243 GEOMETRY. TAYLOR (Thomas, *the Platonist*). THE ELEMENTS OF A NEW METHOD OF REASONING IN GEOMETRY. 4to, 7 plates, some folding, calf. *For the Author*: *London*, 1780

⁂ FIRST EDITION. Thomas Taylor's earliest publication.

THE ELEMENTS OF A NEW METHOD OF REASONING IN *GEOMETRY*:

APPLIED TO THE RECTIFICATION OF THE CIRCLE.

By *THOMAS TAYLOR.*

———*Nec altum sapiamus, nec ultra sobrium, sed veritatem in charitate colamus.*

BACON.

LONDON:

PRINTED FOR THE AUTHOR; AND SOLD BY J. DENIS AND SON, N° 2, NEW BRIDGE-STREET, BLACK-FRIARS. MDCCLXXX.

[Price Two Shillings and Six-pence.]

ERRATUM,

Page 7. Line 5. *Instead of* "the χ Arch," *read* "the Arch,"

ADVERTISEMENT.

THE Author of the following ſmall Tract is not aſhamed to confeſs, that it has been the employment of his leiſure hours for a conſiderable time. If he has failed in the execution, he can, however, ſafely affirm he has not been wanting in the moſt earneſt endeavours towards the completion of his purpoſe. He conſidered that the object of his ſearch and enquiry, although arduous, was at the ſame time glorious, and that the Diſcovery of Truth is always a ſufficient recompence for the difficulty attending its Inveſtigation. As he cannot, by the moſt impartial ſcrutiny, detect any falſe Reaſoning in his Demonſtrations, he flatters himſelf they will not be found altogether deſtitute of ſupport, nor wholly unworthy the

appro-

approbation of the Public. However, ſenſible of his own weakneſs, he would not too confidently preſume on ſucceſs, ſince in this caſe to DESIRE is not ſufficient TO OBTAIN.

In ſhort, animated by a ſincere Love of TRUTH, he flatters himſelf the integrity of his Intentions will in ſome meaſure atone for his want of greater genius and abilities.

Indeed, as an elegant Writer obſerves, ſince Mediocrity is now become a Protection, he has probably obtained THAT PROTECTION in ſpite of himſelf. He only adds (with the ſame Author) and would wiſh the Reader to remember, that while Error ſinks into the abyſs of forgetfulneſs, TRUTH alone ſwims over the vaſt extent of ages.

T H E

E L E M E N T S, &c.

L E M M A.

IF there be any three Quantities, X, Y, Z, the former of which X, is greater than either of the other two, Y, Z, then it cannot be ſaid Z is greater than Y, by the exceſs of X above Y; but if it is at all greater, it muſt be by ſome Quantity leſs than the exceſs of X above Y.

For if it is denied, Z ſhall be equal to X, which is abſurd.

PROPOSITION I. *Fig.* 1.

If from the ſame Centre s, any two Arches tR, $a\chi$, are deſcribed as in the Figure, and with the Radius $s\,a$, of the greateſt $a\chi$,

 the

the Arch tz is drawn touching the Arch Rt, in the Point t, then the Arch Rt, ſhall not be greater than the Arch tz.

From the Point z, draw the Line $z\rho$ parallel to the Line sa; then, as is well known, the Arch ρa ſhall be equal to the Arch zt. Suppoſe the Arch $\chi\rho$ to be divided by a continual Biſection into an infinite number of equal Parts, at the points o, o, o, &c. and let $o\chi$ repreſent one of ſuch parts: From o draw the right line oo, parallel to as, and with a radius equal to as deſcribe the arch λo, and from the centre s, the arch λs. Now let (M) repreſent the infinitely ſmall quantity χo, then if the arch λs exceed the Arch λo, by the quantity M, it ſhall be equal (from the Lemma) to the Arch $a\chi$, which is abſurd. The ſame conſequences will evidently follow, if a Quantity leſs than χo is aſſigned, and ſo the Arch λs is not greater than the Arch λo.

Again, Suppoſe the Arch λo, to exceed the Arch $m\lambda$ by the like Quantity (M), then if the Arch $m\kappa$ exceed $m\lambda$, by (M), it ſhall be equal to the Arch λo. But the Arch λs, is not greater than λo from the Demonſtration, and ſo in this caſe is not greater than the Arch $m\kappa$, which is evidently abſurd. In like manner proceeding infinitely, we ſhall at laſt prove, that the Arch tR is not greater than the Arch tz. Q. E. D.

Fig. 1.

Fig. 1. It muſt be carefully obſerved that this method of Reaſoning will not take place, if the Angle $a\,s\,\pi$ is aſſumed greater than a right Angle, (ſuppoſe the Angle $a\,s\,G$). For ſince it is always required to the Demonſtration that an infinitely ſmall Part of the Arch $a\,G$ muſt be taken, and a Line drawn parallel to $s\,a$. If from the Point F, which terminates the Quadrant $a\,F$, an infinitely ſmall Arch $F\,o$ is taken, and a Line drawn parallel to $a\,s$, the Line ſo drawn ſhall be a Tangent to the Arch FG, and ſo fall without the Curve.——In like manner, any Line drawn from a Point below F, parallel to $a\,s$, ſhall fall without the Curve towards G; and therefore in this caſe the preceding Demonſtration cannot take place.

