WATSON'S
TUTOR'S ASSISTANT;
OR, COMPLETE
SCHOOL ARITHMETIC:
ON A PLAN PECULIARLY CALCULATED TO
FACILITATE THE IMPROVEMENT OF THE PUPIL, AND REDUCE THE LABOUR OF THE TEACHER;
TO WHICH IS ADDED, AN IMPROVED RULE FOR THE
EXTRACTION OF THE CUBE ROOT,
BY WHICH THE OPERATION IS PERFORMED
As easily, accurately, and expeditiously, as that of the Square Root;
TOGETHER WITH THE
USE OF THE CARPENTER'S SLIDING RULE;
AND A FULL AND FACILE
INTRODUCTION TO PRACTICAL MENSURATION;
ALSO,
AN APPENDIX,
Exhibiting various methods of 'Ready Reckoning,' chiefly intended as
EXERCISES IN MENTAL CALCULATION.

BY W. WATSON,
AUTHOR OF 'AN EASY AND COMPREHENSIVE INTRODUCTION TO ALGEBRA.'

FOURTH EDITION, MUCH IMPROVED.

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PREFACE TO THE FIRST EDITION.

In no department of science, for the last half century, has the press teemed with a greater number of publications, than on the subject of Arithmetic:—almost every succeeding writer has discovered, in the imperfections of former works, a sufficient motive for presenting a new treatise to the public.

Without disparaging the merits of others, to the Author of the following pages there still appears room for improvement. Comparatively few are even tolerably calculated for the purposes of tuition, especially in Seminaries for young ladies and Classical Academies, where the time devoted to Arithmetic is necessarily limited. In the works alluded to, the prevailing fault appears to be the want of simplicity; even the Questions comprising the leading Exercises are often involved and intricate, and so obscurely expressed as to baffle the attempts of a youthful inquirer fully to comprehend them; this obscurity tends to deter from inquiry, and check the mental efforts of young beginners. In the following treatise, the Author has endeavoured to avoid these imperfections: the Questions are nearly all new, and their arrangement and gradation such, as to enable the Pupil to prosecute his studies with but very little help from his teacher; Examples are given to illustrate the operation of each Rule, and to every *two* preceding elementary Rules, Exercises are annexed, so arranged as to encourage thought, and habituate the mind to comparison.

For the convenience of Teachers, all the Examples in Simple and Compound Addition are so contrived, that the *Sum* of each is equal to three times the last line; and those of the Exercises are placed beneath each Question. Experience has convinced the Author of the utility of the latter provision to the learner, who is encouraged to new exertions, by seeing that he has solved a question correctly. The doubt and uncertainty inseparable from the plan of withholding the results is avoided, together with the inevitable consequences of mental weariness and aversion.

In order to render the present Treatise an acceptable and useful addition to the library of the Working Mechanic, it is hoped the Rules of Practical Mensuration, the great number of Exercises, and Explanatory Notes, with a familiar description of the Sliding Rule, the construction of its lines, its application to the solution of Arithmetical Questions, Timber Measure, &c. will not be deemed out of place.

To promote the advancement of the Pupil, the Author would recommend, as of paramount importance, frequent catechetical examinations and cyphering in class; the *former*, although it affects inquiry only, tends to impress the rules on the memory, and affords the Tutor a sort of intellectual *barometer;* the *latter*, by exciting emulation, elicits the powers of the mind, and thus greatly accelerates the process of calculation.

ADVERTISEMENT TO THE FOURTH EDITION.

The present edition has been carefully revised and corrected, and such additions made as the Author trusts will be deemed an improvement. With this object in view, he has complied with the expressed wishes of several eminent Teachers, by adding a number of Geometrical Problems, which renders the Mensuration unique and complete.

W. WATSON.

Beverley, January, 1845.

CONTENTS.

EXPLANATION

OF THE

CHARACTERS USED IN ARITHMETIC.

$+$ *Plus, or more.*	The sign of Addition.
$-$ *Minus, or less.*	The sign of Subtraction.
$\times$ *Into, or Multiplied by.*	The sign of Multiplication.
$\div$ *Divided by.*	The sign of Division.
$=$ *Equal to.*	The sign of Equality; as, $4+2=6$.
: *is to* :: *so is* :	The sign of Proportion.
$\surd$	The sign of the Square Root.

Thus $\sqrt{5}$ or $5^{\frac{1}{2}}$ are expressions for the square root of 5, and $\sqrt[3]{5} = 5^{\frac{1}{3}}$ are expressions for the cube root of 5.

Also, $5^{\frac{2}{3}}$ is the cube root of the square of 5. And 5^2, 5^3, and 5^4, is the square, cube, and biquadrate of 5.

$\therefore$ *Ergo, or therefore.* $>$ *Angle.*

DEFINITIONS.

1. *A prime number,* is one that has no divisor between itself and unity.

2. *A composite number,* is one that has two or more divisors, commonly called factors.

3. *A perfect number,* is equal to the sum of all its quotients.

4. *The figures* or *digits* by which all Arithmetical operations are performed are 1, 2, 3, 4, 5, 6, 7, 8, 9, 0; the last is called a cypher or naught, and is of no value when it stands alone, but when put to the right of any integral number increases its value ten-fold.

5. *The unit,* or first digit, can neither multiply nor divide.

6. *A common measure* between two or more numbers, is a number that will divide them all without a remainder.

7. *Multiple,* a number is said to be a multiple of a less number, when it contains it a certain number of times.

8. *Numbers* are said to be *prime* to each other when they admit not of a common divisor: and if *one* be divided by *any* number, the quotient is the reciprocal of that number.

9. *Ratio* is the relation of two quantities of the same kind with respect to quantity, or the fraction which is equal to the quotient of one divided by the other, is called their ratio; hence a ratio can only subsist between quantities of the *same kind.*

The ratio of one quantity with another is expressed by placing two points vertically between them, or by writing the quantity compared above a line, and that with which it is compared below it: thus, the ratio of 3 to 4 is expressed either by 3 : 4, or $\frac{3}{4}$; the first term is called the *antecedent*, and the last the *consequent.*

A ratio, is called a ratio of *equality, majority* or *minority*, according as the antecedent is *equal* to, *greater*, or *less* than the consequent: thus, 3 : 3 is a ratio of equality; 5 : 3 a ratio of majority; and 3 : 5 a ratio of minority. When the terms of a ratio are multiplied or divided by the same number, the ratio itself is not altered.

Ratios may be compared by reducing the fractions which represent them to a common denominator: thus, 3 : 4 is a less ratio than 4 : 5, because $\frac{3}{4}$ is less than $\frac{4}{5}$, or $\frac{15}{20}$ less than $\frac{16}{20}$ by $\frac{1}{20}$.

Ratios of minority *increase*, and ratios of majority *decrease*, by adding the same quantity to each term: thus, if to 3 : 5 there be added 3, the resulting ratio is 6 : 8, or 3 : 4, which is greater than 3 : 5. Also, if to the ratio 5 : 3, there be 2 added, the resulting ratio is 7 : 5, which is less than 5 : 3.

Ratios are compounded by multiplying their antecedents together, and their consequents together: thus, the ratio of 3 : 5, compounded with the ratio of 4 : 3, is 3 × 4 : 3 × 5, or 12 : 15, or 4 : 5.

10. *Proportion*, is the comparison of two ratios.

11. *Progression*, is either Arithmetical or Geometrical. Arithmetical Progression is when three or more numbers increase or decrease by a constant number, as 1, 3, 5, 7, &c. which is an increasing progression, the common difference of which is 2. Also, 20, 17, 14, 11, &c. is a decreasing progression, the common difference of which is 3. Whence Arithmetical Progression is formed by *addition* or *subtraction.*

12. *Geometrical Progression* is when three or more numbers have successively the same ratio, as 1, 2, 4, 8, &c. which is an increasing progression, its ratio or common multiplyer being 2. Also, 16, 8, 4, 2, &c. is a decreasing progression, the ratio of which is ½, whence Geometrical Progression is formed by *multiplication* or *division*.

13. *Harmonical Proportion*, is when the first term is to the third as the difference between the first and second is to the difference between the second and third; or in four terms, when the first is to the fourth as the difference between the first and second is to the difference between the third and fourth; or the reciprocals of numbers in Arithmetical Proportion are in Harmonical Proportion.

MULTIPLICATION.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

OF MONEY.

¼ Farthing.	4 Farthings	make	1 Penny	*marked.* *d.*
½ Half-penny.	12 Pence	——	1 Shilling	*s.*
¾ Three Farthings.	20 Shillings	——	1 Pound	£.

Farthings
4 = 1 Penny.
48 = 12 = 1 Shilling.
960 = 240 = 20 = 1 Pound.

N. B. A Moidore, 27*s.*—A Guinea, 21*s.*—A Mark, 13*s.* 4*d.*—An Angel, 6*s.* 8*d.*—A Crown, 5*s.*—Half-Crown, 2*s.* 6*d.*

Farthing Table.		*The Pence Table.*						*The Shilling Table.*					
qrs.	*d.*	*d.*	*s.*	*d.*	*d.*	*s.*	*d.*	*s.*	*£.*	*s.*	*s.*	*£.*	*s.*
4 ..	1	20 ..	1	8	90 ..	7	6	20 ..	1	0	140 ..	7	0
8 ..	2	24 ..	2	0	96 ..	8	0	30 ..	1	10	150 ..	7	10
12 ..	3	30 ..	2	6	100 ..	8	4	40 ..	2	0	160 ..	8	0
16 ..	4	36 ..	3	0	108 ..	9	0	50 ..	2	10	170 ..	8	10
20 ..	5	40 ..	3	4	110 ..	9	2	60 ..	3	0	180 ..	9	0
24 ..	6	48 ..	4	0	120 ..	10	0	70 ..	3	10	190 ..	9	10
28 ..	7	50 ..	4	2	130 ..	10	10	80 ..	4	0	200 ..	10	0
32 ..	8	60 ..	5	0	132 ..	11	0	90 ..	4	10	210 ..	10	10
36 ..	9	70 ..	5	10	140 ..	11	8	100 ..	5	0	220 ..	11	0
40 ..	10	72 ..	6	0	144 ..	12	0	110 ..	5	10	230 ..	11	10
44 ..	11	80 ..	6	8	150 ..	12	6	120 ..	6	0	240 ..	12	0
48 ..	12	84 ..	7	0	156 ..	13	0	130 ..	6	10	250 ..	12	10

TROY WEIGHT.

			marked.
24 Grains *(gr.)*	make	1 Pennyweight	*dwt.*
20 Pennyweights	——	1 Ounce	*oz.*
12 Ounces	——	1 Pound	℔.

Grains
24 = 1 Pennyweight.
480 = 20 = 1 Ounce.
5760 = 240 = 12 = 1 Pound.

By this weight are weighed Gold, Silver, and Jewels only.

N. B. The Standard for Gold Coin is 22 Carats of fine Gold, and 2 Carats of Copper, melted together. For Silver, it is 11*oz.* 2*dwts.* of fine Silver, and 18*dwts.* of Copper.

AVOIRDUPOIS WEIGHT.

			marked.
16 Drams *(dr.)*	make	1 Ounce	*oz.*
16 Ounces	——	1 Pound	℔.
28 Pounds	——	1 Quarter	*qr.*
4 Quarters or 112*lb.*	——	1 Hundred Weight	*cwt.*
20 Hundred Weight	——	1 Ton	*ton.*

Drams
16 = 1 Ounce.
256 = 16 = 1 Pound.
7168 = 448 = 28 = 1 Quarter.
28672 = 1792 = 112 = 4 = 1 Hund. Weight.
573440 = 35840 = 2240 = 80 = 20 = 1 Ton.

One Ounce Avoirdupois = 437½*gr.* Troy.

There are several other denominations in this Weight, that are used in some particular Goods, viz.

	lb.		lb.
A firkin of butter....	56	A stone of iron, shot, or horseman's weight	14
——— soap.....	64	Butcher's meat	8
A barrel of anchovies	30	A truss of straw......	36
——— soap.....	256	——— new hay ...	60
——— raisins ...	112	——— old hay	56
A puncheon of prunes	1120	36 trusses a load.	
A fother of lead, 19½ *cwt.*			

APOTHECARIES' WEIGHT.

				marked.
20 Grains *(gr.)*	make	1	Scruple	*sc.* or ℈
3 Scruples	——	1	Dram	*dr.* or ʒ
8 Drams	——	1	Ounce	*oz.* or ℥
12 Ounces	——	1	Pound	*lb.* or ℔

Grains
20 = 1 Scruple.
60 = 3 = 1 Dram.
480 = 24 = 8 = 1 Ounce.
5760 = 288 = 96 = 12 = 1 Pound.

The Apothecaries mix their medicines by this weight; but they buy and sell by Avoirdupois weight.

The *pound* and *ounce* in this weight are the same as those in Troy Weight, but the *smaller divisions* are different.

CLOTH MEASURE.

				marked.
2¼ Inches	make	1	Nail	*nl.*
4 Nails	——	1	Quarter Yard	*qr.*
3 Quarters	——	1	Flemish Ell	F. E.
4 Quarters	——	1	Yard	*yd.*
5 Quarters	——	1	English Ell	E. E.
6 Quarters	——	1	French Ell	Fr. E.

LONG MEASURE.

				marked.
12 Inches *(in.)*	make	1	Foot	*f.*
3 Feet	——	1	Yard	*yd.*
6 Feet	——	1	Fathom	*fath.*
5½ Yards	——	1	Rod, Pole, Perch	*r. p.*
40 Poles	——	1	Furlong	*fur.*
8 Furlongs	——	1	Mile	*mi.*
3 Miles	——	1	League	*lea.*

60 Geographical Miles, or } make 1 Degree *deg.* or °
69½ British Miles * }

Inches					
12 =	1 Foot.				
36 =	3 =	1 Yard.			
198 =	16½ =	5½ =	1 Pole.		
7920 =	660 =	220 =	40 =	1 Furlong.	
63360 =	5280 =	1760 =	320 =	8 =	1 Mile.

NOTE.—4 Inches make a hand.——7 yards a rood of fencing or ditching.

LAND AND SQUARE MEASURE.

30¼	Yards	make	1 Perch.
40	Perches	——	1 Rood.
4	Roods	——	1 Acre.
640	Acres	——	1 Square Mile.
144	Square Inches	——	1 Square Foot.
9	Square Feet	——	1 Square Yard.
100	Square Feet	——	1 Square of Flooring, &c.

Inches					
144 =	1 Foot.				
1296 =	9 =	1 Yard.			
39204 =	272¼ =	30¼ =	1 Pole or Perch.		
1568160 =	10890 =	1210 =	40 =	1 Rood.	
6272640 =	43560 =	4840 =	160 =	4 =	1 Acre.

By this measure, all things that have length and breadth are measured; as land, paving, plastering, roofing, tiling, flooring, glazing, &c. *Land* is measured by a chain, called Gunter's Chain, which is 4 poles or 22 yards long; and consists of 100 links. Ten chains in length, and one in breadth make an acre.

NOTE.—49 square yards make a rood of digging.

CUBIC, OR SOLID MEASURE.

1728 Inches	=	12^3	=	1 Solid Foot.
27 Feet	=	3^3	=	1 Solid Yard.

By this, all things that have length, breadth, and thickness, are measured.

LIQUID MEASURE.

			marked.
2 Gills *(gls.)*	make	1 Pint	*pt.*
2 Pints	——	1 Quart	*qt.*
4 Quarts	——	1 Gallon	*gal.*

* This is on a *great* circle of the earth; that is, a circle that divides the earth into two equal parts. *Every* circle contains 360° degrees, each degree, 60' minutes, each minute, 60'' seconds, &c.

NOTE.—277,274 cubic inches make a gallon, which is equal to 10lbs. of distilled water. But 7½lbs. of oil is accounted a gallon.

SPIRIT AND WINE MEASURE.

10 Gallons	make	1 Anker.
18 Gallons	——	1 Runlet.
42 Gallons	——	1 Tierce.
63 Gallons	——	1 Hogshead.
84 Gallons	——	1 Puncheon.
126 Gallons	——	1 Pipe.

A cask of Rum, containing about 100 gallons, is *usually* called a puncheon. A foreign pipe of wine varies from 110 to 140 gallons, and it is customary to charge exactly what the vessel contains.

NOTE.—Before the enactment of the bill for the *uniformity of weights and measures*, in 1824, the wine gallon contained 231, and ale gallon 282 cubic inches.

ALE AND BEER MEASURE.

			marked.
9 Gallons	make	1 Firkin	*fir.*
2 Firkins	——	1 Kilderkin	*kil.*
2 Kilderkins	——	1 Barrel	*bar.*
3 Kilderkins	——	1 Hogshead	*hhd.*
2 Hogsheads	——	1 Butt	*bt.*
2 Butts	——	1 Ton	*ton.*

Pints

2 = 1 Quart.
8 = 4 = 1 Gallon.
72 = 36 = 9 = 1 Firkin.
144 = 72 = 18 = 2 = 1 Kilderkin.
288 = 144 = 36 = 4 = 2 = 1 Barrel.
432 = 216 = 54 = 6 = 3 = 1½ = 1 Hogshead.
864 = 432 = 108 = 12 = 6 = 3 = 2 = 1 Butt.

DRY MEASURE.

			marked.
2 Gallons	make	1 Peck	*pk.*
4 Pecks	——	1 Bushel	*bu.*
8 Bushels	——	1 Quarter	*qr.*
4 Quarters	——	1 Chaldron	*ch.*
5 Quarters	——	1 Wey	*wey.*
10 Quarters	——	1 Last	*la.*

Pints						
8 =	1 Gallon.					
16 =	2 =	1 Peck.				
64 =	8 =	4 =	1 Bushel.			
512 =	64 =	32 =	8 =	1 Quarter.		
2560 =	320 =	160 =	40 =	5 =	1 Wey.	
5120 =	640 =	320 =	80 =	10 =	2 =	1 Last.

This measure is used for all dry goods; such as Corn, Seeds, Roots, Coals, &c.

The Winchester bushel, enacted by William III. contains $2150\frac{2}{5}$ cubic inches, it is $18\frac{1}{2}$ inches diameter, and 8 inches deep. The Imperial bushel contains $2218\frac{1}{5}$ cubic inches, or $18\frac{1}{2}$ inches diameter, and $8\frac{1}{4}$ inches deep; but for heaped measure (by Act 5, Geo. IV. Sess. 1824) its diameter must, from outside to outside, be $19\frac{1}{2}$ inches.

NOTE.—At London, a chaldron of Coals is 36 bushels.

TIME.

			marked.
60 Seconds	make	1 Minute	*mi.* or ʹ
60 Minutes	——	1 Hour	*ho.*
24 Hours	——	1 Day	*dy.*
7 Days	——	1 Week	*wk.*
4 Weeks	——	1 Month	*mo.*
13 Months, 1 day, 6 hours, or 12 Calendar months, or 52 Weeks	——	1 Year	*yr.*

Seconds					
60 =	1 Minute.				
3600 =	60 =	1 Hour.			
86400 =	1440 =	24 =	1 Day.		
604800 =	10080 =	168 =	7 =	1 Week.	
2419200 =	40320 =	672 =	28 =	4 =	1 Month.
31557600 =	525960 =	8766 =	$365\frac{1}{4}$ =	1 Year.	

The calendar months are January, February, March, April, May, June, July, August, September, October, November, and December.

To know the days in each month, observe,

Thirty days hath September, April, June, and November;
February hath twenty-eight alone; all the rest have thirty-one;
Except in Leap Year, at which time February's days are twenty-nine.

The year consists of 365 days 6 hours; which 6 hours become a day in 4 years, and is then added to February.

ARITHMETIC.

ARITHMETIC is the science of Numbers, and teaches the art of computing by them.

The fundamental rules are Notation, or Numeration, Addition, Subtraction, Multiplication, and Division, by which all its operations are performed.

NOTATION AND NUMERATION.

Notation is the *writing* of numbers by figures, and *Numeration* is the art of reading them correctly.

The value of figures depends upon the place in which they stand, as may be seen from the following Table:

hundreds of thousands of millions	tens of thousands of millions	thousands of millions	hundreds of millions	tens of millions	millions	hundreds of thousands	tens of thousands	thousands	hundreds	tens	units
1	2	3	4	5	6	7	8	9	1	2	3

Which Table the learner should read from *right* to *left*, thus:—units, tens, hundreds, thousands, tens of thousands, hundreds of thousands, millions, tens of millions, hundreds of millions, thousands of millions, tens of thousands of millions, hundreds of thousands of millions.

And then from *left* to *right*, thus:—one hundred and twenty-three thousand, four hundred and fifty-six millions, seven hundred and eighty-nine thousand, one hundred and twenty-three.

ROMAN NOTATION.

I.	=	1	XV.	=	15	C.	=	100
II.	=	2	XVI.	=	16	CC.	=	200
III.	=	3	XVII.	=	17	CCC.	=	300
IV.	=	4	XVIII.	=	18	CCCC.	=	400
V.	=	5	XIX.	=	19	D.	=	500
VI.	=	6	XX.	=	20	DC.	=	600
VII.	=	7	XXX.	=	30	DCC.	=	700
VIII.	=	8	XL.	=	40	DCCC.	=	800
IX.	=	9	L.	=	50	DCCCC.	=	900
X.	=	10	LX.	=	60	M.	=	1000
XI.	=	11	LXX.	=	70	MDC.	=	1600
XII.	=	12	LXXX.	=	80	MDCL.	=	1650
XIII.	=	13	XC.	=	90	MDCCXXVIII.	=	1728
XIV.	=	14	XCV.	=	95	MDCCCXXXIII.	=	*1833

SIMPLE ADDITION.

SIMPLE ADDITION teaches the method of finding the *sum* of two or more numbers.

RULE 1. Place the numbers under each other so that units may stand under units, tens under tens, &c.

2. Add up the figures in the row of units, set down what remains *above* the *even tens*, or if nothing remains, a cypher; and carry one for every ten to the next row.

3. Add up the other rows, in the same manner, and under the *last* row or column, set down the *whole sum* contained in it.

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* Those who wish to be acquainted with the different *Scales* of Notation, and the rise and progress of Arithmetic, may amply gratify their curiosity by consulting 'Barlow's Theory of Numbers,' and 'Leslie's Philosophy of Arithmetic.'

PROOF 1. Cut off the upper line and add up the rest as before, and set the sum under the lower line.

2. Add this second sum to the upper line, and if it be the same as the first sum, the work is presumed to be right.

EXAMPLES.

(1)		(2)	(3)	(4)	(5)
244		396	362	463	587
345		257	434	245	644
463		157	476	364	759
526		405	636	536	995
1578	*Sum.*				
1334					
1578	*Proof.*				

(6)	(7)	(8)	(9)	(10)
1033	2346	2456	4567	3572
2345	3489	7884	3456	4686
6410	3443	2450	2647	6678
4894	4639	6395	5335	7468

(11)	(12)	(13)	(14)	(15)
5476	5326	6387	7654	4403
3758	2457	3558	5432	5566
2485	2326	5947	1236	3354
3565	3457	3674	5018	6387
7642	6783	9783	9670	9855

(16)	(17)	(18)	(19)	(20)
56738	19764	37678	44864	48464
47669	27683	44735	38892	37584
25484	35486	21097	28997	23464
45797	46765	29258	19685	23800
87844	64849	66384	66219	66656

(21)	(22)	(23)	(24)	(25)
55757	25768	15466	12765	27938
45644	57667	34561	26789	49688
31063	65576	45678	33564	20789
26892	45685	57889	45636	37669
79678	97348	76797	59377	68042

(26)	(27)	(28)	(29)	(30)
3340	1345	1675	1737	1387
2635	2566	4531	2608	1765
3291	4735	2867	3059	1908
1370	2944	3574	4360	1375
3674	1734	4923	6586	1099
7155	6662	8785	9175	3767

(31)	(32)	(33)	(34)	(35)
55757	57667	20846	26789	25102
31063	25768	15466	12765	49688
45644	25102	34561	33564	20789
25102	45685	57889	45636	37669
26892	15576	45678	20846	27938
92229	89899	87220	69800	80593

(36)	(37)	(38)	(39)	(40)
13638	15576	45678	33564	27938
35063	45685	13638	12765	20789
41644	25102	57889	13638	37669
25102	25768	34561	26789	49688
26892	57667	15466	45636	25102
55757	13638	20846	20846	13638
99048	96718	94039	76619	87412

(41)	(42)	(43)	(44)	(45)
267805	267623	198595	198753	168776
178264	228594	277378	299684	297685
379638	154384	396435	198763	185498
156456	378548	165986	299674	258679
105777	237898	382454	397683	376787
390583	367638	296785	186385	288698
268389	259743	279897	398764	397775
873456	947214	998765	989853	986949

SIMPLE SUBTRACTION.

Subtraction is the method of finding the *difference* between two numbers.

RULE 1.—Place the less number under the greater, units under units, tens under tens, &c.

2.—Begin at the right hand or unit's place, and take each figure in the lower line from the figure above it, and set down the remainder or difference.

3.—If the figure in the lower line be greater than the upper one, borrow ten to the upper figure, for which add one to the next figure in the lower line, and proceed as before.

PROOF.—Add the remainder to the less number, and their sum, if equal to the first, shews the operation is right.

EXAMPLES.

	(1)	(2)	(3)
From	1237645882	98765432152	4971234987
Take	756534727	873716743	2345127263
Rem.	481111155	97891715409	
Proof	1237645882	98765432152	

	(4)	(5)	(6)
From	5643487678	3245670137	7812345678
Take	2345127263	438342673	123456789
Rem.			

	(7)	(8)	(9)
From	1234567890	3456700543	1234567890
Take	53274387	3374832517	78901234
Rem.			

	(10)	(11)	(12)
From	9876543210	3001223344	4801110002
Take	9798641283	78654321	4800978313
Rem.			

	(13)	(14)	(15)
From	3123003784	1112233440	1000113357
Take	7891946	7891234	999887776
Rem.			

	(16)	(17)	(18)
From	6432000001	1000011234	1000012345
Take	555777899	999987468	7894684
Rem.			

	(19)	(20)	(21)
From	7021237395	8234567594	1100323598
Take	876838895	5986327	946312853
Rem.			

	(22)	(23)	(24)
From	3212345691	9000012384	1234567839
Take	946312873	768452835	987654395
Rem.			

	(25)	(26)	(27)
From	9876543295	1000000000	1000000000
Take	1234567809	876543253	999999399
Rem.			

EXERCISES IN

SIMPLE ADDITION AND SUBTRACTION.

1. How many strokes does the hammer of a clock strike in twelve hours? Ans. 78.

2. A person born in 1820, in what year will he be twenty-one? Ans. 1841.

3. Edinburgh is 201 miles north of York, and London 198 miles south of it—what is the distance from London to Edinburgh, by way of York?
Ans. 399 miles.

4. If I travel on Monday 25 miles, on Tuesday 31, on Wednesday 37, on Thursday 18, on Friday 42, and on Saturday 49, how far do I travel in the six days?
Ans. 202 miles.

5. How many Protestants suffered by fire in the reign of Queen Mary; there being 5 bishops, 21 clergymen, 8 gentlemen, 84 tradesmen, 100 labourers, 55 women, and 4 children? Ans. 277.

6. From the year 1649, the Commonwealth lasted 11 years; after which Charles II. reigned 25 years; James II., 5; William and Mary, 14; Ann, 12; George I., 12; George II., 33; and George III. 59; in what year did George III. die? Ans. 1820.

7. King Charles I. was born in 1600, and beheaded in 1649—what age was he when decapitated? Ans. 49.

8 Henry Jenkins died in 1670, at Ellerton-upon-Swale, Yorkshire, aged 169 years; in what year was he born. Ans. 1501.

9. John has 70 marbles and James 37—how many has John more than James? Ans. 33.

10. How long did state lotteries exist in England, the first being in 1569 and the last in 1826?
Ans. 257 years.

11. When Calcutta, the capital of Bengal and of the British possessions in the East-Indies, was taken by a native chief, 146 English prisoners were put into a small prison called the black hole; the following morning only 23 were living—query, how many had died? Ans. 123.

12. Thomas Parr was born in 1483, did penance in Alderbury Church in 1588, and lived 47 years after; what age was he when he did penance, and how old when he died? Ans. 105 when he did penance, 152 when he died.

SIMPLE MULTIPLICATION.

Multiplication is the method of finding the amount of a number, when repeated a certain number of times.

CASE 1.—When the multiplier does not exceed 12.

RULE.—Begin at the *right hand* and multiply *every* figure in the multiplicand; consider how many *tens* there are in each product, the remaining *units* set down under the figure multiplied, and carry *one* for each *ten* to the next product.

The last product is to be wholly set down.

EXAMPLES.

	(1)	(2)
Multiplicand	765432143544	345678901
Multiplier	3	2
Product	2296296430632	

	(3)	(4)	(5)
Multiply	325672132	84321567	765432178
By	3	4	5
Product			

	(6)	(7)	(8)
Multiply	234567898	765432124	234567897
By	5	6	6
Product			

	(9)	(10)	(11)
Multiply	876543210	234567891	763805685
By	7	7	8
Product			

	(12)	(13)	(14)
Multiply	879422345	467845670	84347777
By	8	9	9
Product			

	(15)	(16)	(17)
Multiply	765849769*	349898090	543276820
By	10	11	12
Product			

CASE II.—When the multiplier consists of several figures.

RULE.—Multiply the multiplicand by each figure in the multiplier separately, beginning with the right hand figure, and let the first figure of every product stand exactly under the figure you multiply by. Add these products together, and their sum will be the answer, or whole product required.

EXAMPLES.

	(1)		(2)
Multiply	12345678921	*Multiply*	3507640
By	14		307
	49382715684		24553480
†	12345678921		105229200
Product	172839504894	*Product*	1076845480

3. Multiply 2347683 by 13. Ans. 30519879.
4. Multiply 648370 by 15. Ans. 9725550.
5. Multiply 284567 by 17. Ans. 4837639.
6. Multiply 3286732 by 19. Ans. 62447908.
7. ‡Multiply 54783670 by 20. Ans. 1095673400.

* Multiplying by 10 is the same as annexing a cypher.

† It is not necessary to put both products down as in this example, because by multiplying by the units figure, and adding the back figure of the dividend, the product is got in one line. Or if the *tens* figure of the multiplier be 2, add twice the back figure; if 3, three times the back figure, and so on. This method is very expeditious, and a little practice soon makes the pupils familiar with it.

‡ When the multiplicand and multiplier have both cyphers to the right, they may be counted and put down first, and the product arising from the other parts of the multiplicand and multiplier put down on the *left* of them. The pupil here with advantage might be requested to multiply 20 by 10, by 20, 30, &c.

8. Multiply 62378430	by 24.	Ans. 1497082320.
9. Multiply 3267843	by 27.	Ans. 88231761.
10.*Multiply 10076847	by 35.	Ans. 352689645.
11. Multiply 32786548	by 46.	Ans. 1508181208.
12. Multiply 78432568	by 58.	Ans. 4549088944.
13. Multiply 38067643	by 67.	Ans. 2550532081.
14. Multiply 32568378	by 74.	Ans. 2410059972.
15. Multiply 36423845	by 86.	Ans. 3132450670.
16. Multiply 7325674	by 134.	Ans 981640316.
17. Multiply 3867234	by 275.	Ans. 1063489350.
18. Multiply 5735783	by 309.	Ans. 1772356947.
19. Multiply 3564387	by 5007.	Ans. 17846885709.
20. Multiply 3870030	by 60200.	Ans. 232975806000.

SIMPLE DIVISION.

By Division we find how often one number is contained in another.

CASE I.—*When the divisor does not exceed* 12.

RULE.—Draw a curve line and write the divisor on the left hand of the dividend: then see *how many times* the *divisor* is contained in the first figure or figures of the dividend, and put the quotient under it; and for every unit *over*, carry *ten* to the next figure of the dividend.

PROOF.—Multiply the quotient by the divisor, and to the product add the remainder, if any; the product will be equal to the dividend if the work is right.

EXAMPLES.

(1) *Divisor* 3)743625798 *Dividend*

247875266 *Quotient*

3

743625798 *Proof*

* When the multiplier is resolvable into factors, the pupil should occasionally prove the result by multiplying the multiplicand by one factor, and the product arising by the other.

(2) 2)1234567890

(3) 2)9087654321

(4) 3)1234567809

(5) 3)9876054321

(6) 4)1234539084

(7) 4)7623454780

(8) 5)1325476985

(9) 5)5107552010

(10) 6)1111237856

(11) 6)997432567

(12) 7)113254789

(13) 7)787643278

(14) 8)123456789

(15) 9)987654321

(16) 10)123457642*

(17) 11)987650011

(18) 12)1011135

(19) 12)99765485

CASE II.—*When the divisor consists of several figures.*

RULE I.—Draw a curve line on the right and left of the dividend, and write the divisor on the left.

2.—Find how many times the divisor is contained in as many figures of the dividend as is just necessary, and place the number on the right for a quotient.

3.—Multiply the divisor by the quotient figure, and set the product under the figures of the dividend before mentioned.

* When the divisor is 10, cut off the right hand figure of the dividend for the remainder, the other part of the dividend is the quotient. If the divisor be 100, cut off two figures for the remainder, &c. If the divisor be 20, 30, &c. cut off the last figures as before, and divide the other part of the dividend by 2, 3, &c. and if anything remains it must be placed before the figure cut off for the whole remainder. Generally, cut off as many figures to the right of the dividend as the divisor has cyphers to the right.

4.—Subtract the product from that part of the dividend under which it stands, and bring down the next figure of the dividend to the right of the remainder.

5.—Divide the remainder, so increased, by the divisor as before, for the second figure of the quotient: observing, if it be not contained therein, to put a cypher in the quotient, and bring down another figure to the right of the last brought down.

6.—Proceed with this result as with the former, and so on till all the figures of the dividend are brought down.

PROOF.—As before.

EXAMPLES.

```
       Divis.  Divid.        Quot.
    1.  26 ) 26357      (    1013
             26                26
             ----            ----
               35            6078
               26            2026
               --              19  Rem.
               97           -----
               78           26357  Proof.
               --           =====
        Rem.   19
               ==
```

2. Divide 1378650	by 13.		Ans. 106050.
3. Divide 7867831	by 14.*		Ans. 561987—13.
4. Divide 3567855	by 15.		Ans. 237857.
5. Divide 2345672	by 21.		Ans. 111698—14.
6. Divide 7832856	by 27.		Ans. 290105—21.
7. Divide 1230468	by 35.		Ans. 35156—8.
8. Divide 10123678	by 42.		Ans. 241039—40.

* When the divisor is a composite number, the learner should frequently prove the result, by dividing by the factors of the divisor; that is, dividing the dividend by one factor, and the quotient arising by the other, and if there is a remainder at each division, the whole remainder must be found thus:—Multiply the *first* divisor by the *last* remainder, and to the product add the first remainder. Put down each remainder at the end of the line it arises from, then the whole remainder will stand at the end of the quotient, thus:—

```
        ( 2 | 7867831—1
  14 = {  × | ---------
        ( 7 | 3933915—6
             ---------
              561987—13
              =========
```

9. Divide	3230768	by 56.	Ans. 57692—16.
10. Divide	8732168	by 64.	Ans.
11. Divide	56743632	by 132.	Ans.
12. Divide	84326616	by 216.	Ans.
13. Divide	12327336	by 378.	Ans.
14. Divide	10276416	by 864.	Ans.
15. Divide	105678978	by 3456.	Ans.
16. Divide	6181476969	by 5007.	Ans.

EXERCISES IN

SIMPLE MULTIPLICATION AND DIVISION.

1. If a person travels 27 miles a day, how far will he travel in 8 days? Ans. 216 miles.

2. If a person travels 216 miles in 8 days, how far does he travel per day? Ans. 27 miles.

3. A window having 12 panes in length and 8 in breadth, how many does it contain?* Ans. 96.

4. What number is a twentieth part of 500? Ans. 25.

5. How many soldiers are there in an army consisting of 130 battalions, each 380 men? Ans, 49400.

6. In an army consisting of 49400 soldiers, how many battalions of 380 men each? Ans. 130.

7. If 15 degrees of longitude be equal to one hour of time, how many degrees are equal to 12 hours? Ans. 180.

8. The circumference of the earth is 360 degrees, and it turns round every 24 hours, how many degrees is that per hour? Ans. 15.

9. Sound flies at the rate of 1142 feet per second, how far does it move in 30 seconds.? Ans. 34260 feet.

10. In what time after firing a cannon from Hull Garrison, could it be heard on the Lincolnshire coast opposite; the Humber being 26266 feet over? Ans. 23 seconds.

11. How many square yards are there in a garden that is 100 yards long and 50 broad? Ans. 5000.

12. From Gibraltar to Malta is 1106 miles, how many hours would a ship be in sailing from one place to the other, at the rate of 7 miles an hour? Ans. 158.

---

* The number of panes in length multiplied by the number of panes in breadth, gives the number of panes in the window.

13. Light flies 12 million miles in a minute, how long will it be in passing from the sun to the earth, the distance being 96 millions of miles? Ans. 8 minutes.

14. The sum of two numbers is 487, and their difference 139; required the numbers? Ans. 174 and 313.

15. How many potatoes are there in a garden containing 40 rows, each row 50 roots, and each root 10 potatoes? Ans. 20000.

16. A man travelling 3 miles an hour, for 9 hours a day, how long will he be in travelling 513 miles? Ans. 19 days.

COMPOUND ADDITION.

Compound Addition is the method of collecting several numbers of different denominations into one sum.

RULE I.—Place the numbers so that those of the same denomination may stand *under each other*, and draw a line below them.

2.—Add the numbers in the lowest denomination, and find how many of the next *higher denomination* are contained in their sum.

3.—Set down the remainder, and carry *one* for *each* of the next higher denomination to the next column, which add up as before, and so on to the end.

PROOF.—As in Simple Addition.

MONEY.

	(1)				(2)	
£.	*s.*	*d.*		£.	*s.*	*d.*
21	5	4		36	13	10
13	7	3		19	14	$8\frac{1}{2}$
17	4	7		132	17	$11\frac{1}{4}$
32	2	6		25	9	$2\frac{3}{4}$
12	9	2		7	11	7
15	6	1		12	5	3
111	14	11	*Sum*	234	12	$6\frac{1}{2}$
90	9	7		197	18	$8\frac{1}{2}$
111	14	11	*Proof*	234	12	$6\frac{1}{2}$

(3)

£.	s.	d.
8	3	1
3	2	2
5	4	3
2	5	4
9	7	5

(4)

£.	s.	d.
3	6	2
8	4	3
9	7	6
10	1	7
15	9	9

(5)

£.	s.	d.
6	6	10
7	8	5
3	5	10
9	6	7
13	3	10

(6)

£.	s.	d.
28	6	9
14	9	4½
35	4	6½
35	5	0
56	12	10

(7)

£.	s.	d.
17	9	10
15	6	10½
17	7	9½
28	6	8
39	5	7

(8)

£.	s.	d.
125	10	10½
103	9	9½
137	8	8½
148	5	7½
257	7	6

(9)

£.	s.	d.
10	7	4½
23	9	7½
27	11	3
36	12	8
32	10	2
65	5	6½

(10)

£.	s.	d.
26	12	3
13	4	11
27	6	3½
30	13	10
19	7	1½
58	12	2½

(11)

£.	s.	d.
74	12	6
15	13	2½
16	11	7½
37	17	6½
8	16	2½
76	15	6½

(12)

£.	s.	d.
32	7	2¼
17	10	4¼
43	10	6¼
12	10	5¼
22	11	7¼
20	12	8¼
74	11	4¾

(13)

£.	s.	d.
56	7	0½
18	8	10¼
46	12	9½
36	13	8¼
13	14	7¾
25	15	6¼
98	16	3¼

(14)

£.	s.	d.
30	13	3
15	15	5¾
64	14	2¼
4	7	5½
19	18	6½
24	9	7
79	19	3

(15)

£.	s.	d.
56	9	2
32	11	7¼
8	15	3¾
25	6	10½
46	17	11½
22	15	1½
96	8	0¼

(16)

£.	s.	d.
56	7	6½
13	14	6½
14	9	3¾
13	13	2½
64	18	7½
28	19	1¼
96	1	2

(17)

£.	s.	d.
36	7	5
23	12	9½
29	18	4½
14	13	6¼
16	10	6¼
25	15	4½
73	9	0

(18)

£.	s.	d.
230	5	2½
125	15	7½
526	8	2¼
603	5	10
162	5	4¾
154	2	5
102	12	4½
952	7	6¼

(19)

£.	s.	d.
235	0	2
172	5	4
205	13	2¼
329	2	10½
474	16	11½
503	17	1½
12	12	2¾
966	13	11¼

(20)

£.	s.	d.
174	5	6
307	4	4
131	17	2¾
208	5	7¾
437	12	2
125	18	4¾
366	19	7¾
876	1	5½

TROY WEIGHT.

(21)

lbs.	oz.	dwt.
2	5	4
10	7	10
3	6	8
1	7	9
14	4	3
16	3	7

(22)

oz.	dwt.	gr.
3	12	11
5	7	8
7	13	16
2	5	12
5	15	7
12	7	3

(23)

lbs.	oz.	dwt.	gr.
11	11	11	11
15	8	13	22
7	10	17	23
3	5	13	2
6	2	14	12
22	7	15	11

AVOIRDUPOIS WEIGHT.

(24)

ton	cwt.	qr.
5	12	1
4	2	3
2	13	3
7	14	0
2	7	3
11	5	1

(25)

lb.	oz.	dr.
2	11	0
5	4	12
6	8	6
6	14	13
8	12	11
15	1	13

(26)

ton	cwt.	qr.	lb.	oz.	dr.
5	10	2	6	7	3
3	8	1	14	13	14
1	6	0	12	12	12
11	15	1	7	14	6
13	5	1	8	12	7
17	12	3	11	6	5

APOTHECARIES' WEIGHT.

(27)

lb.	oz.	dr.
5	8	3
6	6	6
2	5	2
7	10	4
6	11	5
14	9	2

(28)

oz.	dr.	scr.
5	5	2
2	7	1
6	5	2
2	4	1
1	5	2
9	6	1

(29)

lb.	oz.	dr.	scr.	gr.
5	7	6	2	12
2	5	2	1	15
1	5	4	0	6
3	6	7	2	14
2	6	5	1	11
7	9	6	1	9

CLOTH MEASURE.

(30)

yds.	qr.	n.
6	1	2
5	3	3
3	2	1
2	3	3
4	0	3
11	2	0

(31)

E. E.	qr.	n.
12	4	2
16	3	2
28	0	3
35	2	1
74	1	2
83	3	3

(32)

F. E.	qr.	n.
27	2	3
14	2	2
15	1	0
34	2	3
17	1	2
55	0	3

LONG MEASURE.

