LESSONS

ON

ARITHMETIC;

IN

PRINCIPLE, AND IN PRACTICE:

FOR THE

INSTRUCTION OF YOUTH OF BOTH SEXES;

AND MORE ESPECIALLY

FOR THAT OF YOUNG MERCHANTS, TRADESMEN,

SEAMEN, MECHANICS AND FARMERS.

BY THOMAS SMITH.

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PREFACE.

To furnish a clear and a familiar description of those Rules of Arithmetic which are generally useful, and to introduce the student to this pleasing and very valuable Art, by gradually unfolding to him the modes of practice, and the principles on which the several Rules proceed, and, to do these in correct, as well as in intelligible English, is my chief purpose in this undertaking.

How far I have succeeded in this purpose, my readers will judge. To enable those who are already versed in the subject, and who, therefore, cannot be induced to read more than parts of the work; to enable those persons to form a ready judgment, I would, respectfully, point out to their attention: first, the manner in which, in the Introduction, I have opened the subject; second, to the writing and the reading of Figures, as taught in Notation and Numeration; then let me beg of them to turn to the manner in which, in Addition, paragraphs 40 to 43, I have explained the principle of "carrying" the tens; and then to the principle of "borrowing," and of repaying, by "carrying," as practiced in Subtraction, paragraphs 49 to 58.

After these, the manner in which I have described the peculiar use, and the distinguishing character of Multiplication, in the three first paragraphs on that Rule; and then, the mode in which, in paragraphs 79 to 82, the principle on which Multiplication by the larger numbers, is carried on. These will, I hope, be found worthy of attention.

Before I entered on Compound Arithmetic, I found the next step must be, on the nature of Fractions. Will the reader, who may be desirous of forming an opinion of the Work, do me the honour to read the whole of the short chapter on this branch of the subject? and, if it would not be too much, I would further request his attention to the mode in which I have developed the principle of working on the various quantities with which it is the business of compound Arithmetic to treat.

These, or a part of these, passages, will, I flatter myself, do more than justify me, in thus urging on the attention of the Public, another Book, in addition to, or rather, with a view to supersede, the many already before it, on the subject of elementary Arithmetic.

In this work, I have, I trust, not only furnished a Book, which, according to its title, will instruct the untaught Artizan, and the lesser Tradesman, but which will, also, in no small degree, serve to relieve the Teacher from a laborious and irksome part of his duties, by enabling his pupils to acquire this branch of education, with little or no assistance from him; and which will, thereby, set both parties free, at an earlier period than usual, to pursue other, and higher branches of knowledge.

The useful, the generally useful branches of Arithmetic, are all that I here profess to teach. But, then, I hope that I do teach them, and the very principles on thich they rest; and, that prepared by these, the learner, who may have occasion, or inclination, for higher attainments, in this art, will find his progress not a little facilitated by the easy and sufficient introduction which he will obtain from this Book.

I have not thought it advisable to enlarge the Book by a great number of examples; nor by a long array of sums, and questions. The work is intended chiefly for those who have passed the age of childhood, and who bring to the study a little power of thought. For these, I think the questions, &c. which will be found in the body of the work, will prove quite sufficient. But, as it is my wish to render it as generally useful as possible, I intend to follow up the publication with a Supplement, to consist of numerous additional and select sums, for the practice of younger pupils.

Besides a Key, as it is commonly called, or answers, to the several questions proposed in this volume, I shall insert, at the end thereof, some few tables, and other matters, not strictly connected with the subject, but so frequently useful to all parties who read and write, as to render it desirable that they should be, at all times, close at hand.

It would be affectation to conceal an opinion which I entertain, as to the merits of all the Books on this subject, in our language at least, that I have been able to lay my hands on; it would be affectation to conceal this opinion, that there is not one amongst them in which is to be found any thing like a rational and perspicuous unfolding of the little intricacies of the Art. And, amongst those designed for the instruction of the young, besides every defect of style, I have marked an entire absence of all that is worthy of the name of principle: a total want of what may properly be called, the REASON of the thing. And, whilst there is in them, nothing to satisfy and to inform the understanding of the learner, there is every thing to puzzle and discourage him.

Similar to these are the defects of every work that I have found on the subject of Book-keeping. It is not that the authors of these works do not themselves understand the subjects; but, it is simply because they have not the faculty of communicating, with the pen, the knowledge they possess.

Tradesmen without number, the most industrious and meritorious of men, carry on their business with great difficulty, become involved and ruined, merely from the want of a simple system of keeping their accounts.

To furnish them with such a system, and to communicate to them a knowledge thereof, in a very small Pamphlet, shall be my next endeavour.

THOMAS SMITH.

Liverpool,

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For the purpose of easy reference, the Paragraphs, and not the Pages, are numbered.

INTRODUCTION.

1. Arithmetic, in its practice, is the art of reckoning, or, of calculating numbers. An art which is constantly in use in the transactions of trade, whether in rude or in civilized states of society; and, besides being employed in numerous operations of manufactures and of science, it is a principal means by which the mariner regulates his course over the ocean. This art, indeed, is of almost universal use in the affairs of civil society, and a correct, and a dexterous practice of it, must always be extremely valuable to the man of business, to the gentleman, and to the student.

2. Such is the value, such the importance of the art I propose to treat of; an art which, valuable as it is, is scarcely more useful than it is pleasing to its possessor, in the power which it gives of calculating with accuracy and despatch. Nor will it be foreign to my purpose to state, that this desirable art is easy of attainment; that is to say, easy, when the principles and the practice thereof are properly placed before the attentive learner.

3. Arithmetic, I have said, is the art of reckoning, or, of calculating numbers. Before we enter on the study of it, therefore, we must devote a little atten-

tion to the mode of writing down, and of reading these numbers, being, as they are, the materials with which we have to perform our work.

4. This writing down, or *note-ing*, the numbers, used in Arithmetic, is called NOTATION; and the reciting, or reading of them, in words, is called Numeration. These are very simple matters, quickly learned; they are necessary as a first step in our proceedings; and a clear knowledge thereof will greatly facilitate the learner's progress.

5. Here, however, let me state, that, at this stage of our proceeding, I shall trouble the learner with no more of this branch of our subject than is necessary to the step we are taking. Numeration and Notation include the reading and the writing of all sorts of numbers; that is to say, of whole numbers, both simple and compound, of fractions, both vulgar and decimal; but, as, in the commencement of our study, we have to deal with simple whole numbers only, so it is to these that I shall now confine myself, reserving what is to be said of other numbers, until we come to treat of the working of them. And this is the course intended to be pursued throughout this work, in which I shall carefully abstain from perplexing the learner with unnecessary matter, and shall study to lead him on in the easiest and most pleasing manner, to a knowledge of the uses and the powers of figures, so far as they are required for the ordinary business of life, or as a preparation for higher mathematical and other studies.

NOTATION;

Or, the Art of Writing Numbers in Figures.

6. For this purpose there are ten several forms, or figures, in use amongst us. And with these ten figures only, but differently placed, and, as occasion may require, repeated, can any number, however large, be most clearly and conveniently expressed.

7. The ten Figures are as follow. The value, or amount, which each of them serves to express, is written underneath, in words.

1	2	3	4	5	6	7	8	9	0
One,	two,	three,	four,	five,	six,	seven,	eight,	nine,	nought, or cypher.

8. Of the last figure, called nought, or cypher; its name, *nought*, comes from the Saxon word, *nothing*; and nothing does it express; that is to say, nothing in amount. However, it is useful; and, indeed, it cannot be dispensed with, when we come to write down the higher numbers, as will appear hereafter.

9. Of the other nine figures, be it observed, that each of them, when standing alone, and unconnected with any of the others; or, when standing the last, on the right hand, of a series of figures; each of these nine, when so standing, expresses merely the number which we have written underneath it: but the highest, even, of these figures, in this its single and separate state, expresses a number no higher than *nine;* and, yet, as the learner knows, hundreds, thousands, and yet higher numbers, are continually required to be expressed.

10. To do this, that is, to express large numbers, by figures, each figure differing in form from every other figure, and, thus to have a different form for every different number; to do this, would be an endless, and an impracticable task. So, another, and a very convenient and neat mode has been adopted for accomplishing this purpose; of which mode we now proceed to treat.

11. The manner, then, in which this important purpose is accomplished, is, a *changing* of the place or places of these figures; by which changing, the value, or the amount, severally expressed by them, becomes *changed*.

12. This change of place, however, of which we have to speak, is *not a mere removal* of the figure, or figures; for a mere removal, as has before been said, would not alter the value of any of them: but the change in the value is produced by ranging two or more of the figures together, so that they stand in a kind of rank, one before another, and it is according to the RANK or STATION which it occupies, that the value of any, and of every figure is estimated.

13. In order to fix in the mind of the young student the effect of this changing of the situation of figures, let him write down, neatly and clearly, on a card, the ten figures; then cut the card into square pieces, each containing one of the figures. Having the figures so prepared, let the student take one of them, the first, for instance, and place it right before him on the table: thus standing, singly, alone, the figure expresses the number, *one*. Then let him take another of the figures, (the second, let it be) and place it over, or under, or to the left of the first, and this first figure still expresses only one; its value is still unaffected by the approach of the other

figure. But place this second figure to the *right* of the first, and then is the value of the first changed; standing *before* another figure, it is advanced, what may be called, one step, by which advance its value is increased tenfold; so that, instead of *one*, it now stands for *ten*. Advance this figure another step, by placing a third after it, suppose you take the 3, and then the 1 stands for *one hundred;* being ten times as much as it represented before the addition of the last figure. Another figure joined to the line, in the same manner, increasing all the figures which stand before it, tenfold, makes the first figure stand for *one thousand*. So much for the effect of a change in the station of a figure.

14. We have now four figures placed in a line, before us. Let them be these four, 1 2 3 4; the first standing for *one thousand*, the second for *two hundred*, and the third for *thirty*, to which amounts they have been raised, from their simple value of one, two, and three, merely by the circumstance of there being other figures placed after them; the fourth figure, having none after it, and standing in the first and lowest station, represents merely its simple number, four; the whole line representing *one-thousand*, *two-hundred*, and *thirty-four*. Such is the manner in which large sums are represented by a few simple figures.

15. It is, then, the rank, or station, in which the figure is placed, that determines its value; and the next thing the learner has to do, is, to fix in his mind the exact value, together with the proper name, of each of these ranks or stations; or, as they are usually called, of these PLACES of figures, a clear understanding of which, will greatly assist him in all which is to follow.

16. In the last paragraph but one, there are four

figures, namely, 1, 2, 3, 4, severally occupying the *first*, *second*, *third*, and *fourth* stations, or PLACES, as we will henceforth call them. Now, the *first* place to the *right* hand, for it is there we begin to count, this *first* place, in which any figure represents only so many *ones* as it is written down for; this is called *the place of* UNITS; (that is, of *ones*); the *second* is called *the place* of TENS; the *third* is, for a similar reason, called *the place of* HUNDREDS; and the *fourth* is called *the place of* THOUSANDS; representing, as every figure written in this place does, just so many *thousands* as it would represent *units*, if written in the first place. Thus we have ascertained the four first places of figures, and have, thereby, learned to write UNITS, TENS, HUNDREDS, and THOUSANDS.

17. Just in the same manner do we proceed to write down yet larger numbers. Every place of figures being ten times the value of that next before it. The *fourth* being *the place* of THOUSANDS, the *fifth* is *the place* of TENS of THOUSANDS, the *sixth* that of HUNDREDS of THOUSANDS, and the *seventh* is *the place* of MILLIONS, the *eighth* is that of TENS of MILLIONS, and the *ninth* is *the place* of HUNDREDS of MILLIONS. And this is going quite high enough, unless we be about to make some rare and curious calculations in Astronomy.

18. One other particular remains to be noticed before we conclude this branch of the subject, and that is, the use of the cypher, or nought.

19. The use of this figure, for *figure* it is, though it does not represent any value; the use of this figure is, to fill up the place, or places that would oftentimes, but for some such thing, be left vacant by the advancement of any other figure, or line of figures. As, for instance, we have occasion

to write down the number *ten*. This number is represented by the figure 1 advanced one step. This advance, however, cannot be made otherwise than by placing some other figure after it. Now, if we write after this 1, a figure which represents a number, we then, not merely advance the *one* into *ten*, but add thereto the number, or value of the additional figure. Suppose this figure to be 2, and that we write them thus, 12; we have then the numbers *ten*, and *two*, or *twelve;* and not *ten*, the number we want to express. But, if we add the *cypher* to the 1, writing them thus, 10, then have we the number desired. *Twenty* is thus expressed, 20; *thirty*, thus, 30. An additional cypher raises these numbers to *hundreds*, as 200, two hundred; 300, three hundred, and so on.

20. To satisfy himself that he thoroughly understands the Notation of figures, which the learner will find of the utmost value in his future study, let him write down, *in figures*, the following sums.

EXERCISES.

21. Four hundred and seventy nine.

Seven thousand, two hundred and sixty one.

Thirty-eight thousand, five hundred and sixteen.

One hundred and sixty-two thousand, seven hundred and forty.

Two hundred and forty-nine thousand, six hundred and five.

Two million, five hundred and ninety thousand, three hundred and forty-one.

Six million, two hundred and fifty-nine thousand, three hundred and eighty-four.

Seventeen million, two hundred and ninety-five thousand, seven hundred and eight.

Sixty-four million, three hundred and fifty thousand, four hundred and sixty-one.

One hundred and forty-eight million, three hundred and ninety-five thousand, two hundred and sixty.

Three hundred and fourteen million, six hundred and eighty thousand, five hundred and twenty-seven.

Six hundred and two million, five hundred and ninety-three thousand, eight hundred and forty-one.

Two hundred and seventy-four million, thirty-nine thousand, five hundred and eighty-six.

One thousand, seven hundred and twenty-five million, three hundred and eighty-four thousand, nine hundred and thirty-seven.

Eight hundred and fifty million, nine thousand, five hundred and fifty.

Five thousand, three hundred and sixteen million, eight hundred and twenty thousand, four hundred and thirteen.

NUMERATION,

22. As I have said, is the art of reciting, or of enumerating, in words, any number, or series of figures; and a due attention to what has been said in treating of NOTATION, will render this next step, and, indeed, every future step of our study, comparatively easy.

23. Numeration, then, is the art of reciting any number, or numbers. To proceed to an example; let it be required to numerate the following: 5836. The Scholar will recollect, that the value of every figure depends almost entirely on the place in which it stands with regard to other figures; so the first thing here to be done is, to ascertain the place, or

rank, of each figure, in order that, on reading it, we may know whether to call it, *units*, *tens*, *hundreds*, &c. And this important point is ascertained in the following easy manner:—We begin with the figure on the right hand of the series, and count on to that on the left; saying, *units*, *tens*, *hundreds*, *thousands*; hereby ascertaining, that the figure on the left represents *thousands*, we read thus, five thousand, eight hundred, and thirty-six. If another figure be added to the series, it raises the highest to tens of thousands: and if to these yet another be added, we have hundreds of thousands, and so on. Now let us add two more, and let them be placed at the left hand of the series, thus:

4 7 5 8 3 6

Counting back, from the place of units, as before, we find, that the figure 4 is in the place of *hundreds* of *thousands*, so we read the series thus: *four hundred seventy-five thousand*, *eight hundred and thirty-six*: another figure would lead us into *millions*, and another after that, into *tens* of *millions*; and so on.

24. It remains with us to see, how the *cypher*, or *nought*, operates in this branch of our subject. Let us, then, take out the figure 7 from the above series, and insert, in the place thereof, a cypher; we then have 405836, which is to be read, *four hundred* and *five thousand* &c. instead of *four hundred seventy-five thousand*, &c: if we insert the cypher in the place of the 5, we have 470836: which is, *four hundred* and *seventy thousand*, &c.; by which examples the attentive reader will scarcely fail to be fully acquainted with the use of this figure.

25. Besides the denomination of figures of which we have already treated, namely, UNITS, TENS,

HUNDREDS, and THOUSANDS, the learner will have heard of some of yet higher denominations; that is to say, of MILLIONS, BILLIONS, TRILLIONS, &c. These vast numbers are not wanted in affairs of business, nor will they now be worth much attention from the student. But as this division of our work would not be complete without some notice of them, I shall here insert the table of numbers, from *units* up to *billions*, and the learner can refer to it whenever occasion or curiosity may lead him so to do.

NUMERATION TABLE;

Showing the Places of

1	Units.
10	Tens.
100	Hundreds.
1000	Thousands.
10000	Tens of Thousands.
100000	Hundreds of Thousands.
1000000	Millions.
10000000	Tens of Millions.
100000000	Hundreds of Millions.
1000000000	Thousands of Millions.

26. Here, also, we have the use of the cypher, and the value and importance of the several places of figures, clearly shown; the unit *one*, being advanced first to ten, then to one hundred, and onwards, until, by the addition of nine cyphers, it expresses one thousand millions.

27. But, to read these large amounts requires a kind of spelling; which is to be avoided by dividing the long lines of figures into portions, by a comma, or other mark, as is here shown, 1000,000,000. By this means we see, at a glance, which are hundreds, which thousands, and which millions; and begin at once to name them.

Second Numeration Table.

5 ,	3	8	2	6 ,	4	1	9 ,	7	4	1
Billions	Thousands of Millions	Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Ten Thousand	Thousands	Hundreds	Tens	Unit

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OF THE WORKING OF FIGURES.

28. Addressing myself, in future, directly to the learner, I proceed to this branch of our subject; by which we are enabled to accomplish all the purposes, whether of business or of curious inquiry, for which the art of Arithmetic is adapted.

29. This working of figures, this reckoning, and calculating, and adjusting of numbers, consists merely of joining and of separating them; that is, of joining them in certain ways, and of separating them into other portions, as occasion may require; and this is the whole art of Arithmetic.

30. To *join*, and to *separate*, are but two modes of proceeding, you will say; yes, and by two modes, all the ends of Arithmetic may be accomplished

even to the largest and most important of its purposes. These two modes, by which it is possible to accomplish every object of the art, are, first, ADDITION, or the joining together of different numbers, and, second, SUBTRACTION, or the separating of them. However, though every thing may be accomplished by these two processes, it would, in most cases, be very tedious, unpleasant, and a great waste of time, to confine ourselves to the use of them merely. So another mode of joining numbers together, than that of Addition, is, in certain cases, adopted; and another mode of separating them, than that of Subtraction. These other modes are called, the first, MULTIPLICATION, and the other DIVISION. Thus, although every thing may be accomplished by TWO modes of proceeding, TWO OTHER modes, for the purpose of abridging the labour, in certain cases have been adopted; and these, together, making four processes, are called the four fundamental, or foundation Rules of Arithmetic. To proceed to the first rule.

ADDITION.

31. This rule, as its name imports, is the art of joining numbers together; the art of collecting the AMOUNT, the SUM; or, as it is often called, the TOTAL, of several numbers. And the process is thus performed:

32. Suppose it be required to add together the following figures or numbers; 3, 2, 1, 4, 5. You may begin at either end of the line; or, indeed, with any of the figures; all that is required is, that you

overlook none of them, and that you count each of the figures only once. Beginning at the left hand, we say, four and five are nine, one makes ten, two make twelve, and three, fifteen; which is the amount required to be ascertained: and in figures, as you know, it is written thus, 15.

33. You will come to the same conclusion, begin at which end you please, and join the numbers by any process you please, so that you join them correctly. And this uniformity of result, this certainty of the same conclusion, proceed to it how you may, so that you proceed correctly; this uniformity of result constitutes a great charm in the study and practice of Arithmetic.

34. But the manner in which we have just added together a few figures, although answering our purpose in this instance, is not, as you will learn, the manner to be employed in the addition of large, and of numerous sums. To effect such a purpose, to do it conveniently and with ease, it is usual to place the numbers to be added, in a pile, or upright row, as thus; then to draw a line under them, and under the line, to place the amount.

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 2
 3
 1
 4
 5
—
15
—
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35. Let us, in this manner, try the addition of other numbers, of some of larger amount, such as the following; which we will, at once, range under each other, in the proper order, placing, as they always must be placed, the units right under each other, and the tens in their proper station.

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36
14
 9
25
40
—
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36. Now, these sums may be added thus, beginning at the bottom; forty and twenty-five are sixty-five, nine make seventy-four, fourteen make eighty-eight, and thirty-six make one hundred and twenty-four.

37. In this manner, you will observe, these sums may be added. But this is not the best; an easier manner is to be adopted. It is not, to be sure, very difficult to add 40 and 25 together, or 88 and 36. But the numbers to be added are often greater than these, being hundreds, and thousands, and upwards; and, in such cases, we should find this mode of adding difficult, and, in using it, we should be very liable to err; so we have, for this, as for almost every other case of difficulty, a very simple and easy mode of proceeding: we go to work thus:

38. Beginning at the bottom, with the column of units, we add them upwards, thus: the first figure being a cypher, we pass on to the next; and so, saying, five and nine are fourteen, and four make eighteen, and six make twenty-four; which sum we may write down underneath, and then begin with the next column; saying, four and two are six, and one makes seven, and three make ten; which sum, also, we must then write down under the other; only, observe, and be careful to remember, that the figures in this last column represent, not *units*, but *tens;* so that the sum of them is ten tens, or one hundred; which is the sum to be written down; and the matter would then stand thus; giving us, as the amount of the sums to be added, 24 and 100, which two sums may be brought into one, by drawing a line beneath them, and adding them up, as we did the others.

```
 36
 14
  9
 25
 40
 --
 24
100
 --
124
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39. This is the manner in which this matter *may* be done; but here is a double adding, and this, too, to find the amount of a few small sums.

40. This double addition is avoided, and the whole amount of any number of sums, however large, is to be obtained by one process of addition, and in a

single line, by a very neat and simple mode of proceeding; a mode to which I must now request your rather particular attention.

41. Let us just look back to the last example, in which we had five sums to add, amounting, together, to 124. Now you will observe, that, in these five sums, the first row of figures to our right represent UNITS; whilst those in the other row represent TENS: bearing this in mind, let us proceed to add them as before. Twenty-four is the amount of the first column; that is, two tens and four. Now you see, if we set down the 4 in its proper place, and, instead of setting them down as above, we add the two tens to the next column, proceeding thus, two and four make six, and two make eight, one makes nine, and three make twelve; and then set this 12 down to the left of the 4; if we do this, then we have 124, the amount, in one line, as we proposed to obtain it.

42. One other example of working a sum in this rule, will make this practice of setting down the units, and of adding, or, as it is called, of "CARRYING" the tens, to the next column: one other example will, doubtless, make the practice thereof clear; and, having acquired a clear insight into it, you will have achieved an important step in the learning of Arithmetic.

43. This example shall be formed of larger sums than those we have hitherto treated; and, learning to add these sums, you may master any mass of figures, however large, that may be placed before you. Here then we begin as before, on the column at the right hand; that is to say, nine and two make eleven, 4 make 15, 5 make 20, and 1 makes 21. So we set down the 1 in its

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 2831
 6405
  974
 8152
 5279
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23741
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proper place; that is, in that of units; and, as we did before, add, or carry the two tens to the next column. Thus, 2 and 7 make 9, 5 make 14, 7 make 21, and 3 make 24; which are, as you will say, 24 tens. Set down the 4 in the place of tens, as is done in the example, and add the 2, as before, to the next column. Thus, 2 and 3 make 5, 1 makes 6, 9 make 15, 4 make 19, and 8 make 27. Set down the 7, which is 7 hundred, in its proper place, and, adding the 2 to the next column, which is the last, proceed thus; 2 and 5 make 7, 8 make 15, 6 make 21, and 2 make 23; which 23 you set down, because you have no more columns to add. And thus you have the amount of the whole.

44. Having, now, as I believe, said enough to enable any student, with but ordinary attention, to make himself master of this rule; I shall, after setting down a few columns, or piles of figures, for the further practice of the learner, proceed to treat of the next rule.

(1)	(2)	(3)	(4)
31752	15036	52074	40736
24061	37204	28413	85195
13590	81752	16257	57308
60342	49036	40815	26951
42716	51874	31528	72534

(5)	(6)	(7)	(8)
802631	402735	526130	350742
154379	257134	357042	583403
634015	371508	135174	350931
219531	158062	248031	173642
528109	435207	702538	825096
370523	630421	485207	927407

SUBTRACTION:

45. Or, Substraction; for either is correct: though the first mode of writing the word is most in use. This word, which comes from the Latin, means to separate: a process the reverse of the last we have been learning, but a process equally useful.

46. As Subtraction is the separating, or the withdrawing of one thing, or number, from another, it will, of course, always be necessary, that the quantity to be withdrawn be less than that from which it is to be subtracted. And, this being borne in mind, we thus proceed: writing down the figure, or figures, expressing the larger quantity, we write, immediately underneath, that which, or those which express the lesser, or quantity to be subtracted, and, drawing a line close below, we write under the line whatever quantity may remain, after the lesser has been taken from the greater; and this, which is the difference between the two quantities, is called the REMAINDER; as thus, we have to subtract 5 from 8, and 3 is the difference, or remainder.

47. But this example is too simple to require a minute's attention: we must proceed to the subtraction of larger numbers; that is, of more figures from each other. And, in order to insure a safe progress, we must proceed step-by-step, as we have done before.

48. Let, then, the next step we take, be the subtraction of 11 from 15, it will stand thus.
Now, we can all of us, without any particular learning, very readily say, in this case, that 4 is the remainder; yet, as it would not

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 15
 11
 —
  4
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always save us from error, so to proceed, let us adopt the proper method, which is this: beginning with the first figure, we say, one from five, and four remain; which 4 we write down, as is here shown, in its proper place; then, looking to the next figures, and finding that they are of equal amount, that there is no difference, we see that the work is done; and so 4 is the remainder between the two sums with which we set out.

49. However, let us take another step, let us subtract 6 from 15. Now, proceed as we did in the last example, we cannot; we cannot subtract the first figure below from that immediately above it, we cannot take 6 from 5, a greater from a less; but we can take 6 from the whole sum, 15; and this is what we do. And the remainder, which is 9, we set down, accordingly.

50. Something resembling this, is the course we take in all similar cases of subtraction; that is to say, when the lower figure is larger than that immediately above it, so that it cannot be subtracted therefrom, we add ten to the upper figure, in order to make it large enough, and this ten, you must observe, we BORROW for the occasion; and we *borrow* it, and *repay* it in the manner described below.

51. In the first place, you will not fail to bear in mind, that the upper sum, from which you are about to subtract, is always to be larger, on the whole, than that below it. Well, now, bearing this in mind, suppose that you have two different quantities, let them be two sums of money, as 1319 pounds, and 2135 pounds, and that you have to subtract the former from the latter. The difference, or remainder, as you will see, is 816 pounds. But, to ascertain this, in an easy, clear, and certain manner, is the object.

52. In order to do this, that is, to subtract sums, one from the other, where the amount of both is too large to be taken at once; and when, in addition to this, some of the figures in the lower line, are larger than those immediately over them; in order to do this in an easy, clear, and certain manner, arithmeticians have adopted this expedient.

53. Knowing that, when there are two different quantities of any thing; as, for instance, the two sums of money, 2135 pounds, and 1319 pounds, between which, the difference, as I have stated, is 816 pounds. Knowing, that if they add ten pounds to one of these sums, and then, ten pounds, also, to the other, the difference between them will still be 816: knowing, as they do, this circumstance, this is the principle on which they proceed: when they come to a figure, in the lower line, that is larger than that over it, they *borrow* ten, which they add to the upper figure; and, in order to restore the balance, they *add* ten to the lower line also; and thus all is kept right.

54. However, the mode of adding this ten to the lower line, must have a moment of your attention; and, having that, you become thoroughly master of subtraction.

55. In order to see this mode, and, indeed, to see further into the whole process of subtraction, let us here state the example of which we have been speaking.

2135
1319
816

56. Beginning with the first figures on the right, as before directed, and using the words of Arithmeticians, we say, "*nine from five, I cannot, but, nine from fifteen, and six remain; set down* 6, *and carry one;*" now, observe, the next figure in the lower line is 1; but this 1 is, you see, ten, standing, as it does, in the

place of tens; so if we add one to it, and thus call it two, that is two tens, we do, in fact, add ten to this line, as we had just before done to the upper line. To proceed with the process; taking up the words where we last left off; that is, at "*carry one*," we thus proceed, "*one to one, are two; two from three, and one, set down* 1." Then begin again, "*three from one, I cannot, but, three from eleven, and eight remain; set down* 8, *and carry one*." Now, here, again, you must observe, that the ten we borrowed last, was nothing less than ten hundred, seeing that it made the figure 1, to which it was added, and which is 100, into 1100; and so we repay it, or, rather, we restore the balance between the two sums, by adding 1 to the next figure in the lower line, which, by this means, is raised from ten, to twenty, hundred. And thus is this sum finished; for the two remaining figures are now alike.

57. Another example, freed from the explanations inserted in the last paragraph, will, I think, clear this matter up; let it be the following.

$$\begin{array}{r} 6425 \\ 4197 \\ \hline 2228 \end{array}$$

58. Having written the figures in the proper order, we proceed. "Seven from five I cannot, but, seven from fifteen, and there remain eight," set down 8, and carry one; one to nine makes ten, ten from two I cannot, but ten from twelve, and there remain two, set down 2, and carry one; one to one makes two, two from four, and there remain two, set down 2; then, four from six, and two remain, set down 2, and the work is finished.

59. More, it is unnecessary to say on this Rule of Subtraction. The learner will, I trust, now be able to work the few sums which are here set down for his practice, and then will proceed with advantage to the next Rule.

(1) From 42365 Sub. 31273	(2) 62853 16427	(3) 57190 35724
(4) From 163547 Sub. 15263	(5) 516036 160274	(6) 735641 489135
(7) From 6243150 Sub. 3561274	(8) 3725892 1350436	(9) 7342019 4136275
(10) From 4835712 Sub. 3128145	(11) 6213585 4125793	(12) 4725183 3258427

(13)	From	3517	Subtract	1624
(14)	,,	4932	,,	2651
(15)	,,	6352	,,	3516
(16)	,,	74538	,,	41843
(17)	,,	85702	,,	7931
(18)	,,	50743	,,	9562

On the Methods of Proof,

In Addition and Subtraction.

60. To err, in his reasoning, is, proverbially the lot, but we should rather say, the propensity of man. And there are few things in which we are more liable to err, than in Arithmetical calculations. Some thought arises, foreign to the matter before us; some momentary dullness, every now-

and-then, passes over the mind, and occasions an oversight, or a miscalculation, which, in matters of Arithmetic, would often, if not discovered and rectified, produce the most substantially injurious, or even ruinous consequences. So deeply impressed with this, their liability to fall into error, in matters of this nature, are all Accountants and Arithmeticians, that it may be averred, that no man, making any pretensions to the character of either, would think of suffering an account, or calculation, however trifling in the amount, to pass through his hands, until it had been once, at least, very carefully examined, or proved, in order to ascertain its correctness.

61. When such is the practice of the most experienced, and even of the most able Arithmeticians, it would be an act of carelessness, or of presumption, such as I trust no pupil of mine will be guilty of, to neglect a rule so very reasonable, and so indispensable.

62. The customary mode with Accountants, of examining their calculations, is, just to go through the process again, varying a little in the mode of proceeding; as, for instance; if it be addition that they would examine, having, in the first process, added up the columns, as I have described in the rule, from the bottom to the top, they *check*, or *prove* this addition, by very carefully reckoning them over again, beginning at the top of each column, and reckoning downwards. After this, if they find the amount of each column the same as that produced by the first process, they reasonably conclude that their additions are right.

63. This is the mode of what is called PROOF, in Addition. Of this I have deferred speaking until now, partly because I did not wish to interrupt the

course of the learner, by any thing not quite essential to his progress, but chiefly have I reserved it until now, when, having passed through another Rule, he may refresh his mind, and confirm his knowledge of Addition, by returning to the few sums set down for him in that rule; by working them over again; which I recommend to him to do; and by further improving himself, by practising the mode of proof which I have here described.

64. As to the method of PROOF in Subtraction, it is simply this. Having completed your sum, you have three lines of figures; that is to say, first, tne the larger sum; second, the sum you are to subtract therefrom; and, third, the remainder, or difference. Now, on a moment's consideration you will perceive, that the two latter sums must always, when added together, exactly amount to the larger sum: twenty, for example, is the larger sum, from which you have subtracted five, and there is a remainder of fifteen: the two latter make up the former sum. And thus it always will be. So, that, to put any process of Subtraction to the proof, all you have to do is, to draw a line under the remainder, and add it and the next sum together, and then your fourth line of figures ought to be same as your first.

MULTIPLICATION.

65. This name, too, comes from the Latin. And the meaning of it is so familiar, that a definition is scarcely called for. But, as a definition, if duly attended to, will forward us in the knowledge of our art, we must not withhold it.

66. To multiply, as you know, is to increase; and as you likewise know, MULTIPLICATION is, the act of increasing. But so, also, is ADDITION the act of increasing. Let us, then, mark the difference.

67. Addition, as defined in paragraph 31, is the art of joining together numbers, or sums of various amounts; whereas, Multiplication increases by a joining together of the same sum, several times repeated: as, twice 4 are 8; 4 times 4 are 16; 6 times 12 are 72, and so on. This joining together of the same amount, several times repeated, may, of course, be accomplished by Addition; first writing down the figures in a sort of column, in the manner directed in Addition, paragraph 34. But the end is attained much more quickly, more pleasantly, and with less liability to error, by the Rule we are now treating of; and herein consists its value.

68. In this Multiplication, two sums are multiplied together, and they produce a third: now, these three sums have each an appropriate name, with which you will find it useful to be familiarly acquainted. When we say, sums are multiplied together, or, that one is multiplied by the other; we mean the same thing. And when two sums are to be multiplied, it is immaterial, as to the result, whether we multiply by one, or by the other, of them: as, for instance, we have to multiply 6 and 12 together, the result is the same, whether we say six times twelve, or twelve times six. And so it would be with sums of any amount.

69. But, though the result is the same, it is more easy to multiply the larger sum by the smaller; and this, as you will soon find, is more especially the case, when either of the sums has several figures in it.

70. Having ascertained, then, that it is better, because it is easier, to multiply by the smaller sum, when two sums are to be multiplied together, we call the smaller one, the multiplier, whilst the larger, we call the multiplicand, and the sum produced by the process, is termed, the product; and these are the terms used in this rule of Multiplication: namely, MULTIPLICAND, MULTIPLIER, and PRODUCT.

71. However, before we proceed further with this rule, it will be well to have the very useful Table which has been constructed for the purpose of enabling us, with ease and correctness, to work sums in it. This Table, it is customary for learners to commit to memory; and this is but a trifling task, when compared with its great value, in facilitating the operations of Multiplication, and, indeed, of almost all operations in Arithmetic. To learn the Table, to get it off by rote, to make it familiar on the tongue, and as it were, a part of the mind, so that, when we would multiply any one figure by another, the product shall, instantly, and without the trouble of thinking about it, rise in the mind, be breathed from the lips, and, if pen or pencil be held, traced by the fingers; this is what ought to be done; and thus familiar may the Table be made, with but a very small degree of labour, indeed, compared with that which it would save to every one who has any thing to do with Arithmetic.

72. Then, too, there is the certainty, in our reckonings of all sorts, which, without having this Table in us, there are scarcely any other means of attaining. In short, it is profitable, pleasant, and I may add, indispensible, thus to have it. And, as there is not any other thing in Arithmetic, which demands from the learner so irksome an employment, as this of getting off by rote, so, I hope he will not demur thus to learn this Table: the very

best mode of doing which, at the same time that it will be improving to him in writing figures, is, not to say it merely with the lips and tongue, but to write the figures down, as he speaks them ; and, to do this in as neat and as regular a manner as possible, thereby training up, and enlisting in the useful office, the hand and the eye, as well as the voice.

Multiplication Table.

Twice			*3 Times*			*4 Times*			*5 Times*		
2	are	4	2	are	6	2	are	8	2	are	10
3	..	6	3	..	9	3	..	12	3	..	15
4	..	8	4	..	12	4	..	16	4	..	20
5	..	10	5	..	15	5	..	20	5	..	25
6	..	12	6	..	18	6	..	24	6	..	30
7	..	14	7	..	21	7	..	28	7	..	35
8	..	16	8	..	24	8	..	32	8	..	40
9	..	18	9	..	27	9	..	36	9	..	45
10	.	20	10	..	30	10	..	40	10	..	50
11	..	22	11	..	33	11	..	44	11	..	55
12	..	24	12	..	36	12	..	48	12	..	60

6 Times			*7 Times*			*8 Times*			*9 Times*		
2	are	12	2	are	14	2	are	16	2	are	18
3	..	18	3	..	21	3	..	24	3	..	27
4	..	24	4	..	28	4	..	32	4	..	36
5	..	30	5	..	35	5	..	40	5	..	45
6	..	36	6	..	42	6	..	48	6	..	54
7	..	42	7	..	49	7	..	56	7	..	63
8	..	48	8	..	56	8	..	64	8	..	72
9	..	54	9	..	63	9	..	72	9	..	81
10	..	60	10	..	70	10	..	80	10	..	90
11	..	66	11	..	77	11	.	88	11	..	99
12	..	72	12	..	84	12	..	96	12	..	108

10 Times			*11 Times*			*12 Times*		
2	are	20	2	are	22	2	are	24
3	..	30	3	..	33	3	..	36
4	..	40	4	..	44	4	..	48
5	..	50	5	..	55	5	..	60
6	..	60	6	..	66	6	..	72
7	..	70	7	..	77	7	.	84
8	..	80	8	..	88	8	..	96
9	..	90	9	..	99	9	..	108
10	..	100	10	..	110	10	..	120
11	..	110	11	..	121	11	..	132
12	..	120	12	..	132	12	..	144

73. The above I deem the more intelligible form of arrangement for the Table, and, therefore, I give it the precedence. There is, however, another form in use, somewhat more compact than this. And, as some teachers carry on the reckoning so high as twenty times twenty, I will do so in this other Table; but, without urging on the learner the task of getting it higher than twelve times twelve.

2nd Multiplication Table.

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

74. In the last table, you see, that outside the line which encloses the block of figures, there is, in a row above, and also in a column down the left-hand side; in each of these situations, there is a regular series of figures rising from 2 to 20. Now, the mode of reading this table is this; beginning with the 2, in the upright row, and reading on, towards the right hand, you find, right under the 2 in the upper line, the figure 4; under the 3 you find 6, and under the 4, is 8: and these tell you, that twice 2 are 4; twice 3 are 6; twice 4 are 8, &c. To go on, you find three times 2 are 6; three times 3 are 9, and so on; just as you had them in the former table.

75. The process of Multiplication is thus carried on: suppose we would know the product of 9 and 4 multiplied together. Writing the figures down thus, and drawing a line under them, we say, four times nine are thirty-six, and then write down the 36, beneath the line; which 36 is the product.

$$\begin{array}{r} 9 \\ 4 \\ \hline 36 \end{array}$$

76. But let it be another figure; let it be 4 times 39, that we would have the product of. Writing the figures down, according to the first example; that is, placing the larger sum for the multiplicand, the smaller for the multiplier, and drawing a line, we say, four times nine are thirty-six, and write down the figure 6 only, bearing, or carrying in mind, the thirty, until we have the product of the 3 multiplied by the 4, and then adding it to that product; thus, four times three are twelve, and remembering that these are twelve TENS, we call the thirty which we have carried, three, (that is three tens,) and say, "four times three are twelve, and three make fifteen;" which being fifteen tens, and written down in their proper place, that is, on the left of the figure 6, make the product 156; which is the product required.

$$\begin{array}{r} 39 \\ 4 \\ \hline 156 \end{array}$$

77. In this manner are any sums, and thus are any number of figures, to be multiplied by a single figure: that is to say, by any figure from 1 to 10; and, indeed, we go on to 12, obtaining the product of 11, or 12 times any line of figures whatever, in a manner equally simple; that is, beginning to multiply the first figure on the right hand, and so proceeding, taking each figure in its turn, through all the figures of the multiplicand, or sum to be multiplied: so that millions are as easily multiplied as are hundreds, or tens; as, for example, extending the line of figures to millions, thus, we say, four times nine are thirty-six, set down 6, and carry 3; then, 4 times 3 are 12, and the 3 carried, make 15, set down 5, and carry 1; then, 4 times 1 are 4, and the 1 makes 5, which 5 is, of course, to be set down, leaving nothing to be carried; so we begin anew, saying, 4 times 4 are 16, set down 6, and carry 1; 4 times 7 are 28, and 1 makes 29, set down 9, and carry 2; 4 times 2 are 8, and 2 make 10; here, we set down the 0, and, according to our rule, carry forward the 1, saying, 4 times 5 are 20, and 1 makes 21, which sum, as the process is now finished, is to be set down. And thus have we, as the product of our multiplicand and multiplier, the sum of twenty-one million, ninety-six thousand, five hundred and fifty-six.

$$\begin{array}{r} 5274139 \\ 4 \\ \hline 21096556 \end{array}$$

78. In this easy manner, do we multiply any line of figures whatever, by any number under ten, bringing out the product thereof in a single line; and, indeed, to 12 do we go, which is the extent of our Table, bringing out the result in a single line; as thus, 12 times 2 are 24, set down 4, and carry 2; 12 times 1 are 12, and the 2 carried make 14, set down 4, and carry 1; and so we proceed to the end.

$$\begin{array}{r} 653412 \\ 12 \\ \hline 7840944 \end{array}$$

79. The few propositions, or sums, as they are commonly called, which here follow, will exercise and improve the learner, and prepare him for the practice of Multiplication in higher numbers.

[1] Multiply 32514 by 3	[2] 25361 5	[3] 63527 4
[4] 57386 7	[5] 20758 8	[6] 83752 11
[7] 75652 9	[8] 45387 12	[9] 99175 6

[10] Mult. 35802 by 7. [11] Mult. 57416 by 8.

[12] „ 68431 „ 5. [13] „ 35922 „ 11.

[14] „ 73518 „ 9. [15] „ 84335 „ 12.

80. Beyond 12 times, however, we do not attempt to multiply in a single line: for, although it may be done, it would be attended with needless difficulty, and liability to error. So, when we have to multiply by any larger number, as, for instance, by 27, we multiply first by the 7, writing the product down in the manner before shown, and then by the 20, the product of which, we write immediately under the other line, as it is shewn by the example; and then, drawing a line underneath, and adding the two lines of products together, we have the product of our multiplicand, when multiplied by 27.

