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ELEMENTS

OF

ARITHMETIC,

COMPRISING

LOGARITHMS,

AND THE

COMPUTATION OF ARTIFICERS,

&c.

BY THE

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PREFACE.

THE object of this Treatise is to furnish a Course of Arithmetic, combining the *Theory* and *Practice* of the science, in a concise and simple form. With this view, the Author has introduced nothing but what is absolutely necessary, and in each rule, he has endeavoured to present a statement of the Rule in the plainest manner—one or more model Examples, in a form which, it is hoped, the pupil may advantageously follow—an explanation of the process, when necessary—and a Proof, shewing the principle upon which the Rule is founded.

By thus directing attention in each case to the *reason* of the rule, it is hoped that the subject may be rendered intelligible and attractive: and a science, which is too often degraded into a mere practical art, may become the means of calling forth and improving the reasoning powers of the pupil.

STUBBINGTON, HANTS,
1st July, 1842.

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PART I.

PART II.

ARITHMETIC.

Part I.

Arithmetic is the science that treats of the nature and use of *Numbers*.

Numbers are represented by *Figures*.

A copious collection of Examples, adapted to this work, will shortly be published.

[illegible] these, with 0, *nought* or *cipher*, all numbers whatever may be represented.

A *unit* denotes *one* individual of its kind: as, *one* pound, *one* hour.

NUMERATION.

Numeration teaches the meaning of figures.

1. *A single figure.*

A figure standing *alone* means so many *units:* thus,

1, Means One Unit:	4, Four Units:	7, Seven Units:
2, Two Units:	5, Five Units:	8, Eight Units:
3, Three Units:	6, Six Units:	9, Nine Units.

2. *Several figures.*

When figures are placed together, they have, besides their own value, a value depending on their *place*.

The value of every figure is increased ten-fold for every place that it stands to the left of the unit's place.

A. *Two figures.*

1. **A figure placed to the left of units represents *Tens*:** thus, 12 means ***one Ten,*** and ***two Units.***

2. When there is no unit in the number, 0 set on the right of tens, marks the place of units: thus,

10, One Ten or Ten:	60, Six tens or Sixty:
20, Two Tens or Twenty:	70, Seven Tens or Seventy:
30, Three Tens or Thirty:	80, Eight Tens or Eighty:
40, Four Tens or Forty:	90, Nine Tens or Ninety.
50, Five Tens or Fifty:	

3. Where there are units in the number, the figure representing them is put in the unit's place: thus,

11, Ten & one or Eleven:	16, Ten & six or Sixteen:
12, Ten & two or Twelve:	17, Ten & seven or Seventeen:
13, Ten & three or Thirteen:	18, Ten & eight or Eighteen:
14, Ten & four or Fourteen:	19, Ten & nine or Nineteen.
15, Ten & five or Fifteen:	

4. Twenty, Thirty, Forty, Fifty, Sixty, Seventy, Eighty, Ninety, keep their names before the units; thus,

21, Twenty-one:	76, Seventy-six:
22, Twenty-two:	83, Eighty-three:
34, Thirty-four:	99, Ninety-nine.

5. By the means of *two* figures, we may express any number from *Ten* to *Ninety-nine*, both inclusive.

B. *Three figures.*

1. A figure placed to the left of tens represents *Hundreds*: thus 112 means *one Hundred, one Ten,* and *two Units.*

2. When there are neither Tens nor Units in the number, two ciphers, placed to the right of Hundreds, mark the places of Tens and Units: thus,

100, One Hundred:	600, Six Hundred:
200, Two Hundred:	700, Seven Hundred:
300, Three Hundred:	800, Eight Hundred:
400, Four Hundred:	900, Nine Hundred.
500, Five Hundred:	

3. When there are Tens and Units in the Number, the figures representing them, are put in the Ten's and Unit's places: thus,

101, One hundred and one.
210, Two hundred and ten.
845, Eight hundred and forty-five.
999, Nine hundred and ninety-nine.

4. By means of *three* figures, we may express any number from *One hundred* to *Nine hundred and ninety-nine,* both inclusive.

C. *Any Number of Figures.*

1. Beginning at the Unit's place and reckoning to the left, the figure in the

1st place means so many Units.
2nd .... Tens.
3rd .... Hundreds.
4th .... Thousands.
5th .... Tens of Thousands.
6th .... Hundreds of Thousands.
7th .... Millions.
8th .... Tens of Millions.
9th .... Hundreds of Millions.
10th .... Thousands of Millions.
11th .... Tens of Thousands of Millions.
12th .... Hundreds of Thousands of Millions.
13th .... Billions.

as in the

NUMERATION TABLE.

I. UNITS	1 Units. 2 Tens. 3 Hundreds.
II. THOUSANDS........	4 Units of Thousands. 5 Tens of Thousands. 6 Hundreds of Thousands.
III. MILLIONS	7 Units of Millions. 8 Tens of Millions. 9 Hundreds of Millions.
IV. THOUSANDS OF MILLIONS........	10 Units of Thousands of Millions. 11 Tens of Thousands of Millions. 12 Hundreds of Thousands of Millions.
	13 Units of Billions, &c.

2. If we divide this table into periods of three figures each, beginning from the right, we find that the *first* period contains, Units, Tens, and Hundreds of *Units*; the *second* period, Units, Tens, and Hundreds of *Thousands*; the *third*, Units, Tens, and Hundreds of *Millions*; the *fourth*, Units, Tens, and Hundreds of *Thousands of Millions*.

3. In like manner we may extend the notation to *Billions*, *Thousands of Billions*, *Trillions*, &c.

4. *To numerate any number of Figures.*

Rule. Divide the figures into periods of three, beginning from the right. The *first* period will be *Units*: the *second* period, *Thousands*: the *third*, *Millions*; the *fourth*, *Thousands of Millions*, and so on; then, commencing at the left, annex to the value expressed by the figures of each period, except that of units, the name of the period.

Ex. 1. Numerate 1235.

Th. Units.

Dividing 1235 into periods, we have 1, 235: the *first* period is 235, the *second*, 1, and the number is therefore, *One thousand, two hundred and thirty-five.*

Ex. 2. Numerate 329246789025.

Ths. of Millions. Millions. Ths.

Dividing into periods, we have 329, 246, 789,

Units.

025, and the number is, *Three hundred and twenty-nine thousand two hundred and forty-six millions, seven hundred and eighty-nine thousand, and twenty-five.*

NOTATION.

Notation, the converse of Numeration, teaches how to express numbers by figures.

RULE. Place the *Units,* the *Thousands,* the *Millions,* &c., in their respective periods, as laid down above.

Ex. 1. Express in figures *Thirty-five thousand, six hundred and twenty-four.*

There is 35 to be placed in the *second* period, being the period of *Thousands,* and 624 in the *first* period, the period of *Units,* and therefore the number is 35624.

Ex. 2. Express in figures *Two hundred and twenty millions, five hundred and four.*

There is 220 to be placed in the *third* period, being the period of *Millions :* there are no *Thousands,* the period of which, the *second,* must therefore be marked by 000 : 504 is in the *first* period, the period of *Units,* and the number is 220,000,504.

ADDITION.

Addition is the collecting of two or more numbers into one sum.

1. *To add Units.*

1. By adding, we find that
One added to *One* makes the sum *Two.*
One added to *Two* makes the sum *Three.*
One added to *Three* makes the sum *Four,* and so on.

Thus we form the

ADDITION TABLE.

1 and	2 and	3 and	4 and	5 and
1= 2	1= 3	1= 4	1= 5	1= 6
2= 3	2= 4	2= 5	2= 6	2= 7
3= 4	3= 5	3= 6	3= 7	3= 8
4= 5	4= 6	4= 7	4= 8	4= 9
5= 6	5= 7	5= 8	5= 9	5=10
6= 7	6= 8	6= 9	6=10	6=11
7= 8	7= 9	7=10	7=11	7=12
8= 9	8=10	8=11	8=12	8=13
9=10	9=11	9=12	9=13	9=14

6 and	7 and	8 and	9 and
1= 7	1= 8	1= 9	1=10
2= 8	2= 9	2=10	2=11
3= 9	3=10	3=11	3=12
4=10	4=11	4=12	4=13
5=11	5=12	5=13	5=14
6=12	6=13	6=14	6=15
7=13	7=14	7=15	7=16
8=14	8=15	8=16	8=17
9=15	9=16	9=17	9=18

This Table is used thus; 1 and 1 are 2; 1 and 2 are 3; 1 and 3 are 4, &c. 2 and 1 are 3; 2 and 2 are 4: 2 and 3 are 5, &c.

2. Similarly, we may add units to higher numbers: thus,

11 and 1 are 12:	101 and 5 are 106:
12 and 2 are 14:	123 and 6 are 129:
21 and 4 are 25:	121 and 9 are 130.

2. *To add numbers of many figures.*

Rule. Set the numbers one *under* another placing units under units, tens under tens, hundreds under hundreds, &c.

Draw a line under the columns.

Add up the columns one after another, beginning from the right.

Set the sum of each column beneath.

The sums of the several columns, thus placed, will be the sum required.

Ex. Add 123, 231, 432.

	Explanation of Operation.
123....	1 hundred, 2 tens, 3 units.
231....	2 hundreds, 3 tens, 1 unit.
432....	4 hundreds, 3 tens, 2 units.
786....	7 hundreds, 8 tens, 6 units.

We set the numbers down, placing the units 3, 1, 2 : the tens 2, 3, 3 : the hundreds 1, 2, 4, severally under each other.

We add up each column : the sum of the 1st column, 2, 1, 3 is 6 : that of the 2nd, 3, 3, 2 is 8 : that of the 3rd, 4, 2, 1 is 7, and the sum required is 786.

Proof. The operation is as follows : 2, 1 and 3 units are 6 units, put 6 in the unit's place : 3, 3 and 2 tens are 8 tens, put 8 in the ten's place : 4, 2 and 1 hundreds are 7 hundreds, put 7 in the hundred's place : the sum is 7 hundreds, 8 tens and 6 units, or 786, as by the above rule.

If the sum of any column has more than one figure, set under that column the figure of *lower* place, and carry the figure of *higher* place to the *next* column: at the last column set down the entire sum.

Ex. Add 768, 975, 864, 891.

Explanation of Operation.

768	7 hundreds, 6 tens, 8 units.
975	9 hundreds, 7 tens, 5 units.
864	8 hundreds, 6 tens, 4 units.
891	8 hundreds, 9 tens, 1 unit.
3498	3 thousands, 4 hundreds, 9 tens, 8 units.

The sum of the 1st column 1, 4, 5, 8 is 18, which has *two* figures: we set down 8 and carry 1 to the 2nd column: the sum of 1, 9, 6, 7, 6 is 29: we set down 9 and carry 2 to the 3rd column: the sum of 2, 8, 8, 9, 7 is 34, which we set down.

PROOF. 1, 4, 5, 8 units are 18 units, or 1 ten and 8 units: put down 8 in the unit's place and carry 1 ten to the tens: 1, 9, 6, 7, 6 tens are 29 tens, or 2 hundreds and 9 tens: put down 9 in the ten's place and carry 2 hundreds to the hundreds: 2, 8, 8, 9, 7 hundreds are 34 hundreds, or 3 thousands, and 4 hundreds, we set down 4 in the hundred's place, and put 3 in the thousand's place: and the sum is 3 thousands, 4 hundreds, 9 tens and 8 units, or 3498, as by the rule.

The correctness of the work, may be proved, by working the sum over again in any two parcels.

Thus, in the above example,

768	864	1743
975	891	1755
1743	1755	3498

whence we conclude that 3498 is the correct sum.

The sign +, plus means addition: thus $2+3$ means 3 added to 2: and if = means "equal to", $2+3=5$, means 3 added to 2 equals 5.

SUBTRACTION.

Subtraction is taking a less number from a greater.

1. *To subtract Units.*

By subtracting, we find that
One taken from *Two* leaves *One*.
One taken from *Three* leaves *Two*.
and so on: in this way we form the

SUBTRACTION TABLE.

1 from	3=1	6=3	5 from	7 from
1=0	4=2	7=4	5=0	7=0
2=1	5=3	8=5	6=1	8=1
3=2	6=4	9=6	7=2	9=2
4=3	7=5		8=3	
5=4	8=6	4 from	9=4	8 from
6=5	9=7	4=0		8=0
7=6		5=1	6 from	9=1
8=7	3 from	6=2	6=0	
9=8	3=0	7=3	7=1	
	4=1	8=4	8=2	9 from
2 from	5=2	9=5	9=3	9=0
2=0				

This table is thus used, 1 from 1 is 0 : 1 from 2 is 1, &c. : 2 from 2 is 0 : 2 from 3 is 1, &c.

Similarly, we may take units from higher numbers : thus,

1 from 19 is 18. 6 from 25 is 19.
2 from 24 is 22. 8 from 37 is 29.
9 from 29 is 20. 5 from 91 is 86.

Since the difference of two numbers is found by taking the less from the greater, the less added to the difference must equal the greater.

Thus 2 from 13 is 11 : and 2 the less added to 11 the difference equals 13, the greater.

In like manner, the difference taken from the greater equals the less : thus, in the above example, 11 taken from 13 equals 2.

If the upper figure in any column be less than the lower, add ten to it: from this sum subtract the lower figure: put down the remainder, and carry *one* to the next figure of the lower line.

Ex. From 52043 take 36835.

Explanation of Operation.

52043.. 5 tens of thousands, 2 thousands, 0 hundreds, 4 tens, 3 units.
36835.. 3 tens of thousands, 6 thousands, 8 hundreds, 3 tens, 5 units.

15208.. 1 tens of thousands, 5 thousands, 2 hundreds, 0 tens, 8 units.

5 from 3 cannot be taken, add 10 to 3, and 5 from 13 is 8: set down 8 and carry 1 to the next figure 3: 4 from 4 is 0: 8 from 0 cannot be taken, add 10 to 0, and 8 from 10 is 2: set down 2 and carry 1 to the next figure 6: 7 from 2 cannot be taken, add 10 to 2 and 7 from 12 is 5, set down 5 and carry 1: 4 from 5 is 1: and the whole difference is 15208.

Proof. 5 units cannot be taken from 3 units: add 10 units (taken from 4 tens or 40 units) to 3 units, and 5 units from 13 units is 8 units, put down 8 in the unit's place: 3 tens from 3 tens (for the 4 tens with 1 ten taken from it has become 3 tens) is 0 ten, put 0 in the ten's place: 8 hundreds from 0 hundreds cannot be taken, add 10 hundreds (taken from 2 thousand or 20 hundreds) and 8 hundreds from 10 hundreds is 2 hundreds, set down 2 in the hundred's place: 6 thousands from 1 thousand (for the 2 thousands with 1 thousand taken from it has become 1 thousand) cannot be taken, add 10 thousands (taken from the 50 thousands, or 5 tens of thousands) and 6 thousands from 11 thousands is 5 thousands, put 5 in the thousand's place: 3 tens of thousands from 4 tens of thousands is 1 ten of thousands: and the difference is 1 ten of thousands, 5 thousands, 2 hundreds, and 8 units, or 15208, as by the rule.

3. Subtraction may be proved, by adding the less number to the difference, and the sum, if the work be correct, will equal the greater: thus,

```
52043
36835
-----
15208 the difference
36835 the less
-----
52043 the greater.
```

4. —, *minus*, means subtraction : thus, $3 - 2 = 1$ means 2 subtracted from 3 equals 1.

MULTIPLICATION.

MULTIPLICATION is a short way of *finding the sum of a number added any number of times.*

The number to be so added is called the *multiplicand*, the number of times it is to be added, the *multiplier*, and the result, the *product*.

(1) *To multiply by Units.*

1 multiplied by 2 (which means the sum of 2 *ones*) is 2.

2 multiplied by 3 (which means the sum of 3 *twos*) is 6.

3 multiplied by 3 (which means the sum of 3 *threes*) is 9, and so on.

Thus we form the

MULTIPLICATION TABLE.

Twice	3 times	4 times	5 times	6 times	7 times
1= 2	1= 3	1= 4	1= 5	1= 6	1= 7
2= 4	2= 6	2= 8	2=10	2=12	2=14
3= 6	3= 9	3=12	3=15	3=18	3=21
4= 8	4=12	4=16	4=20	4=24	4=28
5=10	5=15	5=20	5=25	5=30	5=35
6=12	6=18	6=24	6=30	6=36	6=42
7=14	7=21	7=28	7=35	7=42	7=49
8=16	8=24	8=32	8=40	8=48	8=56
9=18	9=27	9=36	9=45	9=54	9=63
10=20	10=30	10=40	10=50	10=60	10=70
11=22	11=33	11=44	11=55	11=66	11=77
12=24	12=36	12=48	12=60	12=72	12=84

8 times	9 times	10 times	11 times	12 times
1= 8	1= 9	1= 10	1= 11	1= 12
2=16	2= 18	2= 20	2= 22	2= 24
3=24	3= 27	3= 30	3= 33	3= 36
4=32	4= 36	4= 40	4= 44	4= 48
5=40	5= 45	5= 50	5= 55	5= 60
6=48	6= 54	6= 60	6= 66	6= 72
7=56	7= 63	7= 70	7= 77	7= 84
8=64	8= 72	8= 80	8= 88	8= 96
9=72	9= 81	9= 90	9= 99	9=108
10=80	10= 90	10=100	10=110	10=120
11=88	11= 99	11=110	11=121	11=132
12=96	12=108	12=120	12=132	12=144

The Table is read thus,

Twice 1 are 2, twice 2 are 4, &c.; 3 times 1 are 3, 3 times 2 are 6, &c.

The sign $\times$, *into*, expresses multiplication, thus $2 \times 3 = 6$ means that 2 multiplied by 3 equals 6.

The numbers multiplied together are called *factors*: thus in 2×5, 2 and 5 are the factors.

(2) *To multiply numbers of many figures.*

Rule. Set the multiplier under the multiplicand, units under units, &c.

Multiply each figure of the multiplicand in succession, beginning with the units.

Ex. Multiply 1243 by 2.

	Explanation of Operation.
1243........	1 thousand, 2 hundreds, 4 tens, 3 units.
2	
2486........	2 thousands, 4 hundreds, 8 tens, 6 units.

We set 2 under 3, the unit's place of the multiplicand: then 2 into 3 is 6: 2 into 4 is 8: 2 into 2 is 4: 2 into 1 is 2, and the required product is 2486.

Proof. Twice 3 units are 6 units, place 6 in the unit's place: twice 4 tens are 8 tens, place 8 in the ten's place: twice 2 hundreds are 4 hundreds, place 4 in the hundred's place: twice 1 thousand are 2 thousands, place 2 in the thousand's place.

If any of the products be of more than one figure, put down the lower figure and carry the higher to the next product.

Ex. Multiply 6257 by 2.

Explanation of Operation.

6257........	6 thousands, 2 hundreds, 5 tens, 7 units.
2	2
12514........	12 thousands, 5 hundreds, 1 ten, 4 units.

2 into 7 is 14, set down 4 and carry 1 : 2 into 5 is 10, and 1 is 11, set down 1 and carry 1 : 2 into 2 is 4, and 1 is 5, set down 5 : 2 into 6 is 12.

Proof. 2 into 7 units is 14 units, or 1 ten and 4 units, put 4 in the unit's place and carry 1 ten: 2 into 5 tens is 10 tens, and 1 ten makes 11 tens, or 1 hundred and 1 ten: put 1 in the ten's place and carry 1 hundred: 2 into 2 hundreds is 4 hundreds and 1 hundred makes 5 hundreds, set 5 in the hundred's place: 2 into 6 thousands is 12 thousands, set 12 in the thousand's place.

