · 67	3 · 00	33 ♯.
Acute Second	7 : 8	2 · 31	2 · 00	31 ♭.
Acute Third	7 : 9	4 · 35	4 · 00	35 ♭.
Acute Fourth	7 : 10	6 · 17	6 · 00	17 ♭.
Acute Sixth	7 : 12	9 · 33	9 · 00	33 ♭.

3. Augmented and Diminished Intervals.

		True.	Equal.	Error.
Augmented Sixth	14 : 25	10 · 04	10 · 00	4 ♭.
Augmented Fifth	16 : 25	7 · 73	8 · 00	27 ♯.
Augmented Fourth	18 : 25	5 · 69	6 · 00	31 ♯.
Augmented Second	21 : 25	3 · 02	3 · 00	2 ♭.
Diminished Seventh	25 : 42	8 · 98	9 · 00	2 ♯.
Diminished Fifth	25 : 36	6 · 31	6 · 00	31 ♭.

4. Semitones.

		True.	Equal,	Error.
Diatonic Semitone	15 : 16	1 · 12	1 · 00	12 ♭.
Chromatic Semitone	24 : 25	· 71	1 · 00	29 ♯.
Hemitone Major	14 : 15	1 · 20	1 · 00	20 ♭.
Hemitone Minor	20 : 21	· 84	1 · 00	16 ♯.

VULGAR TEMPERAMENT.

1. Intervals of the Diatonic Scale.

		True.	Vulgar.	Error.
Ninth	4 : 9	14 · 04	13 · 93	11 ♭.
Octave	1 : 2	12 · 00	12 · 00	0.
Seventh Major	8 : 15	10 · 88½	10 · 83	5½ ♭.
Seventh Minor	5 : 9	10 · 18	10 · 07	11 ♭.
Sixth Major	3 : 5	8 · 84½	8 · 90	5½ ♯.
Sixth Minor	5 : 8	8 · 14	8 · 14	0.
Fifth	2 : 3	7 · 02	6 · 96½	5½ ♭.
Fourth	3 : 4	4 · 98	5 · 03½	5½ ♯.

VULGAR TEMPERAMENT.

		True.	Vulgar.	Error.
Third Major	4 : 5	3 · 86	3 · 86	0.
Third Minor	5 : 6	3 · 15½	3 · 10	5½ ♭.
Tone Major	8 : 9	2 · 04	1 · 93	11 ♭.
Tone Minor	9 : 10	1 · 82	1 · 93	11 ♯.

2. INTERVALS OF THE DOMINANT HARMONY.

		True.	Vulgar.	Error.
Grave Seventh	4 : 7	9 · 69	10 · 07	38 ♯.
Grave Fifth	5 : 7	5 · 82	6 · 20	38 ♯.
Grave Third	6 : 7	2 · 67	3 · 10	43 ♯.
Acute Second	7 : 8	2 · 31	1 · 93	38 ♭.
Acute Third	7 : 9	4 · 35	3 · 86	49 ♭.
Acute Fourth	7 : 10	6 · 17	5 · 79	38 ♭.
Acute Sixth	7 : 12	9 · 33	8 · 90	43 ♭.

3. AUGMENTED AND DIMINISHED INTERVALS.

		True.	Vulgar.	Error.
Augmented Sixth	14 : 25	10 · 04	10 · 07	3 ♯.
Augmented Fifth	16 : 25	7 · 73	7 · 73	0.
Augmented Fourth	18 : 25	5 · 69	5 · 80	11 ♯.
Augmented Second	21 : 25	3 · 02	3 · 10	8 ♯.
Diminished Seventh	25 : 42	8 · 98	8 · 90	8 ♭.
Diminished Fifth	25 : 36	6 · 31	6 · 20	11 ♭.

4. SEMITONES.

		True.	Vulgar.	Error.
Diatonic Semitone	15 : 16	1 · 11½	1 · 17	5½ ♯.
Chromatic Semitone	24 : 25	· 71	· 76	5 ♯.
Hemitone Major	14 : 15	1 · 19	· 76	43 ♭.
Hemitone Minor	20 : 21	· 85	1 · 17	32 ♯.