```
 3164251
      27
--------
22149757
6328502
--------
85434777
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81. Now, it is very important that you observe the method pursued in the ranging of these two lines of products. In the example above, you will see, that the second line, that is, the product by the 2, begins on the right hand, not immediately under the first line, but that it starts one station higher: and so it must do, as you will recollect, for this 2 is TWO TENS, and its product takes its station accordingly; commencing in the place of tens.

82. This is the most important, nay, it is almost the only thing of moment to be learned in this rule. And to make it clear and familiar to you, I must press your attention to it, somewhat particularly, as exhibited in the annexed example, which is the same as the last, only another figure added to the multiplier.

```
   3164251
       327
----------
  22149757
  6328502
 9492753
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1034710077
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83. On a little attention you will perceive, that each successive line of products in the above example, is not merely one station higher than that which goes before it, but, likewise, that on writing it down, we commence each line by placing the first figure exactly under that with which we multiply. And this, do you observe, IS THE RULE; namely, to place the figure, which, on multiplying, we have first to write down, directly under the figure with which we multiply.

84. One other little matter, on this head, remains to be noticed, after a little consideration of which, I do not doubt but that you will be able, without further instruction, to work any sums whatever in this rule. The point to which I would here call your attention, is, the manner in which the process is carried on, when a cypher occurs amongst the figures of the multiplier.

85. In paragraph 19, the use and the power of the cypher are explained; and it is there shown, that this figure is employed to advance or to increase the value, tenfold, of all figures to the right of which it is placed: recalling this to your mind, you will not only understand the operation of the cypher, as again shown in the three following examples, but will likewise perceive, that whenever you have to multiply any sum by ten, you have only to write a cypher to the right of that sum, that to multiply by a hundred, you have so to write down two cyphers; that the addition of three cyphers will make any sum a thousand times its former amount, and so on.

86. The examples to which I refer, in the last paragraph, as exhibiting the operation of the cypher, are the following; in which, taking the same multiplicand as that last used, I change the multiplier only just to introduce cyphers. And you will, here, clearly see, that, following THE RULE laid down, in paragraph 83, for commencing each line of product, right under the figure of the multiplier by which it is produced, all you have to do, when you come to a cypher in the multiplier, is, to write a cypher underneath it, in the product, and then to go on to multiply with the next figure of the multiplier.

3164251	3164251	3164251
320	3200	207
63285020	632850200	22149757
9492753	9492753	63285020
1012560320	10125603200	654999957

87. After this, all you will have to do, in order to become a proficient in this rule, is, to practice the table, by writing and reciting it repeatedly; and carefully to work the sums here set down for practice.

Examples, &c. in Multiplication.

(1) Mult. 5316428 by 152	(2) 7452640 364	(3) 4720659 580
(4) Mult. 39752140 3526	(5) 85103426 7043	(6) 50917842 4630

(7) Multiply 29852073 by 55612

(8) „ 51603548 „ 52630

(9) „ 62394260 „ 74802

(10) „ 374065324 „ 60437

(11) „ 8539226807 „ 37427

(12) „ 37493403126 „ 53860

DIVISION.

88. As Subtraction is the reverse, or counterpart of Addition, so is this rule the reverse of that of which we have just treated. To multiply is to increase; but as it is to increase by the bringing together of *similar* things, so Multiplication, as we before observed, is, in Arithmetic, the bringing together of *similar*, or *equal*, sums; and just the reverse of this is Division: it is, the separation of larger sums into smaller ones; the smaller being *equal* one to another; as twenty, divided into four equal parts, gives us five for each part: and this is a simple example of the rule of which we are now to treat.

89. Amongst the uses of this rule, take the fol-following as examples. Have you purchased a quantity of goods, say a cask of rice, containing six hundred weight, for four guineas, and you would know how much it cost you per hundred weight; you divide the cost, that is 84 shillings, by the number of hundred weights, and you find that it cost 14 shillings per hundred weight: would you know how much is the cost per pound, then divide the 14 shillings by the number of pounds in the hundred weight, and you find that each pound cost three halfpence. Again, have you a certain quantity of work to perform, a certain journey to accomplish, a certain sum of money to expend, each to be done within a given time, then, divide the work to be performed, the miles to be travelled, or the money to be expended, by the given number of hours, or of days, and you learn what portion of either you must assign to each hour or day, in order to accomplish your task with regularity.

90. Now, in this, as in other rules, there are certain terms employed, with which it is useful to be acquainted; these terms are the four following, namely: DIVISOR, DIVIDEND, QUOTIENT, and REMAINDER. And, first, as to the DIVISOR, this is the sum with which we divide, and it expresses the number of parts into which we are to separate the DIVIDEND, or sum to be divided; QUOTIENT expresses the size, the price, or the quantity of each part; and, REMAINDER, as the term signifies, describes any thing which may be left, too small to make one of the parts; as, for example, suppose that you had to divide thirty-one shillings and six-pence amongst fifteen men. You would find, that two shillings and a penny each, would come to thirty-one shillings and three-pence of the money; so there would yet be three-pence undivided; and, as you could not divide that into fifteen parts, so as

to give each of the men one of them, you would have to call the three-pence, Remainder: so, REMAINDER is that which is left, when you have divided as far as you can divide.

91. The method of stating sums, and of working them, in this rule, is, as hereafter described.

92. To divide twenty by five, or to do any simple thing of that kind, is an easy matter. Any body can do it without any schooling about Arithmetic. But there are large and various sums, to be variously divided, the mere thought of having to perform which, without the proper method, would frighten any rational person.

93. The difficulties, however, in this case, vanish before the proper methods of treating them, as I trust you have found them vanish in all preceding cases.

94. This proper method consists, mainly, in a due statement of the case to to be worked; and, where the sum to be divided is too large to be managed at once, it further consists, in a due separation thereof, into the parts most convenient for the operation. As, for instance, suppose that I have to divide 75 by 3. I do not, at once, attempt to say how many times three there are in the whole sum, seventy-five, but I divide, first, the seventy, and then I deal with the rest in this manner.

95. Writing down, first the dividend, 75, and drawing a small line down before, and carrying it along, underneath the figures, I prefix the divisor, 3: as you see in the example annexed.

3) 75

And, proceeding to work, I say, "*three in seven are twice*," (meaning that three are found twice in seven,) then I set down 2, right under the 7. But, in the 7 there are 2 threes, and one over; that is to say, *in the seventy*, there are *twice three tens*, and *one ten* over; which ten, being only 1, is not, nor cannot be, divided by the 3: so then the manner of proceeding is this. We have 5 yet undivided, besides the ten; so I add these two small sums together, and then divide them, saying, "carry ten to five, which make fifteen; the threes in fifteen are five," so I write down 5, after the 2, thus, and the work is done. And thus, I find, that three are found in 75, 25 times

$$3\,)\,\underline{75}$$
$$25$$

96. To this example, let us add another figure; let us divide 755 by 5, and it will stand thus: we then say, "five in seven, once," setting down 1, under the 7; we proceed, "carry two, the fives in twenty-five are five;" so we set down the 5; and there now being nothing to carry, we have only to say, "five in five, is once," and so we set down 1, and thus complete our work; finding that 755 divided by 5, gives 151; that is, that the fives in 755 are 151.

$$5\,)\,\underline{755}$$

97. However, as has been observed, sums do not always divide in this equal manner; that is, it is not always that the sum we divide contains just a certain number of the divisor, without leaving a remainder. As, for instance, were it, in the last example, 758 that we had to divide, instead of 755, then there would be a remainder of 3: and the sum would stand thus: the remainder being, in this simple sort of division, to be written down after the quotient, with a short line between them.

$$5\,)\,\underline{758}$$
$$151\text{-}3$$

98. Here, again, we must pay our respects to that *insignificant* figure, the *cypher*. To distinguish

them from this, mathematicians call the others *significant* figures; because they, each of them, from 1 to 9, *signify a number*, whilst this 0 signifies *nothing*. But, devoid of importance as it is, in one respect, this figure is, like many other little things, the value of which we are apt to overlook, of vast and indispensable use. And, here, again, in this last of the four fundamental rules of our art, we are called on, as we have been in each of the foregoing, to pay special attention to the influence, and the uses of this figure.

99. But, now that you are pretty well acquainted with the cypher, what remains to be said of its influence and its uses, is easily understood, although these will require a rather full description.

100. The first, and simplest cases of its occurrence, are those in which you find it in the annexed example. And, to give the words which we use, in working this sum, will sufficiently describe the use, and the power of the cypher.

$$5\,)\,\underline{7046}$$
$$1409\text{-}1$$

Commencing the operation, you say, "five in seven is once, set down 1, and carry 2; two to nought make twenty," (*for* 20, *it is, and, indeed,* 20 *hundred*) "fives in twenty are four, set down 4"—Then, there being no remainder, nothing to carry, you begin anew, saying, "five in four, none, set down nought 0." Here you see the use of the cypher. Four cannot be divided by five, so you have nothing to set down; but *the place must not be left vacant*, but must be filled up, in order to keep the two preceding figures, the 14, in their proper stations; so you, having no *significant* figure to set down, fill up the place with a *cypher*.

101. And thus it always must be. In dividing, you take each figure of your Dividend in succession,

one at a time, and if, joined to any remainder, that you may have, such figure do not make you a sum large enough to be divided, you set down a cypher in the quotient, and "*carry*," to the next, whatever figure, or remainder, you may have, that has not yet been divided; as is shown in this example: with which, returning to where we left off, and, seeing that the 4, which is our last figure, stands for forty, joining it to the next figure we thus proceed, "Fives in forty six, are nine, and one over;" set down 9, and carry out the 1, as a final remainder.

102. The next and only remaining case in which I have to speak of the cypher, is this. In paragraph 88, you were told, that Division is the reverse of Multiplication: and in paragraph 85, you find, that when you would multiply any sum by ten, you have only to annex a cypher to the right hand of the figure, or figures, describing such sum; that, to multiply by a hundred, you have just to annex *two* cyphers; for a thousand, *three* cyphers; and so on. And it is just the reverse of this, that you have to do, in Division, when you would divide by ten, by a hundred, by a thousand, &c. that is, you reverse this process; and, instead of annexing figures, you cut them off, and thereby reduce them to a tenth, to a hundredth, or to a thousandth part of the sum they were before. Thus, let us have to divide 628513 by 10. This is done at once by cutting off the 3; and letting it stand thus, 62851 - 3. To divide it by 100, cut off two figures, thus, 6285 - 13. In short, by bringing them down to a lower station, you reduce the value of the figures of your dividend, to a tenth, to a hundredth, or to whatever you please, in even tens, of their former value. And the figures which you cut off, being always less than the divisor, form a remainder.

103. But, observe, that it is only when you have

cyphers on the lowest place, or places, of the divisor, that you can adopt this mode of dividing, by cutting off figures. And then observe, also, that the number of figures which you can cut off the dividend, is exactly the same as that of the cyphers at the end of the divisor, but not more. It signifies not, what the figures at the end of your dividend are; if you have cyphers at the end of your divisor, you may cut off just the same number of figures from the dividend.

104. But, again, observe: I have said that you may, in this easy and simple manner, divide any large sum by 10, by 100, by 1000, and so on: in this manner you *may* do it, in order to abridge your labour, if you please so to do; but it is no RULE, it is merely an expedient to save trouble; you may, if you please, divide as already shown, and as hereafter to be taught, without regard to the cyphers; and the result, provided that you divide correctly, will be just the same.

105. However, there is yet another particular to be noticed, before we dismiss this expedient of dividing by cutting off figures. When we have to divide by *ten*, by *one*-hundred, *one* thousand, or by any number, having a *single unit* followed by a cypher, or cyphers; when we have a division of this sort to make, it is done, at once, just by cutting off, as I have described. But, if we have to divide by two, three, four, five, by twenty-five, by thirty-five, by forty-eight, or, in short, by any number of tens, of hundreds, or of thousands *more* than ONE, we do it by cutting off figures from the dividend, cor-corresponding in number with the cyphers in the divisor, and then cutting off these cyphers, also, we proceed to divide the rest of the dividend, with the remaining figure, or figures, of the divisor, thus: let there be 865327 to be divided by 10, by 100, by

1000, it is done at once, as you see here, in example 1, 2, and 3. But, if we have to divide the same sum, say, by 50, by 200, then by 300, then by 4000, then by 7000, as you see it done here, in examples 4 to 8; in these cases, after cutting off the cyphers, together with a corresponding number of figures in the dividend, we then, with whatever sum may remain in the divisor, proceed to divide the remaining number of figures in the dividend; and, then, carrying out any remainder, as before directed, we bring down, as you see in these examples, whatever figures have been cut off in the dividend; and these united, form the remainder.

(1) .... 86532·7

(2) .... 8653·27

(3) .... 865·327

(4) 5'0) 86532'7
17306·27

(5) 2'00) 8653'27
4326·127

(6) 3'00) 8653'27
2884·127

(7) 4'000) 865'327
216·1327

(8) 7'000) 865'327
123·4327

106. In the simple examples in this rule, of which I have hitherto treated, we have found, that in every case, the *significant* figure in the divisor was smaller than the *first* figure in the dividend; and we could, therefore, begin the operation of dividing, at once, on that figure: but this is not always the case; the first figure sometimes being insufficient, as in the annexed example, we join thereto, the next figure, and then proceed to divide: as, in this case we say "nine in twelve, once, and three over, set down 1, and carry 3," which 3, being in the higher place, make, with the next figure, 36; so we proceed, "nines in thirty-six

9) 12658
1406·4

are four," set down 4; then "nine in five, none;" set down 0; and, then, having 58 yet to divide, we say, "nines in fifty-eight, are six, and four over," set down 6, and carry out the 4, as a remainder; and thus is the sum finished.

107. The few examples which follow will further illustrate this part of the rule. In working them, and, indeed, in working all sums in Division, you will find a familiar habit of using the Multiplication Table, not only useful, but indispensible. As, for instance, when you have to divide, as you have, in the following examples, 65 by 7, the table tells you that 9 times 7 are 63, which is the number you want. Then there comes 97 to be divided by 11: your table tells you, that 9 times 11 are 99, which sum is too much, so you take the next lower, that is, 8 times 11. In the 6th sum, comes 41, to be divided by 12; then 57; then 92; then 80; and then 83; each to be divided by 12; and the table, almost at once, tells you how many times 12 make the nearest approach to these several sums. So that you see how truly, in paragraphs 71 and 72, I have described the uses, and the indispensible importance of this table.

108. You will do well to trace over the work as you see it done in these examples, stating them on paper, or on your slate, and then working them yourself, until you find that you can do them correctly. And, then, the propositions and questions, which are placed immediately after, for your practice, if attentively worked by you, will mature and confirm the knowledge you will have acquired.

EXAMPLES.

[1] 4) 5831	[2] 5) 7046	[3] 8) 9325
1457-3	1409-1	1165-5
[4] 7) 23065	[5] 11) 537104	[6] 12) 417203
3295	48827-7	34766-11

PROPOSITIONS AND QUESTIONS.

[1] It is proposed to divide 625073 by 5.

[2] „ „ 2934068 „ 9.

[3] „ „ 7302216 „ 12.

[4] How many times will 11 divide 36127458 ?

[5] How often is 8 to be found in 4725018 ?

[6] What is the quotient of 5370280 divided by 10 ?

[7] How often is 6 contained in 37920514 ?

[8] Say how many times does 935047261 contain 12.

[9] What is the quotient of 60835724 divided by 9 ?

[10] If 85310 crowns be to be equally divided between seven persons, how many will this be for each person ?

[11] From a field of eight acres, a farmer has reaped 11523 quarts of wheat : how many quarts are there, in this number, for each acre ?

109. Thus we have treated of the manner in which Division is carried on in the more simple operations in which it is employed; that is to say, in the division of any sum, by any figure from 2 to 12, both inclusive. In every process of this sort, the purpose is accomplished in a single line; as it is in Multiplication, when we multiply by a sum not larger than 12. Multiplication and Division are, as before observed, the counterpart or reverse of each other: and it holds good in this, that when we work with a multiplier, or with a divisor, not exceeding 12, we accomplish our object in a single line: which working is, on this account, called *short;* that is, SHORT MULTIPLICATION, and SHORT DIVISION.

110. But, as there is Short Division, so, also, is there LONG DIVISION; for so is the process called when the divisor exceeds 12. And of this we now proceed to treat.

LONG DIVISION.

111. In order to enter with advantage on this branch of our subject, I must recall to your mind, how very much depends on the manner of stating the sums to be worked with; and must apprise you particularly, that in Long Division, the mode of statement requires a little more of our attention than do any of the examples heretofore treated of. Observe, therefore, carefully, the mode of making the statements, and then the mode of working the sums, as exhibited in the following examples.

112. Let 85434777 be to be divided by 27. Here, instead of drawing a line down, and then carrying it immediately underneath the dividend, as we do in Short Division, we, in cases of this kind, state, or place, the sums thus: that is to say, writing down

the divisor, towards the left hand of the paper, we draw a short upright curve after it, then write the dividend, as a continuation of the line, and then another short upright curve, thus:

27) 85434777 (

113. Now, how to begin to divide with these Long Divisors, is the question. In Multiplication we begin with the figures on the right hand of the line, but in this rule, we begin at the other end; *this*, indeed, is a sort of unravelling of *that* process; and here, as was shown in Short Division, we begin at the left end of the line.

114. But, how many of these figures of the dividend are we to work upon at once? This is the next question. The sum to be divided by 27 is 85 millions, &c.; a sum not to be comprehended, much less divided, at once, by any mind, however clear and powerful. So, as in the examples before given in this and in other rules, when large sums are to be dealt with, we take this dividend into portions, and thus, with comparative ease, accomplish our work.

115. But, as I have just said, how many of the figures, what portion of this vast sum do we venture on at once? It is manifest, in the first place, that the portion to be divided must exceed the number with which we are about to divide; we can divide no sum with one larger than itself. More than the divisor then, we must take; and, how much, how many of the figures of the dividend ought we to take at once? Ease, in the accomplishment of any work, you know, is a thing to be desired; and more especially, when along with ease, we can, by any course, lessen the risk of falling into miscalculation or error. Now, although, in this instance, we might

with a little thought, aided, perhaps, by a few trials with the pen, find out how many times 27 would go in 854, which are the three first figures in our dividend, yet it is easier to find out, how many times this 27 is contained in 85; and in this more easy, and more simple calculation, we are, also, less liable to err. Here, therefore, it is, that we find our RULE: that is to say, the guide, or law, for our proceeding. We must have, at the least, as much of the dividend as equals the divisor; but, when we have taken figures enough for this purpose, it is well to stop; to divide that portion first, then to take another portion, and so to proceed to the end of the dividend.

116. This is a Rule on which we proceed in the working of sums in Long Division; that is to say, to take for the operation a due portion of the dividend at a time. And, having done so, the dexterity of the Arithmetician is sometimes called into play, in order to determine, in certain cases, how many times the divisor is contained in that portion. As, for instance, even in the example which here follows, it requires some little thought to discover, without a trial, how many times 27 is contained in 84, which is the first step in the operation; as to the second step, it requires nothing worth calling thought, to see, that the divisor is contained only once in 44: but, to divide 173 by 27, presents a somewhat greater difficulty than did the former; and it is in determining with dexterity, as I have just stated, the fitting number of times, in order to write that number down in the quotient, that evinces the talent of the calculator. As to rules or directions on this point, there are none worth burthening the mind with, save such as will naturally arise, on a little experience. But it must not be forgotten, that the most experienced and able have frequent occasions to make a trial or two, before they can determine on

the proper number. Do you, therefore, never scruple to resort to the same very proper method. So bearing these things in mind, let me call your attention to the following examples, in which you see the whole process of Long Division.

FIRST STEP.

```
27 ) 85434777 (3
     81
     —
      4
```

SECOND STEP.

```
27 ) 85434777 ( 3
     81·
     —
      44
```

THIRD STEP.

```
27 ) 85434777 ( 31
     81··
     —
      44
      27
      —
      173
```

THE WHOLE PROCESS.

```
27 ) 85434777 ( 31 2 5 1
     81······
     —
      44
      27
      —
      173
      162
      ———
       114
       108
       ———
         67
         54
         ———
         137
         135
         ———
           27
           27
           ———
```

117. Now look attentively at the foregoing examples; attending to the first step, and quitting it not for the second until you see clearly the mode of proceeding; and, in like manner, mastering each step ere you leave it.

118. In the first step, we consider, or try with a pen, or pencil, what number of times the divisor is contained in the first convenient portion of the divi-

dend: and, having ascertained this, we write down that number as the first figure of the quotient. In this example, you see, the first figure is 3; with which we multiply the divisor, and write the product of such multiplication, right under that portion of the dividend which we are dividing. Now here, the product of the divisor 27, by the quotient 3, is 81, which is 4 less than the portion we are dividing; that is to say, in this portion of 85, there are 3 times 27, and 4 over. Now, observe again, this 4 is not yet divided: it is like an odd part, too small to make one of the quantities into which we are dividing the large sum; so, as we have yet a sort of store of this large sum left, we join a portion of it to the 4, by bringing the next figure down to it, as you see is done in the second step; and, observe, also, that in order to avoid the mistake of bringing this figure down a second time, we mark it with a dot underneath, as you see in the example. So much for the first and second steps in this process.

119. The third step consists of a dividing of the two figures we have now down; that is, the 44; in which, seeing that there is but once 27, we write this 1 in the quotient, after the 3, then place the 27 underneath, and subtract it from the 44. This leaves a remainder of 17, another odd part, which is too small to make one of the portions; so we proceed as we did before, we dot, and bring down the next figure of the dividend. This gives us 173 for our next division.

120. A few trials, with pen or pencil, may here, again, be necessary, to find how many times the divisor is found, or, as we familiarly call it, how many times it will *go*, in this 173. We soon find that it goes 6 times; so placing the 6 after the 1, in the quotient, we proceed to multiply the divisor with it, to place the product underneath the 173, to

subtract, and to bring down, as before, until we have brought down, and divided the whole of the dividend; as is shown in the example.

121. And, so you find, that in this sum of 85434777, which forms the dividend, the divisor 27 is contained 3164251 times: or, if you choose to put it this way, that this dividend contains 27 times 3164251.

122. At the conclusion of a sum in Long Division, it is customary to draw a curve at the bottom, as you see in the example; which curve, enclosing the last remainder, or, if there be no remainder, a dot or two, as in this example, or a cypher, in order to signify that the work is finished.

123. In the example just given, it has happened, that, at each step of the process, it was found sufficient to bring down a single figure only, and then to proceed again to divide; but this, you will find, is not always the case; the remainder may be so small, that the addition of the next figure will not make it equal to the divisor, in which case you write a cypher in the quotient, and then bring down another figure, as in the following brief example, until you make it large enough to be divisible by the divisor; that is to say, you are so to proceed, if there be figures in the dividend yet to bring down. And, you must further bear in mind, that, on all occasions, for *each* figure you so bring down, of the dividend, you write one figure in the quotient.

124. This is shown in the three following examples, which are constructed merely for the purpose of illustrating this single point: but, carefully to trace over the working

34) 7038 (207
68 · ·
——
238
238

of them, and then to state and work them himself, cannot fail to be highly useful to the learner.

```
45 ) 4862753 ( 108061      53 ) 16414570 ( 309708
     45·····                    159·····
     ——                         ——
      362                        514
      360                        477
      ——                         ——
        275                        375
        270                        371
        ——                         ——
          53                         470
          45                         424
          —                          ——
           8                          46
```

125. To give an example somewhat longer, as the annexed: in which the sum 1647035670 is divided by 327.

```
327 ) 1647035670 ( 5036806
      1635······
      ——
       1203
        981
       ——
        2225
        1962
        ——
         2636
         2616
         ——
           2070
           1962
           ——
            108
```

126. These examples, duly attended to, are sufficient to enable you to work the following sums; and, afterwards, to work any sums in this Rule that are likely to occur in the course of general business.

SUMS, OR QUESTIONS AND PROPOSITIONS.

(1) What number of times is 137 contained in 5201632?

(2) Required 3620751 to be divided by 216.

(3) In 7580962, how many times is 237 to be found?

(4) Give the quotient of 6351704 divided by 318.

(5) How many times will 284 divide 35170842?

(6) How many times is 315 to be fonnd in 74106582?

(7) What is the quotient of 164953208 divided by 275?

(8) There being 365 days in a year, how many years are there in 350724168 days?

(9) How many reams of paper will 75036542 sheets make, 468 sheets to each ream?

(10) There being 825461 pounds of meal to be served out to a number of men, at the rate of 56 pounds for each man, how many men will that quantity serve at this rate?

(11) A certain reservoir contains 153724 gallons of water, how many hours will this water be of running off, through a pipe that discharges it at the rate of 43 gallons to the hour?

(12) Light being ascertained to pass from its source, at the rate of about 286 leagues in a second, how many leagues will it travel in 9507326840 seconds?

On the Methods of Proof,

In Division and Multiplication.

127. This is a very simple affair, requiring scarcely a word beyond the few in which I have to remind you, that, as these two rules are the reverse of each other; that, as Multiplication brings into one large sum, the amount or product of a smaller sum repeated, or multiplied; and, as it is the business of Division, to separate large sums into any number of smaller equal sums, so the two rules are calculated, reciprocally, to PROVE each other. As, for instance, in the last example in this rule of Division, paragraph 125, we have 1647035670 divided by 327, giving, for a quotient, 5036806, and leaving a remainder of 108.

128. Now, to PROVE the correctness of this, or of any sum in DIVISION, take and multiply the divisor and the quotient together, and add to the product the remainder, when there is a remainder; that is, in fact, *undo* the work of dividing, and you will find, when all is correct, that the figures of the dividend are exactly restored.

129. As to the PROOF OF MULTIPLICATION, all you have to do, in order to effect this, is, to divide the product by the multiplier, and this ought to restore to you the figures of the multiplicand.

130. There is another mode used by some persons for proving sums in these two rules, a mode, by adding together the figures, and *casting out the nines*, as it is called. But this mode of *trying* a sum, for I will not call it *proving*, is founded on no true principle, and, as it gives no *certain* evidence of correctness, I shall not trouble my pupils with it.

CONCLUDING LESSON,

ON THE FOUR FUNDAMENTAL RULES: AND ON CERTAIN PROPERTIES AND POWERS OF NUMBERS; WITH THEIR TERMS, &c.

131. It now becomes my duty to describe to you certain modes of rendering some of the processes, in some of the foregoing rules, yet more easy; and, likewise, to teach you, in some measure, to shorten the process in the working of sums in all of the rules.

132. And, first, for the latter of these purposes; that is, the shortening of the processes. All that I mean to propose here, on this matter is, that, as you become practised and expert in the working of figures, you should gradually discontinue, as much as you safely can, the use of the *words* by which we teach learners to recite the process of their work; as, for instance, to return to the example in addition, in paragraph 43, which example I here annex; instead of saying, even to yourself, as I there taught you, "*nine and two make eleven*, 4 *make* 15, 5 *make* 20, *and* 1 *makes* 21," instead of using all these words, even to yourself, learn to say, or rather to think, for as we become experienced, we drop the *saying;* learn first to *say*, and then only to *think*, on beginning to add up the columns, and casting the eyes on the 9 and the 2, say "*eleven*," then, looking on the next figure, say "*fifteen*," then on the next, say "*twenty*." And so you may, as you become practised, learn to proceed through sums in Addition, steadily running your eye over the figures, and collecting their amount.

2831
6405
974
8152
5379
23741

133. In Subtraction, to take the example in paragraph 56, which is, also, here inserted; instead of saying, "*nine from five, I cannot, but nine from fifteen, and six remain; set down six, and carry one.*" You will learn to say, or to think, "*nine from fifteen, six; two from three, one; three from eleven, eight.*" And thus may you learn, as you become practised, to drop words that will then become unnecessary; as the scaffolding of the builder becomes useless, when he has raised his structure.

```
 2135
 1319
 ----
  816
 ----
```

134. With regard to the other object, that is to say, the rendering of some of the processes of Arithmetic more simple, or more easy of execution; this is the method I have in view; but it is applicable only to the larger sums in Division, and applicable, indeed, only to some of them. The thing is this, suppose you have to divide some large number, by some other considerable number, for instance, let it be 31957431 by 324. This, you know, in the usual mode, is a sum for Long Division: and that is, certainly, the more masterly method of doing it. But, if you would have an easier, or, for any reason, be inclined to work in another manner, it may be done thus: The Divisor is 324. Find out what smaller numbers you can divide this sum into; numbers the *product* of which, and *not the addition*, will make it; and then divide with these smaller numbers, one after the other, and these will bring out your quotient, the same as you would bring it out by Long Division.

135. For instance: suppose you have to divide 312 by 24. This is a simple instance, and you see at once that 4 and 6, or 3 and 8, or, 2 and 12, multiplied together, will make the divisor 24. So

```
4 ) 312
    ---
6 )  78
    ---
     13
```

take either of these pairs of numbers, as you see it here done, and divide, first the 312 with one, and then divide the quotient of that operation with the other number, and this will give you 13; which is the number of times that 24 is found in 312; just the same as if you had sought it by Long Division.

$$\begin{array}{r} 3\,)\,\underline{312} \\ 8\,)\,\underline{104} \\ 13 \end{array} \qquad \begin{array}{r} 2\,)\,\underline{312} \\ 12\,)\,\underline{156} \\ 13 \end{array}$$

136. It must not be overlooked, however, that it is only when your divisor can thus be subdivided without leaving a remainder, that you can resort to this mode of proceeding. For, were the divisor, for instance, 23, or, 29, or 31, or any *odd*, and *indivisible* number; or, if it were a large number, which would divide, but which you could not bring down into parts small enough for divisors, in the manner of short division, that is to say, if you could not bring it down into parts small as 12, or smaller, then, also, could you not resort to this mode of dividing.

137. In the example just given, you saw, that the numbers 4 and 6; 2 and 12; or 3 and 8; produce, when multiplied together, the number 24. Now, these several numbers stand in a certain relationship towards each other, which relationship is described by certain terms, in use amongst arithmeticians; and which, therefore, it may be useful to you to be acquainted with.

138. The number 24, being one which can be produced by the *multiplication* of a due portion of the numbers 2, 4, 6, 8, and 12, it is called A MULTIPLE of these numbers, whilst these are called THE SUBMULTIPLES of it. And this relationship subsists between, and these terms are applicable to, all numbers that are mutually capable of producing, and of being produced by each other.

139. This *relationship*, however, of multiple and submultiple, is but a sort of theoretical bearing of the numbers one towards the other. It means, merely, that the smaller *may* be found in the larger; and that the larger *may* be produced by a multiplication of a due portion of the smaller numbers, without there being either surplus or deficiency.

140. I say, that these terms of multiple, and submultiple, applied to certain numbers, mean, simply, to describe that which *may* take place between those numbers; that is to say, that they are mutually *capable* of producing and of being produced. But, when the operation actually takes place; when smaller numbers are multiplied together, for the purpose of producing, and when, in fact, they have produced a larger, then both numbers change their names; and that which is produced is called THE PRODUCT, whilst those which *produced* it, or *made* it, are called ITS FACTORS.

141. To discover, by a ready mode, when this relationship subsists between numbers; or, rather, readily to discover, what are the submultiples of a given sum, with which you may have to work, or which you may have occasion to divide without leaving a remainder; to do this, is sometimes desirable. And, therefore, I will point out some of the means.

142. In the first place, then, *all even* numbers, as you will instantly see, are divisible by 2; then, all numbers ending with 5, or 0, may be divided by 5: then, the even number, 4, will divide any sum, however large, the two last figures of which are *evenly* divisible by that number; and the even numbers, 6, and 8, possess the same power.

143. The power which the other small numbers

possess, of dividing large ones, cannot be so simply classed; and as it is foreign to my purpose, which is only to teach that which is generally useful in Arithmetic, I shall not here enter into the intricacies thereof; especially as the scholar who may only occasionally have need for such things, will find it less troublesome, when the occasasion may occur, to make trial of the few odd numbers which remain, as 3, 7, 9, and 11; but as to 3, the sum which can be divided by 9, can, also, be divided by 3.

144. It belongs essentially to this matter, to state, that the *quotient*, on every division, is a *submultiple* of the *dividend;* that is, in all cases in which the division is complete without leaving a remainder. And, as, in these cases of even dividing, the divisor will, also, be a submultiple, so you have, at once, after *one* such operation, *two* of the submultiples of any sum: if you can then evenly divide the *quotient*, by *another* number, you then have *two* more submultiples; that is, in strictness, you have two more, unless your quotient then prove to be the same number as one of the submultiples you before discovered: for we count one number, in this case, however frequently it may occur, only as *one :* for instance, in stating thus, 2, 3, 4, 6, 9, 12, and 18, the submultiples of 36, we state each figure only once, although 6 times 6 may be employed to produce the number,

145. In the annexed you have a short example of the mode of proceeding to find the submultiples of 945, which are, you see, beginning, as is proper, with the smallest, 3, 5, 7, 9, 21, and 189.