3. *When the Multiplier ends in Noughts.*

Rule. Multiply the figures, and place as many noughts to the right of the product as there are in the multiplier.

Ex. 1. Multiply 243 by 20.

```
 243
  20
----
4860
```

Ex. 2. Multiply 243 by 7000.

```
    243
   7000
-------
1701000
```

PROOF. In *Ex.* 1. $243 \times 2 = 486$, and 243×20 must be 10 times as great and therefore $=4860$: since 0 to the right removes each figure one place to the right, and increases the number 10 times.

(4). *When the Multiplier is of many figures.*

RULE. Multiply by each figure of the multiplier separately, placing the first figure of each product under the figure you multiply by.

Add together the products thus found.

Ex. Multiply 5839 by 756.

```
   5839
    756
-------
  35034
 29195
40873
-------
4414284
```

PROOF. The multiplier is 700 and 50 and 6.

5839×6 is 35034
5839×50 is 291950
5839×700 is 4087300

and the sum 4414284 is the product.

N. B. To save trouble, the noughts are, as above, left out in every product, except the first.

(5). *When the Multiplier is the exact product of two or more numbers.*

RULE. Multiply by the factors in succession.

Ex. Multiply 3472 by 24.

$$24 = 4 \times 6.$$

$$\begin{array}{r} 3472 \\ 4 \\ \hline 13888 \\ 6 \\ \hline 83328 \\ \hline \end{array}$$

DIVISION.

Division finds the number of times that one number is contained in another.

In Division two numbers are given, that which is to be divided, called the *Dividend*, and that which divides, called the *Divisor*; the number found by dividing is called the *Quotient*.

1. *To Divide by Units.*

The *Quotient,* or *number of times* the divisor is contained in the dividend, will evidently be found by subtracting the divisor from the dividend, and from succeeding remainders.

Ex. Divide 4 by 2.

From 4	
(1) Subtract.. 2	
From remainder 2	
(2) Subtract.... 2	
Remainder 0	

Here as 2 is subtracted twice, it is contained 2 times in 4: or 2 is the quotient of 4 divided by 2.

Similarly, 6 divided by 3 is 2, and so on. In this way we might form a *Division Table,* but as division is the converse of multiplication, from the table for the latter we may find the quotients: thus, as $3 \times 7 = 21$, 21 divided by $3 = 7$, and 21 divided by $7 = 3$.

Division is marked by a line with the dividend above it and the divisor below: thus $\frac{10}{5} = 2$ means 10 divided by 5 equals 2.

It is sometimes marked by $\div$: thus $10 \div 5$, means 10 divided by 5.

(2). *To Divide a number of many figures by a number less than* 12.

Rule. Place the divisor to the left of the dividend, with a line between them.

Divide each figure of the dividend in succession, beginning at the highest.

Set the quotients underneath.

Ex. Divide 4684 by 2.

Explanation of Operation.

2)4684	....	2)4 thousands, 6 hundreds, 8 tens, 4 units.
2342	....	2 thousands, 3 hundreds, 4 tens, 2 units.

We place the divisor 2 to the left of the dividend 4684 : then 4 by 2 is 2 : 6 by 2 is 3 : 8 by 2 is 4 : 4 by 2 is 2 : and the quotient is 2342.

Proof. 4 thousands by 2 is 2 thousands, put 2 in the thousand's place : 6 hundreds by 2 is 3 hundreds, put 3 in the hundred's place : 8 tens by 2 is 4 tens, put 4 in the ten's place : 4 units by 2 is 2 units, put 2 in the unit's place.

If after dividing any figure, there be a remainder, carry it, as so many tens, to the next figure: divide that sum and set down the quotient as before.

Ex. Divide 7739 by 3.

Explanation of Operation.

3)7739 ...... 3)7 thousands, 7 hundreds, 3 tens, 9 units
2579—2 rem... 2 thousands, 5 hundreds, 7 tens, 9 units—2 units rem.

Here 7 by 3 is 2 and 1 over: add 1 ten or 10 to the next figure 7, and 17 by 3 is 5 and 2 over: add 2 tens or 20 to 3 and 23 by 3 is 7 and 2 over: add 2 tens or 20 to 9, and 29 by 3 is 9 with remainder 2.

Proof. 7 thousands by 3 is 2 thousands and 1 thousand over, set down 2 in the thousand's place and carry 1 thousand or 10 hundreds to 7 hundreds: 17 hundreds by 3 is 5 hundreds and 2 hundreds over, set down 5 in the hundred's place and carry 2 hundreds or 20 tens to 3 tens: 23 tens by 3 is 7 tens and 2 tens over, set down 7 in the ten's place, and carry 2 tens or 20 units to 9 units: 29 units by 3 is 9 units and 2 units over.

(3). *When the Divisor is greater than* 12.

Rule. Set down the divisor to the left of the dividend. See how often the first figure of the divisor is contained in the first, or, if that be not large enough, in the first two or three figures of the dividend, and set the result in the quotient.

Multiply the divisor by this quotient figure.

Subtract the product thus found from the dividend.

To the right of the remainder bring down the next figure in the dividend.

Proceed with this remainder as a dividend, and so on, till all the figures are brought down.

Ex. Divide 34674378 by 951.

```
951)34674378(36460
    2853
    ----
     6144
     5706
     ----
      4383
      3804
      ----
       5797
       5706
       ----
         918 rem.
```

We put 951 to the left of the dividend: 9 is contained 3 times in 34, place 3 in the quotient: multiply the divisor by it, and subtract the product 2853, the remainder is 614: annex 4, the next figure of the dividend. Again 9 is contained 6 times in 61: place 6 in the quotient and proceed as before.—The quotient is 36460 with remainder 918, which may be written $36460\frac{918}{951}$.

Proof. The operation at full length would stand thus

```
951)34674378(30000
    28530000
    --------
 951)6144378(6000
     5706000
     -------
  951)438378(400
      380400
      ------
   951)57978(60
       57060
       -----
         918
       -----
```

Here we find the first part of the quotient 30000, the product of which and the divisor is 28530000, this taken away leaves 6144378 to be divided by 951, which gives for the quotient 6000, and so on.

4. Division may often be shortened by using the factors of the divisor.

Ex. Divide 3268 by 72.

$$72 = 8 \times 9.$$

```
8)3268
  ----
 9)408—4, 1st rem.
   ---
    45—3, 2nd rem.
```

To find the remainder in this case, we notice that the dividend 3268 being units, the *first* remainder 4 is units: but the *second* dividend 408 being the result of units divided by 8, must be so many *eights*, and the *second* remainder 3 is 3 eights or $3 \times 8 = 24$, and the whole remainder is $24 + 4 = 28$ Hence the *Rule. The whole remainder is equal to the first remainder added to the product of the second remainder and the first divisor.*

REDUCTION.

Quantities expressed by numbers are of many names, as 2 *pounds*, 3 *ounces*, 4 *miles*.

Names are distinguished as Higher and Lower.

MONEY.

The names of English Money are four, Pound, Shilling, Penny, Farthing. £ marks Pounds: *s*, Shillings: *d*, Pence: *q*, Farthings: thus £2 13*s*. 3*d*. 42*q*., stands for 2 pounds, 13 shillings, 3 pence, 42 farthings.

Table of Money.

4 Farthings make 1 Penny.
12 Pence . . . . 1 Shilling.
20 Shillings . . . . . . 1 Pound.

1 Farthing is written $\frac{1}{4}d$.: 2 Farthings, or one Half-penny, $\frac{1}{2}d$.: 3 Farthings, $\frac{3}{4}d$.

A Groat = 4*d*.: a Crown = 5*s*.: a Noble = 6*s*. 8*d*.: a Angel = 10*s*.: a Mark = 13*s*. 4*d*.: a Sovereign = 20*s*.: a Guinea = 21*s*.: a Moidore = 27*s*.

WEIGHTS AND MEASURES.

Measure of Weight.

TROY WEIGHT.

24 Grains	=	1 Pennyweight (*dwt.*)
20 Pennyweights	=	1 Ounce (*oz.*)
12 Ounces	=	1 Pound (*lb.*)

Used for gold, silver, jewels and costly articles.

Particular Weights.

20	Grains	= 1	Scruple.
3	Scruples	= 1	Dram.
8	Drams	= 1	Ounce.

Used by Apothecaries in compounding medicines.

Carat = $3\frac{1}{6}$ Grains.

Used for weighing diamonds.

In gold that has alloy with it, 22 *carats* fine or 18 *carats* fine, means 22 or 18 parts of pure gold in 24 : the former is the old, the latter the new standard.

AVOIRDUPOIS WEIGHT.

16	Drams	= 1	Ounce (*oz.*)
16	Ounces	= 1	Pound (*lb.*)
28	Pounds	= 1	Quarter (*qr.*)
4	Quarters	= 1	Hundred Weight (*cwt.*)
20	Cwt.	= 1	Ton.

This weight is used in almost all commercial transactions.

Particular Weights.

14	Pounds	= 1	Stone.
2	Stone	= 1	Tod.
$6\frac{1}{2}$	Tod	= 1	Wey.
2	Weys	= 1	Sack.
12	Sacks	= 1	Last.

Used in the wool trade.

Measures of Length.

12 Inches	=	1 Foot.
3 Feet	=	1 Yard.
$5\frac{1}{2}$ Yards	=	1 Rod or Pole.
40 Poles	=	1 Furlong.
8 Furlongs	=	1 Mile.
$69\frac{1}{15}$ Miles	=	1 Degree.

Among mechanics the inch is divided into *eighths*. By the officers of the revenue, and by scientific persons into *tenths, hundredths*, &c. Formerly it was divided into 12 *lines*.

Particular Measures of Length.

$2\frac{1}{4}$ Inches	=	1 Nail.
4 Nails	=	1 Quarter.
4 Quarters	=	1 Yard.
5 Quarters	=	1 Ell English.
3 Quarters	=	1 Ell Flemish.
6 Quarters	=	1 Ell French.

Used in measuring cloth.

A Hand	=	4 Inches.
A Fathom	=	6 Feet.
A Link	=	7 Inches 92 hundredths.
A Chain	=	100 Links.
10 Square Chains	=	1 Acre.

Measure of Surface.

144	Square Inches	= 1	Square Foot.
9	Square Feet	= 1	Square Yard.
$30\frac{1}{4}$	Square Yards	= 1	Perch.
40	Perches	= 1	Rood.
4	Roods	= 1	Acre.
640	Acres	= 1	Square Mile.

Measure of Solidity.

1728	Cubic Inches	= 1	Cubic Foot.
27	Cubic Feet	= 1	Cubic Yard.

Measure of Capacity.

4	Gills	= 1	Pint.
2	Pints	= 1	Quart.
4	Quarts	= 1	Gallon.
2	Gallons	= 1	Peck.
8	Gallons	= 1	Bushel.
8	Bushels	= 1	Quarter.
5	Quarters	= 1	Load.

This is the Imperial Measure for all liquids and most dry goods. The last four names are used for dry goods only.

Particular Measures of Capacity.

3 Bushels	=	1	Sack.
12 Sacks	=	1	Chaldron.
1 Firkin	=	9	Gallons.
1 Kilderkin	=	18	Gallons.
1 Barrel	=	36	Gallons.
1 Hogshead	=	54	Gallons of Beer.
1 Butt	=	108	Gallons.
1 Anker	=	10	Gallons.
1 Runlet	=	18	Gallons.
1 Tierce	=	42	Gallons.
1 Hogshead	=	63	Gallons of Wine.
1 Puncheon	=	84	Gallons.
1 Pipe	=	126	Gallons.
1 Tun	=	2	Pipes.

The Imperial Gallon contains 10 lb. Avoirdupois of pure water.

Measure of Time.

60 Seconds	=	1 Minute.
60 Minutes	=	1 Hour.
24 Hours	=	1 Day.
7 Days	=	1 Week.
28 Days	=	1 Lunar Month.
28, 29, 30 or 31 Days	=	1 Calendar Month.
365 Days	=	1 Year.

Angular Measure.

60	Seconds (″)	= 1	Minute (′).
60	Minutes	= 1	Degree (°).
90	Degrees	= 1	Quadrant.
4	Quadrants	= 1	Circumference.

Particular Names.

24	Sheets of Paper	= 1	Quire.
20	Quires	= 1	Ream.
2	Reams	= 1	Bundle.
5	Bundles	= 1	Bale.

Rule I.

To reduce a quantity of a higher name to an equivalent quantity of a lower name.

Rule. Multiply the higher name by the number which shows how many of the lower are contained in one of the higher.

Ex. 1. Reduce 24 pence to farthings.

$$\begin{array}{r} 24d. \\ 4 \\ \hline 96q. \\ \hline \end{array}$$

$$\therefore 24d = 96q.$$

Ex. 2. Reduce £24 to shillings, pence and farthings.

$$\begin{array}{r} \pounds \\ 24 \\ 20 \\ \hline 480s. \\ 12 \\ \hline 5760d. \\ 4 \\ \hline 23040q. \\ \hline \end{array}$$

$\therefore$ £24 = 480*s.* = 5760*d.* = 23040*q.*

PROOF. Since £1 = 20*s.*: £24 must be 24 times as much, or 20*s.* × 24 = 480*s.*

If there be any quantities of lower name in the given number, add them in at the successive reductions.

Ex. 1. In £5 6*s.* $7\frac{1}{4}d$, how many farthings?

$$\begin{array}{lll} \pounds. & s. & d. \\ 5 & 6 & 7\frac{1}{4} \\ 20 & & \\ \hline 106s. & & \\ 12 & & \\ \hline 1279d. & & \\ 4 & & \\ \hline 5117q. & & \\ \hline \end{array}$$

$\therefore$ £5 6*s.* $7\frac{1}{4}d.$ = 5117*q.*

Here, in the multiplication by 20 we add 6*s.*, in that by 12, we add 7*d.*, and in that by 4, we add 1*q.*

Ex. 2. Reduce 1 lb. 5oz. 12dwt. 13grs. to grains.

```
  lb.  oz.  dwt.  gr.
   1    5    12   13.
  12
  ——
  17oz.
  20
  ——
 352dwt.
  24
 ————
 1421
 704
 ————
 8461 gr.
 ————
```

∴ 1 lb. 5oz. 12dwts. 13gr. = 8461gr.

Here we multiply by 12, 20, 24, because 12oz. = 1 lb : 20dwt. = 1oz. and 24gr. = 1 dwt., and we add in the 5oz., 12dwt., 13 grs. at the several multiplications.

Rule II.

To reduce a quantity of lower name to an equivalent quantity of higher name.

Rule. Divide the lower name by the number, which shows how many of the lower are contained in one of the higher.

To each quotient annex the remainder, if any, giving to it the name of the quantity from which it arises.

Ex. 1. In 1200*d.* how many shillings?

$$\begin{array}{r} 12)1200d. \\ \hline 100s. \\ \hline \end{array}$$

$\therefore$ 1200*d.* = 100*s.*

Ex. 2. In 30026 farthings, how many pounds?

$$\begin{array}{l} 4)30026q. \\ \hline 12)7506d.-\frac{1}{2}d. \\ \hline 2,0)62,5s.-6\frac{1}{2}d. \\ \hline \text{£}31\ \ 5s.\ \ 6\frac{1}{2}d. \\ \hline \end{array}$$

$\therefore$ 30026*q* = £31 5*s.* 6½*d.*

Ex. 3. Reduce 111111 square yards to acres.

$$\begin{array}{l} \text{sq. yds.} \\ 111111 \\ \quad\quad\ \ 4 \\ \hline 121\left\{\begin{array}{l} 11)444444\text{qrs.} \\ \hline 11)40404 \end{array}\right. \\ \hline 4,0)367,3\text{p. } 11\text{qrs. or } 2\frac{3}{4}\text{ yds.} \\ \hline 4)91\text{r } 33\text{p. } 2\frac{3}{4}\text{yds.} \\ \hline 22\text{a. } 3\text{r. } 33\text{p. } 2\frac{3}{4}\text{yds.} \\ \hline \end{array}$$

Here we reduce the yards to quarters of yards, and divide by 121, the number of quarters in a perch, and so on.

Proof. The farthings in Ex. 2 are divided by 4, because there are 4 times as many farthings as pence; similarly, the pence are divided by 12, to bring them to shillings, and the shillings by 20 to bring them to pounds.

In some cases Rules I and II are both employed.

Ex. In £253 9*s.* 10*d.* how many half-crowns?

```
          £.   s.  d.
          253  9   10
           20
          -----
          5069s.
           12
          -----
     ( 6)60838d.
  30 {    -------
     ( 5)10139—4d.
         --------
           2027—4 six-pences, or 2s.
         --------
```

∴ £253 9*s.* 10*d.*=2027 half-crowns—2*s.* 4*d.* rem.

We reduce the money to pence by Rule 1, and those pence to half-crowns by Rule II.

By Reduction we may readily form the following Tables, which will be useful in the subsequent Rules.

PENCE TABLE.

d.		*s.*	*d.*	*d.*		*s.*	*d.*
12	=	1	0	84	=	7	0
20	=	1	8	90	=	7	6
24	=	2	0	96	=	8	0
30	=	2	6	100	=	8	4
36	=	3	0	108	=	9	0
40	=	3	4	110	=	9	2
48	=	4	0	120	=	10	0
50	=	4	2	130	=	10	10
60	=	5	0	132	=	11	0
70	=	5	10	140	=	11	8
72	=	6	0	144	=	12	0
80	=	6	8				

POUNDS TABLE.

s.		£.	*s.*	*s.*		£.	s.
20	=	1	0	90	=	4	10
30	=	1	10	100	=	5	0
40	=	2	0	110	=	5	10
50	=	2	10	120	=	6	0
60	=	3	0	130	=	6	10
70	=	3	10	140	=	7	0
80	=	4	0	150	=	7	10

ADDITION OF MONEY, &c.

RULE. Set the numbers under each other, farthings under farthings, pence under pence, &c.

Find the sum of the several names, beginning with the lowest, and reduce each sum to its next highest name.

Set the remainders, if any, under the respective columns, and carry the quotients to the higher names.

Ex. Add £14 3*s.* $7\frac{1}{2}$*d.*, £160 19*s.* $8\frac{1}{4}$*d.*, £12 2*s.* $8\frac{3}{4}$*d.*

£.	s.	d.
14	3	$7\frac{1}{2}$
160	19	$8\frac{1}{4}$
12	2	$8\frac{3}{4}$
187	6	$0\frac{1}{2}$

After setting the numbers down, the work is, 3, 1, 2 farthings are 6 farthings or 1*d.* and 2 farthings: put down $\frac{1}{2}$*d.* and carry 1*d.* 1, 8, 8, 7 pence are 24 pence or 2 shillings and 0 pence, set down 0*d.* and carry 2 shillings: 2, 2, 19, 3 shillings are 26 shillings or £1 and 6 shillings, put down 6*s.* and carry £1: 1, 12, 160, and 14 pounds are £187.

Ex. 2. Add together

tons.	cwt.	qr.	lbs.
35	16	0	20
42	14	2	18
18	9	1	16
17	18	3	7
31	5	3	19
45	12	2	5
191	17	2	1

Here, the sums of the lbs. qrs. cwts. are severally divided by 28, 4, 20, the remainders are set down, and the quotients carried to the next column.

SUBTRACTION OF MONEY, &c.

Rule. Set the lesser quantity under the greater, farthings under farthings, &c.

Subtract the several names, beginning with the lowest.

If in the upper line a name be less than the same name in the lower, add to it as many as make one of the next higher name.

From this sum subtract the name in the lower line: set down the difference, and carry 1 to the next higher name.

Ex. 1. From £176 9*s*. $7\frac{1}{2}$*d*. take £20 19*s*. $3\frac{3}{4}$*d*.