COROLLARY I.

Fig. 1. No. II. It may not, perhaps, be unneceſſary to obſerve, that after the ſame manner as in this Propoſition, it is eaſy to prove that in the Quadrant $a\,K$, if any right line $\varphi\,o$ is drawn parallel to $\chi\,a$, then any other right Line $\varphi\,\rho$ ſhall not be greater than $\varphi\,o$.

COR. II.

Fig. 1. No. III. If Concentric and Tangent Arches are deſcribed as in the Figure terminated by the Quadrantal Arch $\Pi\,s$,

 then

then from this Propoſition, (if we ſuppoſe the concentric Arches from s towards π continually increaſe) any Concentric Arch $\chi\gamma$ ſhall not be greater than it's Tangent Arch $\sigma\varepsilon$.

Cor. III.

Hence, (the ſame Things remaining as in Cor. II.) if $m\theta$ is deſcribed exceeding $\Delta\sigma$ by (M), and if $m\lambda = m\theta$, then all the concentric Arches above Δo ſhall equal their correſpondent Tangent Arches, deſcribed with the Radius $s\varphi$ as per Scheme.

For ſuppoſe $\chi\varepsilon$ to exceed $m\theta$ by (M), then if $m\lambda = m\theta$, the Arch $\chi\gamma$ ſhall be greater than $m\theta$, and not greater than $\chi\varepsilon$ (Pr. I.) Therefore $\chi\gamma$ cannot be leſs than $\chi\varepsilon$, and ſo muſt be equal to it. Q. E. D.

It were eaſy to infer a Variety of other Concluſions from this Propoſition, but as they will naturally preſent themſelves to the Reader's obſervation, without much enquiry, they are omitted in this place.

Propo-

PROPOSITION II. *Fig.* 2.

IF from the Center ϵ, with the Radius $\epsilon \kappa$, the Arch $\kappa \lambda$ is deſcribed, and from the Centers a, c, δ, the reſpective Tangent Arches, $\kappa \mu$, κx, κp, &c. Then the Arch $\kappa \lambda$ ſhall be leſs than the Arch κx.

For from PROP. I. $\kappa \lambda$ is not greater than κx, and from the ſame PROP. $\kappa \mu$ is not greater than κo; $\kappa \mu$ is therefore leſs than κx. But $\kappa \lambda$ is likewiſe not greater than $\kappa \mu$; it is therefore either equal to, or leſs than $\kappa \mu$, in each of which caſes it ſhall be leſs than κx. Therefore I conclude, univerſally, that any concentric Arch $\kappa \lambda$ is leſs than it's Tangent Arch $\kappa \mu$, terminated by the ſame right Line $\epsilon \mu \varphi$. Q. E. D.

COR. I.

Hence we have an eaſy method of approximating continually nearer and nearer to an Equality between two Arches, κp, and $\kappa \lambda$. Draw the right Line $a \mu x$, interſecting the Arch κp in s; then becauſe $\kappa \mu$ is leſs than κs, and $\kappa \lambda$ leſs than $\kappa \mu$, $\kappa \lambda$ ſhall be leſs than κs; through the Point o, Draw the right Line $\epsilon o w$, then κo

ſhall

ſhall be leſs than $\kappa\pi$: But $\kappa\mu$ has been proved to be leſs than κo, and $\kappa\lambda$ leſs than $\kappa\mu$, therefore $\kappa\lambda$ ſhall be leſs than κn, and ſo on infinitely.

Cor. II.

Hence likewiſe, we may eaſily approximate to an Equality between a right Line, and any circular Arch. For it is well known that any Arch $\kappa\lambda$ is leſs than its Tangent $\kappa\varphi$; the Arch $\kappa\mu$ therefore is leſs than κx. But from this Prop. $\kappa\lambda$ is leſs than $\kappa\mu$, and conſequently leſs than the right Line κx. Through the Point o draw the right Line $\epsilon o w$, as per Scheme, then κo is leſs than κw, and ſo $\kappa\lambda$ is leſs than κw. In like manner $\kappa\lambda$ is leſs than κv; and ſo we may proceed infinitely, the Arch $\kappa\lambda$ approaching nearer and nearer to an Equality with ſome part of the Line $\kappa\varphi$, in proportion as the Diſtances κa, $\kappa\epsilon$, $\kappa\delta$, &c. increaſe, that is, in proportion as the Angle of Contact $\varphi\kappa\lambda$ is diminiſhed. But it is to be obſerved, that theſe Approximations infinitely continued will never make one Tangent Arch equal to another, or to a Part of a Tangent right Line: For if a Tangent Line $\kappa\lambda$ be drawn to the Arch $\kappa\lambda$, and from the Center a an Arch $\lambda\varsigma$ is deſcribed, it is ſufficiently evident, that a Line drawn from the Center ι, of the Arch $\kappa\lambda$ ſhall touch the Arch $\lambda\varsigma$ in λ, and a Line drawn from the

Center

Center of the Arch $\kappa\mu$ to the Point μ, ſhall likewiſe be a Tangent to the Arch λs. But the Arch $\kappa\lambda$ is ſtill leſs than the Arch $\kappa\mu$, as may be inferred from PROP. I. and ſo by this Method alone we can never come nearer to an Equality between the Arch $\kappa\lambda$, and any ſuperior Tangent Arch, or Tangent right Line, than the χ Arch, or Tangent Line terminated by the Arch λs.