```
5 ) 945
9 ) 189
 7 ) 21
      3
```

146. In paragraph 134, I have proposed, as an example of the mode of dividing with the submultiples of a divisor, that the operation should be

exhibited on these two numbers; that is to say, on 31957421, divided by 324. Let us, then, go through this operation.

147. On trying the numbers that will evenly divide this 324, we find that they are 2, 3, 4, 6, and 9, and the larger numbers 18, 36, 54, 81, and 162. Now, we would perform the work by Short Division, so let us take some of the smaller of these numbers; let them be the 9 and the 6, which, multiplied together, give us 54; and this again multiplied by the 6, gives us the whole divisor, 324. And then, as you see in the example here annexed, that on the division being performed with these numbers, 9, 6, and 6, the quotient is correctly brought out; namely, 98634, with a remainder of 5 when the division was made by 9. And, then, too, you see, that by taking these small divisors, inversely, and multiplying the quotient with them, back again, each line of the operation, also, comes back again; and that, when you come to multiply by the 9, and take in the remainder 5, which was left when you made the first division by that 9; when you do this, you find that you have the original dividend exactly restored, together with a proof of the correctness of the work, both in the Multiplication, and in the Division. And, it is further worthy of observation, that this process very pleasingly illustrates all that has been said of the mutual operations of these two rules of Multiplication and Division: showing very clearly, that they are the reverse of each other.

Divide by 9	) 31957421	
" by 6	) 3550824	-5
" by 6	) 591804	
	98634	
Multiply by	6	
	591804	
" by	6	
	3550824	
" by	9	
	31957421	

And here I have to observe, that in paragraph 134, wherein I stated, that the mode of using the submultiples of numbers is applicable to certain operations in Division only, I ought to have added, as the preceding illustration of the process fully shows, that the method is applicable, and is sometimes very useful, in the working of sums in Multiplication likewise.

148. In paragraph 136, you are reminded, that it is not all numbers, that can be *evenly* divided. Numbers of this kind, as well as those which may be evenly divided, have a name; and as these are termed multiples, so the *odd* and *indivisible* numbers are distinguished by the name PRIMES, or PRIMARY NUMBERS: that is, they are *primary, or original*, numbers, and not *producible* by the multiplication of other numbers.

149. Whilst I am on this subject, of the Terms by which various numbers are distinguished by mathematicians, I am tempted to make the list a little more complete, than, in the outset, I deemed requisite to my plan. To proceed a little further, then:

150. As the numbers, which, *multiplied* together, producing a certain sum, are called the FACTORS of that sum; so numbers *added* together, and thereby *composing* a sum, are called the COMPONENTS of that sum: and, as these numbers are called its components, so the sum itself, when required to be distinguished by a term, is called, A COMPOSITE NUMBER. So that the use of these Terms is, to distinguish numbers when employed for one purpose, from numbers employed for another purpose: and the use of the *numbers, themselves*, lies here; suppose you have to multiply, or to divide, a certain sum by several small numbers, as by these, 6, 9, 6; instead of making three operations, you

add the numbers together, and multiply, or divide by the sum of them, which is 21. And, if you have to speak of the manner of the operation, you say, that you multiply, or divide, by 21, the *composite* number of the *components* 6, 9, 6.

151. Of other terms, which I have not, before, found it necessary to notice, we have, in Subtraction, those of MINUEND, which describes the sum to be made less, by being subtracted from; and, SUBTRAHEND, which describes the lower, or sum to be subtracted. There is some sense in using the terms, multiplicand, multiplier, and product; in divisor, dividend, and quotient; because they are either good English words, or they are used to describe things, for which we have no short words of our own. But as to *minuend*, and *subtrahend*, we have the words, *greater*, and *lesser;* words of our own, fully and clearly conveying to our minds, without any waste of time in unnecessary study or recollection, all that these two foreign words are required, in ordinary calculations, to express. And, *greater sum*, and *lesser sum*, therefore, I would advise you, on such occasions, always to use; pitying the pedant who resorts to words in other languages to express his meaning, when that meaning may be quite as well expressed by those of his own.

152. So much for certain of the TERMS employed by arithmeticians; of several of which, were it not that a knowledge of them is required, in order to enable us to understand the writings of others, I should be inexcuseable in occupying a moment of the time of my pupils. There is, however, yet one other word, a most barbarous, and, at the same time, a most conceited word, of which I must speak, in order to enable those of my pupils who may not otherways have become acquainted with it, to understand any of the books, on this subject of Arith-

metic, that have already been written. This word is DIGITS: I pray you, my pupils, never to utter it, nor to write it! But, this being a part of the jargon by which the subject has been darkened by dulness and pedantry; as you will find the word, when you look into them, pretty plentifully sprinkled over the pages, I believe, of all former works of this sort, so I must bestow a little time in speaking of it.

153. The word comes from the Latin, *digitus;* that is, *finger*. Savages, who have no better mode, they say, *count* by their *fingers;* and in this manner, say the learned, our ancestors counted, before they had acquired a knowledge of numerical figures, to which figures it appears, that they, very naturally, and it must be allowed, very excusably in them, transferred the word. But is this any reason, I ask, for us to defile our language with the heterogeneous and barbarous term, even were they our own immediate ancestors, instead of the ancestors of the ancient Greeks, or Romans, who had first so applied it? That its use is not necessary, in order to describe either the meaning or the uses of numbers, has, I hope, been made to appear in these pages; through which I have used, when speaking of the *forms* by which we express numbers, the word *figure;* and, when speaking of the *numbers themselves*, I have used *the word itself;* to be sure. And this proper use of words, gives clearness, simplicity, and certainty to writing. Whilst the darkness, and confusion which have been spread over this subject of Arithmetic, must be the inevitable consequence of a casual and *senseless* use of the terms *number*, *digit*, and *figure*.

154. With regard to the *origin* of this word *digit*, which I would expunge from our books, for in our language it has never had a place; with regard to the *origin* of this word, I may be told,

that the word *calculate*, which I have used, has but a similar origin; this word coming from *calculi*, that is, *stones*, with which, also, it is said that savages reckoned up their accounts. But, to this I answer, that the word, whatever be its origin, is become perfectly naturalized amongst us. That this is as much our own, as any word we have; and that, whilst its use is quite familiar, its meaning is understood with perfect precision. But as to the word *digit*, which appears only in books on Mathematics, and appears there only to darken and to confuse; why use it instead of the words *figure* and *number*? For it is used, indiscriminately, for either; and the tantalized reader, in the midst of various other perplexities, has to discover in which of these two meanings he is to take it, on each particular occasion.

155. There are a few other Terms, which we shall have occasion to notice, and to use, hereafter, but with the exception of one, I think it will be best to explain them as they arise. This one is INTEGER, a good Latin word, unaltered, signifying *intire*, or *whole*. We sometimes speak of halves, quarters, and so on; which parts have another name, of which I shall have to treat in the next lesson, and this word, INTEGER, is used in contradistinction thereto, and signifies a whole, intire, and unbroken, number

Of certain SIGNS or Marks used in Arithmetic.

156. You have now learned the four fundamental rules of Arithmetic; you have learned to add, to subtract, to multiply, and to divide numbers; and the next thing I have to tell you is, that arithmeticians have adopted certain marks, or SIGNS, as they are called, by the use of which, they express, not only these four processes, but by which they also write down, with great neatness and brevity, in a straight line, like other writing, all manner of arithmetical propositions, describe arithmetical processes, and state the results; or, as we commonly call them, the answers: as, for example;

$$5 + 9 - 7 \times 8 \div 4 = 14$$

$+$ The first of these signs, is the mark of Addition, and is called PLUS; which is the Latin word for MORE.

$-$ The second, merely a short horizontal line, signifies Subtraction, is called MINUS, the Latin for LESS.

$\times$ The third is the sign of Multiplication, it is a small cross, but differs from that for Addition, being formed of two transverse sloping lines; it has not a name, bnt it signifies that the two numbers between which it is placed, are to be multiplied *into* each other, and it therefore, means "*into.*"

$\div$ The fourth signifies Division; it is, also, without a name, but it is used for the word "*by,*" meaning, when placed between two numbers, that the larger is to be divided by the smaller.

$=$ The fifth is the sign of *equality*, and is used instead of the words, "*equal to.*"

157. And thus, the short line of figures and signs stated above, is to be read thus: 5 *added to* 9, from which, *subtract* 7, then *multiply* what remains *into*, or by 8, *divide* the product by 4, and the result is 14.

158. Or, in a more scholar-like way, it is read thus;

5 plus 9 minus 7 into 8, by 4, is 14.

159. Thus, you see the uses, and the advantages of these SIGNS; and more is unnecessary.

OF FRACTIONS:

THEIR NATURE, MODE OF STATEMENT, &c. &c.

160. This word, too, like many other of the terms used in our art, comes from the Latin: and it is useful, thus, occasionally to attend to the derivation of a word, because it oftentimes, not only gives us a clear insight into its meaning, but, also, fixes that meaning permanently in the mind. FRACTION, comes from the Latin word *fractus*, that is, broken; or, in other words, a part. And, as it is used to describe a part, merely, of any thing that may be the subject of consideration, so, if we be speaking of certain weights, as three ounces, and a quarter; (that is, a quarter of an ounce,) this quarter, being but *a part*, is called a *fraction*: do we speak of of seven pounds and a quarter, then, this "quarter" is also a *fraction*: only observe, that, meaning, as it would, a quarter of a pound, so it would be called a fraction of a pound, whilst the other mean as fraction of an ounce.

161. Thus, PARTS of any thing, whether of weights, of measures, of money, or of periods of time; parts, whether large, or small; as halves, quarters, eighths, sixteenths; or, in short, any portion short of whole, is a fraction; and the treatment, or the working of these parts of numbers, is called the working of fractions; whilst, in order to distinguish them from these FRACTIONS, the numbers of which we have heretofore treated, are called INTEGERS, or WHOLE NUMBERS.

162. Now, as to the working of fractions, every body knows, that four quarters make a whole; that three thirds, that two halves, that five fifths, or six sixths; every body knows that each of these make a whole: and it requires very little more knowledge to enable us to say, that three halves make one and a half; that five quarters make one and a quarter; that seven quarters make one and three quarters; that nine quarters make two and a quarter; and so on. Well, but all these are examples, though very simple examples, certainly, of the ADDITION OF FRACTIONS.

163. To give a few examples of the SUBTRACTION of FRACTIONS: every one knows, that if we take one half from three quarters, that one quarter will remain; that if we take a half from five quarters, then three quarters remain; and it requires but very little more knowledge to say, that if one half be taken from five eighths, that, then, one eighth will remain; that a quarter from seven eighths, and five eighths remain.

164. As to the MULTIPLICATION OF FRACTIONS, every one knows, that six halves make three wholes; and, that twelve quarters come to the same thing; that fifteeen quarters make three and three quarters, and that seven thirds make two and a third.

165. For examples of Division; who does not know, that if we divide three quarters by two, we have, for each portion, three eighths; that, if we divide seven halves by two, that then we have, as the result, seven quarters, or one and three quarters; that nine tenths, divided by three, would give us three tenths, for each portion. And this is a simple exhibition of the DIVISION OF FRACTIONS.

166. To complete the view, I think it just now requisite to take on this subject of Fractions, attend to the mode of stating them, and to some other particulars respecting them, as here described.

167. FRACTIONS, as I have stated, are PARTS: and, as hath, likewise been stated, parts of every size, as, one half, one third, one fourth, three fourths, four fifths, seven eighths, or, in short, any other conceivable quantity, either small or large, as one thousandth part; or, as nine hundred and ninety-nine such parts, each of these quantities is a Fraction. Now, then, if you look attentively at the words in which the above fractions are described, you see that there are TWO TERMS employed in each of them; that is, *one*, and, *half*; then, *one*, and, *third*; then come, *three*, and *fourths*; and so on. And just so are these several quantities expressed in figures; only with this slight addition, that a small line is drawn between the two figures; and, with this further observation, that the two figures be written, not after one another, thus, 1-3, but that they be written smaller than your other figures, and the first of them over the other, thus, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{1000}$, $\frac{999}{1000}$. And, in this manner may every arithmetical fraction be expressed.

168. There being, however, no figure called *half*, *one half* is expressed thus, $\frac{1}{2}$; or, a half being two quarters, it may, very correctly, be thus expressed $\frac{2}{4}$.

169. Some of the other Fractions recited above are thus written, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{8}$. Now, for example, to observe on the last of these, that is to say, on the $\frac{7}{8}$. Each of the figures by which it is expressed fulfils a certain office, and has its appropriate name: a quantity of any kind, as a yard, a pound, or, an hour, may be divided into any number of parts: and, in this Fraction, it is stated to be divided into eight parts; and this is denoted or denominated, by the figure 8, which is, therefore, called the DENOMINATOR. Of eighths, then, it is, that this fraction speaks. But what number of these eighths, are they, that constitute the quantity described by this fraction? The other figure tells us this; the figure 7 gives us this number; and it is, therefore, called the NUMERATOR. And, so these are the terms, these are the uses of the two figures employed to describe every fraction; that is to say, NUMERATOR, which describes *the number* of parts, and, DENOMINATOR, which denotes *the quantity* of those parts.

170. Another point to be observed here, is, that Arithmetical Fractions, speaking, as they do, of parts of any thing, are always understood to be speaking of EQUAL PARTS. But a thing may be divided into UNEQUAL and irregular parts; as, for instance, a pound weight may be broken into eight parts, each different from all the rest in quantity. And, if a person were to speak of three, five, or seven, parts of this sort, we should have no certain, no clear comprehension of the quantity he might mean; and certainty, and clearness, are the very life and soul of Arithmetic. This clearness and certainty is attained, by its being always intended, and always understood, that the parts spoken of in any fraction, are all equal, one with another.

171. As I have stated, a quantity may be divided into any conceivable number of parts; and it is the

proper office of a Fraction, to speak of some quantity less than the whole, even though it be but one thousandth, or, one millionth part less. However, it is, sometimes convenient to speak of things, and even to write them down otherwise; as, instead of saying, a penny halfpenny, we say three halfpennies. Drapers talk of their cloths being four or five quarters wide, of blankets, and counterpanes, and table cloths, being twelve, fourteen, fifteen, &c. quarters square; meaning, by these, quarters of the yard. And this mode of speaking and of writing may be useful in other affairs. Now, although we do not write down three half-pence thus, $\frac{3}{2}$, but do it thus, $1\frac{1}{2}$d., that is, one penny and a half, yet the Drapers write the quantities I have spoken of thus, $\frac{4}{4}$, $\frac{5}{4}$, $\frac{12}{4}$, $\frac{14}{4}$, $\frac{15}{4}$. And this, as stated above, is more convenient to them, being, not only conformable to their mode of speaking, but also shorter than writing, as they otherwise would have to do, in the two latter cases, $3\frac{1}{2}$ yds. $3\frac{3}{4}$ yds.

172. On looking at the Fractions written down in the last paragraph; on looking at $\frac{4}{4}$, $\frac{5}{4}$, $\frac{12}{4}$, $\frac{14}{4}$, $\frac{15}{4}$, you again observe, that these express something *more* than the part, and all of them except the first, something more than a whole, although, in the outset of the same paragraph, I have stated, that it is the proper office of a Fraction to speak of some quantity *less* than a whole. And so it is. And, in conformity therewith, these, only, are called PROPER FRACTIONS, and the others are called IMPROPER FRACTIONS; that is to say, proper Fractions are those which describe less than a whole; and in which, therefore, the numerator is less than the denominator: whilst improper Fractions are those which describe as much as, or more than, a whole; and in which, therefore, the numerator is equal to, or greater than the denominator.

173. One other particular remains to be observed on, before we dismiss this article of Fractions; and this is, to impress on your mind, the propriety of confining yourself to the use of the smallest figures, or terms, by which you can express the quantity you speak of. Three-sixths, for instance, is one half; six-eighths, are merely three quarters, as are nine-twelfths, and fifteen-twentieths. Do you, therefore, always, in such cases, say, and write down, $\frac{1}{2}$, and $\frac{3}{4}$, and leave it to those who know no better, to talk about, and to write, $\frac{3}{6}$, $\frac{6}{8}$, $\frac{9}{12}$, $\frac{15}{20}$; leave this mode of speaking and of writing, to the silly people who talk of *three parts*, and so on, without telling us what is the quantity of the *parts* about which they are babbling.

OF DECIMALS:

THEIR NATURE, USES, AND MODE OF STATEMENT.

174. These, also, are Fractions, or parts of whole numbers. And the difference between these and the fractions treated of in the foregoing lesson, consists in this; that, whereas those express parts, as halves, thirds, fourths, fifths, and so on, in *figures* closely conformable to the *words* by which we describe the same parts, these, Decimals, always describe the parts of which they treat, as *tenths*, *hundredths*, *thousandths*, and so on; that is, indeed, as TENTHS: and, hence their name, which comes from the latin *decem*, ten; or from *decimare*, to divide, or to separate into tenths. Again; the Fractions treated of before, being noted down in figures so conformable to the words by which they are ordinarily described, and, being, therefore, more readily applicable to ordinary affairs, they, when we would distinguish them from the fractions of which we are now to treat, are called, VULGAR FRACTIONS; whilst these, for the purpose of distinction, are called DECIMAL FRACTIONS.

175. Besides Decimal Fractions, there are DUODECIMALS; named, also, from a latin word, *duodecem*, that is, *twelve*. And these, which describe, and enable us to work *twelfths*, are admirably adapted for the calculation of all manner of articles or things that are measured by our scale of

feet and inches, as timber and stone; bricklayers, plumbers, painters, &c. work; and, accordingly, for such purposes are Duodecimals used Of these I propose to treat, under the name of TWELFTHS, at the end of the lessons on Compound Arithmetic.

176. These Decimal Fractions; or, as we will now call them, Decimals, admit, in most cases, of a much simpler and readier mode of statement, and of a much simpler and readier mode of being worked, than do the vulgar fractions; and they are, in such cases, on these accounts, to be preferred.

177. The purposes, moreover, for which they are more particularly useful, are those more nice, and extensive processes of calculation, to which the astronomer, the geographer, the engineer, the surveyor, and the chemist, have frequent occasion to resort. The notice I shall, in this work, think proper to take of Decimals, will serve to prepare persons for any of these professions; will serve the man of business, who may occasionally employ them in his reckonings; and will be further useful, as a means of explaining some few points, on other matters, of which I shall hereafter treat.

178. With regard to the greater simplicity of statement, of which I have spoken above, it consists in this, that, whilst, in the notation of vulgar fractions, we have to write down two lines of figures, the one descriptive of a numerator, and the other of a denominator, in the notation of a decimal it is sufficient to write down the numerator only, the *parts* enumerated thereby, being, as I have said, always *tenths*, or *ten tenths*, or *ten times ten tenths*, or so on. In short, the denominator of a decimal being always *understood*, it is unnecessary to write it down; the value being expressed by the numerator alone.

179. There is yet, however, to be noticed, something in the situation, and in the manner of writing down the figures expressive of a decimal; and I go into the notice of these particulars with great pleasure, because they serve to exemplify still further, the nice order, and the great beauty of the science of numbers.

180. In the notation of whole numbers, as the learner knows, the figures placed next to the right hand represent units, those next before them representing tens; and, so proceeding, they rise in value, ten times in every step; and, now, mark the neatness and beauty with which PARTS of these numbers are expressed by decimals. Take a figure, or line of figures descriptive of whole numbers, make a point, a full stop, after the figure which occupies the units place, and, after this point, that is, lower down even than units, in the scale of places, write down the numerator of your fraction; that is to say, so write it down if it be a decimal; which is a fraction, the denominator of which is ten, or some multiple of ten. And the numerator so written below the units place, however large and numerous the figures of which it may be composed, represents something less than a whole; it represents merely a fraction; and it represents this decimally.

181. For example. Let the number, with its fraction expressed in the ordinary mode, be this, 2517 $\frac{7}{10}$; the same is expressed thus, 2517·7; the vulgar fraction being stated as a decimal. Were the fraction $\frac{17}{100}$, it would be thus, 2517·17. Were it $\frac{176}{1000}$, it would stand thus, 2517·176. And as the value of any decimal figure is determined by its approach to, or its distance from, the place of units, so cyphers placed after the units, and in the higher places of decimals, and thereby driving the significant figures lower down; cyphers so placed diminish

the value of such significant figures. As, to recur, for instance, to the first of the foregoing examples, had the decimal .7 stood thus .07, instead of *seven tenths*, $\frac{7}{10}$, it would have been only *seven hundredths*, $\frac{7}{100}$. And the second example, instead of being *seventeen hundredths*, would, if written thus, with a cypher before it, .017, be reduced to only *seventeen thousandths*, $\frac{17}{1000}$; for so these decimals are called: and, when written as vulgar fractions, it is done as above. However, and do you be careful to mark it, important as the cypher is, in decimals, when placed *above* any of the significant figures, it would be wholly unimportant were it to occur at the end of, or *below* such figures. Between such figures it fulfils its office, and has its effects, as .107, which is $\frac{107}{1000}$; but below them, thus, 170, it would be a mere incumbrance, and ought not to be suffered to remain.

182. As you may thus turn certain vulgar fractions into decimals, by simply discarding the denominator, and writing the numerator after, or below, the units' place, so the contrary change is to be effected, and decimals may be turned into vulgar fractions, by drawing a line underneath the figures thereof, and under this, writing the requisite denominator; which denominator, as before stated, must always be 10, or some multiple of 10; and, as to which of these it is to be, that is determined as I am now about to describe.

183. A fraction, you know, is a part; and, properly speaking, a fraction is always but a part; that is, something short of a whole. To describe such a part, you were shown, in paragraph 172, that the denominator must always be something more than the numerator. Now, on converting a decimal into a vulgar fraction, you have to affix a suitable denominator thereto, and, if you be confined, as you

must, in this case, to the use of *ten*, or of some *multiple of ten*, you affix the proper denominator at once, by writing an unit, followed by just as many cyphers as there are figures in the numerator. This you see done in the foregoing examples, in which the decimal .7, is expressed $\frac{7}{10}$; .07, thus, $\frac{7}{100}$; the decimal .17, thus, $\frac{17}{100}$; and .107 and .176, thus, $\frac{107}{1000}$, and $\frac{176}{1000}$. And THIS IS THE RULE to be followed in affixing the denominator to a decimal; that is to say, you annex to an unit the same number of cyphers as there are figures in the decimal.

184. And, now, as to the value of a decimal. Turning to paragraph 180, you are reminded, that in the notation of whole numbers, at each step which any figure is advanced *above* the unit's place, its value becomes increased tenfold, and, in neat accordance with this principle, do figures, as they take their stations *below* this place, decrease at each step, to just one tenth of their former value. Hence it is that .7 is seven *tenths;* .17 are seventeen *hundredths;* .107 a hundred and seven *thousandths*, and so on: the value of each number being only one tenth of what it would be, were the figures describing it placed in the unit's place.

185. In paragraph 174, having reminded you, that vulgar fractions describe all manner of parts in figures conformable to the words by which they are ordinarily described, I proceeded to state, that decimals always describe such parts, as tenths, and so on. And thus it is done; thus are halves, quarters, thirds, and, in short, all manner of parts described by decimals. *Five* being the half of *ten*, the figure 5 written after what is termed the decimal point, thus, .5, expresses a half; for written as a vulgar fraction, it stands thus, $\frac{5}{10}$. A quarter of *ten*, which is two and a half, we cannot express in figures, otherwise than thus, $2\frac{1}{2}$, which will not serve for a decimal,

but a quarter of a hundred, that is, 25, being a whole number, will serve the purpose; so, be it quarter of what it may, a quarter, in decimals, is thus expressed, .25; that is, twenty-five hundredths. Three quarters, of course, then, are to be expressed .75: and, if you describe these as vulgar fractions, they stand thus, $\frac{25}{100}$, $\frac{75}{100}$. And here, be it observed, is shown THE REASON OF THE RULE laid down towards the close of paragraph 183; which rule is this, that on changing a decimal into a vulgar fraction, the denominator shall be an unit, followed by just as many cyphers as there are figures in the decimal.

186. Halves, and quarters, as expressed by decimals, we have thus described; and, now for other parts. A tenth, is, of course, expressed thus, .1; a fifth, thus, .2; the first of these being a tenth, and the other a fifth part of ten. A twentieth, thus, .05; a fiftieth, thus, .02: for, of these, the first is the twentieth, and the latter, the fiftieth part of a hundred.

187. But, how may we express a third, a sixth, a seventh, an eighth, or a ninth? These are questions of moment. The fact is, that as neither 10, nor any of its multiples, can be evenly and completely divided by 3, 6, 7, or 9, so neither can these parts be exactly expressed by decimal figures. However, though they cannot be expressed with *perfect exactness*, we can come within a thousandth part, a millionth, or, in short, we can make as near an approach to the exact quantity as we may please to make. And thus it is done. Suppose we would have the decimal for $\frac{1}{3}$, we take the 1, that is, the numerator of the fraction, and annexing thereto, one or more cyphers, as, 10, 100, 1000, 10000, then divide by the denominator 3, and the nearer we would come to the exact third, the more cyphers do we annex. Now, you know, that when you

divide 10 by 3, you have a remainder of one; thus, you come at once to the third, within one tenth; add another cypher, making the numerator 100, and divide again, and again have you one left; but this one is only a hundredth; another cypher would bring you within a thousandth part of the perfect third; and thus may you make as near an approach as you please, to the quantity sought. Now let it be that we would come within a ten thousandth part: annexing four cyphers to the numerator, and dividing by the 3, it would stand thus,
and the quotient, consisting of four threes, is the decimal expression of a third: but, indeed, one, or two, or three threes, will generally prove sufficient for the purpose.

$$3 \,)\, \underline{10000} \quad .3333$$

The decimal for a ninth, is a line of units; that for a sixth, is an unit followed by one or more of the figures 6; a seventh, if you would come within a millionth part, you will find to be .142857.

188. Now the decimals of which I have thus far spoken, are such as, when expressed as vulgar fractions, have for their numerator an unit merely; and those spoken of in the last paragraph, as a third, a sixth, a seventh, and a ninth, are likewise of that description of decimals, that, divide the numerator, with its cyphers, as often as you please, the same figures keep recurring. To go on dividing in this manner, coming again, and again to the same point, is like travelling in a circle, and, from this circumstance, it is, that decimals of this description are called, CIRCULATING, or, RECURRING decimals: but of distinctions of this sort, it is not now necessary to speak further.

189. With regard, however, to Fractions, the numerators of which are other than units, you arrive at the decimal expression of them by just the same process as that which I have described in

paragraph 187; that is, you take the numerator of the fraction to be turned into a decimal, for a dividend, and, adding thereto as many cyphers as you choose, you divide by the denominator, and the quotient will be the decimal. As, for example: Let the decimal expressive of $\frac{3}{8}$ be required. It is done thus.

$$\begin{array}{r|l} 8 & 3000 \\ \hline & .375 \end{array}$$

Is it required to find the decimal of $\frac{5}{9}$? Thus it is done.

$$\begin{array}{r|l} 9 & 5000 \\ \hline & .555 \end{array}$$

Again: suppose we would have the decimal expressive of $\frac{7}{18}$. Instead of doing this by Long Divison, the better mode will be to divide by some multiples of the denominator 18; and this, in such cases, where it can be done, will always be the better mode. Let us divide, then, by the multiples 6 and 3; and it will stand thus.

$$\begin{array}{r|l} 6 & 70000 \\ \hline 3 & 11666 \\ \hline & .3888 \end{array}$$

And here it may just be worth our observation, that whilst, in the first of these three examples, we come, after three divisions only, to the perfect decimal expression sought for, there being no remainder after those three divisions, the latter eludes our search, by presenting a continued repetition of the same remaiuder. But this is a matter of little, or of no practical importance: we have already arrived at something less than one eight hundredth part of the exact quantity, and were it the fraction of a hundred weight that we were thus calculating, we are within about two ounces of the precise quantity, and, were not this near enough, we come within a dram and a half by only one division more.

190. One other particular, relating to decimals, remains here to be noticed; and this, whilst it will yet further exemplify their nature, shows the ready usefulness with which they present themselves, on certain occasions.

191. In paragraph 101, it is shown, that when we

have to divide a sum by 10, by 100, or, in short, by any multiple of 10, that the operation is at once effected by a simple cutting off, on the right hand of the dividend, of just so many figures as there are cyphers in the divisor; and the sum so divided, as an example, is 628513, from which, when you divide by 10, you cut off the 3; when by 100, you cut off the 13; and, had you to divide by 1000, you would have to cut off the three last figures, thus, 628·513. Now, observe; these figures, so cut off, are decimals; we are dividing by a thousand, we find in the dividend 628 thousand, and a remainder of 513; that is to say, a five hundred and thirteenth part of a thousand, and, written as a vulgar fraction, this remainder would stand thus, $\frac{513}{1000}$.

192. Decimals, of course, like other numbers, may be added together, subtracted from each other, multiplied, and divided; and it is the great ease and readiness with which, by the use of these numbers, parts of all sorts can be reckoned, that constitutes the great value of decimals. A description of these processes, sufficient now for my pupils, will be but a light matter, but as it would rather interupt the course I have marked out for my lessons, I shall, except incidentally, and as occasion may arise, defer the further treatment of these, and of vulgar fractions, likewise, until the more suitable opportunity, which will present itself towards the conclusion of the work.

OF

COMPOUND ARITHMETIC;

193. Or, of the working of figures employed to express quantities of different denominations, as, bushels, pecks, and gallons; hundred weights, quarters, and pounds; leagues, miles, and furlongs; yards, feet, and inches; years, months, and days; for describing things of different values, too; that is to say, money of various sorts, as pounds, shillings, pence, and farthings. To work figures descriptive of the several portions into which time is divided; and, of all the various quantities used in measuring and weighing; and, figures descriptive of sums of money, of all the various sorts used throughout the world: to work figures employed in these most useful purposes, is the next object of your attention. And, as the quantities and values are so various, and, as several of them are occasionally joined together in one sum, or *compounded*, so, the treatment, or the working of figures thus employed, is called Compound Arithmetic.

194. Let not the young student, however, be alarmed, because of the extent, or the apparent intricacy of this branch of our study. He has, I trust, now found it easy to express and to manage sums of the largest amount, by making himself acquainted with the principles on which they are stated, and by which they are worked; and, by a similar attention to a very simple principle or two, he will, I also trust, see every difficulty vanish here; and find it easy to work figures, however new and strange to him, the several quantities and values they may be employed to express.

195. Throughout his progress hitherto, the learner has experienced the advantages of an acquaintance with the principles of notation; nay, he must have felt, not only the advantages, but the necessity of an acquaintance with these principles.

196. To enter with advantage on this new course, it will be necessary, also, to attend to the sort of Notation which is used in Compound Arithmetic; and, indeed, this is all that the scholar has now to learn, in order to perform every operation in this new and essential branch of the art.

197. In the Notation of which we have before treated, the student knows, that every advance in the value, or in the station of figures, is by tens; that is to say, that in that Notation, we count from one to nine, and, that the next higher number is expressed by one with a cypher after it: that, in short, we therein count by a sort of *tallies* of TEN; and that, as soon as we have completed a tally, we write it down by the figure 10; that two of these tallies are thus written, 20; that three of these are thus noted down, 30, and so on, until we come to ten tallies, when, again, as before we did, on reaching ten, we begin with ten, only that we annex another cypher to it; that we then, again, renew our counting until we reach another ten, and so on, to millions, &c. In short, in the former part of Arithmetic, in that which we call *Simple* Arithmetic, in contradistinction to this, which we call *Compound*, we count as the common porters employed by merchants count, that is, by tallies of ten; and, that, indeed, on this principle, is all Numeration, all Notation, and all calculation carried on in simple, or, as the more learned call it, in Abstract Arithmetic.

198. But, it is in Simple, or Abstract Arithmetic

only, that this mode of counting is used; or, rather, it is, when this mode can be used, that we are entitled to call it Simple, or Abstract. For instance, when we come to the compound, as the counting over of our money. Say that we begin with farthings; as soon as we have four, *(only four, mind, and not ten)* we come to one; that is, one penny; then we count on to the next sort or denomination of our money, which is twelve pennies, and we call it one, that is, one shilling. Then, again, we count on to twenty, when it becomes one again, that is, one pound. So that, in the notation of our money, instead of counting on to ten, and then changing, we count first to 4, then to 12, and then to 20. Have we to count by our measure, of inches, feet, and yards, we count first, 12 inches make one foot, then, 3 feet make one yard. Have we to count by our ordinary weights, we count 16, 28, and 4; that is to say, 16 ounces make one pound, 28 pounds make one quarter of a hundred weight, and 4 quarters make one hundred weight.

199. In the United States of America, a nation founded after the experience of the old world had taught men, how great must be the convenience of keeping their accounts in a sort of money, the lower denomination of which should be a perfect decimal part of the higher: in those States, profiting by experience, they adopted money of this description.

200. The money they adopted, was the dollar of Spain, already current over the continent of America, north and south, and worth about four shillings and two pence of our money. The smaller denomination which they choose, was a hundredth part of this dollar; to which they gave the name of cent., a contraction of the latin word, *centum*, a hundred. And, following the example of the United States, France, with various other coins in its circulation,

divided its livre, or franc, into centimes, that is, hundredths, and thus reckons its money accounts in the same manner.

201. The people of the United States have all sorts of money in circulation amongst them, but in these dollars, and cents, only, do they keep their accounts. And, with money of these denominations; with money, the higher denomination of which rises exactly to ten-times, or to ten-times-ten, the value of the lower, we may at all times make our calculations by Simple Arithmetic only; having, as in the case of the money in question, nothing to do, in adding, or in subtracting, but to place the figures representing dollars, right under each other, and after the point, or line of separation, those representing cents, in like manner; and then to proceed as I have stated. But, indeed, with money of this very convenient sort, we may, if we please, do without the point, or line of separation, the only distinction which its use makes, being in the name, and not in the value expressed by any line of figures; as, for example, 730841 cents are of the same value as 7308 dollars, and 41 cents; so that the insertion, or the omission of the separating point, merely changes the denomination of certain of the figures; a very simple and convenient mode of bringing cents into their equivalent value in dollars, or the contrary: but a mode, as you see, which can be employed only in the accounts of money, each higher denomination of which, as I have stated, rises in value above the lower, as do these which have been adopted in the United States, and in France.

202. Thus, you see, that each sort of measures, each sort of weights, and each sort of money, has its peculiar notation. And this peculiarity requires to be known, and kept in mind, when we work figures descriptive of any of these several quantities.

As, for instance; have I to add together three or four sums of our money, each sum containing pounds, shillings, pence, and farthings. I must, as you will suppose, begin with the farthings; and, counting how many of them there are, and finding that there are 9, I do not, of course write down $\frac{9}{4}$; but, knowing that nine farthings make twopence farthing, I write down $\frac{1}{4}$, and carry 2 to the pence column: let us, then, suppose, that, on being added up, the pence amount to twenty-six. I do not here write down 26; but, knowing that 26 pence are two shillings and two-pence, I set down the two-pence, and add 2 to the shillings. Proceeding with my addition, and finding that the shillings amount to 45, which I know make two pounds five shillings, I set down the 5 shillings, and carry the two to the pounds, which being the highest denomination of our money, and being, therefore, changeable into nothing higher, I have done with the peculiar mode of notation, and add up, as I would add in Simple Addition; that is to say, I resort to the system of carrying the tens, and write down the amounts accordingly.

203. Let us here state the example, in figures, which is described above. But, before I do so, I must apprise you, that we divide, or separate, by means of a dot or two, the figures descriptive of the different sorts of money, as you here see it done: that, in order to show that certain figures are employed to describe our pounds in money, or, as they are otherwise called, pounds sterling, we write, before, or over the figures, the capital letter £, with two short strokes across it; that over figures descriptive of shillings, we write a small *s;* and over those for pence, we write a small *d.* The example will then stand thus:

£		s.		d
160	..	7	..	7 ¾
2525	..	15	..	5 ¼
57	..	6	..	3
228	..	8	..	4 ½
3163	..	7	..	5 ¾
£ 6135	..	5	..	2 ¼

204. A word as to the mode of writing down the smaller sort of our money, that is, halfpennies, and farthings: to these, although real coins, a separate column is not assigned, but they are described, as you see above, merely as the fractions of the penny.

205. This description and example of the notation of our money, contains, and illustrates the principle on which the notation of all money, of whatever country, of all measures, and of all weights is conducted.

206. The PRINCIPLE is this, and, having once made it your own, you are ready and able to calculate every description of money, weight, and measure. You recollect that fractions are parts of whole numbers, and you recollect, *(see paragraph* 172*)* that, when you have parts enough to make a whole, that it is IMPROPER to write them down, or to speak of them as fractions, and that you must, as far as you can, carry them on in whole numbers, and state the odd parts, only, as fractions. Now, observe THE PRINCIPLE, in an instance from our own money. The POUND STERLING is the WHOLE, the SHILLINGS are regarded merely as PARTS OF IT, that is, *twentieths*, whilst the PENNIES, as you know, are *twelfths* of the shilling. Hence it is, that when you have twelve pennies, you call them a shilling; and, when you have reckoned up twenty shillings,

you call them a pound: but, the pound, being THE WHOLE, you count pounds, and reckon them, as I have just stated, and shown in the example, as you learned to treat whole numbers in Simple Arithmetic: and, thus it is to be in all your reckonings; that is to say, when you deal with the highest denomination, whether of money, of weights, or of measures, you treat it, as you would treat the same sum in Simple Arithmetic.

207. Whenever, therefore, you have to state, or to reckon, any sort of money, weights, or measures, the principle on which you have to proceed, is this; to ascertain what is the whole number, and what are the parts, into which the money, the weights, or the measures, of which you have to treat, have been divided: and then you know how to proceed. You have seen how to proceed in the notation, and addition of our money. Take an example of the treatment of our larger WEIGHTS.

208. The TON is the largest weight to which we reckon; it is, therefore, the whole number, and it is broken down into HUNDRED WEIGHTS, twenty of which make one ton. The hundred weight, being composed of a hundred and twelve pounds, a sum inconveniently large, it is broken down into QUARTERS; these are, of course, twenty-eight pounds each; so that the numeration of our larger weights is, 28 pounds make one quarter, 4 quarters make one hundred weight, and 20 hundred weights make one ton. And the marks used to describe these several weights, are, *ton, cwt. qr. lb.*

209. Now, if we have to add together two lines of figures descriptive of these weights, we proceed as you see in the following example; that is, adding the pounds together, we set down the odd ones, and carry forward the 28, as one quarter; the quarters,

except the odd one, we carry forward to the hundred weights, and, every twenty of these, we, in like manner, carry forward to the tons.

Ton.	*Cwt.*	*Qr.*	*lb.*
28	. 15	. 1	. 19
56	. 7	. 3	. 12
85	. 3	. 1	. 3

210. The above, as I have stated, is an example of the notation and addition of our larger weights. They are those by which heavy goods, such as provisions, groceries, and hardware, are weighed; and they are called, Avoirdupoise weights. Besides these, there are the smaller weights, as, Apothecaries, by which medicines are compounded; and Troy weights, used by goldsmiths and others, in weighing the precious metals, jewels, &c. Now these several sorts of weights are differently divided, and have names varying from each other.

211. Then, there are the measures. There are measures for dry goods, and others for liquors; and then, again, measures for land, others for timber, stone, &c., and others for cloth, each differing, in some respects, from the rest.

212. But, be not startled at the multiplicity of these things; for, besides that any one of you, my pupils, will, in all probability, have occasion, for the purposes of his business, to attend only to one or two of these various sorts of weights and measures; besides this, you will recollect, that you are furnished with *the principle* on which they are all to be treated and managed. You have seen, that all you have to do, on proceeding to work figures descriptive of any of them, is to know, first, what is its whole number, or, as it is commonly

called, its largest denomination, and then to know, how that is divided. These things are all stated in certain tables, which, as far as they relate to the weights, measures, &c. used in this country, I here insert, and after these I must subjoin a few sums and questions, in the different rules, for your practise, preceding some of them by an example or two.

TABLES

OF MONEY, WEIGHTS, AND MEASURES,

Used in England; with the marks by which each of them is commonly distinguished.

CURRENT MONEY.

4 Farthings make 1 Penny
12 Pence .... 1 Shilling
20 Shillings, 1 Pound or Sovereign

PENCE.				SHILLINGS.			
d.		*s.*	*d.*	*s.*		*£.*	*s.*
20	are	1	8	20	are	1	0
24	..	2	0	30	..	1	10
30	..	2	6	40	..	2	0
36	..	3	0	50	..	2	10
40	..	3	4	60	..	3	0
48	..	4	0	70	..	3	10
50	..	4	2	80	..	4	0
60	..	5	0	90	..	4	10
70	..	5	10	100	..	5	0
72	..	6	0	110	..	5	10
80	..	6	8	120	..	6	0
84	..	7	0	130	..	6	10
90	..	7	6	140	..	7	0
96	..	8	0	150	..	7	10
100	..	8	4	160	..	8	0
108	..	9	0	170	..	8	10
120	..	10	0	180	..	9	0

AVOIRDUPOISE WEIGHT.

16 drams	make	1 ounce
16 ounces	....	1 pound
14 pounds	....	1 stone
28 pounds	....	1 quarter
4 quarters	....	1 hundred wt.
20 hun. wt.	....	1 ton

dr. drachm	qr. quarter of a
oz. ounce	cwt. hundred weight
lb pound	T. ton

TROY WEIGHT.

24 grains	make	1 penny wt.
20 penny wts.	....	1 ounce
12 ounces	....	1 pound

gr. grain, dwt. penny weight, oz. ounce

APOTHECARIES' WEIGHT.

20 grains	make	1 scruple
3 scruples	....	1 dram
8 drams	....	1 ounce
12 ounces	....	1 pound

gr. grain	ʒ dram
℈ scruple	℥ ounce

The other terms, throughout the Tables, are expressed by contractions, merely; as, *in.* for inches, *ft.* for foot, *yd.* for yard, *pt.* for pint, *qt.* for quart, &c. &c.

WOOL WEIGHT.

7 pounds	make	1 clove
2 cloves	....	1 stone
2 stones	....	1 tod
6½ tods	....	1 wey
2 weys	....	1 sack
12 sacks	....	1 last

ALE AND BEER

2 pints	make	1 quart
4 quarts	....	1 gallon
9 gallons	....	1 firkin
2 firkins	....	1 kilderkin
2 kilderkins	....	1 barrel
1½ barrel	....	1 hogshead
2 barrels	....	1 puncheon
3 barrels	....	1 butt

WINE MEASURE

2 pints	make	1 quart
4 quarts	....	1 gallon
10 gallons	....	1 anker
63 gallons	....	1 hogshead
2 hogsheads	....	1 pipe
2 pipes	....	1 tun
42 gallons	....	1 tierce

DRY MEASURE

2 gallons	make	1 peck
4 pecks	....	1 bushel
4 bushels	....	1 sack
8 bushels	....	1 quarter
4 quarters	....	1 chaldron
10 quarters	....	1 last

SOLID MEASURE

1728 inches	....	1 solid foot
27 feet	....	1 yard

LONG MEASURE.

3 barley corns	make	1 inch
12 inches	....	1 foot
3 ft. or 36 inches	....	1 yard
6 feet	....	1 fathom
5½ yards	....	1 pole
40 poles	....	1 furlong
8 furlongs	....	1 mile
3 miles	....	1 league
69½ miles	....	1 degree

LAND MEASURE.

9 feet	make	1 yard
30¼ yards	....	1 pole
40 poles	....	1 rood
4 roods	....	1 acre

COAL MEASURE

3 bushels	....	1 sack
36 bushels	....	1 chaldron

CLOTH MEASURE.

2¼ inches	make	1 nail
4 nails	....	1 quarter
4 quarters	....	1 yard
3 quarters	....	1 Flemish ell
5 quarters	....	1 English ell
6 quarters	....	1 French ell

TIME.

60 seconds	....	1 minute
60 minutes	....	1 hour
24 hours	....	1 day
7 days	....	1 week
4 weeks	....	1 month

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213. The foregoing Tables are all that I deem it advisable for my pupils to attend to, at present; containing, as they do, the regular denominations of money, of weights, and of measures, in which accounts are generally kept throughout this kingdom. Some few other denominations are in use in particular districts, but as a man soon learns them,

when he comes to have occasion for them, it would be a mere waste of time and of attention, to regard them here.

214. Entertaining a great aversion to parrot-work, believing that few things tend more to check the natural growth of the mind, than the practice of getting off things by rote, as it is called, and persuaded, that when the reason is informed, and the memory kept clear from incumbrance, the things learned will rise with facility to the recollection, when they are needed, and that it is well to dismiss them when they have served their office, persuaded of these things, I have urged on the learner only one little task of this sort, and that task, the learning of the multiplication table, I have spoken of as the only one which the practice of arithmetic absolutely demands.

215. More, indeed, in this way, is not absolutely demanded from the learner, but it would greatly facilitate his progress in learning, and his practice afterwards, were he to, learn also, the Pence Table; and, therefore, I must recommend the committing of that to memory. As to the other Tables, they are here for reference, and any of them that may be useful in our respective avocations, we soon learn, in the easy and proper manner, that is, by practice; and, when they are not useful, the mind is clearer without them: but the Pence Table, being continually useful to every one, ought to be in the memory. However, if the learner choose to decline the task, he must do as to this table, as I must advise him to do as to the others; that is, whenever he have summed up a number of pence, greater than he is accustomed to reduce into shillings, at a word, he should write them down, and divide them by twelve: of shillings, too, when he has more than he can, with a certainty of doing it correctly,

resolve into pounds, these he will, of course, write down, and divide by twenty: and, in this safe manner, he will do wisely to treat all considerable numbers that he may have to deal with.

216. Now, quantities, of these various denominations, may be added together, subtracted from each other; may be multiplied, and, divided: that is to say, you may add together various sums of money; you may add various quantities in weight, and various quantities in measure; only, mind, that those sums, or quantities, that you put together, must be of the same description: you must not, for instance, seek to add together various quantities in weights of different descriptions, as a quantity of things stated in the Troy, to a parcel stated in the Avoirdupoise, or in the Apothecaries' weight; nor a quantity taken in liquor measure, to a quantity taken in dry measure; neither must you think of adding a parcel of weights to a parcel of money, nor to do any incongruous, or nonsensical thing of this sort. No; the quantities, be they weights, or be they measures, must be of a similar description, or the joining of them together would, as, on a moment's reflection you will perceive, produce nothing but irregularity and confusion.

217. As it is, in these matters, with addition, so is it with subtraction. You may subtract a smaller sum of pounds, shillings, and pence, from a larger, but it is scarcely necessary to observe, that you cannot subtract pounds, shillings, and pence, from bushels, pecks, and gallons, nor either of these from hundred weights, quarters, and pounds; nor can you do any thing of such a discordant nature. In short, when you add quantities together, or subtract them from each other, those quantities must be of the same family, as it were; they must belong to the same Table: or, if you have to join together, or

to deduct one from another, as you sometimes may have, quantities of things of the same nature, but weighed, or measured, or valued, by different sorts of weights or measures; as, for example, you may have to ascertain the united weight of two or more quantities of goods, some of them being weighed by the avoirdupoise, and others by the apothecaries' weights; or, which not unfrequently occurs, you may have to put into one sum, quantities of money of the sorts used in different nations, and, therefore, differing in denominations, and value. In cases of this kind, there is nothing irregular or incongruous; the things to be joined or separated, are of the same nature, only measured by different sorts of measures; and all that is to be done, previously to their being so joined or separated, is, that the quantities, be they what they may, be brought under the same Table, be stated in similar quantities, or, as Arithmeticians have it, be reduced to like quantities and denominations. To accomplish which purpose, we have a rule, called Reduction, of which I have to treat hereafter.

218. As to Compound Multiplication, and Compound Division, they consist, not in multiplying, nor in dividing, a parcel of money, by a parcel of money; nor a quantity in weight, or in measure, by a quantity in weight or in measure, but in multiplying or in dividing any of these parcels or quantities by a number, in order to discover, how much any such parcel or quantity may be, when multiplied two-fold, four-fold, a hundred-fold, or, in short, any given number of times; or to discover, how much such quantity yields when divided by any given number. So we have Compound Addition, and Subtraction, then comes Reduction, and afterwards, Compound Multiplication, and Division.

OF

COMPOUND ADDITION;

Or, Addition of Money, Weights, and Measures.

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219. This branch of our subject, after what we have learned, ought to be a very simple and easy matter. Calling to mind what is said in the last lesson, paragraphs 196 to 210, on the notation of compound numbers, and especially to what is said in paragraph 207, on the necessity of ascertaining, first of all, the manner in which the money, the weights, or the measures, which you have to calculate, are divided; calling these things to mind, and, if you yet feel any uncertainty on the subject, clearing it up by reading over those paragraphs, beginning with that numbered 196; doing this, you will require no further instructions to enable you to work correctly the few following sums; or, indeed, any other, coming under this rule, which may elsewhere present themselves.

(1) What is the amount of the following sums of money, £23 . 5 . 3½, £13 . 11 . 9, £45 . 9 . 2, £8 . 12 . 5?

£.	s.	d.
23	5	3½
13	11	9
45	9	2
8	12	5

(2) I have paid, on several accounts, these sums, £15 . 7 . 6, £29 . 14 . 3½, £75 . 16 . 4¾, £130 . 6 . 9; what is the total amount paid?

(3) The yearly expenses, in the undermentioned articles, of a certain family, were as here specified; groceries, £75 . 18 . 3½; butcher's meat, £82 . 3 . 4; meal, or flour, £30 . 14 . 8¾; malt and hops, £15 . 19 . 7; clothes, £105 . 17 . 6; assessed taxes and parish rates, £53 . 12 . 2: now, what was the total expenditure of these items?

(4) Add together the following weights, (*avoirdupoise*): 15cwt. 1 qr. 3 lbs; 3qrs. 19 lbs.; 12 cwt. 2 qrs. 11 lbs.; 19 cwt. 1qr. 25 lbs.; & 8 cwt. 3 qrs. 7lbs.

cwt.	*qrs.*	*lbs.*
15	1	3
0	3	19
12	2	11
19	1	25
8	3	7

(5) A grocer has received five butts of currants, severally weighing, 18 cwt. 2 qrs. 3 lbs.; 12cwt. 1qr. 12 lbs.; 17 cwt. 3 qrs. 16 lbs.; 18 cwt. 0qr. 27 lbs.; 19 cwt. 0 qr. 2 lbs. nett; what is the whole weight of currants, in cwts. qrs. and lbs.?

(6) Received by waggon, 7 packages, severally weighing as here stated; what is the total weight thereof? 3 cwt. 2 qrs. 15 lbs.; 5 cwt. 1 qr. 22 lbs,; 2cwt. 3qrs. 19 lbs; 4cwt. 2qrs. 12 lb; 1cwt. 0qr. 11 lb; 7 cwt. 1 qr. 3 lb.; 6 cwt. 2 qrs. 14 lbs.

(7) How many bushels, pecks, and gallons, are there in the following quantities; 5b. 1p. 2g; 3b. 2p. 0g; 6b. 3p. 0g; 3p. 1g; 7 b. 2 p. 1 g?

b.	*p.*	*g.*
5	1	2
3	2	0
6	3	0
	3	1
7	2	1

(8) What is the amount in quarters, bushels, pecks, and gallons, of the quantities of grain here stated?

qr.	*b.*	*p.*	*g.*
3	2	0	1
7	6	3	0
7	4	3	1
4	4	2	1

(9) In six weeks, a stable-keeper used, week by week, the undermentioned quantities of oats; what was the total quantity used in the time? 6q. 4b. 2p. 0g.; 9q. 2b. 1p. 1g.; 7q. 5b. 3p. 1g.; 5q. 6b. 2p. 0g.; 8q. 3b. 1p. 1g.; 10q. 2b. 0p. 1g.

(10) A maltster sold the undermentioned quantities of malt: what were the total sales? 15q. 4b. 3p.; 7b. 5p. 2p.; 12q. 3b. 1p.; 9q. 7b. 2p.; 5q. 5b. 2p.; 4b. 1p. 5q. 2b. 0p.

(11) Purchased the several parcels of wheat here specified, at the prices and sums severally annexed to each parcel; what is the total quantity of wheat, and what the amount of money paid for the whole?

q.	*b.*	*p.*		*s.*	*d.*		*£*	*s.*	*d.*
53	7	2	at	48	..	per qr.	129	9	0
16	3	1	..	52	..	..	42	13	1½
27	5	3	..	50	6	..	69	19	9½
36	2	0	..	49	..	..	88	16	3
19	4	0	..	46	6	..	45	6	6
41	6	0	..	51	..	..	106	9	3

(12) Sold six hogsheads of fine Jamaica sugar, the nett weight, and the prices of which are as here stated: what is the total weight, and what the total sum for which they are sold?

Cwt.	*qr.*	*lb.*		*s.*	*d.*		*£*	*s.*	*d.*
17	1	13	at	64	..	per cwt.	55	11	5
18	0	7	..	63	6	..	57	6	11½
16	3	24	..	62	..	..	52	11	9
17	2	5	..	60	..	..	52	12	8½
16	1	8	..	61	..	..	49	15	7
15	0	3	..	62	6	..	46	11	8

COMPOUND SUBTRACTION.

220. In stating the sums to be worked in this rule; in writing the smaller quantity underneath that from which it is to be subtracted, always be careful to place each figure right under those of the same denomination. And then, beginning at the right hand, with the lowest denomination, you see whether the quantity in the upper line be large enough to admit of that in the lower being taken from it. If, as in the first example here given, there be no figure, or only a cypher, under the first of the upper line, you then, of course, bring down the upper figure, undiminished. If there be a figure in the lower line, smaller than that above it, you then, as in the second example, write down the difference.

221. But, if the lower figure be the larger, you must borrow, as you do in simple Subtraction, and as you see here done, in the third example: ONLY, OBSERVE, and this, mind, is THE PRINCIPLE always to be acted on, that whilst, as you remember, in simple arithmetic, the sum you borrow, in these cases, is, always, ten, which is but one of the next higher station, so the sum you borrow here, is just as many of the denomination you may, at the time be working on, as make one of the next higher denomination, whatever that may be: thus; if they be farthings that you have to subtract from

the upper line, and there be none, or not a sufficient number in the upper line to admit of the subtraction, the sum you borrow is 4; that is, 4 farthings, which, making one penny, is repaid, by carrying one to the pence in the lower line; and, if you then find them too many to be subtracted from those in the upper, the sum you have to borrow is twelve, that is, the number of pennies in a shilling, and this you repay, in like manner, by adding one to the shillings in the lower line. And so you go on, if there be occasion, borrowing again in the shillings, in which case, of course, you borrow twenty, and pay by adding one to the lower line of pounds; which, being the highest denomination of the series of which you treat, you proceed as in simple Arithmetic; that is, if you have occasion to borrow, the number you borrow is ten, and you pay by carrying one, as before.

EXAMPLES.

FIRST.			SECOND.			THIRD.		
Cwt.	*qr.*	*lb.*	£	*s.*	*d.*	£	*s.*	*d.*
16	3	$14\frac{1}{2}$	115	14	$6\frac{3}{4}$	3163	5	2
9	2	0	96	0	$5\frac{1}{4}$	2785	12	$6\frac{3}{4}$
7	1	$14\frac{1}{2}$	19	14	$1\frac{1}{2}$	377	12	$7\frac{1}{4}$

222. And now, my pupils, you are prepared, I trust, to work the following sums; and to do this with ease, on looking to the manner in which the foregoing are performed.

SUBTRACTION OF MONEY.

(1) From £217 12*s.* $4\frac{1}{2}$*d*, subtract £83 15*s.* 2*d.*

(2) „ £4703 8*s.* 5*d.* „ £1235 13*s.* $7\frac{1}{2}$*d.*

		£	s.	d.		£	s.	d.
(3)	From	13058	17	$3\frac{1}{2}$	Sub.	5739	4	7
(4)	„	41836	9	2	„	8379	12	6
(5)	„	293750	3	11	„	175803	7	3
(6)	„	1530827	14	5	„	953642	17	8

(7) Having received £1375 8*s.* 9*d.*, and paid £862 13*s.* 5*d.*: I desire to know what is the remainder, or balance.

(8) What is the difference between £1236 5*s.* 2*d.*, and £975 19*s.* 6*d.*?

(9) If I receive £1583 6*s.* $4\frac{1}{2}$*d.*, and pay £1375 15*s.* $8\frac{3}{4}$*d.* what will remain?

(10) Having sold goods amounting to £3157 12*s.* and received on account thereof £2798 15*s.* 10*d.*, what is the balance yet to be received?

(11) The annual income of a gentleman being £3215 7*s.* 6*d.*, of which he spends £2732 18*s.* 9*d.*, what is the sum reserved?

(12) A tradesman having sold, from his stock, goods for £15396 9*s.* 3*d.* which goods cost him £14789 15*s.* 7*d.*; I desire to know what were his gains?

(13) Received, on various accounts, the following sums: £512 6*s.* 3*d.*, £73 5*s.* 1*d.*, £362 9*s.* 7*d.*, and £19 17*s.* 8*d.*; and, out of the amount thus received, having paid the several following, that is to say, £27 14*s.* 6*d.*, £157 11*s.* 8*d.*, £275 9*s.* 4*d.*, and £98 7*s.* 11*d.*; I would know what is the balance remaining?

223. Now, the method of stating sums of this sort is the most material thing, therefore, let me beg your particular attention to it. The sums received, as you here see, are stated first, then added up, and the amount carried out, clear of the other figures; then, a little below the first sums, ranged, also, in nice order, are placed the several sums paid, and to be subtracted from the former. These are then added up, and the amount carried out, right under the former; and thus, the two amounts, drawn out, clear and distinct from other figures, are prepared for the work of subtraction, which you will perform as in the other examples. This, which will serve as an example in any similar cases that may follow, whether of money, of weights, or of measures, ought, also, to serve as an example of clearness of statement on any future occasion.

£	s.	d.			
512	6	3			
73	5	1			
362	9	7			
19	17	8			
			£967	18	7
27	14	6			
157	11	8			
275	9	4			
98	7	11			
			£559	3	5

WEIGHTS. (AVOIRDUPOISE.)

		Ton.	*cwt.*	*qr.*	*lb.*		*Ton.*	*cwt.*	*qr.*	*lb.*
(14)	From	175	12	2	13	(15)	364	9	1	11
	Take	98	14	1	16		169	13	2	15

(16) From 732 *tons.* 14 *cwt.* 0 *qr.* 17 *lb.* take 13 *tons.* 18 *cwt.* 2 *qrs.* 7 *lb.*—29 *tons.* 13 *cwt.* 2 *qrs.* 12 *lb.*, and 115 *tons.* 15 *cwt.* 3 *qrs.* 9 *lb.*

(17) From 1240 *tons.* 11 *cwt.* 7 *lb.*, and 918 *tons.* 13 *cwt.* 16 *lb.*, subtract 1153 *tons.* 14 *cwt.*

(18) From 763 *tons.* 12 *cwt.* 6 *lb.*, 163 *tons.* 2 *cwt.* 14 *lb.*, and 1571 *tons.* 9 *cwt.* 13 *lb.*, subtract 130 *tons.* 15 *cwt.* 2 *lb.*, 572 *tons.* 11 *cwt.* 16 *lb.*, and 369 *tons.* 16 *cwt.* 9 *lb.*

DRY MEASURE.

		Qrs.	*b.*	*p.*		*Qrs.*	*b.*	*p.*
(19)	From	257	5	3	(20)	349	3	1
	Take	189	7	1		260	5	2

(21) From 823 *qrs.* 5 *bus.* 1 *p.*, take 156 *q.* 3 *b.* 2 *p.*, 217 *qrs.* 7 *b.* 1 *p.*, and 97 *qrs.* 4 *b.* 0 *p.*

(22) From 317 *qrs.* 6 *bus.* 3 *pecks*; 123 *qrs.* 5 *bus.* 2 *pecks*, and 427 *qrs.* 3 *bus.* 1 *peck*, subtract 613 *qrs.* 5 *bus.* 1 *peck.*

TROY WEIGHT.

The weight of the the precious metals, however large the quantity, is always stated, as I have stated them, in ounces, penny weights, and grains.

(23) From 1217 *ounces*, 16 *penny weights*, and 7 *grains*, take 827 *ounces*, 12 *penny weights*, and 17 *grains.*

(24) A gentleman has agreed with a silversmith for a service of plate, the weight of which is 5614 *oz.* 9 *dwts.* 12 *grs.*, from which is to be deducted an old service, weighing 3259 *oz.* 12 *dwts.* 15 *grs.*, which the silversmith takes in part payment; how many ounces will the gentleman have to pay for afterwards.

The foregoing examples will prove sufficient for the learner; an application of the principle before laid down, and a reference to the proper table, being quite sufficient to enable him to work sums in any of the other denominations of weights or measures.

REDUCTION.

224. In Compound Arithmetic we are dealing with numbers employed to describe *certain* and *various quantities* of THINGS: that is to say, with numbers describing sums of money, quantities in weight, and quantities in measure; matters of which, until now, we had not to treat, our reckonings being confined to mere numbers; to numbers which were, indeed, ready to be applied to the measuring, or to the describing of any of these quantities, but which, until now, we did not so apply. Now, then, therefore, that we are thus applying numbers, we are entered on the useful, or practical part of Arithmetic. The former, although very pleasing, and, on account of its great simplicity, and perfect order, even very beautiful, is valuable only, inasmuch as it aids us in this useful branch of the art, which consists in the reckoning of certain and express quantities of things.

225. The quantities thus to be expressed or reckoned, are, as before stated, of every description; the MEASURES employed being MONEY, by which, amongst us, the value of things is measured; WEIGHTS, by which their heaviness is ascertained, RULES or GUAGES, for measuring their length, breadth, and thickness, and MEASURES of capacity for determining the bulk of things. Such are the means by which things are measured; and numbers are employed to count up, and otherwise to reckon up those measures.

226. But measures, as you have seen, are, and must be various, and, consequently, have various names, or, as we will call them, denominations, whereby to distinguish their several weights, their dimensions, or their value: we have, in our common weights, tons, hundreds, quarters, and pounds. Now, it is out of these various measures, that has grown this rule of Reduction, the purpose of which is, to reduce quantities which you may, already, have stated in one measure, into equal quantities in some other measure; a purpose which, as you will see, there is frequent occasion to effect.

227. To proceed to an example of this Reduction, and of its uses. Suppose you have 5 *cwt.* 2 *qr.* 14 *lb.* of Gum Senegal, costing you £441, and you are desirous of knowing how much each pound cost you. To accomplish this, you have, first, to ascertain the number of pounds in these 5 *cwt.* 2*qrs.*, and finding them to be 616, which, when joined to the 14, make 630 pounds of gum, you proceed to inquire, how much each pound cost you, by ascertaining how much is a six hundred and thirtieth part of £441, which you ascertain by dividing the money by the number of pounds of gum; and the quotient, which tells you, how many times 630 are found in 441, tells you the the cost of each pound. But, as you will truly observe, the number 630 being larger than 441, will not divide it; and, therefore, you can have no quotient. Exactly so; you cannot so divide your money whilst it remains in whole pounds, but, change it into shillings, and, if need be, into pence, and, even into farthings, and then, you find that you can divide it. And this changing is called Reduction, and you have here the use of the rule in the cases of reducing hundred weights and quarters, into pounds; and pounds sterling, into shillings, pence, and farthings.

228. There is yet, however, another mode of Reduction. That of which we have just seen examples, consists in the bringing down of *larger* denominations into *smaller*, and it is, therefore, called, *Reduction Descending:* but it may be, that you have the quantity of goods stated in pounds, and the cost thereof in shillings; thus, 630 *lb.* of gum cost 883 shillings, and you are desirous of having the weight stated in *cwts. qr. lb.*, and the cost in *pounds sterling.* Now this, requiring a process the contrary of the former; this being a changing of the money, and of the weights, from *lower* into *higher* denominations, is called *Reduction ascending:* and, so, there are Reduction ascending, and Reduction descending.

229. This rule, which consists merely of certain methods of changing the denominations of quantities, for the purpose of more conveniently working to some particular end; this Reduction I am here introducing into a somewhat new situation. And it may, therefore, be proper to say a few words, to account for so introducing it. By all other authors, I believe it is inserted either *before*, or *after*, the four rules, of addition, subtraction, multiplication, and division, in compound Arithmetic; and, in thus placing it, as if they could not determine which is its proper situation, the authors seem to be pretty equally divided. I, following the course which I have heretofore pursued in these lessons, that is, to treat of the several branches of the art as they naturally arise, and as they become useful to the learner, determine to introduce Reduction here, after Addition and Subtraction, which, in strictness, are the fundamental rules; before which rules, I have found no occasion for it, and, immediately after which, I find that we shall have that occasion.

REDUCTION DESCENDING.

230. This, as I have stated, is the method by which quantities of any description are brought down from any higher denominations in which they may be described, into other denominations of lower quantity or value: as, in money, it is the bringing of pounds into shillings, these into pence, and pence, into farthings; in weights, it is the bringing of tons into hundred weights, these into quarters, and quarters into pounds; and so on.

231. Now, to effect this reduction; the thing you first do, is, to notice the denomination in which the quantity to be reduced is already expressed; as, suppose it were a sum of money, you notice that it is expressed in pounds; and, pounds, therefore, is the denomination in which the money stands; next, you consider, into which denomination you would reduce it; that is to say, would you bring the sum into shillings, into pence, or would you bring it into farthings? If you would bring it into shillings, you, then, of course, multiply the number of pounds by 20; that is, by the number of shillings in a pound; if you would bring the shillings into pence, you multiply them by 12; or, had you to reduce the pounds at once into pence, you might, if you chose, multiply by 240, which is the number of pennies in a pound. To bring tons into hundred weights, you, in like manner, multiply the number in which you have them stated, by 20; to bring the hundred weights into quarters, you multiply them by 4; and, to bring these into pounds, you, of course, multiply by 28.

232. All this is very simple. But there is another point or two to notice, before we pass on to the few examples which I have to give. The case above stated, that is, the reduction of pounds into lower denominations, gives us no instruction as to the mode of proceeding when the sum to be reduced consists of pounds, shillings, and pence. Let us here, then, annex some figures for shillings and pence, and attend to the method of working.

233. In this manner may this sum of £5 7*s*. 6*d*. be reduced into shillings, and into pence: that is, by writing underneath, and so adding in the odd money at each step. But the established manner is, not so to bring down, and so to add in, the odd, or fractional parts, but to add them into your sum thus. On multiplying by 20, you remember that you write down a cypher in the unit's place, and then multiply by the 2; here, however, you have another figure to take this place, that is, the 7, which, therefore, you write down as you see it here done, and then proceed to multiply by the 2. Next, you reduce the shillings into pence, by multiplying them by 12; saying, "twelve times seven are eighty-four, and the sixpence, to bring down, make ninety," write down nought, and carry nine, "twelve times nought is nothing," so write down the nine; then, "twelve times one are twelve." And so the sum is done.

£	*s*.	*d*.	
5	7	6	
20			
100			shillings in £5.
7			brought down.
107			total shillings.
12			
1284			pence in 107*s*.
6			brought down.
1290			

£	*s*.	*d*.
5	7	6
20		
107		
12		
1290		

234. It must be unnecessary to increase the number of examples. I will here work one or two more; the manner of doing which you will carefully observe; always bearing in mind, that the principle on which this rule proceeds is, the multiplying the sum, or quantity to be reduced, by such number as the denomination into which you would reduce it, is contained in one of the higher denomination from which it is to be reduced, as is shown in the foregoing, and in the following examples; bearing this principle in mind, and, with a little care and attention, working, the few sums which follow, you will become the easy master of this rule.

(2) Reduce £1763 . 14s. . $2\frac{3}{4}$d. into farthings.

