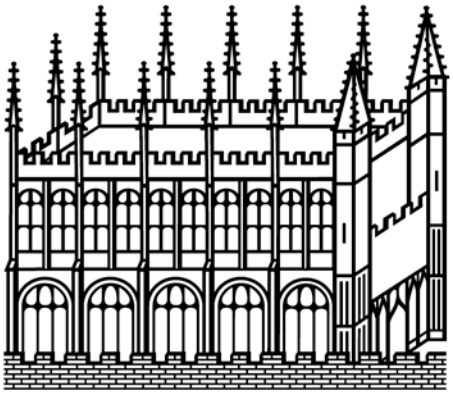

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IN

THEORY AND PRACTICE.

BY

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SCIENCE AND ART COLLEGE, PLYMOUTH.
AUTHOR OF "STEAM AND THE STEAM ENGINE," "NAUTICAL ASTRONOMY, ETC. ETC.

LONDON AND GLASGOW:
WILLIAM COLLINS, SONS, & COMPANY.
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184. g. 45.

PREFACE.

THIS work is intended to give the student a clear insight into the theory and practice of Navigation. It has been the aim of the writer to make the subject as easy, practical, and perspicuous as possible, by presenting the definitions, illustrations, etc., in every variety of aspect.

The study of Navigation, after long neglect, is now beginning to constitute an important part in the education of many of our youths; thanks to the action of the Science and Art Department, and the increased energy of Science Teachers.

The author has keenly felt the want of such a work which he here endeavours to supply, and, at the same time, is desirous to aid the Publishers in producing a set of Text-Books, which places the study of Science within easy reach of every Student in the Kingdom.

The figures necessary to the solution of the following problems would fill a space equal to five or six such volumes, so that with the greatest care, errors may have escaped the author's notice; in which case, he will be greatly obliged to his fellow teachers or others who detect such, to inform him of the same.

H. E.

SCIENCE AND ART COLLEGE, PLYMOUTH,
December, 1874.

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NAVIGATION.

CHAPTER I.

DEFINITIONS AND PRELIMINARY ILLUSTRATIONS.

Definition—Mechanical and Theoretical Navigation—Celo-Navigation—The Earth is a Sphere—Diameter—Poles—Equator—Latitude and Longitude—The Circle—Sphere—Use of Latitude and Longitude—Examples—Longitude in time and arc.

To the boldness of her sailors, to their indomitable perseverance and daring courage, Britain owes much of her supremacy. Doubtless this daring is due to the almost intuitive knowledge of nautical affairs possessed by the Briton.

The theory and practice of navigation, which enables the mariner to understand the mathematical and scientific part of his profession, is a study of great interest and importance.

Before the mariner can enter upon the study of this subject, it is necessary that he should be acquainted with the form of the earth, the lines of the sphere, with their relation to each other, the situation of the continents, with the different countries in each, and the principal seaports, with the direction in which they lie from any given place.

1. Navigation is the art of conducting a ship from one port to another. It is frequently divided into two parts.

(1.) *Mechanical, Practical, or Working Navigation,* which comprehends the art of working, managing, or sailing a ship, so that it shall move in any assigned direction.

(2.) *Theoretical Navigation,* which is the art or science of discovering in what particular direction a vessel must be steered to reach any given place. It relates to the various methods by which the navigator discovers the track on which

he must sail, in what direction to guide his ship when in or out of sight of land, and is that division of navigation which is now to engage our special attention.

The sailor calls to his aid two mathematical sciences, and employs them to assist him in directing his vessel. (*a*) Navigation proper, *Geo-Navigation*, which is the application of our knowledge of the sphere on which we live, to assist him on his voyage.

(*b.*) Nautical Astronomy, or *Celo-Navigation*, which is the application of our knowledge of the celestial sphere to the purposes of the sailor; or it is the application of astronomical facts and deductions to nautical purposes.

In the majority of cases the mariner assumes that the earth is a plane flat surface, hence many of the rules here laid down are based upon that assumption. Every seaman, before going to sea, must be furnished (1) with a correct table of the latitude and longitude of every important seaport, island, rock, shoal, etc. (2) With charts of the seas to be visited, on which are marked the variation of the compass, the correct relative position of places, the depths of the water, etc. (3) With a good set of apparatus, consisting of log, compass, sextant, etc.

2. The Earth is a Sphere.—No simpler illustration of the sphericity of the earth can be given than the appearance of a ship as it approaches and leaves the shore. When a ship sets sail, if a spectator on the sea shore, close to the water's edge, watch it as it recedes from the land, he will perceive that the *large* hull first disappears from view, and not the smaller masts and sails, as should be the case by the laws of optics. Hence, if it be "hull down" first, something must intervene between the spectator and the vessel. This something is the *roundness of the earth.* After the hull has disappeared, next the lower sails are lost to view; then the upper sails and top mast. On the contrary, as the vessel nears the shore her sails first come in sight, and the hull last. The sailor ascends the mast to catch his first sight of land; by so doing, he is enabled to look over the *hill of waters* intervening between the ship and the land.

Vessels, by keeping continually on an eastern or western course, have returned to the same port. For instance, a

vessel may leave Melbourne, in Victoria, sail E. to Cape Horn, next E. to Cape Town, and then E. towards Kerguelen Island, and lastly, E. to Melbourne, the place from which the ship started. These are but two of the many proofs that might be given of the earth's sphericity.

But it is preferred to give an extract from Mr. Plummer's excellent little work called *Introduction to Astronomy.**

"That its surface is curved, and its whole figure more or less spherical, appears from the following considerations:—(1.) When a ship is leaving port, the mariner sees the erections upon the shore gradually sink below the sea-offing or horizon, the ground and lower stories disappearing first, until the whole is hid. Should he now climb to the mast-head, he will see the whole reappear in the opposite order. The simple effect of distance, however, is to diminish the size and the brightness of objects, in no way altering their forms. It is, therefore, only to be explained by the fact, that the surface of the ocean is curved, and rises up between the ship and the shore. From the height of the mast it is possible to

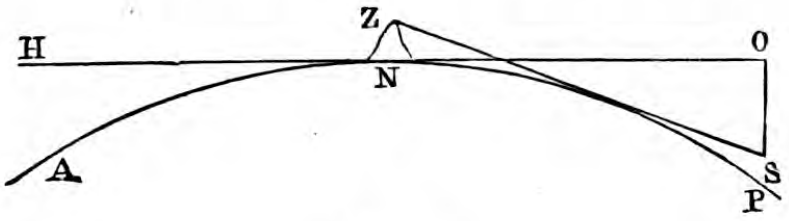


Fig. 1.

look over this curvature; yet, as the distance becomes still greater, the buildings on the coast will sink below the visible horizon, notwithstanding the altitude of the spectator. While the view of an observer at the sea level is limited by a plane passing through the eye parallel to the surface of smooth water, one situated at a height, as on the deck or mast of a ship, is able to see somewhat below this. The depression of the sea-offing is known as the **Dip of the Horizon**, and forms a constant correction for all observations made at sea. Thus, if A N P represent a portion of the earth, the view of an observer at N is bounded by the line H N O; while an observer on a mountain at Z would see all the first observer

* William Collins, Sons, & Co., London and Glasgow, 1873.

could, and in addition that portion of the heavens between O and S—the arc S O, measured on the celestial sphere, being the dip of the horizon at the altitude Z N.

"(2.) A similar proof of the earth's roundness is found in a phenomenon that has been sometimes observed by aëronauts, when ascending very soon after sunset. As they rise rapidly in the air, the sun comes again into view, rising gradually on their western horizon, while the earth below them is in profound shadow. If the earth were a flat plane, it is clear that even a slight depression of the sun below the horizon would cast in shadow the highest altitudes which the aëronaut could reach.

"(3.) If the level of a surveyor be employed upon the surface of the still water of a canal, it will be found that at the distance of one mile the water is depressed below the level of the instrument about eight inches. This affords not only a proof of the earth's form, but some idea of its size. A globe will require to have a diameter of about 7920 miles to have a curvature equal to that observed on the canal, and this may be taken as a very fair approximation to the diameter of the earth.

"(4.) Beyond this there is the evidence of lunar eclipses, which phenomena are palpably caused by the shadow of the earth falling upon the full moon. The outline of the shadow is *always* circular, and no other figure than a sphere will cast a circular shadow in whatever direction the light may fall upon it.

"(5.) There is the not unimportant fact of the circumnavigation of the globe in every direction which the configuration of the land will allow."

To obtain more *accurate* information of the earth's form and dimensions, it is necessary to determine the distance between two remote points on its surface which are upon the same meridian, or due north and south of each other, with the greatest possible care, and then by celestial observation to learn what fractional part of the earth's circumference has actually been measured—an operation of the greatest difficulty in practice, but achieved in the following manner:—A portion of country is selected where the ground is very nearly level, and a base line is marked out. Bars of metal are prepared

as measuring rods with every care, and these are placed end to end along the base line, accurately levelled. They are not allowed to touch, lest some accident or expansion of the bars should cause them to jolt one against another, but the intervals are measured by microscopes, and the temperature of the bars observed, in order that their expansion from heat may be calculable; they are also protected from the rays of the sun by tents when being used. This process is repeated, again and again, along the line with the same rods, till a distance of a few miles is measured with great accuracy, the two extremities being rigorously marked. A third station is now taken into account, generally selected upon the summit of a conspicuous hill, and its bearing is found by means of theodolites stationed at either extremity of the base line. It will be seen that this line and the lines joining each extremity of it to the third station will form a triangle, of which the length of one side is known, and also the amplitude of two of the angles. It becomes then a very simple problem in trigonometry to calculate the lengths of the remaining sides.* Other stations are now chosen, and the newly-found distances of the third station are in their turn taken as the bases of other larger triangles to be treated in the same manner. A net-work of triangles is thus formed, spreading over the country in a northern and southern direction, till the exact distance between two remote stations upon the same meridian of longitude is ascertained. In Britain, **trigonometrical surveys** in this manner have been several times performed, the object being the distance in feet between a point on Shanklin Down, Isle of Wight, and one on the island of Balta, in Shetland. This is called the great meridional arc of England; and several base lines have been measured at different times for its verification—notably at Lough Foyle in Ireland—for it should be remarked that very large triangles are often chosen, and a considerable arm of the sea is passed

* Those unacquainted with the principles of the mathematical solution of triangles will obtain some aid in understanding the above, by drawing triangles with the aid of compasses, etc., of which the base and two angles are given quantities. It will at once be seen that these data determine the magnitude of the triangles, and that the trigonometrical solution is nothing more than a reduction to calculation of a simple geometrical problem.

over without difficulty. We have now to find what fractional part of the earth's circumference has been measured. The direction of the plumb-line is not parallel to itself at different places, but is always towards the earth's centre. It may therefore be considered as a prolongation of the earth's radius at any point. Suppose that at one extremity of the meridional arc it points to a particular star, and at the other extremity it is found to point 10° to the south of that star, then the angle contained between the radii of the earth at these points is 10°, or $\frac{1}{36}$ part of the earth's circumference. An instrument, called a zenith sector, is used to ascertain the direction of the plumb-line, in space or with reference to the stars, at the two stations. In this manner the length of 1° of latitude is found to be about 69·1 miles; and hence the whole circumference will equal 24,876 miles, and the diameter 7918 miles, roughly.

"The importance of the problem has led to similar measurements being made in various parts of the world. The most celebrated arcs are those of Russia, France, England, India, Peru, and the Cape of Good Hope—places differing greatly in latitude—which brings to light this important fact. The length of a degree of latitude in feet always decreases slightly as the equator is reached, showing that the curvature there is greatest, or, in other words, that the earth is not a perfect sphere, but slightly flattened at the poles. Its figure is therefore that of an **oblate spheroid.** The numerous results have been thoroughly investigated by the accomplished mathematicians, Airy and Bessel. They agree in giving the following as the true dimensions of the earth:—

Polar diameter, ...	7899·1 miles
Equatorial diameter, . . .	7925·6 ,,
Difference or polar compression, . .	26·5 ,,
Proportion of diameters, . . .	298 to 299

"These results are further confirmed by similar trigonometrical surveys, carried in an east and west direction, the difference of longitude of the extreme stations being determined by methods which admit of ready explanation. It is only necessary to state that similar dimensions and a like compression is found by these operations."

3. Definitions connected with the Sphere.—A sphere is a

solid, bounded by a surface, every point of which is equally distant from a fixed point within it, this fixed point is named the centre of the sphere.

The **diameter** is a straight line passing through the centre, and terminated at both ends by the surface. The greatest diameter of the earth is 7925·6 miles.

The earth is not exactly a sphere, but is flattened at the top and bottom, so much so that its diameter in that direction is $7899\frac{1}{10}$, or nearly 27 miles less. Reckoning this ellipticity as $26\frac{1}{2}$ miles to 7926, it is about $\frac{26\frac{1}{2}}{7926} = \frac{1}{298}$ of the whole less a difference inappreciable on the largest globe that could be made.

The **axis** of the earth is an imaginary line passing through the centre from north to south. It is this axis or polar diameter that is $26\frac{1}{2}$ miles shorter than the equatorial diameter.

Poles.—The ends of the axis are the poles.

The earth has two motions: (1) the one round the sun in the course of a year, causing the seasons, called its *annual* motion; (2) the other round its own axis every twenty-four hours, causing day and night, termed its diurnal motion.

Equator.—The equator is a line drawn round the middle of the earth at equal distance from the poles. The circumference of the earth is 25,000 miles in round numbers. Its greatest circumference is $7925{\cdot}6 \times 3{\cdot}14159 = 24{,}898{\cdot}671$ miles.

The **Equinoctial** is the great circle in the heavens, drawn where the plane of the equator would cut the celestial sphere if produced.

Latitude is the distance of a place north or south of the equator reckoned in degrees, it is measured as an angle; or latitude is the complement of the angle subtended by two lines, one drawn from the pole, the other from the spectator, to the centre of the earth. The greatest latitude a place can have is 90°, one pole being 90° N. from the equator, the other 90° S.

Longitude is the distance of a place east or west from Greenwich. Like latitude, it is an angular measurement reckoned in degrees. Or longitude is the angle at the pole enclosed by the meridian of Greenwich and that of the

observer. The English assume that the first meridian runs through the Royal Observatory at Greenwich, other nations do not always follow their example.

Parallels of Latitude and Meridians.—The lines drawn on the sphere to show latitude running east and west are called *parallels of latitude*, because they are all parallel to the equator. The lines marking longitude on the sphere run north and south, and all pass through both poles. They are called *meridians of longitude*, and are all *great circles;* for planes cutting the earth where they pass would divide the earth into two equal parts. Latitude is marked on the meridians, longitude on the parallels of latitude, or on the equator.

4. Division of the Circle of the Sphere.—All circles are divided into 360°; hence, as circles can be drawn of any size, the *length* of a degree on one circle does not coincide with that on another, they vary in length according to the radii of the circles. On the sphere, the greatest latitude that a place can have is 90°, because the equator divides every meridian into two equal parts, and each meridian passes through the two poles; hence they are all divided into four parts, each containing $\frac{360°}{4} = 90°$. Latitude is reckoned or marked on the meridians, from the equator to the poles is 90°, the greatest latitude a place can therefore have is 90°, and there are only two places in this extreme latitude, the N. and S. poles.

The *greatest longitude* a place can have is 180°, or no two places on the earth's surface can be more than 180° asunder. Longitude is marked on the equator and parallels, and is reckoned from the first meridian. If two persons were to start from Greenwich to go half round the world, were it possible to travel due east and west, they would meet at 180° E. and 180° W. of Greenwich, which is the same meridian, each having performed a journey of 180°. As soon as each passes the other they go from E. to W., or from W. to E. longitude respectively. The greatest longitude that a place can have is therefore 180°; for if, as hinted above, the one who travelled E. passes the other, he will be in west longitude, and the one who travelled west in east longitude; this gives a meaning to the common expression

used by nautical men, "too far east is west, or too far west is east."

5. Great and Small Circles.—A *great circle* is one that divides a sphere into two equal parts or hemispheres. All meridians are great circles; the equator is the only circle of latitude that is a great circle, all the parallels of latitude are *small circles;* if we notice them on a globe it is seen that the parallels of latitude become smaller and smaller the nearer they approach the poles. The chief of the small circles or parallels are the two tropics and the polar circles.

The tropic of Cancer at 23°½ north of the equator.
,, Capricorn ,, south ,,
The Arctic circle at 23°½ from the north pole.
,, Antarctic ,, ,, south ,,

It will be observed that the two latter are 66°½ from the equator, north and south respectively.

Latitude from is the latitude of the place from which the ship sails.

Latitude in is the latitude of the place at which the ship has arrived.

Difference of Latitude is the difference between "the latitude from" and "the latitude in."

Middle or Mid-Latitude is the latitude of the place midway between "the latitude from" and "latitude in."

Co-mid-latitude is the distance of *mid-latitude* from the nearest pole.

Longitude from is the longitude of the place from which the ship sailed.

Longitude in is the longitude of the place at which the ship has arrived.

Difference of Longitude is the difference between the longitude from and the longitude in.

6. Use of Latitude and Longitude.—The use of latitude and longitude is to determine the precise position of a place on the earth's surface.

Let it be supposed that this small sphere represents the earth; W E the equator; N S the first meridian, or a line passing through Greenwich and the poles; N A D S a meridian drawn through all places 35° E.; A B a parallel of

latitude drawn through all places 20° N., and C D the parallel at 40° S.

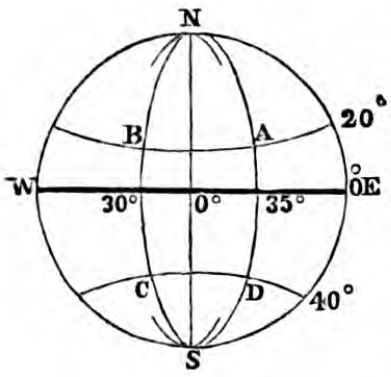


Fig. 2.

The point A is evidently 20° N., so is every point on the same line, hence, if this be the only condition, the point A is indefinite; but if we take the second supposition concerning A, that it is 35° to the east of the line N S, it is seen at once that A is determined, for there can be but one point that satisfies the condition of being 20° N. of W E, and 35° E. of N S, and that is A.

Again, the point B is the only spot on the sphere that can be 20° N. of the equator W E, and 30° E. of the first meridian N S. The difference between points D and A may be realised thus:—

A is 20° N. latitude and 35° E. longitude.
D is 40 S. ,, ,, ,,

Between C and D the difference may be thus exemplified—

C is 40° S. latitude and 30° W. longitude.
D is 40 S. ,, 35 E.

Between A and C the difference is more marked—

A is 20° N. latitude and 35° E. longitude.
C is 40 S. ,, 30 W. ,,

The same facts may be simply and expeditiously illustrated on the black board or slate, by drawing a horizontal line, and a second line across it and at right angles to it, then asking the student to find a point 1, 2, or more inches above or below the horizontal line, and 2, 3, or 1 to the right of left of the perpendicular line. Point out definitely and distinctly, that in any case *there is only one point that answers to the same two conditions.*

We will conclude this chapter by examples and illustrations, showing how to find the differences of latitude and longitude, the distances of places from the equator, mid-latitude and co-mid-latitude, with the method of converting degrees into miles, and miles into degrees.

7. To Convert Miles into Degrees.—Some of the answers

in the ensuing pages will be in miles; these frequently will have to be converted into arc, degrees, minutes, etc. The process is very simple. Let it be required to convert 2741 miles into degrees.

6,0)274,1′
45° 41′ *Ans.*

Convert 5701·3 miles into arc.

6,0)570,1·3
95° 1′·3
60
18·0
Ans. 95° 1′ 18″

The following simple rule is derived from the preceding process.

To convert miles into degrees or arc.—Divide the whole number by 60, and multiply the decimal place by the same, omitting one decimal place in the latter.

1. Express 17·8; 481·5; 2172·2; and 951·25 miles as arc.
Ans. 17′ 48″; 8° 1′ 30″; 36° 12′ 12″; 15° 51′ 15″.

8. To Convert Degrees into Miles.—The usual method is to multiply the degrees by 60, adding in the minutes, and to turn the seconds into decimals.

2. Cape Cornwall is in latitude 50° 7′ 42″ N. How many nautical miles is it from the equator?

50° 7′ 42″ 60)42″
60 ·7
Ans. 3007·7 miles N.

3. Europa Point is in latitude 36° 6′ 45″ N.; find its distance from the equator in nautical miles.

36° 6′ 45″ 60)45″
60 ·75
Ans. 2166·75 miles N.

4. Reduce 154° 14′ 17″ to minutes.

154° 14′ 17″ 60)17
60 ·283̇
Ans. 9254·283̇ miles.

We may observe that one decimal place, and that the

nearest, is sufficient. Sailors are constantly in the habit of calling minutes miles; thus 45° 18′ they read forty-five degrees eighteen miles, instead of eighteen minutes. When latitude is converted into minutes, we may consider that as the distance of any given place in nautical miles from the equator; but when longitude is converted into minutes, it gives but a faint idea how far the places are asunder, or east or west of Greenwich—in miles.

5. Express 75° 4′ 18″; 125° 14′ 10″; 49° 7′ 12″; 170° 17′ 24″; 10° 18′ 25″; 59° 16′ 3″; and 17′ 18″ in miles.

Ans. 4504·3; 7514·16; 2947·2; 10217·4; 618·416; 3556·05; and 17·3.

Next will be solved a general question set in the Science Examination Papers, May 1873, and then we proceed to give other illustrations, with the necessary rules for solving the examples under consideration.

6. (*a.*) Define meridian of the earth, equator, parallel of latitude. Which of these are great circles, and why?

(*b.*) What is the difference of latitude between two places, one of which is on the tropic of Cancer, the other on the tropic of Capricorn?

(*c.*) What is the difference of longitude between Ferro Isles (lon. 17° 58′ W.) and St. Petersburg (lon. 30° 10′ E.)?

For (*b*) latitude of tropic of Cancer is 23° 30′ N.
,, ,, Capricorn ,, 23 30 S.
∴ Difference of latitude ,, 47 0

Cancer being 47° N. of Capricorn, or Capricorn 47° S. of Cancer.

For (*c*) longitude of Ferro Isles is 17° 58′ W.
,, St. Petersburg ,, 30 10 E.
∴ Difference of longitude ,, 48 8

Ferro being 48° 8′ W. of St. Petersburg, or St. Petersburg is situated 48° 8′ to the east of the Ferro Islands.

7. The latitude of A is 45° 15′ 51″ N.; of B 20° 10′ 14″ N.; find the difference of latitude.

Latitude of A is 45° 15′ 51″ N.
,, B is 20 10 14 N.
Difference of latitude is 25 5 37 A is north of B.

8. The latitude of a place A is 20° 14′ 11″ N.; of another, B, it is 19° 14′ 16″ S.; find how far apart these two parallels are:—

Latitude of A is 20° 14′ 11″ N.
,, B is 19 14 16 S.
Difference of latitude is 39 28 27 B is south of A.

These are the only two cases that can occur in finding the difference of latitude. *When the two places have the same name, both north or both south, subtract to find the difference of latitude; but when one is north and the other south, add to find their difference of latitude.*

The teacher must illustrate these points by drawing three lines on the board, one for the equator and two for the parallels of latitude, when it is seen in a moment whether to add or subtract to find the difference of latitude or the distance apart of the two places.

There are three different cases in finding the difference of longitude: (1) when both are east or both west; (2) when one is east and the other west; (3) when the sum in the second case is greater than 180°.

9. Find the difference of longitude between A in longitude 45° 17′ 18″ E., and B in longitude 74° 20′ 48″ E.

Longitude of A is 45° 17′ 18″ E.
,, B is 74 20 48 E.
Difference of longitude is 29 3 30 A being west of B.

10. Find the difference of longitude between two places, one on the meridian 27° 18′ 45″ E. of Greenwich, the other on that 110° 14′ 15″ W. of Greenwich.

Longitude of A is 27° 18′ 45″ E.
,, B is 110 14 15 W.
Difference of longitude is 137 33 0 A being east of B.

11. Find the difference of longitude between A in longitude 121° 17′ 24″ E., and B in longitude 95° 16′ 18″ W.

Longitude of A is 121° 17′ 24″ E.
,, B is 95 16 18 W.
216 33 42
360 0 0
Difference of longitude is 143 26 18 A being west of B.

By a careful perusal of these three cases it is seen that, to find the difference of longitude, we must use the following rules:—

When the two longitudes have the same name, we must subtract; when they have different names, we must add; if, after we have added, the sum is greater than 180°, we subtract it from 360°, and this gives the difference of longitude.

The teacher should accustom the pupils to demonstrate

these facts by illustrations, simply drawing a horizontal line on the paper or slate to represent the equator, and a vertical line crossing it at right angles for the first meridian; he will then see that if the two longitudes represented by two other straight lines are both on the right or both on the left side of the first meridian, *i.e.*, both west or both east, he has to subtract to find their distance apart; while if they be one on each side of the first meridian, *i.e.*, one east and the other west, he must add to find the difference of longitude. The case in which the sum is greater than 180° is best illustrated by a circle, on which it must be shown that the distance apart of two places is the smaller arc of the circle, not the larger one.

In middle latitude sailing, the mid-latitude, or the parallel half way between any two given parallels, will have to be found; it will be better to show now how this is done by a few simple illustrations:—

12. Find the mid-latitude between Cape Cornwall, lat. 50° 8′ 0″ N., and Duncansby Head in latitude 58° 39′ 45″ N.

Latitude of Cape Cornwall,		50° 8′ 0″ N.
,, Duncansby Head,		58 39 45 N.
	2)	108 47 45
Mid-lat. between the two is		54 23 52½ N.

We might have taken one from the other, then half of the difference, and added it to the lower or least latitude, that would have given us the exact middle point between the two, and would have been the simplest way of reasoning; but it admits of an easy algebraical proof, that if b be greater than a—

$$\frac{a+b}{2} \text{ is the same } a+\frac{b-a}{2}$$

$\frac{a+b}{2}$ being the method we have employed.

$a+\frac{b-a}{2}$ the method referred to in the text above.

13. Find the middle latitude between Cape Lopez 0° 36′ 10″ S. and the Cape of Good Hope 34° 22′ 0″ S.

Latitude of Cape Lopez,.....	0° 36 10″ S.
,, Cape of Good Hope,	34 22 0 S.
	2)34 58 10
Mid-latitude is.....	17 29 5 S.

14. Find the mid-latitude between Gibraltar, in latitude 36°6′20″N., and the Cape of Good Hope.

Latitude of Gibraltar,.....	36° 6′ 20″ N.
,, Cape of Good Hope,	34 22 0 S.
	2) 1 44 20
Mid-latitude is.....	0 52 10 N.

15. Find the mid-latitude between Gibraltar and Cape Cornwall. *Ans.* 43° 7′ 10″ N.

16. Find the mid-latitude between Cape Lopez and Duncansby Head. *Ans.* 29° 1′ 47½″ N.

17. Find the mid-latitude between Singapore, latitude 1° 16′ N., and Kandy in Ceylon, latitude 7° 20′ N. *Ans.* 4° 18′ N.

18. In the latter case find the difference of latitude. *Ans.* 6° 4′.

19. Find the mid-latitude between Pekin, latitude 39° 54′ N., and Rio Janeiro, lat. 22° 54′ S. *Ans.* 8° 30′ N.

20. Find the mid-latitude between Plymouth and Manchester, when Plymouth is in latitude 50° 23′ N., Manchester 53° 23′ N. *Ans.* 51° 53′ N.

21. Find the mid-latitude between Lisbon, which is situated in latitude 38° 12′ N., and Bergen in Norway, latitude 60° 24′ N. *Ans.* 49° 18′ N.

22. In the last question find the difference of latitude. *Ans.* 22° 12′.

In finding longitude presently, it will be frequently necessary to find the *co-mid-latitude*, that is, *the distance of the middle latitude between two given places from the nearest pole.*

23. Required the co-mid-latitude between Cape Cornwall and Duncansby Head.

Latitude of Cape Cornwall is.....	50° 8′ 0″ N.
,, Duncansby Head,.....	58 39 45 N
	2)108 47 45
Mid-latitude,.....	54 23 52½
	90 0 0
Co-mid-latitude is	35 36 7½

Having found the mid-latitude as in previous questions, it is evident that the distance of this parallel from the pole is found by subtracting it from 90°, for as from the equator to

the poles there are 90°, and as the mid-latitude brings us a certain distance from the equator, the distance of this place from the pole is its defect from 90°.

24. Find the co-mid-latitude of Cape Lopez and the Cape of Good Hope.

Cape Lopez is in latitude	0° 36′ 10″ S.
Cape of Good Hope ,,	34 22 0 S.
	2)34 58 10
Mid-latitude is.....	17 29 5
	90 0 0
Co-mid-latitude is.....	72 30 55

25. Find the co-mid-latitude of Gibraltar and the Cape of Good Hope.

Gibraltar is in latitude,.....	36° 6′ 20″ N.
Cape of Good Hope ,,	34 22 0 S.
	2) 1 44 20
Mid-latitude is.....	0 52 10 N
	90 0 0
Co-mid-latitude.....	89 7 50

26. Find the co-mid-latitude between Pekin, latitude 39° 54′ N., and Rio Janeiro, latitude 22° 54′ S. *Ans.* 81° 30′.

27. Find the co-mid-latitude between Plymouth and Manchester, when Plymouth is in latitude 50° 23′ N., Manchester 53° 29′ N. *Ans.* 38° 7′.

28. Find the co-mid-latitude between Lisbon and Bergen in Norway, when Lisbon is situated in latitude 38° 12′ N., and Bergen in latitude 60° 24′ N. *Ans.* 40° 42′.

29. Required the difference of latitude and the difference of longitude between the two places, A and B, given (1863).

	Latitude.	Longitude.
A	31° 15′ 25″ N.	17° 30′ 27″ E.
B	45 18 17 N.	23 14 3 W.

Ans. Difference of latitude 14 2 52 B north of A.
Difference of longitude 40 44 30 B west of A.

30. Give definitions of the earth's axis, poles, meridians, and equator. How is the situation of a place on the earth's surface determined? What are the difference of latitude and the difference of longitude between A and B? (1869.)

Latitude of A 13° 18′ N.	Longitude 123° 12′ E.
Latitude of B 33 17 S.	Longitude 74 36 E.

Ans. Difference of latitude 46° 35′ B south of A.
Difference of longitude 48 36 B west of A.

31. Canton is in latitude 23° 7′ 10″ N., longitude 113° 14′ 0″ E.; Cape Mendocino is in latitude 40° 29′ 0″ N., longitude 124° 29′ 0″ W.;

find the difference of latitude and longitude between the two places, and the position of Canton in reference to Cape Mendocino.

Ans. Difference of lat. 17° 21′ 50″ Canton south of Mendocino.
Difference of long. 122 17 0 Canton west of Mendocino.

32. Find the difference of latitude and longitude between Valdivia and Otago, and the relative position of Valdivia to Otago. Given

Lat. of Valdivia 39° 50′ 7″ S. Long. 73° 34′ 35″ W.
,, Otago 45 46 28 S. ,, 170 36 45 E.

Ans. Difference of lat. 5° 56′ 21″ Valdivia north of Otago.
Difference of long. 115 48 40 ,, east of Otago.

33. What is the difference of latitude and longitude between the Cape of Good Hope and Cape Horn? Given

Cape Horn, lat. 55° 58′ 40″ S. Long. 67° 12′ 25″ W.
Cape of Good Hope, lat. 34 22 0 S. Long. 18 24 24 E.

Ans. Difference of lat. 21° 36′ 40″ Cape Horn south.
Difference of long. 85 36 49 Cape Horn west.

34. Sombrero Rock is in latitude 10°45′0″ N., longitude 121°38′0″ E.; the north point of Owhyhee is in latitude 20° 18′ 30″ N., long. 155° 58′ 0″ W.; find their distance apart in degrees, etc., of latitude and longitude.

Ans. Difference of latitude 9° 33′ 30″.
Difference of longitude 82 24 0.

9. To convert Longitude into time and time into Longitude.—It is often very useful to know the difference of time at various places. When a ship sails east, at the places she arrives the time is later in the day, and when west earlier, than at the meridian from which she sailed. For instance, a ship leaving Plymouth and arriving at St. Petersburgh, would find the time at St. Petersburg more than two hours later in the day; on the contrary, if a ship cross the Atlantic to New York, the time will be found five hours earlier.

To find the difference in time at various places is a very simple problem.

35. When it is noon at Greenwich, what time is it at Moscow, latitude 55° 45′ N., longitude 37° 37′ E?

Longitude of Moscow = 37° 37′ E.

$$6,0\overline{)15,0°\ 28'}\ 4$$

2h. 30m. 28sec.

It is 2h. 30m. 28 sec. after noon, or half past two o'clock at Moscow when it is twelve o'clock in London.

When we go east we meet the sun. As the earth turns

round from west to east, any place to the east of our meridian will see the sun first, or to people so situated the sun rises first, and so the day begins earlier; on the contrary, those who travel westward go away, as its were, from the sun, and alter their time accordingly.

In the sum worked out immediately above, the longitude is multiplied by 4 and divided by 60, which produces the same result as if the longitude had been divided by 15. The reason for the process is this: In the circumference of the earth there are 360°, and as the earth turns on its axis once in 24 hours, the 360° divided by 24 gives 15° as the number of degrees that must sweep over the sun in the course of 1 hour; therefore every 15° of longitude coincide with 1 hour of time. If the place be *east* it is *later* in the day, if west, earlier.

Take another example:

36. The meridian of 176° E. passes through New Zealand; what is the relation between the time in New Zealand and London (Greenwich)?

Longitude of New Zealand = 176° E.

4

6,0)70,4

11h. 44 m.,

or when it is noon at London it is 11h. 44m. after noon, or 16m. to 12 at night.

37. Suppose it to be 3 o'clock in the afternoon at Odessa, latitude 46° 28′ N., longitude 30° 44′ E.; what time is it at the Madeiras, latitude 33° 30′ N., longitude 16° 54′ west?

Longitude of Odessa, ...	30° 44′ E.
,, the Madeiras, ...	16 54 W.
Difference of Longitude, ...	47 38 W.

4

6,0)19,0 32

3h. 10m. 32sec.

	h.	m.	sec.	
Time at Odessa, ...	3	0	0	P.M.
Difference of time, ...	3	10	32	
Time at the Madeiras, ...	11	49	28	A.M.,

or it wants about 10 minutes to 12 in the morning.

Rules for Turning Longitude into Time.—1. The difference of longitude between Odessa and the Madeiras was found.

2. This difference of longitude was converted into time by multiplying by 4 and dividing by 60.

3. The difference of time was subtracted from the time at Odessa which gave the time at the Madeiras.

The reasons for these rules are the following:—

(1.) The time of any place depends upon its longitude, or the time at which the sun comes to the meridian of that place.

(2.) There are 15° to every hour of time, or the sun moves through 15° of the ecliptic in one hour. Therefore, if we divide the degrees by 15, they will be converted into hours; but it is a more easy plan to multiply by 4 and divide by 60, for $\frac{60}{4} = 15$ gives at once the reason for the rule.

(3.) When we know the time at one place and difference of longitude between it and another. We proceed thus:—

(*a.*) With difference of longitude and time at place given.

When the place is W., and the time in this W. longitude is known, to find the time at place in E. longitude *add the* difference of longitude in time.

When the place is in W. longitude, and the difference of longitude in time is known, to find the time at a place farther W. subtract the time, adding, when necessary, 12 hours if you want the *civil* time, 24 if the *astronomical.*

(*b.*) When the place is E., and the time in this E. longitude is known, to find the time at place in W. longitude subtract the difference of longitude in time, mentally, adding 12 or 24 when necessary, according to the preceding rule.

When the place is E., and the time is required farther E., add the longitude in time to the first place.

38. The difference of longitude between two places is 98° 41′ 36″; what is the difference in time? *Ans.* 6h. 34m. 46·4sec.

39. Convert the arc 68° 27′ 18″ into time. *Ans.* 4h. 33m. 49·2sec.

40. Given the difference of longitude between Plymouth and Bermuda 60° 30′ 24″; find the difference in time. *Ans.* 4h. 2m. 1·6sec.

41. Plymouth is in longitude 4° 7′ 16″ W., and Iquique Island in 70° 13′ W.; find the difference in time. *Ans.* 4h. 24m. 22·9sec.

42. One place is in lon. 125° 16′ 24″ E., another in 143° 18′ 36″ W.; find the difference in time between the two places.

Ans. 6h. 5m. 40sec.

10. To Convert Time into Arc.—This is the reverse of the

former process; or if we remember that 15° go to the hour, then 60 minutes divided by 15° equals 4, or every degree of longitude corresponds to *four* minutes. From these statements we obtain the following rule for converting time into arc.

Bring the time into minutes and seconds, then divide by 4, *and the answer will be in degrees, etc.*

43. You are required to convert 14h. 10m. 45sec. into arc.

h.	m.	sec.
14	10	45
	60	
4)850	45	

212° 41′ 15″ *Ans.*

44. The difference in time between Valparaiso and the Cape of Good Hope is 6h. 0m. 19·26sec.; what is the difference of longitude?

h.	m.	sec.
6	0	19·26
	60	
4)360	19	15·6

90° 4′ 48″·9 *Ans.*

45. What is the difference of longitude between two places when one is 4h. 10m. 12sec. east of Greenwich, and the other 5h. 7m. 3·5sec. west of the same meridian?

	h.	m.	sec.
Times	4	10	12
	5	7	3·5
	9	17	15·5
		60	
4)557		15	30

139° 18′ 52″·5 *Ans.*

46. When the clock shows 7h. 3m. 50 sec. in the morning at New York, it is 2h. 2m. 53·46sec. in the afternoon at Odessa; find the difference of longitude.

	h.	m.	sec.
Noon,	12	0	0
New York,	7	3	50
	4	56	10
Odessa,	2	2	53·46
	6	59	3·46
		60	
4)419		3	27·6

104° 45′ 51·9″ *Ans.*

47. You are requested to convert the arc 138° 17′ 0″ into time by the preceding method. *Ans.* 9h. 13m. 8sec.

48. The difference of longitude between two places is 73° 1′ 0″; what is the difference in time? *Ans.* 4h. 52m. 4sec.

49. Find the difference in time between two places when the difference of longitude is 155° 32′ 10″. *Ans.* 10h. 22m. 8·6sec.

50. When it is 2h. 10m. 15sec. in the afternoon at A, it is six o'clock in the afternoon at B; what is the difference of longitude? *Ans.* 57° 26′ 15″.

51. To what time does the arc 17° 18′ 41″ correspond? *Ans.* 1h. 9m. 14·75sec.

52. It is required that 11h. 18m. 19sec. of time be converted into arc; find the arc. *Ans.* 169° 34′ 45″.

53. When it is 10h. 30m. A.M. at a place in west longitude, it is 5h. 9m. 10sec. P.M. at one in east longitude; by what number of degrees of longitude are the places separated? *Ans.* 99° 47′ 30″.

54. Convert 9h. 11m. 17·3sec. into arc. *Ans.* 137° 49′ 19″·5.

55. What number of degrees, minutes, and seconds of arc correspond with 15h. 11m. 18sec.? *Ans.* 227° 49′ 30″.

56. It is six o'clock in the morning at Constantinople, in longitude 28° 59′ 2″ E.; what time is it at Cape Farewell, latitude 59° 49′ 12″ N. longitude 43° 53′ 40″ W.?

The whole difference of longitude is 28° 59′ 2″ E.
43 53 40 W.
72 52 42
4
6,0)29,1 30 48

Difference in time is..... 4h. 51m. 30·8sec.
Time at Constantinople,..... 6h. 0m. 0
,, Cape Farewell is..... 1h. 8m. 29·2sec.

Ans. 1h. 8m. 29·2sec. in the *morning*.

57. Considering the above question, find what time it is at Bombay in longitude 72° 54′ 24″ east.

Longitude of Constantinople,... 28° 59′ 2″ E.
Longitude of Bombay,..... 72 54 24 E.
Difference in longitude,..... 43 55 22
4
6,0)17,5 41 28

Difference in time,..... 2h. 55m. 41·4sec.
Time at Constantinople, ... 6h. 0m. 0
Time at Bombay, ... 8h. 55m. 41·4sec. in the morning.

Thus it is seen that, when it is one o'clock in the morning at Cape Farewell, it is six in the morning at Constantinople, and nearly nine at Bombay; while at Aurora Island, as shown by the next illustration, it is about three o'clock in the afternoon of the same day.

58. It is 8h. 55m. 41·4 sec. in the morning at Bombay, in longitude 72° 54′ 24″ E.; what time is it at Aurora Island in the New Hebrides, latitude 14° 53′ S., longitude 168° 16′ E.?

Longitude of Bombay, ...	72°	54′	24″	E.
Longitude of Aurora Island, ...	168	16	0	E.
Difference of longitude, ...	95	21	36	
			4	
6,0)38,1	26	24		
6h. 21m. 26·4sec.				

	h.	m.	sec.
Time at Bombay, ...	8	55	41·4
Difference in time, ...	6	21	26·4
Time at Aurora Island, ...	15	17	7·8

= 3h. 17m. 7·8sec. of the afternoon of the same day, reckoning the day as beginning at midnight, or the civil day.

Before concluding this branch of our subject, let a few hints be repeated as regards the time and place when one place is east, and the other west of Greenwich. If we imagine that, as the earth turns round from west to east, the places to the east come opposite the sun before those lying west, we shall realise all that is necessary. Bombay will turn towards the sun before Aden, Aden before Alexandria, Alexandria before Constantinople, Constantinople before Paris, Paris before New York, etc. Hence it is evident that if one place is east of another, at the place farthest east it is later in the day; while if another be to the west, there it is earlier in the day than at the station where the spectator is supposed to be standing. Bear this well in mind, and no difficulty will arise when we wish to apply the longitude in time to the time at place.

CHAPTER II.

THE COMPASS AND ITS DECLINATION.

Binnacle—Needle—Card and Bowl—Boxing the Compass—Table of Points and Quarter Points—Variation or Declination—Isogonic and Agonic Lines—Annual, Diurnal, and Secular Variation—Dip—Local Attraction—Deviation—Permanent and Sub-Permanent Magnetism—Vertical and Horizontal Induction—How Deviation is ascertained—Reciprocal Bearings—Deviation Table—Examples to find Deviation of Half and Quarter Points—Leeway—How to allow for Leeway—Examples and Illustrations to find True Course and Compass Course—Azimuth Compass.

THE mariners compass is employed to enable the sailor to guide the course of the ship when out of sight of land, and to show him in what direction he is steering his vessel.

11. Binnacle.—On board ship the compass is enclosed in a square box which is placed inside a larger box, called the *binnacle.* The binnacle is fixed on to the after part of the deck, close to the helmsman. To keep the compass at all times perfectly upright, it is supported on *gimbals,* which consist of two concentric rings, each being sustained on an axis at right angles to the other.

12. The Needle.—The essential part of every compass is a well-magnetised needle, freely balanced upon a fine agate point. The form of the needle varies, but a good needle is not too fine at the ends, and of medium size. Its magnetism is permanent, rendered so by being made of well-hardened homogeneous steel. It should be powerful enough to lift, by its own magnetism, a compass card and needle of its own size. The earth is a magnet, in consequence of this all bars or pieces of soft cast or malleable iron, that have an elongated form, become inductive magnets when placed so that the direction of their length is more or less parallel to the magnetic meridian. When perfectly parallel to the magnetic meridian, the more intense the magnetism.

13. The Card and Bowl.—The needle is fixed to a circular card of thin uniform mica covered with paper, upon which are marked the thirty-two points of the compass. The names of these points must be learnt by every seaman or person studying navigation. To be able to repeat them consecutively is called "Boxing the Compass." The instant a point is mentioned the learner must be able to tell how far it is to the right or left of N. or S. The system upon which the points are named is an ingenious but simple one, and immediately discovered by inspection. It may be illustrated and explained thus: Draw two lines at right angles to each other, and mark the ends respectively, N. S. E. W. Next between these two lines draw two lines equidistant from the first two,

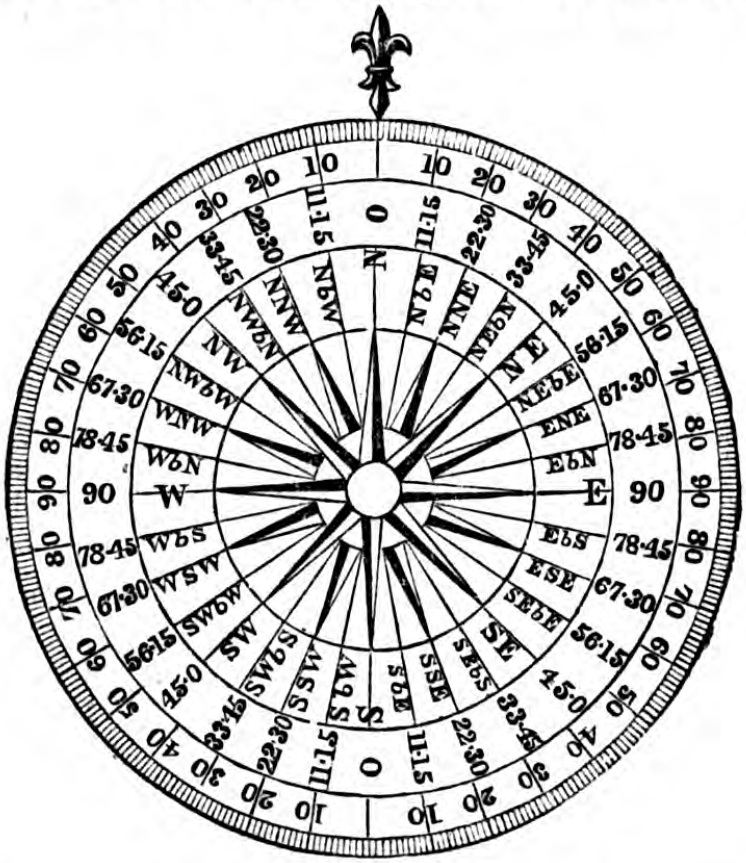


Fig. 3.

and at right angles to each other. Take the letters from the two ends of the lines between which any one lies, and attach them to the ends of the second pair of lines, and we have N.W., S.W., S.E., N.E. Now draw four lines, one between each pair that we have already, and as before, take the letters

from the ends of the nearer lines and *join them*, placing them at the ends of the last lines to give N.N.E., N.N.W., E.N.E., S.S.E., etc. We have again to draw lines between, and our last divisions are obtained; we now use the word *by*, as N. by W. means one point from the N. towards the W.; N.W. by N. means one point from the N.W. towards the N.; W. by N. means one point from the W. towards the N., etc.

TABLE WITH EVERY POINT AND QUARTER POINT OF THE COMPASS,
Giving the Angle with the Meridian.

North.	South.	Points.	°	′	″	Points.	South.	North.
		0 ¼	2	48	45	0 ¼		
		0 ½	5	37	30	0 ½		
		0 ¾	8	26	15	0 ¾		
N. by E.	S. by W.	1	11	15	0	1	S. by E.	N. by W.
		1 ¼	14	3	45	1 ¼		
		1 ½	16	52	30	1 ½		
		1 ¾	19	41	15	1 ¾		
N.N.E.	S.S.W.	2	22	30	0	2	S.S.E.	N.N.W.
		2 ¼	25	18	45	2 ¼		
		2 ½	28	7	30	2 ½		
		2 ¾	30	56	15	2 ¾		
N.E. by N.	S.W. by S.	3	33	45	0	3	S.E by S.	N.W. by N.
		3 ¼	36	33	45	3 ¼		
		3 ½	39	22	30	3 ½		
		3 ¾	42	11	15	3 ¾		
N.E.	S.W.	4	45	0	0	4	S.E.	N.W.
		4 ¼	47	48	45	4 ¼		
		4 ½	50	37	30	4 ½		
		4 ¾	53	26	15	4 ¾		
N.E. by E.	S.W. by W.	5	56	15	0	5	S.E. by E.	N.W. by W.
		5 ¼	59	3	45	5 ¼		
		5 ½	61	52	30	5 ½		
		5 ¾	64	41	15	5 ¾		
E.N.E.	W.S.W.	6	67	30	0	6	E.S.E.	W.N.W.
		6 ¼	70	18	45	6 ¼		
		6 ½	73	7	30	6 ½		
		6 ¾	75	56	15	6 ¾		
E. by N.	W. by S.	7	78	45	0	7	E. by S.	W. by N.
		7 ¼	81	33	45	7 ¼		
		7 ½	84	22	30	7 ½		
		7 ¾	87	11	15	7 ¾		
East.	West.	8	90	0	0	8	East.	West.

There are also half and quarter points, N.$\frac{1}{2}$-E. means half a point from the N. towards the E. E.$\frac{3}{4}$-N. means three-quarters of a point from the E. towards the N.

Bowl.—The card and needle are placed within the *bowl.* Inside the bowl are drawn two black vertical lines, which, when the compass is properly fixed, are in a fore and aft line with the stem and stern post. These are the lubber's lines or points, and when we wish to ascertain the position of the ship's head, we must look to see to which point of the compass the line next to the bow of the vessel is nearest.

In the table on the preceding page the opposite points are placed in juxtaposition; for instance, N. by E. and S. by W. are the opposite points of the compass, 180° apart; so are N.N.E. and S.S.W., etc.

It was said above, "the instant a point is mentioned, the learner must be able to tell how far it is to the right or left of N. or S." The E. point is eight points to *right* of N., and eight to the left of S. To comprehend this statement, the reader must suppose himself to stand in the *middle* of the compass, and looking towards the N.; he will understand that N.E. is four points to the right of N., and N.E. by E. five to the right, whilst N.W. by N., and W.N.W. are respectively three and six points to the left. Now turn towards the south; then S.S.W. is two points to the right of S., and S. by E. one point to the left.

14. Variation or Declination of the Compass.—The compass does not (as we are generally led to believe) point exactly N. or S.; in other words, the magnetic meridian does not coincide with the geographical. *The angle at which the magnetic cuts the geographical meridian is the variation* of the compass. It is termed E. or W. declination, according as the pole of the needle points to the E. or W. of true north. The compass at Cape Horn, instead of pointing due N., points nearly *N.N.E.;* the variation is thus two points E. At St. John's (Newfoundland), the needle points N.W. by N., or the declination is three points westerly; but we must be careful to remember that if the bearing of a distant object by a compass on board is too much *easterly*, the north end of the needle is not drawn towards the east; but, on the contrary, the variation is westerly. By this means the E.

point on the compass card is drawn towards the direction of the body, the bearing of which is observed, this makes it appear eastward by compass.

Variation differs in various parts of the world. At the present time the declination of the compass at London is about 20° W.; at New York, 6° W.; at Hobart Town, 10° E.; at Hyderabad and Cuba, *nil;* and the Cape of Good Hope, 29° W.; at Point Barrow, 40° E. Generally in Europe, Africa, and the Atlantic, the variation is westerly, while in America, Asia, and the Pacific, it is easterly.

15. Secular Variation.—In the same place the declination of the compass differs in the course of time, or changes year by year; this is secular variation. In 1580, the declination in London was 11° 36′ E.; in 1663 it was *nil;* in 1818, it was at its maximum angle towards the west, viz., 24° 41′, which has decreased in the last fifty years to 20° W.

16. Isogonic and Agonic Line.—Isogonic lines are imaginary lines connecting all places on the earth's surface that have the same magnetic declination. The places that have the same magnetic variation as London are situated on a line drawn through Brest, Lisbon, Madeiras; then across the Atlantic to Nova Scotia, East Maine, and Hudson's Bay. This is the N. Isogonic line of 20° W. The S. Isogonic line of 20° W. is more irregular, running through the east central part of the South Atlantic, the Gulf of Guinea, across Africa, from C. Lopez to Sofala, and the south of Madagascar.

Agonic lines are lines connecting the places where there is *no variation.* There are two, the American and Asiatic agone; to these may be added a secondary agone, enclosing an oval in Eastern Siberia, China, Tartary, and the Pacific Ocean outside the Japan Islands.

17. Annual and Diurnal Variation.—Cassini was the first to show that the compass declination is subject to an annual variation, and that this annual declination is at a maximum near the vernal equinox, and diminishes till midsummer to a minimum, afterwards increasing to the spring equinox. *Diurnal variation* was first shown to exist by Professor Graham. In England the N. pole moves every day from E. to W., from sunrise till the hottest part of the day; it then tends towards the E., and regains its original position

about 10 P.M. During the night it remains stationary. The annual variation does not exceed 18′, the diurnal averages about 11′ or 12′ per day.

18. Dip and Dipping Needle.—A Magnetised needle—we have already alluded to the fact—does not remain perfectly horizontal, if movable on a horizontal axis passing through its centre of gravity. In the northern hemisphere its north pole dips or inclines towards the earth, while in the southern hemisphere the opposite phenomena takes place. A needle suspended so as to be capable of freely moving on a horizontal axis, is termed a *dipping neddle*, and the angle it makes is called the *dip* or *inclination*.

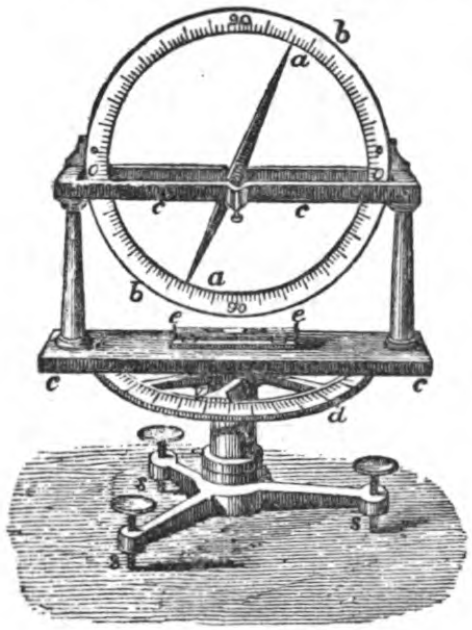


Fig. 4.

In the figure, *aa* represents the dipping needle balanced on its axis, as seen in the centre of the circle. If placed in the magnetic meridian in our latitude it dips or inclines towards the north instead of remaining horizontal. The angle *ob*, or the opposite vertical angle, shows the dip which is now about 67°. At the magnetic poles the dipping needle becomes vertical. If a line be drawn through all places where the dip is *nil*, we shall have the magnetic equator or *aclinic line;* it does not coincide with the geographical equator in more than three or four places; it inclines to the equator at an angle of about 12°. It has been proved most conclusively that—

Tan. lat. from magnetic equator = ½ tan. dip in the place of observation.

Like declination, it is subject to *secular variation;* in London it has been gradually decreasing about 2·81′ annually for the last sixty years.

19. Deviation or Local Attraction.—These terms are frequently used synonymously, but a distinction should really be made between them. *Deviation* is the declination of the compass caused by the iron used in building the ship, and

by the magnetism hammered into it whilst being built. *Local attraction* is the declination caused by the local influences external to the ship, as pieces of iron, iron mooring post and chains, the neighbourhood of iron ships, etc. The extensive introduction of iron into shipbuilding has greatly increased the causes and amount of compass errors. A ship visiting St. Helena receives a practical example of natural local attraction.

It is found by experiments that the deviation is different when the vessel heels over on either side, to what it is when she is on an even keel. We may thus illustrate and explain the phenomena.* It differs from local attraction and variation in that its intensity and direction depends upon the position of the ship's head. Take a small delicate compass, and place a long bar of soft iron in a horizontal position, at right angles to the magnetic meridian, it will not affect the needle; now slightly elevate the farther end, and the south point of the needle will be drawn towards the bar; lower the farther end and the N. point will be attracted, and the S. repelled. This is what happens in N. latitude; in S. latitude the opposite phenomena takes place. Now apply this fact to a ship, and considering that the compass is always horizontal, we shall immediately see that the iron in the ship, the tanks, pillars, etc., are raised or depressed (like our iron bar), according as the vessel heels, and thus the heeling of a ship affects the compass frequently in a very material manner.

Deviation depends upon the position in which a ship, especially an iron or armour-plated ship, *has been built.* Take a bar of iron and hammer it, when a large amount of induced magnetism may be driven into it, which can be subdued by an equal amount of hammering. This is what takes place when a ship is being built; and in proportion as the iron approaches the quality of steel does the induced magnetism, once hammered in, retain its power over the compass. This magnetism, once driven in, may be increased or diminished by grating against piers, striking sunken rocks, or by being struck by a heavy sea.

20. Permanent Magnetism and Sub-permanent Mag-

* *The Magnetism of Ships and the Mariners' Compass,* by Captain Walker, R.N.

netism.—That portion of a ship's induced magnetism which is never destroyed by any subsequent violence is called the *permanent magnetism* of the ship, whilst that portion which can be so removed is termed the *sub-permanent magnetism.*

The compass may be said to be subject to three distinct disturbing influences—

(*a*) Vertical induction.
(*b*) Horizontal induction.
(*c*) Permanent and sub-permanent magnetism.

On no account whatever (however high the authority to the contrary) should any attempt be made to compensate for these errors; for what will compensate for the errors in one hemisphere will double them in the other. Compensation only renders the error more uncertain. The best plan is to find the whole amount of error as frequently as possible, by methods to be presently indicated, and allow for it in the usual way.

Vertical Induction.—If an iron bar, a common needle, a poker, or even a pair of tongs be placed or suspended by a thread for a length of time in the direction of the magnetic meridian, so that one end inclines at an angle of 67° (in our latitude), it will become magnetised by induction. "A bar of very soft iron, placed vertically, may be freely magnetised by induction from the earth, and if reversed in position may be demagnetised and its polarity reversed in a few seconds; whilst a bar of hardened steel is scarcely at all susceptible of the earth's induction."* Vertical induction has the greatest effect when the ship's head is N. or S., and least when E. or W.; that is, lying across the magnetic meridian.

Horizontal Induction.—Horizontal pieces of iron, such as deck beams, engine shafts, etc., affect the compass differently to vertical masses. Take a compass and lay a bar of iron E. and W., the bar will not affect the needle of the compass; now take another bar of iron and place it at right angles to the first, letting the second bar touch the farther end of the first, it will immediately cause the compass needle to deviate from its position, however long the first bar may be. But separate the two bars by ever so small a space, the needle will begin to oscillate and return to its original position.

* Brooke's *Elements of Natural Philosophy*, p. 335.

The following extract, in reference to the point under discussion, is taken from a most useful little book, on *The Magnetism of Ships and the Mariners' Compass*, by the late W. Walker, Captain, R.N.

"This is an important fact, because great magnetic energy may arise and influence a steering compass from the arrangement of the iron in a vessel. If, for example, a merchant vessel have a cargo of iron in her hold, or even iron tanks, steam boilers, or cylinders so stowed in her hold as to be in contact with an iron knee or iron truss bolted to the ship's side and running upwards to the upper deck beams, such a piece of iron being in contact with large masses of metal in the hold would conduct or transfer the magnetism from below, and certainly derange the magnetic needle and cause the compass to indicate a wrong course.

"It is on this principle of magnetic conduction that separate pieces of iron, when brought into actual contact, act magnetically as a single mass. The water tanks in a ship-of-war, if stowed in actual contact, will act on the compass as if a single tank of the same size as the aggregate number of small ones in the hold occupied their places. But if the tanks be kept separate by thin slices of board, then each separate tank, etc., will retain its natural quantity of inductive magnetism, and the place of its poles will change with the motion of the ship."

(*a.*) One powerful compass needle at the distance of two feet from another of the same kind will produce an error in *each compass* of four degrees, it is therefore prudent and proper to have only one compass in a ship's binnacle.

(*b.*) In order to avoid unequal attractions of iron quarter-davits, iron knees, and bolts at the side of the ship, let the binnacle compass be placed on a midship fore and aft line.

(*c.*) See that the lubber's point in your compass bowl and the upright pivot for the card are in a line with the ship's stem and stern post, and prove this by stretching a thread over them in an exact fore and aft direction.

(*d.*) Take care that no iron of any kind be near your compass, either on the deck or under it, for arms are sometimes stowed below, and cabin stoves and funnels with iron linings are sometimes too near the steering compass.

(*e.*) The standard compass should be always used at one

convenient place on a ship's upper deck, on a midship fore and aft line.

Iron and other ships are generally swung on an even keel. Hence a Deviation Table is not so useful or practical as would at first sight appear. The table being of little value when heeling over from the effects of a strong breeze, etc. It has been suggested that three Deviation Tables should be constructed:—

(1) When the ship is on an even keel.
(2) When the ship is heeled over to starboard.
(3) When the ship is heeled over to port.

21. How Deviation of the Compass is Ascertained.—The process of finding the deviation of a ship's compass on every tack is called "swinging the ship." One method of swinging a ship is as follows :—

Place the ship in such a position that some fixed object at a distance of eight or ten miles can be clearly seen, and record its bearing as well as the bearing of the ship's head. The correct bearing of the fixed object, which may be a church spire or a high hill, can be accurately determined from a good, correct chart; but the true bearing is not wanted—all we want is the correct *magnetic* bearing, which is easily found by the method we shall now indicate.

Write down the points of the compass in one, two, or four lines, whichever may be most convenient; then, as the ship swings by wind or tide, or is compelled to move round from one point to another, write down the compass bearing of the distant object opposite the point of the compass towards which the ship's head is directed. If the compass have any error arising from deviation, local attraction, etc., the bearing of the distant object will vary as the ship swings from point to point. After the table is completed, if carefully examined there will be found two points nearly opposite to each other on the compass where the bearings agree; *these two points give the correct magnetic bearing of the distant object*, and on those two points there is no error caused by deviation or local attraction.

It is a remarkable circumstance that if we look for two other points in the table just made, where the bearings most widely differ, these will also be found nearly opposite each other.

Now, if we find the difference between the correct *magnetic* bearing and the incorrect bearing placed opposite each point of the compass in our table, we shall have the deviation on these points. It is thus that the majority of deviation tables are constructed.

22. **Reciprocal Bearings** is a second method, simpler, but not always more accurate. It is used when no suitable object can be seen at a sufficient distance for the effectual employment of the first method. Two compasses are employed; the standard compass on board ship, and one on shore which is assumed to show correct magnetic bearings. The ship is swung round so as to bring her head to each point of the compass. As it turns round, the bearing of the compass on shore is observed from the ship, and at the same instant, by preconcerted signals, the bearing of the ship's compass is observed from the land. Each is written down in its proper order in a table similar to the one indicated in the first method. The observations on shore are first reversed to bring them into the same direction as those taken from the ship, and are compared with the latter; the difference, as before, is the deviation, which is east or west according to circumstances; but we will explain in a moment by the following practical illustration from an examination paper:—

The standard compass on board makes S.W.$\frac{1}{2}$-W., and that on shore N.E. by E.$\frac{1}{2}$-E.; what is the deviation?

Inverting the latter, or shore observation, we compare the two—

Observed (ship),	True (shore inverted),
S.W.$\frac{1}{2}$-W.	S.W. by W.$\frac{1}{2}$-W.

We have now to find the amount of deviation, and whether it be east or west. S.W.$\frac{1}{2}$-W., the observation made on board ship, is $4\frac{1}{2}$ points right of south; while the shore observation, S.W. by W.$\frac{1}{2}$-W., is $5\frac{1}{2}$ points right of south. Their difference shows *one* point deviation, and as the shore or true observation is to the *right* of the ship's, the *deviation* is easterly. Had the true or shore observation been to the left of the ship's, the deviation would have been westerly. Every pair of observations is treated in the same way, and the error, properly marked, is placed opposite each point of

the compass to form the deviation table. As this point is important, it will be illustrated by another example.