£.	*s*.	*d*.
176	9	$7\frac{1}{2}$
20	19	$3\frac{3}{4}$
155	10	$3\frac{3}{4}$

3*q*. from 2*q*. cannot be taken; take 1*d*. or 4*q*. from the 7*d*. and 4*q*. and 2*q*. are 6*q*.: then 3*q*. from 6*q*. leaves 3*q*., set down $\frac{3}{4}$*d*. and carry 1*d*.: 1*d*. and 3*d*. are 4*d*., and 4*d*. from 7*d*. leaves 3*d*., set down 3*d*.: 19*s*. from 9*s*. cannot be taken; take £1 or 20*s*. from £176: 9*s*. and 20*s*. are 29*s*. and 19*s*. from 29*s*. leaves 10*s*., set down 10*s*. and carry £1: £21 from £176 leaves £155.

	days.	hrs.	min.	sec.
Ex. 2. From	5	10	27	15
Take	2	4	13	29
	3	6	13	46

To the 15 seconds we add 60 sec. borrowed from the 27min.

Proof. In Ex. 1, when we added 1*d*. to $\frac{1}{2}$*d*. we took it from 7*d*., which therefore became 6*d*.: and we should have to take 3*d*. from 6*d*.; now taking 3*d*. from 6*d*. is the same as taking 4*d*. from 7*d*.: and so on, in the other names.

MULTIPLICATION OF MONEY &c.

1. *When the multiplier does not exceed* 12.

Rule. Multiply the several names successively, beginning with the lowest.

Reduce each product to its higher name, set down the remainder, if any, and carry the quotient to the product of the next higher name.

Ex. 1. Multiply £2 7*s.* 6½*d.* by 5.

£.	*s.*	*d.*
2	7	6½
		5
£11	17	8½

Here, 5 times 2*q.* are 10*q.* or 2*d.* and 2*q.*, set down ½*d.* and carry 2*d.*: 5 times 6*d.* are 30*d.*, which with 2*d.* is 32*d.* or 2*s.* and 8*d.*, set down 8*d.* and carry 2*s.* : 5 times 7*s.* are 35*s.*, which with 2*s.* is 37*s.*, or £1 and 17*s.*, set down 17*s.* and carry £1 : 5 times £2 are £10, which with £1 is £11.

Ex. 2. Multiply 33 cwt. 2 qrs. 22 lbs. by 7.

cwt.	qr.	lb.
33	2	22
		7
235	3	14

Here, the product of the lbs. 154lbs. is divided by 28, and gives 5 qrs. with remainder 14 lbs.: the product of the qrs., with 5 qrs. added is 19 qrs., which divided by 4, gives 4 cwt. with remainder 3 qrs.: the remainders are put down, and the quotients carried.

2. *When the Multiplier is greater than* 12.

Rule. Multiply by the *factors* of the multiplier.

Ex. Multiply 5*s.* $3\frac{1}{2}$*d.* by 15.

$15 = 5 \times 3.$

	s.	*d.*
	5	$3\frac{1}{2}$
		5
1	6	$5\frac{1}{2}$
		3
£3	19	$4\frac{1}{2}$

If the multiplier cannot be broken exactly into factors, add to the product of the factors of part of it, the upper line multiplied by the remaining part.

Ex. Multiply 5*s.* $3\frac{1}{2}$*d.* by 17.

$17 = 5 \times 3 + 2.$

	s.	*d.*	
	5	$3\frac{1}{2}$	
		5	
1	6	$5\frac{1}{2}$	
		3	
3	19	$4\frac{1}{2}$	
	10	7	$=$ upper line $\times 2$
£4	9	$11\frac{1}{2}$	

N. B. The same result would have been obtained, had we multiplied by 9 and 2 and subtracted the upper line, for $17 = 9 \times 2 - 1$.

Ex. What is the price of 2485 yards of cloth at 15*s.* 7½*d.* per yard?

	s.	*d.*		
	15	7½	=	price of 1 yard.
		10		
7	16	3	= ..	.. 10 yds.
		10		
78	2	6	= ..	.. 100 ..
		10		
781	5	0	= ..	.. 1000 ..
		2		
1562	10	0	= ..	.. 2000 ..
312	10	0	= ..	.. 400 ..
62	10	0	= ..	.. 80 ..
3	18	1½	= ..	.. 5 ..
£1941	8	1½	= ..	.. 2485 yards.

Here we find the prices of 10, 100, 1000 yards: we multiply the price of 1000 by 2, of 100 by 4, of 10 by 8, and of 1 by 5, and hence have the prices of 2000, 400, 80 and 5 yards, or 2485 yards.

DIVISION OF MONEY, &c.

Rule. Divide each name successively, beginning with the highest.

Set down the quotient.

Reduce the remainder, if any, to its next lower name, and add it, so reduced, to the next lower name in the dividend.

Divide that sum, and proceed as before.

Ex. Divide £61 8*s*. 9½*d*. by 4.

	£.	*s*.	*d*.
4)	61	8	9½
	15	7	2¼—2*q* rem.

Hence, £61 by 4 is £15 and £1 or 20*s*. over: set down £15, and add 20*s*. to 8*s*.: 28*s*. by 4 is 7*s*., set down 7*s*.: 9*d*. by 4 is 2*d*. and 1*d*. or 4*q*. over: set down 2 and add 4*q*. to 2*s*.: and 6*q*. by 4 is 1*q*. and 2*q*. over.

Divide 3qr. 17lb. 4oz. by 9.

	qr.	lb.	oz.
9)	3	17	4
		11	4

Here the qrs. are brought to lbs. and added to 17 lb. and then divided by 9: the remainder 2 lb. is brought to oz. and added to 4 oz., and then divided by 9. Thus

3qrs.	2 lb.
28	16
84 lb.	32 oz.
17 lb. added	4 oz. added
9)101 lb.	9)36 oz.
11lb. and 2lb. over.	4 oz.

2. *When the Divisor is greater than* 12.

Rule. Divide by the Factors of the divisor, if it can be broken into factors, if not, proceed by Long Division.

Ex. Divide £59 13*s.* $3\frac{1}{2}$*d.* by 66.

```
       £.  s.   d.
     6)59  13   3½
       ----------
   11)  9  18  10½.. 2q. ⎧
       ----------        ⎨ 56q. or 1s. 2d. rem.
           18   0¾.. 9q. ⎩
           --------
```

Here the remainder after dividing by 6 is 2*q.* and by 11 the remainder is 9*q.* and therefore the true remainder is $9 \times 6+2=56q.$ or 1*s.* 2*d.*

N. B. In Long Division, reduce each remainder to the next lower name, and add in that name.

Ex. Divide £367 13*s.* $6\frac{3}{4}$*d.* by 23.

```
      £.   s.  d.  £.  s.  d.
   23)367  13  6¾(15  19  8½—13q. rem.
      23
      --
      137
      115
      ---
       22
        20
       ---
   23)453(19s.
      23
      ---
      223
      207
      ---
       16
        12
       ---
```

```
23)198(8d.
   184
   ---
    14
     4
   ---
23)59(½d.
   46
   --
   13
   --
```

We bring the remainder £22 into shillings and add in 13*s.*: we bring the remainder 16*s.* into pence and add in 6*d.*: we bring the remainder 14*d.* into farthings and add in 3*q.*

Though it more properly belongs to fractions, we may here give examples of Multiplication and Division, in which a fraction occurs in the multiplier and divisor.

Ex. Multiply £1 16*s.* 6*d.* by 22¾.

	£.	s.	d.
	1	16	6
			2
	3	13	0
			11
	40	3	0
¾	1	7	4½
	£41	10	4½

Here we multiply by 22, as shewn in page 42. Then for $\frac{3}{4}$, we multiply in a separate place £1 16*s*. 6*d*. by 3 and divide the product by 4, and add the result £1 7*s*. 4$\frac{1}{2}$*d*. to the product by 22.

Ex. Divide £30 2*s*. 10$\frac{1}{2}$*d*. by 22$\frac{3}{4}$.

22$\frac{3}{4}$=$\frac{91}{4}$, as will be proved in fractions.

```
        £.  s.  d.
        30  2  10½
                4
       ----------- £.  s.  d.
   91)120  11   6 (1   6   6
        91
        --
        29
         20
        ---
    91)591(6s.
        546
        ---
         45
         12
        ---
    91)546(6d.
        546
        ---
        ...
        ---
```

Here, we multiply 22 by 4, the lower term of the fraction: to the product 88, we add 3, the upper term: then we multiply the dividend by 4 and divide by 91.

RULE OF THREE.

In this Rule three things are given, and a fourth is to be found, between which and one of the three, there shall be the same proportion as there is between the other two. Thus, in the question, if 2 *lbs.* of tea cost 8*s*, how much will 10 *lbs.* cost? three things are given, viz. 2*lbs.*, 8*s*, 10*lbs.*, and a fourth is to be found, viz. the price of 10*lbs*: this price having the same proportion to 8*s*. that 10*lbs*. has to 2*lbs*

RULE. *For the statement.* Of the three given quantities put down as the third term that which is of the same kind as the one required: in the second term, put the greater or less of the other two quantities, according as, from the nature of the question, the quantity sought should be greater or less than the third term: put the remaining quantity in the first term.

For the operation. Multiply the second and third terms together and divide by the first; the quotient will be the quantity required, in the same name as the third term: it may be reduced to a higher name, if necessary.

Ex. 1. If 2*lbs* of tea cost 8*s* how much will 10*lbs*. cost?

lb.		*lb.*		*s.*
2	:	10	: :	8
				10
				2)80*s*.
				40*s*. = the price of 10*lbs*.

Here, the thing sought is *money*, the price of 10*lb* : we therefore put *money*, 8*s*. in the third term: and as the price of 10*lb*. will be *greater* than that of 2*lb*. we put 10*lb*., the *greater* quantity in the second term, and 2*lb*. in the first.

We multiply 8*s* by 10 and divide by 2 : the result 40*s*. is the price of 10*lbs*.

PROOF. If 2*lbs*. cost 8*s*., 1*lb*. must cost 4*s*., and therefore 10*lbs*. must cost 10 times 4*s*., or 40*s*., as by the Rule.

N. B. A more general proof will be given in Proportion.

Ex. 2. If 8 men do a piece of work in 12 days, how many men will do it in 16 days?

days		days		men
16	:	12	: :	8
		8		
		16)96 men.		
		6 men		

Here, as the number of men required to do a piece of work in 16 days is *less* than that in 12 days, we put 12 the *lesser* number in the second place. This is called *Inverse* Proportion.

If the quantities be compound, reduce the first and second terms to the same name, and the third to the lowest name contained in it: then proceed as above.

Ex. If 2*cwt.* 3*qrs.* 14*lbs.* cost £6 14*s.* 2*d.*, what will 12*cwt.* 3*qrs.* cost?

```
cwt. qrs. lbs.        cwt. qrs.           £.  s.  d.
  2   3   14    :      12   3      : :     6  14   2
  4                     4                 20
 --                    --                 --
 11                    51                134
 28                    28                 12
 --                    --                ----
322lbs.               408               1610 pence
---                  102                1428
                     ----               ----
                     1428lbs.          12880
                     ----              3220
                                      6440
                                     1610
                                     ---------12)
                                 322)2299080 (7140d.
                                     2254     ----
                                     ----    2,0)59,5s.
                                       450      ----
                                       322    £29 15s.
                                       ----    ----
                                       1288
                                       1288
                                       ----
                                       ...0
                                       ----
```

Here we reduce the 1st and 2nd terms to the same name, *lbs.*, and the 3rd to *pence*. The quotient being *pence*, is reduced to £. *s.* *d.*

If the *first* term and either of the other two can be divided by the same number, divide them by that number, and the work will be shortened.

Thus, in the above *Ex.*

lb.		*lb.*		*d.*
322	:	1428	: :	1610

Dividing the 1st and 3rd terms by 161,

2	:	1428	: :	10

and dividing the 1st and 2nd terms by 2,

1	:	714	: :	10
		10		

12)7140*d.*

2,0)59,5*s.*

£29 15*s.*

which is shorter than the former operation.

Proof. If a divisor and dividend be both divided by the same number, the quotient is not changed: thus 24 divided by 8 is the same as 12 divided by 4.

Part II.

MEASURES.

A *measure* is a number which will divide another number without remainder: thus 3 is a measure of 15.

A *common measure* of two or more numbers, is that number which will divide them all without remainder: the *greatest* number which will do so, is the *greatest common measure*: thus, of 12, 18, 30 the *common measures* are 2, 3 6; and the *greatest common measure* is 6.

To find the greatest common measure of two numbers.

Rule. Divide the greater by the less, and the divisor by the remainder, and so on till nothing remains. The last divisor is the greatest common measure.

Ex. Find the greatest common measure of 63 and 168.

```
63)168(2
   126
   ---
    42)63(1
       42
       --
       21)42(2
          42
          --
          ..
          --
```

∴ 21 is the greatest common measure.

PROOF. The proof of this rule depends upon the principle, that *if a number measure two others, it will measure their sum, their difference, and any multiple of each.*

Thus 4 is a common measure of 20 and 12.

$$\left.\begin{array}{lll} \text{Their sum} & = 20+12 = 32 = 4\times 8 \\ \text{Their difference} & = 20-12 = 8 = 4\times 2 \\ \text{A multiple of } 20 & = 20\times 5 = 100 = 4\times 25 \end{array}\right\}\text{are all measured by 4.}$$

Now, applying this principle to the above example, we see that 21 is a measure of 42, and therefore of 42+21 or 63 : 21 therefore measures 63×2 or 126, and therefore 126+42 or 168 ; and hence it is a common measure of 168 and 63.

It is also the *greatest* common measure. For suppose a number greater than 21 to measure 168 and 63 : it would therefore measure 168—126 or 42, and therefore 63—42 or 21 ; which is impossible, because no number greater than 21 can measure 21 ; therefore 21 is the greatest common measure of 63 and 168.

To find the greatest common measure of three or more numbers.

RULE. Find the greatest common measure of two of the numbers: then that of this common measure and the third number, and so on.

Ex. Find the greatest common measure of 63, 168 and 27.

As above, 21 is the greatest common measure of 63 and 168.

```
21)27(1
   21
   --
    6)21(3
      18
      --
       3)6(2
         6
         -
         .
         -
```

∴ 3 is the greatest common measure of 63, 168 and 27.

MULTIPLES.

A *multiple* of a number is a number which can be divided by it without a remainder: thus 12 is a multiple of 4.

A *common multiple* of two or more numbers is that number which can be divided by them all without remainder, the *least* number which can be so divided, is the least common multiple: thus, 24 and 12 are *common multiples* of 3 and 4, and 12 is the *least common multiple.*

To find the least common multiple of two numbers.

RULE. Multiply the two numbers together, and divide their product by their greatest common measure.

Ex. Find the least common multiple of 48 and 81.

```
48)81(1
   48
   --
   33)48(1
      33
      --
      15)33(2
         30
         --
          3)15(5
            15
            --
            ..
            --
```

∴ 3 is the greatest common measure, and $\frac{48 \times 81}{3} =$ $16 \times 81 = 1296$, the least common multiple required.

To find the least common multiple of three or more numbers.

RULE. Find the least common multiple of the first two: then of that common multiple and the third, and so on.

Ex. Find the least common multiple of 48, 81, and 75.

As above, 1296 is the least common multiple of 48 and 81.

```
75)1296(17
   75
   ---
   546
   525
   ---
    21)75(3
       63
       --
       12)21(1
          12
          --
           9)12(1
              9
              --
              3)9(3
                9
                -
                .
                -
```

$\therefore \dfrac{1296 \times 75}{3} = 1296 \times 25 = 32400$, the least common multiple of 48, 81 and 75.

Another Method.

RULE. Range the numbers in a line, rejecting all those contained in any higher number.

Divide by the common factors of any two or more of them, put down the quotients, and the numbers which are not divisible.

The product of the divisors and remainders is the least common multiple.

Ex. Find the least common multiple of 6, 15, 4, 3, 5.

$$\begin{array}{r|l} 2) & 6.\ \ 15.\ \ 4. \\ \hline 3) & (3).\ \ 15.\ \ 2. \\ \hline & 5.\ \ 2. \\ \hline \end{array}$$

$\therefore\ 2 \times 3 \times 5 \times 2 = 60$, the least common multiple.

As 3 is contained exactly in 6, and 5 in 15, we reject 3 and 5: we divide 6, 15, 4, by 2 and have 3, 15, 2: as 3 is contained exactly in 15 we reject it, and dividing by 3 have 5 and the undivided number 2.

PROOF. The proof of this rule depends upon the principle, that *the least common multiple of several numbers is formed by the multiplication of their simple factors, omitting one of them so long as it occurs in any two of the numbers.*

Thus resolving 6, 15, 4, 3 into factors, we have,

$$\begin{aligned} 6 &= 3 \times 2 \\ 15 &= 3 \times 5 \\ 4 &= 2 \times 2 \\ 3 &= 3 \times 1 \end{aligned}$$

and leaving out the 3 and 2, when they occur in more than one number, we have

$5 \times 2 \times 2 \times 3 = 60$, the least common multiple.

FRACTIONS.

A FRACTION is a part or parts of a unit: as *one-half of* 1*s*., *two-thirds of* 1£.

A fraction is represented by two numbers placed one over the other with a line between them: as $\frac{1}{2}$, *one-half*, $\frac{2}{3}$, *two-thirds*.

The *lower* number is called the *denominator*, as giving *name* to the fraction, and shews into how many parts the unit is divided: the *upper* number, called the *numerator*, shews the *number* of such parts taken: thus, $\frac{2}{3}$ of 1*s*. which is read *two-thirds of* 1*s*., means that 1*s*. is divided into 3 parts and that 2 such parts are taken, and therefore equals 8*d*.

A *proper* fraction is one whose numerator is less than its denominator, as $\frac{1}{6}$, $\frac{2}{3}$.

An improper fraction is one whose numerator is equal to or greater than its denominator, as $\frac{2}{2}$, $\frac{7}{2}$: in the first case, the fraction equals a unit, thus, $\frac{2}{2}$ of 1*s*. = 1*s*.: in the *latter* case, it is greater than unity: thus, $\frac{7}{2}$ of 1*s*. =3*s*. 6*d*.

A *mixed number* is composed of a whole number and a fraction: as $1\frac{1}{6}$, $5\frac{3}{9}$.

A *compound fraction* is a fraction of a fraction: as $\frac{2}{3}$ of $\frac{1}{2}$.

REDUCTION.

Lemma 1. A fraction is multiplied by a whole number, by multiplying the numerator by it: thus $\frac{2}{3}\times5=\frac{10}{3}$.

Proof. $\frac{10}{3}$ is 5 times as great as $\frac{2}{3}$, since 10 is 5 times as great as 2, and the divisor 3 is unchanged.

Lemma 2. A fraction is divided by a whole number, by multiplying the denominator by it: thus $\frac{2}{3} \div 5 = \frac{2}{15}$.

Proof. $\frac{2}{15}$ is 5 times as small as $\frac{2}{3}$, since 2 remains the same, whilst the divisor is 5 times as great as 3.

Lemma 3. Similarly, a fraction may be multiplied by a whole number, by dividing the denominator, and divided, by dividing the numerator by it: thus, $\frac{4}{6} \times 3 = \frac{4}{2}$, and $\frac{6}{4} \div 3 = \frac{2}{4}$.

Lemma 4. If the numerator and denominator of a fraction be both multiplied or both divided by the same quantity, the value of the fraction is not altered: thus, $\frac{2}{3} = \frac{6}{9}$: for, when we multiply the *numerator* by 3, we *multiply* the fraction by 3: and when we multiply the fraction *denominator* by 3, we *divide* the fraction by 3: hence, as we *multiply* and *divide* the fraction by the same number 3, we do not alter its value. Conversely, $\frac{6}{9} = \frac{2}{3}$.