```
   20
 -------
  35274  the 14 shillings brought down.
     12
 -------
 423290  the 2d. brought down.
      4
 -------
1693163  the ¾d. brought down.
 -------
```

(3) Reduce 837 Tons . 16 cwt. . 2 qr. . 19 lb. into pounds.

```
     20
 -------
  16756  the 16 cwt. brought down.
      4
 -------
  67026  the 2 qrs. brought down.
     28
 -------
 536227  the 19 lb. brought down.
134052
 -------
1876747  total number of pounds.
 -------
```

(4) In 513 French ells, how many nails?

(5) In 213 *quarters*, 5 *bushels*, and 2 *pecks*, how many pecks?

(6) Reduce £1268 13*s.* 5¼*d.* to shillings, pence, and farthings?

(7) In £1932 17*s.* 6*d.* how many farthings?

(8) Reduce 627 *tons* 13 *cwts.* 3 *qrs.* and 23 *lbs.* to pounds.

(9) In 975 *tons* 16 *cwts.* 15 *lbs*, how many pounds?

(10) In 16951 *ounces*, 18 *pennyweights*, and 15 *grains*, *(troy weight)* how many grains?

(11) Reduce 109 *pounds*, 7 *drams*, 2 *scruples*, and 15 *grains*, *(apothecaries)* to grains?

(12) In 68 years, how many weeks, days, hours, and minutes?

(13) Five statute acres form the Necropolis, a very fine burying ground, near Liverpool. In this piece of land, how many roods, poles, yards, and feet?

(14) The other splendid and romantic burying ground, at the south end of the town, called St. James's Cemetery, formed in the ravine of a former stone quarry, and calculated to divide into about fifteen thousand graves, besides upwards of one hundred catacombs or vaults, hewn out of the solid rock; this interesting place, with its exquisitely chaste and beautiful temple, its plantations, its winding paths, and varying acclivities, spread over about eight statute acres of land. In this space, how many roods, poles, and yards are there?

(15) In the degree, which is 69½ miles, how many poles, feet, inches, and barleycorns?

(16) The circumference of the earth is divided into 360 degrees; in this circumference, how many miles, how many furlongs, and how many inches?

(17) In 17 butts of ale, how many kilderkins, gallons, and quarts?

(18) In 26 pipes of wine, how many gallons and quarts?

(19) The diameter of the earth, from the north to the south pole, is estimated at 2633 leagues; in this length, how many miles, furlongs, and yards?

REDUCTION ASCENDING.

235. As Reduction Descending is performed by *Multiplication,* so this, our present rule, is performed by *Division.* For instance: In the example first given, in the foregoing rule, we reduced a certain sum into shillings, and then into pence; let us here reduce the pence back again, first into shillings, and then into pounds; which is to be done by dividing the pence, first by 12, *(the number of pennies in a shilling)* and then by 20, *(the number of shillings in a pound.)* And in this manner, which, as you see, is the *reverse* of that in the last rule, are all sums in Reduction Ascending, to be worked.

236. The process will be well learned by working, amongst others, some of the sums given in the last rule back again, and this, therefore, I propose to have done in the following

EXAMPLES, QUESTIONS, &c.

(1) Reduce 1290 pence into pounds, &c.

```
  12 ) 1290
 2'0 ) .10'7 . 6
      £ 5 . 7 . 6
```

(2) In 1693163 farthings how many pounds?

The chief matter to be noted in these first two sums, is the manner of carrying out the remainders; all of which will now be understood, on mere inspection, by the attentive learner.

```
   4 ) 1693163
  12 ) .423290 ¾
 2'0) ..3527'4 . 2
     £1763 . 14 . 2¾
```

(3) Reduce 1876747 *lb.* into tons, &c.

```
28 ) 1876747 ( 67026 number of quarters to be divided by 4.
     168····
     ----    2'0)1675'6 . 2 cwts. and qrtrs.
      196
      196         837 . 16 . 2 . 19 tons, cwts, qrs. and lbs.
     ----
      ···74
         56
        ---
        187
        168
        ---
         19 remaining pounds carried up.
```

(4) In 304481 pennies, how many pounds?

(5) Bring 14688 quarts into butts of ale.

(6) Reduce 24624 nails, *(cloth measure,)* into French ells.

(7) Reduce 716927 grs. *(troy weight)* into ounces.

(8) In 628215 grains *(apothecaries)* how many pounds?

(9) In 437 French ells of cloth, how many English ells?

(10) Reduce 3681 Flemish ells into French ells, and then into English yards, quarters, and nails.

The two last questions require a little of that consideration which questions occurring in real business, must be expected to demand. Thus, with regard to the ninth, you consider what relation, as to quantity, the French and English ells bear to each other. On referring to the table, you find, that 5 quarters of an English yard, make an English ell, and, that there are 6 quarters in the French ell, the method of proceeding, in this question, then, is, to reduce the French ells into quarters, by multiplying by 6; and then to bring the quarters into English ells, by dividing them by 5.

The four following propositions, also, are such as occur in real business, and require a little thought. To look at the first of them . you will soon see that all you have to do is to multiply the number of dollars by 50, which is the number of English pence stated to be in each dollar, and then, having their value in pence, you know how to bring them into pounds.

(11) Reduce 62875 dollars into pounds sterling, *(that is, English pounds,)* each dollar being rated at 4*s*. 2*d*.

(12) What is the value, in English money, of 7435 Napoleons, *(French)* each Napoleon being worth 15*s*. 10½*d*.?

(13) In 4175 American eagles, how much English money, the eagle being 5 dollars, and the dollar rated at 4*s*. 4¼*d*.?

(14) Reduce 5273 new Louis pieces (*French*) into English money, at 15*s*. 10½*d*. each piece.

COMPOUND MULTIPLICATION;

Or Multiplication of Money, Weights, & Measures.

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237. Again, it may be said, as I have said in the commencement of Compound Addition, that "this branch of our subject, after what we have learned, ought to be a very easy matter. Calling to mind what is said in paragraph 196 to 210, on the notation of compound numbers, and especially what is said in paragraph 207, on the necessity of ascertaining, in the first instance, the manner in which the money, the weights, or the measures, which we have to calculate, are divided;" doing this, you will enter with great ease and pleasure on the practice of this rule.

238. To multiply, you recollect, is, in a certain sense, to *repeat*, that is, to *re-tell*, or *re-count*, and numbers may be said to be re-told, when they are multiplied: as, for example, a number may very properly be said to be *twice-told*, or *thrice-told*, when it is multiplied by two, or by three. And, as it is thus with mere numbers, so, also, is it with sums of money, with weights, and with measures, of any description; any, or all of which, may, by this rule, be multiplied, or told over, twice, or any larger number of times, and the amount, or product of such multiplication, be thereby stated in a single line.

239. Let it be that we have to multiply £2 10*s*. 6*d*. by 2. We place the multiplier, as we do in simple arithmetic, right under the lowest figure of the mul-

tiplicand, and, beginning, also, as we do there, to multiply that lowest figure *first*, we say, in this sum, "*twice six are twelve*," but, and mark this distinction, these *twelve* are pence; were it *seven pence* that we were multiplying by two, we should then have fourteen pence, which, being one shilling and two pence, we should, as you will recollect, according to instructions in paragraph 202, set down only the 2, in the place of pence, and carry on the shilling to *its proper place*. However, to return exactly to our sum: it is *twelve* pence that we have, and nothing over: naught, therefore, that is, cipher in the pence place do we set down, and, bearing the shilling in mind, we proceed with our work, saying, *twice-ten are twenty*, "*and one makes twenty-one*;" but neither are we to write down the 21 shillings: no; we set down the odd shillings; we should set down 19, if there were nineteen of them; but, as you recollect, we only set down the *odd* money, always carrying on, from the pence, as many shillings as we can make up; and from the shillings, as many pounds as we can complete, thereby keeping our accounts clear, by preventing the pence from exceeding eleven in number, and the shillings from exceeding 19, however numerous, and however large the sums may be that we are adding or multiplying together. But, again to return to our work; we set down the odd shilling; and *carrying* the pound, we say, "*twice two are four, and one makes five*," and so we set down 5 in the place of pounds, and the sum is completed. You will now be able to work any of the following sums, without further instructions: for, as to the pounds, did they, in the multiplicand, run up to thousands, or to millions, you would treat them as you do simple numbers, multiplying them and carrying the tens, until you should have disposed of them all. And, as to weights and measures, of any description whatever; keeping your eye on the peculiar notation of those that you may be multiplying; beginning with those of the lowest denomination, and *carrying on* all that you can to the next higher, doing this, and setting down the odd numbers, as I have just shown, and you will find no difficulty in these sums.

(1) Multiply £135 12*s.* 7*d.* by 3.

£	s.	d.
£135	12*s.*	7*d.*
		3
£406	17	9

(2) What is the product of £214 6*s.* 9*d.* multiplied by 5?

(3) A farmer has expended, in money, on his household £217 15*s.* 8*d.* per year, for seven years; what is the total money so spent in the 7 years?

(4) A field of nine acres, sown with wheat, has produced a crop of 2*qrs.* 7*bus.* 2*pks.* to the acre; what is the total produce on the 9 acres?

(5) Eight pieces of cloth contain 28*yds.* 2*qrs.* 3*nails.* each. What is the whole quantity of cloth?

(6) At £18 17*s.* 6*d.* each, how much will eleven oxen come to?

(7) Twelve men are engaged by the year, on wages of twenty-five and a half-guineas each; what is the total amount for the year.

This sum you must, of course, commence, by stating, in pounds, shillings, and pence, the year's wages for one man; and then multiply by the number of men.

In the following, I propose to multiply a compound sum by 10. You will recollect that an abstract, or simple number is thus multiplied, merely by annexing a cipher to the right-hand thereof, as explained in paragraph 85. But we do not so multiply a compound sum: on the contrary, we proceed, with *ten times*, just as we do with any other number.

(8) Ten hogsheads of sugar, each weighing, on the average 15*cwt.* 3*qrs.* 17*lb.* what is the total weight of the ten hogsheads?

240. Thus you have learned to multiply compound sums by numbers from two up to twelve, but not yet higher. To multiply by higher numbers requires a somewhat different mode of proceeding. One of the methods is, to reduce the compound sum to its lowest denomination, you then have it in the shape of a simple number, and may multiply it by any long number. The other method is, to find *submultiples* of your multiplier; submultiples none of which exceed *twelve*, and, with these smaller and more convenient numbers in succession, to multiply the compound sum. I will here exhibit both methods of working on one sum; proposing that it shall be, the multiplication of £16 13*s*. 7*d*. by 225. You will see, in the first method, that I reduce the sum of money from its compound terms into pence, then apply the whole multiplier to it; and, having found the product, reduce that into pounds &c. Whilst, in the second, having found that 5 × 5 × 9 = 225, I multiply the money by these numbers, successively, and thus find the same product. Of these two methods we may say, that one is a *check* on the other; that one *proves* the other; and that both are, therefore, good and useful methods.

First Method.

	£.	s.	d.
	16	13	7
	20		
	333		
	12		
	4003		
	225		
	20015		
	8006		
	8006		
12)	900675		
2'0)	·7505'6-3		
	£3752-16-3		

To find submultiples, or factors: see paragraph 141, *&c.*

5) 225

5) 45

9

Second Method.

£.	s.	d.
16	13	7
		5
83	7	11
		5
416	19	7
		9
£3752	16	3

241. One other point remains to be spoken of, and then I leave you to finish yourself in this rule, by the practice which you will find in the few sums which follow. The point I have to explain is, the method to be adopted, when you would do the work by submultiples, and yet have a number wherewith to multiply that cannot be thus evenly divided: as, for example; let us have to multiply the same sum of money, as this on which we have just been working, by 221, which is a number not to be divided in the manner I have just proposed. This 221, as you will observe, is less by 4 than the former multiplier; but if you choose you may use the former multiplier, or rather, the submultiples thereof, as is done in the last case of working, and from the product subtract 4 times £16 13*s.* 7*d.* and thus will you have a correct result: or, you may adopt this method; unable to find submultiples of 221, you take them of the even number next below it, that is, of 220; these are 5, 4, 11, which produce £3669 8*s.* 4*d.* being £16 13*s.* 7*d.* × 220, or *once* £16 13*s.* 7*d.* short of the sum to be found: but you have that sum by adding this once £16 13*s.* 7*d.* to the product; that is £3669 8*s.* 4*d.* + £16 13*s.* 7*d.* = £3686 1*s.* 11*d.*

(9) Multiply £23 15*s.* 4. by 164.

(10) What is the product of £31 12*s.* 2*d.* multiplied by 221?

(11) What is the weight of 463 Hogsheads of Sugar, the average weight of each Hogshead being 14 *cwt.* 3 *qrs.* 23 *lb.*

(12) Multiply 18 *cwt.* 2 *qrs.* 19 *lb.* by 672.

OF

COMPOUND DIVISION;

Or Division of Money, Weights, and Measures.

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242. This is, of course, the reverse, exactly, of the last rule. The sums, generally, are to be stated in the same manner, and, frequently to be worked in the same manner, as are the sums with which you are now familiar in the division of whole numbers: that is, when the divisor does not exceed 12, when the sum is within the rule of short division, you state the dividend just as you happen to have it, in its several denominations, whether of money, of weights, or of measures, and, drawing a line before and carrying it underneath, you prefix the divisor, as you see it done in the annexed example, and so divide the sum as you see it here done; that is, divide it thus when the several denominations happen thus evenly to fit, or to be divisible by your divisor.

	£	s.	d.
7)	1652	14	7
	236	2	1

243. But, it is scarcely necessary to say that, this *even fitting* of the divisor and dividend, is a thing not to be expected. In this example, I have purposely framed these terms thus to fit, in order that you might, in the first instance, see the process in its clearest form. Let us now make a little alteration in these terms, and let the dividend stand as you see here. In this case, you see, that having divided the pounds by the 7, we have *one* remaining, that is, one not yet

	£	s.	d.
7)	1653	3	7
	236	3	$4\frac{1}{4}$ $\frac{5}{7}$

divided. But we have undertaken to *divide the whole*, and it is to be done thus: the *one remaining* being one *pound*, *reduced* into shillings, *becomes twenty;* and having it thus, we can proceed with our work. But, looking a little before us, we find that there are three more shillings to be divided, so we add the twenty and the three together, saying, "*seven in twenty-three? three times, and two over.*" And, setting down the three, which are, of course, *shillings*, in the shillings' place, we call to mind, that the *two* which we have now remaining, being shillings, when reduced, make 24 pence; and, again looking before us, and finding the 7 pence waiting to be divided, we put them to the 24, saying, "*seven in thirty-one? four times, and three remain;*" which three, pence, reduced to farthings, and divided, give us the quotient which you see above.

244. One other example, or two, will suffice to make this matter easy to you. Let us have one in cloth measure, and one in our avoirdupoise weight. And then, calling to mind, or, unless you have a perfectly clear recollection of it, reading again what is taught in paragraphs 205, 6 and 7, concerning the notation of money, weights and measures, you will trace the working of these, and will, with ease, follow the example given in them, in your working of the few sums which immediately follow.

		Yds.	*qrs.*	*n.*			*Cwt.*	*qrs.*	*lbs.*
(1)	9)	2351	3	2	(2)	12)	5077	2	0
		261	1	$1\frac{1}{9}$			423	0	14

After dividing the cwts. you have one left, which, being four quarters, do not, even with the two quarters which follow, amount to a number sufficient for your divisor; you, therefore, write a cipher in the place of qrs.; and then, with your pencil, on a vacant part of your slate or paper, reducing the quarters to pounds, you find you have 168, which, divided by 12, give the quotient you see.

(3) A tradesman expends in his domestic affairs, in seven years, £1524 9*s.* 8*d.* What is the yearly expenditure?

(4) Eleven oxen cost £207. 12*s* 6*d.* What is the average price of each ox?

(5) Eight pieces of cloth contain 229*yds.* 2*qrs.* What is the length of each piece?

(6) A crop of wheat being 211*bus.* 2*pks.* grown in a field of nine acres. I desire to know, what this is per acre?

(7) Ten hogsheads of sugar weigh, together, 159*cwt.* 0*qrs* 2*lb.* What is the average weight per hogshead

(8) Twelve butts currants weighing 227*cwt.* 3*qrs.* 16*lb.* What is the average weight of each hogshead?

245. So much for the division of compound terms, by numbers not exceeding 12; that is, so much for, *short* division, in compound numbers It yet remains for us to speak of the methods of proceeding when such terms are to be divided by higher numbers. Some of these methods, as you will anticipate, are just the counterpart pursued under correspondent circumstances, in the multiplication of compound numbers, and described in paragraph 239. There you learn, that you may proceed by reducing the several terms to their lowest denomination. And, as it is thus in multiplication, so it is in division; in which, having your terms so reduced, you divide as in simple division: or, when your divisor is a multiple of some suitable small numbers, but only then, you may, if you prefer it, divide the terms as they may happen to stand, in their several denomina-

tions. These two methods of treatment are exhibited below, in the second and third examples respectively. But the simplest, on most occasions, is the method shown in the first of these three examples. In it you see, that the sum to be divided is written down, and the whole divisor applied to it at once, that the pounds are divided as far as they will allow of division, and that the remainder is then reduced into shillings, the shillings in the dividend being brought down at the same time, and that, then the shillings are divided. But a mere inspection of the example will put you into possession of the entire process. It will, however, be well to work the example yourself, on your slate; after which, you will confirm yourself in the understanding of the rule, by carefully stating and working the few sums afterwards proposed.

(8) Divide £3752 . 16*s*. , 3*d*. by 225.

First Method.

```
       £     s.  d.   £    s.   d.
225 ) 3752 . 16 . 3 ( 16 . 13 . 7
      225·   ··   ·
      ----
      1502
      1350
      ----
       152
        20
      ----
    ) 3056 (
      225·
      ----
       806
       675
      ----
       131
        12
      ----
    ) 1575 (
      1575
      ----
      ····
```

Second Method.

```
         £     s.  d.
        3752 . 16 . 3
          20
        -----
        75056
           12
        ------
225 )   900675 ( 4003 = £16 . 13 . 7
        900···
        ---
        ···675
           675
           ---
           ···
```

Third Method.