(33)

lea.	m.	fur.
5	2	5
7	2	7
8	0	2
6	2	6
3	2	0
16	0	6

(34)

m.	fur.	po.
3	6	26
6	7	32
6	2	12
8	5	11
9	2	5
17	4	3

(35)

yds.	ft.	in.
3	2	6
5	1	8
7	0	10
12	1	8
18	0	10
23	1	9

LAND MEASURE.

(36)

a.	r.	p.
6	3	8
5	2	8
8	2	3
7	1	2
6	2	3
17	1	12

(37)

a.	r.	p.
22	1	13
7	2	16
7	1	2
8	2	17
9	0	0
27	1	24

(38)

a.	r.	p.
18	3	39
17	2	36
16	2	25
7	0	36
10	2	32
35	2	24

LIQUID MEASURE.

(39)

gals.	qts.	pts.
12	3	1
7	2	1
9	1	0
7	3	1
7	3	1
22	3	0

(40)

gals.	qts.	pts.
35	3	1
27	2	$1\frac{1}{2}$
27	3	$1\frac{1}{2}$
38	1	1
44	2	1
87	1	0

(41)

gals.	qts.	pts.
26	1	1
13	0	$1\frac{1}{2}$
23	3	1
44	2	1
19	1	$1\frac{1}{2}$
63	3	0

DRY MEASURE.

(42)

lasts	qrs.	bus.
16	4	4
23	8	5
17	5	3
21	9	6
17	3	4
48	5	7

(43)

qrs.	bus.	pks.
33	5	2
23	6	3
15	7	3
36	7	2
34	5	2
72	4	2

(44)

qrs.	bus.	pks.	gals
7	7	2	1
8	6	1	1
19	5	1	0
16	1	0	1
11	3	1	1
32	1	1	1

TIME.

(45)

wks.	days.	ho.
7	5	12
3	4	15
6	4	13
8	3	17
2	6	21
14	5	15

(46)

wks.	days.	ho.	mi.	sec.
2	3	5	12	13
2	4	13	7	21
1	5	21	35	30
4	6	12	11	12
5	4	13	14	14
8	5	8	40	15

COMPOUND SUBTRACTION.

Compound Subtraction is the method of finding the *difference* between *any* two given numbers of different denominations.

RULE 1.—Place the less number under the greater, so that the parts which are of the *same denomination* may stand directly under each other: then, beginning at the right hand, subtract each number in the lower line from that above it, and set down the remainder.

2.—When any of the lower numbers are *greater* than the *upper*, as many must be added to the *upper* number as makes one of the next higher denomination, from the sum of which take the lower number, put down the *difference*, and carry one to the next number in the lower line, which subtract from that above it as before.

PROOF.—As in simple subtraction.

MONEY.

	(1) £.	s.	d.	(2) £.	s.	d.
From	120	13	6½	137	9	2¼
Take	75	12	7½	9	17	4½
Diff.	45	0	11	127	11	9¾
Proof	120	13	6½	137	9	2¼

(3) £.	s.	d.	(4) £.	s.	d.	(5) £.	s.	d.	(6) £.	s.	d.
20	13	7	13	2	3	16	18	2	32	11	7
17	11	8	7	15	2	14	9	7	17	12	6

(7) £.	s.	d.	(8) £.	s.	d.	(9) £.	s.	d.	(10) £.	s.	d.
100	8	11	50	13	4	35	14	8	356	11	4
19	12	2	21	17	1	32	18	5	276	14	10

(11) £.	s.	d.	(12) £.	s.	d.	(13) £.	s.	d.	(14) £	s.	d.
1237	9	6½	3127	17	11¼	111	11	1½	361	1	8½
766	4	8¼	1273	19	4¼	10	15	4½	94	18	2¼

(15) £.	s.	d.	(16) £.	s.	d.	(17) £.	s.	d.	(18) £.	s.	d.
121	6	1¾	121	4	2½	37	11	6	160	3	4¼
17	12	3½	87	12	1¼	19	12	5¾	21	18	6¾

(19)			(20)			(21)			(22)		
£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
1000	10	6	106	6	6¼	110	6	10½	10	0	0
27	13	6¼	99	12	10½	28	18	11¾	9	17	5¾

TROY WEIGHT.

	(23)			(24)			
	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.
From	24	8	20	17	8	10	12
Take	17	12	9	5	7	18	21
Rem.							

AVOIRDUPOIS WEIGHT.

	(25)			(26)				
	lb.	oz.	dr.	ton.	cwt.	qr.	lb.	oz.
From	316	15	12	300	12	2	25	6
Take	68	14	15	278	5	2	25	13

APOTHECARIES' WEIGHT.

	(27)			(28)				
	lb.	oz.	dr.	lb.	oz.	dr.	scr.	gr.
From	13	5	6	121	1	6	2	15
Take	8	5	2	76	7	2	2	17

CLOTH MEASURE.

	(29)			(30)			
	yds.	qrs.	nls.	E. E.	qr.	nls.	in.
From	17	3	2	125	1	1	2
Take	8	2	3	123	2	3	2

LONG MEASURE.

(31)

	mls.	*fur.*	*po.*
From	112	7	35
Take	37	6	36

(32)

mls.	*fur.*	*po.*	*yds.*	*feet.*	*in.*
50	3	19	3	2	10
29	7	25	2	1	11

LIQUID MEASURE.

(33)

	gals.	*qts.*	*pts.*
From	136	2	0
Take	25	3	1

(34)

gals.	*qts.*	*pts.*
718	1	1
312	2	1

DRY MEASURE.

(35)

	qrs.	*bus.*	*pks.*
From	341	4	2
Take	74	7	3

(36)

la.	*qrs.*	*bus.*	*pks.*	*gals*
17	5	3	1	1
9	7	6	3	1

TIME.

(37)

	yrs.	*wks.*	*days.*
From	5	32	4
Take	3	18	6

(38)

days.	*ho.*	*mi.*	*sec.*
6	13	10	5
3	16	4	20

EXERCISES IN

COMPOUND ADDITION AND SUBTRACTION.

1. A farmer received of a corn-factor for wheat, £27. 13s. 6d.; for barley, £13. 12s. 0d.; for oats, £17. 17s. 9d.; and for beans, £9. 19s. 3d.; how much did he receive in the whole? Ans. £69. 2s. 6d.

2. I owed a person £5. towards which I paid 3 guineas; what remains to pay? Ans. £1. 17s. 0d.

3. A boy gave 2s. 6d. for a penknife; 1s. 3d½. for fruit to treat his brothers and sisters with; and 6d. to a poor old man, and then had 8½d. left; what had he at first? Ans. 5s.

4. In the reign of Henry VI. the revenue or amount of taxes averaged £64976. 13s. 6d.; in the year 1815 they amounted to £71153142. 0s. 0d.; how much does the latter exceed the former? Ans. £71088165. 6s. 6d.

5. Paid for the repairs of my house as follows: the mason, £3. 7s. 6d.; the bricklayer, £7. 4s. 4d.; the painter, £5. 8s. 3d.; and glazier, £1. 17s. 6d.; what did the whole of the repairs cost me? Ans. £17. 17s. 7d.

6. A farmer owed to a schoolmaster for a quarter's education to three children £1. 18s. 6d. besides 13s. 8d. for books; what change had he to receive against a £5 bill? Ans. £2. 7s. 10d.

7. The wife of a mechanic took a guinea to the market; what had she left after paying the butcher 5s. 6½d; grocer, 6s. 10d.; flour-dealer, 4s. 6d.; and 2s. 4½d. for trifling articles? Ans. 1s. 9d.

8. From a piece of cloth 31 yards long, the owner sold 6½ yards to one man, and 9¼ to another; how much had he then left? Ans. 15¼ yards.

9. Two of the greatest bells in the world are at Moscow, in Russia, one weighing 192tons, 17cwt. 0qr. 16lb., and the other 128tons, 11cwt. 1qr. 20lb.; what is the weight of both, and the difference between them?
Ans. 321tons, 8cwt. 2qr. 8lb.; diff. 64tons, 5cwt. 2qr. 24lb.

10. From £100. I paid to A, £15. 12s. 6d.; to B, £23. 13s. 8½d.; to C, £30. 18s. 9d.; and to D, £12. 8s. 7½d.; what had I left? Ans. £17. 6s. 5d.

COMPOUND MULTIPLICATION.

Compound Multiplication is the method of finding what any given number of different denominations will amount to, when repeated any proposed number of times.

CASE I.—When the multiplier does not exceed 12.

RULE.—Multiply the lowest denomination of the multiplicand by the multiplier, dividing the product by as many of that as makes one of the next, set down the remainder, and add the quotient to the next greater after it is multiplied.

EXAMPLES.

	(1)			(2)		
	£.	s.	d.	£.	s.	d.
Multiply	7	7	6	18	13	6½
By			3			5
Product	22	2	6	93	7	8½

	(3)			(4)		
	cwt.	*qr.*	*lbs.*	*mls.*	*fur.*	*pls.*
Multiply	32	1	12	13	5	30
By			6			7
Product	194	0	16	96	0	10

EXERCISES.

	£.	s.	d.	
1. Multiply	3	7	6	by 2.
2. Multiply	13	6	8	by 3.
3. Multiply	15	9	7	by 4.
4. Multiply	27	10	9	by 5.
5. Multiply	35	11	3	by 6.
6. Multiply	56	12	5½	by 7.
7. Multiply	48	13	10½	by 8.
8. Multiply	39	14	11½	by 9.
9. Multiply	78	15	9½	by 10.
10. Multiply	136	16	2½	by 11.
11. Multiply	167	17	10¼	by 11.
12. Multiply	139	18	4¼	by 12.
13. Multiply	367	19	11¾	by 12.

14. Multiply 53cwt. 1qr. 7lb. 8oz. by 8.
15. Multiply 9mls. 7fur. 14pls. by 9.
16. Multiply 5a. 3r. 17p. by 10.
17. Multiply 2lb. 7oz. 12dwt. 13gr. by 11.
18. Multiply 4hhd. 1kil. 1fir. 5gal. by 12.
19. Multiply 5yds. 3qrs. 2nls. 1½in. by 12.
20. Multiply 7qrs. 5bush. 3pks. 1½gal. by 12.

CASE II.—When the multiplier is greater than 12.

RULE 1.—Find two numbers which multiplied together will produce the multiplier, then multiply by one number, and the product again by the other.

2.—When no two numbers produce the multiplier, take two whose product is near it, adding as many times the top line as it is short of the multiplier, or subtracting as many as it is above.

EXAMPLES.

	£.	s.	d.	
Mult.	15	12	6	*by* 27.
			3 × 9 = 27	
	46	17	6	
			9	
£	421	17	6	

	£.	s.	d.	
Mult.	3	5	6½	*by* 17.
			2 × 8 + 1 = 17	
	6	11	1	
			8	
	52	8	8	
	3	5	6½	
£	55	14	2½	

EXERCISES.

	£.	s.	d.			£.	s.	d.
1. Multiply	5	3	2	by 14.	Ans.	72	4	4.
2. Multiply	7	11	3¼	by 15.	Ans.	113	9	0¾.
3. Multiply	32	13	7½	by 16.	Ans.	522	18	0.
4. Multiply	72	14	9¼	by 17.	Ans.	1236	11	1¼.
5. Multiply	9	16	5	by 25.	Ans.	245	10	5.
6. Multiply	17	17	7	by 31.	Ans.	554	5	1.
7. Multiply	24	18	3	by 35.	Ans.	871	18	9.
8. Multiply	11	19	4	by 21.	Ans.	251	6	0.
9. Multiply	17	18	6½	by 24.	Ans.	430	5	0.
10. Multiply	19	13	8½	by 28.	Ans.	551	3	10.
11. Multiply	52	12	9¼	by 42.	Ans.	2210	16	4½.
12. Multiply	8	19	10¾	by 48.	Ans.	431	15	0.

COMPOUND DIVISON.

Compound Division is the method of finding how often a *given number* is contained in another *given number* of different denominations.

CASE I.—When the divisor *does not* exceed 12.

RULE.—Place the divisor on the *left hand* of the dividend. Divide the *highest* denomination of the dividend by the divisor, and write down the *quotient*; reduce the *remainder* (if any) into the *next lower* denomination, and add it to the number in *that* denomination, which divide as before.

PROOF.—By compound multiplication.

EXAMPLES.

(1)

	£.	s.	d.
3)	22	2	6
Quotient	7	7	6
			3
Proof	22	2	6

(2)

	£.	s.	d.
5)	93	7	8½
	18	13	6½
			5
	93	7	8½

(3)

	cwt.	qrs.	lb.
6)	194	0	16
Quotient	32	1	12

(4)

	mls.	fur.	po.
7)	96	0	10
	13	5	30

EXERCISES.

	£.	s.	d.	
1. Divide	7	15	2	by 2.
2. Divide	13	11	3	by 2.
3. Divide	40	0	0	by 3.
4. Divide	51	4	3¾	by 3.
5. Divide	21	18	4	by 4.
6. Divide	132	19	5	by 4.
7. Divide	37	13	9	by 5.
8. Divide	137	3	7¾	by 5.
9. Divide	63	7	7½	by 6.
10. Divide	256	17	9	by 6.
11. Divide	102	6	11	by 7.
12. Divide	317	19	0¾	by 7.
13. Divide	198	10	10	by 8.

		£.	s.	d.		
14.	Divide	541	9	6	by	8.
15.	Divide	235	7	2¼	by	9.
16.	Divide	879	16	10½	by	9.
17.	Divide	787	17	11	by	10.
18.	Divide	1130	18	3½	by	11.
19.	Divide	6394	10	3	by	12.
20.	Divide	4459	19	9	by	12.

21. Divide 426cwt. 2qrs. 4lb. by 8.
22. Divide 89mls. 2fur. 6pls. by 9.
23. Divide 58a. 2r. 10p. by 10.
24. Divide 17lb. 17dwt. 23gr. by 11.
25. Divide 55hhd. 0kil. 6gal. by 12.

CASE II.—When the divisor is greater than 12.

RULE. 1.—If the divisor be a composite number, resolve it into its factors, then divide by one factor, and the quotient again by the other.

2.—If the divisor be a prime number, divide by it at once as in long division.

EXAMPLES.

(1)

Divide £421 17s. 6d. *by* 27.

		£.	s.	d.	
27 = {	3	421	17	6	
	9	140	12	6	
		£15	12	6	*Ans.*

(2)

Divide £55 14s. 2½d. *by* 17.

```
      £.  s.  d.
17)55  14  2½(3  5  6½
   51
    4
   20
17)94(5 Shillings
   85
    9
   12
17)110(6 Pence
   102
     8
     4
17)34(2 Farthings
   34
```

(3)

Divide £141 10s. 4d. *by* 112

		£.	s.	d.	
112 = {	4	141	10	4	
	4	35	7	7	
	7	8	16	10¾	
		1	5	3¼	*Ans.*

EXERCISES.

		£.	s.	d.						
1.	Divide	72	4	4	by	14.	Ans.	5	3	2
2.	Divide	113	9	$0\frac{3}{4}$	by	15.	Ans.	7	11	$3\frac{1}{4}$
3.	Divide	522	18	0	by	16.	Ans.	32	13	$7\frac{1}{2}$
4.	Divide	1236	11	$1\frac{1}{4}$	by	17.	Ans.	72	14	$9\frac{1}{4}$
5.	Divide	245	10	5	by	25.	Ans.	9	16	5
6.	Divide	554	5	1	by	36.	Ans.	15	7	11
7.	Divide	871	18	9	by	35.	Ans.	24	18	3
8.	Divide	251	6	0	by	21.	Ans.	11	19	4
9.	Divide	64	2	6	by	27.	Ans.	2	7	6
10.	Divide	494	14	6	by	28.	Ans.	17	13	$4\frac{1}{2}$
11.	Divide	112	2	$2\frac{1}{4}$	by	105.	Ans.	1	1	$4\frac{1}{4}$
12.	Divide	111	17	8	by	112.	Ans.	0	19	$11\frac{3}{4}$

EXERCISES IN

COMPOUND MULTIPLICATION.

Find the values of the following quantities:—

1. 6 yards of cloth, at 2*s*. 6*d*. a yard. Ans. 15*s*. 0*d*.
2. 8 bushels of barley, at 5*s*. 3*d*. a bushel. Ans. £2. 2*s*. 0*d*.
3. 14 gallons of ale, at 1*s*. 8*d*. a gallon. Ans. £1. 3*s*. 4*d*.
4. 15 bushels of oats, at 3*s*. 6*d*. a bushel. Ans. £2. 12*s*. 6*d*.
5. 16 quarters of wheat, at £3. 5*s*. 6*d*. a quarter. Ans. £52. 8*s*. 0*d*.
6. 18lbs. of tea, at 8*s*. 6*d*. a lb. Ans. £7. 13*s*. 0*d*.
7. 22lbs. of tobacco, at 3*s*. 4½*d*. a lb. Ans. £3. 14*s*. 3*d*.
8. 24 lbs. of candles, at 8½*d*. a lb. Ans. 17*s*. 0*d*.
9. 25 yards of silk, at 6*s*. 10½*d*. a yard. Ans. £8. 11*s*. 10½*d*.
10. 27 yards of superfine blue cloth, at £1. 3*s*. 6*d*. a yard. Ans. £31. 14*s*. 6*d*.
11. 28 yards double milled drab, at 17*s*. 9*d*. a yard. Ans. £24. 17*s*. 0*d*.
12. 30 yards of kerseymere, at 7*s*. 10*d*. a yard. Ans. £11. 15*s*. 0*d*.

13. 35 weeks' wages, at 13s. a week.* Ans. £22. 15s. 0d.

14. 42 thousand bricks, at 8s. 4d. a thousand. Ans. £17. 10s. 0d.

15. 48 sheep, at £2. 7s. 6d. each. Ans. £114. 0s. 0d.

16. 56 lambs, at 17s. 6d. each. Ans. £49. 0s. 0d.

17. 64 stones of wool, at 12s. 6d. a stone. Ans. £40. 0s. 0d.

18. 72 pair of stockings, at 2s. 4¼d. a pair. Ans. £8. 9s. 6d.

19. 108 pair of gloves, at 2s. 7¾d. a pair. Ans. £14. 5s. 9d.

20. 17 tons of hay, at £4. 12s. 6d. a ton. Ans. £78. 12s. 6d.

21. 23 stones of bacon, at 7s. 3½d. a stone. Ans. £8. 7s. 8½d.

22. 34 ells of cambric, at 10s. 8½d. an ell. Ans. £18. 4s. 1d.

23. 43 feet of mahogany, at £1. 3s. 2d. a foot. Ans. £49. 16s. 2d.

24. 51 dozen of wine, at £1. 17s. 6d. a dozen. Ans. £95. 12s. 6d.

25. 65 stone of Spanish wool, at £1. 8s. 9d. a stone. Ans. £93. 8s. 9d.

26. 75 stone of Saxony wool, at £2. 12s. 6d. a stone. Ans. £196. 17s. 6d.

27. 103 lbs. of hyson tea, at 9s. 1½d. a lb. Ans. £46. 19s. 10½d.

28. 235 lbs. of leather, at 1s. 2½d. a lb.† Ans. £14. 3. 11½d.

29. 357 yards of cloth, at 16s. 6¼d. a yard. Ans. £294. 17s. 11¼d.

30. 6½ cwt. of cheese, at £3. 0s. 8d. a cwt.‡ Ans. £19. 14s. 4d.

31. 7½ quarters of wheat, at £2. 14s. 6d. a quarter. Ans. 20. 8s. 9d.

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* When the price is in *one* name, the quantity may be multiplied by the price, and the product will be the answer in the same name as the price.

† When the number is large, multiply the price of *one* by 10, and that product by 10, which last multiply by the number of hundreds, to which add the second line multiplied by the figure in the tens place, and top line multiplied by the figure in the units place.

‡ To multiply by ½, is to divide by 2; to multiply by ¼, is to divide by 4; to multiply by ¾, first take ½, then ½ of that *half*, which add together.

32. 13½ cwt. of iron, at 14s. 9d. a cwt.
Ans. £9. 19s. 1½d.

33. 16¼ cwt. of cheese, at £3. 7s. 8d. a cwt.
Ans. £54. 19s. 7d.

34. 28¼ yards of velvet, at 12s. 6d. a yard.
Ans. £17. 13s. 1½d.

35. 120¼ acres, at £50. 10s. 6d. an acre.
Ans. £6075. 12s. 7½d.

36. 112¾ tons of slate, at £1. 10s. 0d. a ton.
Ans. £169. 2s. 6d.

37. 109¾ cwt. of hops, at £8. 12s. 0d. a cwt.
Ans. £943. 17s. 0d.

38. 356¾ yards of Irish cloth, at 3s. 6d. a yard.
Ans. £62. 8s. 7½d.

MISCELLANEOUS EXERCISES IN

COMPOUND MULTIPLICATION & DIVISION.

1. If 1lb. of sugar cost 6d., what will 5lbs. cost?
Ans. 2s. 6d.

2. If 10lbs. of sugar cost 5s. 10d., what will 1lb. cost?
Ans. 7d.

3. If a yard of cloth cost 15s. 6d., what will 15 yards cost? Ans. £11. 12s. 6d.

4. If 15 yards of cloth cost £11. 12s. 6d., what is that per yard? Ans. 15s. 6d.

5. Bought 16 quarters of wheat for £52. 8s. 0d., what is the worth of 1 quarter? Ans. £3. 5s. 6d.

6. The clothing of 24 poor widows cost £30. 12s. 0d., what was the expense of each? Ans. £1. 5s. 6d.

7. At 2½d. per lb., what is the value of 1 cwt.?
Ans. £1. 3s. 4d.

8. If 1s. gain 2½d., what will £1. gain? Ans. 4s. 2d.

9. What has that clerk per week, who receives £70. per annum? Ans. £1. 6s. 11d.—4

10. Bought cheese at £3. 7s. 8d., per cwt., what is that a lb.? Ans. 7¼d.

11. A pensioner having 2s. 3d. a day, what is that a year? £41. 1s. 3d.

12. At 1s. $1\frac{1}{2}$d. per mile, what will be the expense of a chaise for 32 miles? Ans. £1. 16s. 0d.

13. If £1. 16s. 0d. be paid for the hire of a chaise 32 miles, what is that a mile? Ans. 1s. $1\frac{1}{2}$d.

14. If I pay £1. 16s. 0d. for a chaise, at 1s. 6d. a mile, how far do I travel?* Ans. 24 miles.

15. Five persons bought a stack of hay, weighing 14tons, 17cwt. 2qrs., what weight had each?
Ans. 2tons, 19cwt. 2qrs.

16. How far did that person travel per day, who went 231 miles in 15 days? Ans. 15mls. 3fur. 8po.

17. In the reign of Henry III, the price of a Bible was £20., at which time the wage of a labouring man was $1\frac{1}{2}$d. a day; how many days would he have to labour for a Bible? Ans. 3200 days.

18. A club of 35 persons bought a lottery ticket, which came a prize of £20,000., what had each to receive?
Ans. £571. 8s. $6\frac{3}{4}$d.—3.

19. A person having £41. 1s. 3d. a year, what may he spend per month? Ans. £3. 8s. $5\frac{1}{4}$d.

20. If £20. be divided equally among 13 persons, what is the share of each? Ans. £1. 10s. $9\frac{3}{13}$d.

21. Bought 2 dozen of hats for £18. 9s. 6d., and sold them for £19. 15s. 0d., what did I gain by the whole, and by each? Ans. £1. 5s. 6d. 1s. $0\frac{3}{4}$d.

22. If 48 cost £47. 17s. 0d., what is the price of one?
Ans. 19s. $11\frac{1}{4}$d.

23. If 120 quarters of wheat cost £331., what is the price of a quarter? Ans. £2. 15. 2d.

24. If I buy 210 quarters of oats for £262. 10s. 0d., how must I sell them per quarter to gain 20 guineas by them? Ans. £1. 7s. 0d.

25. If a cwt. of leather cost 7 guineas, what is that a lb.? Ans. 1s. $3\frac{3}{4}$d.†

26. At £5. 7s. 4d. a cwt., what is that a lb.?
Ans. $11\frac{1}{2}$d.

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* The divisor and dividend must be brought into the same denomination.

† An useful contraction in questions of this kind, is to multiply the price in shillings by 3, and divide by 7, the quotient is the answer in farthings.

27. At £4. 1s. 8d. a cwt., what is the value of a lb.?
Ans. $8\frac{3}{4}$d.

28. If a man travel 3mls. 5fur. 30p. 4yds. in one hour, how far will he travel in 7 hours?
Ans. 26m. 0f. 15p. $0\frac{1}{2}$yds.

29. A certain garden contains 2r. 35p. 11yds., what quantity is there in 22 such gardens? Ans. 15a. 3r. 18p.

30. If a person travels 52mls. 0fur. 30p. 1yd. in 14 hours, how far does he travel in 1 hour?
Ans. 3mls. 5fur. 30p. 4yds.

31. If a common, containing 47a. 2r. 14p., be awarded to 66 cottagers, what will be the share of each?
Ans. 2r. 35p. 11yds.

32. The hour hand of a clock or watch moves round the dial plate in 12 hours, how many degrees does it pass through in one hour? Ans. 30°.

33. The mariners' compass is divided into 32 points, how many degrees are there in each point? Ans. 11°, 15′.

34. Botany Bay is situate in 150 degrees of *east* longitude from London, what is the hour at that place when it is noon at London?* Ans. 10 o'clock at night.

35. Grand Cairo is situate in 31°, 15′ of east longitude, what is the hour there when it is noon here?
Ans. 5′ past 2 in the afternoon.

36. Washington is situate in 77°, 15′ *west* longitude, what is the hour there when it is noon with us?
Ans. 51′ past 6 in the morning.

37. Quebec is situate in 70°, 45′ of west longitude, what is the hour there when it is noon here?
Ans. 17′ past 7 in the morning.

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* When the longitude is *east*, divide it by 15, and the quotient is the hour in the *afternoon*. When it is *west*, divide by 15, the quotient is the time *before* noon; or, if the quotient be subtracted from 12, the remainder is the hour in the morning.

It is said to be noon at any place in north latitude, when the sun is directly *south* of that place; and at any place in south latitude, when the sun is directly *north* of that place.

Most nations reckon longitude from the meridian passing over their own metropolis; we reckon from the meridian of Greenwich Observatory, east and west, each 180 degrees; and as the earth performs its diurnal revolution in 24 hours, or apparently the sun round the earth in that time, it is manifest that he passes over 15 degrees in every hour, so that a place situate 15 degrees east and another 15 degrees west of London, the first will have the sun on its meridian one hour before, and the second one hour after, it is on the meridian of London.

BILLS OF PARCELS.

A HOSIER'S BILL.

1. *Mr. John Potter,*

Bought of John Whitley, June 1st, 184

	s.	d.				
6 Pairs of stockings ....at	3	8 a pr.				
8 Pairs of worsted ditto......at	3	10 a pr.				
12 Pairs of silk ditto..........at	8	6 a pr.				
7 Pairs of cotton ditto ......at	3	4 a pr.				
5 Pairs of thread ditto ......at	3	9 a pr.				
4 Pairs of lamb's wool ditto ..at	4	6 a pr.				
			£	10	14	9

A MERCER'S BILL.

2. *Mr. John Langdale,*

Bought of Samuel Stead, July 2nd, 184

	s.	d.				
6 Yards of cambric ....at	8	6 a yd.				
14 Yards of silk ....at	9	6 a yd.				
15 Yards of lace ....at	5	10 a yd.				
16 Yards of calico ....at	0	11½ a yd.				
27 Yards of shalloon ....at	2	1½ a yd.				
25 Yards of lawn ....at	2	7½ a yd.				
			£	20	9	10

A GROCER'S BILL.

3. *Mr. W. Hops,*

Bought of J. D. Sollitt, Aug. 3rd, 184

	s.	d.				
11 Pounds of lump sugar ...at	0	11½ a lb.				
16 Pounds of brown ditto ...at	0	5 a lb.				
17 Pounds of rice...........at	0	2½ a lb.				
7½ Pounds of raisins ....at	0	8½ a lb.				
15 Pounds of currants......at	0	5½ a lb.				
9½ Pounds of cocoa ....at	2	6½ a lb.				
			£	2	17	1

A WINE MERCHANT'S BILL.

4\. *Mr. B. S. Booth,*

Bought of Henry Wild, Sept. 4th, 184

		s.	d.			
10	Gallons of claret.......at	12	6 a gal.			
13	Gallons of port.........at	16	3 a gal.			
14	Gallons of sherry......at	17	4 a gal.			
7½	Gallons of brandy......at	28	6 a gal.			
16	Gallons of hollands.....at	17	6 a gal.			
25	Gallons of rum.........at	17	0 a gal.			
			£	74	17	8

A CORN FACTOR'S BILL.

5\. *Mr. W. Grainger,*

Bought of Richard Fewson, Oct. 5th, 184

		s.	d.			
35	Quarters of wheat ......at	56	6 a qr.			
67	Quarters of oats.........at	25	0 a qr.			
28	Quarters of beans ......at	39	6 a qr.			
19	Quarters of peas ....at	42	6 a qr.			
108	Quarters of barley......at	30	0 a qr.			
17	Quarters of rye ....at	29	6 a qr.			
			£	465	7	6

Leeds, Nov. 6th, 184

6\. *Mr. Wm. Settle,*

Bought of Richard Hiley.

		s.	d.			
2¼	Yards of superfine black at	26	6 a yd.			
3½	Yards of blk. kerseymere at	9	9 a yd.			
11¾	Yards of Welsh flannel..at	2	3 a yd.			
17½	Yards of green baize....at	3	2 a yd.			
9¾	Yards of serge ....at	1	10 a yd.			
2⅝	Yards of superfine blue..at	26	0 a yd.			
2⅜	Yards of woollen cord ..at	12	6 a yd.			
			£	14	11	5

Liverpool, Dec. 1*st*, 184

7. *Messrs. Fenby & Ranson,*

To John Parker & Co.

		s.	d.
5 doz. of worsted stockings	at	16	6 a doz.
7 doz. of thread ditto	at	21	9 a doz.
6 doz. of cotton ditto	at	17	6 a doz.
8 doz. of lamb's wool ditto	at	18	6 a doz.
9 doz. of ditto ditto	at	22	6 a doz.
10 doz. of silk ditto	at	36	0 a doz.
11 doz. of socks	at	5	6 a doz.
12 doz. of ditto	at	6	9 a doz.

£ 59 11 9

Manchester, Feb. 2*nd*, 184

8. *Mr. Timothy Twibill,*

Bought of Nesbit, Little, & Co.

20 Pieces of cotton, each 20 yds. at 5½d. a yd.
20 Pieces of ditto, each 30 yds. at 5½d. a yd.
10 Pieces of ditto, each 28 yds. at 7½d. a yd.
15 Pieces of ditto, each 27 yds. at 13d. a yd.
16 Pieces of ditto, each 26 yds. at 10½d. a yd.
25 Pieces of ditto, each 30 yds. at 11½d. a yd.
12 Pieces of cord, each 32 yds. at 16½d. a yd.
20 Pieces of fustian, each 45 yds. at 13½d. a yd.

£ 184 15 4

REDUCTION

Is the method of converting numbers from one denomination to another of the same value.

I.—To bring great names into small names.

RULE.—Multiply by as many of the *less name* as make one of the *greater name*.

II.—To bring small names into great names.

RULE.—Divide by as many of the *less name* as make one of the *greater name*.

EXAMPLES.

In £7. 3s. $2\frac{1}{4}$d., how many farthings?

20			
143 *shillings*		4	6873 *farthings*
12		12	$1718\frac{1}{4}$
1718 *pence*		20	143 $2\frac{1}{4}$
4		*Proof*	£ 7. 3s. $2\frac{1}{4}$d.
6873 *farthings.*			

The former of these is called *reduction descending,* and the other *reduction ascending.*

EXERCISES.

1. In £7. 0s. 0d., how many shillings and pence? Ans. 140 shillings, 1680 pence.
2. In £7. 5s. 6d., how many pence? Ans. 1746.
3. In £8. 6s. 8d., how many farthings? Ans. 8000.
4. In £9. 7s. $6\frac{1}{2}$d., how many farthings? Ans. 9002.
5. In £27. 10s. $11\frac{3}{4}$d., how many farthings? Ans. 26447.
6. In £30. 11s. $11\frac{1}{2}$d., how many farthings? Ans. 29374.
7. In £35. 13s. $7\frac{1}{2}$d., how many farthings? Ans. 34254.
8. In £300. 15s. $6\frac{1}{2}$d., how many half-pence? Ans. 144373.
9. In 1680d., how many pounds? Ans. £7. 0s. 0d.
10. In 1746d., how many pounds? Ans. £7. 5s. 6d.
11. In 7976 farthings, how many pounds? Ans. £8. 6s. 2d.
12. In 9002 farthings, how many pounds? Ans. £9. 7s. $6\frac{1}{2}$d.
13. In 26447 farthings, how many pounds? Ans. £27. 10s. $11\frac{3}{4}$d.
14. In 29374 farthings, how many pounds? Ans. 30. 11s. $11\frac{1}{2}$d.
15. In 34254 farthings, how many pounds? Ans. £35. 13s. $7\frac{1}{2}$d.
16. In 144373 half-pence, how many pounds? Ans. £300. 15s. $6\frac{1}{2}$d.

17. In £25., how many crowns, half-crowns, and sixpences?

Ans. 100 crowns; 200 half-crowns; 1000 sixpences.

18. In 21 guineas, how many pence? Ans. 5292.

19. In 5040 farthings, how many guineas? Ans. 5.

20. In 5760 farthings, how many sixpences, half-crowns, crowns, and pounds?

Ans. 240 sixpences; 48 half-crowns; 24 crowns; £6.

21. In £10. 17s. 6d., how many sixpences and half-crowns? Ans. 435 sixpences; 87 half-crowns.

22. What is the value of 150 half-crowns?

Ans. £18. 15s. 0d.

23. In 20 guineas, how many half-crowns and two-penny-pieces?

Ans. 168 half-crowns; 2520 twopenny-pieces.

24. In £50. how many half-crowns and shilling?

Ans. 400 half-crowns; 1000s.

25. How many French franks, of 10d. each, are there in £20? Ans. 480.

26. How many Portuguese moidores of 27s. each, are equivalent to 630 guineas? Ans. 490.

27. How many Spanish piastres, of 3s. 7d. each, are equivalent to £21. 10s.? Ans. 120.

28. In 560 rubles, at 2s. 9½d. each, how many pounds sterling? Ans. £78. 3s. 4d.

29. How many Russian rubles, at 2s. 9½d. each, are equal to £78. 3s. 4d.? Ans. 560.

30. How many Venician ducats, at 3s. 11½d. each, are equivalent to 380 sovereigns? Ans. 1920.

31. In £840., how many guineas? Ans. 800.

32. In 1680 guineas, how many pounds? Ans. 1764.

33. In 20 guineas, how many half-sovereigns and half-crowns? Ans. 42 h. s. 168 h. c.

34. How many Portuguese milreas, each worth 5s. 7½d., are equivalent to £37. 13. 9d.? Ans. 134.

35. How many Dutch florins, at 1s. 8¼d. each, are equivalent to 1200 milreas, each 5s. 7½d.? Ans. 4000.

36. A debt of £12. was paid with an equal number of crowns and shillings, required the number?

Ans. 40 each.

37. What number of sixpences, shillings, half-crowns, and crowns, will it take to discharge a debt of £50. 8s., the number of each being equal? Ans. 112 of each.

TROY WEIGHT.

1. In 2lb. 5oz. 7dwts. 12grains, how many grains? Ans. 14100.

2. In 14100grains, how many pounds? Ans. 2lb. 5oz. 7dwt. 12gr.

3. How many grains in a gold cup, weighing 11oz. 17dwts.? Ans. 5688.

4. In 20378grains, how many pounds? Ans. 3lb. 6oz. 9dwt. 2gr.

5. How many grains in half a dozen silver spoons, each weighing 1oz. 16dwt. 13gr.? Ans. 5262.

6. From an ingot of silver, weighing 3lb. 7oz. 17dwt., how many spoons can I have made, each weighing 1oz. 16dwt. 13gr.? Ans. 24.

7. In one of the churches at Moscow is a chandelier of massy silver, weighing 2940lbs.; how many dollars might be coined from it, each dollar weighing 16dwt., and what would it amount to, the dollar being valued at 4s. 6d.? Ans. 44100 dollars; worth £9922. 10s. 0d.

AVOIRDUPOIS WEIGHT.

1. In 5tons, how many ounces? Ans. 179200.

2. In 3tons, 2cwt. 2qrs. 14lb., how many pounds? Ans. 7014.

3. In 1cwt. 3qrs. 15lb. 13dr., how many dr.? Ans. 54029.

4. In 179200oz., how many tons? Ans. 5.

5. In 7014lbs., how many tons? Ans. 3ton, 2cwt. 2qr. 14lb.

6. In 54253drams, how many cwt.? Ans. 1cwt. 3qr. 15lb. 14oz. 13dr.

7. In 12tons, 8cwt. 1qr. of cheese, how many cheeses, each 10½lbs.? Ans. 2648.

8. The brazen collossus which stood across the harbour at Rhodes, weighed 321tons, 8cwt. 2qr. 6lb. 8oz.; how many brass candlesticks might have been made from it, each weighing 12oz. Ans. 959998.

CLOTH MEASURE.

1. In 20 yards, how many quarters and nails?
Ans. 80qrs.; 320n.
2. In 320 nails, how many yards? Ans. 20.
3. In 150 English ells, how many yards? Ans. 187½.
4. In 180 Flemish ells, how many yards? Ans. 135.
5. In 275 Flemish ells, how many English ells? Ans. 165.
6. In 756 English ells, how many Flemish ells? Ans. 1260.
7. How many coats, each containing 2¾ yards, might be cut from a piece of cloth 66 yards long? Ans. 24.

LONG MEASURE.

1. In 25 miles, how many furlongs and poles?
Ans. 200fur.; 8000p.
2. In 8000 poles, how many miles? Ans. 25.
3. In 50 miles, how many inches? Ans. 3168000.
4. In 63360 feet, how many miles? Ans. 12.
5. How often will a wheel, 15 feet in circumference, turn round in 30 miles? Ans. 10560.
6. How many steps will a person take in walking 20 miles, each step being 2½ feet? Ans. 42240.
7. A man and a boy walk together 7 miles, the man stepping 3 feet, and the boy 2, each step; how many steps does the boy take more than the man? Ans. 6160.
8. In 5mls. 3fur. 20po. 4yds. 2ft., how many feet?
Ans. 28724.

LAND MEASURE.

1. In 20acres, 2roods, 31perches, how many roods and perches? Ans. 82roods; 3311perches.
2. In 3311perches, how many acres? Ans. 20a. 2r. 31p.
3. A person has 3 fields; the first of which contains 7acres and 12perches; the second, 11acres, 1rood, 18 perches; and the third, 3roods, 39perches; how many perches are there in the whole. Ans. 3109.
4. How many gardens of 32 perches each, might be made out of a five acre field? Ans. 25.
5. An acre contains 4840 square yards; how many feet does it contain? Ans. 43560.
6. In an estate containing 4734acres, 2roods, 20perches, how many farms, each 96acres, 2roods, 20perches? Ans. 49.

LIQUID MEASURE.

1. In 25 gallons, how many pints? Ans. 200.

2. In 400 pints, how many gallons? Ans. 50.

3. In 5 pipes of port wine, how many gallons and pints? Ans. 630 gallons; 5040 pints.

4. In 12 hogsheads of ale, how many pints? Ans. 5184.

5. In 1728 pints, how many firkins of ale? Ans. 24.

6. In 15 ankers of brandy, how many pints? Ans. 1200.

7. How many bottles, each containing $1\frac{1}{2}$ pint, are there in a pipe of wine? Ans. 672.

8. In 20 puncheons of rum, how many runlets?
Ans. 93run. 6gal.

DRY MEASURE.

1. In 27 quarters, how many bushels and pecks?
Ans. 216bushels; 864pecks.

2. In 30 quarters, how many pints? Ans. 15360.

3. In 352 pecks, how many quarters? Ans. 11.

4. In 15360 pints, how many quarters? Ans. 30.

5. In 17qr. 5bus. 2pks, how many quarts? Ans 4528.

6. In 9056 pints, how many quarters?
Ans. 17qrs. 5bus. 2pks.

7. In 15lasts, 8qrs. 5bush. 3pks. 1gal. how many gallons? Ans. 10159.

8. In 40636 quarts, how many lasts?
Ans. 15lasts, 8qrs. 5bush. 3pks. 1gal.

TIME.

I. In 43days, 19hours, 47minutes, 1second, how many seconds? Ans. 3786421.

2. In 3786421 seconds, how many days?
Ans. 43days, 19hours, 47minutes, 1second.

3. How many days since the birth of Chirst, to Christmas, 1840, reckoning $365\frac{1}{4}$ days to a year? Ans. 672060.

4. How many hours has that boy lived, who is just 12 years old? Ans. 105192.

5. How many days in a solar year, which consists of 31556929 seconds? Ans. 365days, 5hours, 48′ 49″.

6. Gunpowder plot was discovered November 5, 1605; how many days is it since, to November 5, 1845?
Ans. 87660.

RULE OF THREE DIRECT

Teaches from three numbers given to find a fourth, which shall have the same proportion to the second as the third has to the first.

RULE I.—Sate the question; that is, place the numbers so that the first and third may be of the same name, and the second the same as the number sought.*

2.—Bring the first and third into one name, and the second into the lowest name mentioned in it.

3.—Multiply the second and third terms together and divide the product by the first, the quotient will be the answer to the question in the same name the second term was brought into.

The method of proof is by inverting the question.

EXAMPLES. †

If 3 yards of cloth cost £1. 16s. 6d., what will 5 yards cost?

	yds.		*£.*	*s.*	*d.*		*yds.*	
As	3	:	1	16	6	::	5	
					5			
		3)	9	2	6			
			3	0	10	Ans. ‡		

If 15 yards cost £2. 12s. 6d., what will 16 yards cost?

	yds.		*£.*	*s.*	*d.*		*yds.*
As	15	:	2	12	6	::	16
					4	× 4 = 16	
			10	10	0		
					4		
15 = {	3		42	0	0		
	×		14	0	0		
	5		2	16	0	Ans.§	

* When the question is stated, the *first* and *second* terms are one the value of the other, and the *third* is that of which the value is wanted: the first is callsd the term of supposition, and the *third* that of demand.

† Previous to the pupil entering upon the exercises, he ought to make himself acquainted with the method of each example.

‡ When the first and third terms are of the same name and not greater than 12, the question may be solved *without* reducing the second term.

§ When the *first* and *third* terms are composite numbers, the question may be solved by compound multiplication and division, as in the first example.

Bought 18lbs. of tea for £2. 16s. 6d., what would be the price of 54lbs.?

	lb.		*£.*	*s.*	*d.*		*lb.*
As	18	:	2	16	6	::	54
Or, as	1	:	2	16	6	::	3 *
					3		
			8	9	6 Ans.		