Case I.

To reduce a fraction to its lowest terms.

Rule. Divide the numerator and denominator by their greatest common measure.

Ex. 1. Reduce $\frac{10}{15}$ to its lowest terms.

The greatest common measure is 5:

$$\therefore \frac{10}{15} = \frac{2}{3}.$$

Ex. 2. Reduce $\frac{3094}{3042}$ to its lowest terms.

```
3042)3094(1
     3042
     ----
       52)3042(58
          260
          ---
           442
           416
           ---
            26)52(2
               52
               --
               ..
```

The greatest common measure is 26.

```
26)3094(119          26)3042(117
   26                   26
   --                   --
    49                   44
    26                   26
    --                   --
    234                  182
    234                  182
    ---                  ---
    ...                  ...
```

$$\therefore \frac{3094}{3042} = \frac{119}{117}.$$

The same may be done, by dividing the numerator and denominator by any numbers in succession that will divide them both without a remainder.

Ex. Reduce $\frac{144}{240}$ to its lowest terms.

$$\frac{144}{240} = \frac{72}{120} = \frac{36}{60} = \frac{12}{20} = \frac{6}{10} = \frac{3}{5}.$$

where the divisors are 2, 2, 3, 2, 2.

PROOF. *Lemma* 4.

CASE II.

To express a whole number by an equivalent fraction with a given denominator.

RULE. Multiply the whole number by the proposed denominator, which also place under the product.

Ex. Express 5 as a fraction whose denominator is 4.

$$5 = \frac{5 \times 4}{4} = \frac{20}{4}.$$

PROOF $5 = \frac{5}{1} = \frac{5 \times 4}{1 \times 4} = \frac{20}{4}.$ *Lem.* 4.

CASE III.

To reduce a mixed number to an improper fraction.

RULE. Multiply the whole number by the denominator and to the product add the numerator: under this sum place the denominator.

Ex. Reduce $3\frac{4}{5}$ to an improper fraction.

$$3\frac{4}{5} = \frac{3 \times 5 + 4}{5} = \frac{15 + 4}{5} = \frac{19}{5}.$$

PROOF. $3 = \frac{15}{5}$ (CASE II) $\therefore 3\frac{4}{5} = \frac{15}{5}$ and $\frac{4}{5} = \frac{19}{5}.$

Case IV.

To reduce an improper fraction to a whole or mixed number.

Rule. Divide the numerator by the denominator, the quotient will be the whole number required: and if there be any remainder, place it over the denominator for the fractional part.

Ex. 1. Reduce $\frac{32}{8}$ to a whole number.

$$\begin{array}{r} 8)32 \\ \hline 4 \\ \hline \end{array}$$

$$\therefore \frac{32}{8} = 4.$$

Ex. 2. Reduce $\frac{981}{16}$ to a mixed number.

$$\begin{array}{r} 16)981(61 \\ 96 \\ \hline 21 \\ 16 \\ \hline 5 \\ \hline \end{array}$$

$$\therefore \frac{981}{16} = 61\tfrac{5}{16}$$

Proof. $\frac{981}{16} = \frac{976+5}{16} = \frac{976}{16} + \frac{5}{16} = 61\tfrac{5}{16}.$

Case V.

To reduce fractions to others of equal value having a common denominator.

Rule. Multiply the numerator and denominator of each fraction, by all the other denominators.

Ex. Reduce $\frac{2}{3}, \frac{4}{7}, \frac{5}{4}$ to a common denominator.

$$\frac{2}{3} = \frac{2\times7\times4}{3\times7\times4} = \frac{56}{84}$$

$$\frac{4}{7} = \frac{4\times3\times4}{7\times3\times4} = \frac{48}{84}$$

$$\frac{5}{4} = \frac{5\times3\times7}{4\times3\times7} = \frac{105}{84}$$

$\therefore \frac{56}{84}, \frac{48}{84}, \frac{105}{84}$ are the fractions required.

To reduce fractions to others having the *least* common denominator, find the least common multiple of all the denominators: then see what the denominator of each fraction must be multiplied by to produce that number, and multiply the numerator and denominator by it.

Ex. Reduce $\frac{8}{3}, \frac{5}{9}, \frac{7}{12}$ to their least common denominator.

36 is the least common multiple of 3, 9, 12.

$$\frac{8}{3} = \frac{8 \times 12}{3 \times 12} = \frac{96}{36}$$

$$\frac{5}{9} = \frac{5 \times 4}{9 \times 4} = \frac{20}{36}$$

$$\frac{7}{12} = \frac{7 \times 3}{12 \times 3} = \frac{21}{36}$$

$\therefore \frac{96}{36}, \frac{20}{36}, \frac{21}{36}$ are the fractions required.

PROOF. *Lemma* 4.

CASE VI.

To reduce a compound fraction to a simple one.

RULE. Multiply the numerators together for a new numerator, and the denominators for a new denominator.

Ex. Reduce $\frac{2}{9}$ of $\frac{9}{22}$ of $\frac{11}{12}$ to a simple fraction.

$$\frac{2}{9} \text{ of } \frac{9}{22} \text{ of } \frac{11}{12} = \frac{2 \times 9 \times 11}{9 \times 22 \times 12} = \frac{198}{2376} = \frac{1}{12}.$$

This work is much shortened by striking out factors common to both numerator and denominator, where such are found: thus

$$\frac{2}{9} \text{ of } \frac{9}{22} \text{ of } \frac{11}{12} = \frac{2 \times 9 \times 11}{9 \times 22 \times 12} = \frac{1 \times 1 \times 1}{1 \times 1 \times 12} = \frac{1}{12}.$$

Here, we divide both numerator and denominator by 9: then we divide the numerator by 2×11, and the denominator by 22.

Case VII.

ADDITION OF FRACTIONS.

Rule. Reduce the fractions, if necessary, to a common denominator: add together the numerators and under their sum place the common denominator.

Ex. 1. Add $\frac{5}{12}$ and $\frac{9}{12}$.

$$\frac{5}{12} + \frac{9}{12} = \frac{14}{12} \text{ the sum required.}$$

Ex. 2. Add $\frac{2}{3}$, $\frac{4}{7}$, $\frac{5}{4}$.

$$\frac{2}{3} = \frac{2 \times 7 \times 4}{3 \times 7 \times 4} = \frac{56}{84}$$

$$\frac{4}{7} = \frac{4 \times 3 \times 4}{7 \times 3 \times 4} = \frac{48}{84}$$

$$\frac{5}{4} = \frac{5 \times 3 \times 7}{4 \times 3 \times 7} = \frac{105}{84}$$

$$\therefore \frac{56}{84} + \frac{48}{84} + \frac{105}{84} = \frac{209}{84}, \text{ the sum required.}$$

Proof. In *Ex.* 1, unity is divided into 12 parts and to 5 of such parts 9 are to be added, therefore the sum is 14 such parts, or $\frac{14}{12}$.

Mixed numbers must be reduced to improper fractions, and compound fractions to simple ones.

Ex. Add $4\frac{1}{3}$ and $\frac{2}{3}$ of $3\frac{3}{4}$.

$$4\tfrac{1}{3} = \frac{13}{3}: \ \frac{2}{3} \text{ of } 3\tfrac{3}{4} = \frac{2}{3} \text{ of } \frac{15}{4} = \frac{5}{2}.$$

$\therefore \frac{13}{3}$ and $\frac{5}{2}$ are the fractions to be added.

$$\frac{13}{3} = \frac{13 \times 2}{3 \times 2} = \frac{26}{6}$$

$$\frac{5}{2} = \frac{5 \times 3}{2 \times 3} = \frac{15}{6}$$

$\therefore \frac{26}{6} + \frac{15}{6} = \frac{41}{6} = 6\frac{5}{6}$, the sum required.

Mixed numbers may often be more conveniently added, by adding the fractional parts, and to their sum annexing the sum of the integral parts.

Ex. Add $7\frac{2}{7}$ and $4\frac{7}{8}$.

$$\frac{2}{7} = \frac{2 \times 8}{7 \times 8} = \frac{16}{56}$$

$$\frac{7}{8} = \frac{7 \times 7}{8 \times 7} = \frac{49}{56}$$

$$\therefore \frac{2}{7} + \frac{7}{8} = \frac{65}{56} = 1\tfrac{9}{56},$$

and $11 + 1\frac{9}{56} = 12\frac{9}{56}$, the sum required.

Case VIII.

SUBTRACTION OF FRACTIONS.

Rule. Reduce the fractions as in Addition, subtract the numerators, and under their difference, place the common denominator.

Ex. 1. From $\frac{9}{10}$ take $\frac{7}{10}$.

$$\frac{9}{10} - \frac{7}{10} = \frac{2}{10} \text{ or } \frac{1}{5}, \text{ the difference required.}$$

Ex. 2. From $2\frac{3}{5}$ take $\frac{2}{3}$ of $\frac{4}{7}$.

$$2\tfrac{3}{5} = \tfrac{13}{5} : \tfrac{2}{3} \text{ of } \tfrac{4}{7} = \tfrac{8}{21}$$

$$\frac{13}{5} = \frac{13 \times 21}{5 \times 21} = \frac{273}{105}$$

$$\frac{8}{21} = \frac{8 \times 5}{21 \times 5} = \frac{40}{105}$$

$$\therefore \frac{273}{105} - \frac{40}{105} = \frac{233}{105}, \text{ the difference required.}$$

Mixed numbers may often be more conveniently subtracted, by subtracting the fractions and to their difference annexing the difference of the integral part.

N. B. If the fraction of the subtrahend be greater than that of the minuend, the latter must be increased by borrowing 1 from its integral part.

Ex. From $5\frac{1}{3}$ take $1\frac{4}{7}$.

$$\frac{1}{3}=\frac{1\times 7}{3\times 7}=\frac{7}{21}$$

$$\frac{4}{7}=\frac{4\times 3}{7\times 3}=\frac{12}{21}$$

$$\therefore\ 5\tfrac{1}{3}-1\tfrac{4}{7}=5\tfrac{7}{21}-1\tfrac{12}{21}=4\tfrac{28}{21}-1\tfrac{12}{21}=3\tfrac{28}{21}-\tfrac{12}{21}=3\tfrac{16}{21}.$$

Here as $\frac{12}{21}$ is greater than $\frac{7}{21}$, we add to $\frac{7}{24}$, $\frac{21}{21}$ or 1 borrowed from 5, and instead of $5\frac{7}{21}$ write its equivalent $4\frac{28}{21}$.

PROOF. Similar to that in Case VII.

CASE IX.

MULTIPLICATION OF FRACTIONS.

RULE. Reduce mixed numbers to improper fractions, and compound fractions to simple ones.

Multiply the numerators together for a new numerator, and the denominators for a new denominator.

Ex. 1. Multiply $\frac{2}{3}$ by $\frac{5}{7}$.

$$\frac{2}{3}\times\frac{5}{7}=\frac{2\times 5}{3\times 7}=\frac{10}{21},\text{ the product required.}$$

Ex. 2. Multiply together $2\frac{3}{5}$, $\frac{8}{9}$, $\frac{15}{4}$, $\frac{3}{26}$.

$$2\tfrac{3}{5}=\frac{13}{5}.$$

$$\frac{13}{5}\times\frac{8}{9}\times\frac{15}{4}\times\frac{3}{26}=\frac{13\times 8\times 15\times 3}{5\times 9\times 4\times 26}=\frac{1\times 1\times 1\times 1}{1\times 1\times 1\times 1}$$

$=1$, the product required.

PROOF. In *Ex.* 1. $\frac{2}{3} \times 5 = \frac{10}{3}$ (*Lem.* 1): but we had to multiply not by 5, but by $\frac{5}{7}$, a number 7 times less than 5: $\frac{10}{3}$ is therefore 7 times too large: hence the true product is $\frac{10}{3} \div 7 = \frac{10}{21}$ (*Lem.* 2).

CASE X.

DIVISION OF FRACTIONS.

RULE. Invert the divisor, and proceed as in multiplication.

Ex. Divide $\dfrac{2}{3}$ by $\dfrac{5}{7}$.

$\dfrac{2}{3} \div \dfrac{5}{7} = \dfrac{2}{3} \times \dfrac{7}{5} = \dfrac{14}{15}$, the quotient required.

Ex. 2. Divide $5\frac{8}{15}$ by $2\frac{1}{6}$.

$$5\tfrac{8}{15} = \tfrac{83}{15} : \ 2\tfrac{1}{6} = \tfrac{13}{6},$$

and $\dfrac{83}{15} \div \dfrac{13}{6} = \dfrac{83}{15} \times \dfrac{6}{13} = \dfrac{83}{5} \times \dfrac{2}{13} = \dfrac{166}{65} = 2\frac{36}{65}$.

PROOF. In *Ex.* 1. $\frac{2}{3} \div 5 = \frac{2}{15}$ (*Lem.* 2): but we had to divide not by 5, but by $\frac{5}{7}$, a number 7 times less than 5: $\frac{2}{15}$ is therefore 7 times too small: hence the true quotient is $\frac{2}{15} \times 7 = \frac{14}{15}$. (*Lem.* 2).

By the above rules complex fractions may be simplified.

Ex. 1. Reduce $\dfrac{1}{2} + \dfrac{2}{3} - \dfrac{1}{6} + \dfrac{3}{8} - \dfrac{1}{12}$ to its simplest form.

$$\frac{1}{2} + \frac{2}{3} - \frac{1}{6} + \frac{3}{8} - \frac{1}{12} = \frac{12}{24} + \frac{16}{24} - \frac{4}{24} + \frac{9}{24} - \frac{2}{24}$$

$$= \frac{37}{24} - \frac{6}{24} = \frac{31}{24} = 1\frac{7}{24}.$$

Ex. 2. Reduce $\frac{2\frac{2}{3}}{4\frac{6}{7} \text{ of } \frac{4}{3}}$ to its simple form.

$$\frac{2\frac{2}{3}}{4\frac{6}{7} \text{ of } \frac{4}{3}} = \frac{\frac{8}{3}}{\frac{34}{7} \text{ of } \frac{4}{3}} = \frac{\frac{8}{3}}{\frac{34 \times 4}{7 \times 3}} = \frac{8}{3} \div \frac{34 \times 4}{7 \times 3} =$$

$$= \frac{8 \times 7 \times 3}{3 \times 34 \times 4} = \frac{1 \times 7 \times 1}{1 \times 17 \times 1} = \frac{7}{17}.$$

Case XI.

To find the value of the fraction of a unit in terms of a lower name, and vice versâ.

Rule. Multiply or Divide, as the case may require, by as many of the lower as make one of the higher.

Ex. Reduce $\frac{1}{7}$ yard to inches.

$$\frac{1}{7} \text{ yd.} = \frac{1}{7} \times \frac{3 \times 12}{1} \text{ inch} = \frac{36}{7} \text{ inch} = 5\tfrac{1}{7} \text{ inch.}$$

Ex. 2. Reduce $\frac{2}{3}$ of 1*d.* to the fraction of £1.

$$\frac{2}{3}d. = £\frac{2}{3} \times \frac{1}{20 \times 12} = £\frac{1}{3 \times 10 \times 12} = £\frac{1}{360}.$$

Proof. Same as in Reduction, Pp. 33, 37.

Case XII.

To find what fraction any compound quantity is of a given unit.

Rule. Reduce the compound quantity and the given unit to the lowest denomination mentioned, and place the latter under the former in the form of a fraction.

Ex. What fraction of £1 is 10*s*. $5\frac{3}{4}$*d*.?

s. *d*.	£
10 $5\frac{3}{4}$	1
12	20
125*d*.	20*s*.
4	12
503*q*.	240*d*.
	4
	960*q*.

$$\therefore 10s.\ 5\tfrac{3}{4}d. = £\frac{503}{960}.$$

Case XIII.

To find the value of the fraction of a unit.

Rule. Multiply the numerator by as many of the lower as make one of the higher, and divide by the denominator.

Ex. What is the value of $\frac{5}{6}$ of 1 yard.?

$$\overset{\text{yd.}}{\frac{5}{6}}=\overset{\text{ft.}}{\frac{5\times 3}{6}}=\overset{\text{ft.}}{\frac{15}{6}}=\overset{\text{ft.}}{2\tfrac{1}{2}}.$$

$$\overset{\text{ft.}}{\frac{1}{2}}=\overset{\text{in.}}{\frac{1\times 12}{2}}=\overset{\text{in.}}{\frac{12}{2}}=\overset{\text{in.}}{6}.$$

$\therefore\ \frac{5}{6}$ yd. = 2ft. 6in.

The work may be done more simply, thus

```
   5yd.
   3
   —
6)15ft.
   —
   2ft.—3ft. rem.
        12
        —
     6)36 in.
        —
        6 in.
        —
```

$\therefore\ \frac{5}{6}$yd. = 2ft. 6in.

Hence we may find the sum or difference of fractions of quantities of the same kind.

Ex. Find the amount of £$\frac{3}{10}$ and $\frac{5}{6}$*s*.

$$\overset{\pounds.}{\frac{3}{10}} = \overset{s.}{\frac{3 \times 20}{10}} = 6s.$$

$$\overset{s.}{\frac{5}{6}} = \overset{d.}{\frac{5 \times 12}{6}} = \qquad 10d.$$

6*s*. 10*d*. the sum required.

or thus,

£.	s.
3	5
20	12
10)60*s*.	6)60*d*.
6*s*.	10*d*.

∴ 6*s*. 10*d*. the sum required.

RULE OF THREE IN FRACTIONS.

The Statement and Operation are the same as in whole numbers: but the terms are reduced, multiplied and divided by the rules for fractions.

Ex. 1. If $3\frac{4}{7}$ cwt. cost £$17\frac{4}{15}$, how much will be bought for £$4\frac{1}{7}$?

$$\pounds 17\tfrac{4}{15} : \pounds 4\tfrac{1}{7} :: 3\tfrac{4}{7} \text{ cwt.}$$

or, reducing the mixed number to improper fractions,

$$\pounds \tfrac{259}{15} : \pounds \tfrac{29}{7} : \tfrac{25}{7} \text{ cwt.}$$

$$\therefore \frac{25}{7}\text{ cwt.} \times \frac{29}{7} \div \frac{259}{15} = \frac{29 \times 25 \times 15}{7 \times 7 \times 259}\text{ cwt.}$$

$$= \frac{10875}{12691}\text{ cwt.} = 3\text{ qrs. } 11\tfrac{1765}{1813}\text{ lbs.}$$

Ex. 2. If $\frac{5}{6}$ lb. cost 6*s.* 8*d.*, what will $\frac{7}{9}$ lb. cost ?

$$\frac{5}{6}\text{ lb.} : \frac{7}{9}\text{ lb.} :: 6s.\ 8d.$$

$$\therefore 6s.\ 8d. \times \frac{7}{9} \div \frac{5}{6} = 6s.\ 8d. \times \frac{7 \times 6}{9 \times 5}$$

$$= 6s.\ 8d. \times \frac{7 \times 2}{3 \times 5} = 6s.\ 8d \times \frac{14}{15} = 6s.\ 2\tfrac{2}{3}d.$$

or since 6*s.* 8*d.* = (by Case XII) £$\frac{1}{3}$

$$\frac{5}{6}\text{ lb.} : \frac{7}{9}\text{ lb.} :: £\frac{1}{3}$$

$$\therefore £\frac{1}{3} \times \frac{7}{9} \div \frac{5}{6} = £\frac{6 \times 7}{3 \times 9 \times 5} = £\frac{14}{45} = 6s.\ 2\tfrac{2}{3}d.$$

PRACTICE.

Practice is a ready way of finding the value of any quantity, when the price of a unit of it is given.