LEMMA.

Fig. III.

IF the Arch $\beta\lambda$ is ſuppoſed equal to the Tangent Arch βp, and to the Tangent right Line $\beta\varphi$, then a Circle may be deſcribed through the Points λ, p, φ, concave towards the Line $\beta\mu$, at right Angles to the Line $\beta\varphi$; and if concentric Arches are deſcribed from the Point μ, (the Center of the Arch $\beta\lambda$) as far as to the Point z, which terminates the Quadrant νz, they ſhall continually exceed each other.

The firſt Part of the Lemma is ſufficiently evident from Calculation, and may be eaſily proved to extend itſelf to any two Tangent Arches and a Tangent right Line; and the ſecond Part is likewiſe evident from this, that the ſame Arch of a Circle has every where the ſame Curvature, and conſequently, ſuppoſing the *Tangent Arch* βp, equal to $\beta \lambda$, to be drawn infinitely near $\beta \lambda$, then a concentric Arch deſcribed from μ, and paſſing through p, ſhall be evidently greater than $\beta \lambda$, and ſo if beyond the Point p, we ſuppoſe an Arch $p t$ taken, equal to the infinitely ſmall Arch $p \lambda$, the Inclination or Curvature of the Arch $t p$ ſhall be the ſame with that of the Arch $p \lambda$; and therefore I conclude the concentric Arch paſſing through p, ſhall be leſs than the concentric Arch paſſing through t, &c.

PROPOSITION III. *Fig.* 3. N° I.

Things remaining as in the Lemma, all the Tangent Arches deſcribed beyond the Arch βp, and terminated by the Arch $\lambda \varphi$, ſhall be reſpectively equal to each other, and to the Tangent Line $\beta \varphi$.

* From the Point z, which determines the Quadrant γz, ſuppoſe an Arch to be deſcribed concentric to the Arch $\beta \lambda$, and with the ſame Radius let a Tangent Arch $\beta \epsilon$, from the Point β be deſcribed,

* It is neceſſary to obſerve, that if the Arch $\gamma \varphi$ is greater than a Quadrant, it ſhall be greater by an Arch leſs than One Half of the Arch $\lambda \varphi$, as is evident; and therefore it will univerſally follow that the Arch $\beta \lambda$ ſhall not be greater than a Tangent Arch drawn infinitely near βp, which is all that is requiſite to the Demonſtration.

ſcribed, then (from PROP. I. and preceding LEMMA) $\beta\lambda$ is not greater than $\beta\epsilon$, and after the ſame Manner it may be ſhewn $\beta\lambda$ is not greater than any Tangent Arch falling within $\beta\lambda$, $\beta\epsilon$. It is therefore either equal to or leſs than the Arch βo. Let then $\beta\lambda$ be leſs than βo, and make $\beta n = \beta\lambda$, then through the three Points λ, n, φ, (from preceding LEMMA :) Let a Circle λ, n, φ, be deſcribed, this ſhall fall within the Arch $\lambda\varphi$, and conſequently ſhall make βs, leſs than βp. But if the Arch $\beta n = \beta\lambda$, then from the Principles already eſtabliſhed, $\beta\lambda$ ſhall not be greater than βs, which is manifeſtly abſurd. Every Tangent Arch therefore, within the Arch $\beta\epsilon$ is equal to the Arch $\beta\lambda$, and after the ſame Manner we prove every Tangent Arch beyond the Arch $\beta\epsilon$, is equal to the Arch $\beta\lambda$, or the Tangent Line $\beta\varphi$. For ſuppoſe an Arch βv, deſcribed infinitely near the Arch $\beta\epsilon$, then $\beta\epsilon$ ſhall not be greater than βv, and if it is ſuppoſed leſs, the ſame Abſurdity will enſue, as we have noted in the former Part of this Propoſition; and ſo we may proceed infinitely. All the Tangent Arches therefore beyond the Arch $\beta\lambda$, terminated by the Arch $\lambda\varphi$, are equal to each other, and to the right Line, $\beta\varphi$. Q. E. D.

COROLLARY.

Hence if any three Tangent Arches (each not exceeding a Quadrant) are aſſumed equal to each other, a Circle deſcribed through them

them, ſhall cut off a Part of the common tangent right Line equal to each, and to any Tangent Arch that can be poſſibly drawn; for if this is denied, one Circle ſhall cut another in three Points, which is abſurd.

Fig. III. No. 2.