If $4\frac{3}{4}$ yards of blue cloth cost £3. 15s. 6d., what would $13\frac{1}{2}$ yards cost?

	yds.		*£.*	*s.*	*d.*		*yds.*	
As	$4\frac{3}{4}$	:	3	15	6	::	$13\frac{1}{2}$	
	4		20				4	
quarters	19		75				54	*quarters*
			12					
			906	*pence*				
			54					
			3624					
			4530					

```
19)48924  | 12 | 2574¾ Ans. in pence
   38     | 2,0|  21,4  6¾
   109    |   £ 10  14  6¾
    95
   142
   133
     94
     76
     18
      4
  19)72(3 farthings
     57
     15 remainder.
```

* When either the first and second, or first and third terms will divide by the *same* number, they may be divided, and the quotients will be in the same proportion as the original terms. Here the first and third are divided by 18, and the quotients are 1 and 3; the ratio of which is the same as 18 to 54.

As ratio in a strict mathematical sense can only subsist between quantities of the *same* kind, some teachers place the term of demand second, and that of the same name as the answer, last; but by far the greater portion think it more intelligible to a child to compare *cause* with *effect*, agreeably to the rule.

EXERCISES.

1. If 5 yards of cloth cost £3. 0s. 10d., what will 3 yards cost? Ans. £1. 16s. 6d.

2. If 6lbs. of tea cost £1. 5s. 6d., what will 8lbs. cost? Ans. 1. 14s. 0d.

3. If 4 quarters of wheat sell for £10. 10s. 0d., what is the value of 9 quarters? Ans. £23. 12s. 6d.

4. If 7 pairs of stockings cost £1. 3s. 4d., what will 11 pairs cost? £1. 16s. 8d.

5. If 8 quarters of oats sell for £11. 4s. 0d., what is 14 quarters worth? Ans. £19. 12s. 0d.

6. If 14 quarters of oats cost £19. 12s. 0d., what is the price of 8 quarters? Ans. £11. 4s. 0d.

7. Bought 8 quarters of oats for £11. 4s. 0d., how many will £19. 12s. 0d. buy? Ans. 14 quarters.

8. Gave £19. 12s. 0d. for 14 quarters of oats, what quantity can I buy for £11. 4s. 0d.? Ans. 8 quarters.

9. If 24lbs. of soap cost £1. 1s. 6d., what will 112lbs. cost? Ans. £5. 0s. 4d.

10. If 1cwt. of soap cost £5. 0s. 4d., what will 24lbs. cost? Ans. £1. 1s. 6d.

11. Bought 24lbs. of soap for £1. 1s. 6d., how much can I buy for £5. 0s. 4d.? Ans. 112lbs.

12. If 2cwt. of sugar cost £10. 11s. 6d., what is the price of 39cwt.? Ans. 206. 4s. 3d.

13. If I give £10. 11s. 6d. for 2cwt., what weight ought I to have for £206. 4s. 3d.? Ans. 39cwt.

14. How much calico can I buy for 2 guineas, when 32 yards cost £1. 4s. 0d.? Ans. 56 yards.

15. If 56 yards of cotton cost £2. 2s. 0d., what is the price of 32 yards? Ans. £1. 4s. 0d.

16. Sold $3\frac{1}{2}$ yards of velvet for £2. 12s. 6d., how much must $10\frac{1}{2}$ yards be sold for at the same rate? Ans. £7. 17s. 6d.

17. Gave 16s. 6d. for $5\frac{1}{4}$ yards of cotton cord, how much would $15\frac{3}{4}$ yards cost? Ans. £2. 9s. 6d.

18. Gave £17. 6s. 2d. for $7\frac{3}{4}$ cwt. of cheese, what would 1ton, 11cwt. cost at the same rate? Ans. £69. 4s. 8d.

19. If 31cwt. of cheese cost £69. 4s. 8d., what will $7\frac{3}{4}$cwt. cost at the same rate? Ans. £17. 6s. 2d.

20. If £17. 6s. 2d. will buy 7cwt. 3qrs. of cheese, how much will £69. 4s. 8d. buy? Ans. 1ton 11cwt.

21. When $9\frac{1}{4}$ are sold for £2. 4s. 1d., what ought to be charged for $27\frac{3}{4}$? Ans. £6. 12s. 3d.

22. A servant engaged a year for 15 guineas, but left at the end of 5 months, what had he to receive?

Ans. £6. 11s. 3d.

23. A servant girl who engaged for £8. 10s. a year, left at the end of 15 weeks, what wage was due to her?

Ans. £2. 9s. $0\frac{1}{4}$d.—44

24. What had that servant to receive who was engaged for £10. 10s. a year, but discharged at the end of 117 days? Ans. £3. 7s. $3\frac{3}{4}$d.—45.

25. If the shadow of a man 6 feet high be 4 feet, what height is that steeple whose shadow is 112 feet?

Ans. 168 feet.

26. If the shadow of a man 5 feet high be 7 feet, what height is that tree whose shadow is 140 feet?

Ans. 100 feet.

27. Wanting to know the height of Beverley Minster, —I measured its shadow, and found it 132 feet; I then erected my walking stick, which is just a yard, perpendicular to the horizontal plain, and found its shadow to be 2 feet,—required the height of the Minster? Ans. 198 feet.

28. Bought 32 yards of broad cloth for £24. 16s. 0d., what was the prime cost of one yard?* Ans. 15s. 6d.

29. At £1. 8s. 6d. per quarter, what is the value of 43 quarters 5 bushels of oats? Ans. £62. 3s. $3\frac{3}{4}$d.

30. If a person spend 6d. a day, what is that a year?

Ans. £9. 2s. 6d.

31. Bought 15 English ells 3 qrs. for £2. 12s. 0d., what is the value of 50 yards? Ans. £6. 13s. 4d.

32. What is the rent of a field containing 8a. 3r. 30p., at £3. 10s. 0d. per acre? Ans. £31. 5s. $7\frac{1}{2}$d.

33. Suppose a shopkeeper to gain $2\frac{1}{2}$d. per shilling profit, what does he gain by £10.? Ans. £2. 1s. 8d.

---

* When the *third* term is unity, it may be solved by compound division; when the *first* term is unity, by compound multiplication.

34. A person who owed £625. 10s. 0d. made an assignment to his creditors, who sold his effects for £225. 8s. 9¾d. what did they receive per £.? Ans. 7s. 2½d.

35. A person failing in business, paid only 7s. 2½d. per £., what did he owe, his creditors having shared £225. 8s. 9¾d.? Ans. £625. 10s. 0d.

36. The rental of a parish is £3600. and the assessment for the poor, £86. 8s. 6d., what is that person's rate, whose rent is £22. 10s. 0d.? Ans. 10s. 9½d.

37. If I buy 3 yards of cloth for £1., how many English ells can I have for £360. 16s. 8d.? Ans. 866 Eng. ells.

38. If 3⅜ yards of cambric cost £1. 2s. 6d., what ought to be charged for 16⅞ yards? Ans. £5. 12s. 6d.

39. Gave £12. 7s. 1d. for 35yds. 3qrs. 3nls., what ought 7yds. 0qr. 3nls. to cost? Ans. £2. 9s. 5d.

40. If the repairing of a mile of road cost £31. 12s. 6d., what length will £300. repair?

Ans. 9mls. 3fur. 35pls. 3yds.—115½

41. If 3mls. 5fur. 30pls. 4yds. of road cost £72. 15s. 3d., what will 22mls. 2fur. 24pls. 2yds. cost at the same rate?

Ans. £436. 11s. 6d.

42. If 5cwt. 3qr. 18lb. of old iron cost £3. 15s. 3d., what weight should I have for £5?

Ans. 7cwt. 3qr. 11lb. 11oz.—675.

43. If a person whose rent is £12. 10s. 0d. pays £1. 13s. 2½d. poors' rate, what is that person's rent whose rate is £24. 18s. 1½d.? Ans. £187. 10s. 0d.

44. Bought 1200 pecks of potatoes for £25., how must I sell them per bushel to gain £10.? Ans. 2s. 4d.

45. What is the value of 3½ fodders of old lead, each fodder 19½cwt. at 1s. 9d. a stone? Ans. £47. 15s. 6d.

46. If when wheat sells at £2. 8s. 0d. per quarter, bread sells for 2d. per lb., what should it sell for when wheat is £3. 6s. 0d. per quarter? Ans. 2¾d.

47. If bread sells at 2¾d. per lb. when wheat sells at £3. 6s. 0d. per quarter, what did it sell for in 1800, when wheat was a guinea a bushel? Ans. 7d.

48. A draper buys 164 yards of linen at 2s. 3½d. per yard, and sells it at 3s. 6d., how much does he gain by the whole? Ans. £9. 18s. 2d.

49. At £2. 13s. 6d. per cwt., what is the value of 7cwt. 3qrs.? Ans. £20. 14s. 7½d.

50. What is the value of 6cwt. 3qrs. 14lbs., when 1cwt. cost £2. 13s. 6d.? Ans. £18. 7s. 9¾d.

51. What may a person spend weekly out of an income of £500. per annum, besides giving £5. a month to the poor? Ans. £8. 9s. 2¾d.—$\frac{1}{13}$.

52. If 1cwt. 7lbs. cost £19. 10s. what will 3cwt. 3qrs. 26lbs. cost? Ans. £73. 1s. 8d.—20.

53. If £18. 7s. 9¾d. buys 6cwt. 3qrs. 14lbs., what weight can I have for £10.? Ans. 3cwt. 2qrs. 26lbs.—12210.

54. A grocer bought 46cwt. 3qrs. 14lbs. of sugar for £125. 15s. 7½d., how must he sell it per cwt. to gain £27. 6s. 10½d. by the whole? Ans. £3. 5s. 4d.

55. A grocer sold 46cwt. 3qrs. 14lbs. of sugar for £3. 5s. 4d. per cwt., and gained £27. 6s. 10½d. by the whole, what did it cost him? Ans. £125. 15s. 7½d.

RULE OF THREE INVERSE.

Inverse proportion is when *more* requires *less*, and *less* requires *more;* that is, *two* of the numbers *increase* in the same proportion as the *other* two *decrease.*

RULE.—State the question as in the Rule of Three Direct, then multiply the *first* and *second* terms together, and *divide* their product by the third; the quotient will be the answer to the question, and will bear the same proportion to the *second* as the *first* does to the third.*

EXAMPLE.

If 21 men can dig a field in 6 days, how many days will it take 7 men to do the same?

```
         men   days   men   days
† As     21  :  6  ::  7  :  18
               21
             7)126
        Ans.   18 days.
```

---

* If the term of *demand* be placed *first*, that of the same name *second*, and the remaining one *third*, they will be in *direct* proportion.

† As 7 men : 21 men :: 6 days : 18 days.

EXERCISES.

1. If 12 men can reap a field in 18 days, in what time would 36 men do it? Ans. 6 days.

2. If 7 men dig a field in 18 days, in what time would 21 men dig it? Ans. 6 days.

3. If I lend my friend £20. for 6 weeks, how long ought he to lend me £10., to requite my kindness? Ans. 12 weeks.

4. If 12 inches long require 12 inches broad to make a square foot, what length will 8 inches broad require? Ans. 18 inches.

5. Suppose 12 yards of silk, 18 inches wide, will make a lady a gown, what length of stuff would she require to make one, that is 24 inches wide? Ans. 9 yards.

6. A person wanting to cut exactly a square yard of mahogany from a plank 2 feet broad, what length must he have? Ans. $4\frac{1}{2}$ feet.

7. How many yards of carpet, 3 feet wide, will cover a room 20 feet long and 15 feet wide? Ans. $33\frac{1}{3}$ yards.

8. If 220 yards in length and 22 in breadth make an acre, what must be the length when the breadth is 16 yards? Ans. $302\frac{1}{2}$ yards.

9. If for a certain sum I have 3cwt. 1qr. carried 15 miles, how far can I have 1cwt. 2qr. 14lb. carried for the same money? Ans. 30 miles.

10. If a person, by travelling 12 hours a day, can perform a certain journey in 6 days, how long would he be on the road, if he travelled only 8 hours a day? Ans. 9 days.

11. What length of print, 5qrs. wide, will it require to hang a bed that takes 45yds., 3qrs. wide? Ans. 27yds.

12. When wheat is £2. a quarter, if a sixpenny loaf weigh $3\frac{1}{2}$lb., what should it weigh when wheat is sold for £3. 10s.? Ans. 2lb.

13. How much money at $3\frac{1}{2}$ per cent. will produce as much interest as £500. at $4\frac{1}{2}$ per cent.? Ans. £642. 17s. $1\frac{1}{2}$d.—$\frac{6}{7}$.

14. If the penny loaf weigh 7oz. 14dr. when wheat is £3. per quarter, what would it weigh in the year 1800, when wheat was 8 guineas a quarter? Ans. 2oz. 13dr.

15. What quantity of water must be added to 100 gallons of double rum, at a guinea per gallon, to reduce the price to 17s. 6d. per gallon? Ans. 20 gallons.

DOUBLE RULE OF THREE

Is the method of solving at one operation, such questions as by the common Rule of Three would require two or more statings.

RULE I.—1st. Write down that number which is of the *same name* as the number sought, for the middle term.

2nd. Take any two of the other numbers, being of the *same* name, and consider whether *more* or *less* is required; if *more*, place the *less* number *first*, and the greater one *third;* but if *less*, place the *greater* number *first*, and *less* one *last*.

3rd. Multiply the middle term by the *product* of the *two last*, and divide by the *product* of the *two first*.

RULE II.—1st. Let the principal *cause* of loss or gain increase or decrease, action or passion, be put in the *first* place.

2nd. Let that which betokeneth *time*, distance of *place*, and the like, be in the *second* place, and the remaining one in the third.

3rd. Place the other terms under their like in the supposition.

4th. If the *blank* falls under the *third term*, multiply the *first* and *second* terms for a divisor, and the *other three* for a dividend, and the quotient will be the answer; but if the *blank* falls under the *first* or *second* term, multiply the *third* and *fourth* terms for a divisor, and the *other three* for a dividend, the quotient will be the answer.

EXAMPLES.

If 6 horses plough 20 acres in 2 weeks, how many acres will 15 horses plough in 5 weeks?

```
         horses   acres   horses
      As   6   :   20   ::  15
   weeks   2                 5 weeks
          --                --
          12                75
                            20
                        12)1500
                       Ans. 125 acres.
```

The same by the 2nd Rule.

horses		*weeks*		*acres*
6	:	2	::	20
15	:	5	::	—

Here the blank falls under the *third* term.

∴ $6 \times 2 = 12$ the divisor.

And $15 \times 5 \times 20 = 1500$ the dividend.

12)1500

125 Ans.

The same by two statings.

	horses		*acres*		*horses*
As	6	:	20	::	15

15

6)300

50 acres.

That is, 15 horses plough 50 acres in the time 6 horses plough 20 acres.

		weeks		*acres*		*weeks*
∴	As	2	:	50	::	5

5

2)250

125 acres Ans.

EXERCISES.

1. If 6 men mow 18 acres in 5 days, how many acres will 10 men mow in 12 days? Ans. 72 acres.

2. If 10 men can reap 72 acres in 12 days, how many acres will 6 men reap in 5 days? Ans. 18 acres.

3. If the carriage of 4cwt. 42 miles be 7s. 6d., what will 12cwt. cost for 21 miles? Ans. 11s. 3d.

4. If the carriage of 12cwt. 21 miles cost 11s. 3d., what will the carriage of 4cwt. 42 miles cost? Ans. 7s. 6d.

5. If the wages of 18 carpenters for 9 days be £28. 7s., what will be the wages of 12 for 20 days? Ans. £42.

6. Suppose 6 men mow 18 acres in 5 days, how many men will it require to mow 72 acres in 12 days? Ans. 10.

7. If 18 gallons of ale serve a family of 6 persons 20 days, how much would 8 such persons consume in 50 day?
Ans. 60 gallons.

8. If £120. gains £4. 16s. in a year, how much would £520. gain in 5 years? Ans. £104.

9. If a school containing 40 writers, cut up 200 quills in 3 weeks, how many would serve 30 writers for a year?
Ans. 2600.

10. If 242 horses consume 211qrs. 6bus. of corn in 42 days, how much will serve 12 horses for 7 days?
Ans. 1qr. 6bus.

11. If with a capital of £1000. a tradesman gains £100. in seven months, in what time will he gain £60. 10s. with a capital of £385.? Ans. 11 months.

12. In what time will £700. at 4 per cent. yield as much interest as £500. for 220 days, at 5 per cent.?
Ans. $196\frac{3}{7}$ days.

PRACTICE

Is the method of calculating the value of goods, by taking aliquot or even parts, instead of computing by compound multiplication, or the rule of three.

TABLE OF ALIQUOT OR EVEN PARTS.

Of a Pound.

s.	d.		
10	0	is	$\frac{1}{2}$
6	8	..	$\frac{1}{3}$
5	0	..	$\frac{1}{4}$
4	0	..	$\frac{1}{5}$
3	4	..	$\frac{1}{6}$
2	6	..	$\frac{1}{8}$
2	0	..	$\frac{1}{10}$
1	8	..	$\frac{1}{12}$

Of a Shilling.

d.		
6	is	$\frac{1}{2}$
4	..	$\frac{1}{3}$
3	..	$\frac{1}{4}$
2	..	$\frac{1}{6}$
$1\frac{1}{2}$	..	$\frac{1}{8}$
1	..	$\frac{1}{12}$

Of a Penny.

2 farthings	$\frac{1}{2}$
1 farthing	$\frac{1}{4}$

Of a Ton.

cwt.		
10	is	$\frac{1}{2}$
5	..	$\frac{1}{4}$
4	..	$\frac{1}{5}$
$2\frac{1}{2}$	..	$\frac{1}{8}$
2	..	$\frac{1}{10}$

Of a Cwt.

qr.		lb.		
2	or	56	is	$\frac{1}{2}$
1	or	28	..	$\frac{1}{4}$

Of a Quarter.

lb.		
14	is	$\frac{1}{2}$
7	..	$\frac{1}{4}$
4	..	$\frac{1}{7}$
$3\frac{1}{2}$	..	$\frac{1}{8}$

RULE I.—*When the price is less than a penny.*

Divide the given number by the aliquot parts that are in a penny; then by 12 and 20 for the answer.

RULE II.—*When the price is less than a shilling.*

Divide the given number by the aliquot part or parts of a shilling, add them together, and divide by 20 for the answer.

(1)

$\frac{1}{2}$	$\frac{1}{2}$	1234 at $\frac{1}{2}$
	12	617
	2,0	5,1 5
	£	2 11 5

(2)

$\frac{1}{2}$	$\frac{1}{2}$	4321 at $\frac{3}{4}$
$\frac{1}{4}$	$\frac{1}{2}$	2160$\frac{1}{2}$
		1080$\frac{1}{4}$
	12	3240$\frac{3}{4}$
	2,0	27,0 0$\frac{3}{4}$
	£	13 10 0$\frac{3}{4}$

(3)

1	$\frac{1}{12}$	1235 at 1$\frac{1}{4}$d.
$\frac{1}{4}$	$\frac{1}{4}$	102 11
		25 8$\frac{3}{4}$
	2,0	12,8 7$\frac{3}{4}$
	£	6 8 7$\frac{3}{4}$

(4)

4	$\frac{1}{3}$	1234 at 5$\frac{1}{2}$d.
1	$\frac{1}{4}$	411 4
$\frac{1}{2}$	$\frac{1}{2}$	102 10
		51 5
	2,0	56,5 7
	£	28 5 7

(5) 1235 at $\frac{1}{2}$d. Ans. £2 11 5$\frac{1}{2}$	(11) 2002 at 2d. Ans. £16 13 8	(17) 754 at 4d. Ans. £12 11 4
(6) 1234 at $\frac{3}{4}$d. Ans. £3 17 1$\frac{1}{2}$	(12) 684 at 2$\frac{1}{2}$d. Ans. £7 2 6	(18) 855 at 4$\frac{1}{4}$d. Ans. £15 2 9$\frac{3}{4}$
(7) 1324 at 1d. Ans. £5 10 4	(13) 685 at 2$\frac{3}{4}$d. Ans. £7 16 11$\frac{3}{4}$	(19) 1076 at 4$\frac{3}{4}$d. Ans. £21 5 11
(8) 1612 at 1$\frac{1}{4}$d. Ans. £8 7 11	(14) 688 at 3d. Ans. £8 12 0	(20) 1124 at 5d. Ans. £23 8 4
(9) 2000 at 1$\frac{1}{2}$d. Ans. £12 10 0	(15) 568 at 3$\frac{1}{4}$d. Ans. £7 13 10	(21) 1134 at 5$\frac{1}{4}$d. Ans. £24 16 1$\frac{1}{2}$
(10) 2001 at 1$\frac{3}{4}$d. Ans. £14 11 9$\frac{3}{4}$	(16) 956 at 3$\frac{3}{4}$d. Ans. £14 18 9	(22) 1153 at 5$\frac{3}{4}$d. Ans. £27 12 5$\frac{3}{4}$

(23)
1156 at 6d.
Ans. £28 18 0

(24)
2267 at $6\frac{1}{4}$d.
Ans. £59 0 $8\frac{3}{4}$

(25)
4376 at $6\frac{3}{4}$d.
Ans. £123 1 6

(26)
3671 at 7d.
Ans. £107 1 5

(27)
2572 at $7\frac{1}{4}$d.
Ans. £77 13 11

(28)
2624 at $7\frac{1}{2}$d.
Ans. £82 0 0

(29)
3456 at $7\frac{3}{4}$d.
Ans. £111 12 0

(30)
3457 at 8d.
Ans. £115 4 8

(31)
4569 at $8\frac{1}{2}$d.
Ans. £161 16 $4\frac{1}{2}$

(32)
4670 at $8\frac{3}{4}$d.
Ans. £170 5 $2\frac{1}{2}$

(33)
4681 at 9d.
Ans. £175 10 9

(34)
4782 at $9\frac{3}{4}$d.
Ans. £194 5 $4\frac{1}{2}$

(35)
4635 at 10d.
Ans. £193 2 6

(36)
2560 at $10\frac{1}{4}$d.
Ans. £109 6 8

(37)
2112 at $10\frac{3}{4}$d.
Ans. £94 12 0

(38)
7211 at 11d.
Ans. £330 10 1

(39)
7654 at $11\frac{1}{4}$d.
Ans. £358 15 $7\frac{1}{2}$

(40)
9871 at $11\frac{3}{4}$d.
Ans. £483 5 $4\frac{1}{4}$

RULE III.—*When the price is more than one shilling, and less than two.*

Take the aliquot part or parts for so much of the given price as is more than a shilling, which add to the given quantity, and divide by 20 for the answer.

(1)

$\frac{1}{4}$	$\frac{1}{48}$	1827 at 1s. $0\frac{1}{4}$d.
		38 $0\frac{3}{4}$
	2,0	186,5 $0\frac{3}{4}$
	£	93 5 $0\frac{3}{4}$

(2)

$\frac{1}{2}$	$\frac{1}{24}$	1827 at 1s. $0\frac{1}{2}$d.
		76 $1\frac{1}{2}$
	2,0	190,3 $1\frac{1}{2}$
	£	95 3 $1\frac{1}{2}$

(3)

$\frac{3}{4}$	$\frac{1}{16}$	1827 at 1s. $0\frac{3}{4}$d.
		114 $2\frac{1}{4}$
	2,0	194,1 $2\frac{1}{4}$
	£	97 1 $2\frac{1}{4}$

(4)

2d.	$\frac{1}{6}$	1060 at 1s. 2d.
		176 8
	2,0	123,6 8
	£	61 16 8

(5) 1002 at 1s. 1½d. Ans. £56 7 3

(6) 1003 at 1s. 1¾d. Ans. £57 9 3¼

(7) 1004 at 1s. 2d. Ans. £58 11 4

(8) 1005 at 1s. 2¼d. Ans. £59 13 5¼

(9) 1110 at 1s. 2½d. Ans. £67 1 3

(10) 1234 at 1s. 2¾d. Ans. £75 16 9½

(11) 2132 at 1s. 3d. Ans. £133 5 0

(12) 2222 at 1s. 3¼d. Ans. £141 3 9½

(13) 3516 at 1s. 3½d. Ans. £227 1 6

(14) 2772 at 1s. 3¾d. Ans. £181 18 3

(15) 6721 at 1s. 4d. Ans. £448 1 4

(16) 5427 at 1s. 4¼d. Ans. £367 9 0¾

(17) 5271 at 1s. 4½d. Ans. £362 7 7½

(18) 5523 at 1s. 4¾d. Ans. £385 9 2¼

(19) 2345 at 1s. 5d. Ans. £166 2 1

(20) 4566 at 1s. 5½d. Ans. £332 18 9

(21) 5678 at 1s. 5¾d. Ans. £419 18 8½

(22) 3456 at 1s. 6¼d. Ans. £262 16 0

(23) 826 at 1s. 6¾d. Ans. £64 10 7½

(24) 697 at 1s. 7½d. Ans. £56 12 7½

(25) 1031 at 1s. 8d. Ans. £85 18 4

(26) 1234 at 1s. 8¾d. Ans. £106 13 9½

(27) 1562 at 1s. 9¼d. Ans. £138 6 0½

(28) 6342 at 1s. 10d. Ans. £581 7 0

(29) 3345 at 1s. 11d. Ans. £320 11 3

(30) 3456 at 1s. 11¼d. Ans. £334 16 0

(31) 1111 at 1s. 11¾d. Ans. £109 18 10¼

RULE IV.—When the price consists of any *even* number of shillings under 20, multiply the quantity by *half* the number, doubling the first figure of the *product* for shillings, and the rest will be pounds.

RULE V.—When the price is an *odd* number of shillings, multiply the quantity by the price, and divide the product by 20.*

* When the price is in *one* name, the quantity being multiplied by *it*, gives the answer in *that* name.

(1)

1264 at 4s.
2
£ 252 16

(2)

1264 at 5s.
5
2,0)632,0
£ 316

(3) 3206 at 2s. Ans. £320 12	(9) 1111 at 8s. Ans. £444 8	(15) 276 at 14s. Ans. £193 4
(4) 3541 at 3s. Ans. £531 3	(10) 564 at 9s. Ans. £253 16	(16) 546 at 15s. Ans. £409 10
(5) 5642 at 4s. Ans. £1128 8	(11) 489 at 10s. Ans. £244 10	(17) 321 at 16s. Ans. £256 16
(6) 6009 at 5s. Ans. £1502 5	(12) 345 at 11s. Ans. £189 15	(18) 457 at 17s. Ans. £388 9
(7) 6009 at 6s. Ans. £1802 14	(13) 267 at 12s. Ans. £160 4	(19) 139 at 18s. Ans. £125 2
(8) 6600 at 7s. Ans. £2310 0	(14) 189 at 13s. Ans. £122 17	(20) 777 at 19s. Ans. £738 3

RULE VI.—1st. When the price is the *aliquot part* of a pound, divide the quantity by *that part*, and the *quotient* will be pounds.

2ndly.—When the price is *shillings* and *pence*, and they *not* an aliquot part, multiply by the shillings, and take parts for the pence as before.

(1)

3s. 4d.	$\frac{1}{6}$	2054 at 3s. 4d.
		£ 342 6 8

(2)

6d.	$\frac{1}{2}$	1346 at 7s. $10\frac{1}{2}$d.
		7
		9422
3d.	$\frac{1}{2}$	673
$1\frac{1}{2}$d.	$\frac{1}{2}$	336 6
		168 3
	2,0	1059,9 9
		£529 19 9

(3)
1234 at 10s.
Ans. £617.

(4)
1234 at 5s.
Ans. £308 10 0.

(5)
1234 at 6s. 8d.
Ans. £411 6 8.

(6)
1000 at 3s. 4d.
Ans. £166 13 4.

(7)
1342 at 2s. 6d.
Ans. £167 15 0.

(8)
2345 at 1s. 8d.
Ans. £195 8 4.

(9)
2350 at 4s. 6d.
Ans. £528 15 0.

(10)
1234 at 17s. 6d.
Ans. £1079 15 0.

(11)
1324 at 16s. 8d.
Ans. £1103 6 8.

(12)
1432 at 18s. 9d.
Ans. £1342 10 0.

(13)
1423 at 19s. 4½d.
Ans. £1378 10 7½.

(14)
1431 at 8s. 9d.
Ans. £626 1 3.

(15)
4131 at 9s. 4½d.
Ans. £1936 8 1½.

(16)
789 at 5s. 9d.
Ans. £226 16 9.

(17)
6741 at 3s. 10d.
Ans. £1292 0 6.

(18)
3222 at 7s. 7d.
Ans. £1221 13 6.

(19)
1823 at 11s. 11d.
Ans. £1086 4 1.

(20)
3748 at 15s. 2½d.
Ans. £2850 0 10.

(21)
7411 at 13s. 3¾d.
Ans. £4932 18 11¼.

(22)
3234 at 14s. 1¼d.
Ans. £2280 12 10½.

(23)
1234 at 12s. 10d.
Ans. £791 16 4.

(24)
896 at 3s. 9¾d.
Ans. £170 16 0.

(25)
538½ at 6s. 8¾d.
Ans. £180 12 5¼.

(26)
432½ at 9s. 11½d.
Ans. £215 6 11¾.

RULE VII.—When the price is pounds, shillings, and pence, multiply the *quantity* by the pounds, and take parts for the rest; their sum will be the answer.

(1)

2s.	$\frac{1}{10}$	567 at £2	3	7½
		2		
		1134		
1s.	$\frac{1}{2}$	56	14	
6d.	$\frac{1}{2}$	28	7	
1½d.	$\frac{1}{4}$	14	3	6
		3	10	10½
		£ 1236	15	4½

(2)

10s.	$\frac{1}{2}$	786 at £1	17	11½
5s.	$\frac{1}{2}$	393		
2s. 6d.	$\frac{1}{2}$	196	10	
5d.	$\frac{1}{6}$	98	5	
½d.	$\frac{1}{10}$	16	7	6
		1	12	9
		£ 1491	15	3

(3)
2345 at £1. 7s. 6d.
Ans. £3224 7 6.

(4)
1232 at £2. 6s. 8d.
Ans. £2874 13 4.

(5)
1321 at £3. 3s. 4d.
Ans. £4183 3 4.

(6)
961 at £4. 2s. 6d.
Ans. £3964 2 6.

(7)
872 at £5. 12s. 6d.
Ans. £4905.

(8)
824 at £6. 16s. 8d.
Ans. £5630 13 4.

(9)
825 at £7. 13s. 4d.
Ans. £6325.

(10)
483 at £1. 17s. 6d.
Ans. £905 12 6.

(11)
438 at £1. 15s. 10d.
Ans. £784 15 0.

(12)
384 at £1. 9s. 4½d.
Ans. £564.

(13)
348 at £1. 14s. 4½d.
Ans. £598 2 6.

(14)
176 at £11. 19s. 8¼d.
Ans. £2109 5 0.

(15)
286 at £17. 18s. 9d.
Ans. £5130 2 6.

(16)
2579 at £1. 16s. 2½d.
Ans. £4669 1 3½d.

(17)
3210 at £1. 18s. 6¾d.
Ans. £6189 5 7½.

(18)
1234 at £1. 19s. 6d.
Ans. £2437 3 0.

(19)
237 at £2. 14s. 3½d.
Ans. £643 7 1½.

(20)
325 at £3. 11s. 10¾d.
Ans. £1168 6 1¾.

(21)
846 at 19s. 4½d.
Ans. £819 11 3.

(22)
632 at £1. 19s. 8¼d.
Ans. £1254 2 6.

Rule VIII.—When both the *price* and *quantity* are of several denominations, multiply the price by the *highest* denomination, and take parts for the *lower* denominations; these added together will be the answer.

1. At £7. 12s. 6d. per cwt. what is the value of 21cwt. 3qrs. 21lbs. of hops?

		£.	s.	d.	
2qr.	½	7	12	6	
				3	× 7 = 21
		22	17	6	
				7	
		160	2	6	
1qr.	½	3	16	3	
14lb	½	1	18	1½	
7lb.	½		19	0¾	
			9	6¼	
	£	167	5	5½	Ans.

2. What is the value of 15gals. 2qts. 1pint of rum, at 18s. 6d. per gallon? Ans. £14. 9s. 0¾d.

3. At £3. 15s. 6d. per cwt., what is the value of 25cwt. 2qrs. 14lb.? Ans. £96. 14s. 8¼d.

4. At £2. 10s. 6d. per cwt., what is the value of 27cwt. 3qrs. 7lb. of cheese? Ans. £70. 4s. 6¼d.

5. What is the value of 7cwt. 3qrs. 10½lb. of butter, at £2. 18s. 6d. per cwt.? Ans. £22. 18s. 10d.

6. What is the value of a stack of hay, weighing 37tons 7cwt. 2qrs., at £4. 16s. 6d. per ton? Ans. £180. 6s. 8¼d.

7. If I buy superfine blue cloth at £1. 4s. per yard, what shall I have to pay for 6yds. 3qrs. 2nls.? Ans. £8. 5s. 0d.

8. At £4. 16s. 9d. per cwt., what is the worth of 11cwt. 0qr. 14lb. of double refined sugar? Ans. £53. 16s. 4d.

9. Sold a field of turnips, which measured 13a. 2r. 30p. for £12. 15s. 6d. per acre, what had I to receive? Ans. £174. 17s. 1¾d.

10. What is the value of 9qrs. 6bus. of beans, at £2. 3s. 6d. per quarter? Ans. £21. 4s. $1\frac{1}{2}$d.

11. What is the value of 13cwt. 3qrs. 16lbs. of hops, at £4. 12s. 6d. per cwt.? Ans. £64. 5s. 1d.

12. What is the value of 7cwt. 3qrs. 12lbs. of clover seed, at £2. 16s. 6d. per cwt.? Ans. £22. 3s. $10\frac{3}{4}$d.

13. What is the value of 5cwt. 2qrs. 10lbs. of coffee, at £3. 15s. per cwt.? Ans. £20. 19s. $2\frac{1}{4}$d.

14. If the repairing of a road cost £20. 12s. a mile, what would be the expense of repairing 9mls. 5fur. 25pls.? Ans. £199. 17s. $8\frac{1}{4}$d.

15. If 1cwt. cost £2. 10s. 10d., what will 29cwt. 2qrs. 13lbs. cost at the same rate? Ans. £75. 5s. $5\frac{1}{2}$d.

16. What is the value of 132tons, 15cwt. 3qrs. 21lbs. at £45. 6s. 8d. per ton? Ans. £6020. 2s. 6d.

17. 121oz. 16dwt. 18gr. was sold for 1s. $10\frac{1}{2}$d. per oz., what did it amount to? Ans. £11. 8s. 5d.

18. What is the value of 4hhd. 1kil. 1fir. 5gals. of ale, at £2. 5s. per hogshead? Ans. £10. 6s. 8d.

INTEREST.

Simple Interest is the *profit* arising from the lending of a sum of money for a certain time.

The principal is the money lent.

The *rate per cent.* is the sum agreed on between the borrower and the lender, to be paid for every £100. for the use of the principal *one year*; that is, 3 per cent. means that £3. interest shall be paid for every £100. borrowed; $3\frac{1}{2}$ per cent. £3. 10s.; 5 per cent. £5. &c.

Amount is the *interest* ADDED to the *principal.*

Interest is also applied to commission, brokerage, purchasing of stock, and insurance.

CASE I.—When the interest is wanted for one year.

RULE I.—Multiply the *principal* by the *rate* per cent. and divide the *product* by 100, the quotient is the *interest* required.*

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* This rule is in effect the same as the *Rule of Three*, a £100. being the *first* term, the *rate second*, and *principal third*.

CASE II.—When the *interest* is required for several years.

RULE 2.—Multiply the *principal* by the *rate*, and that product by the *time*, which last divided by 100 gives the *interest*.

EXAMPLE.

1. What is the interest of £425. 10s. 6d. for a year, at $4\frac{1}{2}$ per cent.?

	£.	s.	d.
$\frac{1}{2}$ | $\frac{1}{2}$ |	425	10	6
			$4\frac{1}{2}$
	1702	2	0
	212	15	3
*	£19,14	17	3
	20		
shillings	2,97		
	12		
pence	11,67		
	4		
farth.	2,68		

or thus:—†

		£.	s.	d.
£4.	$\frac{1}{25}$	425	10	6
10s.	$\frac{1}{8}$	17	0	5
		2	2	$6\frac{1}{2}$
		£19	2	$11\frac{1}{2}$

£19. 2s. $11\frac{1}{2}$d. Ans.

2. What is the interest of £350. 10s. 0d., at 4 per cent. for a year? Ans. £14. 0s. $4\frac{3}{4}$d.

3. What is the interest of £150. for a year, at 5 per cent.? Ans. £7. 10s. 0d.

4. What is the interest of £20. 12s. 6d., at 5 per cent., for a year? Ans. £1. 0s. $7\frac{1}{2}$d.

5. What is the interest of £252. 10s. 6d., at $4\frac{1}{2}$ per cent., for a year? Ans. £11. 7s. $3\frac{1}{4}$d.

6. What is the interest of £800., at $4\frac{3}{4}$ per cent., for a year? Ans. £38.

7. What is the interest of £315. 15s. 0d., for 2 years, at 5 per cent.? Ans. £31. 11s. 6d.

---

* The 100 is not put down, as dividing by it is only cutting off two figures to the right hand of the dividend.

† $4\frac{1}{2}$ per cent. is the same as £4. 10s. per cent., £4. being the 25th of a £100., and 10s. the 8th of £4.

8. If I lend 50 guineas, at 5 per cent., what interest shall I receive for 4 years? Ans. £10. 10s.

9. What is the interest of £56. 10s. 6d., at 3½ per cent., for 10 years? Ans. £19. 15s. 8d.

10. What is the interest of £1000., at 4½ per cent., for 4½ years? Ans. £202. 10s. 0d.

11. What is the interest of £78. 15s., at 2¾ per cent., for 2 years and 6 months?* Ans. £5. 8s. 3¼d.

12. What is the interest of £576. 15s. 6d., at 5 per cent., for a year and 7 months? Ans. £45. 13s. 2½d.

13. What is the interest of £420. 10s. 0d., for 3 months, at 4½ per cent.? Ans. £4. 14s. 7¼d.

14. What is the amount of £30., at 5 per cent., for 20 years? Ans. £60.

15. What is the amount of £500. for 3 years and 8 months, at 5 per cent.? Ans. £591. 13s. 4d.

16. If a gentleman buy a farm for £3320., how should he let it, to pay him 5 per cent.? Ans. £166. per annum.

17. What is the interest of £97. 10s. 6d., at 3⅓ per cent., for 9 months? Ans. £2. 8s. 9d.

18. What is the interest of £136. 12s. 6d., at 3 per cent., for 10 months? Ans. £3. 8s. 3¾d.

19. What will £364. 15. 6d. amount to in a year and 2 months, at 3⅓ per cent.? Ans. £378. 19s. 2½d.

20. What is the amount of £68. 10s. 6d., at 4½ per cent., for a year and 11 months? Ans. £74. 8s. 8¼d.

CASE III.—When the interest is required for any number of weeks.

RULE.—Find the interest of the given sum for *one year;* then, as 52 weeks are to that interest, so are the weeks given to the interest required.

CASE IV.—When the interest is required for any number of days.

RULE.—Find the interest of the given sum for a year; then, as 365 days are to that interest, so are the days given to the interest required.

* To find the interest for months, take aliquot parts of the year, or multiply the interest for a year by the months, and divide by 12.

EXAMPLE.

1. To find the interest of £256. 10s., at 5 per cent., for 3 years and 25 weeks?

£.	s.
256	10
	5
12,82	10
20	
16,50	
12	
6,00	

	£.	s.	d.	
	12	16	6	year
			3	
3 years	38	9	6	
25 weeks	6	3	$3\frac{3}{4}$	
Ans.	44	12	$9\frac{3}{4}$	

	wks.		£.	s.	d.		wks.
As	52	:	12	16	6	::	25

20
256
12
3078
25
15390
6156
52)76950 | 12 | $1479\frac{3}{4}$
52 | 2,0 | 12,3 $3\frac{3}{4}$
249 | | 6 3 $3\frac{3}{4}$
208
415
364
510
468
42
4
52)168(3 farthings.
156
12 remains.

2. Required the interest of £250. 10s. 0d., for 16 weeks, at 5 per cent.? Ans. £3. 17s. $0\frac{3}{4}$d.

3. Find the interest of £500. for 20 weeks, at $4\frac{1}{2}$ per cent.? Ans. £8. 13s. $0\frac{3}{4}$d.

4. What is the interest of £50. for a year and 6 weeks, at 5 per cent.? Ans. £2. 15s. 9d.

5. What is the interest of £580., for 2 years and 25 weeks, at $4\frac{1}{2}$ per cent.? Ans. £64. 14s. $11\frac{1}{2}$d.

6. Required the interest of £750. 10s. 0d., for three years and ten weeks, at $3\frac{1}{2}$ per cent.? Ans. £83. 17s. $0\frac{1}{4}$d.

7. What is the interest of £500., for 60 days, at 5 per cent. per annum? Ans. £4. 2s. $2\frac{1}{4}$d.

8. Find the interest of £70. 16s. 6d., at 5 per cent., for 30 days? Ans. 5s. $9\frac{3}{4}$d.

9. Required the interest of £2050. for a year and 15 days, at $4\frac{1}{2}$ per cent.? Ans. £96. 0s. $9\frac{3}{4}$d.

10. What is the amount of £515., at $4\frac{1}{2}$ per cent., for 7 years and 73 days? Ans. £681. 17s. $2\frac{1}{4}$d.

11. What is the interest of 20 guineas, at 5 per cent., for 7 years and 120 days? Ans. £7. 13s. $10\frac{3}{4}$d.

12. Required the interest of £1264. 11s. 8d. for 72 days, at 4 per cent.? Ans. £9. 19s. $6\frac{1}{2}$d.

COMMISSION AND BROKERAGE.

CASE V.—Is an allowance of so much per cent. from merchants to bankers, brokers, agents, &c. for transacting business for them.

RULE.—If the commission be *above one per cent.* multiply by the *rate*, and divide by 100, as in case I. If *under one per cent.* divide the *given sum* by 100, the quotient is equal to one per cent., of which take aliquot parts with the *rate*.

EXAMPLE.

1. If a broker sells goods to the amount of £976. 12s. 6d., at 6s. 6d. per cent., what is due to him?

1,00 | 9,76 12 6	5s.	$\frac{1}{4}$	9 15 $3\frac{3}{4}$	
9 15 $3\frac{3}{4}$ = £1. per cent.	1s.	$\frac{1}{5}$	2 8 $9\frac{3}{4}$	
	6d.	$\frac{1}{2}$	9 9	
			4 $10\frac{1}{2}$	
		£	3 3 $5\frac{1}{4}$	Ans.