All the examples that occur are similar to one or other of the five following, and may readily be done by using the tables. The operations will sufficiently explain themselves.

Tables of Aliquot Parts.

Parts of a Pound.			Parts of a Shilling.		
10*s*.	is	$\frac{1}{2}$.	6*d*.	is	$\frac{1}{2}$.
6*s*. 8*d*.	..	$\frac{1}{3}$.	4*d*.	..	$\frac{1}{3}$.
5*s*.	..	$\frac{1}{4}$.	3*d*,	..	$\frac{1}{4}$.
3*s*. 4*d*.	..	$\frac{1}{6}$.	2*d*.	..	$\frac{1}{6}$.
2*s*. 6*d*.	..	$\frac{1}{8}$.	$1\frac{1}{2}$*d*.	..	$\frac{1}{8}$.
2*s*.	..	$\frac{1}{10}$.	1*d*.	..	$\frac{1}{12}$.
1*s*. 8*d*.	..	$\frac{1}{12}$.	$\frac{3}{4}$*d*.	..	$\frac{1}{16}$.
1*s*.	..	$\frac{1}{20}$.	$\frac{1}{2}$*d*.	..	$\frac{1}{24}$.
			$\frac{1}{4}$*d*.	..	$\frac{1}{48}$.

Parts of a cwt.			Parts of a qr.			Parts of a lb.		
2qrs.	is	$\frac{1}{2}$.	14lb.	is	$\frac{1}{2}$.	8oz.	is	$\frac{1}{2}$.
1qr.	..	$\frac{1}{4}$.	7lb.	..	$\frac{1}{4}$.	4oz.	..	$\frac{1}{4}$.
14lb.	..	$\frac{1}{8}$.	4lb.	..	$\frac{1}{7}$.	2oz.	..	$\frac{1}{8}$.
8lb.	..	$\frac{1}{14}$.	$3\frac{1}{2}$lb.	..	$\frac{1}{8}$.	1oz.	..	$\frac{1}{16}$.
7lb.	..	$\frac{1}{16}$.						

Ex. 1. What is the value of 846 oz. at $\frac{3}{4}$*d*. per oz.?

$\frac{1}{2}$*d*. is $\frac{1}{2}$	846	= the value at 1*d*. per oz.	
$\frac{1}{4}$*d*. is $\frac{1}{4}$	423	=	$\frac{1}{2}$ ......
	$211\frac{1}{2}$	=	$\frac{1}{4}$ ......
	12)$634\frac{1}{2}$	=	$\frac{3}{4}$ ......
	20)52 $10\frac{1}{2}$		
	£2 12 $10\frac{1}{2}$		

Here, the price of 846*oz.* at 1*d.* per *oz.* is evidently 846*d.*: and the price at $\frac{1}{2}$*d.* per *oz.* must be half that at 1*d.* or 423*d.*: similarly the price at $\frac{1}{4}$*d.* per. *oz.* must be half that at $\frac{1}{2}$*d.* or 211$\frac{1}{2}$*d.*: and therefore the price at $\frac{3}{4}$*d.* (or $\frac{1}{2}$*d.* and $\frac{1}{4}$*d.*,) must be 634$\frac{1}{2}$*d.*, the sum of 423*d.* and 211$\frac{1}{2}$*d.* The 634$\frac{1}{2}$*d.* is then reduced to £. *s.* *d.*

Ex. 2. Find the value of 2150 at 9$\frac{3}{4}$*d.*

	s.		
6*d.* is $\frac{1}{2}$	2150		= the value at 1*s.*
3*d.* is $\frac{1}{2}$	1075		= .... 6*d.*
$\frac{3}{4}$*d.* is $\frac{1}{4}$	537	6	= .... 3*d.*
	134	4$\frac{1}{2}$	= .... $\frac{3}{4}$*d.*
	20)1746	10$\frac{1}{2}$	= .... 9$\frac{3}{4}$*d.*
	£87 6	10$\frac{1}{2}$	

Ex. 3. Find the value of 3271 at 5*s.* 9$\frac{1}{2}$*d.*

5*s.* is $\frac{1}{4}$	£3271			= value at £1.
6*d.* is $\frac{1}{10}$	817	15		= ...... 5*s.*
3*d.* is $\frac{1}{2}$	81	15	6	= .... 6*d.*
$\frac{1}{2}$*d.* is $\frac{1}{6}$	40	17	9	= .... 3*d.*
	6	16	3$\frac{1}{2}$	= .... $\frac{1}{2}$*d.*
	£947	4	6$\frac{1}{2}$	= ...... 5*s.* 9$\frac{1}{2}$*d.*

Ex. 4. Find the value of 2710 at £2 3*s*. 7½*d*.

2*s*. 6*d*. is ⅛	£2710			= value at £1.	
	2				
	5420			=........£2.	
6*d*. is ⅕	338	15		=........	2*s*. 6*d*.
6*d*. is ⅕	67	15		=.	6*d*.
1½*d*. is ¼	67	15		=........	6*d*.
	16	18	9	=........	1½*d*.
	£5911	3	9	=........	£2 3*s*. 7½*d*.

Ex. 5. What is the value of 25cwt. 2qrs. 14lb. at £3 17*s*. 6*d*. per cwt?

	£.	*s*.	*d*.	
2 qrs. is ½	3	17	6	=value of 1 cwt.
			5	
	19	7	6	=...... 5 cwt.
			5	
	96	17	6	=...... 25 cwt.
14 lb. is ¼	1	18	9	=............ 2 qrs.
		9	8¼	=................ 14lb.
	99	5	11¼	=...... 25cwt. 2qrs. 14lb.

Part III.

DECIMALS.

1. Extending the notation in Page 6, and taking the unit as the first figure and proceeding to the right, the

2nd figure means so many tenths.
3rd.................... hundredths.
4th.................... thousandths.
5th.................... tens of thousandths.
&c. &c.

2. The unit's place is noted by a dot placed to the right. Thus, 213.2156, stands for 2 hundreds, 1 ten, 3 units, 2 tenths, 1 hundredths, 5 thousandths, 6 tens of thousandths, or $200+10+3+\frac{2}{10}+\frac{1}{100}+\frac{5}{1000}+\frac{6}{10000}$.

To represent a decimal by a fraction.

Rule. Place in the numerator the decimal with the point removed, and in the denominator 1 with as many noughts as there are decimal places in the given decimal.

Ex. Represent .5, 1.5, 25.212 as fractions.

$$.5=\frac{5}{10}\ :\ 1.5=\frac{15}{10}\ :\ 25.212=\frac{25212}{1000}.$$

PROOF. $25.212=25+\frac{2}{10}+\frac{1}{100}+\frac{2}{1000}$ (by Art. 2) $=25+\frac{212}{1000}=\frac{25212}{1000}$.

Conversely, *a fraction whose denominator is* 10, 100, 1000, *&c., may be changed into a decimal, by pointing off from the numerator, as many decimal places as there are noughts in the denominator.*

$$\text{thus, } \frac{25}{100}=.25\ :\ \frac{25}{1000}=.025\ :\ \frac{3425}{10}=342.5.$$

Ciphers placed to the right of a decimal do not alter its value: thus .5=.50=.500=.5000=&c.: for they are respectively equal to $\frac{5}{10}, \frac{50}{100}, \frac{500}{1000}, \frac{5000}{10000}$ &c., which fractions are all equal.

Every cipher placed to the left of a decimal decreases its value ten-fold, thus, .3, .03, .003, are respectively equivalent to $\frac{3}{10}, \frac{3}{100}, \frac{3}{1000}$.

ADDITION OF DECIMALS.

RULE. Place the numbers under each other, so that the decimal points may be in the same column.

Add up as in whole numbers, and place the decimal point in the sum, under those in the numbers to be added.

Ex. Add .57, 1.352, 1024.35, 10.0042.

$$
\begin{array}{r@{.}l}
 & 57 \\
1 & 352 \\
1024 & 35 \\
10 & 0042 \\
\hline
1036 & 2762 \\
\hline
\end{array}
$$

PROOF. $.57 + 1.352 + 1024.35 + 10.0042$

$$= \frac{57}{100} + \frac{1352}{1000} + \frac{102435}{100} + \frac{100042}{10000}$$

$$= \frac{5700}{10000} + \frac{13520}{10000} + \frac{10243500}{10000} + \frac{100042}{10000}$$

$$= \frac{10362762}{10000} = 1036.2762.$$

If any of the numbers contain circulating decimals, let these be carried out to as many places as there are in the longest of the finite decimals.

Ex. Add $3.\dot{7}\dot{3}$, .873, $51\dot{7}$, 108.2 and 73.463128.

$$
\begin{array}{r@{.}l}
3 & 737373 \\
 & 873 \\
51 & 777778 \\
108 & 2 \\
73 & 463128 \\
\hline
238 & 051279 \\
\hline
\end{array}
$$

Here the first and third numbers are carried out to 6 places.

N.B. If we wrote only 4 figures of the decimals of the answer, we should write 238.0513, increasing the last figure 2 by a unit, since 30 is nearer 27 than 20 is.

SUBTRACTION OF DECIMALS.

Rule. Place the lesser number under the greater, so that the decimal points may be in the same column.

Subtract as in whole numbers, and place the point in the difference, under those in the numbers to be subtracted.

Ex. From 47.439 take 5.9375.

$$\begin{array}{r} 47.439 \\ 5.9375 \\ \hline 41.5015 \\ \hline \end{array}$$

Proof. $47.439 - 5.9375$

$$= \frac{47439}{1000} - \frac{59375}{10000}$$

$$= \frac{474390}{10000} - \frac{59375}{10000} = \frac{415015}{10000}$$

$$= 41.5015.$$

Ex. From $3.\dot{5}\dot{4}$ take 1.35265.

$$\begin{array}{l} 3.5454545 \\ 1.34265 \\ \hline 2.2028045 \\ \hline \end{array}$$

Here $3.\dot{5}\dot{4}$ is extended, and the remainder is $2.20280\dot{4}\dot{5}$.

MULTIPLICATION OF DECIMALS.

RULE. Multiply as in whole numbers.

Mark off in the product as many decimal places as there are in the multiplier and the multiplicand together.

Ex. 1. Multiply 3.143 by 3.

$$\begin{array}{r} 3.143 \\ 3 \\ \hline 9.429 \\ \hline \end{array}$$

Ex. 2. Multiply 4.735 by .74.

$$\begin{array}{r} 4.735 \\ .74 \\ \hline 18940 \\ 33145 \\ \hline 3.50390 \\ \hline \end{array}$$

PROOF. $3.143 \times 3 = \frac{3143}{1000} \times 3 = \frac{9429}{1000} = 9.429.$

If there be not a sufficient number of decimal places in the product, supply the deficiency by placing *noughts* to the left of it.

Ex. Multiply .4735 by .074.

```
   .4735
    .074
   -----
   18940
  33145
 -------
.0350390
```

DIVISION OF DECIMALS.

Rule. Add ciphers, if necessary, to the right of the dividend.

Divide as in whole numbers.

Cut off from the quotient as many decimal places, as the number by which those in the dividend exceed those in the divisor.

Ex. Divide 3·5039 by 47·34.

```
47·34)3·5039000(·07401
       33138
       -----
        19010
        18936
        -----
           7400
           4734
           ----
           2666
```

Here, we add three ciphers to the dividend: we have 5 decimal places in the quotient, because there are 7 in the dividend and 2 in the divisor.

PROOF. $3{\cdot}5039 \div 47{\cdot}34 = \frac{35039000}{1000000} \div \frac{4734}{100} =$

$\frac{35039000}{4734} \times \frac{1}{100000} = \frac{7401}{100000} = .07401.$

2. If the number of decimal places be the same in both dividend and divisor, the quotient will be a whole number.

Ex. Divide 4381.3 by 7.2

```
7.2)4381.3(608
    432
    ---
      613
      576
      ---
       37
      ---
```

PROOF. $4381.3 \div 7.2 = \frac{43813}{10} \div \frac{72}{10} = \frac{43813}{72} =$
608 — 37 rem.

3. If the number of decimal places in the divisor be greater than that in the dividend, place to the right of the quotient as many ciphers, as the number by which the places in the divisor exceed those in the dividend.

Ex. Divide 450000 by .2134

```
.2134)450000(2100000
      4268
      ----
       2320
       2134
       ----
        1860
       ----
```

Here, as there are 4 decimal places in the divisor and none in the dividend, we place 4 ciphers to the right of the quotient 210.

PROOF. $450000 \div .2134 = 450000 \div \frac{2134}{10000} =$

$\frac{450000 \times 10000}{2134} = 210 \times 10000 = 2100000.$

REDUCTION OF DECIMALS.

To reduce a vulgar fraction to a decimal.

RULE. Add decimal ciphers to the numerator, and divide by the denominator.

Ex. 1. Reduce $\frac{7}{8}$ to a decimal.

$$\frac{7}{8} = \frac{7.0000}{8} = .875.$$

Ex. 2. Reduce $\frac{5}{191}$ to a decimal.

$$\frac{5}{191} = \frac{5.0000}{191} = \cdot 0261 \text{ \&c.}$$

```
191)5·0000(.0261 &c.
     382
     ----
     1180
     1146
     ----
      340
      191
      ---
      14
```

Ex. 1. 7÷ [illegible] 8 $= \frac{7\cdot0000}{8} = \cdot875.$

1. When the divisor *terminates*, as in *Ex.* 1, the decimal is called a *terminating* decimal.

2. When the same figures recur uniformly the decimal is called a *recurring* decimal, as in $\frac{1}{37}=\cdot027027027$ &c.

3. When only *part* of the figures recur the decimal is called a *mixed recurring* decimal, as ·0075555 &c.

4. When only one figure recurs it is marked by a point over it: as $\cdot007\dot{5}$ for ·00755 &c.: if several figures recur, by a point over the first and the last, as $\cdot\dot{0}2\dot{7}$ for ·027027 &c.: $\cdot5\dot{6}3\dot{7}$ for ·563737 &c.

To find the value of a recurring decimal.

RULE. Multiply by 1 with as many noughts after it as there are recurring figures.

Subtract the given decimal from this product.

Divide the difference by the above multiplier with one taken from it.

Ex. 1. Find the value of $.\dot{3}$.

$$
\begin{aligned}
.\dot{3} &= \quad 3333 \text{ &c.}\\
10\times\dot{3} &= 3.3333 \text{ &c.}\\
\hline
9\times.3 &= 3\\
\therefore\ .3 &= \frac{3}{9}=\frac{1}{3}.
\end{aligned}
$$

Ex. 2. Find the value of $.02\dot{6}7\dot{3}$.

$$.02\dot{6}7\dot{3} = \quad .02673673 \text{ \&c.}$$
$$1000 \times .02\dot{6}7\dot{3} = 026.73673673 \text{ \&c.}$$
$$999 \times .02\dot{6}7\dot{3} = 26.71$$
$$\therefore .02\dot{6}7\dot{3} = \frac{26.71}{999} = \frac{2671}{99900}.$$

To reduce a quantity to the decimal of another quantity of a higher name.

Rule. Add ciphers to the lowest name, and divide successively by as many of the less as make one of the greater.

Ex. Reduce 18*s.* $9\frac{3}{4}$*d.* to the decimal of £1.

4)3·000*q.*

12)9·750*d.*

20)18·8125*s.*

£·940625.

∴ 18*s.* $9\frac{3}{4}$*d.* = £·940625.

Proof. The same as in page 36.

Conversely, to find the value of a decimal, multiply successively by as many of the less as make one of the greater.

Ex. Find the value of £·940625.

$$\begin{array}{r} \pounds. \\ \cdot 940625 \\ 20 \\ \hline 18\cdot 812500s. \\ 12 \\ \hline 9.7500d. \\ 4 \\ \hline 3.00q. \\ \hline \end{array}$$

$$\therefore\ \pounds.940625 = 18s.\ 9\tfrac{3}{4}d.$$

Here, multiplying by 20, we have £.940625 = 18.812500*s.* or 18*s.* +.812500*s.* Then multiplying .8125*s.* (leaving out the noughts, see page 79) by 12, .8125*s.* = 9.7500*d.* or 9*d* +.7500*d.*: multiplying ·75*d.* by 4, ·75*d.* = 3*q.*

Proof. The same as in page 34.

Ex. 2. Find the value of $\cdot 80\dot{5}$ yards in long measure.

$$\begin{array}{l} \cdot 805\dot{5}\text{yd.} \\ \quad 3 \\ \hline 2\cdot 4166 \text{ ft.} \\ \quad 12 \\ \hline 4\cdot 9999 \text{ in.} \\ \hline \end{array}$$

$$\therefore\ .80\dot{5}\text{yd.} = 2 \text{ ft. } 5\text{in. nearly.}$$

Here we carry 1 to the product of 3 and 5, because had the decimal been carried farther, 1 must have been carried from the preceding product; also 7 is carried to the product of 12 by 6.

Examples in Proportion may be worked by decimals, by reducing the quantities to decimals and proceeding according to Rules in page 48.

Ex. If 1cwt. cost £2 16*s*., what will $40\frac{1}{2}$ cwt. come to?

$$2)1{\cdot}0 \qquad\qquad 20)16{\cdot}0$$

$$40{\cdot}5 \text{ cwt.} = 40\tfrac{1}{2} \text{ cwt.} \qquad £2{\cdot}8 = £2{\cdot}16s.$$

cwt.		cwt.		£.
1	:	40·5	: :	2·8
		2·8		
		3240		
		810		
		£113·40		
		20		
		*s*8·0		

∴ £113 8*s*. = price required.

Part IV.

RATIO.

1. *Ratio* is the relation of one number to another, and is found by dividing one by the other: thus, the ratio of 2 to 3 (written 2 : 3) is expressed by $\frac{2}{3}$.

2. Of the two numbers compared, and called the *Terms* of the ratio, the *former* is called the *Antecedent*, and the *latter* the *Consequent*.

3. To compare ratios, reduce the fractions which express them to a common denominator, and compare the numerators.

Thus, to compare the ratios of 2 : 3 and 4 : 5.

$$\frac{2}{3} = \frac{10}{15}, \text{ and } \frac{4}{5} = \frac{12}{15}$$

and as 10 is less than 12, the ratio 2 : 3 is less than the ratio 4 : 5.

4. A ratio of greater inequality is diminished, and a ratio of less inequality increased, by adding the same quantity to both its terms.

Take the ratio 7 : 5 of *greater* inequality: adding 1 to both its terms, it becomes 8 : 6.

Since $\frac{7}{5} = \frac{42}{30}$ and $\frac{8}{6} = \frac{40}{30}$, it follows that the ratio 8 : 6 is less than the ratio 7 : 5.

Take the ratio 8 : 11 of *lesser* inequality; adding 1 to both its terms, it becomes 9 : 12.

Since $\frac{8}{11} = \frac{96}{132}$ and $\frac{9}{12} = \frac{99}{132}$, it follows that the ratio 8 : 11 is greater than the ratio 9 : 12.

5. If the terms of a ratio be both multiplied or divided by the same quantity, the magnitude of the ratio is not altered.

Take the ratio 3 : 8 ; its magnitude is $\frac{3}{8}$, which $= \frac{6}{16}$ or $\frac{9}{24}$ or $\frac{12}{32}$, &c., and therefore the ratio 3 : 8 is equal to each of the ratios 6 : 16, or 9 : 24, or 12 : 32, &c.

PROPORTION.

1. *Proportion* is the *equality* of ratios : thus, as the ratio 2 : 4 is equal to the ratio 3 : 6 ; 2, 4, 3, 6 are in *Proportion*. This is expressed by 2 : 4 : : 3 : 6 or 2 : 4 = 3 : 6.

2. If four quantities are proportional the product of the first and fourth terms = that of the second and third,

Thus, since 2 : 4 : : 3 : 6.