Suppose the standard compass on board a ship shows the bearing of a compass on shore to be N.E., and the bearing of the ship compass from shore is S.W.$\frac{1}{2}$-S.; we will find the deviation.

Observed bearing.	*True bearing.*
Ship,	Shore inverted,
N.E.	N.E.$\frac{1}{2}$-N.

The difference is half a point, and the true bearing is to the left of the observed, therefore the deviation is half a point westerly.

23. Deviation Table.—Below is given a specimen of a deviation table.

Ship's Head.	Deviation.	Ship's Head.	Deviation.	Ship's Head.	Deviation.	Ship's Head.	Deviation.
North.	2° 20′ E.	South.	2° 45′ W.	East.	6° 40′ E.	West.	8° 0′ W.
N. by E.	3° 40′ ,,	S. by W.	3° 50′ ,,	E. by S.	5° 40′ ,,	W. by N.	7° 40′ ,,
N.N.E.	5° 40′ ,,	S.S.W.	5° 20′ ,,	E.S.E.	4° 40′ ,,	W.N.W.	6° 40′ ,,
N.E. by N.	6° 50′ ,,	S.W. by S.	6° 0′ ,,	S.E. by E.	3° 40′ ,,	N.W. by W.	5° 40′ ,,
N.E.	8° 0′ ,,	S.W.	6° 30′ ,,	S.E.	2° 0′ ,,	N.W.	4° 30′ ,,
N.E. by E.	8° 10′ ,,	S.W. by W.	7° 30′ ,,	S.E. by S.	1° 0′ ,,	N.W. by N.	3° 10′ ,,
E.N.E.	7° 20′ ,,	W.S.W.	7° 40′ ,,	S.S.E.	0° 30′ W.	N.N.W.	1° 50′ ,,
E. by N.	7° 35′ ,,	W. by S.	8° 10′ ,,	S. by E.	1° 30′ ,,	N. by W.	0° 40′ E.

This means that if the ship is sailing N.N.E. by compass, the error of the compass, caused by the iron in the ship, etc., is 5° 40′ E.; and that before the *true course* can be found, this error must be allowed on the *compass course.* So, also, if a true course be known, the error must be applied to it in a contrary direction to find the compass course.

Rule for finding the deviation from two reciprocal bearings:—

(*a.*) Invert the shore or true observation, and find the difference between it and the observation from ship.

(*b.*) Draw two lines at right angles to each other, and mark the letters indicating the four cardinal points at their extremities; then draw lines to the point of the compass to represent the true and ship's bearing.

Looking from the centre of the compass at the two lines, observe whether the true or shore observation be to the right

or left of the other; if it be right, the deviation must be allowed for as east; if left, west.

The deviation for the points can be taken immediately from the table, but it is sometimes wanted for the half and quarter points. We now show how to find it:—

59. Find the deviation from the table to correspond to N.½-E.,

When the ship's head is N. the deviation is 2° 20′ E.
" " N. by E. " 3 40 E.

sum 2)6 0

∴ when the ship's head is N.½-E., or half way between these two points, the deviation is 3 0 E.

60. Find the deviation when the ship's head is E. by S.¼-S.

When the ship's head is E.S.E. the deviation is 4° 40′ E.
" " E. by S. " 5 40 E.

2)10 20

" " E. by S.½-S. " 5 10 E.

Again, " " E. by S.½-S. " 5 10 E.
" " E. by S. " 5 40 E.

2)10 50

" " E. by S.¼-S. " 5 25 E.

It is first found for half way between those points between which the required deviation lies, and then for half way between the one now found, and the nearest one to that required.

61. The ship's head is E.¾-N., the deviation for the east point is 6° 40′ E., and for E. by N. 7° 35′ E.; find the deviation for the ship's head. *Ans.* 7° 21′ 15″ E.

62. Suppose a ship's head be W.¼-S.; find the deviation for that position, when for W. it is 8° 0′ W., and for W. by S. 8° 10′ W. *Ans.* 8° 2′ 10″ W.

63. Given the deviation for S.W. 6° 30′ W., and for S.W. by W. 7° 30′ W.; find it when the ship's head is S.W.¼-W. *Ans.* 6° 45′ W.

64. Given the deviation for N.N.W. is 1° 50′ W., for N. by W. 0° 40′ E.; find the deviation when the ship's head is N. by W.½-W. *Ans.* 35′ W.

65. The ship's head is S.E. by S.½-S.; find the deviation for that position, when it is known that the deviation for S.E. by S. is 1° 0′ E. and S.S.E. is 0° 30′ W. *Ans.* 15′ E.

66. Find the deviation when the ship's head is S.W.½-W.; given deviation for S.W. 6° 30′ W., and S.W. by W. as 7° 30′ W. *Ans.* 7° W.

67. The deviation for N.E. is 8° 0′ E., for N.E. by N. 6° 50′ E.; find the deviation when the ship's head is N.E.¾-N. *Ans.* 7° 7′ 45″ E.

68. The ship's head is W.$\frac{1}{2}$-N.; find the deviation, when it has been ascertained that the deviation, when the ship's head lies W., is 8° W., and when on the W. by N. rhumb it is 7° 40′ W.

Ans. 7° 50′ W.

69. The same conditions existing, suppose a current causes the ship's head to be at W.$\frac{3}{4}$-N., what is the deviation then? *Ans.* 7° 45′ W.

70. It having been ascertained that the deviation for S.S.W. is 5° 20′ W., and for S. by W. 3° 50′ W., and that the ship's head lies S. by W.$\frac{1}{4}$-W.; find the deviation. *Ans.* 4° 12′ 30″ W.

24. Course.—Course is the angle which the ship's track makes with the meridian, and is always reckoned from the north and south in degrees or points.

25. Compass Course.—Compass course is the angle which the ship's track makes with the meridian as indicated by the compass in the binnacle.

26. True Course.—True course is the corrected angle which the ship's track makes with the meridian. A compass course has to be corrected for declination, deviation, and leeway.

27. Starboard is the right hand side of the ship, looking towards the bow or towards the direction in which she is moving.

28. Port or Larboard is the left hand side, looking in the same direction.

29. Starboard Tack.—A ship is said to be on the starboard tack when sailing with the wind blowing on the right hand side of the ship; and on her

30. Port Tack, when sailing with the wind on the left side of the ship.

When the wind is exactly behind the ship it is said to be abaft, hence we explain the nautical term, "the wind being abaft." When the wind is perpendicular to the side of the ship it is said to be on or across the beam; when between these two positions it is indicated by saying the wind is so many degrees abaft the beam; these points will be again referred to under the heading, *Windward Sailing.*

31. Rhumb Line.—A rhumb line is a line that intersects every meridian which it crosses at the same angle. If a ship were to sail from Plymouth to New York, cutting every meridian at the same angle, it would sail on a *rhumb or loxodromic line.*

32. Leeway.—Leeway is the angle made by the ship and the direction in which she moves. This is estimated by the force of the wind, by the rate of the ship, or by the sails she carries. It may be explained thus:—

Let A be the position of a ship, and suppose a wind coming from E.N.E. would drive the ship to B, 4 miles, in a given time, whilst in the same time the vessel would progress to D, 12 miles; by the well-known proposition in mechanics, if we complete the parallelogram A B C D, the ship will neither be found at B nor D, but will sail in the direction indicated by the resultant A C, and will be found at C. The leeway is the angle C A D, it is very evident that the more headway the vessel makes in a given unit of time, the smaller will be the angle C A D; hence the rate of the ship, or the sails she carries, may both together or singly be taken as an index to the amount of leeway. When the wind is on the right hand side of a ship, as the observer looks towards the bow, she is said to be on the *starboard tack;* and when it is on the left hand side of the ship, on the *port tack;* and when the helm is so directed as to place the ship as near the wind as she will lie, she is said to be *close hauled.*

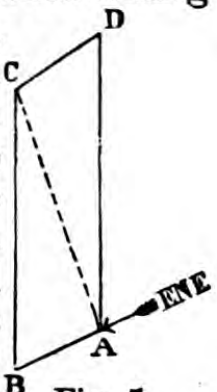


Fig. 5.

Leeway must always be allowed *from* the wind; if the wind be on the left or port side, call the leeway *right*, if on on the starboard or right side, call it left.

Suppose a vessel at O, ready to sail in the direction S. by E., and on the port tack, the wind being across her beam, and coming from E. by N., causing leeway to the amount of *one* point, then the vessel evidently will not sail in the direction shown by the S. by E. rhumb, but would be compelled to move due S., in the direction O S, and would be found at S., so that if the wind be on the port tack or *left* side of the ship, the leeway must be allowed to the right. With a little consideration, the student will perceive that if the ship had been on the starboard tack,

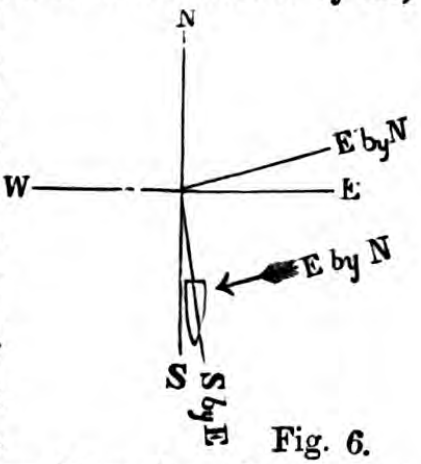


Fig. 6.

she would have sailed S.S.E., and the leeway would have been allowed to the *left*, *i.e.*, in both cases from the wind.

From all this we can see that if the ship's head is pointed in any particular direction, on account of the action of the wind upon the side of the ship, and upon the sails, it does not move exactly in that direction, but is driven out of its course, and the amount it is so driven, we term *leeway*. Precisely defined, it would run thus: *Leeway is the angle between the course and the direction in which the ship is actually moving;* or leeway is the angle which the rhumb line on which the ship really sails, makes with the rhumb line on which the master endeavours to sail her. When the vessel is close hauled, the pressure of the wind and the surge of the sea cause her to fall off sideways to *leeward*, hence the result is named *leeway*. In addition to the circumstances previously mentioned, the amount depends upon the lines and trim of the vessel, the depth of the water drawn, whether the ship be as near the wind as she will lie, etc. It is very evident that if the wind be right behind the ship, or *abaft*, no allowance must be made for leeway; and as the wind comes more or less perpendicularly to the side of the vessel, the leeway must vary accordingly.

33. To find the True Course, when Compass Course is known.—RULE—(*a.*) Put down the compass course in degrees or points, and mark it right or left of N. or S., as the case may be.

(*b.*) Allow easterly variation and deviation to the right, and westerly to the left.

(*c.*) Apply the leeway *from* the wind—that is, if the wind be on the starboard or right hand side of the ship, call it left; if on the port side, call it right.

(*d.*) When both are right or both left (or of the same name), add, and keep the same name right or left; but when the two are of different names, or one left and the other right, take the difference and mark the remainder right or left, according as the greater is right or left.

Or, better still, draw a figure similar to our last, and consider whether the wind drives the ship nearer to or further from the north or south, and subtract or add according as the leeway makes the angle smaller or larger.

71. A schooner sails N.N.E.$\frac{1}{2}$-E. by compass, the variation is 21° 14′ E., deviation as recorded in the table, p. 42. Wind is S.E., and leeway one point; find the true course.

The compass course is N.N.E.$\frac{1}{2}$-E. or	28°	7′	30″	r. of N.
Variation E.,	21	14	0	r.
	49	21	30	r. of N.
Deviation for N.N.E.$\frac{1}{2}$-E., from table,	6	15	0	r.
	55	36	30	r. of N.
Leeway, as wind is on r. side,	11	15	0	l.
	44	21	30	r. of N.
∴ True course is N.	44	21	30	E.

It may be observed that the deviation for N.N.E. is 5° 40′ E., and for N.E. by N., 6° 50′ E.; to obtain the deviation for N.N.E.$\frac{1}{2}$-E., we add these together and take half the sum as previously explained.

72. The same vessel sails E.$\frac{1}{2}$-N. by compass, the declination of the needle is 23° 4′ W., the deviation from the table as before; wind N. by E., giving $1\frac{1}{4}$ points leeway; what is the true course?

The compass course is E.$\frac{1}{2}$-N., or	84°	22′	30″	r. of N.
Variation W.,	23	4	0	l.
	61	18	30	r. of N.
Deviation for E.$\frac{1}{2}$-N. from table,	7	7	30	r.
	68	26	0	r. of N.
Leeway, as wind is on port side,	14	3	45	r.
∴ True course is N.	82	29	45	E.

73. We will take the same question, but suppose the compass course is W.$\frac{1}{2}$-N., and leeway one point.

The compass course is W.$\frac{1}{2}$-N. or	84°	22′	30″	l. of N.
Variation W.,	23	4	0	l.
	107	26	30	l. of N.
Deviation of W.$\frac{1}{2}$-N. from table,	7	50	0	l.
	115	16	30	l. of N.
Leeway, ship on starboard tack	11	15	0	l.
	126	31	30	l. of N.
Subtract from,	180	0	0	
	53	28	30	r. of S.
∴ True course is S.	53	28	30	W.

By applying variation, deviation, and leeway, the ship has gone round (as it were) from the N.W. quarter to the S.W., and because from N. to S. is 180°, we subtract from 180° to obtain the angle the ship track makes from the S.

74. A vessel steers by compass E.S.E., the variation and deviation

amount to $2\frac{3}{4}$ points E., the wind is N. by W., causing $\frac{3}{4}$ of a point leeway; determine the true course.

The compass course is E.S.E., or	6	points	l. of S.
Variation and deviation $2\frac{3}{4}$ points,.....	$2\frac{3}{4}$	,,	r.
	$3\frac{1}{4}$	,,	l. of S.
Leeway,.....	$\frac{3}{4}$	,,	r.
	$2\frac{1}{2}$	,,	l. of S.
∴ True course is.....S.S.E.$\frac{1}{2}$-E.			

Whenever it is convenient it is always better to solve the problems, adhering to points, as in these examples.

75. A ship is steering by compass N.W.$\frac{3}{4}$-W., the variation of the the compass is $2\frac{1}{4}$ points W., the wind is S.S.E., leeway $\frac{1}{4}$ point, deviation as in table; find the true course.

The compass course is N.W.$\frac{3}{4}$-W., or...	$4\frac{3}{4}$	points			l. of N.
Variation $2\frac{1}{4}$ W., ...	$2\frac{1}{4}$	,,			l.
	7	,,			l. of N.
Leeway, on port tack,.....	$\frac{1}{4}$	,,			r.
	$6\frac{3}{4}$	,,			l. of N.
or	75°	56′	15″		l. of N.
Deviation from table,.....	5	22	30		l.
	81	18	45		l. of N.
∴ True course.....N.	81	18	45	W.	

An observation or two may be made on the method of solution employed in this last question.

1. We have solved it by using points as far as possible for the sake of simplicity.

2. Having turned the $6\frac{3}{4}$ points into degrees 75° 56′ 15″ we apply the deviation obtained from the table.

3. The deviation cannot be taken directly from the table, but is found as already explained.

The student is now prepared to exercise himself in solving the problems immediately following, where the compass courses, variation, wind, and leeway are given to find the true course; *in all* cases the *deviation must* be taken from the table on page 42.

34. To Find Compass Course.—When the master of a vessel has found his true position, or sighted a point of land, it is his duty to lay down the course for the next point he wishes to reach. To do this is the converse of the last rule; for,

by examining the chart, he can find the *true course*, when he has next to apply the various errors and leeway to obtain the *compass course*, and thus ascertain the direction in which he must place the ship's head (by compass) to reach the port.

RULE.—Allow easterly variation and deviation to the left, westerly to the right, and leeway against the wind; *i.e.*, if ship be on the port tack, call the leeway left; if upon the starboard, call it right.

76. A vessel's true course is N.N.E., she is on the port tack with the leeway calculated at one point, and the variation from the chart $1\frac{1}{2}$ point W.; find the compass course, using the Table of Deviation.

True course is N.N.E. 2 pts., or	22° 30′ 0″ r. of N.
Variation is W.	16 52 30 r.
	39 22 30 r. of N.
Wind on the port side,	11 15 0 l.
	28 7 30 r. of N.

Now 28° 7′ 30″ r. of N. is N.N.E.$\frac{1}{2}$-E., the deviation for which is 6° 15′ E. Applying this to the angle found, we have

	28° 7′ 30″ r. of N.
Deviation,	6 15 0 l.
True course	N. 21 52 30 E.

77. A ship's true course is W.S.W.; she is on the port tack, with leeway calculated at $1\frac{1}{4}$ point, and declination ascertained to be $2\frac{1}{2}$ points W.; find the compass course.

True course is W.S.W., or	67° 30′ 0″ r. of S.
Variation $2\frac{1}{2}$ pts. W.,	28 7 30 r.
	95 37 30 r. of S.
Leeway wind on left side $1\frac{1}{4}$ pt.,	14 3 45 l.
This is about W.$\frac{3}{4}$-S.,	81 33 45 r.
Deviation for W.$\frac{3}{4}$-S. is W.	8 7 30 r.
Compass course is	S. 89 41 15 W. *Ans.*

CORRECTIONS FOR TRUE COURSE.

78. The compass course is S., the variation 21° E., the deviation as in the table previously given, the ship is on the port tack, the leeway 10° 20′; find the true course. *Ans.* S. 28° 35′ W.

79. Suppose the compass course is E.S.E.$\frac{3}{4}$-S., the variation 14° 30′ E., the deviation as in the table previously given, the ship on the port tack, the leeway 24° 20′; find the true course.

Ans. S. 16° 18′ 45″ E.

80. A master sails E.$\frac{1}{2}$-N., by compass variation 34° 16′ W., deviation as in the table previously given, the ship on the starboard tack, leeway 9° 16′; find the true course. *Ans.* N. 47° 58′ E.

81. If the compass course is N.W., variation 18° W., the deviation as in the table previously given, the wind is south, the leeway 10°; find the true course. *Ans.* N. 57° 30′ W.

82. A ship sails W.$\frac{3}{4}$-N., the variation is $\frac{1}{2}$ pt. E., the deviation as in the table previously given, the wind is N.N.E., the leeway 2 pts.; find the true course. *Ans.* S. 73° 48′ 45″ W.

83. The compass course is W., variation 1 pt. E., the deviation as in the table previously given, the wind is S., leeway 3 pts.; find the true course. *Ans.* N. 53° 0′ 0″ W.

84. A master steers E., variation 5° 10′ W., the deviation as in the table previously given, the wind is W., leeway *nil;* find the true course. *Ans.* S. 88° 30′ E.

85. A ship's compass is set S.W., variation 1 pt. E., the deviation as in the table previously given, the wind is N. by W., leeway $\frac{1}{4}$ pt.; find the true course. *Ans.* S. 46° 56′ 15″ W.

86. If the compass course is W. by S., the variation 15° 20′ W., the deviation as in the tables previously given, the ship is on the starboard tack, the leeway 19° 25′; find the true course. *Ans.* S. 35° 50′ W.

87. A ship sails N.E. by N., variation 23° 10′ W., deviation as in the table previously given, wind blowing S.E., leeway 1 pt.; find the true course. *Ans.* N. 5° 50′ 0″ E.

88. A ship's compass is set S.S.E., variation 12° 16′ W., deviation as in the table previously given, the ship on the starboard tack, leeway 11° 15′; find the true course. *Ans.* S. 46° 31′ 0″ E.

89. A master steers S.E. by E., variation 18° 45′ W., deviation as in the table previously given, the ship on the starboard tack, leeway 28° 16′; find the true course. *Ans.* N. 80° 24′ 0″ E.

90. A ship sailing S.E., variation 1$\frac{1}{2}$ pt. W., deviation as in the table previously given, the ship on the port tack, leeway 1 pt.; find true course. *Ans.* S. 48° 37′ 30″ E.

91. If the compass course be S.W.$\frac{1}{2}$-S., variation 12° 0′ W., deviation as in the table previously given, the ship on starboard tack, leeway 16° 18′; find the true course. *Ans.* S. 4° 49′ 30″ W.

92. A ship's compass course is N., variation 19° 20′ E., deviation as in the table previously given, the ship on the starboard tack, leeway 2 pts.; find the true course. *Ans.* N. 0° 50′ 0″ W.

93. A ship sails N.E. by compass, variation *nil*, deviation as in the table previously given, the ship on the port tack, leeway 1$\frac{1}{2}$ pt.; find the true course. *Ans.* N. 69° 52′ 30″ E.

94. If the compass course be N.W.$\frac{1}{2}$-W., variation 4° 35′ E., deviation as in the table previously given, the wind is S.S.W., leeway 27° 30′; find true course. *Ans.* N. 23° 37′ 30″ W.

95. A master sails E.$\frac{1}{4}$-S., variation 1$\frac{3}{4}$ pt. W., deviation as in the table previously given, wind N. by W., leeway $\frac{3}{4}$ pt.; find the true course. *Ans.* N. 87° 58′ 45″ E.

In the following examples to find the *compass course*, when the true course is given, you are to use the Deviation Table which is here reprinted for the sake of easy reference in working the exercises on these two pages.

Ship's Head.	Deviation.	Ship's Head.	Deviation.	Ship's Head.	Deviation.	Ship's Head.	Deviation.
North.	2° 20′ E.	South.	2° 45′ W.	East.	6° 40′ E.	West.	8° 0′ W.
N. by E.	3° 40′ ,,	S. by W.	3° 50′ ,,	E. by S.	5° 40′ ,,	W. by N.	7° 40′ ,,
N.N.E.	5° 40′ ,,	S.S.W.	5° 20′ ,,	E.S.E.	4° 40′ ,,	W.N.W.	6° 40′ ,,
N.E. by N.	6° 50′ ,,	S.W. by S.	6° 0′ ,,	S.E. by E.	3° 40′ ,,	N.W. by W.	5° 40′ ,,
N.E.	8° 0′ ,,	S.W.	6° 30′ ,,	S.E.	2° 0′ ,,	N.W.	4° 30′ ,,
N.E. by E.	8° 10′ ,,	S.W. by W.	7° 30′ ,,	S.E. by S.	1° 0′ ,,	N.W. by N.	3° 10′ ,,
E.N.E.	7° 20′ ,,	W.S.W.	7° 40′ ,,	S.S.E.	0° 30′ W.	N.N.W.	1° 50′ ,,
E. by N.	7° 35′ ,,	W. by S.	8° 10′ ,,	S. by E.	1° 30′ ,,	N. by W.	0° 40′ E.

96. Suppose the true course of a ship is N. 86° E., the variation of the compass 18° E., and the wind is N., the leeway 6°; find the compass course. *Ans.* N. 54° 15′ 0″ E.

97. Allowing the true course of a schooner to be S. 64° W., the variation of the compass 18° E., the wind N.W. by W., leeway 5°; find the compass course. *Ans.* S. 58° 0′ 0″ W.

98. If the true course of a man-of-war be N. 16° W., and the variation of the compass 18° E., the wind being S. with 8° leeway; find the compass course. *Ans.* N. 37° 50′ 0″ W.

99. The true course of a brig is S. 52° W., variation 18° E., wind N. by E., and the leeway 8°; find the compass course.

Ans. S. 48° 22′ 30″ W.

100. The true course of a ship is N. 29° E., the variation of the compass 18° E., the wind W. by N., leeway *nil;* find the compass course.

Ans. N. 7° 20′ E.

101. If the true course of a brig is N. 67° W., the variation of the compass 18° E., the wind is N. by W., the leeway is 6°; find the compass course. *Ans.* N. 71° 20′ W.

102. Allowing the true course of an East Indiaman to be S. 55° E., the variation of the compass is 18° E., the wind is N.E.$\frac{1}{2}$-N., leeway 6°; find the compass course. *Ans.* S. 84° 40′ E.

103. Let the true course of a ship be N. 78° W., the variation of the compass is 18° E., the wind is S.E., the leeway is *nil;* find the compass course. *Ans.* N. 87° 55′ W.

UNDER THE FOLLOWING CONDITIONS FIND THE COMPASS COURSES:

	True Course.	Variation.	Deviation.	Wind.	Leeway.	Compass Course.
1.	E.	2 points W.	5° W.	W.	*Nil.*	S. 62° 30′ 0″ E.
2.	S. by W.	1½ points E.	9° 30′ W.	E.	1 point.	S. 7° 22′ 30″ E.
3.	N.W. by W.	20° E.	2° W.	E.	1½ points.	N. 57° 22′ 30″ W.
4.	N.E.	1¾ points W.	13° E.	S.W.	*Nil.*	N. 51° 41′ 15″ E.
5.	E.S.E.	*Nil.*	13° E.	N.	1¼ points.	N. 85° 26′ 15″ E.
6.	W. by S.	11° 15′ E.	9° W.	N.N.E.	2 points.	N. 81° W.

104. How are courses corrected for variation and deviation? Give the rule for applying the variation and deviation to a true course to obtain the magnetic course.

105. Having found by Mercator's sailing that my course must be true S. 70° E., find the compass course, with variation 34° W., deviation 8° E., and leeway (supposed) ¼ pt., wind W.N.W.

Ans. S. 41° 11′ 15″ E.

106. I have found by middle latitude sailing that my course must be true N., find the magnetic course when the wind is E., variation 25° E., and deviation 4° W., and leeway 2 pts. *Ans.* N. 1° 30′ E.

107. Find the compass course from Southampton to Jersey, when by the chart the bearing of Jersey from Southampton is S. 30° W., the wind N., and the variation 20° 8′ W., and deviation 9° W., leeway 1 point.

Ans. S. 70° 23′ W.

35. Azimuth Compass.—We deem it right to give a short description of the azimuth compass, although it is an instrument employed chiefly in nautical astronomy. Azimuth compasses are used to measure the angular distance of the sun or other celestial body from the south in north latitude, or from the north in south latitude; and also to measure its distance from the east when rising, or the west when setting. It is in reality an instrument for measuring horizontal angles, hence a prismatic compass is an azimuth compass. It resembles an ordinary compass as to its needle and the manner in which the needle is balanced; the card has two circles divided into degrees, one circle commencing its graduation from the north and south to measure azimuths proper, the other from the east to west to measure the amplitudes.

Affixed to the compass are two sight vanes—one the sight vane proper, having a fine thread stretched along its opening; by this thread the object observed must be bisected; the other has in it a slit through which the observer looks at the object. One is exactly opposite the other. The sight vanes are mounted with hinge joints, so that they can be turned down flat out of the way when not in use. A mirror is attached to the instrument immediately behind the sight vane proper, in which the celestial object is reflected. Let us suppose we are in north latitude, and going to take the bearing of the sun from the south in a morning or in the afternoon. The sight vanes are turned up so as to stand vertically, and the instrument is placed in a horizontal position; the observer then looks through the slit exactly opposite the thread stretched down the sight vane proper, or else through a prism, and then turns the instrument round until the sun is observed reflected in the mirror, and the thread exactly bisects the sun; he then touches a spring to steady the card and reads off the division upon the card, which appears to coincide with the prolongation of the thread; this is the *magnetic* azimuth of the sun, or its bearing from the south. Nautical astronomy teaches us to calculate the *true* azimuth, when, by comparing the true and magnetic azimuth, we can determine the variation of the compass.

x is the sight vane proper; y is the second vane through which the observer looks while he moves the compass until

the fine wire stretched across x bisects the object the angular distance of which is required. In our figure the mirror is omitted for simplicity, but it stands immediately behind x, or towards the spectator. One of the graduated circles is

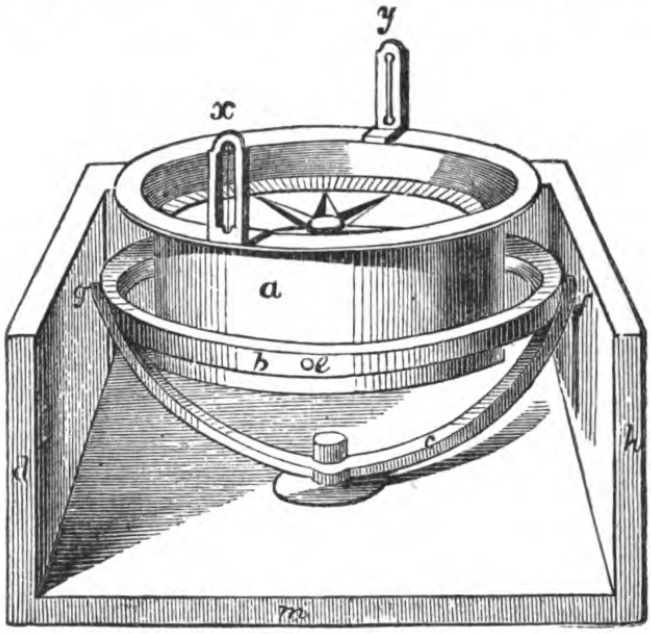


Fig. 7.—Azimuth Compass.

seen drawn on the card, and the gimbals on which the whole is swung are shown at g, e, and f, and the point behind immediately opposite e. The compass case is of brass, and mounted in a box dh.

CHAPTER III.

THE LOG, LOG LINE, AND LOG GLASS.

The Log—Log Glass—Stray Line—Heaving the Log—Patent Log—The Reel—Length of a Nautical Mile and Knot—Table of Lengths of Knots—Examples—General Formula for True Distance—Examination.

THE instrument used by the navigator to guide him as to the direction in which he must go is the **Compass;** those by which he estimates the rate of progress of the ship through the water are the **Log, Log Line,** and **Log Glass.** We shall treat of each separately.

36. The Log.—This instrument by which, as has just been stated, the rate of the ship is measured, consists of a piece of light wood (generally teak) in the shape of a sector of a circle. It is attached by two strings to the *log line* proper. When thrown overboard it immediately assumes an upright position in the water, like an ordinary float, in consequence of being weighted in the arc with lead, and presents its flat surface at right angles to the ship's course, so that it offers a maximum amount of resistance to the water, and theoretically remains stationary.

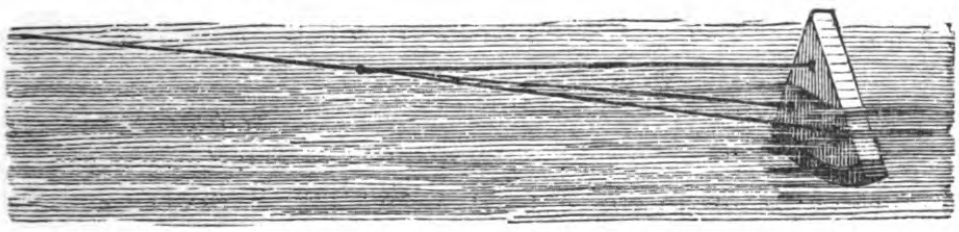


Fig. 8.—THE LOG SHIP OR LOG.

Our figure will give a fair idea of the log, and the method by which the line is attached to the log ship. The upper cord of the three is fastened in its hole by a peg, and this is the cord that is unfastened when the sailor gives the line a

smart jerk to make the log fall flat on the water, and allow itself to be drawn on board more easily.

37. Log Glass.—The log glass is a common sand glass constructed to run out in half a minute, by it the time is measured. Some log glasses are marked 14 seconds, but this is much too short a time to occupy in heaving the log.

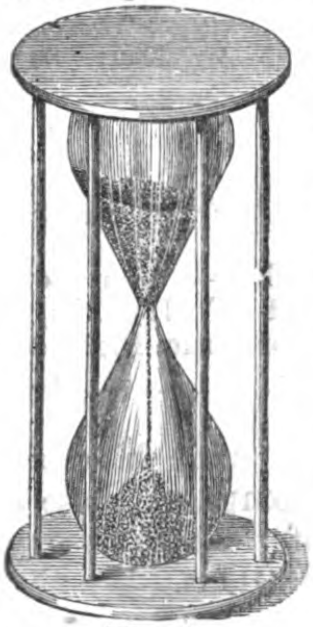


Fig. 9.

38. Log Line, Stray Line, Heaving the Log.—The log line consists of an ordinary piece of cord about 150 fathoms long, wound upon a reel. It is divided by certain marks (bits of bunting, red rag, worsted, etc.) into lengths of 50 feet each, termed a knot; each knot is subdivided by either a piece of twine, with knots in it reeved between the strands of the line, or some other marks, into tenths, which sailors term fathoms. When thrown overboard, the line is unwound from the *reel* as fast as the ship moves. No two log lines are alike, as masters generally buy a piece of cord, and put on it marks to suit their own fancy, simply following a general plan. The *log* is attached to the line and thrown overboard; at first a certain portion of the line is allowed to run out, so that the log may get as steady as possible out of the wash of the water made by the ship. The end of this portion is frequently marked by a red rag, and its length is about the same as that of the ship. This *part* is named the **Stray Line.** As soon as the stray line is run out, the man having the log calls out, when another turns the sand glass, and the operator observes how many of the divisions on the line, or knots, run overboard, stopping the instant it is intimated that the sand glass has run down. If four or five, he concludes at once that the ship is going four or five knots or miles per hour. The moment the glass has run out, the person who is conducting the operation, which is technically termed **heaving the log,** gives the line a sharp pull, thereby detaching one of the strings of the log, and it falls flat upon the water, and is

easily drawn into the ship again. Not less than half a minute should be the time allowed for making the necessary experiment to measure the ship's rate.

39. Patent Log.—The log and log line, which are known to have been in use as long ago as 1570, are now almost entirely superseded by Massey's and other patent logs. These patent logs are a modification of the screw-propeller, and revolve more or less rapidly according to the speed of the ship. A train of wheel work, acting like an ordinary tell-tale or turn-table, registers the number of revolutions or speed of the ship. It will register long distances, but must of necessity be taken out of the water as often as the course is changed.

It has generally four flanges, and the revolutions are registered on three faces, as seen in the next figure; the first registers the quarter, half, three-quarters, and one mile; the second from *one* to *ten* miles; and the third from ten up to one hundred.

40. Walker's Patent Harpoon Ship Log.—The ship log chiefly used now is Walker's Patent Harpoon Ship Log. It is a very convenient and compact instrument, with all the modern improvements.

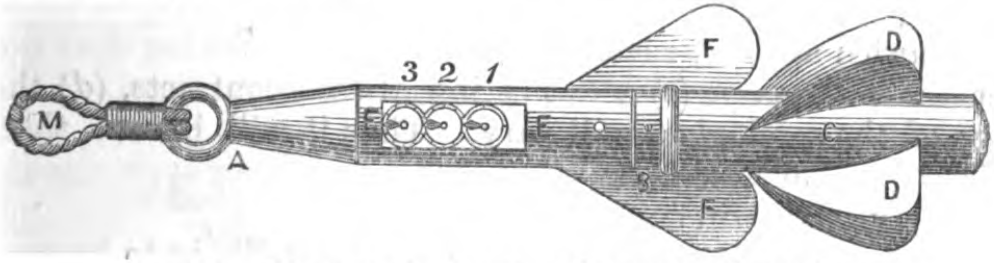


Fig. 10.—WALKER'S PATENT HARPOON SHIP LOG.

The illustration above ought to convey a good idea of it. When used, a tow-line, about 50 fathoms long, is attached at M, the hands being previously adjusted as seen in the figure, so that as soon as the apparatus begins to register it does so correctly.

The part marked A B does not revolve when in the water. It contains the train of wheels employed in registering the number of revolutions made by the *rotator* C, which is compelled to revolve as it is dragged through the water by the water acting on the vanes D and D, which are set on C in a

similar manner to the blades of the screw-propeller. The angle at which the vanes are set is so adjusted to the wheel work of the register, that when the machine has passed through one mile of water, the finger of the first dial will have made one revolution, the finger of dial 2 will have made one-tenth of a revolution, and the index finger of number 3 will have made the one-hundreth part of a revolution; or, in other words, the first index finger goes round in one mile, the second in ten, and the third in one hundred miles. In this log the rotator C and register E E are combined, the rotator being a continuation of the part A B that holds the wheel work. By this arrangement the machine is rendered very compact, and the rotator protected from all front end pressure, and the principal cause of friction is removed, so that the log registers with equal accuracy at all speeds. F is a float plate to compel the machine to move and travel horizontally, for by this modification the friction on the rotator spindle is reduced to a minimum. The case E E turns round so as to cover the dials and the oil hole seen to the right of E, when dirt, etc., is prevented from getting into the dial and wheel work.

41. Uncertainty of the Log and Log Line.—In heaving the log there are many sources of error or contingencies to be guarded against, such as (*a*) currents, (*b*) the log does not remain stationary, (*c*) a wet string always contracts, (*d*) the rate of the sand glass will vary. Thus it will be seen that, upon the whole, heaving the log gives but an approximate and very uncertain result.

42. We have indicated how the string is likely to become a source of error; the glass is no less subject to uncertainty, chiefly through variations of temperature. Glass follows the nearly universal law of contraction by cold and expansion by heat. In hot weather, by the expansion of the glass, and consequent slight enlargement of the neck of the glass, the sand will run out too quickly; whilst, on the contrary, in cold weather the contraction of the same will retard the rate at which the sand runs.

43. The Reel.—The reel is a simple wooden structure, as seen in the figure. It is held in the two hands by the handles H H, when the cord C being wound round it, and the log L

thrown overboard or heaved, the resistance offered by the water to the log causes the cord to run off the reel.

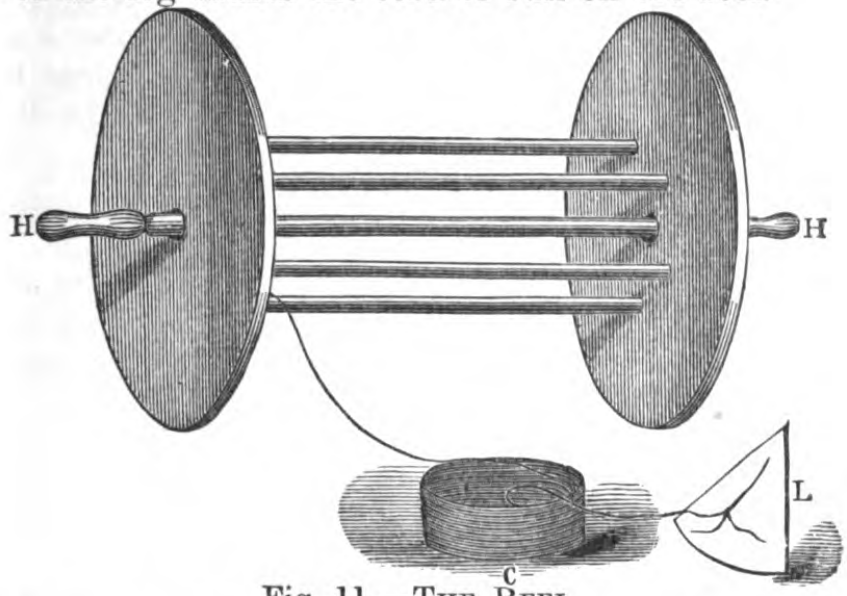


Fig. 11.—THE REEL.

44. Length of a Nautical Mile.—A nautical mile is the sixtieth part of a degree. We will therefore first find the length of a degree in feet on a great circle of the sphere. The radius of the earth, according to the most recent authorities, is 7912·4 miles.

$$\begin{aligned}
\therefore \text{ Circumference of the earth} &= \pi d = 2\pi r \\
&= 2 \times 3{\cdot}1416 \times 7912{\cdot}4 \\
&= 24857{\cdot}59784 \text{ miles} \\
\therefore \text{ Length of one degree.....} &= \frac{24857{\cdot}59784}{360} \\
&= 69{\cdot}04887 \text{ miles} \\
\therefore \text{ Length of a nautical mile} &= \frac{69{\cdot}04887 \times 5280}{60} \\
&= 6076{\cdot}3^{*} \text{ feet.}
\end{aligned}$$

The nautical mile is generally taken as 6080 feet, while a statute mile is 5280 feet, so that—

$$\begin{aligned}
\text{A statute mile : a nautical mile} &:: 5280 : 6080 \\
&:: \quad 33 : \quad 38 \\
\text{(nearly)} &:: \quad 6 : \quad 7
\end{aligned}$$

This proportion shows that, if a vessel is making 6 knots an hour, her rate is about 7 English miles; or 12 knots per hour, 14 English miles.

* Bessel gives the length of a mile as 6087 feet, and the whole circumference of the earth as 131,579,200 feet.

45. Length of a Knot.—The log glass runs out in half a minute; now half a minute is the one hundred and twentieth part of an hour; a knot is taken as the one hundred and twentieth part of a nautical mile. It is evident that if the $\frac{1}{120}$ part of a mile, on the log line, run out in the $\frac{1}{120}$ part of an hour, the ship is going 1 mile per hour. If $\frac{3}{120}$ run out in the same time, the rate of the ship is 3 miles per hour.

We now see the force of the expression, "the ship is going *four* and *five knots* per hour. It really means that she is running at the rate of four or five nautical *miles* per hour.

Since a nautical mile = 6076 ft.

$$\therefore \text{ a knot measures } \frac{6076}{120} = 50{\cdot}6 \text{ ft.}$$

The length of the knot is generally taken as 50 ft.

46. Rate Varies.—The rate of progress varies directly as the time, if the knot is unaltered, but inversely as the length of the knot, if the time is unaltered, when a distance is calculated by the *log* and log line.

(*a.*) *Suppose the glass incorrect and the knot correct.*—If the glass run out too slowly, *more* knots will run overboard in the time than should properly do so, consequently the estimated distance will be *too great;* but if the glass run out too fast, then *too few* knots will run out, and the calculated distance will be *too short. Hence, if the knot be correct, the rate will vary directly as the time.*

(*b.*) *Suppose the glass correct but the knot incorrect.*—If the glass be a true half-minute glass, and the knot by contraction or otherwise *too short*, then in the given time too many knots or too great a length will run out, and the estimated distance will be *too great;* whilst if the knots were *too long*, a sufficient number will not run out in the time, and consequently the estimated distance will be *too little*, or *the estimated distance will vary inversely as the length of the knot when the glass is correct and knot incorrect.*

It should be observed that any length of knot may be used, if we have a corresponding sand glass.

108. A sand glass runs out in 20 seconds; find the correct length of the knot.

Here in 3600sec. or *one hour*, 6080 ft. run out.

Then in 1sec..... $\frac{6080}{3600}$,,

Therefore in 20sec..... $\frac{6080 \times 20}{3600} = 33\frac{7}{9}$ for *Ans.*

109. If a log line be knotted at every 48 ft., what time should the sand glass run?

Here 6080 ft. run out in 1 hour, or 3600sec.

Then 1 ft. runs out in..... $\frac{3600}{6080}$

Therefore 48 ft. run out in ..... $\frac{3600 \times 48}{6080} = 28\frac{8}{19}$. *Ans.*

110. I use a 20 seconds' glass, and the length of the knot is 30 ft. After running an estimated distance of 120 miles, find the true distance the ship has gone.

True length of knot to a 20sec. glass $= 33\frac{7}{9}$ ft. The glass is correct, and the length of the knot incorrect; therefore the distance varies inversely, or

As $33\frac{7}{9} : 30 : : 120 :$

$$= \frac{120 \times 30 \times 9}{304} = 106\frac{11}{19} \text{ miles. } \textit{Ans.}$$

111. What is the length of a knot to a 15 seconds' glass?

Ans. $25\frac{1}{3}$ ft.

112. Required the length of a knot to a 24 seconds' glass.

Ans. $40\frac{8}{15}$ ft.

113. My glass runs out in $26\frac{1}{2}$ seconds; at what distances must I knot my log line? *Ans.* $44\frac{3}{4}\frac{4}{5}$ ft.

114. The length of my knot is 47 ft. 3 in.; how long should the glass run? *Ans.* 28 sec. nearly.

115. The log line is knotted at every 26 ft. 2 in.; what glass must be used? *Ans.* $15\frac{1}{2}$ sec. nearly.

116. Suppose the log line is knotted at every $33\frac{4}{9}$ ft.; find the time the glass must run. *Ans.* 19·8 sec.

117. Find the length of a knot to a glass that will run out in 14 seconds. *Ans.* $23\frac{2}{4}\frac{9}{5}$ ft.

118. Required the length of a knot when using a sand glass that will run out in $20\frac{3}{4}$ seconds. *Ans.* $35\frac{2}{4}\frac{2}{5}$ ft.

119. A glass runs out in 25 seconds; at what distance apart must the knots be placed? *Ans.* $42\frac{2}{9}$ ft.

120. I use a 20 seconds' glass, and the length of the knot is 34 ft.; find the true distance in an estimated run of 120 miles. *Ans.* $120\frac{15}{19}$.

121. Suppose a 48 ft. knot is used with a 28 seconds' glass; what is the true distance, when the estimated distance is 80 miles?

Ans. $81\frac{27}{19}$.

122. A 15 seconds' glass is used with a $24\frac{1}{2}$ ft. knot; what is the error in an estimated run of 100 miles? *Ans.* $3\frac{1}{3}\frac{1}{8}$.

123. In the last example suppose the knot to be $25\frac{1}{2}$ ft.; what is the error? *Ans.* $\frac{2}{3}\frac{5}{8}$ miles.

*General Formula.**—If either the line or the glass, or both, be erroneous, the true distance may be found generally, whilst the following formula will give the true distance *nearly*.

Let k be the true length of the knot; t the seconds in the same part of an hour that k is of a mile; k' the length of the erroneous knot; t' the seconds run by the erroneous glass; d the distance as determined by these erroneous instruments.

It is required to find D the true distance.

If the log line only be erroneous, the distance will vary inversely, and we have—

$$k : k' :: d : \frac{dk'}{k} \text{ the true distance in this case.}$$

If the log and glass be both erroneous, we have

$$t' : t :: \frac{dk'}{k} : \frac{dk't}{t'k} \text{ or D.}$$

From this formula we can deduce rules for finding the true distance, thus:—

a. Let $t = t'$, that is, let the glass be correct, but the knot erroneous, then the expression becomes $\frac{dk'}{k}$.

b. Let $k = k'$, or let the knot be correct and the glass only erroneous, the expression becomes $= \frac{dt}{t'}$.

c. Let $t = 30$ seconds and $k = 50$ feet, then the general formula becomes $\frac{30dk'}{50t'} = \frac{3}{5}\frac{dk'}{t'}$.

This shows that the true distance is nearly equal to three-fifths of the erroneous distance multiplied by the erroneous knot, and the product divided by the erroneous time in seconds.

This general formula may be obtained more simply, thus: A glass runs out in t seconds, the line is wrongly knotted, and its length is l. Find the true distance (D) when it has been calculated that the ship has gone d miles.

(1.) When the knot is correct and the glass wrong: we find the correct glass to the given knot by direct proportion,

* Adapted from Edward Riddle's *Navigation and Nautical Astronomy*. Baldwin. 1824.

remembering that this is the case where the distance varies directly as the time.

The first step here is to find the length of the knot to the sand glass that will run out in t seconds of time.

Sec. Sec. Ft. in a knot.

$$\text{As } 3600 : t :: 6080 : \frac{t \times 6080}{3600} \text{ correct length.}$$

(2.) When the log line varies and the glass remains the same: we have then an inverse proportion, and to find the true distance, knowing the erroneous distance and the lengths of the knots in the two cases. The following inverse proportion exists according to previous reasoning:—

$$\frac{\text{True knot}}{\text{Erroneous knot}} = \frac{\text{Erroneous distance}}{\text{True distance}}$$

Or, true knot : erroneous knot :: erroneous distance : true distance.

Substituting in this proportion the true length of knot just found—

$$\frac{6080 \times t}{3600} : l :: d : \mathrm{D}$$

$$\therefore \mathrm{D} = \frac{ld}{t} \times \frac{3600}{6080}$$

$$= \frac{3}{5} \times \frac{dl}{t}$$

since $\frac{3600}{6080} = \frac{3}{5}$ very nearly.

124. Find the length of a knot to a glass that runs out in 25 seconds.

One hour Sec. One mile.

$$\text{As } 3600 : 25 :: 6080 : \textit{Ans.}$$

$$= 42\tfrac{2}{9} \text{ ft.}$$

125. Find what glass I must use with a line knotted at every 55 ft.

One mile Ft. One hour.

$$\text{As } 6080 : 55 :: 3600 : \textit{Ans.}$$

$$= 32{\cdot}56 \text{ sec.}$$

126. The apparent distance run at sea is 48 miles when the glass runs out in 33 sec.; what is the true distance run, length of knot being 54 ft.? *Ans.* 47·1 or 46·5.

127. Explain the mode of estimating the distance a ship has run by the log. Supposing the log line is wrongly divided, obtain a formula for finding the true distance run by it. (1863.)

Ans. $\mathrm{D} = \frac{8}{5} \frac{dl}{t}$

128. How is a ship's rate of sailing estimated at sea? Describe the log fully. (1864.)

129. Given that the knot as shown by the log line is erroneous, show how to correct the estimated distance run by a ship. (1864.)

130. What are the instruments used at sea to estimate the distance run by a ship during any time? Describe them fully, and show how to allow for any known errors in them. (1866.)

131. How is the distance run at sea found? If the length of the knot or the time in which the glass runs out be incorrect, how must the true distance be found?

132. The apparent distance run is 37 miles, the length of the knot is 53 ft., the glass runs out in 28 sec.; what is the true distance run? (1865.) *Ans.* 41·4 or 42·021 nearly.

133. Explain the log line, and show to what errors it is liable. (1865.)

134. What is the true distance run by a ship when the distance by the log is 78 miles, the length of the knot is 47 ft., and the number of seconds the sand glass takes to run out is 27? (1865.)
Ans. 80·4 or 81·46 nearly.

135. Describe the instrument commonly used for estimating the distance run at sea.

136. What should be the length of the knot on the log line when the sand glass runs out in 21 sec., a nautical mile being assumed to be 6080? (1871.) *Ans.* $35\frac{7}{15}$ ft.

137. A log line is knotted every 45 ft.; find what glass must be employed. *Ans.* 26·6 sec.

138. A glass that runs out in 35 sec. is employed on board a ship; what length of knot is used to correspond with it? *Ans.* 59·1 ft.

139. Describe the log, and show how it is employed to find the distance run. If the distance run by log is 315 miles, and the length of the log line is 47 ft., and the glass runs out in 32 sec., what is the true distance run? (1866.) *Ans.* 277·59 or 273·9.

140. How is the distance run by a ship at sea found? If the length of a knot on the log line is 47 ft., and the distance run is apparently 18 miles; what is the true distance run (glass runs out in 27 sec.)? (1866.) *Ans.* 18·8 or 18·5.

141. Describe the log line, and show how it is divided. (1867.) The apparent distance run at sea is 48 miles when the glass runs out in 33 sec.; what is the true distance run (length of knot 56 ft.)? (1866.) *Ans.* 48·87 or 48·2.

142. What is meant by the terms *log ship, log line?* How is the log line attached to the log ship? What is the object of this mode of attachment? What is the stray line? Suppose a nautical mile to be 6080 feet; what must be the length of a knot on the log line when the glass runs out in 28 sec? (1869.) *Ans.* 47·3 ft. nearly.

To many of these questions two answers are given; the reason for such a proceeding is this:—One answer is obtained by substituting in the formula $D = \frac{3}{5} \cdot \frac{dl}{t}$ which is not

exactly correct, but very near the true answer, as previously explained; the other answer has been found by first obtaining the proper length of knot to the glass given, and then the true distance by the proper proportion existing between true distance, erroneous distance, true knot, and erroneous knot; or, more rapidly, the answers may be found out by substituting in the formula

$$\frac{45}{76} \cdot \frac{d\,l}{t}$$

which was shown to be very nearly equal to

$$\frac{3}{5} \cdot \frac{d\,l}{t}$$

143. What do you mean by the terms (1) log, (2) log line, (3) stray line, (4) log glass, (5) heaving the log?

144. Give a description of the (1) common log and log line, (2) of Massey's Patent Log, (3) of Walker's Harpoon Log.

145. To what errors are we liable in finding the distance by the log and log line?

146. How do the atmosphere and its temperature affect the log, log line, and log glass?

147. Show how the length of a nautical mile and knot are found?

148. Show that the relation between a nautical mile and a statute mile is as 7 : 6 nearly?

149. Prove that true distance is very nearly equal to $\frac{3}{5}$ of the product of the erroneous distance and the erroneous knot divided by the erroneous time.

TABLE OF LENGTHS OF KNOT TO DIFFERENT GLASSES.

Seconds.	K. in ft.	Seconds.	K. in ft.
15	$25\frac{1}{3}$	31	$52\frac{16}{45}$
16	$27\frac{1}{15}$	32	$54\frac{2}{45}$
17	$28\frac{32}{45}$	33	$55\frac{11}{15}$
18	$30\frac{2}{5}$	34	$57\frac{19}{45}$
19	$32\frac{4}{45}$	35	$59\frac{1}{9}$
20	$33\frac{7}{9}$	36	$60\frac{4}{5}$
21	$35\frac{7}{15}$	37	$62\frac{22}{45}$
22	$37\frac{7}{45}$	38	$64\frac{8}{45}$
23	$38\frac{38}{45}$	39	$65\frac{32}{45}$
24	$40\frac{8}{15}$	40	$67\frac{5}{9}$
25	$42\frac{2}{9}$	41	$69\frac{11}{45}$
26	$43\frac{41}{45}$	42	$70\frac{14}{15}$
27	$45\frac{3}{5}$	43	$72\frac{28}{45}$
28	$47\frac{13}{45}$	44	$74\frac{14}{45}$
29	$48\frac{44}{45}$	45	76
30	$50\frac{2}{3}$	46	$77\frac{31}{45}$

CHAPTER IV.

PLANE SAILING.

Definition—Distance—Difference of Latitude—Departure—Course—Illustrations and Examples—Rules.

We are now in a position to begin the practical part of Navigation. Having shown what instruments the sailor requires and employs, it remains to illustrate and explain how he makes the various calculations that determine, with sufficient accuracy, the position of the ship.

47. Plane Sailing.—Plane sailing is sailing a ship, or making the arithmetical calculations for so doing, on the assumption that the earth is perfectly flat, that all meridians are right lines parallel to each other and perpendicular to the equator, while the "parallels of latitude" are right lines equal and parallel to each other, and parallel to the equator; the lengths of the degrees on the meridians, equator, and parallels of latitude being consequently assumed as all equal. It is not a strictly correct supposition to take any part whatever of the earth's surface as a plane; yet, when the vessel goes on short voyages, the results obtained by plane sailing will be sufficiently correct to serve every useful purpose. Plane sailing may be considered as the application of plane trigonometry to the solution of the several cases of plane triangles, where the hypothenuse or longest side is the rhumb line on which the vessel sails. A rhumb line has been previously defined, but it is important to remember that it is really a *curved* line which cuts all the meridians it passes at the same angle, or it is the track of the ship so long as she keeps on the same course. The lines proceeding from the centre of the compass card to the points are frequently termed ***rhumbs.***

Plane sailing cannot always be advantageously employed, although in practice scarcely any other rules are used but those derived from plane sailing. It should not be employed in long voyages unless frequent astronomical observations are made to correct the errors introduced by its false assumptions. But it can always, as previously hinted, be freely used when the distances are short; when near the equator, where the differences of latitude and longitude are very nearly equal; and when the angle which the rhumb line makes with the meridian is small. Plane sailing, considering the length of voyages now-a-days, ought to go out of fashion; but it never will until our seamen and their officers are more highly educated than at present, nor till the nation insist that a thorough *mathematical* education shall be given to officers both in our royal and mercantile navies. The great and serious objection to plane sailing is, that longitude cannot be found by it *accurately*, although in practice it is more frequently found by it than by any other method.

Let us imagine that the circle in the adjoining figure represents the horizon of the place O, where we will suppose a ship to lie ready to proceed on her voyage. O A, O B, O C, or O D may be taken as the rhumb on which our ship sails; A, B, C, or D the place arrived at; then W E represents the "latitude from," while A S H, B N I, C G K, and D F L represent in each case the "latitude in." Perhaps it will conduce to simplicity if we take one of the four triangles and show what each part of the triangle stands for, and then generalise.

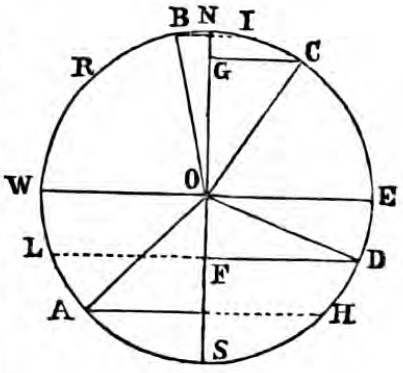


Fig. 12.

In any right-angled triangle the course may be represented by one of the acute angles; the side opposite to that angle will be the departure; the adjacent side the difference of latitude; and the hypothenuse the distance.

Let us take the triangle A O S. Suppose a ship start from O and sails S.W., or at an angle of 45°, and arrives at the place A. From A draw a perpendicular A S upon the line

S G, which represents the meridian sailed from, or passing through O.

Then A S is the *departure.*
S O ,, *difference* of latitude.
A O ,, *distance* or rhumb line.
A O S ,, *course.*

48. Distance is the number of nautical miles a ship sails in any given direction; or it is the number of nautical miles on the rhumb line between the points of departure and arrival.

49. Course.—Course is the angle the ship's track makes with the meridian, it is measured from *north* or *south towards* east and west, either in points of the compass or degrees. The course is, in working out problems, put down in so many points or degrees *right or left* of south or north, and right or left to a person standing in the centre of the compass, looking N. when the course is north, but looking S. when the course is south.

50. Departure.—Departure is the distance a ship has made good, due east or west from the meridian it started from.

The above definition of departure is not mathematically accurate, we will just allude to a more precise one which will be fully exemplified under the heading Mercator's Sailing. If we imagine that the rhumb line, intercepted between any two points on the earth's surface, is divided into an indefinite number of small equal parts, and that each part is the hypothenuse of a right-angled triangle, one side of which is the arc of a meridian, and the other side the arc of a parallel, then the sum of the arcs of a parallel, forming the second side of the triangle, is called the departure.

We will now take the four triangles A O S, B O N, C O G, and O D F, and point out the distance, departure, difference of latitude, and course.

The four distances are represented by the four lines O A, O B, O C, O D.
,, departure, ,, ,, A S, B N, C G, D F.
,, differences of latitude, ,, O S, O N, O G, O F.
,, course, ,, ,, angles AOS, BON, COG, DOF.

In Navigation we are compelled to assume that the

learner has a knowledge both of trigonometry and logarithms, we therefore proceed at once on that assumption without stopping to explain anything connected with those two studies.

For illustration, triangle O G C is taken; we have constructed angle C O G three points, or 33° 45′; angle D O F six points, or 67° 30′; angle A O S four points, or 45° 0′; angle B O N one point, or 11° 15′.

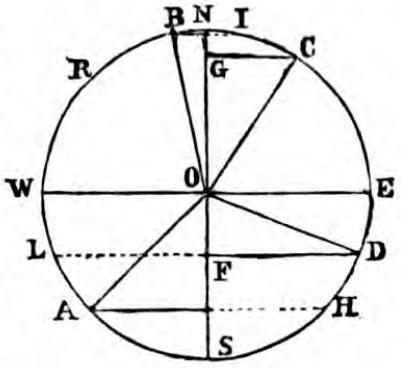


Fig. 13.

Now in triangle G O C.

$$\text{Sin. } O = \frac{C\,G}{C\,O}, \text{ or sin. course} = \frac{\text{departure}}{\text{distance}}$$

$$\therefore \text{Departure} = \text{distance} \times \text{sin. course}$$

$$\therefore \text{Distance} = \frac{\text{departure}}{\text{sin. course}} = \text{departure} \times \text{cosec. course}$$

$$\text{Cos. } O = \frac{O\,G}{O\,C} \quad \text{cos. course} = \frac{\text{difference of latitude}}{\text{distance}}$$

$$\therefore \text{Difference of latitude} = \text{distance} \times \text{cos. course}$$

$$\therefore \text{Distance} = \frac{\text{difference of latitude}}{\text{cos. course}} = \text{diff. of lat.} \times \text{secant course}$$

$$\text{Tan. } O = \frac{C\,G}{O\,G}, \text{ or tan. course} = \frac{\text{departure}}{\text{diff. of lat.}}$$

As an exercise, let the student write all the above formulæ over again until thoroughly conversant with them, using the triangles A O S, B O N, and D O F.

For a more general proof of these things the learner is referred to the proofs of middle latitude sailing which he will now be able to comprehend, and he will see that all rules depend on the simple definitions now being given. The formulæ given above can be put into eleven or twelve different forms to find course, distance, and departure; but as it would practically serve no good purpose to give them in detail, we leave it for an exercise of the student's ingenuity to write them down for himself, and to give the form for logarithmic computation at the same time.

In solving some of the examples that follow the illustra-

tions, it will perhaps be well to point out that when a vessel sails due north or south, she only alters her latitude, when due east or west the departure or longitude only is affected. Should the vessel sail on one of the four courses, making an angle of 45° with the meridian, her difference of latitude and departure are the same.

150. A ship sails from latitude 21° 16′ 45″ N., in the direction O C, that is, on a N.E. by N. course, for 70 miles; find (*a*) the departure, (*b*) the difference of latitude, and (*c*) the latitude in.

The triangle O G C will represent all that is wanted: the distance O C is 70 miles, the course is N.E. by N., or angle G O C is 33° 45′. The quantities represented by G C the departure, and O G the difference of latitude have now to be found.

(*a.*) *To find departure: given course* N. 33° 45′ E., *and distance* 70 *miles.*

$$\text{Sin. course} = \frac{\text{G C}}{\text{O C}} = \frac{\text{departure}}{\text{distance}}$$

$$\therefore \text{Departure} = \text{sin. course} \times \text{distance}$$
$$= \text{sin. } 33° 45′ \times 70$$

Log. sin. 33° 45′ course = 9·744739
Log. .....70 distance = 1·845098
Log. 38·89.....departure = 1·589837

(*b.*) *To find the difference of latitude: gvien course* N.E. *by* N., *and distance* 70 *miles.*

$$\text{Cos. course} = \frac{\text{O G}}{\text{O C}} = \frac{\text{diff. lat.}}{\text{dist.}}$$

$$\therefore \text{Diff. lat.} = \text{cos. course} \times \text{distance}$$
$$= \text{cos. } 33° 45′ \times 70$$

Log. cos. 33° 45′ course = 9·919846
Log. .....70 distance = 1·845098
Log. 58·21 diff. of lat. = 1·764944

60
12·60

$$\therefore \text{Diff. lat.} = 58′ 12″ \text{ N.}$$

(*c.*) *To find the latitude in.*

Latitude from is..... 21° 16′ 45″ N.
Difference of latitude..... 58 12 N.
∴ Latitude in is..... 22 14 57 N.

Ans. Dep. = 38·89 miles; diff. lat. = 58′ 12″; lat. in 22° 14′ 57″ **N.**

RULES.—*To find departure, difference of latitude and latitude in, when course, distance, and latitude from are known.*

(*a.*) *To find departure.*—To the logarithm of sine course add log distance; this sum, omitting 10 from the index of the log, is the logarithm of the departure.

(*b.*) *To find difference of latitude.*—To the logarithm of cosine course add log distance; this sum, omitting 10 from the index, is the logarithm of the difference of latitude in miles.

(*c.*) *To find latitude in.*—To the latitude from apply the difference of latitude.

151. A ship sails from latitude 21° 16′ 45″ N., till she is in latitude 21° 56′ 45″ N. on a N. by W. course; find the departure and distance run.

Triangle O B N will represent the figure we require, B N I represents latitude 21° 56′ 45″ N., and W O E latitude 21° 16′ 45″ N., the course is angle B O N or N. 11° 15′ W. = N. by W. We know O N by subtracting one latitude from the other, when O B and B N the departure and distance have to be found.