2. What is the commission on £320. 10s. 0d., at $1\frac{1}{2}$ per cent.? Ans. £4. 16s. $1\frac{3}{4}$d.

3. What is the brokerage on £764. 10s. 0d., at $\frac{1}{3}$ per cent.? Ans. £2. 10s. $11\frac{1}{2}$d.

4. What is the commission on £1000. at $3\frac{3}{4}$ per cent.? Ans. £37. 10s.

5. What is the commission on £2450. 18s. 0d., at 1s. 6d. per cent.? Ans. £1. 16s. 9d.

6. What is the commission on £998. 12s. 6d., at $\frac{5}{8}$ per cent.? Ans. £6. 4s. 9$\frac{3}{4}$d.

7. My factor has bought goods on my account to the value of £1100. 16s. 0d., what is he entitled to at 3s. 6d. per cent.? Ans. £1. 18s. 6d.

8. What is due to my agent for selling on my account goods to the amount of £1720. 10s. 0d., at 3s. 3d. per cent.? Ans. £2. 15s. 10$\frac{1}{2}$d.

9. If I am allowed 3$\frac{1}{4}$ per cent., what is my commission on £273. 4s. 6d.? Ans. £8. 17s. 7d.

10. Sold 325 quarters of wheat, at £3. per quarter, what is my commission at 1 per cent.? Ans. £9. 15s. 0d.

11. A banker discounts bills for a merchant to the amount of £7860. 16s. 10d., what is his commission at $\frac{3}{4}$ per cent.? Ans. £58. 19s. 1$\frac{1}{2}$d.

12. Shipped by order of my employer, goods to the amount of £972. 12s. 6d., incidental charges £7. 12s. 0d. commission on the whole $\frac{7}{8}$ or 17s. 6d. per cent., what is the amount of the invoice? Ans. £988. 16s. 0$\frac{1}{4}$d.

PURCHASING OF STOCKS.

CASE VI.—Stock is the name given to the debt of government created by loans; As also to the property of trading companies. The rules for buying or selling shares in which are as follow:

RULE 1.—If the given sum be *above par*, (i. e. above 100) multiply the sum to be purchased by the excess above 100, divide the product by 100, and add the quotient to the given sum.

2nd. If the given sum be *under par*, multiply it by the

The mode of supplying the exigence of the state by borrowing money, and levying taxes for the payment of the interest, is called the *Funding System;* and the money thus borrowed constitutes the *National Debt;* the public creditors being the stock-holders. Persons buying or selling stock, usually do it through the medium of a broker, who receives 2s. 6d. per cent. on all he buys and sells.

The capitals of joint stock companies, such as the Bank of England, East-India Company, &c. are denominated stock, and is transferable like the government funds.

price or rate per cent. and divide by 100, the quotient will be the answer. Or,

3rdly.—Instead of multiplying, take parts for the whole price.

EXAMPLES.

1. What is the purchase of £791. 15s. 0d. East-India stock, at $107\frac{1}{2}$ per cent.?

	£.	s.	d.	
$\frac{1}{2}$ | $\frac{1}{2}$ |	798	15	0	
			$7\frac{1}{2}$	
	5591	5	0	
	399	7	6	
1,00)	59,90	12	6	
	59	18	$1\frac{1}{2}$	
	798	15	0	
£	858	13	$1\frac{1}{2}$	Ans.

or thus:—

		£.	s.	d.	
£5.	$\frac{1}{20}$	798	15	0	
£$2\frac{1}{2}$.	$\frac{1}{2}$	39	18	9	
		19	19	$4\frac{1}{2}$	
	£	858	13	$1\frac{1}{2}$	Ans.

2. What is the value of £400. stock, at £120. 5s. per cent.? Ans. £481.

3. What is the purchase of £1340. 12s. East-India stock, at $110\frac{1}{4}$ per cent.? Ans. £1478. 0s. $2\frac{3}{4}$d.

4. Sold £2560. 10s. 6d. India stock, at $106\frac{5}{8}$ per cent. what did I receive? Ans. £2730. 3s. $2\frac{1}{4}$d.

5. What is the purchase of £860. 10s. consols, at $84\frac{1}{4}$ per cent.? Ans. £724. 19s. 5d.

6. At $96\frac{3}{8}$ per cent. what is the purchase of £550. 10s. 6d. bank annuities? Ans. £530. 11s. $4\frac{1}{4}$d.

7. What is the cost of £800. three per cent. consols at $72\frac{5}{8}$ per cent., brokerage 2s. 6d. per cent.?* Ans. £582.

---

* The brokerage, $\frac{1}{8}$ per cent., must be added to the price of £100. stock, when a stock is bought, and subtracted when it is sold.

Here £$72\frac{5}{8}$ =	72	12	6
Brokerage =		2	6

∴ As 1,00 : 72 15 0 :: 8,00

Premium is the money paid for insuring.

Policy is the paper containing the contract, the stamp duty on which is 2s. 6d. per cent. within the United Kingdom, but 5s. on Foreign policies.

Commission is an allowance of, usually, 10s. per cent. to Agents.

If the rate be given in guineas, work as with pounds, and add a twentieth of the result, the sum will be the answer.

8. What cost £760. stock in the three per cents. bought at $64\frac{1}{4}$ per cent., brokerage $\frac{1}{8}$ per cent.? Ans. £489. 5s. 0d.

9. What cost £800. bank stock, at $200\frac{1}{2}$ per cent., brokerage $\frac{1}{8}$ per cent.? Ans. £1605.

10. How much stock in the three per cents. at $72\frac{5}{8}$ can be bought for £582., brokerage $\frac{1}{8}$ per cent.? Ans. £800. stock.

11. How much stock can be purchased at $62\frac{1}{2}$ per cent. with £250. 10s., brokerage $\frac{1}{8}$ per cent.? Ans. £400. stock.

12. What quantity of stock, at $80\frac{1}{8}$ per cent. must be sold to produce £500., brokerage $\frac{1}{8}$ per cent.? Ans. £625.

INSURANCE.

CASE VII.—Insurance is a contract of indemnity, by which one party engages, for a stipulated sum, to indemnify another party against risk to which he is liable.

RULE.—Multiply the sum to be insured by the rate, and divide the product by 100, the quotient is the *premium* to be paid.

EXAMPLE.

1. What will be the expense of insuring £840. at 7 per cent. including policy, 2s. 6d. per cent. and 10s. per cent. commission?

				£.	s.	d.	
2s. 6d.	$\frac{1}{8}$	840					
10s.	$\frac{1}{2}$	7					
		58,80	=	58	16	0	*premium.*
1,00		1,05	=	1	1	0	*policy.*
		4,20	=	4	4	0	*commission.*
		64,05	=	64	1	0	*whole cost.*

2. What premium shall I have to pay, if I insure property worth £860. at $10\frac{1}{2}$ per cent.? Ans. £90. 6s. 0d.

3. What is the insurance of £850., at $12\frac{3}{4}$ per cent.? Ans. £108. 7s. 6d.

4. What is the annual expense of insuring a house, worth £500. at 4s. 6d. per cent.? Ans. £1. 2s. 6d.

5. What is an underwriter to receive for insuring £350. at 6 guineas per cent.? Ans. £22. 1s. 0d.

6. What will be the expense of insuring £185. at 2½ per cent., commission ¼ and policy ⅛ per cents.?
Ans. £5. 6s. 4½d.

7. What will be the expense of insuring £432. at 6 per cent., policy ⅛ and commission ½ per cent.?
Ans. £28. 12s. 4¾d.

8. What is the cost on an insurance effected on £2500. at 6 per cent. on goods from Archangel to Hull, policy ¼ and commission ½ per cent.? Ans. £168. 15s. 0d.

9. What is the expense of insuring £320. worth of goods from London to Liverpool, at 4½ per cent., policy ⅛ per cent.? Ans. £14. 16s. 0d.

10. What is the premium, brokerage, and policy, on an insurance of £5800. from Leeds to London, at 5 per cent., brokerage 5s. and policy 2s. 6d. per cent.?
Ans. prem. £290.; brok. £14. 10s.; policy, £7. 5s.

11. What is the expense of an insurance effected on £180. at 6 per cent. on goods from Leith to London, brokerage ½ and policy ⅛ per cent.? Ans. £11. 18s. 6d.

12. What should an under-writer receive for insuring goods worth £3672. 15s. 0d. from Hull to Hamburgh, at 4 per cent., policy 5s. and commission 7s. 6d. per cent.?
Ans. £169. 17s. 3½d.

COMPOUND INTEREST

Is that which arises from both principal and interest; that is, the interest due at *each* payment is added to the principal, to bear interest for the next payment.

RULE 1.—Find the first year's interest, to which add the principal; then find the interest of that sum, which add to the last principal, and so on for the number of years required.

2nd. Subtract the first *principal* from the last *amount*, and it will give the compound interest required.

EXAMPLES.

1. What is the compound interest of £755. at 4 per cent. for three years?

£.
755
4
30,20
20
4,00

£.	
755	
30	4
785	4
	4
31,40	16
	20
	8,16
	12
	1,92
	4
	3,68

£.	s.	d.
785	4	0
31	8	$1\frac{3}{4}$
816	12	$1\frac{3}{4}$
		4
32,66	8	7
20		
13,28		
12		
3,43		
4		
1,72		

	£.	s.	d.	
	816	12	$1\frac{3}{4}$	
	32	13	$3\frac{1}{4}$	
	849	5	5	amount.
	755	0	0	1st principal.
£	94	5	5	interest.

or thus:—

25	755			1st principal.
	30	4		1st year's interest.
25	785	4		2nd year's principal.
	31	8	$1\frac{3}{4}$	2nd year's interest.
25	816	12	$1\frac{3}{4}$	3rd year's principal.
	32	13	$3\frac{1}{4}$	3rd year's interest.
	849	5	5	amount.
	755	0	0	1st principal.
£	94	5	5	Ans.

The same when the payments are half-yearly.

	£	s.	d.	
50	755			given principal.
	15	2		1st half-year's interest.
50	770	2		2nd half-year's principal.
	15	8	10	2nd half-year's interest.
50	785	10	10	3rd half-year's principal.
	15	12	$2\frac{1}{2}$	3rd half-year's interest.
50	801	3	$0\frac{1}{2}$	4th half-year's principal.
	16	0	$5\frac{1}{2}$	4th half-year's interest.
50	817	3	6	5th half-year's principal.
	16	6	$10\frac{1}{4}$	5th half-year's interest.
50	833	10	$4\frac{1}{4}$	6th half-year's principal.
	16	13	$4\frac{3}{4}$	6th half-year's interest.
	850	3	9	6th half-year's amount.
	755	0	0	given principal.
	£ 95	3	9	Ans. *

2. What is the amount of £220. 5s. 0d., for 2 years, at 4 per cent. per annum? Ans. £238. 4s. 5d.

3. What is the compound interest of £530., for 3 years, at 5 per cent. per annum? Ans. £83. 10s. $9\frac{3}{4}$d.

4. What is the amount of £500., for 4 years, at 3 per cent. per annum? Ans. £562. 15s. $0\frac{3}{4}$d.

5. What is the amount of £325. 10s. 0d., at 5 per cent. per annum, for $3\frac{1}{2}$ years? Ans. £386. 4s. $6\frac{1}{4}$d.

6. What is the amount of £700., for $2\frac{1}{4}$ years, at $3\frac{1}{2}$ per cent. per annum? Ans. £756. 8s. $4\frac{1}{4}$d.

7. What is the amount of £500., for 8 years, at 5 per cent. per annum? † Ans. £738. 14s. $6\frac{1}{4}$d.

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* From the above example it appears, that shorter the periods of payment, the greater is the interest of the lender.

† When there is a large number of payments, the operation is much shortened by using logarithms, thus: representing the principal by *p.*, the amount of £1. for one year by *r.*, time by *t.*, and amount by *a.*

log. *p.* + log. *r.* × *t.* = log. *a.*, or taking the last question,

log. 500 =	....	2,698970
+ log. *r.* × *t.* = log. 1.05 × 8 = .021189 × 8 =		.169512
£738.725	=	2.868482

DISCOUNT

Is the allowance made to persons for paying money before it is due, and is as much as that money, if put to interest, would gain in the same time, and at the same rate; thus allowing 5 per cent. discount, £100. will discharge now, a debt of £105. due a year to come.

RULE 1.—*To find the discount.* As £100. with its interest for the given time, is to that interest, so is the given sum to the discount required.

2nd. *When the present worth is required,* put £100. for the second term, the first and third remaining as before.

EXAMPLES.

1. What discount at 5 per cent. should be allowed for present payment of a debt of £300., due 10 months hence?

mo.			£.	s.	d.	
6 }	= {	½	5	0	0	interest of £100. for 12 months.
4 }		⅓	2	10	0	for 6 months.
			1	13	4	for 4 months.
			4	3	4	for 10 months.
			100	0	0	

	£.	s.	d.		£.	s.	d.		£.
Then as	104	3	4	:	4	3	4	::	300
	20				20				20
	2083				83				6000
	12				12				12
	25,000				1000				72,000
					72				

$$25 = \begin{cases} 5 \,|\, 72000 \\ 5 \,|\, 14400 \end{cases}$$

12 | 2880

2,0 | 24,0

£ 12 0 0 Ans. *

* In business, it is usual to allow a penny a pound, per month.

2. If I hold a bill of £175. 10s. 0d., payable in 3 months, what will be the banker's charge for cashing the same at 5 per cent.? Ans. £2. 3s. 4d.

3. What is the discount of £74. 10s. 0d., for 4 months, at $4\frac{1}{2}$ per cent.? Ans. £1. 2s. 0d.

4. Sold goods to the value of £90. 12s. 6d., to be paid in 6 months, what must be discounted for present payment at 4 per cent.? Ans. £1. 15s. $6\frac{1}{4}$d.

5. What is the present worth of £170., payable in 9 months, discounting at $3\frac{1}{2}$ per cent.? Ans. £165. 13s. $0\frac{1}{4}$d.

6. What is the discount of £250., for 10 months, at $4\frac{1}{4}$ per cent.? Ans. £8. 11s. $0\frac{1}{4}$d.

7. What is the present worth of £210., payable in 15 months, at 4 per cent.? Ans. £200.

8. If I have £500. to receive, 2 years hence, what is its present worth, allowing 5 per cent.?

Ans. £454. 10s. $10\frac{3}{4}$d.

9. What present money is equivalent to a bill of £50. which has 5 months to run, discounting at 5 per cent.?

Ans. £48. 19s. 7d.

10. What is the discount of £150., at 5 per cent., for 73 days? Ans. £1. 9s. $8\frac{1}{4}$d.

11. What is the present worth of 300., one half to be paid at 3 months and the other half at 6 months, discount 5 per cent.? Ans. £294. 9s. $9\frac{1}{2}$d.

12. What is the present worth of £180., payable as follows: £60. at 3 months, £60. at 6 months, and £60. at 9 months, discounting at 6 per cent.?

Ans. £174. 15s. $7\frac{1}{2}$d.

EQUATION OF PAYMENTS

Is the rule by which when several sums are due at different times, we find a *meantime* for the payment of the whole at once.

RULE.—Multiply *each* sum by the *time* it has to run, and *divide* the sum of the product by the whole debt; the quotient is accounted the equated time.

EXAMPLES.

1. A bought of B £30. worth of goods, at 3 months' credit, £40. worth at 6 months' credit, and £50. at 9 months' credit; but it being afterwards agreed to have but one payment of the whole, the time of such payment is required?

£.				
30	× 3	=	90	
40	× 6	=	240	
50	× 9	=	450	
12,0	)		78,0	
			$6\frac{1}{2}$	months, or 6 months and 15 days.*

2. A person who owes £200., agrees to pay £50. every 3 months for a year; but it being afterwards resolved upon to have but one payment, the equated time is required? Ans. $7\frac{1}{2}$ months.

3. £62. is due as follows: £18. in 5 months, £21. in 7 months, and £23. in 9 months; in what time ought the whole to be paid at once? Ans. 7 months 4 days.

4. †A person owes a certain sum; $\frac{1}{3}$ of which is to be paid in 3 months, $\frac{1}{4}$ in 4 months, and the remainder in 8 months; at what time ought the whole to be paid at once? Ans. 5 months 10 days.

5. I have to pay £120. in 3 months, £150. in 6 months, and £180. in 9 months; in what time should the whole be paid at once? Ans. 6 months 12 days.

6. Having to pay £130. in 2 months, £180. in 3 months, £200. in 4 months, and £76. in 5 months; what is the equated time for only one payment? Ans. 3 months 11 days.

7. I owe £60. to be paid as follows: $\frac{1}{6}$ at 6 months, $\frac{1}{3}$ at 3 months, and $\frac{1}{2}$ at 12 months; but wishing to pay the whole at once, what ought to be the time? Ans. 8 months.

8. Having to pay to a tradesman £25. 10s. in 2 months, £30. 10s. in 4 months, £35. 10s. in 6 months, and £45. 10s. at 8 months; in what time ought he to have it all at once? Ans. 5 months 14 days.

---

* 30 days are accounted a month in calculating interest.

† Any number may be assumed for the sum to be paid.

9. I have a bill for £20. 12s. 9d. at 2 months, one for £36. 15s. 6d. at 3 months, and another for £50. 10s. 10d. at 4 months; at what date ought a bill to be given for the whole? Ans. 3 months 8 days.

10. What is the equated time for the payment of the following sums: £65. 10s. 6d. due in 3 months, £73. 3s. 3d. due in 6 months, and £105. 13s. 4d. due in 10 months? Ans. 6 months 27 days.

BARTER

Is the rule by which tradesmen *exchange one commodity for another*.

RULE 1.—Find the *value* of that commodity whose *quantity* is given, which divide by the *given price* of the other commodity, the quotient will be the required quantity.

2.—As the *ready money* price of one commodity is to its *bartering* price, so is the *ready money* price of the other, to its *bartering* price.

EXAMPLES.

1. What quantity of chocolate at 4s. a lb. must be given for 1cwt. 2qrs. 12lbs. at 4s. 6d. a lb.?

```
              cwt. qr. lb.
               1   2   12
               4
              ---
               6
              28
              ---
             180
              4½ = 4s. 6d.
            -----
             720
              90
        s.   ---
        4  | 810 = value of given quantity in shillings.
 lb.  ( 4  | 202½ lbs...2½ )
 28 = { ×  | ----           } over
      ( 7  |  50       ..  )
        4  |   7  6½                cwt. qr. lb.
           |  1  3  6½    that is   1    3   6½ Ans.
```

2. A has 60 yards of superfine blue cloth at 21s. a yard; but in barter advances it to 24s.; B has velveteen at 3s. 11¼d. a yard ready money; to what must he advance it to be on an equality with A, and how many yards must A have for his cloth?

s. *s.* *s.*	*s.*
As 21 : 24 :: 3 11¼	24
	s. d. *s.* 60
or as 7 : 8 :: 3 11¼	4 6 = 4½ | 1440
8	2 | 2
7)31 6	9 | 2880
Advance to 4 6	A receives 320 yds.

3. A exchanges 30 yards of Irish linen, at 2s. 6d. a yard, for oats at 3s. a bushel; what quantity should he receive? Ans. 25 bushels.

4. How many yards of cloth, at 12s. a yard, must be given in barter for 40 yards at 7s. 6d. a yard? Ans. 25yds.

5. How much wheat, at £2. 10s. a quar. must be given in barter for 5 acres of land, at £50. an acre? Ans. 100 quar.

6. A farmer exchanged 28 stones of wool, at 12s. 6d. a stone, with a manufacturer for 21 yards of cloth; what was the cloth a yard? Ans. 16s. 8d.

7. Gave 100 quarters of wheat for 5 acres of land, at £50. per acre, what was the wheat valued at a quarter? Ans. £2. 10s. 0d.

8. How many gallons of rum, at 16s. 8d. per gallon, are equivalent to 20 gallons of brandy, at £1. 4s. 2d. a gallon? Ans. 29 gallons.

9. A farmer exchanged 12 quarters of barley, at £1. 7s. per quarter, with a brewer for ale, at 1s. 6d. per gallon; how many gallons should he receive? Ans. 216.

10. How much tobacco, at £6. 18s. 6d. per cwt., is equivalent to 5cwt. 3qrs. 14lbs. of tea, at 4s. 6d. per lb.? Ans. 21cwt. 1qr. 14lbs.

11. Bought 24 quarters of barley, at £1. 8s. per quarter, for which I paid £13. 12s. in cash, and the remainder in peas, at 5s. per bushel; required the quantity of peas. Ans. 10 quarters.

12. If I give 21cwt. 1qr. 14lbs. of tobacco, at £6. 18s. 8d. per cwt., for 5cwt. 3qrs. 14lbs. of tea, what does it stand me to per lb.? Ans. 4s. 6d.

13. If I exchange 30 yards of superfine blue cloth, at £1. 2s. 6d. per yard, for equal quantities of kerseymere at 7s. 6d. per yard, swansdown at 3s. 6d., and toilonet at 4s., how much shall I have of each? Ans. 45 yards.

PROFIT AND LOSS

Is a rule by which traders regulate their prices so as to gain or lose a certain rate per cent.

CASE I.—*To find the gain or loss per cent.*

RULE.—As the *prime* cost is to the *gain* or *loss*, so is 100 to the rate per cent. Or, as prime cost : selling price :: 100 : 100 + gain per cent.

CASE II.—*At a given gain or loss per cent. to find the selling price.*

RULE.—As 100 is to the prime cost, so is 100 + gain, or, — loss, to the *selling price.*

CASE III.—When the *selling price*, and *gain per cent.* upon it are given, to find the gain per cent. at a proposed price.

RULE— As the *selling price* is to 100 + given gain, so is the *proposed price* to 100 + required gain.

EXAMPLES.

1. Bought cloth at 6s. 8d. per yard, and sold it for 8s. what was the gain per cent.?

```
           s. d.
           8  0
 s.  d.    6  8
As 6  8 :  1  4 :: 10,0
  12      12
  8,0     16
          10
        8)160
   Ans.   20 per cent.
```

or thus:—

```
s. d.          s. d.
1  4 | 1/6 of 6  8 |100
              Ans.  20
```

2. Bought stockings at 3s. 4d. per pair, how must I sell them to gain 20 per cent.?

```
      s.  d.
10,0 : 3   4 :: 12,0
          12
   10)40   0
       4   0
```

or thus:—

```
per cent.   s.  d.
20 = 1/5 | 3   4
         |     8
           4   0 Ans.
```

3. If I clear 10 per cent. by selling rum at 17s. 6d. per gallon, what shall I gain per cent. by selling it at a guinea?

```
     s.  d.           s.
As  17   6 : 110 :: 21
     2                 2
    35                42 sixpences.
                     110
           35 = { 5 | 4620
                { 7 |  924
                       132
                       100 subtract.
                  Ans.  32 per cent.
```

4. If I buy for 4s. and sell for 5s., what shall I gain per cent.? Ans. 25.

5. If I buy at 4s. 8d. per lb. and sell at 5s. 3d., what is my gain per cent.? Ans. 12½.

6. Bought a horse for £25. and sold him for £31. 10s. 6d., what is my gain per cent.? Ans. £26. 2s. 0d.

7. If I buy tea at 4s. per lb., how must I sell it to gain 25 per cent.? Ans. 5s. per lb.

8. If I buy at 4s. 8d. per lb., at what must I sell it to gain 12½ per cent.? Ans. 5s. 3d.

9. If by selling tea at 5s. per lb. I gain 25 per cent., what was the prime cost? Ans. 4s.

10. If by selling at 5s. 3d., per lb., I gain 12½ per cent. what did it cost me per lb.? Ans. 4s. 8d.

11. If a tradesman gains 5s. 6d. on an article which he sells for 22s., what does he gain per cent.? Ans. £33. 6s. 8d.

12. Bought tea at 6s. 8d. per lb. which proved to be damaged, I sold it at 6s. per lb., what was the loss per cent.? Ans. 10.

13. Bought bacon at 6½d. per lb. and sold it at 8d., what did I gain by the sale of 3cwt. 2qrs. 5lb.?
Ans. £2. 9s. 7½d.

14. By selling cloth at 15s. per yard, I lost 10 per cent., what was the prime cost? Ans. 16s. 8d.

15. If I give £3. 10s. for 1 cwt. of cheese, and retail it at 10d. per lb. what do I gain by the whole; and what is the rate per cent. Ans. £1. 3s. 4d. gain, and 33⅓ rate per cent.; or, £33. 6s. 8d. per £100.

16. By selling cloth at 12s. per yard I gain 8 per cent., at what rate per cent. shall I gain by selling it at 14s?
Ans. 26 per cent.

17. By selling cloth at 13s. 4d. per yard, I gained 12½ per cent., what would be the gain or loss per cent. by selling the same at 10s. 8d. per yard? Ans. 10 per cent loss.

18. Sold hats at 18s. 6d. and cleared 20 per cent., required the prime cost. Ans. 15s. 5d.

19. Bought a quantity of old milk cheese at 3¾d. per lb., which I sell at 5½d. per lb.; what is my gain per cent.?
Ans. £46. 13s. 4d.

20. By selling at 4s. 7½d. I clear 10½ per cent., what should I gain by selling at 5s. 2d. Ans. £23. 8s. 9¼d.—87.

21. Bought 200 quarters of beans for £250., *half* of which I sold at a loss of 10 per cent.; how much a quarter should the other half be charged to clear £10. by the bargain? Ans. £1. 9s. 6d.

22. A huckster bought 150 eggs at 2 for a penny, and 150 at 3 for a penny, and sold the whole at 5 for two-pence; what did he gain or lose, and how much per cent.?
Ans. 5d. loss, 4 per cent. loss.

23. Bought 160 quarters of wheat, *one fourth* of which being damaged on board, I sold for £70., losing thereby £16. 13s. 4d. per cent.; how must I sell the remainder per quarter to gain 5 per cent. upon the whole?
Ans. £2. 7s. 1⅗d.

24. By selling one sort of tea at 7s. 6d. a lb. I gain 20 per cent, and by selling another sort at 8s. 8d. a lb. I gain 30 per cent.; now if I mix 3 lbs. of the former with 2 lbs. of the latter, how must I sell the mixture to gain 25 per cent.? Ans. 8s. $0\frac{1}{4}$d per lb.

SINGLE FELLOWSHIP

Is the rule by which we can divide *any number* or *sum* into *any number* of parts, having a *given* proportion.

RULE.—As the *sum* of the *numbers* to which the parts are to be proportional, is to the *number* to be divided, so is each number to its proportional part.

EXAMPLES.

1. Divide £25. 12s. 6d. into two parts, which shall be in proportion to each other as 3 to 5.

		£.	*s.*	*d.*		
	3					
	5					
As	8 :	25	12	6	:: 3	
				3		
	8)	76	17	6		
		9	12	$2\frac{1}{4}$		= less part.

	£.	*s.*	*d.*		
Again, as 8 :	25	12	6	:: 5	
			5		
8)	128	2	6		
	16	0	$3\frac{3}{4}$		greater part.

	£.	*s.*	*d.*	
	9	12	$2\frac{1}{4}$	add *
	16	0	$3\frac{3}{4}$	
Proof	25	12	6	

* The sum of the parts must be equal to the whole sum to be divided.

2. Two merchants trade together: A puts into stock £250. and B £140., and they gain £78.; what is each person's share of the gain? Ans. A £50. B £28.

3. A and B trade together: A puts into stock £20. and B £40., and they gained £50.; what was each person's share? Ans. A £16. 13s. 4d. B £33. 6s. 8d.

4. Three persons took a pasture for £20., each agreeing to pay in proportion to the stock he put in; what had each to pay, A having had 3 head, B 4 head, and C 5 head of cattle in it?

Ans. A £5. 0s. 0d. B £6. 13s. 4d. C £8. 6s. 8d.

5. A gentleman who had three sons at a boarding school, sent a sovereign to be divided among them in proportion to their ages; what had each to receive, A being 8, B 10, and C 12 years of age?

Ans. A 5s. 4d. B 6s. 8d. C 8s. 0d.

6. Four merchants enter into partnership: A advances £225., B £280., C £325., and D £420., with which they gain £500.; what is each person's share?

Ans. A £90. B £112. C £130. D £168.

7. A, B, and C make a stock of £840., A advancing $\frac{1}{4}$, B $\frac{1}{5}$, and C the remainder, with which they gain £420. 10s. 0d.; what is each person's share of the gain?

Ans. A £105. 2s. 6d. B £84. 2s. 0d. C £231. 5s. 6d.

8. A, B, and C bought some houses and land for £3700. whereof A paid £1000., B £1500., and C the rest; they afterwards sold it for £4500.; required the gain of each.

Ans. A £216. 4s. $3\frac{3}{4}$d.—21. B £324. 6s. $5\frac{3}{4}$d.—13.
C £259. 9s. $2\frac{1}{4}$d.—3.

9. Four persons made a joint stock of £456., from the profits of which A received £25. 10s., B £30. 15s., C £45. 5s., and D £50. 10s.; what was each person's stock?

Ans. A £76. 10s. B £92. 5s. C £135. 15s.
D £151. 10s.

10. A person failing in business, owes to A £512., to B £708. 10s., to C £880. 10s., and to D £909.; but his effects are found only to be worth £1500.; what is each person's share? Ans. A £255. 2s. $11\frac{3}{4}$d.—1570.
B £353. 1s. $5\frac{1}{2}$d.—500. C £438. 15s. $8\frac{3}{4}$d.—2650.
D £452. 19s. $9\frac{1}{2}$d.—1300.

DOUBLE FELLOWSHIP

Is when the *different* shares are employed in trade *different* terms of time.

RULE.—As the *sum* of each man's *stock* multiplied into his *time*, is to the whole *gain* or *loss*, so is the product of each man's stock and time, to his share of the gain or loss.

EXAMPLES.

1. A and B enter into partnership; A puts into stock £362. 10s. for 8 months, and B £450. for 6 months, and they gain £320.; what is each person's share of the gain?

£.	s.		£.
362	10	× 8 =	2900
450	0	× 6 =	2700

As 56,00 : 320 :: 29,00
* or, as 7 : 40 :: 29

40

7)1160

A's share 165 14 $3\frac{1}{4}$—5

Also, as 56,00 : 320 :: 27,00
or, as 7 : 40 :: 27

40

7)1080

154 5 $8\frac{1}{2}$—2 B's share.

A's share	165	14	$3\frac{1}{4}$—5	add.
B's share	154	5	$8\frac{1}{2}$—2	
Proof	320	0	0 †	

* The *first* and *third* are abridged by 100, then the *first* and *second* by 8.

† The sum of the remainders, 5 and 2, being equal to the divisor 7, is equivalent to a farthing.

2. A and B enter into partnership; A puts into stock £750. for 5 months and B £600. for six months, and they gain £294.; what is each man's share?

Ans. A £150. B £144.

3. Two merchants trade together; A puts into stock £300. for 8 months, and B £565. for 10 months, and they gain £170.; what is each man's share?

Ans. A £50. 13s. 7¾d.—725. B £119. 6s. 4d.—80.

4. Three merchants trade together; A puts in £300. for 9 months, B £450. for 12 months, and C £500. for 15 months, and they gain £251.; what is each person's share?

Ans. A £43. 8s. 10d.—96. B £86. 17s. 8¼d.—36. C £120. 13s. 5½d.—24.

5. Three graziers hire a pasture for £40. into which A puts 12 oxen for 36 days, B 20 oxen for 27 days, and C 32 oxen for 45 days; what is each person's proportion of the rent? Ans. A £7. 3s. 3¼d.—1476. B £8. 19s. 1¼d.—36. C £23. 17s. 7¼d.—900.

6. Three merchants joined in trade; A had in stock £540. for 3¼ years, B. £500. for 3½ years, and C £448. for 3¾ years; they lost £450. 10s.; what was each person's share of the loss? Ans. A £152. 9s. 8d.—1360. B £152. 0s. 11¾d.—1105. C £145. 19s. 4d.—2720.

7. A gentleman bequeathed £300. to his 3 servants, to be paid in proportion to their wages and length of servitude; now the footman had £30. a year, and had lived with him 5 years; the cook £20. a year, and 7 years' servitude; and the housemaid £12. a year, and 9 years' servitude; what had each to receive?

Ans. Footman £113. 1s. 3½d.—284. Cook £105. 10s. 6½d.—212. Housemaid £81. 8s. 1½d.—300.

8. A puts into stock £712. 18s. 6d. for 3 months, B £528. 13s. 9d. for 4 months, and C £1000. 12s. 10d. for 5 months; they lose £420. 15s. 10d.; required each person's oss? Ans. A £97. 4s. 5¾d.—562400. B £96. 2s. 7½d. —1804224. C £227. 8s. 8¼d.--2076608.

COMMON MULTIPLE.

To find the least common multiple of two or more numbers.

RULE 1.—Write the given numbers in a straight line, with commas between them.

2.—Divide by *any number* that will divide two or more of them, and set down the quotients, and the undivided numbers below.

3.—Divide the second as before, and so on till all the numbers are *prime to each other;* then multiply continually together the *divisors* and *quotients* for the multiple required.

EXAMPLE.

Find the least common multiple of 2, 3, 4, 5, 6, 7, 8, or the least whole number divisible by each of them.

$$\begin{array}{r|l} 2 & 2,3,4,5,6,7,8 \\ \hline 2 & 1,3,2,5,3,7,4 \\ \hline 3 & 1,3,1,5,3,7,2 \\ \hline & 1,1,1,5,1,7,2 \end{array}$$

$\therefore 2 \times 2 \times 3 \times 5 \times 7 \times 2 = 840$. Ans.

EXERCISES.

1. A clock that has two hands, one of which goes round in 60 minutes and the other in 63 minutes; now supposing them together at 12 o'clock, when would they be next together? Ans. 21 hours.

2. A clock having 4 hands, one of which goes round in an hour, another in three quarters of an hour, another in half an hour, and the other in a quarter of an hour, in what time after being together, will they be together again? Ans. 3 hours.

3. Three persons start together to walk the same way round an island 60 miles in circumference; when will they be together again, A travelling 4, B 5, and C 6 miles per hour? Ans. 60 hours.

4. A boy agreed to give a crown for as many oranges as he could tell over by 2, 3, 4, 5, and 6 at a

time, without remainder; what number had he, it being the least? Ans. 60.

5. Seven schoolfellows started together to walk round the play-ground; one walked round in 3 minutes, another in 4, another in 5, and so on; supposing they had continued at the same rate till they all came together again, how long would it have been after the time of starting?

Ans. 42 hours.

6. It is required to find the least whole number, which being divided by each of the nine digits shall leave no remainder? Ans. 2520.

7. The periodic times of four bodies being 24, 22, 20, and 18 days respectively, in what time, after having a conjunction, will they all be together again, and what number of revolutions will each have made?

Ans. 3960 days from one conjuction to another; and 165, 180, 198, and 220, the number of revolutions made by each respectively.

VULGAR FRACTIONS.

A vulgar fraction is a part or parts of an unit; as $\frac{1}{4}$, $\frac{3}{4}$, $\frac{2}{3}$, is *one-fourth*, *three-fourths*, and *two-thirds of one*.

The *top* figure is called the *numerator*, and the *bottom* figure the *denominator*.

The *denominator* shews how many parts the unit is divided into; and the numerator, how many of these parts are to be taken.

A SINGLE *fraction* is when the numerator and denominator is each a simple number, as $\frac{3}{5}$, $\frac{14}{17}$, $\frac{18}{13}$, &c.

A PROPER *fraction* is when the numerator is *less* than the denominator, as $\frac{3}{5}$.

An IMPROPER *fraction* is when the numerator is *greater* than the denominator, as $\frac{5}{3}$.

A COMPOUND *fraction* is the fraction of a fraction, and is known by the word of, as $\frac{2}{3}$ of $\frac{3}{4}$, &c.

A MIXED *number*, or *fraction*, is a whole number and a fraction, as $7\frac{3}{4}$, $17\frac{11}{12}$, &c.

A COMPLEX *fraction* has a fraction or mixed number for its numerator or denominator; or both, as $\frac{\frac{1}{2}}{3}, \frac{5}{4\frac{1}{2}}, \frac{3\frac{1}{2}}{7\frac{1}{4}}$, &c.

When the numerator and denominator of several fractions are the same, the fractions are equal to one another and to unity, as $\frac{3}{3} = \frac{5}{5} = \frac{7}{7} = 1$.

To find the greatest common measure of the terms of a fraction.

RULE.—Divide the *greater* term by the *less*, and the last *divisor* by the last *remainder*, and so on till nothing remains, then the *last divisor* will be the common measure required.

EXAMPLE.

1. Required the greatest common measure of $\frac{162}{1080}$.

```
162)1080(6
     972
     ---
     108)162(1
         108
         ---
          54)108(2
             108
             ---
             ===
```

∴ 54 is the greatest divisor. *

2. Find the greatest common measure of $\frac{246}{372}$. Ans. 6.
3. Find the greatest common measure of $\frac{748}{916}$. Ans. 4.
4. Find the greatest common divisor of $\frac{4626}{1944}$. Ans. 18.
5. Find the greatest common divisor of $\frac{54320}{78645}$. Ans. 35.

To reduce fractions to their lowest or simplest terms.

RULE.—Find the greatest common measure, by the last rule; by which divide both the *numerator* and *denominator*, and it will give the required fraction.

---

* The *greatest* common measure, is the greatest number that will divide *both* terms of the fraction without a remainder; but when the terms of the fraction are small, it may be reduced to its lowest terms, by dividing the *numerator* and *denominator* by *any* number that will divide them *both* without a remainder—thus, $\frac{204}{300} = \frac{51}{75} = \frac{17}{25}$ by dividing first by 4, and then by 3. If there be an equal number of cyphers to the right of each term, they may be cut off. Numbers ending with *even* numbers are divisible by 2; those ending with 0 or 5 are divisible by 5.

EXAMPLE.

1. Reduce $\frac{204}{300}$ to its lowest terms.

```
204)300(1
    204
     96)204(2
        192
         12)96(8
            96
```

$\therefore$ 12)$\frac{204}{300} = \frac{17}{25}$ Ans.

2. Reduce $\frac{540}{612}$ to its lowest terms. Ans. $\frac{15}{17}$.
3. Reduce $\frac{136}{248}$ to its lowest terms. Ans. $\frac{17}{31}$.
4. Reduce $\frac{252}{336}$ to its lowest terms. Ans. $\frac{3}{4}$.
5. Reduce $\frac{72}{288}$ to its lowest terms. Ans. $\frac{1}{4}$.
6. Reduce $\frac{192}{576}$ to its lowest terms. Ans. $\frac{1}{3}$.
7. Reduce $\frac{432}{864}$ to its lowest terms. Ans. $\frac{1}{2}$.
8. Reduce $\frac{208}{684}$ to its lowest terms. Ans. $\frac{52}{171}$.
9. Reduce $\frac{394}{784}$ to its lowest terms. Ans. $\frac{197}{392}$.
10. Reduce $\frac{3460}{7940}$ to its lowest terms. Ans. $\frac{173}{397}$.
11. Reduce $\frac{3042}{3094}$ to its lowest terms. Ans. $\frac{117}{119}$.
12. Reduce $\frac{13667}{14186}$ to its lowest terms. Ans. $\frac{79}{82}$.

To reduce an improper fraction to a whole or mixed number.

RULE.—Divide the *numerator* by the *denominator* for the integral part, and put the remainder, if any *over* the denominator, for the fractional part.

EXAMPLE.

1. Reduce $\frac{11}{2}$ to its proper terms.

$$\frac{11}{2} = 11 \div 2 = 5\frac{1}{2}. \text{ Ans.}$$

2. Reduce $\frac{15}{4}$ to a mixed number. Ans. $3\frac{3}{4}$.
3. Reduce $\frac{47}{6}$ to its proper terms. Ans. $7\frac{5}{6}$.
4. Reduce $\frac{155}{12}$ to its proper terms. Ans. $12\frac{11}{12}$.
5. Reduce $\frac{297}{17}$ to its proper terms. Ans. $17\frac{8}{17}$.
6. Reduce $\frac{492}{25}$ to its proper terms. Ans. $19\frac{17}{25}$.
7. Reduce $\frac{736}{41}$ to its proper terms. Ans. $17\frac{39}{41}$.
8. Reduce $\frac{1401}{19}$ to its proper terms. Ans. $73\frac{14}{19}$.

To reduce a mixed number to an improper fraction.

RULE.—Multiply the *whole number* by the denominator of the fraction, and to the product add the numerator; under which place the denominator.

EXAMPLE.

1. Reduce $5\frac{1}{2}$ to an improper fraction.

$$5\tfrac{1}{2} = \frac{5 \times 2 + 1}{2} = \frac{11}{2} \text{ Ans.}$$

2. Reduce $3\frac{3}{4}$ to an improper fraction. Ans. $\frac{15}{4}$.
3. Reduce $7\frac{5}{6}$ to an improper fraction. Ans. $\frac{47}{6}$.
4. Reduce $12\frac{11}{12}$ to an improper fraction. Ans. $\frac{155}{12}$.
5. Reduce $17\frac{8}{17}$ to an improper fraction. Ans. $\frac{297}{17}$.
6. Reduce $19\frac{17}{25}$ to an improper fraction. Ans. $\frac{492}{25}$.
7. Reduce $17\frac{39}{41}$ to an improper fraction. Ans. $\frac{736}{41}$.
8. Reduce $73\frac{14}{19}$ to an improper fraction. Ans. $\frac{1401}{19}$.
9. Reduce $327\frac{3}{5}$ to an improper fraction. Ans. $\frac{1638}{5}$.
10. Reduce $100\frac{11}{30}$ to an improper fraction. Ans. $\frac{3011}{30}$.

To reduce a compound fraction to a single one.

RULE.—Multiply all the *numerators* together for a new numerator, and all the *denominators* for a new denominator; then reduce the new fraction to its lowest terms.

EXAMPLE.

1. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a single fraction.

$$\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{2 \times 3 \times 4}{3 \times 4 \times 5} = \frac{24}{60} = \frac{2}{5} \text{ Ans. *}$$

2. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ to a single fraction. Ans. $\frac{6}{12} = \frac{1}{2}$.
3. Reduce $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{9}{10}$ to a single fraction. Ans. $\frac{135}{240} = \frac{9}{16}$.
4. Reduce $\frac{2}{3}$ of $\frac{3}{7}$ of $\frac{7}{9}$ to a single fraction. Ans. $\frac{2}{9}$.
5. Reduce $\frac{7}{8}$ of $\frac{9}{10}$ of $\frac{11}{12}$ to a single fraction. Ans. $\frac{231}{320}$.
6. Reduce $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{9}{10}$ of $\frac{11}{12}$ to a single fraction. Ans. $\frac{77}{160}$.

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* The operation may be abreviated by striking out all the like quantities from the numerators and denominators. Here the 3 and 4 in the numerator cancel the 3 and 4 in the denominator. Mixed numbers must be reduced to improper fractions.