Another Method of demonſtrating the preceding Propoſition.— Let a Part *a d* of the Arch χ *a* be aſſumed infinitely ſmall, and let a Line be drawn from *a* to φ, the Center of the Arch χ *m*. Then becauſe every Point of the Line φ *m* beyond *m* falls without the Arch ϵ *m*, the right Line φ *a* ſhall interſect the Arch *m* ϵ in one Point only, *viz.* in the Point *o*. Through the Point *o* deſcribe an Arch touching the Line $\chi\,\epsilon$ in χ, then the Arch χ *o* ſhall not be greater than the Arch χ *d* (the Arch χ *o* being leſs than the Arch χ *a*, as is eaſily inferred from Prop. I.) for if it is greater by an infinitely ſmall Quantity, it ſhall be equal to the Arch χ *a*, which is manifeſtly abſurd. In like manner we may proceed with the reſt, and demonſtrate that every inferior tangent Arch is not greater than it's ſuperior terminated by the ſame Arch *m* ϵ, till at length the Arch χ *m* is proved not greater than ſome Tangent Arch drawn infinitely near it; ſuppoſe the Arch χ *t*; for the Arch χ *n* is leſs than χ *t*, and if it were poſſible that by proceding in this

this manner, the infinitely ſmall Part of ſome Arch falling without the Arch $\epsilon\, m$, ſhould never be in the right Line $\varphi\, m\, \kappa$, an infinitely ſmall Part (ſuppoſe $m\, n$) muſt at length remain, or by continuing the Proceſs, a Part ſtill leſs than that, and ſo on, *ad infinitum*. $\chi\, m$, therefore, is not greater than $\chi\, t$, and $\chi\, t$ is not greater by the above Method than $\chi\, d$; therefore $\chi\, t$ and $\chi\, m$ are equal, and ſo of the reſt.

The Two preceding Methods of Demonſtration might have been omitted on Account of the ſuperior Accuracy and Elegance of the following; however as they ſerve to illuſtrate the Method of Reaſoning adopted in this Work, they are on that account retained.

Fig. III. No. 1.

If the Arch $\beta\, o$ is ſuppoſed equal to the Arch $\beta\, \epsilon$, equal to the right Line $\beta\, \varphi$, then any other Arch $\beta\, s$, falling within $\beta\, o$, $\beta\, \epsilon$ ſhall be equal to the right Line $\beta\, \varphi$.

Let a Line be drawn from the Point o, meeting the Arch $\beta\, s$ produced in a, and ſuppoſe the Arch $s\, a$ to be divided by a continual Biſection into an infinite Number of equal Parts, and let $a\, b$ be one of thoſe Parts; from the Point b, let the Line $b\, \mu$ be drawn, this ſhall interſect the Arch $\lambda\varphi$ in ſome Point i: Through the Point i let the tangent Arch βi be deſcribed, then the Arch βo ſhall not be greater than the Arch βi, for if it is greater by a Quantity equal to the Arch $i p$, it ſhall be equal to the Arch βy, that is (becauſe $a b$ is infinitely ſmall, and $i y$ is leſs than $a b$, as is eaſy to infer from Prop. I.) if it is greater by a Quantity leſs than one infinitely ſmall, an Abſurdity ſhall enſue; in like manner the Arch βi is proved not greater than βt, by ſuppoſing $b d$, in the Arch $s a$, equal to $a b$, and ſo we may proceed infinitely, till at length the Arch βo is demonſtrated not greater than the Arch βs. In like Manner the Arch βs is not greater than $\beta \epsilon$, and ſo muſt be equal to it and to the right Line $\beta \varphi$. Q. E. D.

Proposition IV. *Fig.* 4.

TO Cut any given Angle $s \varphi R$, not exceeding a right Angle, in any given Ratio.

Let it be required to be divided in the Ratio of the Line φQ to the Line φs. Deſcribe from the Center φ, the Arches $Q o$, $s R$, and

and bisect the Line $\mathfrak{Q}\,\varphi$, in the Point λ, and from the same Center φ, describe the Arch $\lambda\,\kappa\,a$. At the Point λ, with a Radius equal to φ-$\mathfrak{Q}$, describe the Tangent *Arch* as in the Figure, and at the same Point with a Radius double to $\varphi\,\mathfrak{Q}$, another Tangent Arch, then $\frac{1}{2}$ of the Tangent Arch $\lambda\,\kappa$ shall be equal to $\frac{1}{4}$ of the Second, and $\frac{1}{8}$ of the Third, as is evident. Through the Points of Equality, describe a Circle; this shall make $\frac{1}{4}$ of the Arch Q *o* equal to some Part of the Tangent Arch described with the Radius $\varphi\,s$, and so a Part of the Arch *s R* may be taken equal to the Arch $\mathfrak{Q}\,o$, which was required to be done.

COROLLARY.

Hence, by a double Operation, any obtuse Angle may be cut in a given Ratio; and hence, by the Way, it is easy to find a right Line equal to a given circular Arch.

SCHOLIUM.