(*a.*) *To find difference of latitude.*

Latitude from is.....	21° 16′ 45″
Latitude in is	21 56 45
∴ Difference of lat. is	40

(*b.*) *To find the distance.*

We have now found difference of latitude, and we know that difference of latitude is connected with the cosine. Hence from triangle O B N

$$\text{Cos. course} = \frac{\text{O N}}{\text{O B}} = \frac{\text{diff. of lat.}}{\text{distance}}$$

$$\therefore \text{Distance} = \frac{\text{diff. lat.}}{\text{cos. course}}$$

$$= \text{diff. of lat.} \times \text{sec. course}$$

$$= 40 \times \text{sec. } 11° 15′$$

Log. secant 11° 15′ course	= 10·008426
Log.....40 diff. of lat.	= 1·602060
Log. 40·78 miles distance	= $\overline{1}$·610486

(*c.*) *To find the departure.*

We have now to find departure, knowing distance and course. Departure is connected with the sine. We use the same rule as in the previous question.

$$\text{Sin. course} = \frac{\text{B N}}{\text{O B}} = \frac{\text{departure}}{\text{distance}}$$

$$\therefore \text{Departure} = \text{sin. course} \times \text{distance}$$

$$= \text{sin. } \textit{one} \text{ point} \times 40{\cdot}78$$

Log. sin. 11° 15′ course = 9·290236
Log. 40·78 distance = 1·610486
Log. 7·95 miles departure = 0·900722

Ans. Distance = 40·78 miles; departure = 7·95 miles.

RULES.—*To find distance and departure when course and difference of latitude are known.*

(*a.*) *To find distance.*—To log. secant course add log. difference of latitude, omitting ten from the sum, we have the logarithm of the distance.

(*b.*) *To find departure.*—The same rule as in last example.

152. A ship sails from latitude 21° 16′ 45″ N. on an E.S.E. course till she has made a departure of 39 miles; required the distance run and the latitude in.

The triangle O D F will apply to this case. Angle F O D represents 6 points or E.S.E., the departure is the line F D = 39 miles; the distance O D and the difference of latitude O F are to be found.

(*a.*) It will be convenient to find the distance first, which, as course is given, depends upon the departure, which, as observed before, determines the sine of the angle.

$$\text{Sin. F O D} = \frac{\text{D F}}{\text{O D}} = \frac{\text{departure}}{\text{distance}}$$

$$\therefore \text{Distance} = \frac{\text{departure}}{\text{sin. course}}$$

$$= \text{departure} \times \text{cosec. course}$$

$$= 39 \times \text{cosec. 6 pts.}$$

Log. cosec. 6 points course = 10·034385
Log. 39m. departure = 1·591065
Log. 42·21 distance = 1·625450

(*b.*) The distance now is given with the course to

find the difference of latitude. The latter represents the cosine.

$$\therefore \text{ Cos. course} = \frac{\text{O F}}{\text{O D}} = \frac{\text{diff. of lat.}}{\text{distance}}$$

$$\therefore \text{ Diff. lat.} = \text{cos. course} \times \text{distance}$$
$$= \text{cos. 6 points} \times 42{\cdot}21.$$

Log. cos. 6 points course = 9·582840
Log. 42·21 m. distance = 1·625450
Log. 16·15 miles diff. lat. = 1·208290

60
9·00

(*c.*) *To determine latitude.*

Latitude from is ..... 21° 16′ 45″ N.
Difference of latitude..... 16 9 S.
∴ Lat in 21 0 36 N.

Ans. Distance 42·21 m.; diff. lat. 16′ 9″ m.; and latitude in 20° 0′ 36″ N.

By this problem we are supplied with a second rule for ascertaining the distance.

RULE.—*To find distance and difference of latitude when the departure and course are given.*

(*a.*) *To find distance.*—To log. co-secant course add log. departure, omit 10 from the index, and the result is log. distance.

(*b.*) *To find difference of latitude.*—Rule as in the first illustration.

153. A ship sails from latitude 21° 16′ 45″ N., between S. and W., for 45 miles, till she finds herself in latitude 20° 43′ 51″ N.; what course has she steered, and what departure has she made?

Triangle A O S is adapted for this example. The distance O A is known to be 45 miles, and the difference of latitude is found thus—

Latitude from is ..... 21° 16′ 45″ N.
Latitude in is .... 20 43 51 N.
∴ Difference of lat. is ... 0 32 54 S. = 32·9 miles.

(*a.*) Since difference of latitude depends upon the cosine, we start thus:—

$$\text{Cos. course} = \frac{\text{O S}}{\text{O A}} = \frac{\text{diff. lat.}}{\text{dist.}} = \frac{32{\cdot}9}{45}$$

Log. 32·9, diff. of lat., is = 1·517196
Log. 45 m. distance = 1·653213
Log. cos. 43° 1′ course = 9·863983

(*b.*) With the course and distance it is now easy to determine the departure as before.

$$\text{Sin. course} = \frac{\text{A S}}{\text{A O}} = \frac{\text{departure}}{\text{distance}}$$

$$\therefore \text{Departure} = \text{sin. course} \times \text{distance}$$
$$= \text{sin. } 43° \, 1' \times 45 \text{ miles.}$$

Log. sin. 43° 1′, course, = 9·833919
Log. 45 m. distance = 1·653213
Log. 30·7 miles, departure, = 1·487132
Ans. Course S. 43° 1′ W.; departure 30·7 miles.

RULE.—From hence we obtain a rule for finding the course.

To find course and departure when distance and difference of latitude are known.

To find course.—From log. difference of latitude subtract log. distance, mentally adding 10 to the index of the first logarithm, the remainder is log. cosine course.

To find departure.—Proceed as before.

154. A ship sails from latitude 44° 30′ N., between N. and W., for 200 miles, when her departure is found to be 155 miles; find the true course on which the vessel sailed, and the difference of latitude. Suppose variation is 18° 15′ E., and deviation 5° W.; find the compass course.

(1.) The distance is given 200 miles.
(2.) The departure is given 155 miles.

(*a.*) Distance and departure are connected by sine, we therefore commence with

$$\text{Sine B O N} = \frac{\text{B N}}{\text{B O}} = \frac{\text{departure}}{\text{distance}} = \frac{155}{200}$$

Log. 155 m. departure = 2·190332
Log. 200 m. distance = 2·301030
Log. sin. 50° 48′ course = 9·889302

(*b.*) We now proceed to find difference of latitude, which is connected with the cosine.

$$\text{Cos. course} = \frac{\text{O N}}{\text{O B}} = \frac{\text{diff. lat.}}{\text{distance}}$$

$$\therefore \text{Diff. lat.} = \text{cos. course} \times \text{distance}$$
$$= \text{cos. } 50° \, 48' \times 200$$

Log. cos. 50° 48′ course = 9·800737
Log. 200 distance = 2·301030
Log. 126·4 diff. of lat. = 2·101767

60

24·0

(*c.*) To find latitude arrived at, we have

Latitude from.....	44°	30′	0″	N.
Difference of lat.	2	6	24	N.
Latitude in	46	36	24	N.

Ans. Course N. 50° 48′ W.; diff. of lat. 126′ 24″; lat. in 46° 36′ 24″ N.

We have next to find compass course.

True course is	50°	48′	l. of N.
Variation E.....	18	15	l.
	69	3	l.
Deviation W.	5	0	r.
Compass course .....N.	64	3	W.

RULES. —*To find course and difference of latitude when distance and departure are given.*

To find course.—From log. departure subtract log. distance, mentally adding 10 to the index of the departure log., the remainder is the logarithm of sin. course.

Difference of latitude and latitude in are found as in the previous questions.

155. A ship sails from latitude 44° 30′ N., between S. and E., till her departure is 80 miles, and difference of latitude 169 miles; find the true course on which she sailed, the distance, and the compass course, with 31° 20′ E. variation, and 9° W. deviation.

(*a.*) The triangle O D F will do for our figure. We have given

Departure 80 miles
Diff. of lat. 169

On these two depend the tangent, hence:—

$$\text{Tan. F O D} = \frac{\text{D F}}{\text{O F}}$$

$$\text{Tan. course} = \frac{\text{departure}}{\text{diff. of lat.}} = \frac{80}{169}$$

Log. 80 departure $= 1{\cdot}903090$
Log. 169 diff. lat. $= 2{\cdot}227887$
Log. tan. 25° 20′ course $= \overline{9{\cdot}675203}$

(*b.*) As we have now course, and both difference of latitude and departure, we may use either sin. or cosine.

$$\text{Sin. course} = \frac{\text{D F}}{\text{O D}} = \frac{\text{dep.}}{\text{dis.}}$$

$$\therefore \text{Dis.} = \frac{\text{dep.}}{\text{sin. course}}$$

$$= \text{dep.} \times \text{cosec. course}$$

$$= 80 \times \text{cosec. } 25° \ 20'$$

Log. 80 departure	= 1·903090
Log. cosec. 25° 20′	= 10·368674
Log. 186·9 distance	= 2·271764

$$\text{Cos. course} = \frac{\text{O F}}{\text{O D}} = \frac{\text{diff. lat.}}{\text{dist.}}$$

$$\therefore \text{Dis.} = \frac{\text{cos. course}}{\text{diff. lat}}$$

$$= \text{diff. lat.} \times \text{sec. course}$$

$$= 169 \times \text{sec. } 25° \ 20'$$

Log. 169 diff. lat.	= 2·227887
Log. sec. 25° 20′	= 10·043911
Log. 186·9 distance	= 2·271798

(*c.*) To find latitude in, we have—

Latitude from	44° 30′ 0″ N.
Diff. of latitude 169	2 49 0 S.
Lat. arrived at	41 41 0 N.

(*d.*) To find compass course.

True course is	25° 20′ 0″ l. of S.
Variation E.	31 20 0 l.
	56 40 0 l.
Deviation W.	9 0 0 r.
Compass course ... S.	47 40 0 E.

Ans. True course = S. 25° 20′ E.; distance = 186.9 miles.
Lat. in = 41° 41′ N.; compass course = S. 47° 40′ E.

RULES.—*Another rule for course is deduced from this problem.*

To find course when departure and difference of latitude are given. From log. departure subtract log. difference of latitude, mentally adding 10 to the index of the upper logarithm, the remainder is the logarithm tan. true course.

51. Rules for Plane Sailing.—The whole of the rules in plane sailing are here collected for the sake of simplicity, and to render them easily accessible.

RULES FOR COURSE.

(1.) When distance and difference of latitude are given. From log. difference of latitude subtract log. distance, mentally adding 10 to the index of the former logarithm, the remainder is log. cosine course.

(2.) When departure and distance are given. From log.

departure subtract log. distance, mentally adding 10 to the index of the former logarithm, the remainder is log. sine course.

(3.) When departure and difference of latitude are given. From log. departure subtract log. difference of latitude, mentally adding 10 to the former logarithm, the remainder is the logarithm of the tangent of the course.

RULES FOR DISTANCE.

(1.) When course and difference of latitude are given. To log. secant course add log. difference of latitude, omitting 10 from the index of the sum, gives the logarithm of the distance. Or, subtract log. cosine course from log. difference of latitude, mentally adding 10 to the upper logarithm, the remainder is the logarithm of the distance.

(2.) When departure and course are given. To log. cosecant course add the logarithm of the departure, omitting 10 from the index, the sum is the logarithm of the distance. Or, from log. departure subtract log. sine course, mentally adding 10 to the index of the upper log., the remainder is the logarithm of the distance.

RULES FOR DIFFERENCE OF LATITUDE.

(1.) When course and distance are given. To the logarithm of cosine course add the logarithm of the distance, the sum, omitting 10 from the index, is the logarithm of the difference of latitude in miles.

(2.) When departure and course are given. To the logarithm of the co-tan. course add log. departure, the sum, omitting 10 from the index, is the logarithm of the difference of latitude in miles. Or, from log. departure subtract log. tan. course, mentally adding 10 to the former log., the remainder is log. diff. of lat. in miles.

RULES FOR DEPARTURE.

(1.) When course and distance are given. To the logarithm of sine course add the logarithm of the distance, omitting 10

from the index, the sum is the logarithm of the departure in miles.

(2.) When course and difference of latitude are given. To logarithm of tan. course add logarithm of difference of latitude, the sum, omitting 10 from the index, is the departure in miles.

MISCELLANEOUS EXAMPLES IN PLANE SAILING.

156. Required the compass course and distance from A to B.
Lat. A 50° 20′ N.; var. 23° 16′ W.; lon. A 5° 15′ W.
Lat. B 51° 50′ N.; dev. 8° 4′ E.; lon. B 5° 15′ W.
Ans. Dist. 90 m.; C. C. N. 15° 12′ E.

157. Required the magnetic course and distance from A to B.
Lat. A 50° 22′ N.; var. 20° 4′ W.; lon. A 4° 8′ W.
Lat. B 48° 46′ N.; dev. 7° 3′ W.; lon. B 4° 8′ W.
Ans. Dist. 96 miles; M. C. S. 27° 7′ W.

158. Required the compass course and distance from A to B.
Lat. A 57° 18′ S.; var. 1½ E.; lon. A 18° 19′ E.
Lat. B 63° 19′ S.; dev. 3° W.; lon. B 18° 19′ E.
Ans. 361 miles; C. C. S. 13° 52′ 30″ E.

159. Cape Finisterre (lat. 42° 54′ N., lon. 9° 16′ W.) bears from a ship N.E.½-E. distant 120 miles; what is her latitude? (1873.)
Ans. 41° 37′ 54″ N.

160. Show that course, distance, difference of latitude, and departure may be represented by an angle and three sides of a right-angled triangle. Hence obtain formulæ connecting (1) course, departure, and diff. lat.; (2) course, distance, and departure; (3) course, distance, and diff. lat. (1869.)

161. A ship sails from Odessa, lat. 46° 30′ N., lon. 30° 45′ E., and, after 24 hours, is in lat. 43° 18′ N., having made 135 miles of easting; required her compass course and distance. Variation 1¼ points E.; deviation 6° 25′ E.; leeway 1¼ points; wind S.S.W. (1869.)
Ans. Dist. 234·7 miles; comp. course S. 41° 32′ E.

162. A ship is known to be sailing due S., but the compass course steered is S. by W.¾-W. The variation of the compass is 1½ points W.; what is the deviation? If the latitude left is 5° 18′ N., and latitude arrived at is 7° 44′ S.; what is the distance run? (1871.)
Ans. Deviation 2° 48′ 45″ W.; distance 782 miles.

163. Find the compass course and distance from A to B. Given:
Lat. A 36° 18′ N.; variation ¾ pts. E.; lon. A 45° 18′ E.
Lat. B 13° 27′ N.; deviation 4° E.; lon. B 45° 18′ E.
Ans. Distance 1371 m.; C. C. S. 12° 26′ 15″ E.

164. A vessel sails from Cowes, latitude 50° 45′ 37″ N., longitude

1° 16′ 15″ W., on a true S. course for 54 miles; required the latitude and magnetic course when variation is 21° W. and deviation 12° 10′ E.

Ans. Lat. in 49° 51′ 37″ N.
Mag. course S. 8° 50′ W.

165. A ship sails from latitude 41° 15′ N. for 125 miles on a course N. 3½ points W.; find the difference of latitude, departure, and latitude in. *Ans.* Diff. lat. 96·6; dep. 79·3.
Lat. in 42° 51′ 36″ N.

166. A vessel is taken from latitude 44° 30′ N. on N.E.½-E. course till she comes to latitude 47° 40′ 18″ N., what distance has been run, and what departure made? *Ans.* Dis. 300 m.; dep. 231·9 m.

167. A ship from latitude 44° 30′ N. sails S.S.E.¼-E. till she has made 103 miles departure, what distance has she run, and what is the latitude in? *Ans.* 241 miles dis.; diff. lat. 217·9 m.
Lat. in 40° 52′ 6″ N.

168. Suppose a ship to sail from latitude 44° 30′ N. between S. and W. for 270 miles till she is in latitude 40° 25′ 54″ N., what is the true course steered and departure made good?

Ans. Course S.S.W.¼-W.; dep. 115·3 miles.

169. A ship sails from latitude 44° 30′, between N. and W., for 290 miles, when she finds her departure is 161·1 miles; find the true course on which she has sailed, and difference of latitude, and latitude in. *Ans.* Course N.W. by N.
Diff. lat. = 241·1 m.; lat. in 48° 31′ 6″ N.

170. Suppose a ship to sail from latitude 44° 30′ 0″ N., between S. and E., till her departure is 148 miles, and difference of latitude 98·9 miles; find the true course on which she sailed, with the distance and latitude in.

Ans. Course S.E. by E.; distance 178 miles.
Latitude in 42° 51′ 6″ N.

171. Required the compass course and distance from A to B. Given:

Lat. of A 51° 25′ N.; var. 19° 20′ W.; lon. 9° 29′ W.
Lat. of B 49° 16′ N.; dev. 3° 16′ E.; lon. 9° 29′ W.

Compass course S. 16° 4′ W.
Distance 129 miles.

172. Required the magnetic course and distance from A to B. Given:

Lat. of A 49° 53′ 37″ N.; var. 2 points E.; lon. 6° 20′ W.
Lat. of B 49° 0′ 37″ N.; dev. 4° 15′ E.; lon. 6° 20′ W.

Mag. course S. 26° 45′ E.
Distance 53 miles.

173. Give a demonstration of the principles of plane sailing, with the definition of departure (1862). Explain the terms course and departure, and show that diff. lat. = dis. × cos. course. (1863 and 1868.)

174. Prove that departure = distance × sin. course. (1863.)

175. Required the compess course and distance from A. to B. Given:

Lat. A 3° 18′ N.; var. 2½ pts. E.; lon. A 30° 25′ W. (1864.)
Lat. B 12° 51′ S.; lon. B 30° 25′ W.

Ans. Course S. 28° 7′ 30″ E.; distance 933 miles.

176. Draw a figure illustrating the definitions of the following terms in navigation:—(1) Rhumb line; (2) course; (3) distance; (4) difference of latitude; (5) departure. (1864.)

177. Prove that distance = departure × cosec. course. (1864.)

178. Show by a figure what is meant by the terms distance, course, and departure. (1865.)

179. What is meant by the departure?

How is the departure estimated when a vessel sails on a given course from one point to another on the earth's surface? (1868.)

180. Find the compass course and distance between A and B.

Given latitude A 23° 18′ S.; longitude A 18° 20′ W.
,, ,, B 31° 53′ S.; longitude B 18° 20′ W.
Variation 1½ point W.; deviation 3° W. (1868.)

Ans. Course S. 19° 52′ 30″ W.; distance 515 miles.

181. Required the compass course and distance from A to B.

Lat. A 73° 30′ N.; var. 1½ W.; lon. A 43° 20′ E.
Lat. B 78° 41′ N.; dev. 3° W.; lon. B 43° 20′ E. (1865.)

Ans. Course N. 19° 52′ 30″ E.; distance 311 miles.

182. How must I steer by compass when my true course is S.W. by W.¾-W., variation 2½ points E., deviation ¾ point W., leeway ¼ point, wind S. by E.? (1870.) *Ans.* S.W.

183. Define course, distance, and departure; express the departure in terms of the course and distance. (1871.)

184. If a ship sail from Oporto in latitude 41° 9′ N. 275 miles on a N.W.½-W. course; required her departure and the latitude she has arrived at. *Ans.* Departure 212·6 miles; lat. in 44° 3′ 30″ N.

185. A ship sailed from latitude 41° 3′ S., W.N.W., till her departure was 145·1 miles; required the nautical distance she sailed and the latitude in.

Ans. Distance 157 miles; latitude in 40° 2′ 54″ S.

186. A ship being in latitude 10° 4′ S. sailed between N. and E. till she was in latitude 8° 12′ S., she made 80 miles departure, what was the true course and distance? (2) find the magnetic course, with variation 2 points E., deviation 9° W.

True course N. 35° 32′ E.; distance 137·6
Mag. course N. 22° 2′ E.

187. Suppose a vessel to sail south-westward till her departure is twice her difference of latitude, find the true course; (2) find magnetic course, with variation and deviation amounting to 1¾ points E.

Ans. True S. 63° 26′ W.
Mag. S. 43° 44′¾ W.

188. Define the terms departure, nautical distance, true difference

latitude, and difference longitude, and show that departure = true difference latitude × tan. course. (1872.)

189. A ship sails from latitude 38° 4′ N., S.E. by S., till her departure is 50 miles; required the distance she has sailed and her latitude. Construct a figure. (1872.)

Distance 90 miles; difference of latitude 74·8 miles.
Latitude in 36° 49′ 12″ N.

190. A ship is in latitude 45° 18′ S., longitude 44° 25′ E., and sails in a W.S.W. direction until she is in latitude 47° 14′ S.; required the distance run and the departure. (1872.)

Distance 303·1; departure 280 miles.

191. A ship sailed from Cape Finnisterre, latitude 42° 56′ 10″ N., longitude 9° 16′ 15″ W., on a course between W. and S. till her departure was 100 miles and difference of latitude 19·9; required the true course and distance.

Find the compass course and distance if the variation is 25° 10′ W., deviation 4° E.

True course W. by S.; distance 102 miles.
Compass course N. 80° 5′ W.

CHAPTER V.

TRAVERSE SAILING.

Definitions—Log-Board—Traverse Tables—Resolving a Traverse by Construction and by the Tables—Examples—Illustrations and Exercises.

52. Traverse Sailing or Compound Courses.—A ship can seldom, on account of land, rocks, shoals, etc., make straight for the port to which it is bound. It more frequently happens that the wind is directly or partly against a ship's straightforward course to the place to which she is going. In making any place she reaches it by a zig-zag course, which is made by sailing with the wind first on one side of the ship, then on the other. *Traverse sailing is the art of finding the one course and distance that would bring a ship into the position it has been brought by two or more courses and distances.* It is plane sailing extended to several courses and distances, where the several courses, departures, and differences of latitudes are thrown into one course, one departure, one difference of latitude, and one distance.

53. Log-Board.—In the steerage or some convenient place in the ship is kept a ruled board, termed the *Log-Board*, divided into several columns, on which the various courses, distances, winds, leeway, occurrences, etc., are entered by the master for the day. Generally in the first left-hand column are the hours of the day, in the second and third the knots and miles, in the fourth the winds, in the next the leeway, and in the extreme right hand column any occurrence connected with wind, rain, storm, ships seen, etc. This is transcribed into the log-book each day at noon; but the subject will receive more attention under the heading of the *Day's Work.*

54. Traverse Table.—All operations and calculations in

navigation are greatly facilitated by the use of logarithmic and other tables; the table most frequently employed is the *traverse table.* Its construction requires no great amount of mathematical knowledge, while its usefulness is very great. All distances from 1 to 300 miles are taken, and the departure and difference of latitude calculated for every course, from $\frac{1}{4}$ point to 8 points, and from 1° to 45° by the two formulæ.

Departure = distance × sin. course
Diff. lat. = distance × cos. course.

The courses, distances, and results obtained are all tabulated and called the traverse table. Inasmuch as the sine of an angle is equal to the co-sine of its complement, the difference of latitude for 30° will be the departure for 60°, and the difference of latitude for 60° will be the departure for 30°. The difference of latitude and departure for 1° will be the departure and difference of latitude for 89°. Hence, on the first page of the traverse tables, we have 1°, with distance, difference of latitude, and departure at the top, and 89° at the bottom; on the second page 2°, with distance, etc., at the head, and 88° at the foot; but at the bottom of departure column for 1°, 2°, etc., is *latitude,* and latitude column has *departure* at the bottom. Hence 45° pages complete the traverse table. A traverse table of points, half and quarter points, etc., is also calculated, and is of great convenience. As a matter of fact sailors seldom reckon their courses in degrees, but always put down the course in points, and half and quarter points. The great utility of these tables is, that *all questions in plane sailing* can be solved by looking out for the quantities in the table-book; as we proceed it will be shown how the examples and illustrations from p. 78 to 81 may be solved by the use of the tables alone. Their usefulness is not confined to plane sailing; it will be our duty to show, when we come to them, that parallel sailing and middle latitude sailing may be worked out by them; while, under the Day's Work, a method will be given of finding longitude by these same tables. In the day's work, when the totals of the differences of latitude and departure have been found, the course and distance are found in the tables, but generally in employing the tables, it is the course and dis-

tance that are known, and the difference of latitude and departure that have to be found. From the very beginning of navigation it is a very good custom to find every answer in plane sailing from the table-book *as a check upon* the working by logarithms.

To resolve a traverse or compound course is then to reduce several courses and distances to a single course and distance.

First, we will show how to resolve a traverse geometrically, and next by the usual method of employing the tables.

192. A ship sails from latitude 51° 25′ N., longitude 8° 12′ W.; S.S.E. 30; E.byS. 18; S.W.byW. 36; W.$\frac{3}{4}$-N. 14; and S.E.byE.$\frac{1}{4}$-E. 46 miles: required the equivalent course and distance, and the latitude and longitude of the place arrived at. (1872.)

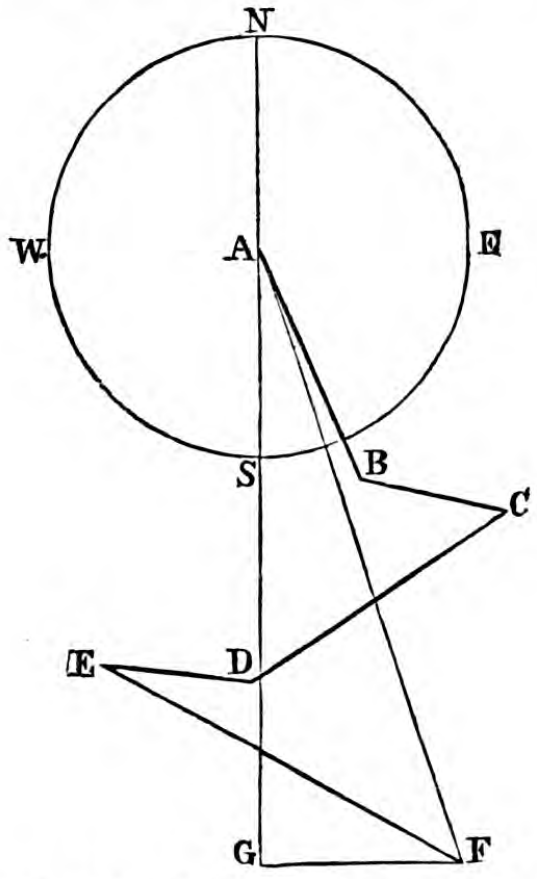


Fig. 14.—*Scale*, 36 miles to 1 inch.

Let A be the centre of the compass. (1) The ship sails S.S.E. 30 miles. The line A B is drawn S.S.E., so that angle G A B is 22° 30′, then A B is marked off as 30 miles.

The scale used is 36 miles to the inch. (2) Then she goes E.byS. 18 miles, so B C is drawn E.byS., and B C cut off 18 miles from the same scale. (3) The ship now sails S.W.byW. 36 miles, therefore C D is drawn five points right of S., and C D taken by scale as 36 miles. (4) The ship next goes W.$\frac{3}{4}$-N. 14, so we draw the line D E at an angle W.$\frac{3}{4}$-N., or 81° 33′ 45″ left of N., and cut off D E 14 miles by scale. (5) Lastly, the ship turns and sails S.E.byE.$\frac{1}{4}$-E., or $5\frac{1}{4}$ points left of S., for 46 miles, so we draw E F in that direction, and make E F equal to 46 miles. This leaves the ship at F, now drop a perpendicular from F upon the meridian left, A G, when F G will be the departure, which is by scale 25 miles. A G is the difference of latitude, by scale 72 miles, while G A F is the course. After measuring this angle we find it to be S. 19° E., and the distance A F is from the scale 77 miles.

Results: Difference of latitude, 72 miles; course, S. 19° E.
Departure,..... 25 miles; distance, 77 miles.

We have now to show how to work out this traverse by the tables.

Courses.	Dist.	Diff. of Lat.		Departure.	
		B.	S.	E.	W.
S.S.E., ... 2 pts.	30		27·7	11·5	
E.byS., ... 7 pts.	18		3·5	17·7	
S.W.byW., ... 5 pts.	36		20		29·9
W.$\frac{3}{4}$-N., ... $7\frac{1}{4}$ pts.	14	2·1			13·8
S.E.byE.$\frac{1}{4}$-E., ... $5\frac{1}{4}$ pts.	46		23·6	39·5	
		2·1	74·8	68·7	43·7
			2·1	43·7	
	Diff. lat. S.		72·7	25	Dep. E.

(*a.*) Draw lines forming six columns: in the first, or left hand column, place the courses in their proper order, and one under the other. In the second column place distances opposite their proper courses. At the head of the next two

columns, put difference of latitude and N. and S., to indicate north and south; over the two remaining put departure E. and W., for east and west.

(*b.*) Open the table-book at page headed "Difference of latitude and departure for 2 points;" look down the distance or first column on the left hand for 30; opposite 30 is found 27·7 and 11·5, which are the difference of latitude and departure for 30 when sailing 2 points from N. or S.; the 27·7 must be placed in the column headed S., and the 11·5 in the column headed E., because the course is S. and E.

For the *second course* E.byS., turn to the page 7 points (marked 1 point at top). Look in the first column on the left hand for distance 18, where 17·7 and 3·5 are placed opposite the 18; but as your course is at the bottom you must read these 3·5 difference of latitude, and 17·7 departure, and place the 3·5 under S. and 17·7 under E.

Third course.—Turn to page 5 points, and opposite the 36 we read the 20 and 29·9 from right to left, not as they are placed in the tables, because the course is at the bottom of the page; place the 20 under S. and 29·9 under W.

Fourth course.—Because beginners are so apt to make mistakes in taking out the difference of latitude and departure when the course is at the bottom of the page, we repeat the process at the risk of monotony. Turn to $7\frac{1}{4}$ points at bottom of page, where, in first left hand columns opposite 14, is seen 13·8 and 2·1, *which we read* 2·1 *and* 13·8, *since the course is at the bottom of the page,* and place them under N. and W. respectively.

Fifth course, $5\frac{1}{4}$ points, is also taken out from bottom, and opposite 46 we read from right to left 23·6 and 39·5, and place them under S. and E.

(*c.*) Add up each column, and subtract the less from the greater in each pair. In this case subtract the 2·1 N. from 74°·8 S., leaving on the whole a total difference of latitude of 72°·7 S. Now subtract the 43·7 from 68·7, leaving 25 miles as the departure towards the E.

(*d.*) Turn to the traverse table again, and find the two numbers 72·7 and 25 opposite each other in the difference of latitude and departure columns. A little practice will soon accustom one to turn to the proper page directly, but we

may always notice this: if the *difference of latitude* is the *greater* of the two numbers, the course is at the *top* of the page; while if *departure* is *greater*, the course is at the bottom of the page. Looking in the traverse table, then, we find 72·7 and 25, or the nearest numbers to them 72·8 and 25·1, together on the page for 19°, and immediately to the left of them is 77 in the distance column. We therefore write down

Course,..... S. 19° E.; distance, 77 miles.
Difference of latitude, 72·7 miles; departure, 25 miles.

The student must notice how far this solution agrees with the previous one.

We have now to find the latitude and longitude. Latitude is easily found; it is rather unusual to find the longitude at this early stage, but the process is so simple and mechanical that a child can comprehend it; therefore it will be introduced here.

Latitude.		Mid-latitude.	Longitude.
Lat. from	51° 25′ 0″ N.	51° 25′ 0″ N.	Lon. from 8° 12′ W.
Diff. lat. 72·7 =	1 12 42 S.	50 12 18 N.	Diff. of lon. 40 E.
Lat. in	50 12 18 N.	2)101 37 18	Lon. in .....7 32 W.
		50 48 39	
		90 0 0	
	Co-mid-latitude,	39 11 21	

Apply the difference of latitude 72·7 to the latitude from, and we have the latitude in 50° 12′ 18″ N.

We next add the latitude *from* and *in* together, and take half the sum, this gives us the middle latitude; we subtract it from 90°, which gives the co-mid-latitude.

Co-mid-latitude is the distance of a place midway between two given latitudes from the nearest pole.

With this co-mid-latitude 39°, we go to the traverse tables, and on page 39° find the departure 25 *miles in the departure column* (nearest is 25·2); corresponding to this is 40 in the distance column: this we apply to the longitude eastwards, because the departure is east, and it gives us longitude in 7° 32′ W.

193. A ship from Rockal, latitude 57° 39′ 32″ N., longitude 13° 31′ 6″ W., ran the following courses and distances; required the compass course and distance, and latitude and longitude in, with variation 17° W., deviation 9° 1′ E.

(*a.*) N.W.byN.....20 miles. S.E.....55 miles.
(*b.*) S.W.....40 ,, W.byS.....41 ,,
(*c.*) N.E.byE.....60 ,, E.N.E.....66 ,,

Courses.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
N.W.byN.,	20	16.6			11·1
S.W.,	40		28·3		28·3
N.E.byE.,	60	33·3		49·9	
S.E.,	55		38·9	38·9	
W.byS.,	41		8·0		40·2
E.N.E.,	66	25·3		61·0	
		75·2	75·2	149·8	79·6
				79·6	
		Dep. 70·2 E.			

True Course E.; distance, 72 miles.

Having filled up the above traverse table, the sum of the northings and southings are equal, so the latitude remains unaltered, or the ship, after six courses and distances, has returned to the same parallel. Altogether, the vessel has gone 149·8 towards the east on three of the courses, while on the other three she has made 79·6 miles to the west, leaving 70·2 miles of progress towards the east, consequently the course made good is due east, and distance 70·2 miles. We will now solve the same by construction, on a scale of 40 miles to the inch.

A is the starting point of the ship. The first course is laid down from a scale of 40 miles to the inch, with A*a* as 20 parts or half an inch, the line A*a* making an angle of three points to the left of N. Next *a b* is drawn one inch long in the direction *a b* making an angle of four points with the meridian. Third, *b c* is drawn one and a half inches long at an angle of five points to the right of N.; *c d* is then drawn S.E. one and three-eighths of an inch in length, after which *d e* is drawn on the W.by S. rhumb line, a trifle over an inch long; and lastly, *e* C is drawn on the

E.N.E. rhumb $1\frac{13}{20}$ of an inch in length, which brings the ship to the point C; take the length of A C from the scale of equal parts of 40 miles to the inch, it is found to be 70 miles in length, and due E., which gives the same course and distance by construction as was previously found by calculation. Note that the parts are given in inches for convenience in this exercise, although in practice they seldom adapt themselves so readily to the parts of an inch, but such an example is selected for simplicity of illustration.

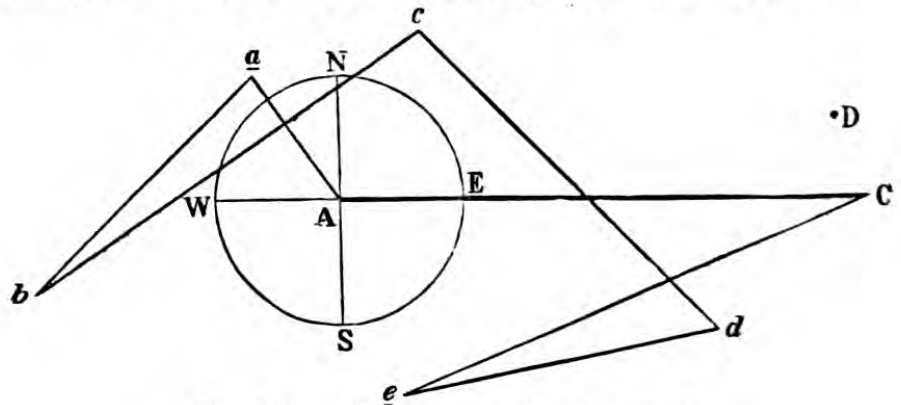


Fig. 15.—*Scale*, 40 miles to 1 inch.

To find latitude and longitude.

Latitude.—The ship making no difference of latitude remains in the parallel of Rockal.

The Longitude.—The middle latitude is here the place started from and arrived at 57° 39′ 32″ N. Therefore co-mid-latitude 32° 20′ 28″.

With this co-mid-latitude we go to the traverse tables, and on page 32° we find departure 70·2 in the departure column; opposite to it in the distance column we have 132m.

This is applied as the difference of longitude E. because the departure is E.

Longitude from,.....	13°	31′	6″	W.
Diff. longitude 132m. =	2	12	0	E.
Longitude in,.....	11	19	6	W.

To find compass course:

True course,.....	90°	0′	0″	l. of S.
Variation W., ...	17	0	0	r.
	73	0	0	l. of S.
Deviation E., ...	9	1	0	l.
Compass course, ...	82	1	0	l. of S.
Compass course,....S.	82	1	0	E.

194. A vessel sails on the following courses and distances from lat. 46° 27′ 36″ N., lon. 61° 56′ 8″ E.; find course, distance, lat. and lon. in.: E. 45 m.; E.byS. 50 m.; E.byN. 30 m.; N.E. 45 m.; N.byW. 30 m.; and N.E.byE. 27 m.

Courses.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
E.	45			45	
E.byS.	50		9·8	49	
E.byN.....	30	5·9		29·4	
N.E.....	45	31·8		31·8	
N.byW.	30	29·4			5·9
N.E.by.E	27	15		22·4	
		82·1	9·8	177·6	5·9
		9·8		5·9	
		72·3 diff. lat. N.		171·7 Dep. E.	

Course, N. 67° E.; distance, 186 miles.

	Latitude.	Mid-latitude.	Longitude.
	46° 27′ 36″ N.	46° 27′ 36″ N.	61° 56′ 8″ E.
Diff. lat.	1 12 18 N.	47 39 54 N.	4 12 0 E.
Lat.	47 39 54 N.	2)94 7 30	Lon. 66 8 8 E.
		47 3 45	
		90 0 0	
		Co-mid-lat. 42 56 15	

With 43° as a course and 171.7 in the departure column, gives 252 in the distance column for difference of longitude.

There is nothing particular to notice in the above example, except the two courses E.byN. and N.byW. 30 miles, where the numbers coincide but interchange, as difference of latitude and departure.

This example is now worked out by construction. Let N.E.S.W. represent the compass. Suppose our ship sails from C.

The 1st course is E.,..... 45 m. = C*a* or 45 parts by scale.
,, 2nd ,, E.byS.,..... 50 ,, = *ab* ,, 50 ,,
,, 3rd ,, E.byN.,..... 30 ,, = *bc* ,, 30 ,,
,, 4th ,, N.E., ... 45 ,, = *cd* ,, 45 ,,
,, 5th ,, N.byW.,..... 30 ,, = *de* ,, 30 ,,
,, 6th ,, N.E.byE.,..... 27 ,, = *e*A ,, 27 ,,

Measuring angle ACK by the protractor, or by a common scale ruler, it is found to be 23°. Therefore course from north is 90° – 23° = N. 67° E.

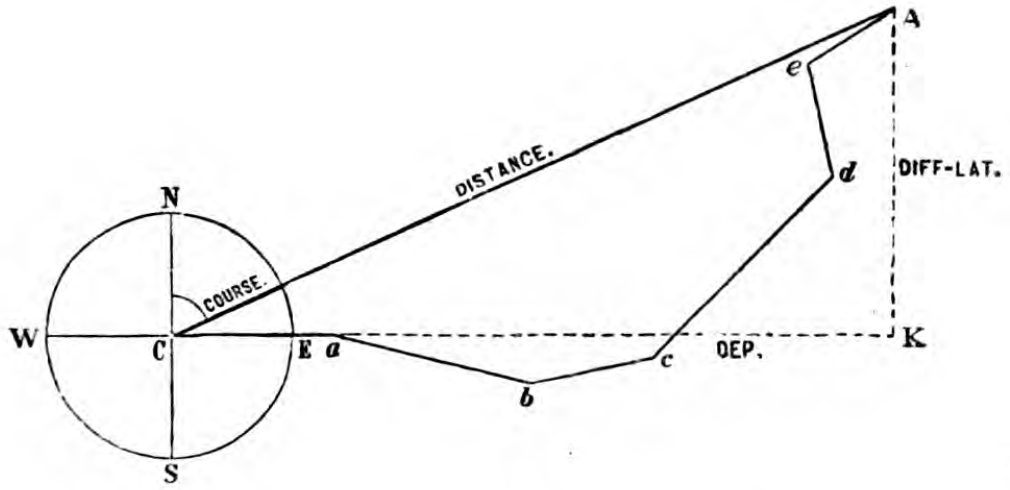


Fig. 16.—*Scale*, 80 miles to the inch.

Measuring the distance by the same scale of eighty parts to an inch gives the length of C A as 186 miles, which answer corresponds exactly with that found before.

In the same way C K departure = 172 miles.
,, A K diff. lat., = 72 miles.

There are still two or three points to which we will call attention in the next example.

195. A vessel sails S. 65° W. 25·7 miles, N. 80° W. 14·7 miles, N. 27° W. 30·6 miles, W. 42·8 miles, N. 18° W. 34·9 miles, S. 72° W. 35 miles, N. 21° W. 24·5 miles; find the course and distance, also latitude and longitude in, supposing the ship to start from Cape Ortegal, lat. 43° 46′ 40″ N., lon. 7° 50′ 30″ W.

(1) It must be observed in this example that the difference of latitude and departure are taken from the traverse table of degrees, the three previous examples from that of points.

(2) In taking out the difference of latitude and departure for 25·7, 14·7, and 24·5, we take them out from opposite the distances 257, 147, and 245, mentally shifting the decimal places one point towards the left hand.

Courses.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
S. 65° W.....	25·7		10·8		23·2
N. 80° W.	14·7	2·5			14·4
N. 27° W.	30·6	27·2			13·8
W.....	42·8				42·8
N. 18° W.....	34·9	33·1			10·7
S. 72° W.	35		10·8		33·3
N. 21° W.....	24·5	22·8			8·7
		85·6	21·6		146·9
		21·6			
		64			

Course, N. 66° W.; distance, 160 miles.

	Latitude.	Mid-latitude.	Longitude.
Lat. from	43° 46′ 40″ N.	43° 46′ 40″ N.	7° 50′ 30″ W.
Diff. lat. 64 =	1 4 0 N.	44 50 40 N.	3 24 0 W.
Lat. in	44 50 40 N.	2)88 37 20	Lon. in 11 14 30 W.
Mid.-lat.		44 18 40	
		90 0 0	
Co-mid-lat.		45 41 20	

With 46° as a course and 146·9 in its proper column, gives 204 in the distance column for difference of longitude, 204 = 3° 24′.

In taking out 30·6 we take out for 30, then go to distance 6, and mentally shifting the decimal place one point to the left, add to the difference of latitude and departure for 30. The same thing is done with 34·9.

In working this and the preceding examples a small scale has been adopted, but learners are advised to draw their figures at first to a larger scale, 20 or 30 miles to the inch, by so doing they will obtain more satisfactory results. The last example by geometrical construction will stand thus—

The 1st course is	S. 65° W.	25·7 = A*a*	or	25·7	parts	by scale
,, 2d ,,	N. 80 W.	14·7 = *ab*	,,	14·7	,,	
,, 3d ,,	N. 27 W.	30·6 = *bc*	,,	30·6	,,	
,, 4th ,,	W.	42·8 = *cd*	,,	42·8	,,	
,, 5th ,,	N. 18 W.	34·9 = *de*	,,	34·9	,,	
,, 6th ,,	S. 72 W.	35 = *ef*	,,	35	,,	
,, 7th ,,	N. 21 W.	24·5 = *f*C	,,	24·5	,,	

It will be found by measurement that the angle NAC or course is N. 66° W., and distance AC 160 miles; while the departure AK and difference of latitude are 64 and 147 respectively, agreeing with the preceding calculations.

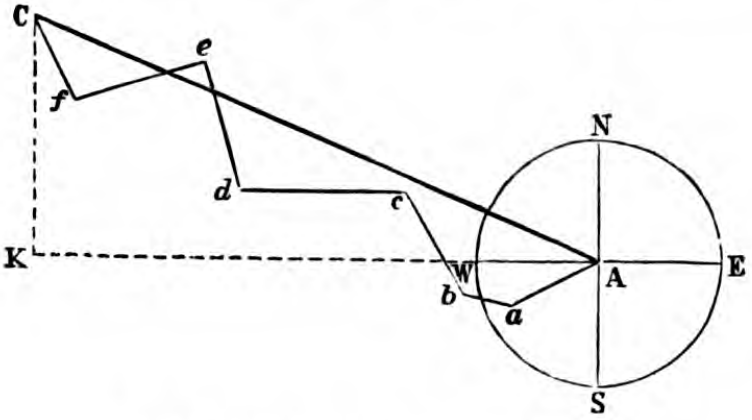


Fig. 18.—*Scale*, 80 miles to the inch.

55. *The method of finding longitude given here is called obtaining the longitude by* **Inspection.** There is a test of accuracy of work which the student should never neglect to employ. As soon as he has taken out his difference of latitude and departure for every course, let him run down the columns, and see that all his northings and southings are in their proper columns; the same with the departure E. and W. Again, notice when the course is greater than 45° that the departure is the greater number of the two taken out, and when the course is less than 45° that the departure is the less number of the two put down opposite the course. These are but simple things, but attention to little points frequently ensures a student's success.

196. A ship sailing from latitude 24° 32′ N. runs the following courses and distances, viz., S.W.byW. 45 miles, E.S.E. 50 miles, S.W. 30 miles, S.E.byE. 60 miles; required the latitude she came to, and the direct course and distance thither. If variation be 20° W. and deviation 9° E., find the compass course.

Draw a circle to represent a compass.

(1.) From A, the rhumb line, A B is drawn S.W.byW., and A B is cut off 45 miles (from a scale of 45 miles to the inch).

(2.) Draw B C at an angle of 6 points, *i.e.*, the rhumb E.S.E., and make B C 50 from the same scale.

(3.) Draw C D on the rhumb line S.W., marking off 30 miles by scale.

(4.) Draw D E in a S.E.byE. direction, and make its length 60 miles by scale.

(5.) Then E is the position of the ship and the place we have to find.

Join A E and draw E F perpendicular to the meridian A F, then A E is the distance, F E is the departure, A F is the difference of latitude, $\angle$F A E is the course.

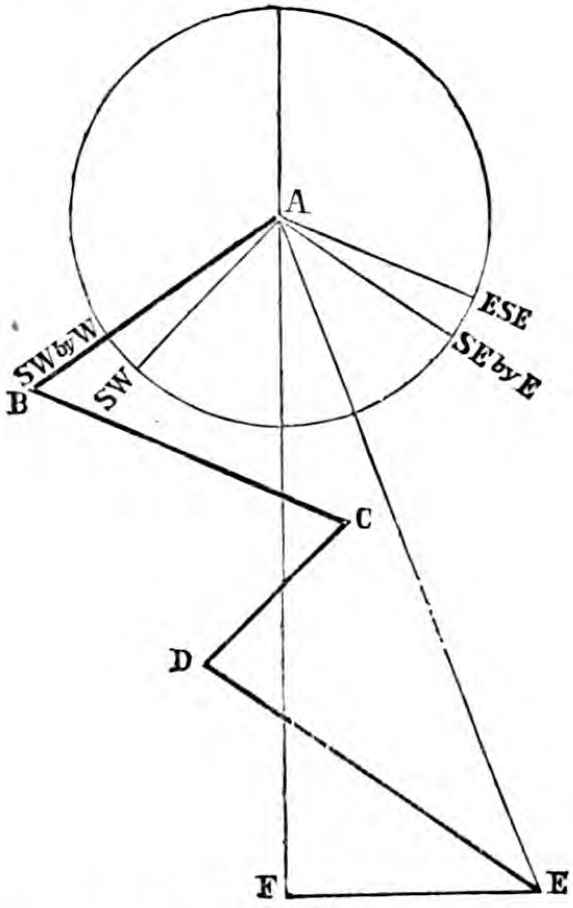


Fig. 16.—*Scale*, 45 miles to 1 inch.

(6.) From the scale of 45 miles to the inch, measure off the length of A E, it will be found to be 105·5 miles, F E is 37·5 miles, and A F about 99 miles, while the angle (course) F A E is S. 21° E.

Ans. Course S. 21° E.; diff. lat. 99 miles; dep. 37·5. Distance 105·5 miles.

The same problem shall now be solved by the tables, so that a comparison of the two methods may be instituted.

Courses.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
S.W.byW.	45		25		37·4
E.S....E.	50		19·1	46·2	
S.W.	30		21·2		21·2
S.E.byE.	60		33·3	49·9	
			98·6	96·1	58·6
				58·6	
				37·5	

From the traverse tables we find that the course and distance answering to these are—

Course S. 21° E. (more exactly S. 20° 49′ E.)
Distance 105·5 miles.

For latitude in.		For course.	
Latitude from	24° 32′ 0″ N.	True course	20° 49′ l. of S.
Diff. lat.	1 38 36 S.	Variation	20 0 r.
Latitude in	22 53 24 N.		0 49 l.
		Deviation	9 0 l.
			9 49 l.

Compass course S. 9° 49′ E.

It will perhaps be wise to illustrate one or two points by further examples.

197. Suppose the distance is 546 miles, with course N.W.byN., take out the difference of latitude and departure from the traverse table.

The traverse table only extends to 300 miles.

We must divide the distance, therefore, into two parts, or either take the half, third, fourth parts, etc., of 546 miles, and then, having taken out the difference of latitude and departure for this half, third, fourth part, etc., multiply them by 2, 3, or 4; or, we must take out for 300 miles and then the 246, and add the departures and differences of latitude together, thus—

COURSE THREE POINTS.

(1.) $546 = 3 \times 182$; or $546 = 300 + 246$.

Dist.	Diff. lat.	Dep.	Dist.	Diff. lat.	Dep.
182	151·3	101·1	300	249·4	166·6
3	3	3	246	204·5	136·7
Totals, 546	453·9	303·3	Totals, 546	453·9	303·3

198. A ship sails on an E.N.E. course 29·5, find the difference of latitude and departure.

In this case take out for 295, and shift the decimal place one to the left.

Thus for 6 points and distance

295 = 112·9 diff. lat., 272·5 dep.
∴ 29·5 = 11·3 diff. lat., 27·2 dep.

199. Find from the table the difference of latitude and departure for 485·7 miles on a course N. 37° W.

Course N. 37° W.

Dist.	Diff. lat.	Dep.
300	239·6	180·5
185	147·7	111·3
·7	·5	·4
485·7	387·8	292·2

The decimal 7 we take out for 7, which is 5·6 and 4·2, and shift the decimal place one point towards the left, calling the number ·5 and ·4. (It is not a bad custom to increase the number by *one* when the decimal place is greater than 5, *i.e.*, we might call 5·6 in this case ·6.) As above, 112·9 we called 11·3 not 11·2, when we shifted the decimal point.

There is another point which will greatly conduce to accuracy if attended to; we have purposely passed it over in the previous pages to avoid puzzling the student. Take the annexed illustration.

200. Suppose with course N. 31° E., we have a difference of latitude of 216·5, take out the departure and distance.

Looking in the traverse table on page 31°, we find the nearest difference of latitude is 216 or 216·9. We will take out the 216 with its distance, etc., and put it down as under.

Dist.	Diff. lat.	Dep.
252	216·0	129·8
·6	·5	·3
252·6	216·5	130·1

The 216 was found in the tables, the decimal 5 is found in its proper difference of latitude column as 5. So we take out all the terms | 6 | 5·1 | 3·1 | and write them as we have done above, mentally shifting the decimal point to | ·6 | ·5 | ·3 | .

201. Two ships A and B part company in latitude 51° 30′ N., and meet together again at the end of two days, having run as follows:—

A.	B.
(*a.*) N.N.E. ..... 96 miles	(*a.*) N.N.W. ..... 96 miles
(*b.*) W.S.W. ..... 96 ,,	(*b.*) E.S.E. ..... 96 ,,
(*c.*) E.S.E. ..... 96 ,,	(*c.*) W.S.W. ..... 96 ,,
(*d.*) N.N.W. ..... 96 ,,	(*d.*) N.N.E. ..... 96 ,,

Required the position of each ship at the end of the second day, and their direct course and distance.

We will make one figure serve for the traverse made by the two ships.

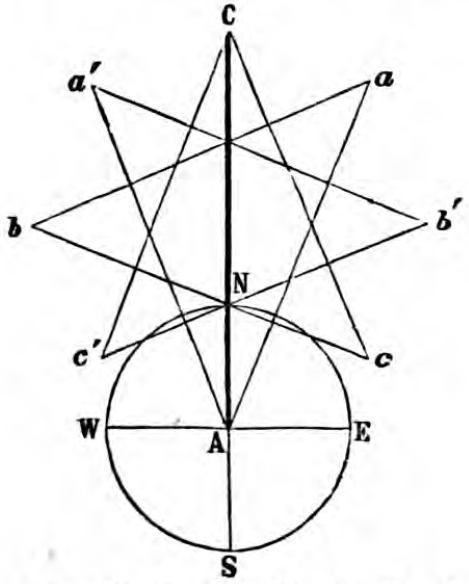


Fig. 19.—*Scale*, 80 miles to the inch.

For ship A.	For ship B.
1st course is N.N.E. ∴ Aa = 96 m.	1st course is N.N.W. ∴ Aa' = 96 m.
2d ,, W.S.W. ∴ ab = 96 m.	2d ,, E.S.E. ∴ $a'b'$ = 96 m.
3d ,, E.S.E. ∴ bc = 96 m.	3d ,, W.S.W. ∴ $b'c'$ = 96 m.
4th ,, N.N.W. ∴ cC = 96 m.	4th ,, N.N.E. ∴ $c'C$ = 96 m.

The courses and distances having been laid down, it is found that the two ships have arrived at the same place C, and therefore there is no departure in either case, only a difference of latitude directly N. 104 miles.

For latitude in we have

Latitude from	51° 30′	N.
Diff. lat. 104 m. ..... =	1 44	N.
Lat. in	53 14	N.

The traverse tables for the two vessels will show thus—

Course.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
N.N.E.	96	88·7		36·7	
W.S.W.	96		36·7		88·7
E.S.E.	96		36·7	88·7	
N.N.W.	96	88·7			36·7
		177·4	73·4	125·4	125·4
		73·4			
		104 N.			

Course.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
N.N.W.	96	88·7			36·7
E.S.E.	96		36·7	88·7	
W.S.W.	96		36·7		88·7
N.N.E.	96	88·7		36·7	
		177·4	73·4	125·4	125·2
		73·4			
		104 N.			

It will be perhaps noticed that the second table is the first inverted, and that the results are the same as obtained by construction.

True course due N.; distance 104 miles; latitude in 53′ 14″ N.

202. Given latitude of A, 49° 35′ N. and longitude 24° 50′ W.; latitude of B, 43° 37′ N., longitude 20° 17′ W.; variation of compass 27° W.; find the compass course from A to B by the traverse table only.

	Diff. lat.	Mid-lat.	Diff. lon.
Lat. A	49° 35′ N.	49° 35′ N.	24° 50′ W.
B	43 37 N.	43 37 N.	20 17 W.
	5 58	2)93 12	4 33
	60	46 36	60
	358 S.	90	273 E.
		Co-mid-lat. 43 24	

We have to work by the traverse table only, and therefore go to page 43°, and find distance 273 in the distance column, then in the departure column is found 186·2 for the departure. (This is simply reversing the process by which the difference of longitude was found on page 63.)

Now with difference of latitude 358, and departure 186·2, go to the tables; but first, these are too great to be found there, we therefore take their halves,

179 diff. lat.; 93·1 dep.,

which are found under 27°, with distance 201.

∴ True course is S. 27° E.; distance 402 miles.

To find compass course:

True course, ...	S. 27° E.	or l. of S.
Variation, ...	27 W.	or r.
Compass course, ...	0	or due S.

203. Show that course, distance, departure, and difference of latitude may be represented by an angle and three sides of a right-angled triangle; hence obtain formulæ connecting (1) course, departure, and difference of latitude; (2) course, distance, and departure; (3) course, distance, and difference of latitude. (1868.)

204. Why can I not determine fully the place of my ship by help of the formulæ used in plane sailing only? What additional formula will enable me to do this? Show that I can use a traverse table instead of directly applying this formula. A ship sails from lat. 23° 18′ N., lon. 54° 27′ W., to lat. 27° 19′ N., lon. 51° 59′ W.

Ans. Course N. 29° E.; dist. 276 miles.

205. Draw a diagram showing the course, distance, departure, and difference of latitude, obtaining the departure directly from the traverse table. (1868.)

206. What is meant by the traverse? Show how it is to be constructed. (1865.)

207. A ship in lat. 13° 5′ N., lon. 123° 23′ E., sails 225 m. S.W.½W. by compass; deviation of the compass 6° 35′ W.; variation 1½E.; find the latitude and longitude arrived at. (1867.)

Ans. Lat. 11° 15′ 54″ N.; lon. 120° 1′ E.

208. On June 12, at noon, in lat. 18° 15′ S., lon. 73° 15′ W., during the next 24 hours sailed 35 miles E.S.E., 30 miles W.byS., 44 miles S.W.byW., 29 miles N.byE., in what latitude and longitude am I on June 13, at noon? (1868.) *Ans.* Lat. 18° 30′ 18″ S.; lon. 73° 45′ W.

209. What do you mean by the traverse table? Explain its use, and describe how it is constructed. Show clearly how it is that you can use it to find by inspection the longitude in. (1870).

210. What do you mean by dead reckoning? Describe the instruments made use of in this method of navigating a ship. Why are we unable to trust wholly to it? (1870.)

211. Leaving a place in lat. 37° S.; lon. 151° E., I sail E.N.E. by compass 39 miles, with wind S.E., making 1½ points leeway, variation 9° 20′ E., deviation 5° W. The wind now shifts to the E., and I change my course S.E.byS., and make 2 points leeway, and the deviation for this position of the ship's head is 3° E., variation remaining the same. I sail for 6 hours at the rate of 5 miles; find my lat. and lon. in. (1870.) *Ans.* Lat. 37° 7′ 30″ S.; lon. 151° 52′ E.

212. What is meant by resolving a traverse? How would you prepare a table for this purpose? What is meant by the traverse table in the book of tables? On what formulæ is it based? (1871.)

213. A ship sailing from lat. 27° 30′ N., lon. 14° 20′ W., arrives at a place in lat. 29° 45′ N., after making 66 miles easting: what is the lon. arrived at? (1871.) *Ans.* Lon. in 13° 5′ W.

214. In the right-angled triangle ABC, A=90°, C=67° 30′, and CB=16·8; find by the traverse table the sides AB and AC. (1873.)

Ans. AB=6·43; AC 15·52.

215. A ship sails from A (lat. 50° 30′ N., lon. 15° 45′ W.) to B as follows:—S.W.½-S., 40 miles; N.byW., 15 miles; S.S.E., 55 miles; W.byN., 70 miles; find the latitude and longitude of B.

Also find what course she would have to take from A to B to arrive at the latter place without altering her direction, and the distance she would have to go. (1873.)

Ans. Course S. 55° W.; dist. 93 miles.
Lat. in 49° 36′ 42″ N.; lon. in 17° 43′ W.

216. A ship in latitude 50° 12′ S., lon. 33° 14′ W., runs due N. 185 miles, what are her latitude and longitude in? What will be her compass course allowing 21° easterly variation, 4° 20′ westerly deviation, and one point leeway, wind being S. by E.

Ans. Lat. in 47° 7′ 0″ S.; lon. in 33° 14′ W.
Compass course N. 16° 40′ W.

217. A ship in lat. 38° 44′ N., lon. 18° 33′ W., sails by compass E.N.E. 70 miles; required the lat. and lon. in given variation of compass ¾ pts. W deviation 8° E., leeway one point, direction of wind E.S.E. *Ans.* Lat. in 39° 10′ 48″ N.; lon. 17° 10′ W.

218. A ship sails from lat. 51° 25′ N., lon. 8°12′ W.,

S.S.E.	30 miles.
E.byS.	18 ,,
S.W.byW.	36 ,,
W. $\frac{3}{4}$ N.	14 ,,
S.E.byE. $\frac{1}{4}$ E.	46 ,,

Required the equivalent course and distance, and the latitude and longitude of the place arrived at.

Ans. Course S. 19° E.; dist. 77 miles.
Lat. 50° 12′ 18″ N.; lon. 7° 32′ W.

219. Two ships A and B start on the 22nd of December, from the same place, they sail for 60 hours in the directions indicated below; find by construction the distance they are apart at the end of that time, and the course and distance from B to A.

A sails		B sails	
N.W.byN.	20 miles.	N.E.byN.	65 miles.
S.W.	40 ,,	S.E.byS.	53 ,,
N.E.byE.	60 ,,		
S.E.	55 ,,		
W.byS.	41 ,,		
E.N.E.	66 ,,		

Ship B will be at point D in fig. 15, page 89, and the course from ship B to ship A at C will be S.S.E. 11·5 miles.

CHAPTER VI.

CURRENT SAILING.

Definition—Set and Drift of Currents—Examples.

56. Current Sailing is but an indifferent name for what sailors or book-makers include under the term *current sailing*. It is not sailing at all, but simply an allowance made for the setting and drift of currents, or it may be defined as the allowance made for the effects of currents and tides upon the distance a vessel has run.

57. Set and Drift of Currents.—The *set* of a current is that point of the compass towards which the water runs; the *drift* is the velocity of the current expressed in miles per hour. If we say a current is drifting the vessel E.N.E., we mean to say that the water is coming from W.S.W. and going towards the E.N.E. Whenever a master is sailing his vessel he looks out for tides and currents, and judges of their effects upon the course on which he is steering his vessel, and allows for them accordingly. This allowance is generally made once a day, and is treated as if it were an additional course. It is evident that if the set of the current is in the same direction as the ship is sailing, the velocity of the current must be added to that of the vessel, but if it set in an opposite direction it will be subtractive; but if, as is generally the case, the current be oblique to the ship's course, the influence upon the ship will be compounded with the drift and the angle at which it impinges upon the ship. The ship will sail one way, the current will drive her another, and she will therefore move in a direction compounded of the two velocities.

The succeeding example will show how the current course is allowed for.

220. A ship sails N.E. by E. 40 miles in a current that sets N. by E. 16 miles in the same, what course and distance has she made good.

First Method.

Courses.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
N.E.byE.	40	22·2		33·3	
N.byE.	16	15·7		3·1	
		37·9		36·4	

∴ Course made good from tables is N. 44° E.; distance 52 miles.

The problem is simply treated as one with two courses and solved accordingly. It might have been done directly by construction, thus:—

Second Method.—Let N E S W represent the horizon, and let it also be supposed that the ship starts from A.

From A draw A a' on the N.E. by E. rhumb, on a scale of 80 miles to an inch, as A a' is 40 miles, make it half an inch long. Next draw a' C on the N. by E. rhumb, making a' C 16 miles or parts on the same scale. Join A C and measure it on the scale, it will be found to be about 52 miles long, while the course N A C is N. 43° 45′ E.

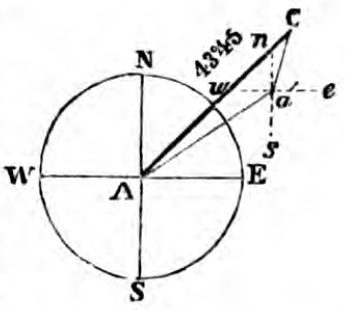


Fig. 20.—*Scale*, 80 miles to 1 inch.

Third Method.—By trigonometry it is worked thus:—

$Aa' = 40$
$a'C = 16$

$\angle Aa'C = Aa'w + wa'n + na'C$
$= a'AE + wa'n + na'C$
$= 3 \text{ pts.} + 8 \text{ pts.} + 1 \text{ pt.}$
$= 12 \text{ pts.}$

The case of triangles is now given to solve with two sides and the included angle.