7. Reduce $\frac{7}{12}$ of $\frac{5}{7}$ of $\frac{3}{4}$ of 12 to a single fraction. Ans. $\frac{15}{4}$.

8. Reduce $\frac{2}{5}$ of $1\frac{1}{2}$ of $2\frac{1}{4}$ of $5\frac{1}{3}$ to a single fraction. Ans. $\frac{36}{5}$.

9. What is the value of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of 20? Ans. 14.

10. What is the value of $\frac{1}{12}$ of $2\frac{1}{2}$ of $3\frac{3}{5}$ of 20? Ans. 15.

To reduce a fraction of one denomination to the fraction of another, but greater, *retaining the same value.*

RULE.—Multiply the *denominator* by all the denominations from *that given* to the *one sought.*

EXAMPLE.

1. Reduce $\frac{3}{4}$ of a penny to the fraction of a £.

$d.\ \frac{3}{4} \times \frac{1}{12} \times \frac{1}{20} = \frac{3}{960} = \frac{1}{320}$ £. Ans. *

2. Reduce $\frac{3}{4}$ of a penny to the fraction of a shilling. Ans. $\frac{1}{16}s.$

3. Reduce $\frac{2}{3}$ of a farthing to the fraction of a £. Ans. $\frac{1}{1440}$.

4. Reduce a farthing to the fraction of a £. Ans. $\frac{1}{960}$.

5. Reduce $\frac{3}{8}$ of a crown to the fraction of a guinea. Ans. $\frac{5}{56}$.

6. Reduce $\frac{4}{5}$ of a shilling to the fraction of a guinea. Ans $\frac{4}{105}$.

7. Reduce $\frac{3}{5}$ of 6s. 8d. to the fraction of 7s. 6d. Ans. $\frac{8}{15}$.

8. Reduce $\frac{3}{4}$ of an oz. to the fraction of a cwt. Ans. $\frac{3}{7168}$.

9. Reduce $\frac{5}{8}$ of a pint to the fraction of a gallon. Ans. $\frac{5}{64}$.

10. Reduce $\frac{3}{4}$ of a perch to the fraction of an acre. Ans. $\frac{3}{640}$.

11. Reduce 50 seconds to the fraction of a day. Ans. $\frac{1}{1728}$.

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* To bring pence or parts of a penny to the fraction of a £. it is only dividing by 12 and 20. To $\div$ by 12, is to $\times$ by $\frac{1}{12}$.

To reduce a fraction of one denomination to another, but less, *retaining the same value.*

RULE.—Multiply the *numerator* by all the denominations, from *that given* to the *one sought.*

EXAMPLE.

1. Reduce $\frac{1}{320}$ of a £. to the fraction of a penny.

   £ $\frac{1}{320} \times \frac{20}{1} \times \frac{12}{1} = \frac{240}{320} = \frac{3}{4}$d. Ans. *

2. Reduce $\frac{1}{16}$ of a shilling to the fraction of a penny. Ans. $\frac{3}{4}$d.
3. Reduce $\frac{1}{1440}$ of a £. to the fraction of a farthing. Ans. $\frac{2}{3}$.
4. Reduce $\frac{5}{56}$ of a guinea to the fraction of a crown. Ans. $\frac{3}{8}$.
5. Reduce $\frac{4}{105}$ of a guinea to the fraction of a shilling. Ans. $\frac{4}{5}$.
6. Reduce $\frac{3}{7168}$ of a cwt. to the fraction of an oz. Ans. $\frac{3}{4}$.
7. Reduce $\frac{3}{108}$ of a yard to the fraction of a nail. Ans. $\frac{4}{9}$.
8. Reduce $\frac{1}{8640}$ of a lb. troy, to the fraction of a grain. Ans. $\frac{2}{3}$.
9. Reduce $\frac{4}{15}$ of a £. to the fraction of a crown. Ans. $\frac{16}{15}$.
10. Reduce $\frac{2}{7}$ of a guinea to the fraction of a £. Ans. $\frac{3}{10}$.
11. Reduce $\frac{3}{4}$ of a moidore to the fraction of a guinea. Ans. $\frac{27}{28}$.
12. Reduce $\frac{3}{5}$ of 6s. 8d. to the fraction of $\frac{4}{5}$ of a crown. Ans. $\frac{6}{5}$.

To reduce fractions to their proper quantities.

RULE.—Multiply the numerator by the next lower denomination, and divide by the denominator; if there be a remainder, reduce it again, and divide as before, and so on to the lowest name.

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* This rule is the reverse of the last, therefore the multipliers will be inverted.

EXAMPLES.

1. Reduce $\frac{2}{3}$ of a £. to its proper quantity.

$$\begin{array}{l} 2 \\ \underline{20} \\ 3)\underline{40s.} \\ \underline{\underline{13\ 4d.}}\ \text{Ans.} \end{array} \quad \text{or, £}\ \frac{2}{3} = \frac{2 \times 20}{3} = \frac{40}{3}\text{s.} = \begin{array}{cc} s. & d. \\ 13 & 4 \end{array}$$

2. What is the value of $\frac{2}{3}$ of a shilling? Ans. 8d.
3. What is the value of $\frac{3}{4}$ of a £.? Ans. 15s.
4. What is the value of $\frac{7}{12}$ of a guinea? Ans. 12s. 3d.
5. What is the value of $\frac{3}{8}$ of a £. Ans. 7s. 6d.
6. What is the value of $\frac{2}{3}$ of $\frac{3}{4}$ of a guinea? Ans. 10s. 6d.
7. What is the value of $\frac{3}{5}$ of 7s. 6d. Ans. 4s. 6d.
8. What is the value of $\frac{3}{4}$ of a guinea? Ans. 15s. 9d.
9. What is the value of $\frac{3}{4}$ of half-a-crown? Ans. 1s. $10\frac{1}{2}$d.
10. Reduce $\frac{5}{8}$ of an acre to its proper quantity. Ans. 2r. 20p.
11. Reduce $\frac{5}{8}$ of a gallon to its proper quantity. Ans. 2qts. 1pt.
12. Reduce $\frac{3}{5}$ of a lb. troy, to its proper quantity. Ans. 7oz. 4dwts.
13. Reduce $\frac{3}{10}$ of a day to its proper quantity. Ans. 7h. 12m.
14. Reduce $\frac{2}{3}$ of a ton to its proper quantity. Ans. 13cwt. 1qr. 9lb. 5oz. $5\frac{1}{3}$dr.
15. Reduce $\frac{26}{27}$ of a butt to its proper quantity. Ans. 1hhd. 2kil. 1fir. 5gal.

To reduce a given quantity to its equivalent fraction.

RULE.—Reduce the *given quantity* to the lowest denomination mentioned, for a *numerator*, and the specified integer into the same name for a *denominator;* this fraction reduced to its lowest terms will be the required answer.

EXAMPLE.

1. Reduce 12s. 8½d. to the fraction of a £.

s.	d.		
12	8½	20s. = £1.	
12		12	
152		240	
4		4	
610 numerator.		960 denominator.	∴ $\frac{610}{960}=\frac{61}{96}$ Ans.

or thus, 12s. 8½d. $=\dfrac{(12\times 12+8)\times 4+2}{20\times 12\times 4}=\dfrac{610}{960}=\dfrac{61}{96}$

2. Reduce 4s. 8½d. to the fraction of a £. Ans. $\frac{113}{480}$.
3. Reduce 8d. to the fraction of a shilling. Ans. $\frac{2}{3}$.*
4. Reduce 15s. to the fraction of a £. Ans. $\frac{3}{4}$.
5. Reduce 12s. 3d. to the fraction of a guinea. Ans. $\frac{7}{12}$.
6. Reduce 7s. 6d. to the fraction of a £. Ans. $\frac{3}{8}$.
7. Reduce 10s. 6d. to the fraction of a guinea. Ans. $\frac{1}{2}$.
8. Reduce 4s. 6d. to the fraction of 7s. 6d. Ans. $\frac{3}{5}$.
9. Reduce 15s. 9d. to the fraction of a guinea. Ans. $\frac{3}{4}$.
10. Reduce 1s. 10½d. to the fraction of half-a-crown. Ans. $\frac{3}{4}$.
11. Reduce 2r. 20p. to the fraction of an acre. Ans. $\frac{5}{8}$.
12. Reduce 7oz. 4dwts. to the fraction of a lb. Ans. $\frac{3}{5}$.
13. Reduce 7ho. 12mi. to the fraction of a day. Ans. $\frac{3}{10}$.
14. Reduce 13cwt. 1qr. 9lb. 5oz. 5⅓dr. to the fraction of a ton. Ans. $\frac{2}{3}$.
15. Reduce 1hhd. 2kil. 1fir. 5gal. of ale, to the fraction of a butt. Ans. $\frac{26}{27}$.

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* When the *given number* is in *one name*, put the integer under it reduced to the *same name*, and reduce the fraction to its lowest terms.

*To reduce fractions to a common denominator.**

RULE.—Multiply *each numerator* separately, *into all* the denominators *except its own* for a new numerator, and *all* the *denominators* together for a *common denominator.*

EXAMPLE.

1. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to *equivalent* fractions, having a common denominator.

$$\left.\begin{array}{l} 2 \times 4 \times 5 = 40 \\ 3 \times 3 \times 5 = 45 \\ 4 \times 4 \times 3 = 48 \end{array}\right\} \text{new numerators.}$$

$$3 \times 5 \times 4 = 60 \quad \text{common denominator.}$$

∴ The required fractions are $\frac{40}{60}$, $\frac{45}{60}$, and $\frac{48}{60}$.

2. Reduce $\frac{3}{5}$ and $\frac{2}{3}$ to a common denominator.
Ans. $\frac{9}{15}$ and $\frac{10}{15}$.

3. Reduce $\frac{3}{7}$ and $\frac{5}{9}$ to a common denominator.
Ans. $\frac{27}{63}$ and $\frac{35}{63}$.

4. Reduce $\frac{3}{5}$, $\frac{3}{7}$, and $\frac{3}{9}$ to a common denominator.
Ans. $\frac{189}{315}$, $\frac{135}{315}$, and $\frac{105}{315}$.

5. Reduce $\frac{6}{7}$, $\frac{7}{8}$, and $\frac{8}{9}$ to a common denominator.
Ans. $\frac{432}{504}$, $\frac{441}{504}$, and $\frac{448}{504}$.

6. Reduce $\frac{3}{5}$, $\frac{8}{9}$, and $\frac{7}{10}$ to a common denominator.
Ans. $\frac{270}{450}$, $\frac{400}{450}$, and $\frac{315}{450}$; or, $\frac{54}{90}$, $\frac{80}{90}$, $\frac{63}{90}$.

7. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to a common denominator.
Ans. $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, and $\frac{48}{60}$.

8. Reduce $\frac{12}{17}$, $\frac{15}{23}$, and 7 to a common denominator.
Ans. $\frac{276}{391}$, $\frac{255}{391}$, and $\frac{2737}{391}$.

9. Reduce 3, 5, and $\frac{7}{3}$ to fractions, having 9 for their denominator.
Ans. $\frac{27}{9}$, $\frac{45}{9}$, and $\frac{21}{9}$.

10. Reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$ to fractions, having 6 for their denominator.
Ans. $\frac{3}{6}$, $\frac{2}{6}$, and $\frac{1}{6}$.

---

* Fractions are reduced to a common denominator, in order to prepare them for *addition* and *subtraction;* they are of the *same value* after they have a common denominator as they were before; for it is nothing more than multiplying each *numerator* and its *denominator* by such a number as will make all the denominators alike; as an illustration, let us find equivalent fractions to $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$, each having the same denominator: multiplying the *first* by 30, second by 20, *third* by 15, *fourth* by 12, and *fifth* by 10, there results $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, and $\frac{50}{60}$. Fractions are said to have a *common denominator when the denominators* are alike; and it is the *least* when the terms of the fractions admit not of a common divisor.

ADDITION
OF VULGAR FRACTIONS.

RULE.—If the fractions have *not* a common denominator, reduce them to one by the last rule: then add all the numerators together; under which, place the *common denominator;* the result will be the sum of the required fractions.

EXAMPLE.

1. Add $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ together.

$$\left.\begin{array}{r} 2 \times 4 \times 5 = 40 \\ 3 \times 3 \times 5 = 45 \\ 4 \times 4 \times 3 = 48 \end{array}\right\} \text{new numerators.}$$

$$\frac{\text{sum of numerators } 133}{\text{common denominator } 3 \times 4 \times 5 = 60} = 2\tfrac{13}{60}. \text{ Ans.}$$

or, $\frac{2}{3} + \frac{3}{4} + \frac{4}{5} = \frac{40}{60} + \frac{45}{60} + \frac{48}{60} = \frac{133}{60} = 2\frac{13}{60}$.

2. Add $\frac{1}{2}$ and $\frac{1}{3}$ together. Ans. $\frac{5}{6}$.
3. Add $\frac{2}{3}$ and $\frac{3}{4}$ together. Ans. $\frac{17}{12} = 1\frac{5}{12}$.
4. Add $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ together. Ans. $\frac{13}{12} = 1\frac{1}{12}$.
5. Add $\frac{7}{13}$ and $\frac{6}{13}$ together. Ans. 1.
6. Add $\frac{1}{2}$ of $\frac{2}{8}$ to $1\frac{1}{2}$. * Ans. $1\frac{5}{8}$.
7. Add $\frac{3}{7}$ and $\frac{5}{14}$ together. Ans. $\frac{11}{14}$.
8. Add $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{7}$ together. Ans. $1\frac{103}{105}$.
9. Add $\frac{3}{7}$ of 14 to $\frac{1}{3}$ of 12. Ans. 10.
10. Add $\frac{3}{4}$ of $\frac{3}{5}$ to $\frac{2}{3}$ of $\frac{4}{5}$. Ans. $\frac{59}{60}$.
11. Add $\frac{15}{17}$, $\frac{16}{17}$, and $\frac{3}{17}$ together. Ans. 2.
12. Add $2\frac{1}{2}$, $3\frac{1}{2}$, and $4\frac{1}{4}$ together. Ans. $10\frac{1}{4}$.
13. Add $\frac{3}{2}$, $\frac{4}{3}$, and $\frac{7}{4}$ together. Ans. $4\frac{7}{12}$.
14. Add $12\frac{1}{2}$, $14\frac{1}{3}$, and $16\frac{1}{4}$ together. Ans. $43\frac{1}{12}$.
15. Add $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{3}{5}$ and $\frac{3}{4}$ of $\frac{8}{9}$ together. Ans. $1\frac{23}{30}$.
16. Add $\frac{4}{5}$, $\frac{13}{16}$, and $\frac{17}{30}$ together. Ans. $2\frac{43}{240}$.
17. Add $\frac{1}{2}$, $7\frac{1}{2}$, $8\frac{3}{4}$, $2\frac{4}{5}$, and $\frac{3}{7}$ together. Ans. $19\frac{137}{140}$.
18. Add $\frac{3}{4}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{2}{7}$, and $\frac{3}{14}$ together. Ans. $2\frac{1}{2}$.
19. Add $\frac{2}{7}$, $\frac{2}{14}$, $\frac{3}{21}$, $\frac{4}{28}$, $\frac{5}{35}$, and $\frac{6}{42}$ together. Ans. 1.
20. Add $\frac{9}{72}$, $\frac{11}{36}$, $\frac{4}{9}$, $\frac{7}{18}$, $\frac{5}{24}$, and $\frac{19}{36}$ together. Ans. 2.

---

* *Compound fractions* must be reduced to *single* ones, and *mixed numbers* (if not of high value) to *improper fractions*. When there are several mixed numbers to add together, it is best to add the whole numbers together first; to which add the sum of the fractions.

SUBTRACTION

OF VULGAR FRACTIONS.

RULE.—Reduce the fractions to a *common denominator*, (if they have not one,) as in the last rule, then subtract the *less* numerator from the *greater*, and under the difference put the common denominator.

EXAMPLE.

1. From $\frac{7}{12}$ take $\frac{3}{8}$.

$$\left.\begin{array}{r} 7 \times 8 = 56 \\ 12 \times 3 = 36 \end{array}\right\} \text{numerators.}$$

$$\text{difference of numerators } \frac{20}{\text{com. denom. } 12 \times 8 = 96} = \frac{5}{24} \text{ Ans.}$$

$$\text{or thus, } \frac{7 \times 8 - 12 \times 3}{12 \times 8} = \frac{56 - 36}{96} = \frac{20}{96} = \frac{5}{24}$$

2. From $\frac{4}{5}$ take $\frac{3}{4}$. Ans. $\frac{1}{20}$.
3. From $\frac{3}{4}$ take $\frac{2}{3}$. Ans. $\frac{1}{12}$.
4. From $\frac{7}{8}$ take $\frac{2}{7}$. Ans. $\frac{33}{56}$.
5. From $\frac{7}{12}$ take $\frac{5}{12}$. Ans. $\frac{1}{6}$.
6. From $\frac{9}{10}$ take $\frac{2}{3}$ of $\frac{3}{4}$. Ans. $\frac{2}{5}$.
7. From $9\frac{1}{4}$ take $6\frac{3}{4}$. Ans. $2\frac{1}{2}$.
8. From 8 take $\frac{2}{3}$. Ans. $7\frac{1}{3}$.
9. What is the difference between $\frac{3}{13}$ and $\frac{4}{39}$? Ans. $\frac{5}{39}$.
10. What is the difference between $\frac{1}{2}$ and $\frac{1}{3}$? Ans. $\frac{1}{6}$.
11. What is the difference between $\frac{9}{63}$ and $\frac{32}{28}$? Ans. 1.
12. From $\frac{2}{3}$ of $\frac{6}{7}$ take $\frac{7}{8}$ of $\frac{4}{7}$. Ans. $\frac{1}{14}$.
13. From $6\frac{2}{3}$ take $4\frac{3}{4}$. Ans. $1\frac{11}{12}$.
14. From $8\frac{5}{8}$ take $7\frac{3}{8}$. Ans. $1\frac{1}{4}$.
15. From $15\frac{7}{16}$ take $14\frac{11}{16}$. Ans. $\frac{3}{4}$.
16. What is the difference between $\frac{3}{14}$ of 7 and $\frac{5}{14}$ of 21? Ans. 6.
17. From $\frac{12}{13} + \frac{3}{4}$ take $\frac{17}{26} - \frac{3}{13}$. Ans. $1\frac{1}{4}$.
18. What number added to $\frac{14}{17}$ will make $2\frac{3}{5}$? Ans. $1\frac{66}{85}$.
19. What number taken from $2\frac{7}{8}$ will leave $\frac{7}{29}$? Ans. $2\frac{147}{232}$.

MULTIPLICATION

OF VULGAR FRACTIONS.

RULE.—Multiply all the numerators together for a new numerator, and all the denominators for a new denominator.

EXAMPLE.

1. Multiply $\frac{7}{12}$ of $\frac{6}{7}$ by $\frac{3}{4}$.

   * $\frac{7}{12} \times \frac{6}{7} \times \frac{3}{4} = \frac{126}{336} = \frac{21}{56} = \frac{3}{8}$. Ans.

2. Multiply $\frac{1}{2}$ by $\frac{1}{2}$. Ans. $\frac{1}{4}$.
3. Multiply $\frac{3}{4}$ by $\frac{4}{3}$. Ans. 1.
4. Multiply $\frac{3}{4}$ by $\frac{3}{4}$. Ans. $\frac{9}{16}$.
5. Multiply $\frac{3}{5}$ by $\frac{6}{7}$. Ans. $\frac{18}{35}$.
6. Multiply $\frac{7}{8}$ by $\frac{2}{3}$ of $\frac{3}{4}$. Ans. $\frac{7}{16}$.
7. †Multiply $5\frac{4}{7}$ by 9. Ans. $50\frac{1}{7}$.
8. Multiply 7 by $\frac{7}{9}$. Ans. $5\frac{4}{9}$.
9. Multiply $\frac{7}{10}$ by $\frac{2}{3}$ of 7. Ans. $3\frac{4}{15}$.
10. Multiply $\frac{9}{10}$ by $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$. Ans. $\frac{3}{8}$.
11. Multiply $\frac{3}{4}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{1}{8}$. Ans. $\frac{1}{24}$.
12. Multiply 24 by $\frac{2}{3}$ of $\frac{3}{4}$. Ans. 12.
13. Multiply 6 by $\frac{2}{3}$ of 5. Ans. 20.
14. Multiply the *sum* of $\frac{1}{2}$ and $\frac{1}{3}$ by 30. Ans. 25.
15. Multiply $17\frac{11}{13} - 16\frac{12}{13}$ by $8\frac{2}{3}$. Ans. 8.
16. Multiply $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and 6 together. Ans. 1.
17. Multiply $1\frac{1}{2}$, $\frac{1}{2}$, $2\frac{1}{2}$, and $\frac{1}{3}$ together. Ans. $\frac{5}{8}$.
18. Multiply $\frac{13}{15} + \frac{7}{10}$, $\frac{3}{7} - \frac{1}{10}$, and $1\frac{7}{23}$ together. Ans. $\frac{47}{70}$.

DIVISION

OF VULGAR FRACTIONS.

RULE.—Prepare the given fractions if necessary, then *invert* the divisor and proceed as in multiplication.

EXAMPLE.

1. Divide $\frac{4}{7}$ by $\frac{2}{3}$.

   $\frac{4}{7} \div \frac{2}{3} = \frac{4}{7} \times \frac{3}{2} = \frac{12}{14} = \frac{6}{7}$.

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* Here, by striking out the 7's, and dividing the 6 and 12 by 6, then multiplying the quotients, the answer is obtained.

† A fraction is multiplied by an integer, either by multiplying the numerator, or dividing the denominator; but it is usual to express the integer fractionally, by putting an unit under it.

2. Divide $\frac{3}{7}$ by $\frac{2}{7}$. Ans. $\frac{3}{2}$.
3. Divide $\frac{1}{2}$ by $\frac{1}{2}$. Ans. 1.
4. Divide $\frac{1}{4}$ by $\frac{1}{2}$. Ans. $\frac{1}{2}$.
5. Divide $\frac{1}{2}$ by $\frac{1}{4}$. Ans. 2.
6. Divide 4 by $\frac{3}{4}$. Ans. $5\frac{1}{3}$.
7. Divide $\frac{3}{4}$ by 4. Ans. $\frac{3}{16}$.
8. Divide $3\frac{1}{6}$ by $9\frac{1}{2}$. Ans. $\frac{1}{3}$.
9. Divide $7\frac{1}{3}$ by $9\frac{5}{9}$. Ans. $\frac{33}{43}$.
10. Divide 12 by $\frac{3}{4}$ of $\frac{4}{5}$. Ans. 20.
11. Divide $\frac{7}{8}$ of 16 by $\frac{9}{10}$ of 20. Ans. $\frac{7}{9}$.
12. Divide $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{7}{6}$ by 14. Ans. $\frac{2}{45}$.
13. Divide $\frac{5}{6} + \frac{6}{7}$ by $\frac{7}{12}$. Ans. $2\frac{44}{49}$.
14. Divide $\frac{6}{7} - \frac{5}{6}$ by $\frac{3}{7}$ of $\frac{1}{24}$. Ans. $1\frac{1}{3}$.
15. Divide $\frac{7}{16} + \frac{5}{12}$ by $\frac{7}{16} - \frac{5}{12}$. Ans. 41.

RULE OF THREE.

RULE.—State the question as in whole numbers, then *invert* the divisor, and multiply it by the product of the other two.

EXAMPLE.

1. If $1\frac{1}{2}$ yards of kerseymere cost 13s. 6d., what will $4\frac{3}{4}$ yards cost?

$$\begin{array}{lcccccc} & \textit{yds.} & & \textit{s.}\ \textit{d.} & & \textit{yds.} \\ \text{As} & 1\frac{1}{2} & : & 13\ \ 6 & :: & 4\frac{3}{4}. \\ \text{Or, as} & \frac{3}{2} & : & \frac{27}{40} & :: & \frac{19}{4}. \end{array}$$

$$\therefore \frac{2}{3} \times \frac{27}{40} \times \frac{19}{4} = \frac{1026}{480} = \frac{171}{80} = £2.\ 2s.\ 9d.\ \text{Ans.}$$

2. If $\frac{3}{4}$ of a yard cost £$\frac{1}{3}$., what will $\frac{1}{2}$ a yard cost at that rate? Ans. 4s. $5\frac{1}{3}$d.

3. If $5\frac{3}{4}$ yards cost £$\frac{3}{4}$., what will $2\frac{1}{2}$ yards cost? Ans. 6s. $6\frac{1}{4}$d... $\frac{1}{23}$.

4. If $\frac{1}{16}$ of a ship be worth £20. 5s. 0d., what is the value of $\frac{5}{8}$? Ans. £202. 10s. 0d.

5. If $\frac{3}{16}$ of a ship be worth £252. 12s. 6d., what is the value of $\frac{21}{96}$ of her? Ans. £294. 14s. 7d.

6. If three yards cost £$2\frac{4}{5}$., what will $10\frac{2}{7}$ yards cost at the same rate? Ans. £9. 12s. 0d.

* To divide a fraction by an integer—divide the numerator, if it be divisible; if not, multiply the denominator.

7. If $\frac{1}{8}$ of a steam boat be worth £73. 1s. 3d., what part of her is worth £250. 10s. 0d.? Ans. $\frac{3}{7}$.

8. By working 12 hours in the day, A can do a piece of work in 7 days, and B can perform the same in 5 days, how long will it serve them when they work together?

Ans. 2 days 11 hours.

9. Divide a guinea among A, B, and C, so that their shares may be in proportion of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ respectively.

Ans. A = 9s. $8\frac{1}{4}$d. . $\frac{3}{13}$. B = 6s. $5\frac{1}{2}$d. . $\frac{2}{13}$.
C = 4s. 10d. . . $\frac{8}{13}$.

10. Supposing a brewer to have 4 syphons, one of which will empty his cooler in a *quarter* of an hour, another in *half* an hour, one in *three quarters*, and the other in *an hour*, in what time would they empty it, all running together? Ans. $7\frac{1}{5}$ minutes.

11. How many yards, at 3s. 3d. a yard, are equivalent to $60\frac{1}{4}$ yards, at 4s. 4d. a yard? Ans. $80\frac{1}{3}$ yards.

12. What quantity of shalloon $\frac{3}{4}$ of a yard wide, will line $11\frac{3}{4}$ yards of cloth, $1\frac{1}{2}$ yard wide? Ans. $23\frac{1}{2}$ yards.

13. If $174\frac{6}{7}$ yards of cloth cost £43. 13s. 4d., what should be charged an ell English to gain £4. 7s. 4d. by the whole? Ans. 6s. $10\frac{1}{2}$d.

14. If $6\frac{5}{8}$ yards of cloth $\frac{9}{8}$ wide will make a cloak, how much at $\frac{3}{4}$ wide will line it? Ans. $9\frac{15}{16}$ yards.

15. If $\frac{5}{6}$ of a crown buy $\frac{5}{9}$ of an ell English, how many yards will $7\frac{3}{4}$ guineas buy? Ans. $27\frac{1}{8}$.

DECIMAL FRACTIONS.

A decimal fraction is a fraction whose denominator is 10, 100, 1000, &c., or an unit with as many cyphers after it as there are figures in the numerator, which numerator only is put down with a point before it. *Thus:*

$\frac{5}{10}$ = .5 five tenths. $\frac{25}{100}$ = .25 twenty-five hundredths, &c.

Decimals express the parts of an unit in the same manner as whole numbers express the number of units, as may be seen by the following table:

Whole numbers.							Decimal parts.					
7	6	5	4	3	2	1	.2	3	4	5	6	7
Millions.	C of Thousands.	X of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Parts of Tens.	Parts of Hundreds.	Parts of Thousands.	Parts of X Thousands.	Parts of C Thousands.	Parts of Millions.

1. From the above table it appears that as whole numbers *increase* in a ten-fold proportion to the *left hand*, decimal parts *decrease* in a ten-fold proportion to the *right hand*.

2. Cyphers placed before decimal parts, decrease their value, by removing them farther from the decimal point, or unit's place; thus, .5 is 5 parts of 10, or $\frac{5}{10}$; but .05 is 5 parts of 100, or $\frac{5}{100}$; .005 is 5 parts of 1000, or $\frac{5}{1000}$, &c.

3. Cyphers after decimal parts do not alter their value, for $.5 = .50 = .500 = \frac{5}{10}$.

To transform a Vulgar Fraction to an equivalent Decimal Fraction.

RULE.—Annex *cyphers* to the *numerator*, and divide by the denominator, the quotient will be the decimal fraction required.

EXAMPLES.

1. Reduce $\frac{3}{8}$ to a decimal.

   8)3.000

   .375 Ans.

2. Reduce $\frac{1}{16}$ to a decimal.

   16)1.0000

   .0625 Ans.

3. Reduce $\frac{1}{2}$ to a decimal. Ans. .5. *

4. Reduce $\frac{1}{3}$ to a decimal. Ans. $.333\dot{3}$. †

* A *finite* decimal is one that ends at a certain place.

† An *infinite* decimal is one that does not terminate; and if the same figures are repeated from the decimal point, it is called a *pure repeater*, the last of which is marked with a dot. It is not necessary to take them farther than one figure, because $.\dot{3}$ is equı alent to .3333, &c.

5. Reduce $\frac{1}{4}$ to a decimal. Ans. .25.
6. Reduce $\frac{3}{4}$ to a decimal. Ans. .75.
7. Reduce $\frac{7}{12}$ to a decimal. Ans. $.58\dot{3}3$. *
8. Reduce $\frac{5}{8}$ to a decimal. Ans. .625.
9. Reduce $\frac{4}{9}$ to a decimal. Ans. $.44\dot{4}$.
10. Reduce $\frac{7}{10}$ to a decimal. Ans. .7.
11. Reduce $\frac{5}{11}$ to a decimal. Ans. $.45\dot{4}\dot{5}$. †
12. Reduce $\frac{11}{12}$ to a decimal. Ans. $.916\dot{6}$.
13. Reduce $\frac{9}{16}$ to a decimal. Ans. .5625.
14. Reduce $\frac{7}{20}$ to a decimal. Ans. .35.
15. Reduce $\frac{4}{21}$ to a decimal. Ans. .190476. ‡
16. Reduce $\frac{51}{55}$ to a decimal. Ans. $.\dot{9}2\dot{7}$. +
17. Reduce $158\frac{1}{2}$ to a decimal. Ans. 158.5.
18. Reduce $\frac{5}{4}$ of $\frac{8}{9}$ to a decimal. Ans. $1.1\dot{1}\dot{1}$.
19. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ to a decimal. Ans. .375.
20. Reduce $\frac{7}{12}$ of $\frac{15}{7}$ to a decimal. Ans. 1.25.

ADDITION IN DECIMALS.

RULE.—Set down the numbers under each other according to the value of their places, and the decimal points will stand exactly under each other: then add them up as in whole numbers.

EXAMPLE.

1. Add 7.53 + .527 + 25.036 + 1.1111 + 100 together.

```
  7.53
   .527
 25.036
  1.1111
100.
--------
134.2041 Ans.
```

* When there are figures before the repeater, it is called a mixed repeater.

† When two or more figures repeat, it is called a circulating decimal.

‡ If there is a figure or figures between the decimal point and the circulating figures, it is called a *mixed circulate*; every infinite decimal would circulate, were it carried out far enough, but when the circle does not readily appear, the 5th or 6th figure is usually marked with a +

2. Add 1.64 + 32.543 + .023 + .2576 + 324 together. Ans. 358.4636.

3. Add 643 + .037 + 543.4 + 2.76543 + .4 together. Ans. 1189.60243.

4. Add 75.0756 + 3254.072 + 3.543 + .576 + .004 together. Ans. 3333.2706.

5. Add 3.20076 + 27 + .0058 + 7238.5 + 12.75 + .54344 together. Ans. 7282.

SUBTRACTION

OF DECIMAL FRACTIONS.

RULE.—Place the numbers as in addition, having the decimal points under each other, and subtract as in whole numbers.

EXAMPLE.

1.	From	72.0300	
	Take	7.2943	
		64.7357	Remains.
	*	72.0300	Proof.

2. From 764.23 take .76432. Ans. 763.46568.
3. From 10.0954 take 9.007. Ans. 1.0884.
4. From 960.2076 take 74.32. Ans. 885.8876.
5. From 100 take 35.076. Ans. 64.924.
6. From 5 take .0003. Ans. 4.9997.
7. From .0052 take .00489. Ans. .00031.
8. From 1 take .9876543. Ans. .0123457.
9. From 7 take 6.9988. Ans. .0012.
10. From 2 take 1.9876. Ans. 0124.

MULTIPLICATION OF DECIMALS.

RULE.—Place the factors and multiply them as in whole numbers, and from the product to the right hand, point off as many figures for decimals, as there are decimals in *both factors;* but should there not be as many figures in the product, prefix as many cyphers to the left hand as are wanted.

* Cyhers to the right of decimals are of no value.

EXAMPLES.

1. Multiply	3.75	2. Multiply	.0123
By	2.5	By	.0203
	1875		369
	750		2460
Product	9.375		.0024969

3. Multiply 79.347 by 23.15. Ans. 1836.88305.
4. Multiply .6348 by .8204. Ans. .52078992.
5. Multiply 3.56 by .764. Ans. 2.71984.
6. Multiply .3854 by .0012. Ans. .00046248.
7. Multiply 762 by .007. Ans. 5.334.
8. Multiply 5.103 by 10. * Ans. 51.03.
9. Multiply 5.103 by 100. Ans. 510.3.
10. Multiply 5.103 by 1000. Ans. 5103.
11. Multiply 3.25 by .001. Ans. .00325.
12. Multiply 13.107 by .00791. Ans. .10367637.
13. Multiply $132.\dot{4}$ by .8 † Ans. $105.9\dot{5}$.
14. Multiply $321.\dot{6}$ by 2.5. Ans. $804.1\dot{6}$.

DIVISION OF DECIMALS.

RULE 1.—Divide as in whole numbers, and point off from the quotient as many figures for decimals as the *dividend* has more than the divisor.

2. If the dividend has not so many places of decimal parts as are in the divisor, annex as many cyphers as will make them equal; the quotient will then be a *whole* number.

3. If the quotient has not as many *figures* as it should have decimal places, then as many cyphers must be put on the *left hand* as there are places wanting.

---

* Multiplying by 10, 100, &c. it is only removing the decimal point as many places to the right as the multiplier has cyphers.

† The *repeaters* being *ninths*, and the *finite* decimal *tenths*, the first product must be divided by 9. And when there are two or more figures in the multiplier, the remainder must be put down twice in the second product, three times in the third, &c. so that each product may end together.

EXAMPLES.

1. Divide .1239 by 7.

```
7).1239
   .0177 quotient.
```

2. Divide 8658.276 by 243.21.

```
243.21)8658.276(35.6 Ans.
       7296 3
       1361 97
       1216 05
        145 926
        145 926
```

3. Divide 75.32 by 8.	Ans. 9.415.
4. Divide 25.12 by 11.	Ans. $2.2\dot{8}3\dot{6}$.
5. Divide 25.12 by 1.1.	Ans. $22.\dot{8}3\dot{6}$.
6. Divide 42.576 by 1.2.	Ans. 35.48.
7. Divide 425.76 by .12.	Ans. 3548.
8. Divide 21321.9 by .9.	Ans. 23691.
9. Divide 1000.08 by .08.	Ans. 12501.
10. Divide 10.94 by 100 *	Ans. .1094.
11. Divide 2.71984 by .764	Ans. 3.56.
12. Divide 1836.88305 by 23.15.	Ans. 79.347.
13. Divide 123.70536 by 54.25.	Ans. 2.2802.
14. Divide .5942 by 176.	Ans. .003376+.
15. Divide 762 by 97.	Ans. 7.85567+.
16. Divide 376 by 1.28.	Ans. 293.75.
17. Divide 500 by .3123.	Ans. 1601.0246.
18. Divide .568 by 52.	Ans. .010923+.
19. Divide 1.324 by 36.2	Ans. .03657+.
20. Divide 762.15 by .325.	Ans. 2345.0769.

* When the divisor is 10, 100, &c. it is only removing the decimal point as many places to the *left* as the divisor has cyphers.

REDUCTION OF DECIMALS.

To reduce Money, Weights, and Measures, to their equivalent Decimals.

RULE 1.—Write the *given* numbers *under* each other for dividends: then divide each line, (beginning with the least name) by as many as make one of the next greater; the last quotient will be the required decimal.

Or 2ndly.—Reduce the given quantity to a vulgar fraction (page 100) and that fraction to a decimal.

EXAMPLES.

1. Reduce 12s. $6\frac{3}{4}$d. to the decimal of a £.

4	3	far.		*s.* *d.*		*s.*
12	6.75	d.	Or thus:	12 $6\frac{3}{4}$		20
2,0	12.5625	s.		12		12
£	.628125	Ans.		150		240
				4		4
				603		960

$\therefore \frac{603}{960} = \frac{201}{320} = .628125$ Ans.

2. Reduce 10s. to the decimal of a £. Ans. .5.
3. Reduce 6d. to the decimal of a shilling. Ans. .5.
4. Reduce 6s. 8d. to the decimal of a £. Ans. $.3\dot{3}$.
5. Reduce 4d. to the decimal of a shilling. Ans. $.\dot{3}$.
6. Reduce 8d. to the decimal of a £. Ans. $.0\dot{3}$.
7. Reduce 2s. 6d. to the decimal of a £. Ans. .125
8. Reduce 3s. 6d. to the decimal of a crown. Ans. .7.
9. Reduce 3s. $4\frac{1}{4}$d. to the decimal of a £. Ans. .16875.
10. Reduce 5s. $4\frac{1}{2}$d. to the decimal of a £. Ans. .26875.
11. Reduce 12s. $8\frac{1}{4}$d. to the decimal of a £. Ans. .634375.
12. Reduce 11s. $9\frac{1}{2}$d. to the decimal of a £. Ans. $.58958\dot{3}$.
13. Reduce 1s. $11\frac{3}{4}$d. to the decimal of a £. Ans. $.098958\dot{3}$.
14. Reduce 18s. $11\frac{3}{4}$d. to the decimal of a £. $.948958\dot{3}$.

15. Reduce 2 roods and 20 perches to the decimal of an acre. Ans. .625.

16. Reduce 2qrs. 14lb. to the decimal of a cwt. Ans. .625.

17. Reduce 2ft. 4in. to the decimal of a yard. Ans. $.\dot{7}77$.

18. Reduce 5bush. 3pks. to the decimal of a quarter. Ans. .71875.

19. Reduce 8½ inches to the decimal of a foot. Ans. $.708\dot{3}$.

20. Reduce 13oz. 12dr. to the decimal of a lb. Ans. .859375.

To find the value of a Decimal Fraction.

RULE.—Multiply the number of decimals by the number of parts in the next inferior denomination, cutting off the decimals from the product; then multiply the *remaining decimals* by the next *inferior* denominator, and cut off the decimals as before, thus proceeding till you have brought in the least known parts of the integer, the figures cut off on the left hand will be the answer.

EXAMPLES.

1. Value .628125 of a £.

```
    .628125
         20
  ---------
  12.562500
         12
  ---------
   6.750000
          4
  ---------
   3.000000 or 12s. 6¾d.
```

2. Value $.\dot{3}33$ of a £.*

```
  .333
    20
 -----
 6.666
    12
 -----
 8.000 or 6s.8d
```

3. What is the value of .5 of a £? Ans. 10s.
4. What is the value of .5 of a shilling? Ans. 6d.
5. What is the value of .125 of a £? Ans. 2s. 6d.
6. What is the value of .7 of a crown? Ans. 3s. 6d.
7. What is the value of .16875 of a £? Ans. 3s. 4½d.

* In valuing a repeating decimal, observe to carry, at the last or right hand, by 9 instead of 10; and in multiplying by 20, or any number with cyphers, put down the first product as often as there are cyphers in the multiplier.

8. What is the value of .26875 of a £? Ans. 5s. $4\frac{1}{2}$d.
9. What is the value of .634375 of a £? Ans. 12s. $8\frac{1}{4}$d.
10. What is the value of $.58958\dot{3}$ of a £? Ans. 11s. $9\frac{1}{2}$d.
11. What is the value of $.098958\dot{3}$ of a £?
Ans. 1s. $11\frac{3}{4}$d.
12. What is the value of $.948958\dot{3}$ of a £?
Ans. 18s. $11\frac{3}{4}$d.
13. What is the value of $1.\dot{3}$ of a £? Ans. £1. 6s. 8d.
14. What is the value of $.\dot{3}$ of a shilling? Ans. 4d.
15. What is the value of $.0\dot{3}$ of a £? Ans. 8d.
16. What is the value of .625 of an acre? Ans. 2r. 20p.
17. What is the value of .625 of a cwt? Ans. 2qr. 14lb.
18. What is the value of $.\dot{7}$ of a yard? Ans. 2ft. 4in.
19. What is the value of .859375 of a ℔ troy?
Ans. 10oz. 6dwts. 6gr.
20. What is the value of .6319755 of a cwt?
Ans. 2qr. 14lb. 12oz. 8dr.

To transform Decimals to their equivalent Vulgar Fractions, when the Decimal is finite.

RULE.—Place the given decimal for the numerator, and an unit with as many cyphers annexed as there are figures in the given decimal for a denominator, then reduce the fraction to its lowest terms.

EXAMPLES.

1. Reduce .125 to a vulgar fraction.

$$.125 = \frac{125}{1000} = \frac{1}{8}. \text{ Ans.}$$

2. Reduce .7 to a vulgar fraction. Ans. $\frac{7}{10}$.
3. Reduce .25 to a vulgar fraction. Ans. $\frac{1}{4}$.
4. Reduce .75 to a vulgar fraction. Ans. $\frac{3}{4}$.
5. Reduce .625 to a vulgar fraction. Ans. $\frac{5}{8}$.
6. Reduce .775 to a vulgar fraction. Ans. $\frac{31}{40}$.
7. Reduce .5625 to a vulgar fraction. Ans. $\frac{9}{16}$.
8. Reduce .05 to a vulgar fraction. Ans. $\frac{1}{20}$.
9. Reduce .9375 to a vulgar fraction. Ans. $\frac{15}{16}$.
10. Reduce .00224 to a vulgar fraction. Ans. $\frac{7}{3125}$.

When the Decimal is a pure circulate.

RULE.—Place the given decimal for the numerator with as many 9's as there are places in the circulate, for a denominator.

EXAMPLES.

1. Reduce $.\dot{6}$ and $2.\dot{0}6\dot{3}$ to vulgar fractions.

Here $.\dot{6} = \frac{6}{9} = \frac{2}{3}$. Ans. And $2.\dot{0}6\dot{3} = 2\frac{63}{999} = 2\frac{7}{111}$. Ans.