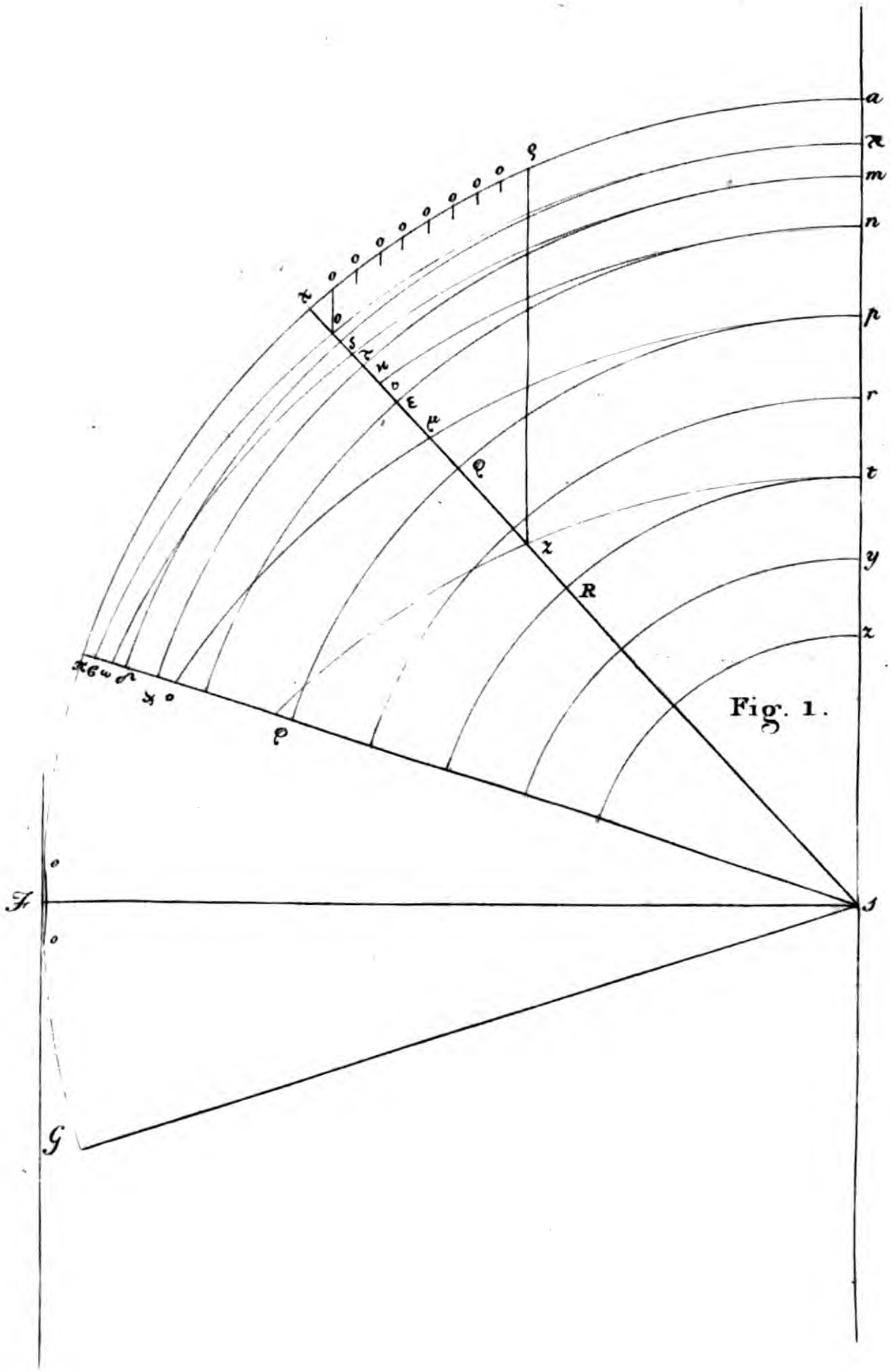
THE Quadratix and Spiral, it is well known, have been severally applied, by very able Mathematicians, to a Solution of the above Proposition; however, not with sufficient Success to stop all future Attempts; the Construction of each depending on the

continual

continual Biſection of an Arch, which although very ſimple in Theory, yet when continued beyond a certain Degree of Minuteneſs is obnoxious to the greateſt Errors in Practice; not to mention that after all, the Rectification of the Circle cannot be geometrically obtained by theſe Inventions, and that in the Opinion of the beſt Judges they are mechanical Curves. Perhaps when the Errors ariſing from ſuch a Conſtruction are fairly eſtimated, the preceding Method will appear no contemptible Subſtitute for this Purpoſe; ſince if the Principles on which it is founded are once admitted, the Solution will be natural and eaſy, and at the ſame time conſiſtent with the greateſt Rigor of Mathematical Demonſtration.

It likewiſe ſeems probable, that the Rectification of other Curves may be obtained by a ſimilar Method of Reaſoning, as the Lemma at the beginning of this Tract extends itſelf to all kind of Quantities whatever:—But a Critical Examination of this Point would not only exceed the Limits, but likewiſe be foreign from the original Deſign of this Work.

THE END.