To find the angles.

$$\text{Tan. } \frac{C-A}{2} = \frac{c-a}{c+a} \text{ cot. } \frac{a'}{2}$$

$$= \frac{40-16}{40+16} \text{ cot. 6 pts.}$$

$$= \frac{24}{56} \text{ cot. 6 pts.}$$

Log. 24	=	1·380211
Log. cot. 6 pts.	=	9·617224
		10·997435
Log. 56	=	1·748188
$\therefore$ Log. tan. $\frac{C-A}{2} = 10° 4'$	=	9·249247

$$\frac{C-A}{2} = 10° 4' \qquad \therefore\ C = 32° 34'$$

$$\text{and } \frac{C+A}{2} = 22° 30' \qquad \therefore\ A = 12° 26'$$

A and C are found by taking the sum and difference of $\frac{C-A}{2}$ and $\frac{C+A}{2}$

It now remains to show how the side A C is found; according to the simple formula in plane trigonometry we have—

To find the distance A C.

$$\frac{AC}{Aa'} = \frac{\text{sin. } Aa'C}{\text{sin. } ACa'} = \frac{\text{sin. 12}}{\text{sin. } 32° 34'}$$

$$\therefore\ AC = \frac{40 \times \text{sin. 12 pts.}}{\text{sin. } 32° 34'}$$

Log. 40.....	=	1·602060
Log. sin. 4 pts.....	=	9·849485
		11·451545
Log. sin. 32° 34'	=	9·731009
Log. 52·54 AC.....	=	1·720536

$\therefore$ Distance by calculation is 52·5.

To find angle N A C *or course.*

$$NAC = NAa' - a'AC$$

$$= 56° 15' - 12° 26' = 43° 49''$$

The student must be careful to note how much more tedious this process is than the first. No more striking instance can be

given of the very great value of the Traverse Tables when this difficult problem can be so easily and readily solved by them.

221. A ship is sailing north 5 miles an hour, and a current from the E. sets her towards the W. one and a half miles an hour, what progress has she made in 10 hours?

As the current is at right angles to the ship's course, we have simply a right-angled triangle to solve, which can be done by (1) geometry, (2) construction, (3) by the traverse tables, (4) by calculation. It is here done by the first and second methods only. The student must draw a triangle for himself.

(1) *By Geometry.*

If AB be the distance made good, and C the right angle,
Then BC represents the current
and AC the distance.

$\therefore AB^2 = BC^2 + AC^2$
$= 5^2 + 1{\cdot}5^2$
$= 27{\cdot}25$

$\therefore$ AB = 5·22 miles per hour.
$\therefore$ In 10 hours distance made good is 52·2 miles.

(2) *By the Traverse Tables.*

The departure is 1·5 miles per hour
,, difference of latitude is 5 miles per hour
$\therefore$ from table course made good is N. 16 W.
Distance, 5·2 miles per hour.

222. A ship sailed N.N.W. 4 miles, while a current sets her E.N.E. 3 miles; how far is the ship from her starting point, and what course has been made good?

Examining the conditions of the question, it will be seen that the ship makes good a course direct for the north, with a distance of 5 miles: for the angle between the current and the course is 8 points, and therefore the triangle is right-angled.

CHAPTER VII.

THE DAY'S WORK.

Definition—Sea Journal—Departure-course—Dead Reckoning—Illustrations and Examples.

It is very unusual in works on navigation to enter so early upon the Day's Works; but, as the chief preliminaries have been cleared away, it is perhaps unwise to defer so important a subject to the end of the book.

58. Definition.—The *Day's Work* is that part of a ship's reckoning written down, and made up from day to day, in order to determine the position of the ship. A proper account is kept of the courses and distances run by the ship during the day and night on a black board, called the **log-board,** properly ruled and divided, and hung up in some convenient place. The first column on this log-board shows the hours; the second and third, the knots and tenths the ship sails per hour; the fourth shows the courses, by compass, on which the ship has sailed; the fifth, the quarter from which the wind blows.

59. The log ought to be heaved every hour, but this is seldom the case, except in H. M. ships.

The courses marked on the log-board being those steered by compass; in order to obtain the true course it is necessary to allow for variation, deviation, and leeway upon each course.

Every day at noon the log-board ought to be transcribed into the **log-book** (a book ruled exactly like the log-board, but with columns for leeway, remarks, etc.), then the courses should be corrected, and the distances summed up, and a traverse table formed with the corrected courses and distances; this table, completed and resolved by the methods

given in traverse sailing, will give the total difference of latitude, departure, course, distance, and latitude and longitude in. These obtained, it is usual to find the position of the ship, when possible, by astronomical observation; if the dead reckoning agrees with the place of the ship as shown by these observations, so far good, but if they disagree, then the place of the ship, as found by the celestial observations, must be considered its true place.

60. A Sea Journal is a book wherein is entered regularly each day's work, or the result of it, viz., the position of the ship for every day, with any remarkable occurrence that may have been noticed during the voyage, relating to atmospheric phenomena, objects seen, etc.

Departure Course.—When a ship is leaving the land, the master notices the last cape or headland in sight, and the direction it lies from his ship; he also judges how far it is away, and puts these down in his log as the first course and distance, calling them the departure course and distance; this is also known as *taking the departure.* The *departure course* must always be inverted, because the master puts down the latitude and longitude of the headland, and considers that his starting point. Hence, as to go to the place where the ship is from the headland, he would have to sail on the exact opposite course to that which the cape bears from him, he inverts it.

Suppose, for instance, that the Lizard bears N.E.byN., 7 miles away, the master takes his departure from the Lizard, putting down the latitude and longitude of that headland as his starting point; then, to bring his ship into its present position from the Lizard, he knows that he will have to sail on a S.W.byS. course for 7 miles. Hence the rule, *always invert* the *departure course.* This course and distance is the *first on the voyage* that he puts down on his log-board, and subsequently enters in his log-book.

61. Dead Reckoning.—Dead Reckoning is calculating the position of the ship from the distances as given by the log, log-line, and log-glass, the course as allowed for from the chart, and the estimation for leeway and currents. The expression is met with "according to the dead reckoning," *i.e.*, according to the rough estimate made from the log of the

ship's position. The exact position of the ship when out of sight of land can only be found by celestial observations, for the dead reckoning is but a rough estimate of the ship's true position: it is liable to many errors. These may arise from a miscalculation of the distance run—erroneous judgment as to the effects of tides and currents—mistakes in heaving the log; as the wind varies, insensibly the speed of the ship is continually varying, the sailing master has to give a guess as to its whole effect during the hour. It is impossible for any one to tell within one or two degrees on what rhumb the ship is sailing, and the course is usually estimated in points and quarter points; this is, therefore, a source of error, as well as the constantly varying declination and deviation, as the ship heels over, or passes from one magnetic meredian to another. It will be seen that with guesses for leeway, effects of currents, velocity, etc., the sailor's calculations are at the mercy of many contingences.

62. To find Longitude in the Day's Work.—In the chapter on Traverse Sailing we gave a mechanical method of finding longitude by the use of the traverse tables. As the latter are founded on mathematical formulæ, it is quite possible to find a proof of the method employed. Because of its great importance we repeat the rule. *Find the departure in its proper column, with the co-mid-latitude as a course,* then the distance in the distance column is the difference of longitude to be applied east or west according as the departure is east or west.

The proof alluded to above is as follows:—

63. To find Longitude by Inspection.—Proof of rule used for finding longitude in the chapter on Traverse Sailing, and in the Day's Work.

RULE.—Using the co-mid-latitude as a course, find the departure in its proper column, then opposite to the departure is found the distance, this distance is the difference of longitude.

In parallel sailing it is proved that—

$$Departure = diff.\ of\ lon. \times cos\ lat.$$

But in middle latitude sailing the cosine of the middle latitude is used, which gives the answer nearly, hence we may write—

$$Departure = diff.\ lon. \times cos\text{-}mid\text{-}lat. \quad (1)$$

$$Cos\text{-}mid\text{-}lat. = sin.\ (90° - mid\text{-}lat.) = sin.\ co\text{-}mid\text{-}lat.$$

$$Sin.\ course \text{ or } cos.\ mid\text{-}lat. = \frac{departure}{distance}$$

$$\text{From (1) } cos\text{-}mid\text{-}lat. = \frac{departure}{diff.\ of\ lon.} = \text{sin. course}$$

$$\therefore \frac{Departure}{Distance} = \frac{departure}{diff.\ of\ lon.}$$

$$\therefore \text{Distance} = difference\ of\ longitude.$$

Therefore the distance is equal to the difference of longitude when we consider that co-mid-latitude is the course, and take departure in its proper column.

Current Course.—During the day the ship may get into a current, the master must allow for its effect, or else at the end of the day he will not correctly ascertain the place of his vessel. It is customary to complete the day's work at noon, because then it is very easy to ascertain the latitude of the ship by celestial observations, and the variation of the compass mechanically.

Notice particularly how the day's work is tabulated. The hours are numbered from noon of one day to noon of the next in the first column. In the second and third columns are placed the knots and decimals of a knot. Next come, in their proper order, the courses, with the leeway and direction of the wind, so that it may be known which way to apply the leeway to the course. Lastly, come the deviation and variation, both of which affect the compass in the same manner. It is not a bad plan to mentally add or subtract the deviation, according as they are of the same or different names, to or from the variation before applying the latter. It will save a good number of figures.

223. Suppose a vessel is lying 10 miles off the Lizard, the master, in putting down his departure course, says: Lizard, lat. 49° 57′ N. lon. 5° 14′ W., being N. 35° E., distant 10 miles, deviation 8° 10′ W. Suppose he then sails by the following log account, his departure being taken at noon, 1st June, 1872, required his latitude and longitude in at noon, 2nd June, 1872.

Hours.	Knots.	Tenths.	Compass Courses.	Leeway points.	Direction of Wind.	Deviation.	Remarks.
1	5	3	S.S.W.	½	E.S.E.	8° 30′ W.	P.M.
2	4	9					
3	6	1					
4	5	4					
5	4	4	N. by E.	2	N.E. by E.	12° 40′ E.	
6	3	5					Variation 23° 30′ W.
7	2	9					
8	3	0					
9	2	4					
10	4	3	S.S.E. ¾-E.	1	W.S.W.	12° 50′ E.	
11	3	4					
12	4	4					
1	3	2					A.M.
2	2	8					
3	4	3					
4	5	4					
5	6	0					
6	4	5	E. by S.	1½	N.N.E.	5° 30′ E.	
7	3	5					A current set the ship, the last 3 hours, 2½ miles per hour N.E. by compass.
8	5	2					
9	4	8					
10	5	4					
11	4	3					
12	3	6					

(May 1869 partly.)

Departure Course.

S. 35° 0′ 0″ W.
or 35° 0′ 0″ r. of S.
Deviation 8° 10′ 0″ l.
26° 50′ 0″ r. of S.
Variation 23° 30′ 0″ l.
3° 20′ 0″ r. of S.
or S. 3° W.

Current Course.

N.E. 45° 0′ 0″ r. of N.
Variation 23° 30′ 0″ l.
21° 30′ 0″ r. of N.
or N. 22° E. for 7·5 miles.

1st Course.

S.S.W. is 22° 30′ 0″ r. of S.
Leeway.... 5° 37′ 30″
28° 7′ 30″ r. of S.
Deviation. 8° 30′ 0″ l.
19° 37′ 30″ r. of S.
Variation 23° 30′ 0″ l.
3° 52′ 30″ l. of S.
or S. 4° E.

3rd Course.

S.S.E. ¾-E. is 30° 56′ 15″ l. of S.
Leeway.....11° 15′ 0″
42° 11′ 15″ l. of S.
Variation.....23° 30′ 0″ l.
65° 41′ 15″ l. of S.
Deviation.....12° 50′ 0″ r.
52° 51′ 15″ l. of S.
or S. 53° E.

2d Course.

N. by E. is 11° 15′ 0″ r. of N.
Leeway.....22° 30′ 0″ l.
11° 15′ 0″ l. of N.
Deviation...12° 40′ 0″ r
1° 25′ 0″ r. of N.
Variation ...23° 30′ 0″ l.
22° 5′ 0″ l. of N.
or N. 22° W.

4th Course.

E. by S. is 78° 45′ 0″ l. of S.
Leeway.....16° 52′ 30″
61° 52′ 30″ l. of S.
Deviation.... 5° 30′ 0″ r.
56° 22′ 30″ l. of S.
Variation ...23° 30′ 0″ l.
79° 52′ 30″ l. of S.
S. 80° E.

The distance on which the ship has sailed are obtained from the second and third left-hand columns, thus—

k. t.	*k. t.*	*k. t.*	*k. t.*
5·3	4·4	4·3	4·5
4·9	3·5	3·4	3·5
6·1	2·9	4·4	5·2
5·4	3·0	3·2	4·8
21·7	2·4	2·8	5·4
	16·2	4·3	4·3
		5·4	3·6
		6·0	31·3
		33·8	

The 10 is from the heading, and the 7·5 is $3 \times 2\frac{1}{2} = 7{\cdot}5$ from the current course in the right-hand column. Collecting

these, and tabulating them with the corrected courses, we have—

Courses.	Dist.	Diff. lat.		Dep.	
		N.	S.	E.	W.
S. 3° W.	10		10		·5
N. 22° E.	7·5	6·9		2·8	
S. 4° E.	21·7		21·6	1·5	
N. 22° W.	16·2	15·0			6·0
S. 53° E.	33·8		20·3	27	
S. 80° E.	31·3		5·4	30·8	
		21·9	57·3	62·1	6·5
			21·9	6·5	
			35·4	55·6	

Course S. 57° E; distance 66 miles.

Latitude.		Mid-latitude.	Longitude.	
Lat. from	49° 57′ 0″ N.	49° 57′ 0″ N.	Lon. from	5° 14′ 0″ W.
Diff. lat.	0 35 24 S.	49 21 36	Diff. lon.	1 26 30
Lat. in	49 21 36 N.	2)99 18 36	Lon. in	3 47 30 W.
		Mid-lat. 49 39 18		
		90 0 0		
		Co-mid-lat. 40 20 42		

In the departure course, let it be observed that 3° 20′ we call 3°, the nearest degree; in the current course 21° 30′ we have called 22°, but with equal reason we might have called it 21°. It is customary, when there are minutes, to take the nearest degree for the course: 14° 45′ we call 15°, but 14° 25′ we call 14°, and so on. In the current course the distance 7·5 is found by multiplying $2\frac{1}{2}$ miles by the 3 hours. *Observe that the variation,* TAKEN *from the remarks, is applied both to the departure and current courses.*

In the first course we have S. 4° E. as the course, and distance 21·7. The 21·7 is thus obtained: As the course is not changed till the beginning of the fifth hour, we reckon up the distances run during the first four hours, 5·3 + 4·9 + 6·1 + 5·4 = 21·7. The course is changed at the beginning of the fifth hour, and continues till the end of the 9th, adding up the distances we get 4·4 + 3·5 + 2·9 + 3 + 2·4 = 16·2, and so on with the other two, 33·8 and 31·3.

224. June 27, being yesterday, noon, 7 miles to the S.W. of Angra Pequena, in lat. 26° 38′ 30″ S., and lon. 15° 2′ 30″ E., we have sailed till this day, at noon, as per log. account below, in a current setting S.S.W. $2\frac{1}{2}$ miles per hour. The variation $1\frac{1}{2}$ points W., required the ship's place.

Hours.	Knots.	One-tenths.	Courses.	Leeway.	Wind.	Deviation.	Remarks.
1	6	0	S.W.	1	W.N.W.	6° 30′ W.	
2	6	2					
3	6	4					
4	6	0					
5	5	3	S. by W.				
6	6	0		1	W. by S.	3° 50′ W.	
7	5	1					Deviation for ship's head on departure course must be allowed as 6° W.
8	5	4					
9	5	2					
10	5	3					
11	5	0					
12	5	2	S.S.W.	$1\frac{3}{4}$	West	5° 20′ W.	
1	5	2					
2	5	0					
3	4	6					
4	5	0					
5	5	1					
6	5	2	S.W. by S.	$\frac{3}{4}$	S.E.	6° W.	
7	5	4					
8	5	4					
9	6	0					
10	6	0					
11	5	4					
12	5	5					

Departure Course.

S.W., ...	45°	0′	0″	r. of S.
Variation, ..	16	52	30	l.
	28	7	30	r. of S.
Deviation, ..	6	0	0	l.
S.	22	7	30	W.

1st Course.

S.W., ...	45°	0′	0″	r. of S.
Variation, ..	16	52	30	l.
	28	7	30	r. of S.
Deviation, ..	6	30	0	l.
	21	37	30	r. of S.
Leeway, ...	11	15	0	l.
S.	10	22	30	W.

2d Course.

S. by W.,...	11° 15′ 0″	r. of S.
Variation,..	16 52 30	l.
	5 37 30	l. of S.
Deviation,..	3 50 0	l.
	9 27 30	l. of S.
Leeway,.....	11 15 0	l.
S.	20 42 30	E.

3d Course.

S.S.W., ...	22° 30′ 0″	r. of S.
Variation,..	16 52 30	l.
	5 37 30	r. of S.
Deviation,..	5 20 0	l.
	0 17 30	r. of S.
Leeway,.....	14 3 45	l.
S.	13 46 15	E.

4th Course.

S.W. by S.,.	33° 45′ 0″	r. of S.
Variation,..	16 52 30	l.
	16 52 30	r. of S.
Deviation,..	6 0 0	l.
S.	10 52 30	l. of S.
Leeway,.....	8 26 25	
S.	19 18 55	W.

Current Course.

S.S.W., ...	22° 30′ 0	r. of S
Variation,..	16 52 30	l.
S.	5 37 30	r. of W.

Collecting these, and tabulating in the usual manner, we have the course and distance thus :—

Courses.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
S. 22° W.....	7		6·5		2·6
S. 10° W.....	29·9		29·4		5·2
S. 21° E.	32		29·9	11·5	
S. 14° E.	30·1		29·2	7·3	
S. 19° W.....	38·9		36·7		12·6
S. 6° W.....	60		59·7		6·3
			191·4	18·8	26·7
			18·8		18·8
			172·6		7·9

∴ Course is S. 3° W., and distance made good 173 miles.

Lat. from.....	26°	38′	30″	S.	Lon. from	15°	2′ 30″ E.
Diff. of lat. (172·6)	2	52	36	S.	Diff. lon. (9).....	9	0 W.
Lat. in.....	29	31	6	S.	Lon. in	14	53 30 E.
Sum	2)56	9	36				
Mid-lat.....	28	4	48				
	90	0	0				
Co-mid-lat.....	61	55	12				

With this 61° as a course, and departure 7·9 in its proper column, we find difference of longitude 9 in the distance column.

In this example the departure course has not been inverted, the student must look carefully for the reason, and understand why it is dealt with differently in this one case.

225. Dec. 24, the south-east point of Minorca (in lat. 39° 47′ 0″ N., lon. 4° 21′ 22″ E) bore N.N.W. 9 miles away, the ship afterwards sailed as per the following log. account; make up the dead reckoning with no allowance for deviation.

Hours.	Knots.	One-tenths.	Courses.	Wind.	Leeway.	Remarks.
1	5	6	S.W.	W.N.W.	2	
2	6	0				
3	6	0				
4	6	0				
5	5	2				
6	3	4	S.W.½-S.	W.N.W.½-W.	1½	Variation allowed 20° W.
7	3	4				
8	3	4	S.W. by S.	W. by N.	1	
9	3	0				
10	2	2				
11	2	2				
12	1	6	S.W. by W.	N.W. by W.	1	
1	2	0				
2	1	6				
3	2	0				
4	2	0				
5	2	0				
6	2	0				
7	2	6				
8	2	6	N.W.½N.	W.S.W.½-W.	1½	A current set the ship the whole time 1 mile per hour toward the W. by compass.
9	2	0				
10	2	0				
11	4	3				
12	4	3				

Departure Course.

S.S.E., ...	22°	30′	0″	l. of S.
Variation, ..	20	0	0	l.
S.	42	30	0	E.

Current Course.

W., ...	90°	0′	0″	r. of S.
Deviation, ...	20	0	0	l.
S.	70	0	0	W.

1st Course.

S.W., ...	45°	0′	0″	r. of S.
Variation, ..	20	0	0	l.
	25	0	0	r. of S.
Leeway, ...	22	30	0	
S.	2	30	0	W.

2d Course.

S.W.½-S., ...	39°	22′	30″	r. of S.
Variation, ..	20	0	0	l.
	19	22	30	r. of S.
Leeway, ...	16	52	30	
S.	2	30	0	W.

3d Course.

S.W.byS. ...	33°	45′	0″	r. of S.
Variation, ..	20	0	0	l.
	13	45	0	r. of S.
Leeway, ...	11	15	0	
S.	2	30	0	W.

4th Course.

S.W.byW.,	56°	15′	0″	r. of S.
Variation, ...	20	0	0	l.
	36	15	0	r.
Leeway, ...	11	15	0	
S.	25	0	0	W.

5th Course.

N.W.½N. ...	39°	22′	30″	l. of N.
Variation, ..	20	0	0	l.
	59	22	30	l.
Leeway, ...	16	52	30	
N.	42	30	0	W.

Courses.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
S. 42° ½-E.	9		6·7	6·0	
S. 2° ½-W.	28·8		28·7		1·2
S. 2° ½-W.	6·8		6·7		·3
S. 2° ½-W.	10·8		10·7		·47
S. 25° W.	15·8		14·3		6·6
N. 42° ½-W.	15·2	11·2			10·2
S. 70° W.	24		8·2		22·6
		11·2	75·3		41·3
			11·2		6·0
			64·1		35·3

Course S. 29° W.; distance 73 miles.

Latitude from	39°	47′	0″	N.	Lon. from	4°	21′	22″ E.
Diff. lat. 64·1	S. 1	4	6	S.	Diff. lon.	0	56	0 W.
Lat. in.....	38	42	54	N.	Lon. in	3	25	22 E.
Sum. 2)	78	29	54					
Mid. lat.	39	14	57					
	90	0	0					
Co-mid-lat.	50	45	3					

In taking out the quantities for the half degrees, we take them out interpolating between the others thus: in looking for difference and latitude and departure for N. 42° ½-W. 15·2 miles, first turn to page 42° and 152 miles (not 15), and opposite the 152 miles we see 113·0 and 101·7; on next page 43°, opposite 152 miles, is found 111·2 and 103·7. These we write thus—

11·3	and	10·17
11·12		10·37
2)22·42		2)20·54
11·2		10·2

which gives the 11·2 and 10·2 used in the traverse above. S. 2° ½-W. 10·8 will afford a better illustration. On page 2°, opposite 108, we have 107·9 and 3·8; on page 3°, opposite 108, we have 107·9 and 5·7 which we write thus—

10·79	·38
10·79	·57
2)21·58	2)·95
10·79	·47

and so obtain what is used above.

226. May 15, 1872, at noon, a point of land in latitude 58° 20′ S. and lon. 74° 30′ E. bore by compass E. by N. ½-N. distant 18 miles, ship's head E.S.E., deviation 5° 35′ E., afterwards sailed as by the following log account; find the latitude and longitude in on May 16th at noon. (40.)

Hours.	Knots.	One-tenths.	Course.	Wind.	Leeway.	Remarks.
1	4	5	S.S.E.	S.W.	1½	Deviation, 0° 56′ W.
2	3	5				
3	6	0				
4	5	5				
5	4	6				
6	5	4				
7	3	0				
8	4	6	W.N.W.	S.W.	1½	Deviation, 6° 50′ W.
9	5	4				
10	6	5				Variation of compass 2¼ pts. W.
11	7	0				
12	6	5				
1	6	0	S. by E.	S.W. by W.	1¼	Deviation, 2° 20′ W.
2	5	5				
3	4	5				
4	7	0				
5	6	5				
6	4	5				
7	6	0				
8	4	8	W. by N.	S.W.	1⅜	Deviation, 8° 10′ W.
9	5	5				A current set the
10	7	0				ship the last three
11	6	5				hours 2½ miles an
12	6	0				hour N.E. by compass.

Lat. 58° 58′ 54″ S.
Lon. 74° 10′ 0″ E.

Course S. 14° W.; distance 40 miles.

227. Jan. 17, 1871, at noon, a point of land in latitude 49° 30′ S., and lon. 74° 20′ E., bore by compass E.S.E., distant 14 miles (ship's head S., deviation 3° 0′ W.), afterwards sailed as by the following log account; find the latitude and longitude in on Jan. 18, 1871, at noon. (40.) (1871.)

Hours.	Knots.	One-tenths.	Courses.	Wind.	Leeway.	Remarks.
1	6	5	N.N.E.	E.	1¼	Deviation, 10° E.
2	5	0				
3	7	5				
4	4	5				
5	8	0				
6	6	5				
7	4	5	S. by E.	S.W.	1½	Deviation, 4° 20′ W.
8	7	0				
9	5	5				
10	4	5				Variation of the compass, 2¼ points E.
11	5	0				
12	6	0				
1	8	0	N.W.	N.N.E.	¾	Deviation, 1° 40′ W.
2	5	5				
3	4	5				
4	6	0				
5	7	5				
6	8	0				
7	6	5	W. by S.	S. by E.	½	Deviation, 7° 27′ W.
8	5	5				
9	7	5				A current set the ship the last four hours 2½ miles an hour S.E. by compass.
10	4	0				
11	6	5				
12	7	5				

Ans. Lat. in 48° 52′ 24″ N.

,, Lon. in 73° 30′ 0″ E.

Course N. 41° W.; distance 50.

228. April 15, 1863, at noon, a point of land in latitude 49° 55′ N., and longitude 5° 15′ W., bore by compass N.N.E., 10 miles distant, and then we sailed as follows; required the latitude and longitude in on April 16th, at noon. (1863.)

Hours.	Knots.	One-tenths.	Courses.	Wind.	Leeway.	Remarks.
1	4	5	S.W. b^{y}W.	S.	1½	Deviation, 7° 27′ W.
2	4	5				
3	4	5				
4	5	0				
5	5	0				
6	5	5	S.S.W.	S.E. ½ S.	2	Deviation, 5° W.
7	5	5				Variation of compass
8	5	5				22° 30′ W.
9	4	7	W.S.W.	S.	1	Deviation, 7° 50′ W.
10	4	7				
11	4	7				
12	4	5				
1	5	0	W. by S.	S. by W.	1½	Deviation, 8° 20′ W.
2	5	0				
3	5	5				
4	5	5				
5	5	5	W.	S.W. by S.	2½	Deviation, 8° 50′ W.
6	6	0				
7	6	0				
8	6	7				Deviation, 8° 30′ W.
9	6	7	W. ½ N.	S.W. by S.	2	A current set the ship
10	6	7				the last four hours
11	6	5				S.E. ½-E. 2 knots an
12	6	5				hour.

Departure course, corrected for deviation, is S. 5° W.

Lat. 48° 46′ N.
Lon. 7° 44′ W.

Course S. 55 W.; distance 120 miles.

229. September 10th, at noon, a point of land in lat. 69° 30′ S., and lon. 63° 15′ E., bore by compass N.N.W., distant 20 miles, afterwards sailed as by the following log account; find the latitude and longitude in, at noon, on September 11th. (1863.)

Hours.	Knots.	One-tenths.	Courses.	Winds.	Leeway.	Remarks.
1	6	5	S.E. by E.	N.E.	2	Variation of compass $1\frac{3}{4}$ points W.
2	4	6				
3	7	0				
4	5	5				
5	4	6				
6	6	4				Leeway is estimated in points.
7	4	5				
8	5	6				
9	6	0				
10	7	4	N.E.	S.E.	$2\frac{1}{4}$	
11	5	5				
12	6	5				Midnight.
1	7	0				
2	5	5				
3	6	0				
4	4	6				
5	5	5	N.W.byW.	W.S.W.	$2\frac{1}{2}$	
6	6	5				
7	4	6				
8	3	5				A current sets the ship during the last three hours $4\frac{1}{2}$ miles an hour S.W. by compass.
9	6	0				
10	7	0				
11	4	5				
12	5	5				

Lat. 69° 15′ 48″ S.
Lon. 64° 1′ E.
Course N. 49° E.; distance 22 miles.

230. April 9, 1864, at noon, a point of land in latitude 54° 20′ S., and longitude 44° 50′ E., bore by compass N.N.E., distant 20 miles, ship's head being S.E., deviation 1° 50′ E.; afterwards sailed as by the following log account; find the latitude and longitude in on April 10th, at noon. (1864.)

Hours.	Knots.	One-tenths.	Courses.	Wind.	Leeway.	Deviation.	Remarks.
1	4	6	N.N.W.	W.S.W.	1½	1° 40′ W.	
2	5	3					
3	3	0					
4	4	4					
5	2	6					
6	4	3					Variation of compass, 1¾ W.
7	5	0					
8	3	3					
9	5	4					
10	5	0	S. by W.	W.	2	4° 20′ W.	
11	4	6					
12	5	3					
1	6	0					
2	3	5					
3	4	4	E.S.E.	S. by W.	1¼	5° 35′ E.	
4	5	0					
5	3	8					
6	4	0					
7	5	6					A current set the ship the last four hours 8½ S. by E. by compass.
8	4	5					
9	5	4	W. by N.	S.S.W.	1¾	8° 10′ W.	
10	2	8					
11	4	0					
12	5	0					

Lat. in 54° 30′ 6″ S.
Lon. in 45° 6′ 0″ E.

Course S. 43° E.; distance 14 miles.

Courses corrected.

S. 5° W. 20; N. 27° W. 37·9; S. 35° E. 24·4; N. 85° E. 27·3; N. 87° W. 17·2; S. 21° E. 8·5.

231. May 2d, 1873, at noon, a point of land in latitude 34° 1′ S., longitude 25° 41′ E., bore by compass N.W.$\frac{1}{2}$-W. 7 miles distant, variation 1$\frac{3}{4}$ points E., deviation for ship's head S.E., 1° 50′ E.; afterwards sailed as by the following log account; find the latitude and longitude on May 3d at noon.

Hours.	Knots.	One-tenths.	Course.	Wind.	Leeway.	Remarks.
1	4	4	S.byE.	E.S.E.	1$\frac{1}{2}$	Deviation, 2° 20′ W.
2	6	2				
3	5	3				
4	3	5				
5	6	6				
6	7	2				
7	4	8				
8	5	3	E.S.E.	N.E.byN.	$\frac{1}{2}$	Deviation, 6° 35′ E.
9	7	2				
10	6	4				
11	3	1				
12	2	5				Variation, 1$\frac{3}{4}$ pts. E.
1	6	3	N.W.	N.E.byN.	$\frac{3}{4}$	Deviation, 4° 50′ W.
2	7	1				
3	5	4				
4	3	7				
5	4	5				
6	6	1	S.S.W.$\frac{1}{4}$-W.	West.	1$\frac{1}{2}$	Deviation, 5° W.
7	5	0				
8	4	3				A current set the ship the last four hours E.S.E. by compass 3$\frac{1}{2}$ knots an hour.
9	4	7				
10	5	4				
11	6	0				
12	3	5				

Courses and distances, S. 29° E. 7 m.; S. 23° W. 38 m.; S. 36° E. 24·5 m.; N. 39° W. 27 m.; S. 23° W. 35 m.; S. 48° E. 14 m.

Ans. Course S. 12° W.; distance 83 miles.

Lat. in 35° 22′ 30″ S.; lon. in 25° 20′ E.

232. March 1st, 1874, at noon, a ship was in lat. 37° 20′ S., and lon. 0° 52′ E., afterwards sailed as by the following log account, work up the reckoning till March 2nd, at noon.

Hours.	Knots.	One-tenths.	Courses.	Leeway.	Wind.	Deviation.	Remarks.
1	4	2	S.E. by E.	1	S by W.	9°E.	
2	4	0					
3	3	2					
4	3	0					
5	3	6					
6	3	0					
7	1	0	S.W.	2	S.S.E.	6°W.	
8	1	0					
9	1	0	N.N.E.	2	East.	8°E.	
10	1	0					Variation of Compass, 19°W.
11	2	4	North.	2	E.N.E.	2°E.	
12	3	2					
1	4	4					
2	4	6					
3	5	0					
4	5	2	N.W.½W.	1½	N.N.E.	7°W.	
5	5	4					
6	6	0					
7	6	4					A current set the ship, the whole time, 1 mile per hour S.W. by compass.
8	6	0	West.	0	N.N.W.	14°W.	
9	6	0					
10	5	4					
11	5	6					
12	5	0					

S. 77½ E., 20·4 m.; S. 42½ W., 2 m.; N. 11 W. 2 m.;
N. 39½ W., 19.6 m.;
S. 86½ W., 23 m.; S. 57° W., 28 m.; S. 26° W., 24 m.
Course, S. 62° W.; distance, 58 miles.
Lat. in 37° 47′ 0″ S.
Lon. in 0° 13′ 0″ W.

233. Nov. 8th, a ship was in latitude 40° 30′ S., and longitude, by account, 0° 10′ W.; afterwards sailed as by the following log account. Find her place at noon, November 9th, by dead reckoning.

Hours.	Knots.	One-tenths.	Courses.	Leeway.	Wind.	Deviation.	Remarks.
1	4	2	SbyW½-W	½	West.	6°W.	
2	5	6					
3	4	0					
4	4	2					
5	5	0					
6	5	2	ESE½-E	½	NEbyN.	8°E.	
7	5	8					
8	4	2					
9	3	5					
10	5	4					
11	6	2					Variation, 15°W.
12	4	2					
1	3	3					
2	3	4	SSW½-W	¾	WbyN.	7°W.	
3	4	2					
4	6	6					
5	6	0					
6	5	0					
7	5	4					
8	5	0	SbyE¼-E	0	West.	3°E.	A current set the ship 12 miles to the SSW. by compass.
9	4	3					
10	3	5					
11	4	4					
12	2	8					

S. 10° E. 23 m.; S. 74½ E. 37·8 m.; S. 2° W. 30.6 m.; S. 26 E. 20 m.; S. 7½ W 12.

Ans.—Course, S. 26° E; distance, 185 miles.
Latitude in, 42° 3′ 18″ S.; lon., 0° 52′ E.

234. April 10th, 1874, a ship was in latitude 50° 30′ N., and longitude 14° 10′ W.; afterwards sailed as by the following log account; work up the reckoning to noon, April 11.

Hours.	Knots.	One-tenths.	Course.	Leeway.	Wind.	Deviation.	Remarks.
1	2	5	S.E.byS.	2	S.W.	3° E.	
2	2	4					
3	3	3					
4	4	3					
5	3	6					
6	4	5	W.N.W.	1¼	S.W.	7° W.	
7	4	0					
8	5	5					
9	5	5					
10	4	0					
11	4	4					Var. allowed.
12	5	6	S.W.	0	S.E.byS.	7° W.	28° W.
1	6	5					
2	7	0					
3	7	3					
4	6	6					
5	5	4	S.byE.	1	S.W.byW.	1° E.	
6	4	0					
7	5	5					
8	3	6					
9	4	0					
10	6	5	N.W.byW.	¼	S.W.byS.	6° W.	
11	7	5					
12	8	0					

S. 81° E. 16·1 m.; N. 88° W. 27·9 m; S. 10° W. 33 m.; S. 49½ E. 22·5; N. 87° W. 22 m.

Ans.—Course, S. 26° W.; distance made good, 53 miles.

Latitude in, 49° 42′ 42″ N.

Longitude in, 14° 45′ W.

235. At noon, March 21, a ship in latitude 33° 24′ N., and longitude 170° 15′ E., afterwards sailed as by the following log account; work up the reckoning to noon, March 22.

Hours.	Knots.	One-tenths.	Course.	Leeway.	Wind.	Deviation.	Remarks.
1	5	0	E½-N.	¼	N.byE.	3° E.	
2	5	5					
3	5	5					
4	6	0					
5	6	0					
6	6	0					
7	6	0					
8	6	0					
9	5	0	East.	½	N.N.E.	4° E.	
10	4	0					
11	3	5					Var. allowed.
12	3	0					10° E.
1	2	5	N.E.	2	W.N.W.	3° E.	
2	2	5					A current
3	2	5					*from* N.W. set
4	2	0					the ship 2
5	2	0					miles per hour
6	3	0					for 4 hours in
7	3	4					the afternoon,
8	3	4					and 3 miles
9	3	6					per hour for 3
10	3	6					hours in the
11	4	8	N.W.	1	N.E.byN.	3° W.	morning.
12	3	0					

S. 80°E. 46 m.; S. 70°E. 15·5 m.; N. 80½-E. 28·5 m.; N. 49° W. 7·8 S. 35°E. 8 m.; S. 35°E. 9 m.

Ans.—Course, S. 79°E.; distance, 94 miles.

Latitude in, 33° 6′ 30″ N.; longitude in, 172° 5′ E.

CHAPTER VIII.

PARALLEL SAILING.

Definitions—Proof of Rule—Rules—Examples—Traverse Tables used in Parallel Sailing—Table of the Lengths of the Degrees—Exercises:

64. Parallel Sailing.—*Parallel Sailing* is the art of finding what distance a ship runs due east or west in sailing from the meridian of one place to that of another, on a parallel of latitude.

This kind of sailing or piloting is very frequently employed. A master "runs down his latitude," *i.e.*, runs the ship due S. or N. till he is in the parallel of latitude on which the place is situated that he is bound for; the head of the ship is then pointed due E. or W. till the place is reached. Supposing a small island is situated at a good distance from the mainland, this is a certain way of reaching it when a good account is kept of currents and leeway, and the dead reckoning properly worked up to ascertain when the ship is in the exact longitude of the island.

The repetition of one or two preliminary definitions will assist us materially in what follows.

65. Meridian Distance.—*Meridian distance* is the perpendicular distance between the "meridian left" and the meridian arrived at.

66. A great circle is a circle that divides the sphere into two hemispheres. The equator and all the meridians are great circles.

67. A small circle of a sphere is a circle that divides it into two unequal parts, such as the tropics of Cancer and Capricorn, the Arctic and Antarctic circles, and all the parallels of latitude.

68. The tropic of Cancer is a small circle at $23\frac{1}{2}$ degrees N. of the equator; the tropic of Capricorn is the corresponding circle south of the equator.

69. The Arctic Circle is a small circle at $23\frac{1}{2}$ degrees from the N. pole; the Antarctic circle is the corresponding circle round the S. pole.

Between the tropic of Cancer and the Arctic circle lies the north temperate zone; between the tropic of Capricorn and the Antarctic circle lies the south temperate zone.

A consideration of these definitions and a careful study of a terrestial globe will show at once that the equator is the largest circle of latitude, and that the parallels as they near the pole are constantly decreasing circles, becoming smaller and smaller as they approach the pole. The equator is a larger circle than the tropics, the tropics are larger than the polar circles, and the polar circles larger than one drawn at 10° from the poles. Again, as the circles are smaller the degrees must be smaller; for every circle being divided into 360°, it is at once manifest that the degree of longitude on the Arctic circle is less than that on the tropic of Cancer, for it is the $\frac{1}{360}$ part of a much smaller circle. In fact, if a ship sail due E. 55·02 miles at the Tropic, or 23·92 miles at the Arctic circles; in each case the longitude will be changed one degree, although the vessel at the Arctic circle has sailed 31·1 miles less than the other. At the Equator, if a ship go due E. or W. every 60 miles she goes she changes her longitude one degree, but this is nowhere else the case. At 45° N. or S. latitude, if a ship sail due E. or W. 60 miles she changes her longitude nearly 1° 25′. At 60° N. or S. in sailing due E. or W. 60 miles the longitude is changed 2° or 120′, while at 73° if the ship go 60 miles due E. or W. the longitude would be affected 3° 25′. In looking at the globe, it is seen that the meridians approach nearer and nearer to each other as they pass to the poles, where they all intersect and longitude vanishes; it must also be observed that the meridians cut all the parallels at right angles. In Parallel Sailing no false assumption is made concerning the earth, we take it to be a sphere and not a plane, as in Plane Sailing. We grapple with the difficulty that however far apart the meridians may be separated at the equator, they gradually approach nearer and nearer till they meet, and that the number of nautical miles in the degrees of longitude vary very considerably among one another.

70. Proof for Parallel Sailing.—Having given these few definitions and illustrations, we will now prove the fundamental formula in parallel sailing.

Meridian distance = difference of longitude × Cos. latitude.

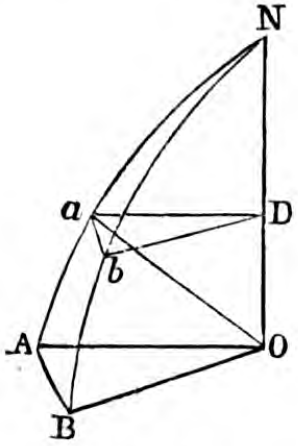


Fig. 21.

Let A B O N represent a portion of the sphere; O the centre of the earth; N the north pole. Let A*a*N and B*b*N be quadrants of two meridians. Let A B be an arc of the equator, and *ab* an arc of a parallel of latitude. Join *a*O: then A*a* or B*b*, or angle A O*a*, is the latitude of *a* or *b*; A B is the difference of longitude, and *ab* is the meridian distance between *a* and *b*; A B O is the plane of the equator, and let *ab*D be a plane parallel to the equator, therefore *a*D is parallel to A O, and *b*D parallel to B O, and angle A O B is equal to *a*D*b*.

Now arcs which subtend equal angles are to one another as the radii of the circle of which they are arcs;

$$\therefore \frac{\text{arc } ab}{\text{arc AB}} = \frac{aD}{AO} = \frac{aD}{aO} \quad \because \text{A O} = a\text{O, both being radii of the sphere.}$$

$$= \text{Sin. } a\text{OD}$$

$$= \text{Cos. } a\text{OA}$$

$$\therefore \frac{\text{arc } ab}{\text{arc AB}} = \text{Cos. } a\text{OA}$$

but arc *ab* is the departure or meridian distance, arc A B is the difference of longitude, and angle *a*O A is latitude,

$$\therefore \text{Cos. } a\text{O A} = \text{Cos. lat.}$$

$$\therefore \frac{\text{mer. dis.}}{\text{diff. lon.}} = \text{Cos. lat.}$$

$$\therefore \text{mer. dis.} = \text{Cos. lat.} \times \text{diff. lon.}$$

$$\therefore \text{diff. lon.} = \frac{\text{mer. dis.}}{\text{Cos. lat.}} = \text{mer. dis.} \times \text{sec. lat.}$$

If M = mer. dis., L = diff. long., we get the formula M = L × cos. *l*, by which all questions in Parallel Sailing may be solved.

$$\therefore \text{M.} = \text{L. Cos. l.}, \quad \ldots\ldots \text{A}$$

$$\therefore \text{L.} = \frac{\text{M.}}{\text{Cos. l.}} = \text{M. Sec. l.}, \quad \ldots\ldots \text{B}$$

$$\therefore \text{Cos. l.} = \frac{\text{M.}}{\text{L.}} \quad \ldots\ldots \text{C}$$

With these three formulæ, or rather with the one formula in its three shapes, all questions in Parallel Sailing may be solved.

The following are a few examples and illustrations, with the necessary rules for Parallel Sailing.

236. A vessel sails from A to B, required the compass course and distance. Given:

Lat. A 38° 42′ N.; var. 21° 10′ W.; lon. A 9° 9′ W.
Lat. B 38 42 N.; dev. 8 12 E.; lon. B 19 10 W.

As both places are on the same parallel of latitude, we have to sail on a parallel and go W. from a place (Lisbon) in W. longitude to one farther west.

Lat. 38° 42′ N. { Lon. A 9° 9′ W. / Lon. B 19 10 W.

Diff. lon. 10 1 W.
60
601

Mer. dis ..... = diff. lon. Cos. lat.
M ..... = 601 × Cos. 38° 42′

Log 601 ..... = 2·778874
Log cos. 38° 42′ ... = 9·892334
Log 469 ..... = 2·671208

∴ Meridian distance is 469 miles.

Now for compass course, true course is due W. 90° 0′ 0″ l. of N.
Variation W. 21 10 0 r.
68 50 0 l.
Deviation 8 12 0 l.
77 2 0 l. of N.
∴ Compass C., N. 77 2 0 W.
Ans. Distance 469 miles C. C., N. 77° 21′ W.

From this problem the annexed rules for Parallel Sailing are deduced.

(1) *To find meridian distance*—

To log difference of longitude add log cosine of the latitude, omitting 10 from the index. The sum is the logarithm of the meridian distance.

From problem (238) following—

(2) *To find latitude* in parallel sailing, given meridian distance and difference of longitude.

From log meridian distance, subtract log difference of

longitude, mentally adding 10 to the index of the upper logarithm. The remainder is log Cos. latitude.

(3) *To find difference of longitude* (as in Ex. 239.)—

To log meridian distance add log secant latitude, omitting 10 from the index. The sum is log. difference of longitude.

237. Find the length of a degree of longitude in latitude 45°.

Here difference of longitude is 60′.

M = L. Cos. *l*.
M = 60 × Cos. 45°
Log 60 = 1·778151
Log Cos. 45 = 9·849485
Log 42·42 = $\overline{1}$·627636
Ans. 42·42 miles.

238. Find in what latitude the length of a degree of longitude is 45 miles.

M = L. Cos. lat.
45 = 60 × Cos. *l*.
$$\text{Cos. } l. = \frac{45}{60}$$
Log 45 ..... = 1·653213
Log 60 ..... = 1·778151
Log Cos. 41° 25′ = $\overline{9}$·875062
Ans., lat. 41° 25

239. In lat. 50° 35′ N. I sail due E. 340 miles, how much do I change my longitude?

M = L. Cos. *l*.
340 = L. Cos. 50° 35′
∴ L = 340 × Sec. 50° 35′
Log 340 ... = 2·531479
Sec. 50° 35′ = 10·197257
Log 535·4 = 2·728736

6,0)535′·9
Ans. 8° 55′ 54″ of longitude.

240. Suppose the longitude from which the ship started is 140° 56′ E., let us find the longitude of the ship.

Lon. from	140°	56′	0″	E.
Diff. of lon.	8	55	54	E.
Lon. in	149	51	54	E.

241. A and B lie on the same parallel of 34° 33′ S.

Give lon. of A 18° 24′ E.
Give lon. of B 2 9 W.

What is the distance between them in nautical miles (1868)?

Lat. 34° 33′ S. Lon. A 18° 24′ E.
B 2 9 W
Diff. lon. 20 33
60
1233

M = L. Cos. *l*.
= 1233 × Cos. 34° 33′

Log 1233 ..... = 3·090963
Log Cos. 34° 33′ = 9·915733
Log 1015 ..... = 3·006696

Distance between A and B is 1015 miles.

242. What are the compass courses and distances of a ship that makes 54 miles westing, then 80 miles northing, from Swan River, lat. 32° 3′ S., lon. 115° 45′ E., var., 18° E., dev. 4° 5′ E., in the first case, and 3° 10′ W. in second.

M = L. Cos. *l*.
54 = L. Cos. 32° 3′
∴ L = 54 × Sec. 32° 3′

Log 54 = 1·732394
Log Sec. 32° 3′ = 10·071817
Diff. lon. 63·71 = 1·804211

Lon. from 115° 45′ 0″ E.
Diff. lon. 1 3 42 W.
Lon. in 114 41 18 E.

Being in lat. 32° 3′ S. and lon. 114° 41′ 18″ E., the ship went 80 miles north, now to find the lat. in

Lat. from 32° 3′ 0″ S.
Diff. lat. 1 20 0 N.
Lat. in 30 43 0 S.

(1) Compass course.					(2) Compass course.				
W. is	90°	0′	0″	r. of S.		0°	0′	0″	from N.
Var. E.	18	0	0	l.	Var. E.	18	0	0″	l.
	72	0	0	r.		18	0	0	l.
Dev. E.	4	5	0	l.	Dev. W.	3	10	0	r.
C. C.	67	55	0	r. of S.	C. C.	14	50	0	l. of N.

Two courses S. 67° 55′ W., and N. 14° 15′ W.

243. With what velocity does the earth travel at the equator

Velocity is generally reckoned in feet per second, but our units in this answer will be *miles* and *hours*.

In 24 hours the earth moves through 360°

In 1 hour, ,, ,, $\frac{360}{24} = 15°$

In 1 degree there are 60 nautical miles.
In 15 degrees there are 60 × 15 = 900 miles.

And this is the velocity of the earth in miles per hour at the equator. If we calculate for latitude 60° we shall have 450 miles, thus—

M = L. Cos. lat.
M = 900 × Cos. 60°
Log 900 = 2·954243
Log Cos. 60° = 9·698970
Vel. is 450 = 2·653213

71. The use of the Traverse Table in Parallel Sailing.—All questions in parallel sailing may be solved by the traverse tables. Let us take the four examples following—

244. A ship in latitude 30° N. sails due E. 60 miles; how much has she changed her longitude.

The complement of 30° is 60°, looking on page 60° of the traverse table for 60 miles distance we find the departure 52 miles. This is the difference of longitude.

245. A ship in latitude 51° N. sails due W. 264 miles; find the difference of longitude.

The complement of 51° is 39°, looking on page 39° for distance 264 miles, we find the departure is 166·1 miles, which is the required answer.

246. A ship in going 90 miles changes her longitude 75·5 miles; find the latitude in. *Ans.* 33° latitude.

In traverse table with distance 90 miles we find departure 75·5 on page 57°; hence the latitude is the complement of 57°.

247. In latitude 23° I sailed till the difference of longitude was 220′·9; find the distance sailed.

Using the complement of 23°, viz., 67°, on that page distance 220·9 has opposite to it the departure 203·3 miles, which is the distance required.

The student should endeavour to get thoroughly conversant with these mechanical methods, for with their aid he can check any number of problems in Parallel Sailing in a very few minutes.

TABLE SHOWING THE LENGTH OF A DEGREE IN NAUTICAL MILES IN EVERY LATITUDE FROM 1° TO 90°.

Lat.		Lat.		Lat.		Lat.		Lat.		Lat.	
1°	59·99	16°	57·67	31°	51·43	46°	41·68	61°	29·09	76°	14·51
2°	59·96	17°	57·38	32°	50·88	47°	40·92	62°	28·17	77°	13·50
3°	59·92	18°	57·06	33°	50·32	48°	40·15	63°	27·24	78°	12·48
4°	59·85	19°	56·73	34°	49·74	49°	39·36	64°	26·30	79°	11·45
5°	59·77	20°	56·38	35°	49·15	50°	38·57	65°	25·36	80°	10·42
6°	59·67	21°	56·01	36°	48·54	51°	37·76	66°	24·41	81°	9·38
7°	59·56	22°	55·63	37°	47·92	52°	36·94	67°	23·44	82°	8·35
8°	59·42	23°	55·23	38°	47·28	53°	36·11	68°	22·48	83°	7·32
9°	59·26	24°	54·81	39°	46·63	54°	35·27	69°	21·50	84°	6·28
10°	59·09	25°	54·38	40°	45·96	55°	34·41	70°	20·52	85°	5·23
11°	58·89	26°	53·93	41°	45·28	56°	33·55	71°	19·53	86°	4·18
12°	58·69	27°	53·46	42°	44·59	57°	32·68	72°	18·54	87°	3·14
13°	58·46	28°	52·97	43°	43·88	58°	31·79	73°	17·54	88°	2·09
14°	58·22	29°	52·47	44°	43·16	59°	30·90	74°	16·54	89°	1·05
15°	57·95	30°	51·96	45°	42·43	60°	30·00	75°	15·53	90°	0·00

This table is exceedingly useful in many ways. To reduce these to English miles, simply multiply by $69\cdot\frac{1}{20}$* and divide by 60.

248. What is the length of a degree in English miles in latitude 51°? In tables the length of a degree in latitude 51° = 37·76 m.

$$\begin{array}{r} 37\cdot76 \\ 69\frac{1}{20} \\ \hline 33984 \\ 22656 \\ 1888 \\ \hline 6,0)260,7\cdot328 \\ \hline \end{array}$$

Ans. 43·45 miles.

If the meridian distance be wanted, it can be found from the table. Suppose the following question be put—

249. A ship in latitude 40° changes the longitude 5° 30′, what is the meridian distance?

Distance for 1° in lat. 40° from table = 45·96 miles
" $5°\frac{1}{2}$ " is = $45\cdot96 \times 5\frac{1}{2}$
= 252·78

* The exact number is 69·048, but we are aware that a slightly different number has been previously given.

250. A ship in latitude 40° 25′ changes her longitude 4° 15′, find the meridian distance?

Distance in 40° parallel = 45·96
„ 41° „ = 45·28
Difference for 60′ ..... = ·68
„ 1′ ..... = $\frac{·68}{60}$
„ 25′ ..... = $\frac{·68 \times 25}{60} = ·28\dot{3}$

45·96
·28
45·68

∴ Distance for 1° in latitude 40° 25′ = 45·68
∴ „ 4°¼ „ = 45·68 × 4¼
= 194·14 miles.

251. In what latitude do 46·08 miles make one degree of longitude?

The nearest in the table is 46·63 for 39° latitude

46·08
·55 is the difference

The ·67 is the difference between 39° = 46·63
and 40 = 45·96
Difference for ..... 60 = ·67

60
·67)33·00(49
268
620
603
17

Hence if ·67 is difference for 60, then ·55 is difference for 49.

Ans. 39° 49′ latitude.

252. Prove that *meridian distance* is equal to *difference of longitude* multiplied by the *cosine of the latitude.* (1863.)

253. Required the compass course and distance from A to B. (1865.) Given:

Lat. A 52° 15′ S.; var. 1½ points W.; lon. A 37° 30′ W.
Lat. B 52° 15′ S.; dev. 8° 50′ W.; lon. B 48° 18′ W.

Ans. Course N. 64° 17′ 30″ W.; distance 396·7 miles.

254. What is meant by a parallel of latitude? and show that the length of a degree of longitude in latitude l is to the length of a degree of longitude at the equator as Cos. l:1. In what latitude are the lengths of a degree of longitude 30 and 20 miles respectively. (1864.) *Ans.* 60° and 70° 31′ 43.

255. Show that in parallel sailing dis. = diff. lon. × Cos. lat. (1865.)

In travelling 35 nautical miles on the parallel 55° 25′ N., how much do I change my longitude? (1867.) *Ans.* 61′·6; or 1° 1′ 36″

256. A and B lie on the parallel of 58° 30′ N
Given lon. A 15° 12′ E.
Given lon. B 13° 18′ W.
What is the distance between them in nautical miles? (1868.)

Ans. 893·4 miles.

257. Define a great circle and a small circle of a sphere, giving an example of each. What connection is there between the tropic of Cancer and the Arctic circle? (1870.)

258. Find the true course and distance from A to B.

Lat. of A 54° 25′ S.; lon. A 15° 30′ E.
Lat. of B 54° 25′ S.; lon. B 9° 15′ W.

Ans. Course W.; distance 864·1 miles.

259. What are the true course and distance of a ship which makes 65 miles southing and 75 miles easting? (1870.)

S. 49° 5′ E.; 99·24 miles.

260. What are the compass course and distance from A to B? Given:

Lat. A 25° 18′ S.; lon. A 145° 12′ E.; var. $1\frac{1}{4}$ pts. E.
Lat. B 25° 18′ S.; lon. B 136° 18′ E.; dev. 11° 25′ W.

Ans. Course S. 87° 21′ 45″ W.; distance 482·7 miles.

261. How are the compass course and distance found when a ship sails (1) on a parallel? (2) on a meridian? Prove the rules where necessary. (1865.)

262. Required the compass course and distance from A to B. (1866.)

Lat. A 28° 40′ N.; var. $1\frac{3}{4}$ pts. W.; lon. A 2° 20′ E.
Lat. B 28° 40′ N.; dev. 8° 50′ E.; lon. B 4° 10′ E.

Ans. Course S. 79° 8′ 45″ E.; distance 96·5 miles.

263. A ship is in latitude 35° 30′ S., longitude 27° 28′ W., and is to sail in a due E. direction 301 miles, what is the compass course she must steer and lon. in, variation $1\frac{3}{4}$ points E., deviation 8° 50′ E? (1866.) *Ans.* Course N. 61° 28′ 45″ E.; lon. in 21° 18′ 18″ W.

264. A ship in latitude 49° 35′ N., longitude 8° 40′ W., sails due S. 15 miles, and then alters her course due E. for 25 miles, what are her latitude and longitude in? What course must she be steered by the ship's standard compass, allowing 25° 49′ W. for deviation in the first position of her head, and 12° 50′ E. in the second? (1869.)

Ans. Lat. in 49° 20′ N.; lon. in 8° 6′ 39″ W.
Courses S. 25° 49′ W. and N. 77° 10′ E.

265. How far must a ship sail due E. in latitude 70° to change her longitude 5°. *Ans.* 102·6 miles.

266. In what latitude is it that every time a ship goes 8 miles she changes her longitude 9·5 miles. *Ans.* 32° 38′ (lat.)

267. A ship changes her longitude 1° when she has only sailed 50 miles, what is the latitude? *Ans.* 33° 33′ (lat.)

268. How many miles answer to a degree of longitude in latitude 48° 26′? *Ans.* 39·77 miles.

269. In what latitude do 46·08 miles answer to one degree of longitude? *Ans.* 39° 49′.

270. A ship sails in latitude 32° N. due E. till her difference of longitude is 384 miles; required the distance she has run.

Ans. 325·6 miles.

271. A ship from latitude 53° 36′ N., longitude 10° 18′ E., sails due W. 236 miles; required her present longitude.

Ans. 3° 40′ longitude E.

272. In what latitude do 384 miles of meridian distance answer to 500 miles of difference of longitude? *Ans.* 39° 49′ latitude.

273. From two ports in latitude 32° 20′ N., distant 256 miles, two ships sail directly N., how far are they distant from one another when they come to latitude 44° 30′ N. *Ans.* 216·1 miles.

274. If two ships in latitude 44° 24′ N., distant from each other 217 miles, should sail directly S. until their distance is 256 miles apart; what latitude are they in? *Ans.* 32° 31′ N. latitude.

275. What is the distance between St. Mary in latitude 37° N., lon. 25° W., and a place in latitude 37° N., lon. 76° 23′ W.

Ans. 2462 miles.

276. A ship from Cape Clear sails 265 miles westward, what is the longitude in? Cape Clear is in latitude 51° 27′ N., longitude 9° 29′ W.

Ans. Longitude 16° 34′ W.

277. A ship having run due E. for three days at the rate of five knots an hour, finds she has altered her longitude 8° 16′; what parallel of latitude did she sail on? *Ans.* Latitude 43° 28′.

278. Suppose two ships in the parallel of 47° 54′ N., their difference of longitude being 9° 35′, should sail directly S. 836 miles; how far were they from each other when they set out, and also after they had run the given distance? *Ans.* 385·5 miles mer. dis.
476·9 ,,

279. What is the difference of latitude between two parallels on different sides of the equator, when between the same two meridians, the meridian distance in the northern parallel is 238 miles, in the southern it is 195 miles, and at the equator 352 miles.

Ans. 103° 49′.

280. A ship sails due W. from Funchal (latitude 32° 37′ N., longitude 16° 58′ W.); how many miles will she go before she arrives at the meridian of Fayal (lon. 28° 43′ W.)? (1873.) *Ans.* 593·8 miles.

281. In sailing on a parallel of latitude I find the distance actually made good is 38 nautical miles, while I have changed my longitude by one degree; on what parallel am I sailing? *Ans.* 50° 42′.

282. Two places on the earth's surface move with velocities in the ratio of 5:2, their difference of latitude is 52°; find the latitude of the two places. *Ans.* Lat. 15° 18′ and 67° 18′.

283. Two ships in latitude 24° 24′ N., distant 215 miles, sail directly S. till their distance apart is 235 miles; what latitude have they come to? *Ans.* 5° 30′ latitude.

284. While one place on the earth's surface moves one mile on account of the rotation of the earth on its axis, another moves $1\frac{5}{9}$, their distance apart is 32° of latitude; find the latitude of the two places. *Ans.* Latitudes 21° 10′ and 53° 10′.

CHAPTER IX.

MIDDLE LATITUDE SAILING.

Definition—Explanation—Proof of Rules—Rules—Examples—Traverse Table applied to Middle Latitude Sailing—Examination Exercises.

72. Middle Latitude Sailing is Parallel and Plane Sailing combined, or it is the principles of Plane and Parallel Sailing used conjointly. The method is not exactly accurate, inasmuch as plane sailing is partly employed in its solution, but the parallel sailing partially corrects the errors introduced by the false assumption made in plane sailing—

(*a.*) When a ship sails due north or south, she, keeping on the same meridian, merely alters her latitude without producing any effect on the longitude; the distance she runs is her difference of latitude, which being properly applied to the "latitude from," gives the "latitude in."

(*b.*) When a ship sails due east or west she keeps upon the same parallel and so alters not her latitude, she only changes her longitude; the difference of longitude she has made is determined by the equation $M = L \text{ Cos. } l$; so, knowing the distance she has sailed we can find the longitude in, or knowing the meridian left and arrived at, we can find the difference of longitude.

(*c.*) When a ship sails neither on a parallel nor on a meridian, but on an oblique course, or on a rhumb line at an angle with the meridians, she alters both her latitude and longitude.

It is the province of Middle Latitude Sailing to show how in this latter case the *course, distance, departure, difference of latitude, latitude in,* and *longitude in,* are found when certain of these quantities are given.

In Parallel Sailing we have had M = L × Cos. *l*, in middle latitude sailing we write this Dep. = diff. lon. × Cos. mid-lat., or mer. dis. = diff. lon. × Cos. mid-lat. The adaptation of the formula is founded on the consideration that the arc of the parallel in the middle latitude of two places, intercepted between their two meridians, is very nearly equal to the departure between the two places.

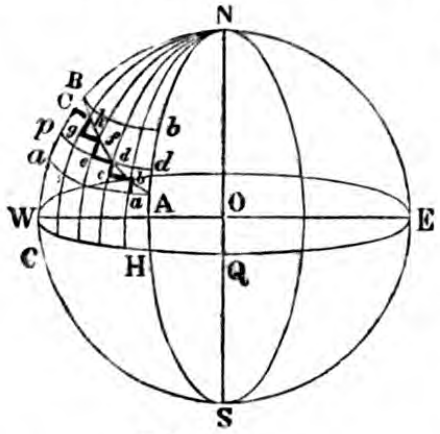


Fig. 22.

Let N be the pole of the earth.

A and B two places on the earth's surface.

N H and N C the meridians passing over the places A and B.

C H the arc of the equator intercepted between the meridians passing over A and B.

And the line joining A and B the rhumb line passing over A and B.

Let A *a* and B *b* be arcs of the parallels of latitude passing through A and B, and *d p* the parallel of latitude passing through the middle place between A and B.

Then, if a ship be supposed to sail on a rhumb line from A to B, her departure will evidently be greater than B *b*, but less than A *a*, and is in many cases very nearly equal to *d p*, the departure in the middle latitude.

Hence *d p* = C H cos. H *d*.

∴ *dep.* or *mer. dis.* = diff. long. × cos. mid-lat.

This formula is not exactly correct, but is sufficiently so for all practical purposes. We will show presently when the greatest amount of error exists. There is, however, some third parallel between those passing through A and B, by using which, to compute the distance between the two meridians, the exact distance required may be obtained. At a great distance from the pole, or near the equator, and when the difference of latitude is small, or where the course is greater than 45°, this third parallel is found nearly in the middle, between the two, but always a little *nearer* the pole. For this reason, and from an inspection of our figure, we come to these conclusions, that middle latitude sailing should not be used

in high latitudes, nor when the course is near the meridian, nor when the places are one on one side of the equator and the other on the other; but it may be employed with exactitude when the places are on the same side of the equator, in low latitudes, and when the course is greater than 45°.