2. Reduce $.\dot{3}\dot{6}$ to a vulgar fraction. Ans. $\frac{4}{11}$.
3. Reduce $.\dot{7}$ to a vulgar fraction. Ans. $\frac{7}{9}$.
4. Reduce $.\dot{7}\dot{2}$ to a vulgar fraction. Ans. $\frac{8}{11}$.
5. Reduce $.\dot{0}\dot{9}$ to a vulgar fraction. Ans. $\frac{1}{11}$.
6. Reduce $.\dot{4}\dot{5}$ to a vulgar fraction. Ans. $\frac{5}{11}$.
7. Reduce $1.\dot{1}$ to a vulgar fraction. Ans. $1\frac{1}{9}$.
8. Reduce $.\dot{1}4\dot{8}$ to a vulgar fraction. Ans. $\frac{148}{999}$.
9. Reduce $.\dot{3}4\dot{2}$ to a vulgar fraction. Ans. $\frac{38}{111}$.
10. Reduce $.\dot{7}073\dot{1}$ to a vulgar fraction. Ans. $\frac{29}{41}$.

When the Decimal is a mixed circulate.

RULE.—Subtract the finite part from the whole mixed decimal, the remainder is the numerator; under which place as many 9's as there are repeating places in the circulate, and annex as many cyphers as there are *finite* places before the circulate.*

EXAMPLES.

1. Reduce $.13\dot{8}$ and $2.4\dot{1}\dot{8}$ to vulgar fractions.

1st. $.13\dot{8} = \frac{138-13}{900} = \frac{125}{900} = \frac{5}{36}$. Ans.

2nd. $2.4\dot{1}\dot{8} = 2\frac{418-4}{990} = 2\frac{414}{990} = 2\frac{23}{55}$. Ans.

2. Reduce $.1\dot{6}$ to a vulgar fraction. Ans. $\frac{1}{6}$.
3. Reduce $.8\dot{3}$ to a vulgar fraction. Ans. $\frac{5}{6}$.
4. Reduce $.58\dot{3}$ to a vulgar fraction. Ans. $\frac{7}{12}$.
5. Reduce $.0\dot{6}$ to a vulgar fraction. Ans. $\frac{1}{15}$.

* Various rules have been divised by writers on circulating decimals, for addition, subtraction, multiplication, and division; but the plainest way is to transform them into vulgar fractions, and to operate by those rules, then transform the results into decimals.

6. Reduce $.00\dot{3}$ to a vulgar fraction. Ans. $\frac{1}{300}$.
7. Reduce $.11\dot{3}\dot{6}$ to a vulgar fraction. Ans. $\frac{5}{44}$.
8. Reduce $.521\dot{7}$ to a vulgar fraction. Ans. $\frac{587}{1125}$.

RULE OF THREE IN DECIMALS.

EXAMPLES.

1. If $6\frac{3}{4}$ yards cost £5. 12s. 6d., what will $2\frac{1}{4}$ yds. cost?

```
          yds.     £.  s.  d.     yds.
      As. 6¾   :   5   12  6  ::  2¼

  Or, as  6.75  :  5.625  ::  2.25 *
                   2.25
                  -----
                  28125
                 11250
                11250         £.
           6.75)12.65625(1.875
                675          20
                ----      ------
                5906      17.500
                5400          12
                ----      ------
                 5062      6.000
                 4725
                 ----
                  3375          £.  s.  d.
                  3375     Or, 1   17  6
                  ====     ============
```

2. If 12lb. cost £4. 8s. 0d., what will 30lb. cost? Ans. £11. 0s. 0d.

3. If $12\frac{1}{2}$ yards cost £3. 10s. 6d., what will $18\frac{3}{4}$ cost? Ans. £5. 5s. 9d.

4. If 1 quarter 4 bushels of wheat cost £4. 1s. 0d., what will 3 quarters 6 bushels cost? Ans. £10. 2s. 6d.

5. If silver be sold at 5s. 3d. per ounce, what is the value of 32oz. 5dwts? Ans. £8. 9s. $3\frac{3}{4}$d.

6. If $12\frac{3}{4}$lbs. cost 10s. $4\frac{1}{2}$d., what is the worth of $127\frac{1}{2}$lbs. at the same rate? Ans. £5. 3s. 9d.

7. If $\frac{3}{8}$ of a yard of velvet cost 8s., what will $\frac{5}{16}$ of a yard cost? Ans. 6s. 8d.

---

* Since the first term is just *three* times the *third*, the answer would be obtained by dividing the *second* term by 3.

8. If I buy $12\frac{5}{8}$ yards of superfine broad cloth for 10 guineas, what quantity can I buy for £52. 10s. 0d?
Ans. 63,125 yards.

9. Paid 12s. 9d. for the carriage of 12cwt. 3qrs., from Leeds to Bradford; what weight ought I to have carried for £1. 16s. $10\frac{1}{2}$d.? Ans. 36cwt. 3qr. 14lb.

10. What is the worth of 1lb. 8oz. 10dwts. of gold, when 10dwts. are sold for £1. 12s. 3d.? Ans. £66. 2s. 3d.

11. How many yards of carpeting, a yard broad, will it take to cover a room that took $18\frac{3}{4}$ yards, $1\frac{1}{2}$ yard broad? Ans. $28\frac{1}{8}$ yards.

12. If £530. 15s. 0d. will gain £17. 10s. 0d. in $10\frac{1}{2}$ months, what principal will gain an equal sum in 15.25 months? Ans. £365. 8s. $8\frac{1}{4}$d.

13. If $3\frac{3}{8}$ yards of cambric cost £1. 2s. 6d., what ought to be charged for $16\frac{7}{8}$ yards? Ans. £5. 12s. 6d.

14. If $7\frac{1}{4}$ yards cost £3. 10s. $8\frac{3}{4}$d., what will $12\frac{1}{2}$ yards cost? Ans. £6. 1s. $10\frac{1}{2}$d.

INVOLUTION

Is the raising of powers from any proposed root, or the method of finding the *square, cube, biquadrate, &c.*, of any number.

RULE.—Multiply the given number into itself for the *square*, twice into itself for the *cube*, three times for the *biquadrate*, &c.

EXAMPLES.

1. Required the 5th power of 12.

12	root.	Or, thus: 12	
12		12	
144	square.	144	= 2nd power.
12		144	
1728	cube.	576	
12		576	
20736	biquadrate	144	
12		20736	= 4th power.
248832	= 5th power.	12	
		248832	= 5th power.

2. What is the square of 17? Ans. 289.
3. What is the square of 24? Ans. 576.
4. What is the cube of 25? Ans. 15625.
5. What is the biquadrate of 20? Ans. 160000.
6. Raise 5 to the 5th power. Ans. 3125.
7. Raise 7 to the 6th power. Ans. 117649.
8. What is the square of .053? Ans. .002809.
9. What is the cube of .005? Ans. .000000125.
10. What is the square of $\frac{3}{7}$? Ans. $\frac{9}{49}$.

EVOLUTION

Is the reverse of Involution, or it is the finding of the square root, cube root, &c. of any number.

SQUARE ROOT.

To extract the *square root* of any number, is to find a number which is the side of a square, or one which being multiplied into itself, produces the first number.

RULE 1.—Begin at the *units* place and point the given numbers into periods of *two* figures each. If the figures consist of whole numbers and decimals, the whole numbers must be pointed from *right* to *left*, the decimals from *left* to *right*.

2.—Find the *greatest square* number that is contained in the *first period* towards the left hand; placing the square under the same, and its *root* in the *quotient*.

3.—Subtract the square number from the first period, and to the remainder bring down the next two figures or period for a dividend.

4.—Double the quotient and place it for a divisor on the left hand of the dividend; see how often it is contained therein, (rejecting the units place) and put the answer in the quotient, and also on the right hand of the divisor.

5.—Multiply the divisor by the last figure put in the quotient, and subtract the product from the dividend; to the remainder bring down the next period, and so on till all the periods are brought down.

6.—If any thing remain, annex two cyphers and repeat the work, and for every two thus added a decimal will be obtained in the root.

Roots	1 .	2 .	3 .	4 .	5 .	6 .	7 .	8 .	9 .
Square	1 .	4 .	9 .	16 .	25 .	36 .	49 .	64 .	81 .

EXAMPLES.

1. What is the square root of $\dot{1}5\dot{1}2\dot{9}$(123 root.

```
      1
22)  51
     44
243) 729
     729
```

3. What is the square root of $9\dot{6}0\dot{4}$(98

```
      81
188) 1504
     1504
```

2. What is the square root of 5(2.23606 +

```
          4
42)     100
         84
443)    1600
        1329
4466)   27100
        26796
447206) 3040000
        2683236
      * 456764
```

4. What is the square root of 441 ? Ans. 21.
5. What is the square root of 1024 ? Ans. 32.
6. What is the square root of 3364 ? Ans. 58.
7. What is the square root of 9409 ? Ans. 97.
8. What is the square root of 12321 ? Ans. 111.
9. What is the square root of 10609 ? Ans. 103.
10. What is the square root of 43681 ? Ans. 209.
11. What is the square root of 974169 ? Ans. 987.
12. What is the square root of 1522756 ? Ans. 1234.
13. What is the square root of 24750625 ? Ans. 4975.

* When the number, whose root is required, is a surd, that is, has not an exact root, the operation may be carried on to 4 or 5 places of decimals, which is generally sufficient for all practical purposes.

14. What is the square root of 128?
Ans. 11.31370849.

15. What is the square root of 9712.71805?
Ans. 98.553 +.

16. What is the square root of .00076128?
Ans. .02759 +.

17. What is the square root of 3? Ans. 1.73205 +.

18. What is the square root of $\frac{16}{49}$? Ans. $\frac{4}{7}$ } *

19. What is the square root of $\frac{539}{1100}$? Ans. $\frac{7}{16}$ }

20. What is the square root of $\frac{5}{8}$? Ans. .79056 +.

21. What is the square root of $8\frac{3}{5}$? Ans. 2.9325 +.

CUBE ROOT.

To extract the cube root of any number, is to find the length of the side of a solid figure, whose length, breadth, and thickness are equal; or a number, which multiplied into itself, and that product again by the same number, will produce the proposed number.

RULE 1.—Divide the given number into periods of *three* figures each, beginning at the units place, pointing to the *left* in integers, and to the *right* in decimals. †

2.—Find the greatest cube in the first period, which subtract, placing to root in the quotient, and bring down another period for a dividend.

3. Multiply the root by 3 for a supplemental divisor, and its square by 3 for the *first* trial divisor, and annex two cyphers.

4.—Divide the dividend by the trial divisor, and the quotient is the next figure of the root, which figure annex to the supplemental divisor.

5.—Then multiply the supplemental divisor thus increased by the last figure, and put the product under the

---

* When the root of a fraction is required, reduce it to its lowest terms; then if each part be a square, its root may generally be found by inspection; but if it be a surd, transform it to a decimal, and operate as before.

† This rule is in principle the same as the one in general use. The method of finding the trial divisors (after the first) without squaring the quotient, constitutes its great advantage.

trial divisor, add the two together for the complete divisor, which multiply by the last figure, and subtract the product from the dividend, and to the remainder bring down another period for another dividend.

6.—Multiply the root thus found by 3 for a *supplemental* divisor, and for the *trial divisor*, add (mentally) the *square* of the last quotient figure to the *sum* of the *two last lines* in the trial divisor, and annex two cyphers as before, and so on.

Roots	1	2	3	4	5	6	7	8	9
Cubes	1	8	27	64	125	216	343	512	729

EXAMPLE.

1. Required the cube root of 14814434816.

```
                                14,814,434,816(2456 root
sup. div.  trial div.            8
             12,00               6814
64 × 4  =   256
            1456 × 4 =           5824
            1728.00              990434
 725 × 5 =  3625
            176425 × 5 =         882125
            180075.00            108309816
7356 × 6 =  441 36
            18051636 × 6 =       108309816
```

EXERCISES.

2. What is the cube root of 9261? Ans. 21.
3. What is the cube root of 50653? Ans. 37.
4. What is the cube root of 110592? Ans. 48.
5. What is the cube root of 912673? Ans. 97.
6. What is the cube root of 1728000? Ans. 120.
7. What is the cube root of 15813251? Ans. 251.
8. What is the cube root of 57960603? Ans. 387.
9. What is the cube root of 204336469? Ans. 589.
10. What is the cube root of 529475129? Ans. 809.

11. What is the cube root of 14437662875? Ans. 2435.
12. What is the cube root of 28991029248? Ans. 3072.
13. What is the cube root of 55088885125? Ans. 3805.
14. What is the cube root of 64144108027? Ans. 4003.
15. What is the cube root of 723183538176? Ans. 8976.
16. What is the cube root of 7? Ans. 1·9129312+.
17. What is the cube root of 21? Ans. 2·7589243+.

DUODECIMALS; OR CROSS MULTIPLICATION.

RULE.—Under the multiplicand write the corresponding denominations of the multiplier, that is, feet under feet and inches under inches, &c. Multiply each term of the multiplicand, beginning at the lowest denomination, by the feet in the multiplier, and write the several products under the respective terms of the multiplicand from which they arise; observing to carry 1 for every 12 from each lower denomination to the next higher. Then multiply by the inches in the multiplier (if any) in the same manner, and write the products each one place on the right hand, carrying as before at 12. Next multiply by the parts, (if any) writing each product another place to the right, and so on; the sum of the several lines will be the answer.

EXAMPLES.

1. Multiply 3 feet 9 inches by 2 feet 6 inches.

	ft.	*in.*		
	3	9		
	2	6		
	7	6		by × by 2
	1	10	6	by × by 6
*	9	4	6	Ans.

---

* The same result may be obtained by *practice*, *decimals*, *vulgar fractions*, and *integers;* by one or other of which methods the learner should occasionally prove the results. The 9 is square feet; the 4 is not square inches, but four twelfths of a foot; the 6, which is called parts, is actually square inches. Every inferior denomination is one twelfth of the next superior denomination.

2. Multiply 7 feet 8 inches 9 parts by 3 feet 4 inches 3 parts.

ft.	in.	pts.			
7	8	9			
3	4	3			
23	2	3			by × by 3 feet
2	6	11	0		by × by 4 inches
	1	11	2	3	by × by 3 parts
25	11	1	2	3	Ans.

	ft.	in.	p.		ft.	in.	p.		ft.	in.	p.	iii	iiii
3. Mult.	4	6		by	3	9		Ans.	16	10	6		
4. Mult.	5	4		by	2	6		Ans.	13	4	0		
5. Mult.	9	3		by	3	5		Ans.	31	7	3		
6. Mult.	6	8		by	5	9		Ans.	38	4	0		
7. Mult.	8	4		by	6	6		Ans.	54	2	0		
8. Mult.	12	7		by	4	11		Ans.	61	10	5		
9. Mult.	15	9		by	8	5		Ans.	132	6	9		
10. Mult.	16	4		by	9	6		Ans.	155	2	0		
11. Mult.	17	1		by	10	7		Ans.	180	9	7		
12. Mult.	18	5		by	11	8		Ans.	214	10	4		
13. Mult.	25	3		by	12	10		Ans.	324	0	6		
14. Mult.	9	2	3	by	4	3	5	Ans.	39	4	4	8	3
15. Mult.	55	6	0	by		9	3	Ans.	42	9	4	6	
16. Mult.	7	8	10	by	6	9	6	Ans.	52	6	5	11	

Practice.

3	9	
	$2\frac{1}{2}$	
7	6	
1	10	6
9	4	6

Decimals.

3.75
2.5
1875
750
9.375
12
4.500
12
6.000 9 4 6

Vulgar Fractions.

$3\frac{9}{12} = \frac{15}{4}$

$2\frac{6}{12} = \frac{5}{2}$ ft. in. p.

$\therefore \frac{15}{4} \times \frac{5}{2} = \frac{75}{8} = 9 \quad 4 \quad 6$

Integerly.

$$\left.\begin{array}{l} 3 \quad 9 = 45 \\ 2 \quad 6 = 30 \end{array}\right\} \textit{inches.}$$

$$\frac{1350}{144} = 9 \quad 4 \quad 6$$

THE CARPENTER's SLIDING RULE.

DESCRIPTION:—This instrument is much used by artificers, in taking dimensions, and computing the contents of their work; it consists of two equal pieces of box, each 12 inches in length, connected by a folding joint, having in one piece a brass slider; on which are two lines marked B. C. and two marked A. D. on the rule. Three of these lines A, B, C, are similar both in number and division; being numbered from 1 to 10 twice over, and the divisions on D from 4 to 40; this line is called the girt line; it is also marked W. G. at 17.15. and A. G. 18.95, the wine and ale gauge points, to make it serve the purpose of a *gauging rule.*

The edge of the rule is usually divided into tenths; namely each foot into 10 equal parts, and each part into 10 other equal parts; by which, dimensions may be taken in feet, tenths, and hundredths of a foot.

On the same side there is generally a table of the value of a load of wood (50 cubic feet) at all prices, from 12d. to 30d. a foot.

Construction of the *lines* A, B, C:—Suppose a scale of half of the length of these lines, to be divided into 1000 equal parts, then because the logarithm of 1 is 0, 1 will stand at the beginning of the lines; also, because the log. of 2 is .301, the distance between 1 and 2 on the *lines* will be = to 301 equal parts of the supposed scale; and because the log. of 3 is .477, the distance between 1 and 3 on the *lines* will be 477 of the same equal *parts,* and for the same reason, the distance between 1 and 4 is 602 parts, between 1 and 5 is 699 parts, between 1 and 6 is 778 parts, between 1 and 7 is 845 parts, &c.; which parts, form the primary divisions of the above *lines,* and the intermediate divisions are formed in a similar manner by taking the logarithms of the intermediate numbers. The line D is constructed in the *same way,* except that the radius of D is double that of A, B, C; that is, the logarithmic distances must be doubled.

To estimate the value of the divisions.

When the 1 on the left is accounted 1, the one in the middle will be 10, and the 10 at the end will be 100; if the 1 at the beginning be accounted 10, 100, 1000, &c. then the integral numbers 2, 3, 4, &c. will represent twice, thrice, four times, &c. as much.

The values of the integral divisions being thus estimated, those of the intermediate divisions may be readily known; thus if the first one on the lines A, B, C, denote 1, then since there are 50 divisions on the lines between 1 and 2, the value of each intermediate will be $\frac{1}{50}$ or .02. On the same supposition, since the second 1 and 2 represent 10 and 20, and the space between them divided into 50 parts, the value of each of which is $\frac{10}{50} = \frac{1}{5} = .2$, and in a similar way may the values of the intermediate divisions between any two adjoining integral numbers be known.

USE OF THE SLIDING RULE.

PROBLEM 1.—*To multiply one number by another.*

RULE.—Set 1 on B to either of the factors on A; then against the other factor on B is the product on A.

EXAMPLES.

1. Multiply 3 by 7, by the sliding rule.

Set 1 on B to 3 on A; then against 7 on B is 21 on A, *

2. Multiply 7 by 9. Ans. 63.
3. Multiply 9 by 12. Ans. 108.
4. Multiply 9 by 16. Ans. 144.
5. Multiply 17 by 25. Ans. 425.

PROBLEM 2.—*To divide one number by another.*

RULE.—Set 1 on B to the *divisor* on A; then against the *dividend* on A stands the quotient on B.

---

* By drawing out the slider till 1 on B is opposite to 3 on A, we obtain the sum of the distances 1 to 3 on A, and 1 to 7 on B, but by the construction of the lines their distances are as

$$\log. 3 + \log. 7 = \log. 3 \times 7 = \log. 21.$$

EXAMPLES.

1. Divide 45 by 5, by the sliding rule.

Set 1 on B to 5 on A; then against 45 on A is 9 on B. *

2. Divide 75 by 5. Ans. 15.
3. Divide 432 by 12. Ans. 36.
4. Divide 560 by 16. Ans. 35.
5. Divide 816 by 17. Ans. 48.

PROBLEM 3.—*To find a fourth proportional.*

RULE.—Set the first term on B to the second on A, then against the third term on B is the fourth on A.

EXAMPLES.

1. If 3cwt. cost £24., what will 7cwt. cost?

Set 3 on B to 7 on A; then against 24 on B is 56 on A. †

2. If 2cwt. 2qrs. cost £16. what will 5cwt. cost? Ans. £32.
3. If 12 yards cost £5. 10s. what will 48 yards cost? Ans. £22.

PROBLEM 4.—*To extract the square root.* ‡

RULE.—Set the middle division, 1 on C to 10 on D; then against the given number on C stands its *square root* on D.

EXAMPLES.

1. Find the square root of 144 by the sliding rule.

Set 1 on C to 10 on D, then against 144 on C is 12, the root on D.

2. Find the square root of 196. Ans. 14.
3. Find the square root of 441. Ans. 21.
4. Find the square root of 1225. Ans. 35.
5. Find the square root of 3025. Ans. 55.

---

* The slider being drawn out till 1 on B is opposite to 5 on A, we obtain the difference of the distances of 1 to 5 on A, also of 1 to 45 on A (expressed on B 1 to 9) but by the construction of the lines, the difference

$$= \log. 45 - \log. 5 = \log. \tfrac{45}{5} = \log. 9.$$

† By drawing out the slider till 3 on B is opposite to 7 on A, we obtain the distance from 1 to 7 + the distance from 1 to 24 on A — the distance of 1 to 3 on B, that is

$$\log. 7 + \log. 24 - \log. 3 = \log. (7 + \tfrac{24}{3}) = \log. (7 + 8) = \log. 56.$$

‡ If the given number consists of an *even* number of figures, it is to be found on the *left* hand part of the line C, and if of an odd number, on the *right* hand part of C

PROBLEM 5.—*To find a mean proportional between two given numbers, or the square root of their product.*

RULE.—Set one of the given number on C to the same number on D; then against the other number on C is the mean proportional on D.

EXAMPLES.

1. Find a mean proportion between 9 and 25.
Set 9 on C to 9 on D; then against 25 on C is 15 on D.

2. Find the mean proportional between 16 and 9, by the sliding rule. Ans. 12.

3. Find the mean proportional between 4 and 36, by the sliding rule. Ans. 12.

4. Find the mean proportional between 16 and 36, by the sliding rule. Ans. 24.

5. Find the mean proportional between 25 and 81, by the sliding rule. Ans. 45.

TIMBER MEASURE.

PROBLEM 1.—*To find the area of a plank.*

RULE.—Multiply the *length* by the *mean breadth*, and the product will be the area.*

EXAMPLES.

1. What is the content of a plank, 14 feet 6 inches long and 15 inches broad?

By Duodecimals.

ft.	*in.*		
14	6		
1	3		
14	6		
3	7	6	
18	1	6	Ans.

By the Sliding Rule.

12 on B : 15 on A :: 14½ on B : 18½ on A.

or, by Reduction.

$$\frac{174 \times 15}{144} = \frac{145}{8} = 18\tfrac{1}{8}. \text{ Ans.}$$

* When the board tapers, add the breadth at each end together, and divide their sum by 2 for a mean breadth; or measure it in the middle.

2. Find the area of a plank 1 foot 6 inches broad and 10 feet 3 inches long. Ans. 15ft. 4in. 6pt.

3. What is the content of a board 15 feet 6 inches long and 16 inches broad? Ans. $20\frac{2}{3}$ feet.

4. What is the content of a plank 15 feet 1 inch long and 17 inches broad? Ans. 21 feet 4′ 5.″

5. What is the content of a plank 12 feet 8 inches long and 11 inches broad? Ans. $11\frac{1}{18}$ feet.

6. What is the content of a plank 24 feet 4 inches long and 14 inches broad? Ans. 28ft. 4in. 8pt.

7. What is the content of a plank 15 feet long, the breadth at one end being 15 inches, and at the other end 13 inches? Ans. $17\frac{1}{2}$ feet.

8. Bought 24 deal planks, 17 feet long and 11 inches broad, at 1s. $1\frac{1}{2}$d. a foot, what did they cost me?

Ans. £21. 0s. 9d.

PROBLEM 2.—*To find the content of squared timber.*

RULE.—Multiply the breadth by the depth, and that product by the length.

BY THE SLIDING RULE.

Find the mean proportional between the breadth and depth, that is the square root of their product; then set the length in feet on C, to 12 on D, and against the mean proportional in inches on D, is the solid content in feet on C.

EXAMPLES.

1. What is the content of a log of timber 16 feet long, its mean breadth and thickness being 18 inches?

$$16 \times 1\tfrac{1}{2} \times 1\tfrac{1}{2} = 16 \times \tfrac{3}{2} \times \tfrac{3}{2} = \tfrac{144}{4} = 36 \text{ cubic feet. Ans.}$$

By the Sliding Rule.

As 16 on C : 12 on D :: 18 on D to 36 on C.

2. Find the content of a log of wood the breadth and thickness each 14 inches, and length 24 feet. Ans. $32\frac{2}{3}$ feet.

3. What number of solid feet are there in a piece of timber 10 feet long, and its breadth and thickness each 15 inches? Ans. $15\frac{5}{8}$ feet.

4. How many solid feet are there in a piece of timber 15 inches broad, 10 inches thick, and 24 feet long? Ans. 25 feet.

5. Find the content of a log of wood, whose length is 18 feet, breadth 18 inches, and depth 12 inches? Ans. 27 feet.

6. Find the content of a piece of timber $18\frac{1}{2}$ feet long, 18 inches broad at one end, and 14 at the other, and the thickness at one end 15 inches, and 11 at the other.

Ans. $26\frac{13}{18}$ feet.

7. Find the content of a piece of timber 20 feet long, 20 inches broad at the greater end, and 12 inches at the less end; its thickness 16 inches at the greater end, and 8 inches at the less end. Ans. $26\frac{2}{3}$ feet.

PROBLEM 3.—*To find the content of unsquared timber.*

RULE.—Multiply the square of $\frac{1}{4}$, of the mean circumference by the length for the content.*

BY THE SLIDING RULE.

Set the length on C to 12 on D, then against the quarter girt in inches on D is the solid content in feet on C.

EXAMPLES.

1. What is the content of a piece of round timber whose length is 20 feet and mean circumference 24 inches?

$$\frac{6^2 \times 20}{12 \times 12} = \frac{720}{144} = 5 \text{ feet. Ans.}\dagger$$

$$\text{Or } \tfrac{1}{2} \times \tfrac{1}{2} \times 20 = \tfrac{20}{4} = 5 \text{ feet. Ans.}$$

By the Sliding Rule.

As 20 on C : 12 on D :: 6 on D : 5 on C. Ans.

---

* This Rule, though very inaccurate, is universally used by the dealers in green wood.

The method of finding the content of round timber by the Landing Waiters in Her Majesty's Ports, is considerably nearer the truth, and is as follows: multiply the square of the quarter girt in inches by the length in feet, and divide the product by 113,081 (or shortly by 113, the area of a circle inscribed in a square whose side is one foot) for the content in cubic feet; or by the sliding rule, thus, set the length on the slide or line C to the guage point 10,635 (the square root of 113,081) on the girt or line D, and against one fourth of the circumference on the girt line, is the content on the slide.

† Here the quarter girt being in inches, the product must be divided by 144 to bring it into feet.

2. Find the content of a tree, its length being 15 feet and mean girt 36 inches. Ans. $8\frac{7}{16}$ feet.

3. Find the content of a tree, the length of which is 25 feet, and mean girt 6 feet. Ans. $56\frac{1}{4}$ feet.

4. Find the content of a tree, 30 feet long and mean girt 4 feet. Ans. 30 feet.

5. Find the content of a tree, its length being 18 feet and mean girt 33 inches. Ans. 8.5078 feet.

6. Find the content of a tree, the length of which is 21 feet and a quarter girt $8\frac{3}{4}$ inches. Ans. 11.165.

7. Find the content of a tree, the length of which is 35 feet and mean girt 48 inches. Ans. 35 feet.

8. Find the content of a tree, of which the length is 20 feet, the girt at the greater end 6 feet, and at the less end 4 feet.* Ans. $31\frac{1}{4}$ feet.

9. Required the cubic feet in an irregularly tapered tree; the length of which is 32 feet, and the girts in the middle of every 8 feet, 78 inches, 62 inches, 52 inches, and 48 inches. Ans. 50 feet.

10. Find the content of a tree, its length being 28 feet, the girt at the greater end 8 feet 4 inches, and four others at 7 feet distance, 7 feet 3 inches, 6 feet 6 inches, 4 feet 1 inch, and 3 feet 10 inches. Ans. 63 feet.

11. Find the content of a tree, the length of its trunk being 50 feet, and its quarter girt in the middle 32 inches; the length of a large bough 15 feet, and its quarter girt 15 inches; the length of another bough 11 feet, and its quarter girt 10 inches. Ans. 386.63 feet.

---

* When a tree tapers *regularly*, the girt may be taken at the middle for the mean girt, or it may be taken at both ends, and half the sum will be the mean girt.

When a tree tapers *irregularly*, being thick in some places and small in others, the girt may be taken at the ends, and at equal distances, then will the sum of the girts, divided by their number, be the mean girt; or the tree may be divided into several lengths, and the content of each part computed separately; then will their sum be the content of the whole tree.

When the tree has its bark on, a deduction is usually made from the quarter girt, according to the thickness of the bark, in order to reduce it to such a circumference as it would have without its bark. For ash, beech, &c. half an inch for every foot of the quarter girt is generally deemed a fair allowance; but for oak, an inch or inch and a half is thought necessary.

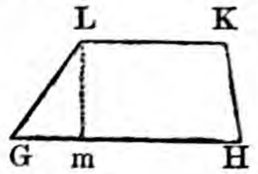
MENSURATION OF SURFACES.

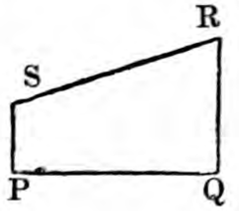
Problem 1.—*To find the area of a square, rectangle, parallelogram, &c.* *

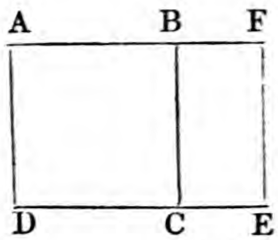
Rule.—Multiply the length by the breadth, and the product will be the area, or multiply half the sum of the parallel sides by their perpendicular distance.

EXAMPLES.

1. What is the area of the square A B C D, whose side is 3 feet 6 inches?







ft.	*in.*	
3	6	
3	6	
10	6	
1	9	
12	3	Ans.

2. What is the area of the square, its side being 21 feet? Ans. 441 feet.

3. What is the area of the rectangle A F E D, its length being 6 feet, and breadth 3 feet 6 inches? Ans. 21 ft.

4. What is the area of a rectangular field, its length being 575 links, and breadth 425 links?† Ans. 2a. 1r. 31p.

---

* A quadrilateral figure is bound by four straight lines: and when the sides are parallel it is called a *parallelogram;* if the angles are right angles, it is called a *rectangle;* if the sides are equal and the angles right angles, it is termed a *square;* when the sides are all equal, but the angles not right angles, it is termed a *rhombus;* when the opposite sides only are equal, and the angles not right angles, it is termed a *rhomboid;* when the sides are unequal, it is termed a *trapezium;* and a trapezium with two of its sides parallel, is called a *trapezoid.*

† Land is measured by a chain 22 yards in length, which is divided into 100 links.

The area in links divided by 100,000 (the square links in an acre) gives acres, then multiplying the remainder respectively by 4 and 40, and dividing as before gives roods and perches; or it is the same thing to cut off 5 figures to the right for decimals.

5. What is the area of a field, its length being 1980 links, and breadth 1125 links? Ans. 22a. 1r. 4p.

6. How many square yards are there in a rectangular field, whose length is 5 chains, and breadth 2 chains?
Ans. 4840 yards = 1 acre.

7. What is the area of a rectangular field, its length being 14.5 chains and breadth 9.75 chains? Ans. 14a. 0r. 22p.

8. The side of a square field is 147 yards, what would it cost digging at 7d. a rood? Ans. £12. 17s. 3d.

9. What is the area of a rhombus, its side being 84, and perpendicular breadth 24? Ans. 2016.

10. A grass-plot in the form of a rhombus cost £3. 9s. 4d. sodding, at 4d. a yard, required its area and length, its perpendicular breadth being 39 feet.
Ans. area 208 square yards, length 48 feet.

11. What is the area of a trapezoid, its parallel sides L K, G H being 25 and 31, and their perpendicular distance L m, 12 yards? Ans. 336.

12. What is the area of a field in the form of a trapezoid, its parallel sides being 700 and 340 links, and their perpendicular distance 250 links? Ans. 1a. 1r. 8p.

13. What would a garden in the form of a trapezoid P Q R S, cost digging at 12s. 6d. an acre, the parallel sides S P, R Q being 436 and 564 links, and their distance P Q, 385 links? Ans. £1. 4s. 0¾d.

14. Required the side of a square field, whose area contains 23049601 square links. Ans. 4801 links.

15. The area of a rectangular field is 2 acres 1 rood 31 perches, what is its length, its breadth being 425 links? *
Ans. 575 links.

16. Required the side of a square court that cost £18. 1s. 3d. paving, at 1s. 3d. yer yard. Ans. 17 yards.

17. Required the area and length of a rectangular piece of land, which cost £3. 1s. 10¾d.—⅗ digging, at £1. 5s. 4d. an acre, its breadth being 425 links.

Ans. { 2a. 1r. 31p. area.
575 links, length.

---

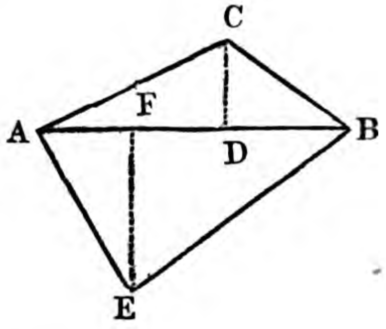
* The area in perches multiplied by 625 (square links in a perch) gives links. Similar areas are to each other, as the square of their like sides.

Problem 2.—*To find the area of a triangle, when the base and perpendicular are given.*

Rule.—Multiply the base by the perpendicular, and half the product will be the area.*

EXAMPLES.