```
9 ) 225          £      s.    d.
5 ) ·25    9 ) 3752 . 16 .  3
      5    5 ) ·416 . 19 .  7
           5 )   83 .  7 . 11
                 16 . 13 .  7
```

(9) Divide £5263 14*s.* 10*d.* + £2307 16*s.* 3*d.* by 306.

(10) Multiply £1730 3*s.* 7*d.* by 93, and divide the product by 274.

(11) Divide 6305 *tons.* 13 *cwt.* 2 *qrs.* 17 *lb.* by 186.

(12) What is the result of 316 *tons.* 17 *cwt.* 1 *qr.* 5 *lb.* multiplied by 27, and divided by 315?

(13) Divide 7360 *quarts,* 7 *bushels,* and 2 *pecks,* by 431.

(14) Multiply 5 *yds.* 2 *qrs.* 3 *nls.* by 840, and divide the product by 732.

(15) Multiply 29 *yds.* 3 *qrs.* by 175; to the product, add 267 *yds.* and divide the amount by 574.

(16) Add 3516 *qrtrs.* 5 *bus.* 3 *pecks,* to 278 *qrs.* 6 *bus.* 2 *pecks.* and divide the sum thereof by 714.

(17) Add 523 *tons* 14 *cwt.* 21 *lb.* to 3125 *tons.* 12 *cwt.* 2 *qrs.* 16 *lb.*, and divide the amount by 483.

(18) What is the five thousandth, two hundredth, and thirtieth part of £27658 12*s.* 3*d.*?

MULTIPLICATION OF FEET, INCHES, &c.

Or the computation of builders', and of other artificers' work, commonly treated of under the names, *Duodecimals,* or *Cross Multiplication;* of this I shall treat at the end of the lesson on the working of Decimals.

OF

PROGRESSION AND PROPORTION,

AND OF THE

RATIOS OF NUMBERS.

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246. It may be advisable to apprise the reader, that this lesson will be directed, not to the *practice*, but rather to the *principles*, or to that which may with propriety be termed the *science*, of this very important part of arithmetic; that is to say, it will be a dissertation on those principles on which calculations proceed, in all those rules which have the *adjustment of proportions* for their object. Rules, which, in the books, are treated of under the several names of Rule of Three, Single, Double, Direct, and Inverse; the Chain Rule, or Compound Proportion; Interest; Exchanges; Fellowship; Brokerage; Barter, and some others, which are mere applications of the rule of Proportion; in none of which rules, if he make himself master of the principles treated of in this lesson, will the learner find any difficulty, even though the name thereof may never before have struck his ear.

247. The lesson, however, being on the *principles*, and, notwithstanding that it is, therefore, if duly conducted, most valuable, and interesting, yet is it one which the learner, who may be desirous to hasten onwards in the mere *practice*, may pass over in the first instance: but I ought to assure him, that it may be well if he take an early opportunity

of returning to it. And I flatter myself that he will not think his attention unrequited. To proceed, then, with the lesson:

248. Progression is advancement; enlargement; increase. And the word is applied to the manner in which any series of numbers increase by any sort of regular and orderly progress; as do 2, 6, 10, 14, 18; or 5, 7, 9, 11, 13; or 1, 2, 3, 4, 5; each of which lines, or series, of figures increases, as you see, at each step, by the repeated addition, in the first, of *four*, in the second, of *two*, and in the third, of *one*. And the word progression is, also, applied, although not quite so properly, to series of numbers which, in a mode equally regular, *decrease* as they proceed, as do 18, 14, 10, 6, 2. And, whilst the former may be called *increasing* series, those of the latter description are called *decreasing* series.

249. But there is another mode of progression, a mode in which the increase is produced, not by the repeated addition of one number, but by a repeated multiplication by one number, as in the following series, 2, 8, 32, 128, 512, and so on; the number with which we multiply, in this instance, being four.

250. Now these two modes of progression have, as different things ought to have, different names. That mode which is produced by *Addition*, is called *Arithmetical Progression*; whilst that which is produced by *Multiplication*, is called *Geometrical Progression*. The reasons for adopting *these* two names are not very obvious; and to inquire into those reasons would lead us out of our course. But it is necessary thus to distinguish the two modes. With regard to the numbers, or, as they are called, TERMS, forming both the modes of progression, they have several very curious, and some of them very valuable

properties; of which properties, however, I shall here, at least, say but little more than I think requisite to lead clearly to the principles of some of the ensuing rules.

251. In the first place, with regard to any series of numbers increasing, or decreasing, in arithmetical progression, thus, 4, 6, 8, or 8, 6, 4, it is to be observed, that the first and the last number in each series, added together, are equal to twice the middle number. And the reason of this, as you cannot fail to see, is, that as the numbers increase, or decrease, by even, or equal steps, so the number on one side is as much *more*, as that on the other side is *less*, than the middle number; so that, put the two together, and they make twice the middle number: and this is true of every such series, whether the steps by which numbers increase or decrease be great or small.

252. But this, which is true with regard to series such as the above, consisting of three numbers, is, as implied above, likewise true of any other regular series of numbers or terms, however extensive the series, and whether the increase, or the decrease, at each step, as I have just stated, be great or small. For, let us suppose a series of terms consisting of a thousand; or rather, in order that we may have *a middle* term, let the series consist of a thousand and one. Is it not evident, that as there will be five hundred steps of increase on one hand of the middle term, and five hundred of decrease on the other, every step being equal; is it not evident, that what is lost on one hand is gained on the other; and that if we add the last term on each hand together, we shall have, as in the above short series of three terms, just twice the middle term. And, on this principle it is, that in measuring the trunks of trees, as timber, the measurement as to thickness, is taken by *girthing* them around the

middle, that is, at an equal distance from each end. And this is regarded as the true measurement, or as it is called, the average thickness; seeing that, whatever the trunk may lose by tapering towards one end, is gained by its increase towards the other.

253. Again, that which is true of the two extreme terms of a series of numbers, whether that series be long or short, is true of any other two terms, taken one on either hand, an equal number of steps from the middle term. And this is true of all such pairs of terms, for the reason above stated; namely, that as much being gained on the one hand, at every step, as is lost by every step on the other, any two terms taken at an equal number of steps from the middle term, will be equal to any two other terms taken in like manner, however near, or however distant from the middle term; and equal, too, for the same reason, to twice the middle term.

254. Yet, again: take a line of four terms; thus, 5, 6, 7, 8. Now here is not *one*, but two middle terms, and the extreme terms being each equally distant from the middle terms, whatever one extreme falls short of the two middle terms added together, is made up by the other extreme; so that, *the two extremes are equal to the two means;* for, by this name, "*means*," are these middle terms called. And this, which is true with regard to this series, is, for the same reason, true with regard to those of any other series; and true, also, with regard to any other two terms of any other series, such terms being taken thus at equal distance from the two middle terms.

255. A consequence of the relationship thus existing between a series of numbers of this description is this; that if we be informed of three, of almost any three particulars respecting such series, we can tell all the other particulars respecting

it. As, for example, if we have the first and the last terms, and the sum of the whole, we can then tell the number of terms, and the rate of increase at each step; or, as it is called, the rate of progression. Again, having the first term, the rate of progression, and the number of terms, we can tell the last term, and the sum of all the terms. But here is quite sufficient for our present purpose, in arithmetical progression. And, so, now for the other, and more important kind of progression; that is, when the increase is produced by a repeated multiplication by one number, which, as before stated, is called, Geometrical Progression; a mode of progression which produces numbers bearing a very different relationship towards each other from that which is produced by the successive addition of the same number to each successive term. And, for the purpose of more distinctly marking this difference, let us here set down a series of terms in each of the modes of progression. And, further, in order that the difference may be most distinctly seen, let us, in each series begin with the same number, and let the increase be made in each by the use of the number three: thus,

5,	8,	11,	14,	17,	20,	23,	Arithmetical Progr.
5,	15,	45,	135,	405,	1215,	3645,	Geometrical Progr.

256. Now, the vast increase which the latter series makes, compared with the former, is not the point on which I have to remark, but, in the first place, the difference in the relationship which the several terms bear one towards another. On looking at the first, or arithmetical series, you can scarcely say that any one of the terms is twice as many, three times as many, four times, or, indeed any number of times as many, as any other term; in short, no one term, scarcely, will help to produce, nor can it be the product of any of the others; or,

if, occasionally, such a relationship do exist between any of the terms, it is quite accidental. Far otherwise, however, is it with the terms which form the other series; amongst these, each term is a *factor* of all that come after it, and a *product* of each and of all that go before it; so that the whole series, from the first to the last, however long it may be, is linked together by a kind of proportion, one term being exactly three times, nine times, or some other number of times, greater, or less, than any of the others. And these, with several other interesting and very useful qualities, belong, not peculiarly to the series of terms that I have here laid before you, but to any other similar series, whatever their length, whatever the terms, or whatever the number by which, at each step, they may be increased.

257. Series of numbers increasing by geometrical progression afford problems similar to those of which we have spoken in the other mode of progression; that is to say, having any three of the properties of any series; as the first, the last, the middle term, or terms, the sum of the whole, the rate of increase, the number of terms; having any three of these properties, we can find the rest. But, again, these are not the important points at which I am aiming; the great matter, those points for which, almost alone, I have introduced this branch of arithmetic into my book, are now to be treated of.

258. On reverting to the geometrical series of terms before given, you see, that whilst 5 multiplied by 3 increases to 15, 135 multiplied, also, by 3, increases to 405. Now this 405, though so much more than 135, bears the same proportion, bears exactly the same proportion, to this latter number, as does 15 to 5; that is to say, each of the larger terms, or numbers, is three times as large as its preceding term; and, of course, the contrary, or in

more scientific phrase, the *converse* of this proposition is correct; that is, each of the smaller of these terms is one-third of that which immediately succeeds it; and the *proportion* between the respective terms subsists in this manner.

259. Now, a consequence of these relative proportions amongst numbers is this, that if you have a number, for which you would find what may be called a *relation*, in *another number*, and, that your wish is, that the relationship between the two numbers shall be of the same degree; that is, that it shall be equally near as the relationship subsisting between certain other numbers, which you have already before you: a consequence of these proportions amongst numbers is this, that you are hereby enabled, with great ease, neatness, and truth, to find out such relations. And this matter, simple as it may yet appear, is the greatest ornament, and may, perhaps, be justly pronounced to be the most valuable application of the art of arithmetic. Of the application we must speak hereafter; at present we must direct our attention somewhat more to the nature of these proportions, and to the mode of discovering, and of forming them.

260. Each series of numbers, whether in arithmetical, or in geometrical progression, has the same properties as any other series increasing in like manner, so that it is immaterial what numbers we take as subjects for our experiments and observations. For the present we will take the geometrical series before given, and here repeated: that is,

5, 15, 45, 135, 405, 1215, 3645.

You observe, here are seven terms, of which 135 is the middle term. Now, amongst the properties of these numbers, we find these: that the product of the middle term multiplied into itself, is equal to the product of the two extreme terms; equal, also, to that arising from the multiplication of the two next terms on either side of it; these are 45 and 405; and equal, also, to the product of the two terms next again to them, that is, to 15 × 1215; and, in short, to whatever length the series were extended, the same property would subsist amongst the terms; that is to say, that the middle term, multiplied into itself, would produce the same sum as would be produced by the multiplication of any two terms in the series, such terms being taken thus, in a sort of pairs, one on each side, at an equal distance from the middle term. And, further, be it observed, that as this is the case with every series, consisting of an *odd* number of terms, the same properties exist, and the same relationship also exists, in series that have an even number of terms, and in which, consequently, we find two in the middle: that is to say, the multiplication of these two terms produces, in like manner, a sum equal to that produced by any other two terms in the series, such two terms being taken on either hand, and at equal distance from the two middle terms.

261. The next property, in numbers of this description, on which I have to remark, is this; I have before stated, that any two adjoining terms, in any regular series, bear the same proportion towards each other, as do any other two terms, similarly situated; that is to say, turning again to the series on which we have been observing, 5 and 15 bear the same proportion towards each other, as do 15 and 45; as 45 and 135; as 135 and 405; and so on, through the series. Nor is this sort of relationship confined to terms immediately adjoining each other, but exists, in like manner, between those taken at similar distances from each other, whatever the distance may be: thus, if we take every other term, 5 and 45 bear the same proportion towards

each other, as do 15 and 135; and as do 45 and 405, and so on; or if you take the terms at a yet greater distance from each other, as 15 and 405, which bear the same proportion towards each other as do 5 and 135; as 45 and 1215, and so on. And thus it is, and thus will it always be in every geometrical series of numbers.

262. Here I find it expedient to state, that numbers employed for these purposes have their proper names. Generally we call them *terms*, merely; when we take *a pair*, or, as they are called by some, a *couplet* of terms, and would distinguish the first term from the last, we call the *first* the *antecedent*; that is, the *before-going*, and that which *comes after* is called the *consequent*; thus, in the two couplets last cited, that is, 5 and 135, and 45 and 1215, the terms 5 and 45 are *antecedents*, and 135 and 1215 are to be called *consequent terms*. And be it further observed, that as these two couplets are said to be *proportionals*, as the latter is *compared* to that which goes before it, so this may be conveniently called the *antecedent proportional*, and the other the *consequent proportional*.

263. In all these cases on which we have been observing, you will not fail to have seen, that the proportion subsisting between the corresponding terms, is of this nature; that, whilst in every instance, the *antecedent* terms in each couplet bear the same proportion towards each other as do their respective *consequents*, so, also, in the terms stated, you will see, that the *consequents* are either three times, nine times, or twenty-seven times, as large as their respective *antecedents*: as, to turn our attention again to the two couplets last cited, whilst 135 is twenty-seven times as much as 5, so 1215 is, also, twenty-seven times as much as 45. And this *twenty-seven*, being the *rate*, of increase, is called

the RATE, or more learnedly, the RATIO, of the proportion subsisting between the two couplets of terms. Again, the proportion may be spoken of in the contrary mode, the terms being reversed, thus, 135 and 5 bear the same proportion towards each other as do 1215 to 45; or 1215 is to 45 as 135 is to 5; in which cases, 27 is the ratio of *decrease*. As to the ratio, whether of increase, or of decrease, it might be ever so large, or ever so small; it might be a million times, or only a millionth part, as great; or it might be so much less; still, in proportional terms and couplets, all the properties of which I have spoken must and will exist.

264. To save the *words* which I have hitherto employed in stating these proportional numbers, and for the purpose, likewise, of presenting such statements more quickly, and more distinctly to the eye, arithmeticians are in the practice of employing a few points, which you will find it useful to be familiar with, and to which, therefore, I now call your attention.—On referring to the last paragraph, you find the following passage "135 and 5 bear the same proportion towards each other as do 1215 to 45; or 1215 is to 45 as 135 is to 5."—Now, by the use of the points which I am about to describe, all this is presented to the eye more readily, and quite as clearly, in this manner.—135 : 5 : : 1215 : 45; or, 1215 : 45 : : 135 : 5. To explain the points:

: *The two points mean* "IS TO;" *or* "IS IN THE SAME PROPORTION."

: : *The four dots, or points, mean,* "AS," *or* "SO IS;" *or may be read in any other words descriptive of that sort of relationship which subsists between couplets of numbers that are proportionals.*

So the foregoing proportional terms, translating the dots into words, are read thus: 135 *is to* 5, *as* 1215 *is to* 45; and so on. And this mode of stating such propositions, with the dots, we must henceforth generally employ.

265. Hitherto I have spoken of these proportional terms, only as of a regularly-continued series; that is to say, as in the series given, beginning with 5, and continuing regularly to increase by a ratio of 3. Nothing, however, of this sort is required to form proportional numbers: numbers or terms may continue to increase, or to decrease, in that continued, and uninterrupted order, in which case they are called, *continued proportionals:* or they may be interrupted, or broken, thus, 15 - 45 :: 405 - 1215; or, 2 - 14 :: 21 - 147; in which case they are called, *interrupted proportionals.*

266. I have shown, in paragraphs 258, and 263, that proportion in numbers consists in the increase, or decrease amongst them, being at the same rate, or in the same ratio: and in paragraph 259, I stated, that the mode of discovering numbers, which shall bear the same ratio towards each other, may be justly pronounced to be the most valuable application of the art of arithmetic. Let us, then, proceed attentively to the consideration of this mode, in order to become masters of a process of so much importance.

267. To make our experiments on the numbers, or terms, already cited: 15 is to 45 as 405 is to 1215. Now, what proportion does 15 bear towards 45? We remember, indeed, that the former term is one-third of the latter; but, suppose we knew nothing of this, how ought we to proceed, in order to discover the proportion, or ratio between the two numbers? The method is, to *divide* the larger by the

smaller, and the *quotient* is the *ratio*. And, were it our object, to find a number that should bear the same proportion to some other number, as 45 does to 15, the course we have to take is, to *multiply* that other number by the ratio, which is 3, and the product is the number sought: as, for instance, let the number for which we would find a similar relation be 405; this, multiplied by 3, produces 1215: but, did we want a number in *decreasing* proportion; did we want a number which should bear the same proportion to 405, as 15 does to 45, then we *divide* the 405 by the ratio, and the *quotient*, 135, is the number we would discover. So, then, in the first case, above stated, 15 : 45 :: 405 : 1215; and, in the second, 45 : 15 :: 405 : 135.

268. These, however, as before stated, are not the only points of resemblance, or of proportion, subsisting between numbers of this description. In the above we have only shown, that the two consequent terms bear a like proportion to their two antecedents; as, that 45 : 15 :: 1215 : 405; whereas 45 is, also, to 1215, as 15 is to 405; that is to say, the two antecedents bear the same proportion towards each other, as do the two consequents. So you will find it to be in every case. But to prove it in this instance, divide the larger antecedent term by the smaller, thus, 1215 ÷ 45 = 27; which 27 you will find, also, is the quotient, when the larger consequent is divided by the smaller.

269. Finding, as we thus easily do, the rate of increase, or of decrease, between any two numbers, and finding that this rate, or ratio, is, in proportional numbers, *the measure* between the antecedent terms and their consequents, we shall, as you will find, have no difficulty in discovering any proportional numbers that we may require.

270. Now of the four terms, 15, 45, 405, and 1215, suppose that we had only the three first, and that it were our wish to find the fourth which, term bears the same proportion to the third, as the second does to the first. The thing we have first to do is, to discover the ratio between the first and second terms, in order to which, as before shown, we divide the larger by the smaller, and this gives us the ratio 3, with which, by multiplying the third term, we produce the fourth; or, let the three terms be these, 405, 1215, 15; and let it be our wish to find a fourth, which shall bear the same relation to the 15 as 1215 does to 405. We divide and multiply as before, and the fourth term is produced. And in this manner, having *two* numbers, or *two* quantities of any kind, bearing a certain proportion towards each other, and *a third*, to which we would find a number or quantity that should bear a like proportion; in this manner do we proceed, and thus easily may we find the number we require; That is to say, thus may we proceed, when the smaller of the first and second terms will divide the larger without leaving a remainder, as in the cases we have thus far tried. But, observe, this is not always, nay, this is seldom the case; and it is never a thing to be calculated on. So that the proper mode is, to proceed in a method that will be clear, whatever the terms may be with which we have to work: and here is that manner of proceeding.

271. In our experiments on the terms 15, 45, 405, we divided, as you recollect, the second by the first, and then multiplied the third by the quotient arising from that division; that is to say, we took a fifteenth part of the second term, (45) wherewith to multiply the third; and thus did we find the term we were in quest of. Now, what would have been the consequence had we multiplied the third term by the whole, instead of doing it by a fifteenth

part, of the second? The consequence would have been, that we should have had a term, or number, fifteen times larger than that required. But this would be a matter of no difficulty; for it would, as you will see, be set right at once, and our purpose be gained, by dividing the over-large product by 15. Let us write this process down: $405 \times 45 = 18225$, and $18225 \div 15 = 1215$. Which 1215 bears the same proportion to 405 as does 45 to 15. AND THIS IS THE RULE; this, when the terms are properly placed, this MULTIPLYING THE SECOND AND THIRD TERMS TOGETHER, AND DIVIDING THE PRODUCT BY THE FIRST, avoids all the difficulties arising from the occurrence of fractions in the course of the process, and gives us, in all cases, any proportional terms we may require. This is the RULE of PROPORTION, commonly called the RULE of THREE; and, in their admiration of it, and in testimony of their sense of its great value, the learned of former times bestowed on it the name of GOLDEN RULE; a title which it richly merits, as you will see, when you become acquainted with its great and various uses.

272. It is almost superfluous to employ another word on this subject, and quite unnecessary to give in this lesson, any further examples, or to make any further experiments. I have stated that the Rule laid down is easily applicable to all cases, to every degree of proportion amongst numbers and quantities; and this will be seen in the ensuing lessons, which, under the names of Rule of Three, Single, Double, and Inverse; of the Chain Rule, or Compound Proportion, apply the principles of proportion to affairs of business.

THE RULE OF THREE;

DIRECT AND INVERSE.

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273. This is the Rule towards which I have been conducting my pupil throughout the whole of the preceding dissertation. And, that this rule, to say nothing of the other rules which immediately follow, and which grow out of the same Principle; that this rule is worthy of such a preparation, worthy of so careful a development of its Principles, will soon become apparent.

274. *Numbers,* which are expressed by *figures,* are employed to describe *quantities.* As, for instance, if we would describe, in writing, a quantity of any thing that is measured by the foot of twelve inches, the party to whom we would describe it, being previously acquainted with the length of the foot, requires only to be informed of the *number* of feet, which number we state in *figures.* So that *figures* describe *numbers,* and *numbers* describe *quantities;* and quantities, too, of every description, whether of weight, of measure, of extent, or of time.

275. This being the office, that is to say, the use of numbers, and having fully learned, in the former rules, not only this use, but how to join together, and to separate these numbers, in every possible mode, almost all that it may be desirable for you to learn further, is the method of duly proportioning quantities towards each other; as, for instance, suppose you have purchased a lot of goods, a piece of land, or any other species of property, for a certain sum, and that you wish to know, at what rate you should retail your purchase, in order to gain a certain sum by the whole transaction; or,

suppose you have to adjust the claims of a number of parties, whether legatees, partners, or creditors, to certain effects, or to their value in money; or, that, if, according to a certain scale, you have to lay in provisions, for a specified time, for a certain number of persons; or, that, having a certain piece of work to get done, requiring a number of men, and that it were your wish to know, according to a certain rate, or scale, with which you may be provided, what number of men ought to be employed, in order to have the work done within a given time; to solve problems of this description, that is, to *adjust proportions*, is almost all that you have now to learn; and these, such problems as these, and a countless number of others of a similar description, which are for ever occurring, you are enabled, by this rule, to treat with ease and success. Having stated this, let us proceed to examples; the several methods of treating which, I shall state as occasion and opportunity occur.

276. First example. A man dies, having, by will, ordered his effects to be sold, and legacies, amounting altogether to £755. to be paid: that is to say, to A he willed £200; to B £270 4*s*. 6*d*.; and to C £284 15*s*. 6*d*. But, after paying necessary expences, it is found, that the effects produce only £604; and the question arises. How much of this £604 should be paid to each of the legatees, in order that each may have his just proportion? And, first, what is A to receive, instead of £200?

277. The proportion in which the money is to be divided, is, of course, the same as that which the produce of the effects bears to the sum of the legacies named in the will; and the question, therefore, must be stated thus: As that sum which ought to be £755, proves to be only £604, what, according to the same proportion, ought £200 to be?

But, stated with the *signs*, instead of *words*, as taught in paragraph 264, the question would stand

£.		£.		£.
755	:	604	: :	200

and the rule, in order to bring out the answer, as shown in the last lesson, paragraph 271, is, "to multiply the second and third terms together, and to divide the product by the first;" and this, in the case before us, gives 160, which is the number of pounds due to the first-named legatee, A. Instead, however, of shares in the effects of a deceased person, these several sums might be shares in a partnership concern, or shares of creditors in an insolvent estate. And this rule would, with perfect correctness, adjust the claims of the several parties.

278. To return to the example: In fixing the first legacy, I purposely avoided the introduction of small money, or fractions of a pound, in order to keep the question as simple as possible. But these simple numbers, as before stated, are things not to be looked for in matters of real business; and, therefore, for cases in which fractions do occur, we must be provided with a suitable method; which method forms a part of the rule of three; and which, therefore, I now proceed to explain, and to join to the great, or main rule already laid down.

279. The first legacy is £200. and the question arising thereon, as before stated, stands thus; 755 : 604 : : 200. Now, you know, by the Rule, that we are to multiply the second and the third terms together; but, observe, if we were to reduce the third term, which now represents pounds, into shillings; that is to say, if we were to multiply it by twenty, and then to multiply this number of shillings by the second term, we should have, as

the product, 2416000, which, divided by the first term, would give 3200, a sum just twenty times as great as it ought to be; which excessive sum, as you will immediately perceive, arises from the circumstance of our having, in reducing it into shillings, made the third term twenty times larger than its just proportion; for the proportion was 755 : 604 : : 4000. But, mark; had we, on reducing the third term to shillings, *reduced the first term, likewise*, to the same denomination; had we done this, we should have preserved the proportion, and the answer would have come out correct; and correct, too, would it have been, had these two terms been reduced into pence, or into farthings; that is to say, if *both* of them had been so reduced, and the proportion between them had thereby been preserved. Hence, then, arises the *second* branch of the rule of which we are treating; that is to say, that *the first* and *third terms* are to be of *the same denomination.* And you must bear in mind, that if they be not so before you begin to work your question, the first thing you have to do is, to reduce these two terms to the same denomination.

280. There remains one other branch of this rule to be observed on here: you will find, if you have not already noticed it, that you cannot multiply sums consisting of pounds, shillings, and pence; or hundreds weight, quarters, and pounds, or any such compound terms, by certain large numbers; neither can you multiply large numbers by compound sums of this description. The consequence is, that when any of your terms in this rule consist of such sums, you must almost always reduce them into one even and simple number, or denomination, as you will see done in the case of the third term in the following example, which term, being £270. 4*s*. 6*d*. is reduced into pence; and, in order to balance it, and preserve the proportion, as I have just taught,

the first term is reduced in like manner. And then, the work of multiplying the second and third terms together, and dividing the product by the first, brings out the answer, not in pence, mind, not in the denomination into which you reduced the first and the third terms, but in pounds; which is the denomination in which you left the second term. And thus it is, that the answer, which is to bear the same proportion to the third, as the second term does to the first, always comes out in the same denomination as that in which the second term stands.

281. This is the third, and concluding condition of the rule. And the Rule of three, the whole principles of which are thus laid before you, now stands complete; that is to say, Having arranged your Terms, in due order, you observe which of them are, if there be any that are, *compound terms;* and, for the convenience of working, you reduce such terms into their *lowest denomination*, taking care, always, TO BRING THE FIRST AND THE THIRD TERMS INTO THE SAME DENOMINATION; then TO MULTIPLY THE SECOND AND THIRD TERMS TOGETHER, AND TO DIVIDE THEIR PRODUCT BY THE FIRST TERM; and THE QUOTIENT, arising from such division, WILL BE THE ANSWER, IN THE SAME DENOMINATION AS THAT IN WHICH YOU LEFT THE SECOND TERM.

282. This is the Rule of Three. Something remains to be said, as we proceed, on the methods of arranging the terms; or, as it is called, of stating the questions, previously to beginning the work. The doing of this requires thought. It is a fine and a pleasing exercise for the mind; similar to that which is required by many favourite games. But the attention it demands, even were it somewhat of a toil, would be amply repaid, by the power of

calculation which this rule gives over numerous questions in arithmetic which arise in the ordinary course of every man's affairs.

283. The Statement, and the Working, in the case of the second legacy, here follow. It will be well if you first trace them attentively over, and then, with your pencil, work the sum yourself; carefully comparing, when you have done.

```
 £.      £.          £.  s.  d.
 755  .  604  :  :  270 - 4 - 6
  20                 20
 ----               ----
15100               5404
   12                 12
------              -----
181200             64854 third term reduced to pence.
                     604 second term, for multiplier.
                  ------
                  259416
                 3891240
                --------
       181200 ) 39171816 ( 216 - 3 - 7 .2  Answer.
                362400 · ·
                ------
                ·293181
                 181200
                -------
                 1119816
                 1087200
                 -------
                 · · 32616 Remainder to be reduced into
                        20     Shillings, and then divided.
                  -------
                ) 652320 ( 3 shillings carried up.
                  543600
                  ------
                  108720 remainder to be reduced into pence
                      12
                 -------
               ) 1304640 ( 7 pence carried up
                 1268400
                 -------
                · · 36240 remainder to be reduced
                        4          into farthings.
                 -------
                 144960 remains, not containing the divisor
```

once, is, therefore, not a farthing. Its value expressed as the decimal of a penny is .2

284. This method of working, with regard to the reducing of the first and the third terms into pence, may be considered as the regular method. But though it is so, there are cases in which it may be departed from, as will be shown in the working of the next example; wherein, taking advantage of the circumstance of the pence being *six*, that is *half* a shilling, the first and third terms are reduced, not into pennies, but into sixpences, the shillings being multiplied by two, instead of by twelve; and, indeed, the first term is brought at once into sixpences, the number of pounds being multiplied by *forty*. This is a saving of figures; the proportion is preserved; and the answer is the same as it would be were these terms reduced, as in the last question, into pence. To go into all the modes of shortening such processes, would too much enlarge the book. Experience will suggest to the ingenious arithmetician, all that can be desirable in the way of shortening; and for him that has not numerous questions to work, here are the regular and safe methods, fitted alike to all circumstances.

285. The adding of the several shares together, as you see it done at the foot of the following sum, *proves* the correctness of the several workings. But we have not, in all cases, an opportunity of thus determining that correctness, and, as we ought, in every case, to be sure that we are right, a proper mode of proving a sum in this rule, is to reverse the question, and so work it, as it were, back again; as, for example, in the case of this next question; say, as £227. 16. 4$\frac{3}{4}$.2, or rather, in order to get rid of these fractions, as £227 16 5 is to £248 15 6, so is £604 to the sum to be found: which sum you will find is £755 and a farthing; this farthing being occasioned by the fraction borrowed above, for the purpose of making the pence even.

The statement and working on the third legacy stand thus.

```
£.       £.      £.  s. d.
755  :  604 : : 284-15-6
 40              20
-----           ----
30200           5695
                   2
                -----
                11391
                  604
                -----
                45564
               683460
               -------
   30200) 6880164 ( 227 - 16 - 4 ¾ .2
          60400
          -----
           84016
           60400
          ------
          236164
          211400
          ------
           24764
              20
          ------
        ) 495280 ( 16 shillings
          30200
          ------
          193280
          181200
          ------
           12080
              12
          ------
        ) 144960 ( 4 pence
          120800
          ------
           24160
               4
          ------
         ) 96640 ( 3
           90600
           -----
            6040 remainder; expressed as the
                 decimal of a farthing is .2.
```

Thus the Shares are

to A.	£160	0	0	
B.	216	3	7	.2
C.	227	16	4	.8
	£604	0	0	

being the sum which was to be divided.

286. E and F enter into business together. E embarks in the concern his fortune of £2570, and F £2180. At the end of seven years trading they agree on a dissolution of the partnership, and find their effects and money amount altogether to £8643. 15. 0. Now, of this sum, what is the portion of each partner; the money being to be divided according to the capital which they respectively embarked in the business? See the note below.

```
        £.       £.   s.  d.      £.
     As 4750 : 8643 - 15 - 0 : : 2570
                 20
              ------
              172875
                2570
              ------
            12101250
            864375
           345750
           --------
  475'0 ) 44428875'0 (93534 - 5 ½
          4275 ····   £4676 - 14 - 5 ½ E's share.
          ----
           1678
           1425
           ----
            2538
            2375
            ----
             1637
             1425
             ----
              2125
              1900
              ----
               225
                12
              ----
            ) 2700 ( 5
              2375
              ----
               325
                 4
              ----
            ) 1300 ( 2
               950
              ----
               350 = .7 of a farthing.
```

Here we have to find, in the first place, what was the whole sum with which the traders set out. This was £4750, the amount of their two fortunes. This sum being increased into the larger one named in the question, the share of each partner is increased in the same proportion, and the statement and the working thereof, on E's share, will stand thus,

287. In paragraph 284 I have spoken of shortening the process, and have given an example of one of the modes in which it is to be done. And here, on the second question of the partnership, is another example of this mode of proceeding. You will recollect, that according to the rule laid down in paragraph 279, you are at all times to be careful to preserve the proportion between the first and the third terms, without which care the whole process would be deranged, and the result, of course, would be erroneous. You were, in the passage referred to, taught, that if you multiply one of these terms, that is, the first, or the third, by 20, or by any other number, you must multiply the other in the like manner; and thus preserve the original proportion. But, as you may thus *multiply*, so may you also *divide* those two terms; and in this manner, when it can be done, you may lessen the labour, by reducing the number of figures: all which is very proper. You have, in the following example, an instance of this mode of proceeding, the first and the third terms being divided by ten, by the cancelling of, or cutting off, the cypher, at the end of each; and, could those terms be evenly divided by any other number than ten; could they be reduced to the smallest numbers, it would be equally correct, and would simplify the process, so to divide them. But, besides this shortening of the first and the third term, you will see, that instead of reducing the middle term into shillings, advantage is taken of the circumstance of the shillings in that term being 15, that is, *three fourths of a pound*, and the whole sum is brought into *fourths;* and then the quotient, according to the rule, comes out in *fourths* of *a pound*, as does the remainder also: which remainder, 50, being in crowns, I should reduce into shillings, and then divide by the first term, but seeing that 5 times 50

would not produce a sum large enough to be divided by the first term, I determine to reduce it at once into pence, and so multiply it by 60, the number of pence in a crown, and then, proceeding, as you see, with the work, find that this remainder gives us 6 ¼ d. and a small fraction.