73. We shall next compare the two kinds of Temperament, by showing the effect of each of them, first upon the more common Intervals, and secondly upon the various Chords of Harmony; the former being necessary, both for the sake of Melody, and for the progression of the parts in Harmony.

INTERVALS.

		Equal.	Vulgar.
Ninth	4 : 9	4 ♭	11 ♭.
Octave	1 : 2	0	0.
Seventh Major	8 : 15	12 ♯	5½ ♭.
Seventh Minor	5 : 9	18 ♭	11 ♭.
Sixth Major	3 : 5	16 ♯	5½ ♯.
Sixth Minor	5 : 8	14 ♭	0.
Fifth	2 : 3	2 ♭	5½ ♭.
Fourth	3 : 4	2 ♯	5½ ♯.
Third Major	4 : 5	14 ♯	0.
Third Minor	5 : 6	16 ♭	5½ ♭.
Tone Major	8 : 9	4 ♭	11 ♭.
Tone Minor	9 : 10	18 ♯	11 ♯.
Semitone Diatonic	15 : 16	12 ♭	5½ ♯.
Semitone Chromatic	24 : 25	29 ♯	5 ♯.

MAJOR CHORDS.

Major Triad :—

		Equal.	Vulgar.
Major Third	4 : 5	14 ♯	0.
Fifth	2 : 3	2 ♭	5½ ♭.

Dominant Harmony :—

Major Third	4 : 5	14 ♯	0.
Fifth	2 : 3	2 ♭	5½ ♭.
Grave Seventh	4 : 7	31 ♯	38 ♯.

Added Ninth :—

Major Third	4 : 5	14 ♯	0.
Fifth	2 : 3	2 ♭	5½ ♭.
Grave Seventh	4 ; 7	31 ♯	38 ♯.
Ninth	4 : 9	4 ♭	11 ♭.

MAJOR CHORDS.

Added Sixth :—

		Equal.	Vulgar.
Major Third	4 : 5	14 ♯	0.
Fifth	2 : 3	2 ♭	5½ ♭.
Major Sixth	3 : 5	16 ♯	5½ ♯.

Suspended Fourth :—

Fourth	3 : 4	2 ♯	5½ ♯.
Fifth	2 : 3	2 ♭	5½ ♭.

Suspended Sixth :—

Major Third	4 : 5	14 ♯	0.
Major Sixth	3 : 5	16 ♯	5½ ♯.

Suspended Seventh :—

Major Third	4 : 5	14 ♯	0.
Fifth	2 : 3	2 ♭	5½ ♭.
Major Seventh	8 : 15	12 ♯	5½ ♭.

Suspended Ninth :—

Major Third	4 : 5	14 ♯	0.
Fifth	2 : 3	2 ♭	5½ ♭.
Ninth	4 : 9	4 ♭	11 ♭.

MINOR CHORDS.

Minor Triad :—

Minor Third	5 : 6	16 ♭	5½ ♭.
Fifth	2 : 3	2 ♭	5½ ♭.

Added Major Sixth :—

Minor Third	5 : 6	16 ♭	5½ ♭.
Fifth	2 : 3	2 ♭	5½ ♭.
Major Sixth	3 : 5	16 ♯	5½ ♯.

Added Minor Sixth :—

Minor Third	5 : 6	16 ♭	5½ ♭.
Fifth	2 : 3	2 ♭	5½ ♭.
Minor Sixth	5 : 8	14 ♭	0.

MINOR CHORDS.

Suspended Sixth :—

		Equal.	Vulgar.
Minor Third	5 : 6	. . 16 ♭ . .	5½ ♭.
Minor Sixth	5 : 8	. . 14 ♭ . .	0.

Suspended Seventh :—

Minor Third	5 : 6	. . 16 ♭ . .	5½ ♭.
Fifth	2 : 3	. . 2 ♭ . .	5½ ♭.
Minor Seventh . . .	5 : 9	. . 18 ♭ . .	11 ♭.

Suspended Ninth :—

Minor Third	5 : 6	. . 16 ♭ . .	5½ ♭.
Fifth	2 : 3	. . 2 ♭ . .	5½ ♭.
Ninth	4 : 9	. . 4 ♭ . .	11 ♭.

COMPOUND DOMINANT HARMONIES.