1. What is the area of a triangular field A B C, whose base A B is 756 links, and perpendicular C D, 345 links?

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      756
      345
     ----
     3780
    3024
   2268
  -------
 2)260820
  -------
  1,30410
        4
  -------
  1,21640
       40              a.  r.  p.
  -------
  8,65600    Ans.      1   1   8
```

2. What is the area of a triangle, whose base is 38 yards and perpendicular 18 yards? Ans. 342 yards.

3. The base of a triangular field measures 1958 links, and perpendicular 994 links, what is its area?

Ans. 9a. 2r. 37p.

4. Required the area of a triangle, its base being 21.24 and perpendicular 12.5? Ans. 132.75.

---

* 1. Double the area divided by the base gives the perpendicular.

2. Twice the area divided by *half* the sum of the sides, gives the diameter of the inscribed circle. ∴ *half* the sum of the sides multiplied by the radius, gives the area.

3. The *diameter* of a circle circumscribing the triangle A B C, is $=$ A C $\times$ B C $\div$ D C, or twice the product of the three sides divided by their *sum*, gives the product of the diameters of the circumscribing and inscribed circles, ∴ if one be known, the other is found by division.

4. The product of any two sides, multiplied into the natural sine of their included angle, gives double the area, ∴ double the area divided by the product of any two sides, gives the natural sine of their included angle.

5. All the angles of every triangle contain 180°, which is equal to two right angles, or half a circle.

5. What is the area of a triangular plot of land, the base of which measures 36 chains, and perpendicular 21 chains? Ans. 37a. 3r. 8p.

6. What is the annual rent of a triangular field, which lets for £2. 11s. 6d. per acre; its base measuring 25, and perpendicular 16 chains? Ans. £51. 10s. 0d.

7. How many square yards are there in a triangle, the base of which measures 28, and perpendicular 18 feet? Ans. 28 yards.

8. Measuring along the base of a triangular field 564 links, I found the place of the perpendicular, and the perpendicular itself, 640 links; the remainder of the base measured 636 links, what is the area?* Ans. 3a. 3r. 14p.

9. Measuring along the base of a triangular field I found the perpendicular to rise at 465, and its length 445 links; the remainder of the base measured 635 links: required the area. Ans. 2a. 1r. 31½p.

10. Measuring from A, along the diagonal A B of the trapezium or four sided field A C B E, the perpendicular F E rose at 300 links, and its length 550 links; the perpendicular C D rose at 620, and its length was 300 links: the remainder of the diagonal measured 480 links; required the area of the field.† Ans. 4a. 2r. 28p.

11. Measuring along the diagonal of a four sided field, the perpendicular to my right rose at 320, and measured 235 links, and the perpendicular to my left rose at 730, and measured 435 links; required the area, the remainder of the diagonal measuring 2 chains. Ans. 3a. 0r. 18p.

PROBLEM 3.—*To find the area of a triangle, when three sides are given.*

RULE. From *half the sum* of the three sides, subtract each side severally, multiply the *half sum*, and the three

* These, and the following, should be laid down with a plotting scale.

† If half the sum of the perpendiculars be multiplied into the diagonal, it gives the area at once.

In measuring land, whatever are the shapes of the fields, practical surveyors always divide them into triangles, lay them down on paper, and find the perpendiculars by the plotting scale. When off-sets are taken from the chain line to the ins and outs of the fence, they straighten the fence by drawing a give-and-take-line upon the plan before they cast it; the method of doing it is by applying a piece of clear lantern-horn, which has been plained even at the edge between two pieces of wood, to the crooked fence, so as to include as much on one side as they exclude on the other.

remainders continually together, and the square root of the product will be the area.

EXAMPLES.

1. Required the area of a triangle, the sides of which are 26, 28, and 30.

26			
28	42	42	42
30	26	28	30
2)84	16 = 1st diff.	14 = 2nd diff.	12 = 3rd diff.
42 = half sum.			

$$\therefore \sqrt{42 \times 16 \times 14 \times 12} = \sqrt{112896} = 336 \text{ Ans.}$$

2. Required the area of a triangle, the three sides being 13, 14, and 15. Ans. 84.

3. Required the area of a triangle, the sides of which are 24, 36, 48. Ans. 418.282.

4. Required the area of an equilaterial triangle, each side being 36 yards. Ans. 561.1844 yards.

5. Required the area of a triangular field, the sides of which measure 15, 20, and 25 chains. Ans. 15 acres.

6. Required the area of a triangular field, the sides of which are 9, 12, and 15 chains. Ans. 5a. 1r. 24p.

7. Required the area of a field in the form of an isosceles triangle; its base being 20, and each of its other sides 15 chains. Ans. 11a. 0r. 28p.

8. What is the area of a triangular field, the sides of which are 760, 628, and 456 links? Ans. 1a. 1r. 28p.

9. In measuring a four sided field, I represented the angles on an eye plan by A, B, C, D, and found A B = 8 chains, B C = 9 chains, C D = 7 chains, D A = 10 chains, and the diagonal A C = 11 chains: required the area.* Ans. 6a. 3r. 36p.

---

* As an additional exercise, lay down the field, and cast the following off-sets taken on the lines, on A B, at the end of the 1st, 2nd, 3rd, 5th, and 6th chains, 25, 40, 50, 60, and 40 links; on B C, at 2, 4, and 7 chains, 35, 40, and 80 links; on C D, at 1, 3, and 5 chains, 60, 25, and 70 links; on D A, at 1, 2, 3, 4, 5, and 7 chains, 20, 40, 60, 30, 25, and 80 links; at the beginning and end of each line, O . Area on A B = 29000, B C = 37000, C D = 28000, D A = 38750 square links, or, 1a. 1r. 12p. to add.

10. Representing the angles of a five sided field by A, B, C, D, E; the side A B measures 720 links, B C 350 links, and diagonal C A 940 links; also, C D 630, D E 600, and diagonal C E, 1030 links; also, A E 600 links: required the area. Ans. 5a. 2r. 18p.

11. The three sides of a triangle are 336, 293, and 85, required its area, and diameter of the circumscribing circle. Ans. area, 11424; diam. 366.25.

12. The sides of a triangle are 52, 56, and 60, required its area, and diameters of inscribed and circumscribing circles.* Ans. area 1344, diam. 32 and 65.

PROBLEM 4.—*Any two sides of a right angled triangle being given, to find a third.*†

RULE 1.—When two legs are given to find the hypotenuse, add the *square* of one leg to the *square* of the other, and the *square root* of the sum will be the hypotenuse.

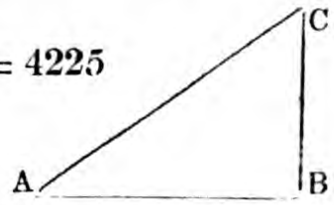
2.—When the hypotenuse and one of the legs are given to find the other leg; from the *square* of the hypotenuse, subtract the *square* of the given leg, and the square root of the remainder will be the required leg.

EXAMPLES.

1. In the right angled triangle A B C are given the base A B, 56, and perpendicular B C, 33, to find the hypotenuse A C.

Here $56^2 + 33^2 = 3136 + 1089 = 4225$

and $\sqrt{4225} = 65 = $ A C.



* Twice the distance between the middle of the base and the point of contact made by the inscribed circle = the *difference* of the sides: and the base multiplied into the difference of the segments of the base made by the perpendicular, gives the *product* of the *sum* into the *difference* of the sides ∴ the *sum* is found by division.

The difference between the radius of the circumscribing circle and the diameter of the inscribed, multiplied by the radius of the former, gives the square of the distance of their centres.

† The base and perpendicular of a right angled triangle are frequently called *legs*. The *right angle* is opposite the hypotenuse.

2. The *difference* between the *hypotenuse* and *sum* of the other two sides is equal to the *diameter* of the inscribed circle.

3. The product of the segments of the hypotenuse made by the contact of the inscribed circle, is equal to the area.

2. If the hypotenuse A C be 53 and base A B 45; what is the perpendicular B C?

$$\text{Here } 53^2 - 45^2 = 2809 - 2025 = 784$$

$$\text{And } \sqrt{784} = 28 = \text{B C.}$$

3. The base of a right angled triangle is 77, and the perpendicular 36; what is the hypotenuse? Ans. 85.

4. The hypotenuse of a right angled triangle is 109, and perpendicular 60; what is the base? Ans. 91.

5. Close by the side of a river, 44 feet broad, is a wall 33 feet high; how long must a ladder be to reach from the edge of the water to the top of the wall? Ans. 55 feet.

6. A boy flying his kite had 220 yards of cord out, when it stuck upon the vane of a steeple; required its height, his distance being the length of a field 8 chains long. Ans. 132 yards.

7. A lamplighter placed his ladder, which is 15 feet long, against a wall so that its top just reached the top of the wall, its bottom projecting 3 feet; how far would the top of the ladder fall, by removing its foot 2 feet further from the wall? Ans. $6\frac{1}{2}$in. +.

8. A ladder 50 feet long being placed in a street, reached a window 40 feet from the ground on one side, and by turning it over without removing the foot, it reached another 30 feet high on the other side; required the breadth of the street. Ans. 70 feet.

9. Two travellers started from the same place at eight in the morning, one travelling *north-west* 6 miles an hour, the other *north-east* 8 miles an hour; how far were they asunder at one o'clock of the same day? Ans. 50 miles.

10. The hypotenuse of a right angled triangle is 780, and the perpendicular 468; find the base, area, and natural sines of the acute angles A, C.*

Ans. base 624, area 146016, sines .6 and .8.

---

* The natural sine of angle A = B C ÷ A C = cosine of angle C. Sine C = A B ÷ A C = cos. A. Angle

11. The hypotenuse of a right angled triangle is 520 and the base 340, find the perpendicular, area, and sines of the acute angles.

Ans. perp. 393.446, area 66885.82, sines .65384 and .75662.

12. The hypotenuse of a right angled triangle is 5, and the sines of the acute angles .6 and .8, find the legs and area. Ans. legs 3, 4, area 6.

13. The natural sines are .6 and .8, and base 48 chains, required the hypotenuse and area.

Ans. hypotenuse 60 chains, area 86a. 1r. 24p.

14. Required the legs of a right angled triangle, its area being 24 and hypotenuse 10.* Ans. 6 and 8.

15. The area of a right angled triangle is 30, and hypotenuse 13; required the legs. Ans. 5 and 12.

16. The area of a right angled triangle is 336, and hypotenuse 50; required the legs. Ans. 14 and 48.

PROBLEM 5.—*To find the area of a regular polygon.*

RULE.—Multiply the sum of the sides by the perpendicular demitted from its centre to one of its sides, and half the product will be the area.

2.—Multiply the square of one side by the number or area opposite to the name of the polygon in the following table, and the product will be the area.

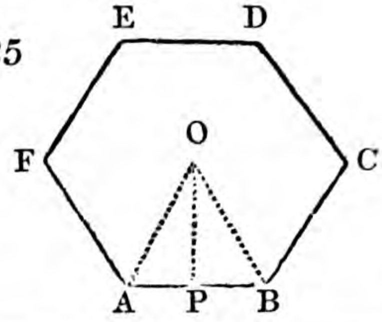
B = 90° = a *right* angle, the sine of which is 1. And as the sine of any angle is to its opposite side, so is the sine of any other angle to its opposite side, and vice versa. The tangent of the angle A = B C ÷ A B = cot. C. Tan. C = A B ÷ B C = cot. A. Also secant of A = A C ÷ A B = cosec. C. Sec. C = A C ÷ B C = cosec. A. And from a table of natural sines the angles themselves may be found.

* When the *area* and *hypotenuse* are given to find the legs.

RULE.—*To* and *from* the *square* of the hypotenuse, *add* and *subtract four times* the area, and the square root of this *sum* and *difference*, gives the *sum* and *difference* of the legs; then half the difference *added* to half the *sum* gives the greater leg, and half the *difference* subtracted from half the *sum* gives the less leg. This is a new rule.

EXAMPLES.

1. Required the area of the regular hexagon A B C D E F; the side of which is 20.5 feet, and perpendicular O P, 17.75 feet.



Here $20.5 \times 6 \times 17.75 = 2183.25$

$\therefore \frac{2183.25}{2} = 1091.625$ feet.

Ans.

Name.	*Number of Sides.*	*One half angle of the Polygon.*	*Area when the side is 1.*	*Perpendicular when the side is 1.*
Equilateral Triangle..	3	30°	0.4330127	0.288675136
Square	4	45°	1.	0.5
Pentagon ..	5	54°	1.7204770	0.6881909602
Hexagon...	6	60°	2.5980762	0.8660254038
Heptagon ..	7	$64°\frac{2}{7}$	3.6339124	1.0382606984
Octagon ...	8	$67°\frac{1}{2}$	4.8284271	1.2071067812
Nonagon...	9	70°	6.1818242	1.3737387097
Decagon ...	10	72°	7.6942088	1.5388417686
Undecagon .	11	$73°\frac{7}{11}$	9.3656399	1.7028436194
Duodecagon	12	75°	11.1961524	1.8660254038

2. Find the area of a pentagon, the side of which is 25 feet, and perpendicular 17.2. Ans. 1075 feet.

3. Find the area of an hexagon, the side of which is 24 yards. Ans. 1496.4918912 square yards.

4. Find the area of an octagon, whose side is 415 links. Ans. 8a. 1r. 10p.

5. Find the area of a nonagon, its side being 280 links. Ans. 4a. 3r. 15p.

6. Find the area of a decagon, its side being 50 yards. Ans. 19235.522 square yards.

7. Find the area of a duodecagon, its side being 100 links. Ans. 1a. 0r. 19p.

8. Find the side of an hexagon, its area being 1496.4918912. Ans. 24.

9. The area and perpendicular of a pentagon are 1075 and 17.2, required the side. Ans. 25.

10. The area of a decagon is 1923552.2, required the side. Ans. 500.

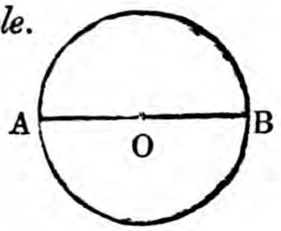
PROBLEM 6.—*To find the area of a circle.**

RULE 1.—Multiply the *square* of the diameter by .7854, or the *square* of the radius by 3.1416, or the square of the circumference by .07958 and the product will be the area.

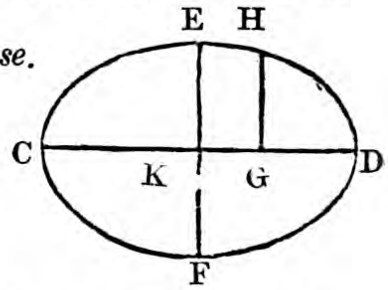
EXAMPLES.

1. If the diameter, A B, or longest straight line that can be drawn in a circle, be 7 yards; what is its area?

Circle.



Ellipse.



Here $7 \times 7 \times .7854 = 38.4846$ square yards. Ans.

2. Find the area of a circle, its diameter being 66 feet. Ans. 3421.2024 square feet.

3. Find the area of a circle, the diameter of which is 100 yards. Ans. 7854 yards.

4. Find the area of a circular table, the diameter of which is 3 feet 6 inches. Ans. 9.62115 feet.

5. Find the area of a circle, the diameter of which is 1612 links. Ans. 20a. 1r. 25p.

6. Find the area of a circular park, the diameter of which is 2754 links. Ans. 59a. 2r. 11p.

7. Find the area of a circle, the circumference of which is 5280 links. Ans. 22a. 0r. 29p.

8. Find the area of a circle, its circumference being 6005 links. Ans. 28a. 2r. 31p

* If the diameter of a circle be 1, its circumference is 3.1416 *very nearly*, and area .7854. AO = O B radius = half the diameter.

9. Find the diameter of a circle, its area being 1 square yard.* Ans. 1.1283 yards.

10. Find the diameter of a circular field, its area being 24200 square yards. Ans. 175.53439 yards.

11. Find the diameter of a circle, its area being 38.4846 square yards. Ans. 7 yards.

12. Find the circumference of a circle, its area being 392.7. Ans. 70.2471.

13. Find the circumference of a circle, its area being .7854. Ans. 3.1416.

14. Find the diameter of a circle, its circumference being 31.416.† Ans, 10.

15. Find the circumference of a circle, its diameter being 14 ‡ Ans. 43.9824.

16. The circumference of the earth being 25000 miles, what is its diameter? Ans. 7957.72854.

17. The diameter of the sun being 883220 miles, what is its circumference? Ans. 2774723.952 miles.

18. The diameters of two concentric circles are 150 and 120, what is the area of the ring included between their circumferences?§ Ans. 6361.74.

19. A circular island, the diameter of which is 50 yards, is surrounded by a moat, the breadth of which is 5 yards: required the area of the water. Ans. 863.94 square yards.

20. A circular field, the diameter of which is 4 chains, is surrounded by a ditch 6 yards broad, what is the area of the ditch and field?

Ans. { 1a. 1r. 1p. field.
0. 1 18 ditch. }

21. There is a circular island the area of which is 50 acres, and it is surrounded by a moat of equal width; the area of which is 10 acres: required the circumference of the island, and the breadth of the moat.

Ans. { 1743.8660316 yards circumference.
26.4902 yards breadth. }

* The area of a circle divided by .7854, the quotient is the square of the diameter.

† The diameter of any circle multiplied by 3.1416, the product is the circumference.

‡ The circumference of any circle divided by 3.1416, the quotient is the diameter.

§ Concentric circles are circles having the same centre; *the difference* of their areas is the area of the ring; or, the difference of the *squares* of their diameters multiplied by .7854, gives the area of the ring.

22. What is the area of an ellipse, the transverse diameter being 20, and conjugate 15?* Ans. 235.62.

23. Find the area of an ellipse, the transverse and conjugate diameters being 600 and 480 links.

Ans. 2a. 1r. 1p.

24. Find the area of an ellipse, the transverse and conjugate diameters being 480 and 360 links.

Ans. 1a. 1r. 17p.

25. An elliptical island is surrounded by a moat 50 links broad, required the area of land and water, the transverse and conjugate diameters of the land being 8 and 6 chains.

Ans. { 3a. 3r. 3p. land.
1. 0. 28. water.

26. What is the circumference of an ellipse, the transverse diameter being 7, and the conjugate diameter 1?

Ans. 15.708.

27. What is the circumference of an ellipse, the transverse and conjugate diameters being 140 and 120?

Ans. 409.6.

28. Find the circumference of an ellipse, the transverse and conjugate diameters of which are 480 and 360.

Ans. 1332.8.

29. The area of an ellipse is 706.86, and its conjugate diameter 25, find the radius of a circle having the same area, together with the area of the ring included between the circles described on the conjugate and transverse diameters.† Ans. rad. 15, area of ring 527.0034.

30. The areas of three circles are 7.0686, 12.5664, and 19.635; find the area of a triangle formed of their diameters, together with the area of its inscribed circle.

Ans. { area of triangle,‡ 6.
——— insc. circle, 3.1416.

---

* The area of an ellipse is found by multiplying the *product* of the two diameters by .7854. The circumference is found by multiplying the square root of *half the sum* of the squares of the two diameters by 3.1416. C D, E F are the transverse and conjugate diameters, C G, G D are the abscisses to the ordinate G H.

† The ellipse is equal to a circle, the diameter of which is a mean proportional between the two axis, that is equal to the square root of their product.

‡ The least whole numbers, forming the sides of a right angled triangle are 3.4 and 5.

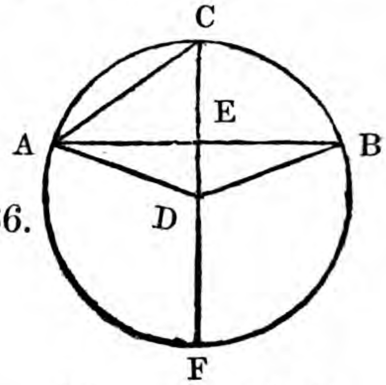
PROBLEM 7.—*To find the length of any arc of a circle.*

RULE 1.—From eight times the chord of *half* the arc subtract the chord of the *whole* arc, and $\frac{1}{3}$ of the remainder will be the length of the arc *nearly*.

RULE 2.—Multiply the number of degrees in the arc by the radius, and by .0174533, the last product will be the length of the arc, very *nearly*. Or, as 360° : circumference, :: degrees in the arc, : length of the arc.

EXAMPLES.

1. Find the length of the arc A C B; A C the chord of *half* the arc being 6, and versed sine C E, 4.*



Here $\sqrt{AC^2 - EC^2} = AE$.

Or, $\sqrt{6^2 - 4^2} = \sqrt{20} = 4.472136$.

$$\therefore \frac{6 \times 8 - 4.472136 \times 2}{3} = \frac{48 - 8.944272}{3} =$$

13.018576 Ans.

2. Required the length of an arc, the chord of which is 16, and the chord of half the arc 12. Ans. $26\frac{2}{3}$.

3. Half the chord of the whole arc is 12, and the chord of half the arc 15: required the length of the arc. Ans. 32.

---

* A B = the chord of the *whole* arc A C B; A C = the chord of the arc A C, or chord of *half* the arc A C B. A E = E B = *half* the chord of the whole arc. C E × E F = A E × E B = A E² = E B². A D = D B = C D = D F = radius. ∴ A E² ÷ C E + C E = C F = A C² ÷ C E. The space bound by A D B C A is called a *sector*, and that bound by A E B C A a *segment*. Join A F, then the triangles A F C, A F E, and A E C are similar: therefore C F : A C :: A F : A E :: A C : E C.

4. What is the length of an arc, the radius being 20, and the chord of the whole arc 24? Ans. 25.73093.

5. The radius is 10, and chord of half the arc 12: required the length of the arc. Ans. 25.6.

6. The versed sine C E is 9, and co-versed sine E F 16: required the length of the arc A C B. Ans. 32.

7. What is the length of an arc, the versed sine being 4 and chord of half the arc 8? Ans. 16.7145.

8. What is the length of the arc, the diameter being 50 and versed sine 8? Ans. 41.113.

9. What is the length of an arc of 83° 24′, the radius of the circle being 32? Ans. 46.5793.

10. Required the length of an arc of 30° 30′, the diameter being 100. Ans. 26.61628.

PROBLEM 8.— *To find the area of any sector of a circle.*

RULE.—Multiply the length of the arc by the radius, and half the product will be the area.*

EXAMPLES.

1. Find the area of a sector, the chord of its arc being 32 feet, and radius 20 feet.

Here $\sqrt{AD^2 - AE^2} = DE$. (see last figure.)

or $\sqrt{20^2 - 16^2} = \sqrt{144} = 12 = DE$.

$\therefore 20 - 12 = 8 = EC$.

and $\sqrt{AE^2 + CE^2} = AC$

or $\sqrt{16^2 + 8^2} = \sqrt{320} = 17.88854 = AC$

$\therefore \dfrac{17.88854 \times 8 - 32}{3} = 37.0361$ = length of the arc, which multiplied by 10, half the radius, gives 370.361 the area.

---

* Or as 360° : area of the circle, :: the degrees in the arc : the area of the sector.

2. Find the area of a sector, the chord of half the arc being 6, and versed sine 3. Ans. 37.6278.

3. Find the area of a sector, the chord of its arc being 40 yards, and the radius 50. Ans. 1028.8.

4. Find the area of a sector, the chord of its arc being 12 chains, and radius 18. Ans. 11a. 0r. 1p.

5. Find the area of a sector, the chord of its arc being 600 links, and the chord of half the arc 500 links. Ans. 1a. 3r. 3p.

6. Find the area of a sector, the arc of which contains 30°, the diameter being 10 yards. Ans. 6.545 yards.

7. Find the area of a sector, the arc of which contains 60° 30′, the radius being 10. Ans. 52.796.

8. Find the area of a sector, its arc containing 80° 40′, the diameter being 16 chains. Ans. 4a. 2r. 0p.

PROBLEM 9.—*To find the area of any segment of a circle.*

RULE.—Find the area of the sector which has the same arc with the segment: find also the area of the triangle formed by the chord of the segment, and the radii of the sector, then the *difference* or *sum* of these areas will be that of the segment, according as it is less or greater than a semicircle.

EXAMPLES.

1. Find the area of a segment, the chord of which is 40, and height 10.

Here $\sqrt{AE^2 + EC^2} = \sqrt{AC^2} = AC$ chord of half the arc, (see last figure.)

or $\sqrt{20^2 + 10^2} = \sqrt{500} = 22.36067$

$\therefore \frac{22.36067 \times 8 - 40}{3} = \frac{178.88536 - 40}{3} = \frac{138.88536}{3}$

$= 46.29512$ length of the arc A C B.

Also $AE^2 \div EC + EC = FC$ the diameter

or $400 \div 10 + 10 = 50$

$\therefore 50 \div 2 = 25$ the radius C D

and $CD - CE = 25 - 10 = 15 = ED$ the perpendicular. Now the area of the sector is

$$\frac{46.29512 \times 25}{2} = 23.14756 \times 25 = 478.689$$

and triangle, A E × D E = 20 × 15 = 300

∴ the area of the segment = 278.689

2. Find the area of a segment, the chord being 12, and height 2. Ans. 16.3274.

3. Find the area of a segment, the diameter being 8, and height 2. Ans. 9.78633.

4. Find the area of the segment, the cord being 20, and height 6. Ans. 85.113198.

5. Find the area of a segment, the diameter being 25, and cord of half the arc 15. Ans. 158.

6. Find the area of a segment, the arc of which contains 90°, the diameter being 20. Ans. 28.54.

7. Find the area of a segment, the arc of which contains 270°, the diameter being 50. Ans. 1785.125.

8. Find the area of a segment, the arc of which contains 30°, the radius being 50.* Ans. 29.5.

9. Find the area of a segment, the arc of which contains 60°, the radius being 12. Ans. 13.0442.

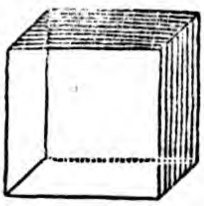
10. The length of the arc is 26.18, and the radius 50: required the area of the segment. Ans. 29.5.

MENSURATION OF SOLIDS.

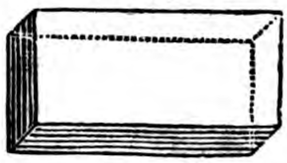
PROBLEM 1.— *To find the solidity of a cube and parallelopipedon.*

RULE.—Multiply the length by the breadth, and that product by the depth.

Cube.



Parallelopipedon.



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* The natural sine of 30° is $\frac{1}{2}$ $= .5$

———————— 60° is $\frac{1}{2} . 3^{\frac{1}{2}} = .8\,66025$

EXAMPLES.

1. What is the surface and solidity of a cube, whose side is 4 feet 6 inches?*

By Decimals.

ft.	*in.*	
4	6	= 4.5
4	6	= 4.5
		225
		180
		20.25
		4.5
		10125
		8100
		91.125
		12
		1.500
		12
		6.000 or 91 *ft.* 1 *in.* 6 *p.*

By Duodecimals.

ft.	*in.*		
4	6		
4	6		
18	0		
2	3		
20	3		× 6 = 121 *ft.* 6 *in.* surface.
4	6		
81	0		
10	1	6	
91	1	6	solidity.

2. What is the surface and solidity of a cube, its side being 18 inches? Ans. sur. 13ft. 6in. sol. 3ft. 4in. 6p.

3. What is the surface and solidity of a parallelopipedon, its length being 20 feet, breadth 10 feet, and depth 5 feet? Ans. surface 700ft. sol. 1000ft.

4. What is the solid content of a gravestone, its length being 6ft. 6in. breadth 4ft. and thickness 6in.? Ans. 13ft.

5. What would a cellar cost digging at 6d. per solid yard, its length being 24ft. breadth 12ft. and depth 8ft.? Ans. £2. 2s. 8d.

6. At 6d. per cubic yard, what would be the expense of digging a cellar, 40ft. 4in. long, 25ft. 7in. broad, and 9ft. 9in. deep? Ans. £9. 6s. 3½d.

* The surface is found by adding together the area of the six faces. And as an additional exercise, the pupil ought to find the surface of each solid as he proceeds. Some schoolmasters have the solid figure cut in wood, which greatly facilitates the comprehension of the pupil. Similar solids are to each other as the cubes of their like sides.

7. What quantity of water will a cistern hold, that is 12ft. 4in. long, 8ft. 6in. deep, and 6ft. 6in. broad?

Ans. 4246.658 gallons.

8. How many cubic yards of marle will be required to cover a field $1\frac{1}{2}$in. deep; the length of which is 10 chains, and breadth 1 chain? Ans. $201\frac{2}{3}$.

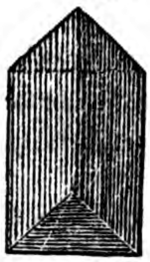
9. How much was a square garden raised, the side of which is 60 yards, by the earth taken from a bed of clay 36 yards long and 24 broad, the earth being 18in. deep?

Ans. 4.32 inches.

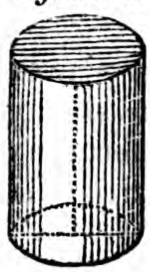
PROBLEM 2.—*To find the solidity of a prism and cylinder.**

RULE.—Multiply the area of the end by the perpendicular height.

Prism.



Cylinder.



EXAMPLES.

1. What is the solid content of a triangular prism, its length being 16 feet, one of the sides of its base being 2 feet, and a perpendicular drawn to that side from the opposite angle 18 inches?

$$\text{Here } \frac{2 \times 18}{2 \times 12} = 1.5 \text{ feet area of the end.}$$

$$\therefore\ 1.5 \times 16 = 24 \text{ cubic feet. Ans.}$$

2. What is the solid content of a triangular prism, its length being 15 feet, one of the sides of its base being 18 inches, and the perpendicular drawn to that side from the opposite angle, 12 inches? Ans. $11\frac{1}{4}$ feet.

---

* 1. Each end of a prism is similar and equal, the ends of a cylinder are equal circles.

2. The surface is found by multiplying the perimeter by the height, then adding the area of the ends.

3. What is the superficial and solid content of a triangular prism; each side of its base being 12 inches, and length 12 feet? Ans. sur. 36.866ft. sol. 5.1961ft.

4. What is the superficial and solid content of a pentagonal prism; its length being 20 feet, and the side of its base 2 feet 6 inches?

Ans. surf. 271.5058ft. sol. 215.0596ft.

5. What is the superficial and solid content of an octagonal pillar; its side being 2 feet, and height 30 feet?

Ans. surf. 518.6274ft. sol. 579.4112ft.

6. Find the content of a stone roller, the diameter of its end being 18 inches, and its length 5 feet.

Ans. 8.8357 feet.

7. The diameter of a well is 3 feet 6 inches, and its depth 18 feet; how many gallons of water will it hold?

Ans. 1079gal. 1qt.

8. The piazza of the East-Riding Sessions House, Beverley, is supported by four cylindrical stone columns; the circumference of each being 9 feet, and height 27 feet; what quantity of stone do they contain?

Ans. 696.16584 cubic feet.

9. A brewer has an elliptical cooler; the transverse diamater of which is 20 feet 6 inches, conjugate 12 feet 6 inches, and depth 10 inches; how many gallons will it hold? Ans. 1045gal. 0qt. 1pt. 1gl.

10. A lead pipe, the diameter of which is 1 inch, just holds a gallon of water; required its length.*

Ans. 29ft. 5in.

PROBLEM 3.—*To find the solidity of a pyramid and cone.*†

RULE.—Multiply the area of the base by the perpendicular height, and $\frac{1}{3}$ of the product will be the content.

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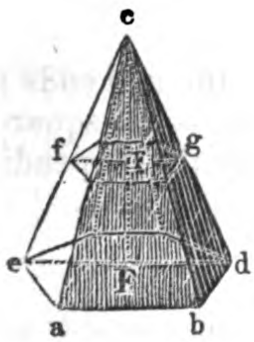
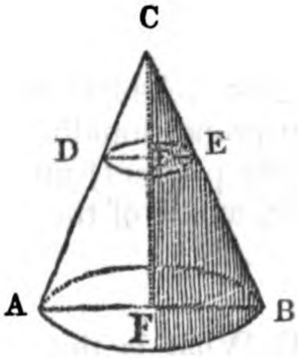
* The solid content divided by the area of the end gives the length, or divided by the length gives the area of the end.

† The base of a pyramid is a polygon, that of a cone a circle: the vertex of each being a point.

To find the surface of a pyramid or cone, multiply the perimeter of the base by the slant height, and half the product is the surface of the sides, to which add the area of the base.





EXAMPLES.

1. What is the solid content of a pentagonal pyramid, the side of its base being 2 feet, and perpendicular height 20 feet?

Here $(1.7204774 \times 4 \times 20) \div 3 = 45.87939$ feet.

2. What is the solid content of a triangular pyramid, each side of its base being 4 feet, and perpendicular height 20 feet? Ans. 46.188 feet.

3. What is the superficial and solid content of a square pyramid, each side of its base being 8 feet, and slant height 20 feet? Ans. sup. 384ft. sol. 418.0458ft.

4. What is the solid content of an octagonal pyramid, the side of its base being 3 feet, and perpendicular height 24 feet? Ans. 347.6467.

5. What is the solid content of a cone, the diameter of its base being 2 feet 6 inches, and perpendicular altitude 12 feet? Ans. 19.635.

6. What is the superficial and solid content of a cone, its slant height being 18 feet, and the diameter of its base 8 feet 6 inches?

Ans. sup. 297.0775ft. sol. 330.8276ft.

7. What is the curve surface and solidity of a cone, its slant height being 25 feet, and the circumference of its base 6 feet 9 inches?

Ans. sur. 84.375ft. sol. 30.186ft.

PROBLEM 4.—*To find the solidity of a frustum of a pyramid and cone.**

RULE.—Add together the areas of the two ends and a mean proportional between them (that is the square root of their product) multiply this *sum* by the perpendicular height, and $\frac{1}{3}$ of the product will be the solidity.

EXAMPLES.

1. What is the solid content of a frustum of a cone, its height being 15 feet, the greater diameter 5 feet, and less 4 feet?

$$5^2 = 25$$
$$4^2 = 16$$
$$5 \times 4 = 20$$
$$\overline{61} \times .7854 \times 5 = 239.547 \text{ feet. Ans.}$$

2. What is the solid content of a frustum of a cone, its height being 30 feet, the greater diameter 6 feet, and less 3 feet? Ans. 494.802.

3. What is the solid content of a frustum of a cone, the diameter of the greater end being 5 feet, and that of the less end 2 feet 6 inches, and its altitude 20 feet? Ans. 229.075.

4. The length of a mast is 60 feet, its diameter at the greater end 20 inches, and that of the less end 12 inches, what is its solidity? Ans. 85.521.

5. How many cubic feet are there in a piece of timber, of which the ends are squares, each side of one end being 15 inches, and each side of the other 6 inches, the length along the side being 24 feet? Ans. 19.497.

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* 1. A frustum is what is left after a piece has been taken from the vertex of a cone or pyramid, the cutting plane passing parallel to the base, as a b d e f g, and A B D E.

2. When the ends are circles, add together the *squares* of their diameters, and the *product* of their diameters, then multiply the *sum* by .7854, and again by $\frac{1}{3}$ of the height, for the solidity.

3. To find the surface, multiply the sum of the perimeters of the two ends by the slant height, and half the product added to the areas of the ends, gives the whole surface.

6. What is the solidity of a frustum of an octagonal pyramid, its height being 9 feet, the side of the greater end 30 inches, and that of the less end 20 inches?

Ans. 191.125.

7. Required the surface and solidity of a frustum of a cone, the diameters being 20 and 6, and slant height 25.

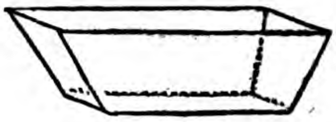
Ans. surf. 1363.4544, sol. 3493.4592.

PROBLEM 5.—*To find the solidity of a prismoid.*

RULE.—To the sum of the areas of the two ends, add *four* times the area of a section parallel to, and equally distant from both; this sum multiplied by one-sixth of the height, gives the solidity.*

EXAMPLES.

1. The length and breadth of a fishpond at the top are 100, and 48 yards; the length and breadth at the bottom 80 and 36 yards; and perpendicular depth 4 yards: how many cubic yards of earth had come out?



$\frac{1}{2}(100+80)=90$, and $\frac{1}{2}(48+36)=42$.

$100 \times 48 = 4800$ area of top.

$80 \times 36 = 2880$ area of bottom.

$90 \times 42 \times 4 = 15120$ area of 4 times the middle section.

$22800 \times \frac{4}{6} = 15200$. Ans.

2. What is the solid content of a block of stone, of which the ends are rectangles, the length and breadth of one end being 8 and 6 feet, and of the other end 6 and 4 feet; and its length 9 feet? Ans. 318 cubic feet.

* This rule gives the true content of all frustums, and of all solids whose parallel sections are similar figures; and is also a good approximation for all other kinds of solids. When the sides of the base are equal, the solid is a frustum.

3. What is the cubic content of a mill-hopper, the sides at the top being 60 and 50 inches, at bottom 12 and 10 inches, and its perpendicular depth 4 feet? Ans. $34\frac{4}{9}$ feet.

4. What is the cubic content of a trough; the top measuring 36 inches by 30; bottom 30 inches by 24; and depth 20 inches? Ans. 10.347 cubic feet.

5. What is the solidity of a log of Memel timber, the ends of which are rectangles; the length and breadth of one end being 18 and 15 inches, and the corresponding sides of the other end 14 and $11\frac{1}{2}$ inches; its length being 18 feet? Ans. 26.6458 cubic feet.

6. How many gallons of water will a washing-tub hold; its length and breadth at the top being 4 and 2 feet, and at the bottom 2 feet 10 inches, and 1 foot 4 inches; its depth being 18 inches? Ans. 53.838 gallons.

PROBLEM 6.—*To find the solidity of a sphere, or globe.*

RULE.—Multiply the *cube* of the diameter by .5236 = (one-sixth of 3.1416); or, the cube of the circumference by .016887, or the surface by one-sixth of the diameter, for the solidity.

EXAMPLES.

1. What is the solidity of a sphere, the diameter of which is 30 inches?

$$\frac{.5236 \times 30 \times 30 \times 30}{.12 \times 12 \times 12} = 8.18125 \text{ cubic feet. Ans.}$$

2. What is the solidity of a sphere, the diameter of which is 3 feet? Ans. 14.1372 cubic feet.

3. What is the solidity of a globe 4 feet in diameter? Ans. 33.5104 cubic feet.

4. What is the solidity of a sphere, the diameter of which is 15 inches? Ans. 1767.15 cubic inches.

5. What is the solidity of a globe, whose diameter is 3 feet 6 inches? Ans. 22.44935 cubic feet.

6. What is the solidity of the earth, supposing it a sphere whose diameter is 7912 miles?

Ans. 259333411782.8608 cubic miles.

7. Required the solidity of a sphere, the surface of which is 31416.* Ans. 523600.

8. The gilding of a globe cost £56. 10s. $11\frac{1}{2}$d.—$\frac{106}{125}$, at 2s. 6d. per foot; required its solidity. Ans. 904.7808 feet.

9. The solidity of a sphere is 113.0976 feet; required the number of square yards in its surface. Ans. 12.5664 yards.

10. What is the solidity of a segment 2 inches high, cut off from a sphere 52 inches diameter?† Ans. 318.348in.

11. What is the solidity of a segment 4 feet high, cut off from a sphere 20 feet in diameter? Ans. 435.635 cubic feet.

12. What is the solidity of a segment 2 feet 6 inches high, cut off from a sphere whose diameter is 7 feet 3 inches? Ans. 54.8141 cubic feet.

13. What is the curve surface and solidity of a segment $3\frac{1}{2}$ inches high, cut off from a sphere, the diameter of which is 8 inches? Ans. surf. 87.9648, sol. 109.0397.

14. If with a pair of compasses, extended 9 inches, a circle be described upon a globe, the diameter of which is 15 inches, what is the *surface* and *solidity* of the segment cut off by that circle? Ans. surf. 254.4696, sol. 522.1716.

15. Required the solid content of a segment of the earth; the circumference of its base being the parallel of latitude 36° 52′, supposing the earth a perfect sphere whose diameter is 8000 miles.‡ Ans. 27880652800 cubic miles.

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* The surface of a sphere = 4 times the area of one of its great circles = convex surface of its circumscribing cylinder = square of diameter × by 3.1416 = diameter × by the circumference.

A sphere = two-thirds of its circumscribing cylinder.

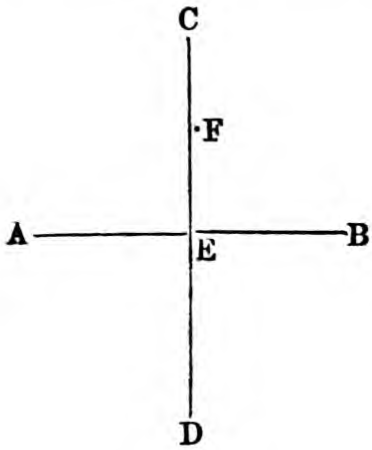
The curve surface of a segment is found by multiplying the circumference of the sphere by the height of the segment; or the square of the chord of half the arc by 3.1416.

† To find the solidity of a segment.—From three times the diameter of the sphere, subtract twice the height of the segment: then multiply the remainder by the square of the height, and the product again by .5236. Or, to three times the square of the radius of the base of the segment, add the square of the height; multiply the sum by the height, and the product by .5236 for the solidity.

‡ The complement of 36° 52′ is 53° 8′, the natural sine of which is .800034; and 1 : 4000 :: .8000 : the radius of the segment's base.

GEOMETRICAL PROBLEMS.

PROBLEM 1.—*To bisect a given line* A. B.



From A and B as centres, with any opening in the compasses as radius greater than half of A B, describe arcs cutting each other at C and D. Through C and D draw the line C D, and A B will be besected in E.

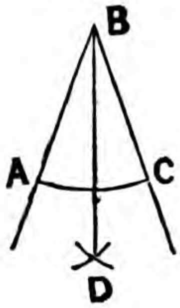
NOTE.—When A B is near the edge of a plane, with centres A and B, and any radius, describe arcs intersecting each other in C as before. Also with centres A and B, and any radius less than the former, describe arcs cutting each other in F. Through C F draw C E, which will divide A B into two equal parts.

3rd method.—By the sector.

Take the length A B in the compasses, open the sector till this extent forms a transverse distance between 10 and 10 on the line of lines. Take the length from 5 to 5 on the same line, and it will be half of A B.

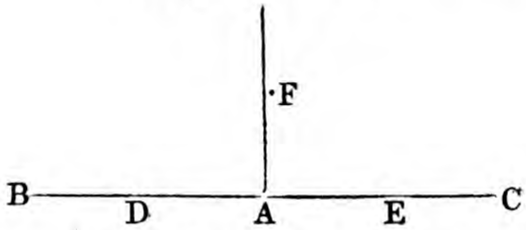
NOTE.—By this method any line may be readily divided into any number of equal parts, by successively dividing each sub-division.

PROBLEM 2.—*To bisect a given angle* A B C.



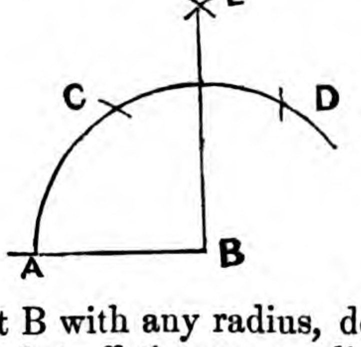
From B as a centre, with any radius, describe the arc A C. From A and C with any radius, describe arcs intersecting each other in D. Then draw B D and it will besect the angle B, as required.

PROBLEM 3.—*To erect a perpendicular from a given point* A, *in the given line* B C.



Set off on each side of the point A any two equal distances A E, A D. From D and E as centres, and with any radius greater than half D E, describe two arcs intersecting each other in F. Through the points A and F, draw the line A F, and it will be the perpendicular required.

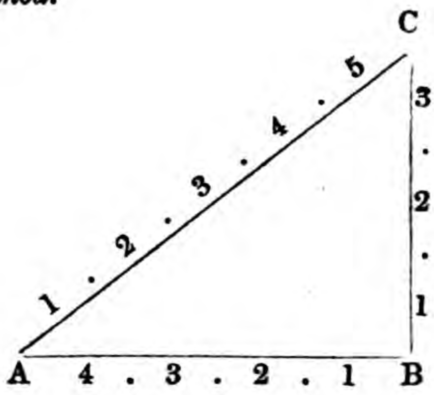
2nd method.—When the point is at, or near, the end of the line.



From the point B with any radius, describe the indefinite arc A C D. Set off the same radius A B on the arc A D, from A to C, and from C to D. From the points

C and D, with any radius, describe arcs cutting each other in E. Through B and E draw B E, and it will be the required perpendicular.

3rd method.



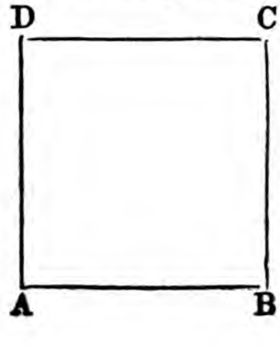
From B to A set off any length 4 times; from B as a centre, with 3 of the same parts as radius, describe an arc at C; and from A, with 5 of those parts as radius, cut the arc C. Through B and C draw the line C B, which will be the perpendicular required.

NOTE 1.—This method is generally called raising a perpendicular by the numbers 3, 4, and 5, which are the least whole numbers forming the sides of a right angled triangle.

2.—Perpendiculars are much more readily raised, or let fall, by means of a square, or a plotting scale with lines across at right angles to the edge.

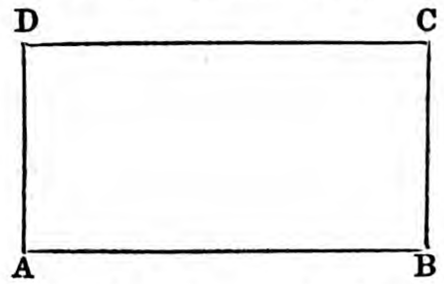
3.—Similarly, may *any* triangle, the sides of which are given, be constructed or laid down; for any one side being made the base, and the other two sides as radius, and the extreme sides of the base as centres; if arcs be made to intersect, the point of intersection is the vertical angle.

PROBLEM 4.—*To construct a square upon a given line* A B.



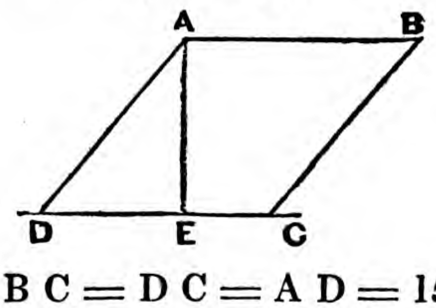
From the point B draw B C perpendicular and equal to A B; on A and C, with radius A B, describe arcs cutting each other in D; draw D A, D C, and A B C D will be the required square.

Problem 5.—*To construct a parallelogram or rectangle, the length and breadth of which are given.*



On the given length A B, erect B C, the given breadth, perpendicular to A B. With A as a centre, and radius B C, describe an arc; and with C as a centre, and radius A B, describe another arc cutting the former in D. Draw the lines A D and C D, and A B C D is the required rectangle.

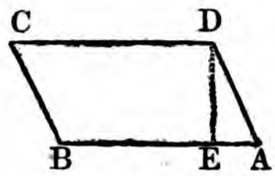
Problem 6.—*To construct a rhombus, its side and perpendicular height being given.*



Let A B = B C = D C = A D = 15, and A E the perpendicular = 12. From A demit the perpendicular A C = 12, and through E with a parallel ruler draw the indefinite line D E, and from A and B as centres, with radius, make arcs cutting D C in D and C; join A D, B C, and A B C D is the required rhombus.

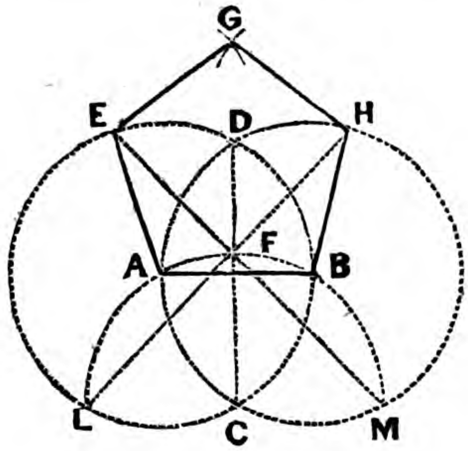
Problem 7.—*To construct a rhomboid, the base, perpendicular, and place of perpendicular being given.*

Let the base A B = D C = 30, perpendicular D E = 12, and distance A E = 10.



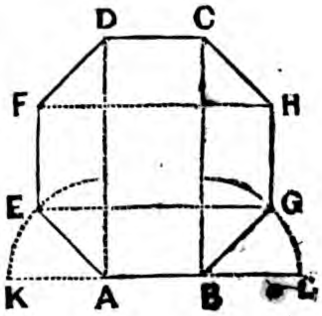
Make A B = 30, and A E = 10, at E erect the perpendicular E D, which make = 12, and join A D; with radius A B, and D as a centre, describe an arc; and with B as a centre, and radius A D, describe another arc, cutting the former in C. Draw the lines D C and B C, and A B C D is the rhomboid required.

Problem 8.—*To describe a regular pentagon on a given line* A B.



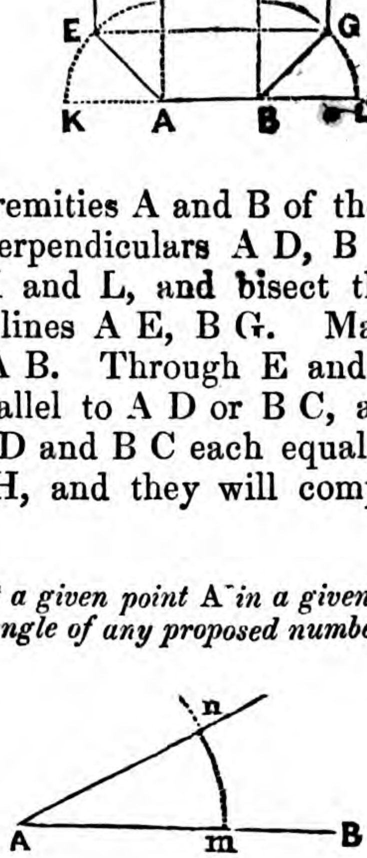
From the points A and B, with A B as radius, describe two circles intersecting each other in C and D; join C D, and from their intersection C, with the same radius A B, describe the arc L A B M, cutting the two circles in L and M, and the line C D in F. Draw the lines L F, M F, which produce to meet the circumference in H and E. From the points E and H, with the radius A B, describe arcs crossing each other in G, (or from the points A and B as centres, and A H or B E as radius, describe arcs cutting each other in G); then join A E, E G, G H, H B, and they will complete the pentagon required.

PROBLEM 9.—*On a given line* A B *to construct a regular octagon.*



From the extremities A and B of the given line, erect the indefinite perpendiculars A D, B C. Produce A B both ways to K and L, and bisect the angles D A K, C B L, by the lines A E, B G. Make A E and B G each equal to A B. Through E and G draw the lines E F, G H, parallel to A D or B C, and each equal to A B. Make A D and B C each equal to E G, and join D F, D C, C H, and they will complete the required octagon.

PROBLEM 10.—*At a given point* A *in a given straight line* A B, *to make an angle of any proposed number of degrees.*



From A with a radius equal to 60°, taken from a scale of chords, describe an arc cutting A B in m; then take in the compasses the proposed number of degrees from the same scale of chords, and apply that extent from m to n; through the point n draw A n, and it will make the angle A of the number of degrees proposed.

NOTE.—If the required angle be *obtuse*, that is, greater than 90°, set off its supplement n A m, or what it wants of 180° as above, then produce m A to p, and with radius A m = A p, and centre A describe a semicircle m a p, and p A n will be the required angle.

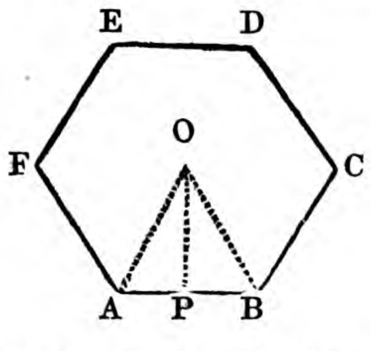
The same by the protractor.

Apply the diameter of the protractor to the line A B, so that the centre may coincide exactly with the point A. Make a mark against the given number of degrees on the edge of the protractor at n. Then remove the instrument, and draw a line from A through the point n, and the angle n A B will contain the number of degrees required.

The same by the line of chords on the sector; when the given degrees are under 60, *say for instance* 40°.

From the point A, with any radius A m describe the arc m n. Open the sector till the same radius A m make a transverse distance between 60 and 60 on the line of chords. Take with the compasses the transverse distance from 40 to 40 on the same line of chords, and set it off from m to n; then through n draw A n, and n A B will be the required angle.

PROBLEM 11.—*Upon* A B, *a given line, to make a regular polygon, of any proposed number of sides.*



Divide 360° by the number of sides, subtract the quotient from 180°, and divide the remainder by 2, the quotient is the number of degrees made by the radius of the circumscribing circle and the given side A B (see table prob. 5.) Set off this angle O A P and O B P, from each end of A B, and the point O where they meet, is the centre of the circumscribing circle, which describe with the radius O A or O B, and apply the chord or given line A B to the circumference the proposed number of times, and it will be the required polygon.

MISCELLANEOUS EXERCISES
IN MENSURATION.

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1. Westminster Hall is the largest room in the world unsupported by pillars, being 275 feet long and 74 feet broad; what would be the expense of laying the floor with marble, at 12s. 6d. a yard? Ans. £1413. 3s. 10½d.

2. What will be the expense of a ceiling 43 feet 10 inches, by 25 feet 6 inches, at 1s. 6d. a yard? Ans. £9. 6s. 3½d.

3. Find the expense of a brick floor, measuring 25 feet 6 inches by 15 feet 8 inches, at 2s. 3d. a yard. Ans. £4. 19s. 10½d.

4. Find the expense of painting 5 windows, each 6 feet 3 inches by 3 feet 8 inches, at 7d. a yard. Ans. 7s. 5d.

5. What is the expense of glazing a window at 1s. 6d. a foot, measuring 5 feet 6 inches by 3 feet 4 inches? Ans. £1. 7s. 6d.

6. In a wall 80 feet long, 6½ feet high, and 2½ bricks thick, how many standard roods of 272 square feet?* Ans. 3 roods, 50⅔ feet.

7. Find the expense of a stone roller, its diameter being 2 feet and length 5 feet, at 2s. 6d. a solid foot. Ans. £1. 19s. 3d.

8. Find the expense of paving a circular court at 2s. 6d. a yard, its diameter being 40 feet. Ans. £17. 9s. 0¾d.

9. Find the expense of paving a triangular court at 2s. 3d. a yard, its base being 84 feet and perpendicular 46 feet. Ans. £24. 3s. 0d.

10. What would be the expense of digging a well, the diameter of which is 4 feet and depth 32 feet, at 4s. a solid yard? Ans. £2. 19s. 6¾d.

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* *Standard brick work* is a brick and half thick. Walls are reduced to standard thickness by multiplying their superficial contents by the number of half bricks in thickness, and di, iding the product by 3.

11. How many flags, 30 inches long and 20 broad, will be required to flag an area 30 yards long and 20 broad?
Ans. 1296.

12. How many bricks will lay a floor, that is 20 feet 8 inches long, and 16 feet 6 inches broad, allowing 32 bricks to a square yard? Ans. $1212\frac{4}{9}$.

13. If a ploughman turns up a nine inch fur, how far will he travel in ploughing an acre? Ans. 11 miles.

14. If 20 yards of small thread be wound round a vertical staff 4 inches round, how far would a person travel to unwind the same?* Ans. 6 miles $749\frac{3}{4}$ yards.

15. How far would the person travel, supposing the same to be wound round a vertical staff, the circumference of which is 1 inch? Ans. 25 miles 1239 yards.

16. What length of cord will be required to tether a horse in the corner of a square field, so as to allow him to graze over a rood of land only? Ans. $39\frac{1}{4}$ yards nearly.

17. What length of cord will be required to tether a horse in the corner of a field in the form of an equilateral triangle, so as to allow him to graze over a rood only?
Ans. 48.07 + yards.

18. If a cubic foot of iron be beat into a square rod, the side of which is just half an inch, what will be its length? Ans. 192 yards.

19. If the same piece of iron be beat into a circular rod, whose diameter is half an inch, what will be its length? Ans. $244\frac{1}{2}$ yards nearly.

20. If a cube of copper, whose side is 3 inches, be made into a square plate, whose thickness is $\frac{1}{4}$ of an inch, what will be the length of its side? Ans. 10.3923 inches.

21. A silversmith has a globe of silver, whose diameter is 6 inches, which he wishes to make into a circular salver $\frac{3}{16}$ of an inch thick, what must be its diameter?
Ans. 27.712 inches.

22. If two sides of a triangle, the area of which is 8 acres, be 12 and 16 chains, what is the third side?
Ans. 13.7017 chains.

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* The *square* of the length of the string divided by the diameter of the staff (in the same denomination) gives the length of the spiral or track of the person.

23. Wanting to part off 2 acres from the vertex of a triangular field, the sides of which are 12, 15, and 17 chains, by a *fence* parallel to the longest side; required its length. Ans. 8.116 chains.

24. Required the side of an equilateral triangle, the area of which is just an acre. Ans. 4.8057 chains.

25. Required the area of a circle inscribed in a triangle, the sides of which are 30, 40, and 50. Ans. 314.16.

26. In an isosceles triangle, two circles are inscribed touching each other and the sides of the triangle; the diameters of the circles are 12 and 48, required the sides of the triangle. Ans. 80, 80, and 96.

27. If a heavy sphere, whose diameter is $1\frac{1}{2}$ inch, be dropped into a conical glass full of water, whose diameter is 2 inches and slant side 3 inches, how much water will run over? Ans. 1.7030595 cubic inches.

28. In measuring a rectangular field, I found that a perpendicular fell from one of the angles upon the diagonal, at the distance of 4 chains from one end, and 9 from the other; required the area of the field. Ans. 7a. 3r. 8p.

29. Required the area of 3 rectangular fields, whose diagonals are 15, 17, and 25 chains respectively, and the area of the greatest circle that can be inscribed in each, between the diagonal and sides is 2.82744 acres.

Ans. { 10a. 3r. 8p.
12 0 0
16 3 8

30. In a right angled triangle, the segments of the hypotenuse made by the contact of the inscribed circle, are 8 and 12; required the area of the circle. Ans. 50.2656.

31. A cone, the diameter of which is 20 inches, and altitude 3 feet, is to be divided into two equal parts, by a section parallel to the base; required the height of each part. Ans. 28.595 and 7.405 inches.

32. In a right angled triangle, the greater segment of the base made by the contact of the inscribed circle is 150, and the area of the triangle 15000; required the area of the inscribed circle. Ans. 7854.

33. The hypotenuse is 39, and radius of the inscribed circle 6; required the two legs. Ans. 15 and 36.

34. The area of a circle, circumscribing a right angled triangle, is 490.875, and the area of the inscribed circle 78.54; required the sides of the triangle. Ans. 15, 20, & 25.

35. The area of a right angled triangle is 2100, the area of its inscribed circle 706.86, and the distance from the centre of the circle to the greater acute angle 25; required the sides of the triangle. Ans. 35, 120, and 125.

36. A rectangular field, the diagonal of which is 22.5 chains, cost £60. 15s. 0d. draining and digging at £2. 10s. 0d. an acre, what would it cost fencing at 3s. 6d. a chain? Ans. £11. 0s. 6d.

37. If a horse be tethered at one angle of a triangular plantation, the sides of which are each 2 chains, how much ground will he be able to graze over, the tether being 3 chains long? Ans. 2a. 2r. $10\frac{1}{2}$p.

38. The equal sides of an isosceles triangle are 20, and the base 24; required the area of the inscribed and circumscribing circles. Ans. $\begin{cases} 12^2 \times .7854. \\ 25^2 \times .7854. \end{cases}$

39. A string being wound round an equilateral triangle, the side of which is 2 chains, what area would a person inclose by unwinding the same, supposing one end to be fastened to an angle, and its last position parallel to the side first cleared, or coincident with the side produced; and how far would the person travel?

Ans. 6a. 0r. 6p. area 553 yards nearly.

40. Given the product of the two segments of the base of a triangle made by the perpendicular = 576, and the segments of the same made by the contact of the inscribed circle, 20 and 30, to find the area of the triangle and radius of the circle. Ans. area 600, radius 10.

41. Given the difference of the sides 6, the greater segment of the base made by the contact of the inscribed circle 48, and the distance from the point of contact to the foot of the perpendicular $2\frac{2}{5}$, to find the radius of the circle and area of the triangle. Ans. rad. 24, area 3024.

APPENDIX.

THE following epitome of Arithmetic, though chiefly intended as exercises for Pupils in mental calculation, will nevertheless be found deserving the attention of Accountants, as well as Preceptors, and men of business generally.

When the price is in pence.

RULE.—Consider the number of articles as so many *pence*, which transform mentally into £. *s. d.* which multiply by the price of one article.

EXAMPLES.

Find the value of the following quantities:—

1. $46\frac{1}{2}$ at $4\frac{1}{2}$d. 2. $1369\frac{3}{8}$ at 8d.

	s.	d.			£.	s.	d.	
$46\frac{1}{2}$d. =	3	$10\frac{1}{2}$		$1369\frac{3}{8}$ =	5	14	$1\frac{3}{8}$	
		$4\frac{1}{2}$					8	
	15	6			45	12	11	Ans.
	1	$11\frac{1}{4}$						
	17	$5\frac{1}{4}$	Ans.					

			d.	£.	s.	d.				£.	s.	d.
3.	24	at	2*	0	2	0	×	2	=	0	4	0
4.	36	at	3	0	3	0	×	3	=			
5.	48	at	4	0	4	0	×	4	=			
6.	60	at	7	0	5	0	×	7	=			
7.	72	at	5	0	6	0	×	5	=			
8.	96	at	7	0	8	0	×	7	=			

* It is the same as reckoning each penny in the price a shilling, and multiplying by the number of twelves in the quantity; therefore if the number is not a certain number of twelves, add or subtract as many times the price of one, as it wants or is above.

No.	Quantity	at *d.*	£.	*s.*	*d.*			£.	*s.*	*d.*
9.	99	at 10	0	8	3	×10	=	4	2	6
10.	108	at 9	0	9	0	× 9	=			
11.	120	at $11\frac{1}{2}$	0	10	0	×$11\frac{1}{2}$	=			
12.	27	at 11	0	2	3	×11	=			
13.	35	at 5	0	2	11	× 5	=			
14.	56	at $7\frac{1}{2}$	0	4	8	× $7\frac{1}{2}$	=			
15.	66	at $8\frac{1}{2}$	0	5	6	× $8\frac{1}{2}$	=			
16.	74	at $9\frac{1}{2}$	0	6	2	× $9\frac{1}{2}$	=			
17.	$86\frac{1}{2}$	at $10\frac{1}{2}$	0	7	$2\frac{1}{2}$	×$10\frac{1}{2}$	=			
18.	$186\frac{1}{2}$	at $4\frac{1}{2}$	0	15	$6\frac{1}{2}$	× $4\frac{1}{2}$	=			
19.	240	at $3\frac{1}{2}$	1	0	0	× $3\frac{1}{2}$	=			
20.	480	at $4\frac{3}{4}$	2	0	0	× $4\frac{3}{4}$	=			
21.	720	at $7\frac{1}{4}$	3	0	0	× $7\frac{1}{4}$	=			
22.	960	at $15\frac{1}{2}$	4	0	0	×$15\frac{1}{2}$	=			
23.	365	at 5	1	10	5	× 5	=			
24.	360	at $7\frac{3}{8}$*	7	7	6	× $1\frac{1}{2}$	=			
25.	365	at $17\frac{3}{4}$	17	15	0	× $1\frac{1}{2}$+$17\frac{3}{4}$d.×5	=	26	19	$10\frac{3}{4}$
26.	1433	at $7\frac{1}{8}$	7	2	6	× 6 — $7\frac{1}{8}$ ×7	=	42	10	$10\frac{1}{8}$

No.	Quantity	at *s.*	*d.*	£. *s.* *d.*	*s.* *d.*		£.	*s.*	*d.*
27.	1431	at 2	$6\frac{1}{4}$	30 5 0×6—2	$6\frac{1}{4}$×9	=	180	7	$3\frac{3}{4}$
28.	1681	at 5	$9\frac{1}{4}$	69 5 0×7+5	$9\frac{1}{4}$	=	485	0	$9\frac{1}{4}$
29.	1440	at 1	$4\frac{3}{8}$	.. .. ..	..	Ans.	98	5	0
30.	$959\frac{7}{8}$	at 8	8	.. .. ..	..	..	415	18	11
31.	1431	at 2	$6\frac{1}{4}$	.. .. ..	..	..	180	7	$3\frac{3}{4}$
32.	1451	at 3	$4\frac{1}{2}$	.. .. ..	..	..	244	17	$1\frac{1}{2}$
33.	$1679\frac{7}{8}$	at 9	$5\frac{3}{4}$	.. .. ..	..	..	796	3	10
34.	1672	at 2	$1\frac{1}{4}$	.. .. ..	..	..	175	18	2
35.	$1919\frac{3}{4}$	at 1	$5\frac{5}{8}$	.. .. ..	..	..	140	19	$7\frac{3}{4}$
36.	1681	at 25	$9\frac{1}{4}$	.. .. ..	..	..	2166	0	$9\frac{1}{4}$
37.	2940	at 1	$11\frac{5}{8}$	.. .. ..	..	..	289	8	$1\frac{1}{2}$
38.	2688	at 13	$2\frac{1}{8}$	.. .. ..	..	..	1771	0	0
39.	$39473\frac{1}{2}$	at 1	$5\frac{1}{2}$†	.. .. ..	..	..	2878	5	$6\frac{1}{4}$
40.	$85632\frac{3}{4}$	at 1	$10\frac{1}{2}$	.. .. ..	..	..	8028	1	$4\frac{3}{4}$

* If each penny in the price be reckoned a £, it gives the value of 240, and by adding or subtracting, as in the preceding note, we obtain the value of any intermediate number.

† Reckoning the pence in the price as shillings, it is 17s. 6d., half a crown short of a £; therefore dividing the quantity considered as £'s by 12, we have £3289. 9s. 2d. from which deducting $\frac{1}{8}$, gives the answer.

When the price is in shillings.

RULE.—Consider the number of articles as so many shillings, which transform mentally into pounds, then multiply by the price of one article.

EXAMPLES.

Find the value of the following articles.

1. 80 at 3s.

80s. = £4.
3
£ 12. Ans.

2. $37\frac{1}{2}$ at £0 3s. 6d.

	£.	s.	d.	
$37\frac{1}{2}$s. =	1	17	6	
			$3\frac{1}{2}$s.	= 3s. 6d.
	5	12	6	
		18	9	
£	6	11	3	Ans.

			s.	d.		£.	s.	d.				£.	s.	d.
3.	20	at	3	0	..	1	0	0	×	3	= Ans.	3	0	0
4.	40	at	3	4	..	2	0	0	×	$3\frac{1}{3}$	=	6	13	4
5.	60	at	7	6	..	3	0	0	×	$7\frac{1}{2}$	=	22	10	0
6.	80	at	9	3	..	4	0	0	×	$9\frac{1}{4}$	=	37	0	0
7.	100	at	4	6	..		..	..			=	22	10	0
8.	120	at	4	3	..		..	..			=	25	10	0
9.	126	at	2	4	..	6	6	0	×	$2\frac{1}{3}$	=			
10.	$164\frac{1}{2}$	at	4	9	..	8	4	6	×	$4\frac{3}{4}$	=			
11.	$182\frac{1}{2}$	at	3	9	..	9	2	6	×	$3\frac{3}{4}$	=			
12.	260	at	12	6	..		..	..			=	162	10	0
13.	380	at	7	3	..		..	..			=	137	15	0
14.	365	at	7	0	..		..	..			=	127	15	0
15.	365	at	2	6	..		..	..			=	45	12	6
16.	365	at	2	3	..		..	..			=	41	1	3
17.	168	at	3	2	..	8	8	0	×	$3\frac{1}{6}$	=	26	12	0
18.	248	at	12	1	..	12	8	0	×	$12\frac{1}{12}$	=	149	16	8
19.	288	at	7	8	..	14	8	0	×	$7\frac{2}{3}$	=	110	8	0
20.	$42\frac{1}{2}$	at	11	6	..	2	2	6	×	$11\frac{1}{2}$	=			
21.	$765\frac{3}{8}$	at	9	3	..	38	12	6	×	$9\frac{1}{4}$	=			
22.	$1340\frac{5}{8}$	at	2	9	..	67	12	6	×	$2\frac{3}{4}$	=			

When the price is shillings and pence, or £. s. d.

RULE.—Find the value of each separately, which add together.

EXAMPLES.

Find the value of the following quantities.

1. 23 at 3s. 5d.											
		£.	s.	d.		£.	s.	d.	
First	23s. × 3 =	1	3	0	× 3 =	3	9	0	
and	23d. × 5 =	0	1	11	× 5 =	0	9	7	
						3	18	7	Ans.

2. 124½ at 4s. 7d.									
First	124½s. × 4 =	6	4	6	× 4 =	24	18	0	
and	124½d. × 7 =	0	10	4½	× 7 =	3	12	7½	
						28	10	7½	Ans.

3. 67¾ at £2 3s. 4d.								
£. 67¾ × 2 =	67	15	0	× 2 =	135	10	0	
67¾s. × 3 =	3	7	9	× 3 =	10	3	3	
67¾d. × 4 =	0	5	7¾	× 4 =	1	2	7	
					146	15	10	Ans.

			£.	s.	d.			£.	s.	d.
4.	95	at	0	3	8	.. ..	Ans.	17	8	4
5.	95½	at	0	3	10	.. ..	Ans.	18	6	1
6.	130	at	0	2	1	.. ..	Ans.	13	10	10
7.	57	at	0	2	11	.. ..	Ans.	8	6	3
8.	116	at	0	7	2	.. ..	Ans.	41	11	4
9.	112½	at	0	9	7	.. ..	Ans.	53	18	1½
10.	131¼	at	0	11	5	.. ..	Ans.	74	18	5¼
11.	137¾	at	0	8	8	.. ..	Ans.	59	13	10
12.	74½	at	1	5	1	.. ..	Ans.	93	8	8½
13.	92¼	at	3	9	5	.. ..	Ans.	320	3	8¼
14.	126	at	1	12	6	.. ..	Ans.	204	15	0
15.	156½	at	2	13	4	.. ..	Ans.	417	6	8
16.	268¼	at	2	14	6	.. ..	Ans.	730	19	7½

		£.	s.	d.				£.	s.	d.
17.	365 at	1	15	8	..	..	Ans.	650	18	4
18.	$376\frac{3}{8}$ at	2	3	8	..	..	Ans.	821	15	$0\frac{1}{2}$
19.	$763\frac{1}{8}$ at	2	9	4	..	..	Ans.	1882	7	6
20.	$1326\frac{5}{8}$ at	1	12	6	..	..	Ans.	2155	15	$3\frac{3}{4}$
21.	$2457\frac{7}{8}$ at	1	6	2	..	..	Ans.	3215	14	$4\frac{3}{4}$

To find the amount of tons, cwts., qrs. and lbs.

RULE.—Call the tons pounds, and the cwts. shillings, then to three times the quarters, add the ninth part of the lbs. for pence, which gives the value at £1. a ton.

EXAMPLES.

1. 56 tons, 18cwt. 1qr. 18lbs. at £1. 5s. 6d. a ton.

		£.	s.	d.	
5s.	$\frac{1}{4}$	56	18	5	value at £1.
6d.	$\frac{1}{10}$	14	4	$7\frac{1}{4}$	
		1	8	$5\frac{1}{2}$	
		72	11	$5\frac{3}{4}$	Ans.

2. 75 tons, 12cwt. 2qrs. 27lbs. at £3. 10s. a ton.

75	12	9	price at £1.
		$3\frac{1}{2}$	= £3. 10s.
226	18	3	
37	16	$4\frac{1}{2}$	
264	14	$7\frac{1}{2}$	Ans.

3. 85 tons, 17cwt. 3qrs. 9lbs at £3. 5s. a ton.
Ans. £279. 2s. $11\frac{1}{2}$d.
4. 17 tons, 9cwt. 2qrs. 18lbs. at £9. 6s. 8d. a ton.
Ans. £163. 3s. 6d.
5. 91 tons, 6cwt. 1qr. at £7. 5s. a ton.
Ans. £662. 0s. $3\frac{3}{4}$d.
6. 25 tons, 16cwt. 3qrs. 19lbs. at £3. 7s. 6d. a ton.
Ans. £87. 4s. 7d.
7. 832 tons, 11cwt. 2qrs. 27lbs. at £6. 17s. 6d. a ton.*
Ans. £5724. 0s. $9\frac{1}{2}$d.

* When the difference between the given price and the next greater number of pounds, is an aliquot part of a pound, multiply by the next greater number, and deduct the aliquot part.

8. 431 tons, 13cwt. 1qr. 10lbs. at £10. 2s. 6d. a ton.
Ans. £4370. 12s. 6d.

9. 971 tons, 15cwt. 1qr. 9lbs. at £4. 13s. 4d. a ton.
Ans. 4534. 18s. 3d.

10. 696 tons, 15cwt. 1qr. 12lbs. at £7. 10s. a ton.
Ans. £5225. 15s. $1\frac{3}{4}$d.

To find the amount of cwts. qrs. and lbs.; also the price of a lb., that of a cwt. being given.

RULE.—Reduce them into lbs., and for every 9s. 4d. in the given price, reckon 1d.; for 7s., $\frac{3}{4}$d.; for 4s. 8d., $\frac{1}{2}$d.; and for 2s. 4d., $\frac{1}{4}$d.*

EXAMPLES.

1. 5cwt. 1qr. 18lbs. at 18s. 8d. per cwt. = 2d. a lb.

112 + 46

2d. = $\frac{1}{120}$ 606 = lbs.

£5 1 0 Ans.

2. 3cwt. 3qrs. 4lbs. at 6 guineas a cwt. = $13\frac{1}{2}$d. a lb.

112 + 88

1s.	$\frac{1}{20}$	424		
$1\frac{1}{2}$d.	$\frac{1}{8}$	21	4	0
		2	13	0
		£23	17	0 Ans.

3. 1cwt. 1qr. 7lbs. at 2s. 4d. a cwt. = $\frac{1}{4}$ a lb.

112 + 35

147 lbs. at $\frac{1}{4}$d. = 3s. $0\frac{3}{4}$d. Ans.

4. 2cwt. 1qr. 14lbs. at 4s. 8d. a cwt. = $\frac{1}{2}$ a lb.

112 + 42

266 lbs, at $\frac{1}{2}$d. = 11s. 1d. Ans.

* If the price of a lb. be given, and that of a cwt. required, it is immediately found by reckoning every penny in the price 9s. 4d., or which is the same thing, every farthing 2s. 4d.

5. 2cwt. 2qrs. 14lbs. at 7s. a cwt. = $\frac{3}{4}$d. a lb.
112 + 70
294 lbs. at $\frac{3}{4}$d. = 18s. 4$\frac{1}{2}$d. Ans.

6. 2cwt. 3qrs. 14lbs. at 9s. 4d. a cwt. = 1d. a lb.
112 + 98
322 lbs. at 1d. = £1. 6s. 10d. Ans.

7. 3cwt. 1qr. 0lb. at 11s. 8d. a cwt. = 1$\frac{1}{4}$d. a lb.
112 + 28
364 lbs. at 1$\frac{1}{4}$d. = £1. 17s. 11d. Ans.

8. 4cwt. 2qrs. 7lbs. at 14s. a cwt. = 1$\frac{1}{2}$d. a lb.
112 + 63
511 lbs. at 1$\frac{1}{2}$d. = £3. 3s. 10$\frac{1}{2}$d. Ans.

9. 5cwt. 2qrs. 14lbs. at 16s. 4d. a cwt. = 1$\frac{3}{4}$d. a lb.
112 + 70
630 lbs. at 1$\frac{3}{4}$d. = £4. 11s. 10$\frac{1}{2}$d. Ans.

10. 6cwt. 1qr. 12lbs. at 18s. 8d. a cwt. = 2d. a lb.
112 + 40
712lbs. at 2d. = £5. 18s. 8d. Ans.

11. 9cwt. 1qr. 17lbs. at £1. 5s. 8d. a cwt. = 2$\frac{3}{4}$d. a lb.
112 + 45
1053 lbs. at 2$\frac{3}{4}$d. = £12. 1s. 3$\frac{3}{4}$d. Ans.

12. 12cwt. 3qrs. 25lbs. at £1. 17s. 4d. a cwt. = 4d. a lb.
112 + 109
1453 lbs. at 4d. = £24. 4s. 4d. Ans.

13. 8cwt. 1qr. 10lbs. at £2. 16s. 0d. a cwt. = 6d. a lb.
112 + 38
934 lbs. at 6d. = £23. 7s. 0d. Ans.

14. 64cwt. 2qrs. 13lbs. at 4 guineas a cwt. = 9d. a lb.
* 2 + 69
7237 lbs. at 9d. = £271. 7s. 9d. Ans.

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* Multiplying cwts by 2, and adding two figures to the right, is the same as multiplying by 112 in one line.

Discount.

RULE.—Multiply the *principal* by double the *rate;* cut off the units figure, the rest is the answer in shillings.* Or if the rate be high, take aliquot parts.

EXAMPLES.

Find the discount, or interest, on the following sums.

1. £. 78 1 3 at $4\frac{1}{2}$ per cent.
   9 = double the rate.
   70,2 11 3 = £3 10 3 Ans.

2. £. s. d.

		£.	s.	d.	
20	$\frac{1}{5}$	576	8	9	at $22\frac{1}{2}$ per cent.
$2\frac{1}{2}$	$\frac{1}{8}$	115	5	9	
		14	8	$2\frac{1}{2}$	
		£129	13	$11\frac{1}{2}$	Ans.

3. Deduct $1\frac{1}{2}$d. on the shilling from £17. 12s. 6d.

$1\frac{1}{2}$d.	$\frac{1}{8}$	17	12	6	
		2	4	$0\frac{3}{4}$	discount.
		£15	8	$5\frac{1}{4}$	Ans.

	£.	s.	d.				£.	s.	d.
4.	51	5	6	at $5\frac{1}{2}$	per cent.	Ans.	2	16	5
5.	211	15	9	at $3\frac{1}{2}$	per cent.	Ans.	7	8	3
6.	185	11	7	at $2\frac{1}{2}$	per cent.	Ans.	4	12	$9\frac{1}{2}$
7.	137	4	6	at $1\frac{1}{2}$	per cent.	Ans.	2	1	2
8.	731	6	6	at $\frac{1}{2}$	per cent.	Ans.	3	13	$1\frac{1}{2}$
9.	153	13	10	at $2\frac{1}{4}$	per cent.	Ans.	3	9	2
10.	17	5	6	at 2d.	on the shilling.	Ans.	2	17	7
11.	25	14	10	at 3d.	on the shilling.	Ans.	6	8	$8\frac{1}{2}$
12.	365	15	6	at $7\frac{1}{2}$	per cent.	Ans.	27	8	$7\frac{3}{4}$
13.	285	13	6	at 9	per cent.	Ans.	25	14	$2\frac{1}{2}$
14.	156	12	6	at 10	per cent.	Ans.	15	13	3
15.	61	7	8	at 15	per cent.	Ans.	9	4	$1\frac{1}{2}$
16.	784	10	6	at $17\frac{1}{2}$	per cent.	Ans.	137	5	$9\frac{3}{4}$
17.	362	10	4	at 20	per cent.	Ans.	72	10	$0\frac{3}{4}$
18.	980	16	6	at 25	per cent.	Ans.	245	4	$1\frac{1}{2}$

* The figure cut off is tenths of a shilling, for every *one* in which reckon $1\frac{1}{4}$d.; for 10s. $\frac{1}{2}$d.; and for 5s. $\frac{1}{4}$d. This rule is equivalent to finding the interest of a given principal at a given rate for a year.

	£.	s.	d.			£.	s.	d.
19.	203	13	6	at 27 per cent.	Ans.	54	19	10
20.	100	15	6	at $27\frac{1}{2}$ per cent.	Ans.	27	14	$2\frac{3}{4}$

EXERCISES.

1. I have goods which cost £100.; how should I sell them to gain 20 per cent. profit, and allow a discount of 30 per cent.?

Ans. 100 — 30 : 100 :: 100 + 20 : £171. 8s. $6\frac{3}{4}$d.

2. If gin is sold for 15s. a gallon, when it is $15\frac{1}{2}$ under proof, what should be charged when it is reduced to $25\frac{1}{2}$ below proof? Ans. 13s. $2\frac{1}{2}$d.—$\frac{134}{169}$.

3. Bought a gig for £55., for which I was allowed 10 per cent. discount. I sold it again for what it cost; what was my gain per cent? Ans. £11. 2s. $2\frac{2}{3}$d.

4. Bought cloth at 15s. a yard, on which I laid 15 per cent. profit, but allowed 5 per cent. for ready money; what was my gain per cent.? Ans. $9\frac{1}{4}$ per cent.

5. Bought Goods for £120., for which I was allowed 25 per cent. for cash; I sold them for what they cost, and allowed $12\frac{1}{2}$ per cent. discount; what did I gain, and what per cent? Ans. £15. gain, $16\frac{2}{3}$ per cent.

6. Sold rum 5 per cent. above proof for 18s. a gallon, what should it sell for when reduced 10 per cent. below proof? Ans. 15s. $5\frac{1}{7}$d.

7. I have articles which cost 5s. each, on which I lay 10 per cent. profit, but allow 5 per cent discount; what do I gain per cent? Ans. $4\frac{1}{2}$ per cent.

8. Sold goods for 5 guineas, but allowed 5 per cent. discount, by which I gained 5 per cent.; what did the goods cost me? Ans. £4. 15s.

9. By selling cloth at a guinea a yard, allowing 5 per cent. discount for cash, I gain 5 per cent.; what do I gain by 5 yards? Ans. 4s. 9d.

10. I have cloth which cost 12s. 6d. a yard manufacturing, on which I lay $12\frac{1}{2}$ per cent. profit, but allow 5 per cent. discount, or 12 months' credit. Now, one customer A takes the discount, and another B the credit; what do I gain per cent. by each?

Ans. A £6. 17s. 6d. B £7. 2s. $10\frac{2}{7}$d.

To find the Interest of any sum for any number of months at 5 per cent.

RULE.—Account each pound in the principal a penny, which multiply by the months.*

At 6 per cent.

RULE.—Multiply the principal by the months, cut off the units figure, the rest is the answer in shillings.

At 4 per cent.

RULE.—Calculate at 6 per cent. and deduct $\frac{1}{3}$.†

At $3\frac{1}{3}$ per cent.‡

RULE.—Calculate at 5 per cent. and deduct $\frac{1}{3}$.

Or Generally.

Multiply the principal by the months, and the product by double the rate, cut off one figure to the right, the rest give the answer in pence.

EXAMPLES.

Find the interest of the following sums at 5 per cent.

	£.	s.	d.		£.	s.	d.		£.	s.	d.
1.	60	0	0	for 2 months.	0	5	0	× 2 =	0	10	0
2.	72	0	0	for 3 months.	0	6	0	× 3 =	0	18	0
3.	84	0	0	for 4 months.	0	7	0	× 4 =	1	8	0
4.	96	0	0	for 5 months.	0	8	0	× 5 =			
5.	117	0	0	for 6 months.	0	9	9	× 6 =			
6.	136	10	0	for 7 months.	0	11	$4\frac{1}{2}$	× 7 =			
7.	174	15	0	for 8 months.				=	5	16	6
8.	220	5	0	for 9 months.				=	8	5	$2\frac{1}{4}$
9.	275	2	6	for 10 months.	1	2	$11\frac{1}{8}$	× 10 =			
10.	356	15	0	for 11 months.				=	16	6	$6\frac{1}{4}$

Find the interest of the following sums at 6 per cent.

1. £123 4 8 for 5 months.

$$\begin{array}{rrr} £123 & 4 & 8 \\ & & 5 \\ \hline 61,6 & 3 & 4 \end{array} = £3.\ 1s.\ 7\tfrac{1}{2}d.\ \text{Ans.}$$

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* Reckon for 15s. $\frac{3}{4}$d.; for 10s. $\frac{1}{2}$d.; and for 5s. $\frac{1}{4}$d.; any thing lower may be neglected.

† If the interest be wanted at 3 per cent. calculate at 6, and take $\frac{1}{2}$; at 2 per cent. take $\frac{1}{3}$.

‡ This is the rate allowed by the Savings' Banks, and is obviously $\frac{2}{3}$ of 5. At $2\frac{1}{2}$ per cent. calculate at 5, and take $\frac{1}{2}$.

	£.	s.	d.			£.	s.	d.
2.	30	0	0 for 3 months.	..	Ans.	0	9	0
3.	40	0	0 for 4 months.	..	Ans.	0	16	0
4.	50	0	0 for 5 months.	..	Ans.	1	5	0
5.	60	0	0 for 6 months.	..	Ans.	1	16	0
6.	70	0	0 for 7 months.	..	Ans.	2	9	0
7.	87	10	0 for 8 months.	..	Ans.	3	10	0
8.	135	0	0 for 9 months.	..	Ans.	6	1	6
9.	47	10	0 for 10 months.	..	Ans.	2	7	6
10.	38	10	0 for 11 months.	..	Ans.	2	2	4

Find the interest of the following sums at 4 per cent.

1.	£25	4	6 for 5 months.
			5
$\frac{1}{3}$)	126	2	6
	42	0	10
	8,4	1	8 = 8s. 5d. Ans.

	£.	s.	d.			£.	s.	d.
2.	60	0	0 for 2 months.	..	Ans.	0	8	0
3.	40	0	0 for 3 months.	..	Ans.	0	8	0
4.	53	5	0 for 4 months.	..	Ans.	0	14	$2\frac{1}{2}$
5.	174	0	0 for 5 months.	..	Ans.	2	18	0
6.	61	10	6 for 6 months.	..	Ans.	1	4	$7\frac{1}{2}$
7.	128	14	6 for 7 months.	..	Ans.	3	0	$0\frac{1}{2}$
8.	135	15	0 for 8 months.	..	Ans.	3	12	5
9.	375	12	8 for 9 months.	..	Ans.	11	5	$4\frac{1}{2}$

Find the interest of the following sums at $3\frac{1}{3}$ per cent.

1.	75	15	0 for 2 months.	or thus, $75\frac{3}{4}$ =	6	$3\frac{3}{4}$
			2			2
3)	151	10	0	3)	12	$7\frac{1}{2}$
	50	10	0		4	$2\frac{1}{2}$
	101d.	0	0 = 8s. 5d. Ans.		8	5

	£.	s.	d.			£.	s.	d.
2.	50	0	0 for 3 months.	..	Ans.	0	8	4
3.	38	5	0 for 4 months.	..	Ans.	0	8	6
4.	74	8	0 for 5 months.	..	Ans.	1	0	8
5.	97	15	0 for 6 months.	..	Ans.	1	12	7
6.	110	12	6 for 7 months.	..	Ans.	2	3	$0\frac{1}{4}$

	£.	s.	d.			£.	s.	d.
7.	124	10	6 for 8 months.	..	Ans.	2	15	4
8.	164	13	4 for 9 months.	..	Ans.	4	2	4

Find the interest of

	£.	s.	d.			£.	s.	d.
9.	56	12	6	at 2 per cent. for 2 months.	Ans.	0	3	$9\frac{1}{2}$
10.	142	10	6	at $2\frac{1}{2}$ per cent. for 5 months.	Ans.	1	9	$8\frac{1}{4}$
11.	172	6	6	at $3\frac{1}{2}$ per cent. for 7 months.	Ans.	3	10	$4\frac{1}{4}$

Interest for days.

Rule.—Multiply the pounds by the days, and that product by *double* the rate; then divide by 3, and cut off two figures to the right, the rest is the answer in pence, deducting $\frac{1}{73}$ or 1d. for 6s.

EXAMPLES.

1. Find the interest of