```
        £        £    s.  d.        £
  As 4750  :  8643 - 15 - 0  :  :  2180
                 4
              -----
             34575 reduced into crowns.
               218 third term without the 0.
              -----
            276600
            34575
           69150
           -------
  475 )    7537350 ( 15868 No. of crowns
           475····   -----
           ----      £ 3967 - 0 - 6¼  F's share.
           2787
           2375
           ----
            4123
            3800
            ----
             3235
             2850
             ----
              3850
              3800
              ----
                50 crowns remaining
  Multiply by   60 to bring them into pence
              ----
             ) 3000 ( 6
               2850
               ----
                150 pence remaining
             by   4 to bring into farthings.
               ----
             ) 600 ( 1
               475
               ---
               125 = .2
```

288. Further to multiply examples, in this place, were almost useless, except for the purpose of pointing out to the learner the mode of stating, or, in other words, the mode of placing the Terms of questions. This is a matter, as I have before intimated, requiring some ingenuity, and one on which of course every thing depends, seeing that, having properly placed the terms, we are to multiply and to divide those terms agreeably to the places in which they severally stand.

289. In this matter, the mode of statement, the following question, being of a description somewhat different from the former, will be useful. If by 12 furnaces 8060 tons of Iron can be smelted in 6 days, how many tons can be smelted by the same number of furnaces in 21 days?

Now, our rule enables us, having three terms, or quantities given, to find a fourth. But here are already four terms. So, how are we to treat them; how are we to state this question? We, must note on which of the four terms the question turns; that is to say, *which of the terms are compared;* or, *which are to be compared* with *any other terms.* And these, the *compared,* and the *comparing* terms, are those which we are to use in the statement; and, for the rest, they are to be dropped, however numerous they may be in any such questions. As to the terms in this question, the 6 days, and the 21 days, are compared together; and, seeing that 8060 tons are smelted in 6 days, we are to find, how many tons, without any change in the number of furnaces, can be smelted in 21 days. On these three terms, then, the question rests. The number of furnaces being in both cases the same, nothing need be said about them: and the question may be stated thus; as 6 : 8060 : : 21; and the answer is 28210: which number bears, of course, the same proportion to 21, as 8060 does to 6.

290. Before, however, we proceed further in the mode of stating questions, it is necessary to remark, that besides those of which we have hitherto treated, there are questions of another description, coming within the operation of this Rule; questions which require a somewhat different mode of treatment, and which, therefore, it will be necessary to learn to separate from those of which we have hitherto spoken.

291. Of the questions we have already employed as examples, you will observe, that the three first allot, in just proportions, to three several persons, a sum of money, which, being divided amongst them, leaves for each of the parties, in the same proportion, a sum *smaller* than that intended for him; whilst the next two questions, being employed to divide between two persons, a sum which has been *increased*, gives, according to the rate of increase, a *larger* portion to each of the parties. The questions being of this description; the sum to be divided by the first of the questions, being *less* than the intended sum, they are said to be of a description in which "*less requires less;*" whilst, in the two latter questions, the sum to be divided being *more* than the original sum, and each share, in consequence, being to be *more*, these questions are said to be of a description in which "*more requires more.*" That is, in short, the proportions between the respective sums being in *right* proportion; the questions being of that description in which "less requires less, and more requires more," the proportions are said to be *direct;* and, so, all questions of this sort are said to belong to the Rule of Three Direct; a rule which comprehends by far the greater portion of questions arising in the usual course of affairs.

292. However, seeing that such a distinction is necessary, as this of *direct proportion*, there is, of

course, a class of questions in which the proportion is in some way different from this. If 12 men require 100 quartern loaves in 30 days, how many such loaves will 18 men require in the same time? This, like all the former, is a question in direct proportion; for here, *more requires more*; that is, 18 men require more loaves than do 12 men, in the same space of time. And, if we put the converse of this question; that is, if we say. If 18 men require 150 loaves, how many will 12 men require in the same time? It is, also a question for the rule of three direct. But, if, on the contrary, the question be, If 12 men require 100 loaves in 30 days, what number of days will the same quantity serve 18 men? Then the question is of a very different nature. For, the quantity of loaves being fixed, it is obvious that a *greater* number of men will require *less* time in which to consume them. Thus, then, we have a case in which *more requires less*." And, the number of loaves being the same, were the number of men greater, the time, of course, would be still less. And similar to this would be the question, were it as to *the time* that a fixed quantity of provisions of any sort, would serve a greater, or a smaller number of persons. Were the number of persons smaller, the time which the provisions would serve them, would, as you see, be longer; and this would be a case in which "*less requires more*."

293. Again, in the same indirect proportion must the answer be, were the question relating to a quantity of work to be performed by a given number of persons, instead of a quantity of provisions to be consumed. As, for instance, were the question, If 12 men can do a certain piece of work in 150 days, in how many days will 18 men do the same? Here the answer must be, that the more men would require the less time. And, similar, in

principle, are the two following questions. If 50 yards of carpet one yard wide, will cover a certain floor, how many yards of carpet three fourths of a yard wide will cover the same? Here the less width requires the more yards. In the relations of price and of quantity, too, the same seemingly *indirect* proportion (for it is only *seemingly* so) sometimes arises, as in the question which follows, in which the *greater price* gives the smaller quantity. If, when wheat is 7s the bushel, we have 6*lb* of bread for a shilling, how many pounds ought we to have for the shilling when wheat is 10s 6d the bushel? we must, of course, have less weight for the same money; seeing that the price of the article is, in the latter case, advanced.

294. Thus, these five latter cases are, in a certain sense, *contrary* to proportion, and they are, therefore, said to belong to the Rule of Three Inverse; the word "*inverse*" coming from the latin, *inversus*; that is, upside-down, or contrarywise.

295. With regard to the mode of treating questions in Rule of Three Inverse; We have seen, that when the proportion is direct, we state the question in a direct, and in something like the natural manner in which the terms present themselves to the mind; saying, for instance; If 18 men require 150 loaves, how many loaves will 12 men require? or, If 12 men require 100 loaves, how many will 18 men require? (the time in these cases being the same, need not be stated) And, having so stated the questions, we have seen, that to multiply the second and third terms together, and to divide the product by the first, brings forth the proper answer. BUT, WHEN THE PROPORTION IS INVERSE; HOW ARE WE TO PROCEED? It is pretty clear that in such case we must *invert* the terms; that is, place them *contrarywise*, the first last, and the last

first; *or, we must alter our mode of treating them.* And this latter, which is generally adopted, being, as I think the more eligible course, shall have our preference.

296. In all questions in the rule of three, whether direct or inverse, you will find, that there are *two terms* laid down, as bearing a certain proportion towards each other, and that the question is asked on *the other term:* Now, *this odd term,* which *wants its proportional, or mate,* is always, in the mode of statement which we have determined on, to stand *third,* in the statement of the question. Having thus fixed on the *third* term, you can scarcely fail to place the other two correctly. Calling to mind, that the object of your proceeding is to find a fourth term, that shall bear the same relation towards the third, as the second does to the first, you will place the term *first* which corresponds with the *third;* and the middle term will then correspond with, and ought always, whether the question be inverse or direct, to describe quantities of the same kind of things, as the fourth is to describe, whether those things be money, weights, measures, or any other thing.

297. Having thus stated your Terms, you consider, whether the proportional sought be *direct,* or *inverse;* according to the explanation just given. And, having determined this point, you work accordingly. If the proportional be to be *direct,* you proceed to find it according to the rule already so frequeutly laid down. But, if it be to be *inverse,* that is to say, if it be one of these questions in which more requires less, or less requires more, you then, having reduced, if necessary, your first and third terms to the like denomination, proceed to multiply, not the second and third together, but THE FIRST AND SECOND, AND

TO DIVIDE THE PRODUCT BY THE THIRD: and thus do you find the answer. AND THE REASON, why this mode of proceeding brings out the proportional sought, is manifest: For, let us look again at the first question of this description, stated above, and we see, that if 12 men consume a certain number of loaves in 30 days, it is evident that the same loaves would serve *one man twelve* times as long; that is 360 days. Here, you see, we have multiplied the first and second terms together, and have found how many days the proposed quantity would serve *one* man; but the question is, how many days that quantity would serve 18 men. Is it not evident that it would serve them *one eighteenth* part of the time? *Divide the time*, then, that is the 360 days by 18, which is the *third term*, and you have the proportional sought.

298 For the same reason do we thus proceed to find the number of yards of carpet three-quarters wide, required to cover a floor which takes 50 yards of yard-wide to cover it. The question stated stands thus; if 50 yds. : 1 yd. : : 3 qrs. Here, reducing the first term to the same denomination as is the third, that is, to quarters of yards, we should multiply it by the second, but that second being only one, which has no power of multiplication, the question stands thus; If 200 quarters of a yard will cover the floor, how many times 3 quarters will do the same? The answer is found, of course, by dividing by the 3, which is the third term.

299 To lead you to the practice of distinguishing questions, as they may occur in business, one from another, in direct, or in indirect proportion, I shall here intermix them. And, not only shall I intermix them as to direct and inverse, but, for the purpose of exercising the attention and the judgment of the learner, somewhat in the manner in which

these faculties are to be exerted in affairs of actual business, I shall mingle, along with questions as to quantities, and prices and numbers of things; with these I shall occasionally intermingle a question on Interest of money; on Fellowship, Brokerage, Barter &c. accompanying any question that may seem to require particular directions, with such directions. And I doubt not but, by a careful attention to these exercises, you will soon become a proficient in this Golden Rule of Arithmetic.

QUESTIONS, &c.

In Rule of Three, Direct and Inverse; in Interest, Discount, Brokerage, Fellowship and Barter.

(1) If a silver cup weighing 11 oz. cost £5 16 0 what will a tankard, weighing 28 oz. cost, at the same rate?

(2) When 17 horses are required to draw, up a certain brow, a steam boiler weighing $15\frac{1}{2}$ tons; how many horses will be required to draw a boiler of 35 *tons* 11 cwt. up the same brow?

(3) A certain pipe will discharge from a reservoir, in 17 hours, 315 hogsheads of water; how many hogsheads will the same pipe discharge in $47\frac{1}{4}$ hours?

(4) Two merchants, C. and D. carrying on business in partnership, find, at the end of their engagement, that they have gained £7305. 18. 3., which is to be divided according to the capital embarked by each; C's capital was £13850, and D's £11730; what is the share of the gain for each merchant?

(5) Three men can forge a chain cable 95 *fathoms* long, in 21 days; what time will these men require to forge a similar cable 125 *fathoms* long?

(6) If 1572 *gallons* of water will serve 25 men for 84 days; how hany *gallons* will serve the same number of men 215 days?

(7) When 75 loaves are provision for 13 men for 12 days; how many days would the same number of loaves serve 23 men?

(8) If 29 barrels of beef, each containing 200 pounds, will serve a ship's company of 26 men, on a certain voyage; how many barrels will serve 47 men during a voyage of the same duration?

On proceeding to work this sum, you will perceive, that you will have to begin by ascertaining the whole weight of the beef allowed for the 26 men; which, of course, is to be done by multiplying the weight in one barrel, by the number of barrels. Having found the quantity for 26 men, you will know how to state and to work, in order to find that for 47 men.

(9) If 86 *cwt.* 1 *qr.* 16 *lb.* of biscuit will serve 49 men on a voyage of 92 days; how many days will the same quantity serve 63 men?

(10) When 16 men can build a wall, of certain dimensions, in 23 days, how many men will it require to build the same wall in 12 days?

(11) If 1935 yards of cloth, ell wide, will clothe 360 men, how many yards, of yard and a half wide, will it require to clothe the same?

State, If 5 quarters take 1935, what will 6 quarters take?

(12) A Broker has sold merchandise to the amount of £4753. 7. 6., on which he is to charge a commission of a half *per cent;* that is 10s on each hundred pounds: what is the amount of commission?

This *per cent*, is in constant use in business, and must be explained; and no better opportunity than the present can occur. *Per* is the latin preposition answering to our *by*; but it is used, likewise, to express *for*, and *in*. And in some one of these meanings it is, as I have said, in constant use in business: and occasionally so used, also, in scientific disquisitions. *Centum*; or, cut short, *cent.* is the latin for *a hundred*, or *the hundred*; so that, when we say so much *per cent*, we say, so much *in*, or *for*, or *by* the hundred; as in the above question, which is to be stated thus, as 100. is to have 10s, how much is £4753. 7. 6. to have? and the answer will, of course, come in shillings.

There is another matter connected with this phrase *per cent*, which it is important to have well understood; and, as I have found persons of passable education deficient in the knowledge of it, I must make it clear. We speak of so much *per cent*; as, *one per cent*, *two per cent*; *two and a half*, *three*, *five*, *seven and a half*, *ten*, *twenty*, *and thirty*, *per cent*; and so on. Now all these mean, *one*, *two*, *two and a half*, and so on, *in*, or *for*, or *by*, *the hundred*; that is to say, a hundred of the things spoken of; be they what they may. When applied to *money* of this country, it means a hundred pounds sterling; which word, *sterling*, we use to distinguish the pound in *money* from the pound in *weight*. *Two per cent*, then, speaking of money, means 40s. *in*, or *for*, or *by* the £100. 2½ means two pounds and half a pound, that is £2. 10. 0; 7½ means £7. 10. 0; 7¾ is £7. 15. 0. So that, a *half per cent*, speaking of money, means half a pound, or 10s.; and not, as I really have seen some persons fancy, half of the hundred: for half of the hundred is, of course, *50 per cent*, and so is it described. The usual commission charged by bankers in the country, for the exchange and transfer of money is a ¼ *per cent*; that is 5s. for the hundred pounds; so 2s. 6d. *in*, or *for*, or *by* the hundred pounds is called an *eighth per cent*.

In speaking of other articles than money, as of goods that lose weight, or otherwise diminish in value by keeping, we say they *lose so much per cent*, naming the amount of the loss in every hundred; some few articles, indeed, as *salt*, *gain* by being kept, in certain situations; and then it is said that the *gain is so much per cent*: whether, by the word "*cent*," we mean the hundred *bushels*, the hundred *tons*, the hundred *pounds*, or the hundred *articles*. In speaking of the profits, or losses, in trade, we commonly say, that they are so much *per cent*, meaning, merely, *at the rate of* so much in the hundred pounds, without any reference whatever to the amount of purchases, or sales.

Thus applied, *per cent*, is a very useful and very proper phrase; being neat, compact, and universally understood by experienced people, all the world over. And there can be no impropriety in using it, as it frequently is used, in chemical, mechanical, or indeed, in any *merely physical* calculations, and statements: but, to apply it to human beings! To talk of so many *per cent* being afflicted with some fever or other calamity; to talk of so many *per cent* dying! or, indeed, to apply the huckstering term at all, in any shape, to any thing immediately appertaining merely to the persons or condition of beings so sensitive, with capabilities, and with duties so vast and so various, and with aspirations so exalted and so refined; to apply this *truckster's* phrase to such beings, as it has sometimes been of late applied, by individuals of a certain school of economists and would-be philosophers; what does this evince, but great and unfeeling insolence, united with innate vulgarity of disposition, and shallowness of understanding.

To return to the foregoing question (12) which is to be stated thus,

£100 : 10s. : : £4753 7s. 6d.

(13) What is the brokage on £15864. 10s. at a quarter per cent?

(14) A certain Common is about to be enclosed from the occupation of the common people, and divided amongst the landowners of the township, in the proportion of their several possessions: The Common contains 173 acres, and the other land of the township is 1695 acres. Now what share of this Common will 'Squire Graspwell, a stock exchange broker, have, his land in the township being 358 acres?

(15) If the floor of a drawing-room require 120 yards of Brussels carpet $\frac{3}{4}$ wide; how many square yards will there be in a Turkey carpet, large enough to cover the floor?

Turkey carpets are made altogether in a piece. A square yard is merely a square piece, measuring a yard each way. So that this question is, how many yards of 4-fourths carpet will cover this floor?

(16) A dealer in carriages is in treaty with a dashing draper, for a curricle in exchange for cloth. The curricle is offered for 100 guineas, in *money*, but in barter for cloth the dealer demands 125 guineas. Now, how much must the draper add to the money-price of his cloth, which is 19s. per yard, in order to be even with the carriage man?

If 100 gs. : 125 gs. : : 19s.

(17) A cheese-factor is bartering with a corn-merchant, a fine dairy of Cheshire cheese, in exchange for barley. The weight of the dairy is 5 *tons*, 3 *cwt*, 2 *qrs*. and the price 65s. *per cwt.* how much barley, at 4s. 6d. per bushel, must be given for the cheese?

Find the value of the cheese, by compound multiplication; and then state, If 4s. 6d. purchase 1 bushel, what will the value of the dairy purchase?

(18) If three *yds.* and seven-eighths of cloth, seven-fourths of a yard wide, will make a suit of clothes, how much cloth of a yard wide ought the tailor to have, in order to make a similar suit?

It would be tight work to restrict your tailor to the precise quantity; because he could not cut out so advantageously from a narrow cloth, as he could out of a broad one. But a question of this sort has its use; and it is to be thus stated, if 7-fourths : 3 & 7-eighths : : 4-fourths; when reduced, the terms stand thus 7 : 31 : : 4, the first and the third terms being quarters, and the middle term eighths of a yard. And, in eighths, therefore, the answer will come.

(19) If 8 men can cut down a field of corn in $3\frac{1}{2}$ days; how many men will it require to do the same in a day and a quarter?

The answer to this question being to give us the number of men, may say, so many men and a fraction; as, 22 and two-fifths, or some such thing. Now, although it may be very well to talk of four-fourths, and two-fifths, and three-eighths of a yard, or a pound, yet it is not so seemly to talk thus of splitting up men into fractions. You will recollect, however, and console yourself with the recollection, that the splitting up goes no further than the figures, arising unavoidably out of useful questions of this description.

(20) If I have the use of £165 of my friend's money, for 5 months, how long ought he to have £93 of mine, to requite the favour?

(21) What is the heighth of a steeple, which, when the sun is up, casts a shadow 317 *feet* along the ground, whilst a staff, standing 8 *feet high*, directly upright, on the same level, casts a shadow 5 *ft.* 7 *in.*?

By this method you may ascertain the heighth of a tree, of a pillar, or of any such object without actually measuring it; that is, by setting up a staff, the length of which you know, measuring the shadow which it casts, and then, as the shadow is to the length of the staff, so is the shadow of the object to the heighth thereof.

(22) A ship's company of 52 men have provision, in biscuit, for 30 days, at 1*lb.* 14*oz.* *per* man, *per* day: what must be the allowance to each man *per* day, in order to make this stock serve the company 50 days?

(23) A garrison of 3729 men, having provision sufficient for 11 weeks, at the rate of $1\frac{1}{4}$ *lb.* beef, $1\frac{3}{4}$ *lb.* biscuit, and $2\frac{1}{2}$ *quarts* of water, per man, per day; to what quantity of each of these articles must the men be reduced, in order to make the supply serve them for 16 weeks?

Here will be three statements, and three workings; an answer being required for each of the articles. The first statement will be, 11 : $1\frac{1}{4}$: : 16. You will consider whether the proportion be direct, or inverse?

(24) What is the interest, that is, the charge for the use of £3150 for a year; if the interest on £100 be £4. 15s. for the same time?

This is a very simple question; to be stated thus, 100 : 4. 15. 0 : : 3150. The question which follows is a little more difficult. However, a moment's reflection will tell you, that having ascertained the interest for *one* year, you have only to multiply that interest by 5, the number of years, and, to the product, to add half a year's interest, and then you have the answer. *Per cent.* I have explained; *per annum*, or *per ann.* is *by the year*, *annum* being the latin for *a year*.

(25) At $4\frac{1}{2}$ *per cent. per annum*, what is the interest on £5217 for $5\frac{1}{2}$ years?

(26) What is the compound interest on £5217, for $5\frac{1}{2}$ years, at $4\frac{1}{2}$ *per cent. per annum*?

This, you will see, is the same question as the last, except that I have made it a question of *compound* interest; that is, *interest, on* *interest*, added to the principal sum, and accumulating every year. As, for instance, if the interest on the above-named sum of money for one year be £234 15s. $3\frac{1}{2}$d., *compound interest* means, that this interest, not being paid to the lender, becomes a *new*, or *additional* loan, to be *added* to the former principal sum; which principal, being thus increased for the second year, the interest is to be calculated thereon, by an entirely new statement and working: the third term being the original loan augmented by the year's interest. The result of this working, that is, the interest for the second year, will then, again, be to be added, and a new statement, and new working take place, for the third year; and, of course, for each of the succeeding years, until you come to the half year, when you will state if £100 produce 45s. that is, half a year's interest, what will the augmented principal sum produce? And thus do you find the compound interest on any sum, and at any rate per cent. per annum, for any number of years.

(27) What is the compound interest on £13620 for three years and nine months, $3\frac{3}{4}$ *per cent*, per annum?

To find the interest for the three quarters of a year, you will perceive, that you may adopt one of these two methods; that is, you may go on with the statement and working, to a fourth year; and, having ascertained that, by deducting from it *one quarter*, you have the interest for *the three quarters* of the year: the other method is, that which is pursued in the former question; namely; take the interest on £100 for the three quarters of a year, as the middle term, and state the question thus, if £100 : £2 16s. 3d., what will the augmented principal sum produce?

(28) What is the compound interest on £5217 for $2\frac{1}{4}$ years; interest at $4\frac{1}{2}$ *per cent. per annum*. payable half yearly?

Here is the same principal sum, and same rate of interest, as in questions 25 and 26, but for just half the time, and the interest made payable every half year; which, is, in other words, a stipulation that the interest shall be calculated, and added to the principal, at the end of each half year; and not at the end of the year, as in the former case; a mode of reckoning which, as you will find, makes a difference in favour of the lender of the money, of no less than £2 12s. $9\frac{1}{4}$d. in the first year; a trifle which people who put out money to interest, are by no means in the habit of disregarding; and the consequence is, that the interest on money so hired, is generally reckoned and added to the principal at the end of each half year, and so it goes, rolling on, enriching the already wealthy lender, and impoverishing the indiscreet, or unfortunate borrower.

The statements will, of course, be to be made in this manner, as £100 is to half a year's interest, that is, to £2 5s. 0d., so is the principal sum lent, to its interest. And so you go on for each half year, increasing the principal each time by the addition of the interest.

(29) What is the compound interest on £13620, for two years, at $3\frac{3}{4}$ *per cent, per annum*, payable half yearly?

This question, as to principal and rate of interest, is the same as that numbered 27. And thus I use the same sums, nearly, in order that the difference between calculating interest yearly, and half yearly, may be clearly seen. This difference will be manifest on a comparison of the answer to this question, with the result of the second operation on question 27.

(30) What is the interest on £4217 12s. 6d., for 60 days, at 5 per cent, per annum?

Five per cent per. ann. That is, £5 for £100, for the term of 365 days. But we would know, what is the interest on this money, at this rate, for 60 days? Let us find the interest on £100 for this time, and then we shall find the rest. If 365 days have £5, what will 60 days have? Suppose the answer be, that 16s $5\frac{3}{4}$d is the interest of £100, at this rate, for 60 days. We may then say, If £100 : 16s $5\frac{3}{4}$d :: £4217 12s 6d, and this brings the answer to the question.

In like manner, may you find the interest on any sum, at any rate, for any number of days; and this is the Rule-of-Three method, the basis of all the other methods of computing such things. There are shorter methods; but, besides that it would take a considerable space in the book to describe them, my pupils would, I hope, forget them, in the pursuit of more valuable knowledge, and in the practice of more useful occupations than that of calculating interest of money; for the doing of which, there are Tables ready constructed, for those who have frequent occasion for such things. Another question, or two may be useful.

(31) What is the interest on £3715, for 54 days, and for 73 days, at $4\frac{1}{2}$ per cent, per annum?

These questions on interest are useful, not only because there is no settling affairs of our own, nor those of others, without an occasional recourse to them; but they are useful, too, in this place, because they afford very fine practice for the learner, in this most valuable rule. I must not, however, close this series of instructions on the reckoning of interest, without giving, on the space I have here to spare, some intimation of my serious disapprobation of the practice of lending out money on such terms. The practice is so general, that we think not of the consequences, and have all forgotten that it has ever been interdicted. Were accumulation, indeed, the sole end of our existence; did virtue consist in "the heaping up of riches," and true piety in the worshipping of them and their possessors, we should all now be right, in this country; the wisest, the most virtuous, most pious people that the sun ever shone upon; and the "grinding of the faces of the poor," by the already over-grown capitalist, would be meritorious. But, as wealth will neither save its possessors from private, nor from public calamities; as it will secure to a people neither power nor freedom; nor health nor happiness to an individual; but, on the contrary, as it invites the conqueror, and avenges the wrongs of those from whom it is unduly drawn, by afflicting the greedy with numerous ailments, both of body and of mind; as it is manifestly the will of the Most High, that such shall be the consequences of excessive accumulation; so has usury, that is, "the lending for gain," which is the great means of accumulation, been forbidden, and held in well-merited abhorrence, by the wise and the good, in all ages and nations.

(32) What is the present payment on £6075 10s. payable, that is, due in twelve months, discount at the rate of 5 *per cent, per annum*?

Now, mark, this *discount* is very like *interest;* so like, indeed, that they are very commonly confounded, and, as *interest* is a little *more* than *discount*, so, those who *pay* money, generally avail themselves of the near resemblance, and take that which is a little more advantageous to them. There is, however, this difference; INTEREST is a charge for the use of a sum of money, and is to be paid AFTER *the use of it has been had*, at the stipulated time; whilst DISCOUNT is an allowance made for it, on its being paid BEFORE it becomes due: that is to say, it is a payment of interest beforehand; which payment, therefore, according to the rules of usury itself, ought to be something *less* than when it is deferred. The difference, for instance, between interest and discount, on £100, payable in one year, at 5 per cent, per annum, is this; the interest to be paid at *the end* of the year, being £5, the discount paid on *the commencement* is £4 15s. 2$\frac{3}{4}$d.: that is 4s. 9$\frac{1}{4}$d. less: being exactly the interest on the discount, for the time that it is paid in advance; and this difference, as you see, amongst those who love money, is a thing not to be overlooked.—To find this discount on £100 payable in a year, at 5 per cent. per ann. according to our rule, you state it thus, If £105, due after the lapse of a year, may now be paid by £100, what is the present worth of £5? And the foregoing question, that is, No. 32, is to be stated thus,

If £105 : £100 : : 6375 10s 0d.

(33) What is the present payment for £5760, payable in *half a year*, discount at 7$\frac{1}{2}$ *per cent, per annum*?

Do not fail to observe, that the question here is, not as to the amount of money to be paid a year before it becomes ude, but only half a year before; we must, therefore, take only half of the rate per cent per ann. that is £3 15s, and adding it to £100 for the first term, we state the question thus,

As £103 15s due in six months, may now be paid by £100, what is the present payment for £5760?

OF

DOUBLE RULE OF THREE,

Or Rule of Five;

AND

OF COMPOUND PROPORTION,

Or the CHAIN-RULE.

301. The proportional numbers of which we have yet treated, exist in pairs, or, as we have called them, couplets; and the business of the Rule of Three, as has been stated, is this; that having one pair of terms, and a single, or odd term, we thereby find a fourth, which, bearing a certain proportion to the others, completes the second couplet. And this, which is almost the simplest office of the rule of proportion, is called, when it is to be distinguished from more complex operations of a similar kind, and of which I am now about to treat; this more simple form of the rule of proportion is called, Single Rule of Three.

302. As there is *Single*, so you will justly infer that there is DOUBLE RULE OF THREE. And, as the office of the single consists in finding a fourth term, and thus completing a single pair of proportional terms, so that of the double rule, is to find a sixth term, which shall bear a required proportion to five terms previously ascertained. For example, the questions which this rule determines are of this nature;—If 12 men can build a wall 200 yards long, and 16 feet high in 20 days, how many men can build a similar wall, but 300 yards long, in 30 days?

Or; if a family of 14 persons expend £42 per month, of thirty days; how much, at the same rate, will a family of 21 persons spend in 45 days?—Questions of this sort, and a great variety of others, having, as these have, *five* terms, which five terms *form an essential part of the question*; with questions of this sort, it is, that the Double Rule of Three; or, as it is sometimes called, the Rule of Five, has to deal. And it is by a sort of blending of several terms together, into two distinct sums, and a division of one of those sums by the other, that the question is resolved: much in the same manner as in the Rule of Three, in which, by multiplying, we blend certain of the terms together, and divide the product thereof by the other term, and so obtain the term sought.

303. The questions I have stated above are of that very simple form, and I have purposely chosen them so, in order that you may see the process more clearly; they are of so simple a form, that you can answer them without working them with a pen: and this simplicity will enable you to see, that the process I am about to describe, leads you to the right answer. Let us, then, work the second of them, and trace the working in such a manner, as to lead clearly into the principle on which the rule is founded, by which rule such questions as these are stated and worked. The question is; If a family of 14 persons expend £42 in 30 days, how much, at the same rate, will a family of 21 persons spend in 45 days?

304. Now, in the first place, let us see how much £42 allows to each of the 14 persons. We find that it is £3 for each. Next, this money is to serve each person 30 days; and this, we find, is 2s. per day for each person; and this we have found, you must bear in mind, by dividing the money, which,

by the very words, and according to the sense of the question, is to be thus divided, amongst the parties and amongst the 30 days. And so 2s. *each day, for each person; is the portion.* But the thing we want to know is; how much 21 persons should have in 45 days? Give to each of the 21 persons 2s. for each day; that is, multiply the 2s. by the number of persons, and then by 45, the number of days, and, of course, you have the sum required, which is 1890s. or £94. 10s. How easy, then, is this matter; how very simple this *double rule,* when it is properly looked at; we have merely divided and multiplied, and here is the answer.

305. But, in this example we divided first by 14, and then by 30. I need not say, that it would be the same thing had we divided first by 30, and then by 14; but it may not occur to the learner that instead of doing it thus by two processes, had we divided the money at once by the *product* of these two numbers, that is, by 420, the result would have been the same; and, again, it may not occur to him, that instead of proceeding in the order in which we have done, that is, *dividing in the first instance,* and *multiplying* in the second; it may not occur to him, that, were we to reverse this order of proceeding, that is to say, were we to *multiply first,* and to *divide afterwards*; it may not occur to him that the answer would be equally correct. Let us, however, try this matter.

306. The terms with which we multiplied are these, 21 and 45; and with these we multiplied the money; but we did so *after* it had been divided by 14×30. Now, however, we propose to multiply it *before,* and to divide it *afterwards*; and the process will stand thus, £$42 \times 21 \times 45 = 39690$, which £39690 $\div 14 \times 30 =$ £94. 10s. which is the same result as that to which we were conducted by the other process.

And this is the approved and established method, of the merits of which I shall hereafter have to speak.

307. The *principle* on which stands the mode of proceeding in this rule is this: we have £42 given as the allowance for 14 persons for 21 days; that is, 14×21 times as much as the share of one person for one day: the share for one person for one day is, therefore 2s.; and, if we multiply, as we did in the former case, this 2s. by 21, we have one day's allowance for 51 persons; which is something towards an answer to our question; and, then, again; if we multiply this product by the number of days, that is, by 45, we have the complete answer; that is to say, if we thus multiply the 2s., we have the answer; but, on the contrary, if we thus multiply the £42, which is 14×30 times as much as 2s., we then, of course, have 14×30 times as much as we seek for; we have, in short, as I have shown above, £39690; which excessive sum, however, as you have seen, is easily reduced to the right amount, by being divided by the 14×30. And this is the double rule of three; this discloses all the reason, all the principle, all the science that there is in this apparently formidable rule; with which principle, having once made your mind familiar, you may master and play with the rule, and with all questions that come within its operation, as you would play with the lightest object of your amusement.

308. But, although we may, for ourselves, and when the calculations are only for ourselves, dispense with all formality of statement, and almost make play with our calculations, yet, clearness of statement, and regularity in our method of proceeding we must observe, if others be to look at, and to understand our work; and, besides that it is a great means of avoiding error, we must observe such form

if we ourselves, would, at any future time, understand our own work. Therefore, let me here engage your attention to the proper mode of stating questions in this rule, and yet more explicitly than I have hitherto done, to the more approved mode of working them.

309. First; with regard to the mode of *statement*. In the questions now under consideration there are *five* terms that are essential to the statement. In the foregoing rule we have dealt with questions that have only *three* such terms, and the considerations by which we were guided in the statement of those three terms, will serve, with very few additions, not only to guide us in the statement of questions of *five terms*, but in the statement, also, of questions composed of yet more numerous terms; which questions come under the rule of compound proportion; or, as it is called by some, the chain-rule; owing this name, no doubt, to the manner in which the several terms are linked together.

310. As the considerations by which we were governed in stating the terms in rule of three, are to serve as the ground-work of our statements in this somewhat more complex rule, we cannot do better than, after reading paragraph 282, turn to, and read that numbered 290. In the latter paragraph we have a question on the quantity of iron that may be smelted under certain given circumstances; and, in it, we have three essential points; expressed by the terms of which the statement is formed: let us now, by the addition of two other points, or conditions, make it a question for double rule of three. Thus;—if, at 12 furnaces, 30 men can smelt 8060 tons of iron in 6 days; how many tons can 45 men smelt in the same furnaces in 21 days: it being understood that the quantity of iron smelted will increase at the same rate as the number of men and the number of days are increased?

311. Now you will recollect, that this question, when it had but three terms, was stated thus, 6 : 8060 : : 21 ; that is, if 6 days will smelt 8060 tons, how many tons will 21 days smelt? We found that, in this time, 28210 tons might be smelted. But this, we also found, could be done by 30 men: and it is asked, with the same means, and in the same time, how much 45 men can smelt. Hence, then, arises the second question; which, when stated in form, stands, thus, 30 : 28210 : : 45; that is to say, if 30 men can smelt 28210 tons, what will 45 men smelt? To which question, the answer, as you will find, if you work the sum, is 42315. And this, you see, is finding the answer by a double statement, and by a double working; and, hence the name of double rule of three. In this method, that is, by a double statement, and double working, you may state and resolve all questions in proportion, having *five* terms, for which you want a sixth. However, before I leave this matter, it may be well to apprise you, as it may render the statements less difficult, that it is not of any consequence, in this, nor in any such questions, whether you put the terms thereof in this order; making first a statement of the *days*, and then of the *men*, or you put the first of these last, and the last first; you may just as well say, 30 : 8060 : : 45; and, having worked this, and brought out the answer, which, observe, is 12090, you take *this answer* for the middle term of the second question; thus, 6 : 12090 : : 21; that is, as 6 days are to 12090 tons, so are 21 days to the number of tons to be found; which number, as found by the other method, is 42315.—So much for the working of these sums by two statements.

312. As to the mode of proceeding, in order to do them by *one* statement, and by one process of working: We have seen above, in the question of

a family expending a certain sum in a certain time, how exactly, and how easily, we find what ought to be the expenditure of a greater number of persons, in a longer time; and this, too, by a very simple process of multiplication and division. The question was, if a family of 14 persons, expend £42 in 30 days, how much, at the same rate, will a family of 21 persons expend in 45 days? And the answer was found by multiplying the £42 by the two latter terms, and then by dividing the product by the two former terms. Now, just read paragraph 297, and then let us look carefully into the relationship which these several terms bear towards each other. And, first; it is laid down, that a certain proportion does already exist between the 14 persons and the sum of money mentioned; that is to say, that 14 persons expend £42. And then comes the question,—How much 21 persons may expend? This 21, then, is *the odd term*, as described in the paragraph you have just been reading; it is the term which "wants its proportional, or mate;" and it is, therefore, of these three terms, to stand last. The two other terms take their stations as in single rule of three; that is to say, the middle term is to be that which speaks of the same kind of thing as the answer is to speak of, and is always the term which bears the same relation to the first, as the fourth is to bear towards the third. The three terms will then stand thus, 14 : 42 : : 21.

313. Thus disposed you will note, that the three terms are, in a certain sense, balanced; in the centre stands *the odd term*, and on either hand stands a term descriptive of similar things. And now come the other two terms, that is to say, the *fourth* and *fifth*, which are to be placed, one on either hand, beneath the first and third terms. These two terms, in this question, speak, both of them, of *days*; in all questions, the *fourth* and the *fifth* terms, like

the first and the third, will speak of the same kind of things; so they, also, like the first and third, are to be placed so as to balance each other, and, as in the case of the first and third, likewise, that term on which the question is asked takes the right hand, or third place. So that you see, there is no mystery in all this; no conjuration. It is a very simple, rational, and even a very pleasing, as well as a very useful affair. I here state the question of which we have been speaking. In the second line, the centre place, as you see, stands vacant, but it is customary to put a small mark of some sort in it, such as a small cross, in order to show, that it has not been left vacant through an oversight. And the five terms of this question, as here stated, will serve as a guide and example in the statement of all sums in double rule of three; and, of sums, not in this rule only, but likewise, as I have before intimated, in sums containing the largest number of terms.

per.	£.	*per.*
14 :	42 : :	21
30 :	× : :	45

314. However, confining our attention, at present, to this question of five terms, which is stated above in due form, let us turn back to paragraphs 304, 305,-6, in which we traced the working of this very sum. In the two first of these paragraphs, different modes of working are suggested, but it is in the latter of them, that is to say, in 306, that the approved and established mode is fully traced and described. And this mode is approved and established, because, in pursuing it, we keep clear, during the process, of the fractions, in which, in most questions, by either of the other modes, we should be entangled.

315. And what is this approved mode of proceeding, as exemplified in this question? In the

paragraph last referred to, we have it thus stated, "$42 \times 21 \times 45 = 39690$; which £39690 $\div 14 \times 30 =$ £94. 10s."—And, what is this process, but a *multiplication* of the middle, or second term, by the terms standing in the third column; and a *division* of the product thereof, by those in the first column; a process which is in exact conformity with that pursued in single rule of three direct.

316. However, this is only the method of treating questions in *direct* proportion. The terms being read in the order in which they are stated, we multiply the *second* and *third* together *and the term, that stands under the third;* and the product becomes the *dividend;* and, for a *divisor*, we multiply the the *first, and the term standing under it;* and, as in rule of three, "the quotient is the answer to the question, in the same denomination as that in which we leave the second number." And, in this method, too, are all questions in direct proportion, however numerous and compounded their terms; in this method, of multiplying the middle, or odd term, as I have found it convenient to call it, by all the terms in the right hand column, and dividing by all those that stand on the left; in this method are all questions in direct proportion to be resolved.

317. But in these more compound questions, the proportions, as in single rule of three, are not always *direct;* and, when they are otherwise, a suitable change in the method of treating them is, of course, to be pursued. As to the mode of statement, it may be the same, whether the question be direct or inverse; so that it may be as well not to think of this point until the statement be completed. But, having done this, you look at the several terms, as instructed in paragraph 297 and, confining your attention to the first, second, and third terms, you

determine whether the question they ask be to be answered in *direct* or in *inverse* proportion. And, having determined this, you mark accordingly; that is to say, you mark the term with which you are to divide. If the question be *direct*, you divide by the *first* term; *if indirect*, you know you are to divide by the *third* term. And whichever of these terms you have to divide by, mark it, with a small star, placed distinctly by the side of it. Having done this, you proceed to examine the two terms next below; and mark, in the same manner, the term which, according to the question asked by them, is to be the divisor. And, having the terms thus marked, your work becomes very simple indeed; for, all you have to do, in the most difficult and most complex question that can be conceived, is, to *multiply* all the terms together that are *not marked*, and to *divide* the product by those that *are marked*; "and the quotient will be the answer to the question, in the same denomination as that in which you left the second term." Precepts without examples, are of little avail, whether it be in morals or in arithmetic; so let me engage your attention to the questions which follow; and especially to the method of statement, and of preparation for working.

(1) If 16 men can build a wall, of certain dimensions, in 23 days, when the days are 9 hours long; how many men will build the same wall in 12 days, when the days are 11 hours long?

This is the same question as that numbered 10 in rule of three, but with the two additional terms as to the length of the days. Remembering, now, that every thing depends on the statement, and calling to mind, or re-perusing the instructions on this subject, given in paragraph 296, you may state three of the terms of this question, just as you did the same terms when it was a rule of three question, thus; and then place the two additional terms, according to instructions in paragraph 313, as you here see it done.

days		*men*		*days*
23	:	16	: :	12
hours				*hours*
9	:	⋈	: :	11

Or you may say, in the first line, as you see it here done; If 9 hours a day will suffice 16 men to do the work, how many men will 11 hours suffice; and then the two other terms take their places, in their proper stations beneath; and, this important part of the work is completed.

hours		*men*		*hours*
9	:	16	. .	11
days				*days*
23	·	×	: .	12

But now come the queries: How are these terms, as to *direct*, or *inverse?* One thing at a time is the way to do work well, and most especially it is the way in which the mind works with the most pleasing, and most beneficial effect; so let us here confine our attention to one line of terms at a time. And, first, let us determine as to the line in which the *days* are stated. If 23 days require 16 men, 12 days will require *more* men; but 12 days are less time than 23 days, that is, "*less requires more,*" therefore this line is *inverse*, the *third* term is to be the *divisor;* and, according to instructions in paragraph 317, we mark it with a small star, as you see it done in the annexed statement. Now for the next line. If 9 hours require 16 men, how many men will 11 hours require? Fewer men, it is manifest will be required for the longer hours. This, therefore, is a case in which "*more requires less;*" and this part of the question, also, is in *inverse* proportion, and the term in the third column here, too, must be marked for a divisor. All is now clear before you. Following the rule given above, you multiply together the Terms that are not marked, for a dividend; and, for a divisor, you take the product of the terms that are marked; or, if it will not entangle you in fractions, you may divide by the marked terms one after the other, as you were taught to divide by submultiples, one after the other.

days		*men*		*days*
23	:	16	: :	12 *
hours				*hours*
9	:	×	. .	11 *

(2) If a party of 9 persons, on an excursion of pleasure, in the space of three weeks, expend £134 5*s.* 4½ *d.*, how much, at the same rate, may a party of 13 persons expend on a like affair, in 25 days?

persons		*£.*	*s.*	*d.*			*persons*
*9	:	134	5	4½	:	:	13
days							*days*
*21	:		×		:	:	25

(3) A body of troops, in garrison, consisting of 1253 men, have provision for 63 days, at the rate of 25 oz. each man per day, and a reinforcement of 295 men, without any provisions, join them; to what quantity per day must each man be reduced,

in order to make the provisions serve the whole garrison 72 days?

Add the reinforcement to the first body of troops × 1253 295 = 1548; then state the terms thus. I leave you to mark them for *direct* or *inverse*, yourself.

men		*oz.*				*men*
1253	:	25	:	:		1548
days						*days*
63	:	×	:	:		72

(4) What is the interest on £ 4217. 12*s.* 6*d.* for 60 days, at 5 per cent. per annum?

Another question on *interest!* and this exactly the same as that numbered (30) in rule of three!—I cannot well devise a better means of showing the respective powers, of each of these valuable rules, than this of applying them to the same question. In the former rule you learned how to resolve this question by two statements, and two workings; here we do it at once, but according to the rule we are in, we do it by a double statement, which, together with the working, exhibit that beautiful coincidence, and harmony with the former mode of treating the question, which is always to be found in truth of statement, and correctness of reasoning, whatever difference there may be in the mere manner of conducting either.

£.		£.			£.	*s.*	*d.*
100	:	5	:	:	4712	12	6
days					*days*		
365	:	×	:	:	60		

This is so neat a method of computing interest for a number of days, and the treatment of the same question, by the two modes, of single, and of double proportion, tend so much to illustrate each other, that I must propose the two other questions in one, for your practice.

(5) What is the interest on £ 3715, for 54 days, and for 73 days, at 4½ per cent. per annum?

(6) If a locomotive steam engine, by the consumption of 12 *cwt.* of coals, propel a train of carriages, and their loads, weighing altogether 63 *tons.* 7 *cwt.* along a rail-road, a distance of 32 miles, how many coals ought to be consumed, at the same rate, in order to drive a train of carriages and their loads, weighing 82 *tons.* 11 *cwt.* on a similar road, 89 miles?

(7) If an engine, as above, by the consumption of 12 *cwt.* of coals, propel 63 *tons.* 7 *cwt.*, a distance of 32 miles, how many miles ought 82 *tons.* 11 *cwt.* to be propelled by 17 *cwt.* 3 *qrs.* coals?

(8) If an engine, as before, by the consumption of 12 *cwt.* of coals, propel to a distance of 32 miles, a train of carriages weighing 63 *tons.* 7 *cwt.* what weight of a train ought 19 *cwt.* 2 *qrs.* of coal to propel along the road a distance of 39 miles?

(9) If 634 tiles, each 10 *inches long*, and 7½ *inches* broad, will pave a certain floor, how many bricks, 9 *inches* long, and 4½ *inches* broad, will pave the same?

(10) If an iron bar 3 *feet* long, 1½ *inch* thick, and 3½ inches broad, weigh 52½ *lb.* how much will be the weight of a bar of the same breadth, but 4 *feet* 7 *inches* long, and 2¾ *inches* thick?

(11) An iron bar 3¾ *in.* broad, 1¾ *in.* thick, and 8 feet long, weighs 75¾ lb. what, then, will be the weight of a bar of the same thickness, but 4⅝ *inch* broad, and 11¾ feet long?

(12) At 5 per cent per annum, how long will £179 be, at simple interest, before it amount to £300?

	£.		*days*			£.
If	100	:	365	:	:	105
	179	:	×	:	:	300

(13) On the birth-day of a child, just 12 years of age, the father proposed to put out a sum of money, which, at 4½ per cent. per annum, simple interest, should increase to £500, just as the child should complete its twentyfirst year: Now what is the sum to be put out, in order to accomplish the father's purpose?

This is but a question in rule of three; but, requiring, as it does, some little skill in the statement, it may suit the learner better, in this more advanced lesson.

(14.) If 6 men sink a foundation, 17 *yards* long, 7 *yards* broad, and 6 *feet* deep in $5\frac{1}{2}$ days, how many days ought the same men to be allowed, in which to extend the foundation 6 *yards* in length, and $2\frac{1}{2}$ *yds.* in breadth?

(15) A garden wall, measuring 297 *yards* round, 12 *feet* high, and brick and a half thick, has been built by 19 men, in 27 days; how many men can build a wall the same height, and same thickness, but 315 yards long, in 11 days?

COMPOUND PROPORTION;

OR, THE CHAIN-RULE.

318. Let us now enter on compound proportion. In the foregoing we have dealt with questions that have five terms essential to their statement, and that require a sixth, for the answer. Here I propose, to treat of such as have yet more terms, and I expect you will be delighted with the simplicity and ease with which propositions most frightfully complex and unanswerable in their appearance to an unlearned eye; propositions, too, often of very great importance; I expect you will be delighted with the ease with which you will learn to manage propositions of this nature. And, that, when you shall have seen them so managed, you will agree with me in the opinion, that the rule by which we learn so dexterously to treat them, may very justly be regarded, both on account of its beauty and its usefulness, as the great ornament of commercial and mechanical arithmetic.

THE CHAIN-RULE.

319. To enter on this beautiful rule in the manner we have before occasionally pursued, let us take the first proposition in the rule of five terms, and to it, let us add two other conditions, or terms, thereby bringing it into this rule. The proposition was, to find an answer to the question; "If 16 men can build a wall of certain dimensions, in 23 days, when the days are 9 hours long, how many men will build the same wall in 12 days, when the days are 11 hours long? To these five let us add the conditions, that in the first instance the wall is proposed to be 180 yards long, and in the second that it be 205 yards long: and so we have seven terms in the question, waiting for an eighth.

320. You know how to state the five terms; and recollect that they were placed as you see the first five terms in the annexed statement. And they may be read thus, *If* 23 *days will suffice* 16 *men, how many men will* 12 *days suffice;* and, then, *if* 9 *hours, under the above conditions, will serve, what will* 11 *hours do?*

days		*men*		*days*
23	:	16	: :	12
hours				*hours*
9	:	×	: :	11
yards				*yards*
180	:	×	: :	205

And now appear the two additional terms; which merely come to ask, "If, under the foregoing conditions 180 yards of wall can be built, under what conditions; or, rather, what additional number of men must be set to work, in order to build such a wall 205 yards long, within the given number of days and hours? and these two new terms are placed underneath the former, as you see it done above. And, so, all the difficulty is over. You have, to be sure, just to consider, whether the questions be *direct*, or *inverse*, and to mark them accordingly, before you begin the working, which working, you know, after the statement is completed, is mere child's play.

321. However, now that we have the mystery stripped off this once tantalizing, and somewhat formidable rule, let us play with it, and become yet more familiar. Let us add two other conditions to the question; let us say that, in the first instance, the wall was proposed to be 13 inches thick, and, that now it is proposed to be 22 inches thick. I need not tell you how to state the question, nor how to mark the terms; and, as to the working! that is nothing. You now know all about these things, as well as the most experienced and able mathematician. You know how to state, and how to work questions of nine terms; and can, with just the same ease, state and work such questions, should they consist of yet more terms. And this knowledge you have acquired, and acquired it, as I trust, with great ease and pleasure, merely by beginning each lesson at the right end; by having, in fact, the right end offered to you; and, with this additional circumstance, that you have been led to take one thing at a time, and thus to master all the difficulties, one after another.

322. It is not teaching arithmetic, but it is doing something even more valuable, to embrace this opportunity, to enforce on your careful, and everlasting attention, the advantages to be derived, from beginning any study you may undertake, at the *right end*, as I have called it. Never mind the homeliness of the phrase. Fine words, I could give you, were it the time to give them; but you have heard the saying, of the inefficacy of "*fine words*," in all cases where *substantial service* is required. And here it is substantial knowledge that we are in pursuit of. So let me impress on your attention the advantages, and the pleasures too, to be derived from commencing your study at the right end: then take the several parts in their due order, attending to one part at a time, only;

and, if you pursue this course, you will, without pretensions to extraordinary powers of any sort, master subject after subject, and science after science; and all this, too, not only with ease, but with pleasure.

323. This beginning, however, at the right end, this taking of the matter in due order, and attending to it, one thing at a time: How is this to be done? will you learn it from books? will you learn it from masters? Very few of either, are there, that are able thus to teach. And hence the pain, the difficulty, the labour of study. Hence, very naturally, the dislike in which it is often held by men of good sense and talent. Hence it is, that such men, who have avoided what is called study, are every day seen to outstrip the scholar, who has suffered the natural brightness of his mind to be quenched and obscured, by attending to the lectures of stupid professors, and by poring over treatises, most of which treat of things that are of no use; and, treat of what they may, almost all of which are wholly destitute of the *principles* of the subject of which they treat, besides having the fault of beginning at no end, of trailing their fatigued and disgusted reader through a labyrinth of dulness, of quitting him, stupified, and of ending without a conclusion.

324. Such are the books, and such are the professors of learning, generally. But, how are you to avoid them; how to learn without them? Without books, and teachers, you cannot learn many things, which will conduce greatly to your happiness, to your usefulnes, and to your rank in society. Read, therefore, and listen. Learn where you can. And, whenever you meet with a book, or with a teacher whom you can understand, and from which, or from whom, by moderate attention, you

can acquire useful knowledge, do not fail to remember, that "wisdom is the principal thing; therefore, get wisdom; get understanding." But, on the contrary, if you find your teacher dull, and your study irksome and profitless; if wearied nature shrink from the task, vindicate her sacred rights; throw up your book, or quit your professor, and solace and refresh your mind amid the scenes of nature, or in the society of your friends For be assured, that the teacher who is not to be understood by you without difficulty, whatever may be his reputation for learning, has undertaken an office for which he is unqualified; that he knows not where to begin, nor how to carry on his lessons; that, in fact, he has, almost to a certainty, no comprehension of the principles of the subject of which he professes to treat; and that, therefore, the sooner you withdraw from his tuition and the better.

325. "Useful Knowledge;" but what is this? On your decision as to this point, more than on all others put together, depends, not only the estimation in which you will be held by the circle in which you move, but on it mainly depends the circle itself to which you may be raised or sunk; and, therefore it is, that I venture on this apparent digression. For digression it scarcely is; seeing that I shall have to conclude with a piece of advice relating to the study of this subject of arithmetic, and to the higher branches of learning to which it leads; a piece of advice, which to some ingenious and aspiring young men, may be of more value than all that I attempt to teach throughout the rest of the book.

326. What, then, is this useful knowledge? Undoubtedly, after the knowledge of our private and our public duties, *useful knowledge* consists in an acquaintance with those things which are

serviceable in our several stations and employments. And, after these, as we all have, or ought to have, leisure and inclination for some other pursuit, in order to relieve, and to embellish our minds, and to save us from sinking into little better than mere animal existences, useful knowledge will consist, in the first place, of an acquaintance with the history, the geography, the natural productions and other resources, of our own country; of its laws and institutions, and of the effects of those laws and institutions; of the character and influence of our country abroad, and of its interests, both foreign and domestic. Then come a knowledge of foreign countries, together with their natural history. And these are useful, these are ornamental branches of knowledge, and constitute, undoubtedly, much of that "wisdom and understanding," the value of which the inspired teacher so impressively enforces.

327. But here is nothing said in recommendation of the study of mathematics; nothing in favor of a further prosecution of those sciences to which this book may serve as an introduction, and of which sciences, indeed, arithmetic forms a part. Exactly so, and it is chiefly for the purpose of advising my pupils, to be wary how they be tempted to prosecute studies of this kind, further than their particular professions and employments require, that I have ventured thus far out of the mere office I had professedly undertaken.

328. To the architect, the surveyor, and the engineer, arithmetic and geometry are "*the useful knowledge*." And, applied by them, as this knowledge continually is, to the measurement and formation of solid matter, and for useful purposes, the study of these sciences is healthful and invigorating to the mind. But woe be to the mind of him who indulges himself in the assiduous study of them,

separate from their uses. They are attractive, they are fascinating, when once a man has given himself up to them; and they have even been recommended by some men of great name, as a means of improving the mind. But judge you by the result; look at the men who have devoted themselves to these studies, forgetting almost all the substantial duties and enjoyments of life, and dreaming themselves away in mere abstractions; and beware how you surrender yourself a victim to the idle contemplation of mere lines and figures, however varying and amusing their forms and relations. And beware, likewise, of thinking yourself superior to your fellows, merely on account of your knowledge of things so unsubstantial, and to you so useless.

329. To resume our lesson on the foregoing question, which, as before observed, from three, we have now increased to nine terms; it was, at first, somewhat in this form: If 16 men can do a certain piece of work in 23 days, how many men can do the same work in 12 days? So there was one pair of terms relating to the number of days; and the odd term gave us the number of men. To these we added a pair of terms fixing the number of hours in each term of days; next we added two other terms, specifying the length of each wall; and, after these, by two other terms, descriptive of their respective thicknesses, we increased the number of terms to four pairs and one. Let us now, in order to make the question consist of eleven terms, propose two terms on the heights of the two walls; and place them as we have done the others, one on the right, and the other on the left, as you see them in the annexed statement.

330. Those terms which affirm, that in 23 days, of 9 hours long each, a wall, in length 180 yds , in thickness 13 inches, and in height 14 feet, can be built, by the given number of men ; that is to say, those terms which affirm that so-and-so *is, has been, or may be*, stand in column on this side ; and,

The terms which, taking all that is said on the other side for granted, ask how many men, then, in 12 days, each day 11 hours long, can build 205 yards of wall, 22 inches thick ; and 17 feet high ? These, which are, in fact, the *terms of the question*, are ranged in like order on this side ; each term opposite to, and balancing its corresponding term on the other hand ; with the odd term placed in the middle.

days		*men*		*days*
23	:	16	: :	12*
hours				*hours*
9	:	×	: :	11*
yards				*yards*
*180	:	×	: :	205
inches				*inches*
*13	:	×	: :	22
feet				*feet*
*14	:	×	: :	17

331. There are other methods of statement used by other persons, but this method is in accordance with that which I have pursued heretofore. I have adopted it, after very mature consideration ; thinking that it grows quite as naturally out of the principles on which the process is founded, if not more naturally, and more simply, than any of the others. The other methods I would give, were it not from some apprehension that, in the minds of learners, they might become somewhat mixed, and a trouble, rather than a service. However, give them, or not, I must prepare my pupils to meet those who prefer, and who, as my pupils will find, contend very pertinaciously for other methods of

statement. Recollect, that the object of the rule is, to bring out, with certainty and ease, the answers to the several questions. Each person will pursue his own method, and will understand it best; let him do so; and let my pupils beware how they employ any part of their time and attention, in arguing about the matter, until they find, by actual performance, that some person, neither more experienced, nor otherwise more able than themselves, can, by a different method of statement, work out the answer, with greater ease and rapidity. But this is wholly out of the question. It cannot signify whether the *odd* term, as I have chosen to call it, be placed in the middle, as I have placed it, or, as others place it, some at the top, some at the bottom, and others to the right, of the right-hand column. It signifies not, where it be placed, provided that we know, without trouble, where to find it: nor does it signify much, whether the other terms be placed as I have placed them, that is to say, the terms of *affirmation*, or of *supposition*, on *the left*, and those of *the question* on the *right*, and to be *marked*, as I mark them, afterwards, for *divisors*; or, as others place them; the *divisors* all on *one side*, and the *factors* of the *dividend on the other*; it signifies no great deal which of these methods be adopted. Let each person, without good reason to the contrary, and I am persuaded that no good reason can be assigned for being at the trouble of making a change; let each, I say, adhere to the method he may have learned; and, to my pupils I say, whenever you have to check, or to prove, the working of another person, in rules where the mode of statement forms an important part of the work, do you always prefer to state and work the matter yourself; thereby keeping your mind clear of the useless labour of tracing out the peculiar method by which the question may otherwise be treated.

332. To return to our question, as stated before. The terms which are to produce the *divisor* being *marked*, as you see, all you have to do, in order to find the answer, is, as I have before taught, to multiply them together; and, for the *dividend*, to multiply together the terms that are not *marked*. And the answer you will find, when the fraction is reduced tells us that $58\frac{197}{273}$ men will build the proposed wall, in the proposed time.

333. Thus is a question, comprising eleven terms, some of them requiring answers in *direct*, and others of them, answers in *inverse* proportion; thus, by a single working, is a question, so complex, to be answered. Something is said, in paragraph **311**, about working questions in double rule of three by a double statement; those questions have *two* pairs of terms, and an odd one. Now this last question, of eleven terms, having *five pairs* and an odd one, may be worked by *five single* statements, bringing out, of course, precisely the same result. I have so worked it, and will here give the statements, with the answers, each of which *answers*, as shown in paragraph **311**, becomes the *middle term* of the next statement. In working the question thus, by so many distinct processes, the quantities become so much broken into fractions, that, until I have given them some little further instructions on the working of this sort of numbers, my pupils will scarcely be able to manage them. I shall, in all probability, revert to these very statements, and to their management, when I come to treat of the working of fractions. In the mean time the five statements serve the purpose for which they are intended; which is simply that of exhibiting to the learner, the perfect concurrence, as to the answer, arising from the two different methods of working.

		days		men		days		men
Question	1.	23	:	16	: :	12	Answer	$30\frac{2}{3}$
		hours				hours		
..	2.	9	:	$30\frac{2}{3}$	: :	11	,,	$25\frac{1}{11}$
		yards				yards		
..	3.	180	:	$25\frac{1}{11}$	: :	205	,,	$28\frac{19}{33}$
		inches				inches		
..	4.	13	:	$28\frac{19}{33}$	: :	22	,,	$48\frac{14}{39}$
		feet				feet		
..	5.	14	:	$48\frac{14}{39}$	: :	17	,,	$58\frac{197}{273}$

Determining the time, and the work to be done, we have asked, how many men are required to do it. Let us now have one question as to *the time*, and another, all other circumstances determined on, as to *the length* of the work; and, using the same terms nearly, as this, on which we have been working, varied only to keep a little clear of fractions, we shall become yet more familiar with the nature, and the power of our rule.

(2) If 58 men can build a wall 205 yards long, 17 feet high, and 22 inches thick, in 12 days, when the days are 11 hours long, in how many days can 16 men build a wall 180 yards long, 14 feet high, and 13 inches thick, when the days are 9 hours long?

(3) If 16 men, in 23 days, of 9 hours long, can build a wall 180 yards long, 14 feet high, and 13 inches thick; what length of a wall, 17 feet high, and 22 inches thick, will 58 men build, in 12 days, when the days are 11 hours long?

(4) The hides of cattle, which are a great article of commerce in South America, are sold in Buenos Ayres by the *pesado*, being 35 Portuguese pounds weight, for 32 of the depreciated paper dollars of that country, one of which dollars is valued at $8\frac{1}{2}$d. of our money; but 97 of their pounds are equal in weight to 100 of ours. Now, under these circumstances, what is the value, in English money, *First*, of 120 English pounds? *Second*, what is the value of 112 lb.? *Third*, what the value of 100 lb.? and *Fourth*, what the value of 1 lb.?

334 Here are four questions together; which, to save room, I propound on the same series of conditions. But we must take them, one at a time. And, first, with regard to the statement of questions of this description.

335 There is a mode of statement, which, for the purpose of distinction, I will call the *continental* method; being, so far as my observation has gone, practised and taught by persons from the continent of Europe. This method, which is peculiarly well suited for resolving questions of foreign commerce, is, to set down *all the terms* of the question, without stopping to examine, as we have done in ours, which are *essential* to the working, and so omitting the rest; this method, which, with admirable ease, takes in all the terms, arranges them in a manner so simple, that a child is capable of the work, and, having done so, begins to reduce and to cancel those terms, when they are capable of being reduced and cancelled, until it brings them down, generally, to numbers so few, and so small, as to require no more labour in the working, than do the fewer terms in which, with much more exertion of mind, we contrive to state questions of the same description, in our method.

In this continental method, the terms are ranged in two columns, beginning with that on the left-hand, at the head of which is placed that term, the *value*, the *price*, or the *produce* of which we have to discover; then, on the line next below it, but on the left-hand column, comes a term which speaks of a quantity of the *same denomination*, as does this first term; then, opposite to this *second* term, and underneath the first, is placed a term, which, in the question, you will find expressed or understood, to be *equal* to the *second* term; then, again, crossing to the left, is written a term of the same denomination as the *third*, which, again, is balanced on the right, by a term which, although *in another denomination*, always expresses a thing of the *same kind*, and *equal* in *quantity*: thus do they commence with the *odd term*, the *equal*, or *balance* of which the answer is to give; thus do they commence with the odd term, and then proceed downwards, *not changing* the *denomination* on the *left-hand*, but only on the right, and, on this hand, too, *balancing*, always, in quantity, or in value, the term last written on the left, just as you see them in the statement which follows, until all the terms be set down.

In this statement the words required to explain the terms, are inserted; it is a safe mode, and I would advise you not lightly to omit to follow the example,

What is the value of..............120℔ English

100℔ English being equal to .... 97℔ Buenos Ayres

35℔ Buenos Ayres selling for ...... 32. Buenos Ayres Dollars

1 Dollar being valued at .... 8½d. English.

Very similar to this is another method of stating the question. The difference, indeed, consists merely in this, that the *odd term* is placed at the *foot*, instead of being at the *head* of the right hand column. But this difference of statement can make none in the result, seeing that, the terms in the first column are to be multiplied together for a divisor, and all the other terms, place the odd term where you may, are to be multiplied together for a dividend; that is so say, such must always be the case in the Continental method, which method, so well adapted for questions of this description, in which the proportion is always *direct*, does not, so far as I have looked at it, appear to be fitted to work questions in *inverse* proportion with any peculiar advantage.

(5) Carolina Rice sells in Hamburg at 100lbs. of the weights used there, for 12 marcs, 8 schellings; how much, in English money, will 112lb. English weight cost; 112 lb. English being equal to 105 lb. Hamburg; 16 schellings being one marc, and 13 marcs equal to 20 shillings?

What will be the cost of 112lb English;

.. ..

You cannot fail to do the rest right. Observe, for one of the terms you will have 12 *marcs*, 8 *schellings*; now, need I remind you, that this being a *compound term*, must be reduced to one denomination. There are 16 schellings, you saw, in one marc, so that it is 12½ marcs, and it will suffice to reduce the term to *half marcs*; but, then, observe, and never fail to bear this in mind, that when you reduce a term on *one side*, you must, in order to preserve the proportion, as shown in paragraph 179, reduce, in like manner, *the corresponding term on the other side*. So be careful to have this done in all cases.

(6) The price of Wheat at Hamburg is 120 *dollars, currency, per last*; what will be the cost of one *English Quarter*?

To complete this question, it is necessary to know, that in Hamburg, as in many other countries, the *current money* is sunk in value below the *standard* or, as they call it, in the German States, *Banco*. In Hamburg, it requires 125 *marcs currency* for 100 *marcs banco*: then 1 *dollar* is equal to 3 *marcs, and* 13 *marcs* is equal to 20 *shillings English*: and for the measures, 11¼ English *qrs.*, that is, 90 *bushels*, are equal to 1 *last*, Hamburg measure. As to the finishing of the statement, I will leave you to do that, but you may begin it in either of these methods.

1 Qr.	8 Bushels.
11¼ English Quarters=1 Last.	90 English Bushels=1 Last.

(7) Brazil Coffee, in Hamburg, is worth 4 *schillings per pound*, in the money and weight of that city: at this rate, what will be the cost of our *cwt.* or 112lbs. English?

You have the money, and the weights of Hamburg compared with those of England, in the last note.

(8) Coffee, in Rio de Janeiro, is 3 *milrees per aroba*, which is equal to 32lbs. English; a milree is equal to 2*s*: what is the price, at Rio, of 1 *cwt.* in English money?

(9) At Bahia the price of Sugar is 2000 *rees per aroba;* 1000 *rees* are valued at 32*d*. *English;* what would be the cost in English money, of 1 *cwt?*

(10) Cotton is selling in Pernambuco at 3400 *rees per aroba;* the exchange there is 1000 *rees* for 44d. *sterling;* what is this *per pound, English?*

(11) In Havanna, the price of Sugar is 12 *reals per aroba,* how many *grots banco,* will 1lb. cost Hamburg money and weight?

The Spanish aroba—24lb Hamburg
8 reals equal to 1 Spanish dollar
444 dollars equal to £100 sterling
£1 equal to 13⅓ marcs banco
1 marc equal to 32 grots.

(12) Coffee, in Havanna, is 7 Spanish *dollars per quintal,* to how many *schillings, banco, per pound, Hamburg* will the same amount?

1 Quintal is equal to 4 arobas.

(13) The price of Indigo in Calcutta, is 160 *sicca rupees per factory maund;* 1 *sicca rupee* being equal to 22d. and 1 *factory maund* to 74½lb. *English,* what will be the price *per pound,* in our money?

(14) In the city of Hamburg, the price of Wheat is 100 *dollars banco per last,** expenses of freight &c. on bringing it to England are 22 *per cent;* what, then, will be the cost *per quarter* of 8 *bushels*, in England?

Here is a new circumstance in this question, that is, the cost of 22 *per cent.* on bringing the wheat here; and this, after all the other conditions are stated, will merely make an additional line; thus, 100 *will, with the charges, come to* 122. How very simple is this rule! However, I will give the whole statement; for these occasional helps to the learner cannot do otherwise than smooth, and light up the path to learning.

	8 *bushels*
90 *bushels*	1 *last*
1 *last*................	100 *dollars*
1 *dollar*.............	3 *marcs*
13 *marcs*..............	20s. *sterling*.
100, *with charges*	122.

(15) Brazil Sugar at Hamburg, selling for 8 *grots banco per pound*, 105 of which are equal to 112lb. English; 32 *grots* are *one marc*, and $13\frac{1}{2}$ *marcs* equal to 20*s*. sterling; what will be the price in England, *per cwt*, with expences $17\frac{1}{2}$ *per cent?*

The few remaining questions in this rule will be on the ARBITRATION that is, on the ADJUSTING of "EXCHANGES," which rule enables a merchant, or dealer in money, to calculate the actual, or the probable loss or gain, on sums of money, transmitted under the peculiar circumstances of the case, from one commercial city to another.

(16) A merchant in London has to receive 2,000,000 rees from Oporto: for this sum, how much will he receive in London, if he order it to be remitted to his Agents, first to Hamburg, thence to St. Petersburg, and thence home, the exchanges being, 42 *schellings banco*, at Hamburg, for 1000 rees; at St. Petersburgh, one *paper ruble* for $8\frac{1}{2}$ *schellings banco;* and, at London, one pound sterling for 27 paper rubles?

To be stated thus, ...	2,000,000 *rees*
1000 *rees*	42 *schellings banco*
$8\frac{1}{2}$ *schellings banco*....	1 *paper ruble*
27 *paper rubles*	1 *pound sterling.*

(17) A London merchant remits £1000 to Lisbon, thence it is remitted to Hamburg, from which place it is returned to him. The courses of exchange being as stated below; will this merchant gain or lose by this transaction?

45 pence, English.—1 milree of Portugal.
1 milree, ... —44 schillings at Hamburg
16 schilling .... — 1 marc
$13\frac{1}{2}$ marcs ...... — 1 pound sterling.

It is understood, not that the £1000 is *cased-up*, and so carried about from one of these cities to the other, and then returned; for then there would be no question about its either gaining or losing; it is understood that the £1000, on its arrival in Portugal, is *exchanged* into the money of that country. And, if it be, as such remittances generally are, a *Bill of Exchange*, the Bill is, in fact, *sold* for so much money; which is called EXCHANGING. And, in the above question it is proposed to sell this Bill of £1000 value, first in Lisbon for *milrees;* at 45*d*. each; then in Hamburg the *milrees* for 44 *schillings* each; the *schillings* to be changed into *marcs* at 16 *schillings* each, of which *marcs* $13\frac{1}{2}$, it appears are worth *one pound sterling*. Now, this selling of Bills of Exchange, is as regular, and as frequent a matter of business, in commercial cities, and must almost of necessity be so, as the selling of any other articles of traffic. For it arises out of the sales; thus. A London merchant receives an order for £1000 worth of Goods, to be shipped to a merchant at Lisbon: now, how is the London merchant to get paid for his goods? To go over himself, for the money, or for the Lisbon merchant to bring it to him, is out of the question, and there would be great expense, loss of time, and some risk, in any other method of making the payment direct, from the Lisbon to the London merchant. But, observe, if the London merchant, having shipped off the goods, write out an order on the Lisbon merchant for the money; and, then, going on the "EXCHANGE" at London, and finding another person who, having to make a remittance of such a sum to Lisbon, is in want of some safe, and inexpensive method of doing it; if the London merchant, with his *order*, or *draft*, or *Bill* of *Exchange*, for so the order is called, if he meet such a person, with £1000, which he is thus seeking an opportunity to send to Lisbon, how pleasantly can these two parties settle their accounts with their Lisbon friends; the London merchant hands over his Bill of Exchange, receives the £1000, which pays him for his goods; and the other has only to enclose the Bill in a letter to his correspondent in Lisbon. There are no *freight charges*, no *insurance* required, for he receives a *duplicate*, and a *triplicate* Bill, so that if one miscarry, another reaches its destination; and, on receiving it, the Lisbon correspondent presents it to the merchant on whom it is drawn, receives the money, and so, almost without risk, or expense, or delay, all the parties settle their accounts. This is the manner in which merchants in different nations pay and receive, and settle their accounts with each other. And this is the chief and legitimate use of Bills of Exchange, which form not only a very important object of attention to the merchant, but require, for their regulation and security, a system of laws extremely exact and comprehensive.

Now these Bills of Exchange, being an article of sale, rise and fall in price, as do other articles of traffic. They are, or ought to be, orders for payment, in return for "*value received;*" which very words, or some words equivalent thereto, must be on the face of a Bill, or it is incomplete. They are orders for payment for goods, sent from one place to another. And, by way of illustration, if we take any two commercial cities, as London and Hamburg; when more shipments of goods are making from the former of these cities, to the latter, than from the latter to the former, there will be more Bills of Exchange drawn in London, than are required to balance the accounts; that is, there will be more Bills on Hamburg, than are wanted by merchants in London to pay their correspondents in Hamburg, a consequence of which will be, that Bills on Hamburg will not sell so well in London. The seller, rather than go, or send over to Hamburg for *the money*, will take something less for his Bill, than the sum named, in it, and the difference between the sum named, and that for which the Bill sells, is called the *rate of exchange*, and, in this case, it is said to be *against Hamburg*, or *in favour* of *London:* and, as in the case supposed, Hamburg Bills on London will be selling for more than their nominal amount, so there, also, the *rate* of exchange is *in favour* of *London*. This *rate of exchange*, being then, an allowance made for payment of the Bill on the spot, may approach, but never can exceed the expense of bringing over the money from the country on which it is drawn. It is, to the party that has to allow it, the *charge* on obtaining a settlement; just the same as any freightage, insurance, or other expense, is a charge on him.

(18) A merchant in St. Petersburg has to pay 10,000 *rubles* at Oporto. Which will be more advantageous to him, to make the remittance by way of Hamburg, or by way of London?

Here will be two statements, and two workings, in order to find what will be gained or lost by exchanging the money through either of these courses. Both workings will end in giving the produce of the 10,000 *rubles* in *milrees*, at Oporto. And then the advantage of either course will appear, on comparing these results. As there is, frequently, an advantage to be gained by sending a payment thus through one course, rather than through another, this is a description of questions which merchants, engaged in extensive concerns, have frequent occasion to consider. After the exchanges, I will insert the two statements; having to make use of the latter of the two, for the purpose of tracing out, and illustrating the principle on which all these statements and workings are founded.

At Hamburg, 8 rubles .. = 71 schillings
,, Oporto .. 42 schillings = 1 milree
,, London .. 25 rubles .. = 1£ sterling
,, Oporto .. 43d. English = 1 milree.

STATEMENTS:

Exchange through Hamburg.	*Exchange through London.*
10,000 rubles.	10,000 rubles.
8 ruble .. = 71 schillings	25 rubles .. = 1£ sterling.
42 schillings = 1 milree	1£ ...... = 240d.
	43 pence .. = 1 milree

336. Now, mark the process: In the latter of these statements, and you will see the PRINCIPLE, that is to say, HOW, and WHY this mode of treating questions of this description, brings out the required answer. Here we have 10,000 *rubles* to be exchanged into *pounds sterling;* at 25 *rubles per pound,* they exchange for £400; these *pounds,* for convenience of reckoning in this case, we change into pence; at 240 for each pound, this gives us 96000 *pennies,* 43 of which, according to the existing state of exchange, will purchase one *milree;* divide the pennies, then, by 43, and you have the number of *milrees,* which, by this course, the 10,000 *rubles* will produce. Nothing can be more simple, nothing more natural than this: it is

merely changing your money as you go from one country into another, into the money of the country you are in; a process which, without the use of figures, is continually carrying on by persons who travel with a little money in their pockets. Now, bearing this in mind, look well at the statement. And what do you see in it, but such an arrangement of the numbers, descriptive of the *amount to be exchanged*, and then of *the several sorts of money* into which it is to be exchanged, as leaves you nothing to do, in order to come to a conclusion, but to divide those numbers that stand in the right-hand column, by those that stand in the left: only, in order to avoid the trouble which *fractions* might occasion at every division, we make, in this, as in the other rules of proportion, only one division of the matter, by first multiplying all the numbers together that are to be divided, and then, all those together with which we have to divide.

337. To complete the instructions on the methods of cancelling, and of shortening numbers, as spoken of in paragraph 335. The terms being set down freely, and becoming sometimes numerous, as they do in this *continental* method of statement, it is important to be expert in the practice of reducing them. The principle on which it is to be done, and something of the method, are clearly shown in paragraph 287; which, therefore, I must beg you to turn to, and to read with attention.

338. Having read that paragraph, you have learned, that it is by dividing terms on each side of the statement, by the same number, and the doing of this before you begin to multiply the terms together, that the shortening is to be effected. You recollect, that the object of all operations in this rule is, to discover proportionals of numbers. For instance, in question 16, we have to discover a *proportional to* 2,000,000, *under these conditions*, viz:

that, $\frac{1000}{2000000}$ *be equal to* 42, *that* $\frac{84}{42} = 1$, *and* $\frac{27}{1} = 1$. And such proportional is discovered, by multiplying the numerators of these fractions together for one sum, and the denominators together for another, and, then, by dividing the larger sum by the smaller. Now, these numerators are the terms on the left hand of the foregoing statement, and the denominators are those on the right, and, when multiplied together, without being reduced, they would make very untoward sums; but, strike out the three ciphers on each side, and the number of figures become greatly reduced, even in this question, which happens to be rather unfavourable for our purpose. The terms on each side of a statement thus stand, as divisors and dividends to each other. And, may be reduced by any of these means; by striking out ciphers, an equal number on each side; or, if there happen to be on one side, a term, just the same as some term on the other side, by cancelling both terms; or, by dividing a larger term on one side, by a smaller on the other, when the larger will evenly divide, and then placing the quotient in the place of the larger, and an unit in that of the smaller, as 8.... 56 may be reduced to 1....7; and 56....8 to 7....1: For there were, before this reduction took place, one eight on one side, and seven eights on the other; that is to say, seven-times as many on one side as on the other, and according to a law in this rule, we still preserve the proportion between the terms. And, were the terms these, 800 .... 5600, by striking out the ciphers, and dividing as before, you bring them down to 1 .... 7. Again, were the terms 910....2730, cancel the cipher on each side, and divide the larger term by the smaller, and you have the proportion, and all the powers of these two large terms perfectly preserved, for every purpose of this rule, in the simple terms 1....3. And, lastly:

When you have done all you can to reduce the terms, by cancelling ciphers, and by dividing terms on one side, by some of those on the other, there is another method to which you have recourse, when large numbers yet remain, and this is, to divide terms on each side, by any numbers that will evenly divide them: and, by these means you will sometimes reduce the terms of a long and heavy statement so much, as to have nothing but single units left in the left-hand column; and these having no power of division, the answer to the question will be found in the *product* of a few remaining terms in the right-hand column.

72 8	*divided by*	8 =	9 1
39 .. 78	„	13 =	3 6
3 6	„	3 =	1 2
18 6	„	6 =	3 1

339. You will now understand, thoroughly, all about this rule: and especially if, after what you have just learned, you will work the foregoing questions again. For, you will find the processes surprisingly simplified, by the instructions which I have just given; instructions which, had they been given earlier, would rather have tended to render the subject intricate. I shall yet, however, have an opportunity of throwing additional light on the method of stating and of working questions in Compound Proportion, when I come to treat of Reduction of Fractions; and to this lesson I refer my pupil. On closing the lesson, I have to urge you, at all times, in every part of the process, to be careful to make your statements in *clear* figures, as well as to make them correct; and to advise you, that in *cancelling* a term, you do not *rub out*, nor otherwise *obliterate* a figure, but merely draw your pen or pencil *across* it, leaving it still legible, so that you may easily revise your work; a precaution against error which you should never fail most rigidly to observe.

PRACTICE.

340. This is the Tradesman's, that is, the *Buyer* and *Seller's* Rule; if *rule* it be called; which it scarcely can, for there is no rule in it, except that by it you are enjoined to do every thing in the quickest, and shortest manner; an injunction which able and complete tradesmen steadily adhere to, and assiduously inculcate. And, in doing which, they not only show their wisdom, but find their advantage. "Time is the stuff of which life is made," says some one; and very truly. And this precious time, "of which life is made," it is the settled, and habitual practice of the accomplished tradesman, neither to waste, nor to suffer to be wasted by others, if he can prevent it. Hence comes his success, for every one likes to deal with him; and hence his leisure, too; his means of dispensing good; his enjoyments, and those of his family; and his repose.

341. Practice, then, means any of the modes of expeditiously calculating, in matters of trade; that is to say, any of the methods, or expedients, by which the processes taught in the foregoing lessons may be materially abridged, in those numerous but very simple calculations which an active tradesman finds himself continually called on to make.

342. Again, the foregoing lessons disclose much of the science, as well as teach much of the practice, of this very valuable branch of knowledge. And teach these things, so far as the lessons proceed, as they are required and practised, by the man of business, the scholar and the statesman. But this PRACTICE, which deals altogether in calculations of money, weights, and measures, although it has its eye on *principle*, drives right at its mark

by the *shortest route;* each branch of trade having its peculiar course, and each individual tradesman his by-paths, and short-cuts. However, I shall put my pupil into the broader and safer paths only; into those along which he may see his way clearly; and, as he may proceed in life, having his eye on his fellows, he will not fail to profit by their example, in the peculiar line of business into which his fortune may lead him.

343. "Practice, then, means any of the methods of expeditiously calculating in matters of trade," as I have said above; and here is an example of it. Suppose I would know how much money a considerable number, or quantity, of anything would come to, at 2s. each; let it, for example be, 1362 yards of linen. The mere arithmetician might multiply the number of yards, by the number of shillings, as you see it done here, and thus, having the cost of the whole in shillings, would divide those shillings by 20, and then he has the money in pounds. This is the method in which the mere arithmetician might do it; and very proper, too, for him. But the tradesman, always, on the alert for saving time, sees in an instant, that 2s. is the *tenth* part of a pound, so he divides the number of yards by ten, by cutting off the last figure; thus, 136'2, and, seeing that this last figure represents so many yards, at 2s., each; that, in short, it is 2 *yds.* at 2s. each, he multiplies this figure by 2; and so he has, in this shorter method, the same result as is obtained by the more formal, and longer method.