We may suppose the rhumb line A B to consist of the hypothenuses of an indefinite number of small triangles, and that the other sides of the triangles are represented by the dark lines, these latter will evidently stand for the meridian distances and differences of latitudes; the lines parallel to the equator represent the former, and those perpendicular to the same the differences of latitude. Let us suppose that the triangles formed by the meridians and parallels drawn on our figure are indefinitely small or infinite in number, so that the small portions of the curve lines which constitute them may, without sensible error, be taken for straight lines. These small triangles will be all similar to each other, since they each contain a right angle, for the meridians cut the parallels at right angles, and the angles to the left or the upper angle in each triangle is the course, therefore they are all equal, and the third or remaining angle must then be equal; therefore the triangles are similar to each other. Hence, by *Euclid* V. 12, the departure being represented by the base of the infinitely small triangle, the sum of the bases represents the whole departure, the sum of the hypothenuses represents the total distances, and the sum of the third sides represents the total difference of latitude.

Now considering the small triangles as right-angled triangles.

(*a*) $\text{Cos. course} = \frac{ab}{Ab}$ $\therefore$ $ab = Ab$ cos. course
$cd = bd$ cos. course
$ef = df$ cos. course
$gh = fh$ cos. course
etc. = etc.

$\therefore ab + cd + ef + gh + \text{etc.} = (Ab + bd + df + fh + \text{etc.})$ cos. course.

$\therefore$ *Diff. of Latitude* = *Distance* × *cos. course.*

$$(b) \qquad \text{Sin. course} = \frac{Aa}{Ab} \quad \therefore \begin{array}{l} Aa = Ab \text{ sin. course} \\ bc = bd \text{ sin. course} \\ de = df \text{ sin. course} \\ fg = fh \text{ sin. course} \\ \text{etc.} = \text{etc.} \end{array}$$

$$\therefore Aa + bc + de + fg + \text{etc.} = (Ab + bd + df + fh + \text{etc.}) \text{ sin. course.}$$

$$\textit{Departure} = \textit{distance} \times \textit{sin. course.}$$

$$(c) \qquad \text{Tan. course} = \frac{Aa}{ab} \quad \therefore \begin{array}{l} Aa = ab \text{ tan. course} \\ bc = cd \text{ tan. course} \\ de = ef \text{ tan. course} \\ fg = gh \text{ tan. course} \\ \text{etc.} = \text{etc.} \end{array}$$

$$\therefore (Aa + bc + de + fg + \text{etc.}) = (ab + cd + ef + gh + \text{etc.}) \text{ tan. course.}$$

$$\therefore \textit{Departure} = \textit{diff. lat.} \times \textit{tan. course.}$$

We now repeat succinctly much that has been said; and let the student, if he does not thoroughly understand what we now say, begin the chapter again. No time will be lost by so doing.

The departure, made by sailing on a rhumb line between two places on the earth's surface, is equal to the distance of the meridians of the two places reckoned on the parallel of latitude, which lies midway between the parallels on which the two places lie. And although this supposition is not strictly correct, for the mean of the cosines of the two latitudes is not exactly the cosine of the arithmetic mean of those latitudes, nor is the departure on the rhumb cutting the two meridians precisely equal to that in the mean position, but it is equal to that in a somewhat higher latitude; yet, when the places are in low latitudes, or not far from the equator, and when the parallels are close to each other, the error arising from the assumption made in middle latitude sailing is very small, and does not sensibly affect the nautical conclusions drawn from it.

We will now indicate the chief formulæ used in middle latitude sailing, and then give a few illustrations drawn from examination papers. Before proceeding, let us insist upon the necessity, in all cases, of drawing the triangle before any attempt is made to solve the problems; and that it is most needful, in order to have a firm grounding in this subject,

for the student to pick out the rules from the triangle and not from his memory.

$$\text{Mer. dis.} = \text{diff. lon.} \times \text{cos. lat.}$$
$$\text{Dep. or mer. dis.} = \text{diff. lon.} \times \text{cos. mid-lat.} \qquad \text{(Z)}.$$

By plane sailing,

$$\text{Tan. course} = \frac{\text{dep.}}{\text{diff. lat.}}$$

$$\therefore \text{Tan. course} = \frac{\text{diff. lon.} \times \text{cos. mid-lat.}}{\text{diff. lat.}} \qquad \text{(A)}.$$

$$\text{From (Z) diff. lon.} = \text{dep.} \times \text{sec. mid-lat.}$$
$$\text{By plane sailing, dep.} = \text{dis.} \times \text{sin. course.}$$
$$\therefore \text{diff. lon.} = \text{dis.} \times \text{sin. course} \times \text{sec. mid-lat.} \qquad \text{(B)}.$$

Every problem in middle latitude sailing will resolve itself into one of the three cases, Z, A, or B, but the most important are A and B. It is as well to call attention to the circumstance, that the aim in all we have under consideration is to show how the longitude may be obtained with the greatest accuracy.

A proper and judicious selection of examples in Mid-Latitude Sailing should always be made. Careful attention should be given, as it shows a want of thought, and a due appreciation of the rules and distinctions in Navigation, to work out certain classes of problems by Middle Latitude Sailing. In the government examinations, it is a very common practice to ask for the course and distance from a certain latitude and longitude to another differing both in latitude and longitude, leaving it to the option of the student whether he will work by Middle Latitude or Mercator's Sailing. This is really a test of the superior training of the student. The safest way in all these cases is to use Mercator's Sailing; but it is open to the student to exercise a little discrimination. Generally, if the latitudes are high, use Mercator's Sailings; but if low, and the difference of longitude is greater than the difference of latitude, or it would have been more correct to say, the meridian distance greater than the difference of latitude, the student may, if he prefer, solve the question by Middle Latitude Sailing.

285. Required the compass course and distance from Cape Negro, latitude 15° 50′ S., longitude 11° 30′ E., to Cape St. Roque, latitude

5° 15′ S., longitude 34° 30′ W. (1864). This question is incomplete; we complete it thus: variation 19° 10′ W., deviation 7° 15′ W.

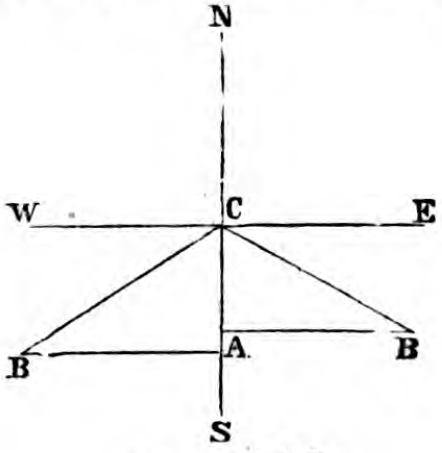
		Diff. lat.	Mid-lat.		Diff. lon.
Cape Negro,	lat.	15° 50′ S.	15° 50′ S.	Longitude	11° 30′ E.
Cape St. Roque,	,,	5 15 S.	5 15 S.	,,	34 30 W.
		10 35	2)21 5		46 0 W.
		60	10 32½ mid-lat.		60
	Miles,	635 N.		Diff. lon.	2760 W.

To find course—

See figure, but remember the course is between N. and W

$$\text{Tan. } C = \frac{AB}{AC} = \frac{\text{dep.}}{\text{diff. lat.}} = \frac{\text{diff. lon.} \times \text{cos. mid-lat.}}{\text{diff. lat.}}$$

Log 2760 diff. lon.	3·440909
Log. cos. 10° 32½′ mid-lat.....	9·992608
	13·433517
Log 635 diff. lat.	2·802774
Log tan. 76° 49′ course.....	10·630743



$$\text{Cos. } C = \frac{AC}{BC}$$

$$\therefore \text{ Cos. } C = \frac{\text{diff. lat.}}{\text{dis.}}$$

$\therefore$ distance = diff. lat. × sec. course.

Log 635 diff. lat.	2·802774
Sec. 76° 49′ course,.....	10·641936
Log 2784 dist.....	3·444710

True course N. 76° 49′ W. is..	76° 49′ l. of N.
Variation, ...	19 10 r.
Deviation,.....	57 39 l. of N.
	7 15 r.
	50 24 l. of N.

Ans. Compass course N. 50° 24′ W.; distance, 2784 miles.

RULES FOR MIDDLE LATITUDE SAILING.

To find course.—To log difference of longitude add log cos. mid-latitude, from this subtract log difference of latitude, the remainder is log tan. course.

To find departure.—To log difference of longitude add log cos. mid-latitude, the sum, omitting 10, is the log of the departure.

To find distance.—To log difference of latitude add log secant course, the sum, omitting 10, is log distance.

To find difference of longitude.—Add together log distance, log sine course, and log secant mid-latitude, omitting 10, the sum is log difference of longitude.

286. Required the compass course and distance from A to B. (1866.)

From A to B would be across the land, but it was set by the Science and Art Department in 1866 as one of the questions in Navigation.

Lat. A 48° 15′ } N.; variation 1½ pts. W.; lon. A 12° 23′ } E.
Lat. B 43 18 } N.; deviation 5° 35′ E.; lon. B 30 15 } E.

Diff. lat.	Mid-lat.	Diff. lon.
A is in lat. 48° 15′ N.	48° 15′ N.	Longitude 12° 23′ E.
B is in lat. 43 18 N.	43 18 N.	,, 30 15 E.
4 57	2)91 33	17 52
60	45 46½ mid-lat.	60
Diff. lat. 297 miles S.		Diff. lon. 1072 E.

$$\text{Tan. course} = \frac{AB}{AC} = \frac{\text{dep.}}{\text{diff. lat.}} = \frac{\text{diff. lon.} \times \text{cos. mid-lat.}}{\text{diff. lat.}}$$

Log 1072 diff. lon.	3·030195
Log cos. 45° 46½′ mid-lat.	9·843531
	12·873726
Log 297 diff. lat.	2·472756
Log tan. 68° 20′ course.....	10·400970

For distance.

$$\text{Cos. } C = \frac{AC}{CB} = \frac{\text{diff. lat.}}{\text{distance}}$$

$\therefore$ dis. = diff. lat. × sec. C.

Log. 297 diff. lat.	2·472756
Log. sec. course	10·432731
Log. 804·4 dist.	2·905487

For compass course.

True course S. 68° 20′ E. is 68° 20′ l. of S.

Variation	16 52 30″ r.
	51 27 30 l.
Deviation	5 35 0 l.
	57 2 30 l.

Ans. Compass course S. 57° 2′ 30″ E.; distance 804·4 miles.

287. Required the course and distance from A to B. Given:
Lat. A 45° 18′ N.; var. of compass 1½ pts. E.; lon. A 36° 15′ E.
Lat. B 29 27 N.; deviation 2° 30′ E.; lon. B 57 18 E. (1866).

Diff. of lat.	Mid-lat.	Diff. lon.
Lat. of A 45° 18′ N.	45° 18′ N.	Lon. 36° 15′ E.
Lat. of B 29 27 N.	29 27 N.	Lon. 57 18 E.
15 51	2)74 45	21 3 E.
60	37 22½ mid-lat.	60
Miles 951 S.		Diff. lon. 1263 E.

$$Tan.\ course = \frac{A\,B}{A\,C} = \frac{dep.}{diff.\ lat.} = \frac{diff.\ lon.\ cos.\ mid\text{-}lat.}{diff.\ lat.}$$

Log 1263 diff. lon.....	3·101403
Log cos. 37° 22′½ mid-lat.....	9·900192
	3·001595
Log 951 diff. lat...	2·978181
Log tan. 46° 32′ course.....	10·023414

For distance.

$$Cos.\ course = \frac{A\,C}{A\,B} = \frac{diff.\ lat.}{distance}$$

$\therefore$ *distance* = *diff. lat.* × *sec. course.*

Log 951 *diff. lat.* .....=	2·978181
Log 46° 32′ *sec. course* =	10·162454
Log 1382·1 m. distance =	3·140635

For compass course.

True course S.	46°	32′	0″	E. l. of S.
Variation	16	52	30	l.
	63	24	30	l. of S.
Deviation ...	2	30	0	l.
	65	54	30	l. of S.

Ans. Compass course S. 65° 54′ 30″ E.; distance 1382 miles.

288. A ship leaves a place in latitude 12° 33′ S., and longitude 94° 20′ E., and sails by compass E.S.E. 98 miles, variation 8° E., deviation 7° 20′ E., leeway ¾-point, direction of the wind S.byE.; find the latitude and longitude in. (1873.)

This example can be worked out by the rules given under Traverse Sailing, but it will also well illustrate our work under Middle Latitude Sailing.

To find true course—

Compass course, E.S.E.....	67°	30′	0″	l. of S.
Variation and deviation.....	15	20	0	r.
	52	10	0	l. of S.
Leeway	8	26	15	l.
True course ..... S.	60	36	15	E.

To find difference of latitude—

$$Cos.\ course = \frac{diff.\ lat.}{distance.}$$

$$\therefore\ diff.\ lat. = distance \times cos.\ course.$$

Log 98 m.	1·991226
Log *cos.* 60° 36′¼	9·690940
Log 48·1 m. diff. lat.	1·682166

To find latitude in—

Latitude from	12°	33′	0″	S.
Difference of latitude		48	6	S.
Latitude in	13	21	6	S

To find mid-latitude—

Latitude from	12°	33′	0″	S.
Latitude in	13	21	6	S.
	2)25	54	6	
Mid-lat.	12	57	3	

To find difference of longitude and the longitude in—

$$Tan.\ C = \frac{diff.\ lon. \times cos.\ mid\text{-}lat.}{diff.\ lat.}$$

$$\therefore\ diff\ lon. = \frac{tan.\ C \times diff.\ lat.}{cos.\ mid\text{-}lat.}$$

Log tan. course 60° 36′¼	10·249201
Log diff. lat. 48·1	1·682166
	11·931367
Log cos. mid-lat. 12° 57′ 3″	9·988810
Diff. lon. 87·61	1·942557

Longitude from	94°	20′	0″	E.
Diff. lon. 87·61	1	27	36	E.
Longitude in	95	47	36	E.

Ans. Lat. in 13° 21′ 6″ S.; longitude in 95° 47′ 36″ E.

289. A ship sails from A (latitude 50° 30′ N., longitude 15° 45′ W.) to B as follows: S.W.½-S. 40 miles, N.byW. 15 miles, S.S.E. 55 miles, W.byN. 70 miles; find the latitude and longitude of B.

Also find what course she would have to take from A to B to arrive at the latter place without altering her direction, and the distance she would have to go.* (1873.)

* See first observation concerning last illustration.

Courses.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
S.W.½-S.	40	...	30·9	...	25.4
N.byW.	15	14·7	...	...	2·9
S.S.E.	55	...	50·8	21	...
W.byN.	70	13·7	...	...	68·7
		28·4	81·7	21	97·0
			28·4		21
			53·3		76

To find latitude in—

Latitude from	50°	30′	0″	N.
Diff. of lat.	0	53	18	S.
Lat. in	49	36	42	N.

To find mid-latitude—

Latitude from	50°	30′	0	N.
Latitude in	49	36	42	N.
Sum 2)	100		6·42	
Mid-lat.	50		3 21	

To find difference of longitude—

Departure = diff. lon. × cos. mid-lat.

76 = diff. lon. × cos. 50° 3′ 21″

∴ diff. lon. = 76 × sec. 50° 3′ 21″

Log 76	1·880814
Log sec. 50° 3′ 21″	10·192437
Log 118·3 diff. lon.	2·073251

To find longitude in—

Longitude from	15°	45′	0″	W.
Diff. lon. 118·3	1	58	18	W.
Longitude in	17	43	18	W.

The course and distance from traverse tables are S. 55° W. 93 miles.

By middle latitude sailing they are found thus—

$$Tan.\ course = \frac{diff.\ lon. \times cos.\ mid\text{-}lat.}{diff.\ latitude.}$$

Log diff. lon. 118·3	2·073251
Cos. mid-lat. 50° 3′ 21″	9·807563
	11·880814
Diff. of latitude 53·3	1·726727
Log. tan. course 54° 57′ 26″	10·154087

To find distance—

$$Cos.\ course = \frac{diff.\ lat.}{distance.}$$

$$\therefore distance = diff.\ lat. \times sec.\ course.$$

Log diff. lat. 53·3	1·726727
Log sec. 54° 27′ 26″	10·235591
Log 91·68 m. dist.	1·962318

Ans. Lat. in 49° 36′ 42″ N.; course S. 54° 57′ 26″ W.
Lon. in 17° 43′ 18″ W.; distance 91·68 miles.

290. Explain the principle and prove the rules for middle latitude sailing. (1861.)

291. Required the compass course and distance from A to B.
Lat. A 55° 40′, Lat. B 57° 50′ } N.; var. 1½ E.; lon. A 29° 39′, lon. B 34° 15′ } W. (1863.)
Ans. Compass course N. 66° 12′ 30″ W.; distance 199·4 miles.

292. Required the compass course and distance from A to B.
Lat. A 9° 30′, Lat. B 14° 10′ } N.; var. 1¾ pts. E.; lon. A 72° 15′, lon. B 65° 40′ } E. (1863.)
Ans. Compass course N. 73° 46′ 15″ W.; distance 477·3 miles.

293. What is meant by middle latitude sailing? obtain the formulæ required for it.

294. Required the true course and distance from A to B, the latitude of A 57° 35′ N., and longitude 17° 16′ W., and latitude of B 55° 25′ N., longitude 20° 57′ W. (1863.)
Ans. True course S. 43° 10′ W.; distance 178·2 miles.

295. Find the compass course and distance between A and B.
Lat. A 33° 18′ S.; lon. A 72° 0′ W.; var. 16° E.
Lat. B 42° 3′ S.; lon. B 173° 30′ E.; dev. 9° 25′ E. (1870.)
Ans. Course S. 59° 4′ W.; distance 5461 miles.

296. Write down the formulæ for middle latitude sailing. Under what limitations would you apply this method? (1865.)

297. Required the compass course and distance from A to B.
Lat. of A 15° 20′, Lat. of B 19° 40′ } S.; var. 2½ pts. W.; dev. 7° 50′ W.; lon. A 4° 10′ E., lon. B 1° 20′ W. (1855).
Ans. Compass course S. 86° 23′ 30″ W.; distance 408·1 miles.

73. Traverse Tables used in Middle Latitude Sailing.—The next example will be worked out mechanically to show that, with the aid of the Traverse Tables, it is a simple matter to resolve problems in Middle Latitude Sailing. The problem is afterwards worked by the usual method, and a comparison can be instituted between the two results.

The following nine examples are adapted from Robertson's *Navigation.* London, 1786.

298. What is the course and distance from Cape Clear in Ireland, latitude 51° 24′ 55″ N., longitude 9° 29′ W. to the Island of St. Mary's in the Azores, latitude 36° 58′ N., longitude 25° 12′ 33″ W.

	Latitude.		Mid-lat.		Longitude.
Cape Clear	51° 24′ 55″ N.		51° 24′ 55″		9° 29′ 0″ W.
St. Mary	36 58 0 N.		36 58 0		25 12 33 W.
Diff. lat....	14 26 55		2)88 22 55	Diff. lon.	15 43 33
	60	Mid-lat.	44 11 27·5		60
	S. 866·9 m.		90		W. 943·5 miles.
		Co-mid-lat.	45 48 32·5		

(*a.*) With the co-mid-latitude you go to the Traverse Table and find 46° at the bottom of the page, then look for 943·5 in the distance column; but as 943·5 is a much larger number than is given on any page for distance, the greatest being 300, we take an aliquot part, say $\frac{1}{5}$ or $\frac{943\cdot5}{5} = 188\cdot7$; opposite this distance 188·7, or the nearest 189, is found departure 136. Multiplying 136 by 5 gives 680, which is the proper departure.

(*b.*) We now take the difference of latitude 866·9 and departure 680, and look for them together in the table; as before, being too great, we use a part only, say $\frac{1}{4}$ of 866·9 and 680, or 216·7 and 170—the nearest are 216·7 and 169·3—these are found together under 38° opposite the distance 275 miles, this 38° is therefore the course, and distance 275 miles is multiplied by the 4 = 275 × 4 = 1100, which is the distance sought. *Ans.* Course S. 38° W.; distance 1100 miles.

By calculation the course and distance are found thus—

$$\text{Tan. C} = \frac{\text{diff. lon.} \times \text{cos. mid-lat.}}{\text{diff. lat.}}$$

Log diff. lon. 943·5	2·974742
Log cos. mid-lat. 45° 48′ 32″·5	9·843266
	12·818008
Log diff. lat. 866·9	2·937969
Log tan. course 37° 11′ 8″	9·880039

Dist. = diff. lat. × sec. course.

Log diff. lat. 866·9.....	2·937969
Log sec. course	10·098718
Log distance 1088·14 miles.....	3·036687

Ans. Course S. 37° 11′ 8″ W.; distance 1088·14 miles.

299. A ship had taken her departure from a place (St. Michael in the Azores) 37° 45′ 10″ N., 25° 41′ 30″ W., steers N. 33° 19′ E., bad weather coming on, no observation was obtained for eight days, the master then found the vessel was in latitude 51° 48′ N.; required the true distance and present longitude.

Latitude.		Mid-latitude.	
Latitude from	37° 45′ 10″ N.		37° 45′ 10″ N.
Latitude in	51 48 0 N.		51 48 0 N.
	14 2 50		2)89 33 10
	60		44 46 35
miles	842·8 N.		90
		Cos. mid-lat.	45 13 25

(1.) We take the course 33° among the degrees, and find the difference of latitude 842·8 in its proper column, and take out the departure and distance that accompany the difference of latitude. Using one-fourth of 842·8 gives, after multiplication, distance 1004, departure 546·8.

(2.) With co-mid-latitude among the degrees and departure in its own column, we find the difference of longitude among the distances. Using 45° and one-third of 546·8 gives, after multiplication, distance = 774, which we use as the difference of longitude.

Longitude from is	25° 41′ 30″ W.
Difference of longitude (774)....	12 54 0 E.
Longitude in	12 47 30 W.

By direct computation the answers would be obtained thus—

Dis. = *diff. lat. sec. course*

Log. diff. lat. 842·8...	2·925725
Log cosec. C. 33° 19′	10·077977
Log dis. 1008·56 m....	3·003702

$$Diff.\ lon. = \frac{tan.\ course \times diff.\ lat.}{cos.\ mid\text{-}lat.}$$

Log tan. C. 33° 19′...	9·817759
Log diff. lat. 842·8 ..	2·925725
Log sec. 45° 13′ 25″	10·152217
Diff. lon. 786·5.....	2·895701

Longitude from	25° 41′ 30″ W.
Diff. lon. 786·5	13 6 30 E.
Longitude in	12 35 0 W.

The difference in the answers is caused by taking the minutes into account in the calculation, and neglecting them in the work by inspection. Had we chosen to interpolate between the degrees in the Traverse Table, the answers would agree more nearly.

In the working of the above, it is well to observe that the secant of the mid-latitude is employed and added instead of subtracting the cosine. This is always the best method to employ, and saves both time and space.

300. A ship takes her departure from a place in latitude 51° 18′ N., longitude 9° 50′ W., and steers S. 33° 8′ W. for 1024 miles; required the present latitude and longitude. *Ans.* Lat. in 37° 0′ 30″ N.
Lon. in 22° 50′ 6″ W.

301. A vessel in latitude 51° 18′ N., longitude 22° 6′ W., is bound to a place in the S.E. quarter distant 1024 miles, the place bound for is in latitude 37° N.; what is the course and longitude arrived at?
Ans. S. 33° 5′ E; longitude 9° 6′ 48″ W.

302. A ship from latitude 37° N., longitude 48° 20′ W., sails between N. and E., and arrives at latitude 51° 18′ N., having made 564 miles departure; find the course, distance run, and present longitude. *Ans.* Course N. 33° 19′ E., or N.E.byN. nearly;
Dist. 1027; longitude in 35° 14′ W.

303. A schooner leaves latitude 51° 18′ N., longitude 9° 50′ W., and sails S. 33° 19′ W. till her departure is 564 miles; find present latitude and longitude and the distance run.
Ans. Latitude in 37° N.; distance 1026 miles.
Longitude in 22° 55′ 54″ W.

304. Suppose a vessel leaves latitude 37° N., longitude 9° 2′ W., and sails between N. and W. for 1027 miles, with a departure of 564 miles; required the course with latitude and longitude in.
Ans. Course N. 33° 19′ W., or N.W.byN. nearly.
Lat. in 51° 18′ N.; lon. in 22° 8′ W.

305. A ship from latitude 51° 18′ N., longitude 22° 6′ W., having sailed between S. and E. for several days, made 564 miles departure, which was equivalent to 786 miles of longitude; what is the latitude and longitude of the place arrived at? Find also course and distance.
Ans. Lat. in 37° N.; lon. 9° W.
Course S. 33° 19′ E., or S.E.byS. nearly; distance 1026 m.

306. From a place in north latitude a ship sails S. 33° 15′ E. until 564 miles of departure are made, and 786 miles difference of longitude; required the latitude sailed from and arrived at by the formulæ employed in Middle Latitude Sailing.

This last example is solved thus—

$$(1.)\ \textit{Cos. mid-lat.} = \frac{\textit{departure}}{\textit{diff. lon.}} = \frac{564}{786}$$
$$= 44° 7′ 49″$$

Again, $$\frac{\textit{Sin. course}}{\textit{departure}} = \frac{\textit{cos. course}}{\textit{diff. lat.}}$$

$$\therefore \textit{diff. lat.} = \frac{\textit{cos. course} \times \textit{departure}}{\textit{sin. course}}$$

$$= 860{\cdot}2 \text{ miles}$$

$$= 14^\circ\ 20'\ 12''$$

$$\therefore \text{Half} = 7^\circ\ 10'\ 6''$$

$$\therefore 44^\circ\ 7'\ 49'' + 7^\circ\ 10'\ 6'' = \text{lat. from} = 51^\circ\ 17'\ 55''\ \text{N.}$$

$$44^\circ\ 7'\ 49'' - 7^\circ\ 10'\ 6'' = \text{lat. in} = 36^\circ\ 57'\ 43''\ \text{N.}$$

Ans. Lat. from 51° 17′ 55″ N.; lat. in 36° 57′ 43″ N.

307. A master took his departure from a point in lat. 51° 35′ N., longitude 8° 30′ W., bearing N.N.W. distant 20 miles, and then sailed as by the following log account. Correct the courses for variation and leeway, and find the course and distance from the given point, and latitude and longitude in by Middle Latitude Sailing; from thence find course and distance to Ushant.

Hours.	Courses.	Knots.	Tenths.	Wind.	Leeway.	Remarks.
1	S.S.W.	6	6	N.W.	4°	
2		8	4			
3		4	7			
4		7	5			
5		5	8			
6	S.	3	2	N.W.	5°½	Variation, 22° 30′ E.
7		8	4			
8		4	4			
9		7	5			
10		11	1			
11		5	4			
12	S.E.byS.	4	5	N.W.	3°	
1		2	5			
2		3	5			
3		3	1			
4		4	2			
5		5	2			
6	S.S.E.	4	4	W.	8°	
7		4	4			
8		3	4			A current lasted from 7 P.M. to 4 A.M., and carried the ship 2½ miles per hour S.E.½-S. by compass.
9		4	4			
10		5	4			
11		8	5			
12		3	5			

Correcting the courses and tabulating as below, we have—

Courses.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
S.	20		20		
S. 41° W.	33		24·9		21·6
S. 17° W.	40		38·3		11·7
S. 14° E.	23		22·3	5·6	
S. 8° E.	34		33·7	4·7	
S. 1½ pts. E.	22½		21·5	6·5	
			160·7	16·8	33·3
					16·8
					16·5

To find latitude in—

Latitude from	51°	35′	0″ N.
Diff. of lat. 160·7	2	40	42 S.
Lat. in	48	54	18 N.

To find mid-latitude—

Latitude from	51°	35′	0″ N.
Latitude in	48	54	18 N.
2)100	29	18	
50	14	39	N.

To find course—

$$Tan.\ course = \frac{departure}{diff.\ of\ lat.} = \frac{16{\cdot}5}{160{\cdot}7}$$

Log 16·5	1·217484
Log 160·7	2·206016
Log tan. 5° 51′ 44″	9·011468

To find distance—

$$Cos.\ course = \frac{diff.\ lat.}{distance}$$

$\therefore$ *dis.* = diff. lat. × sec. course

Log diff. lat.	2·206016
Log sec. course	10·002277
Log dist. 161·5	2·208293

To find difference of longitude—

$$diff.\ lon. = \frac{Tan.\ course \times diff.\ lat.}{cos.\ mid\text{-}lat.}$$

Log tan. course	9·011468
Log diff. lat.	2·206016
Log sec. mid-lat.	10·194147
Log diff. lon. 25·806	1·411631

To find longitude—

Longitude from	8° 30′ 0″ W.
Diff. lon. 25·8	0 25 48 W.
Longitude in	8 55 48 W.

It remains now to find the course and distance to Ushant.

	Latitude.	Mid-latitude.	Longitude.
Lat. from ...	48° 54′ 18″ N.	48° 54′ 18″	8° 55′ 48″ W.
Lat. Ushant	48 28 8 S.	48 28 8	5 3 6 W.
	26 10	2)97 22 26	3 52 42 E.
	= 26·1 miles S.	48 41 13	60
			232·7

To find course—

$$Tan.\ course = \frac{diff.\ lon. \times cos.\ mid\text{-}lat.}{diff.\ of\ lat.}$$

Log diff. lon. ... 232·7	2·366796
Log cos. mid-lat. 48° 41′ 13″	9·819658
	12·186454
Log diff. lat. 26·1	1·416641
Log tan. course 80° 21′ 27″	10·769813

To find distance—

$$Cos.\ course = \frac{diff.\ lat.}{distance}$$

$\therefore$ distance = *diff. lat.* × sec. course.

Log. diff. lat.	1·416641
Log sec. C.	10·775984
Log dis. 155·8	2·192625

Ans. Course S. 80° 21′ 27″ E.; distance 155·8 miles.

In high latitudes it is very unwise to employ middle latitude sailing. An example will be worked out to show that in such a case, by taking the cosine of the middle latitude, a considerable error remains, which might, uncorrected, lead to fatal results.

308. A ship from latitude 67° 30′ N., longitude 8° 46′ W., sailed in four days according to the following log account:—N.E. 64 miles, N.W.byN. 58 miles, W.N.W. 72 miles, W. 48 miles, S.byE. 45 miles, E.S.E. 40 miles; find the present latitude.

Traverse Table.						Longitude Table.					
Course.	Distance.	Diff. lat.		Departure		Successive latitudes.	Sum of latitudes.	Mid-latitudes.	Co-mid-latitudes.	Difference of longitude.	
		N.	S.	E.	W.					E.	W.
						67° 30′					
N.E.....	64	45·3		45·3		68 15	135° 45′	67° 52′	22° 8′	120	
N.W.byN..	58	48·2			32·2	69 3	137 18	68 39	21 21		91
W.N.W....	72	27·6			66·5	69 31	138 34	69 17	20 43		190
W.....	48				48·0	69 31	139 2	69 31	20 29		137
S.byE.....	45		44·1	8.8		68 47	138 18	69 09	20 51	25	
E.S.E.	40		15·3	36·9		68 32	137 19	68 39	21 21	101	
		121·1	59·4	91·0	146·7					246	418
		59·4			91·0						246
		61·7			55·7						172

To find latitude—

Latitude from	67°	30′	0″ N.
Diff. lat.	1	1 42	N.
Latitude in	68	31 42	N.
Sum of latitudes.....	)136	1 42	
Mid-latitude.....	68	0 51,	and its complement is 21° 59′ 9″.

To find longitude—

Longitude from.....	8°	46′	0″ W.
Diff. lon. 172.....	2	52	0 W.
Longitude in	11	38	0 W.

If the co-mid-latitude 22° be used to find the longitude, opposite departure 55·7 we have distance 149 in the distance column. Employing this as the difference of longitude, the longitude becomes

Longitude from.....	8° 43′ W.
Diff. lon. 149.....	2 29 W.
Lon. in	11 15 W.

between the 11° 15′ and 11° 38′ W. exists a difference of 23 miles, a very large amount, showing very clearly that the errors arising from the assumption made in mid-latitude sailing may amount to something considerable in latitudes beyond 45° or 50°. In the right hand columns of the preceding page something slightly different to anything previously exhibited may be seen; but a little patience will show that there is nothing novel in it. All that is done is this:—

(1.) With the difference of latitude found from the traverse table, the latitudes from and in are obtained at the end of each course and distance respectively. These successive latitudes fill up the first column. In the second column is given the sum of each latitude and the one preceding it; in the third, the half of these or the mid-latitudes at the end of each course and distance, with the co-mid-latitude in the next column; in the two extreme right hand columns are given the differences of longitude obtained by using these co-mid-latitudes; and the departures in the last column of the left hand table: putting each in its proper column, the east and west are added up, and the difference taken, which, on the whole, shows that the vessel has altered her longitude 172′.

Briefly, then, the reason why one result differs so widely from the other is, that in the one case the departures are all reckoned upon one parallel of latitude (68°), namely, the middle parallel between the place left and that at which the vessel has arrived. We have intimated plainly that when a ship runs but a short distance, and when the latitude is not very high, then the departure reckoned on the parallel midway between the places come to and started from will give the difference of longitude sufficiently near the truth to be relied on. But this refers only to single courses, and not to several. For it is evident that greater accuracy will be

insured when there are several courses, if we calculate the departure on each course by using the mid-latitude between the commencement and termination of that course. It is evident from Parallel Sailing that the same departure, calculated with two different parallels of latitude, will give unequal differences of longitude. It can seldom happen when two departures are compounded together, and calculated on the same parallel, that the difference of longitude resulting from the compounding together will be the same as the aggregate of the differences of longitude resulting from the calculations made with the two departures with separate mid-latitudes. Hence it will appear where mid-latitude sailing is used, that, in high latitudes, we must be careful to find the difference of longitude on each course and distance, or otherwise a serious source of error will be introduced.

CHAPTER X.

MERCATOR'S SAILING.

Mercator's Sailing—History, Charts—Proof of Rules—Examples and Illustrations—To construct a Mercator's Chart—To lay down a Course and Distance on a Mercator's Chart—Examination—A Day's Work—How to use the Chart in finding Course, Distance, Latitude, and Longitude—Comparison of Mercator's and Middle Latitude Sailing—A Question—Meridional Parts and Proofs of Formulæ.

74. Mercator's Sailing.—*Mercator's* and *Plane Sailing* both proceed upon the assumption that the meridians are parallel right lines, and the parallels of latitude are everywhere equal and parallel to each other and the equator. But they differ in this particular:—In *Plane Sailing* the parallels of latitude are placed as they really are (except near the poles) at equal distances one from the other, but are increased in length; while in *Mercator's Sailing* the distances apart are increased in the same ratio as their lengths. *Middle Latitude Sailing* was shown to possess certain disadvantages. When running long distances it is unadvisable to employ it, particularly in high latitudes, and courses less than 45°. *Mercator's Sailing* supplies a remedy for this, and may be employed almost without limit. It gives us formulæ for finding the course, distance, and difference of longitude under such conditions that we may take them as correct; not perfectly so, but yet so near the truth that there never need be any doubt as to the accuracy of the results obtained.

Mercator's Sailing is sailing a ship on the principles laid down by Gerard Mercator; or, perhaps more strictly, we may define it as finding a ship's course, distance, and longitude on the principles laid down by Mercator.

Gerard Kauffman, who is better known by his Latinised name Gerard Mercator, was a Fleming, and lived in the seventeenth century. He was the first to show the method of constructing charts, in which the parallels, meridians, and rhumb lines are all represented by straight lines. This distortion causes lands within the Arctic and Antarctic circles to appear much larger than they really are; so that with a large map of the world, drawn on Mercator's projection, the eye will *not* give a correct idea of the relative sizes of different countries. To an Englishman, Mr. Edward Wright, a well-known tutor in his day, the honour is due of first showing the true principles on which the lines in these charts should be laid down with mathematical accuracy. Mercator himself seems to have employed some mechanical method fairly correct; but Wright's method for the construction of such charts immediately superseded the original inventor's. Mr. Wright's method not being strictly and geometrically correct, induced many eminent mathematicians to investigate the subject; among whom Gregory, in 1668, and Dr. Halley, in 1690, solved the problem, and gave the correct formulæ for the calculation of meridional parts. Wright enlarged the meridians, always increasing them *equably* for each minute of latitude; whereas they alter at every increase or point of latitude, the cosines of any two parallels, however near together, not coinciding. It was casually discovered that a certain analogy exists between the divisions of the meridian line and the logarithmic tangents. If in our common tables of logarithmic tangents the indices alone be considered as integers, and all the rest of the places as decimals, then the difference between the logarithmic radius and the logarithmic tangent of half the complement of any latitude, divided by ·00012633, gives the meridional parts for that latitude. Presently the mathematical formulæ for the computation of meridional parts will be investigated.

In Mercator's charts the meridians are all drawn parallel to each other. The consequence of this is, that their distances asunder are nowhere correct but at the equator; in all other parts of the world, as we proceed farther and farther from the equator, the error increases more and more. Except at the equator, as we said before, the meridians are too far apart;

to compensate for this the degrees of latitude are made to increase in length from the equator towards the poles.

It has been shown in Parallel Sailing that

$$\textit{Meridian distance} = \textit{diff. lon.} \times \textit{cos. latitude.}$$
$$\therefore \textit{diff. of lon.} = \textit{meridian dis.} \times \textit{sec. lat.}$$

Hence it is evident that if we make

$$\textit{Mer. dis.} = \textit{diff. of lon.}$$

we must increase the meridian distance in the ratio of the secant of the latitude. Now, when the meridians are drawn parallel to each other, the degrees or minutes of longitude are everywhere the same length, being increased in the ratio of the secant of the latitude, therefore the lengths of the degrees of latitude measured on the meridians must be increased by a proportionate length. Hence the first mile of latitude from the equator will be represented by $1' \times$ sec. $1'$; the second by $1' \times$ sec. $2'$; the third by $1' \times$ sec. $3'$; the nth by $1' \times$ sec. n'.

Therefore the distance of any parallel from the equator is given by taking the sum of all the natural secants,

$$\text{Sec. } 1' + \text{sec. } 2' + \text{sec. } 3' + \text{to } n \text{ terms,}$$

and multiplying by $1'$. This method of finding the meridional parts is not strictly accurate, because the earth is not a perfect sphere, and the degrees of latitude measured on the meridians become a little longer as we approach the poles, but the error is of no consequence whatever in those latitudes in which it is practicable to sail a ship.

75. Meridional Difference of Latitude *is the true difference of latitude increased in the ratio of the secant of the latitude.*

We repeat a little. At the earth's equator a degree of longitude is equal in length to a degree of latitude; but as we recede from the equator and approach the poles, the lengths of the degrees of longitude become less and less, while the degrees of latitude, supposing the earth to be a perfect sphere, remain the same. Now, in Mercator's projection of the earth on a plane, the degrees of latitude are severally increased in the same proportion as the degrees of longitude have been.

Suppose a ship to start from A and run to a place P.

Now on a sphere the distance between A and P would be AP, the difference of latitude AC, and the departure CP.

We must now show how to represent a triangle similar to this on a Mercator's chart. Since the latitude and longitude on the chart are proportionally increased, the space required to represent the 180° of latitude and 360° of longitude on a plane would be greater than the surface required to represent the same extent of latitude and longitude on a sphere, by the difference between the areas of the requisite rectangular spaces on the chart, and the areas of two inscribed circles, touching each other and three sides of the rectangle. For this reason, the line representing the difference of latitude on a sphere is less than a corresponding line on Mercator. Also, the line representing the departure on the sphere becomes the difference of longitude on Mercator's charts; for since departure may be assumed as the sum of the infinitely small parts of parallels which form the sides of right-angled triangles, the other sides of which are infinitely small parts of the rhumb line and arcs of meridians; and since in Mercator the degrees of longitude may be measured along any parallel, the sum of such arcs of the parallels on a sphere would in Mercator be represented by the sum of the differences of longitudes taken at any distance from each other; that is, the departure on the sphere is represented by the difference of longitude on Mercator. That the distance on Mercator would be represented by a longer line than when represented on the sphere, is shown by reason of the difference of area required for the representation of the whole surface of the earth on a plane and on a sphere.

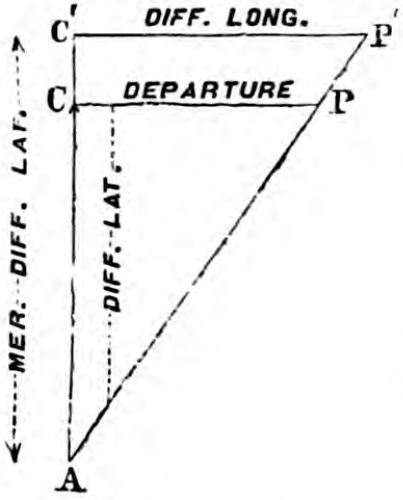


Fig. 28.

Let the foregoing figure represent a portion of the surface of the earth on Mercator's projection, where ACC′ is an arc of the meridian, and AP the distance sailed by a ship on a rhumb line. Let CP be a parallel of latitude passing over

the place P. Then will angle CAP be the ship's course, AC the true difference of latitude made by the ship, and CP the departure.

Let AC′ represent the meridional parts corresponding to the true difference of latitude AC, and complete the figure by drawing C′P′ parallel to CP, meeting AP produced in P′. It can easily be proved that if C′P′ represent the difference of longitude, AP and PP′ form one straight line, when the former is joined to P′.

AC′ is the meridional difference of latitude.
AC ,, true ,, ,,
CP ,, departure.
C′P′ ,, difference of longitude.

$$Tan\ course = \frac{P'C'}{AC'} = \frac{difference\ of\ longitude.}{meridional\ diff.\ of\ lat.} \qquad \text{I.}$$

$$\therefore\ diff.\ lon. = mer.\ diff.\ lat. \times tan.\ course. \qquad \text{II.}$$

When the course is found we use the true difference of latitude to find distance in the common formula.

$$Cos.\ course = \frac{diff.\ lat.}{distance.}$$

$$\therefore\ dist. = diff.\ lat. \times secant\ course.$$

These are all the formulæ necessary for Mercator's Sailing.

309. March 12, at 5.20 p.m., I leave St. Ann's Head in lat. 51° 41′ N., lon. 5° 9′ W., for Land's End, in lat. 50° 4′ N., lon. 5° 42′ W., how must I steer by compass? allowing 23° westerly variation, 3° W. for deviation, and ¼ point leeway, the wind being E.S.E. What distance shall I run, and when shall I arrive, supposing that my rate of sailing is 10 knots an hour? (1869.)

	Diff. lat.	M. P.	Diff. lon.
St. Ann's Head,.....Lat.	51° 41′ N.	3635	Lon. 5° 9′ W.
Land's End, ...Lat.	50 4 N.	3481	Lon. 5 42 W.
True diff. of lat.	1° 37′ S.	154	diff. lon. 33′ W.
	60		
	97 S.		

$$Tan.\ C. = \frac{diff.\ lon.}{mer.\ diff.\ lat.}$$

Log 33, diff. lon.....1·518514
Log 154, mer. diff. lat.....2·187521
Log tan. 12° 5′ course = 9·330993

Dist. = *true diff. lat.* × *sec. course.*

Log 97, diff. lat..... 1·986772
Log sec. 12° 5′ course...10·009730

Log 99·19 miles..... 1·996502

True course S. 12° 5′ W.; distance 99 miles.

Compass Course.

True course...	12°	5′	0″	r. of S.
Variation.....	23	0	0	r.
	35	5	0	r. of S.
Deviation.....	3	0	0	r. of S.
	38	5	0	r. of S.
Leeway	2	48	45	l.
	35	16	15	r. of S.

Compass course S. 35° 16′ 15″ W.

$$\text{Time of sailing} = \frac{99{\cdot}19}{10}$$

$$= 9{\cdot}9 \text{ hours.}$$

Started on March 12 at 5h. 20m.
Time going.....9h. 55m.

March 12 at...15h. 15m.

Or ¼ past 3 in the morning.

We will now endeavour to fully explain the methods by which this problem is worked.

Rules—(1.) Put down the latitude and longitude as above, and find the differences of latitude and longitude, and take out the meridional parts from the table book, putting them opposite the latitude to which they correspond.

(*a.*) To find the difference of latitude when both are N., or both S., subtract for the difference; but if one be north and the other south, add for the difference.

Note.—If you add the latitudes together, add the meridional parts together; if you subtract one pair, subtract the other.

(2.) *To find the course.*—From logarithm difference of longitude in miles, subtract the logarithm of the meridional difference of latitude, mentally adding 10 to the index of the first logarithm; the remainder is the tangent of the course which must be taken out from the tables.

(3.) *To find distance.*—To the logarithm of the true difference of latitude add log secant course (omitting 10 in the index), the sum gives log distance to be found in the tables.

(4.) Find the compass course as before, remembering to allow leeway *towards* the wind.

310. Find the compass course and distance from A to B. Given:

Lat. A 30° 18′ S.; var. 1 pt. E.; lon. A 17° 18′ W.
Lat. B 21° 33′ N.; dev. 6° 15′ E.; lon. B 3° 17′ E. (1868.)

	Diff. lat.	M. P.		Diff. lon.
Lat. A	30° 18′ S.	1909	Lon. A	17° 18′ W.
„ B	21 33 N.	1325	„ B	3 17 E.
	51° 51′	3234		20° 35′
	60			60
True diff. lat.	3111 N.		Diff. lon.	1235 E.

$$\text{Tan. C.} = \frac{\text{diff. lon.}}{\text{M. D. of lat.}}$$

Dis. = true diff. lat. × sec. C.

Log. 1235 diff. lon.	3·091667	Log. 3111 diff. lat.	3·492900
„ 3234 M. D. of lat.	3·509740	„ sec. 20° 54′ course	10·029558
Log. tan. 20° 54′ course	9·581927	Log. 3330 dist.	3·522458

True course, N. 28° 54′ E.; distance, 3330 miles.

Compass Course.

True course	20°	54′ 0″	r. of N.
Variation	11	15 0	l.
	9	39 0	r. of N.
Deviation	6	15 0	l.
Compass course,	3	24 0	r. of N. or N. 3° 24′ E.

Observations on this question.—It will perhaps be noted that this question would leave the vessel in the S. of the Sahara desert! So it must have therefore been simply given as an illustration of the rules.

One latitude being N. and the other S., they are added together to find the difference of latitude, hence the meridional parts are added.

The longitudes being one east and the other west, are also added to find the difference.

The course is marked N.E., because the ship goes from S. to N. latitude and from W. to E. longitude.

Course, distance, and compass course are found as in the previous question.

311. A ship sails from latitude 37° 28′ N., longitude 49° 27′ W., in a direction N.N.W. 470 miles; what are her latitude and longitude in? and what course must she now steer for New York, lat. 40° 42′ N., longitude 73° 59′ W.? (1869.)

Here we must first find difference of latitude by Plane

Sailing, and then latitude in; then difference of longitude and longitude in; next course to New York.

Diff. lat. = dis. × cos. course; diff. lon. = mer. diff. lat. × tan. C.

		Lat. in.	M. P.
Log 470 dis.	2·672098	Lat. from.....	37° 28′ 0″ N. 2415
Cos. 2 pts. course	9·965615	Diff. lat. (434·2)	7 14 12 N.
Log 434·2 diff. lat.	2·637713	Lat. in.....	44 42 12 N. 2991
			576

		Lon. in.	
Log 576 mer. diff. lat.	2·760422	Lon. from.....	49° 27′ 0″ W.
Log tan. 2 pts. course	9·617224	Diff. lon. (238·5)	3 58 30 W.
Log 238·5 diff. lon. ...	2·377646	Lon. in	53 25 30 W.

We have now to find course from latitude 44° 42′ 12″ N., longitude 53° 25′ 30″ W., to New York, lat. 40° 42′ N., longitude 73° 59′ W.

Diff. lat.		M. P.	Diff. lon.	
Lat. from.....	44° 42′ 12″ N.	2991	Lon. from.....	53° 25′ 30″ W.
Lat. N. Y.	40 42 0 N.	2664	Lon. N. Y.	73 59 0 W.
	4 0 12	327		20 33 30
	60			60
True diff. lat.	240·2 S.		Diff. lon.	1233·5 W.

$$\text{Tan. C.} = \frac{\text{diff. lon.}}{\text{mer. diff. lat.}}$$

Distance.
Dis. = diff. lat. sec. course.

Log 1233·5 diff. lon.	3·090980	Log 240·2 diff. lat....	2·380573
Log 327 M. D. lat....	2·514548	Log sec. 75° 9′.....	10·591269
Log tan. 75° 9′ C. ...	10·576432	Log 937·2 dist.....	2·971842

True course to New York, S. 75° 9′ W.; distance 937 miles.

The next is given as a simpler illustration:—

312. Find the course and distance from the Cape of Good Hope, latitude 34° 22′ S., longitude 18° 24′ E., to the Falkland Isles, latitude 52° 21′ S., longitude 59° 18′ W., by Mercator's Sailing.

Diff. lat.		M. P.	Diff. lon.
Lat. of Cape of Good Hope,	34° 22′ S.	2198	Lon. 18° 24′ E.
,, Falkland Isles, ...	52 21 S.	3699	,, 59 18 W.
	17 59	1501	77 42
	60		60
	1079 S.		4662 W.

$$\text{Tan. C.} = \frac{\text{diff. lon.}}{\text{M. D. of lat.}}$$

Log 4662	3·668572	Log 1079	3·033021
Log 1501	3·176381	Log sec. 72° 9′	10·513533
Log tan. 72° 9′	10·492191	Log 3520	3·546554

Dis. = diff. lat. × sec. course.

Ans. True course, S. 72° 9′ W.; distance, 3520 miles.

By Middle Latitude Sailing the answers were—

True course, S. 72° 20′ W.; distance, 3588 miles.

313. Find the course and distance by Mercator's Sailing from C Farewell, latitude 59° 49′ N., longitude 43° 54′ W., to C. Wrath, latitude 58° 38′ N., longitude 4° 57′ W.

	Diff. lat.	M. P.	Diff. lon.
Cape Farewell, lat.	59° 49′ N.	4505	lon. 43° 54′ W.
Cape Wrath, ,,	58 38 N.	4367	,, 4 57 W.
	1 11	138	38 57 F
	60		60
	71 S.		diff. lon. 2337 E.

$$\text{Tan. C.} = \frac{\text{diff. lon.}}{\text{M. D. of lat.}}$$

Log 2337	3·368659	Log 71	1·851258
Log 138	2·139879	Log sec. 86° 37′	11·229030
Log tan. 86° 37′	11·228780	Log 120·3	3·080288

dist. = diff. lat. × sec. course.

Ans. Course S. 86° 37′ E.; distance, 1203.

314. If the variation be 43° W., and deviation 9° E., and leeway 2 pts. with wind from N.; find the compass course.

True course,	86° 37′ 0″	l. of S.
Variation,	43 0 0	r.
	43 37 0	l. of S.
Deviation,	9 0 0	l.
	52 37 0	l. of S.
Leeway,	22 30 0	
Compass course,	75 7 0	l. of S. or S. 75° 7′ E.

The following answers will vary a little, according to the tables used:—

315. Find the course and distance by Mercator's Sailing from Start Point, latitude 50° 13′ N., longitude 3° 38′ W., to Ushant, latitude 48° 28′ N., longitude 5° 3′ W.?

Ans. Course S. 27° 58′ W.; distance 118·8 miles.

316. In what direction must I place my ship's head, by compass,

to reach Kinsale light, starting from the Longship's light, the variation being 2 pts. W., and deviation ¾ pt. W.?

Latitude of the Longship's light, 50° 4′ N., lon. 5° 44′ W.
" Kinsale " 51 37 N., lon. 8 27 W.
Ans. Course N. 17° 0′ 45″ W.; distance 138·8 miles.

317. What is the course and distance by Mercator's Sailing from Heligoland, latitude 54° 12′ N., longitude 7° 53′ E., to the Inch Cape light, latitude 56° 27′ N., longitude 2° 23′ W.
Ans. Course N. 68° 57′ W.; distance 375·8 miles.

318. Find the compass course and distance from Plymouth (Pen-lee), latitude 50° 19′ N., longitude 4° 11′ W., to Jersey, latitude 49° 15′ N., longitude 2° 2′ W., variation 21° W., deviation 7° E.
Ans. S. 38° 29′ E.; distance 105 miles.

319. What is the true course and distance by Mercator's sailing from Cape Verde, latitude 14° 45′ N., longitude 17° 32′ W., to Monte Video, latitude 34° 53′ S., longitude 56° 16′ W.
Ans. Course S. 36° 44′ W.; distance 3716 miles.

320. What is the course and distance by a compass with 17° E. variation and ¾ pts. E. deviation, from Cape Mendocino in latitude 40° 29′ N., longitude 124° 29′ W., to the Solomon Isles, latitude 7° 15′ S., longitude 157° 45′ E.
Ans. Course S. 31° 8′ W.; distance 5198 miles.

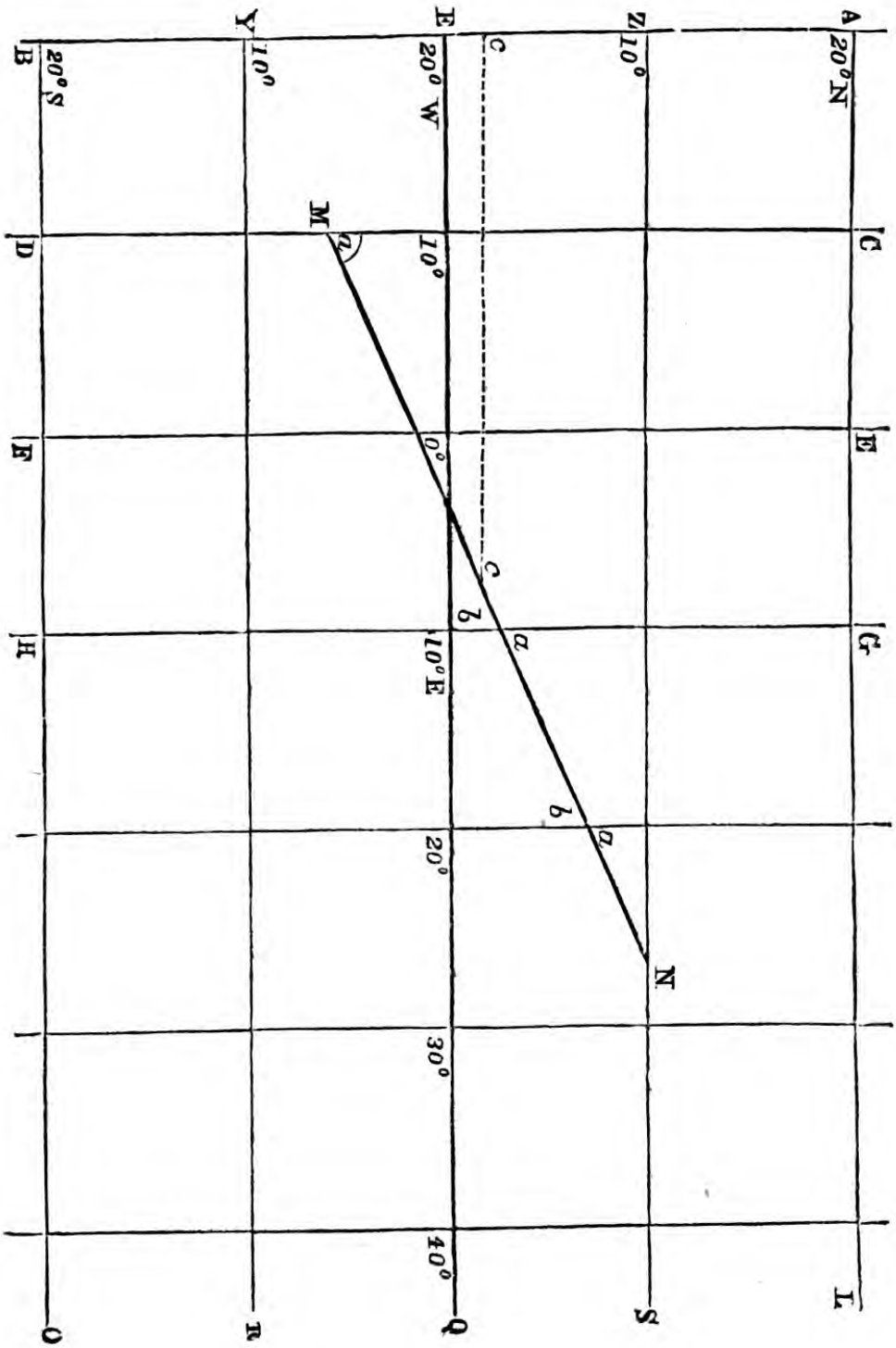
321. Find the course and distance from Valparaiso, lat. 33° 2′ S., longitude 71° 40′ W., to Foveaux Straits, latitude 47° 16′ S., longitude 167° 15′ E. *Ans.* C. S. 81° 14′ W.; dist. 5603 miles.

322. By Mercator's Sailing find the true course and distance from Cape Horn, latitude 55° 59′ S., longitude 67° 12′ W., to the Cape of Good Hope, latitude 34° 22′ S., longitude 18° 24′ E.
Ans. Course N. 70° E.; distance 3792 miles.

323. A vessel sails from Bourbon, 430 miles N. by E.: find the course and distance thence to Point de Galle, variation of compass 5° W., deviation 2 pts. W. Bourbon, latitude 20° 52′ S., longitude 35° 27′ E.; Point de Galle, latitude 6° 1′ N., longitude 80° 18′ E.
Ans. Course S. 87° 7′ E.; distance 2859 miles.

76. Mercator's Charts.—We have now to show how Mercator's charts are practically constructed, and why they are used by mariners in preference to every other kind of map, and then point out a few simple rules to guide the student in using them.

The best way to construct a chart is as follows:—First draw a line across the middle of the paper, from right to left, to represent the equator, E Q. Second, draw a few vertical lines at *equal* distances apart, and cutting the equator at right angles at A B, C D, E F, G H, etc., let these lines represent the meridians everywhere the same distance apart.


A
20° N
Z
10°
E
20° W
Y
10°
B
20° S
C
E
G
L
M
N
S
Q
R
O
D
F
H
10°
0°
10° E
20°
30°
40°
a
b
c

Third, suppose the meridians we have drawn are 10° apart, or 600 miles at the equator (the student may suppose them any distance apart he pleases, 1°, 2°, 3°, etc.).

Then from 0° to 10° is 600 miles.
,, 10 to 20 ,,
,, 20 to 30 ,,
,, etc., etc. ,,

Fourth, go to the table of meridional parts, and as we have supposed the meridians are 10° asunder, we will suppose the parallels are also 10° apart and take out the meridional parts for—

10° = 603
20 = 1225 1225 - 603 = 622
30 = 1888 1888 - 1225 = 663
40 = 2623 2623 - 1888 = 735
etc., etc., as many as are required.

Now, take in the compasses 603 parts, remembering that from 0° to 10° is 600, and measure them up and down from the points E and Q, from E to Z and Y, and Q to S and R; through Z and S and Y and R, draw the lines Z S and Y R; we have now the first two parallels, one 10° N. of the equator, the other 10° S. of the equator.

For the next two parallels, either measure up and down from E and Q 1225 parts (of which 0° to 10° are 600), or measure 622 up from the line Z S and down from Y R; through the points thus obtained draw the lines A L and B O, which are the parallels of 20° S. and N. of the equator. The next pair of parallels of 30° must be drawn at 1888 parts from the equator, or 663 parts from the lines A L and B O; unfortunately the page will not admit of showing these, or it would be seen how the distances apart of the parallels gradually increase. The student must not rest until he can readily draw a Mercator's chart.

The great advantage of Mercator's chart is this, that if you draw a line from any one place to another, the angle at which that line cuts any meridian is the course from the one place to the other.

Let M be one place on the earth's surface and N another. Join them by the straight line M N; then the angle *a* is the course from M to N, and *b* the course from N to M.

The master of the vessel generally has a large parallel

ruler, which he places on the two places, and then moves the ruler up to the centre of the compass, of which there are generally more than one marked on the chart. As soon as he has his parallel ruler on the compass, he sees in a moment what the course is, as the ruler is pointing in that direction.

Another advantage is this: that you can tell with a pair of compasses in a moment what is the distance between two places. Suppose *c* is the middle point of M N. Carry point *c* to the side of the chart; then measure half the length of M N up and down from this point, when the number of degrees lying on the meridian between the two points thus found will give the distance between the two places when brought into nautical miles. The latitude of a place is found by simply bringing the eye in a line from the place to the side of the chart, and the longitude by bringing the eye straight down to the bottom or up to the top of the chart.

Having constructed a Mercator's chart, we next proceed to draw two others, laying down on the chart the several courses a ship has sailed.

324. Construct a Mercator's chart, lay down the following five courses and distances that the ship sailed on during three days, and find the latitude and longitude in:—S.byW. 90 miles, W.S.W. 70 miles, S.E.by.S. 100 miles, W.byS. 120 miles, S.E. 60 miles. Latitude from 40° 20′ S., longitude from 59° 40′ E. To be drawn to a scale of ·8 inches to a degree of longitude.

The chart will extend from lat. 40° S. to 45° S., and from longitude 56° E. to 60° E.

(1.) Construct a scale of equal parts similar to the one shown in the figure. Take 0 – 1, 1 – 2, 2 – 3, etc., each eight-tenths of an inch, because the scale is ·8, then each of these parts will represent 60′, or one degree of longitude; divide the one at the upper end into any number of convenient parts, as 6, 10, 20, etc., when each part will = $\frac{60}{6}$, or $\frac{60}{10}$, or $\frac{60}{20}$, etc., = 10′, or 6′, or 3′ of longitude.

(2.) As the latitude is south, and the ship goes south, the line AB is drawn at the top of the page for the first parallel of latitude; had the ship sailed N., or the course been generally N., it would have been drawn at the bottom. Again, if the vessel is in N. latitude, and the ship sail S., draw the first parallel at the top of the page, and *vice versâ*.

(3.) From the problem suggested, it appears that on the whole the ship goes from E. to W.; the vertical line AC is first drawn, which is the extreme meridian towards the E. in that part of the chart where the ship is sailing. Always consider whether the ship goes east or west; if west, draw the first meridian on the right side of the page, if east, draw it to the left.

(4.) Next mark off AD, DE, EF, FB, each ·8 of an inch, or one division from the scale, then AC, DT, EU, FV, and BR, represent five meridians, viz., 60°, 59°, 58°, 57°, and 56° east, drawn perpendicular to the parallel AB, and everywhere the same distance apart.