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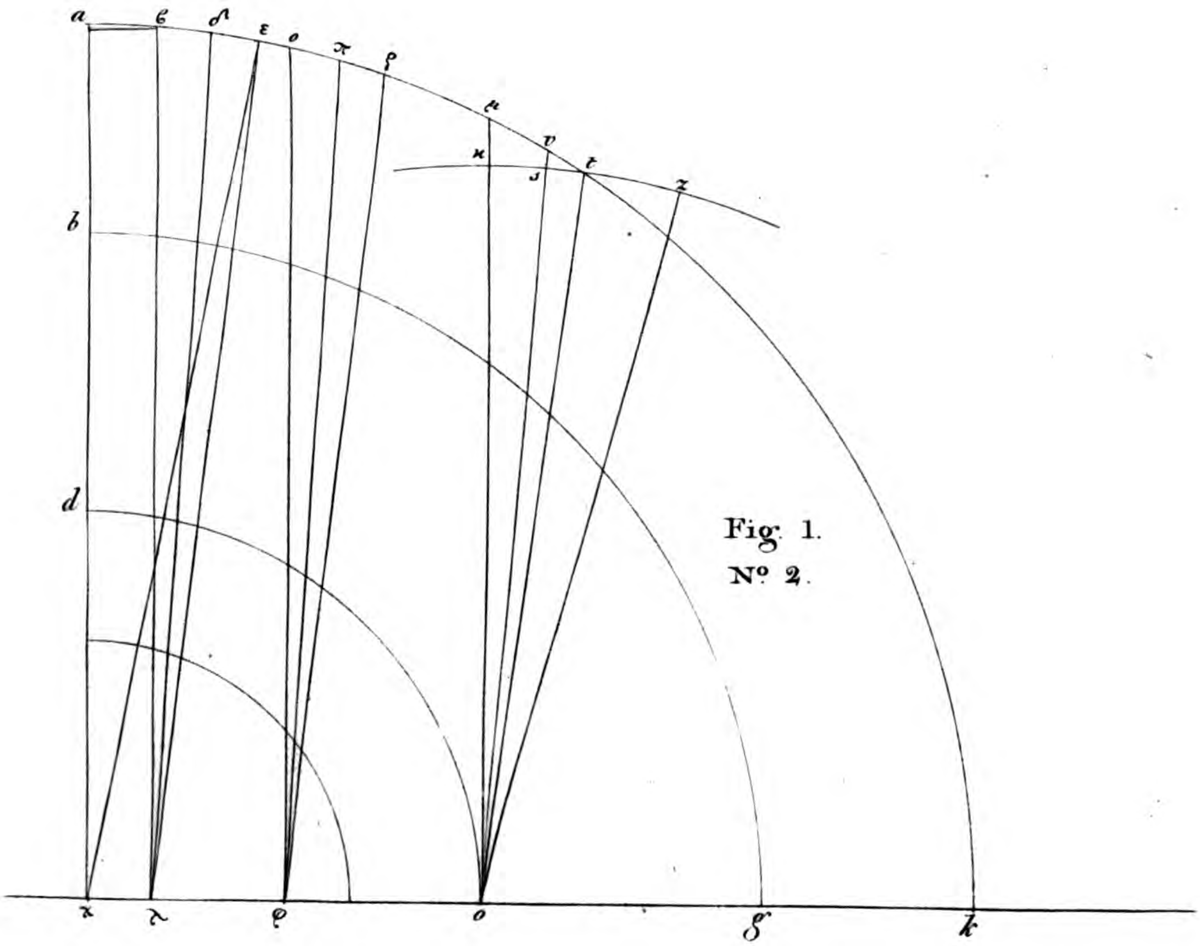

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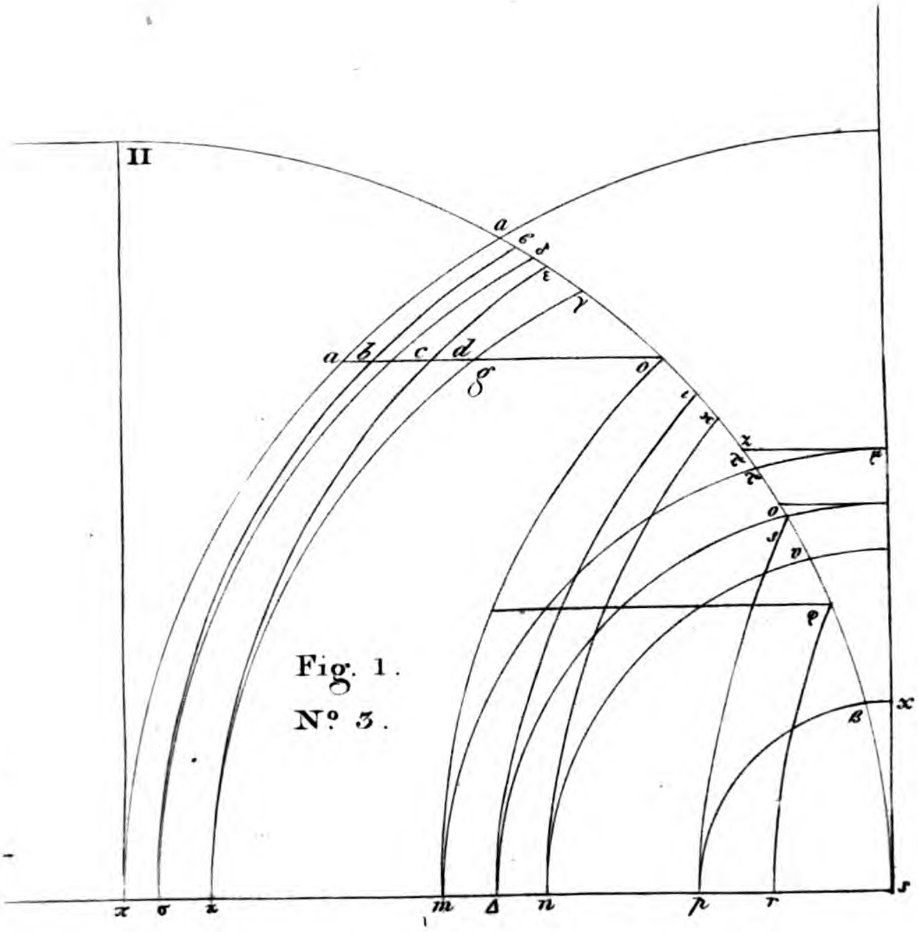