Diminished Seventh :—

Minor Third	5 : 6	. . 16 ♭ . .	5½ ♭.
Grave Fifth	5 : 7	. . 17 ♯ . .	38 ♯.
Diminished Seventh .	25 : 42	. . 2 ♯ . .	8 ♭.

Italian Sixth :—

Major Third	4 : 5	. . 14 ♯ . .	0.
Augmented Sixth . .	14 : 25	. . 4 ♭ . .	3 ♯.

French Sixth :—

Major Third	4 : 5	. . 14 ♯ . .	0.
Acute Fourth	7 : 10	. . 17 ♭ . .	38 ♭.
Augmented Sixth . .	14 : 25	. . 4 ♭ . .	3 ♯.

German Sixth :—

Major Third	4 : 5	. . 14 ♯ . .	0.
Fifth	2 : 3	. . 2 ♭ . .	5½ ♭.
Augmented Sixth . .	14 : 25	. . 4 ♭ . .	3 ♯.

74. In reviewing these Tables, the first remark that occurs, is the great sacrifice of good harmony that is made, in adopting the Equal

Temperament for the organ; in place of the Vulgar Temperament which has, for centuries past, been considered the best for that instrument. The beauty of the Harmony depends more upon the Major Third, than upon any other Interval. In the Vulgar Temperament, great ingenuity was displayed in contriving to obtain eight Major Thirds, with their Inversions the Minor Sixths, in perfect tune: for the sake of which, it was reckoned expedient to submit to great harshness in the remaining four, which were called Wolf Intervals. In the Equal Temperament, these Intervals are all tuned alike; but differ from the true pitch, by about two thirds of a Comma, which was formerly considered intolerable.

75. The only other remark that I shall make, is the false tuning of the Dominant Harmonies, which in most cases approaches, and in some even exceeds, that of the Wolf Intervals. This, of course, depends on the pitch of the true Dominant Seventh; which appears to me, beyond all doubt, to be the Harmonic Seventh, produced by the seventh part of the string; that is, in the second Octave below. Even on a bass string of the pianoforte, that Harmonic is as distinct to my own ear, as any of the others, especially while the sound of the string is dying away.

76. The Subdominant, or Fourth of the Scale, is usually supposed to be the Seventh of the Dominant Harmony. It makes, with the Dominant, an Interval of 9 : 16, equal to Minor Seventh, less Comma. This Interval differs from the Harmonic Seventh, by Komma Major, 63 : 64; an Interval greater than the Comma, and less than the Diesis.

77. The substitution of the Subdominant for the Harmonic Seventh, is attended with the following difficulties:—

1. The Dominant Harmony, which, of all Chords, is next in importance to the fundamental concord, or Major Common Chord, is derived from two unconnected Triads:—

Triad of F . .	F A C F . .	4 . 5 . 6 . 8	× 8	=	32 . 40 . 48 . 64.
Triad of G . .	G B D . . .	4 . 5 . 6 –	× 9	=	36 . 45 . 54 . –

Dominant Seventh . . .	G B D F . . .	36 . 45 . 54 . 64.

2. In the Dominant Chord, so constructed, the Seventh is out of tune with the other three notes of the Chord; which contains the following false Intervals :—

G F	36 : 64 . . .	Minor Seventh, less Comma.
B F	45 ; 64 . . .	Diminished Fifth, less Comma.
D F	54 : 64 . . .	Minor Third, less Comma.

3. The Compound Dominant Harmonies are much worse; being derived from Triads still more remote :—

Diminished Seventh . .	G♯ B D F . . .	75 . 90 . 108 . 128.
Italian Sixth	F A — D♯ . .	128 . 160 . — . 225.
French Sixth	F A B D♯ . .	128 . 160 . 180 . 225.
German Sixth	F A C D♯ . .	128 . 160 . 192 . 225.

4. The regular progression of the Harmonic Intervals (1, 2, 3, 4, 5, 6, 7, 8, 9, 10) is awkwardly interrupted. This may be seen by the Frontispiece to my Elements of Musical Science, in which the Harmonic curve is drawn to a scale, and the place of the substituted Seventh (9 : 64, instead of 9 : 63 (say) = 63 : 64) is shown by a mark on the line E♭. It is evident to the eye, that the curve, if drawn through that mark, would be distorted. In like manner, a correct ear will detect the false tuning of the Chord, if the Subdominant be taken as a Dominant Seventh.