$\frac{2}{4} = \frac{3}{6}$ and, multiplying by 24,

$2 \times 6 = 4 \times 3.$

3. In a proportion the fourth term = the product of the second and third divided by the first.

Thus, since $2 : 4 :: 3 : 6$

$$2 \times 6 = 3 \times 4, \text{ and } 6 = \frac{3 \times 4}{2}.*$$

4. Similarly, either of the mean terms equals the product of the extremes divided by the other mean.

5. One of the extremes and either of the means in a proportion may be divided by the same number, and the resulting numbers will be in proportion.

Thus, since $2 : 3 :: 4 : 6$

$\therefore \frac{2}{3} = \frac{4}{6}$ and, dividing both fractions by 2,

$\frac{1}{3} = \frac{2}{6}$

whence $1 : 3 :: 2 : 6$.

This rule is of practical use in diminishing the number of *figures* employed in Proportion, see page 51.

Ratio and Proportion are practically applied in the following Rules: *Rule of Proportion, Interest, Stocks, Discount, Equation of Payment, Fellowship, Alligation, Exchange, &c.*

RULE OF PROPORTION.

Assuming that *Cause* and *Effect* are proportionate to each other, we shall have first cause : second cause :: first effect : second effect, and any three of these terms being given, the fourth may be found, by Rules 3 and 4 in Proportion.

N. B. Call, for convenience, the required quantity in each case, x.

* Hence the Rule for the operation in Rule of Three, see page 48.

Ex. 1. If 5 men mow 12 acres in a certain time, how many acres will 16 men mow in the same time?

Here, $\left.\begin{matrix} 5 \text{ men} \\ 16 \text{ men} \end{matrix}\right\}$ are the 1st and 2nd causes.

$\left.\begin{matrix} 12 \text{ acres} \\ x \text{ acres} \end{matrix}\right\}$ are the 1st and 2nd effects.

Whence $\overset{\text{men}}{5} : \overset{\text{men}}{16} :: \overset{\text{acres}}{12} : \overset{\text{acres}}{x}$.

$\because 5 \times x = 16 \times 12$

and $x = \dfrac{\overset{\text{ac.}}{16 \times 12}}{5} = \overset{\text{ac.}}{38} . \overset{\text{ro.}}{1} . \overset{\text{po.}}{24}$.

Ex. 2. If 60 bushels of corn feed 6 horses for 50 days in how many days will 15 horses consume 75 bushels?

Here, $\left.\begin{matrix} \overset{\text{horses}}{6} \times \overset{\text{days}}{50} \\ 15 \times x \end{matrix}\right\}$ are the 1st and 2nd causes.

$\left.\begin{matrix} \overset{\text{bushels}}{60} \\ 75 \end{matrix}\right\}$ are the 1st and 2nd effects.

Whence $6 \times 50 : 15 \times x :: 60 : 75$

$\therefore 15 \times x \times 60 = 6 \times 50 \times 75$

and $x = \dfrac{6 \times 50 \times 75}{15 \times 60} = 25$ days.

Ex. 3. If a man walk 240 miles in 7 days of 16 hours each, in how many days of 12 hours each can he do the same?

$$\text{Here, } \left.\begin{matrix} \text{dys.} & & \text{hs.} \\ 7 & \times & 16 \\ x & \times & 12 \end{matrix}\right\} \text{ are the 1st and 2nd causes.}$$

and the 1st and 2nd effects are equal.

$$\therefore\ 7 \times 16 : x \times 12 :: 1 : 1$$

$$x = \frac{7 \times 16}{12} = 9 \text{ days } 4 \text{ hours.}$$

INTEREST.

1. *Interest* is the sum paid for money lent. The money lent is called the *Principal*: the money agreed to be paid for £100 lent for one year is the *Rate per cent*: the Principal and Interest together make the *Amount*.

SIMPLE INTEREST.

2. *Simple Interest* is when interest is paid only on the money lent.

To find the Simple Interest of a given sum for a given time at a given rate.

Rule. Multiply the principal by the rate per cent, and divide by 100: that will give the interest for one year, which multiplied by the number of years, will give the Interest required.

Ex. Find the interest of £537 10*s*. for $3\frac{1}{2}$ years at 4 per cent.

```
          £.  s.
         537  10
               4
        ---------
   100)21,50   0
          20
        -------
        10,00
        -------
```

```
     £.  s.
 ½   21  10 = Interest for 1 year.
          3
    --------
     64  10 = ........ 3 ....
     10  15 = ........ ½ ....
    --------           --
    £75   5 = ........ 3½ years.
    --------           --
```

Proof. By proportion, we should have £100 : principal : : rate per cent : interest for 1 year.

$$\therefore \text{ Interest for 1 year} = \frac{\text{Principal } \times \text{ rate per cent}}{100}$$

$$\text{and Interest for } 3\tfrac{1}{2} \text{ years} = \frac{\text{Principal} \times \text{rate per cent} \times 3\tfrac{1}{2}}{100}$$

To find the Amount.

Ex. What is the amount of £537 10*s*. for 3½ years at 4 per cent?

```
                £    s.
   As above,   75    5 = Interest.
              537   10 = Principal.
              ---------
             £612   15 = Amount.
              ---------
```

When the time involves months or days, the interest may be found by Rule of Three or Practice, reckoning 12 months or 365 days to a year.

Ex. What is the interest of £825 13*s.* 8*d.* at $4\frac{3}{4}$ per cent, in 3 years and 5 months?

	£.	s.	d.
$\frac{1}{2}$	£825	13	8
			4
	3302	4	8
$\frac{1}{2}$	412	16	10
	206	8	5
	39,21	9	11
	20		
	4,29		
	12		
	3,59		
	4		
	2,36		

∴ £39 4*s.* $3\frac{1}{2}$*d.* = Interest for 1 year.

1 year : 3 years 5 months, : : £39 4*s.* $3\frac{1}{2}$*d.*

which worked out would give the answer, or by Practice,

months		£.	s.	d.	
4	$\frac{1}{3}$	39	4	$3\frac{1}{2}$	= Interest for 1 year.
				3	
1	$\frac{1}{4}$	117	12	$10\frac{1}{2}$	= ....3 ....
		13	1	5	= ....4 months.
		3	5	$4\frac{1}{4}$	= ....1 ....
		£133	19	$7\frac{3}{4}$	= ....3 years 5 months.

This Rule may be worked by decimals by reducing the principal, as also the rate per cent. divided by 100, and the time, each to a decimal, and multiplying together the results.

Ex. Find the interest of £537 10*s*. for $3\frac{1}{2}$ years at 4 per cent.

20)10.0 — £537.5 $\frac{4}{100} = .04$ 2)1·0 — 3·5 years.

£.		£. s.
537·5	=	537 10
·04	=	4 ÷ 100
21·500	=	Interest for 1 year.
3·5	=	$3\frac{1}{2}$ years.
107500		
64500		
75·2500		
20		
5.0	∴	£75 5*s*. = Interest required.

Ex. 2. Find the interest of £47 10*s*. for 4 years and 52 days at $4\frac{1}{2}$ per cent.

```
20)10.0          2)1.0
 ————            ————
 £47.5            4.5
 ————            ————
```

$$\frac{4\frac{1}{2}}{100} = \frac{4.5}{100} = .045$$

```
365)52.0000(.1424            £.       £.  s.
     365                   47.5  = 47   10
     ———                   .045  = 4½ ÷ 100
     1550                  ————
     1460                  2375
     ————                 1900
       900                ——————
       730               2.1375 = Interest for 1 year.
       ———               4.1424 = 4 years + 52 days.
       1700              ——————
       1460               85500
       ———               42750
        240             85500
                       21375
                      85500
                     ————————
                   £8.85438000
                            20
                     ————————
                    17.0876s.
                          12
                     ——————
                     1.0512d.
                     ——————
```

∴ £8 17*s*. 1*d*. .0512 = the Interest required.

The same will apply to *Discount*, *Purchasing of Stock*, *Commission*, *&c*.

COMPOUND INTEREST.

If the interest, when due, be added to the principal and interest calculated up on the whole amount, it is called *Compound Interest.*

Ex. Find the compound interest and amount of £250 for 2 years at 5 per cent.

£.			
250	£.	s.	
5	250	6	= Principal.
——	12	10	= Interest for 1 year.
12,50	——		
20	262	10	= Amount for 1 year.
——	——		
10,00			

£. s.				
262 10				
5	£.	s.	d.	
——	13	2	6	= Interest for 2nd year.
13,12 10	12	10	0	= .... 1st year.
20	——			
——	25	12	6	= Interest for 2 years.
2,50	262	10	0	= Amount for 1 year.
12	——			
——	288	2	6	= Amount for 2 years.
6,00	——			

DISCOUNT.

Discount is the allowance made for money paid before it is due, and the Discount subtracted from the Amount gives the *Present Worth.*

Rule. To find the Discount, use the Proportion, as the amount of £100 for the given time : the given sum : : the interest of £100 : discount required.

Ex. What is the discount of £75 due 15 months hence, at 5 per cent per annum?

The amount of £100 for 15 months (as found by Rule Page 96) is £106 5*s.*

£.	s.		£.		£.	s.		£
106	5	:	75	: :	6	5	:	x
20					20			
2125					125			

$$\therefore x = \frac{125 \times 75}{2125} = \frac{75}{17} = £4\ 8s.\ 2\tfrac{3}{4}d.\ \tfrac{5}{17}$$

the discount required.

Hence we may find the *Present Worth.*

£.	s.	d.	
75	0	0	= money due 15 months hence.
4	8	$2\frac{3}{4}$	= discount.
70	11	$9\frac{1}{4}$	= present worth.

We may also find the Present Worth by the Proportion, as the amount of £100 : the given sum : : £100 : the present worth.

Thus, in the above *Ex*. £106 5*s*. : £75 : £100 : £x which worked out, would give x = £70 11*s*. 9¼*d*.

This method finds the *true* present worth, and is founded on the consideration that £100 is the present worth of its amount considered as a debt, and then, the proportion is simply this, as the amount of £100 considered as a debt is to £100 the present worth of that debt, so is any other debt to its present worth.

The usual method of finding the present worth is to find the interest of the debt for the given time, to consider this as discount, and subtract it from the debt.

Thus, in the example above, the discount thus found would be £4 13*s*. 9*d*., which is too much by 5*s*. 6¼*d*, and consequently the present worth is too little by that sum.

When the time is short, as is generally the case in real business, the error is inconsiderable.

N. B. *Three* days, called days of grace, are allowed after a bill is nominally due, before it is *legally* due. Thus, if a bill were drawn on the 8th of April at 3 months, it would be due, not on the 8th, but the 11th of July.

Ex. Find the present worth of a £77 bill drawn 8th of March, at 6 months, and discounted 3rd of June at 5 per cent.

Counting forward 6 months and 3 days grace, the bill would be due on 11th September, the number of days to which date from 3rd June would be 100, and the interest of £77 for 100 days at 5 per cent, is £1 1*s*. 1¼*d*., and therefore the present worth is £75 18*s*. 10¾*d*.

COMMISSION, BROKERAGE, INSURANCE.

Commission is the sum paid to a merchant for buying or selling goods for another.

Brokerage is a smaller allowance paid usually for transacting money concerns.

Insurance is a payment of so much per cent to secure property from losses which may occur by fire, or otherwise.

RULE. As the charge is in each case so much per cent, the rule will be the same as that for finding the Simple Interest for one year, viz, multiply the sum by the rate per cent, and divide by 100.

Ex. 1. What is the commission on £754 16*s*. at $2\frac{1}{4}$ per cent.?

£.		£.		£.	s.		£.
100	:	$2\frac{1}{4}$	: :	754	16	:	x

which worked out would give the answer, or more briefly thus,

	£.	s.
$\frac{1}{4}$	754	16
		2
	1509	12
	188	14

$$
\begin{array}{r}
100)16{,}986 \\
20 \\
\hline
19{,}66 \\
12 \\
\hline
7{,}92 \\
4 \\
\hline
3{,}68 \\
\hline
\end{array}
$$

∴ £16 19*s*. 7¾*d*. = commission required.

Ex. 2. Find the insurance on £512 9*s*. 4*d*. at £6 16*s*. 6*d*. per cent.

	£.	*s*.	*d*.
	512	9	4
			6
12*s*. = $\frac{1}{10}$ of £6	3074	16	0
4*s*. = $\frac{1}{3}$ of 12*s*.	307	9	7¼
6*d*. = $\frac{1}{8}$ of 4s.	102	9	10½
	12	16	2¾
	100)34,97	11	8½
	20		
	19,51		
	12		
	6,20		

∴ £34 19*s*. 6*d*. = Insurance required.

STOCKS.

When a Government borrows money, it gives the lenders *Bonds*, implying that the nation is indebted to them the sum borrowed, and, on that sum, interest is paid at the time fixed upon. These bonds are transferable from one person to another.

Each bond is called £100 stock, bearing interest at a certain rate per cent. Thus, in what are called the 3, $3\frac{1}{2}$ or 4 per cents, one of these bonds entitles its owner to £3, £3 10*s*. or £4 yearly interest.

The value of these bonds fluctuates according to the credit of the country; thus, a bond for £100 stock at 3 per cent may be worth £95 sterling, or £85 10*s*. or £83 17*s*. 6*d*. or any other sum.

All transactions in the stocks depend on the Rule of Proportion, as will appear from examples.

Ex. 1. How much money must be paid for £2400 in the 3 per cents, at $89\frac{1}{2}$?

stock		stock		£.	s.		£.
£100	:	£2400	: :	89	10	:	x

whence the required sum may be found, or more briefly, thus

$$\begin{array}{r} \pounds. \\ \tfrac{1}{2}\quad 2400 \\ 90 \\ \hline 216000 \\ 1200 \\ \hline 100)214800 \\ \hline \pounds 2148 \\ \hline \end{array}$$

∴ **£2148 sterling will buy £2400 stock.**

Here we multiply by 90, and subtract ½, which is the same as multiplying by 89½.

Ex. 2. What quantity of stock in the 3½ per cents at 89½ may be brought for £2148. ?

$$\overset{\pounds}{89\tfrac{1}{2}} \text{ or } \frac{179}{2} \ : \ \overset{\pounds}{2148} \ :: \ \overset{\text{stock}}{100} \ : \ \overset{\text{stock}}{x}$$

$$\begin{array}{r} 100 \\ \hline 214800 \\ 2 \\ \hline \end{array} \quad \pounds.$$

$$\begin{array}{l} 179)429600(2400 \\ \quad\ \ 358 \\ \quad\ \ \overline{\ \ 716} \\ \quad\ \ \ \ 716 \\ \quad\ \ \overline{\ldots 00} \end{array}$$

∴ **£2400 = stock required.**

Ex. 3. A man invests £2000 in the 3 per cents at $90\frac{1}{5}$, what interest will he receive?

For $90\frac{1}{5}$ he receives £3 interest, hence

£.		£.		£.		£.
$90\frac{1}{5}$	:	2000	: :	3	:	x

whence, x = £66 10*s.* $4\frac{1}{2}d.$ the interest required.

Ex. 4. A man transfers £1000 stock in the 4 per cents at 90 to the 3 per cents at 72: how much of the latter stock will he receive, and what will be the change in his income?

The values of the stock bought and sold must be equal;

hence, £1000 stock × 90 £. = stock required × 72 £.

$$\therefore \text{ Stock required} = \frac{1000 \times 90}{72} = £1250.$$

Also, £1000 in the 4 per cents gives £40 interest.

and £1250 .... 3 .... £37 10*s.*

∴ £2 10*s.* a year is lost by the transfer.

Dealings in stocks are made through brokers, who charge £$\frac{1}{8}$ or 2*s.* 6*d.* per cent. They are paid by both buyers and sellers, and therefore brokerage is to be added to the price of stock bought, and subtracted from that sold: thus, if the stock be transferred at $85\frac{1}{2}$, brokerage being £$\frac{1}{8}$, the buyer gives $85\frac{1}{2} + \frac{1}{8}$ or $85\frac{5}{8}$ for £100, and the seller receives $85\frac{1}{2} - \frac{1}{8}$ or $85\frac{3}{8}$ for £100.

EQUATION OF PAYMENTS.

Equation of Payments finds the just time for the payment *at once* of several debts due at *different* times.

RULE. Multiply each debt by its time, and add the products together.

Divide their sum by the sum of all the debts.

Ex. If £100 be due in 3 months, £210 in 2 months, and £160 in 5 months: find the equated time.

£.		months.		
100	×	3	=	300
210	×	2	=	420
160	×	5	=	800
470	the sum of the debts.			1520

$$\therefore \frac{1520}{470} = 3\tfrac{11}{47} \text{ months, the equated time.}$$

PROOF. The interest of all the debts should equal the interest of their sum for the equated time, and as the interests are proportional to the sums × times, we have $(100 \times 3) + (210 \times 2) + (160 \times 5) = 470 \times$ equated time: whence the equated time $= \frac{1520}{470}$, as by the Rule.

BARTER, PROFIT AND LOSS.

The method of working these Rules will be readily suggested by the questions.

Ex. 1. How much tea at 9*s.* per lb. must I receive for 4 cwt. 2 qr. of chocolate at 4*s.* per lb.?

1 *lb.* : 4 *cwt.* 2 *qr.* : 4*s.*

```
4
———
18
28
———
144
36
———
504
4
———
2016s. = the price of the chocolate.
```

9*s.* : 2016*s.* : 1*lb.*

```
      1
   ———
 9)2016
   ———
112)224 lb.
   ———
      2 cwt. = quantity of tea.
```

We first find the price of the chocolate, and then find how much tea could be bought for that amount.

Ex. 2. If nutmegs are bought at 17*s*. 6*d*. per lb., how must they be sold to gain 16 per cent.?

£100 : £116 : : 17*s*. 6*d*.

```
             12
            ----
            210
            116
            ----
           1260
          231
          -----
     100)24360
         ------
        12)243 3/5
         ------
         20)20 3 3/5
         ------
          £1.0.3 3/5
```

Here, as £100 is to become with the profit on it £116, so 17*s*. 6*d* must be increased in the same proportion.

FELLOWSHIP.

Fellowship is the rule by which two or more persons having a Joint Stock, determine their shares in the Profit or Loss.

RULE. Add together all the stocks, and as the whole stock is to each man's share of it, so is the whole profit or loss to each man's share of it.

Ex. Three persons, A, B, C, trade together, A puts in £700, B £1000, and C £1600. They gain £880, what is each man's share?

£	
700	
1000	
1600	
3300	joint stock.

£.		£.		£.		
3300	:	700	: :	880	:	A's share.
3300	:	1000	: :	880	:	B's share.
3300	:	1600	: :	880	:	C's share.

Which proportions, worked out, will give

£.	*s.*	*d.*	
186	13	4	A's share.
266	13	4	B's share.
426	13	4	C's share.

When each man's portion of the Profit or Loss depends upon the Principal and Time jointly, multiply the two together and proceed as above.

Ex. A and B enter business together, A contributes 280 for 6 years, and B £170 for 8 years. They gain £250. What is each man's gain?

£.		yrs.			
280	×	6	=	1680	product of A's capital and time.
170	×	8	=	1360	.... B's ....
				3040	

				£		
3040	:	1680	: :	250	:	A's gain.
3040	:	1360	: :	250	:	B's gain.

Which proportions, worked out, will give

£.	s.	d.		
138	3	$1\frac{3}{4}$	$\frac{11}{19}$	A's gain.
111	16	10	$\frac{8}{19}$	B's gain.

ALLIGATION.

Alligation finds the rate or quality of a mixture, from the rates or qualities of the ingredients of which it is made up.

Rule. Multiply the ingredients by their respective rates.