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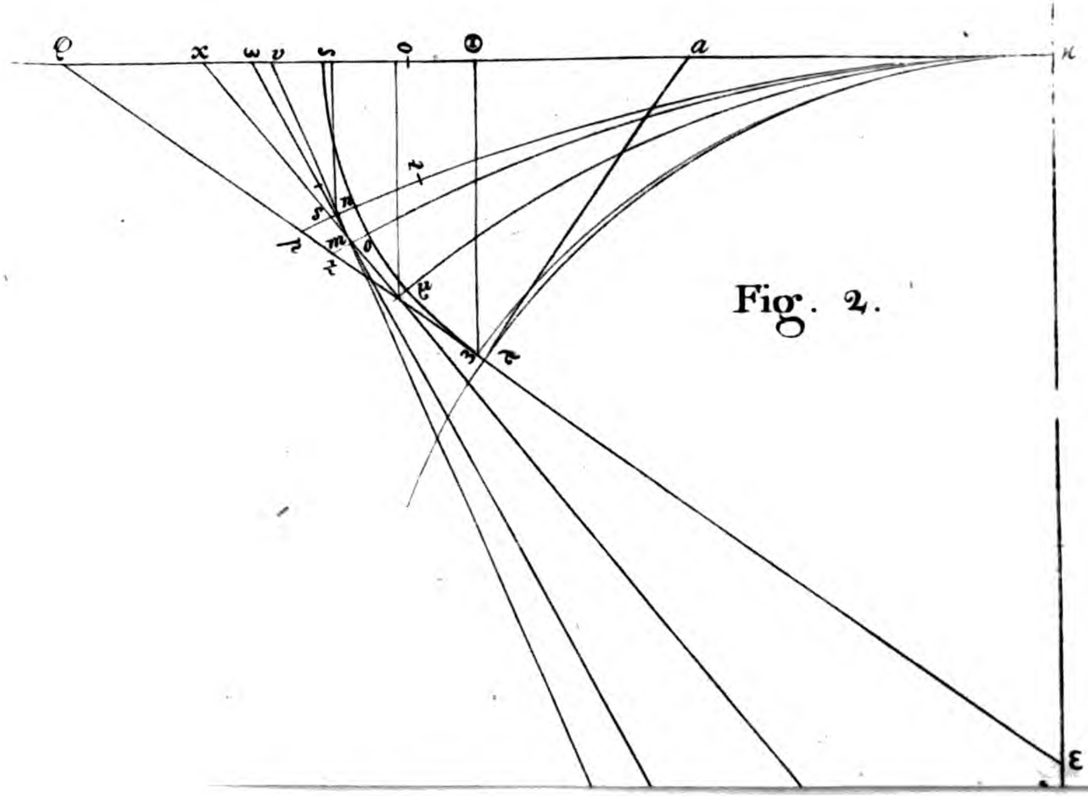