```
      1362
         2
2'0) 272'4
    £136-4
```

344. This first example, to be sure, as it should be, is a very striking instance of *practice;* and it

exhibits very clearly, and very forcibly, the method to be pursued in this rule; and cannot fail, with the aid of another example or two, to lead you into a pretty thorough knowledge of the method of practice in most cases. Having to ascertain the cost of a quantity of things, then, of any sort, the cost of which, singly, is 2s, we divide by 10, because 2s. are the tenth part of a pound; so, of course, if we have to ascertain the cost of a number of things which are 5s. each, we divide that number by 4, because 5s. is the fourth part of a pound; if they be 2s. 6d. each, we divide by 8; if 3s. 4d. by 6; because 2s. 6d, is the eighth, and 3s. 4d. is the sixth, of a pound; and thus do we get the cost in pounds: in short, we consider the *number* of things in each of these cases, as so many *parts* of a pound, and this practice is a mere *reduction* of those parts into pounds. When the price of each article, each pound, or each yard, is less than a shilling, we generally *reduce those parts* into shillings, and then into pounds; so that the *principle* of this rule of practice, is *reduction*. To return, however, to our example.

345. The cases I have yet cited are amongst the very simplest that occur; for prices, as you know, do not run thus conveniently on *tenths*, on *fourths*, on *eighths*, on *sixths*, and on such even parts, neither of the pound, nor of the shilling. However, these, as I have said, show the principle on which we proceed. When we can thus do the thing at once, *at once* we do it; but, if we cannot, we do it at twice, or oftener, as the occasion may require. For instance, suppose the price were 2s. 8d. *per yard*, for 1362 yards of linen; cutting off the last figure, as shown before, we have the amount at 2s. per yard; then, knowing that 8d. is a third of 2s. we know that a third of the amount already obtained, will be the amount of the eight-pences; and so we

divide that amount by 3, writing the quotient immediately underneath, then add the two together, and thus have we the amount of the given number of yards. at 2 shillings and 8 pence per yard, as shown here. But suppose the price were 2s. 9d.

```
     136'2
  3) 136'4
      45.8-0
  £ 181.12-0
```

per yard. We may proceed as before, to find the amount at 2s. and, then, seeing that 6d. is a *fourth* of 2s. and that 3d. is the half of 6d. we divide the amount of the 2s. by 4, for the six-pences, and the amount produced by the six-pences, we divide by 2 for the three-pences, writing each line close under the foregoing, and, adding all together, as you see it done here, we have the amount of 1362 at 2s. 9d. Thus, I say, we may proceed. But we may, also, get the amount at 2s. 6d. *per yard*, first; in the manner before intimated; that is, 2s. 6d. being the *eighth* part of a pound, if we divide he number of things by 8, we have their cost in pounds, at 2s. 6d. each: and, then, for the 3d. yet wanting, to make up the 2s. 9d. seeing that 3d. is a *tenth* part of 2s. 6d. divide the amount last obtained by 10, as you see it done here, add the two quotients together, and you have, as you see, the same result as by the former method.

```
 1'0| 136'2
   4| 136.4
   2|  34.1
       17.0-6
   £ 187.5-6
```

```
   8| 1362
 1'0|  170.5
        17.0-6
   £   187.5-6
```

346. In like manner do we proceed, be the price what it may, finding the amount in parts, and adding them together. The examples, however, thus far given, are cases in which the prices are *more than a shilling*, which prices, therefore, we treat as *parts* of *a pound*; as, in the case in which the price is 2s. 6d. we divide by 8, and so have the price of the whole in pounds: but, you will observe,

that the price may be *less than a shilling* for each yard, pound, or whatever it may be; as, for instance, it may be 6d. that is, half a shilling, in which case, dividing by 2, we get the amount in shillings; but a more masterly method is this, as you see it done in the margin; that is, consider the number of things, whatever they be, as so many sixpences, and *reduce* them at once into pounds &c. by dividing by 40, the number of six-pences in a pound. If the price be 7d. take parts for the 6d., and then for the 1d. thus. For the 6d. divide the number of things by 2, and, for the 1d. divide the sum of the six-pences by 6; 1d. being a *sixth* of 6d. then add these parts together, and so you have the amount in shillings; and I need not tell you how to deal with them: or, but this is the more intricate method, because of the fractions, and, therefore, the less safe; however, as it throws some light on the matter, besides serving as a proof to the other method of working, I will give it. The method is, to divide the number of things by 40, as before shown, for the 6d. and then, for the penny, divide the sum of the sixpences by 6, as you here see it done, and the amount of the two quotients is, of course, the amount of the given number of things at 7d. each, in pounds, &c.

```
4'0) 136'2
     34.1
```

```
2) 1362
6)  681
    113.6
    794.6
```

```
4'0) 136'2
  6)  34.1
       5.13-6
   £ 39.14-6
```

347. To proceed in this manner, and in this manner we shall unravel the whole affair; and to proceed, too, without changing the number of things, for, when we can manage *one* number, we can manage *any* number; to proceed in this manner. Suppose the price be 7½d. Seven-pence half-penny, is 6d. and a *fourth* of 6d. Reduce the

number of articles, then, into shillings, by dividing by 2; then take a *fourth* of this quotient, for the three-halfpence; thus, and then you have the amount of the whole in shillings and pence. Or, suppose the price $4\frac{3}{4}$d. each; that is 4d, $\frac{1}{2}$d, and $\frac{1}{4}$d. Now fourpence being a *third* of a shilling; and a half-penny being an *eighth* of four-pence; and a farthing the *half* of a halfpenny, we proceed thus. Consider the number of articles as so many four-pences, and reduce them into shillings, by dividing them by 3; then take an *eighth* part of the shillings for the half-penny; and then a half of the sum of half-pennies for the farthings; and so you have, when those three parts are added together, the amount of so many things at $4\frac{3}{4}$d each.

2)	1362
4)	681
	170. 3
s.	851. 3d.

3)	1362
8)	454
2)	56. 9
	28. $4\frac{1}{2}$
s.	539. $1\frac{1}{2}$

348. It would really be a waste of time, with my pupils, whom I have all along regarded as capable of thinking, and of reasoning, to say any more on this rule, as to the methods of working, so far as *prices* are concerned; except, that in order to leave nothing material, in this useful rule, unexplained, I may observe, that when the price of a number, or quantity, of things consists of so many pounds, shillings, and pence, you, as you will suppose, multiply the number, or quantity by the number of pounds in the price, and take parts, as shown above, for the odd money, and then add all together. As, for example, let it be that we would find the value of 1362 ounces of gold, at the mint price, which is £3. 17s. $10\frac{1}{2}$d per oz. Multiplying the number of ounces by 3, gives us the value at £3. Then, for the rest, we take parts, as you here see it done, and adding the product &c. together, we find the amount required.

For the 2s. 6d. divided by 8	1362........	Number of ounces	
	3........	,, pounds sterling	
...... 15s. $\frac{1}{4}$ £3 4	4086........	Value @ £3. 0s 0d	
	1021 10 0..	,,	@....15. 0
...... 3d. $\frac{1}{10}$ of 2s. 6d. 10	170 5 0..	,,	@....2. 6
...... 1½d. $\frac{1}{2}$ 3d. .. 2	17 0 6..	,,	@....0. 3
	8 10 3..	,,	@....0. 1½d.
	£5303 5 9		£3 17 10½

349. However, let us try another method, thus, £3 17s. 10½d. is only 2s. 1½d. short of £4: let us see what the quantity comes to at £4: then, finding the product of 1362 at 2s. 1½d. and deducting it from the amount at £4, we have the required amount; thus,

1362 oz.	1362 oz. at 2s. = 2724 0
4£	$\frac{1}{8}$) 1362 ,, ,, 0 1½ = 170 3
£5448 0 0	2'0) 289'4 3
144 14 3	£144 14 3
£5303 5 9	

One good example, well considered, is better than a number imperfectly understood; so let us work this question in another method or two.

1362 oz.	For 2s. divide by......1'0)136'2
4£	136 4 0
5448 0 0	For 1½d divide by 8) 136'2s
144 14 3	2'0) 170 3 = 8 10 3
£5303 5 9	£144 14 3

1362	2'0)136'2 at 1s.
4	68 2 0
5448	8) 68 2 0
144 14 3	8 10 3 @ 1½d. that is $\frac{1}{8}$ of a shilling.
5303 5 9	£144 14 3

350. Now, these PARTS, of which I have been speaking, and which we have been using pretty freely; these *parts* have a very suitable name. We use them, as you must have observed, as we use *factors*, and *submultiples*; that is, for the purpose of avoiding *long multiplication*, and *long division*,

on occasions in which it is more convenient to work with two, or more, small numbers, rather than with one large one. But, and mark the neatness of the distinction; *these parts* with which we work in Practice, differ from those, in this respect; that whilst, in the use of factors and submultiples we care not how many we may have to work with; because, if there be many, they are so much the smaller, and therefore, more easily worked; whilst this is the case with those parts, the contrary is the case as to the parts used in practice; for here, simplicity, and ease, and clearness of statement, are all promoted by working our sums by as *few parts* as possible, and hence comes the name *aliquot;* which is a latin word signifying *some few;* so, *some few parts,* which are the things we ought to work with, in practice, are called, ALIQUOT PARTS.

351. One other point remains to be noticed before I dismiss this rule, and leave you to perfect yourself in it, by practising on the few sums which follow. This point is, the method of proceeding when we would ascertain the total cost of a number, or quantity of things, stated in weights or measures of *several denominations;* as in *cwts, qrs, lbs: bushels, pecks, gallons;* and so forth. As to the method of treating such quantities, it is very simple, and need scarcely more than to be mentioned, in order to be understood by the attentive pupil. The method is, to take parts in these quantities, as shown in the *first* of the two following examples, and then to add the product of those parts to any other amount you may have; or, another method may sometimes be adopted with advantage, and this is, to find the amount of the *whole number,* next above the quantity in question, as in the case of the *second* working which here follows, and from such whole number, to subtract the value of the part, or parts, by which the whole number exceeds the given quantity.

(1) Find the value of 13 *cwt.* 3 *qrs.* 14*lb.* @ 84s.

The quantity for which we find the value by these operations being only 14*lb.* short of 14 *cwt.* and, as it is a simple matter to find the cost of the *whole number*, 14 *cwt.* this method is, in the second working, adopted; and 14*lb* being an eighth of a *cwt.* an eighth of 84s. is subtracted from the cost of the whole quantity, and thus have we the same result as that obtained by the first process.

	cwt.	*qrs.*	*lb*			
5)	13	3	14			
	4					
13×4=	52	0	0	13 *cwt.* @ £4		0
„ ÷5=	2	12	0	„ „	0	4
	2	2	0	2 *qrs.* „	4	4
	1	1	0	1 „ „	4	4
	0	10	6	14*lb*	4	4
	£58	5	6			

(2) *cwt.* *qrs.* *lb.* 13 3 14 = 14 *cwt.* less 14 *lb*

	14		
	4		
14*cwt* @ £4 = ...	56		
„ „ „ 4*s*=⅕ of £14=	2	16	
	58	16	0
⅛ deduct for 14*lb*		10	6
	£58	5	6

As it is chiefly thus by PARTS that we work in this rule, it will be useful to have Tables of such as are most frequently in use. I will, therefore, here give you such Tables of the Parts of our money, and of our Avoirdupoise weights. And, as to those of other measures, the Tradesman who has occasion, will be at no loss for them.

TABLES OF ALIQUOT PARTS.

OF A POUND.

s.	*d.*		£
10	0	are	½
6	8	....	⅓
5	0	....	¼
4	0	....	⅕
3	4	....	⅙
2	6	....	⅛
2	0	....	1/10
1	8	....	1/12
1	0	....	1/20

OF A SHILLING.

6*d.*	are	½
4	....	⅓
3	....	¼
2	....	⅙
1½	....	⅛
1	is	1/12
½	..	1/24

OF A CWT.

qrs.		*lb.*		
2	or	56	are	½
1	..	28	is	¼
0	..	16	are	1/7
0	..	14	..	⅛
0	..	7	..	1/16

OF A PENNY.

far.		*d.*
2	are	½
1	is	¼

OF A TON.

10	*cwt.*	½
5	..	¼
4	..	⅕
2½	..	⅛
2	..	1/10

OF A QUARTER.

lb.		
14	are	½
7	....	¼
4	....	1/7
3½	....	⅛

(3)	1217	@	7½d	(28)	25871	@	2	0	(53)	72930	@	7s	2d
4	4305	„	0¾	29	42890	„	2	0¼	54	64079	„	7	6
5	1618	„	1¼	30	712864	„	2s	2d	55	80753	„	7	9½
6	2571	„	1½	31	692351	„	2	3	56	29651	„	8	4
7	7230	„	1¾	32	875364	„	2	3½	57	37295	„	8	8
8	5290	„	2¼	33	793251	„	2	4¼	58	58392	„	9	2½
9	6375	„	2½	34	538972	„	2	6¾	59	84127	„	9	0
10	8521	„	2	35	380379	„	2	10½	60	38705	„	10	6
11	9728	„	3¼	36	290275	„	3	1½	61	63542	„	11	6½
12	10352	„	4¾	37	863092	„	3	2¼	62	75063	„	12	3
13	35780	„	5½	38	732051	„	3	4	63	138941	„	13	6
14	27305	„	6¾	39	185902	„	3	5¼	64	37095	„	13	2
15	93654	„	7½	40	963201	„	3	7	65	58230	„	13	4
16	52730	„	8¼	41	53721	„	4	6	66	83054	„	14	6½
17	71028	„	8¾	42	39654	„	4	7	67	62971	„	15	3
18	63527	„	9½	43	73015	„	4	8½	68	317065	„	15	4
19	27835	„	10¼	44	64912	„	4	10	69	83706	„	16	8
20	38721	„	11	45	48176	„	5	2½	70	43072	„	16	6
21	12530	„	12¾	46	83403	„	5	4¾	71	68504	„	17	6
22	52370	„	13¼	47	217518	„	5	8	72	39071	„	18	0
23	87653	„	14½	48	59304	„	5	11	73	27306	„	18	6
24	53072	„	16¾	49	92531	„	6	2¼	74	43851	„	19	0
25	83057	„	19½	50	37042	„	6	6	75	59632	„	21	0
26	52703	„	21¼	51	52709	„	6	8	76	75063	„	24	0
27	72035	„	22¾	52	48321	„	6	10½	77	93217	„	25	0

No.	Cwt.	Qrs.	lb		s.	d.
(78)	215	2	14	@	10	0
79	197	3	7	„	9	4
80	372	1	9	„	10	6
81	430	0	15	„	12	0
82	263	2	11	„	12	8
83	514	3	4	„	14	0
84	792	1	21	„	16	6
85	605	0	19	„	17	6
86	378	2	17	„	18	0
87	1351	1	23	„	19	0
88	740	3	15	„	23	0
89	409	2	12	„	27	0
90	537	1	19	„	32	0
91	1508	2	11	„	45	6
92	527	1	18	„	63	0

No.	Tons	Ct.	Qrs.		s.	d.
(93)	35	12	2	@	66	0
94	27	15	3	„	69	6
95	43	12	1	„	74	0
96	74	3	2	„	95	0
97	112	1	0	„	112	6
98	135	17	3	„	123	0

No.	Qrs.	Bsls.	Pks.		s.	d.
(99)	53	7	2	@	42	0
100	75	5	1	„	45	6
101	93	2	3	„	57	0
102	112	6	0	„	64	0
103	135	5	4	„	66	6
104	142	7	2	„	68	0
105	231	2	3	„	72	0

OF FRACTIONS,

Their Origin, Reduction, Addition, Subtraction, Multiplication, and Divison.

352. Already, in paragraphs 160 to 173 I have treated of the nature of Fractions, and of their mode of statement; and have slightly glanced at the method of working them. Let me advise the learner to read those few paragraphs again, as a very suitable preparation for this lesson.

353. Fractions present themselves under various forms. They sometimes appear amongst the terms of a question or proposition; but in ordinary concerns of business, saving the simple fractions, halves, quarters, and eighths, seldom present themselves except as remainders, in operations of division; as, to give a brief and simple instance; have we to divide £109. 10s. 2d. by 6; we have, for the quotient £18. 5s. 0$\frac{1}{4}$d., and a remainder of two farthings yet undivided, the sixth of which we must annex to the quotient, before the division be complete. Now, how to do this, is a point in fractions; this remainder is but 2 *fourths*, and how shall we divide it by 6? The fact is, we cannot so divide it: but we can do something else, equally efficient; we can express 2 sixths of a farthing, as we have just expressed the fourth of a penny, by writing it after the farthing thus $\frac{1}{4}$ $\frac{2}{6}$. But, bear in mind, that although this may be done, it is not the proper method of stating such a matter, for we have now *two* of those broken quantities which arithmeticians very wisely seek to avoid; and, *two fractions*, with *different denominators;* and, to complete the difficulty, one of these fractions is *a fraction of the other!* a thing never to be permitted! A single fraction we cannot avoid, in such a

case; but a single one is nothing; it is the *two;* and especially, the having *one* of them *a fraction of the other*, that would make an arithmetician start from the work. So, how to avoid it is the question. Very easily! Having reduced the two pence into farthings, and finding, that although you have thus reduced, to the *lowest denomination*, your divisor will not *evenly* divide it, you express the quotient of 2 pence divided by 6, thus, $\frac{2}{6}$d. or, rather ^{1}d. and then you have, as the exact quotient £18. 5s. $0\frac{1}{3}$d., which, like all quotients, if multiplied by the divisor, will reproduce the dividend.

354. This is saying a great deal about a trifle, it may be said. But I am not saying all this about the third of a penny; it is on the origin, and on the treatment of arithmetical fractions, that I am speaking. And, besides laying a solid foundation for more important matters, we have, in this trifling instance seen, not only how fractions commonly originate, and what is to be done with them in certain cases, but, likewise, what is never to be done with them in any case: a kind of learning which is never to be despised.

355. These fractions will occur. You can seldom divide any sum, except those that are fabricated for the purpose of being evenly divided, without these troublesome guests. They will intrude in reckonings in real business. And, as it is for real business, and not for that kind of *showy, sleight-of-hand arithmetic*, which is taught by itinerant professors, that I would prepare my pupil; as I would clear and strengthen his mind, by the infusion of principles; and not burthen his memory with paltry tricks and expedients, so I must teach him to deal with these troublesome visitors, called fractions, which present themselves in various shapes; even the same value appearing under an

endless variety of forms. As, for example, *a half* is expressed by any fraction, the *denominator* of which is *twice the* numerator; and a *third* is described by any fraction, the *denominator* of which is *thrice its numerator*, let these denominators and numerators be what they may.

356. Did fractions occur, only as halves, quarters, fifths, eighths, and so on; did they appear with only one figure for a numerator, and one for a denominator, we should easily manage them; but as they arise from the operations of division, and, as the numerator is the remainder, and the denominator the divisor. So, having sometimes a *long* divisor, we have, in such cases a *long denominator;* and, as the remainder may be but a little less than the divisor, so we sometimes find ourselves with a fraction described in two *long* lines of figures.

357. But the same value, as I have said, is often expressed by a short fraction, as by a long one; as, for example, $\frac{1}{5}$ describes as much as $\frac{23357}{116785}$: for, in both these expressions, the numerator is *one fifth* of the denominator; and the quantity described by each is, consequently, a fifth, merely, of one whole. But, how much more convenient is the shorter, than the longer expression! Hence, in all cases, to employ the shorter expression, as inculcated in paragraph 173, is a settled law amongst arithmeticians.

358. Fractions are sometimes to be added together; as in the case of the operations immediately following paragraph 283. In the first of these operations we have a remainder 144960, which, with its divisor, stated fractionally, is $\frac{144960}{181200}$; and, in the second operation we have $\frac{6040}{30200}$. And each of these is a fraction of a farthing, merely. In the cases from which I have taken them, I reduced

these remainders to *decimals*, that being a convenient form for the occasion; but of these hereafter. At present we have to do with *vulgar fractions* only, and here are two pretty long ones, and to be added together, in order to prove the truth of the two operations from which they have arisen.

359. Now, how to add these two fractions together, is the question. To do so with them in their present form is impossible. A description of them in *words*, would be more puzzling than are the figures, so let us use some fractions of a simpler form, but presenting a similar difficulty, by way of illustration; let it be that we have to add together $\frac{4}{5}$ and $\frac{3}{15}$. Four fifths we could add to two fifths, or to three fifths, or, in short, to any number of *fifths*, for it would but be the *adding* together of so many *fifths*. But to add *fifths* to *thirds*, to *fourths*, to *fifteenths*, or to *any other parts than fifths*, is an impracticable incongruity; that is to say, to add *fractions* together of *different denominations*, is impracticable. Yet *quantities*, however different in size, may be added together, and the smaller may be subtracted from the larger; but, how are the fractions, descriptive of these quantities, to be added or subtracted? Thus it is to be done. They are to be brought into *like denominations;* that is to say, *the parts*, of which two, or more fractions speak, are to be brought, or *reduced, to the same size, or same value;* and then you find no incongruity in adding *those parts* together, as $\frac{1}{5}$ and $\frac{4}{5}$ make $\frac{5}{5}$, or one whole. Nor anything difficult in subtracting the smaller from the larger, and having $\frac{3}{5}$ as the difference. And yet these fractions $\frac{1}{5}$ and $\frac{4}{5}$ are in value, not only the same as $\frac{3}{15}$ and $\frac{4}{5}$; but the same precisely as the *two long* fractions stated in the last paragraph.

360. Thus, having fractions of *one and the same denomination*, we can work them together. In

addition, we add the numerators together, retaining the denominator; which, belonging, in common, to the two fractions, is called *the common denominator*. And, in *subtraction*, also, as in whole numbers, you subtract the smaller number from the larger; that is, the smaller numerator from the larger, and, to *the difference*, you put the common denominator. But, to bring fractions of different denominators thus to have a common denominator. This is called

REDUCTION of FRACTIONS.

361. This is every thing, in these numbers. As to their addition and subtraction, you have just seen, that these are merely the addition and subtraction of *numerators;* and the multiplication and division of them are much the same; and, so far as matters of business require, they are quite as simple. It is the *reduction* of fractions that is every thing; the reduction of them in *two* ways, and, for two different purposes. *First*, there is the bringing of fractions of *different denominations*, into the *same* denomination, in order, as you have seen, to prepare them for addition and subtraction; and, *second*, there is the reduction of them, from long and inconvenient numbers, or terms, to their shortest and most compact form.

362. A fraction expresses a part, or parts, of a whole; and its numerator always bears the same proportion to its denominator, as the part, or parts described by the fraction, bears to the whole thing spoken of. This point needs no illustration here. The numerator, therefore, and the denominator of a fraction, bear a proportion towards each other. But, the proportion of numbers is not altered, as shown in paragraphs 258-9, by their being multiplied, or divided; *provided* that they be multiplied,

or divided, *by the same* number. As, if we multiply both the terms of the fraction, $\frac{1}{2}$, by 8, it remains unaltered in its value; it being then $\frac{8}{16}$; the numerator half of the denominator; so, if we divide these two terms by 4, we have $\frac{2}{4}$, which expresses the same thing. And thus it is with any fraction, whether the figures by which it is described be few or many. This point, of the value of a fraction being unaltered by the *equal* division or multiplication of its numerator and denominator, being established, we are led very easily to the two methods of *reducing fractions.*

363. And, first, as to the reduction of fractions to the *same denomination.* Let it be, that we would thus reduce $\frac{1}{2}$ and $\frac{2}{6}$. Beginning with the first of them, let us multiply both its terms by 6, and we have $\frac{6}{12}$: and now, let us take the other, that is the $\frac{2}{6}$ and multiply both its terms by 2, and we have $\frac{4}{12}$. But the fractions are now both of *one denomination,* and their value is unaltered. And, how has this been accomplished? Look at the terms, before, and after, they were reduced, and you will see, that it has been accomplished by multiplying both the terms of each fraction by the *denominator of its neighbour.* And thus it is, that any two fractions, without altering their value, are to be reduced to the same denomination; that is to say, by multiplying the two terms of each fraction, by the denominator of the other.

$$\frac{1}{2} \quad \frac{2}{6} \quad : \quad \frac{6}{12} \quad \frac{4}{12}$$

364. Nor is it with *two* fractions merely, that this method is to be pursued. The same treatment will reduce any number of fractions to a common denomination; as, for example, in order to keep the matter as simple as possible: let us add one other small fraction to the former; let us take these three to be reduced to a common denominator $\frac{1}{2}$ $\frac{2}{6}$ $\frac{3}{5}$.

Now this is the method of stating such an operation with clearness. And, when you have the fractions thus reduced, if you have done it for the purpose of adding them together, which is almost the only purpose for which you can have thus to reduce *three* or more fractions, you state them as you see here below; the numerators in a line, with the sign of addition between them, and, below the line, and about the middle, you write *the common denominator, once only*, however many the fractions may be. And then, if you would bring the whole into *one* fraction, it is done, as you see, by adding the line of numerators together, and by writing the *common* denominator, in the proper form, under the sum of them.

For the new denominator.

$$2 \times 6 \times 5 = 60$$

For the numerators.

$$1 \times 6 \times 5 = 30$$
$$2 \times 2 \times 5 = 20$$
$$3 \times 2 \times 6 = 36$$

$$\frac{30+20+36}{60} = \frac{76}{60}$$

365. As to the PRINCIPLE of this reduction, it has already been adverted to; but to state it more explicitly. For a common denominator, we *multiply all the denominators* together; that is, we multiply the denominator of the *first* fraction, by those of the *second* and *third;* and, were there yet more, we should go on, thus multiplying them. Having done this, having thus multiplied the denominator by certain numbers, *in order to keep the value of the fraction unaltered*, we multiply the numerator by *the same* numbers. Now these numbers are, *the denominators of the other fractions:* and thus it is, that, multiplying all the denominators together for a common denominator, and the numerator *of each* fraction by the denominators *of the others*, brings any number of fractions into *one*, or *common*, denomination, *without altering their value.*

(1) Reduce $\frac{7}{8}$, $\frac{2}{5}$, and $\frac{4}{7}$ to one denominator.

(2) Reduce $\frac{12}{17}$, $\frac{3}{5}$, and $\frac{2}{9}$ to a common denominator; and state the sum of them.

(3) What is the sum of $\frac{13}{15}$, $\frac{7}{9}$, $\frac{3}{8}$ and $\frac{2}{5}$?

You will, of course, first reduce these fractions to a common denominator; and then, adding the numerators together, will place the sum of them, in due form, over that denominator.

(4) Add these fractions together, $\frac{135}{271}$, $\frac{72}{90}$, $\frac{31}{43}$.

(5) Reduce $\frac{23}{30}$, and $\frac{3}{8}$, and subtract the smaller from the larger.

(6) What is the difference between $\frac{17}{20}$, and $\frac{43}{97}$?

(7) Subtract $\frac{25}{62}$ from $\frac{19}{20}$.

(8) Subtract $\frac{33}{70}$ from $\frac{33}{47}$.

366. As for the *reduction* of fractions, from long and inconvenient terms, such as you will find on your hands after each of the foregoing operations, it is done thus. We have fully established the principle, that the division of the terms of a fraction, *provided that both terms be divided by the same number*, makes no alteration in its value. On this principle, then, it is, that the terms of fractions are reduced. We divide those terms; when they will *evenly* divide, until we bring them to their *shortest numbers*. Some terms, or numbers, will not divide by any other number; such terms according to paragraph 148, are called *primary*, or *primes*. And, when both the terms of a fraction, or, indeed, when *the larger term* of a fraction is a *prime number;* there is no reducing that fraction. For instance, here is a fraction, the *larger* term of which is of this description $\frac{32}{223}$. As for the smaller of its terms, no number whatever will divide more easily; but, because the larger will not divide, this fraction cannot be reduced: But make this stubborn number the *smaller term* of a fraction, and let it be one of the *submultiples* of that larger term, and then no fraction will reduce better. For, let the fraction be $\frac{223}{669}$; and, seeing that the smaller of these terms is *one third* of the larger, $\frac{1}{3}$ is the proper representative of this long fraction.

367. When the numerator of a fraction will thus evenly divide its denominator, we deem it fortunate; for it brings the denominator at once to its lowest possible term, and makes the numerator, only a single unit; $\frac{3}{27}$, so reduced, is $\frac{1}{9}$. And, were the terms inverted thus; $\frac{27}{3}$, that is, were it an *improper fraction*, dividing the larger term by the smaller gives us $\frac{9}{1}$, or rather 9: $\frac{27}{135}$ becomes $\frac{1}{5}$, and $\frac{2781}{15124} = \frac{1}{4}$.

368. In fractions such as the shorter of these, we can see at a glance, what numbers will divide them; and a little consideration will sometimes enable us to discover, almost without trial, what number will divide the terms of a long fraction. But it would not do to rely thus on our sagacity; we must have a *rule*; and, accordingly, we have one, by which, *to a certainty*, we can find the best, that is, *the largest* divisor for the two terms of a fraction: or find, also to a certainly, if the terms cannot be divided. *The rule* is this; *Divide the larger term by the smaller*, and, if it leave a remainder, bring down the smaller term, that is, the last divisor, and divide it by the remainder; and if, again, this leave a remainder, bring down the former remainder, that is, again, the last divisor, and divide it with the last remainder; and so you proceed, *dividing the larger term by the smaller, and the last divisor, by the last remainder, until you have no remainder left;* (for to this it will come) *and your last divisor, which is, also, your last remainder, will divide both terms of your fraction:* that is to say, it will divide them, if it be a number that has the power to divide: but, if the process bring you down to a *single unit*, which has no power of division in it, then you find that your fraction stands already in its shortest terms.

A few examples will be useful, therefore, I annex them. In case (1) the smaller term divides the larger at once; it is, therefore, the number we seek. In (2) we have a similar case, but stated in a form which more fully, and clearly, shows *how* such a fraction is so completely reduced. In the other cases, which here follow, the division is carried further; the *third* gives us 1812, as the last remainder, and that therefore, is the divisor for which we are seeking; the *fourth* gives us 906; the *fifth* brings us down to 2; and the *sixth* process spins out until it brings us to 1; an assurance that the fraction is, already in its lowest terms.