Divide the sum of the products by the sum of the ingredients.

Ex. A wine merchant mixes together 20 gallons of wine at 12*s.* per gallon, 25 gallons at 14*s.*, and 36 gallons at 16*s.*: what will be the price of a gallon of the mixture?

$$\left.\begin{array}{l} 20 \times 12 = 240 \\ 25 \times 14 = 350 \\ 36 \times 16 = 576 \end{array}\right\}\text{product of each kind of wine and its price.}$$

81 gallons in the whole mixture. 1166*s.* value of the mixture.

$$\therefore \frac{1166}{81} = 14\text{s. } 4\tfrac{1}{2}\text{d. price of a gallon of the mixture.}$$

Proof. The price of one gallon of the mixture must equal the value of the whole mixture, divided by the number of gallons in it.

TARE AND TRET.

Tare is an allowance for the weight of the box, barrel, &c. in which goods are conveyed.

Tret is an allowance of 4*lb.* in every 104*lb.* for waste, dust, &c.

Cloff is an allowance, after Tare and Tret are deducted of 2*lb.* in every 3*cwt.*, that the weight may hold good, when sold by retail.

When Tare only is deducted from the *gross* weight, the remainder is called *suttle.*

When all allowances have been made, the remainder is called the *neat* or *net* weight.

Ex. 1. What is the neat weight of 7 barrels of potash, each weighing 201*lb.*, tare being 10*lb.* per cwt ?

lb.
201
7
——
1407 *lb.*
——

The neat weight of 112*lb.* is 112*lb.* − 10*lb.* = 102*lb.*

lb.		*lb.*		*lb.*
112	:	1407	: :	102

whence 1281*lbs.* 6*oz.* the neat weight.

Ex. 2. Find the neat weight of 39cwt. 3qr. 21lb., tare 18lb. per cwt., tret and cloff. as usual.

	cwt.	qrs.	lb.
	39	3	21
16lb. = $\frac{1}{7}$ of 1 cwt.	5	2	23
2lb. = $\frac{1}{8}$ of 16lb.		2	24
Tare,	6	1	19
Suttle,	33	2	2
4lb. = $\frac{1}{26}$ of 104lb.	1	1	4
	32	0	26
2lb. = $\frac{1}{168}$ of 3cwt.			22
Neat Weight	32	0	4

DIVISION INTO PROPORTIONAL PARTS.

This Rule shews how to divide a given quantity into parts which shall have given ratios to each other.

Rule. As the sum of the numbers expressing the ratios is to any one of these numbers, so is the quantity to be divided to the part corresponding to the number in the second place of the proportion.

Ex. Divide 398 into 3 parts which shall be to one another as 5, 7, and 11.

The sum of 5, 7 and 11 is 23, then

$$23 : 5 :: 398$$
$$23 : 7 :: 398$$
$$23 : 11 :: 398$$

whence the parts are 86 $\frac{12}{23}$, 121 $\frac{3}{23}$ and 190 $\frac{8}{23}$.

CHAIN RULE.

This Rule solves questions, in which several quantities are so connected that a certain number of the first, equals a certain number of the second, and so on, and it is required to find how many of the last equal a given number of the first.

Rule. First put down the term of demand: then arrange under it all the other quantities as equations, observing that each term on the left must always be of the same kind as the one on the right next above it.

Divide the product of all the terms on the right, by the product of those on the left.

Ex. 1. If 16 plums are worth 12 pears, and 15 pears are worth 10 apples, how many apples ought I to have for 72 plums?

$$\begin{aligned} & 72 \text{ plums.} \\ 16 \text{ plums} &= 12 \text{ pears.} \\ 15 \text{ pears} &= 10 \text{ apples.} \end{aligned}$$

$$\frac{72 \times 12 \times 10}{16 \times 15} = \frac{9 \times 4 \times 10}{2 \times 5}$$
$$= 36 \text{ apples.}$$

Ex. 2. If 3 yards of cloth be worth 45s, what is the value of 7 yards?

$$7 \text{ yds.}$$
$$3 \text{ yds.} = 45\text{s.}$$
$$\frac{7 \times 45}{3} = 105\text{s.}$$

PROOF. 3yds. × its price must = 7yds. × 45s. ∴ the price $= \frac{7 \times 45}{3}$, as by the rule.

EXCHANGE.

Exchange is the act of paying or receiving the money of one place for its equivalent in the money of another.

A bill of exchange is a written order for the payment of a certain sum of money at an appointed time.

The *rate or course of exchange* is the *uncertain* sum of money of one place, which is given for a *certain* sum of that of another. Thus, London gives to Paris the *certain* for the *uncertain*, *i. e.*, the £ sterling for an uncertain number of francs, varying according to the circumstances of trade.

The *par of exchange* is such a quantity of the money of one place as is equal to a certain quantity of the money of another.

Agio in Amsterdam and other places, is the difference between their current money, called the *currency*, and the money of exchange, or that issued from the bank, called *banco*, the latter being usually finer than the former.

Arbitration of exchange is a comparison between the courses of exchange of several places.

London and Dublin.

Accounts are kept in Dublin as in London, in pounds shillings and pence, but the value of these denominations is $\frac{1}{12}$ greater in Dublin than London. Thus the English shilling is worth 1*s.* 1*d.* Irish, and the English £ is worth £1 1*s.* 8*d.* Irish. The *par of exchange* is therefore $8\frac{1}{2}$ per cent.

Ex. Reduce £159 12*s.* 6*d.* Irish to British Money, exchange at $6\frac{1}{2}$ per cent.

$$2)1.0 \qquad\qquad 12)6.$$

$$£106.5 = £106\tfrac{1}{2}. \qquad 20)12\cdot5 \quad £.\ s.\ d.$$

$$159.625 = 159\ 12\ 6.$$

$$\begin{array}{ccccc} £. & & £. & & £. \\ 106.5 & : & 100 & : & 159.625. \end{array}$$

whence £149 17*s.* $7\frac{3}{4}$*d.* the answer.

London and France.

Accounts are kept in France in *Francs* and *centimes*, or in *livres*, *sous*, and *deniers*.

10 centimes	= 1 decime.	12 deniers	= 1 sou.
100 centimes	= 1 franc.	20 sous	= 1 livre.
		3 livres	= 1 ecu.
		80 francs	= 81 livres.

Ex. 1. Reduce 4305 francs 95 centimes into sterling money, exchange at 24 francs 25 centimes per £ sterling.

fr.		fr.		£.
24.25	:	4305.95	:	1.

whence £177 11*s.* 3½*d.* the answer.

Ex. 2. Reduce £675 18*s.* 3*d.* to francs and centimes, exchange at 23 francs 15 centimes per £ sterling.

12)3.00
20)18.25
675.9125 = £675 18*s.* 3*d.*

£.		£.		fr.
1	:	675.9125	: :	23.15

whence 15647fr. 37cent., the answer.

Ex. 3. Reduce 37823fr. 59cent. to Irish Money, exchange between England and Ireland at 7½ per cent, and between England and France at 24fr. 51cent. per £ sterling.

37823.59fr.
24.51 fr. = £1 English.
£100 Eng. = £107.5 Irish.

whence, $\frac{37823.59 \times 107.5}{24.51 \times 100}$ = £1658 18*s.* 7*d.* Irish, the answer.

London and Amsterdam.

Accounts are kept at Amsterdam in *florins, stivers,* and *pennings*, or in *pounds, shillings,* and *pence* Flemish.

16 pennings	=	1 stiver.
20 stivers	=	1 florin or guilder.
12 pence Flemish, or 6 stivers	=	1 shilling Flemish.
20 shillings Flemish, or 6 florins	=	1 pound Flemish.
$2\frac{1}{2}$ florins, or 50 stivers	=	1 rix dollar.

Ex. Reduce 219 flor. 17 stiv. currency, to banco, agio at $3\frac{3}{4}$ per cent.

£. s.		£.		flor. stiv.
103 15s.	:	100	: :	219 17.

or, by Decimals,

£.		£.		flor.
103.75	:	100	: :	219.85

whence, 211flor. 18stiv. 1p., the answer.

By aid of the requisite tables we may solve any cases similar to the above. For this purpose the following tables are given.

Portugal. 1000 reis = 1 milrei: 400 reis = 1 crusado: 4800 reis = 1 moidore: $2\frac{1}{2}$ crusados of exchange = 1 milrei.

Spain. 34 maravedies = 1 real: 8 reals = 1 piastre or dollar of exchange: 4 piastres = 1 pistole: 375 maravedies = 1 ducat of exchange.

Leghorn and Genoa. 12 denari = 1 soldo: 20 soldi = 1 pezza or dollar of exchange.

Naples. 10 grains = 1 carlin: 10 carlins = 1 ducat regno.

Vienna. 4 pfennings = 1 creutzer: 60 creutzers = 1 florin or gulden: 90 creutzers or $1\frac{1}{2}$ florin = 1 rix dollar.

Petersburgh. 100 copecs = 1 ruble.

Stockholm. 12 pfenings = 1 skilling: 48 skillings = 1 rix dollar.

Copenhagen. 12 pfennings = 1 skilling: 16 skillings = 1 mark: 6 marks = 1 rix dollar.

N. B. The American dollar, and the rix dollars of Vienna, Copenhagen and Stockholm are each equal to about 4*s.* 6*d.* British, at par.

The East India coin is the rupee = about 2*s.*: a *lack* of rupees = 100,000 rupees or about £10,000: and a *crore* = 100 lacks.

ARBITRATION OF EXCHANGE.

Arbitration of Exchange determines the most advantageous mode of drawing and remitting bills.

Ex. A merchant has to remit a sum of money to Amsterdam: the direct exchange is 37*s.* Flemish per £ sterling, but between London and Paris the exchange is 24 francs per £ sterling, and between Paris and Amsterdam it is 54*d.* Flemish for 3 francs: had he better remit direct, or through Paris?

We have to find the arbitrated price of £1 sterling through Paris.

£1 sterling.
£1 sterling = 24 francs.
3 francs = 54*d.* Flemish or 4.5 shillings.

$$\frac{24 \times 4.5}{3} = 8 \times 4.5 = 36s. \text{ Flemish.}$$

or £1 sterling through Paris = 36*s.* Flemish.

It is therefore better to remit direct.

Part V.

INVOLUTION.

If a number be multiplied by itself, and the product by the same, and so on to any assigned number of products, the process is called *Involution*.

The number of times which the number has been used in the multiplication, expresses the *power* to which it is raised. Thus

$$2 \text{ or } 2 = \text{the 1st power of 2.}$$
$$2\times2 \text{ or } 4 = \text{the 2nd power of 2, or the square of 2.}$$
$$2\times2\times2 \text{ or } 8 = \text{the 3rd power of 2, or the cube of 2.}$$
$$2\times2\times2\times2 \text{ or } 16 = \text{the 4th power of 2.}$$
$$2\times2\times2\times2\times2 \text{ or } 32 = \text{the 5th power of 2,}$$

and so on.

This operation is concisely denoted by small figures, called *indiccs*, placed to the right of the numbers, thus 2^1, 2^2, 2^3, 2^4, &c., severally denote the 1st, 2nd, 3rd, 4th, &c., powers of 2.

By Multiplication, we find the following Tables.

1. *Table of Squares.*

$1^2=1.$	$5^2=25.$	$9^2=81.$
$2^2=4.$	$6^2=36.$	$10^2=100.$
$3^2=9.$	$7^2=49.$	$11^2=121.$
$4^2=16.$	$8^2=64.$	$12^2=144.$

2. *Table of Cubes.*

$1^3=1.$	$4^3=64.$	$7^3=343.$
$2^3=8.$	$5^3=125.$	$8^3=512.$
$3^3=27.$	$6^3=216.$	$9^3=729.$

To raise a *fraction* to any power, raise both the numerator and the denominator to that power.

Ex. Raise $\frac{2}{6}$ to the 3rd power.

$$(\tfrac{2}{6})^3 = \frac{2^3}{6^3} = \frac{8}{216}.$$

By raising a *decimal* to the 2nd and 3rd powers, we shall find that the square of a decimal will have 2, 4, 6, 8, &c. decimal places, and the cube 3, 6, 9, 12, &c. decimal places, according as the given decimal has 1, 2, 3, 4, &c. decimal places.

EVOLUTION.

Evolution is the reverse of involution, and means *the extracting of the root of a number*, or finding what number raised to a certain power will produce a given number.

The roots are usually represented by $\sqrt{}$, or by fractional indices ; thus,

$\sqrt{}$ 9 or $9^{\frac{1}{2}}$ means the 2nd, or square root of 9.
$\sqrt[3]{27}$ or $27^{\frac{1}{3}}$. . . . . 3rd, or cube root of 27.
$\sqrt{81}$ or $81^{\frac{1}{4}}$. . . . . . . 4th root of 81.

SQUARE ROOT.

The square root of a number is that number which multiplied by itself will produce the number. Thus, the square root of 9 is 3 ; of 64 is 8.

To find the square root of any number.

Rule. Put a point over the unit's place and over every alternate figure.

Find the greatest square number contained in the first period to the left, and put the square root of that number on the right as in division.

Square the root thus found, and place that square under the first period : subtract, and to the remainder bring down the next period for a dividend.

Double the root already found, and place the product for a divisor.

Divide the dividend exclusive of the figure on its right hand by the root thus doubled, and place the quotient in the root and in the divisor.

Multiply and subtract as in division : bring down the next period, and so on, till the work is completed.

Ex. 1. **Find the square root of 1444.**

$$1\dot{4}4\dot{4})38$$

9

---

68)544

544

. . .

Ex. 2 **Find the square root of 288369.**

$$2\dot{8}8\dot{3}6\dot{9})537$$

25

---

103)383

309

1067)7469

7469

. . . .

Here, we put a point over the unit 9 and over the 3 and 8. The greatest square number contained in 28, the first period to the left, is 25, the square root of which 5 we put in the root: we square 5, and put its square 25 under the 28: we subtract, and to the remainder 3 annex 83 the next period: we then double 5 and place 10 in the divisor: 10 is contained 3 times in 38: we place 3 in the root and divisor: we then multiply 103 by 3, and so on.

To extract the square root of decimals.

RULE. Point off every alternate decimal figure, adding a nought if the number of decimals be odd.

The number of decimals in the root will equal the number of decimal periods.

Ex. Extract the square root of 22.09 and of 104.793.

```
   2̇2.0̇9(4.7              1̇0̇4.7̇9̇3̇0̇(10.23
   16                      1
   —                       —
 87)609               202)479
    609                   404
    —                     —
    ...              2043)7530
    —                     6129
                          ————
                          1401
                          ————
```

To extract the square root of a fraction.

RULE. Find the square root of its numerator and denominator separately.

Ex. Find the square root of $\frac{144}{169}$.

$$\sqrt{\left(\frac{144}{169}\right)} = \frac{\sqrt{144}}{\sqrt{169}} = \frac{12}{13}.$$

If the numerator and denominator be not complete squares, reduce the fraction to a decimal, and find the square root of that decimal.

Ex. Find the square root of $\frac{31957}{25}$.

$$\sqrt{\frac{31957}{25}} = \sqrt{1278.28} = 35.753 \text{ \&c.}$$

The reason of the pointing will appear, if we consider that

$$1^2 = 1$$
$$10^2 = 100$$
$$100^2 = 10000 \text{ \&c.}$$
$$(.1)^2 = .01$$
$$(.9)^2 = .81$$
$$(.01)^2 = .0001$$
$$(.99)^2 = .9801 \text{ \&c.}$$

Hence, we see that the square of one figure cannot exceed two figures, and the square of two figures cannot exceed four figures, and so on: also, that the square of every decimal consists of 2, 4, 6, &c. number of figures: so that by putting a point over the unit's place, and over every alternate figure to the right and left, we divide the given number into periods of two figures, each period answering to a single figure in the root.

The Proof of the rule for square root is best understood from Algebra, being derived from the expression $(a + b)^2 = a^2 + (2a + b)b.$, according to which the working of *Ex.* 1. is as follows.

$$\sqrt{1444} = \sqrt{900 + 480 + 64}$$

```
         900 + 480 + 64(30+8
         900
      ---------------
60 + 8)480 + 64
       480 + 64
       --------
        .     .
       --------
```

CUBE ROOT.

The cube root of a number is that number which multiplied by itself twice, will produce the number. Thus the cube root of 27 is 3 : of 125 is 5.

To extract the cube root of a given number.

RULE. Point the unit's place and every third figure.

Find the greatest cube contained in the first period on the left, and place its root on the right.

Put the cube of this root under the first period, subtract, and to the remainder bring down the next period for a dividend.

To find a divisor, square the root thus found and multiply that square by 300.

Determine by trial with the divisor what the next figure must be, and put it in the root.

Multiply the divisor by the last figure in the root.

Square the last figure, multiply it by 30, and by the preceding root.

Cube the last figure.

Add up, and subtract, and bring down the next period and so on, till the work is completed.

Ex. **Extract the cube root of 76765625.**

$$\dot{7}6\dot{7}65\dot{6}25(425$$

$$4^3 = 64$$

$$4^2 \times 300 = 4800)12765$$

$$4^2 \times 300 \times 2 = 9600$$
$$2^2 \times 30 \times 4 = 480$$
$$2^3 = 8$$
$$10088$$

$$(42)^2 \times 300 = 529200)2677625$$

$$(42)^2 \times 300 \times 5 = 2646000$$
$$5^2 \times 30 \times 42 = 31500$$
$$5^3 = 125$$
$$2677625$$

To extract the cube root of a decimal.

Add ciphers, if necessary, to make the decimal consist of three, six, nine, &c. places, and point off every third figure to the right of the unit's place.

The number of decimals in the root will equal that of the decimal periods.

Ex. Find the cube root of 28·25

$$
\begin{array}{rl}
 & \dot{2}8.25\dot{0}00\dot{0}(3.04 \\
3^3 = & 27 \\
\hline
3^2 \times 300 = 2700) & 1250 \\
\hline
(30)^2 \times 300 = 270000) & 1250000 \\
\hline
(30)^2 \times 300 \times 4 = & 1080000 \\
4^2 \times 30 \times 30 = & 14400 \\
4^3 = & 64 \\
\hline
 & 1094464 \\
\hline
 & 155536000
\end{array}
$$

and so on.

To extract the cube root of a fraction.

Find the cube root of its numerator and denominator separately, or, if they be not complete cubes, reduce it to a decimal, of which extract the root as above.

The reason for pointing every third figure is as follows:

$$1^3 = 1$$
$$10^3 = 1000$$
$$100^3 = 1000000 \text{ \&c.}$$
$$(\cdot 1)^3 = \cdot 001$$
$$(\cdot 9)^3 = \cdot 729$$
$$(\cdot 01)^3 = \cdot 000001$$
$$(\cdot 99)^3 = \cdot 970299 \text{ \&c.}$$

Hence, we see that the cube of one figure cannot exceed three figures, and the cube of two figures cannot exceed six figures, and so on: also that the cube of every decimal consists of three, six, or nine, &c. places: so that by putting a point over the unit's place, and over every third figure to the right and left, we divide the given number into periods of three figures, each period answering to one figure in the root.

Proof. The proof depends on the formula $(a+b)^3 = a^3 + (3a^2 + 3ab + b^2)\,b$, and is difficult and complicated, if investigated by numbers.

LOGARITHMS.

By Involution we find that 2^0, 2^1, 2^2, 2^3, 2^4, 2^5 &c. are severally equal to 1, 2, 4, 8, 16, 32, &c. Here, the indices 0, 1, 2, 3, 4, 5, &c. are called the logarithms of 1, 2, 4, 8, 16, 32 &c. to base 2. Similarly, 10^0, 10^1, 10^2, 10^3, 10^4, 10^5, &c. are equal to 1, 10, 100, 1000, 10000, 100000 &c. Here, 0, 1, 2, 3, 4, 5 &c. are the logarithms of 1, 10, 100, 1000, 10000, 100000, &c. to base 10. Hence (to base 10) log. 1 = 0: log. 10 = 1: log. 100 = 2: log. 1000 = 3: log. 10000 = 4: log. 100000 = 5, &c.