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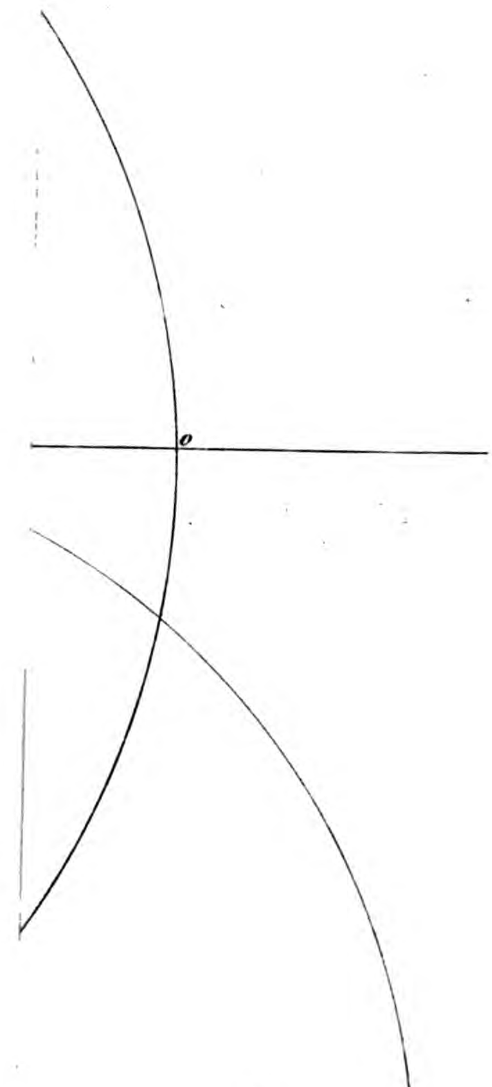
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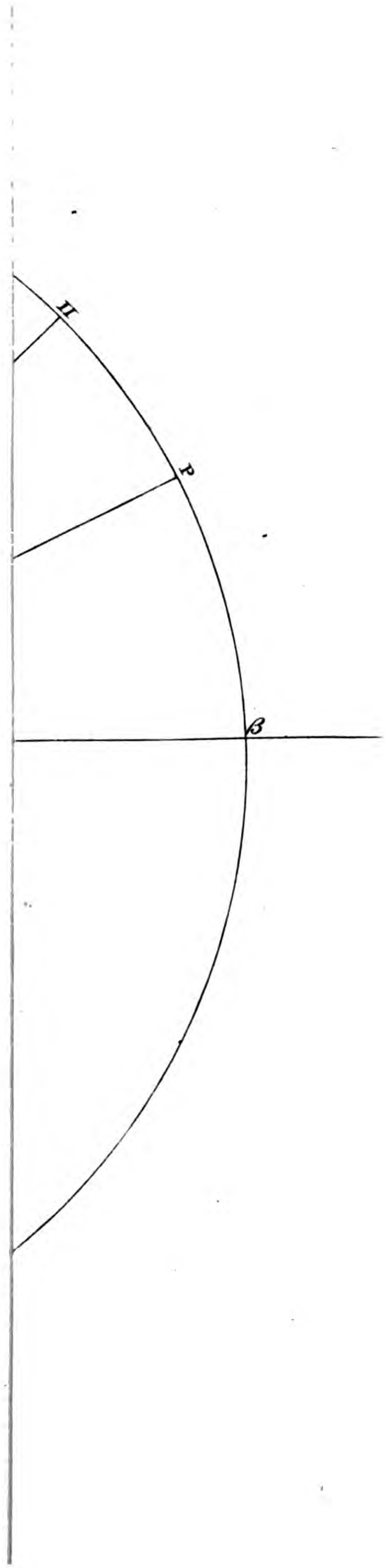

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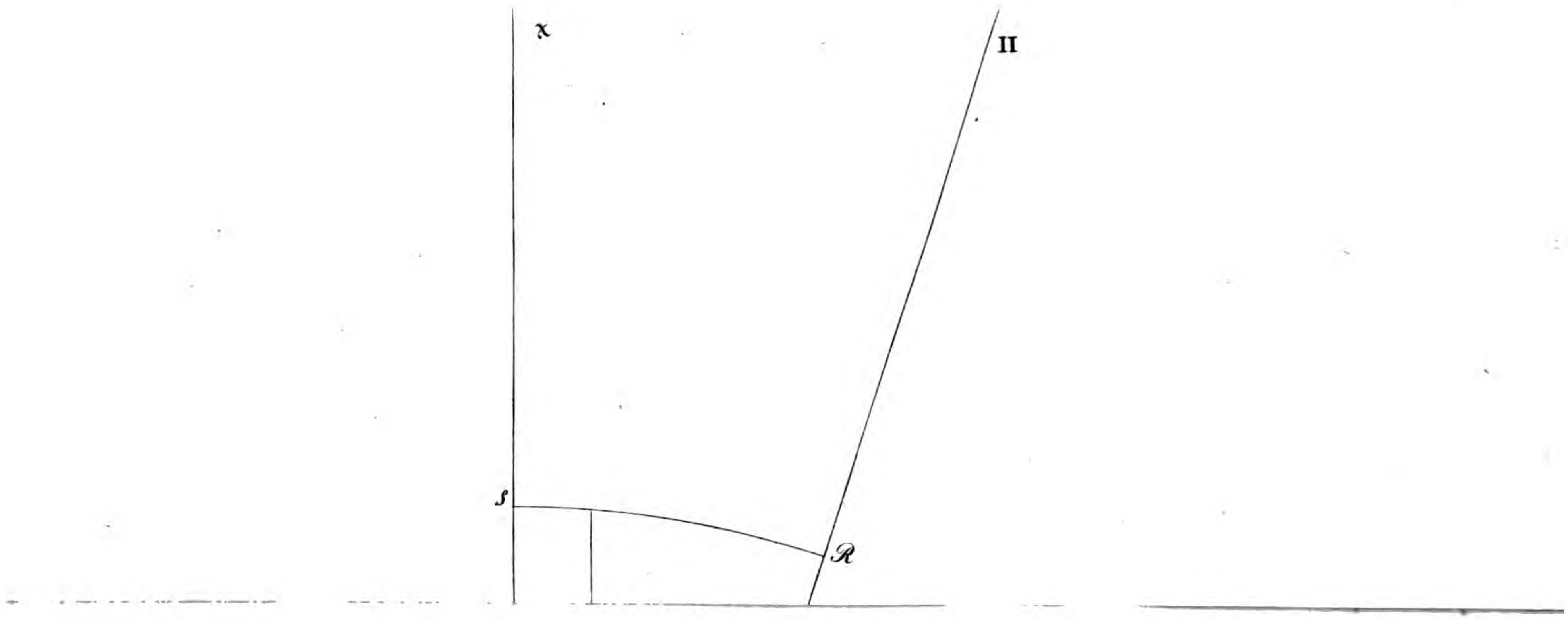
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