(5.) The parallels are found thus: Take from the tables the meridional parts corresponding to each degree of latitude, among which the vessel is working, and place them on the side of the page on which the chart is being drawn, thus—

The meridional parts for		40°	are	2609	
,,	,,	41	,,	2688 difference	= 79
,,	,,	42	,,	2767 ,,	= 79
,,	,,	43	,,	2848 ,,	= 81
,,	,,	44	,,	2931 ,,	= 83
,,	,,	45	,,	3015 ,,	= 84

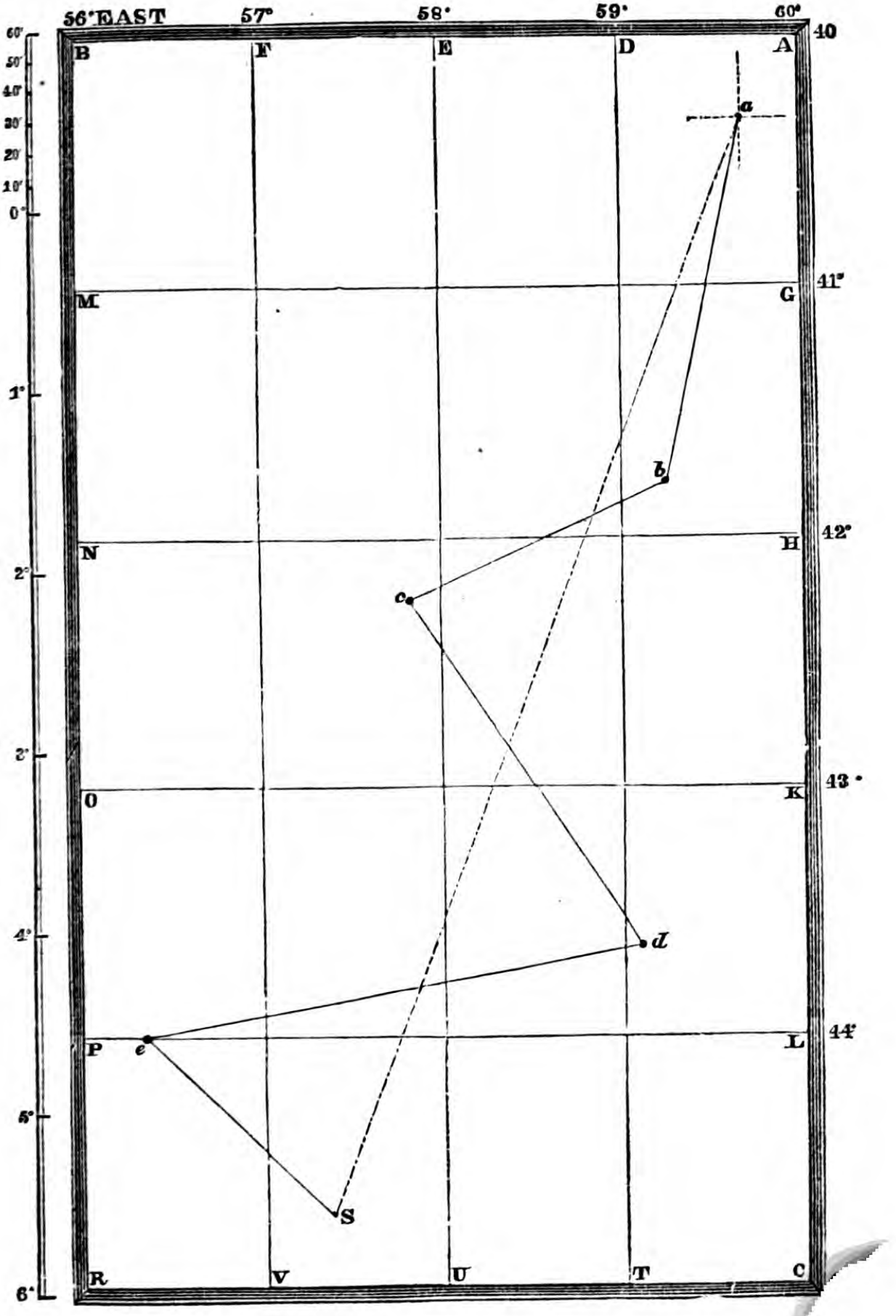
When they are in order, subtract each number from the one immediately below it, and place them in order, as shown in the example.

This shows that the first parallel is to be drawn at 79′ division from AB, so, placing the compasses on point 1°, they are opened out to 20′ = 60′ + 20′ = 80′, allowing a minute portion for 1′, we have the 79′ required, which are marked from A to G. As the next difference is also 79, the same length is repeated. The reason why they coincide is because the sum of the seconds in the lower latitudes differ very little.

In the same manner take off 81′ from the scale, and mark them from H to K; next mark off KL = 83, and LC = 84. Then draw the parallels GM, HN, KO, LP, and CR.

(6.) It remains now, having shown how to lay down the parallels and meridians on a Mercator's chart, to exhibit the manner in which the courses and distances on the same are graphically represented.

(*a.*) Find the position from which the vessel started by


56°EAST
57°
58°
59°
60°
60'
50'
40'
30'
20'
10'
0°
1°
2°
3°
4°
5°
6°
B
F
E
D
A
40
M
G
41°
N
H
42°
O
K
43°
P
L
44°
R
V
U
T
C
a
b
c
d
e
S

measuring off 20′ for lat. 40° 20′ S., and drawing a faintly dotted line parallel to AB. Next measure off 40′, because the longitude is 59° 40′ E., and draw a faintly dotted line at right angles to the former, or parallel to AC, at the distance 40′ from DT, the 59th parallel. Where they intersect at *a* is the place on the chart from which the vessel sails. *The* 20′ *for latitude are obtained by considering* BM = 60′, and taking 20 of those parts, or $\frac{1}{3}$ of BM. Thus *a* is found 20′ south of AB, and in longitude 59° 40′, the 40 being $\frac{2}{3}$ of AD to the right hand.

(*b.*) From *a* draw *ab* at an angle of one point S.byW. 11° 15′ with the meridian, and mark off *ab* = 90′, take the 90′ from BN as near the part, opposite to the middle of *ab*, as you possibly can. Hence it is necessary to divide BM, MN, NO, OP, and PR, each into 60 equal parts, in fact, make BR a scale; but it will be seen that it is not a scale of equal parts, for BM, MN, NO, etc., are *not equal.*

(*c.*) From *b* draw *bc* = 70′ towards the W.S.W. point of the compass, measuring the 70 from N. up and down, *i.e.*, 35 towards O and 35 towards M.

(*d.*) Now from *c* draw *cd* S.E.byS., with a distance of 100 miles, taking 50 up and down from O.

(*e.*) From *d* draw *de* W.byS., and 120 miles long.

(*f.*) *e*S is now drawn S.E. 60 miles, the 60 being the length from P to R.

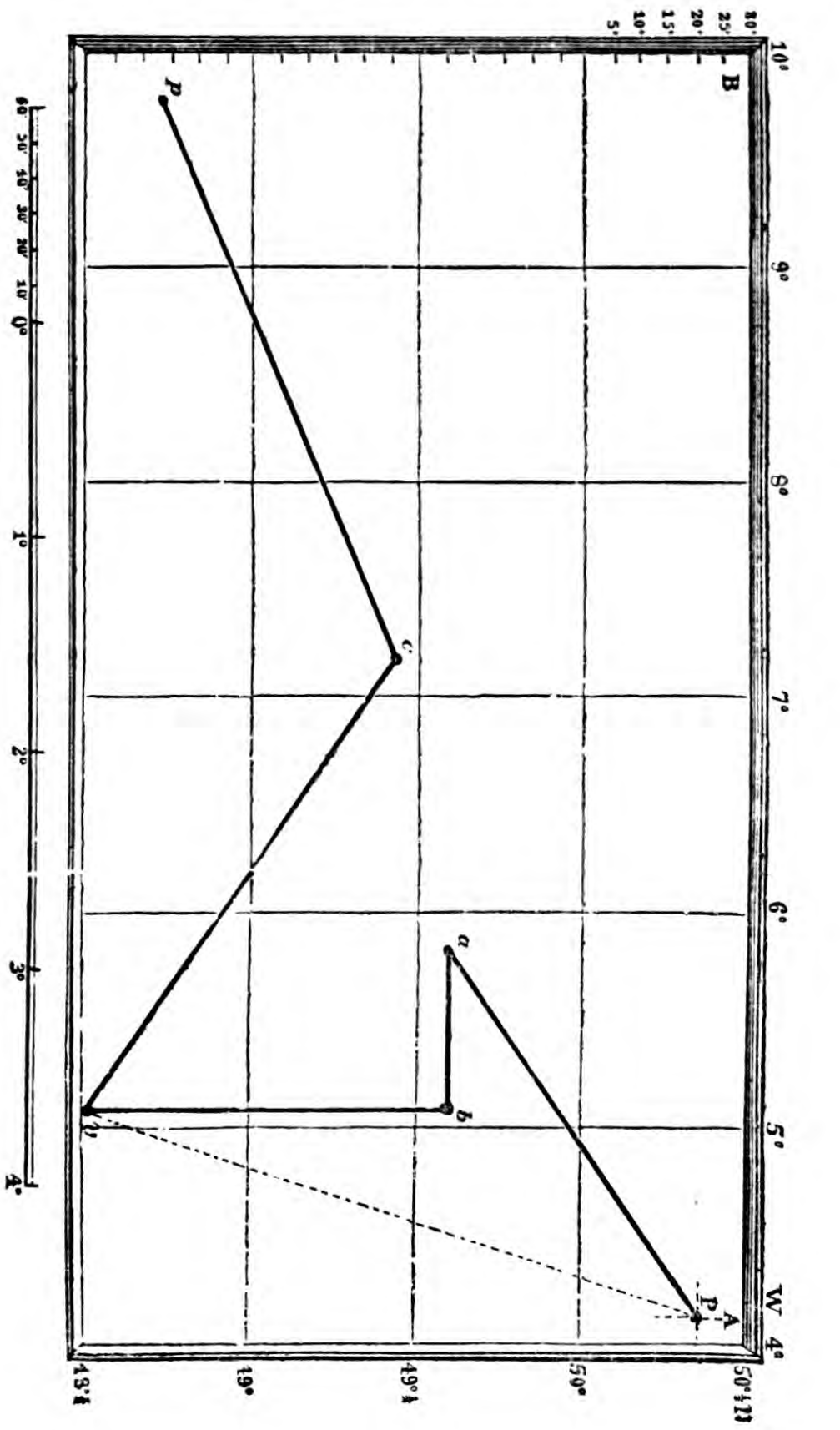
(7.) Next the latitude and longitude of the ship at S. are taken from the chart and found to be

Latitude in 44° 44′ S.
Longitude in 57 21 E.

The course and distance made good are found by joining *a*S, measuring the distance from *a* to S, along the side BR, and taking the angle formed by *a*S with any meridian as DT. They are respectively distance 282 miles; course S. 20° 30′ W.

325. A ship left Plymouth harbour, latitude 50° 22′ N. longitude 4° 7′ W., and sailed about the channel on the following courses and distances:—S.W. for 80 miles, E. for 28 miles, S. for 65 miles.

Lay down a Mercator's chart on a scale of ·85 inches to a degree of longitude, show the courses and distances on it, and state what one course and distance would have brought the ship to the same place in the neighbourhood of Ushant, lat. 48° 30′ and lon. 5° 3′ W. Sup-


B
P
C
a
b
D
A
P
W
10'
9°
8°
7°
6°
5°
4°
0°
1°
2°
3°
4°
50°11'
50°
49°½
19°
43'½

pose that afterwards the vessel sails on two courses, N.W.byW. for 100 miles, W.S.W. for 121 miles into the Atlantic find the latitude and longitude in.

(1.) Draw line AB as the first parallel of latitude 50°½ towards the top of the page. The scale being ·85 inches to the degree of longitude, mark off points as in the figure for 4°, 5°, 6°, 7°, 8°, 9°, 10° W. longitude, each ·85 inches apart.

(2.) For latitude taking out the meridional parts below, and finding the differences we have—

Meridional part for	50°	30′	= 3505				
,,	,,	50	= 3458	difference	47		
,,	,,	49	30 = 3412	,,	46 =	93	
,,	,,	49	= 3366	,,	46 =	139	
,,	,,	48	30 = 3321	,,	45 =	184	

This shows that the parallel of 50° is to be drawn at 47′ from AB.

,,	,,	,,	49°	30′	,,	93 ,,
,,	,,	,,	49		,,	139 ,,
,,	,,	,,	48	30	,,	184 ,,

(3.) Now fix on the position of Plymouth P, by drawing two dotted lines, one in latitude 50° 22′ N., the other in longitude 4° 7′ W., the intersection of these lines gives the position of Plymouth.

(4.) Draw the three lines

	Pa	to the S.W.	for 80 miles	to	*a*.
	ab	,, E.	,, 28	,,	*b*.
and	*bv*	,, S.	,, 65	,,	*v*.

This brings the ship into the neighbourhood of Ushant, in latitude 48° 30′ S., longitude 5° 3′ W. The one course and distance to *v* is S. 18° W.; distance 118 miles.

(5.) From this point the vessel runs on two courses into the Atlantic; to represent these we draw

	vc	to the N.W. by W.	for 100 miles.	
and	*cp*	,, W.S.W.	,, 121	,,

which brings the ship to *p*.

The latitude of the ship is then 48° 47′ N.
,, longitude ,, 9 46½ W.

326. Construct a Mercator's chart, and, laying down the courses, find lat. and lon. in, from the following day's work:—Lat. from 63° 10′ N., lon. from 20° 40′ E.; Cor. courses, E. by N. 80′, S.W. 90′, E. 100′, W.S.W. 90′, E. by S. 80′, S.W. by W. 60′. To be drawn on a scale of ·9 inches to the degree of longitude. *N.B.*—The chart will extend from 60° to 64° N., and from 20° to 24° E.

327. Construct a Mercator's chart, and, laying down the courses, find lat. and lon. in; from the following day's work:—Lat. from 61° 15′ S. lon. from 49° 20′ W.; correct courses are E.S.E. 90′, N. 140′, S.S.W. 65′, W.byS. 70′, N.E.byN. 95′, N.N.W. 50′. To be drawn on a scale of 1·2 inch to a degree of longitude. *N.B.*—The chart will extend from 58° to 62° S., and from 46° to 50° W.

328. Construct a Mercator's chart, on a scale of 1·2 inch, extending from 52° to 56° S., and from 20° to 24° W. A ship sailed from latitude 55° 40′ S., longitude 23° 20′ W., as follows:—Compass courses, E. by N. ½-N. 123′, S.W. ½-S. 80′, N. by W. 150′, N.E. by E. 100′, S. by E. ½-E. 72′, N.W. ½-W. 115′; variation 1½ points W. Lay down the true courses, and find the latitude and longitude in.

329. Construct a Mercator's chart, on a scale of 1·1 inch, extending from 53° to 57° S., and from 137° to 141° W. A ship sailed from latitude 53° 15′ S., longitude 139° 38′ W., as follows:—Compass courses, S.E.byE. 110′, W.byN. ¼-N. 115′, S.E. 142′, S.W. ¾-S. 58′, N.E. ¼-E. 105′, S. ¾-W. 65′; variation 1¾ point E. Lay down the true courses, and find the latitude and longitude in.

330. Construct a Mercator's chart, on a scale of 1·1 inch to a degree of longitude, extending from 54° to 58° S., and from 32° to 37′ E. A ship sailed from latitude 54° 25′ S., longitude 36° 10′ E., as follows:—Compass courses, S.W. ½-S. 120′, N.E. ½-N. 100′, N.W. byW. ½-W. 135′, S.S.W. ½-W. 155′, S.E. ½-E. 120′, N. 240′; variation 3 points W. Lay down the true courses, and find the latitude and longitude in.

331. Construct a Mercator's chart, on a scale of 1·2 inch to a degree of longitude, extending from 60° to 64° N., and from 10° W. to 15° W. A ship sailed from latitude 60° 50′ N., longitude 11° 10′ W., as follows:—Compass courses, N. 100′, N.E. 85′, S.E.byE. 50′, S.W.byW. 140′, N.W. 35′, S.byW. 45′; variation 3 points W. Lay down the true courses, and find the latitude and longitude in.

332. Construct a Mercator's chart, on a scale of 1·3̇ inch to a degree of longitude, extending from 64° to 67° 30′ N., and from 0·0 to 5° W. A ship sailed from latitude 64° 30′ N., longitude 3° 30′ W., as follows:—Compass courses, N.E. ½-E. 145′, N.W.byW. 66′, S.S.W. ½-W. 53′, S.E.byE. ½-E. 83′, S.W. 75′, N.W. 60′; variation 2½ points W. Lay down the true courses, and find the latitude and longitude in.

333. Draw a Mercator's chart, on a scale of 1·8 inch to the degree of longitude, extending from lat. 64° to 68° S. and 73° W., 78° E.; lay down on its two points of land—one in lat. 60° 20′ S., lon. 75° 30′ E.; and the other in lat. 67° 34′ S., lon. 76° 15′ E. Lay down the position of a ship which bore by compass from the former of these rocks E. by N. ½-N., distant 18 miles (ship's head E.S.E., deviation 5° 35′), and find its distance and compass-bearing from the second rock; the variation allowed being 2½ pts. W. *N.B.*—This chart should be drawn with neatness as well as accuracy. (Honours, 1872.)

334. On June 25th, 1869, at noon, a ship was in lat. 49° 30′ N., and lon. 5° 53′ W., and afterwards sailed as by following log account. Work up the reckoning to noon, June 26th.

Hours.	Knots.	Tenths.	Standard Compass Course.	Leeway Points.	Winds.	Deviation of Standard Compass.	Remarks.
1	1	8	S.S.E.	½	E. by N.	2° W.	P.M.
2	2	5					
3	2	0					
4	2	3					
5	1	0					
6	5	5	N.	0	S.S.W.	1° E.	
7	6	0					
8	5	2					
9	4	5					
10	3	5	N. by E.	0	S.W. by S.	2° E.	
11	3	2					
12	4	0					Midnight.
1	3	5					
2	4	5					
3	4	0					
4	3	0					
5	3	0					
6	3	5	N.E. by N.	2	N.W.	8° E.	Variation allowed 23° W.
7	2	5					
8	3	6					
9	3	2					
10	4	8	N.E. by E.	1	N.W. by N.	9° E.	
11	6	0					
12	5	5					

The true latitude and longitude found by observation at noon, June 26th, was lat. 50° 55′ N., lon. 5° 35′ W. Find the set and drift of the current which has been affecting the ship during the last four hours. Lay down the above day's work on a Mercator's chart on a scale of 1·2 inches to the degree of latitude. The chart is to extend from lat. 49° N. to lat. 51° N., and from lon. 4° W. to 6° W. Lay down also a rock in lat. 49° 50′ N., lon. 5° 10′ W., and give (from the chart) the bearing and distance of the rock from the ship at noon, June 26th. (Honours, 1869.)

***Ans.* Set N. 28° W., drift 7 miles per hour.**

335. Define meridional difference of latitude. What is the meaning of meridional parts?

$$\text{Prove that mer. diff. lat.} = \frac{\text{diff. lon.}}{\text{Tan. course.}}$$

Write down formulæ connecting course, difference of longitude, distance, and middle latitude. (1873.)

336. Required the compass course and distance from A to B.

Lat. A 37° 44′ N.; lon. A 25° 40′ W.
Lat. B 28° 28′ N.; lon. B 16° 15′ W.
Variation $2\frac{1}{4}$ pts. W.; deviation 2° 20′ W. (1873.)

Ans. Course S. 12° 50′ 12″ E.; distance 731·45 miles.

337. Draw the framework of a Mercator's chart, extending from 179° E. to 178° W. longitude, and from 49° S. to 52° S. latitude, on scale of 1·6 inches to a degree of longitude.

338. Find the course and distance from A to B. Given:

Lat. A 53° 18′ S.; lon. A 76° 14′ E.
Lat. B 56° 25′ S.; lon. B 78° 13′ E.
Variation $1\frac{1}{4}$ E.; deviation 8° E. (1873.)

Ans. Course S. 42° 13′ 45″ E.; distance 199·2 miles.

339. State fully the principles of Mercator's projection. What is the chief value of a chart constructed on this principle? (1871.)

340. What are the compass course and distance between New York and Cape Palmas? Given:

New York.....Lat. 40° 40′ N.; lon. 74° 0′ W.
Cape Palmas...Lat. 3° 58′ N.; lon. 7° 22′ W.
Variation $1\frac{1}{2}$ W.; deviation 7° 50′ W. (1871.)

Ans. Course S. 34° 3′ 30″ E.; distance 4246 miles.

The following are the examples in Middle Latitude Sailing, from page 149. You are now required to work them out by Mercator's Sailing, and to compare the answers.

341. Explain the principle and prove the rules of Mercator's Sailing. (1861.)

342. Required the compass course and distance from A to B.

Lat. A 55° 40′ } N.; var. $1\frac{1}{2}$ E.; lon. A 29° 39′ } W. (1863.)
Lat. B 57° 50′ } lon. B 34° 15′ }

Ans. Compass course N. 66° 20′ 30″ W.; distance 200 miles.

343. Required the compass course and distance from A to B.

Lat. A 9° 30′ } N.; var. $1\frac{3}{4}$ pts. E.; lon. A 72° 15′ } E. (1863.)
Lat. B 14° 10′ } lon. B 65° 40′ }

Ans. Compass course N. 73° 52′ 15″ W.; distance 478·4 miles.

344. What is meant by Mercator's Sailing? obtain the formulæ required for it. Required the true course and distance from A to B, the latitude of A 57° 35′ N., and longitude 17° 16′ W., and latitude of B 55° 25′ N., longitude 20° 57′ W. (1863.)

Ans. True course S. 43° 14′ 0″ W.; distance 178·4 miles.

345. Find the compass course and distance between A and B.

Lat. A 33° 18′ S.; lon. A 72° 0′ W.; var. 16° 0′ E.
Lat. B 42° 3′ S.; lon. B 173° 30′ E.; dev. 9° 25′ E. (1870.)
Ans. Course S. 59° 5′ 0″ W.; distance 5477 miles.

346. Write down the formulæ for Mercator's Sailing. Under what limitations would you apply this method? (1865.) Required the compass course and distance from A to B.

Lat. of A 15° 20′ } S.; var. 2½ pts. W.; lon. A 4° 10′ E.
Lat. of B 19° 40′ } S.; dev. 7° 50′ W.; lon. B 1° 20′ W. (1855.)
Ans. Compass course S. 86° 34′ 30″ W.; distance 409·7 miles.

347. What is the compass course and distance between A and B? Given:

Lat. A 35° 18′ S.; lon. A 171° 20′ W.; var. of compass 1½ pts. W.
Lat. B 32° 14′ S.; lon. B 175° 18′ E.; dev. 10° E. (1871.)
Ans. Compass course N. 67° 43′ 30″ W.; distance 692·8 miles.

348. Explain the principles of Mercator's chart popularly, as to an elementary navigation class. (1861.) Describe a Mercator's chart. (1863.)

349. Required the compass course and distance from A to B.

Lat. A 55° 40′ } N.; var. 1½ E.; lon. A 29° 30′ } W.
Lat. B 57° 50′ } N.; var. 1½ E.; lon. B 32° 15′ } W.
Ans. Compass course N. 51° 49′ 30″ W.; distance 158·6 miles.

350. Explain how to draw a Mercator's chart, and show how to find the latitude and longitude of a place on the chart. (1864.)

351. Write down the formulæ used in Mercator's Sailing. Find the true course and distance from A to B. (1864.)

Lat. A 27° 18′ N.; lon. A 3° 15′ W.
Lat. B 15° 16′ S.; lon. B 27° 3′ W.
Ans.. S. 28° 38′ W.; distance 2910 miles.

352. Required the compass course and distance from A to B.

Lat. of A 37° 27′ N.; var. 2 pts. E.; lon. A 54° 19′ W.
Lat. of B 14° 17′ S.; dev. 1° 50′ E.; lon. B 30° 15′ W. (1865.)
Ans. Compass course S. 48° 8′ E.; distance 3392 miles.

353. A ship in lat. 13° 5′ N., lon. 37° 18′ W., is to make a point of land in lat. 33° 10′ N., lon. 54° 33′ W.; find the compass course she must steer and the distance. Given variation 1½ points W., deviation 8° 12′ W. (1867.)
Ans. Compass course N. 21° 21′ 30″ W.; distance 1534 miles.

354. What do you mean by meridional parts? What is the mer. diff. lat. of two places whose latitudes are 12° 18′ N. and 8° 12′ S. respectively? (1867.) *Ans.* 1238.

355. Show how to calculate approximately a table of proportional parts. Given the meridional parts for 28° = 1741′. Calculate the meridional parts for 28° 15′. (1865.)

The first part of the question is answered in any book on trigonometry; the latter part may be done by the following approximate method—

The meridional parts for 28° (or $28 \times 60 = 1680'$) being 1741; what is the meridional parts for 28° 15′ or 1695′

As 1680 : 1695 : : 1741
or, as 112 : 113 : : 1741

113
5223
19151
112)196733(1756, *Ans.*
112
847
784
633
560
733
672
61

This is not strictly accurate, but near enough for the purpose required, and is what the examiner asked for.

356. Explain how a Mercator's chart may be constructed, and show how to find the course and distance from two points on it. (1865.) Find the compass course and distance between A and B.

Lat. A 33° 18′ S.; lon. A 72° 0′ W.; var. 16° 0′ E.
Lat. B 42° 3′ S.; lon. B 173° 30′ E.; dev. 9° 25′ E. (1870.)
Ans. Compass course S. 59° 5′ W.; distance 5461 miles.

357. What is meant by Mercator's projection and meridional difference of latitude? Given meridional parts for lat. 35° = 2233; find by calculation meridional parts for 35° 18′. (1860.) *Ans.* 2251.

HOW TO USE A MERCATOR'S CHART.

77. To find the Course from the Chart.—Take a parallel ruler, and lay it down on the chart, so as to bring the point showing the position of the ship and the place to which you are going on to the edge of the ruler. Now move the ruler, parallel to itself, to one of the compasses marked on the chart, then the rhumb line corresponding to the direction of the ruler is the course. In other words, if the two places be joined by a straight line, the angle with which that line cuts any of the meridians is the *course*.

78. To find the Distance between two places on a Chart.—Join the two points between which the distance is sought, and take the middle point of the line; bring that point by a line parallel to the equator on to the side of the chart. Now measure one-half the length of the line upward from the point, and the other downward, when the sum of the miles intervening between the extreme points is the distance sought.

79. To find Latitude on the Chart.—Lay your parallel ruler through the two places parallel to the equator, or to the lines of latitude, being careful that one end of your ruler reaches the graduated meridian on the side of the chart, where the edges of the ruler meet this graduated meridian the latitude is indicated, and also the difference of latitude.

80. To find Longitude on the Mercator's Chart.—Proceed the same way as in latitude, but have the parallel ruler perpendicular to the equator; then if the edges of the ruler run one through each place where the edges of the ruler reach the top or bottom of the chart, the graduated parallel will show the longitude and difference of longitude.

358. A ship sails from latitude 67° 30′ N., longitude 8° 46′ W., and in four days has made the following six courses and distances; find the position of the ship at the end of the time by Mercator's Sailing:—N.E., 64 miles, N.W.byN., 58 miles, W.N.W., 72 miles, W. 48 miles, S.byE., 45 miles, E.S.E., 40 miles.

This is the same question as proposed in Middle Latitude Sailing, p. 155; it is introduced here for the sake of comparison, to show the difference in the two methods, and to prove that if a master will be careful with his middle latitude sailings, he may find his position with correctness.

A.—First course and distance, N.E. 64 miles.

To find difference of latitude—

Diff. of lat. = *dis.* × *cos. course.*

Distance 64.....log =	1·806180
Course 4 points .....cos. =	9·849485
Diff. of lat. 45·25.....log =	1·655665

Latitude from.....	67° 30′ 0″ N.	MP = 5532
Diff. of latitude 45·25	0 45 15 N.	,, = 5651
Latitude in	68 15 15 N.	119

To find difference of longitude—

Diff. lon. = mer. diff. lat. × tan. course.

Mer. diff. lat. 119.....log	=	2·075547
Course 4 points.....tan.	=	10·000000
Diff. of lon. 119.....log	=	2·075547

Longitude from.....	8°	46′	0″ W.
Diff. lon. 119.....	1	59	0 E.
Longitude in.....	6	47	0 W.

B.—Second course and distance, N.W.byN., 58 miles.

To find difference of latitude—

Diff. lat. = dis. × cos. course.

Distance 58.....log	=	1·763428
Course 3 points.....cos	=	9·919846
Diff. lat. 48·22.....log	=	1·683274

Latitude from	68°	15′	15″ N.	MP	= 5651
Diff of lat. 48·22 N....	0	48	13 N.	,,	= 5784
Latitude in.....	69	3	28		133

To find difference of longitude—

Diff. lon. = mer. diff. lat. × tan course.

Mer. diff. lat. 133.....log	=	2·123852
Tan. 3 points.....tan.	=	9·824893
Diff. lon. 90·94.....log	=	1·948745

Longitude from.....	6°	47′	0″ W.
Diff. of longitude.....	1	30	56 W.
Longitude in.....	8	17	56 W.

C.—Third course and distance, W.N.W., 72 miles.

To find difference of latitude—

Diff. of lat. = dis. × cos. course.

Distance 72.....log	=	1·857332
Course 6 points.....cos.	=	9·582840
Diff. of latitude 27·55...log	=	1·440172

Latitude from.....	69°	3′	28″ N.	MP	= 5784
Diff. of lat. 27·55	0	27	33 N.	,,	= 5862
Latitude in.....	69	31	1 N.		78

To find difference of longitude—

Diff. lon. = Mer. diff. lat. × tan course.

Mer. diff. lat. 78	log	=	1·892095
Course 6 points	tan.	=	10·382776
Diff. lon. 188·3	log	=	2·274871

Longitude from	8°	17′	56″ W.
Diff. lon. 188·3	3	8	18 W.
Longitude in	11	26	14 W.

D.—Fourth course and distance, W. 48 miles.

To find difference of longitude (there is no difference of latitude)—

Diff. lon. = mer. dis. × sec. lat.

Mer. dist. 48	log	=	1·681241
Lat. in 69° 31′ 1″	sec.	=	10·456013
Diff. lon. 137·1	lon.	=	2·137254

Longitude from	11°	26′	14″ W.
Diff. lon. 137·1	2	17	6 W.
Longitude in	13	43	20 W.

E.—Fifth course and distance, S. by E. 45 miles.

To find difference of latitude—

Diff. of lat. = dist. × cos. course.

Distance 45	log	=	1·653213
Course 1 point	cos.	=	9·991574
Diff lat. 44·13	log	=	1·644787

Latitude from	69°	31′	1″ N.	MP = 5862
Diff. lat. 44·13	0	44	7 S.	,, = 5736
	68	46	54 N.	126

To find difference of longitude—

Diff. lon. = mer. diff. lat. × tan. course.

Mer. diff. lat. 126	log	=	2·100371
Course 1 point	tan.	=	7·298662
Diff. lon. 25·06	log	=	1·399033

Longitude from	13°	43′	20″ W.
Diff. lon. 25·06	0	25	3 E.
Longitude in	13	18	17 W.

F.—Sixth course and distance, E.S.E. 40 miles.

To find difference of latitude—

Diff. lat. = distance × cos. course.

Distance 40.....log = 1·602060
Course 6 points.....cos. = 9·582840
Diff. lat. 15·3.....log = 1·184900

Latitude from ..... 68° 46′ 54″ N. MP = 5736
Diff. of lat. 15·3..... 0 15 18 S. ,, = 5696
Latitude in..... 68 31 36 40

To find difference of longitude—

Diff. lon. = mer diff. lat. × tan course.

Mer. diff. lat. 40.....log = 1·602060
Course 6 points .....tan. = 10·382776
Diff. lon. 96·57.....log = 1·984836

Longitude from ..... 13° 18′ 17″ W.
Diff. of longitude 96·57 ..... 1 36 34 E.
Longitude in ..... 11 41 43 W.

This longitude, 11° 41′ 31″, is found to agree within two miles and a half with that found by Middle Latitude Sailing, 11° 38′ 0″ W.

The following question was proposed at the May Examinations, 1872; the solution appeared in *Naval Science*, 1872.

359. A right cylinder of indefinite length touches the earth, supposed spherical, along the equator, and an eye is placed at the centre, and the earth's surface projected on this cylinder, which is afterwards developed: Show (1) that all the meridians are projected into straight lines parallel to one another; and (2) that the length of a small arc of any meridian in latitude L on the projection is equal to the true length of the arc multiplied by sec. latitude.

In the annexed figure, supposing that the circle whose centre is C represents the sphere, then E B may be taken for the equator, while the lines B l and E K will represent the cylinder. If L and L′ be two points on the surface of the earth, join them with the centre C, and produce them to l and l', then l and l' will be the projection of these points on the cylinder, while the line $l\,l'$ will be the projection of the arc L L′.

(1.) If the eye be placed at the centre of the earth, it will be at once evident that each meridian will be projected as a

straight line on the cylinder, and that all these meridian lines will be parallel one to the other. It will also be seen that an arc of the meridians, when projected on the cylinder, will increase very rapidly as the eye passes from B towards P, in fact they increase in the ratio of the square of the secant of the latitude, which may be thus proved—

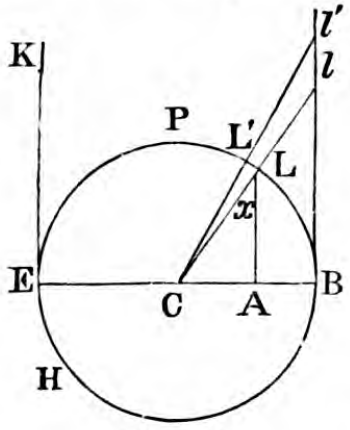


Fig. 29.

(2.) Let us suppose that B L H represents a meridian.

That C is the centre of the earth.

That L L′ are two points on the surface of the earth, situated on the same meridian, and whose latitudes differ very slightly.

Suppose also that Bl' is a tangent to the meridian, touching at the equator, representing the cylinder in the question.

The angle BCL is the latitude of L; the angle BCL′ the latitude of L′; the angle LCL′ may be considered as an infinitesimal increment to L.

$$\text{Tan. BC}l = \frac{\text{B}l}{\text{CB}} \quad \therefore \text{B}l = \text{BC tan. BC}l$$

$$\text{Tan. BC}l' = \frac{\text{B}l'}{\text{BC}} \quad \therefore \text{B}l' = \text{BC tan. BC}l'$$

Now evidently the projection of the arc LL′ on the circumscribing cylinder is—

$$\begin{aligned} ll' &= \text{B}l' - \text{B}l \\ &= \text{BC tan. BC}l' - \text{BC tan. BC}l \\ &= \text{BC (tan. BC}l' - \text{tan. BC}l) \\ &= \text{BC tan. (BC}l + l\text{C}l') - \text{tan. BC}l \end{aligned}$$

Calling angle BCl, θ and lCl' $d\theta$ we have—

$$\begin{aligned} ll' &= \text{BC} \left\{ \text{tan. } (\theta + d\theta) - \text{tan. } \theta \right\} \\ &= \text{BC} \left\{ \frac{\text{tan. } \theta + \text{tan. } d\theta}{1 - \text{tan. } \theta \text{ tan. } d\theta} - \text{tan. } \theta \right\} \\ &= \frac{\text{BC (tan. } \theta + \text{tan. } d\theta - \text{tan. } \theta + \text{tan. }^2\theta \text{ tan. } d\theta)}{1 - \text{tan. } \theta \text{ tan. } d\theta} \\ &= \frac{\text{BC tan. } d\theta \text{ (1} + \text{tan. }^2\theta)}{1 - \text{tan. } \theta \text{ tan. } d\theta} \\ &= \frac{\text{BC tan. } d\theta \text{ sec. }^2\theta}{1 - \text{tan. } \theta \text{ tan. } d\theta} \end{aligned}$$

When the angle $d\theta$ is infinitely small, the expression, since the tangent and the *arc* will very nearly coincide, becomes,

$$ll' = \text{BC} \sec.^2\theta \times d\theta$$

which adapted to radius unity is written,

$$ll' = d\theta \times \sec.^2\theta$$
$$= \text{LL}' \times \sec.^2(\text{latitude of place})$$

Hence each arc of the meridian, projected on a cylinder whose side is tangential to the equator, increases in the ratio of the secant square of the latitude.

It can readily be proved that L'x represents the arc of Mercator's projection, corresponding to the arc of the meridian LL'. Any one wishing to pursue the subject further, and to learn how to draw a Mercator's chart on this principle, may study with advantage an article in *Naval Science*, Vol. I.

81. Meridional Parts.—We now proceed to show how the formulæ are obtained by which the tables of meridional parts are calculated.

The two formulæ, or the two expressions for the same formula, are—

$$\text{M} = \log_{\epsilon} \cot. \frac{c}{2} \text{ where } c = 90° - \text{latitude}$$
$$= \log \cot. \tfrac{1}{2} \text{ co-lat.}$$

Or,

$$\text{M} = 3{\cdot}8984895 + \log (\log \cot. \tfrac{1}{2} \text{ co-lat.} - 10)$$

We repeat a little of what has gone before. The main feature in Mercator's projection is that every point of the globe is projected on a plane surface, and that since the meridians are all drawn parallel to each other, an additional illustration of this fact has been given in the general problem just solved, therefore their distance apart is increased in every position except at the equator: this is compensated for by increasing the distances at which the parallels of latitude are drawn.

It has been shown that—

$$\textit{Diff. lon.} = \textit{mer. dis.} \times \textit{secant. lat.}$$

Or that to make the departure equal to the difference of longitude, it must be increased in the ratio of the secant of the

latitude. For instance one mile of departure in latitude 45° becomes $1 \times \sec. 45° = 1 \times \sqrt{2} = 1{\cdot}414$ miles, or it is increased ·414 miles. Again, take latitude 70° in this latitude, one mile becomes $1 \times \sec. 70° = 2{\cdot}923$ miles, or in latitude 70°, on a Mercator's chart, the departure is nearly doubled, to make it correspond with the difference of longitude. Any small division is always lengthened in the ratio of the latitude near which it is situated.

The definition previously given of the meridional difference of latitude will now be better understood. *It is a portion of a meridian increased in the ratio of the secant of its latitude.*

Beginning at the equator we see from what precedes that

1 mile of the meridian in lat.			1′	is equivalent to	$1' \times \sec. 1'$
1′	,,	,,	2′	,,	$1' \times \sec. 2'$
1′	,,	,,	3′	,,	$1' \times \sec. 3'$
1	,,	,,	4′	,,	$1' \times \sec. 4'$
...			...		...
...			...		...
...			...		..
1′	,,	,,	$(n-1)'$	,,	$1' \times \sec. (n-1)'$
1′	,,	,,	n'	,,	$1' \times \sec. n'$

Adding all these together, it is evident that in n miles of latitude, reckoning from the equator, the meridional difference in latitude is —

$$= 1' (\sec. 1' + \sec. 2' + \sec. 3' + \sec. 4' + \ldots\ldots + \sec. (n-1)' + \sec. n'$$

These are the natural secant (not the logarithmic), and their sum tabulated for every latitude forms the table of meridional parts found in all good books of logarithmic tables. If the distances apart be taken smaller, we shall have a nearer approximation to the correct meridional parts for any given latitude. Again, as the earth is *not* a *perfect sphere*, allowance has to be made for its spheroidal form, or the *depression* must be taken into the calculations. It will be found that if different tables of meridional parts be examined they will be seen to differ; the reason for this circumstance is, that the depression allowed for is not the same, there being among astronomers and mathematicians a dispute as to the exact difference between the equatorial and polar diameters. The dispute, or rather difference in calculation, is less than a mile!

The annexed method of calculating the true meridional parts for any latitude, is due to the celebrated astronomer Dr. Halley, and published in the *Philosophical Transactions.*

Let dq represent a very very small portion of the equator.

Let dp represent the corresponding portion of any parallel in latitude l.

On the sphere we have proved, substituting the assumption made above, that

$$dp = dq \cos. l,$$

this upon Mercator's projection becomes

$$dp' = dq,$$

where dp' represents a corresponding infinitesimal portion of a *parallel* on a Mercator's chart; *i.e.*, dp' on the Mercator's chart represents dp on the globe. Now let dl represent a very very small part of a *meridian* on the *globe* equal to dq and adjacent to dp. Hence

$$dp' = dl.$$

But according to the assumption we have made regarding the projection of the meridians, whether they are projected on the plane chart or on to a cylinder touching the earth at the equator and tangential to it, *the portion of the meridian which represents* dl must be larger than dl' actually is, in the ratio that dp' exceeds dp; that is, if dl' represents dl on the chart, then

$$\frac{dl'}{dl} = \frac{dp'}{dp}, \text{ but since } dp' = dq \text{ and } dp = dq \cos. l.$$

$$\therefore \frac{dp'}{dp} = \frac{1}{\cos. l.}$$

$$\therefore \frac{dl'}{dl} = \frac{1}{\cos. l.} = \frac{\cos. l.}{\cos.^2 l.}$$

$$= \frac{\cos. l.}{1 - \sin.^2 l} = \frac{\cos. l.}{(1 + \sin. l)\,(1 - \sin. l).}$$

$$= \tfrac{1}{2} \left\{ \frac{\cos. l}{1 + \sin. l} + \frac{\cos. l}{1 - \sin. l.} \right\}$$

$$\therefore dl' = \tfrac{1}{2} \left\{ \frac{\cos. l dl}{1 + \sin. l} + \frac{\cos. l dl}{1 - \sin. l} \right\}$$

Integrating both sides of the equation

$$l' = \tfrac{1}{2}\left\{\log_\epsilon (1 + \sin.\ l) - \log_\epsilon\ (1 - \sin.\ l)\right\}$$
$$= \tfrac{1}{2}\ \log_\epsilon \frac{1 + \sin.\ l}{1 - \sin.\ l}.$$

Converting this into sin. $\frac{l}{2}$, or half angles, and reducing to tangents, it becomes

$$l' = \log_\epsilon (\tan.\ 45° + \tfrac{l}{2})$$
$$= \log_\epsilon (\cot.\ \tfrac{1}{2}\ (90 - l),$$
$$\text{or mer. parts} = \log_\epsilon\ \cot.\ \tfrac{1}{2}\ \text{co-latitude}.$$

As the result now stands, we have proved that the sum of all the infinitely small parts lying between latitude l and the equator, is equal to

$$\log_\epsilon \text{co tan. } \tfrac{1}{2} \text{ co-latitude},$$

but this logarithm has ϵ for its base; to make it of more practical value it must be adapted to the common logarithms with base 10. To do this it must be multiplied by 2·302585, which is $\log_\epsilon 2$. Then it must be adapted to the radius of the earth and expressed in nautical miles, which is effected by multiplying by the number of nautical miles in the radius of the earth, which is 3437·75 nearly.

Hence,

$$\log_\epsilon \text{cot. } \tfrac{1}{2} \text{ co-lat.} = 2{\cdot}302585 \times 3437{\cdot}75 \times \log \text{cot. } \tfrac{1}{2} \text{ co-lat.},$$
$$\text{or mer. parts} = \log. 2{\cdot}302585 + \log 3437{\cdot}75 + \log (\log \text{cot. } \tfrac{1}{2} \text{ co-lat.} - 10)$$
$$= 3{\cdot}8984895 + \log (\log \text{cot } \tfrac{1}{2} \text{ co-lat.} - 10).$$

M. Delambe has shown that if the spheroidal figure of the earth be taken into account, the formula becomes

$$\text{mer. parts} = \log_\epsilon \tan. (45 + \tfrac{1}{2} \text{ geocentric co-lat.}),$$
$$= \log_\epsilon \cot. \tfrac{1}{2} (\text{geocentric co latitude}),$$

which is the same expression.

CHAPTER XI.

GREAT CIRCLE SAILING.

Definition—Spherical Triangle—Vertex—Course, or Angle of Position—Distance—Geerkin's Method of Laying Down the Great Circle Track—Professor Airy's Method—To Draw the Great Circle Curve on a Mercator's Chart—To compute the Shortest Distance—Formulæ and Rules—To find the First and Last Course with Formulæ—Rules and Examples—To find the Vertex of the Great Circle (Two Methods)—To Find a Succession of Points on a Great Circle (Two Methods)—Illustrations and Examples—The Rhumb Line and Great Circle Track—Composite Sailing—Limitations of Great Circle Sailing.

82. A Great Circle has been previously defined as a circle that divides the sphere into two hemispheres. Its plane will necessarily pass through the centre of the earth. All meridians are great circles of the sphere. It is generally advantageous, except when it is necessary to sail into certain latitudes, to meet with winds or currents that are particularly favourable and local, for a ship to reach her port by the shortest route. The distance as shown by Mercator, though always trustworthy, is very seldom the shortest. The least distance between any two places on the earth's surface is the arc of a great circle lying between them, this is a difficult assertion to prove mathematically, but easily proved by example and illustration. In consequence of what is now said, the path a ship describes in going from one place to another, being a rhumb line, will always exceed the arc of a great circle passing over the places, except when the rhumb line happens to coincide with a great circle, and this is the case when the ship sails upon a meridian or along the equator.

To find upon what track a ship sails, and to lay that path down on a chart, it is necessary to know the course; that the course may be known she must sail on the same for

some time, and as long as a vessel continues on the same course, the line she describes will be a rhumb line. It is almost impossible to keep the ship upon any assigned line that is not a rhumb line, and since the meridians and equator are the only great circles on the globe that are rhumb lines, a ship can be made to describe no other great circle. But although a vessel cannot be kept exactly on any assigned great circle, she can be frequently brought upon it and always kept near it, sufficiently near to answer all the purposes of Great Circle Sailing.

The solution of problems in great circle sailing depends upon spherical trigonometry, hence to rightly comprehend the whole subject, the student must be well versed in the solution of right-angled and oblique spherical triangles. In the preceding pages we have dealt entirely with right-angled plane triangles. Great Circle Sailing admits of many varieties of problems which are exercises in trigonometry, hence *Spherical Trigonometry and Navigation* convey the same meaning to a student of Great Circle Sailing.

After what has now been said, we may more accurately define our subject.

83. Great Circle Sailing.—*Great Circle Sailing* is the art of finding what places a vessel must pass through, and what courses she must steer, that her track may coincide as nearly as possible with the arc of a great circle passing through the places sailed from and bound to.

It seems probable that steam ships will ultimately displace nearly all sailing vessels, and as sailing vessels depend upon tides, winds, and currents, and often take long and circuitous routes to meet them, while steamers take the most direct course: Great Circle Sailing must receive much greater attention at the hands of our mercantile marine of the future than it has hitherto, not only on this account, but because vessels frequently run up into much higher latitudes than formerly.

84. Definitions.—A *spherical triangle* is formed by the intersection of three great circles.

The **distance** is the arc of a great circle, and forms the third side of the spherical triangle, or it is the track on which the ship runs when sailing on a great circle.

Vertex.—The *vertex* of a great circle is that point of the great circle which is farthest from the equator.

The **course.**—The *course* is the angle included between the ship's track and the meridian.

Angle of Position.—The angle which a great circle passing through any place makes with the meridian of that place is called its *angle of position*, because it shows the angular position of that place from any other place through which the given circle passes.*

In navigation the sides of the spherical triangle are the two meridians passing over the two places between which the ship's track lies, the third side is the ship's track. The lengths of the two sides formed by the meridians are always known from the latitudes of the places to and from one of the angles, that at the pole is found from the difference of longitude between the two places. Hence it is evident we have to solve a spherical triangle; given two sides and the included angle. To enable us to do this, spherical trigonometry supplies several rules depending upon the mode in which we commence the solution of our triangle.

For instance, some writers

1. Find the distance.
2. The latitude and longitude of the vertex.
3. A succession of points on the great circle.
4. The course and distance from point to point.

Others—

1. Find the first and last course
2. The distance.
3. The position of the vertex.
4. A succession of points on the great circle.

We will find—

1. The distance.
2. The first and last course.
3. The position of the vertex.
4. A succession of points.

It is very important that seamen should have an easy and

* Wilson's *Navigation*, p. 336. Edinburgh, 1773. Robertson's *Navigation*, p. 191, Vol. II. London, 1786.

quick method—accurate, within well defined limits—of laying down the great circle track on an ordinary Mercator's chart. Such a method follows immediately below. It is well to bear in mind, as already suggested, that the intervention of land, ice, currents, weather, and various other influences, considerably limit the adoption of Great Circle Sailing in the royal and mercantile navies.

85. Captain Geerken's Method.—Captain Ludvig Geerken, of the Bremen steamer *Hansa*, in 1857, suggested a ready method of describing the curve on a Mercator's chart, which pretty fairly represents the corresponding arc of a great circle on the globe. It is as follows:—

"Take the middle latitude between the place of departure and the place of destination; subtract the result from 90° for the complement of the middle latitude; double this, it gives a number of degrees (on the equator) as the radius for the great circle track between the places in question. If on opposite sides of it, the equator is always cut in a straight line between the two places.

"In tracing a track on a chart, the compasses are to be placed on the equatorial side of the place.

"This problem does not require any table or book, but only a pair of large compasses, or a sliding scale, with a point and pencil at right angles to the scale, or a pair of beam compasses, and it gives great power to a Mercator's chart, which has all the elements in itself for tracing a great circle track."

This is very simple, and approximates to the truth. It also possesses this advantage, that after the great circle track has been traced on the chart, the mixed or composite courses may easily be laid down by the eye.

86. Professor Airy's Method.—Captain Geerken's method having been submitted to the Astronomer Royal, he suggested the following mode of laying down the track; it approaches even nearer than the preceding to the correct projection of the great circle.

"(1.) Join the two places (on the chart) by a straight line. Find its middle. Draw thence a perpendicular to that line on the side next the equator, and, if necessary, continue it beyond the equator.

"(2.) With the middle latitude (between the two places) enter the following table, and take out the corresponding parallel.

"(3.) The centre of the required sweep will be the intersection of this parallel with the perpendicular (first drawn)."

Suppose a vessel has to sail on the great circle track between two places, one in 50° N. the other 30° N., the two places are first joined by a straight line, and a perpendicular erected from its centre *towards* the equator; then taking $\frac{50° + 30°}{2} = 40°$, we go to the annexed table and find opposite 40° the latitude 41° 18′. Where the perpendicular already drawn intersects the parallel of 41° 18′, is the centre from which the arc of the approximate great circle must be struck.

TABLE FOR FINDING CENTRE OF ARC OF GREAT CIRCLE.

Middle latitude.	Corresponding parallel.	Middle latitude.	Corresponding parallel.
20°	81° 13′	52°	11° 33′
22	78 16	54	6 24
24	74 59	56	1 13
26	71 26		
28	67 38	58	4 0
30	63 37	60	9 15
32	59 25	62	14 32
34	55 5	64	19 50
36	50 36	66	25 9
38	46 0	68	30 30
40	41 18	70	35 52
42	36 31	72	41 14
44	31 38	74	46 37
46	26 42	76	52 1
48	21 42	78	57 25
50	16 39	80	62 51

These tentative methods do not apply in all cases; it cannot be fairly expected that they should. When the distance between the two places is required with greater accuracy, and more correct than can be measured off from the chart, recourse must be had to direct computation. The above is taken from *Great Circle Sailing*, by Admiral Fitzroy, a tract published by the Board of Trade.

87. To Draw the Great Circle Curve on a Mercator's Chart.—The following is partly extracted from a small pamphlet, published by the Board of Trade, *Passage Tables and General Sailing Directions*:—

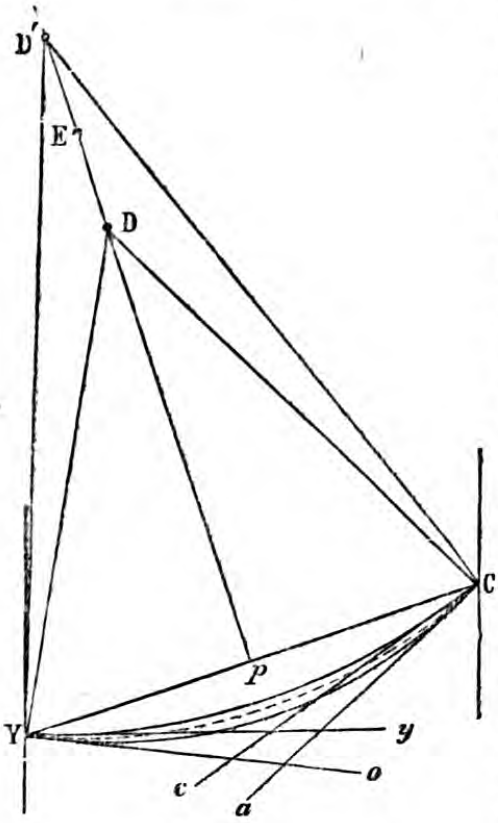


Fig. 30.

Suppose it be desirable to draw on a chart the great circle track between Cape Clear and New York. First join the two places by the rhumb line YC. Now find the first and last courses by the usual method in great circle sailing; these will be found to be 77° 25′ and 53° 24′. Next find the course by Mercator's principles, which is 76° 30′ at both places. The differences between the great circle course and Mercator's are, say, 1° and 23°. At each end of the rhumb line at C and Y make the two angles pYy and pCc equal to 1°, and pYo and pCa equal to 23°. Erect YD and CD perpendiculars to Ca and Yo, and YD′ and CD′ perpendiculars to Yy and Cc, meeting in D and D′ respectively. From centres D and D′, with radii D′C and DC, describe the two arcs: then the great circle track lies between them, and should be drawn by hand, as shown by the dotted line, avoiding, if possible, rocks or land, and even going a little out of the way to meet with favourable currents. In drawing this figure, we have encountered the great difficulty of the method, the angle pCc being only 1°, we have been compelled to distort it and make it 16° for the sake of perspicuity, so that when drawing the perpendicular CD′ it might fall on the chart. Frequently this is the case, one of the courses differs so little from the Mercator's course, that the first angle is small, consequently the centre of the inner arc will not fall on the chart.

Joining the two places by a rhumb line, and at each end

constructing angles equal to the differences of the Mercator's and great circle courses, and erecting perpendiculars to these lines, two points are found from which arcs may be described between which the great circle lies, when it may be approximately drawn by hand.

If we bisect the line DD′ in E, and take that as the centre, describing an arc through the points C and Y, it will approximate to the form of the great circle tract. In fact, a shorter method is as follows:—At either end, say C, make the two angles as above, and erect perpendiculars from the lines C*c*, C*a*. Now bisect the rhumb line by a perpendicular as shown in the figure, intersecting the first two lines in D and D′; bisect DD′ on E, from centre E at distance C describe the arc as represented by the dotted line. Then that line is what we require; if it pass through an island, or is in too high a latitude, draw a curve by hand, within or near, to suit the various circumstances.

88. To Compute the Shortest Distance.—Let A and B be the two places between which the ship is to sail on the great circle track.

A is the nearer of the two to the equator.

Let BPB′ represent the meridian passing through B.

Let PA*o* represent the meridian passing through A.

Let AB represent the arcs of the great circle passing through A and B.

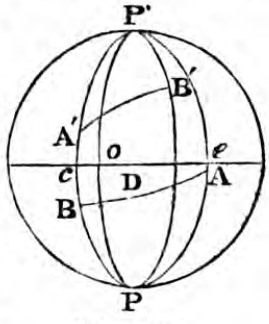


Fig. 31.

We have to find the length of AB, a side of the spherical triangle ABP.

The following are the known elements in the triangle:—

APB, which is the difference of longitude between A and B.

BP, which is 90° – latitude of B, or 90° – B*c*.

AP, which is 90° – latitude of A, or 90° – A*e*.

Always let A be the place nearer to the equator, then AP will be greater than BP, or 90 – A*e* greater than 90 – B*c*.

Call BP, the side opposite A, 90° – B, where B is the latitude of B.

Call AP, the side opposite A, 90° – A, where A is the latitude of A.

Call AB, the side opposite APB, D, the *distance*.

From spherical trigonometry, we have

$$\text{Cos. APB} = \frac{\text{cos. AB} - \text{cos. BP cos. AP}}{\text{sin. BP sin. AP}},$$

$$\therefore \text{Cos. AB} = \text{cos. AP cos. BP} + \text{sin. AP sin. BP cos. APB.}$$

Substituting the terms assumed above, and remembering that $\text{cos. APB} = 1 - 2\ \text{sin.}^2 \frac{\text{APB}}{2}$,

$$\therefore \text{Cos. D} = \text{cos. (90} - \text{A) cos. (90} - \text{B)} + \text{sin. (90} - \text{A) sin. (90} - \text{B)} - 2\ \text{sin. (90} - \text{A) sin. (90} - \text{B) sin.}^2 \frac{\text{APB}}{2},$$

$$\therefore \text{Cos. D} = \text{sin. A sin. B} + \text{cos. A cos. B} - 2\ \text{cos. A cos. B sin.}^2 \frac{\text{APB}}{2}.$$

Adding unity to each side, and remembering that

$$1 + \text{cos. D} = 2\ \text{cos.}^2 \frac{\text{D}}{2} \text{ and } 1 + \text{cos. (B} - \text{A)} = 2\ \text{cos.}^2 \frac{\text{B} - \text{A}}{2},$$

$$\therefore 2\ \text{cos.}^2 \frac{\text{D}}{2} = 2\ \text{cos.}^2 \frac{\text{B} - \text{A}}{2} - 2\ \text{cos. A cos. B sin.}^2 \frac{\text{APB}}{2}.$$

Dividing by 2, and assuming that

$$\text{Cos. A cos. B sin.}^2 \frac{\text{APB}}{2} = \text{cos.}^2 \text{H}, \qquad (\alpha).$$

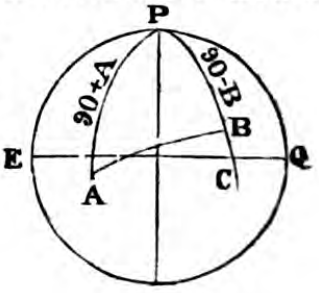
we have

$$\text{Cos.}^2 \frac{\text{D}}{2} = \text{cos.}^2 \frac{\text{B} - \text{A}}{2} - \text{cos.}^2 \text{H}.$$

Therefore by the usual formula in trigonometry,

$$\text{Cos.}^2 \frac{\text{D}}{2} = \text{sin.} \left(\text{H} + \frac{\text{B} - \text{A}}{2}\right) \text{sin.} \left(\text{H} - \frac{\text{B} - \text{A}}{2}\right) \qquad (\beta).$$

Equations α and β give the necessary rules for finding the distance on a great circle. But before putting the formulæ into words, they require to be adapted to the case where one place is north of the equator and the other south. Until the present work we are not aware that the formulæ now under discussion have been made applicable to all cases of the solution of spherical triangles.



Let APB be the spherical triangle, with A on one side of the equator and B on the other, then the side

$$\text{AP} = 90 + \text{A}$$
$$\text{BP} = 90 - \text{B}$$
$$\angle \text{APB} = \text{polar angle},$$

in fact all the conditions remain the same, excepting for A we have 90 + A.

Now $$\text{Cos. APB} = \frac{\text{cos. AB} - \text{cos. BP cos. AP}}{\text{sin. BP sin. AP}},$$

$$\therefore \text{Cos. AB} = \text{cos. BP cos. AP} + \text{sin. BP sin. AP cos. APB.}$$

Substituting the terms assumed above, we have

$$\text{Cos. D} = \text{cos. (90+A) cos. (90-B)} + \text{sin. (90+A) sin. (90-B)} - 2 \text{ sin. (90+A) sin. (90-B) sin.}^2 \frac{\text{APB}}{2},$$

$$\therefore \text{Cos. D} = -\text{sin. A sin. B} + \text{cos. A cos. B} - 2 \text{ cos. A cos. B sin.}^2 \frac{\text{P}}{2}.$$

Adding 1 to each side, and substituting

Cos. (A+B) for (cos. A cos. B − sin. A sin. B),

$$1 + \text{cos. D} = 1 + \text{cos. (A+B)} - 2 \text{ cos. A cos. B sin.}^2 \frac{\text{P}}{2}$$

Remembering that

$$\text{Cos. D} = 2 \text{ cos.}^2 \frac{\text{D}}{2} - 1,$$

$$\text{Cos. (A+B)} = 2 \text{ cos.}^2 \frac{\text{A+B}}{2} - 1,$$

also dividing by 2, and assuming that

$$\text{Cos. A cos. B sin.}^2 \frac{\text{P}}{2} = \text{cos.}^2 \text{H}, \qquad (\alpha')$$

we obtain

$$\therefore \text{Cos.}^2 \frac{\text{D}}{2} = \text{cos.}^2 \frac{\text{A+B}}{2} - \text{cos.}^2 \text{H},$$

which, from the formula used before, becomes

$$\text{Cos.}^2 \frac{\text{D}}{2} = \text{sin.} \left(\text{H} + \frac{\text{A+B}}{2}\right) \text{sin.} \left(\text{H} - \frac{\text{A+B}}{2}\right) \quad (\beta').$$

This completes the formulæ, and, with the two found above, includes every case which can arise in Great Circle Sailing.

89. Rules for Great Circle Sailing—Distance:—

$$(\alpha) \quad \text{Cos.}^2 \text{H} = \text{cos. lat. A cos. lat. B sin.}^2 \frac{\text{diff. lon.}}{2}.$$

A *is taken as the place nearer the equator.*

(1.) Take the difference of longitude and divide it by 2, this is $\angle \frac{\text{APB}}{2}$, or half diff. of long.

Take out log sine half diff. of longitude, and multiply it by 2.

To this add the logarithms of the consines of the two latitudes. Half this sum is the logarithm of cosine H.

$$(\beta)\ \text{Cos.}^2 \frac{P}{2} = \text{sin.}\left(H + \frac{B-A}{2}\right) \text{sin.}\left(H - \frac{B-A}{2}\right),$$

where A and B stand for the latitudes of the two places.

When the latitudes have the same name.

(2.) Having found H, take half the *difference* between the latitudes of A and B, add it to H, and find the difference between it and H.

(3.) Take out the logarithms of the sines of these two arcs, and add them together, half their sum is the log cosine of half the distance.

(4.) Multiply this arc by 2, and reduce it to miles, and the distance (D) is found.

$$(\beta')\ \text{Cos.}^2 \frac{D}{2} = \text{sin.}\left(H + \frac{A+B}{2}\right) \text{sin. } H - \left(\frac{A+B}{2}\right).$$

(2′.) *When the latitudes have different names,* Rule 2 becomes—

Having found H, take half the *sum* of the latitudes of A and B, and add it to and subtract it from H, etc.

360. Required the shortest distance, or distance on the great circle, between Cape Clear in Ireland, latitude 51°26′ N., longitude 9°29′ W., and New York, latitude 40° 43′ N., longitude 74° 7′ W.

RULE *a*.

$$\text{Cos.}^2 H = \text{cos. lat. A cos. lat. B sin.}^2 \frac{APB}{2}.$$

Lat. A = 40° 43′ N. Lon. 74° 7′ W.
Lat. B = 51° 26′ N. Lon. 9 29 W.

2)64 38

32 19 $= \frac{APB}{2}$.

$\frac{ABP}{2}$ = 32° 19′.....Sin. = 9·728027

2

19·456054

Lat. A = 40 43 .....Cos. = 9·879637
Lat. B = 51 26 .....Cos. = 9·794784

2)19·130475

H = 68° 26′ 22″ = Cos. 9·565237

RULE β.

$$\text{Cos.}^2 \frac{D}{2} = \text{sin.} \left(H + \frac{B-A}{2}\right). \text{ Sin.} \left(H - \frac{B-A}{2}\right).$$

A = 40° 43′
B = 51 26
B − A = 10 43
$\frac{B-A}{2}$ = 5 21 30
H = 68 26 22

$\therefore H + \frac{B-A}{2} = 73° 47′ 52″$

$\therefore H - \frac{B-A}{2} = 63\ 4\ 52$

$H + \frac{B-A}{2} = 73° 47′ 52″$ Sin. = 9·982398

$H - \frac{B-A}{2} = 63\ 4\ 52$ Sin. = 9·950192

2)19·932590

$\frac{D}{2} = 22° 16′ 57″$ Cos. = 9·966295

2

D = 44 33 54

60

Distance = 2673·9 miles.

The figure on page 197 has the spherical triangle A′P′B′ corresponding to the position of the two places A and B, in the example here worked.