```
(1)  3781) 15124 ( 4
           15124
           .....

(2)  36240) 36240 ( 1
            )181200 ( 5
```

```
(3) 3624 ) 19932 ( 5               (5) 3626 ) 19942 ( 5
           18120                              18130
           1812 ) 3624 ( 2                    1812 ) 3626 ( 2
                  3624                               3624
                  ....                               ...2 ) 1812
                                                            906

(4) 3624 ) 20838 ( 5               (6) 3625 ) 19937 ( 5
           18120                              18125
           2718 ) 3624 ( 1                    1812 ) 3625 ( 2
                  2718                               3624
                  906 ) 2718 ( 3                        1
                        2718                         ....
                        ....
```

369. With regard to the PRINCIPLE on which this *common divisor for two numbers* is discovered, it is this. Taking the *third* of the foregoing cases for our remarks: In 19932 we find, that 3624 is contained 5 times, with a remainder of 1812. This *remainder* we afterwards find, is *half* of the *divisor*; but that divisor was found 5 times in the larger number, and this remainder over: then, *this remainder*, which proves to be half of the divisor, will be found *ten times* in the large number, *and, once over*; ten and one are eleven, therefore, this remainder is to be found *eleven times in the larger number*, and being the half, that is, being found twice in the smaller term, without leaving a remainder, it is a common divisor for both terms, and, applied to those terms, it reduces the fraction as you here see.

$$1812)\frac{3624}{19932}(\frac{2}{\ }$$

370. I now entertain none of the apprehensions expressed in paragraph 333, as to the inability of my pupils to work the five several statements arising out of the first question in compound proportion. In the statement of fractions, the *numerator* being understood to be a *dividend* to the *denominator*, so this form has been adopted, as a very neat and compact one, for stating the terms of questions in compound proportion: the terms to be divided being written, like numerators of fractions, *above*, and those with which the division is to be effected, as denominators, *below* a line. The terms being thus placed, the resolution of the question becomes a mere reduction of so many fractions into one; the terms *above* the line, being multiplied together for a numerator, and those *under* the line, for a denominator; and *this large fraction, reduced to its shortest terms,* to whole numbers, &c. is an answer to the question. I will here repeat the several statements, spoken of above, with the *signs*, descriptive of the operations.

First, $\dfrac{23 \times 16}{12}$ reduced to $\dfrac{23 \times 4}{3} = \dfrac{92}{3} = 30\frac{2}{3}$.

Second, $\dfrac{9 \times 30\frac{2}{3}}{11} = \dfrac{9 \times 92}{33} = \dfrac{828}{33} = \dfrac{276}{11} = 25\frac{1}{11}$.

3rd, $\dfrac{205 \times 25\frac{1}{11}}{180} = \dfrac{41 \times 25\frac{1}{11}}{36} = \dfrac{41 \times 276}{396} = \dfrac{11316}{396} = 28\frac{19}{33}$.

4th, $\dfrac{22 \times 28\frac{19}{33}}{13} = \dfrac{20746}{429} \div 11 = \dfrac{1886}{39} = 48\frac{14}{39}$.

5th, $\dfrac{17 \times 48\frac{14}{39}}{14} = \dfrac{32062}{546} \div 2 = \dfrac{16031}{273} = 58\frac{107}{273}$.

6th, $\dfrac{23 \times 16 \times 9 \times 205 \times 22 \times 17}{12 \times 11 \times 180 \times 13 \times 14}$ reduced to $\dfrac{23 \times 1 \times 1 \times 41 \times 1 \times 17}{3 \times 1 \times 1 \times 13 \times 7}$

is equal to $\dfrac{16031}{273} = 58\frac{107}{273}$, which is the answer.

Placed in this form, the terms stand well for being *reduced*, or *cancelled*. In the *first* statement, you find a small specimen of this reduction, 16 and 12, being divisible by 4, are reduced to 4 and 3. The other statements, having *fractions*, require, according to rule, paragraph 280, to be reduced to a simple number; which is done by multiplying the integer by the denominator of the fraction: and, then, in order to preserve the proportion, the denominator, or term with which the division is to be made, which in this statement is 11, this is to be reduced in like manner. As $\frac{30\frac{2}{3}}{11}$, reduced to *thirds*, becomes $\frac{92}{33}$; and, in the *third* statement, $\frac{25\frac{1}{11}}{36}$ reduced to *elevenths*, becomes $\frac{276}{396}$; and so in each of the other cases. The *sixth* statement, having *all* the *terms together*, affords more scope for the cancelling and reducing; and to that, therefore, I particularly refer you.

371. On the MULTIPLICATION, and the DIVISION OF FRACTIONS, I have to add a few words. The *proportion* which the numerator of a fraction bears to the denominator, being the index of its value; is it not manifest, that, to *alter* that proportion, will *alter* the value of the fraction? Now, a fraction expressive of *a half*, is any fraction, the numerator of which is *half* the denominator. Well, then, suppose we have to multiply such a fraction by any number, say by 3; is it not plain, that this multiplication, that is to say, this *increase of the value* of the fraction will be effected, either by *multiplying* the *numerator*, or by dividing the denominator by the 3; for either of these *so alter the proportion* between the terms of the fraction, as to make it express three times its former value? And Division of Fractions is, of course, merely the *reverse* of this. *Divide* the *numerator*, or *multiply* the *denominator*, and the operation is effected. As to Multiplication, and Division of Fractions *by* Fractions, they are useless; or worse than useless.

OF DECIMALS:

Of their Addition, Subtraction, Multiplication, and Division.

372. With any taste for *order*, and *simplicity*, and *exactness* of statement, it is impossible to think of this mode of expressing, and of working fractional numbers, without pleasure. And on sitting down to treat of the working of decimals, and to conclude what remains to be said on the subject, I abstain with difficulty, from launching out into a new and eulogistic description of these numbers.

373. But, indeed, this is unnecessary. The description which I have given of them; of their nature, uses, and mode of statement, in the lesson beginning with para-

graph 174, is quite sufficient. And, begging the learner to turn to that paragraph, and to read on to the end of that numbered 184, I shall proceed to teach the working of these numbers, for the learning of which, my attentive pupils will then be well prepared.

374. Sufficiently for our purpose, the notation of decimals is shown in the few paragraphs referred to; their value is spoken of in the last of those paragraphs; and our next step will be their Addition, Subtraction, Multiplication, and Division.

375. With regard to these operations. They are all of them performed precisely the same as in whole numbers, except that, in stating the lines of figures for addition, and subtraction, we are to range them, not as in whole numbers, the last figures on the right hand, in each line, right over each other; but to be guided by the decimal points, which, being the index of value, the figures are to be very carefully written, so, that these points all fall *right under each other;* as you will see that they do in each of the following examples.

376. Adding up these figures in the usual manner, you see, that those in the column of the highest value amount to 17; that is, *seventeen tenths* of an unit. Now, *seventeen tenths* are *ten tenths*, and *seven tenths; ten tenths*, are *one whole;* and, as such, it is set down, above the decimal point. And thus it is with these numbers. You have no *improper fractions.* As soon as, by addition or multiplication, you have parts *sufficient* to make a whole number, those parts, without any of the troublesome reduction required in vulgar fractions, quietly take their station on the other side of the decimal point, as whole numbers; and are thus in readiness, either to remain, or for any further operations. In this second example you see the *whole* number arising from the addition of the decimals, carried on, and added to the integers. In short, it is altogether simple addition, only that we have the decimal point to be preserved in its proper place.

	.99
	.753
	.014
1	.757

7	.325
19	.753
12	.53
39	.608

377. As to SUBTRACTION. It is nothing! You write the numbers to be subtracted, underneath those from which they are to be taken, taking care to keep the decimal points right under each other; and then work as in simple subtraction. See two or three examples.

.835	15 .327	1732 .1693
.619	9 .74315	598 .782516
.216	5 .58385	1133 .386784

378. In MULTIPLICATION we pay no regard to this matter, of ranging the decimal *point* of the multiplier *under* that of the multiplicand, but write these two terms under each other, as in simple multiplication; and, as you see them stand in the annexed example. Thus multiplying, we find the product as in whole numbers. Which, having done, we count the *number* of *decimals* that there are, in *both* the *multiplicand* and *multiplier*, and then, this number, of both together, we mark off for decimals in the product, and the figures to the left of the point, are, of course, integers

```
      75. 5943
         32.16
  ------------
      4535658
      755943
     1511886
    2267829
  ------------
   2431.112688
```

379. DIVISION, you know, is exactly the reverse of *multiplication*. It is calculated to *undo*, exactly, that which is effected by multiplication. As, for example, suppose we would *undo* the work in the foregoing example; that is, suppose we would divide 2431. 112688 by 32. 16. We write down these two sums, and divide just as we do in simple division: only, with regard to the *decimal point;* having found the quotient to be 755943, we observe this rule in *fixing* that point in its *proper place:* we count the number of decimal figures in the *dividend;* we see, how many *more* there are in it, than in the *divisor,* and *this difference* is the number to be cut off for decimals in the quotient. And this proceeding, as you will observe, restores to us the original multiplicand; and proves, not only that both operations are right; but, that *the rules* by which we work them are right, likewise.

We must not, however, omit to observe here, that cases arise, in which the quotient is so small, that there may not be decimal figures so many in the quotient, as there are in the dividend more than in the divisor. When a case of this kind occurs, you put, *before* the figures of the quotient, *ciphers*, sufficient to make up the required number; and then you set the decimal point.

380. Here I might finish on this rule of division of decimals; but that there yet remains this point to be noticed. It is not, as you know, the *quantity of figures*, that determines exactly the *value* of any number; but the individual value of each figure, and the station in which it stands. The figure 9 expresses a higher number than 8, when spoken of without regard to station; but, place 9 *below* the decimal point, and 8 above it, and 8 is then the higher number. In like manner 2.9756, is a number of less value, than is 3.12; and, as a smaller number will always divide a larger, we might have to divide these three figures, by the five. How then, are we to act in a case like this. To divide the smaller number of figures by the larger, without some *abatement* of the number in the larger, or some *increase* of the smaller, is manifestly impracticable. One of these courses must, then, be adopted, and it must be one which will *not alter* the *relative* value of the two numbers. Now what method is there of doing this? Look at the *two last sentences* in paragraph 181, and there you see, that you may put any number of ciphers *below*, that is *after* the figures of a *decimal*, *without altering its value*. Adopt this course, in this instance with your short dividend, and having completed your division, you fix the decimal point as before instructed.

381. Such ease in working, such simplicity of statement, such readiness for every operation, as we find in decimal numbers, cannot fail to suggest to every mind, engaged on the subject, this question: How happens it, that this neat and beautiful method of stating *parts* of whole numbers, has not entirely supplanted the method by vulgar fractions? The answer to this question leads us to the last rule that remains to be spoken of in these numbers. Decimals have not sup-

planted vulgar fractions, *because remainders,* whence almost alone come fractions, always present themselves, in the usual operations of arithmetic, as the *nominators* of vulgar fractions, the divisor being the *denominator.* Remainders, after divison, thus present themselves, and they require, before they can be stated, or worked as decimals, to be *reduced* to these numbers, an operation which it is not always worth our while to perform.

382. This reduction of vulgar fractions to decimals, together with every thing else relating to these numbers, that I deem useful to men of business, are taught in the few remaining paragraphs of the former lesson; refering the learner to those paragraphs, beginning with that numbered 185, and recommending his attention to them, I conclude the subject.

DUODECIMALS,

TWELFTHS, OR CROSS MULTIPLICATION;

Being the Calculations of Measurements taken in Feet, Inches, &c.

383. These duodecimals are briefly spoken of in paragraph 175; the use of them is described above; and, in conformity with my uniform plan, I have now to unfold the Principle, and to describe some of the processes, by which calculations in these numbers are to be performed.

384. Duodecimals, then, are used to calculate quantity; quantity in *superficial extent* and in *bulk;* superficial, such as the measurement of boards, of the work of painters, glaziers, and plaisterers; and quantity in *bulk*, as in a piece of timber, or a block of stone. All which things are usually measured by the foot of 12 *inches;* which inches, for nice purposes, are each divided into *parts;* and hence the name, and the use of duodecimals, or twelfths, by means of which we ascertain the superficial, or the solid, contents of anything measured by this *foot rule* and its parts. The contents are usually brought into square, or cubic feet; that is, into squares measuring a foot on each side; or into solid blocks,

called *cubes*, the dimensions of which are a foot each way, in length, breadth, and thickness; and, having the quantity stated in these dimensions, they are easily, when required, reduced into yards.

385. As an example of the calculation of superficial quantity. Here is a wall, 8 *feet* long, and 6 *feet* high. And we would know, how many feet, that is to say, how many squares, measuring a foot each way, there are in the whole. The wall is 8 feet long: let us divide the length into 8 equal parts, as you see in figure 1. But it is, also, 6 feet high. Let us, then, taking the same figure, divide the *height* thereof into six parts, and ruling the divisions *across*, as we ruled the former downwards, we have it divided as Fig. 2. Now count the squares; and what number do you find? 48, which number, you know, is the *product* of 8 multiplied by 6; that is to say, the *product* of the *length* of the wall multiplied by the *height*. Knowing this, knowing that the length, multiplied by the breadth, will give the superficial contents, we have, in future, no occasion, when we would know the number of square feet in anything, to divide the thing, thus into squares, and then to count them; we have only, having the number of feet in length, or, indeed, the number in yards either, and the number in breadth; we have only to multiply these two numbers together and the product is the number of square feet, or of square yards, just as the denominations may be.

Fig. 1

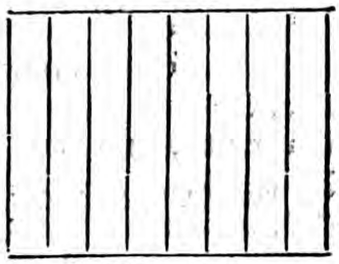
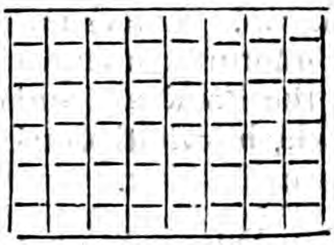


Fig 2.



386. Again, suppose that it were a solid body, such as a block of stone, measuring, in length and breadth just what we have stated. And, in thickness, 6 feet. Dividing this thickness into layers of one foot thick each, and then cutting each layer across, in the lines marked in figure 2, would give us 48 small blocks in

each of the 6 layers, that is, 6 times 48 blocks, which make 288; which number of blocks is, as you see, the product of the length, breadth, and thickness, multiplied successively into each other.

387. But suppose we come to *inches*. Let us take one of these small blocks, measuring a foot on every side, and called, therefore, a cubic foot. It is 12 inches long, 12 inches broad, and the same in thickness. Now, how many solid, or cubic inches are there in such a block? We need not, now, cross it with lines, in order to ascertain this; for, multiplying these three dimensions into each other, gives us the number of solid inches. And, to finish this matter of *cubic* measurement, in order that we may afterwards proceed without interruption in the consideration of that of superficies, let us take as many of these blocks, of one cubic foot each, as will make a cubic yard. Laying the blocks together, we must have, for the first layer, three of them each way, that is, 9 blocks; these will make a layer, of due length and breadth, and *one foot thick;* but we are to have it 3 *feet* thick, so that we must have 3 times 9 blocks, which are 27, which is the number of *cubic feet* in a *cubic* yard.

388. Thus is there *square* measure, and *solid*, or *cubic* measure. *Square* measure applies only to *surfaces*, and the quantity in anything is attained, by multiplying the *length* into the *breadth;* whilst the quantity in *cubes* is found, by multiplying the *length*, *breadth*, and *thickness* into each other. And the quantity thus found, whether the thing measured be square or cube, is called the *contents* of that thing. To return to the measurement of superficies.

389. For our first experiment, let us take a surface 9 *feet* 7 *inches* long, and 8 *ft.* 3*in.* broad; and let the contents be found in feet.

390. Now in this, as in all other proceedings, it is desirable that we have a clear and distinct idea of the matter of which we are treating. So let us fix, very stedfastly in our minds, *what quantity* of surface is meaned to be described, by each of these denominations,

feet, inches, and *parts.* As to the quantity measured by a *foot,* here it means, as before explained, a *foot square;* because we are about to accertain the contents in *feet:* which, therefore, are the *highest denomination;* which, therefore, are the WHOLE NUMBER of which *inches* are 12 *ths,* and parts, 12 *ths* of 12 *ths.*

391. But, now mark: what is *an inch* in this measure; what is a *twelfth* of a square foot; is it a *square inch?* By no means. A square inch requires to be *extended,* or *multiplied,* twelve-fold, when, becoming as long as a foot, and one-twelfth the breadth, it becomes the *twelfth* of a *foot;* that is to say, it becomes an *inch,* in this sort of measure. In like manner it is with a *part.* A PART multiplied by, that is, extended to the length of a *foot,* is a *part;* multiplied by an inch, therefore, it is only the *twelfth* of a *part;* and *parts* multiplied together are, of course, one remove *yet lower,* in the scale of denominations; they are only the 12*ths.* of 12*ths.* of *parts.* Thus, *feet* multiplied by *feet,* are *feet; inches* multiplied by *feet,* are *inches;* and *parts* multiplied by feet are parts: the rest you will see. And, seeing these matters clearly, and bearing them in mind, you will find no difficulty in understanding, and in working all manner of propositions in this very useful rule.

392. To return, now, to our case, as stated in the last paragraph but one. Writing down the dimensions, the different denominations right under each under, as follows, you begin to multiply by the feet in the lower line; saying, "8 times 7 are 56." But, now, what are these 56? They are the product of inches multiplied by feet; they are, therefore, *inches;* divided by 12 they give 4 *feet* 8 *inches;* the inches being set down in their proper place, the feet are carried forward. Then, multiplying the feet, saying, "8 times 9 are 72, and 4 carried are 76." And now to multiply by the 3 inches. *Inches* you have seen, multiplied together, produce *parts;* so 3 times 7 = 21 *parts* = 1 *in* 9 *pts.* Proceeding next to multiply the 9 feet by the 3 inches, we have 27, which,

ft.		*in.*		
9	:	7		
8	:	3		
76	:	8		
2	:	4	:	9
79	:	0	:	9

with *one* carried from the last, make 28 inches; which are 2 ft. 4 in. and which, written down, as you see, and the two lines of product added together, we have 79 *ft.* 0 *in.* 9 *pts.* as the product of 9 *ft.* 7 *in.* by 8 *ft.* 3 *in.*

393. To sum up the process in the form of a Rule: Having properly stated the dimensions to be multiplied together, you begin the work by multiplying the lowest denomination in the upper line, by the highest in the lower line. And, from this circumstance, it is, that the name of CROSS MULTIPLICATION comes. Beginning thus, and bearing in mind, that *parts* multiplied by feet, produce parts, you divide the produce of such parts by 12, set down the remainder, *as parts*, and carry the quotient forward, as inches; then, multiplying the inches by the feet, you have *inches*, which divided by 12, are carried onwards to the feet. The *feet* are multiplied together, like any other integers, and we set down the product, with the addition of anything that may have been carried. So much for the Multiplication *by* FEET.

394. And, for the Multiplication by INCHES. Multiplying the parts by inches give, as you recollect, *twelfths of parts;* the product here, therefore, you must divide by 12, set down the *remainder, one station below the parts,* and carry on the quotient as parts; then, multiplying the *inches* together, you have parts for the product, which, added to whatever may have been carried, and divided by 12, give inches, which inches, added to the product of the feet, and divided by 12, produce feet, and this finishes the multiplication by *inches.*

395. Then, to multiply by PARTS, if you go to such minute dimensions, which you scarcely will, unless you be computing the value of plates of fine glass, or cloth of gold, or some such costly article: if you multiply by these small parts, you proceed on the same principle. On this principle, *parts* multiplied by *parts;* produce *twelfths* of *twelfths* of *parts;* and these will be a denomination *two* places *below parts.* All this might be made palpable, to the eye, by a diagram, which it was my design to give, but I relinquish the design on finding, that in order to show the *minute* divisions, and the larger

ones likewise, such a figure must be inconveniently large. So I rely a little on the imagination of the ingenious student, who will be at no loss to understand the matter, after the description just given.

396. Another proposition, a little nice in its measurements, will assist yet further to clear up this matter, and to confirm right impressions on the mind of the learner. Let us, then, see, what are the superficial contents of a Plate of Glass, measuring 7 *ft.* 5 *in.* 8 *pts.* in length, and 5 *ft.* 3 *in.* 7 *pts.* in breadth.

Worked in the method we have just learned, the proposition is done as you see in the first of these operations. But, in order to show you, beyond all question, that it is correct, let us, discarding this process of multiplying parts by inches, and inches by parts, and so on, from which alone arises any difficulty that may yet hang about the matter; let us, discarding this process of multiplication, and writing all the dimensions down, repeating them, just as when they are repeated by Multiplication; and just, indeed, as if we had the *several spaces* described by the figures, actually spread out before us, in order that we might place them together, in rows, according to their several denominations, and sum them up; let us thus set down these several dimensions, and *add* them together, and you thereby see, not only that the result is the same as that produced by the other operation, but you see, also, the value, that is to say, the *extent* of every space indicated by each figure.

By Multiplication.

ft.		*in.*		*pts.*
7	:	5	:	8
5	:	3	:	7
37	:	4	:	4
1	:	10	:	5
		4	:	4 · 3 · 8
39	:	7	·	1 · 3 · 8

By Addition.

7 · 5 · 8....					The upper lines × by the feet 5 times.
7 · 5 · 8....					
7 · 5 · 8....					
7 · 5 · 8....					
7 · 5 · 8....					
	7 · 5 · 8				By the inches × 3 times.
	7 · 5 · 8				
	7 · 5 · 8				
		7 · 5 · 8			By the parts × 7 times.
		7 · 5 · 8			
		7 · 5 · 8			
		7 · 5 · 8			
		7 · 5 · 8			
		7 · 5 · 8			
		7 · 5 · 8			
39 · 7 · 1 · 3 · 8					

Before we proceed further, let me tell you, that these *small parts* have their proper *names*, and *marks;* which names and marks have been determined by their several distances from the *foot. Inches* being the *first* remove from the foot, are called *primes*, that is *firsts*, and the mark over them is a single comma (,) Those which we have called *parts*, which is the name generally given by workmen, being in the *second* station, are called *seconds*, and have two commas (,,); and the next have three commas (,,,) and are called *thirds*.

397. One point more, and I conclude the lesson.

For the sake of simplicity, I have used, as examples, cases, in each of which there is but *one* figure in each denomination, whether in the multiplicand, or in the multiplier. For example, I have shown you, how to multiply 9 *feet* 7 *in.* by 8 *ft.* and 7 *ft.* 5 *in.* 8 *parts* by 5 *feet;* that is to say, I have taught you *short multiplication* in this rule. There must, of course, be cases, in which the number of feet with which we have to multiply, is expressed by *two* or *more* figures, requiring, in consequence, something like *long* multiplication. For example, let it be that we have to multiply 153 *ft.* 2*in.* by 25 *ft.* 9*in.* Writing these, multiplicand and multiplier, down in the usual manner, you would stop when you come to do the work, seeing that the product, in multiplying by the 25, must be stated in *two lines.* And then come 153 *feet* to be multiplied by 9 *inches,* the product of which, being *inches,* must be written, not in the place of inches, at once, but *beside* the chief working, and when reduced into feet, by being divided by 12, may come into its proper station, as you see it done in the *first* of these two examples. But the better mode of doing it is this, as shown in the second example. In which you will see, that the multiplication by the 25 *ft.* is done like a common sum in compound multiplication; and that, for the 9 *inches,* which are *three quarters of a foot,* we take *aliquot parts* of the multiplicand, as in *practice,* that is to say, *a half,* and then the *half of a half;* and these, written in their proper places, and the whole added together, give us the product of the two dimensions proposed in this example.

(1st.) *ft.* *in.*	*ft.* *in.*	(2nd.)
153 · 2	153 · 2	½) 153 · 2
25 · 9	9	5
765 ·· 10	12) 1378 · 6	765 · 10
3063 ·· 4	114 · 10 · 6	5
114 ·· 10 · 6		3829 · 2
3944 ·· 0 ·· 6		½) 76 . 7
		38 . 3 . 6
		3944 · 0 · 6

PARTICULAR MEASURES OF LENGTH.

(*From the Tradesman's &c. Almanack.*)

A Nail is......2¼ Inches		used for measuring cloths of all kinds.
A Quarter .... 4 Nails		
A Yard ...... 4 Quarters		
An Ell........ 5 Quarters		
A Hand ...... 4 Inches; used for the height of horses.		
A Fathom .... 6 Feet; used in measuring depths.		
A Link ...... 7 Inches, 92-hundredths		used in Land Measure.
A Chain .... 100 Links		
Ten Chains, square, 1 Acre.		

MEASURE OF SURFACE.

144 Square Inches make..........	1 Square Foot
9 Square Feet	1 Square Yard
30¼ Square Yards................	1 Perch or Rod
40 Perches	1 Rood
4 Roods, or 160 Perches........	1 Acre
640 Acres	1 Square Mile

MEASURES OF SOLIDITY AND CAPACITY.

Division I.—Solidity.

1728 Cubic Inches make	1 Cubic Foot
27 Cubic Feet....................	1 Cubic Yard

Division II.

Imperial Measure of capacity for all liquids, and for all dry goods, except such as are comprised in the third Division :—

4 Gills make	1 Pint	= 34⅔	cubic inches, nearly
2 Pints............	1 Quart	= 69⅓	——
4 Quarts..........	1 Gallon	= 277¼	——
2 Gallons..........	1 Peck	= 554½	——
8 Gallons..........	1 Bushel	= 2218½	——
8 Bushels..........	1 Quarter	= 10¼	cubic feet, nearly
5 Quarters	1 Load	= 51⅓	——

Division III.

Imperial Measure of capacity for coals, culm, lime, fish, potatoes, fruit, and other goods, commonly sold by *heaped measure* :—

2 Gallons make	1 Peck	= 704	cubic inches, nearly
8 Gallons..........	1 Bushel	= 2815½	——
3 Bushels..........	1 Sack	= 4 8/9	cubic feet, nearly
12 Sacks............	1 Chaldron	= 58⅔	——

The Goods are to be heaped up in the form of a cone, to a height above the rim of the measure, of at least three-fourths of its depth. The outside diameter of measures, used for heaped goods are to be at least double the depth, consequently not less than the following dimensions :—

Bushel ......19½ inches	Gallon .....9¼ inches
Half-Bushel 15¾ ——	Half-Gallon 7¾ ——
Peck.......12¼ ——	

The Imperial Measure was established by Act 5 Geo. IV. c. 74. Before that time, there were four different measures of capacity used in England :—1st. For wine, spirits, cider, oils, milk, &c.; this was one-sixth less than the Imperial measure. 2nd. For malt liquor, this was one fifty-ninth part greater than the Imperial measure. 3rd. For corn, and all other dry goods not heaped, this was one thirty-third part less than the Imperial Measure. 4th. For coals, which did not differ sensibly from the Imperial Measure.

The Imperial Gallon contains exactly 10 lbs. avoirdupoise of pure water; consequently the pint will hold $1\frac{1}{4}$ lb., and the bushel 80 lbs.

PARTICULAR WEIGHTS.

			cwt.	*qrs.*	*lb.*	
8 Pounds make	1 Stone					used for Meat.
14 Pounds	1 Stone	=	0	0	14	used in the Wool Trade.
2 Stone	1 Tod	=	0	1	0	
$6\frac{1}{2}$ Tod	1 Wey	=	1	2	14	
2 Weys	1 Sack	=	3	1	0	
12 Sacks	1 Last	=	39	0	0	

MISCELLANEOUS.

12 Dozen make a Gross
A Weigh is 256lbs,
12 Barrels make a last
A Quire of Paper is 24 Sheets
A Ream of Paper is 20 Quires
A Bundle of Paper is 2 Reams
A Bale of Paper is 10 Reams
A Roll of Parchment, or Vellum, is five dozen, or 60 skins
A Dicker of Hides is 10 skins
A Last of Hides is 20 Dickers
A Dicker of Gloves is 10 doz. prs
A Firkin of Butter is 56lbs.
A Firkin of Soap is 64lbs.
A Tierce of Rice is about 5 cwt.
A Hogshead of Tobacco is from 9 to 10 cwt.
A Barrel of Gunpowder is 1 cwt.
A Pack of Wool is 240lbs
20 Stones of Flour make a Sack
A load of Timber, unhewed, is 40 ft.
A Load of Bricks, 500 in number
A Load of Tiles, 1000 in number
A Load of Hay, in London, is nearly 18 cwt.
A Load of Straw, 36 Trusses, of 36lbs. each
A Chaldron of Coals, in London, is 36 bushels
A Chaldron of Coals, in Newcastle, is 53 cwt.
A Cart of Coals, in Scotland is 12 cwt
A Deal of Coals, in Scotland is 23 cwt
A Grain of Gold is worth about 2*d.*
An Ounce of fine Silver is worth from 5*s.* to 5*s.* 6*d.*

DIVISIONS OF THE YEAR.

The time in which the Earth performs one complete journey round the Sun, being divided into 12 equal positions, called months, gives, for each month, 30 *days*, $10\frac{1}{2}$ *hours*, within a few seconds; and these months, from their reference to the Sun, are called Solar Months, from *Sol*, the Latin word for *Sun*. In like manner is the same portion of time measured by the motion of the Moon; and the 12 divisions arising therefrom are called Lunar Months; from the name of that planet, which in Latin is *Luna*. These divisions may both be considered as *natural*, or as *astronomical* months. But for legal purposes, and for purposes of business, a somewhat different mode of dividing the year prevails.

This other division of the year is into what are called civil, or Calendar months; *civil*, because they are the divisions adopted by the *civil* governments of Europe; and *Calendar*, from a mode of registering these divisions of the year by the ancient Romans, who called certain days of each month the calends of that month.

The following lines, committed to memory, are in general use, for the purpose of ascertaining, at any time, the number of days in each of the Calendar months.

Thirty days hath September,
April, June, and November;
February hath twenty-eight alone,
And all the rest have thirty-one,
Except in leap year, when in fine,
February's days are twenty-nine.

ROMAN NUMERALS.

The ancient Romans, who enjoyed not the benefit of this beautiful system of notation, which the modern world derived from the Arabians; the Romans employed certain of their Capital letters, wherewith to describe numbers; and, as this mode is still useful, for particular purposes, and still used, it may be well to describe it. It will be sufficient to write some of these numerals, and to place opposite to each, the Arabic figure of the same value.

I	1	XI	11	XL	40	C	100
II	2	XII	12	XLI	41	CL	150
III	3	XIII	13	XLV	45	CC	200
IV	4	XIV	14	L	50	CCC	300
V	5	XV	15	LV	55	CCCC	400
VI	6	XVI	16	LX	60	D	500
VII	7	XX	20	LXV	65	DC	600
VIII	8	XXI	21	LXX	70	DCC	700
IX	9	XXX	30	LXXX	80	M	1000
X	10	XXXIII	33	LC	90	MD	1500

THE KEY; OR ANSWERS.

It was my design to give Answers to the several operations proposed to the learner for his practise, but various reasons have concurred to dissuade me from inserting them here. Amongst these, some considerable, and perhaps sufficient reasons, are these; that such answers, being, not "*leading* strings," but rather a sort of *suspending straps*, now, happily, not much used, I think, in this country; a kind of straps which they called *leading* strings, and by which infants were wont to be *suspended* and *swung* about, partly resting on their feet, and partly on the straps, which, passing under the *arm-pits*, and *across the breast* of the tender creature, must have had a strong tendency to produce a contraction of the chest, and an undue and hulking rise of the shoulders, impairing in after life, the health, and strength, and symmetry of the unfortunate victim.

Such "suspending straps," in some measure would "ANSWERS" be, in this case. So I think it better for my pupil, leading him on, as I trust I have done, to a well-grounded knowledge, and to a pleasing exercise of the art, and thereby preparing him for the practice of business, I think it better to leave him here, as he must be left in real business, to rely on himself for proving the truth of his work; by working each question *back again*, or by working always in *two methods*, a practice which will amply repay him, both in the satisfaction it affords, and in the time it saves; time which, without such a practice, would be to be wasted in *re-calling*, and in *rectifying* accounts, to the great discredit, and to the severe mortification of him, from under whose hands they had passed without due examination.

However, for the use of Teachers, who cannot be expected to devote the time requisite to trace over, and to examine the sums of a number of pupils, I shall prepare a key, and shall make it worth their attention by the addition of a series of new propositions and questions.

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