Logarithms usually consist of a whole number and a decimal: thus the log. of 200 is 2·301030: the whole number 2 is called the *index*; the decimal part ·301030 is called the *mantissa*.

In the tables the decimal part only is inserted, the index being found by the following Rules.

Rule. 1. If the number contain an integer, the index of the logarithm is less by one than the number of integral figures.

Thus, the index of 243 is 2
.... 24.3 is 1
.... 2.43 is 0

Proof. Log. 10=1; log, 100=2; log. 1000=3, &c.
∴ log. of a number between 10 and 100, is 1 and some decimal. And log. of a number between 100 and 1000 is 2 and some decimal.

Rule 2. If the number be a decimal, the index will be $\bar{1}$, $\bar{2}$, $\bar{3}$, &c., according as the first significant figure to the left is in the 1st, 2nd, 3rd, &c. place from the decimal point.

Thus, the index of .234 is $\bar{1}$
.... .0234 is $\bar{2}$
.... .00234 is $\bar{3}$

Proof. $\log. .234 = \log. \frac{234}{1000} = \log. 234 - \log. 1000 =$ 2 and some decimal $- 3 = \bar{1}$ and some decimal.

To find the logarithm of any number.

Find the mantissa by the tables, and annex the index found by the above rules.

Ex. Find the log. of 24, 14.06, 1.406, .024.

log. 24 = 1.380211
log. 14.06 = 1.147985
log. 1.406 = 0.147985
log. .024 = $\bar{2}$.380211

To find the number corresponding to a given logarithm.

Find the number corresponding to the mantissa in the tables, and make it to consist of as many integral or deci-places as the above rules direct.

Ex. 1.380211 is log. of 24
2.394452........ 248
$\bar{2}$.3802111 ...... .024

*The logarithm of a product consisting of any number of factors, is equal to the sum of the logarithms of those factors.** *Thus,*

Log. 1 × 10 = log. 10 = 1 = 0 + 1 = log. 1 + log. 10.

Log. 10 × 100 = log. 1000 = 3 = 1 + 2 = log. 10 + log. 100.

* For a general proof of this and the two following rules, see Algebra.

The logarithm of a fraction is equal to logarithm of the numerator—the logarithm of the denominator. Thus,

$$\text{Log. } \frac{100}{10} = \text{log. } 10 = 1 = 2 - 1 = \text{log. } 100 - \text{log. } 10.$$

The logarithm of any power or root of any quantity is found by multiplying or dividing the logarithm of that quantity by the index of the power or root. Thus,

$$\text{Log. } (10)^2 = \text{log. } 100 = 2 = 2 \times 1 = 2 \text{ log. } 10.$$

$$\text{Log. } (100)^{\frac{1}{2}} = \text{log. } 10 = 1 = \tfrac{1}{2} \times 2 = \tfrac{1}{2} \text{ log. } 100.$$

Hence by logarithms, multiplication and division are reduced to addition and subtraction: involution and evolution to multiplication and division.

Ex. 1. Multiply 7 by 23.

$$\text{log. } (7 \times 23) = \text{log. } 7 + \text{log. } 23$$

$$\text{log. } 7 = 0.8450980$$

$$\text{log. } 23 = 1.3617278$$

$$2.2068258 = \text{log. } 161$$

$$\therefore\ 7 \times 23 = 161.$$

Ex. 2. Divide 324 by 27.

$$\text{log. } \frac{324}{27} = \text{log. } 324 - \text{log. } 27.$$

$$\text{log. } 324 = 2{\cdot}5105452.$$

$$\text{log. } 27 = 1{\cdot}4313639$$

$$1{\cdot}0791813 = \text{log. } 12.$$

$$\therefore\ \frac{324}{27} = 12.$$

Ex. 3. Raise 2 to the seventh power.

$$\log.\ (2)^7 = 7 \times \log.\ 2.$$

$$\log.\ 2 = 0{\cdot}3010300$$

$$\underline{\phantom{0{\cdot}30103}7}$$

$$2.1072100 = \log.\ 128.$$

$$\therefore\ 2^7 = 128.$$

Ex. 4. Extract the seventh root of 128.

$$\log.\ \sqrt[7]{128} = \tfrac{1}{7}\ \log.\ 128.$$

$$\frac{\log.\ 128}{7} = \frac{2{\cdot}1072100}{7}$$

$$= {\cdot}3010300 = \log.\ 2.$$

$$\therefore\ \sqrt[7]{128} = 2.$$

Ex. 5. Find the value of $\dfrac{2^6 \times 25^2}{4^3 \times 10^2}$ ·

$$\log.\ \frac{2^6 \times 25^2}{4^3 \times 10^2} = \log.\ (2^6 \times 25^2) - \log.\ (4^3 \times 10^2).$$

$$= 6\ \log.\ 2 + 2\ \log\ 25 - \{\ 3\ \log.\ 4 + 2\ \log.\ 10\ \}$$

$$6\ \log.\ 2 = 1{\cdot}806180 \qquad 3\ \log.\ 4 = 1{\cdot}806180$$

$$2\ \log.\ 25 = 2{\cdot}795880 \qquad 2\ \log.\ 10 = 2{\cdot}000000$$

$$4.602060 \qquad\qquad 3.806180$$

$$3.806180$$

$${\cdot}795880 = \log.\ 6.25$$

$\therefore$ 6.25 value required.

To find the logarithm of a composite number.

RULE. Resolve the number into its factors, and find, by the above Rules, the log. of those factors.

Ex. 1. Find the log. of 987.

$$987 = 3 \times 7 \times 47$$

$$\therefore \log. 987 = \log. 3 + \log 7 + \log. 47.$$

Ex. 2. Given the log. of 2 and 3, find log. $\frac{9}{16}$.

$$\log. \frac{9}{16} = \log. 9 - \log. 16$$

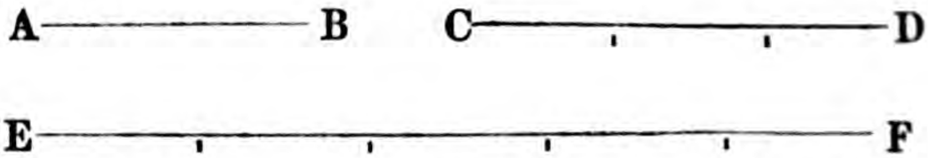
$$= \log. 3^2 - \log. 2^4 = 2 \log. 3 - 4 \log. 2.$$

PART VI.

LINEAL MEASURE.

The *unit* of lineal measure is a straight line of certain length, as *one yard, one foot, one inch.*

By comparison with this unit the magnitudes of other lines may be determined. Thus if AB be the lineal unit,



and denoted by 1 : a line CD which contains AB, 3 times will be denoted by 3 : and EF, which contains it 5 times will be denoted by 5, or if AB=1, CD=3, and EF=5.

RULE. 1. *Right-angled triangle.* The Hypothenuse equals the square root of the sum of the squares of the sides.

2. *Right angled triangle.* One side equals the square root of the square of the hypothenuse minus the square of the other side.

3. *Circle.* The circumference equals twice the radius multiplied by 3·14159, nearly.

4. *Circle.* The radius equals the circumference divided by 6·28318, nearly.

Ex. 1. Find the hypothenuse of a right angled triangle whose sides are 3 and 4 feet.

$$\text{Hypothenuse} = \sqrt{3^2+4^2} = \sqrt{9+16.}$$
$$= \sqrt{25} = 5 \text{ feet.}$$

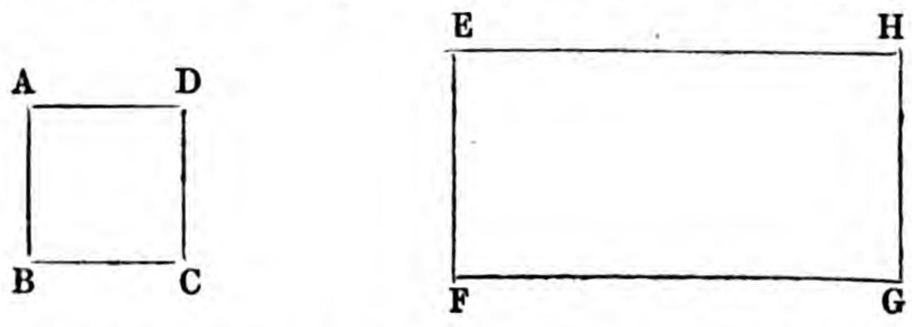
Ex. 2. Find the radius of a circle whose circumference is 40 yards.

$$\text{Rad.} = \frac{40}{6{\cdot}28318} = 6{\cdot}3662 \text{ yards.}$$

SUPERFICIAL MEASURE.

A *superficies* has length and breadth.

The *unit* of superficial measure, is a square surface whereof each side equals in length the lineal unit: thus, if BC, AB, (which may be regarded as the length and



breadth) be each *one yard, one foot,* or *one inch*: the superficial unit ABCD, will be *one yard square, one foot square*, or *one inch square.*

The Area of a rectangular parallellogram will be represented arithmetically by the product of two of its adjacent sides.

Let EFGH be a rectangular parallellogram, and let EF contain AB 5 times, and FG contain it 6 times; by dividing EF into those 5 parts and FG into 6, and drawing through each of the points thus made, lines parallel to EH and HG, EFGH will be divided into 30 figures, each equal to ABCD, and $\therefore$ if we call ABCD $= 1$: EFGH $= 30 \times \text{ABCD} = 30 \times 1 = 5 \times 6$.

Cor. If EF and FG each $= 12$ inches, then EFGH $= 12 \times 12 = 144$ square inches: but EFGH is in this case a square foot.

$\therefore$ One square foot $= 144$ square inches: similarly,

One square yard $= 9$ square feet.

RULES FOR FINDING AREAS.

1. *Parallelogram.* The area equals the base multiplied by the perpendicular altitude.

2. *Triangle.* The area equals half the product of the base and perpendicular altitude.

3. *Triangle.* From half the sum of the three sides subtract each side separately, multiply the half sum by these three remainders, and extract the square root of the product.

4. *Trapezium.* The area equals half the product of either diagonal and the sum of the perpendiculars let fall on it from the opposite angles.

5. *Circle.* The area equals the square of the radius, multiplied by 3.14159, nearly.

6. *Circle.* The area equals half the radius multiplied by the subtending arc.

7. *Ellipse.* The area equals the product of the semi-axes, multiplied by 3.14159, nearly.

Ex. 1. What is the area of a circle whose radius is 2 feet?

$$\text{Area} = 4 \times 3.14159 = 12.56636 \text{ square feet, nearly.}$$

Ex. 2. Find the area of a triangle whose base is 8 feet, and perpendicular altitude is 3 feet.

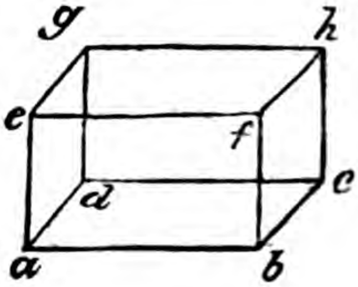
$$\text{Area} = \frac{8 \times 3}{2} = 12 \text{ square feet.}$$

SOLID MEASURE.

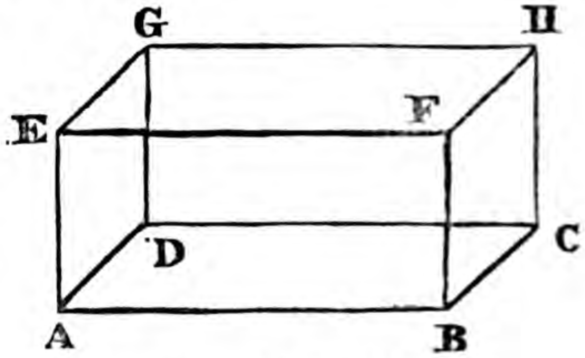
A *Solid* is that which has length, breadth, and height.

A *unit* of solid measure, is a rectangular parallelopiped, whose length, breadth, and height are each one lineal unit, as *one yard, one foot, one inch,* in which case, the parallelopiped is called a *cubic yard,* a *cubic foot,* or a *cubic inch.*

The Solid content of a rectangular parallelopiped will be numerically represented by the product of its length, breadth, and height.



For let ah be the cubic unit, and AH another rectangular parallelopiped, whose length AB = 5 lineal units, breadth AD = 2 lineal units, and height AE = 3 lineal units; by dividing AB into 5 parts, AD into 2, and AE into 3, and completing the figures, we should have 30 parallelopipeds, each equal to ah: and ∴ calling $ah = 1$, we should have AH $= 30 \times ah =$ $30 \times 1 = 5 \times 2 \times 3 =$ length × breadth × height.



Cor. If the sides be each equal to 3 feet, AH $= 3 \times 3 \times 3 = 27$ solid feet; but AH, in this case, = one solid yard,

∴ 1 solid yard = 27 cubic feet.

Similarly, 1 cubic foot = 1728 cubic inches.

RULES FOR FINDING SOLID CONTENT.

1. *Parallelopiped.* The content equals the area of the base multiplied by the perpendicular height.

2. *Prism and Cylinder.* The content equals the area of the base multiplied by the perpendicular height.

3. *Pyramid and Cone.* The content is equal to the area of the base multiplied by one third of the perpendicular height.

4. *Sphere or Globe.* The content is equal to the cube of the radius multiplied by 4.18879, nearly.

Ex. Find the solid content of a rectangular piece of timber, 2 feet long, 3 wide, and 6 long.

Solid content $= 2 \times 3 \times 6 = 36$ solid feet.

DUODECIMALS.

Duodecimals is a convenient method of computing small areas and solid contents, used by artificers to estimate work done.

The dimensions are commonly taken in yards, feet, inches, parts, &c. decreasing from left to right in a twelvefold proportion: the inches, parts, &c. are termed primes, seconds, thirds, &c., and are denoted by $'$, $''$, $'''$, &c.: thus, 20ft, $9'$, $6''$, $4'''$, means 20 feet, 9 inches, 6 parts, 4 thirds.

Glazing, and Masons' flat work, &c. are measured by the square foot: painting, paving, plastering, &c. by the square yard: flooring, rooffing, &c. by the square of 100 feet.

Brick-work is measured by the rod of $16\frac{1}{2}$ ft., and $272\frac{1}{4}$ square feet of brick-work.

MULTIPLICATION.

Rule. Write the terms of the multiplier under the corresponding terms of the multiplicand.

Multiply every term in the multiplicand, beginning at the lowest, by each term in the multiplier, beginning with the highest.

Divide each product, except that which is feet, by 12, and place the remainder under the multiplicand, when the multiplier is feet; one place removed to the right, when it is primes; two places, when it is seconds; three when it is thirds; and so on, observing to add the quotient to the next product.

The sum of these products will be the answer.

Ex. 1. Multiply 8 ft. 6′ 9″ by 7 ft. 3′ 3″.

	ft.					
	8.	6′.	9″			
	7.	3.	3			
	59.	11.	3			
	2.	1.	8.	3		
		2.	1.	8.	3	
sq. ft.	62.	3′.	0″.	11‴.	3⁗	product required.

Proof. $\overset{\text{ft.}}{7} \times \overset{\text{ft.}}{9''} = \overset{\text{ft.}}{7} \times \overset{\text{ft.}}{\frac{9}{144}} = \overset{\text{ft.}}{\frac{63}{144}} = 63'' = 5' + 3''$

$\overset{\text{ft.}}{7} \times \overset{\text{ft.}}{6'} = \overset{\text{ft.}}{7} \times \overset{\text{ft.}}{\frac{6}{12}} = \overset{\text{ft.}}{\frac{42}{12}} = 42' = \overset{\text{ft.}}{3} + 6'$

$\overset{\text{ft.}}{7} \times \overset{\text{ft.}}{8} = \overset{\text{ft.}}{56}$

59. 11′ 3″

Similarly,

$$3' \times 9'' = \frac{3}{12}\text{ft.} \times \frac{9}{144}\text{ft.} = \frac{27}{1728}\text{ft.} = 27''' = 2'' + 3'''$$

$$3' \times 6' = \frac{3}{12}\text{ft.} \times \frac{6}{12}\text{ft.} = \frac{18}{144}\text{ft.} = 18'' = 1' + 6''$$

$$3' \times 8\text{ft.} = \frac{3}{12}\text{ft.} \times 8 = \frac{24}{12} = 2\text{ ft.}$$

2. 1'. 8'' 3'''

and so on.

Ex. 2. Find the solid content of a rectangular solid, whose length is 2 ft. 3 in., width 1 ft. 7 in., and depth 9in. 4''.

	ft.			
	2	3'		
	1	7		
	2	3		
	1	3	9	
sq. ft.	3	6	9	
		9	4	
	2	8	0	9
		1	2	3
cub. ft.	2	9'	3''	0'''

The price of any piece of work may be found by Rule of Three of Practice.

Ex. What is the price of the above solid at 2*s*. 6*d*. per cubic foot?

		s.	*d.*		
		2	6	=	price of 1 cubic foot.
		5	0	=	...... 2 cub. ft.
6	$\frac{1}{2}$	2	6	=	...... 6′
3	$\frac{1}{2}$	1	3	=	...... 3′
3	$\frac{1}{4}$		$3\frac{3}{4}$	=	...... 3″
		*s.*9	$0\frac{3}{4}$	=	...... 2. 9′. 3″.

GAUGING.

Liquids are measured by the imperial gallon, which is equal to 277.274, cubic inches: therefore, to find the number of gallons a vessel contains, divide its solid content by 277.274.

Ex. What number of gallons are there in a cistern, whose length is 40 inches, breadth 24, and depth 16?

Here, solid content $= 40 \times 24 \times 16$
$= 15360$ cubic inches.

and number of gallons $= \dfrac{15360}{277.274}$
$= 55.3964$ gal.
$= 55$ gal. 1 qt. 1 pt.

Malt and corn, &c. are estimated by the Imperial Bushel, containing 2218.192 cubic inches: therefore to find the number of bushels, find the solid content, and divide it by 2218.192.

LAND SURVEYING.

Land is measured by Gunter's chain, which is 4 poles, or 22 yards long, and divided into 100 equal parts, called links.

Since an acre is a rectangular area whose length is 40 poles, and breadth 4 poles, it will equal $40p. \times 4p. = 1000$ links $\times$ 100 links $= 100000$ square links, and hence, if any superficial content be expressed in links, this divided by 100000 will give the number of acres.

A pole $= 5\frac{1}{2}$ yards, or 25 links.

A square pole $= 30\frac{1}{4}$ square yards, or 625 square links.

Rood $= 1210$ square yards, or 25000 square links.

Acre $= 4840$ square yards, or 100000 square links.

Hence a square mile

$$= 1760 \times 1760 \text{ yards} = 3097600 \text{ square yards.}$$

$$= \frac{3097600}{4840} \text{ acres} = 640 \text{ acres.}$$

Ex. Find the area of a rectangular field whose length is 25 chains, 8 links, and breadth 14 chains, 75 links.

```
25ch.    8 links = 2508 links
14ch.   75 links = 1475 links
                   --------
                      12540
                     17556
                    10032
                    2508
                   --------
        100000)3699300 sq. links.
                   --------
                   36.99300 acres
                          4
                   --------
                    3.97200 rods
                          40
                   --------
                    38.88000 poles.
                   --------
```

∴ the area of the field = 36ac. 3 rods, 38 poles, nearly.

FINIS.

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