361. Find the distance between Cape Clear and New York by Mercator's Sailing.

	Diff. lat.	Mid-lat.	Diff. lon.
Lat.	A = 40° 43′ N.	M.P. 2665	74° 7′ W.
,,	B = 51 26 N.	,, 3594	9 29 W.
	10 43	929	64 38
	60		60
	643 diff. lat.		3878

The course must be found first thus—

$$Tan.\ course = \frac{diff.\ lon.}{mer.\ diff.\ lat.} = \frac{3878}{929}$$

Diff. lon. 3878	log = 3·588608
Mer. diff. lat. 929	log = 2·968016
Tan. course 76° 31′ 41″	log = 10·620592

To find distance—

$$Distance = true\ diff.\ lat. \times sec.\ course$$
$$= 643 \times \text{sec. } 76° 31′ 41″$$

True diff. lat. 643	log = 2·808211
Course 76° 31′ 41″	log = 10·632700
Distance 2760	log = 3·440911

It will be seen from this illustration that the distance is less on the great circle by (2760 – 2674 =) 86 miles than by Mercator's Sailing, and that by pointing a ship's head directly towards the point of the horizon where the place lies is not the quickest method of reaching the place.

362. Required the distance on the great circle between A and B when their latitudes and longitudes are as annexed.

Lat. A 12° 45′ S. Lon. A 154° 5′ W.
Lat. B 23° 10′ N. Lon. B 140° 30′ W.

$$2)\overline{13°\ 35'}$$
$$\overline{6°\ 47'\ 30''} = \frac{APB}{2}$$

RULE a or a'.

$$\text{Cos.}^2\ H = \text{cos. lat. A cos. lat. B sin.}^2 \frac{APB}{2}$$

$\frac{APB}{2} = 6°\ 47'\ 30''$	sin. =	9·072836
		2
		18·145672
Lat. A = 12° 45′	cos. =	9·989157
Lat. B = 23° 10′	cos. =	9·963488
		2)18·098317
H = 83° 34′ 13″	cos. =	9·049158

RULE β'.

$$\text{Cos.}^2 \frac{D}{2} = \text{sin.} \left(H + \frac{A+B}{2}\right) \text{sin.} \left(H - \frac{A+B}{2}\right)$$

A = 12° 45′
B = 23 10
A + B = 35 55
$\frac{A+B}{2}$ = 17 57 30

H = 83 34 13

$\therefore$ $H + \frac{A+B}{2} = 101°\ 31'\ 43''$

$H - \frac{A+B}{2} = 65\ 36\ 43$

$H + \frac{A+B}{2} = 101°\ 31'\ 43''$	Sin. =	9·991149
$H - \frac{A+B}{2} = 65\ 36\ 43$	Sin. =	9·959408
		2)19·950557
		9·975278

$\frac{D}{2} = 19°\ 9'\ 0''$ = Cos.

D = 38 18 0
60

Distance 2298 miles.

The figure on page 198 exhibits a spherical triangle as it

would appear if the triangle corresponding to this solution were drawn on the globe.

The same problem is now solved by Mercator's Sailing.

Diff. of lat.	Mer. diff. lat.	Diff. of lon.
Lat. A 12° 45′ S.	M.P. 767	Lon. A 154° 5′ W.
Lat. B 23 10 N.	,, 1421	,, B 140 30 W.
35 55	2188	13 35 E.
60		60
2155		815

To find course—

$$Tan.\ course = \frac{diff.\ lon.}{mer.\ diff.\ lat.} = \frac{815}{2155}$$

Diff. of lon. 815	log = 2·911158
Mer. diff. lat. 2188.....	log = 3·340047
Tan. course 20° 25′ 47″	log = 9·571111

To find distance—

$$Cos.\ course = \frac{true\ diff.\ lat.}{distance}$$

$$\therefore\ Distance = true\ diff.\ lat. \times sec\ course.$$
$$= 2155 \times \sec.\ 20°\ 25′\ 47″$$

True diff. lat. 2155.....	log = 3·333447
Course sec. 20° 25′ 47″	log = 10·028213
Distance 2299·6 miles.....	log = 3·361660

This distance coincides most remarkably with that in the previous answer, being but a mile and a half more than on the great circle track; the reason for this is, the course is near the equator, where the two distances necessarily approach each other. Had the courses been away from the equator, we should expect a greater difference; in fact, the further away the less any two answers so obtained will coincide.

90. To Find the First and Last Courses.—When a ship keeps to the great circle track she has continually to change her course. It will be our duty presently to show how to find a succession of points on the great circle track, and to point out how the vessel is to reach those places. At present we have to find only two of these courses, the first and the last, or by referring back to the two figures we have to find the angles PAB and PBA, which are the angles that the ship's track AB makes with the two meridians PA and PB.

The angles in a spherical triangle are always found by using Napier's analogies.

$$(a.)\quad \text{Tan.}\frac{A+B}{2}=\frac{\text{cos.}\frac{BP-AP}{2}}{\text{cos.}\frac{BP+AP}{2}}\text{cot.}\frac{APB}{2}$$

$$=\text{Sec.}\frac{BP+AP}{2}\text{cos.}\frac{BP-AP}{2}\text{cot.}\frac{APB}{2}$$

$$=\text{Sec.}\frac{a+b}{2}\text{cos.}\frac{a-b}{2}\text{cot.}\frac{APB}{2}\qquad(a)$$

$$(b.)\quad \text{Tan.}\frac{A-B}{2}=\frac{\text{sin.}\frac{BP-AP}{2}}{\text{sin.}\frac{BP+AP}{2}}\text{cot.}\frac{APB}{2}$$

$$=\text{Cosec.}\frac{BP+AP}{2}\text{sin.}\frac{BP-AP}{2}\text{cot.}\frac{APB}{2}$$

$$=\text{Cosec.}\frac{a+b}{2}\text{sin.}\frac{a-b}{2}\text{cot.}\frac{APB}{2}\qquad(b)$$

Here a and b represent the sides of the spherical triangle, or the distances of the places from the pole near B. A and B have been previously used to denote the latitudes of the places, but here we employ them instead of the angles PAB and PBA. The student must guard himself against confusion. From these two equations the two angles PAB (A) and PBA (B) are known for PAB (A) is the sum of the two angles $\frac{A+B}{2}$ and $\frac{A-B}{2}$ and B the difference.

Rules to Find the First and Second Course.

(a.) Find the polar distances of A and B (from the pole near B).

(b.) Take the sum and difference of these two quantities.

(c.) The angle APB is found by taking half the sum of the difference of longitude.

(d.) Add together—	(e.) Add together—
(1.) Log secant half sum.	Log cosec. half sum.
(2.) Log cosine half difference.	Log sine half difference.
(3.) Log. cotan $\frac{APB}{2}$	Log cotan $\frac{APB}{2}$

363. Required the first and last courses on a voyage, made on the great circle track, between Cape Clear in Ireland and New York.

N. York lat. 40° 43′ N. A — $\therefore a = 49°\ 17'$ Lon. 74° 7′ W.
C. Clear lat. 51 26 N. B — $b = 38\ 34$ Lon. 9 29 W.

2)10 43 diff. lat. — $\therefore a+b = 87\ 51$ — 2)64 38 diff. lon.
5 21½ half diff. lat. — $a-b = 10\ 43$ — 32 19 = ½ diff. lon.

$$\therefore \frac{a+b}{2} = 43\ 55\tfrac{1}{2}$$

$$\frac{a-b}{2} = 5\ 21\tfrac{1}{2}$$

$$Tan.\frac{B+A}{2} = sec.\frac{a+b}{2}\ cos.\frac{a-b}{2}\ cot.\frac{diff.\ lon.}{2} \qquad Tan.\frac{B-A}{2} = cosec.\frac{a+b}{2}\ sin.\frac{a-b}{2}\ cot.\frac{diff.\ lon.}{2}$$

A and B are the angles, a and b the co-latitudes.

$\frac{a+b}{2}$	43° 55′ 30″	sec.	10·142517		43° 55′ 30″	cosec.	10·158949	
$\frac{a-b}{2}$	5 21 30	cos.	9·998098		5 21 30	sin.	8·970273	
$\frac{Diff.\ lon.}{2}$	32 19 0	cot.	10·198884		32 19 0	cotan.	10·198884	
	65 24 37	tan.	10·339499		12 1 1	tan.	9·328106	
Sum	65° 24′ 37″				65° 24′ 37″			difference
	12 1 1				12 1 1			
	B = 77 25 38				53 23 36 = A			

N. 77° 25′ 38″ W., course from Cape Clear to New York.
N. 53 23 36 E., ,, ,, New York to Cape Clear.

364. Required the first and last courses from two places A and B.

Lat. A 12° 45′ S. $pd = 102°\ 45'$ — Lon. A is 154° 5′ W.
Lat. B 23 10 N. $pd = 66\ 50$ — Lon. B is 140 30 W.

Sum 169 35 — 2)13 35 E.
Diff. 35 55 — 6 47 30 = $\frac{APB}{2}$
Half sum 84 47 30
Half diff. 17 57 30

Sec. half sum	84° 47′ 30″	log =	11·042020
Cos. half diff.	17 57 30	log =	9·978309
Cot. $\frac{APB}{2}$	6 47 30	log =	10·924107
Tan. $\frac{A+B}{2}$	89 21 4	log. =	11·944436
Cosec.....	84° 47′ 30″	log =	10·001798
Sin.....	17 57 30	log =	9·489009
Cot.....	6 47 30	log =	10·924107
Tan. $\frac{A-B}{2}$	68 57 35	log =	10·414914

$$\left.\begin{matrix} 68°\ 57'\ 35'' \\ 89\ 21\ \ 4 \end{matrix}\right\} = 158°\ 18'\ 39'' = \text{sum B.}$$
$$= 20\ 23\ 29 = \text{diff. A.}$$

∴ Courses are—

N. 20° 23′ 39″ E. = ∠ PAB
N. 158 18 39 W. = ∠ PBA } in fig. p. 198.
or S. 21 41 21 W. = ∠ ABC

91. To Find the Position of the Vertex of a Great Circle.—As already defined, the vertex of a great circle is that part of it which touches the highest latitude; it generally lies between the two places when they are widely separated, but it by no means necessarily does so. The position of the vertex must be known to enable us to calculate the course and distance from one point to another on the great circle. It will be found that if the two angles A and B in the triangle APB be both greater or both less than ninety degrees, the vertex will fall between the two places; but if one be greater and the other less, it will not fall between them, but beyond the angle B away from A.

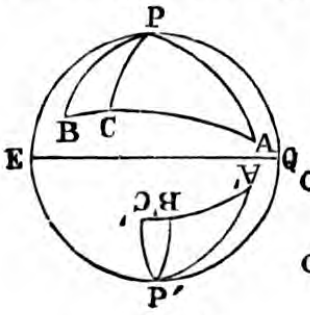


Fig. 32.

Let A B C be a spherical triangle.
A P the polar distance and co-latitude of A.
B P the polar distance and co-latitude of B.
From P draw P C perpendicular to A B or A B produced.
Then C is the vertex of the great circle of which A B is an arc.
ACP is a right-angled triangle.
Then PAC, PCA, and AP are known.

It is required to find PC, the co-latitude of C, and hence its latitude, and CPA the difference of longitude between A and C, and hence the longitude of C.

Napier's circular parts give the following simple formula—

Sin M = *product* of the *cosines* of the *opposite* parts.
∴ *Sin.* PC = *cos.* (90 − PA) *cos.* (90 − PAC).
∴ *Sin.* (90 − *lat. of vertex*) = *cos.* (90 − PA) *cos.* (90 − PAC).
∴ *Cos. lat. vertex* = *cos. lat.* A *sin.* ∠ A. (I.)

Again, from the same circular parts—

Sin. M = *product* of the *tangents* of the *adjacent* parts.
Sin. (90 − PA) = *tan.* (90 − PAC) *tan.* (90 − APC).
∴ *Sin. lat. of* A = *cot.* PAC *cot.* APC.

$$\therefore \textit{Cot.}\ \text{APC} = \frac{\textit{sin. lat.}\ \text{A}}{\textit{cot.}\ \text{PAC}} = \textit{sin. lat.}\ \text{A} \times \textit{tan.}\ \text{PAC.}$$

or *cotan. diff. lon.* = *sin. lat. of* A × *tan.* ∠ A. (II.)

RULES FOR FINDING THE LATITUDE AND LONGITUDE OF THE VERTEX.

I. *Cos. lat. vertex = cos. lat. of* A × *sin.* ∠ A.

Latitude.—To the cosine of the latitude of the place nearer the equator add the sine of the angle A *found according to the preceding formulæ, this gives the cosine of the latitude of the vertex.*

II. *Cotan. diff. of lon. = sin. lat.* A × *tan.* ∠ A.

Longitude.—To the sine of the latitude of place A *add the tangent of angle* A, *this sum is the co-tangent of the difference of longitude,* and must be applied from A towards B.

An examination of spherical trianges A′P′C′ will show that when angle B′ is greater than a right angle, that the perpendicular P′C′ falls beyond B′, and the difference of longitude found is angle A′P′C′; therefore if that difference be applied to the longitude of A, it *must* give the longitude of the vertex C′.

365. Find the latitude and longitude of the vertex of the great circle lying between Cape Clear and New York.

Latitude of A 40° 43′ N. Longitude 74° 7′ W.
,, B 51 26 N. ,, 9 29 W.

The angle of the spherical triangle at A has been found = 53° 23′ 36″
,, ,, B ,, = 77 25 38

To find latitude of vertex.
Cos. lat. vertex = cos. lat. A *sin.* A.

Lat. A = 40° 43′ 0″ cos. = 9·879637
∠ A = 53 23 36 sin. = 9·904579
Lat. ver. 52 31 24 cos. = 9·784216

To find longitude of vertex.
Cotan. diff. lon. = sin. lat. A *tan. lat.* A.

Lat. A 40° 43′ 0″ sin. = 9·814460
∠ A 53 23 36 tan. = 10·129101
Diff. lon. 48 42 45 cotan. = 9·943561

∴ Latitude of vertex is 52° 31′ 24″ N.

Longitude of vertex
Lon. of A..... 74° 7′ 0″ W.
Diff..... 48 42 45 E.
Lon. of vertex is 25 24 15 W.

92. A Second Method of Finding the Latitude and Longitude of the Vertex.—Using the last figure, we have (considering that AB is the *distance* = D),

Latitude of vertex—

$$\frac{Sin.\ PAB}{sin.\ APB} = \frac{sin.\ PB}{sin.\ AB},$$

$$\therefore Sin.\ PAB = \frac{sin.\ PB}{sin.\ AB} \times sin.\ APB,$$

$$= \frac{sin.\ PB}{sin.\ D} \times sin.\ APB.$$

Multiplying both sides by *sine* AP,

$$\therefore Sin.\ PAB \times sin.\ AP = \frac{sin.\ AP \times sin.\ PB \times sin.\ APB}{sin.\ D}.$$

But

$$\frac{Sin.\ PAB}{1} = \frac{sin.\ PC}{sin.\ PA},$$

$$\therefore Sin.\ PC = sin.\ PAB \times sin.\ PA.$$

Substituting

$$\therefore Sin.\ PC = \frac{sin.\ AP \times sin.\ PB \times sin.\ APB}{sin.\ D.},$$

$$\therefore Cos.\ lat.\ vertex = \frac{cos.\ lat.\ A \times cos.\ lat.\ B\ sin.\ diff.\ lon.}{sin.\ D.}$$

Longitude of vertex—

Having found a second formula for the latitude of the vertex, we employ it thus to find the difference of longitude of vertex.

From Napier's circular parts, using the previous spherical triangle, we get

Cos. M = *product* of *tangents* of adjacent parts,

$\therefore$ *Cos.* APC = *tan.* PC × *tan.* (90 − AP);

$\therefore$ *Cos. diff. lon.* = *cotan. lat. vertex* × *tan. lat.* A.

RULES FOR FINDING THE LATITUDE AND LONGITUDE OF THE VERTEX.

Latitude—

$$Cos.\ lat.\ vertex = \frac{cos.\ lat.\ A \times cos.\ lat.\ B \times sin.\ diff.\ lon.}{sin.\ distance.}$$

Add together the logarithms of the cosines of the two latitudes and log sine difference of longitude; from this sum subtract log. sin. distance, the result is the cosine of the latitude of the vertex.

Longitude—

Cos. diff. of lon. = *cotan. lat. vertex* × *tan. lat.* A.

Add together the logarithm of the tangent of the latitude of A *and that of the co-tangent of the latitude of the vertex, the sum is the cosine of the difference of longitude.*

This difference of longitude is to be applied from A towards B.

These formulæ will now be applied to the same problem as before, to show that the results obtained by either method are, as they ought to be, the same.

366. Find the latitude and longitude of the vertex of the great circle lying between Cape Clear and New York.

Lat. A is 40° 43′ N.	Lon. A is 74° 7′ W.
Lat. B is 51 26 N.	Lon. B is 9 29 W.
	64 38 diff. of longitude.

Distance already found on page 201, = 44° 33′ 54″.

Latitude of vertex.

$$\text{Cos. } lat. \text{ vertex} = \frac{\text{cos. } lat. \text{ A cos. } lat. \text{ B sin. diff. lon.}}{\text{sin. D.}}$$

Lat. A	40° 43′ 0″	cos.	9·879637
Lat. B	51 26 0	cos.	9·794784
Diff. lon.	64 38 0	sin.	9·955969
			9·630390
Distance	44 33 54	sin.	9·846162
Lat. ver.	52 31 20	cos.	9·784228

∴ Latitude of vertex is = 52° 31′ 20″ N.

Longitude of vertex.

Cos. diff. lon. = *cotan. lat. vertex* × *tan. lat.* A.

Lat. A	40° 43′	tan.	9·934823
Lat. ver.	52° 31′ 20″	cotan.	9·884632
Diff. lon.	48 42 37	cos.	9·819455

Lon. of New York	74° 7′ 0″ W.
Diff. lon.	48 42 37 E.
Lon. of vertex	25 24 23 W.

In the second example to illustrate our rules and formulæ. Find the latitude and longitude of the vertex of the great circle that connects the two places A and B.

Lat. A..... 12° 45′ S.	Lon. A is 154° 5′ W.
Lat. B..... 23 10 N.	Lon. B is 140 30 W.
	13 35 = diff. of lon.

∠ A as already found = 20° 23′ 29″
∠ B ,, ,, = 158 18 39
Distance = 38° 18′ 0″

First method—

To find latitude of vertex.
Cos. lat. vertex = cos. lat. A × *sin.* ∠ A.

Lat. A	12°	45′	0″	cos.	9·989157
A ...	20	23	29	sin.	9·542117
Lat. ver.	70	8	0	cos.	9·531274

∴ Latitude of vertex is 70° 8′ 0″ N.

To find longitude of vertex.
Cot. diff. lon. = *sin. lat.* A × *tan.* ∠ A

Lat. A	12°	45′	0″	sin.	9·343797
A	20	23	29	tan.	9·570222
Diff. lon.	85	18	36	cot.	8·914019

Lon. of A ..... 154° 5′ 0″ W.
Diff. lon. ..... 85 18 36 W.
Lon. of vertex 68 46 24 W.

Ans. Lat. of vertex 70° 8′ N.; lon. 68° 46′ 24″ W.

It should be observed that, although A is south of the equator and B north, no difference has been made in the formulæ. The fact is, if in the formulæ (90 + lat.) be substituted for *a* or *b*, both formulæ remain the same after reduction.

Second method—

Latitude of vertex.

$$\text{Cos. lat. vertex} = \frac{\text{cos. lat. A cos. lat. B sin. diff. lon.}}{\text{sin. D.}}$$

Lat. A...	12°	45′	0″	cos.	9·989157
Lat. B...	23	10	0	cos.	9·963488
Diff. lon.	13	35	0	sin.	9·370808
					9·323453
Dist.....	38	20	14	sin.	9·792594
Lat. ver.	70	9	9	cos.	9·530859

∴ Latitude of vertex is 70° 9′ 9″ N.

Longitude of vertex.
Cos. diff. lon. = *cot. lat. vertex* × *tan. lat.* A.

Lat. A...	12°	45′	0″	tan.	9·354640
Lat. ver.				cotan.	9·557576
Diff. lon.	85	18	50	cos.	8·912216

Lon. of A ..... 154° 5′ 0″ W.
Diff. of lon. ..... 85 18 50 W.
Lon. of vertex..... 68 46 10 W.

Ans. Lat. of vertex 70° 9′ 9″ N.; lon. 68° 46′ 10″ W.

These answers will be found to coincide very closely.

93. To Find a Succession of Points on the Great Circle.—It has been already intimated that practically it is found impossible to keep a vessel on the great circle track, all that can be done is to confine her as near the great circle as tides, winds, currents, etc., will allow. To accomplish this, it is evident that if a series of points can be laid down on the chart, and the master sail his ship from point to point by the ordinary Mercator's Sailing, he will shorten his route

very considerably. It is exceedingly easy to find a succession of points beginning at the vertex, because we have a right-angled triangle; from the vertex draw a succession of meridians to cut the great circle path, each part of a meridian within the spherical triangle becomes the hypotenuse of a right-angled triangle. It is a custom to select these points at about 5° of longitude from each other, beginning at the vertex. If the vertex be in 40° W. longitude, take the points as

55° 50° 45° 40° 35° 30° 25° etc.

when the vertex falls between the two places A and B; but if the vertex does not fall between the two places, the points must be chosen at equal distances from A towards B, still using the right-angled spherical triangle with the co-latitude of the vertex, and the angle at the poles formed by the meridian through the vertex, and the one under consideration.

Fig. 33.

C being the vertex, let E, F, G, etc., be the points whose latitude it is wished to find.

Then in triangles PCE, PCF, PCG, etc.,

We know angles CPE, CPF, CPG, etc., respectively.

We know angles CE, CF, CG, etc., respectively.

I. *Method.*—In triangle PCE we have from Napier's circular parts —

$$\text{Sin. } (90 - \text{CPE}) = \text{tan. PC tan. } (90 - \text{PE}).$$

$$\therefore \text{Cos. CPE} = \text{tan. PC cot. PE.} \qquad (a.)$$

$$\therefore \text{Tan. PE} = \frac{\text{tan. PC}}{\text{cos. CPE}} = \text{tan. PC} \times \text{sec. CPE.}$$

$$\therefore \text{Cot. lat. of E.} = \text{cot. lat. vertex} \times \text{sec. CPE.} \qquad \text{A.}$$

Where CPE is the difference of longitude between point E. and the vertex, generally 5°, 10°, 15°, etc.

II. *Method.*—From line *a* above we have—

$$\textit{Cos.}\ \text{CPE} = \textit{tan.}\ \text{PC}\ \textit{cot.}\ \text{PE.}$$

$$\therefore \textit{Cot.}\ \text{PE} = \frac{\textit{cos.}\ \text{CPE}}{\textit{tan.}\ \text{PC}}$$

$$= \textit{Cos.}\ \text{CPE} \times \textit{cotan.}\ \text{PC.}$$

$$\textit{Tan. lat. of}\ \text{E.} = \textit{cos. diff. of lon. from vertex} \times \textit{tan. lat. of vertex.} \qquad \text{B.}$$

RULES FOR FINDING A SUCCESSION OF POINTS ON THE GREAT CIRCLE.

A. *First Method*—

Cotan. lat. of E. = *cotan. lat. of vertex* × *sec.* CPE.

To log *cotan.* latitude of vertex add log *secant* difference of longitude between the point and the vertex; the sum gives the *co-tangent* of the latitude of the point sought, and of which the longitude has been assumed.

B. *Second Method*—

Tan. lat. of E. = *cos. diff. lon. from vertex* × *tan. lat. of vertex.*

To log *tan.* latitude of the vertex add log *cosine* difference of longitude between the point and the vertex; the sum gives the *tangent* of the latitude of the point sought.

367. Find a succession of points at every 5° of longitude on the great circle route between Cape Clear and New York.

(1.) Lat. of New York....	40° 43′ 0″ N.	Longitude	74° 7′ 0″ W.
(2.) Lat. of vertex of G.C.	52 31 20 N.	,,	25 24 23 W.
(3.) Lat. of Cape Clear...	51 26 0 N.	,,	9 29 0 W.

By Second Method—

Tan. lat. E. = *cos. diff. of lon. from vertex* × *tan. lat. of vertex.*

Difference of lon. 5°	cos. = 9·998344
Lat. of vertex 52° 31′ 20″	tan. = 10·115386
Lat. E. 52° 25′ 4″	tan. = 10·113730
Difference of lon. 10°	cos. = 9·993351
Lat. of vertex 52° 31′ 20″	tan. = 10·115386
Lat. F 52° 5′ 56″	tan. = 10·108737
Difference of lon. 15°	cos. = 9·984603
Lat. of vertex 52° 31′ 20	tan. = 10·115386
Lat. G 51° 32′ 17″	tan. = 10·099989

By First Method—

Cotan. lat. of E. = *cotan. lat. vertex* × *sec.* CPE.

Difference of lon. 20°	sec. = 10·029848
Lat. of vertex 52° 31′ 20″	cot. = 9·884632
Lat. H 50° 36′ 19″	cotan. = 9·914480
Difference of lon. 25°	sec. = 10·042724
Lat. of vertex 52° 31′ 20″	cot. = 9·884632
Lat I 49° 46′ 11″	cotan. = 9·927356

Difference of lon. 30°	sec.	=10·062469
Lat. of vertex 52° 31′ 20″.....	cot.	= 9·884632
Lat. J 48° 28′ 51″	cotan.	= 9·947101
Difference of lon. 35°	sec.	=10·086635
Lat. of vertex 52° 31′ 20″.....	cot.	= 9·884632
Lat. K 46° 53′ 38″	cotan.	= 9·971267
Difference of lon. 40°	sec.	=10·115746
Lat. of vertex 52° 31′ 20″.....	cot.	= 9·884632
Lat. L 44° 58′ 30″.....	cotan.	=10·000378
Difference of lon. 45°	sec.	=10·150515
Lat. vertex 52° 31′ 20″.....	cot.	= 9·884632
Lat. M 42° 41′ 2″.	cot.	=10·035147

Fig. 34.

	Lon. of Each Point.	Diff. of Lon. between the Point and the Vertex.		Lat. of Each Point.		
New York.....	74° 7′ 0″	48°	42′ 37″	Lat. of New York	40° 43′ 0″	A.
	70 24 23	45	0 0		42 41 2	M.
	65 20 23	40	0 0		44 58 30	L.
	60 24 23	35	0 0		46 53 38	K.
	55 24 23	30	0 0		48 28 51	J.
	50 20 23	25	0 0		49 46 11	I.
	45 24 23	20	0 0		50 36 19	H.
	40 24 23	15	0 0		51 32 17	G.
	35 24 23	10	0 0		52 5 56	F.
	30 24 23	5	0 0		52 25 4	E.
Lon. of vertex	25 24 23	0	0 0	Lat. of vertex.....	52 31 20	C.
	20 24 23	5	0 0		52 25 4	E.
	15 55 23	9	29 0	Lat. of Cape Clear	51 26 0	B.

All the workings are left to show the student how column three is found. They are very simple; and it will be perceived that to find a succession of points on a great circle is not so tedious as would appear at first sight. The master of

the vessel would now have to prick the points on his chart, and find the courses and distances either by calculation or in the usual mechanical way, with a parallel ruler and a pair of compasses or divider.

The courses, distances, etc., may be calculated and tabulated as in the next two examples, which are adapted from two very old works on *Navigation*, Robertson's and Wilson's, showing that our new methods are sometimes old ones resuscitated.

94. Wilson's Example.—Given the latitude and longitude of a place sailed from, and also of a place bound to, to find the several latitudes of those points in a great circle passing through the places which alter in longitude from either of the given places by an assigned quantity, suppose 5°, together with the courses and distances between those points.

(1.) Find the distance of the places and their angles of position (*i.e.*, find D and first and last course).

(2.) Find the perpendicular from the pole to the great circle passing through the given places (*i.e.*, find the latitude of the vertex); also find the several angles at the poles made by the given alterations of longitude between the perpendicular and the successive meridians come to by applying 5°, 10°, 15°, etc., to the longitude of the vertex.

(3.) With this perpendicular, and these polar angles severally, find as many corresponding latitudes by saying—

As tan. perpendicular : rad. :: cos. 1st polar angle : tan. 1st lat.
:: cos. 2nd polar angle : tan. 2nd lat.
etc. etc.,

i.e. (*tan. lat.* of E = *cos. diff. of lon.* × *tan. lat. of vertex*).

(4.) The latitudes of the several points in the great circle being found, and their differences of longitude known, find the several courses and distances between them. These will be the courses and distances the ship must run in sailing between the given places, to keep nearly on the arc of a great circle.

368. A ship bound from the Lizard in lat. 49° 57′ N., lon. 5° 14′ W., to the Island of Bermuda in latitude 32° 25′ N., lon. 66° 38′ W., proposes to sail on a great circle, and to alter her course at every 5° of longitude; required the latitudes of the places where the ship is to alter her course, and also the course and distance between each.

The figure being constructed, and the computation made for distance, AB is found to be 47° 54′,* the angle of position PAB at Bermudas to be 49° 35′, and the angle of position PAB at the Lizard to be 87° 15′,

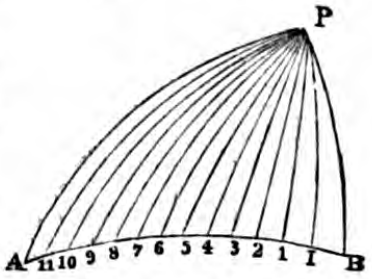


Fig. 35.

Draw PI at right angles to AB, and through P describe the arcs of co-latitude P_{11}, P_{10}, P_9, etc., making the angles of difference of longitude AP_{11}, AP_{10}, AP_9, etc., equal to 5°, 10°, 15°.

To find perpendicular PI.			To find polar angle API.		
As radius.....		10·00000	As cot. PAB...	49° 35′	9·93022
To sin. PA.....	57° 35′	9·92643	To radius.....		10·00000
So is sine PAB	49 35	9·88158	So is cos. PA..	57 35	9·72922
To sine PI.....	40 0	9·80801	To cot. API ...	57 49	9·79900

Now the polar angle API, or the difference of longitude between the perpendicular and the meridian of Bermudas, viz., 57° 49′ being taken from 61° 24′, the whole difference of longitude leaves 3° 35′ for the difference of longitude between the Lizard and the perpendicular.

Also 5°, the proposed alteration of longitude, being subtracted as often as it can be from 57° 49′, leaves the several polar angles, IP_{11}, = 52° 49′, IP_{10} = 47° 49′, IP_9 = 42° 49′, etc., as in the first column of the following table, and with these angles and the perpendicular PI, are found the several latitudes where the ship alters her course, by saying—

Polar angle† IP_1, 2° 49′,	IP_2 7° 49′	IP_3 12° 49′	IP_4 17° 49′	etc.
As tan. IP 40° 0′ 9·92381	9·92381	9·92381	9·92381	etc.
To radius,..... 10,00000	10·00000	10·00000	10·00000	
So is cosin polar angle, ... 9·99947	9.99595	9·98904	9·97865	
To tan. lat..... 10·07566	10·07214	10·06523	10·05484	etc.
at 1, 49° 58′	at 2, 49° 45′	at 3, 49° 18′	at 4, 48° 37′	

* These answers hardly agree with those in Ex. 381, page 224, because our tables are more correct.

† It will be observed that the formula is the same as on p. 212.

TABLE OF POLAR ANGLES.

Polar Angles.	Successive Longitudes.	Successive Latitudes.	Diff. Lon.	Diff. Lat.	Merid. Parts.	Merid. Diff. of Lat.	Course.	Distance.
	5° 14′	49° 57′			3469·8			
IPB ..... 3° 25′	8 49	50 0	215	3	3474·5	4·7	88° 45′	137·5
IP_1 ..... 2 49	11 38	49 58	169	2	3471·4	3·1	88 57	107·6
IP_2 ..... 7 49	16 38	49 45	300	13	3451·2	20·2	86 9	193·6
IP_3 ..... 12 49	21 38	49 18	300	27	3409·6	41·6	82 6	196·4
IP_4 ..... 17 49	26 38	48 37	300	41	3347·2	62·4	78 15	201·4
IP_5 ..... 22 49	31 38	47 42	300	55	3264·7	82·5	74 37	207·3
IP_6 ..... 27 49	36 38	46 31	300	71	3160·4	104·3	70 50	216·3
IP_7 ..... 32 49	41 38	45 3	300	88	3034·2	126·2	67 10	226·8
IP_8 ..... 37 49	46 38	43 17	300	106	2886·4	147·8	63 46	239·8
IP_9 ..... 42 49	57 38	41 10	300	127	2714·9	171·5	60 15	255·9
IP_{10} 47 49	56 38	38 41	300	149	2520·5	194·4	57 3	273·9
IP_{11} 52 49	61 38	35 46	300	175	2300·7	219·8	53 46	296·1
	66 38	32 25	300	201	2058·0	242·7	51 2	319·6
								2872·2

"The sum of the several distances is 2872·2 miles, but the distance on a great circle was found to be 47° 54′ or 2874 miles, which, though the shortest distance, exceeds the distance just now found by 1·8 miles, wherefore some small fractions must have been lost in the work."

By proceeding thus with all the polar angles, the latitudes of the several places where the ship alters her course, will be found to be as in the third column of the preceding table; and the longitudes of these places being also known, the several courses and distances between them are found by Mercator's Sailing. The work at length is here omitted, and only the results tabulated on the opposite page.

95. Robertson's Example.—A ship being bound from a place in latitude 16° S., longitude 6° 15′ W. to a place in latitude 32° 25′ N., longitude 66° 38′ W., it is proposed she shall sail near the arc of a great circle passing through those places, and alter her course at every 5° of longitude; required the latitude where those courses are to be altered, and also the course and distance between each place of alteration.

Latitude of place sailed from 16° 0′ S. Lon. 6° 15′ W.
Latitude of place bound to... 32 25 N. Lon. 66 38 W.
60 23 = diff. lon

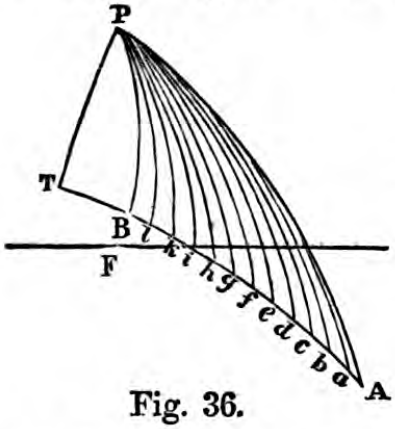


Fig. 36.

The figure in the margin will represent the spherical triangle. Having described the figure, and found the

Angle PBA = 120° 15′ and ∠PAB = 49° 20′.

The distance AB = 75° 19′.

And PB = 57° 35′ (90 − 32° 25′).

To find the perpendicular PT.				To find the polar angle BPT.			
As radius	= 90°	0	= 10·00000	As radius	= 90	0	= 10·00000
To sin. PB....	= 57	35	= 9·92643	To cosin. BP	= 57	35	= 9·72922
So sin. PBT...	= 59	45	= 9·93643	So tan. PBT...	= 59	45	= 10·23419
To sin. PT....	= 46	49	= 9·86286	To cot. BPT...	= 47	25	= 9·96341

Now the several polar angles, longitudes, latitudes, differences of longitude, differences of latitude, meridional parts, meridional differences of latitude, courses, and distances being computed on waste paper, their results are to be entered in a table as follows—

Polar Angles.	Successive Longitudes.	Successive Latitudes.	Diff. of Longitudes.	Diff. of Latitudes.	Meridian Parts.	Mer. Diff. Latitudes.	Courses.	Distances.
	6° 15′	16° 0′ S.			973			
∠TP*l* =102° 48′	11 15	11 45	300	255	709	264	48° 39′	386·0
∠TP*k*= 97 48	16 15	7 16	300	269	437	272	47 48	400·5
∠TP*i*= 92 48	21 15	2 38	300	278	158	279	47 5	408·3
∠TP*h*= 87 48	26 15	2 4 N.	300	282	124	282	46 46	411·7
∠TP*g*= 82 48	31 15	6 43	300	279	404	280	46 58	408·8
∠TP*f*= 77 48	36 15	11 13	300	270	677	273	47 42	401·2
∠TP*e*= 72 48	41 15	15 31	300	258	943	266	48 26	388·8
∠TP*d*= 67 48	46 15	19 32	300	241	1195	252	49 51	374·7
∠TP*c*= 62 48	51 15	23 13	300	221	1433	238	51 34	355·5
∠TP*b*= 57 48	56 15	26 34	300	201	1654	221	53 37	338·9
∠TP*a*= 52 48	61 15	29 34	300	180	1858	204	55 47	320·1
	66 38	32 25	323	171	2058	200	58 14	324·8
								4519·3

"The track of a ship, when directed nearly in the arc of a great circle as in the preceding article, may be delineated on the Mercator's chart by marking on it, by the help of the latitudes and the longitudes, the successive places where the ship is to alter her course, then those places or points being joined by right lines, will show the path along which the ship is to sail under the proposed circumstances."

369. A ship being bound from a place in lat. 37° N., lon. 22° 56′ W. to a place in the same latitude in longitude 76° 23′ W., it is proposed she shall sail as near the arc of a great circle as she can, by altering her course at every 5° of longitude; required the latitude at each time of altering the course, and also the courses and distances between these several latitudes?

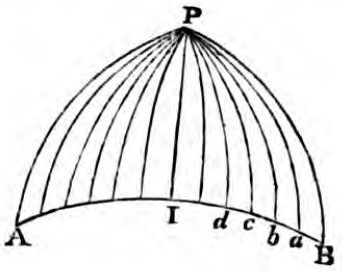


Fig. 37.

Distance = 2526 = 42° 6′.

Courses A = 73° 9′ = ∠B.

The triangle APB is isosceles AI = BI.

∴ If the courses and distances are found on one side of I, they will do for the other.

Polar Angles.	Successive Longitudes.	Successive Latitudes.	Diff. Lon.	Lat. Diff.	M.P.	Courses.	Distances.
∠ IPB 26° 43′½	22° 56′	37° 0′			2392·6		
∠ IP*a* 21 43½	27 56	38 5	5° 0′	65	2474·6	74° 43′	246·6
∠ IP*b* 16 43½	32 56	38 56	5 0	51	2539·8	77 44	240·0
∠ IP*c* 11 43½	37 56	39 33	5 0	37	2587·6	80 57	235·0
∠ IP*d* 6 43½	42 56	39 57	5 0	24	2618·8	84 4	252·2
	49 39½	40 9	6 43½	12	2634·5	87 46	307·9
							1261·9

Hence it appears the ship must first sail from lat. 37° N. to lat. 38° 5′ N., thence to lat. 38° 56′ N., thence to lat. 39° 33′ N., hence to lat. 39° 57′ N., thence to lat. 40° 9′ N., which is the highest latitude the ship must sail into. From thence she must proceed through the latitudes 39° 57′, 39° 33′, 38° 56′, 38° 5′, and so to 37° 0′ the parallel she first set out from, and in which she is to find the place to which she is bound.

The courses the ship must steer are first, N. 74° 43′ W.; second, N. 77° 44′ W.; third, N. 80° 57′ W.; fourth, N. 84° 4′ W.; fifth, N. 87° 46′ W.; sixth, S. 87° 46′ W.; seventh, S. 84° 4′ W.; eighth, S. 80° 57′ W.; ninth, S. 77° 44′ W.; and lastly, S. 74° 43′ W., which last will bring the ship to the given place if she sail the distances placed respectively opposite the courses just found.

This example shows us that the courses and distance correspond on each side the vertex; a little consideration will bring us to the conclusion that they do so in all cases, for each point on one side of the vertex, so far as it goes, corresponds with a point on the other side, so that, as a rule, if we calculate the courses and distances for one side of the vertex, we may write them down for the remaining side.

The next two examples are fully worked out to show the learner how to lay out his work: they are arranged after a

certain method to give greater facility in taking out the logarithms.

370. Find the shortest distance between Raz Point in France and Santiago in the Island of Jamaica.

Santiago, lat. 16° 38′ N. A	$\therefore a$	= 73° 22′	Lon. 70° 55′ W.
Raz Point, lat. 48 2′ N. B	b	= 41 58	Lon. 4 35 W.
2)31 24 diff. lat.	$\therefore a+b$	= 115 20	2)66 20 diff. lon.
15 42 = ½ diff. lat.	$\therefore a-b$	= 31 24	33 10 = ½ ,,
	$\therefore \frac{a+b}{2}$	= 57 40	
	$\therefore \frac{a-b}{2}$	= 15 42	

For the computation of distance.

(α) $Cos.^2 H = Cos.\ lat.\ A.\ cos.\ lat.\ B.\ \sin.^2 \frac{1}{2}\ diff.\ lon.$

½ Diff. lon.	33° 10′ 0″	$\sin.^2$	= 9·738048
			2
			19·476096
Lat. A	16 38 0	cos.	= 9·981436
Lat. B	48 2 0	cos.	= 9·825230
			2)19·282762
H	64 1 37	cos.	= 9·641381

(β) $\text{Cos.}^2 \frac{D}{2} = \text{sin.} \left(H + \frac{\text{diff. lat.}}{2}\right) \left(\text{sin. } H - \frac{\text{diff. lat.}}{2}\right)$

$H + \frac{\text{diff. lat.}}{2} = 64° 1′ 37″ + 15 42 0 = 79 43 37$ sin. = 9·992981

$H - \frac{\text{diff. lat.}}{2} = 64 1 37 - 15 42 0 = 48 19 37$ sin. = 9·873292

2)19·866273

$\frac{D}{2} = 30° 59′$ = cos. = 9·933136

2

$D = 61\ 58$

60

3718 miles.

For computation of first and last courses.

$\text{Tan.}\frac{B+A}{2}=\text{sec.}\frac{a+b}{2}\cos.\frac{a-b}{2}\cot.\frac{P}{2}$ $\quad$ $\text{Tan.}\frac{B-A}{2}=\text{cosec.}\frac{a+b}{2}\sin.\frac{a-b}{2}\cot.\frac{P}{2}$

While A and B are the latitudes, a and b are the *co-latitudes.*

$\frac{a+b}{2}$	57°	40′	0″	sec.	10·271773	57°	40′	0″	cosec.	10·073169
$\frac{a-b}{2}$	15	42′	0	cos.	9·983487	15	42	0	sin.	9·432329
$\frac{\text{Diff. lon.}}{2}$	33	10	0	cot.	10·184721	33	10	0	cot.	10·184721
$\frac{B+A}{2}=$	70	2	40	tan.	10·439981	$\frac{B-A}{2}=$ 26	6	20	tan.	9·690219

Sum $\left\{\begin{matrix}70°\ 2'\ 40'' \\ 26\ 6\ 20\end{matrix}\right.$ $\quad$ $\left.\begin{matrix}70°\ 2'\ 40'' \\ 26\ 6\ 20\end{matrix}\right\}$ difference.

B = 96 9 0 $\quad$ 43 56 20 = A.

1st and last course N. 96° 9′ W., or S. 83° 51′ W., and S. 43° 56′ 20″ W.

For computation of vertex.

Latitude.

$$=\frac{\textit{Cos. lat. ver.}}{\textit{Sin. D.}}=\frac{\textit{cos. lat. A cos. lat. B sin. diff. lon.}}{}$$

Longitude.

$$=\textit{tan. lat. A cotan. lat. vertex.}$$

Lat. A...	16°	38′	0″	cos.	9·981436	16°	38′	0″	tan. 9·475303
Lat. B...	48	2	0	cos.	9·825230				
Diff. lon.	66	20	0	sin.	9·961846				
					19·768512				
Distance	61	58	0	sin.	9·945800				
Lat. vert.	48	19	50	cos.	9.822712	48	19	50	cot. 9·949395
					diff. lon.	74	34	49	cos. 9·424698

Latitude vertex 48° 19′ 50″ N.

Lon. A ..... = 70° 55′ 0″ W.
Diff. lon. .... = 74 34 49 E.
Lon. of vertex 3 39 49 E.

371. The captain of a ship on a voyage from Canterbury in New Zealand to San Francisco, determines to sight Cape St. Lucas; find the first and last courses, the distance and the latitude and longitude of the vertex of the great circle on which he endeavoured to keep his vessel.

Canterbury, lat.....43° 0′ S. = B. ∴ b = 47° 0′ Lon. 173° 0′ E.
Cape St. Lucas, lat. 22 52 N. = A. ∴ a = 112 52 Lon. 109 54 W.

2)65 52 $\quad$ $a+b$ = 159 52 $\quad$ 282 54
Half sum of lat... = 32 56 $\quad$ $a-b$ = 65 52 $\quad$ 360
$\frac{a+b}{2}$ = 79 56 $\quad$ 2)77 6 diff. lon.
$\frac{a-b}{2}$ = 32 56 $\quad$ 38 33 = ½ ,,

For the computation of distance.

(α) $\text{Cos.}^2 \text{H} = \text{cos. lat. A. cos. lat. B. sin.}^2 \frac{\text{diff. lon.}}{2}$

½ Diff. lon.	38°	33′	0″	sin.²	9·794626
					2
					19·589252
Lat. A	22	52	0	cos.	9·964454
Lat. B	43	0	0	cos.	9·864127
					2)19·417833
H	59	13	50	cos.	9·708916

(β') $\text{Cos.}^2 \frac{\text{D}}{2} = \text{sin.} \left(\text{H} + \frac{\text{sum of lats.}}{2}\right) \left(\text{sin. H} - \frac{\text{sum of lats.}}{2}\right)$

$\text{H} + \frac{\text{sum of lats.}}{2}$ =	59°	13′	50″		
	32	56	0		
	92	9	50	sin.	9·999690
$\text{H} - \frac{\text{sum of lats.}}{2}$ =	59	13	50		
	32	56	0		
	26	17	50	sin.	9·646431
					2)19·646121
$\frac{\text{D}}{2}$ =	48°	17′	23″	= cos.	9·823060
			2		
D =	96	34	46		
			60		
			5794·8		

For computation of first and last courses.

$\text{Tan.} \frac{\text{B+A}}{2} = \text{sec.} \frac{a+b}{2} \text{ cos.} \frac{a-b}{2} \text{ cot.} \frac{\text{P}}{2}$ $\quad$ $\text{Tan.} \frac{\text{B-A}}{2} = \text{cosec.} \frac{a+b}{2} \text{ sin.} \frac{a-b}{2} \text{ cot.} \frac{\text{P}}{2}$

Here B and A are the latitudes, a and b the *co-latitudes.*

$\frac{a+b}{2}$	79°	56′	0″	sec.	10·757474	79°	56′	0″	cosec.	10·006738
$\frac{a-b}{2}$	32	56	0	cos.	9·923919	32	56	0	sin.	9·735330
$\frac{\text{P}}{2}$	38	33	0	cot.	10·098617	38	33	0	cot.	10·098617
$\frac{\text{B+A}}{2}$ =	80	34	38	tan.	10·780010	$\frac{\text{B-A}}{2}$ = 34	43	8	tan.	9·840685

Sum $\left\{\begin{matrix} 80° & 34′ & 38″ \\ 34 & 43 & 8 \end{matrix}\right.$ $\quad$ $\left.\begin{matrix} 80° & 34′ & 38″ \\ 34 & 43 & 8 \end{matrix}\right\}$ difference.

$\begin{matrix} 115 & 17 & 46 \end{matrix}$ $\quad$ $\begin{matrix} 45 & 51 & 30 \end{matrix}$

1st Course N. 115° 17′ 46″ E. $\quad$ Last Course. N. 45° 51′ 30″ E.
Or .....S. 64 42 14 E.

For computation of vertex.

Latitude.

$$= \frac{\textit{Cos. lat. vertex} = \textit{cos. lat. A cos. lat. B sin. diff. lon.}}{\textit{sin. distance.}}$$

Longitude.

$$\textit{Cos. diff. lon. of vertex} = \textit{tan. lat. A cot. lat. vertex.}$$

Lat. A.....	22°	52′	0″	cos.	9·964454	22° 52′ 0″	tan.	9·625036
Lat. B.....	43	0	0	cos.	9·864127			
Diff. lon. (P)	77	6	0	sin.	9·988898			
					19·817479			
Dist.....	96	34	46	sin.	9·997130			
Lat. vertex..	48	36	23	cos.	9·820349	48 36 23	cot.	9·945184
						Diff. lon. 68 10 41	cos.	9·579220

∴ Latitude of vertex is 48° 36′ 23″ N.

Longitude of A 109° 54′ 0″ W.
Diff. lon..... 68° 10 41 W.
Lon. of vertex 178 4 41 W.

In one case for sin..... 92° 9′ 50″ take out *cosine* 2° 9′ 50″
In other case for sin.... 96 34 46 ,, ,, 6 34 46

372. Required the shortest distance from Glasgow in latitude 55° 51′ N., longitude 4° 16′ W., to St. Johns (Newfoundland), latitude 47° 33′ N., longitude 52° 43′ W. Give the angles of position, and the latitude and longitude of the vertex of the great circle.

Ans. Distance, 1828 miles; courses, N. 85° 3′ W., and S. 55° 57′ W. Lat. of ver., 56° 0′ N.; lon. of ver., 10° 14′ W.

373. Required the shortest distance from Loop Head in latitude 52° 34′ N., longitude 9° 57′ W., to Cape Cod, latitude 42° 2′ N., longitude 70° 4′ W. Give the first and last courses, and the latitude and longitude of the vertex.

Ans. Distance, 2450 miles; courses, N. 80° 2′ W., and S. 53° 42′ W. Lat. of ver., 53° 14′ N.; lon. of ver., 22° 25′ W.

374. Find the distance on the great circle lying between Charleston and the Land's End. Give the first and last courses, and the latitude and longitude of the vertex.

Charleston, lat. 32° 46′ N.; longitude, 79° 51′ W.
Lizard, ... lat. 49° 57′ N.; longitude, 5° 12′ W.

Ans. Distance, 3366 miles; courses, S. 48° 22′ W., and N. 77° 38′ W. Lat. of ver., 50° 56′ N.; lon. of ver., 21° 21′ W.

375. Find the shortest distance between the mouth of the Coanza, latitude 9° 5′ S., longitude 12° 45′ E., and Staten Island, latitude 54° 42′ S., longitude 63° 40′ W.; find the first and last courses, and the position of the vertex.

Ans. Dist. 4486 m.; courses, S. 35° 36′ W., and S. 95° 50′ W., or N. 84° 10′ W. Lat. of ver., 54° 55′ S.; lon. of ver., 70° 49′ W.

376. Give all the particulars of the great circle track lying between C. St. Antonio, near the mouth of the Rio-de-la-Plata, and Cape Lopez.

Lat. of C. St. Antonio, 36° 30′ S.; longitude, 56° 30′ W.
Lat. of Cape Lopez,.... 0° 36′ S.; longitude, 8° 43′ E.

Ans. Distance, 4196m.; courses, S. 104°52′W., or N. 75°8′W., and S. 51°W.
Lat. of ver., 39° 1′ S.; lon. of ver., 80° 32′ E.

377. Find the latitude and longitude of the vertex of the great circle lying between the Cape of Good Hope and Bahia; required also the distance between the two places, and the first and last courses.

Lat. of the Cape, 34° 22′ S.; longitude, 18° 29′ E.
Lat. of Bahia,.... 12° 57′ S.; longitude, 38° 30′ W.

Ans. Distance, 3342m; courses, N. 98°W., or S. 57°8′W., and S. 51°W.
Lat. of ver., 35° 10′ S.; lon. of ver., 32° 27′ E.

378. What is the length of the arc of the great circle lying between A in latitude 16° S., longitude 6° 15′ W., and B in latitude 32° 25′ N., longitude 66° 38′ W.; find the first course and distance, with the latitude and longitude of the vertex.

Ans. Distance, 4519 miles;
Courses, N. 120° 15′ W., or S. 59° 45′ W., and N. 49° 20′ W.
Lat. of ver., 43° 11′ N.; lon. of ver., 114° 3′ W.
or Lat. of farther vertex, 46° 49′ S.; longitude, 65° 57′ E.

379. What is the least distance between St. Mary's in latitude 37° 0′ N., longitude 25° 0′ W., and Cape Henry in latitude 37° 0′ N., longitude 76° 23′ W.? and find also the first and last course and the position of the vertex.

Ans. Distance 2432 miles; courses N. 73° 51′ W., and S. 73° 51′ W.
Lat. of ver., 39° 54′ N.; lon., 50° 41′½ W.

380. What is the nearest distance from the Isle of St. Thomas in lat. 0° 0′ N., and longitude 1° 0′ E., to reach Port St. Julian in latitude 48° 51′ S., and longitude 65° 10′ W.; find the courses and the latitude and longitude of the vertex.

Ans. Distance 4475 miles; courses S. 71° 36′ W., S. 38° 37′ W.
Lat. of ver., 51° 22′ S.; lon., 17° 23′ W.

381. What is the shortest distance between the Lizard and the Island of Bermudas? also find the angles of position at each place, and the latitude and longitude of the vertex.

Lizard in lat. ..... 49° 57′ N.; lon. 5° 21′ W.
Bermudas in lat.... 32 35 N.; lon. 63 32 W.

Ans. Distance 2744 miles; courses S. 89° 29′ W., and S. 49° 47′ W.
Lat. of ver., 50° 4′ N.; lon., 174° 7′ E.

382. What is the nearest distance from the Island of St. Helena to reach the Island of Bermudas. Give also the angles of position at each place with the latitude and longitude of the vertex.

Island of Bermudas in lat. 32° 35′ N.; lon. 63° 32′ W.
Island of St. Helena in lat. 15 55 S.; lon. 5 49 W.

Ans. Dist. 4406 m.; courses N. 48°1′W., and S. 121°59′W., or N. 58°1′W.
Lat. of ver., 44° 23′ N.; lon. of ver., 101° 57′ W.

383. From a ship's position at Algoa Bay, latitude 34° 2′ S., longitude 25° 36′ E., a master determines to sail his ship on the great circle lying between that place and Cape Comorin, latitude 8° 6′ N., longitude 78° 6′ E.; find the distance and angles of position, with the latitude and longitude of the vertex.

Ans. Distance, 3908 miles.
Courses, S. 120° 2′ E., or N. 59° 58′ E., and N. 46° 26′ E.
Lat. of vert., 44° 10′ S.; lon. of vert., 3° 28′ W.

384. Find the distance and first and last courses on the great circle lying between San Francisco in latitude 37° 40′ N., longitude 122° 24′ W., and Poverty Bay in New Zealand, latitude 39° S., longitude 178° E. Give also the latitude and longitude of the vertex.

Ans. Distance, 5870 miles; courses, N. 42° 40′ E.; N. 42° 48′ E.
Lat. of vert., 57° 32′ N.; lon. of vert., 61° 59′ W.

385. A ship is in Table Bay, latitude 33° 54′ S., longitude 18° 20′ E., she sails to New York, latitude 40° 43′ N., longitude 74° 7′ W. Lay down the great circle track with the first and last courses; find also the latitude and longitude of the vertex; give any remarks you may have to offer as objections to following such a track.

Ans. Distance, 6778 miles.
Courses, N. 55° 22′ W., and N. 115° 44′ W., or S. 64° 16′ W.
Lat. of vert., 46° 56′ N.; lon. of vert., 23° 1′ W.

386. Lay down the great circle track between Cayenne in latitude 4° 55′ N., longitude 52° 21′ W., and the mouth of the Coanza, latitude 9° 5′ S., longitude 12° 45′E. Giving the distance, the latitude and longitude of the vertex, and the two angles of the spherical triangle.

Ans. Distance, 3982 miles; angles, N. 99° 29′ W.; N. 71° 51′ W.
Lat. of vert., 10° 40′ N.; lon. of vert., 15° 6′ W.

387. Find the least distance between the S. point of the island of Japan, in latitude 30° 36′ N., longitude 130° 10′ E., and Arica in latitude 18° 28′ S., longitude 70° 24′ W. At the same time give the latitude and longitude of the vertex and the angles of position.

Ans. Distance, 9466 miles; angles, N. 61° 42′ E. and S. 53° 2′ E.
Lat. of vert., 33° 22′ N.; lon. of vert., 129° 56′ W.

Note.—All these answers are obtained by using the formulæ on page 199 for distance, 204 for the first and last courses, and 207 for the latitude and longitude of the vertex; if other formulæ are employed, the answers will slightly differ.

96. Longitude of Vertex, *when* one angle is greater than 90°.—When one of the two courses is less than 90°, and the other greater than 90°, they are not of the same affection, as it is termed. To find the longitude of the vertex in this case requires a little attention. Take for instance example 378. If we apply the ordinary rule, lat. of vertex comes out

43° 11′ N., lon. 65° 57′. Now this 65° 57′ is the longitude of the vertex farther away from the ship, or 65° 57′ E. This subtracted from 180°, gives the longitude of the vertex nearer the ship, viz., 114° 3′ W., the latitude of the farther vertex is the complement of 43° 11′, viz., 46° 49′. The longitude of the vertex falls between the two places when both angles of position are both greater or both less than 90°. When one angle is greater than 90°, it is a good rule to apply the difference of longitude to the longitude of A, away from that of B; but in all cases the position must be well examined before the longitude of the vertex is determined.

97. The Rhumb Line and the Great Circle Track.—The rhumb line does not by any means coincide with the great circle track; while the former cuts every meridian it crosses at the same angle, the latter crosses every meridian at a different angle.

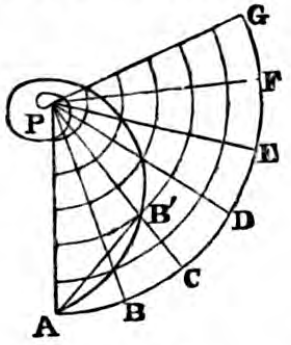


Fig. 38.

The straight lines drawn from P to A,B,C,D, etc., are rhumb lines. In fact, every straight line drawn from the place of the spectator to the boundary of the horizon is named a *rhumb line.* When a master wishes to sail his ship in any assigned direction, the ship's head is put upon that rhumb, and, by keeping to the same course, the ship cuts every meridian at the same constant angle. In so doing this distance forms a spiral continually approaching the pole N. or S.

Suppose a ship sails from A to B′ on the rhumb line, then it cuts all the meridians at the same angle, and if the course be continued as in the figure, we obtain a spiral winding round the pole approaching nearer and nearer to it. It is such a curve as this that we have previously defined as a *loxodromic line,* or *loxodrome.* Sometimes it is termed an equiangular spiral. If the great circle track were drawn on the same figure above, it would be seen that it does not cut every meridian at the same angle, and therefore cannot coincide with the loxodrome or with the rhumb line.

98. Composite Sailing.—For the invention of composite sailing, we are indebted to Mr. Towson of Liverpool, who has published a set of tables to facilitate Great Circle Sailing,

and shown that some of the advantages of the latter may be secured without keeping entirely to the great circle track. Whenever the great circle track reaches too high a latitude, Mr. Towson has shown that the best way to proceed is to first decide upon the highest parallel of latitude it is advisable the ship should sail into; then sail on a great circle till the latitude is reached; next run along the parallel as far as desirable, and last leave the parallel at a tangent, finishing the voyage by returning to the great circle track lying between the two places. We must not omit to mention, that great circle charts have been constructed upon which the distance or path between any two places is shown by a straight line, while the rhumb line is shown by a curve. On such a chart the "composite track" would be the parallel of latitude and two tangents to the parallel.

99. Limitations of Great Circle Sailing. — The difficulties in making the calculations for great circle sailing are sufficient to deter the majority of practical men from adopting it. Again, as before intimated, many impediments, as islands, land, too high a latitude, etc., lie in its way. Several modifications to further extend its use, and mechanical methods already referred to, have been introduced. Theory and practice in this case are often widely separated. The sailing master has to take advantage of winds and currents, and considers how he shall make the *quickest passage*, which is not always the *shortest*. The best way to find out where the quickest passage can be made, is to lay down the great circle on a Mercator's chart, which has the winds and currents marked on it; then with the straight line on the chart joining the two places, first compare the two paths, *i.e.*, the Mercator's and great circle tracks, taking note of what currents of wind or water will assist the vessel; whichever offers the quickest passage is the best route, if not the shortest. Again, if by modifying the great circle track, by keeping to a lower latitude, the ship can be brought into currents in favour of the vessel, that will be the best track. Although the greatest advantages of great circle sailing over the rhumb are obtained when sailing in high latitudes, yet, in consequence of the danger arising from ice and icebergs floating from the North Pole into the North Atlantic, and from the South Pole into

the South Pacific and South Atlantic, navigators are unable to secure these advantages.

From page 193, Vol. I., of *Naval Science*, we extract the following:—

"In the passage from Panama to Australia, the rhumb track would entangle us in the Low Archipelago, in Dangerous Archipelago, and carry us into the very focus of coral reefs, atolls, lagoon islands, and sunken rocks, while the great circle route would take us clear of these dangers. On the other hand, the great circle track from Cape Horn to Cape of Good Hope (were there no other objections), would run the ship on one of the Sandwich group, while the rhumb course would carry her clear of such dangers.

"During the first three months of the year, the great circle track from the Straits of Sunda to Cape of Good Hope, would give a greater immunity from the dangers of cyclones than the rhumb track; but, on the contrary, during the months of August and September, in sailing from the Bermudas to the Land's End, we should expect to meet with expiring hurricanes and boisterous weather more on the great circle track than on the rhumb."

CHAPTER XII.

SAILING TO WINDWARD OR PLYING TO WINDWARD.

Definitions—Illustrations and Examples.

WHEN the wind is unfavourable to the ship's course, or coming directly from or partly from the place to which the vessel is bound, she cannot sail exactly in the teeth of the wind, she has therefore to shape her course in a zig-zag direction, first sailing away from her direct path on one side, and then crossing it and sailing away on the other. Thus the wind first acts on one side of the ship, then on the other. When on board, and looking over or towards the head or stem of the vessel,

Starboard is the right side of the ship.

Port or Larboard is the left side of the ship.

Forwards or **Afore** is towards the stem of the vessel.

Aft or **Abaft** is towards the stern of the vessel.

The Beam means *athwart* or across the middle of the ship.

When a ship sails towards the same point that the wind is blowing, or when the wind is exactly behind the ship coming from the opposite quarter to which the vessel is sailing, the wind is said to be *right aft* or *right astern*, and her course is 16 points from the wind.

When a ship sails with the wind impinging perpendicularly to her side, or when it blows directly across her side, she is said to *have the wind on her beam*, then her course is 8 points from the wind.

When the wind blows from a point of the horizon that lies between the one to which she is going, and that at right angles to her sides, it is said to be *before the beam;* and when it blows from a point of the horizon lying between the same

perpendicular, and the opposite rhumb to that on which the ship is sailing, it is said to be *abaft the beam*, or "when the wind blows obliquely across the ship, it is said to be *abaft the beam*, or *before the beam*, according as her course is more or less than 8 points from the wind."

Plying to Windward.—When a ship endeavours to sail towards that point of the compass from which the wind blows, she is said to *sail on a wind* or to *ply to windward.*

Close Hauled.—When a vessel sails as near as she can to the point of the compass from which the wind is blowing, she is said to be close hauled. The majority of ships lie within 5 or 6 points of the wind, but yachts and fast sailing clippers will often lie nearer. For instance, if the wind is coming from the north, the majority of vessels can make headway towards the N. on a N.W.byW., W.N.W., or N.E.byE., E.N.E. course.

The *weather side* or *windward side* of a vessel is the side on which the wind is blowing, the other side is the *lee* or *leeside.* A few illustrations will perhaps show what is meant better than definitions. Take the following example to give point to what has been said.