```
         £.   s.  d.
         50   10  0 at 5 per cent. for 20 days.*
         20 + 10
      3) 101,00   s.  d.
       33⅔d. = 2   9½d.
                   0½
               2   9 Ans.
```

```
2.      165   17  6 at 4 per cent. for 24 days.†
         24 — 3
       3981
          8 = double the rate.
     3) 318,48   s.  d.
        106d. = 8   10
                     1½
                8    8½
```

* Such parts of the days as the shillings are of the pounds, must be added in, that is, for 10s. add half the days, for 5s. a quarter of them. &c. A day, or a few pence, makes no difference.

† The shillings and pence only wanting half a crown to make them a pound, add 1 to the pounds, and deduct 3, $\frac{1}{8}$ of the days.

	£.	s.	d.					
3.	981	1	8 at $1\frac{1}{2}$ per cent. for 91 days.*					
	91			s.	d.	£.	s.	d.
	892,71		=	74	$4\frac{3}{4}$ =	3	14	$4\frac{3}{4}$
							1	$0\frac{1}{4}$
						3	13	$4\frac{1}{2}$ Ans.

	£.	s.	d.		£.	s.	d.
4.	5	0	0	at 5 p. c. for 100 days. Ans.	0	1	$4\frac{1}{4}$
5.	90	0	0	at $3\frac{1}{2}$ p. c. for 10 days. Ans.	0	1	$8\frac{3}{4}$
6.	124	19	8	at $3\frac{1}{2}$ p. c. for 36 days. Ans.	0	8	$7\frac{1}{2}$
7.	74	5	0	at $2\frac{1}{2}$ p. c. for 40 days. Ans.	0	4	$0\frac{3}{4}$
8.	85	10	0	at $2\frac{1}{2}$ p. c. for 48 days. Ans.	0	5	$7\frac{1}{4}$
9.	110	15	0	at $3\frac{1}{2}$ p. c. for 51 days. Ans.	0	10	10
10.	165	13	4	at 5 p. c. for 110 days. Ans.	2	9	11
11.	63	16	3	at 5 p. c. for 75 days. Ans.	0	13	$1\frac{1}{4}$
12.	51	4	11	at 5 p. c. for 27 days. Ans.	0	3	$9\frac{1}{2}$
13.	42	9	10	at 5 p. c. for 94 days. Ans.	0	10	$11\frac{1}{4}$
14.	127	7	3	at 5 p. c. for 17 days. Ans.	0	5	11
15.	124	19	10	at $3\frac{1}{2}$ p. c. for 277 days. Ans.	3	6	$4\frac{3}{4}$
16.	998	19	0	at $3\frac{1}{2}$ p. c. for 197 days. Ans.	18	17	$5\frac{1}{2}$
17.	974	19	6	at $4\frac{1}{2}$ p. c. for 167 days. Ans.	20	1	$5\frac{3}{4}$
18.	120	12	6	at 4 p. c. for 24 days. Ans.	0	6	4
19.	273	15	0	at 3 p. c. for 69 days. Ans.	1	11	$0\frac{1}{2}$
20.	7650	12	6	at 2 p. c. for 112 days. Ans.	46	19	$0\frac{1}{4}$

Interest for years, months, and days.

RULE.—Multiply the principal by double the rate; cut off *one* figure to the right, the rest of the pounds give the answer in shillings for a year, or in pence for a month, or half-pence for 15 days.†

EXAMPLES.

1. Find the interest of £23. 13s. 10d. for 2 years, 3 months, and 10 days, at $3\frac{1}{2}$ per cent.

* Here double the rate being 3, there is no necessity to multiply by it, as the product would have to be divided by the same.

† Every *one* in the figure cut off is $1\frac{1}{5}$d. or say $1\frac{1}{4}$d., 10s. is $\frac{1}{2}$d., and 5s. $\frac{1}{4}$d. for the year; but for months, every *one* cut off, is $\frac{1}{10}$d.

£. s. d.
23 13 10
7 double the rate.

		s.	d.		£.	s.	d.	
16,5 16 10 × 2 =		16	7	× 2 =	1	13	2	for 2 years.
	10 days $\frac{1}{3}$)	1	$4\frac{1}{2}$	× 3 =	0	4	$1\frac{1}{2}$	for 3 months.
			$5\frac{1}{2}$	=	0	0	$5\frac{1}{2}$	for 10 days.
					1	17	9	Ans.

2. Find the interest of £56. 10s. 6d. for 4 years, 5 months, and 15 days, at $4\frac{1}{2}$ per cent.

£. s. d.
56 10 6
9

				£.	s.	d.	
50,8 14 6 × 4 =	2 10	$10\frac{1}{2}$	× 4 =	10	3	6	for 4 years.
	4	$2\frac{3}{4}$	× 5 =	1	1	$1\frac{3}{4}$	for 5 mths.
	15 days		=	0	2	$1\frac{1}{4}$	
				11	6	9	Ans.

3. Find the interest of £20. 12s. 6d. for 5 years, 2 months, and 5 days, at 5 per cent. Ans. £5. 6s. $9\frac{3}{4}$d.

4. Find the interest of £40. 10s. for 3 years, 3 months, and 20 days, at $2\frac{1}{2}$ per cent. Ans. £3. 6s. 9d.

5. Find the interest of £50. 15s. 6d. for 2 years, 8 months, and 25 days, at 4 per cent. Ans. £5. 11s. $0\frac{3}{4}$d.

6. Find the interest of £138. 12s. 6d. for 4 years, 2 months, and 13 days, at $4\frac{1}{2}$ per cent. Ans. £26. 4s. $3\frac{1}{2}$d.

7. Find the interest of £360. 15s. for 10 years, 10 months, and 10 days, at $3\frac{1}{3}$ per cent. Ans. £130. 12s. 1d.

8. Find the interest of £78. 16s. 5d. for 3 years, 5 months, and 21 days, at 6 per cent. Ans. £16. 8s. $5\frac{1}{4}$d.

9. Find the interest of £286. 8s. 6d. for 4 years, 1 month, and 27 days, at $3\frac{1}{2}$ per cent. Ans. £41. 13s. $8\frac{3}{4}$d.

10. Find the interest of £324. 15s. for 2 years, 2 months, and 15 days, at 3 per cent. Ans. £21. 10s. $2\frac{1}{4}$d.

THE END.

W. B. Johnson, Printer, Market-Place, Beverley.