5. When the Chord changes from the Minor Seventh to the Dominant Seventh, how can the performer alter the Seventh *by the ear*, till it is *in tune* with the new Chord, if the new Chord be a discord? This relates to a conversation which I had some years ago with Emiliani, when I put some questions to him, and noted down his answers. One of the questions referred to the following progression:

A C E G . . .	10 . 12 . 15 . 18 × 2	= 20 . 24 . 30 . 36.
A C♯ E Ĝ . . .	4 . 5 . 6 . 7 × 5	= 20 . 25 . 30 . 35.

Question. When the Chord changes from the Minor Seventh to the

Dominant Seventh, do you make any change on the Seventh? Do you flatten it a little? *Answer.* "Yes, surely." *Q.* I make the difference between the two Sevenths to be as 36 to 35? *A.* "But we alter the Seventh by the ear, till it is in tune with the new Chord."

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78. The following Table may help to explain the effect of the Vulgar Temperament.

SCALE OF VULGAR TEMPERAMENT.

For	B♯	D×	F×	Play	C	E	G.	
„	E♯	G×	B♯	„	F	A	C.	
„	A♯	C×	E♯	„	B♭	D	F.	
„	D♯	F×	A♯	„	E♭	G	B♭.	
„	G♯	B♯	D♯	„	G♯	C	E♭	Great Wolf.
„	C♯	E♯	G♯	„	C♯	F	G♯	Wolf.
„	F♯	A♯	C♯	„	F♯	B♭	C♯	Wolf.
„	B	D♯	F♯	„	B	E♭	F♯	Wolf.
	E	G♯	B.					
	A	C♯	E.					
	D	F♯	A.					
	G	B	D.					
	C	E	G.					
	F	A	C.					
	B♭	D	F.					
	E♭	G	B♭.					
For	A♭	C	E♭	Play	G♯	C	E♭	Great Wolf.
„	D♭	F	A♭	„	C♯	F	G♯	Wolf.
„	G♭	B♭	D♭	„	F♯	B♭	C♯	Wolf.
„	C♭	E♭	G♭	„	B	E♭	F♯	Wolf.
„	F♭	A♭	C♭	„	E	G♯	B.	

Between F♭ and B♯, 21 Triads in all, there are 8 Wolves. Of the rest, 13 in number, the Thirds are perfect; the Fifths are all flat by $\frac{1}{4}$ Comma, except the great Wolf between G♯ and E♭.

79. The Diatonic Scale, with all the Enharmonic notes introduced, is as follows :—

C . . .	·0	
C♯ . . .	·708	Chromatic Semitone. S.
	·408	Enharmonic Diesis. D.
D♭ . . .	1·116	Diatonic Semitone.
	·708	Chromatic Semitone. S.
$\bar{D}$ . . .	1·824	Tone Minor.
	·216	Comma. C.
$\overset{+}{D}$ . . .	2·040	Tone Major.
	·708	S.
D♯ . . .	2·748	
	·408	D.
E♭ . . .	3·156	Third Minor.
	·708	S.
E . . .	3·864	Third Major.
	·708	S.
E♯ . . .	4·572	
	·408	D.
F . . .	4·980	Fourth.
	·708	S.
$\bar{F}$♯ . . .	5·688	
	·216	C.
$\overset{+}{F}$♯ . . .	5·904	
	·408	D.
G♭ . . .	6·312	
	·708	S.
G . . .	7·020	Fifth.
	·708	S.
G♯ . . .	7·728	
	·408	D.
A♭ . . .	8·136	Sixth Minor.
	·708	S.
$\bar{A}$ . . .	8·844	Sixth Major.
	·216	C.
$\overset{+}{A}$ . . .	9·060	
	·708	S.
A♯ . . .	9·768	
	·408	D.
B♭ . . .	10·176	Seventh Minor.
	·708	S.
B . . .	10·884	Seventh Major.
	·708	S.
B♯ . . .	11·592	
	·408	D.
C . . .	12·000	Octave.

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