388. The wind is due S., and a ship is bound to a port 25 miles away, lying exactly south of her position; show how she can reach the port in two tracks, if she can lie within 6 points of the wind. Let the wind be on the starboard side for the first tack.

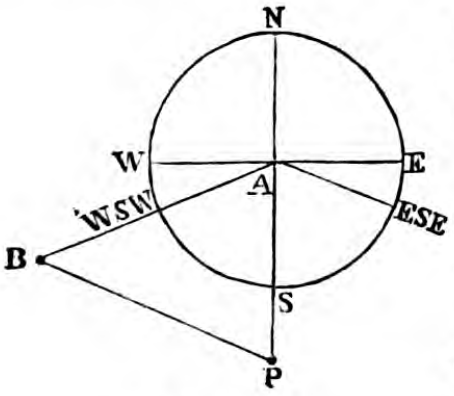


Fig. 39.

As the wind is S., and she can sail within 6 points of the wind, her courses must be E.S.E. and W.S.W.

Let P be the port to which the ship is bound.

Draw the compass marking the cardinal points.

Lay down the lines E.S.E. and W.S.W.

Let AB be on the W.S.W. rhumb, and draw from P the line PB parallel to the E.S.E. rhumb. Then the two lines AB and BP indicate the direction and distance the vessel must sail to reach the port in two tacks. It is evident that

BA = BP, because the angles at BAP and BPA are equal, each of them being 6 points ∴ ∠ABP = 4 points.

$$\frac{AB}{AP} = \frac{AB}{25} = \frac{\text{sin. 6 points.}}{\text{sin. 4 points.}}$$

$$\therefore AB = \frac{25 \text{ sin. 6 points.}}{\text{sin. 4 points.}}$$

Log 25	= 1·397940
Log sin. 6 points	= 9·965615
	11·363555
Log sin. 4 points	= 9·849485
Log 32.66 miles	= 1·514070

∴ She must first sail W.S.W. for 32·66 miles, and then sail E.S.E. for 32·66 miles.

389. A port lies 13 miles to the N.N.E. from a ship, the wind is also N.N.E.; show how a ship can reach the port in three tacks, if she can sail with 5 points of the wind. What will be the courses if the variation be 2 points E., deviation on starboard tack 3° E., and port tack 7° W.? Let the ship sail on the port tack for the first course.

In the annexed figure let it be supposed that A is the position of the ship, and that P is the port on the N.N.E. rhumb, to which the ship is bound, and from whence the wind is blowing directly against the vessel's course.

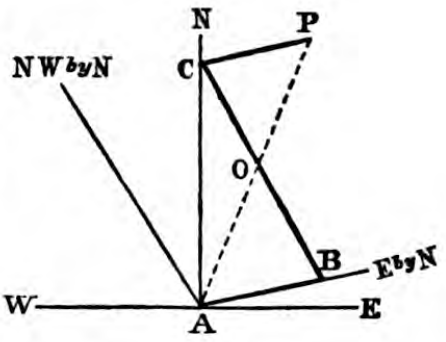


Fig. 40.

Draw the E. by N. rhumb, and another showing the position and direction of the N.W.byN rhumb. The vessel must first sail with the wind on the port side from A to B on a course E.byN.; arriving at B she must tack and sail on her second course N.W.byN. to C, where she must tack a second time and change her course again to the E.byN., upon which she will arrive at her destination P. It is evident from the preceding example that AB, BO, OC, and CP are all equal.

The angle OBA = OCP = 6 points.
The remaining angles BAO, BOA, COP, and CPO all = 5 points.
∴ AB = BO = CO = CP.

Again,

$$AO = OP = \tfrac{1}{2}\,AP = \tfrac{1}{2} \text{ of } 13 = 6{\cdot}5 \text{ miles.}$$

$$\frac{AB}{AO} = \frac{AB}{6{\cdot}5} = \frac{\text{sin. } AOB}{\text{sin. } ABO} = \frac{\text{sin. 5 points.}}{\text{sin. 6 points.}}$$

$$\therefore AB = \frac{6{\cdot}5 \text{ sin. 5 points.}}{\text{sin. 6 points.}}$$

$$= 5{\cdot}849 \text{ miles.}$$

$$\therefore AB = 5{\cdot}849 = CP \quad \therefore BC = 2 \times 5{\cdot}849 = 11{\cdot}698 \text{ miles.}$$

First and Third Course.

True course E.byN.	$= 78° 45'$ *r.* of N.
Variation E.	$22\ 30$ *l.*
	$56\ 15$ *r.* of N.
Deviation W.	$7\ 0$ *r.*
Compass course.....	$63\ 15$ *r.* of N.

or N. 63° 15′ E., 5·849 miles.

Second Course.

True course N.W.byN.	$= 33° 45'$ *l.* of N.
Variation E.	$22\ 30$ *l.*
	$56\ 15$ *l.*
Deviation E.	$3\ 0$ *l.*
Compass course.....	$59\ 15$ *l.* of N.

or N. 59° 15′ W., 11·698 miles.

390. A ship that can lie within 5 points of the wind is compelled to call for orders at a port bearing S.E.byS. 40 miles distant, the wind is S. all the time; what are the two courses and distances she must sail on, supposing, on account of a rock, that during the first course the ship must be on the starboard tack?

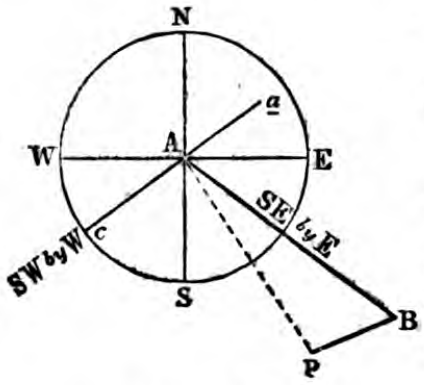


Fig. 41.

Draw rhumbs representing the N.S.E. and W.

Let A be the position of the ship.

Let P be the position of the port.

As the ship can lie within 5 points of the wind, which is S., the courses are S.E.byE., and S.W.byW.

Draw the lines representing these two latter rhumbs. Then AB will represent the first distance; the ship sailing from A to B will have the wind on the starboard, and the course

S.E.byE. Draw BP parallel to the S.W.byW., then BP represents the second distance with the course S.W.byW.

Angle ABP = alternate angle BAa = 6 points.
,, BPA = alternate angle PAc = 8 points = a right angle.
$\therefore$,, PAB = 2 points.

$$\text{Cos. 2 points} = \frac{AP}{AB} \quad \therefore \quad AB = AP \text{ secant 2 points.}$$

$$= 40 \text{ sec. } 22° 30'$$
$$= 43{\cdot}29 \text{ miles.}$$

$$\text{Tan. 2 points} = \frac{BP}{AP} \quad \therefore \quad BP = AP \text{ tan. 2 points.}$$

$$= 40 \text{ tan. } 22° 30'.$$
$$= 16{\cdot}56 \text{ miles.}$$

Suppose the variation had been $1\frac{3}{4}$ points W., and deviation on the first course 10° E., and second 12° W., the compass courses would be found thus—

First course.

True course SE.byE.	= 56° 15′ 0″	l. of S.
Variation W.	= 19 41 15	r.
	36 33 45	l. of S.
Deviation E.	= 10 0 0	l.
Compass course	46 33 45	l. of S.

or S. 46° 33′ 45″ E. for 43·29 miles.

Second course.

True course SW.byW.	= 56° 15′ 0″	r. of S.
Variation W.	= 19 41 15	r.
	75 56 15	r. of S.
Deviation W.	= 12 0 0	r.
Compass course	87 56 15	r. of S.

or S. 87° 56′ 15″ W. for 16·56 miles.

391. The wind is at N.E.byN.$\frac{1}{2}$-E., and a master has to reach a port 80 miles exactly to windward. If his ship can lie within 6 points of the wind, show on what two courses he must sail, and how far.

Ans. Starboard tack, N.N.W.$\frac{1}{2}$-W., 104·5 miles.
Larboard tack, E.S.E.$\frac{1}{2}$-E., 104·5 miles.

392. A ship was in sight of a port which lay N.byE.$\frac{1}{2}$-E., 18 miles away, when a gale began to blow from the N.E., she ran 48 miles on her larboard tack within 6 points of the wind, and then tacked; what was the course and distance to the port?

Ans. Course N. 47° 42′ W.
Distance 52·89 miles.

393. A master wishes to carry his ship into a port that bears

S.W.byS., in two boards, but finds he must double a cape bearing S.byE. The cape lies 25 miles N.E.by E. from the port. The vessel cannot lie nearer the wind than $6\frac{1}{2}$ points; required the course and distance in each tack, variation being 2 points W.

Ans. S.$\frac{1}{2}$-E.; distance 58·51 miles.
N.W.$\frac{1}{2}$-N.; distance 51·85 miles.

394. What time will be required for a ship to make the Eddystone, when it bears N.E. 15 miles away, in two boards, the wind being N.E.byN., and the ship can make 4·5 knots per hour, and lie within $5\frac{1}{2}$ points of the wind.

Ans. Courses E$\frac{1}{2}$-S.; = 17·26.; distances N.N.W.$\frac{1}{2}$-W. = 13·94.
Time, 6·93 hours.

395. A ship wishes to make Looe Island in three boards. It is situated to the N.N.W. of the vessel, 20 miles away, and the wind is N. The ship can lie within 5 points of the wind, and sails 4 knots per hour. Give the courses and distances to reach the island, and the time of day, supposing the first course was laid on at 9 A.M.

Ans. N.E.byE. 6 m.; N.W.byW. 21·2 m.; N.E.byE. 6 m.
Time, 5 h. 18 m. P.M.

396. A ship that can lie within 5 points of the wind is bound for a port bearing N.$\frac{1}{2}$-W. 25 miles, the wind being N.W.; required her course and distance on each tack to reach the port close hauled in two boards.

Ans. W.byS. 7 85 miles.
N.byE. 26·9 miles.

CHAPTER XIII.

OBLIQUE AND CURRENT SAILINGS.

A CURRENT is a body of water, or air, flowing steadily in one direction.

The *set* of a current is its course or direction, or that point of the compass towards which it is moving.

The *drift* of a current is the rate at which it moves, generally reckoned in miles per hour. The drift may be two or two and a half miles per hour, the set to the N., E., W., S., etc. There is this distinction between a current and a wind: a *north-east current* goes *towards* the north-east, but a *north-east wind* comes *from* the north-east.

A ship may sail with a current, against it, or obliquely to it. When a ship sails with the current, her motion is evidently increased by the whole drifts of the current, and her run is equal to the sum of her own proper motion, and that due to the drift of the current. Thus, if a ship sail N.byE. 20 miles in a current that sets N.byE. 4 miles in the same time, her course and distance made good will be N.byE. 24 miles. But if the current sets against the ship, or the ship sail against the current, her motion is diminished by the drift, and the run of the ship in the given time is equal to the difference between her own proper motion and the current's drift. For instance, if the ship sail N.N.E. 48 miles in a current that sets S.S.W. 10 miles in the same time, then the course and distance made good will be N.N.E. 38 miles, or, if in the same time the ship sail N.N.E. 18 miles, and the current set S.S.W. 24 miles, the ship will fall astern, and her course and distance will be S.S.W. 6 miles while contending with the current; suppose the velocity of the current to be equal to that of the ship, then the latter will stand quite still, the one velocity or motion destroying

the other. When the ship sails obliquely to the current, the current will retard or accelerate the motion of the vessel according to its direction in proportion to the angle at which the direction of the ship's head lies across it. The ship will sail along the diagonal of a parallelogram, of which the force or rate of the current forms one side, and the proper velocity of the ship the other.

Currents then are certain motions of the waters of the sea, by means of which all bodies within their influence are compelled to move somewhat in their direction; if it be a vessel it is compelled to alter its course and give way to the motion of the stream. A simple and practical method of finding the *set* and *drift* of a current, is to jump into a boat, and having rowed a short way from the ship, let fall a small anchor or heavy weight, this will keep the boat still; next throw overboard the log and log-line, the rate at which the line runs out will be the *drift* of the current, and the direction in which the log and line lie (taken by a small boat compass) will be its *set*.

Suppose a ship at A ready to go to D, 5 miles away, which it can accomplish in one hour and a half; but a wind arises from the E.N.E., which would carry it to B, three miles in the same time. Then as the vessel is acted on by two forces, it will, if a north course be sailed on, be found neither at B nor D, but at C, and will sail along the line AC, the diagonal of the parallelogram ABCD.

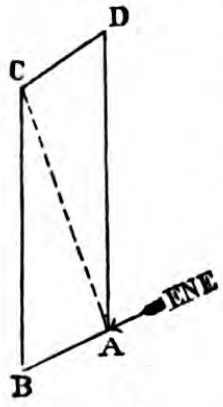


Fig. 42.

100. Oblique Sailing.—It is sometimes found advantageous, by those who possess the necessary knowledge, to solve some of the problems in navigation by the ordinary rules of trigonometry. This is generally termed *oblique sailing;* ordinary navigation is the solution of right-angled triangles, *oblique sailing* the solution of triangles which have no right angle.

To *set* an object is to observe what rhumb or point of the compass is directed towards it.

The *bearing* of an object is the rhumb on which it is seen, and the bearing of one place from another is reckoned by the name of the rhumb passing through the two places.

A vessel in the open ocean, out of sight of land, can seldom or never make use of the rules of oblique sailing; but in coasting, in approaching the land, islands, shoals, etc., oblique sailing is of assistance in a variety of ways. All problems can be solved by construction.

397. A ship sails E.N.E. 49 miles, a current which sets S.W. 13·5 miles in the same time, what is the course and distance made good?

In this figure let AB represent the E.N.E. rhumb, and suppose AB = 49 miles.

Let BC represent a S.W. rhumb, and BC = 13·5 miles.

Join AC, then AC represents the distance the ship has run.

C represents the position of the ship.

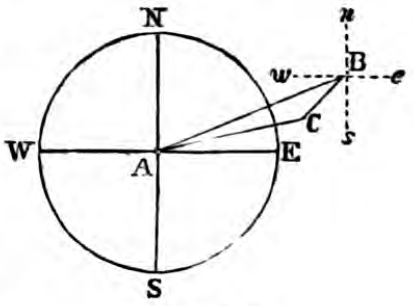


Fig. 43.

NAC represents the course made good.

$\angle wBC = 45°$, and $\angle wBA$ = alternate $\angle BAE$ = 2 points;
and since wBC = 4 pts.,
$\therefore ABC = wBC - wBA.$
= 4 pts. − 2 pts.
= 2 pts. = 22° 30′ = B;

$\therefore$ we know AB = 49 : BC = 13·5, and $\angle ABC$ = 2 pts., *i.e.*, two sides and the included angle.

$$\angle C + A = 180° - 22° 30' \quad \therefore \frac{C+A}{2} = 90° - 11° 15' = 78° 45'.$$

By the usual formula for the solution of triangles, when two sides and the included angle are given, we have

$$\text{Tan. } \frac{C-A}{2} = \frac{c-a}{c+a} \text{ cot. } \frac{B}{2} = \frac{49-13{\cdot}5}{49+13{\cdot}5} \text{ cot. } 11° 15'$$
$$= \frac{35{\cdot}5}{62{\cdot}5} \text{ cot. } 11° 15'$$

Log 35·5.....	=	1·550228	
Log cot. 11° 15′	=	10·701338	$\frac{C+A}{2} = 78° 45'$
		2·251566	
Log 62·5.....	=	1·795880	$\frac{C-A}{2} = 70° 42'$
Log tan. 70° 42′	=	10·455686	$\therefore C = 149° 27'$
			$A = 8\ 3$

To find AC we have

$$\frac{AC}{BC} = \frac{\text{sin. B}}{\text{sin. A}} = \frac{\text{sin. } 22° 30'}{\text{sin. } 8° 3'}$$

$$\therefore AC = \frac{13{\cdot}5 \times \text{sin. } 22° 30'}{8° 3'}$$

Log 13·5..... = 1·130334
Log sin. 22° 30′ = 9·582840
10·713174 ∴ AC = 36·89
Log sin. 8° 3′ = 9·146243
Log 36·89..... = 1·566931

Course from A to C = E.N.E. + ∠ BAC
= 67° 30′ + 8° 3′
= N. 75° 33′ E.

It is a more simple plan to make a small traverse table of the problem thus:—

Courses.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
E.N.E. .... S.W.	49 13·5	18·8	9·5	45·3	9·5
		18·8 9·5	9·5	45·3 9·5	9·5
		9·3		35·8	

∴ From the tables the course is N. 75°, E. 37 miles.

398. Coasting along Devon, I saw a cape bearing N.E.byN. from the ship, we then stood away N.W. 10 miles and observed the same cape bearing E.N.E. from the ship; what distance was the ship from the cope at each station?

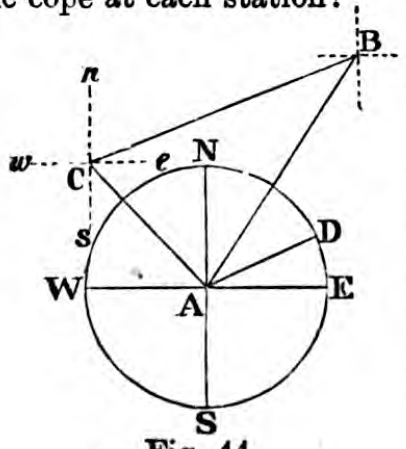


Fig. 44.

Let the annexed circle represent the horizon.

Let A be the position of the ship at the first station, when AB is the N.E.byN. rhumb.

Let C represent the position of the ship at the second station, when AC is the N.W. rhumb line.

Draw AD parallel to CB.

$\therefore$ $\angle$ CAB = 78° 45′ and side AC = 10 miles.

$\angle$ ABC = BAD = 33° 45′.

$\therefore$ BAD = NAD − NAB = 6 points − 3 points = 3 points.

$\therefore$ Since CAB + ABC = 7 pts. + 3 pts. = 10 pts.

$\therefore$ ACB = 16 pts. − 10 pts. = 6 pts. = 67° 30′.

To find AB.

Now $\frac{AB}{AC} = \frac{\text{sin. ACB}}{\text{sin. ABC}}$

$\therefore$ $AB = \frac{10 \times \text{sin. } 67° 30'}{\text{sin. } 33° 45'}$

Log 10.....	= 1·000000
Log sin. 67° 30′...	= 9·965615
	10·965615
Log sin. 33° 45′...	= 9·744739
Log AB 76·62.....	= 1·220876

To find CB.

$\frac{BC}{AC} = \frac{\text{sin. BAC}}{\text{sin. ABC}}$

$\therefore$ $BC = \frac{10 \times \text{sin. } 78° 45'}{\text{sin. } 33° 45'}$

Log 10.....	= 1·000000
Log sin. 78° 45′...	= 9·991575
	10·991575
Log sin. 33° 45′...	= 9·744739
Log BC 17·65.....	= 1·246836

Ans. 17·65 miles at first station, and 16·62 miles at second station.

399. Two ships A and B sail from two islands bearing one from the other N.E. and S.W., their distance being 76 miles. A sails S.byE., and B runs E.byS., at last they meet; required the distance that each ship ran.

Drawing the figure in the margin:—

Let A and B be the positions of the two ships.

A*c* represents the E. by S. rhumb.

BC is drawn parallel to it, and is therefore E.byS.

AC represents the S. by E. rhumb.

AB is given equal to 76 miles.

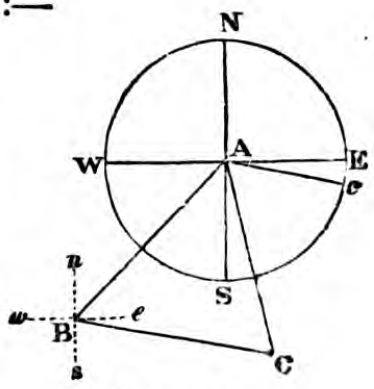


Fig. 45.

$\left.\begin{array}{l}\angle A = CAS + BAS = 5 \text{ points.} \\ \angle B = ABe + eBC = 5 \text{ points.}\end{array}\right\} = 56° 15'.$

$\therefore$ AC = BC.

$\therefore$ $\angle C = 6 \text{ points} = 67° 30' = cAC.$

To find AC and BC.

$\frac{BC}{AB} = \frac{\text{sin. 5 points.}}{\text{sin. 6 pts.}}$ $\therefore$ $BC = \frac{AB \text{ sin. 5 points.}}{\text{sin. 6 points.}}$

Log AB or 76.....	= 1·880814
Log sin. 5 points	= 9·919846
	11·800660
Log sin. 6 points	= 9·965615
Log AC or BC 68·39.....	= 1·835045

Ans. A sails S.byE. 68·39 miles; B sails E.byS. 68·39 miles.

400. Coasting along, two headlands were observed, the first bore N.W.byN., and the second N.N.E., then standing away E.byN.$\frac{3}{4}$-N. 20 miles, it was found that the first bore W.N.W.$\frac{1}{2}$-W., and the second N.byW.$\frac{1}{2}$-W.; required the bearing and distance of the two headlands one from the other.

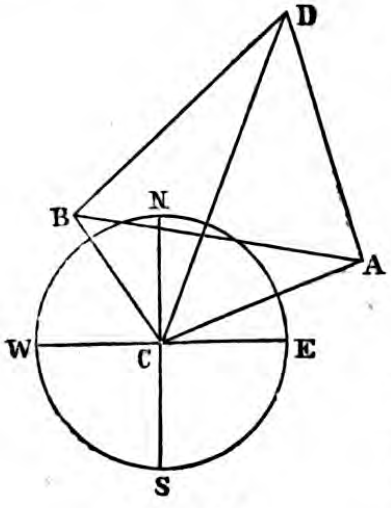


Fig. 46.

Let C represent the first place of the ship.

Draw CB towards the N.W.byN.

Draw CD towards the N.N.E.

Draw CA towards the E.N.E.$\frac{1}{4}$-E. = 20 miles.

Draw AB from A parallel to W.N.W.$\frac{1}{2}$-W., meeting CB in B.

Draw AD parallel to N.byW.$\frac{1}{2}$-W., meeting CD in D, then B will be the first and D the second headland, BD is the distance that we require to find, and CDB the bearing from the N.N.E. line which also must be determined.

In the triangle ABC are given $\angle BAC = 36° 34'$, or the angle between W.S.W.$\frac{1}{4}$-W., and W.N.W.$\frac{1}{2}$-W.

ABC = 39° 22′, and is the angle between E.S.E.$\frac{1}{2}$-E. and S.E.byS.

CA = 20 miles as given in the problem.

To find the side BC.

$$\frac{BC}{AC} = \frac{\text{sin. BAC}}{\text{sin. ABC}} = \frac{\text{sin. } 36° 34'}{\text{sin. } 39° 22'}$$

$$\therefore BC = \frac{20 \times \text{sin. } 36° 34'}{\text{sin. } 39° 22'}$$

Log 20	= 1·301030
Log sin. 36° 34′ ..	= 9·775070
	11·076100
Log sin. 39° 22′ ..	= 9·802282
Log BC 18·78.....	= 1·273818

To find CD in the triangle ACD.

CAD = 92° 49′ = the ∠ between W.S.W.$\frac{1}{4}$-W., and N.byW.$\frac{1}{2}$-W.

CDA = 39° 22′ = the ∠ between S.S.W., and S.byE.$\frac{1}{2}$-E.

CA = 20 miles.

$$\frac{CD}{CA} = \frac{\text{sin. CAD}}{\text{sin. ADC}} = \frac{\text{sin. } 92° 49'}{\text{sin. } 39° 22'}$$

$$\therefore CD = 31·49$$

Now from triangle BCD we have to find BD and the bearing.

BC = 18·78 CD = 31·49.

BCD = 56° 15′ the angle between N.W.byN. and N.N.E.

$$\text{Tan. } \frac{B-D}{2} = \frac{b-d}{b+d} \text{ cot. } \frac{C}{2}$$

$$= \frac{31\cdot49 - 18\cdot78}{31\cdot49 + 18\cdot78} \text{ cot. } 28° 7'\tfrac{1}{2}$$

$$= \frac{12\cdot71}{50\cdot27} \text{ cot. } 28° 7'\tfrac{1}{2}$$

$$= 25° 18' 54''$$

$$\therefore \frac{B+D}{2} = 61° 52' 30''$$

$$\frac{B-D}{2} = 25\ 18\ 54$$

$$\therefore B = 87\ 11\ 24$$

$$D = 36\ 33\ 36$$

∴ The bearing of B from D is 36° 33′ 36″ W. of the S.S.W. line, that is 22° 30′ + 36° 33′ 36″ = S. 59° 3′ 36″ W., or about S.W.byW.$\frac{1}{4}$ W.

For the distance—

$$\frac{BD}{BC} = \frac{\text{sin. BCD}}{\text{sin. BDC}} \therefore BD = \frac{18\cdot78 \times \text{sin. } 56° 15'}{\text{sin. } 36° 33' 36''}$$

$$= 26\cdot21 \text{ miles.}$$

Ans. Course S.W.byW.$\frac{1}{4}$-W.; distance 26·21 miles.

401. Two headlands bore from a ship N. 45° W. and N. 53° E., the ship then sailed S.byW. 18 miles, and they bore N. 27° W. and N. 35° E.; required the bearing and distance of the two headlands from each other.

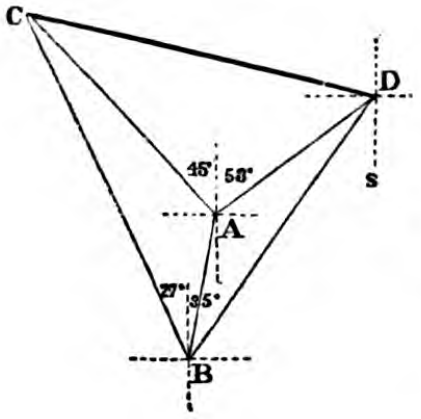


Fig. 47.

At A draw two lines as the dotted lines towards the four cardinal points, suppose A the first position of the ship. Draw the line AD, the rhumb line of 53° from N. towards E., and AC 45° from N. towards W.

From A draw AB to represent the run of the ship 18 miles on the S.byW. rhumb; here draw the dotted lines towards the N.S.E. and W. points of the compass, after which draw BC and BD at angles 35° and 27°; BD N. 35° E., and BC N. 27° W. Then C and D will show the position of the two headlands; we have now to find CD and determine the direction of C from D.

$\angle CAD = 98 \qquad \angle CBD = 62°$
$\angle DAB = 11° 15' + 90° + (90 - 53°) = 138° 15'$
$\angle CAB = (90° 11' 15'') + (90° 45°) = 123° 45'$
$AB = 18$ miles.

(*a.*) $\therefore$ In triangle BAD

$\angle A = 138° 45' \qquad \angle B = (35° - 11° 15') = 23° 45'$
$\therefore D = 17° 30'$ and AB 18 miles.

$$\therefore \frac{BD}{AB} = \frac{BD}{18} = \frac{\text{sin. } 138° 45'}{\text{sin. } 17° 30'}$$
$$\therefore BD = 45{\cdot}34 \text{ miles.}$$

(*b.*) In the triangle ABC we know AB,

$\angle CAB = 123° 45' \qquad CBA = 27° + 11° 15' = 38° 15' \therefore ACB = 18°$

and have to find BC

$$\frac{BC}{AB} = \frac{BC}{18} = \frac{\text{sin. } 123° 45'}{\text{sin. } 18°} \therefore BC = \frac{18 + \text{sin. } 123° 45'}{\text{sin. } 18°}$$
$$\therefore BC = 48{\cdot}43$$

(*c.*) In the triangle DBC, the two sides BC, BD, and the included angle are known.

$$\therefore \text{Tan. } \frac{D-C}{2} = \frac{CB - BD}{CB + BD} \text{ cot. } \frac{CBD}{2}$$
$$= \frac{48{\cdot}43 - 45{\cdot}34}{48{\cdot}43 + 45{\cdot}34} \text{ cot. } 31°$$
$$= \frac{30{\cdot}9}{93{\cdot}77} \text{ cot. } 31°$$
$$= \text{tan. } 3° 8'$$

$$\therefore \frac{D+C}{2} = 59° 0'$$
$$\frac{D-C}{2} = 3° 8'$$
$$\therefore D = 62° 8'$$
$$C = 55° 52'$$

Now $sDC = sDB + BDC$
$= 35° + 62° 8' = 97° 8'$

$\therefore$ C bears from D – S. 97° 8' W., or N. 82° 52' W., which is the bearing required.

(*d.*) Lastly, in the triangle CBD we have to find CD, the distance between the two headlands.

$$\frac{CD}{CB} = \frac{CD}{48{\cdot}43} = \frac{\text{sin. } 62°}{\text{sin. } 62° 8'}$$
$$\therefore CD = \frac{48{\cdot}43 \text{ sin. } 62°}{\text{sin. } 62° 8'} = 48{\cdot}37 \text{ miles.}$$

Ans. Course N. 82° 52' W.; distance 48·37 miles.

In the course of the following problem, we shall have to find the distance between two inaccessible points, when they are visible from two different stations.

I. PROBLEM.—This figure will apply to several of the problems on pages 247-253.

Let us suppose that the points X and Z cannot be reached from a ship at B, and that it is necessary to find their distance apart.

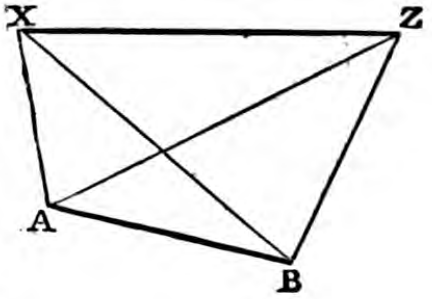


Fig. 48.

Let the two angles XBZ and and XBA be observed, and then suppose the ship sail along BA to A, and the angles XAZ and ZAB are observed, the distance BA being measured by the log. The problem is thus solved:—

(*a.*) In triangle AXB, the side AB and the angles BAX and ABX are known.

$$\therefore \frac{AX}{AB} = \frac{\text{sin. ABX}}{\text{sin. AXB}} \quad \therefore \text{ AX is known.}$$

(*b.*) In the triangle AZB, the side AB and the angles ZAB and ZBA are known.

$$\therefore \frac{AZ}{AB} = \frac{\text{sin. ABZ}}{\text{sin. AZB}} \quad \therefore \text{ AZ is known.}$$

(*c.*) In the triangle ZAX, the sides AX and AZ are known, and the angle XAZ.

$$\therefore \text{Tan.} \frac{X-Z}{2} = \frac{AZ-AX}{AZ+AX} \text{ cot. } \frac{ZAX}{2},$$

$\therefore$ the angles are known in ZAX.

(*c'.*) Then, finally,

$$\frac{ZX}{AZ} = \frac{\text{sin. ZAX}}{\text{sin. AXZ}},$$

$\therefore$ XZ with AZ, BZ, AX, and BX, are known.

II. PROBLEM.—Given the distances of A, B, and C from each other, or given AB, BC, and CA, to find their distance from the given place or station S.

First observe the angles ASB and CSB.

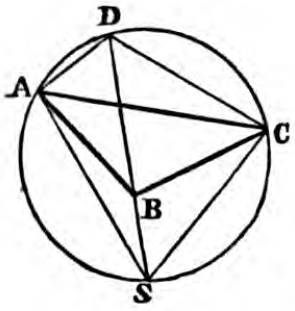


Fig. 49.

At C construct the angle ACD = ASB.

At A construct the angle DAC = BSC.

Describe about the triangle ADC the circle ADC.

Join DB, and produce it to meet the circumference in S, then S is the place where the observer stands, its distance from A, B, and C, respectively, are required.

(*a.*) Find AD from the triangle ADC thus:—

$$\frac{AD}{AC} = \frac{\text{sin. DCA}}{\text{sin. ADC}}$$ (ADC is known, being 2 rt. ∠ − ASC)

∴ AD is known.

(*b.*) Find the angle BAC from the triangle ABC.

$$\text{Cos.}\frac{BAC}{2} = \sqrt{\frac{S(S-BC)}{AB.AC.}}$$ ∴ angles BAC, etc., in triangle ABC are known.

∴ angle BAD is now known.

(*c.*) In triangle BAD, find the angle ABD. Here two sides and the included angle are known.

$$\text{Tan.}\frac{D-B}{2} = \frac{AB-AD}{AB+AD}\ \text{cot.}\ \frac{BAD}{2}$$ ∴ angles in BAD, etc., are known.

∴ angle ABS the supplement of ABD is found.

(*d.*) Find the sides SA and SB from the triangle ABS.

$$\frac{AS}{AB} = \frac{\text{sin. ABS}}{\text{sin. ASB}} \qquad \frac{SB}{AS} = \frac{\text{sin. BAS}}{\text{sin. ABS}}$$ ∴ AS and SB are known.

(*e.*) From the triangle ASC, find the side SC.

$$\frac{\text{Sin. ACS}}{\text{Sin. ASC}} = \frac{AS}{AC} \text{ and } \frac{SC}{AC} = \frac{\text{sin. CAS}}{\text{sin. ASC}}$$ ∴ SC is found.

These five steps give the three distances SA, SB, and SC.

402. My ship sails for 12 hours on a S. by W. course 7 miles an hour

in a current setting E.N.E. 3 miles an hour, required her true course and destination.

Courses.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
S. by W.	84	...	82.4	...	16.4
E.N.E.	36	13·8	...	33·3	...
		13·8	82·4	33·3	
			13·8	16·4	
			68·6	16·9	

Ans. Course S. 14° 16′ E.; distance 70·8 miles.

Here, it will be noticed, that the problem consists of two courses and distances, so a small traverse is made, and the answer obtained in the usual way.

403. A ship sails N.N.E. for 5 hours 4 miles an hour, while a current sets her N.W. one mile an hour, what is the true course and distance she made?

This question, although it may be solved by trigonometry, is a simple problem. We have simply two courses and distances to resolve into one course and distance.

Courses.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
N.N.E.	20	18·5	...	7·7	...
N.W.	5	3·5	...	...	3·5
		22		7·7	3·5
				3·5	...
				4·2	

∴ Course is N. 11° E.; 22 miles.

404. A ship steams 10 knots an hour, and her compass course is

S.byW., what is ner true course and distance in $4\frac{1}{2}$ hours, a current drifting her SW.$\frac{3}{4}$-W. $2\frac{1}{4}$ knots an hour? Variation $2\frac{1}{2}$ points E., deviation on first course 4° 10′ E., and on current course 5° W.

Ship's course.		Current course.	
Compass, ...	11° 15′ 0″ r. of S.	S.W.$\frac{3}{4}$-W. =	53° 26′ 15″ r. of S.
Variation, ...	28 7 30 r.	Variation,	28 7 30 r.
	39 22 30 r.		81 33 45 r. of S.
Deviation, ...	4 10 0 r.	Deviation, ...	5 0 0 l.
	43 32 20 r. of S.		76 33 45 r. of S.
	or S. 44 W.		or S. 77 W.

Courses.	Dist.	Diff. Lat.		Departure.	
		N.	S.	E.	W.
S. 44° W.	45·0	...	32·4	...	31·3
S. 77 W.	10·1	...	2·3	...	9·8
		...	34·7		41·1

Ans. Course S. 50° W.; distance 54 miles.

405. In sailing down channel I observed the Start bearing due N., the ship then sailed S.E. for 12 miles, I then observed that the Start bore N.W.byN.; find the distance from the Start at each observation.

Let S be the Start Point; B the position of the ship at the first observation. Let the ship sail from B to A 12 miles; then the Start bears N.W.byN., or lies in the direction of the line AS.

$\angle$ BAD = ABE = 4 pts. $\therefore$ ship sails S.E.
$\angle$ EAS = 3 ,, $\therefore$ the Start bears N.W.byN.
$\therefore$ $\angle$ BAS is *one* point.
and $\angle$ ABS is *evidently* 12 points.
$\therefore$ $\angle$ BSA is *three* points.

Since

$$\frac{\text{Sin. SAB}}{\text{Sin. BSA}} = \frac{\text{BS}}{\text{AB}} = \frac{\text{BS}}{12} = \frac{\text{sin. 1 pt.}}{\text{sin. 3 pts.}}$$

$$\therefore \text{BS} = \frac{12 \times \text{sin. 1 pt.}}{\text{sin. 3 pts.}}$$

Log 12	1·079181
Log sin. 1 pt. ..	9·290236
	10·369417
Log sin. 3 pts.	9·744739
Log 4·213	0·624678

Again,

$$\frac{\text{Sin. ABS}}{\text{Sin. ASB}} = \frac{\text{AS}}{\text{AB}} = \frac{\text{AS}}{12} = \frac{\text{sin. 12 pts.}}{\text{sin. 3 pts.}}$$

$$\therefore \text{AS} = \frac{12 \times \text{sin. 12 pts.}}{\text{sin. 3 pts.}}$$

Log 12	1·079181
Log sin. 12 pts. (4 pts.)	9·849485
	10·928666
Log sin. 3 pts.	9·744739
Log 15·27	1·183927

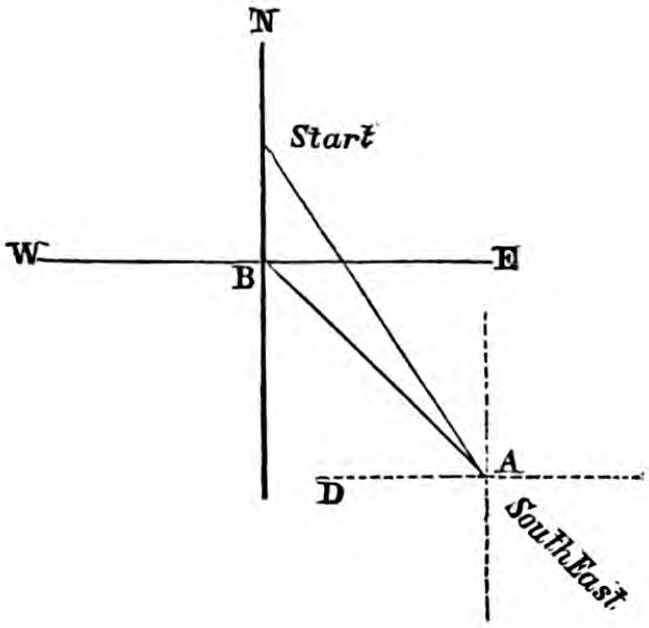


Fig. 50.

At the first observation the ship is 4·213 miles from the Start, at the second 15·27 miles.

406. From my ship a cape bore S.S.E. 4 miles away, I then sailed on a certain course and distance between S. and E. till the same cape bore W.S.W. 3 miles away; find my course and distance.

Ans. Distance 5 miles; course S. 59° 22′ E.

407. Plymouth breakwater light bore from a yacht S.S.W. 3 miles distant, it then sailed 5 miles S.; find the bearing and distance of the light at the second observation.

Ans. N. 27° 15′ W., distance 2·507 miles.

408. A point bears N.W.byW., I sail directly to the point, finding the distance 4 miles, the vessel is then put on a N.E.byN. course for 3 miles; what is now the distance from and course to, the starting point?

Ans. S. 19° 23′ E.; distance 5 miles.

409. Coasting along the shore of the Isle of Wight, I saw St. Catherine's point bearing from me N.E.byN., then I stood away N.W. 20 miles, and observed the point bearing from me E.N.E.; required the distance of the ship from the cape at each station.

Ans. 33·26 and 35·2.

410. At noon a headland bore N.W.byW., then we sailed N.N.E. 5 miles an hour, and anchored at 9 in the evening on a bank 54 miles from the headland; required the distance of the headland from the ship at noon, and its bearing from the bank.

Ans. 39·9 miles; S. 68° 56′ W., about W.S.W.$\frac{1}{4}$-W.

411. Being out at sea in sight of land, I observed two headlands, one bearing N.byW., the other N.W., taking up the chart I find they are 12 miles apart, and one bears E.N.E. from the other; find the distance of the ship from the two headlands.

Ans. 21·18 miles from one, and 19·95 from the other.

412. A ship steams 11 knots an hour, and her apparent course is W.byS.; what is her true course in $4\frac{3}{4}$ hours, suppose a current drifting N.N.E. 2 knots an hour? (1863.) *Ans.* S. 88$\frac{1}{2}$° W. 50 miles.

413. In sailing down the channel I observe the Eddystone Lighthouse due N., after sailing SW. for 8 miles I observe it again, and find its bearing N.N.E.; what was its distance from me at each of the observations? (1865.) *Ans.* 8 miles, and 14·78 miles.

414. How is the effect of a current allowed for? A ship sailing 4 hours on a corrected course S.E.$\frac{1}{2}$-E. over an apparent distance of 75 miles, is set during the time by a current E.byN. by the compass 4 miles an hour, deviation 8° 15′ E.; what are the true course and distance? (1867.) *Ans.* S. 59° E., 86 miles.

Several of the examples following, with one or two of the illustrations preceding, are taken from Robertson's or Wilson's *Navigations*, but corrected and slightly altered in some places.

415. Running along shore a headland bore N.E.byN., the ship then ran E.byN. for 15 miles, when the headland bore W.N.W.; what was the distance from the headland at each observation?

Ans. (1) 8·496 miles; (2) 10·81 miles.

416. In rounding a cape a tower was seen up a river bearing due N., then running N.W. 6 miles, it bore N.N.E.; required the distance from the tower at each observation.

Ans. (1) 14·48 miles; (2) 11·08 miles.

417. A ship after sailing S.E.byS. from her port 62 leagues, being in latitude 29° 16′ N., met another that had run 124 leagues from a place bearing W.byN. from the port left by the first ship; what was the course of the second ship, and the distance between the two ports?

Ans. Distance 72·18 leagues; course S. 58° 3′ E.

418. A merchant ship meeting with a man-of-war, which had sailed S.E.byE. 39 leagues from the latitude 45° 30′ N., informed the captain that she had been plundered in the same latitude, and had since run 67 leagues on a direct course between S. and W., and that the pirates were cruising about the place where they robbed her; required the course that the man-of-war must steer to come up with the pirates? *Ans.* N. 71° 8′ E.

419. Two ships sail from a port at the same time, A goes E.S.E. for 37½ miles, B runs S.S.E. the same distance; required their distance apart and bearing at the end of the time.

Ans. Course N. 45° E.; distance 28·7.

420. A schooner sailed S.W., at the rate of 4½ miles an hour, at 9 in the morning from a cape which bore S.E.byE. distant 15 miles; what was the distance and bearing of the same cape at 4 in the afternoon? *Ans.* Bearing N. 68° 8′ E.; distance 37·44 miles.

421. Wishing to ascertain the breadth of a small bay in which were several islands, I sailed N.N.W. 50 miles from a cliff on the eastern side, and then ran S.S.W. 35 miles and reached the western shore; with these conditions find the breadth of the bay.

Ans. 35·35 miles.

422. From two ports lying on the same meridian, and separated 236 miles apart, two ships sail one south-eastwards the other north-eastwards till they meet, each having run 372 miles; find the courses on which they sailed. *Ans.* S. 71° 30′ E.; N. 71° 30′ E.

423. Two ships meeting at sea, on comparing notes, find that they started from the same latitude from two ports 576 miles apart, one ship ran between the N. and E. 328 miles, the other ran 416 miles; required the courses on which each ship sailed.

Ans. N. 44° 49′ E.; N. 55° 59′ W.

424. A barque in three days coasted round a triangular-shaped island, the first day along one side, and her courses and distances, resolved by a traverse, gave S.W. distance 64 miles as the course and distance made good, the second day's distance was 106 miles between S. and E., and the third day was 83 miles; required the courses of the second and third days.

Ans. Second day's course S. 83° 29′ E.
Third day's course N. 46° 21′ W.

425. In a harbour two landmarks were observed, the one bore E., the other W., and after running S. 3·3 miles, the eastern one bore N. 58° 30′ E., and the western one N. 47° 20′ W.; required the distance from the landmark at each observation.

Ans. 3·58 miles, 4·87 miles, 5·38 miles, 6·31 miles.

426. Coasting along shore two rocks were observed, the one bore E. and the other W., then sailing S.S.E.½-E. 16½ miles, the western rock bore N.W.byW.½-W., and the eastern N.E.½-E.; required their distance at each observation.

Ans. 19·45 miles; 30·88 miles; 25·51 miles; 22·94 miles.

427. Three ships, B, C, and D, sail from the same port A, all bound for the same parallel of latitude, B's course is N.W.byN. distance 12 miles, C's course is N.E.byE., and D's E.N.E.$\frac{1}{2}$-E.; find the distance of C and D from their ports, and also the distance of those ports from each other. *Ans.* AC=17·96; AD=34·36; BC=21·6; BD=39·56.

428. Two ships A and B sail from a port C, A sails S.S.E. for 50 miles, and B, S.W.byS. for 40 miles; required their bearing and distance, and how far A is the southward and eastward of B.

Ans. BA=43·33 miles; BD=12·93 miles; DA=41·36 miles. Bearing of B from A is S. 72° 38′ E.

429. At sunrise, being 4 o'clock, a ship saw a sloop to the W.N.W., and a brig to the E.byN., they all came to anchor alongside one another at 9 o'clock, each having run since sunrise 5 knots per hour, and the ship's course was due N.; required the courses sailed by the sloop and brig, and their distance from the ship in the morning when she observed them.

Ans. 19.13 miles; course N.E.; 9·75 miles; course N.N.W.

430. Having dropped anchor in very deep water in a river, with a buoy and a rope 50 fathoms fixed to it, I fell down the stream till I had veered out 140 fathoms of cable, and then the buoy was 120 fathoms ahead of me; what was the depth of the water?

Ans. 48·71 fathoms.

431. Coming in sight of two headlands bearing N. and S. of each other, the southern one bore due E. from the ship, and the other N.E.byE., after sailing due E. 5 miles, the northern headland bore N.E.byN.$\frac{1}{2}$-E.; required the distance between the two headlands, and their distance from the ship each time they were observed.

Ans. 13·31 miles; 9·56 miles; 6·07 miles; 7·395 miles.

432. Having cast anchor, the bearings of a tower, a mill, and a lighthouse were taken, they were known to be in a line lying due east and west, the tower bore W.S.W., the mill S.W.byS.$\frac{1}{2}$-W., and the lighthouse S.byW.$\frac{1}{2}$-W., the distance of the tower from the mill is 5$\frac{1}{2}$ miles; how far are each of those objects distant from the place where they were seen? *Ans.* 9·021 miles; 4·466 miles; 3·608 miles.

433. A ship proceeds at the rate of 8 knots, and steers S.S.E., she finds at the end of 1$\frac{1}{2}$ hours she has made good 14 knots S.E.$\frac{1}{2}$-E.; what is the force and direction of the current? (1873.)

Ans. Course N. 70° 30′ E.; rate 4·4 miles.

434. A steamer and sailing vessel part company, the steamer proceeds by compass S.W. (variation 1$\frac{1}{2}$ points W., deviation 12° 30′ W.) at the rate of 7$\frac{1}{2}$ knots, the sailing vessel sails by compass E.S.E.$\frac{1}{4}$-S., and makes one point leeway, wind being N.E. (variation 1$\frac{1}{2}$-W., deviation 9° E.) at the rate of 4$\frac{1}{2}$ knots; what distance will they be apart at the end of 3$\frac{1}{4}$ hours? (1873.) *Ans.* 25·4 miles.

435. A ship making for a port which lies N.E.byN. (true), distant [illegible] nautical miles, allows the variation and deviation the wrong way,

the variation being properly 2 points east, and deviation ¼ point west, after sailing on the port tack (wind N.W.byW.) and making ½ point leeway for three hours at the rate of 6·5 knots, the mistake is discovered; how must she now steer by compass (variation as before, deviation ½ point east) to reach her destination, and how much will she have lost supposing her rate to be uniform on the different courses? (Honours, 1873.)

Ans. Course N. 19° 31′½ W.; and the ship has to go 14·92 miles instead of 10·5.

436. A ship wishes to reach a point, the true bearing of which is S.W., and distance 15 miles, a current is running S.E.byE. (true), 4 knots an hour, and the ship sails at the rate of 7·5 knots per hour, she is on the port tack (wind S.S.E.) making half a point leeway, variation 2½ points W., deviation ¼ point E.; what course must she steer by compass, and how long will she be in reaching the port? (Honours, 1873.) *Ans.* Course 80° 46′¾ W.; time 2·67 hours.

437. From a ship at sea three objects on shore are observed, and the angles at the ship subtended by each pair of these objects are measured, the relative positions of the objects are found from the chart; determine the position of the ship (see p. 244).

EXAMPLE.—A, B, C, being the objects, AB=5, BC=6, and AC=7 miles respectively, the angles subtended at the ship by A and B, and B and C respectively, are 28° 34′ and 36° 40′; find the distance of the ship from A, B, and from C. (Honours, 1872).

Ans. A=6 m., B 1·1, and C 6·8.

438. There are three rocks, A, B, and C: B is two miles E.N.E. from A, and C is four miles from A and three miles from B, to the eastward; a ship sails close hauled (5½ points) on the starboard tack, with the wind at west, at the rate of 8 knots an hour, in a current setting E.S.E. 3 knots an hour; required the ship's course and distance made good in two hours, also the bearing and distance of C at the end of that time. (Honours, 1872).

Ans. Course S. 6° 56′¾ W.; dist. 16·4; dist. from C 15¾.

439. Two ships leave a port at the same time, one steers N.byE.¾-E., and the other N.N.W., one sails at the rate of 10·5 knots per hour, and the other at the rate of 8 knots per hour. What will be their distance and bearing from one another in four hours?

Ans. 28·1 miles, N. 69° 18′ E. or 69° 18′ W.

440. I lay to, due S. of a lighthouse, at a distance of 7½ miles, but two hours and a half after I found that a current had set me E.byS.¼-S. Supposing the rate of the current to be two miles an hour, how do I bear in my last position from the lighthouse, which is now invisible, and what is my distance from it? (1871.)

Ans. N. 29° 6′ E., 10 miles.

441. Two ships A and B leave the same port together on different tacks, variation of compass being 2¼ points W.; A is on the port tack, and sails by compass E.½-N. 7·5 knots per hour, deviation of

her compasses being 1¼ E., leeway ½ point; B has to make a true course N.W.½-W. at the rate of 5 knots an hour; she, however, takes this as her compass course, and after one and a half hours is observed by A to bear W.byN. (true). What is the deviation of B's compass, supposing her to have made 1 point leeway? (Honours.)

Ans. 46° 11′ E.

442. In surveying a harbour I place two boats in convenient positions, from each of which the other boat and a house and mill on shore can be seen. The angles at the first boat, between the second boat and the house and mill respectively, were observed to be 58° 20′ and 95° 20′. The angles at the second boat, between the first boat and the house and mill respectively, were 98° 45′ and 53° 30′. What is the distance between the house and mill, that between the two boats being 900 yards? (Honours.) *Ans.* 1493 miles.

443. A ship sailing at the rate of 6 knots close hauled (five points) on the port tack, with wind N., making one point leeway, was eight miles N.byE. of a lighthouse at noon. At 2 P.M., finding the wind had shifted, so as to make her break off two points, she went about keeping 5 points from the wind, but making only ½ point leeway, and going seven knots. At 3 P.M, a steamer passed eight miles W.byN. of the lighthouse, steering due north, and eventually fell in with the sailing vessel. At what rate did she go, and at what time did they meet? (Honours.) *Ans.* 9·6 m.; 5 h. 20 m. P.M.

444. A privateer is lying 12 miles S.W. of a harbour, and observes a merchantman leave it in the direction S. 78° E., at the rate of 8½ miles an hour. In what direction, and at what rate, must the privateer sail in order to come up with the merchantman in one hour and a half? (1868.) *Ans.* N. 74° 27′ W.; rate, 14·5 m.

445. In standing up Channel, I observe St. Catherine's Point N.E.½-E. I afterwards sailed 25 miles due E., and the bearing of St. Catherine's was N.¾-W. What was my distance at each observation? (1867.) *Ans.* 28·8 m. and 17·36.

446. A shoal bears N.W. 24 miles from a ship, the ship sails N.byW. 9·5 miles an hour, and in three hours grounds upon the shoal; what was the strength and direction of the current? (1868.)

Ans. Rate, 5·27; S. 46° 6′½ W.

447. Two headlands bore from a ship N. 54° E., and S. 81° E. After sailing S.E. 12 miles, they bore N. 5° W. and E. 40° N.; required their true bearing and distance from each other. (1868.)

Ans. S. 46°½ E., or N. 46°½ W., 11·4 miles.

448. A steamer running into the Channel at 8 A.M., observes Dover, latitude 51° 8′ N., longitude 1° 19′ E., bearing W.½-S., and Calais (latitude 50° 58′ N., longitude 1° 51′ E.) bearing S.S.E., and at 10 A.M. the bearing of Dover was N.N.W., and of Calais N.E.byN.; find the rate of sailing of the steamer. (Honours, 1869).

Ans. Course S. 10° W.; rate, 13½ miles.

449. Two ships leave a port at the same time, and after one has sailed S.E.byE. 40 miles, and the other S.W. 50 miles, they are equally distant from a rock which lies due S. of a port; required the bearing and distance of each from the rock. (Honours, 1869.)

Ans. 34½ miles; N. 85° E., S. 74° W.

450. Two ships meet at sea bound for two ports, distant from one another 12 miles. If one sail N.W.byN. 6 miles an hour, and the other N.E. 8 miles an hour, they will arrive at their respective ports at the same time; required the distance each ship must run, and the bearings of the ports from each other. (Honours, 1869.)

Ans. 7·8 m. and 10·4; N. 84° 43′½ E., or S. 84° 43′½ W.

451. Sailing on a W.S.W. course past an island, the north-eastern extremity was observed to bear N. 15° E., the western extremity N. 80° W., and the summit of a remarkable peak near the centre N. 35° W. After running 6 miles, the north-eastern extremity bore N. 42° E., the western extremity N. 20° W., and the peak N. 34° F., the elevation of which at this moment was 7° 10′; find the length of the island, its direction from one extremity to the other, and also the height of the peak above the level of the sea. (Honours, 1869.)

Ans. 9·5 miles; S. 62° W.; h. = ·78 miles.

SPECIMEN EXAMINATION PAPERS.

General Instructions.

If the rules are not attended to, the paper will be cancelled.

You are permitted to answer questions from the Elementary Paper, or from the Advanced Paper, or from the Honours Paper, but you must confine yourself to one of them.

In all cases the number of the question must be placed before the answer on the worked paper.

The value attached to each question is shown in brackets after the question. But a full and correct answer to an easy question will in all cases secure a larger number of marks than an incomplete or inexact answer to a more difficult one.

Three hours are allowed for this paper.

FIRST STAGE OR ELEMENTARY EXAMINATION.

Instructions.

You are only permitted to *attempt eight* questions, viz., *four* out of each of the sections into which the paper is divided.

Section I.

1. Define "latitude of a place," "parallels of latitude," "meridian distance," "difference of longitude," "rhumb line," "course." On what curves is a ship sailing when her course is (1) = 0°, and (2) when it is = 90°? (12½).

2. Describe fully the card of the mariner's compass, both the upper surface of it and the under, showing how the needle is placed, and

the arrangements for balancing it on its point. The indications of two compasses, A and B, when together on shore, are the same, the N. point of each being directed to the N.N.W. point of the horizon; B is taken on board a ship, and its N. point is now directed to the N.W.byN. point of the horizon. State, in degrees, minutes, etc., the difference of the two, and account for it, and for neither of them pointing to the true north on shore. (12½).

3. Define what is called the "distance," and show how it is measured. With a glass of 28', what should be the length of a knot on the log-line, taking the nautical mile at 6080 feet? A ship estimated her distance run as 100 miles, using a line of 52 feet, and a glass of 28'; what was the true distance made? (12½).

4. Explain what you mean by the following terms:—Plane sailing, traverse sailing, middle latitude sailing. Prove that diff. lat. = dist. × cos. course; dep. = dist. × sin. course. (12½).

5. Describe fully the construction and use of the traverse table. Show how it is you can apply this table to the solution of such problems as the following:—The height of a ship's mainmast is 180 feet, the angle subtended by it was 4°; required the distance of the observer. Solve this example. (12½).

6. Prove the formula of middle latitude sailing:—Dep. = diff. lon. × cos. mid. lat. Does this give strictly accurate results? Give a reason for your answer. How do you proceed when the two places are situated in different hemispheres? (12½).

Section II.

7. Find the true course in the following:—Compass course N.E. byN., var. 1¾ pts. E., dev. 10° W., leeway 2 pts., direction of wind S.E. byE. Find the compass course in the following:—True course S.S.W., var. 2¼ pts. W., dev. 9° E. (10.)

8. A ship sails 205 miles E.S.E., starting from a place in lat. 32° 22′ N.; what is the latitude of the place arrived at? (10.)

9. How many miles would a ship have to sail from Dornoch Firth (lat. 57° 50′ N., lon. 3° 50′ W.) to arrive at a position due south of the Nase (lon. 7° 3′ E.) on the same parallel? (12½.)

10. Find by middle latitude method the compass course and distance from A to B. Given:

Lat. A 10° 30′ S.; lon. N 50° 20′ E.; var. of compass 1¼ pts. W.
Lat. B 20° 52′ S.; lon. B 55° 30′ E. dev. 9°. W. (12½.)

11. A ship makes good a course S.W.byW.½-W., true, but she steers S.S.W.¾-W.; the variation is 2 pts. E., and the deviation is 8° 30′ W.; what is the leeway, and from what point of the compass is the wind blowing, supposing the ship to be 5½ points off the wind? (12½.)

12. A ship is in lat. 44° 30′ N. and 17° 20′ W., and afterwards sails

on the following traverse:—True courses, S.W.byW., distance 28 miles; W.S.W., 64 miles; S.byW., 53 miles; S.E., 72 miles; N.byW., 25 miles; required the latitude and longitude in. (12½.)

SECOND STAGE OR ADVANCED EXAMINATION.

INSTRUCTIONS.

Read the General Instructions at the head of the Elementary Paper.

You are permitted to *attempt* all the questions.

13. Explain clearly what you mean by "middle latitude sailing," and "Mercator sailing;" and show why the latter is a more perfect method than the former. (20.)

14. Describe the methods of determining the deviation of a ship's compass. A table of deviation is drawn up for an iron ship soon after she is launched; under what circumstances could this not be depended upon? Describe fully the magnetic changes the ship would be subject to. (25.)

15. What do you mean by taking a departure? Give the various methods, both without and with the aid of the chart. (20.)

16. Why cannot a ship always be kept accurately upon a great circle track? Is there any particular case in which it can? Investigate a rule for finding the latitude and longitude of the vertex of a great circle between two places. (25.)

17. What course would a steamer (steaming 6·5 knots an hour) have to steer in order to reach a point 20 miles S.E. of her, a current running N.E.byE. 3 miles an hour? how long would she be in reaching her destination? (20.)

18. A steamer and a sailing vessel leave Cape Clear at the same time for Bermuda, the former proceeding upon the great circle, and and the latter upon the rhumb track; the steamer averages 6 knots an hour, at what average rate must the sailing vessel go to reach Bermuda at the same time as the steamer?

Cape Clear,...Lat. 51° 25′ N. Lon. 9° 29′ W.
Bermuda,.....Lat. 32 22′ N. Lon. 64 30 W. (30.)

19. Required the compass course and distance from Cape Finisterre to Terceira.

Cape Finisterre, lat. 42° 54′ N.; lon., 9° 16′ W.
Terceira, ... lat. 38 39 N.; lon., 27 14 W
Variation 2¼ pts. W., deviation 3° 30′ E. (20.)

20. May 22nd, 1874, at noon, a point of land in latitude 20° 10′ S., and longitude 57° 32′ E., bore by compass S.S.W., distant 16 miles, deviation for ship's head S.S.E., being 1° 20′ W.; afterwards sailed as by the following log account; find the latitude and longitude in on May 23rd at noon. (40.)

Hours.	Knots.	Tenths.	Courses.	Wind.	Leeway.	Remarks.
1	6	5	S.W.byW.	S.S.E.	1	Deviation, 7° W.
2	7	0				
3	4	5				
4	5	4				
5	7	1				
6	3	5				
7	4	8				
8	4	6	N.W.	N.byE.	1½	Deviation, 5° 20′ E.
9	6	0				
10	5	6				
11	7	5				
12	4	5				
						Variation of compass 1½ pts. W.
1	5	6	N.N.W.	N.E.	¾	Deviation, 2° 40′ W.
2	4	5				
3	6	4				
4	3	4				
5	5	3				
6	7	2	E.S.E.	S.	½	Deviation, 5° 30′ W.
7	3	5				
8	5	4				
9	4	5	S.E.	S.S.W.	1	Deviation, 1° 50′ E.
10	7	0				
11	6	4				A current set the ship the last 2½ hours 3 miles an hour, N.W.byW. by compass.
12	6	2				

HONOURS EXAMINATION.

INSTRUCTIONS.

Read the General Instructions at the head of the Elementary Paper.

You are permitted to *attempt* all or any of the questions.

21. In an iron ship, show the necessity for the mariner to be acquainted with the circumstances under which she was built. Describe also the means used by the compass-adjuster to compensate for the deviation. Investigate the theory of the heeling error. (50.)

22. Two ships, A and B, leave two places on the same meridian (15° 30′ W.); A is on the equator and sails due W., and B sails for latitude 50° 30′ N. on the arc of the great circle, joining her position with a place in lat. 25° 33′ S. lon. 35° W.; find the latitude and longitude of the place where their tracks will cross each other. (50.)

23. Having cast anchor in an unknown bay, describe how you would proceed to make a survey of it. Explain the different methods of determining your base line, and the principles you would adopt in the triangulation; also how you would sound, and how you would reduce your soundings before laying them down on your chart. In sounding you suddenly come upon a shoal; what steps would you take? (60.)

24. A ship in north latitude and longitude 4° E. sails W.N.W.$\frac{1}{4}$-N. by compass (variation 2$\frac{1}{2}$ pts. W. and deviation 2° 50′ E.), till she reaches the meridian of 8° W.; she then sails by compass N.E. (variation as before, deviation 5° 30′ W.) until she arrives at the meridian from which she started. The latitude is now 49° N.; required the latitude from which she originally started. (45.)

25. A ship leaving a place in lat. 23° S., lon. 178° 30′ E., sails on the following courses:—N.E. 30′ (var. 1 pt. E., dev. 8° E., leeway $\frac{1}{2}$ pt., wind E.N.E.); E.S.E. 50′ (var. $\frac{3}{4}$ pt. E., dev. 5° W., leeway 1 pt., wind N.byE.); E.byN. 30′ (var. $\frac{3}{4}$ E., dev. 11° E., wind aft); find the distance and true bearing from the ship in the last position of an island in lat. 22° 50′ S. and lon. 180°. (40.)

26. Being uncertain of my position by dead reckoning, on running up channel, I sight a known headland; show how I can determine my position by sailing past it. By what observations of this point could I obtain my distance from it without calculation? (30.)

27. Explain clearly the construction of the table of meridional

parts, investigating the formula used in its computation; find the meridional parts used in the construction of the chart in question 28. (55.)

28. Draw a Mercator's chart on a scale of 1·25 miles to a degree of longitude, extending from 56° to 59° S., and from 29° to 33° W. A headland A (lat. 56° 15′ S., lon. 29° 22′ W.) bore N.E.by.E.-$\frac{3}{4}$E., and a rock B (lat. 56° 28′ S., lon. 29° 45′ W.) bore S.byE.; the ship afterwards sailed as follows:—True courses, S.W.-$\frac{3}{4}$W. 85′, S.S.E. 71′, N.byE. 112′. Lay down the position of the ship at first, and the courses, then find the lat. and lon. in (70.)

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